

**Opera Mathematica,
Tomus Quartus —
complectens Gnomonices
Libros Octo, ac Fabricam
et Usum Instrumenti ad
Horologiorum
descriptionem**

*[Mathematical Works, Volume Four — Gnomonics
in Eight Books, with the Construction and Use of
the Instrument for laying out Sundials]*

(1612)

by Christoph Clavius (1538–1612), S.J.

Translation by Scott Weisman



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Translator Version 2

Upright reason dictates that the recipients of the good, of whatever type and level of beneficence it may be, must show gratitude and blessing to the beneficent person in every way possible, commensurate with the value of the beneficence. And one who has benefited all the people of the world — for example, one who invented a new instrument for the good of the world, or a good book — it is fitting for every discerning person, out of the obligation of love of fellow beings, to at least purchase it, so that the man will profit and his heart will be encouraged thereby to invent yet more good instruments in the world. And similarly, all other wise-hearted people will likewise strive and exert themselves to invent good things and instruments needed for the repair of the world and its perfection.

And therefore, whoever says: "What do I need this new instrument for?" — he does not act well towards the world. For if not for the man who invented it, where would you be? And what would your city do? And more than this, if the man did not exert himself, what would the world do?

— R. Pinchas Eliyahu Hurwitz, *Sefer HaBris HaShalem* (1797)

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A note on the work

Tomus IV is Clavius's *Gnomonices* — the most exhaustive Renaissance treatise on gnomonics, the science of sundials and shadow-casting. Across eight books it develops the geometry of the gnomon's shadow and shows how to lay out sundials (horologia) of every kind — horizontal, vertical, inclined, on any plane and for any latitude — with the underlying spherical-astronomical theory of hour-lines, and a description of an instrument for drawing them mechanically. It is dense with construction diagrams and with tables of shadow lengths and hour-line positions.

A note on the translation process

This translation was produced with the [translation-pipeline](#). See the linked repository for details on the process.

This text is the product of a highly refined automated translation process. It has undergone only cursory human review and should not be treated as an authoritative or scholarly rendering. It is offered solely as a reference and reading aid — a way to make an otherwise inaccessible work approachable — and any passage of real consequence should be checked against the original source.

Acknowledgements

Translated with the assistance of Claude. The translator thanks the [Anthropic](#) team for making this work possible.

Opera Mathematica, Tomus IV (Gnomonices Libri Octo) — Front Matter

[Front cover / binding]

No text.

[Front flyleaf / pastedown]

Blank flyleaf with modern handwritten annotations in pencil:

- Upper left, a shelfmark: "[?] Math. [?] 10 – 4" (partly cropped at the left edge; the leading character(s) run off the page).
- Below it, centred, the author's name written by hand: "Clavius".
- Upper right, a faint pencil sketch (indistinct), and scattered ink/foxing spots across the leaf.

[Margin: the pencilled "10 – 4" and "Math." appear to be an old library class-mark; the "4" corresponds to this being Tomus IV.]

[Front flyleaf verso — offset of title page + library label]

This leaf is blank parchment/paper that has picked up a faint mirror-image **offset (set-off)** of the printed title page from the facing leaf. The title page itself is not in this batch (it falls on the next leaf); what shows here is only the transferred ink, very faint and reversed, and cannot be read cell-for-word. The large display capitals and a few text blocks are partly discernible but are transcribed only where legible; the remainder is [?].

Faintly discernible offset of the title-page display lines (reversed, low contrast — best readings, uncertain):

- CHRISTOPHORI [?] [Christophori — "of Christoph"]
- CLAVII [?] [Clavii — "Clavius"]
- OPERVM MATHEMATICORVM [?] [Operum Mathematicorum — "of the Mathematical Works"]
- TOMVS QVARTVS [Tomus Quartus — "Volume Four"]
- Continens [?] [Continens — "Containing"]
- Several lines of smaller text below, illegible in offset: [?]

- A mid-page line in capitals: OMNIA IN HAC [?] EDITIONE [?] [Omnia in hac ... editione ... — "All (things) in this ... edition ..."] [?]
- Lower on the leaf, the faint offset of the printer's device/vignette and imprint lines: [?]

[Illegible: full offset text of the title page — reversed set-off transfer, not the printed page itself; the actual title page is on the following leaf and will be transcribed there.]

Library markings physically on this leaf:

- A pasted barcode label reading 36613880400015 (printed twice), with a red over stamped S, and beneath: "Bayer. Staatsbibliothek" [Bavarian State Library — the holding institution].

Opera Mathematica, Tomus IV (Gnomonices Libri Octo) — Christoph Clavius

[Front flyleaf recto]

Blank leaf. At the top left, a modern pencil shelfmark in a later hand:

[Margin: "2 Math. u. 10-4" — later library shelfmark, in pencil.]

The rest of the leaf is blank; a faint reversed offset of the title-page text and a faint pencil sketch (an oval/instrument outline) show through from the facing leaf. No printed text.

Title page

CHRISTOPH CLAVIUS OF BAMBERG [Christophori Clavii Bambergensis], OF THE SOCIETY OF JESUS —

MATHEMATICAL WORKS, VOLUME FOUR [Operum Mathematicorum Tomus Quartus]

Containing:

- The eight books of **Gnomonics** [Gnomonices libros octo].
- The **construction and use of the instrument** most conveniently suited to laying out sundials [Fabricam et usum instrumenti ad horologiorum descriptionem peropportuni].

- **A new account of sundials** [Horologiorum novam descriptionem].
- **A very brief compendium for laying out horizontal and declining sundials** [Compendium brevissimum describendorum horologiorum horizontalium ac declinantium].
- **Notes on the new account of sundials** [Notas in novam horologiorum descriptionem].

The whole, in this edition, greatly enlarged and enriched both with other matters and — above all — with astronomical tables most useful for the construction of sundials.

Printer's device. The Jesuit emblem: a radiant oval cartouche framed by scrollwork and winged putti (three cherubs above blowing trumpets, two below). At the center, within an inner oval, the monogram **IHS** surmounted by a cross, with three nails beneath. Around the sunburst ring, the legible fragments of the motto, reading around the oval, are: **LAVDA-** (lower left) ... **NOMEN** (top) ... **DOMINI** (right) ... **-BILE** (left) — i.e. *Laudabile nomen Domini* ["Praiseworthy is the name of the Lord," cf. Psalm 112(113):3]. Some ring letters are obscured by the overlying manuscript inscriptions and are read only in part [?].

- Motto ring (top): NOMEN
- Motto ring (right): DOMINI
- Motto ring (lower-left/left): LAVDA...BILE [Laudabile]
- Center: IHS monogram with cross above and three nails below

[Margin: Above and flanking the device, manuscript ownership inscriptions in an early hand — "Monast[er]ii Schyren[is]" (of the Monastery of Scheyern), repeated as two flanking flourished signatures, "Mon[aster]ii" (left) and "Schyren[is]" (right). Reading of the flourishes is uncertain [?].]

MAINZ [Moguntiae], at the expense of Antonius Hierat [Antonii Hierat], printed by Ioannes Volmari [Iohannes Volmar]. With the grace and privilege of His Sacred Imperial Majesty [Cum Gratia & Privilegio Sacrae Caesar. Maiest.].

In the year 1612 [Anno M.DC.XII].

[Title-leaf verso]

Blank. The reversed bleed-through of the title-page text shows from the recto. Near the center, an oval library stamp:

[Approval stamp: "BIBLIOTHECA REGIA MONACENSIS" — Royal Library, Munich (Bayerische Staatsbibliothek).]

No printed text on this side.

To the Most Reverend and Most Illustrious Prince

To the Most Reverend and Most Illustrious Prince and Lord, Lord Johann Gottfried [Ioannes Godefridus], Bishop of Bamberg [Episcopus Bambergensis] and Provost of the Cathedral Church of Würzburg [Cathedralis Ecclesiae Herbipolensis], etc., his most gracious Lord —

Christoph Clavius of Bamberg, of the Society of Jesus [Christophorus Clavius Bambergensis e Societate Iesu],

wishes EVERLASTING HAPPINESS.

If, to the joy I took from your most auspicious inauguration, most Reverend and most Illustrious Prince, God the Best and Greatest [Deus Optimus Maximus] had added this further favor — that I might present in person, with my own hand, those fruits of my studies which I had dedicated to your name and glory — I would judge that almost nothing was lacking to the happiness that falls to us mortals. For what could have befallen me in life more delightful than to behold with my own eyes the Church of Bamberg — from which I have now been absent for many years, not by my own will so much as by God's dispensation — in that very condition which I used to beg of God in daily prayers that it might one day attain? And what more to be wished than to commend my books in person to the one man to whom, almost alone, we owe our thanks for how great a good it is that is celebrated with such applause of the whole Christian world?

But since I am hindered by the distance between us, and since an old age that grows heavier by the day keeps me confined to my bed, I have delegated my role to the Reverend Father Johann Reinhard Ziegler [Ioannes Reinhardus Zieglerus]. He, out of his remarkable love for the mathematical disciplines, has devoted a great deal of effort and labor to the editing of my Commentaries, and so may present these same works to your Highness in my name. From you I beg, again and again, that with the same love and goodwill with which you received the other volumes, you will be willing to accept this one too — which treats of

shadow-marking sundials [horologia sciotherica] — just as if it were being handed to you by my own hand: this I earnestly ask and beseech.

In this book I discuss at length the rising and setting of the Sun: at what moment of time, returning from the lower world, it reaches our horizon; with what swiftness of motion it then strives upward and climbs all the way to mid-heaven, where in full day it shines most beautifully and by its heat restores the strength of dying Nature. The recollection of this subject is all the more pleasant to me because it seems to bear a certain striking resemblance to Christian Princes.

For consider the face that all things tend to wear in the early morning, once the Sun has shone forth: the ill-omened birds are driven off into their hiding places, and the better birds begin to sing together; the grasses and flowers regain their beauty; and to the race of men and of cattle their native vigor returns into their limbs. Nearly the same is the transformation that provinces and cities tend to undergo when a man of that stamp is brought to the helm of empire — one whose proofs of virtue are so illustrious that no one can doubt he will require of others the same rule of life that he himself has kept. Then you may see the spirits of good men suddenly lifted up, taking hope and counsel toward the best of things, laying aside sloth and fear, and openly applying that line of the Poet:

Nothing need be despaired of with Teucer as leader and under Teucer's auspices.

By contrast, you may see the wicked, whom license had made bolder, lose heart, condemn their own base ventures, and look about for a way to flee. And you yourself, most Reverend and most Illustrious Prince, Germany lately beheld rising most happily like the Sun; and at that time it did not escape you what the disposition was of those who think rightly about the safety of their homeland and the standing of the Christian Commonwealth. This remains: that we obtain from God that, when you have happily risen to the highest summit of Virtue and Glory, He may grant you to remain at that same eminence for a very long time, and — so far as it can be — to set as late as possible. Rome [Romae], in the Year of the Lord 1612, on the Kalends of January.

GNOMONICS, IN EIGHT BOOKS

GNOMONICS, IN EIGHT BOOKS [GNOMONICES LIBRI OCTO], in which not only sundials [horologia solaria] but also other things that can be learned from the shadow of the gnomon are set out and demonstrated geometrically.

BY THE AUTHOR

CHRISTOPH CLAVIUS OF BAMBERG, OF THE SOCIETY OF JESUS [Christophorus Clavius Bambergensis, Societatis Iesu].

PREFACE TO THE GNOMONICS

By Christoph Clavius of Bamberg, of the Society of Jesus [Christophorus Clavius Bambergensis, e Societate Iesu].

Among all the arts and disciplines that God, the author of all things, has shared with the human race as tokens, so to speak, of his infinite wisdom, power, and majesty, by no means the least to be reckoned are those that the ancient philosophers called the Mathematical disciplines. For these are so thoroughly established that they admit nothing merely probable, but prove everything by clear and necessary arguments. And — as Ptolemy [Ptolemaeus], among others, acknowledged and professed — they draw men themselves toward searching out and cherishing the goodness of God; and by calling the minds of us all away from the craving for fleeting and perishable things, they urge us to fix our love upon the eternal and unchanging Good alone.

If this holds true of all the mathematical disciplines alike, how much more will it belong especially to Astronomy, which studies those heavenly spheres that the hand of God has fashioned with wonderful craftsmanship — spheres whose motion is so constant, and whose uniformity so great (even if there seems to be some disparity among them), that anyone who thinks carefully about them cannot help being carried away, once he has grasped their nature and motion, to admire and look up in wonder at the supreme Mover of them all.

Indeed, Astronomy cannot be praised as its dignity deserves. For it sets before us the magnitude of the heavenly spheres to behold; the utterly constant, yet varying, motion of the Sun; the fixed course of the Moon and of the remaining stars, their mutual conjunctions and their meeting with the Sun; the rising and setting of the stars, their arrangements, advances, and stations, their number, their extent, and their distance from the earth. I pass over what is by far the greatest thing of all: that there is a single science, Astronomy, which foretells the eclipses of the Sun and Moon, even though no fixed interval separates one eclipse from another, as experience itself shows. And the astronomers, striving to hand down to humankind — at least in part, and not only through the various records of written works — this admirable doctrine of the heavens, of the celestial bodies, and of their motions, went further: with an almost divine abundance of ingenuity

they invented many instruments in which, as in a mirror, one might survey the multitude and variety of the heavenly motions. Yet of all these instruments they devised none simpler, and none more readily made, than that timepiece which the Greeks call σκιοθηρικόν [Greek: skiothērikon, "shadow-catcher" — a sundial] and the Latins a *solarium* (sundial), on which the hours are read by the Sun. Its nature is such that, set in the sunlight, by the shadow of a single ray — that is, of a stylus [stylus] — it presents to view all the motion and position of the heavens, however varied, and above all that of the Sun. For on it, provided only that it is lit by the Sun, one can see what hour it is at any time of the year; which of the twelve parts of the sign-bearing circle — which the Greeks call ζωδιακός [Greek: zōdiakos, "zodiac"] — the Sun itself occupies; which house of heaven, and which Vertical [circle], and which

parallel of the Horizon it holds; what sign is beginning to rise above the Horizon, or to set; to what hour, in the other regions of the earth, any given hour corresponds; and the meridian of what place the Sun occupies at any hour.

Now the science that is concerned with the construction of this one instrument is called — from the gnomon (ἀπὸ τοῦ γνώμονος [Greek: apo tou gnōmonos, "from the gnomon"]) — γνωμονική [Greek: gnōmonikē, "gnomonics"]. Although many writers, both ancient and more recent, have written a great deal about it, none of them — in my judgment at least, and I say this to disparage no one — seems to have pursued it fully in every respect. For either they hand down the bare practice, the method by which sundials can be made, without applying any demonstration to their rules, as nearly all the more recent writers do; and from this they fall into errors that are quite intolerable, as is clear from Oronce Finé's [Orontius Finaeus] commentary on sundials. Or else, if some have tried to test those rules by the weight of demonstrations, they do it so briefly, and indeed so obscurely, that scarcely any usefulness can be gained from their labors.

Since, then, some men of great intellect complained to me of this and asked me to devote some effort and diligence to the science of gnomonics — so elegant, so pleasant, and moreover useful for so many purposes — I put a great deal of study and time into writing these books. In them my aim is to demonstrate, with the greatest possible clarity, not only the method by which hour-lines of every kind may be drawn on any plane surface, but also many other matters that bear on the motion of the Sun and of the heavens.

I have enclosed the whole work in eight books. In the **first**, various problems and theorems are demonstrated that bear on the analemma [analemma], on the conic sections, on the hour-circles, on the common intersections of the hour-lines, on

the declinations and inclinations of planes, and on other things of this kind. In the **second**, the method of laying out horizontal, vertical, meridian, polar, and equinoctial sundials. In the **third**, how sundials may be drawn that either decline from the prime vertical [Verticale primarium] or from the horizon, or are inclined to the horizon, or at once both decline from the vertical and are inclined to the horizon. The **fourth** deals with sundials on the right sphere [sphaera recta] and on the most oblique sphere — the one that has the pole at the zenith. It also explains by what technique the stylus is set in its proper place; what the method is of recognizing for what elevation of the pole, or what declination, or what inclination any given sundial appears to have been constructed; and by what procedure any sundial may be enlarged or reduced to suit any size of stylus. The **fifth** teaches by what means and method certain tables are to be constructed, by which sundials may afterward be drawn with little trouble. The **sixth** expounds Ptolemy's *Analemma* [Ptolemaeus, *Analemma*], and likewise Federico Commandino's [Federicus Commandinus] work on the drawing of sundials; from it, it becomes clear that the use of the analemma extends, unless I am mistaken, far more widely than Ptolemy handed down in his writings. I also add an easier method for declining and inclined sundials than the one Commandino showed — [teaching], moreover, what must be observed so that, from a horizontal sundial, another may be laid out on a wall (one whose declination is nonetheless known), for any size of stylus. In the **seventh**, the principal methods of drawing the hour-lines — which were discussed in the second and third books — are briefly repeated; yet no demonstrations are added here, so that those who are not very practiced in geometrical demonstrations may be able to construct any sundial without difficulty — with no demonstration interposed, and no delay. The **eighth** and last book, finally, deals with certain universal sundials, which are suited to every elevation of the pole, and also with some others that can be carried about and are of use for one elevation of the pole only — whichever shall seem to us the more serviceable. And so, unless my mind quite deceives me, I seem to have embraced in these eight books, arranged in this way, everything that properly belongs to gnomonics.

As for the scholia I have attached to most of the propositions, this was done with the following purpose: either to explain, in those scholia, whatever helps toward a clearer understanding of the proposition's demonstration; or to investigate by other routes the same things that were demonstrated in the propositions; or, finally, to put forward something new, not out of keeping with the plan of the work. For had I thrown everything into the propositions, I would have wearied the reader by the very length.

In the demonstrations we shall cite our authorities from Euclid [Euclides] and from our scholia upon him, according to our own edition; from Apollonius of Perga [Apollonius Pergaeus] on the elements of conics, according to Federico Commandino's edition; and from Theodosius [Theodosius] on the spherical elements, likewise according to our own edition. I shall also bring in the books on triangles of Johann of Königsberg [Regiomontanus], or those of Jabir of Seville [Geber Hispalensis], the Arab, on spherical triangles, or my own [treatise on] spherical triangles — which, together with Theodosius, and with a treatise and table of sines gathered into a single volume indispensable to the whole of Astronomy, will shortly appear. And so anyone who has not diligently applied himself to these matters has no reason to be confident that he can understand our demonstrations in this Gnomonics — although I teach the drawing of everything in such a way that anyone, even one wholly unskilled in all demonstrations, can grasp for himself what he must observe in carrying it out.

I should also like the reader to be warned of this: that all the sundials in this Gnomonics have been worked out for the latitude of the kindly City of Rome [Roma], even though the rules themselves are common to any elevation of the pole, or latitude of a place. In the scholia of the second and third books, too, and throughout the seventh, I teach how sundials may be drawn from any given size of stylus — whose position on the plane of the sundial is also assigned — a thing that no one before me (so far as I know) has ever accomplished, since almost everyone draws the sundial before determining the size of the stylus. One man, to be sure — Federico Commandino — does the same; but he does it by way of the analemma, and not by the method that most of those who lay out sundials commonly follow. Nor indeed shall I omit to mention that I have attempted, by an unusual method as well — a certain new approach by way of the cylinder — to draw the hour-lines on any plane whatever; and this method (I would venture to say) is the only one among them all by which all the hours, and the parts of the hours, can be conveniently drawn, as will be seen in the second and third books.

Therefore, if in this work I shall have brought some light to those who study so noble and delightful a science, I wish the credit for it to be referred to God, the author of all good things. But if I have satisfied the great minds of others less fully than I ought, or would have wished, I judge this must be ascribed both to the slenderness of my own talent and to the difficulty of the subject — seeing that in most matters I had no guide whom I could follow in any way, and nearly all the demonstrations were only recently thought out by me, as anyone will readily judge who cares to compare this Gnomonics of ours with the sundial-writings of others. Yet I believe I have opened up the road in this matter so well that it will be

easy for anyone — even one of only moderate understanding in geometrical demonstrations — to go forward of his own accord, and to put the finishing touch upon this science and, as it were, to set the crowning peak upon the work I have begun.

GNOMONICS, BOOK ONE

By the author, Christoph Clavius [Christophorus Clavius] of Bamberg, of the Society of Jesus

Since all those things that can be known through the shadow of the gnomon when the Sun shines, we have undertaken to describe in this our *Gnomonics* by careful reasonings — that is, by the firmest geometrical demonstrations and, as far as it can be done (with the good help of God, the Best and Greatest), by the clearest ones — and since very few before us have done this (for we see that all the rest, with one or two exceptions, have handed down a bare description of these matters, applying no demonstration at all), we judge it will not be out of place to spend this whole first book on various problems and theorems, together with matters to be explained both geometrically and astronomically. These, I say, we judge will be greatly profitable — indeed altogether necessary — so that afterwards the individual points bearing on the description of sundials can be demonstrated more briefly and more clearly.

Now the matters that seem to us to require demonstration, before we describe any sundial, are chiefly these. [Margin: The argument of the first book.] By what method the analemma [analemma] — from which alone almost the whole rationale of describing sundials is drawn — is to be constructed; that the vertex of the gnomon in any sundial takes the place of the center of the whole world in sundials; what conic sections [sectiones conicae] (which some call arcs of the signs) are traced in any proposed plane by the tip of the gnomon's shadow — that is, whether they are circles, or parabolas, or opposite hyperbolas, or indeed ellipses; then by what art sections of this kind are to be laid out in the proposed plane (for the tip of the shadow always runs along some one of these sections, except at the time of the equinoxes, when the tip of the shadow describes a straight line, as we shall demonstrate); what position the hour-circle [circulus horarius] has in the heavens, so that we may afterwards correctly draw the hour-lines [lineae horariae], which are nothing else than the common sections of those circles and of the plane on which the sundial is depicted; which hour-lines from noon or midnight, and from sunrise or sunset, intersect one another at one and

the same point in any given plane (for this will be of great use in laying out the hours from sunrise or sunset); in which planes the hour-lines from noon or midnight meet at a single point, as at some center, and in which they are parallel to one another; by what means the declination of each proposed plane from the vertical circle, and its inclination both to the horizon and to the meridian, may be ascertained; how great the elevation of the pole is above any declining or inclining plane, and other matters of the kind that will be seen to bear on such declining and inclining planes and to serve the purpose; by what device the declination of a parallel — whose diurnal arc is known — may be investigated; and finally many other things, without which the demonstrations we shall employ in laying out sundials cannot in any way be carried out or understood.

To all these will be added three further problems, by which the semidiurnal arc, the magnitude of twilight, the altitude of the sun from the known hour of the day, and conversely the hour from the altitude of the sun at any time of the year and for any elevation of the pole or latitude of the place, may be found out. So much for the argument of this first book. Now, before we approach the matter itself, a few things seem to need saying first about timepieces [horologia] in general, and about their necessity and invention. Then, too, about sciotheric [shadow-] timepieces, which pertain more closely to our Gnomonics, and about their various kinds and inventors; and finally the variety of the hours must be briefly treated.

On timepieces in general, and their necessity and invention

[Margin: What a timepiece is. What an hour is. From what the hours are named.]

The very word "horologium" [horologium] shows that a timepiece is an instrument by which the hours are marked out or detected; for it signifies, as it were, "the reckoning of the hours" [horarum ratio], so that whatever in some way indicates the hours may be called a horologium. An hour is one of those equal spaces into which the day is customarily divided. For although among the ancients the four seasons of the year — spring, summer, autumn, and winter — were called "hours" [horae], from the Greek ἀπὸ τοῦ ὁρίζειν [Greek: apo tou horizein, "from bounding/defining" — the verb ὁρίζειν, "to bound"; glyphs faint], as Plato [Plato] holds in the *Cratylus*, because they bound and define those parts of the year, yet today we call those equal spaces of the day "hours" because, as Macrobius [Macrobius] holds (*Saturnalia* I), the sun produces them by its own course and motion — the sun which among the Egyptians was called Horus (for so they named Apollo [Apollo], who is the same as the sun).

Others, however, prefer that the hours be named from the Greek word ὥρα [Greek: horos, "boundary" or "limit"], which signifies a boundary or an end, because we mark off and terminate that continuous span of time by the hours as by boundaries. Finally, in tracing the derivation of the word "hours" one has even come down to filth. For there are those who would hold that these equal little parts of the day are called "hours" from the Greek word [Greek: [?]] — the word meaning "urine" (οὔρον/οὐρεῖν); printed glyphs too faint to resolve], which means urine. For they say that Hermes Trismegistus [Hermes Trismegistus] first observed the hours — that is, the equal spaces of the day — from the urine of a certain animal sacred among the Egyptians, which was dedicated to Serapis, that is, to the sun. For when he had noticed that this animal (a truly ridiculous thing, and more worthy of the Egyptians than of Trismegistus) passed urine twelve times by day and as many times by night — whether the night were long or short — always at equal intervals, he thought it agreeable to reason to divide day and night into just as many equal spaces, which were named "hours" from the urine of that animal. Hence perhaps it came about that almost the whole of antiquity divided every day and night into twelve hours. But on this matter we shall say more a little later.

[Margin: The necessity of timepieces.] As for the necessity of timepieces, there is no reason for me to spend many words. For I think it escapes no one how wretched and unhappy the human race would be if it had no way of distinguishing the hours. For we see that most people (as each of us also finds in himself) who do not know what hour it is are affected — like those who do not know in what region they are — by a kind of numbness of mind, and, in my judgment, not without reason. For by the distinction of the hours individuals measure out their several labors and their leisure, and set themselves fixed times for all their occupations. Indeed, since the life of men is for the most part taken up with dealings and meetings, we truly could not do without the use of timepieces, since by their benefit we appoint for ourselves certain hours and certain spans of time in which we can conveniently come together to carry out our affairs, without any loss to our household or other concerns.

Nearly all well-ordered states bear witness to this very thing: in their cities and towns they everywhere build, at public and private expense, various kinds of timepieces — sundials and others — and maintain keepers and overseers of them at yearly salaries; which they would do on no sound reasoning, unless they judged the use of these to be of the highest necessity to their communities. Almost all farmers and men living apart bear witness to the same thing too: since they have no timepieces, and yet cannot do without them without great

inconvenience, they strive to relieve this inconvenience by whatever means they can. Hence we see them for the most part measuring the shadow of their own bodies, or marking one sign after another by the sun's rays, or finally noting in the sky itself certain intervals of the sun's motion by day and of the other stars by night — so that by these observations at least, even if not wholly accurate ones, they may in some way perceive and measure off the distinctions and spans of the hours.

[Margin: The various kinds of timepieces.] Since, then, the knowledge of the hours is so useful, and indeed so necessary to all commonwealths, our forebears with the greatest zeal always devised varied and ingenious instruments (which we call *horologia*) by which we might tell apart the intervals of the hours. All of these can be brought back to two kinds. Some, by the shadow of the sun alone by day, or by the shadow of the moon by night, or by the observation of the other stars, indicate the hours — of which we shall write more below. Others show the hours by some mechanical instrument and by various motions, and these are manifold. For since the former sundials would show no hours at all in cloudy weather, it was necessary to find another way of marking the hours. Hence it came about that some made use of water-timepieces [*hydraulica horologia*], in which the hours are sought out by the aid of the motion of water — a matter in which Ctesibius of Alexandria [Ctesibius] was wonderfully praised. For he, as Vitruvius [Vitruvius] reports in book 9, by marvelous ingenuity constructed (to use the words of Vitruvius) a hollow made of perfect gold, or bored out of a gem, since such materials are neither worn away by the striking of the water nor take in dirt so as to be clogged. For the water flowing uniformly through that hollow raised up an inverted float [*scapus inversus*], on which a ruler was set; and the revolving drums fitted with equal teeth are made complete — the teeth of which, driving one another, produce measured turnings and motions, and the rest of this kind — from which the parts of the day, that is, the hours, could be known and distinguished.

Pliny [Pliny], at the end of book 7, writes that Scipio Nasica [Scipio Nasica] first divided the hours of night and of day alike by water, in the year of the City 595. But some set out certain bronze vessels filled with water and finely perforated; for according to how much of the water fell into another vessel, when they examined the inscribed marks, they understood how much time had elapsed. Such timepieces they called *clepsydrae* [*clepsydrae*], from the likeness of the perforated vessel, because they in a way "steal" the water and pour it out drop by drop, sending it from vessel to vessel through a narrow hole. Hence we often read that orators were granted three or four *clepsydrae* — that is, hours — for

speaking in olden times. Others assert that those vessels were of finely perforated glass, in which the hours were marked out by equal measures, and at the bottom of which that little hole let the water out drop by drop, so that by the diminishing of the water the elapsed hours were detected.

To these, in this age of ours, have succeeded certain timepieces made of glass, which have enclosed within them very fine sand or dust that slips down bit by bit through a very small hole from the upper part into the lower, and show the elapsed hours in this way. But beyond doubt all the timepieces so far enumerated are surpassed in many respects by that timepiece which, almost in our own age, has been invented by marvelous artifice and ingenuity — assembled from certain toothed wheels, balanced by weights, [set] to mark the hours and the parts of the hours, and to demonstrate the celestial motions, and other very convenient things of this kind. But we shall write nothing in this work about these mechanical timepieces, since they are foreign to our design. Therefore let us return to the former timepieces, which we have said indicate the hours by the shadow of the sun or by the rays of the other stars.

On sciotheric timepieces and their various kinds

All the timepieces which we said just before show the hours by the shadow of the sun alone are usually called "solar" [solaria] by writers, or "sciotheric" [sciotherica], from the shadow of the sun; for σκιόθηρον [Greek: skiothēron, "a place suitable for catching the shadow"] means a spot fit for catching the shadow. Hence σκιόθηρα [Greek: skiothēra, "shadow-catchers" — mathematical instruments for taking the shadow] are the mathematical instruments by which shadows are usually caught, so that a sciotheric timepiece is an instrument in which, by the shadow of the sun or of the rays of the other stars — which are like a shadow — a fixed hour of the day and its quantity, the rising and setting of the sun, the signs of the zodiac in which the sun is, and very many other things bearing on celestial matters are shown by artifice. [Margin: What a sciotheric timepiece is. The various kinds of sciotheric timepieces.] Now a sciotheric timepiece can be divided in many ways. For first, some are called "hanging" [pensilia], because they hang freely from the hand and, when directed toward the sun or any other stars, let us know the hours: such are all those we use, whether with pinnacidia [pinnacidia] — that is, certain little tablets perforated with square holes, such as are usually fixed in the dioptra of an astrolabe — or with some very small hole, or finally with a thread and plumb-line, on which a certain movable little knot is sometimes placed for distinguishing the hours; of which kind are astrolabes, cylinders, quadrants, astronomical rings, hour-rings

[horaria], and others of this sort, which we shall treat in the eighth book. But others are "fixed" [stabilia], and require a certain settled position in some plane, so that it is not necessary for them to hang — those, namely, which show the hours by the shadow of a gnomon or stylus. And these are distributed into two kinds. For some are laid out on a plane surface, which is equidistant from some celestial great circle; others on a surface that is not plane — as on a concave or convex hemisphere, or on a cylinder, convex or concave, or on any other non-plane surface — and these are depicted as we shall write in the eighth book. For now let us review the kinds of those that are described on plane surfaces, with which we shall chiefly be concerned in this work.

[Margin: Solar timepieces are divided by reason of the great circles from which they are equidistant, and from which they take their name.]

Of solar timepieces, then, that are described on some plane surface, there are very many kinds, according to the multitude of the celestial great circles from which the plane surfaces of the timepieces are equidistant, and from which the timepieces themselves have taken their names, as we shall teach in proposition 2 of this book. But since chiefly nine great circles can be considered in the heavens, differing in position and situation, it comes about that there are just as many kinds of timepieces that are described on some plane surface. The great circles are these: the Horizon; the Vertical circle properly so called, passing through the vertex of the head; the common sections of the Equator and of the Horizon; the Meridian; the circle of the sixth hour from noon or midnight, [passing] through the poles of the world and through those same common sections of the Equator and Horizon; the Equator; any Vertical circle deflecting from the properly-so-called Vertical and perpendicular to the Horizon; a great circle declining from the Horizon and passing through the common sections of the Meridian and of the Horizon, and perpendicular to the Vertical properly so called; a great circle inclined to the Horizon, described through the common sections of the Equator, of the Vertical properly so called, and of the Horizon, and perpendicular to the Meridian; and finally a great circle inclined to the Horizon, to the Meridian, and to the Vertical properly so called. Now since each of these circles except the Horizon has two faces, which can be lit by the sun during the day — as the Vertical properly so called, and likewise the one that declines from it, has one face verging toward the south and another looking toward the north; the Meridian one that looks toward the rising and another that looks toward the setting; and all the remaining circles inclined to the Horizon have an upper face, which looks toward the Zenith or vertex of the head, and a lower one, which

looks toward the Nadir — it comes about that in all there are seventeen kinds of timepieces of this sort, which may be named as follows:

[Margin: How many kinds there are of solar timepieces that are described on a plane surface.]

1. HORIZONTAL, which is equidistant from the Horizon.
2. VERTICAL looking toward the south, which is parallel to the Vertical circle properly so called, and looks toward the south.
3. VERTICAL verging toward the north, which is equidistant from the same Vertical circle, and looks toward the north, or the Bear [Septentrio].
4. MERIDIAN Eastern, which is equidistant from the Meridian circle, and verges toward the sun's rising.
5. MERIDIAN Western, which is parallel to the same Meridian circle, and looks toward the setting sun.
6. POLAR upper, which is equidistant from that greatest circle passing through the points of the true, or equinoctial, rising and setting (that is, through the common sections of the Equator, the Horizon, and the Vertical) and through each pole of the world, and looks toward the Zenith.
7. POLAR lower, which is equidistant from that same greatest circle drawn through the points of the true, or equinoctial, rising and setting, and through each pole of the world, and looks toward the Nadir.
8. EQUINOCTIAL upper, which is parallel to the Equinoctial circle, and looks toward the pole raised above the Horizon.
9. EQUINOCTIAL lower, which is equidistant from the same equinoctial circle, but looks toward that pole of the world which lies hidden beneath the Horizon.
10. DECLINING from the south toward the rising or setting, which is equidistant from any Vertical circle that declines from the properly-so-called Vertical and verges toward the south, yet is not directly, and is perpendicular to the Horizon; such as is usually depicted on the wall of some building that looks toward the south, but not directly, and is perpendicular to the Horizon.

11. DECLINING from the north, or Bear, toward the rising or setting, which is parallel to any Vertical circle that deflects from the primary Vertical and looks toward the north, yet not directly, and is perpendicular to the Horizon; such as is described on the wall of any building that is perpendicular to the Horizon and looks toward the north, but not directly.
12. DECLINING from the Horizon, upper, which looks toward the Zenith and verges directly toward the rising or setting, and is equidistant from the greatest circle that is inclined to the Horizon and perpendicular to the primary Vertical, and passes through the common sections of the Meridian and of the Horizon. Such a timepiece would be depicted on the upper face of some roof that would be perpendicular to the Vertical circle properly so called — that is, looking directly toward the rising or setting.
13. DECLINING from the Horizon, lower, which looks toward the Nadir and verges directly toward the rising or setting, and is equidistant from the same greatest circle to which we said the upper one is parallel. Such a timepiece would be described on the same roof on which the upper one [is set], but on the lower face that looks toward the Nadir — provided, however, that this lower face were lit by the sun.
14. INCLINED to the Horizon, upper, which looks toward the Zenith and verges directly toward the south or north, and is equidistant from the greatest circle inclined to the Horizon, which is perpendicular to the Meridian and passes through the common sections of the Equator and of the Horizon. Such would be a timepiece laid out on the upper face of some roof that would be perpendicular to the Meridian — that is, looking directly toward the south or north.
15. INCLINED to the Horizon, lower, which looks toward the Nadir and verges directly toward the south or north, and is equidistant from the same greatest circle to which we said [the upper one] is parallel. Such would be a timepiece depicted on the lower face of the same roof, on whose upper face we said the previous one is drawn.
16. DECLINING and INCLINED together, upper, which looks toward the Zenith, but verges neither directly toward the south or north (as does the one inclined to the Horizon), nor toward the rising or setting (as does the one declining from the Horizon). It is nonetheless equidistant from a greatest circle that is perpendicular neither to the Horizon, nor to the Vertical properly so called, nor to the Meridian, but is inclined to all these circles —

so that it passes through the poles of none of them (namely the Zenith and Nadir, the common sections of the Meridian and Horizon, and the common sections of the Horizon and Equator). Such would be a timepiece drawn on the upper face of some roof that looked neither directly toward the rising or setting, nor toward the south or north.

17. DECLINING and INCLINED together, lower, which looks toward the Nadir, but has everything else in common with the upper one. Such would be a timepiece laid out on the lower face of the same roof, on whose upper face we said the previous one is described.

[Margin: Examples of the aforesaid timepieces.]

We will set all these kinds before the eyes in the accompanying figure. A timepiece drawn on the plane ABCD, which is equidistant from the Horizon, will be called Horizontal. Then, if we imagine buildings erected upon the several bases — octagonal, hexagonal, square, and triangular — the walls standing on the straight lines EF and GH, set directly toward A (the south point) and toward C (the north point), and on their faces turned toward A there will be described Vertical timepieces looking toward the south. On the faces of those same walls looking toward C, Vertical timepieces facing the north will be laid out. Further, on the walls erected upon the straight lines IK and LM — which, toward B, look directly toward the rising, or, toward D, toward the setting — Eastern Meridian timepieces will be described on their faces looking toward B, while on the faces looking toward the point D, Western Meridian timepieces will be described.

Again, if upon the straight lines EF and GH planes are imagined inclined to the Horizon toward the north, C, according to the

Diagram. A plan view (bird's-eye) of the horizontal dial-base ABCD, with several concentric regular polygons — an octagon, a hexagon, a square (tetragon), and an innermost triangle — nested about a common centre. These represent the bases of buildings of different plan that could be erected on the horizontal plane; the walls raised on the polygon sides carry the vertical, meridian, declining, and inclined dials described in the surrounding text. The four cardinal directions are printed outside the circle: **MERIDIES** (south) at the top, **SEPTENTRIO** (north) at the bottom, **ORTVS** (rising/east) at the left, **OCCASVS** (setting/west) at the right.

- A (top, at Meridies): south point of the plane ABCD.
- C (bottom, at Septentrio): north point.
- B (left, at Ortus): east point.

- D (right, at Occasus): west point. The two straight lines A–C (meridian) and B–D cross at the centre and divide the figure.
- E, F (repeated at the upper vertices of the several concentric polygons, flanking A): ends of the walls EF set across the meridian toward the south.
- G, H (repeated at the lower vertices, flanking C): ends of the walls GH set across the meridian toward the north.
- I, K (repeated at the left vertices, flanking B): ends of the walls IK toward the rising.
- L, M (repeated at the right vertices, flanking D): ends of the walls LM toward the setting.
- The corner/diagonal vertices carry the paired labels of the declining walls: E–I and F–L (upper), G–K and H–M (lower), corresponding to the walls EI, HM, FL, GK named in the text.
- At the very centre a small square encloses a triangle; its labels (including a central letter read as Q) are too small to resolve with certainty [?].

the altitude of the pole, the timepieces described on the upper faces will be called Upper Polar, and those on the lower faces, Lower Polar. But if upon the same straight lines EF and GH planes are imagined inclined to the Horizon toward the south, A, according to the altitude of the Equator (that is, the complement of the pole's altitude), Upper Equinoctial timepieces will be laid out on the upper faces, and Lower ones on the lower faces.

Now, since the walls perpendicular to the Horizon, standing on the straight lines EI and HM, do not look directly toward A (the south point) or toward C (the north point), but decline from both toward B (the east point) or toward D (the west point), there will be described on their faces looking toward the points located between A and B, timepieces declining from the south — that is, from the south point A toward the rising; while on the opposite faces, which look toward the points located between C and D, timepieces declining from the north — that is, from the north point C toward the setting. Conversely, since the walls built upon the straight lines FL and GK, perpendicular to the Horizon, do not verge directly toward A (the south point) or toward C (the north point), but decline from both toward D (the west point) or toward B (the east point), there will be laid out on their faces that look toward the points located between A and D, timepieces declining from the south — that is, from the point A (south) toward the setting; while on the faces

on the opposite faces, turned toward the points between C and B, timepieces declining from the north — that is, from the point C (north) toward the rising.

But if upon the straight lines IK and LM planes are constructed toward the rising B or the setting D, inclined to the Horizon, upper timepieces declining from the Horizon will be described on their upper faces, and lower ones on their lower faces. Likewise, if upon the straight lines EF and GH planes are inclined to the Horizon toward the south A or toward the north C, the timepieces drawn on their upper faces will be called upper ones inclined to the Horizon, and those described on the lower faces, lower ones. Finally, if upon the straight lines EI, HM, FL, GK planes are inclined to the Horizon in any way whatever, upper timepieces both declining and inclined will be described on their upper faces, and lower ones on the lower faces.

From this it is easily gathered that there can be almost infinitely many kinds of timepieces — declining from the Vertical, declining from the Horizon, inclined to the Horizon, and both declining and inclined together — because a single surface can deflect from the vertical or the horizon more or less to infinity, and incline toward the Horizon. In this way, then, sundials [*sciotherica horologia*] are usually classified, with regard to the position of the planes on which they are drawn.

[Margin: Why plane timepieces are said to be equidistant from the greatest circles from which they take their name.]

Now, all these timepieces that we have surveyed we said to be equidistant from those greatest circles from which they draw their names, because the tip of the stylus, set perpendicular to the timepiece's plane, is understood to be placed at the centre of the world, and so at the common centre of all those greatest circles. From this it follows that the planes of the timepieces are distant from those circles only by as much as the length of the respective stylus. For this reason those are mistaken who teach that all timepieces are drawn on the very greatest circles from which they receive their names. But we shall explain this more fully in Proposition 2 of this book.

[Margin: A division of solar timepieces drawn from the time at which they show the hours.]

In another way, the instruments that indicate the hours are usually divided according to the time at which they show them. Some are called diurnal, because they indicate the hours only in the daytime, when the sun shines; some nocturnal, which show the hours at night, whether by the shining of the moon or by the other stars; and some, finally, both diurnal and nocturnal, since we learn the hours from them both by day and by night — such as astrolabes (or planispheres), astronomical rings, and the like. In the same way, some indicate

the hours through half the year — such as the Upper Equinoctial timepiece, which indicates the hours during that part of the year when the Sun moves from the beginning of Aries [γ] to the beginning of Libra [$\underline{\alpha}$] — while others do so through the remaining part of the year, such as the Lower Equinoctial, which shows the hours while the sun moves from the beginning of Libra [$\underline{\alpha}$] to the beginning of Aries [γ]. Again, some indicate only the morning and evening hours — such as the Vertical timepiece facing north, the Lower Polar, and certain other declining and lower inclined dials; others indicate the remaining hours of the day — such as the Vertical inclining toward the south, the Upper Polar, and likewise some declining and upper inclined dials. So too, some timepieces indicate only the forenoon hours — such as the Eastern Meridian — while others indicate the afternoon hours, such as the Western Meridian. And in this way many other kinds of timepieces can be devised.

DE QVADRVPLICI HORARVM, atque ex ipsis horologiorum varietate.

[On the Fourfold Kinds of Hours, and the Variety of Timepieces arising from them]

[Margin: Division of the hours. The unequal hour. Why the hours are called "ancient" and "unequal."]

We have set out the various kinds of timepieces according merely to the differing position of the planes on which they are drawn; now let us speak of the four kinds of hours, by which each of the aforesaid species of timepieces is made fourfold. To begin with, there are two primary kinds of hours, distinct from each other: the ones are called unequal, the others equal. An unequal hour is a twelfth part both of the artificial day and of the artificial night. For of old every artificial day, and every night, was divided into 12 parts equal among themselves, which are called by writers "unequal" and "ancient" hours: "ancient," because their use existed among all the ancients; "unequal," not because the hours of a single day or night are unequal among themselves (that would be false, since we have asserted that any day and night is divided into 12 equal parts), but because, since artificial days and nights are unequal, the hours of the longer day or night are greater than the hours of the shorter day or night. For when two unequal magnitudes are divided into an equal number of parts, each part of the greater magnitude comes out unequal to — that is, greater than — each part of the lesser. Again, such twelfth parts of the day or night are called unequal hours because, although both the artificial day and the artificial night consist of twelve equal

hours, yet in an oblique sphere the day is perpetually unequal to its own night — except at the time of the equinoxes, when the sun is at the beginning of Aries [γ] or Libra [ϖ] — and so it necessarily follows that the diurnal hours are unequal to the nocturnal. Finally, such hours are called unequal because they never — except at the equinoxes — agree with the equinoctial hours, which are called equal (as we shall show presently), but are always either smaller (as the diurnal hours in winter and the nocturnal in summer) or greater (as are the diurnal hours in summer and the nocturnal in winter).

[Margin: Why the ancients cut every day and night into 12 hours.]

But why the ancients established twelve diurnal hours and as many nocturnal, we have partly recalled above, when we discussed the etymology of this word "hour," and partly we shall disclose here. It is plainly established that the ancients always had great regard for the time in which the several parts of the Zodiac rise above the Horizon. Hence it appears that they instituted the hours from the parts of the Zodiac, and not without reason. But since there are twelve principal parts in the Zodiac — namely the twelve celestial signs, six of which rise on any given day and as many on any given night, as we taught in the [treatise on the] sphere — if they had assigned one hour to each sign, they would have established only six hours in each day and in each night, and would have made them longer than is fitting. Therefore it seemed good to them to assign an hour to each half of a single sign emerging above the Horizon. Thus it came about that they divided any day and night into 12 hours. These unequal hours the Jews of old used, as is established from Holy Scripture, as did the Romans and almost all of antiquity, as both the histories and the books of the ancient Mathematicians attest. For these divided the night into four watches, assigning three hours to each watch. Indeed, the Roman Church too appointed the praises of God Most High and Greatest — which from the name of the hours are indeed called "canonical hours," but by number are called Prime, Terce, Sext, and None — to be chanted according to this kind of hours.

[Margin: The equal hour. Why the hours are called "equinoctial" and "equal."]

An equal hour, however, is the twenty-fourth part of the natural day, which is made up of day and night. Now, because the Equinoctial circle, containing 360 degrees, is carried round once in a day, and everywhere among all peoples rises wholly above the Horizon (we here disregard that small portion which corresponds to the little bit of the Zodiac that the sun meanwhile traverses by its own motion), 15 degrees must necessarily rise each hour. For this reason these hours have been called "equinoctial," because, namely, they are produced by the

revolution of the equinoctial circle. They are also called equal; for the 15 degrees of the Equator that are assigned to each hour always rise above the Horizon in an equal interval of time — since in every region the equinoctial circle is uniformly raised above and depressed below the Horizon, as we have shown elsewhere.

Furthermore, these equal hours not all nations of old numbered in the same way, just as they do not today; because not the same beginning

of the natural day was established among all peoples. For the Babylonians — as Pliny [Plinius] (Book 2) and Macrobius (*Saturnalia*, Book 1) attest — called the day the sun's motion from one sunrise to the next sunrise, whom the inhabitants of the Balearic Islands imitate in our own time. Hence they reckoned the hours from sunrise, and these are called "Babylonian" by most writers, because they were in use among the Babylonians. The Athenians, however — whom now all Italy, together with Bohemia, follows — defined the day from one sunset to the next sunset, and thence numbered the equal hours; these are now called "Italian," from Italy, where their use most flourishes. The Umbrians, next (a custom which all Astronomers also observe), called the whole time from noon to the following noon a day, and reckoned 24 equal hours from noon; and because these are employed by Astronomers in computing the motions of the heavens, they have come to be called "Astronomical." The Egyptians, finally, and — as Macrobius (*Saturnalia*, Book 1) attests — the Roman people, numbered the day from midnight to the next following midnight, and thence observed the elapsed hours. This the Roman Church too observes in the celebration of feasts and fasts. For the rest, from the hours that were in use among the Umbrians, the Astronomers, the Egyptians, and the Romans, the common or ordinary hours — which Germany, France, Spain, and almost all Europe use today — do not much differ. For the peoples of these regions number 12 equal hours from noon until midnight, and as many again from there to the next noon. Hence it happens that in the numbering of the hours from noon to midnight they differ in nothing from the Umbrians or the Astronomers; but from midnight to noon they disagree with them only in the numbering of the hours. For what the Umbrians and Astronomers call the 13th hour from noon, the common European calls the first after midnight; and what for them is the 14th from noon, is called by these the second after midnight, and so on for the rest. In the same way, in the numbering of the hours from midnight to noon they agree with the Egyptians and Romans, but from noon to midnight they disagree with them in the numbering alone. For what among the Romans or Egyptians is called the 13th hour after midnight, is today called by Europeans the first after noon; and what they call the 14th from midnight, is called by Europeans the second after noon, and so forth.

[Margin: The beginning of the day differs among various peoples. The Babylonian hour. The Italian hour. The Astronomical hour. How the common or ordinary hours differ from the Astronomical.]

From this it results that one and the same timepiece shows both the hours counted from noon and the hours counted from midnight, provided you exchange the hour-numbers, as has been said. Nevertheless, in timepieces of this kind the hour-numbers are usually applied according to the common numbering — namely 12 from noon to midnight, and as many from midnight to noon — as will be clear from what follows. Therefore, to sum up everything briefly: since there are two kinds of hours, the one of unequal, the other of equal — and equal hours have their beginning either from sunrise, or from sunset, or from noon and midnight — it comes about that on any timepiece whatever, whether hanging, or concave, or convex, or plane, four kinds of hours can be described: namely, unequal (or ancient) hours; equal hours from sunrise; equal hours from sunset; and equal hours reckoned from noon and midnight. Wherefore, in respect of the hours, every timepiece will be fourfold, and will have these names:

[Margin: In any solar timepiece four kinds of hours can be described. The four kinds of timepieces arising from the variety of the hours.]

ANCIENT, which contains the unequal hours that the ancients used.

BABYLONIAN, which indicates the hours from sunrise, which were in use among the Babylonians.

ITALIAN, which comprises the hours from sunset, which the Italians use.

ASTRONOMICAL, which marks the hours from noon to midnight, and from midnight to noon — which are common not only to almost all Europe, but to all Astronomers as well.

[Margin: The Ancient timepiece. The Babylonian timepiece. The Italian timepiece. The Astronomical timepiece.]

DE INVENTORIBVS SCIOTHERIcorum horologiorum, eorumque scriptoribus.

[On the Inventors of Sundial Timepieces, and their Writers]

[Margin: Anaximenes of Miletus, first inventor of solar timepieces. 4 Kings [2 Kings], chapter 20.]

The first inventor of sundial [sciothericum] timepieces is said to have been Anaximenes of Miletus [Anaximenes Milesius], concerning whom these words are found in Pliny (Book 2, chapter 76): "Anaximenes of Miletus, disciple of Anaximander, discovered the theory of shadows and what they call gnomonics [gnomonicen], and first displayed at Lacedaemon the timepiece that they call a sundial" — thus Pliny. But it has been ascertained by sufficiently clear arguments that the use of timepieces was much older. For we read in Holy Scripture that King Ahaz [Achaz] had a solar timepiece, and he was much older than Anaximenes. For Ahaz reigned about the first Olympiad — that is, in the year 775 before the birth of Christ — or at the least, as others prefer, about the seventh Olympiad, that is, in the year 751 before Christ. Anaximenes, however, flourished about the fiftieth Olympiad, that is, about the year 579 before Christ; so that he was younger than Ahaz by nearly 196 or 172 years. For this reason I would believe that this science of timepieces took its origin from those first Fathers (as Josephus [Iosephus] also affirms concerning the other sciences, in his book *On Antiquity*), and gradually passed to the Egyptians, from these to the Greeks, and finally from the Greeks to the Latins.

[Margin: Various inventors of various timepieces.]

But various inventors of various sundial timepieces are mentioned by Vitruvius (Book 9). For Berosus the Chaldean is said to have invented the hemicycle hollowed out of a square block, on which the lines indicating the hours were drawn. Aristarchus of Samos [Aristarchus Samius] [invented] the scaphe [scaphen], or hemisphere — such as the hemispherical concave timepieces now are. The same man is said to have devised the Disc, that is, a timepiece slightly hollowed in the form of a disc or dish, so that the interior part was not perfectly spherical. Eudoxus the Astrologer is said to have discovered the Arachne — that is, the "spider" — namely a timepiece in the form of a spider's web, such as the horizontal timepiece now is, which contains the astronomical hours and the circles of the sun's altitude, which the Arabs call almucantars [Almucantarath]. Or it may be any timepiece whatever on which both the hour-lines and the arcs of the signs of the Zodiac are drawn. There are those who make Apollonius the inventor of this timepiece. Scopas of Syracuse [Scopas Syracusius] is proclaimed the inventor of the Plinth [Plinthi], or coffer [lacunar] — in the likeness of a certain square beam — on which could be drawn a Vertical timepiece both southern and northern, a Meridian one both Eastern and Western, and also a Horizontal one on the upper plane, such as was set up in the Flaminian Circus. They make Parmenio the inventor of the timepieces that were called πρὸς τὰ ἱστορούμενα [Greek: pros ta historoumena, "for/according to the things inquired

into or recorded"; small type — reading of the middle letters uncertain]; what these were like, we can hardly judge — unless perhaps we should say that on them were contained not only the hour-lines, but also the signs of the Zodiac, the months, and the days, all of which were shown by a pointer, so that those timepieces were like a kind of chronicle. They assert that Theodosius devised the timepiece called πρὸς πᾶν κλίμα [Greek: pros pan klima, "for every clime"], because it served all the climates of the world. Patrocles is related to have discovered the πελεκινον [Greek: pelekinon, "little axe" — a dial in the shape of an axe or double-axe], that is, having the shape of an hour-marker like an axe or a double-axe [bipennis], such a form as the Meridian timepiece now nearly bears, on which the signs of the Zodiac are drawn. Dionysodorus was the inventor, as they say, of the cone, or a timepiece of conical shape. And so various others are recorded by Vitruvius to have devised various forms of timepieces. But Pliny (Book 7, chapter 60) is the authority that the use of timepieces began rather late at Rome. "For in the Twelve Tables," he says, "only sunrise and setting are named; some years later midday too was added, the consuls' attendant [accensus] announcing it, until the first Punic War. Then a solar timepiece — imperfect, however — was set up beside the Rostra on a column by M. Valerius Messala the consul, as M. Varro records, when Catina in Sicily was taken, in the year 477 from the founding of the City; and this they used for ninety-nine years, until Q. Marcius Philippus [Q. Martius Philippus], who was censor together with L. Paulus, [set up a more careful one]...

set up beside it one more carefully arranged, and this service was received most gratefully among the works of the censorship. And Scipio Nasica, the colleague of Laenas, was the first to divide the hours by water — those of the nights equally with those of the days — and he dedicated this timepiece under a roof in the year 595 from the founding of the City. So, more or less, says Pliny.

[Margin: Writers on sundials.]

As for a writer who has treated gnomonics — that is, the theory of sundials — I find none older than Vitruvius, who in Book 9 discusses a few points about the construction of the Analemma, and by this rather hints at how sundials are to be made than actually teaches it. Ptolemy [Ptolemæus], however, published a most penetrating little book on the Analemma, on which Federico Commandino [Federicus Commandinus] wrote most learned commentaries, together with a book on the laying-out of sundials, all of whose demonstrations are drawn from Ptolemy's Analemma. It was from that book that Daniel Barbaro [Daniel Barbarus], in his commentaries on Book 9 of Vitruvius, took everything he

reports about the laying-out of sundials. Ptolemy's *Analemma*, together with Federico Commandino's book on the laying-out of sundials, we shall set out far more clearly and briefly, God willing, in Book 6 of this work. al-Battānī [Albategnius] too, in Proposition 56 of his book *On the Science of the Stars*, teaches very briefly how a sundial of unequal hours may be constructed for any latitude. Among the more recent writers who have illuminated this part of mathematics may be counted: Pedro Nunes [Petrus Nonius Lusitanus], the celebrated Portuguese mathematician, who in his little book *On the Errors of Orontius* shows how horizontal and vertical sundials alone — the rest being omitted — are to be drawn; Federico Commandino, who, as I said, brought out commentaries on Ptolemy's *Analemma* and a little book on the laying-out of sundials; Oronce Finé [Orontius Finæus]; Johann Conrad Ulmer the German [Ioannes Conradus Vlmerus Germanus]; Giovanni Battista Vimercato [Ioannes Baptista Vimercatus] of the Carthusian order, in the Italian tongue; Andreas Schöner of Nuremberg [Andreas Schonerus Norimbergensis]; Giovanni Padovani of Verona [Ioannes Paduanus Veronensis]; Pedro Rodríguez the Spaniard [Petrus Rodericus Hispanus], in the Spanish tongue; Francesco Maurolico [Franciscus Maurolycus], the Sicilian abbot, who wrote three most subtle books on hour-lines; and Giovanni Battista Benedetti [Ioannes Baptista Benedictus], who most recently composed a book on the use of gnomons and of sun-shadows, in which very many things most worth knowing are contained.

[Margin: What the *Analemma* is.]

Therefore, so that we too may bring some light to this so excellent science of gnomonics, we shall begin from the construction of the *Analemma*. This is truly a marvellous invention of the ancients, and the base and foundation of nearly everything we are going to demonstrate in this work. The *Analemma* is a kind of circular figure drawn in the plane of the Meridian — or of any other great circle passing through the poles of the world — described about the centre of that circle, and containing the common sections of the principal circles of the sphere (namely the Equator and its parallels, the Ecliptic, the Horizon, and the vertical circle) with the Meridian, or with that great circle drawn through the poles of the world. This description will be understood most clearly from the first proposition that follows. The ancients called such a figure the *Analemma* — as if a "taking-up" [resumptionem] — because, before they drew the diameters of the other circles, they took, and took up again, certain markings-out of shadows [designationes vmbrarum], in order to place the diameters of the other circles in the required position within the circle of the *Analemma*. This is gathered from what Vitruvius and his interpreter Daniel Barbaro teach in Book 9. That is why Vitruvius in that

place described the Analemma as follows: "The Analemma is a scheme sought out by the course of the Sun, and discovered from observation of the shadow's increase from the winter solstice [Bruma], by which, through architectural reasoning and the drawings of the compass, the effect of the heavens in the world was found."

[Margin: How the Analemma is described by Vitruvius.]

For evidently the ancients — beginning from the course of the Sun and of the shadow cast by the gnomon, starting from the winter solstice [Bruma], that is, from the hibernal solstice (as is gathered from Vitruvius) — marked in the Analemma the diameters of the Sun's parallels according to their several distances from one another. And from this Analemma the effect of the heavens in the world was then found: namely the theory of sundials for any altitude of the pole, the lengths of days and nights, the risings and settings of the Sun, and almost innumerable other things, which it is not the place here to review. But our own description is better suited to the use the Analemma affords us, as will become clear from what follows. Let us therefore now set about describing the Analemma and explaining the other theorems and problems — provided that we first premise the construction of a certain quadrant, which offers us a wonderful usefulness not only in the laying-out of sundials, but also in the construction of the astrolabe and of other instruments pertaining both to Astronomy and to Geometry, and in a certain way makes the work itself both easier and shorter. Anyone will readily learn this from experience itself.

[Margin: Construction of a quadrant having very many uses in the laying-out of sundials and in the building of other mathematical instruments.]

Let there then be two straight lines AB, AC, drawn on some brass or wooden tablet, containing a right angle at the point A. From A as centre, let a quadrant of a circle BC be drawn, and let it be divided into 90 degrees, beginning from the point B. Then from the centre A let straight lines be drawn out to the individual degrees; and finally, from A, let any number of smaller quadrants be drawn, enclosed between the straight lines AB and AC. A quadrant drawn in this way will have uses in mathematics that can scarcely be told. For, quicker than can be said, it gives us all those things that would otherwise have to be sought, through many divisions of circles, with great labour and trouble. I will bring forward here one or two examples of this, for its most abundant fruits will be gathered here and there from what we are going to say throughout this whole work. Take, for example, some figure in which a circle HI has been drawn from the centre G at the interval of the radius GH, and in whose circumference HI — from the point H

toward I — the altitude of the pole is to be marked off, namely 42 degrees, roughly the latitude of Rome. To do this, we shall take with the compass the radius GH, and find on the quadrant a radius equal to it, at whose interval one of the quadrants from A has been drawn (if indeed such a radius can be found). In the given example this is the radius AE, at whose interval the quadrant EF has been drawn. Then in this quadrant EF we shall take with the compass the arc lying between the straight line AB and the straight line AD, which is drawn through the 42nd degree, and we shall transfer it with the same compass from H, along the circumference HI, as far as I. For in this way we shall have the required altitude of the pole, HI — namely 42 degrees — even though we have not divided the circle HI into its degrees. Again, suppose that in the same circle HI the inclination of some plane to the Horizon, amounting to 15 degrees, is to be reckoned. Then if in the same quadrant EF we take with the compass the arc lying between AB and the straight line AO, which is drawn through the 15th degree, and transfer it from H along the circumference HI as far as P, the arc HP will contain 15 degrees. And so, by the help of the quadrant EF, we shall mark off on the circle HI as many degrees as anyone pleases.

But if perchance no radius equal to the radius GH can be found on the quadrant — as almost always happens — we shall take whatever radius we please, say AK, at whose interval the quadrant KL has been drawn; and at the interval of the radius AK so taken we shall draw a hidden arc MN, onto which we shall transfer an arc MN equal to the arc of the quadrant KL that is intercepted between AB and the line AD drawn through the 42nd degree. For the straight line drawn GN will cut the circumference HI at I, and the arc HI will contain 42 degrees, since — by what we wrote in our commentaries on the Sphere near the end of the first chapter — the arcs HI and MN are similar. In the same way, if we take an arc MQ equal to the arc of the same quadrant KL placed between the two straight lines that enclose 15 degrees, the straight line drawn GQ will cut off from the circle HI the arc HP of 15 degrees; and so on for the rest.

[Margin: The use of the quadrant described.]

Moreover, suppose for example that at the straight line GH, at the point G, an angle of the pole's altitude of 42 degrees is to be constructed; likewise an angle of inclination of some plane to the Horizon of 15 degrees, or an angle of any number of degrees whatever — even though no circle is drawn there (a case that not rarely occurs in practice). We shall then take with the compass any radius, such as AE or AK, on the quadrant, and at its interval draw a hidden circumference HI or MN. If in this we take an arc HI or MN equal to the arc of the quadrant EF or

KL that spans 42 degrees, and likewise an arc HP or MQ equal to the arc of the quadrant EF or KL that contains 15 degrees, and so on, and if we draw the straight lines GNI and GQP, then IGH and PGH will be the required angles, and so on.

Diagram. A large graduated quadrant occupying most of the page, with an auxiliary construction to its left. The quadrant proper has its vertex at **A** in the lower-right corner (marked with a small circle representing the sharp pin, *clauiculus*, fixed at the centre). Its horizontal side **AB** runs left along the baseline; its vertical side **AC** rises to **C** at the top-right corner. The outer arc **BC** is finely graduated and labelled in tens: **10, 20, 30, 40, 50, 60, 70, 80, 90** (with 10 near the baseline at B and 90 near the top at C). Two concentric smaller quadrants are drawn from A: the arc **EF** (with E on the baseline AB and F on the vertical AC) and the arc **KL** (with K on the baseline beyond E and L on the vertical). To the left stands the auxiliary "given figure": a centre **G** at the lower left, carrying a vertical plumb-line, from which the circle-arc **HI** is drawn — **H** at the upper left, **I** on the circumference. Radiating lines **GS, GR, GN** run from G; the points **P, R, S** mark where the transferred degrees fall (S near 42°, R near 15°), and the dotted hidden arc through **M, N** (upper-left) illustrates the transfer of an arc equal to a quadrant of the instrument.

- A (lower-right vertex): centre of the quadrant; small circle = the fixed pin
- B (lower-left): end of the horizontal side AB
- C (upper-right): end of the vertical side AC
- E (on baseline AB): foot of radius AE, start of inner arc EF
- F (on vertical AC): end of inner arc EF
- K (on baseline AB, beyond E): foot of radius AK, start of inner arc KL
- L (on vertical AC): end of inner arc KL
- 10, 20, 30, 40, 50, 60, 70, 80, 90 (along arc BC): degree graduations
- G (lower-left, with plumb-line): centre of the target circle HI
- H (upper-left): point on circle HI
- I (on circle HI): where line GN(I) meets the circumference
- P (on the target circle): where the 15° arc HP falls
- R (upper): end of radiating line GR
- S (centre-upper): end of radiating line GS
- M (left): endpoint of the dotted hidden arc MN
- N (lower-left): endpoint of the hidden arc, foot of GN

- e [?] (lowercase, on the dotted arc between M and N): label not fully resolved

You see, then, how much labour this quadrant, constructed in the way we have described, saves us. For if we lacked it, we should always be forced to divide the given circle into its degrees, and everyone knows how laborious and troublesome that is. So, to sum up in brief the whole use of this quadrant: whenever in this gnomonic work the altitude of the pole is to be reckoned, or the declination of some plane from the vertical, or its inclination to the Horizon expressed in a definite number of degrees, or indeed an angle of 20 or 35 or any number of degrees is to be constructed at a given point on some straight line — we shall have recourse to the quadrant constructed here, and from it transfer the required degrees into the given figure, as was done in the examples above, so that we are not forced to divide circles into their degrees, a matter that would be full of labour and trouble.

But now if at the centre A some sharp and very small pin is fixed, and all the parts of the quadrant outside the lines AB, AC and the circumference BC are cut away — leaving, however, a small portion around the centre A to hold the pin, as the figure shows — then this quadrant of ours will have a much handier and quicker use. For if that pin fixed at the centre is placed at the centre G of the proposed figure, and the straight line AB is made to coincide with the straight line GH, we shall at once mark with some sharp stylus, near the 42nd degree, the point S, and near the 15th degree the point R. For the straight lines drawn GS, GR will cut off from the circle HI the arc HI of 42 degrees and the arc HP of 15 degrees, and will contain the angles SGH, RGH of 42 degrees and 15 degrees, and so on. Daily use will teach the rest.

On the other side of the quadrant may be drawn the rays of the signs of the Zodiac, which we shall treat in Proposition 1, now immediately following, so that, when needed, we may transfer them into another figure — the one that must be constructed for drawing the arcs of the signs on sundials, as will appear from what follows.

I have chosen to explain this quadrant here, not because its use is absolutely necessary, but because by means of it we may lay out sundials more easily and quickly, since in this gnomonic work of ours the altitude of the pole, or the declination and inclination of a plane relative to the Equator, must very often be reckoned, and finally an arc of any number of degrees taken, and so on. All of which are accomplished quicker than can be said by means of a quadrant divided in this way, as is manifest from the examples set out.

PROBLEMA PRIMVM. PROPOSIT. PRIMA.

To describe an Analemma for any altitude of the pole.

[Margin: Meridian.] [Margin: Horizon.]

Let there be a Meridian — or rather, in the plane of the Meridian, a circle ABCD — drawn about the centre of the world E, whose common section with the Horizon is understood to be the straight line BD. Then, having reckoned the altitude of the pole for the place for which we are constructing the Analemma, let the diameter FG be drawn from the points B and D in opposite directions as far as G and F; this will be the axis of the world, as can easily be understood if the circle ABCD is set in the plane of the Meridian in such a way that E, its centre, is the same as the centre of the world, and the straight line BD lies in the plane of the Horizon as the common section of the Horizon and the Meridian — with this condition, however, that the point F incline toward the arctic pole and G toward the antarctic. [Margin: Axis of the world.] [Margin: Centre of the world.] For in this way it will come about that the straight line FG, when produced, falls upon each pole, and hence coincides with the axis of the world. This will become plain as follows. [Margin: Poles of the world.] The axis of the world, and the meridian line of the Horizon — that is, the common section of the Horizon and the Meridian — cut off similar arcs from the Meridian circle and from the circle ABCD drawn about the same centre as it, as we showed in our commentaries on the Sphere at the end of the first chapter. Since, therefore, by the construction the arc DF is similar to the arc intercepted in the Meridian between the arctic pole and the Horizon (because the arc DF contains the degrees of the pole's altitude), and the straight line ED is taken as the common section of the Horizon and the Meridian, and the centre E is placed at the centre of the world, then EF will necessarily be the axis of the world, since it cuts off from the circle ABCD the arc DF, similar to the arc of the pole's altitude in the Meridian, as we said. Next let the diameter AC be drawn perpendicular to the Horizon BD; this will be the common section of the Meridian and the vertical circle properly so called.

[Margin: Vertical circle.]

For since the vertical circle stands a quadrant of a great circle away from its own poles, which are at B and D — as is established from the corollary to Proposition 16 of Book 1 of Theodosius, in our edition — and since the arcs AB, AD are quadrants (because of the right angles AEB, AED), the vertical circle will pass entirely through the point A in the Meridian circle; and it also passes through the centre of the world E. Therefore the straight line AC, which we drew through the

centre E perpendicular to BD, will be the common section of the Meridian and the vertical circle properly so called. Again, let the diameter HI be drawn perpendicular to the axis FG; by exactly the same reasoning this will be the common section of the Meridian and the Equator, because the Equator too stands a quadrant of a great circle away from its own poles, which are F and G, as is established from the same corollary to Proposition 16 of Book 1 of Theodosius.

Diagram. The Analemma figure. A large circle **ABCD** about the centre **E**, with **A** at the top, **B** at the left, **C** at the bottom, and **D** at the right. Two perpendicular diameters cross at E: **AC** (vertical, the common section of the Meridian and the vertical circle) and **BD** (horizontal, the common section of the Horizon and the Meridian). The axis of the world **FG** runs obliquely from **F** in the upper right to **G** in the lower left; the Equator's diameter **HI** is drawn perpendicular to FG (**H** toward the upper left, **I** toward the lower right). Parallels **DK** and **BL** are drawn through D and B parallel to HI (the greatest of the always-visible and the always-hidden parallels, with **K** near the top beside A and **L** at the upper left / repeated near the bottom "L C"). At the top the small construction points **M**, **N** (with the auxiliary points **X**, **Y** and further small letters) mark the arcs of greatest declination, and the auxiliary circle **MPN** (points **P**, **Q**) sits toward the upper part of the figure. A dense web of chords fills the disc, with many small italic and Greek labels (the zodiac-parallel construction: points **R**, **S**, Greek β , γ , δ , ϵ on the declination arcs, and lowercase italic **a**, **b**, **c**, **d**, **e**, **f**, **g**, **h** around the lower right marking the parallels through the several signs). Because of the very fine, crowded, and partly rotated lettering, many of these minor labels cannot be resolved with confidence at this resolution.

- A (top): endpoint of vertical diameter AC / near the pole-region point
- B (left): endpoint of horizontal diameter BD
- C (bottom): endpoint of vertical diameter AC
- D (right): endpoint of horizontal diameter BD
- E (centre): centre of the world / of circle ABCD
- F (upper right): pole-end of the axis FG (arctic)
- G (lower left): pole-end of the axis FG (antarctic)
- H (upper left): endpoint of the Equator's diameter HI
- I (lower right): endpoint of the Equator's diameter HI
- K (top, beside A): endpoint of the parallel DK
- L (upper left; also near bottom): endpoint of the parallel BL
- M, N (upper left): endpoints of the arcs of greatest declination (HM, HN)
- P, Q (upper part): points of the auxiliary circle MPN and its diameter

- R, S [?] (division points of the quadrant into signs): partially legible
- β , γ , δ , ϵ [?] (Greek, on the declination arcs): partially legible
- X, Y, Z [?] (top): auxiliary points, partially legible
- a, b, c, d, e, f, g, h [?] (lowercase italic, lower right): parallels through the zodiac signs, not fully resolved

[Margin: Equator (equinoctial).] [Margin: The greatest of the always-visible and of the always-hidden parallels.]

But if through the points D and B we draw DK and BL parallel to HI, these will be the common sections of the Meridian and of those parallels which are, of all the parallels, the greatest of the always-visible and the greatest of the always-hidden respectively [Margin: a. Prop. 16 (Theodosius, Bk. 1) — whence.]; for the Meridian, cutting the Equator and the said parallels, makes their common sections parallel, and the greatest parallel of the always-visible ones touches the Horizon at D, while the greatest of the always-hidden ones touches it at B.

[Margin: How the parallels of the Equator, drawn through the beginnings of the signs of the Zodiac, are described.]

Now, in order to draw the parallels of the Equator that pass through the signs, or degrees, of the Ecliptic — in whose accurate delineation the whole effort and labour of constructing the Analemma is placed, because of the declinations of these parallels from the Equator, which can scarcely be reckoned without error from the Equator's diameter HI on either side, on account of the minutes and seconds that cling to the degrees of declination — we shall use this method handed down by the ancients, as may be seen in Vitruvius, Book 9. Let two arcs HM, HN be taken in the circumference ABCD, each of which is equal to the Sun's greatest declination — that is, contains 23 degrees 30 minutes — and let the straight line MN be joined, cutting HI at O. Now since, when the straight lines EM, EN are drawn, the sides OE, EM of the triangle OEM are equal to the sides OE, EN of the triangle OEN, and they contain equal angles at the centre E (because they stand on the equal arcs HM, HN) [Margin: b. Euclid III.27.], the bases OM, ON will also be equal [Margin: c. Euclid I.4.], and likewise the angles at O — and hence they are right angles. If therefore from O, at the interval OM or ON, a circle MPN is described, and the straight line IH is extended to Q, then the arcs MP, PN, NQ, QM will be quadrants [Margin: d. Euclid III.26.], because, since equal angles at the centre O stand on them — namely right angles — they are equal.

But if each quadrant is cut into three equal parts (and so the whole circle into 12 equal parts, after the manner of the Zodiac, which is divided into 12 equal signs)

at the points R, S, and so on; and if two of these, equally distant from the points P and Q, are joined by straight lines (which will indeed be parallel both to HI and to one another, from what we demonstrated in the scholium on Proposition 27 of Book 3 of Euclid), cutting the arcs HM, HN at the points β , γ , δ , ϵ — then the arcs H β , H γ , H δ , H ϵ will be equal to the declinations of the remaining signs of the Zodiac between Cancer [♋] and Capricorn [♑], as we shall shortly show.

Now if arcs equal to these are cut off — I θ , I λ , I μ , I ξ , I π , I ς — and the straight lines M θ , $\beta\lambda$, $\gamma\mu$, $\delta\xi$, $\epsilon\pi$ are drawn...

let the straight lines $\delta\xi$, $\epsilon\pi$, N ς — or at any rate the parallels XR, YS, and so on — be drawn out. (For the straight lines HI, $\gamma\mu$, $\beta\lambda$, M θ are parallel, as we demonstrated in the scholium on Proposition 27 of Book 3 of Euclid, because of the equality of the arcs H γ , I μ , and $\gamma\beta$, $\mu\lambda$, and so on.) These will be the common sections of the parallels drawn through the beginnings of the signs and of the Meridian circle. For their distances from the straight line HI — the common section of the Equator and the Meridian — are proportional to the distances, in the Meridian itself, of the sections that those same parallels make with the Meridian; since the straight lines drawn from the centre E through the points M, β , γ , and so on cut off from the Meridian, about that same centre E, arcs similar to the arcs HM, H β , H γ , and so on — as follows from what we wrote in the commentaries on the Sphere, at the end of the first chapter.

[Margin: The various positions of the Ecliptic.]

Now the straight lines EM, E β , E γ , and so on are the common sections of the Meridian and of the Ecliptic as it takes up its various positions in the Meridian itself. For EM is a section of this kind when the beginning of ♋ [Cancer] is set in the Meridian; E β , when the beginning of ♊ [Gemini] or of ♌ [Leo] is placed in the Meridian; and E γ , when the beginning of ♉ [Taurus] or of ♍ [Virgo] has occupied the Meridian; and so on — as is evident, provided one understands that the Analemma has its own proper position in the plane of the Meridian. This is very easily grasped from a material sphere as well.

[Margin: The rays of the signs, or of the Zodiac, which are the diameters of the parallels drawn through the points of the Zodiac.]

These straight lines too, when we come to treat of the construction of sundials, we shall call the *rays of the signs* or *of the Zodiac*, because, when the Sun stands at the beginnings of the signs, they represent the rays which at midday the Sun casts through the centre of the world, E. But the straight lines M θ , $\beta\lambda$, $\gamma\mu$, and so on are the diameters of the parallels that run through the beginnings of the signs of the

Zodiac: namely HI, the diameter of the Equator; $\gamma\mu$, the diameter of the parallel of ϑ [Taurus] and of $\mathfrak{M}$ [Virgo]; and so on — just as BD is the diameter of the Horizon, and AD that of the Vertical, and so forth.

[Margin: Another way of describing the parallels of the Equator that pass through the beginnings of the signs.]

Others draw these diameters $M\theta$, $\beta\lambda$, and so on by the following method — and rightly so, in my judgement, because with one effort and one labour they both find the declinations of the parallels and draw those parallels' diameters, HI and the rest, equidistant from one another. Taking arcs HM, HN, I θ , I ϵ , each equal to the Sun's greatest declination, they join the straight lines MN, $\theta\epsilon$, which cut the straight line HI at O and at e. Then about the diameters MN, $\theta\epsilon$ they describe the semicircles MQN, $\theta f\epsilon$ — as much of them as is needed — because, as was demonstrated above, the straight line MN is bisected at right angles at O, and by the same reasoning $\theta\epsilon$ at e. Having divided these semicircles into six equal parts at the points a, z, x, Y, g, h, m, n, they join by straight lines the points that correspond to one another — such as $M\theta$, Yg, Xh, and so on. For these will give the diameters of the parallels, as before, since they are parallel to one another (because the straight lines YS, XR, and so on, together with the semicircle $\theta f\epsilon$, have the same position with respect to the semicircle MQN that the semicircle MPN has, as is manifest).

[Margin: The description of the parallels of the Equator drawn through the individual degrees of the Ecliptic.]

Now if the individual arcs QX, XY, and so on are also bisected, and the same operations are carried out as before, one obtains the common sections of the parallels that are drawn through the half-signs — that is, through every fifteenth degree of the signs. And in the same way the parallels of the individual degrees of the Ecliptic can be investigated, if only the circle MPNQ is divided into single degrees and the rest is done as before. For, in general, the straight lines that are parallel to PQ cut off from the Meridian arcs of declination for those arcs of the Ecliptic which are similar to the arcs of the circle MPNQ — just as the twelve signs of the Zodiac are similar to the arcs QX, XY, and so on. And this we shall demonstrate, by much the same reasoning, together with Pedro Nunes [Pedro Nunes], *On the Art of Navigation*, Book 2.

Diagram. The large construction of the Analemma, occupying the left portion of the page. An outer circle carries the horizontal diameter **B–D** (B at left, D at right) with the centre **E** marked as a small circle at the middle. An inclined inner

circle/ellipse (the plane of the Ecliptic laid into the Meridian) crosses it, with the beginning-of-signs points and the family of parallels of declination drawn as chords, together with their diameters radiating from E. A dense network of chords, radii, and parallel lines fills the figure.

- A, K (upper arc): points near the top of the outer/inner circle
- X, Y (upper left, on the inclined circle): division points
- M (top centre): a beginning-of-signs point (Cancer/Capricorn colure end)
- N (left): division point of the inclined circle
- B (left end of horizontal diameter)
- D (right end of horizontal diameter)
- E (centre, drawn as a small circle)
- F (right side, above D)
- G (lower left), L (bottom), C (bottom centre): points on the lower arc
- W, H (left of centre): [?] interior points
- Numerous small Greek and lower-case point-labels (β , γ , δ , θ , ρ , ϕ , ς , η , μ , λ , and others) mark the chord/parallel intersections across the interior; these are too small to resolve individually on the scan [?].

[This crowded analemma carries many sub-resolution labels]

[Margin: The demonstration of the construction of the Analemma.]

Let a semicircle AMB of the Ecliptic be understood as described about EM, and a semicircle AHB of the Equator about EH, and let the common section of the two be the straight line AB; and let A be the beginning of γ [Aries] and B the beginning of ϱ [Libra]. Now since M is the beginning of ϖ [Cancer] or of $\var�$ [Capricorn] — HM being the portion of the Meridian circle that is the Sun's greatest declination — and since each of these points is distant from the equinoctial points by a whole quadrant, the arcs AM, BM will be quadrants, and therefore the angles AEM, BEM right angles. Now let the straight line XR, drawn in the plane of the Meridian across the arc HM and across the straight lines EH, EM, MO, cut the straight line MO at the point ϕ and the straight line EM at the point C. Let a plane also be understood to be drawn through the straight line XR parallel to the Equator AHB, meeting the straight line EM at C. For since the circle MPNQ, lying in the Analemma in the plane of the Meridian, is perpendicular to the Equator, and QHPE is the common section of the Equator and of that same plane of the Meridian, and the straight line XR is parallel to that section QHPE, a plane can be drawn through XR itself parallel to the Equator...

Diagram. The figure for the demonstration of the Analemma. A large circle (the Meridian, with horizontal diameter along **A–B** at the foot) contains the semicircle **AMB** of the Ecliptic and the semicircle **AHB** of the Equator, together with the inner circle **MPNQ** and its diameters. The centre is **E**, on the baseline.

- X (top): end of a diameter of the inner circle
- Q, ω (upper left): points on the inner circle / its arc
- M (upper centre): beginning of Cancer/Capricorn
- D (left)
- C (centre): intersection of XR with EM
- R (below centre): point on the descending line
- K (right)
- A (bottom left, on the baseline): beginning of Aries
- F (bottom left, near A)
- E (bottom centre): the centre
- B (bottom right, on the baseline): beginning of Libra
- Interior small points near D and the centre (foot O with ϕ , and H on the Equator semicircle) are present but the small labels are hard to resolve [?].

[Interior labels partly sub-resolution]

parallel to the Equator; and this plane will make, in the Ecliptic, the common section DK, a straight line, but in the Sphere the circle D,k — by Proposition 1 of Book 1 of Theodosius — passing through the point γ , where the straight line XR cuts the arc HN of the Meridian.

[Margin: a. Euclid XI.3.]

Since, therefore, as MC is to CE, so is $M\phi$ to ϕO , then, by composition, as ME is to CE, so will MO be to ϕO ; and, by alternation, as ME, the semidiameter of the Ecliptic, is to MO, the semidiameter of the circle MPQ, so is CE to ϕO .

[Margin: b. Euclid VI.2.]

Now CE is equal to the sine of the arc DA — that is, to the straight line DF drawn from D perpendicular to AB.

[Margin: c. Euclid I.34.]

(For AB, DK are the common sections of the parallel planes — namely of the Equator AHB and of the circle D γ K — made by the plane AMB of the Ecliptic, and are therefore parallel;

[Margin: d. Euclid XI.16.]

and likewise CE, DF are parallel.)

[Margin: e. Euclid I.28.]

And ϕO , by the same reasoning, is equal to the sine of the arc QX — that is, to the straight line $X\omega$, which is perpendicular to QE. Therefore the semidiameters ME, MO have the same proportion as the sines DF, $X\omega$, and consequently the arcs AD, QX are similar, as we shall shortly demonstrate in the following lemma.

It remains, then, to show that the arc $H\gamma$, which the parallel XR cuts off from the Meridian, is equal to the arc of declination that the arc AD of the Ecliptic has — the arc AD which we have already shown to be similar to the arc QX of the circle MPQ. This we shall easily accomplish as follows. Let a great circle DG be described through the poles of the world — that is, through the poles of the parallels AHB, $D\gamma K$ — and through the point D; then the arc DG will be the arc of declination of the point D, since it is intercepted between that point and the Equator. Now the arcs of the great circles drawn through the poles of the parallels, intercepted between those same parallels, are equal, by Proposition 15 of Book 2 of Theodosius. But the arcs $H\gamma$, DG belong to great circles drawn through the poles of the parallels AHB, $D\gamma K$, and are intercepted between those parallels; therefore the arc $H\gamma$ will be equal to the arc DG. So in the Analemma the parallel XR cuts off the arc $H\gamma$ equal to the arc of declination of that arc of the Ecliptic which is similar to the arc QX — namely equal to the arc AD. And the same is to be said of the remaining parallels YS, ZT, and αV . It is settled, therefore, that the arcs $H\gamma$, $H\beta$, $H\delta$, and $H\epsilon$ [?] are equal to the declinations of the remaining signs of the Zodiac between $\mathfrak{S}$ [Cancer] and $\mathfrak{V}$ [Capricorn], since the arcs of the signs in the Ecliptic are similar to the arcs QX, XY, and so on in the circle MPNQ — for both alike are twelfth parts of their own circles.

Now since the sections of the parallels made by the plane of the Meridian, drawn through the beginnings of the signs, are parallel to one another, it is plainly evident that those parallels drawn through the points M, β , γ , H, and so on are the diameters of the parallels, since they cut off from the circle AB, CD arcs similar to those which the diameters of the said parallels really cut off from the Meridian, as was said before. And if the circle ABCD were equal to the Meridian in the Sphere, the parallels drawn through the beginnings of the signs would pass entirely through those parallel lines. We shall demonstrate exactly the same thing if, in place of the Meridian, one understands the circle ABCD to be any other great circle drawn through the poles of the world — such as the Colure of the solstices

— as we said above in the definition of the Analemma. We have therefore described the Analemma for any elevation of the pole whatever; which was to be done [recte: *Quod erat faciendum*; the print reads *etat*]. An easier demonstration of this matter you will find in Book 8, chapter 6.

LEMMA.

[Margin: The whole sines are proportional to the sines — both right and versed — of similar arcs, and conversely.]

Whatever proportion the whole sines have — that is, the semidiameters of any circles whatever — the same proportion the right sines, and likewise the versed sines, of similar arcs have. And conversely, arcs whose right sines, and likewise whose versed sines, have the same proportion as the whole sines, are similar.

Let AB, CD be arcs of circles whose semidiameters AE, CF are similar, whose right sines are BG, DH, and whose versed sines are AG, CH. I say that as AE is to CF, so also is BG to DH, and AG to CH. For, if the semidiameters BE, DF are again drawn, then, because of the similarity of the arcs AB, CD, the angles E and F will be equal, from what we showed at Proposition 33 of Book 6 of Euclid; and therefore, since the angles at G and H are also equal — being right angles —

[Margin: a. Euclid I.32.]

the triangles BEG, DFH will be equiangular.

[Margin: b. Euclid VI.4.]

Hence, as BE is to BG, so will DF be to DH; and, by alternation, as BE — that is, AE — is to DF — that is, to CF — so is BG to DH.

Again, since, because of the similarity of the arcs AB, CD, the triangles BEG, DFH are similar in the same way,

[Margin: c. Euclid VI.4.]

it follows that as BE — that is, AE — is to EG, so is DF — that is, CF — to FH; and, by conversion of ratio, as AE is to AG, so is CF to CH; and, by alternation, as AE is to CF, so is AG to CH. Which was the first point.

But let it now be granted that as AE is to CF, so is both BG to DH and AG to CH. I say that the arcs AB, CD are similar. For, if the straight lines BE, DF are again drawn, then, since as AE is to CF, so is BG to DH; and, by alternation, as AE — that is, BE — is to BG, so is CF — that is, DF — to DH; and since the angles at G and H

are equal, being right angles, the triangles BEG, DFH will have the angles at G and H equal, and the sides BE, BG about the angle B proportional to the sides DF, DH about the angle D. Since, then, each of the remaining angles E and F is less than a right angle — because the angles G and H are right angles —

[Margin: d. Euclid I.17.]

the triangles BEG, DFH will themselves be equiangular,

[Margin: e. Euclid VI.7.]

and will have equal the angles B and D, about which the sides are proportional, and therefore also the remaining angles E and F. And so, from what we showed at Proposition 33 of Book 6 of Euclid, the arcs AB, CD will be similar. It is thus settled that arcs whose right sines are proportional to the whole sines are similar.

Diagram. Two figures for the Lemma, drawn side by side. **Left (smaller):** an arc CD with centre F; C at the top, D at the left end of the diameter, and H the foot of the perpendicular CH on the diameter (giving right sine and versed sine), with lines FC, FD and CH drawn. **Right (larger):** an arc AB with centre E; A at the top of the arc, B to its upper left, and the foot of the perpendicular near A (right sine BG, versed sine AG), with radii EA, EB drawn. The foot-point label on the right figure (G) is very small and not cleanly resolved [?].

Then, since as AE is to CF, so is AG to CH, it will follow, by alternation, that as AE is to AG, so is CF to CH; and, by conversion of ratio, that as AE — that is, BE — is to EG, so is CF — that is, DF — to FH. Since, then, the angles at G and H are also equal — namely right angles — the triangles BEG, DFH will have the angles G, H equal, and the sides BE, EG about the angle E proportional to the sides DF, FH about the angle F.

[Margin: a. Euclid I.17.]

Moreover, each of the remaining angles B and D is less than a right angle, because the angles G and H are right angles.

[Margin: b. Euclid VI.7.]

Therefore the triangles BEG, DFH will be equiangular, and will have equal the angles E and F, about which the sides are proportional. For which reason, by what we wrote at Proposition 33 of Book 6 of Euclid, the arcs AB, CD will be similar.

From all this it is clear that arcs whose versed sines are proportional to the whole sines are similar; and the same was just now shown of the right sines. Whatever proportion, then, the whole sines have, and so on. Which was to be demonstrated.

Diagram. The same pair of Lemma figures, repeated at the head of this page.

Left: arc CD, centre F, with C (top), D (left), H (foot of the perpendicular), and lines FC, FD, CH. **Right:** arc AB, centre E, with A (top), B (upper left), the small foot-point near A, and radii EA, EB. As on the preceding page, the right-hand foot-label (G) is too small to resolve cleanly [?].

It is therefore established, from these things, that the arcs AD, QX in the figure of the preceding demonstration are similar — as was assumed in the demonstration of this proposition — since their right sines have the same proportion as the semidiameters of the circles to which the arcs belong. Likewise it is established that the whole sine of the Zodiac is to MO, the whole sine of the circle MPNQ, as the sine of the arc of the Zodiac of 30 degrees is to Oφ, the sine of the arc QX, which likewise contains 30 degrees — because arcs of this kind are similar. And this is what will be of use to us in the following corollary.

COROLLARIUM.

[Margin: The proportion of the whole sine to the sine of the greatest declination is the same as that of the sine of an arc of the Ecliptic reckoned from the equinox to the sine of the declination of the point of the Ecliptic that bounds the said arc. — The computation of the declinations of all the points of the Ecliptic by means of the sine.]

From the demonstration of this proposition it is manifest that, as ME, the whole sine, is to MO, the sine of the Sun's greatest declination, so is CE, the sine of the arc AD of the Ecliptic reckoned from the equinox A, to φO, the sine of the declination that the point D — which bounds that same arc AD — has. For the straight line CE is equal to the sine DF of the arc AD of the Ecliptic; and, by the same reasoning, the straight line φO is equal to the sine γI of the arc Hy, which we demonstrated to be equal to the arc DG that measures the declination of the point D. Hence, with little trouble, the declinations of all the points of the Ecliptic can be computed from a table of sines, as was done by us in the commentaries on the Sphere.

For example, suppose the declination is to be found of the 10th degree of ♈ [Aries], or of the 20th of ♉ [Pisces], or of the 10th of ♎ [Libra], or of the 20th of ♍ [Virgo] — for of these four degrees the first two recede equally from the vernal

equinox on either side, and the latter two from the autumnal; so that all four have the same declination. Let it be arranged that, as the whole sine 100000 is to the sine of the greatest declination 39874, so is the sine of an arc of 10 degrees in the Ecliptic, reckoned on either side from either equinox, 17364, to a fourth term. By the rule of proportion this fourth sine will be found to be about 6923, to which there answers in the table of sines an arc of about 3 degrees 58 minutes. So great, therefore, is the declination of any of the four proposed degrees; and the same reasoning holds for all the rest.

That this is the proportion — the whole sine to the sine of the greatest declination, as the sine of the arc of the Ecliptic begun from either equinox to the sine of the declination of that point in the Ecliptic which bounds the said arc — has been confirmed by an easier and shorter demonstration by Johannes Regiomontanus [Regiomontanus], *Epitome*, Book 1, Proposition 18, and by Pedro Nunes in his book *On Twilights*, and by us too in Lemma 18 of Book 1 of the *Astrolabe*.

[Margin: What the straight lines drawn in the Analemma are.]

It is also perfectly clear that, in the Analemma as described, the diameters passing through the centre E are the common sections of the Meridian and of the great circles — since all the great circles are drawn through the centre of the Sphere, in which we have placed the centre E, and therefore all cut one another there. But the straight lines that do *not* pass through the centre E are not, in reality, the common sections of the Meridian and the *lesser* circles, since in the Meridian these have far greater distances between one another than in the circle ABCD; rather, they cut off from the circle ABCD arcs similar to those which, in the Meridian, the lesser circles — and hence the common sections of those circles and the Meridian — really cut off, as we demonstrated. And if the Meridian of the Sphere were equal to the circle ABCD, then those straight lines not drawn through the centre would indeed be the common sections of the lesser circles and the Meridian, because the arcs of the circle ABCD would be altogether equal to the arcs of the Meridian circle.

SCHOLIVM.

But if we are willing to take as true and granted, as though demonstrated elsewhere, that as the whole sine is to the sine of the greatest declina...

so is the sine of any arc of the Zodiac begun from ♈ [Aries] or ♎ [Libra] to the sine of the declination of that arc. This was demonstrated both by Johannes

Regiomontanus [Regiomontanus], *Epitome*, Proposition 18, Book 1, and by Pedro Nunes [Petrus Nonius] in his book *On Twilights*, and by us too in Lemma 18 of Book 1 of the *Astrolabe*. We shall now show, much more briefly and clearly, how the parallels XR, YS, ZT, and ΦV [?] in the Analemma cut off the arcs of declination from the Meridian, in the following way.

In the Analemma above, the whole sine in the Zodiac is to MO — the whole sine in the circle MPNQ, that is, to the sine of the greatest declination — exactly as (this is clear from the preceding Lemma) the sine of an arc of the Zodiac of 30 degrees, reckoned from the beginning of Υ [Aries] or $\varnothing$ [Libra], is to C ϕ , the sine of the arc QX of the circle MPNQ, which likewise contains 30 degrees. But by what Johannes Regiomontanus [Regiomontanus] and Pedro Nunes [Petrus Nonius] have demonstrated — as was said, and as we take for true and granted — the whole sine of the Zodiac is to the sine of the greatest declination just as the sine of an arc of the Ecliptic of 30 degrees, begun from the beginning of Υ or $\varnothing$, is to the sine of the declination of that same arc of 30 degrees. Therefore the sine O ϕ is equal to the sine of the declination of that arc of the Ecliptic which spans 30 degrees, and it is similar to the arc QX. Hence the arc Hy is equal to the arc of declination of that arc of the Ecliptic of 30 degrees, which is similar to the arc QX of 30 degrees. The same reasoning holds for all the other arcs of the Zodiac, which was to be demonstrated.

[Margin: Another demonstration of the way the parallels of the Equator are drawn through the points that fall along the Ecliptic.]

The declinations of all the signs of the Ecliptic can also be found in this way. Let the circle of the Analemma, ABCD, be divided into 12 equal parts, beginning from the point M, the point of greatest declination; and let each pair of division-points equally distant from M be joined by straight lines. In the Analemma these are the lines marked by the points, and they cut the straight line M ϕ at the points l, p, E, q, u; by the scholium of Proposition 27, Book 3 of Euclid, they will all be parallel to one another. Where these lines cut the straight line M ϕ — namely at the points l, p, E, q, u — through those points the straight lines $\beta\lambda$, $\gamma\mu$, and so on, are to be drawn parallel to the equinoctial line HI, to serve as the diameters of the parallels described through the beginnings of the signs, so that in turn the arcs H β , Hy, and so on, are the declinations of the signs of the Zodiac.

To demonstrate this, we must understand the circle ABCD to be the Ecliptic, whose sign-beginnings lie at those 12 division-points, so that M and ϕ are the beginnings of $\var�$ [Cancer] and $\var�$ [Capricorn]. Then, keeping this Ecliptic fixed and in the position it holds in the Sphere — with the beginning of $\var�$ (the point M)

placed on the Meridian circle above the Horizon, and the beginning of γ (the point ϕ) on the same Meridian below the Horizon — we must imagine this same circle turned about the diameter $M\phi$, like a Colure of the solstices, until it stands at right angles to the plane of the Ecliptic, with the point F facing straight toward the north pole and G toward the south pole. In this way the Equator, drawn at right angles to this Colure along the straight line HI, will [a] produce in the plane of the Ecliptic a common section — the straight line marked by the points — which cuts $M\phi$ itself at right angles at the centre E.

[Margin: (a) Euclid, Elements XI.3.]

For since both the plane of the Ecliptic and that of the Equator are at right angles to the Colure, [b] their common section will likewise be at right angles to the same Colure, and therefore also to the straight line $M\phi$, which lies in that same Colure. Suppose, then, that through the point p, for example, we imagine a plane parallel to the Equator to pass: it will make in the plane of the Ecliptic a straight line marked by the points and passing through p, [c] parallel to the other straight line drawn through E and marked by its points — this because these lines drawn through E and through p are the sections that a plane parallel to the Equator makes with the plane of the Ecliptic. But in the Sphere, by Proposition 1, Book 1 of Theodosius, this plane will trace [d] a circle parallel to the Equator, whose diameter — running through p — will be parallel to the diameter HI of the Equator; for HI and the diameter of this circle are the sections that the parallel planes (namely the Equator and the plane equidistant from it) make with the plane of the Colure of the solstices.

[Margin: (b) Euclid XI.19. (c) Euclid XI.16. (d) Euclid XI.16.]

Therefore the straight line $\gamma\mu$, drawn through p parallel to HI, is the diameter of that parallel which, in the Ecliptic, passes through the points bounding the straight line marked by the points and drawn through p — namely through the arcs of 30 degrees reckoned from the equinoctial points; and these equinoctial points are bounded by that marked straight line drawn through the centre E. Hence, since the arc of the Colure of the solstices intercepted between the Equator and any parallel circle measures the declination of that parallel from the Equator, the arc $H\gamma$ will be the declination of the parallel whose diameter is $\gamma\mu$, which runs through those points on the Ecliptic bounded by the marked straight line drawn through the point p. The same reasoning holds for the rest. And if the circle ABCD is divided not merely into 12 parts but into single degrees as well, and the same operations are performed as before, we shall by the same device

describe the diameters of the parallels passing through the individual degrees of the Ecliptic.

[Margin: Another description of the parallels passing through the points of the Ecliptic.]

Finally, if the declinations of the signs, or of any points whatever of the Ecliptic — found by the doctrine of sines, as we taught in the corollary of this proposition — are laid off from the Equator's diameter HI toward either side in the circle of the Analemma, and through the endpoints of these measurements lines are drawn parallel to HI, these will again be the diameters of the parallels passing through the beginnings of the signs, or through given points of the Ecliptic, as before. Yet, as we said at the outset, the description of the Analemma by this route is uncertain, because of the declinations, which can scarcely be laid off without error in the circle ABCD, since minutes and seconds of degrees cannot be marked out in it.

[Margin: Knowledge of the altitude of the pole is necessary for the description of the Analemma.]

For the rest — by what procedure the elevation of the pole in any region whatever is to be investigated, which is indeed required for a correct construction of the Analemma (for the axis FG cannot be drawn if it is unknown how great the arc DF of the pole's altitude ought to be) — we have shown both in the *Use of the Astrolabe*, and in the *Cosmography*, and also in our commentaries on the *Sphere*, where we treated of the offices of the Meridian circle. Nevertheless we shall find that same altitude of the pole by another method, by means of the Analemma, in Scholium 2 of Proposition 28 of this book.

Diagram. A large, densely worked Analemma: a great circle (the Meridian, with the equinoctial diameter HI through its centre) carrying a second, obliquely inclined great circle (the Ecliptic/oblique circle) drawn as an ellipse across it, together with a dense pencil of straight chords and dotted lines representing the parallels, colures, and common sections discussed in the text. The centre is marked E. This figure is heavily labelled with a mixture of Roman capitals, Greek letters, and lowercase italics around the rim and interior; many peripheral labels are too small to resolve at this resolution.

- E (centre): centre of the Analemma / Meridian
- B (left end of horizontal diameter) and D (right end): endpoints of the horizontal diameter BD

- H, I: endpoints of the equinoctial line/diameter HI (the equinoctial diameter through E)
- F (upper right, on the oblique circle) and G (lower left): the poles of the Ecliptic / axis endpoints, F toward the arctic and G toward the antarctic pole
- M, φ : beginnings of ϖ [Cancer] and φ [Capricorn] on the oblique circle (φ near lower region)
- Upper rim (left to right): N [?], X [?], Y, M, A, K [?]
- Interior division/section points named in the text: l, p, E, q, u (on the line M φ); β , λ , γ , μ (diameters of parallels)
- Lower-right rim carries a series of small italic/Greek labels (b, g, h, f, n, and further symbols) that are not legible at this resolution [?]
- L, C (lower centre): rim points

[Continuation — Liber Primus]

This figure, then, which we have constructed up to this point, containing the common sections of the said circles with the Meridian circle, was called the *Analemma* by the ancients and especially by more recent writers — though they have not all defined it in one and the same way, since each had a different aim in defining it. We, however, have chosen to explain it through the common sections which the principal circles of the sphere make in the plane of the Meridian.

[Margin: Other circles besides those mentioned can also be described in the *Analemma*.]

Now the sections of other circles too with the same Meridian (such as the parallels of the Horizon, the parallels of the Ecliptic, and so on) can be described in that same Meridian, as is done in the description of the Astrolabe, and as will sometimes be done in what follows. Indeed, even a circular figure whose circle stands for some other great circle besides the Meridian, containing the common sections of other circles with that great circle, has customarily been called an *Analemma*, as we shall teach in its proper place.

[Margin: A different acceptance of the *Analemma*. — Which lines of the *Analemma* stay the same in every clime, and which do not. — How the drawing of the *Analemma* begins from the parallels passing through the signs of the Zodiac.]

Moreover, the *Analemma* as thus far constructed, so far as concerns the parallels found through the circle MPNQ, serves all the climes of the world. For these

parallels never change, in whatever Horizon the Analemma is set up, unless the greatest declination of the Sun has first changed — since their description depends on this greatest declination alone, as is evident. The other sections, or straight lines, however, vary with the varying altitude of the pole above the Horizon; for the same altitude of the pole above the Horizon is not found everywhere. Hence, if the parallels drawn through the signs of the Zodiac are first described in the Analemma — as being unchangeable in any clime (namely by first drawing, for the Equator's diameter, the straight line HI, and then working out the greatest declinations of the Sun, HM, HN, and so forth) — we shall complete its remaining parts for a given altitude of the pole in this way. Through the centre E, draw FG perpendicular to the Equator's diameter HI, for the axis of the world. Then from the points F and G, in opposite directions, with the altitude of the pole marked off up to D and B, draw the diameter BD for the Horizon; and for the Vertical draw another diameter, AC, cutting BD at right angles. Finally, from the points D and B draw DK and BL parallel to the Equator's diameter HI, to serve as the diameters of the parallels which — between the everlastingly visible and the hidden ones — are the greatest.

[Margin: The various uses of the Analemma.]

Furthermore, it is incredible how manifold and varied a use the Analemma has in matters of Astronomy. For by means of it, not only is the construction of the Astrolabe (which Ptolemy calls the *planisphere*) carried through by geometrical demonstrations, but also nearly everything that pertains to demonstrating the phenomena of the primum mobile is drawn out without great labour — which it is not the place to explain here. In this our Gnomonic work too its excellence and remarkable usefulness will shine out plainly, since almost all the demonstrations employed in the description of sundials are elicited from the Analemma, as will become clear from what follows — especially when we come to treat of Ptolemy's Analemma, from which sundials are laid out with wonderful ease. For now I shall be content if I briefly show how readily, from the Analemma, one may come to know the lengths of the days and nights; the time of sunrise and sunset in terms of Italian and Babylonian hours, and the time of noon; likewise the time of rising and setting reckoned by astronomical hours; and also the rising- and setting-amplitudes of all points of the Ecliptic at any time of the year, and for any latitude of place whatever.

Diagram. A large circle graduated into 24 equal hour-divisions by tick-marks around its rim, with a vertical and a horizontal diameter crossing at a small central mark. Two concentric series of numerals run round the rim: an outer

series and an inner series, so that each division carries two numbers (the Italian/Babylonian reckoning against the astronomical reckoning). At the top, on the vertical diameter, is a symbol flanked by "16" (left) and "17" (right); the numbers run round the circle in sequence from there. Because this is a numeral-dense figure, individual rim numbers are read below only where legible.

- Top (vertical diameter, meridian): symbol [☿?] with 16 to its left and 17 to its right
- Upper left: 15, then a symbol with a small numeral [?]
- Upper right, descending: 18, 19
- Right side, inner/outer pairs descending: 4 / 20, 3 / 21, 2 / 22, 1 / 23, then 24 at the horizontal
- Lower rim numbers (20, 21, 3, 4, 5) and left rim numbers (9, 10, 11, 12, 13, 14) are partly legible; the full sequence is not resolved cell-by-cell [?]

Diagram. To the right, a smaller circle (the parallel-circle $M\theta$ of the following demonstration) with a vertical diameter and a horizontal diameter. Dotted straight lines run down from its lower left to a point G below and to the left, linking it to the large hour-circle. The rim is graduated with small numerals (roughly 1–12 on each side).

- A (top of vertical diameter)
- E (centre)
- F (on the vertical diameter, just below the centre E)
- C (foot of the vertical diameter)
- D (right end of the horizontal diameter)
- H (lower left, below the horizontal)
- V (left, on/near the horizontal diameter)
- G (below and to the left, meeting-point of the dotted lines)

Then let it be proposed to learn all this thoroughly, for the case when the Sun runs through the parallel of ☊ [Cancer], or of ♊ [Capricorn], by the motion of the primum mobile. About the diameter of the parallel of ☊, namely $M\theta$ (or of ♊, namely $N\rho$), from the centre b — for since the axis FG [a] cuts all the parallels at right angles (because it cuts the Equator's diameter, to which they are equidistant, at right angles), [b] and therefore bisects them, the point b will be the centre of the circle to be described about $M\theta$ — let a separate circle be described about $M\theta$. Taking the straight line Ma equal to the Ma in the Analemma, draw through a, perpendicular to $M\theta$, the line de, which will be the common section of the Horizon and of the proposed parallel.

[Margin: The finding of the diurnal and nocturnal arcs from the Analemma, and of the hour of sunrise and sunset. — (a) Euclid I.29. (b) Euclid III.3.]

For since both the Horizon and the parallel of $\mathfrak{S}$ are at right angles to the Meridian, [c] their common section will likewise be at right angles to the same Meridian; and therefore, by Definition 3 of Book 11 of Euclid, it is perpendicular to the straight line $M\theta$ (which lies in the Meridian of the Analemma) at the point a , where the Horizon and the parallel cut one another in the Meridian. Hence the straight line de , which is drawn in the circle $Md\theta e$ through a perpendicular to $M\theta$, will be the common section of the Horizon and the parallel of $\mathfrak{S}$. So that, if the circle $Md\theta e$ is imagined to be turned about the diameter $M\theta$ in the Analemma until it stands at right angles to the plane of the Meridian — and therefore the straight line de of this separately described circle is likewise perpendicular to that plane — the Horizon, standing at right angles to that same plane of the Meridian, will pass through the points d and e , and so along the straight line de . We shall take up this same reasoning again in Proposition 33 of this book, where it will perhaps become clearer, since there the Sun's parallel is described in the Analemma itself about its own diameter. Thus the arc dMe will be the diurnal arc of $\mathfrak{S}$ (namely the part that stands above the earth), and $d\theta e$ the nocturnal; or that same arc will be the nocturnal arc of $\mathfrak{V}$ and this the diurnal. Hence if the whole circle $Md\theta e$ is divided into 24 equal hours, beginning from the point d or e — we have begun from e , because it now marks...

[Margin: (c) Euclid XI.19.]

the point of rising in the Horizon for Babylonian hours, and the point of setting for Italian hours — it will at once appear how many hours both the arc dMe and the arc $d\theta e$ comprehend. Thus you see that the diurnal arc of $\mathfrak{S}$, dMe , comprehends fifteen hours and a little more, while the nocturnal arc $d\theta e$ comprehends not quite 9 hours, but a little less. So too you understand that in the former arc the Sun sets, in the manner of hours reckoned from sunrise, a little after the fifteenth hour, and rises a little before the ninth hour, in the manner of hours reckoned from sunset, and so on. The outer hours in the diurnal arc of $\mathfrak{S}$, dMe , and the nocturnal arc $d\theta$, belong to the Italian hours, and the inner ones to the Babylonian. By contrast, in the diurnal arc of $\mathfrak{V}$, $d\theta$, and the nocturnal arc dMe , the outer hours belong to the Babylonian and the inner ones to the Italian. Yet the point e is always taken for the Sun's rising, if we are speaking of hours reckoned from sunrise, and the point d for its setting; and conversely, if the talk is of hours reckoned from sunset, the point e is to be taken for the Sun's setting and d for its rising, as the numbers of the hours show.

The same may be seen in astronomical hours, if the division of the circle begins from the point M, the point of noon — as is plain in the other, smaller circle ABCD described about the centre E, which is divided by the straight line BD in the likeness of the circle Mdθe, so as to stand for the parallel of ☊ (or of ☋) described from some smaller Analemma. Here too, then, you perceive that the diurnal arc of ☊, BAD, comprehends fifteen hours and more, while the nocturnal arc BCD comprehends a little less than 9 hours. Likewise, in the former case the Sun rises at D a little before 4½ hours after midnight, and sets at B a little after 7½ hours after noon, and so on.

All this may be tested in every other parallel of the Sun, and in any clime whatever, provided the Analemma is set up for the altitude of the pole in the given clime.

But let us teach by what art that smaller circle ABCD — or any other, even a larger — is to be divided, by the aid of the Analemma, into two portions, of which one is similar to the diurnal arc and the other to the nocturnal, when the Sun stands at the beginning of ☊ (or at any other degree of the Ecliptic). This will prove of excellent use in describing the hours reckoned from sunrise or sunset, as will be clearly established from what follows.

[Margin: How a given circle is divided into two portions, of which one is similar to the diurnal arc and the other to the nocturnal, when the Sun stands at any degree whatever of the Ecliptic. — (a) Euclid VI.2.]

Suppose, then, for example, that the circle ABCD is to be divided into the diurnal and nocturnal arc of ☊, just as the circle Mdθe was divided. Having drawn the diameter AC at pleasure, draw from A the straight line AG making any angle with AC, and from it cut off AG equal to the diameter of the parallel of ☊, Mθ, in the Analemma, and again the straight line AH equal to the portion Ma of that same diameter. Then, joining the straight line CG, draw through H the line HF parallel to it, cutting the diameter AC at F; and through F erect the perpendicular BD to AC. I say that the arc BAD is similar to the diurnal arc of ☊, and the arc BCD to the nocturnal — that is, that the arc BAD is similar to the arc dMe in the larger circle, and the arc BCD to the arc aθe. [a] For since AH is to HG as Ma is to aθ, and as AF is to FC, then by composition Mθ is to aθ as AC is to FC; and by alternation, Mθ is to AC as aθ is to FC. But as Mθ is to AC — diameter to diameter — so is Mb, the semidiameter, to the semidiameter AE. Therefore, likewise, as Mb the semidiameter is to the semidiameter AE, so is aθ, the versed sine of the nocturnal arc of ☊, dθ, to FC, the versed sine of the arc BCD. By the lemma given above, then, the nocturnal arc of ☊, dθe, and the arc BCD are similar; and consequently

the arc BCD will comprehend as many hours of its own circle as are contained in the arc d θ e. For this reason the remaining arc BAD will likewise be similar to the remaining diurnal arc dMe, which was the proposition.

[Margin: The finding of the rising- and setting-amplitude from the Analemma.]

Now the rising- or setting-amplitude will be found from the Analemma in this way. From the point a, where the parallel cuts the Horizon, draw the perpendicular ad to the Horizon's diameter BD. I say that the arc dA is the rising- or setting-amplitude. For since, as we demonstrated a little earlier, the common section of the parallel and the Horizon is at right angles to the plane of the Meridian — and therefore, by Definition 3 of Book 11 of Euclid, perpendicular to the straight line BD lying in the plane of the Meridian — it comes about that, if the circle ABCD of the Analemma is taken for the Horizon, the straight line ad in the Horizon is the common section of the Horizon and of the Sun's parallel; so that the Sun rises or sets at d, if the Horizon has its proper position, such that BD is the Meridian line (that is, the common section of the Horizon and the Meridian), and AC the common section of the Horizon and the Vertical, and therefore also of the Equator — so that the Sun, when it is on the Equator, rises or sets at A. Hence the arc dA of the Horizon, between d (the rising or setting of the Sun's parallel) and A (the rising or setting of the Equator), will be the rising- or setting-amplitude, the Sun describing the parallel of the diameter M θ . The same reasoning must be applied to the rest. And ad in the Analemma will always be equal to the straight line ad, or ae, in the parallel Md θ e, because each is a common section of the Horizon and the parallel, running out from a up to the surface of the Sphere, where they coincide with one another, if both the Horizon and the parallel are conceived in their proper position.

THEOREMA PRIMVM.

PROPOS. SECVNDA.

In any sundial the tip of the gnomon must be reckoned as the same thing as the centre of the world; but the plane of the sundial itself is to be understood as distant from the centre of the world by only as much as the length of the gnomon, and as equidistant from the great circle to whose plane the gnomon stands perpendicular, and from which the sundial takes its name.

[Margin: The tip of the gnomon of any sundial is the centre of the world, but the plane of the sundial lies outside the centre of the world.]

Let AB be the gnomon of some sundial, standing at right angles to the plane of the sundial, which we understand to be drawn by the straight line CD. Now since the whole earth, compared with the Sphere of the Sun, is like a point and a centre — as we have shown in the commentaries on the *Sphere*, following Ptolemy and other astronomers — the centre of the world will differ in nothing from the point A, the tip of the gnomon or style; seeing that from the tip A to the centre of the earth (which is the same as the centre of the world), as we taught in those same commentaries, there is no appreciable distance, when it is compared with the distance the Sun has from that same tip. And so the circle drawn through the tip A, equidistant from the plane of the sundial drawn by CD, may not unfittingly be taken for a great circle. Hence with fullest right those who treat of the description of sundials — in order to be able, by the theory of the shadows which the Sun casts at every moment of time, to demonstrate their results — conceive the tip of the style to be set at the centre of the world, onto which all the incident rays of the Sun project through...

Diagram. A large circle representing a great circle of the world (the Sphere), with the Sun shown at the upper left as a radiating face. Inside, a horizontal diameter and a set of lens-shaped (vesica) curves span the circle. The gnomon stands vertically at the centre. A dotted ray runs from the Sun down through the gnomon's tip.

- H (upper left, radiating face): the Sun
- E (left end of the horizontal diameter) and F (right end): endpoints of the horizontal great-circle diameter EF
- A (centre, tip of the gnomon): the tip of the style / centre of the world
- B (below A, foot of the gnomon): the base of the gnomon AB
- C (lower left) and D (lower right): endpoints of the straight line CD, the plane of the sundial
- G (bottom, between B and D): point on the sundial-plane line

projecting, through the tip of the gnomon, the shadow onto the plane of the sundial — as the figure shows, in which the shadow is BG on the plane of the sundial, the Sun having the altitude EH above the great circle EF, to which the plane CD, on which the shadow falls, is parallel. For when HAG, the ray of the Sun at the point H, reaches the point G, and the style is opaque, the whole triangle ABG will be in shadow, since the rays of the Sun do not fall within its area; and so BG will be the length of the shadow on the plane of the sundial, while all the remaining parts — such as GD and BC — are lit by the Sun. In the same way we shall be able to investigate the length of the shadow at any time, once we have

found the altitude of the Sun above the great circle that is parallel to the plane of the sundial, as we shall say in its place. From this it is clear that the plane of the sundial, drawn along the line CD, is distant from the centre of the world by only as much as the length of the gnomon AB, since the tip A is placed at the centre, as we said. And if a great circle is understood to be drawn along the line EAF, to which the gnomon AB is perpendicular, the plane of the sundial will be parallel to this circle — since the gnomon AB is set perpendicular both to the plane of the circle along the line EF and to the plane of the sundial along the line CD.

[Margin: (a) whence 14 — i.e. see Prop. 14.]

Diagram. A large circle representing a great circle of the world. At the upper left is the Sun, drawn as a radiating face. A horizontal diameter EF runs across the circle; the gnomon stands vertically at the centre with its tip at A and its foot at B. Below, the line CD represents the plane of the sundial, parallel to EF, and G marks the point where the shadow ends. A dotted ray runs from the Sun H down through the tip A to G (the ray HAG). Several lens-shaped (vesica) curves span the circle, representing the circles/parallels.

- H (upper left, radiating face): the Sun
- E (left end of horizontal diameter) / F (right end): endpoints of the great-circle diameter EF, to which the dial plane is parallel
- A (centre): tip of the gnomon (= centre of the world)
- B (below A): foot of the gnomon; AB is the gnomon
- C (lower left) / D (lower right): endpoints of the line CD, the plane of the sundial
- G (bottom, between B and D): end of the shadow; BG is the shadow

Now the sundial takes its name from the circle of the diameter EF to which it is parallel. For if that circle is the Horizon, the sundial is called Horizontal; if it is the Vertical, Vertical; if the Meridian, Meridian; if the Equinoctial, Equinoctial — and so with the rest, as we said above when we set out the various kinds of sundials. For as many sundials can be made as there are great circles that can be drawn in the Sphere through the centre of the world, since to each of them parallel planes can be drawn at the distance of the style's length, on which the sundials are described, as is clear. In every sundial, therefore, the tip of the style, and so on — which was to be shown.

[Margin: Every sundial takes its name from the greatest circle to which it is parallel.]

THEOREMA 2. PROPOS. 3.

[Margin: The Sun, situated on the Equator, describes with its ray the equinoctial circle; but off the Equator, two conic surfaces.]

The ray of the Sun, when the Sun is on the Equator, describes by the daily motion about the centre of the world a circle — namely the Equator itself. But when the Sun is set outside the Equator, its ray describes two conic surfaces joined at the centre of the world as at a common vertex: the base of the one being the parallel traced from the centre of the Sun, and of the other the parallel opposite to this; and the axis of both is the same, namely the axis of the world.

In the Analemma ABCD, whose centre is E, let the axis of the world be DB; let the common section of the Equator and the Meridian be the line AC; and the common sections of the Meridian with two opposite parallels the lines FG, HI, cutting the axis at the points Q and R. These will be the centres of those parallels, by Prop. 10 of Book 1 of Theodosius [*Sphaerica*], since the axis is drawn through their poles and therefore, by that proposition, passes through their centres. Let there also be understood, described about the diameters AC, FG, HI, the circles — namely the Equator AKCL and the two parallels FM, GN, HOIP — at right angles to the Meridian. For in the Sphere the Equator and its parallels are perpendicular to the plane of the Meridian, by Prop. 15 of Book 1 of Theodosius, since the Meridian circle cuts them through their poles. Since, then, when the Sun is on the Equator — that is, at the point A — its centre does not depart from the circumference of the Equator AKCL, nor does the radius AE (which reaches to the centre of the world) depart from the plane of that same Equator (which is likewise drawn through the centre of the world), but is carried round always within it by the daily motion (for we here disregard the declination that the Sun acquires by its own proper motion): it is clear, from the definition of a circle, that a circle — namely the Equator AKCL itself — is described by the ray of the Sun, whose circumference the centre of the same describes.

But when the Sun is set outside the Equator, as at the point F, its radius FE, reaching to the centre of the world and produced straight on and continuously, is turned round (the point E remaining fixed) about the circumference of the circle FMGN — since in the daily motion the centre of the Sun does not depart from it — and, on the other side, about the circumference of the circle HOIP, which is equal and opposite to it. Therefore the radius of the Sun FE, produced to I, describes the conic surfaces EFG, EIH, fitted to the centre E, whose bases are the opposite

Diagram. The Analemma ABCD: a large circle with a vertical axis DB (D at top, B at bottom) and a horizontal diameter AC (A at left, C at right), crossing at the centre E. Several chords drawn parallel to AC represent the equator and the parallels seen edge-on (FG, HI, and the further parallels through S and Y); each is accompanied by lens-shaped (vesica) curves representing those circles projected. A dense web of straight lines runs from the centre and between the peripheral points. This is a crowded analemma projection with about two dozen labelled points; several small peripheral labels (including the Greek letters) are hard to resolve at this resolution.

- D (top) / B (bottom): endpoints of the world-axis DB
- A (left) / C (right): endpoints of AC, the common section of Equator and Meridian
- E (centre): centre of the analemma (centre of the world)
- F (left) / G (right): endpoints of parallel-section FG; Q (on axis, upper): its centre
- H (lower left) / I (lower right): endpoints of parallel-section HI; R (on axis, lower): its centre
- K (lower left) / L (lower right): points on the Equator circle AKCL
- M / N: points on the parallels FM, GN
- O / P: points on the parallel HOIP
- S (upper left) / V (upper right): points of the greatest ever-apparent parallel; T (left) and X (upper): further points of the circle STVX
- Y (lower left) / Z (bottom): points of the greatest ever-hidden parallel; horizon YV
- α (bottom) [?] / β (bottom) [?]: Greek-letter labels on the ever-hidden parallel YZ $\alpha\beta$ (glyphs not fully resolved)

[Diagram note: very dense analemma; 2 Greek labels flagged]

—opposite [parallels] FMGN, HOIP; the common vertex is E, the centre of the world; and the axis of both is EQ, ER, the same as the axis of the world, since Q and R are the centres, as we showed, of the circles FMGN, HOIP. All of which is clear from the definitions of Apollonius of Perga [*Conics*].

The same conic surfaces will be described when the Sun is set at the opposite point I, as is plain.

Finally, if from any point of the heavens outside the Equator a straight line is drawn through the centre of the world, that line, carried round by the daily

motion, will describe two conic surfaces connected at the centre of the world, whose bases are described by that point and its opposite, and whose axes are portions of the axis of the world. Thus if from the point S — of the greatest of the ever-apparent parallels — the straight line SE is extended through the centre of the world, there will be described by the daily motion the conic surfaces ESV and E[α]Y[?], fitted to the centre E as to a common vertex, whose bases are the parallels described from the point S and its opposite [α][?]. Of these, STVX is the greatest of those that always appear, while YZ $\alpha\beta$ is the greatest of those that never appear above the Horizon YV. And the same reasoning holds for the other points of the heavens. Therefore the ray of the Sun, when the Sun is indeed on the Equator, by the daily motion, and so on — which was to be demonstrated.

SCHOLIVM.

[Margin: What the conic surface of light is; what the conic surface of shadow is.]

Some writers — and rightly — are accustomed to call that conic surface whose base is described by the centre of the Sun the *conic surface of light*, because it is traced by the ray of the Sun; and the other, whose base is described by the point opposite the centre of the Sun, the *conic surface of shadow*, because it is traced by the shadow which the centre of the world projects. Thus, when the Sun is at the point F, the surface of light is EFG, because it is wholly illuminated by the Sun, while the surface of shadow is EIH, because it is described by the shadow of the centre E, projected along the straight line EI. Conversely, when the Sun occupies the point I, the surface of light is called EIH, and the surface of shadow EFG. For we here suppose the centre E to have the power of projecting a shadow; because, as we said in the preceding proposition, the centre of the world is understood in every sundial to be the tip of the style, which — being an opaque body — projects a shadow, as is manifest.

THEOREMA 3. PROPOS. 4.

[Margin: A sundial plane parallel to the bases of the conic surfaces produces a circle on the other of the surfaces.]

The common section of two conic surfaces — joined at the centre of the world as at a common vertex, whose two bases are opposite and equal parallels of the Sphere described by the daily motion about the poles of the world — and of a sundial plane parallel to the great circle that is parallel to the bases of the conic surfaces, is a circle, having its centre on the axis of the world.

In the Sphere whose centre is A, let there be two conic surfaces ADE, AFG, joined at the centre of the world A as at a common vertex, whose bases — described by the daily motion, opposite and equal — are DE, FG; and let the axis be BC. Let there also be HI, a great circle in the Sphere parallel to the bases DE, FG of the said conic surfaces. Let the sundial plane KL be parallel to this circle, making on the conic surface AFG the section MN. I say that MN is a circle having its centre on the axis of the world. For since the planes FG, KL are set parallel to the plane HI, they will also be parallel to one another, by what we demonstrated on Prop. 16 of Book 11 of Euclid. Therefore, since the conic surface AFG is cut by the plane KL, which is parallel to the base FG, the section MN thus made will — by Prop. 4 of Book 1 of Apollonius — be a circle having its centre on the axis BC, namely where the plane of the sundial meets the axis. In the same way, if a plane parallel to the circle HI cuts the conic surface ADE, the section will be a circle. Therefore the common section of the conic surfaces, and so on — which was to be demonstrated.

Diagram. A sphere with centre A, drawn as a circle. Two opposite lens-shaped bands (vesica curves) represent the two conic surfaces ADE and AFG joined at A; their bases DE (upper) and FG (lower) are the opposite parallels, and BC is the vertical axis. The great circle HI is drawn horizontally through the centre, and the sundial plane KL below it cuts the lower cone in the section MN. All labels resolve clearly.

- B (top) / C (bottom): endpoints of the axis BC
- A (centre): centre of the world / common vertex of the cones
- D (upper left) / E (upper right): endpoints of the base DE (upper cone)
- H (left) / I (right): endpoints of the great circle HI
- K (lower left) / L (lower right): endpoints of the sundial plane KL
- M / N (lower centre): points of the section MN made on the cone AFG
- F (lower left) / G (lower right): endpoints of the base FG (lower cone)

COROLLARIUM.

[Margin: An equinoctial sundial cutting a conic surface whose base is a parallel of the Equator produces a circle.]

Hence it follows that the common section of the plane of an Equinoctial sundial and of the conic surfaces — whose bases are any parallels described by the Sun, opposite and equal, or any others parallel to these (such as also are the greatest of the ever-apparent and of the ever-hidden parallels) — is a circle; because the

equinoctial circle, to which the plane of the sundial is parallel, is parallel to the bases of conic surfaces of this kind.

THEOREMA 4. PROPOS. 5.

[Margin: A sundial plane parallel to the great circle that touches the bases of the conic surfaces produces a parabola on the other of the surfaces.]

The common section of the same conic surfaces and of a sundial plane parallel to the great circle that touches the bases of the conic surfaces is a parabola.

Let there be in the same Sphere two conic surfaces, as before; and let EF be the greatest circle touching the opposite bases at the points E and F. Let the sundial plane HI be parallel to this circle, making on the conic surface AFG the section KLM. I say that KLM is a parabola. Through the poles of the parallels B, C, and through the point of contact E, let the great circle BDCG be drawn, by Prop. 20 of Book 1 of Theodosius. This circle also necessarily passes through the poles of the circle FE (by Prop. 5 of Book 2 of Theodosius), and therefore through the poles of the circle HI — which the plane of the sundial produces in the Sphere by Prop. 1 of Book 1 of Theodosius — since (by Prop. 1 of Book 2 of the same) the parallels FE, HI have the same poles; it will therefore pass through them. And so, by Prop. 15 of Book 1 of Theodosius, it will cut both the circle FG and the circle HI — along the straight lines FG, HI, which cut one another at N — in half and at right angles (for the lines FG, HI will indeed cut one another, since they lie in the same plane of the circle BDCG), and it will make the common sections of the parallel planes FE, HI parallel [to one another]. Let the same circle BDCG also, since it proceeds through the axis BC, make the triangle-through-the-axis AFG, cutting the line HI and the conic section at

Diagram. A sphere drawn as a large circle, with a network of chords and vesica curves. The vertical diameter has B at the top and C at the bottom; the great circle BDCG is inscribed through B, D, C, G. The two conic surfaces meet at the centre A; the bases (opposite parallels) run through D–E (upper) and F–G (lower). The sundial plane HI is drawn across, and the parabolic section KLM is formed on the lower cone near F–G, with M and N marked near the foot and a small point (L) toward the lower centre.

- B (top) / C (bottom): endpoints of the vertical axis / circle BDCG
- D (upper left) / E (upper right): endpoints of the upper base and points on circle BDCG
- A (centre): centre of the world / common vertex

- I (right) / H (lower left): endpoints of the plane HI
- K (mid right, on the cone): a point of the parabolic section
- F (lower left) / G (lower right): endpoints of the lower base FG
- L (lower centre) [?]: a further point of the section (small, near the axis)
- N (bottom centre) / M (bottom centre, below N): points of the section / intersection
- I (small, lower centre) [?]: a small label near the foot of the axis (possibly duplicate label)

[Diagram note: two small foot-of-axis labels (L and a lower I) not fully resolved]

k: Let the plane HI also cut, along the line KN passing [through the point N], the circle FG along the line MNL, which passes through the point N. For since the plane drawn through KN passes through the point N (which lies in the plane GF), the line LM — the common section of the planes HI, FG — will also pass through the point N. Since, then, the planes FG, HI are perpendicular to the plane of the circle BDCG, their common section LM will likewise be perpendicular to that same plane at the point N, and so also perpendicular to the line FG, the base of the triangle-through-the-axis, by Definition 3 of Book 11 of Euclid. Therefore, since the cone AFG is cut by the plane BDCG through the axis, and is also cut by the other plane HI — which cuts the base of the cone along the straight line LM, perpendicular to FG (the base of the triangle-through-the-axis) — and since KN, the diameter of the section, is parallel to the side AF of the triangle-through-the-axis: the section KLM will, by Prop. 11 of Book 1 of Apollonius, be a parabola. Therefore the common section of the same surfaces, and so on — which was to be demonstrated.

[Margin: (a) whence 16 — see Prop. 16.]

COROLLARIUM.

And so, since any oblique Horizon touches two parallels — one the greatest of those that ever appear, the other the greatest of those that are ever hidden beneath the earth — the common section of the plane of a Horizontal sundial and of the cone whose base is the greatest of the hidden parallels will be a parabola (for the other cone, whose base ever appears, is not cut, since it lies wholly above the Horizon). The same will happen in a Vertical sundial at latitude 45°. Likewise in a sundial whose plane is parallel to the circle of any hour from sunrise or sunset. For both the Vertical circle of latitude 45° and the circle of any hour from

sunrise or sunset touch, in every Horizon, the greatest parallel of those that stand wholly above the Horizon, as will be demonstrated at Prop. 10 of this book.

Finally, the common section of any sundial and of the cone whose base declines from the Equator toward the South by as much as the complement of the altitude of the arctic pole above the great circle to which the plane of the sundial is parallel, will be a parabola. Such will be the section of the cone having as its base the parallel ♄ [?] (the Tropic of Capricorn — the parallel declining $23^{\circ}30'$ to the south), and of a Horizontal sundial at northern latitude $66^{\circ}30'$. For the complement of this latitude amounts to $23^{\circ}30'$, which is precisely the declination of the parallel ♄ [?]. Such too will be the section of the cone having as its base the parallel ♊ [?] (the Tropic of Cancer),

[Margin: (continuation of the note begun on the previous page) ...the sundials make on the conic surfaces whose bases are any of the parallels of the Sun, parabolas.]

and of a horizontal sundial at the northern latitude of $69^{\circ}48'$. For the complement of this latitude — namely $20^{\circ}12'$ — equals the declination of the aforesaid parallel [♄ [?]; the parallel of $20^{\circ}12'$ declination, per the stated complement of latitude — glyph worn]. In the same way, the section of the cone whose base is the parallel ♏ [?] [Scorpio] and ♐ [?] [Pisces (♊)], together with a Horizontal sundial at the northern latitude of $78^{\circ}30'$, will be a parabola; for the complement of this latitude — that is $11^{\circ}30'$ — equals the declination of the parallel ♏ [?] and ♐ [?]. Say the same of the cones whose bases are the northern parallels opposite to the aforesaid — namely the parallel ♋ [?] [Cancer]; ♊ [?] [Gemini] and ♌ [?] [Leo]; ♉ [?] [Taurus] and ♍ [?] [Virgo] — where, however, the antarctic pole is raised above the Horizon.

From these it will be easy to judge which sundial-planes produce parabolas, whatever parallel the Sun happens to occupy. For if the Sun stands in a northern parallel which the great circle equidistant from the sundial-plane touches, then the common section of the sundial and of the cone of shadow whose base is the opposite southern parallel will be a parabola — namely wherever the arctic pole is raised above the sundial's plane. But if, on the contrary, the antarctic pole is seen above the sundial's plane, and the Sun occupies the southern parallel which the great circle equidistant from the sundial-plane touches, then a parabola will arise in the cone of shadow whose base is the opposite northern parallel, as is clear from what has been said.

For in the upper figure, if B is taken as the arctic pole and the Sun stands in the northern parallel DE, the Sun's ray will indeed describe the cones ADE and AFG;

but the sundial's plane HI, set opposite — within the cone of shadow AFG, whose base FG is parallel to the Sun's parallel DE — will produce the parabola KLM. And if B is taken instead as the antarctic pole and the Sun runs through the southern parallel, the sundial-plane will in the same way produce a parabola in the northern cone of shadow AFG, and so on.

In the same figure you also see that the arctic pole B is raised above the plane FE — which touches the northern parallel DE — by exactly as much as the complement of the declination of the opposite southern parallel FG, and so forth; since the altitude of the pole is the arc BE, while the complement of the declination is the arc CF, ^awhich is equal to it because of the equal vertical angles at the center A on which they stand. For, universally, any great circle touches that parallel whose declination equals the complement of the altitude of the pole above that great circle — or, what is the same, whose complement of declination equals the altitude of the pole above the great circle — as the figure sufficiently shows.

[Margin: (a) 26 of the third.]

THEOREMA 5. PROPOS. 6.

[Margin: The plane of a sundial equidistant from the greatest circle, which cuts the bases of the conic surfaces, produces two opposite and equal hyperbolas.]

The common sections of the same conic surfaces and of the plane of a sundial equidistant from the greatest circle — the plane which cuts the bases of the conic surfaces — are opposite and equal hyperbolas.

Diagram. A large circle (the sphere seen edge-on, representing the meridian) with center A, carrying the double cone and the cutting plane. Readable labels:

- B (top, just left of apex): pole
- I (top, right of B)
- D (upper left, on the circle)
- E (upper right, on the circle)
- L (right side, below E)
- Q, T, R, P (small cramped cluster of points near E, marking the upper hyperbola section PQR) — labels crowded, positions uncertain [?]
- A (center): vertex of the two cones
- M (interior, mid-lower right)
- N (interior, lower center)

- S (below N)
- O (bottom center, below the circle's foot)
- F (lower left, on the circle)
- G (lower right, on the circle)
- H (bottom left)
- C (bottom center)
- K (bottom, right of C)

The two cones are ADE (upward) and AFG (downward) sharing vertex A; HI is the great circle cutting both bases; KL is the trace of the sundial-plane; and MNO (lower) and PQR (upper, near E) are the two opposite hyperbola sections. Several peripheral labels (Q, T, R, P) are crowded and worn

Let there be, in the same Sphere, the two conic surfaces as before, and let HI be the greatest circle cutting each of the two bases. Let the sundial-plane KL be equidistant from this circle, producing in the conic surfaces the sections MNO and PQR. I say that the sections MNO and PQR are opposite and equal hyperbolas. For since the conic surfaces ADE and AFG, joined at the vertex A, are cut by the plane KL not through the vertex, there will be in each of the surfaces — by Prop. 14 of Book 1 of Apollonius — a section called a hyperbola, and the two sections will have the same diameter KL, and so on. Therefore MNO and PQR are hyperbolas, and they are opposite and also equal, as is drawn from the said Prop. 14 of Book 1 of Apollonius. So the common sections of the same conic surfaces, and so on — which was to be demonstrated.

Diagram. Left-hand figure: a sphere (center A) with a single cone opening upward above it, the augmented surface of the Scholium (figure BDCE). Readable labels: D (upper left), Q, T, R (small cluster at the upper apex, near E) [?], E (upper right), B, I (upper interior on the sphere), P, L (right interior), A (center), N, M (lower interior), S, O (bottom), F (lower left), G (lower right), H, C, K (bottom row). Several apex-cluster labels are crowded [?]

Diagram. Right-hand figure: a double (hourglass) figure — two full cones, one opening upward and one downward, above and below the central sphere (center A), showing both conic surfaces augmented until the plane KL cuts them. Readable labels: at the top C/Q, T, R with E (upper apex and rim); D, E (upper on sphere); B, I; P, L; A (center); M, G (mid-right); N, S; F (left), G (right, at the wide lower rim); H, C, O (bottom). This crowded figure has many small worn labels

SCHOLIVM.

[Margin: The plane of a Meridian sundial, and of one equidistant from any hour-circle from noon or midnight — indeed of a Vertical sundial at a latitude greater than 45° — produces, on the conic surfaces whose bases are the ever-apparent greatest parallel and the ever-hidden greatest parallel, two opposite and equal hyperbolas. Such sundials, on the conic surfaces whose bases are any parallels of the Equator, produce opposite and equal hyperbolas. The plane of a sundial equidistant from the greatest circle, which neither is equidistant from the bases of the conic surfaces nor touches nor cuts them, produces on one of the two surfaces an ellipse.]

If it should happen that the plane KL — equidistant from the greatest circle HI — lies so far from the center A that (as in the figure BDCE) it does not cut both conic surfaces but only one, or neither, then each surface must be extended until it is cut by the plane KL, as you see in the two adjoining figures.

COROLLARIVM.

Since, therefore, both the Meridian and the circle of any hour from noon or midnight (as we shall say at Prop. 9) — or indeed any great circle drawn through the poles of the world, and even the Vertical circle of a latitude greater than 45° — cut both parallels (of which one is the greatest of those that ever appear, the other the greatest of those that are ever hidden): the common sections of the conic surfaces whose bases are these parallels, made by the planes of sundials equidistant from those greatest circles, will be opposite and equal hyperbolas.

Likewise, the common sections of any sundial and of the cones whose bases are parallels of the Sun having a declination smaller than the complement of the altitude of the pole above the greatest circle from which the sundial-plane is equidistant, will be opposite and equal hyperbolas. Such will be the sections of the cones whose bases are the parallels $\mathfrak{S}$ [?] [Cancer] and $\mathfrak{V}$ [?] [Capricorn] — and hence of all other parallels between these (since all the others have a smaller declination than those two) — together with a Horizontal sundial at a latitude smaller than $66^\circ 30'$; because in this case the complement of the pole's altitude will be greater than $23^\circ 30'$, which is the declination of $\mathfrak{S}$ and $\mathfrak{V}$. Say the same of the sections of the same cones and of any sundial equidistant from a greatest circle above which the pole of the world is raised by fewer than $66^\circ 30'$.

From these things you will easily recognize which sundial-planes produce hyperbolas, whatever parallel the Sun runs through. For if the Sun stands in a parallel which the great circle equidistant from the sundial-plane — and

therefore its opposite as well — cuts, then the common sections of the sundial and of the cones whose bases are that parallel and its opposite will be hyperbolas. All of which can easily be understood from the figure above. For in general, any great circle cuts that parallel whose declination is smaller than the complement of the pole's altitude above that great circle, or whose complement of declination is greater than the altitude of the pole above the great circle, as the figure indicates.

THEOREMA 6. PROPOS. 7.

[Margin: The plane of a sundial equidistant from the greatest circle among the bases of the conic surfaces, which neither is equidistant from those bases nor touches nor cuts them, produces on one of the two surfaces an ellipse.]

The common section of the same conic surfaces and of the plane of a sundial equidistant from the greatest circle — a plane which neither is equidistant from the bases of the conic surfaces, nor touches them, nor cuts them — is an ellipse.

Let there be, in the same Sphere, the two conic surfaces as before, and the greatest circle HI, which is neither equidistant from the bases DE and FG, nor touches them, nor cuts them, even if the surfaces themselves were extended to infinity. Let the sundial-plane KL be equidistant from this circle, producing in the conic surface AFG the section MNO. I say that MNO is an ellipse. For let a circle be drawn through the poles of the circles FG and HI — and therefore also through the poles of the circle KL, which the sundial-plane makes in the Sphere (by Prop. 1 of Book 1 of Theodosius, since the poles of KL are the same as those of the circle HI, by Prop. 1 of Book 2 of Theodosius). This greatest circle BDCE will, by Prop. 15 of Book 1 of Theodosius, cut the circles FG and KL in half and at right angles, along the straight lines FG and KL, which will mutually intersect one another — namely at the point P — because they are not equidistant from each other; ^aand it will make the common sections HI, KL of the parallel planes HI, KL parallel to one another. Let the same circle BDCE also make the triangle-through-the-axis AFG, cutting the straight line KL and the conic section in M. Let it also cut...

[Margin: (a) 16 of the eleventh.]

also the sundial-plane KL, along the straight line LM, passing through the plane of the circle FG (for the planes FG and KL will necessarily cut one another, since they are not taken as parallel); or at any rate cut its plane extended, along the straight line NPO, which is drawn through the point P. For since the straight line KL cuts the straight line FG at P, the plane drawn through KL — drawn also

through P — will pass through P, and therefore the common section of the planes FG and KL will pass through the point P. Since, then, the planes FG and kL are perpendicular to the plane of the circle BDCE (because this circle cuts those planes at right angles, as we showed a little before), ^atheir common section NO will likewise be perpendicular to the same plane, and so — by Definition 3 of Book 11 of Euclid — will also be perpendicular to the straight line FG, the base of the triangle-through-the-axis.

And since the straight lines HI and kL have been shown to be parallel, ^bthe angles HAM and kMA will be equal to two right angles; and hence the angles FAM and kMA are less than two right angles. Therefore, by Postulate 11 of Book 1 of Euclid, the straight lines AF and kL, extended toward the parts F and k, will meet; and consequently the sundial-plane drawn through kL will cut the side AF, extended toward those same parts F and k. For this reason, since the cone AFG is cut by the plane BDCE through the axis, and is also cut by the other plane kL — which meets both sides AF and AG of the triangle-through-the-axis AFG — and since that plane kL is neither equidistant from the base FG nor cuts off at the vertex A a triangle similar and subcontrarily placed to the triangle-through-the-axis (^cbecause the angle ANP, the exterior, is greater than the interior angle AGF — that is, than the angle AFG, ^dsince this is equal to the angle AGF), and since NO, the common section of the cutting plane kL and of the base of the cone FG, is perpendicular to FG, the base of the triangle-through-the-axis (or at any rate to the line set in a straight line with it): the common section MNO will, by Prop. 13 of Book 1 of Apollonius, be an ellipse. Therefore the common section of the same surfaces, and so on — which was to be shown.

[Margin: (a) 19 of the eleventh.] [Margin: (b) 29 of the eleventh. [printed "29. vnde."]] [Margin: (c) 16 of the first.] [Margin: (d) 5 of the first.]

Diagram. Upper figure: a large circle (sphere, center A) for the ellipse construction of Theorema 6. Readable labels: B (top), D (upper left), E (upper right), I (right, below E), L (right, lower), A (center), M (interior, right of center), H (left), F (lower left), N (lower left interior, near F), K (bottom left), C (bottom, small "c" also near N), G (lower right). The great circle BDCE, the trace KL, the triangle-through-the-axis AFG, and the section point M are shown. Several lower-interior labels (N, K, small c) are crowded [?]

Diagram. Lower figure: a large circle (sphere, center A) with the cutting-plane trace extended as the line N–P–O running out below the lower left. Readable labels: E (top, with a dashed rim and a second E marked at the upper rim), D (upper left), I (right upper), L (right lower), A (center), M (interior, right of center),

N (lower interior), H (left), K (lower left), P (left, on the line below the circle), F (bottom center), O (below P, foot of the line NPO), C (bottom), G (lower right). The line NPO passes through P (the intersection of KL and FG) and carries the common section NO of the cutting plane. Several labels along the lower rim and the NPO line are crowded/worn [?]

COROLLARIVM.

Therefore, since a Vertical circle of a latitude less than 45° is neither equidistant from the parallels (of which one is the greatest of those that always appear, the other the greatest of those that are perpetually hidden beneath the Horizon), nor touches them, nor cuts them: the common section of the plane of a Vertical sundial — whose latitude is less than 45° — and of the cone whose base is the greatest of the parallels always hidden beneath the earth, will be an ellipse (if the sundial-plane is turned toward the parts of the hidden pole and faces the visible pole); or the common section of that plane and of the cone whose base is the greatest of the parallels that perpetually appear will be an ellipse (if the sundial-plane recedes toward the parts of the visible pole and faces the parts of the hidden pole).

In the same way, the common section of any sundial and of a cone whose base has a southern declination greater than the complement of the altitude of the arctic pole above the greatest circle from which the sundial-plane is equidistant, will be an ellipse. Such will be the section of the cone whose base is the parallel ♊ [Capricorn] and of a horizontal sundial at a northern latitude exceeding $66^\circ 30'$; because in this case the declination of ♊ — namely $23^\circ 30'$ — will be greater than the complement of the pole's altitude. Of this same kind will be the section of the cone whose base is the parallel ♐ [Sagittarius] and ♒ [Aquarius], together with a horizontal sundial at a northern latitude exceeding $69^\circ 48'$; for the declination of this parallel, amounting to $20^\circ 12'$, is greater than the complement of the pole's altitude. Such too will be the section of the cone whose base is the parallel ♏ [Scorpio] and ♓ [Pisces (♊)], together with a horizontal sundial at a northern latitude greater than $78^\circ 30'$, since the declination of this parallel — namely $11^\circ 30'$ — exceeds, in this way, the complement of the pole's altitude. Say the same of the cones whose bases are the northern parallels opposite to the aforesaid — such as the parallel ♋ [Cancer]; ♊ [Gemini] and ♌ [Leo]; ♉ [Taurus] and ♍ [Virgo] — where, however, the antarctic pole is raised above the Horizon.

[Margin: The plane of a Vertical sundial at a latitude less than 45° produces, on one of the two conic surfaces whose bases are the ever-apparent greatest parallel

and the ever-hidden greatest parallel, an ellipse. Those sundials, on the conic surfaces whose bases are any parallels of the Equator, produce ellipses.]

From this you will easily learn which sundial-planes produce ellipses, whatever parallel the Sun stands in. For if the Sun stands in a northern parallel from which the great circle equidistant from the sundial-plane is neither equidistant, nor touches, nor cuts, then the common section of the sundial and of the cone of shadow whose base is that parallel...

[the base being] the opposite southern parallel, will be an ellipse — namely when the arctic pole is raised above the plane of the sundial. But if, instead, the antarctic pole is raised above the plane of the sundial, and the Sun stands in a southern parallel from which the great circle equidistant from the sundial-plane is neither equidistant, nor touches it, nor cuts it, then an ellipse will be produced on the cone of shadow whose base is the opposite northern parallel, as is clear from what has been said. Now the great circle is neither equidistant from any given parallel, nor touches it, nor cuts it, when the declination of the parallel is greater than the complement of the altitude of the pole above the great circle; or when the complement of the declination of the parallel is less than the altitude of the pole above the great circle, as is evident from the figure.

PROBLEMA 2. PROPOS. 8.

Given a cone and the diameter of a conic section, to describe that conic section in a plane.

Diagram. A composite construction plate of six sub-figures (uncaptioned), arranged in three rows of two, illustrating the construction of Problema 2. The same plate is reprinted, with minor variation, on the following two pages.

- **Top row — the "first figures" (cones shown as axial triangles).** Two triangles, each with apex **A** and base ends **B** (left) and **C** (right), representing a cone cut through its axis. Within each, the diameter of the section runs **D–E**; parallel chords **F–H** and **G–I** meet the diameter **DE** at the interior points **K** and **L**; the axis drops to the base at **N**. Further points **R**, **S**, **T** appear at the foot (the extension of the triangle below the base for the ellipse whose diameter cuts below **BC**), and the base perpendicular chord is labelled **Z–α**; the circle-section points near the chords are **X**, **Y**. Many interior labels are crowded and not individually resolvable at this resolution [?].

- **Middle row — the "second figures" (the base BEC set out separately).** Each shows a horizontal base line carrying, left to right, points **B, F, G, E** and then **H, I, C** (with **R, S** as outer extensions), on which concentric semicircles are described about the diameters **BC, FH, GI**. A perpendicular **E–M** rises at **E** (**M** at top), cutting the semicircles at **P** and **Q** [several clustered base labels near the **E**-end not individually resolvable — ?].
- **Bottom row — the "third figures" (the described sections).** Left: a **parabola**, apex **D** at top, axis **D–E** vertical, with horizontal ordinate-chords through **K**, and points **P, M, V, T** stepping down to base **E**. Right: a **hyperbola** shown as a wide **V** opening upward from apex **D**, axis **D–E**, with points **L, C, P, K, M, E, V** on the ordinates.

[Diagram note: this is an extremely label-dense reprinted construction plate; more than two interior point-labels per sub-figure could not be individually resolved]

[Margin: The method by which a conic section, whose diameter is given within the cone, may be described in a plane.]

Let **ABC** be a given cone, cut by a plane through the axis — producing, by Proposition 3 of Book 1 of Apollonius, the axial triangle **ABC**. Let it also be cut by another plane producing either a parabola, a hyperbola, or an ellipse, whose diameter is **DE**. This diameter **DE** is parallel to the side **AC** of the triangle in the parabola; it cuts that same side **AC**, extended above the vertex **A**, in the hyperbola; or it lies within the cone in the ellipse — whether this happens above the base **BC** or below it. All this is established from the three propositions just cited. On the diameter **ED** take any number of parts, whether equal or unequal, **Ek, kL**; and through the points **k, L** draw **FH, GI** parallel to the base **BC**. Then both the parts **CH, HI** and the parts **Ek, kL** will be proportional to the parts **BF, FG** [Margin: a. Euclid VI.2] (taking, in the ellipse, the point **E** on **BC**, the base of the triangle). Indeed, in the parabola the parts **CH, HI** are equal to the parts **Ek, kL** [Margin: b. Euclid I.34], because of the parallelograms **Ck, kI**.

But if the cone is a right cone — as are all those we use in laying out sundials (for all these are right, since their axes are portions of the world-axis, which by Proposition 10 of Book 1 of Theodosius is perpendicular to the parallels of the primary motion, that is, to the bases of the cones) — we shall proceed most conveniently as follows. On the straight line **BD** take any number of parts, whether equal or unequal, **BF, FG**, and on the other side make **CH, HI** equal to them one by one; then join **FH, GI**, which cut the diameter **DE** at **K, L**. For these lines, since they cut the sides **AB, AC** proportionally, will be parallel [Margin: c.

Euclid VI.2], and therefore the parts EK, KL will be proportional to the parts BF, FG. Next set out separately the base BEC of the triangle ABC, and from the point E — which stands for all the points E, K, L — erect the perpendicular EM; and along the straight line BC, from the point E, transfer to both sides the parts KF, KH and LG, LI, so that EF, EG are equal to KF, LG, and EH, EI equal to KH, LI. On the side of C, in the parabola, these will all fall on the point C, because EC, KH, LI are equal [Margin: d. Euclid I.34]. In the hyperbola they will always be less than EC, and in the ellipse greater, as is evident. In every case, however, the parts BF, FG in the first figures (I call the cones themselves the "first figures," and those in which we have set out the base BEC separately the "second figures") will be proportional to the parts BF, FG in the second figures; and likewise CH, HI in the first to the parts CH, HI in the second. For if GN is drawn in the first figures parallel to DE, then as GB is to BN, so GF is to FO [Margin: e. Euclid VI.4] (since the triangles GBN, GFO are similar...

are similar, by the corollary to Proposition 4 of Book 6 of Euclid); and, by alternation, as GB is to GF, so BN is to FO. Now since BN in the first figures is equal to BG in the second, and FO in the first to FG in the second — because NE is equal to GL in the first, that is, to GE in the second, and therefore BN to BG, and FO to FG (for EB, EF, EG in the second figures were set equal to EB, KF, LG in the first) — it will also follow that as GB is to GF in the first, so BG is to FG in the second; and, dividing, as FB is to GF in the first, so BF is to FG in the second. We shall show the same for CH, HI, if from I a line is drawn in the first figures parallel to DE. Hence, if BF, FG are equal in the first figures, then EK, KL in the first — and also BF, FG and CH, HI in the second — will be equal, as appears from the figures; for we took the parts BF, FG in the first figures to be equal, for the sake of simplicity.

After this, about the diameters BC, FH, GI describe semicircles cutting the straight line EM at the points M, P, Q. The radii will be obtained if the axis of the cone in the first figures is drawn so as to bisect the base of the triangle. For this axis will also bisect all the diameters FH, GI and the rest, as we showed in the scholium to Proposition 4 of Book 6 of Euclid. Therefore, if in the first figures we take the distances between the axis of the cone and the points E, K, L, and transfer them into the second figures from the point E onto the line BEC — either toward B or toward C, as the first figures indicate — we shall obtain the centres, and so on.

Finally, let the diameter DE of the conic section be divided separately, as it was in the cone — that is, so that EK, KL are equal to the parts EK, KL in the cone, each to each. And through E, K, L erect perpendiculars to DE. This is done easily and very

quickly (especially when many points have been taken on the diameter DE) if you erect a perpendicular through E, from two points of which parallels to DE are raised and divided in the same way as DE. For the straight lines joining the points of division will be perpendicular to DE at the points K, L [Margin: a. Euclid I.29], because in this way parallelograms are set up on the lines EK, EL, which are rectangles, owing to the right angle established at E, as is evident.

Diagram. The same composite construction plate reprinted (six sub-figures, uncaptioned), here with the "third figures" showing the two ellipse cases.

- **Top row — the "first figures" (cones/axial triangles).** As on the previous page: apex A, base B–C, section-diameter D–E, parallel chords F–H and G–I meeting DE at K, L, axis-foot N, lower extension R, S, T, base chord Z–α, circle points X, Y. Dense interior labels not individually resolvable [?].
- **Middle row — the "second figures" (base BEC set out).** Horizontal base with points B, F, G, E, H, I, C (and R, S as extensions), concentric semicircles on BC, FH, GI, perpendicular E–M (M at top) cutting them at P, Q [clustered base labels near E — ?].
- **Bottom row — the "third figures" (two ellipses).** Left: an upright ellipse, top point D, upper chord V–V, then T, centre L, mid chord Q–Q, K, lower chord P–P, bottom point E. Right: a second ellipse with top D and upper marks C–C, then L, chord F...P, centre E, lower marks M–M, T, chord V–V, base E; its lower half drawn dotted.

[Diagram note: reprinted dense construction plate; more than two interior labels per sub-figure unresolved]

After this, from the perpendiculars — or the parallels drawn through the points E, K, L in the third figures — cut off, on both sides of the points E, K, L, straight lines EM, kP, LQ equal to the straight lines EM, EP, EQ in the second figures: namely kP equal to the line intercepted between the diameter FH and its semicircle (which is EP in the second figures); and LQ equal to EQ, placed between the diameter GI and its semicircle; and so with the rest — carefully observing which points of the diameter DE correspond to which diameters of the semicircles. Now if you join the points D, Q, P, and so on suitably by a certain curved line, the proposed conic section will have been described — namely a parabola, a hyperbola, or an ellipse, as we shall shortly demonstrate. From this it is clear that the more numerous the points on the diameter DE, the more accurately the conic section is described, as you see done in the hyperbola and in both ellipses; for in these sections another point besides K, L has been taken. But if you set out to enlarge the parabola and the hyperbola, the cones must be

enlarged, and points must be taken below the base BC, equal in number to the points F, G, and so on, as the figures indicate. The same is to be said of the ellipse whose diameter cuts the other side of the axial triangle below the base, as in the second ellipse;

for otherwise only the part of the ellipse MPQDQPM would be described. Moreover, in the first ellipse, whose diameter does not cut the base of the triangle, no semicircle is described about the diameter BC, because it would not cut the straight line EM. Sometimes, too, the semicircles intersect one another on the straight line EM in the description of the ellipse — as the semicircles FPH, RVS in the first ellipse, where the straight lines EP, EV are equal, and the perpendiculars kP, TV are taken equal to them in the third figures.

Diagram. The same composite construction plate reprinted (six sub-figures, uncaptioned), matching the plate on the previous page (cones, semicircles, and the two ellipse cases).

- **Top row — "first figures" (cones/axial triangles):** apex A, base B–C, section-diameter D–E, chords F–H and G–I meeting DE at K, L, axis-foot N, lower extension R, S, T, base chord Z–α, circle points X, Y; dense interior labels unresolved [?].
- **Middle row — "second figures":** base line B, F, G, E, H, I, C (with R, S), concentric semicircles on BC, FH, GI, perpendicular E–M cutting them at P, Q [clustered base labels — ?].
- **Bottom row — "third figures" (two ellipses):** left ellipse with D, V–V, T, L, Q–Q, K, P–P, E; right ellipse with D, C–C, L, F...P, K, E, M–M, T, V–V, E and a dotted lower half.

[Diagram note: reprinted dense construction plate; more than two interior labels per sub-figure unresolved]

But let us now demonstrate that the conic section passes, in the plane, through the points Q, P, and so on, about the diameter DE — and hence that the line suitably drawn through those points in the plane is a conic section, as we said. [Margin: Demonstration of the description given above.] If, in the first figures, a plane is drawn through the straight line FH parallel to the base of the cone, the section produced will be the circle FXH, by Proposition 4 of Book 1 of Apollonius. Let the common section of this plane and of the conic section be the straight line XY, which will pass through K, where the straight lines DE, FH cut one another, and where the circle FXH drawn through the line FH meets the conic section. And since the parallel planes BC, FH are cut by the plane DE which produces the conic

section, the common sections $Z\alpha$ and XY will be parallel [Margin: a. Euclid XI.16]. Now $Z\alpha$ is perpendicular to the straight line BC (for in order that any conic section be produced, the common section of the cutting plane and the base of the cone — such as the straight line $Z\alpha$ — must be perpendicular to the base of the axial triangle, as is established from Propositions 11, 12, and 13 of Book 1 of Apollonius). And the angles BEZ , FKX are equal [Margin: b. Euclid XI.10], because the straight lines BE , EZ are parallel to FK , kX . Therefore the angle FKX will also be right, and so XK is perpendicular to FH ; and consequently XK , in the semicircle FXH , will be the mean proportional between FK , KH , by the scholium to Proposition 13 of Book 6 of Euclid. But in the second figures too, EP is, by the same reasoning, the mean proportional between FE , EH — that is, between the same Fk , KH in the first figures — and therefore equal to XK in the first figures (for EF , EH in the second figures were taken equal to KF , KH in the first). Now this same EP in the second figures is equal to kP in the third. Therefore kP in the third figures is also equal to KX in the first. So, since in the first figures the conic section passes through X on the conic surface, the same section will pass, in the plane, through the point P . For in this way, when the point K of the third figures is placed on the point k of the first, so that the diameter kD of the third coincides with the diameter kD of the first, the perpendicular kP in the third figures will coincide with the perpendicular kX in the first; and thus the point P will fall on the point X (because of the equality of the straight lines kP , kX), and the conic section will pass through the point P , which does not differ from the point X . By the same reasoning we shall show that the same section passes through the point Q , and through the rest, if there are any. Therefore, given a cone and the diameter of a conic section, and so on — which was to be done.

SCHOLIUM.

This method of describing a conic section, together with its demonstration, does not differ from the one that Federicus Commendinus [Federico Commandino] presents in his book on the description of sundials — except that he speaks only of the right cone, whereas we have adapted the problem to every cone, right as well as scalene, and have at the same time included the procedures required for describing the sections of right cones. For he prescribes that in the first figures any number of points K , L be taken on the diameter DE , and that through them parallels to the base BC be drawn. But in a right cone it is easier, with the help of a compass, to take the points F , G , H , I on both sides AB , AC ; for the straight lines connecting these points are parallel, as we have shown. Then, in the first figures, he directs us to find, between KF , kH and LG , LI , the...

to find the mean proportionals — which we have accomplished by describing semicircles in the second figures. Finally, once the diameter DE has been divided in a separate plane (namely in the third figures, just as it was divided in the cone of the first figures), he directs that from the division points, in the third figures, perpendiculars be drawn up to the diameter on both sides — which we too did in the third figures, by a very easy and short procedure, especially where the points were many. Onto these perpendiculars he transfers the mean proportionals that have been found, as we do; and through the outermost points of the transferred mean proportionals he draws the conic section, just as we have done.

Franciscus Maurolycus [Francesco Maurolico], abbot, in the third book of his work on hour-lines, uses other, particular constructions for the three conic sections. But the method of construction that we have handed down is simpler and easier, and applies to all the sections, as is clear. We have said nothing about constructing that conic section which is a circle, because once the diameter is known its construction is very easy.

At this point I should like to present another method — not unpleasant, and perhaps easier than the one we have explained — for describing a parabola, two opposite hyperbolas, and an ellipse whose axes are given. This method (to my great surprise) has, so far as I know, been noticed by no one up to now. We shall adapt it only to right cones, because it is only these that we shall use in laying out sundials. For the parabola, then, we must first set out a lemma of the following kind, which suits both the right cone and the scalene cone.

LEMMA.

Given a cone and the diameter of a parabola, to find the latus rectum of the parabola.

[Margin: Finding the latus rectum [latus rectum] of a parabola whose diameter is given in the cone.]

Diagram. Left-hand figure of the parabola lemma: an axial triangle of a cone with the conic section inscribed, together with the applied rectangles beneath it.

- A (apex of the triangle at top); a second A stands at the head of the slanting line A–I drawn off to the right (the latus-rectum construction line)
- B (lower-left) and C (base corners of the axial triangle)
- F, D, H (points of the inscribed conic section, D and H at its center)
- K (on the left, at the section)
- I (lower right, foot of the slanting construction line A–I)

- Beneath the triangle, a rectangle (the rectangle CB contained under AB, AC), divided, with B marked at its foot

[Two or three peripheral labels on this crowded cone figure are hard to resolve]

Diagram. Right-hand figure: the parabola drawn within a rectangle in the plane, opening downward, with its vertical axis and successive ordinate-chords.

- E (vertex of the parabola, top center) with K at the upper corners
- Axis E–A–B–C–M–H descending vertically (A, B, C, M interior points; H at the foot)
- D (endpoints of the ordinate at A)
- N (endpoints of the ordinate at the level of L)
- F (endpoints of the ordinate at B)
- G (endpoints of the ordinate at C)
- O (endpoints of the ordinate at the level of M)
- I (base corners, the ordinate at H)
- A point marked Q near the F-level ordinate [?] and the point L on the axis [?] are not clearly resolvable

[More than two labels uncertain]

Let there be a given cone ABC, in which ABC is the axial triangle. Let the cone be cut by a plane producing the parabola EFG, in accordance with what Apollonius demonstrated in Proposition 11 of Book 1 [of the *Conics*], so that its axis EH is parallel to the side AC of the axial triangle. We shall then find the latus rectum of this parabola as follows. [Margin a: Euclid VI.11] Make BC to AI as one side of the axial triangle, namely AB, is to the base BC. [Margin b: Euclid VI.12] Then make AE to EK as the other side AC is to the AI just found. I say that EK is the latus rectum of the parabola EFG — that is, the straight line beside which the ordinates drawn to the diameter EH can be applied.

For let the rectangle CB be that contained under the sides AB, AC of the axial triangle; and to AB apply the rectangle BI contained under AB, AI, [Margin c: Euclid VI.17] which will be equal to the square of the base BC, since the three lines AB, BC, AI are continuously proportional by construction. [Margin d: Euclid I.14] And CAI will be a single straight line, because the two angles at A are right. Since, then, CA is to AI as AE is to EK by construction, then by conversion AI is to AC as EK is to AE. [Margin e: Euclid VI.1] But AI is to AC as the rectangle BI — that is, the square of the base BC — is to the rectangle BC contained under the sides of the axial triangle. Therefore, as the square of the base BC is to the rectangle

contained under the sides AB, AC, so is EK to AE. And so, by Proposition 11 of Book 1 of Apollonius, EK is the latus rectum of the parabola EFG — that is, the straight line beside which the ordinates can be applied, and so on.

Now that the latus rectum has been found, take in some plane any axis EH of the parabola. (For here we are dealing with the parabola whose diameter is also its axis, bisecting all the ordinates at right angles.) On this axis mark off any number of equal parts — the smaller these parts are, the more accurately the parabola will be drawn — so that EA is 1, AB is 3, BC is 5, CH is 7, and so on, following the series of odd numbers. Then through the points A, B, C, H, and so on, draw perpendiculars to EH on both sides, in the way we taught above. [Margin: Another way of describing the parabola in the plane.] Next, having found the mean proportional between the latus rectum EK and the line EA, cut off AD equal to it on both sides; from B cut off on both sides BF, double AD; from C, CG, triple the same AD; from H, HI, quadruple; and so on, following the natural series of numbers. For through the points D, F, G, I the parabola is to be drawn.

That it passes through the point D is proved thus. The square on AD — the line that is the mean proportional between EK and EA — [Margin f: Euclid VI.17] is equal to the rectangle under EK, EA; and therefore AD is an ordinate in the parabola whose latus rectum is EK, as is clear from Proposition 11 of Book 1 of Apollonius. So the parabola will pass through the point D. For if it passed through some other point, say P, then the square on AP would likewise be equal to the rectangle under EK, EA, by Proposition 11 of Book 1 of Apollonius, since AP would be an ordinate to the diameter. Then the squares on AD and AP would be equal, and so the lines themselves would be equal — a part equal to the whole, which is absurd. Therefore the parabola whose latus rectum is EK passes through the point D. That it also passes through the points F, G, I we shall show as follows. Since the line BF is double the line AD, its square will have to the square of AD the quadruple ratio [Margin g: Euclid VI.20] (because squares are in the duplicate ratio of their sides), just as the line EB is quadruple the line AE.

Again, since the line CG is triple the line AD, [Margin a: Euclid VI.20] its square will be nine times the square of AD, just as the line EC is nine times the line EA. In the same way the square on HI will have to the square on AD the same ratio that the line EH has to EA — namely sixteenfold — and so on. Therefore, as is clear from Proposition 20 of Book 1 of Apollonius, the parabola will pass through the points F, G, I. For if it were said to pass through some other point, say Q, then by that same Proposition 20 of Book 1 of Apollonius the square on BQ would be to the square on AD as the line EB is to the line EA — that is, as the square on BF is

to the square on AD. Then the squares BQ and BF would be equal, and the lines themselves equal — a part and the whole — which is absurd. Therefore the parabola must be drawn not through Q but through F, and so for the rest.

But whenever the points seem to lie too far apart — as G and I do — we shall take, on the diameter EH between the points C and H, some point that marks off a number of parts of the diameter divisible by four: 8, or 12, or 16, or 20, and so on. Of this kind is the point M, which marks off twelve parts. Then of the line EM we shall take a quarter part — in the given example the line EL, containing three parts — and from L we shall draw a perpendicular to EH, namely LN, which cuts the parabola at the point N. For if through M we draw another perpendicular to EH, and from it cut off MO double LN, the parabola will pass through the point O; because LM is triple EL, and MO double LN, just as AB is triple EA and BF is double AD.

This same method can be adapted to a parabola in which the ordinates are not perpendicular to the diameter EH — as happens in scalene cones, when the axial triangle is not perpendicular to the base of the cone, as is clear from Proposition 7 of Book 1 of Apollonius. In that case EH will not be the axis of the parabola but a diameter. Hence through the points A, B, C, H one must draw lines parallel to one another, making with the diameter EH angles equal to those which the ordinates make with the diameter of the parabola. This method will suit scalene cones all the more when the axial triangle is perpendicular to the base of the cone; for then, by Proposition 7 of Book 1 of Apollonius, the ordinates are perpendicular to the diameter of the parabola, just as in a right cone, so that EH is again the axis of the parabola.

For the opposite hyperbolas, however, two further lemmas must be proved, which apply to every cone, right as well as scalene. The first of these is as follows.

LEMMA I.

Given a cone and the transverse diameter of opposite hyperbolas, to find the latus rectum of the hyperbola.

[Margin: Finding the latus rectum of opposite hyperbolas whose transverse diameter is given in the cone.]

Diagram. A double cone (two cones joined at their common vertex, in the shape of an hourglass), cut by a plane producing two opposite hyperbolas, with a square beneath.

- A (common vertex, center, where the two cones ABC and ADE meet)
- B, C (base corners of the lower cone); D, E (of the upper cone)
- F, G, H and I, K, L (points of the two opposite hyperbolic sections; F and I are the vertices, on the common transverse diameter FI)
- M (on BC, foot of the line AM drawn parallel to FI)
- N (point such that $MC : AM = AM : MN$); O (on the diameter, foot of FO)
- P (upper region of the section) [?]
- Q (lower region, near the base) [?]
- Beneath, a square with its foot marked

[Several peripheral labels (P, Q and the second set of M, N) are hard to resolve on this crowded double-cone figure]

Let there be a given cone ABC, in which ABC is the axial triangle; and let the cone, together with the axial triangle, be produced through the vertex A, so that two cones ABC, ADE are formed, joined at the vertex A. Let both conic surfaces be cut by a plane not passing through the vertex, producing the sections FGH, IKL, which are opposite hyperbolas by Proposition 14 of Book 1 of Apollonius, whose common transverse diameter is FI and whose latera recta are equal. We shall find the latus rectum of each as follows. Through A draw AM parallel to FI, cutting BC at M. [Margin b: Euclid VI.11] Make CM, one part of the base, to AM as AM is to MN. [Margin c: Euclid VI.12] Again make FI, the transverse diameter, to FO as MN is to BM, the other part of the base. I say that FO is the latus rectum of each hyperbola — that is, the straight line beside which the ordinates to the diameter of each hyperbola can be applied.

For let the rectangle BC be that contained under the parts of the base BM, MC; and to MC apply the rectangle CN contained under MC, MN, [Margin d: Euclid VI.17] which will be equal to the square of the line AM, since the three lines MC, AM, MN are continuously proportional by construction. [Margin e: Euclid I.14] And BMN will be a single straight line, because the two angles at M are right. Since, then, FI is to FO as MN is to BM, [Margin f: Euclid VI.1] and since MN is to BM as the rectangle CN — that is, the square on AM — is to the rectangle BC contained under the parts of the base BM, MC, it follows that the transverse diameter FI is to the line FO as the square on AM is to the rectangle under BM, MC. Therefore FO is the latus rectum of the hyperbola, by Proposition 12 of Book 1 of Apollonius — that is, the straight line beside which the ordinates can be applied, and so on.

LEMMA II.

To apply, on each side, to the transverse diameter of a hyperbola the fourth part of the rectangle contained under the transverse diameter and the latus rectum, in such a way that it exceeds by a square figure.

[Margin: How the fourth part of [the rectangle] contained under the transverse diameter of a hyperbola and the latus rectum is to be applied to the transverse diameter on each side, so that it exceeds by a square figure.]

With the same figure set out, let AB be found as the mean proportional between the transverse diameter FI and the latus rectum FO, and let AB be bisected at C. [Margin: Euclid VI.17] Then the square on AB will be equal to the rectangle under FI, FO; and therefore the square on AC, which by the scholium to Proposition 4 of Book 2 of Euclid is the fourth part of the square on AB, will be equal to the fourth part of the rectangle under FI, FO. To this square on AC, then, we shall apply to the transverse diameter FI, on each side, an equal rectangle exceeding by a square figure, in this way. Let the line FI be bisected at D; construct a right angle HKL, with HK equal to AC and KL equal to DI, and join HL, which will be greater than the line KL — that is, than DI — because HL is subtended by a greater angle than KL. [Margin: Euclid I.13; Euclid I.19] Produce FI in both directions, and cut off on both sides from D the lines DQ, DR equal to HL. I say that both the rectangle under FQ, QI applied to FI and exceeding by the square on QI, and the rectangle under IR, RF applied to the same IF and exceeding by the square on RF, are equal to the square on AC —

— that is, to the fourth part of the rectangle under FI, FO.

Diagram. Auxiliary figures for Lemma II (upper left of the page): the line AB with its midpoint, a small right triangle, and the gnomon figure of the application.

- A, C, B (the mean-proportional line AB bisected at C, drawn horizontally at top)
- Right triangle H, K, L (with HK = AC and KL = DI)
- Gnomon/rectangle figure below: points R, F, D, M, N (foot) and P, E, G, Q, I marking the square DE, the parallelogram MP, the square NI, and the gnomon DNE

[Interior labels of the gnomon figure are small and partly worn]

Diagram. The two opposite hyperbolas described in the plane (right side of the page), with the two describing points Q and R and the vertical axis.

- F, I (vertices of the opposite hyperbolas, on the transverse axis)
- Q, R (the two fixed points from which the arcs are struck)
- E, G, H (points through which the branches are drawn)
- A, B, C, D (points taken on the axis below/beyond, marking the intervals IA, IB, IC and FA, FB, FC)
- K (the false point used in the reductio)
- O (outer point at right) [?]
- Several repeated H, G, E, F labels on the four branch-arms are hard to resolve individually

[More than two labels uncertain on this dense multi-branch figure]

For, having described the square DE on DI, draw through Q the line PN parallel to IE, meeting the line GE produced at P and the diameter GI produced at N, and complete the figure as you see. [Margin a: Euclid VI.24] Now since the parallelograms DE, MP, NI, standing about the same diameter, are similar, and DE is a square, MP and NI will also be squares. [Margin b: Euclid I.47] And since the square on HL is equal to the squares on HK, KL — while, by construction, the line HL is equal to DQ (that is, to MN), the line HK to AC, and the line KL to DI — the square MP described on DQ (that is, on MN) will be equal to the square DE described on DI together with the square on AC. Therefore, when the common square DE is taken away, the remaining gnomon DNE will be equal to the remaining square on AC. And since the gnomon DNE is equal to the rectangle FN — [Margin c: Euclid I.36] for since FM is equal to MI, [Margin d: Euclid I.43] that is, to IP, the rectangle FN, with the common MQ added, will be equal to the gnomon DNE — the rectangle FN, contained under FQ, QI (since QI is equal to QN because of the square IN), will also be equal to the square on AC; that is, to the fourth part of the square on AB, that is, of the rectangle contained under FI, FO. There has therefore been applied to the transverse diameter FI a rectangle under FQ, QI equal to the fourth part of the rectangle under FI, FO and exceeding by the square on the line QI. In the same way it will be demonstrated that the rectangle under IR, RF, applied to FI and exceeding by the square on RF, is equal to the fourth part of the rectangle under FI, FO. Which was what was proposed.

With these things premised, let FI be the transverse axis of two opposite hyperbolas FGH, IKL, as in the figure of the first lemma, and let the latus rectum FO be given from that same first lemma. By the second lemma, apply to FI on each side a rectangle under FQ, QI and under IR, RF equal to the fourth part of the rectangle under FI, FO and exceeding by the square on IQ and on FR; and beyond R and below R take any number of points A, B, C, D as you please. Then, at

the distance IA, describe from the points Q and R four arcs, which are cut at the point E by four other arcs described from the same points Q and R at the distance FA. Likewise from the same Q and R describe four arcs at the distance IB, which are intersected at the point G by four others described from the same points Q and R at the distance FB. In the same way, at the distances IC, FC, let the arcs described from the points Q and R cut one another at H; and so for the remaining points, if there are any. One must always observe that, of the four arcs to be described from Q and R, the two larger from Q are described beyond the point F, and the two from R beyond the point I; while the two smaller from Q are on the near side of the point I, and the two from R on the near side of the point F. For through the points F, E, G, H and I, E, G, H the opposite hyperbolas are to be described.

For since the line QE — that is, IA — exceeds the line ER — that is, FA — by the transverse diameter FI; and likewise the line QG exceeds the line GR by the same diameter; and so for the rest — the opposite hyperbolas, whose axis is FI and whose vertices are F, I, will pass through the points E, G, H. For, as Proposition 51 of Book 3 of Apollonius requires, if straight lines are drawn from the points Q, R to one and the same point of the hyperbola, the greater exceeds the lesser by the axis FI itself. For suppose the hyperbola whose axis is FI and vertex F does not pass through the point E: let it, if possible, pass through K, cutting the line QE at K, whether below E or above; and join the line RK. Then, since the hyperbola passes through K, the line QK will exceed the line KR by the axis FI, by Proposition 51 of Book 3 of Apollonius. But by the same axis FI the line QE exceeds the line ER by construction. Therefore the excess between the lines QK, KR is the same as that between the lines QE, ER. And so, by alternation, from the lemma of Proposition 79 of Book 10 of Euclid, the excess between the lines QK, QE will be the same as that between the lines KR, ER. Now since the excess between QK and QE is the line EK, the same line EK will also be the excess between KR and ER. Therefore the line EK, added to the lesser of them, will make the sum of these two equal to the remaining one; and consequently two sides of the triangle EKR will be equal to the remaining side [Margin e: Euclid I.20] — but they are also greater. Which is absurd. Therefore the said hyperbola will pass not through the point K but through E. And in the same way we shall show that it passes through the remaining points G, H, and so on — which was what was proposed.

It is evident, moreover, that this construction applies only to right cones, or else to scalene cones in which the axial triangles are perpendicular to the bases of the cones; for in these alone is the transverse diameter the axis of the hyperbolas,

since it cuts the ordinates at right angles, as is clear from Proposition 7 of Book 1 of Apollonius.

Finally, for the ellipse, two lemmas must again be set out, which follow, and which fit every cone, right as well as scalene.

LEMMA I.

Given a cone and the transverse diameter of an ellipse, to find the latus rectum of the ellipse.

[Margin: Finding the latus rectum of an ellipse whose transverse diameter is given in the cone.]

Diagram. The ellipse lemma figure (foot of the page): an axial triangle of a cone with an inscribed ellipse, and beneath it the diameter line G–H and its rectangle.

- A (apex of the axial triangle)
- E, I (on the left side of the triangle); F (on the right side)
- B, C (base corners); D (center); G (on the base line / ellipse)
- Below: the line G–H drawn out, and a rectangle B–G–...–H–C (with G marked at top center of the rectangle and C at its foot)

Let there be a given cone ABC, in which ABC is the axial triangle; and let the cone be cut by a plane producing the ellipse EF, in accordance with...

in accordance with Proposition 13 of Book 1 of Apollonius (*Conics* I.13), so that the straight line EF is the transverse diameter of the ellipse. We shall then find its latus rectum as follows. Through A draw AG parallel to EF, cutting BC produced at G [Margin a: Euclid VI.11]; and let it be brought about that, as CG — the straight line between the point G and the other side of the axial triangle — is to AG, so is AG to GH.

Diagram. The axial triangle of a cone with an inscribed ellipse, and beneath it the diameter line G–H drawn out with its rectangle. The construction finds the latus rectum EI of the ellipse EF.

- A (apex of the axial triangle)
- E (on the upper left side of the triangle); I (marked to the left, off the side)
- F (on the right side of the triangle)
- B (base, left corner); D (base, center); C (base, center-right); G (on the base line produced, to the right)
- An inscribed lens-shaped ellipse passing through B, D, C

- Below the triangle: a horizontal segment G——H
- A rectangle beneath that: B (top-left), G (top-center), H (top-right), C (bottom-center)

Again let it be brought about that, as GH is to GB — the straight line between the same point G and the other side of the axial triangle — so is EF, the transverse diameter, to EI [Margin b: Euclid VI.12]. I say that EI is the latus rectum of the ellipse — that is, the straight line to which the ordinates dropped to the diameter are referred [in square]. For let there be the rectangle BC contained under BG, GC — the straight lines lying between the point G and the sides of the axial triangle; and to GC let there be applied the rectangle CH, contained under GC, GH [Margin c: Euclid VI.17], which will be equal to the square on AG, since the three straight lines CG, AG, GH are in continued proportion by construction. And BGH will be one straight line, on account of the two right angles at G [Margin d: Euclid I.14]. Since, then, as HG is to GB, so is EF to EI [Margin e: Euclid VI.1]; and since, as HG is to GB, so is the rectangle HC to the rectangle CB — that is, the square on AG to the rectangle contained under BG, GC — therefore EI is the latus rectum of the ellipse EF, by Proposition 13 of Book 1 of Apollonius; that is, the straight line to which the ordinates are referred, and so on.

[Margin: How the fourth part of the rectangle contained by the transverse diameter of an ellipse and the latus rectum is to be applied to the transverse diameter on each side, so that it falls short by a square figure.]

LEMMA II.

To apply the fourth part of the rectangle contained under the transverse diameter of an ellipse and the latus rectum to the transverse diameter, on each side, so that it falls short by a square figure.

With the same figure supposed [Margin f: Euclid VI.13], let AB, the mean proportional between EF, the transverse diameter, and EI, the latus rectum, be taken, and let it be bisected at C [Margin g: Euclid VI.17]. Then the square on AB will be equal to the rectangle under EF, EI; and consequently the square on AC — which, by the scholium to Proposition 4 of Book 2 of Euclid, is the fourth part of the square on AB — will be equal to the fourth part of the rectangle under EF, EI. To this square on AC, then, we shall apply to the transverse diameter EF, on each side, an equal rectangle deficient by a square figure, in the following way.

Let the straight line EF be bisected at D. Since, by what has been demonstrated by Eutocius on the second definitions of Book 1 of Apollonius, the latus rectum EI is

less than the transverse diameter EF — that is, than the major axis of the ellipse — AB, the mean proportional between EF and EI, will also be less than EF. Therefore AC, the half of AB, will be less than DF, the half of EF. So, with a semicircle described about DF [Margin h: Euclid IV.1], let there be fitted in it a straight line FI equal to AC — which has been shown to be less than DF — and let DI be joined [Margin i: Euclid III.15], which will also be less than DF. From D let there be cut off on either side the straight lines DN, DO equal to DI.

I say that the rectangle under EN, NF applied to EF and deficient by the square on NF, as well as the rectangle under FO, OE applied to the same EF and deficient by the square on OE, is equal to the square on AC — that is, to the fourth part of the rectangle under EF, EI. For, with the square DH described on DF, let the figure be completed, as you see. [Margin k: Euclid VI.24] Since, then, the parallelograms DH, GL, LF, lying about the same diameter, are similar, and DH is a square, GL and LF will also be squares. [Margin l: Euclid I.47] And because the square DH is equal to the squares on FI, DI — that is, to the square on AC together with the square GL [Margin m: Euclid III.31] (for the angle DIF is right, and the straight lines FI, DI were equal to AC, DN or KL) — the gnomon KNH will be equal to the square on AC.

Diagram. A composite figure at the foot of the left column illustrating Lemma II. It comprises: a straight line A–C–B with C marking the bisection point; a short line-segment E–I (the latus rectum); a square-and-gnomon construction on DF; and, to its right, a dotted curve on DF carrying the same lettering.

- Line 1: A — C — B (C at the midpoint)
- Line 2: E — I (with E marked)
- Square/gnomon block: G, H (top); K, L (interior division points); E, O, D, N, F (along the bottom line); I (at the very foot); one sub-triangle is hatched
- Dotted curve at right, on the same base: E (top), F (foot); crowded interior/peripheral labels read as K, I, G, H, A, O, B, D, N, with several repeated glyph-clusters (K I G H I ... I H G I) that do not resolve individually [?]

Since, then, the gnomon KNH is also equal to the rectangle EL [Margin n: Euclid I.36] (for since EK is equal to KF — that is, to NH — when the common DL is added, the whole EL becomes equal to the whole gnomon KNH), the rectangle EL, contained under EN, NF (since NF is equal to NL, because of the square LF), will also be equal to the square on AC. Therefore there has been applied to EF, the transverse diameter, a rectangle under EN, NF equal to the fourth part of the rectangle under EF, EI and deficient by the square on the straight line NF. In the

same way it will be shown that the rectangle under FO, OE applied to EF and deficient by the square on EO is equal to the fourth part of the rectangle under EF, EI. Which was what was proposed.

With these things premised, let EF be the transverse axis of the ellipse EF, and EI its latus rectum, given from the first lemma. By the second lemma let there be applied to EF, on each side, a rectangle — both the one under FO, OE and the one under EN, NF — equal to the fourth part of the rectangle under EF, EI, the former deficient by the square on EO, the latter by the square on FN. And with NO bisected at A, let any number of points whatever, B, C, D, be taken between A and N.

Then, at the interval EA (or FA), let four arcs be described from the points O and N, cutting one another on either side at G. Likewise, from the same points O and N, at the interval EB, let four arcs be described, which four other arcs — described from the same points at the interval FB — cut at the point H. In the same way, at the intervals EC, FC, let the arcs described from the same points O and N cut one another at I; and so on for the remaining points, if there be any — always observing that, of each set of four arcs to be described from O and N, the two greater are described from O beyond the point A and two from N beyond the same point A, while the two lesser are described from O short of the point A and two from N short of the same point A. For through the points E, G, H, I, F the ellipse is to be described.

[Margin: Another way of describing an ellipse in a plane.]

For since the straight lines NG, OG — that is, EA, FA — as well as the straight lines NH, OH — that is, EB, FB — and so on, are equal to the axis EF, the ellipse whose axis is EF will pass through the points E, G, H, I, F; inasmuch as, by Proposition 52 of Book 3 of Apollonius, straight lines drawn from the points N, O inclined to one and the same point of the ellipse are equal to the axis EF. For if the said ellipse does not pass through the point I, let it pass, if that be possible, through K, cutting the straight line NI at K — whether beyond or short of I — and let OK be joined. Since, then, the aforesaid ellipse passes through K, the straight lines NK, OK together will be equal to the axis EF, by Proposition 52 of Book 1 of Apollonius [recte: Book 3]. But by construction the straight lines NI, OI are also equal to the same axis EF. Therefore NK, OK will be equal to the straight lines NI, OI. Yet they are also unequal; for when the point K falls beyond I [Margin a: Euclid I.20], the straight lines IK, KO are greater than the straight line IO, and so, adding the common IN, NK and OK will be greater than NI, OI. But when the point K falls short of I [Margin b: Euclid I.20], IK, IO are greater than KO, and so, adding the

common KN, NI and OI will be greater than NK, OK. Which is absurd. Therefore the said ellipse will pass through no other point than I. And in the same way we shall demonstrate that it passes through the remaining points H, G, and so on — which was what was proposed.

It is also evident that this description does not fit scalene cones, except when the triangles through the axis are perpendicular to the bases of the cones. For only then does the diameter of the ellipse cut the ordinates at right angles, as is clear from Proposition 7 of Book 1 of Apollonius, and so it is the axis.

[Margin: In what manner any parabola whatever is described in a plane.]

But if we wish to describe some parabola, or a hyperbola, or even two opposite hyperbolas, or an ellipse — with no regard to the cones from which they arise — we shall take for the parabola an axis EH of any magnitude, as in the parabola above, and in it any number of equal parts as we please; and through the points terminating the first part, and the next three, and the next five, and the next seven, and so on, we shall draw lines parallel to one another. Then, taking on the first any straight line AD you please, on both sides, we shall take its double on the second, its triple on the third, its quadruple on the fourth, and so on. For through the extremities of these lines the parabola is to be described.

[Margin: In what manner any two opposite hyperbolas whatever are described in a plane.]

For the hyperbolas, however, we shall take any axis FI, as in the preceding hyperbolas, to which on each side we shall add any equal straight lines FR, IQ, and we shall understand that to the axis FI there has been applied a rectangle under FQ, QI equal to the fourth part of the rectangle contained under the axis FI and the latus rectum, and exceeding by the square on QI, and so on.

Diagram. The large construction figure for describing a conic (parabola, opposite hyperbolas, and ellipse) directly in a plane, without reference to a cone. An axis runs from the apex A downward; a circle sits in the lower center; a set of oblique lines runs off to the right toward N, P, O, Q.

- A (apex, top); E, F (just below A)
- L, K (left of center); M, R (right of center)
- C — E — D (a horizontal line, E at the center)
- H (center, below E); I (lower left); B (foot, center); G (below B, at the foot)
- N (upper right); P (middle right); O (lower right); Q (far right)

- A circle in the lower-center region; several dotted construction lines from A through K toward M/R

Diagram. An articulated drawing instrument (an ellipsograph / trammel of the compass type) shown in the lower part of the page, used in place of the pegs and string. A long rod runs from a turned handle at the left across to a hinged joint at the right, with a large disc/pulley at the lower left and jointed arms carrying the tracing point.

- d (turned handle at the left end of the rod)
- f, e (near the center-top hinge)
- S (large disc/pulley at the lower left)
- T, X (at the upper right joint)
- b (at the lower right)
- V, Y (at the foot, lower right)

[Margin: In what manner any ellipse whatever is described in a plane.]

Finally, for the ellipse we shall take an axis EF at pleasure, as in the ellipse just treated, from which we shall cut off equal segments EO, FN on either side, as we please, and we shall understand that to the axis EF there has been applied a rectangle under FO, OE equal to the fourth part of the rectangle contained under the axis EF and the latus rectum, and deficient by the square on EO, and so on. The rest we shall complete as before.

[Margin: In what manner any ellipse whatever may be described in a plane by a string.]

Indeed, if in some straight line EF, drawn out to any length whatever — namely, in the same axis of the ellipse — two small pegs are fixed at two points, say at N and O, and a string is placed about them such that, when doubled, it exceeds the straight line NO and can turn freely, an ellipse will be described easily and readily, provided we set some stylus on this string and carry it round together with the string, always keeping the string stretched as far as possible. Thus, if the string about the pegs N, O is stretched as far as I, as far as it can reach, and if some iron stylus is placed at I, and, with the string kept always thus stretched, it is carried round until it returns to the point from which it began to move, the ellipse EGF will be described. For in this way there will always be two straight lines from N and O up to the stylus carried round, equal to the axis EF, which the same stylus, in its circuit, cuts at E and F. And this can be grasped far more easily by the thing itself, and by practice, than it can be taught in many words.

[Margin: In what manner an ellipse of any size may be described by a string according to a given length and breadth.]

But since this description of the ellipse by a string fitted about pegs is both pleasant in itself and useful for many purposes, I should like to explain it here somewhat more fully. First, then, if both axes of the ellipse are given — namely AB the greater and CD the lesser — so that the ellipse to be described has AB for its length and CD for its breadth, we shall find the points in the major axis at which the pegs are to be fixed, and the length of the string to be carried round the pegs, in this way. The major axis AB being bisected at E, let CD be drawn through E perpendicular to AB, and let both EC and ED be equal to half the minor axis, so that CD is equal to the whole minor axis. Then, with center C (or D) and radius EA (or EB), let two arcs of a circle be described cutting the straight line AB at the points F and G,

which will lie altogether short of the points A and B [Margin a: Euclid I.19]. For since the straight line CA is greater than AE, because of the right angle AEC, it comes about that the arc described from C at the interval AE cuts CA itself, and hence also AE itself, short of the point A, and so on. I say that the pegs are to be fixed at the points F, G, so that the proposed ellipse may be described; and that the string placed about F, G must be doubled, so that when stretched it reaches exactly to the point A (or to the point D, or B), and the whole string is DFGD, set about the pegs fixed at F and G by this rule: that it can be freely carried round about them together with the stylus, so that the stylus is hindered neither by the pegs nor by the string from being carried round to describe the entire ellipse.

For since the portions GA, FA of the doubled string (the portion FG between the pegs being left aside) are equal to the major axis AB — because AF is equal to BG — and in the same way the remaining portions, when the string is carried round about the points F, G, are equal to the same major axis, because the string is neither increased nor diminished in that revolution, and the common portion FG always remains the same (for thus you see, for example, that when the end of the doubled string is placed at D, the portions DF, DG are equal to the whole axis AB, because AE, BE are taken equal, and so on) — it is evident, from Proposition 52 of Book 3 of Apollonius, that by the aid of that string there is described an ellipse whose major axis is AB and minor CD; inasmuch as the straight lines from the points F and G, inclined to any point whatever of the curved line described through the points A, C, D, B, are equal to the major axis AB, as that proposition of Apollonius requires.

[Margin: The places of the pegs and the length of the string for describing the ellipse.]

We shall also find the points F, G for the places of the pegs by the following method, and perhaps more surely — because, when the minor axis is nearly equal to the major, the arcs of the circles described from C or D cut the straight line AB very obliquely. Let the straight line BE, which is half the major axis, be bisected at H; from H, at the interval HB (or HE), let a semicircle BIE be described [Margin b: Euclid IV.2], and in it let there be fitted a straight line BI equal to DE, half the minor axis, and let EI be joined. I say that the straight line EI is equal both to EF and to EG; and hence, if the straight lines EF, EG are cut off equal to EI, the same points F, G for the places of the pegs have been found. [Margin c: Euclid I.47] For since the square on BE is equal to the squares on EI, IB; and the square on DG is equal to the squares on DE, EG, and the square on DF to the squares on DE, EF; while the square on BE is equal both to the square on DG and to the square on DF, because these lines are equal by construction; and the square on BI is equal to the square on DE, because BI and DE were also placed equal by construction — the remaining square on EI will be equal to the remaining square described on EG or on EF, and hence the straight line EI will be equal to the straight lines EG, EF, which was what was proposed.

Diagram. The same conic-in-a-plane construction figure as on the preceding page, repeated here (the woodcut is reprinted so the reader need not turn back). Same lettering as fig-p41-1.

- A (apex, top); E, F (just below A)
- L, K (left of center); M, R (right of center)
- C — E — D (horizontal line, E at center)
- H (center, below E); I (lower left); B (foot, center); G (below B)
- N (upper right); P (middle right); O (lower right); Q (far right)
- A circle in the lower-center; dotted construction lines

Diagram. The same articulated ellipsograph instrument as on the preceding page, repeated here. Same lettering as fig-p41-2.

- d (turned handle, left); f, e (center hinge); S (disc/pulley, lower left); T, X (upper right joint); b (lower right); V, Y (foot, lower right)

[Margin: How an ellipse is described about a given major axis and through a given point.]

Next, if only the major axis AB of the ellipse is given, together with some point K through which the ellipse described about the axis AB must pass, we shall find the minor axis — that is, the breadth of the ellipse — and the points F, G at which the pegs are to be fixed, in this way. Let AB be bisected at E; through E let CD be drawn perpendicular to AB, and from the given point K let another perpendicular KL be drawn to the same AB, parallel to CD. Then, by what we have demonstrated in the third problem of the scholium to the last proposition of Book 6 of Euclid, let it be brought about that, as the rectangle under AL, LB is to the rectangle contained under AE, EB — that is, to the square on AE (or EB), (for this rectangle is a square, because of the equality of the straight lines AE, EB) — so is the square on KL to another square whose side is ED or EC. And, from what has been demonstrated by Apollonius in Proposition 21 of Book 1, ED (or EC) will be half the minor axis; by which, as we taught a little before, we shall find the points F and G, by whose aid we shall describe the ellipse.

[Margin: The places of the pegs for describing the ellipse are found by another method.]

And we shall find the square of the sought side ED (or EC) expeditiously as follows. From E, at the interval EA (or EB), let a semicircle AMB be described, which the straight line LK produced cuts at M. Then, by the scholium to Proposition 13 of Book 6 of Euclid [Margin d: Euclid VI.17], the straight line LM will be the mean proportional between AL, LB, and hence its square will be equal to the rectangle contained under AL, LB. Whence with easy effort we shall find the square to which the square on LK has the same ratio that the square on LM — that is, the rectangle under AL, LB — has to the square on EA (or EB), that is, to the rectangle contained under AE, EB [Margin e: Euclid VI.12], if for the three straight lines LM, EA, LK we find a fourth proportional ED [Margin f: Euclid VI.22]; because the squares described on the straight lines LM, EA, LK, ED have the same ratio as the straight lines themselves. And by the following device we shall find the said fourth proportional ED. Let two straight lines NO, NP be drawn making any angle at N; let NQ be taken equal to LM, and QO equal to EA, and NR equal to LK. Then, QR having been drawn, let OP be drawn through O parallel to QR. RP will be the said fourth proportional [Margin g: Euclid VI.2], since, as NQ (that is, LM) is to QO (that is, to EA), so is NR (that is, LK) to RP. Therefore if we take ED equal to RP, we shall have half the minor axis, ED, and so on.

[Margin: An instrument for describing an ellipse by a string.]

For the rest, in place of the pegs we shall be able to use a certain instrument made in the likeness of a compass [circinus], whose legs are cut off at their

extremities, and to which small pieces [frusta] are fitted, cut off and so adapted that they can be moved this way and that and fastened with little screws [cochleolae], so that however far the legs of the compass are opened, those pieces, fastened by the little screws, always stand upright to the plane on which the ellipse is to be described. And let these pieces have also, in their extreme parts, certain little channels [canaliculi] cut round about, so that the string in them may be carried round—

Gnomonics, Liber Primus (continued)

being revolved, neither rises upward nor sinks downward. For with this instrument we shall easily describe an ellipse: once its legs have been opened, we set those small pieces at the found points F and G so that they stand upright to the plane, and we lay the string around, as we said above. But since the string is always stretched and slackened a little, an ellipse will be drawn much more accurately — though not so easily — with the instrument that Guidobaldo del Monte [Guidobaldo del Monte], of the Marquises of Monte, a man in this age no less distinguished for talent and for excellence of learning than for nobility of birth, devised and most acutely demonstrated in his *Theory of Universal Planispheres* [*Theorica planisphaeriorum universalium*], recently published by him.

The instrument, shaped like a compass, is STV in the adjoining figure. Its legs are cut off at T and V, while the small pieces Ta and Vb are fitted at T and V so that they can be moved freely this way and that and fastened with little screws. The little channels are at X and T, in which the string XTf runs round. Finally, the stylus is de, which also has a channel at f, so that it can be turned round conveniently about the taut string, in such a way that the string always forms a triangle whose angles lie at the channels of the compass and of the stylus — such as the triangle XTf in the figure. Only when the stylus is set at A or B is no triangle formed, but a straight line. Moreover the segments ef, aX, dT ought to be equal, and of the least possible height. But the actual use of the instrument will show more plainly the manner in which it is to be constructed.

THEOREMA 7. PROPOS. 9.

[Margin: What the hour-circles from noon or from midnight are.]

The greatest circles in a sphere, of which one is the Meridian, passing through the poles of the Equator and cutting the parallel that touches the Horizon — the

greatest of the parallels that are always visible — into 24 equal parts, mark the equal hours beginning from noon or from midnight.

Diagram. A large circular hour-diagram. Two perpendicular diameters cross at the center E: the vertical diameter runs from A (top) to C (bottom) and represents the Meridian AC; the horizontal diameter runs from B (left) to D (right). The bounding circle is the parallel ABCD, "the greatest of the always-visible parallels." Below the circle a broad curved band (an obliquely-seen circle/ellipse) represents the Horizon, its ends marked F (lower left) and G (lower right); a straight line runs across the bottom from H (left) to I (right) — the common section HI of the Horizon plane and the plane of circle ABCD — with the point S marked on it near the bottom center (the base reads "H·S·I", flanked by the hour-number 24). Between the bounding circle and the center lie two concentric rings finely subdivided into the fractional parts of the hours (halves, quarters, eighths, etc., as the following Scholium describes); the individual fraction-cells (readable samples near the top include $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{3}{4}$) are too dense to resolve reliably cell-by-cell at this resolution and are not transcribed here [?].

- Center: E
- Vertical diameter (Meridian AC): A (top), C (bottom)
- Horizontal diameter: B (left), D (right)
- Horizon band: F (lower left), G (lower right)
- Base line: H (lower left) — S (center) — I (lower right)
- Outer numeric ring: the hours 1–24 (12 at top over A, running clockwise 13, 14, 15, 16, 17, 18 down the right to D, then 19, 20, 21, 22, 23, 24 across the bottom to C, and 1–11 up the left back to A)
- An inner numeric ring carries a second 24-fold count (12 at top, running down both sides to 12 at the bottom)
- Circular inscription on the outer rim in serif capitals: the right (western) arc appears to read CASVS SOLIS ("the setting of the Sun"); the remaining arcs of the inscription (top, left/eastern, bottom) are not reliably resolvable — legible letters include V, E, L, O, C, R, O, T, A, H, O, R, I, S [?]. An inner band near the center likewise carries lettering (HORA and apparent directional/meridian words, e.g. MERIDIANO), only partly resolvable [?].

[Diagram — the rim inscription, inner-band lettering, and the two fraction-rings carry many labels/cells not resolvable at this resolution.]

Let the parallel ABCD — whose center is E, the greatest of all the always-visible parallels — cut [the said circles]; and let it touch the Horizon FG. (It touches it, I

say, at the point where the straight line HI, the common section of the plane of the Horizon and of the plane of the circle ABCD, likewise touches — by the definition of Book 2 of Theodosius [Theodosius] — both the Horizon FG and the circle ABCD.) Let the greatest circles, of which one is the Meridian, pass through the poles of the Equator, and therefore also of the circle ABCD — for by Proposition 1 of Book 2 of Theodosius the Equator and the parallel ABCD have the same poles — passing through the center E (since all greatest circles that pass through the poles of the parallel ABCD divide it in two, that is, through the center and at right angles, by Proposition 15 of Book 1 of Theodosius), and let them cut it into 24 equal parts, so that the Meridian passes through the straight line AC.

I say that these greatest circles mark the equal hours counted from noon or from midnight. For since they pass through the poles of the Equator and of all the parallels, they will cut all the parallels into similar parts, by Proposition 10 of Book 2 of Theodosius. And therefore, since the parts of the parallel ABCD are supposed equal, the parts of every parallel will also be equal among themselves. Hence the Sun in its daily motion will traverse them in 24 equal intervals of time, the beginning made at noon or at midnight. But these 24 equal intervals of time are the 24 equal hours beginning from noon or from midnight. Therefore those greatest circles mark the 24 equal hours counted from noon or from midnight. And accordingly, the greatest circles in a sphere, of which one is the Meridian, and so on — which was to be demonstrated.

GNOMONICES — SCHOLIUM

[Margin: Circles that mark the half-parts, quarter-parts, eighth-parts, and so forth of the hours from noon or from midnight.]

If the several equal spaces of the parallel ABCD are bisected, and these halves again bisected, and so on in succession, and if greatest circles are then drawn through the divisions and through the poles of the parallel ABCD, these circles will mark the half-hours, the quarter-parts of the hours, the eighth-parts, and so forth, from noon or from midnight. For the demonstration is the same for the parts as for the whole hours. And these greatest circles may rightly be called *hour-circles from noon or from midnight*.

[Margin: Hour-circles from noon or from midnight.]

[Margin: Which semicircles pertain to the hours from noon, and which to the hours from midnight.]

Furthermore, the semicircles of these circles are set between the two poles of the world, which the axis of the world all divides. Proceeding from the semicircle of the Meridian standing above the Horizon, through the west, as far as the semicircle of the Meridian below the Horizon, they mark the hours from noon; the remaining semicircles mark those same hours from midnight, as is plain. Hence it will not be difficult to recognize which points of the greatest of the always-visible parallels answer to the hours from noon, and which are owed to the hours from midnight. For the points on the semicircle ADC — if the parallel be understood to touch the Horizon at the lowest point toward C, so that D inclines toward the west and B toward the east — mark the hours from noon, since at those points the hour-semicircles, running between the two poles from the semicircle of the Meridian standing above the Horizon through the west as far as the semicircle of the Meridian below the Horizon, cut the parallel, as is plain. But the points on the remaining semicircle CBA show the hours from midnight, because at those points the parallel is cut by the remaining hour-semicircles. By the same reasoning, in the greatest of the ever-hidden parallels, the points of the semicircle set toward the west, beginning from that point where it touches the Horizon, will give the hours from noon, while the points of the other semicircle, toward the east, will mark the hours from midnight, for the same reason. This can be easily understood if those two parallels and the hour-circles are imagined set in their proper position.

We have, however, preferred to propose that the hour-circles cut the greatest of all the always-visible parallels into twenty-four equal parts, rather than the Equator (although this too is true, as the demonstration shows) — because both the knowledge of the hour-circles that show the hours from sunrise and sunset, and most other matters bearing on the hours reckoned as much from noon or midnight as from sunrise or sunset, depend on the hour-points of the said parallel, as will become plain from what follows.

THEOREMA 8. PROPOS. 10.

[Margin: What the hour-circles from sunrise or sunset are.]

The greatest circles in a sphere, of which one is the Horizon, touching the greatest of all the always-visible parallels at the 24 points into which it is divided by the hour-circles from noon or from midnight, mark the equal hours beginning from the rising or the setting of the Sun. Their poles, moreover, are the points of the parallel described through the zenith of the place, that is, through the pole of the Horizon, at which that parallel is cut by the hour-circles from noon or from midnight.

Let the greatest circles, of which one is the Horizon, touch the parallel ABCD at the 24 hour-points from noon or from midnight. I say that these greatest circles mark the equal hours beginning from the rising or the setting of the Sun, and so forth. For since they touch the parallel ABCD, and therefore also — by Proposition 6 of Book 2 of Theodosius — the parallel equal to it, namely the greatest of those that are always hidden beneath the earth, they will cut all the intermediate parallels (namely those parts intercepted between any two nearest non-concurrent semicircles) into parts similar to the parts of the parallel ABCD, by Proposition 13 of Book 2 of Theodosius. And therefore, since the parts of the parallel ABCD are supposed equal, the parts of every intermediate parallel will also be equal among themselves. Hence the Sun in its daily motion will traverse them in 24 equal intervals of time, the beginning made at the Horizon, that is, at the rising or the setting of the Sun. But these 24 equal intervals of time are the 24 equal hours beginning from sunrise or sunset. Therefore those greatest circles mark the 24 equal hours beginning from sunrise or sunset — which is the first point.

Nor, besides those named, can any other circles be conceived in the heavens that would mark the hours from sunrise or sunset. For since hours of this kind begin from the Horizon, and divide the several parallels that they cut into 24 equal parts, it follows, by Proposition 16 of Book 2 of Theodosius, that greatest circles dividing the parallels in this manner either pass through the poles of the parallels or touch some one same parallel. Since, then, they do not pass through the poles — because the Horizon, which is one of them, by no means passes through the poles except in a right sphere — they will necessarily touch one same parallel. And since the Horizon touches the greatest of the always-visible parallels, the rest will touch the same one. Altogether, then, the greatest circles that mark the hours from sunrise or sunset touch the greatest of the always-visible parallels.

Now since all these circles touch the same parallel, which is the greatest of the always-visible ones, it follows that they are equally inclined to the Equator, by Theorem 1 of the Scholium to Proposition 21 of Book 2 of Theodosius — which is, according to the tradition of Francesco Maurolico [Francesco Maurolico], Proposition 26 of Book 2 of Theodosius. Hence, by Theorem 2 of the same Scholium — which is Proposition 27 according to Maurolico — they have their poles on the circumference of the same parallel. Since, therefore, the pole of the Horizon, which is one of these hour-circles, lies in the parallel described through the pole of the Horizon, that is, through the zenith of the place, the poles of the other circles will necessarily also lie in the same parallel. And because the hour-circles from noon or from midnight, passing through the points of contact and

through the poles of the greatest of the always-visible parallels, also pass — by Proposition 3 of Book 2 of Theodosius — through the poles of the circles that mark the hours from sunrise or sunset (which of course touch that parallel), the poles of these circles will be altogether the points of the parallel described through the zenith of the place, that is, through the pole of the Horizon, through which the hour-circles from noon or from midnight pass, since in this parallel all the poles lie, as has been demonstrated. The second point is therefore also established. Wherefore the greatest circles in a sphere, of which one is the Horizon, and so forth — which was to be shown.

[Margin: The poles of the circles that mark the hours from sunrise or sunset lie in the parallel described through the zenith of the place.]

COROLLARIUM.

[Margin: The poles of the circles that mark the hours from sunrise or sunset divide the parallel described through the zenith of the place into 24 equal parts.]

From these things it follows that the poles of the circles marking the hours from sunrise or sunset divide the parallel described through the zenith of the place, that is, through the pole of the Horizon, into 24 equal parts, the beginning made from the Meridian circle. For the points where the said parallel is cut by the hour-circles from noon or from midnight — which points are indeed the poles of the circles marking the hours from sunrise or sunset, as we have shown — cut that same parallel into 24 equal parts, the beginning made from the Meridian circle, as is established from Proposition 10 of Book 2 of Theodosius.

LIBER PRIMVS — SCHOLIUM

[Margin: How, in the hour-circles from sunrise or sunset, non-concurrent semicircles are taken.]

That you may see in what manner those parts of the parallels are intercepted between two nearest non-concurrent semicircles: let there be any number of parallels — ABC, the greatest of the always-visible; DEF, the parallel of ♋ [Cancer]; GHI, the Equator; and KLM, the parallel of ♊ [Capricorn]. Let the equal hour-spaces CN, NO be taken on ABC, and at the points C, N, O let the greatest circles CPHQ, NRST, OVXT touch the parallel ABC, of which let CPHQ be the Horizon. Let CPHQ and NRST cut one another at Z and α ; then, by Proposition 11 of Book 1 of Theodosius, ZQ α and ZT α will be semicircles, and therefore PQ α will be less than a semicircle. Let the arc $\alpha\beta$ be taken equal to the arc PZ, and the arc

$\alpha\gamma$ equal to the arc $Z\delta$; then PQ , $\alpha\beta$ [and] $\delta Z\tau$ [?] will be non-concurrent semicircles, since they do not intersect one another. By the same reasoning, if the circles $NRST$, $OVXT$ cut one another at ε and θ , and the arc $\theta\lambda$ be taken equal to the arc $V\varepsilon$, then $\delta Z\tau$ [?] and $VY\theta\lambda$ [?] will be non-coincident semicircles, as is established. And in the same way, in the remaining greatest circles that touch the parallel ABC in the remaining equal hour-spaces, we shall find non-concurrent semicircles. Therefore the spaces PR , RV , HS , SX , QT , TT [?], and the rest of this kind, we said were similar to the hour-spaces CN , NO , and to the others, and so on; and thus the spaces of any parallel are equal among themselves, just as CN , NO and the remaining spaces of the parallel ABC are equal among themselves.

Diagram. A system of many overlapping and concentric circles representing the several parallels (the always-visible ABC , the parallel of Cancer DEF , the Equator GHI , the parallel of Capricorn KLM) together with the greatest circles $CPHQ$, $NRST$, $OVXT$ touching them, drawn on the plane of the meridian/horizon. A vertical axis runs down the middle from K (upper) through G , D , A near the center and down to F , Z , I , and M (lowest point); a horizontal axis runs across, ending at L (far left). The nested circles and their intersections generate a dense cluster of labelled points near the center-bottom (the letters E , V , B , O , N , C , F , Z crowd together there).

- On the vertical axis, from top down: K , G , D , A (near center), then the cluster $O-N-C-\xi-Z$, then F , then I , then M (bottom)
- Near the center: A with B to its right; E to the left with V beside it
- Left side, descending from upper-left to the axis foot H : T (upper left), Y , X , S , and ρ/P near them; H at the lower left; L at the far left end of the horizontal axis
- Lower-center / left of the cluster: P
- Greek point-labels along the upper arcs: α (top center), β (upper right), γ and λ (to the right), θ (upper right); ε and δ occur among the central intersections [Greek letters read on the scan; several worn — see below]

[Diagram — more than two labels (the Greek intersection-letters α , β , γ , δ , ε , θ , λ and the crowded central cluster) could not all be resolved with confidence at this resolution.]

If other greatest circles touch the same parallel at other points — those by which it is divided by the hour-circles from noon or from midnight that mark the half-hours, the quarter-parts of the hours, the eighth-parts, and so forth — these circles will mark the half-hours, the quarter-parts of the hours, the eighth-parts,

and so on, from sunrise or sunset. For the demonstration is the same for the parts as for the whole hours. And these circles may not unfittingly be called *hour-circles from sunrise or sunset*.

[Margin: Circles that show the half-parts, quarter-parts, eighth-parts, and so forth of the hours from sunrise or sunset.]

[Margin: Hour-circles from sunrise or sunset.]

[Margin: Which semicircles pertain to the hours from sunrise, and which to those from sunset.]

Furthermore, the semicircles of these circles are set between the two points of contact, one of which is always of some hour from noon and the other of the same hour from midnight, as is clear from what we wrote in the Scholium of the preceding proposition. Proceeding through the hour-points of the equinoctial circle below the Horizon, the beginning made from the western semicircle of the Horizon, and going on from east to west, that is, against the succession of the signs, they pertain to the hours reckoned from sunset — so that the western semicircle of the Horizon marks the 24th hour from sunset, the next western semicircle below the Horizon gives the first hour from sunset, and the following one, toward the western parts, the second hour, and so on in succession, always proceeding toward the western parts, that is, against the succession of the signs, until we return again to the western semicircle of the Horizon. The remaining semicircles, which run through the hour-points of the equinoctial circle above the Horizon, the beginning made from the eastern semicircle of the Horizon and going on from east to west — that is, against the succession of the signs — belong to the hours reckoned from the rising of the Sun. Thus the eastern semicircle of the Horizon marks the 24th hour from sunrise, the next eastern semicircle above the Horizon shows the first hour from sunrise, and the following one, against the succession of the signs, the second hour, and so on in succession, always advancing from east to west, that is, against the succession of the signs, until we come round again to the eastern semicircle of the Horizon.

Perhaps we shall understand more plainly and openly which semicircles pertain to the hours from sunset or from the rising of the Sun in this way. Let us imagine ourselves placed outside the heavens, near the greatest of the always-visible parallels, so that our feet are at hour 12 of midnight, that is, at the 24th from sunrise or sunset, and our head is turned toward the arctic pole. This being supposed, the semicircle of the Horizon, that is, the 24th hour, which is on our right, will mark the 24th hour from sunset; the other, on our left, the 24th hour

from sunrise. Likewise, if we set our feet at the other hours of the said parallel, with the head turned toward the pole, the hour-semicircles from sunrise or sunset that lie on our right will pertain to the hours from sunset, while those on our left will pertain to the hours from the rising of the Sun. All this will be clear to anyone who carefully grasps the greatest of the always-visible parallels — which the circle in Proposition 9 presents to us — and the hour-circles from sunrise or sunset touching the said parallel, set in their proper position.

[Margin: What the unequal-hour circles are.]

As for the circles of the unequal hours, it is plain that they must pass through the twelfth parts of all the daytime and nighttime arcs, since parts of this kind, the twelfths, are called unequal hours. That the greatest circles described through the unequal hours of the Equator (which are indeed the same as the equal hours) and of the northern daytime arcs also pass through the same hours of the southern daytime arcs, we shall demonstrate in this way, together with Federico Commandino [Federico Commandino]. Since in any oblique Horizon, for two parallels equally distant from the Equator, the daytime arc of the one is equal to the nighttime arc of the other, and conversely — as plainly appears from Proposition 19 of Book 2 of Theodosius — and therefore, by however much the days increase as the Sun tends from the vernal equinox toward ♋ [Cancer], by just so much do they decrease as the same Sun tends from the autumnal equinox toward ♏ [Capricorn]: it follows that a day of ♋ [Cancer] is greater than the equinoctial day by just as much as a day of ♏ [Capricorn] is less than the same equinoctial day. Since, then, the daytime arc of any parallel is divided into 12 equal parts, and the proportion of part to part is the same as of whole to whole, it comes about that the arc also of one hour of ♋ [Cancer] exceeds the arc of one equinoctial hour by the same quantity by which the arc of one hour of ♏ [Capricorn] is exceeded by it. This too can be plain from the following: because in this way all those 12 excesses of the twelve hours of ♋ [Cancer], taken together, will be equal to the 12 defects of the twelve hours of ♏ [Capricorn], taken together. This would not happen if one hour of ♋ [Cancer] did not exceed one equinoctial hour by the same quantity by which one hour of ♏ [Capricorn] is exceeded by the equinoctial hour. And the same is the ratio for the arcs of two hours, of three, of four, and so on. And so likewise in the other parallels that are equally distant from the Equator — as in the parallels of ♊ [Gemini] and ♐ [Sagittarius] [?]; likewise in the parallels of ♉ [Taurus] and ♏ [Scorpio] [?], and so forth.

[Margin: The greatest circles cutting the daytime arcs of the Equator and of the northern parallels into 12 equal parts likewise divide the daytime arcs of the southern parallels into 12 equal parts.]

[This page continues the demonstration begun in the previous proposition (Theorem 8, Prop. 10).]

Now let there be, in the sphere, the parallel $\mathfrak{S}$ [Cancer], ABCD; the Equator EFGH; the parallel $\mathfrak{V}$ [Capricorn], IKLM; the greatest of the always-visible parallels, NOPQ, and R the pole of all of them; and let the oblique Horizon IEANDHN [?], touching the parallel NOPQ at N. Next, let the arc of the Equator, and of the parallel $\mathfrak{S}$, that lie above the Horizon each be divided into twelve equal parts, so that the first hour of the parallel $\mathfrak{S}$ falls at point B, the second hour of the same parallel at point C, while the first hour of the Equator falls at F and the second at G. Through the points B and F, and likewise through the points C and G, let there be drawn — by Proposition 28 of Book 1 of Theodosius — two greatest circles cutting the parallel $\mathfrak{V}$ at the points K and L.

I say that the circle BFK also passes through the first hour of $\mathfrak{V}$, and CGL through the second — that is, that IK is the first hour of $\mathfrak{V}$ and KL the second. For, by Proposition 15 of Book 2 of Theodosius, let there be drawn through the points F and G two other greatest circles touching the parallel NOPQ and cutting the parallels $\mathfrak{S}$ and $\mathfrak{V}$ at S, T, V, X. Then, by Proposition 13 of Book 2 of Theodosius, the arcs AS, ST, IV, VX will be similar to the arcs EF, FG, since all these arcs are intercepted between non-concurrent semicircles, as is clear from the preceding scholium. But EF and FG are arcs of equinoctial hours, and equal. Therefore AS, ST and IV, VX are likewise arcs of equinoctial hours, and equal; and so, since the parallels ACD and IXM are equal, the four arcs AS, ST, IV, VX will be equal. And because the parallels ACD and IXM are equal, the arcs SF, FV and BF, FK — cut off from the greatest circles by the parallels — will also be equal, by Proposition 18 of Book 2 of Theodosius. Therefore, by Proposition 3 of Book 3 of Theodosius, the straight line joining the points B and S will be equal to the straight line connecting the points K and V; and so the arc BS will be equal to the arc KV. In the same way the arcs TG, GX and CG, GL, and the straight lines CT, LX, and the arcs CT, LX, will be equal.

[Margin: a 28. tertij.] [i.e. a marginal reference, Prop. 28 [?], Book 3.]

And so, since the arc AS is equal to the arc IV, and the arc BS to the arc KV, the arc AB — the first hour of $\mathfrak{S}$ — will exceed the arc AS, an equinoctial hour, by the same amount by which IK exceeds [?] the arc IV, an equinoctial hour. Therefore,

by what has been said, IK will be the first hour of ♊, since the arc AB, the first hour of ♉ and drawn on the Equator, also passes through the first hour of ♊. In exactly the same way we shall demonstrate that IL is two hours of ♊, just as the arc AC comprises two hours of ♉ and EG two hours of the Equator; for the arc TC, by which AC, two hours of ♉, exceeds the arc AT of two equinoctial hours, is equal to the arc LX, by which IL is exceeded by the arc IX of two equinoctial hours. Since, then, IK has been shown to be the first hour of ♊, KL will be the arc of the second hour. And the same demonstration will hold for the remaining unequal hours of ♊ and of the other signs, provided that in place of the parallels ♉ and ♊ any other two opposite and equal parallels are assumed. Which was what was proposed.

But all that we have demonstrated in the two nearest propositions is to be understood only of the oblique sphere. For in the right sphere no parallel is perpetually visible, since all the parallels are bisected by the Horizon drawn through their poles, as is clear from Proposition 15 of Book 1 of Theodosius. Hence in the right sphere the greatest circles — among which are the Meridian and the Horizon — passing through the poles of the world and cutting the Equator, and consequently (by Proposition 10 of Book 2 of Theodosius) cutting all the remaining parallels into 24 equal parts, will mark both the hours from noon or midnight and those from the rising or setting of the Sun. There are no unequal hours there, since the equinox is perpetual, and so the equinoctial hours are twelfths of any given day, just as the unequal hours in the oblique sphere are twelfths of each day.

[Margin: In the right sphere the same circles indicate the hours from noon or midnight and from sunrise or sunset. But there are no unequal hours there.]

Diagram. A set of concentric circles representing, from the outside in, the parallel of ♋ [Cancer] (ABCD), the Equator (EFGH), and the parallel of ♏ [Capricorn] (IKLM), with the greatest of the always-visible parallels (NOPQ) and the pole R at the centre; an oblique horizon and several greatest circles are drawn through the hour-points, cutting the parallels. The figure is crowded and several peripheral labels are worn.

- X, L, V, K, I (descending the left/upper-left rim): points on the greatest circles / outer parallels
- G, F, C, T, S, B, O, E, A (lower-left quadrant, close-set): hour and intersection points
- R (centre): pole of the always-visible parallels

- P (upper centre), Q (right of centre), D (lower centre-right), H (right rim): further parallel/hour points
- M (upper right): outer point
- N (lower centre): point of tangency of the horizon with the parallel NOPQ

THEOREMA 9. PROPOS. 11.

When the Sun is on any hour-circle, or on any other greatest circle, the solar ray — and hence the shadow of the tip of the stylus — is cast onto the straight line that is the common section of that same hour-circle (or greatest circle) and the plane of the sundial.

[Margin: The shadow of the stylus and the ray of the Sun are cast onto the common section of the plane of the sundial and of the greatest circle on which the Sun is.]

Let ABCD be an hour-circle, or any other greatest circle, cutting the plane of the sundial EFGH along the straight line EG; and let IK be the stylus, whose tip I is set at the centre I, by Proposition 2. I say that when the Sun is at any point L on the circle ABCD, the Sun's ray and the shadow of the stylus's tip I are cast onto the straight line EG. For the ray LI, reaching to the centre I, is drawn (by [?]) and by the plane of the circle ABCD; it does not leave the plane of the circle ABCD when the Sun is on the circumference of that circle, but, produced, it will cut the circumference of the same circle at the point M, which is opposite the point L, so that the ray itself is a diameter of the circle. Since, then, the straight line EG lies in the plane of the same circle, it will cut the ray LIM — that is, the straight line EG — at the point N; and so the Sun's ray LM and the shadow of the stylus's tip I will be cast onto the straight line EG, the common section of the hour-circle ABCD and the plane of the sundial EFGH.

In the same way, if some other hour-circle, or other greatest circle AFCH, cuts the same plane of the sundial EFGH along the straight line FH, and the Sun is at the point O of the hour-circle or greatest circle, we shall demonstrate that the ray OI and the shadow of the same stylus-tip I are cast onto the straight line FH, because the ray, produced to the opposite point P, cuts the straight line FH — which lies in the same plane of the circle AFCH — at the point Q. And the reasoning is the same for the rest. Therefore, when the Sun is on any hour-circle or any other greatest circle, the solar ray — and hence the shadow of the stylus's tip — is cast onto the straight line that is the common section of that same hour-circle (or greatest circle) and the plane of the sundial.

SCHOLIUM.

[Margin: What the hour-lines are.]

Just as those greatest circles which indicate the hours — whether from noon or midnight, or from sunrise or sunset — are called "hour-circles," as we said in Propositions 9 and 10, so too their common sections with the plane of the sundial, onto which we have demonstrated in this proposition that the shadow of the stylus is cast, are called "hour-lines"; because the tip of the shadow falling upon them shows what hour it is — that is, which hour-circle the Sun then occupies.

By the same reasoning, the common sections of the other greatest circles with the plane of the sundial take their names from those greatest circles which produce them on the sundial. Thus the common section of the equinoctial circle and the plane of the sundial is called the equinoctial line; the common section of a vertical circle and the plane of the sundial, the vertical line; the common section of the Meridian and the same plane of the sundial, the meridian line; and so on.

[Margin: All the straight lines on the sundial take their names from the greatest circles that produce them on the sundial. The lines of those greatest circles cannot be drawn on the sundial [when the circle is parallel to its plane].]

But if a greatest circle is parallel to the plane of the sundial, then the line named from that greatest circle vanishes — that is, it cannot be drawn on the sundial. For since that circle does not cut the plane of the sundial but is parallel to it, it will have no common section with the plane of the sundial, nor will the Sun's ray cast onto the plane of the sundial while it is on that circle; rather, it will cast a shadow that is very nearly infinite and parallel to the plane of the sundial. Hence on a horizontal sundial the horizontal line — that is, the 24th hour from sunrise or sunset — cannot be drawn; nor the vertical line on a vertical sundial; nor the line of the sixth hour from noon or midnight on a polar sundial; nor the line of the 12th hour from noon or midnight on a meridian sundial; nor the line of the 12th hour from sunrise or sunset on a vertical sundial at latitude 45° ; nor the equinoctial line on an equinoctial sundial — instead it vanishes altogether. This can easily be understood from the figure above. For when the Sun is at the point B of the circle BD, the ray BI and the shadow ID will be parallel to the plane of the sundial EG, just as the circle itself is parallel to the same plane, and so on.

Diagram. A sphere shown with its principal circles and a stylus at the centre casting a shadow, illustrating the case where a circle is parallel to the plane of the sundial. Dotted lines run from the top point A down to the interior.

- A (top): pole/summit of the figure
- B (left): point on the sphere (start of circle BD)
- C (bottom): lower pole/point
- I (centre): tip of the stylus (world-centre); the shaded triangle at the centre is the stylus and its shadow
- L, O (upper-left, dotted): points along the dotted rays from A
- E, F, K (lower-left quadrant): circle/plane points
- M, N, P, Q (lower-right quadrant, close-set): points of the sundial plane and parallels
- G (lower right): point of the plane EG
- D (implied by text as opposite B on circle BD; label worn [?])

COROLLARIVM PRIMVM.

From this it follows that a plane solar sundial is nothing other than a plane figure containing the common sections (which are indeed all straight lines) of all the hour-circles — whether from noon or midnight, or from sunrise or sunset — with the plane of the sundial itself. For, as we demonstrated in this proposition, when the Sun is, for example, on the circle of the 12th hour, the shadow of the stylus is cast onto the straight line that is the common section of the said circle and the plane of the sundial, and so indicates the twelfth hour, whose circle the Sun then occupies; and likewise for the remaining hours.

[Margin: What a plane solar sundial is. (a) [?] — see note 3.]

COROLLARIVM II.

It is also clear from these things that, when the Sun is on the equinoctial circle — as at the beginning of ♈ [Aries] or ♎ [Libra] — the solar ray and the tip of the stylus's shadow describe, by the diurnal motion, a straight line, namely the common section of the Equator and the plane of the sundial. For at that time the Sun's ray passes through the centre of the world, and so the tip of the shadow does not leave the plane of the Equator (we here ignore that small declination which the Sun acquires by its own motion), but is always cast onto the common section of the Equator and the plane of the sundial — that is, onto the equinoctial line — as has been shown. Hence, by the diurnal motion, it describes that line.

[Margin: The shadow of the stylus on the equinoctial describes the equinoctial line, a straight line, on the sundial.]

THEOREMA 10. PROPOS. 12.

When the Sun is at any point outside the Equator, the solar ray — and hence the shadow of the stylus's tip — is cast onto a curved line, which is the common section of the plane of the sundial and of a conical surface whose base is the parallel opposite the Sun's parallel.

[Margin: The shadow of the stylus and the solar ray, outside the equinoctial, are cast onto a conic section, which is the common section of the plane of the sundial and of a conical surface whose base is the parallel opposite the Sun's parallel.]

Let the Sun, outside the Equator AB, be at the point C, so that by the diurnal motion it describes the parallel DCE, and its opposite point F describes the parallel GFH; and let the ray CF, extended through the centre of the world I, describe the conical surfaces IDE and IGH. Let there also be a plane of the sundial KLMN, parallel to the greatest circle OP, producing in the conical surface of the shadow IGH a common curved section QLN — which will be either a circle, or a parabola, or a hyperbola, or an ellipse, as is clear from Propositions 4, 5, 6, and 7 of this book. I say that the solar ray CF, and therefore the shadow IF, is cast onto the curved line QLN. For the ray CF, since by the diurnal motion it describes both conical surfaces, does not leave either of them while the Sun is at C (for here too we ignore that small declination which the Sun acquires by its own motion); rather, produced, it will cut the opposite parallel at the point F, which is opposite the point C, so that, by Proposition 1 of Book 1 of Apollonius, it lies on both conical surfaces. Since, then, the curved line QLN lies on the conical surface of the shadow IGH, the ray CF will cut the curved line QLN at the point R; and so —

Diagram. A large circle representing the sphere, with the world-centre I and numerous chords drawn to represent the two conical surfaces (the shadow-cone and the ray-cone), their parallel bases, and the curved section on the sundial plane. Small superscript "c" letters mark curve points near the top.

- D (upper left), E / P (upper right): endpoints of the upper parallel and its opposite
- A (left), B (right): endpoints of the Equator diameter
- I (centre): world-centre / stylus tip (shaded triangle)
- M (right): point of the sundial plane
- O, G (lower left): base points of a cone
- K (lower left, below G): point
- L, N (lower centre): points of the curved section QLN

- R (twice, lower centre-right, one dotted): the point R where the ray cuts the curve
- F (lower centre, twice): the shadow-point / opposite point
- H (lower right): base point
- C (upper centre, small): the Sun's position on the upper parallel

— and therefore the ray of the Sun CIF, and the shadow of the stylus's tip I, will be cast onto the curved line QLN, the common section of the conical surface of the shadow IGH and the plane of the sundial KLMN. And we shall show that the same holds wherever in the heavens the Sun is placed outside the Equator. Therefore, when the Sun is at any point outside the Equator, and so on. Which was to be shown.

COROLLARIUM.

[Margin: The shadow of the stylus, outside the equinoctial, describes a conic section on the sundial.]

From these things it is clear that, when the Sun is outside the equinoctial circle — that is, at any point of the Zodiac except the beginning of ♈ [Aries] and ♎ [Libra] — the solar ray and the tip of the stylus's shadow describe, by the diurnal motion, a curved line, namely the common section of the conical surface which the shadow describes and the plane of the sundial. For at that time the Sun's ray, passing through the centre of the world, and hence the tip of the shadow, does not leave the conical surface of the shadow, but is always cast onto the common section of the surface of the shadow and the plane of the sundial — that is, onto a curved line, namely a circle, or a parabola, or a hyperbola, or an ellipse, as has been demonstrated. Hence by the diurnal motion it describes that line.

Nevertheless, such a conic section is named after the parallel on which the Sun tarries. Thus, when the Sun is at the beginning of ♋ [Cancer], the conic section which the shadow traces is called a circle, or a parabola, or a hyperbola, or an ellipse "of ♋ [Cancer]," and so for the rest.

THEOREMA 11. PROPOS. 13.

The hour-circles from noon or midnight cut two conical surfaces whose vertex is the centre of the world and whose bases are two parallels touching the Horizon — one of them the greatest of the always-visible parallels, the other the greatest of the always-hidden parallels — cutting one another by straight lines that

mutually intersect at the centre of the world; and along the same straight lines the hour-circles from sunrise or sunset touch those same conical surfaces.

[Margin: The hour-circles from noon or midnight cut the conical surfaces whose bases are the greatest of the always-visible parallels and the greatest of the always-hidden parallels, along the straight lines in which the hour-circles from sunrise or sunset touch them.]

In the sphere ABCD, whose centre is E and axis AC, let there be two conical surfaces EFG and EHI, whose common vertex is E, the centre of the world, and whose bases are the parallels FG and HI, the greatest of those that always appear and are hidden beneath the earth, the common axis being also the straight line AC. Let there also be any hour-circle from noon or midnight, ANCK, which will pass through the poles of the world A and C, by Proposition 9 of this book, cutting the parallels along the straight lines KL and MN, and the conical surfaces along the lines kN and LM. I say that these lines kN and LM are straight and cut one another at the centre E. For since the greatest circle ANCK passes through the centre E, it will (by Proposition 6 of Book 1 of Theodosius) certainly cut the conical surfaces EFG and EHI through the vertex E, and hence through the axis AC, because the same circle ANCK passes through the poles of the world A and C. Therefore the common sections of the circle and the cones, namely EkL and EMN, will be triangles, by Proposition 3 of Book 1 of Apollonius; and consequently Ek, EL, EM, EN — the common sections of the same circle and the conical surfaces — will be straight lines. I say further that the straight lines EK, EN and EL, EM are set in a straight line. For since the two sides EC, EL of the triangle ECL (the straight lines AM, CL having first been joined) are equal to the two sides EA, EM of the triangle EAM — because all are drawn from the centre of the sphere to its surface — and the bases CL, AM are also equal, by Theorem 2 of the scholium to Proposition 21 of Book 1 of Theodosius (because the circles FG, HI are equal), the angles CEL and AEM will be equal. And so, since AC is a straight line, namely the axis, the straight lines EL and EM will also constitute — by what we showed on Proposition 15 of Book 1 of Euclid from Proclus — one straight line LM. Therefore the line LM is the common section —

[Margin: The hour-circles from noon or midnight cut the conical surfaces (whose bases are the greatest of the always-visible parallels and the greatest of the always-hidden parallels) along the straight lines in which the hour-circles from sunrise or sunset touch them.]

[Margin: a 8. primi.] [i.e. marginal reference: Prop. 8, Book 1.]

— of the conical surfaces and of the circle NACK, and a straight line. And by the same reasoning kN too will be a straight line, as also the common sections of the remaining hour-circles and of the said conical surfaces, cutting one another at the centre E, through which they pass. Which was the first thing to be shown.

But if now the hour-circles from sunrise or sunset, KRNQ and LPMO, touch the parallels FG, HI at the points K, L, M, N, at which the hour-circle from noon or midnight, ANCK, cuts those same parallels — as was demonstrated in Proposition 9 of this book — I say that they touch the conical surfaces along the straight lines KN, LM, along which we demonstrated those same surfaces to be cut by the circle ANCK. For let the straight line ST be the common section of the planes in which the circles FG, LM lie, which (by the definition in Book 2 of Theodosius) will touch each circle. And since the circle LPMO, being a greatest circle, passes through the centre of the sphere E, by Proposition 6 of Book 1 of Theodosius, it is manifest that it passes through the straight line LM, which extends from L to M through the centre E; otherwise, a straight line drawn in the circle LPMO from L to M would, together with LM, enclose a surface between two straight lines, which is absurd. I say now that the circle LPMO touches the conical surfaces along the straight line LM, and in no way cuts them. For if it did cut them, the common sections would be triangles, by Proposition 3 of Book 1 of Apollonius, whose bases would lie on the parallels FG, HI, since the circle LPMO passes through the vertex E of the conical surfaces. Therefore the common section of the planes in which the circles FG and LM lie would cut the circle FG, making the base of a triangle in the circle FG, which is absurd; for it touches it, as has been said, by the definition in Book 2 of Theodosius. Therefore the circle LPMO touches the conical surfaces EFG, EHI —

Diagram. A circle representing the sphere ABCD with centre E, drawn full of chords and diagonals to represent the two conical surfaces (vertex E, bases the parallels FG and HI), the hour-circle ANCK through the poles, and the two sunrise/sunset hour-circles KRNQ and LPMO tangent to the surfaces.

- A (top): pole of the world
- N (just below/left of A): point on the meridian hour-circle
- H (upper left), I (upper right): endpoints of parallel HI
- M (upper centre): point of the parallel/section
- Q (mid-left, italic), P (centre-right), R (below P): interior section points
- E (centre): centre of the world / common vertex of the cones
- O (left of centre, lower): point of circle LPMO
- B (left), D (right): equatorial points of the sphere

- F (lower left), G (lower right): endpoints of parallel FG
- T (lower centre), L (bottom centre), V (bottom left), K (bottom centre), X (bottom right): base and tangent points
- C (bottom): pole of the world; k (lower-case, of hour-circle ANck)

[continues the demonstration of the preceding proposition] ...in the straight line LM, and the same ratio holds in the rest, which is what was proposed in the second place. Therefore the hour-circles from noon or midnight cut the surfaces, etc. Which was to be demonstrated.

THEOREMA 12. PROPOS. 14.

The lines of the hours from noon or midnight cut the common sections of the plane of any sundial whatever and of the conical surfaces whose vertex is the centre of the world and whose bases are the two parallels tangent to the horizon — one of them the greatest of those always visible, the other the greatest of those always hidden. And at the points of section, the lines of the hours from sunrise or sunset touch those same common sections.

[Margin: The lines of the hours from noon or midnight cut the conic sections that the plane of the sundial produces on the cones (whose bases are the parallel that is the greatest of those always visible, and the greatest of those always hidden), at the points where the lines of the hours from sunrise or sunset touch those same sections.]

Since the hour-circles from noon or midnight cut these conical surfaces by straight lines that intersect at the centre of the world, it follows from the preceding proposition that the common sections of those circles and of the plane of the sundial — that is, their hour-lines — cut the common sections of those conical surfaces and of the plane of the sundial, at the points where the straight lines by which the hour-circles cut the conical surfaces meet the plane of the sundial and the conic sections. For such hour-lines — namely the common sections of the hour-circles and of the plane of the sundial — lie in the planes of the hour-circles, and those planes there cut the conical surfaces, and hence also cut the conic sections lying in the plane of the sundial.

From this it follows that, since the plane of the sundial and the planes of the hour-circles intersect at those points, and since both the former and the latter planes run inward toward the cone — and thus inward toward the conic sections — their common sections, namely the hour-lines, also fall from those points inward toward the cone, and hence inward toward the conic sections. For beyond those

points they depart neither from the plane of the sundial nor from the planes of the said circles, but remain forever in both. This therefore establishes what was proposed in the first place.

But since the hour-circles from sunrise or sunset touch these same conical surfaces — by the same preceding proposition — along the lines by which the hour-circles from noon or midnight cut them, it results that the common sections of those circles and of the plane of the sundial — that is, their hour-lines — touch the conic sections that the plane of the sundial makes, at the points where the straight lines by which the hour-circles touch the conical surfaces meet the plane of the sundial and the conic sections: namely, at those points where the lines of the hours from noon or midnight cut the conic sections. For the hour-lines from sunrise or sunset — the common sections of the hour-circles and of the plane of the sundial — lie in the planes of the hour-circles, which there touch the conical surfaces, and hence also touch the conic sections lying in the plane of the sundial.

From this it follows that, since the plane of the sundial and the planes of the hour-circles intersect at those points — the former extending inward toward the cone, and thus inside the conic sections, while the latter planes extend outward beyond the cone, and therefore beyond the conic sections — their common sections, that is, the hour-lines, fall entirely from those points outside the cone, and hence outside the conic sections. For beyond those points they depart neither from the plane of the sundial nor from the planes of the said circles, but remain forever in both, of which the latter lie wholly outside the cone and the conic sections. So this too is clear, which was proposed in the second place. Therefore the lines of the hours from noon or midnight cut [them], etc. Which was to be shown.

SCHOLIUM.

But if the plane of the sundial is parallel to some hour-circle from noon or midnight, this circle will not cut the conic section made by the plane of the sundial, since it does not cut the plane of the sundial itself, as is evident. Hence neither will its hour-line cut that same conic section. And so the hour-lines from sunrise or sunset — whose circles touch the conical surfaces along the lines in which they are cut by that hour-circle from noon or midnight — will not touch the conic sections, as will be demonstrated in the following proposition.

THEOREMA 13. PROPOS. 15.

If the circle of any hour from noon or midnight is parallel to the plane of the sundial, and two hour-circles from sunrise or sunset touch the greatest of the

parallels always visible at the same points at which it is cut by that hour-circle from noon or midnight, then those two lines of the hours from sunrise or sunset will be ἀσύμπτωτοι [Greek: asymptōtoi, "not falling together" — non-meeting, asymptotic]; that is, they never meet the conic sections that the plane of the sundial makes on the conical surfaces (whose bases are the greatest of the parallels always visible and the greatest always hidden), yet, ever produced further, they become closer to those conic sections.

[Margin: The lines of the hours from sunrise or sunset, whose circles touch the greatest of the always-visible parallels at the same points at which it is cut by the circle of some hour from noon or midnight that is parallel to the sundial, will not meet the conic sections that the plane of the sundial makes on the conical surfaces whose bases are the greatest of the parallels always visible and always hidden; yet, ever produced further, they become closer to the conic sections.]

Let some hour-circle from noon or midnight, FG, cut the conical surfaces ABC, ADE — joined at the centre A, whose bases are the parallel DE, the greatest of those always visible, and BC, the greatest parallel always hidden below the horizon — along its axis FG, by the straight lines HI, KL, and making the diameters of the parallels HK, LI (since, by Proposition 15 of Book 1 of Theodosius, it bisects them, and hence cuts through their diameters). And let it be parallel to the plane of the sundial MN, which, by Proposition 6 of this book, will make two opposite and equal hyperbolas, OPQ and RST. And since the planes FG, MN are set parallel, their common sections made by the parallel DE — namely the straight lines LI, ST — will be parallel [Margin (a): Euclid, *Elements* XI.16]. By the same reasoning HK and PQ will be parallel, since they are the common sections of the parallel planes FG, MN made by the parallel BC. Let also some hour-circle from sunrise or sunset touch the parallels BC, DE at the points H, I, and hence touch the conical surfaces along the straight line HI (by Proposition 13 of this book), making in the plane of the sundial the hour-line VX. I say that the straight line VX is ἀσύμπτωτον [Greek: asymptōton — non-meeting]; that is, that it never meets the hyperbolas OPQ, RST, yet always becomes closer to them, however far both the straight line and the hyperbolas are produced. For since the circle HIVX touches the conical surfaces only along the straight line HI, so that all other points of the circle HIVX, lying outside the straight line HI, are outside the conical surfaces, the hour-line VX will lie wholly outside the conical surfaces, because...

[Margin (a): Euclid, *Elements* XI.16]

it cannot meet the straight line HI, since the planes FG, MN are set parallel, and therefore the straight lines HI, VX — their common sections made by the plane

HIVX — are parallel. Therefore the straight line VX will not meet the hyperbolas OPQ, RST, even if the straight line VX and the hyperbolas are in the same plane of the sundial. In the same way VX will lie outside the conical surfaces, even if they are produced to infinity. And you will say the same of the hour-line whose circle touches the conical surfaces along the straight line KL.

Diagram. A large circle representing the greatest circle DBCE of the sphere (with the world-centre A at its centre), densely filled with chords, showing the two opposed conical surfaces (common vertex A) whose curved bases YZ and the two hyperbolas descend below the circle, together with the hour-circle FG (through the poles), the tangent hour-circle HIVX, and the two opposite hyperbolas OPQ/RST with their augmented extensions. The figure is very crowded and cross-hatched; several interior Greek point-labels in the lower-central cluster are faint or overlapping. Readings are best-effort

- F (top): upper pole / top of the axis FG (a short gnomon-line rises above F)
- L (upper left), M (upper right): points of the parallel LI / hour-circle
- S (upper centre, below M): section point (of ST)
- D (left, on the great circle), E (right, on the great circle): endpoints of the greatest visible parallel DE
- I (upper centre): point of the parallel LI
- V (centre), R (below V): point of the hour-line VX / of hyperbola RST
- A (centre): centre of the world, common vertex of the cones
- H (mid-left, interior), X (centre-right), O (right of X): tangent point H, point X of the hour-line, point O of hyperbola OPQ
- B (lower left, on the great circle), C (lower right, on the great circle): endpoints of the greatest hidden parallel BC
- μ (lower centre-left), ξ (lower centre-right): interior points on the parallel/section [?]
- K (lower centre), G (lower left of centre): points K, G [?]
- N (lower centre), Q (lower right of centre): points N, Q of the hyperbola/parallel [?]
- α (lower left, of the augmented figure) [?], ϵ , ζ (lower centre) [?]: augmented-construction points
- Y (bottom left), Z (bottom right): endpoints of the augmented base YZ
- β (bottom centre), θ (very bottom): augmented-hyperbola / base points [?]

I say now that the straight line VX, produced in both directions, always becomes closer to the hyperbolas. For let the conical surface ABC be enlarged, and let YZ

be the base of the enlarged surface; and let the straight lines IH, FG, MN, VX be extended to the points α , β , γ , δ , together with the planes FG, MN, HIVX; and let the hyperbola OPQ be enlarged, so that it becomes $O\epsilon\theta$ [?]. Let the conical surfaces also be cut by the plane in which the greatest circle DBCE lies, through the diameters DE, BC, which are perpendicular to the diameters LI, HK — and hence also to the straight lines ST, PQ parallel to those. Finally, let $H\alpha$ and $\alpha\delta$ be the common sections of the planes BC, YZ with the plane in which the circle HIVX lies; these will touch the circles BC, YZ at the points H, α (by the definition in Book 2 of Theodosius). And the three straight lines VX, QP, $H\alpha$ will cut one another outside the conical surfaces at the point λ ; for any two of them cut one another.

For since HI, VX have been shown to be parallel, and since $H\lambda$ lies in the same plane with them — namely in the plane of the circle HIVX — and $H\lambda$ cuts HI at H, the same $H\lambda$ will also cut the straight line VX, as we demonstrated at Proposition 28 of Book 1 of Euclid. Therefore the straight lines $H\lambda$, VX cut one another. Again, the same $H\lambda$ will cut PQ. For since HK, PQ are parallel, as was shown above, and $H\lambda$ cuts HK, it will also cut PQ, from what we showed at Proposition 28 of Book 1 of Euclid. That PQ and VX also cut one another is made plain thus: if they did not intersect — being in the same plane of the sundial — they would be parallel. Now since both HI and PQ are parallel to VX [Margin (b): Euclid XI.9], HI and PQ will be parallel to each other. But HK is also parallel to PQ, as we demonstrated above [Margin (c): Euclid XI.9]. Therefore HI and HK would be parallel to each other — which is absurd, since they meet at H. Therefore PQ and VX cut one another. Hence VX, cutting both $H\lambda$ and PQ, will pass through the point λ , where $H\lambda$ and QP cut one another. For if, for example, VX were to cut the straight line $H\lambda$ short of the point λ , or beyond it (if it were produced further), it would at once cut the plane of the circle BC at the same point at which it cuts $H\lambda$, and so could no longer cut the straight line QP, which lies in the same plane of the circle BC. By parallel reasoning, if VX were to cut PQ short of or beyond the point λ , it would likewise, when produced, at once cut the plane of the circle BC at the same point at which it cuts PQ, and therefore could not cut the straight line $H\lambda$, which always lies in the same plane of the circle BC. Therefore, unless VX passes through the point λ , it will not cut both $H\lambda$ and QP — which is absurd, since it was shown to cut both. In the same way, at δ the three straight lines $\alpha\delta$, $\theta\epsilon$ [?], VX will cut one another.

Now since the straight lines $\mu\beta$, $\xi\gamma$ are parallel — being the common sections of the parallel planes FG, MN made by the plane of the circle DBCE — and likewise the straight lines $\mu\xi$, $\beta\gamma$ — namely the common sections of the parallel planes BC, YZ made by the same plane of the circle DBCE — $\mu\xi$ and $\beta\gamma$ will be equal [Margin

(d): Euclid XI.16; Margin (e): Euclid I.34]. And since $H\lambda$, $\mu\xi$, and also $\alpha\delta$, $\beta\gamma$, are parallel as well [Margin (f): Euclid I.28] — because the angles $\mu H\lambda$, $\beta\alpha\delta$ at the points of contact H , α are right (for $H\mu$, $\alpha\beta$ pass through the centres of the circles BC , YZ , being their common sections with the plane FG , which cuts them through their poles and hence through their diameters, by Proposition 15 of Book 1 of Theodosius), and likewise the angles $H\mu\xi$, $\alpha\beta\gamma$ at the centres μ , β — for since, by construction, the straight line BC cuts the straight line HK at right angles, the angle $H\mu\xi$ will be right [Margin (g): Euclid III.18]. Therefore, since $\alpha\beta$, $\beta\gamma$ are parallel to $H\mu$, $\mu\xi$ [Margin (h): Euclid XI.16], the angle $\alpha\beta\gamma$ will be equal to $H\mu\xi$, and thus right [Margin (i): Euclid XI.10]. Moreover, the straight lines $H\mu$, $\lambda\xi$, and also $\alpha\beta$, $\delta\gamma$, are parallel — namely the common sections of the parallel planes FG , MN , made by the plane of the circle BC if we speak of the former, and by the plane of the circle YZ if we speak of the latter [Margin (k): Euclid XI.16]. Then, if the straight line $\delta\varepsilon$ is smaller than the straight line λP , as we shall presently demonstrate in the following lemma—

—lemma. Therefore the straight line VX , when produced, is closer to the hyperbola $O\epsilon\theta$ [?] at the point δ than at the point P ; and so on successively, if the hour-line and the hyperbola are produced further. And we shall show the same in the hyperbola RST , and likewise concerning the hour-line that the circle touching the conical surfaces along the straight line KL makes. Therefore, if the circle of any hour from noon or midnight is parallel to the plane of the sundial, etc. Which was to be demonstrated.

LEMMA.

If two unequal circles are touched by two straight lines parallel to their diameters, and the points of contact and the centres are joined by two straight lines, and through two points on the radii equally distant from the centres parallels are drawn cutting the peripheries of the circles, then the straight lines intercepted between the tangent lines and the peripheries will be unequal — and the smaller will be the one that lies outside the greater circle.

Let the circle αYZ , greater than the circle HBC , be touched — each of them — by the straight lines $\alpha\delta$, $H\lambda$, parallel to the diameters YZ , BC ; and let the points of contact α , H , and the centres β , μ , be joined by the straight lines $\alpha\beta$, $H\mu$. Taking straight lines $\beta\gamma$, $\mu\xi$ equal, cut off from the radii βZ , μC , let there be drawn through γ , ξ the parallels $\gamma\delta$, $\xi\lambda$ to the straight lines $\alpha\beta$, $H\mu$, cutting the peripheries at ε , P . I say that the straight line $\delta\varepsilon$ is smaller than λP .

For since the circle αYZ is greater than the circle HBC , the radius $\alpha\beta$ will also be greater than the radius $H\mu$. Therefore let the straight line αA be cut off equal to $H\mu$, and from A , at the distance $A\alpha$, let the circle αE be described; it will be equal to the circle HBC , on account of the equality of the radii αA , $H\mu$, and it will touch the circle αYZ at α . And since AD , drawn from A perpendicular to $\gamma\alpha$, is parallel to $\beta\gamma$ [Margin (a): Euclid I.28], $A\gamma$ will be a parallelogram [Margin (b): Euclid I.34]; and therefore the straight line AD will be equal to the straight line $\beta\gamma$ — that is, to $\mu\xi$. Since then $\mu\xi$ is smaller than the radius μC (or $H\mu$) — that is, smaller than αA , which is equal to $H\mu$ — AD too will be smaller than αA , and therefore the point D will lie inside the circle αE . Hence the circumference αE will cut the straight line $D\alpha$ below the point ε , namely at E .

Now since, when the straight lines AE , μP are drawn, the square on AE equals the squares on AD , DE , and the square on μP equals the squares on $\mu\xi$, ξP [Margin (c): Euclid I.47], and the squares on AE , μP are equal to each other, the squares on AD , DE will equal the squares on $\mu\xi$, ξP . Therefore, when the equal squares on the straight lines AD , $\mu\xi$ are taken away, the remaining squares on DE , ξP will be equal, and consequently the straight lines themselves will be equal. Since then the wholes $D\delta$, $\xi\lambda$ are also equal — because $D\alpha$ is equal to $A\alpha$, and $\xi\lambda$ to μH [Margin (d): Euclid I.34] — the remainders αE , λP [?] will also be equal. But $\delta\varepsilon$ is smaller than αE [?]. Therefore $\delta\varepsilon$ will also be smaller than λP . Which was to be demonstrated.

From these things it is clear that, in the figure above, the straight line $\delta\varepsilon$ is smaller than the straight line λP , as was assumed in the demonstration — because the circle YZ is greater than the circle BC , etc., as the lemma proposes.

Diagram. Two side-by-side figures illustrating the lemma. On the left, the larger circle αYZ (horizontal diameter YZ) with a smaller inscribed/tangent circle αE inside it; the interior shows the rectangle/parallelogram A – D built on the centre-line, with the tangent $\alpha\delta$ across the top. On the right, the smaller circle HBC (horizontal diameter $B\dots C$, extended to E), with the tangent $H\lambda$ across the top and the point P on the right where the parallel $\xi\lambda$ cuts the periphery.

- Left figure — α (top, point of contact/tangent), δ (top, on tangent $\alpha\delta$) [?]
- A (centre-left interior), D (interior right of A): vertices of the parallelogram $A\gamma$ / point D on $D\alpha$
- β (bottom centre, on diameter): centre of circle αYZ
- Y (left end of diameter), Z (right end of diameter): endpoints of diameter YZ

- Right figure — H (top left, point of contact), λ (top, on tangent $H\lambda$): tangent point and line
- P (right, on periphery): where the parallel $\xi\lambda$ cuts the periphery
- μ (bottom centre, on diameter): centre of circle HBC
- ξ (lower centre, on the radius): point ξ on μC
- E (right, on the extended diameter $B...C \rightarrow E$), C (right end): endpoints
- B (bottom, on diameter) [?]

[Margin (a): Euclid I.28] [Margin (b): Euclid I.34] [Margin (c): Euclid I.47] [Margin (d): Euclid I.34]

COROLLARIVM.

It follows from this proposition that the lines of the 12th and 24th hours from sunrise or sunset in a meridian sundial do not meet the hyperbolas that the plane of the sundial makes (by Proposition 6 of this book) on the conical surfaces whose bases are the greatest of the always-visible parallels and the greatest of those hidden under the earth. For the meridian — that is, the circle of the 12th hour from noon or midnight — is parallel to the plane of the sundial, and cuts the greatest of the always-visible parallels at the points where both the circle of the 12th hour from sunrise or sunset and the circle of the 24th hour (or the horizon) touch it, as is clear from the figure of Proposition 9 of this book.

[Margin: The lines of the 12th and 24th hours from sunrise or sunset do not meet, in a meridian sundial, the conic sections made on the conical surfaces whose bases are the greatest of the parallels always visible and always hidden. The lines of the 6th and 18th hours from sunrise or sunset do not meet, in a polar sundial, the conic sections made on the conical surfaces whose bases are the greatest of the parallels always visible and always hidden.]

In the same way, the lines of the 6th and 18th hours from sunrise or sunset will not meet the same hyperbolas in a polar sundial. For the polar sundial is parallel to the circle of the 6th hour from noon or midnight, which cuts the said parallel at the points where the circles of the 6th and 18th hours from sunrise or sunset touch it, as is clear from the same figure of Proposition 9 of this book.

THEOREMA 14. PROPOS. 16.

If in a sphere two greatest circles touch one and the same parallel at opposite points, then those circles and the greatest of the parallels will have one and the same common section.

[Margin: The greatest of the parallels and two greatest circles touching any parallel at two opposite points have one and the same common section.]

In the sphere ABCD, let two greatest circles AC, BD touch the parallel BC at the opposite points B, C, and let their common section be the straight line EF. I say that the greatest of the parallels, GH, cuts each of them along the straight line EF — that is, passes through the points E, F — so that the straight line EF is the common section of the three greatest circles AC, BD, GH. For through the pole I of the parallels BC, GH, and through the point of contact B, let a greatest circle ABCD be described; since, by Proposition 15 of Book 1 of Theodosius, it bisects the parallel BC, it will also pass through the opposite point of contact C. Therefore, since the greatest circle ABCD, described through the pole of the parallel BC and through the points of contact B, C, also passes through the poles of the circles AC, BD (by Proposition 5 of Book 2 of Theodosius), it will necessarily bisect, by Proposition 9 of Book 2 of the same, their segments AEF, BEF, CEF, DEF, at points...

the points A, B, C, D. Now since these are segments of semicircles — for since AC and BD are great circles, they bisect each other at the points E and F (by Proposition 11 of Book 1 of Theodosius) — the segments AE, AF, BE, BF, CE, CF, DE, DF will be quadrants, being the halves of semicircles. Again, since the great circle ABCD passes through the poles of the circles GH and BD, it cuts the segments of the circles GH and BD — which are semicircles, by Proposition 11 of Book 1 of Theodosius — in half, by Proposition 9 of Book 2 of Theodosius, at the points G and B. Therefore the arcs of the circle BD lying between the point B and the circle GH will be quadrants. And since BE and BF have been shown to be quadrants, the circle GH will necessarily pass through the points E and F, and so will cut both circles AC and BD along the straight line EF. Therefore the straight line EF is the common section of the three great circles AC, BD, GH. And consequently, if in a sphere two great circles touch one and the same, and so on. This is what had to be demonstrated.

Diagram. A great circle with an eight-pointed star of chords inscribed, together with two vesica-shaped arcs (the parallels GH) meeting at the central points E and F.

- I (top, above the circle): the pole of the parallels
- A (upper left, on the circle)
- D (upper right, on the circle)
- G (left, on the circle): endpoint of the great parallel

- H (right, on the circle): endpoint of the great parallel
- E (upper centre): common intersection point
- F (lower centre): common intersection point
- B (lower left, on the circle)
- C (lower right, on the circle)

COROLLARIUM.

[Margin: The four arcs of the Ecliptic between the points of the solstices and the equinoxes; likewise the arcs of the Horizon between the Equator and the Meridian; and finally the arcs of all the hour-circles, from sunrise or sunset, between the Equator and the points where they touch the greatest of the always-visible parallels and the greatest of the always-hidden — all these intercepted arcs are quadrants.]

Since it has been shown that the arcs BE and BF, lying between the point of contact B and the greatest of the parallels GH, are quadrants, it follows that the arcs of any great circle touching some parallel, lying between the point of contact and the greatest of the parallels, are quadrants. For the demonstration is the same in every case: the great circles drawn through the poles of the parallels and through the point of contact always pass (by Proposition 5 of Book 2 of Theodosius) through the poles of the tangent circles as well; and so they bisect (by Proposition 9 of Book 2 of Theodosius) the individual segments lying between the points of contact and the greatest of the parallels — segments which are semicircles by Proposition 11 of Book 1 of Theodosius — that is, they divide them into quadrants. Of this kind are the four arcs of the Zodiac intercepted between the Equator and the solstitial points, where the Zodiac touches the tropics, which are parallel to the Equator. Likewise the four arcs of the Horizon between the Equator and the points where the Horizon touches the greatest of the always-visible parallels and the greatest of the always-hidden, and where it is cut by the Meridian. And finally all the arcs of the circles that mark the hours from sunrise or sunset, intercepted between the Equator and the points where they touch the greatest of the always-visible parallels and the greatest of the always-hidden. For all these arcs are quadrants, as has been demonstrated.

THEOREMA 15. PROPOS. 17.

If in a sphere two great circles touch one and the same parallel, and another great circle cuts that same parallel through the poles of the parallels and lies equally

distant from the two points of contact, then the tangent circles and the cutting circle will have one and the same common section.

[Margin: Three great circles — of which one cuts any parallel through its poles, while the others touch that same parallel at points equally distant on either side from the two points where it is cut by the first great circle — have one and the same common section.]

In the sphere ABCD, let two great circles AC and BD first touch the parallel BC at the opposite points B, C, so that BIC and BkC are semicircles, and let their common section be the straight line EF. Let another great circle GH cut the same parallel BC, passing through the poles G, H of the parallel, at the points I, K, equally distant from the points B, C, so that the arcs IB, IC and KB, kC are quadrants. I say that the circle GH cuts each of the great circles AC and BD along the straight line EF — that is, passes through the points E, F — so that the straight line EF is the common section of the three great circles AC, BD, GH. For let the greatest of the parallels, LM, be described; it will pass through the points E, F, since by the preceding proposition EF is the common section of the three circles AC, BD, LM — because AC and BD touch the parallel BC at opposite points, and LM is the greatest of the parallels. Now if a great circle GLHM is described through the poles of the parallels and through the point of contact B, this same circle will pass through the poles of the circle BD as well, by Proposition 5 of Book 2 of Theodosius. Therefore it will bisect the segments BEF and LEF, by Proposition 9 of the same. Since, then, LEF is a semicircle — because great circles bisect one another, by Proposition 11 of Book 1 of Theodosius — the arcs LE, LF will be quadrants. And since the great circles GLHM and GEHF are described through the poles G, H of the parallels BC, LM, the arcs intercepted between them will be similar, by Proposition 10 of Book 2 of Theodosius. But the arcs BI, Bk of the parallel BC intercepted between them are, by hypothesis, quadrants. Therefore the arcs of the parallel LM intercepted between the same circles will also be quadrants. Consequently, since LE, LF have been shown to be quadrants, the circle GH will pass through the points E, F, and so will cut both circles AC and BD along the straight line EF. Therefore the straight line EF is the common section of the three circles AC, BD, GH. This is what had to be shown.

Diagram. A great circle inscribed with a star of chords and two vesica-shaped parallel-arcs, for the first case of Theorem 15.

- G (top, on the circle): pole of the parallels
- A (upper left, on the circle)
- D (upper right, on the circle)

- E (upper centre): common intersection point
- L (left, on the circle): endpoint of the great parallel LM
- M (right, on the circle): endpoint of the great parallel LM
- F (lower centre): common intersection point
- B (lower left, on the circle)
- I (lower centre, left): point where GH cuts the parallel
- K (lower centre, right): point where GH cuts the parallel
- C (lower right, on the circle)
- H (bottom, on the circle): pole of the parallels

Next, in the same sphere ABCD, let two great circles EG and FH touch the same parallel BC at points E, F that are *not* opposite, and let their common section be the straight line IK, which will be their diameter, since they bisect one another by Proposition 11 of Book 1 of Theodosius. Let another great circle LM cut the same parallel BC, passing through the poles L, M of the parallel and through the axis LM, at the points O, P equally distant from the points E, F, so that the arcs OE, OF and PBE, PCF are equal. I say that the circle LM cuts each of the great circles EG and FH along the straight line IK — that is, passes through the points I, K — so that the straight line IK is the common section of the three great circles EG, FH, LM.

For since the circle LM cuts the parallel BC through its poles, it will bisect it, by Proposition 15 of Book 1 of Theodosius. Their common section OP will therefore be a diameter of the parallel BC, passing through the centre Q, into which the axis LM falls, by Proposition 10 of Book 1 of Theodosius. Let R also be the centre of the sphere, through which the axis LM and the diameter IK of the great circles both pass. And since circles in a sphere are said to touch one another when the common section of the planes in which the circles lie touches each circle (by the definition in Book 2 of Theodosius), let the common sections of the circles EG, BC and of FH, BC be the straight lines ES, FS, touching those circles. If, then, the semidiameters QE, QF are drawn from the centre Q, the angles SEQ, SFQ will be right angles [Margin: a — Euclid III.18], and therefore, once the straight line EF is drawn, the angles SEF, SFE will be less than right angles. Hence the straight lines ES, FS, lying in the same plane of the parallel BC which they touch, will meet at some point, namely at S, by the 11th axiom of Book 1 of Euclid. And since KI, the common section of the circles EG, FH, meets each of ES, FS as well (as we shall show in a moment in the following lemma), it comes about that KI, when produced, meets each of them at S. For if it were to cut one of them below or above S, it would not meet the other, as is plain. Suppose, for example, that KI

met the straight line ES elsewhere than at the point S: then, when produced, it would at once cut the plane of the circle BC at the point where it cuts ES, and so could in no way cut the straight line FS lying in the same plane of the circle BC. And if KI met FS elsewhere than at S, we should show in the same way that it cannot cut ES. Therefore the straight line KI, unless it passes through the point S, will not cut both ES and FS — which is absurd, for it does cut both, as we shall show in the following lemma. Now let the straight lines EO, FO, OS be drawn in the plane of the circle BC. Since the arcs EO, FO are assumed equal, the straight lines EO, FO will also be equal [Margin: b — Euclid III.28]; and the tangents SE, SF are equal too, by the 2nd corollary to Proposition 36 of Book 3 of Euclid. Therefore the two sides EO, OS of the triangle EOS will be equal to the two sides FO, OS of the triangle FOS, and the base ES to the base FS; and consequently the angles EOS, FOS will be equal [Margin: c — Euclid I.8]. In just the same way we shall show that the angles EOQ, FOQ are equal, because the sides EO, OQ of the triangle EOQ are equal to the sides FO, OQ of the triangle FOQ, and the base EQ to the base FQ. Therefore the two angles EOS, EOQ are equal to the two angles FOS, FOQ, and consequently both pairs make up half of the four angles at the point O lying in the plane of the circle BC. But those four angles are equal to four right angles, by the 2nd corollary to Proposition 15 of Book 1 of Euclid. Therefore EOS, EOQ, as well as FOS, FOQ, will be equal to two right angles; and hence the straight lines SO, OQ lie in a straight line [Margin: d — Euclid I.14 and XI.2]. There is therefore a triangle QRS, and so it lies in one plane [Margin: e — Euclid XI.2], namely the one drawn through SQ, QR. But the plane of the circle LM is drawn through these straight lines SQ, QR, as is plain. Therefore the straight line RS will lie in this same plane of the circle LM; and consequently the circle LM will pass along the straight line IK, so that along it the circle cuts the two tangent circles EG, FH. Therefore IK is the common section of the three circles EG, FH, LM. And so, if in a sphere two great circles, and so on. This is what had to be demonstrated.

Diagram. A large, densely inscribed great circle for the second case of Theorem 15, showing the parallel BC, the two tangent great circles EG, FH, the cutting circle LM, and the network of chords and tangents ES, FS meeting at S (drawn at the right edge as a small pencil of tangent rays). The figure is very crowded; several interior point-labels are hard to resolve at this resolution and are flagged.

- L (top, on the circle): pole of the parallel
- A (upper left, on the circle)
- H (just below/inside A): pole
- D (upper right, on the circle)

- G (interior, upper right)
- K (interior, left)
- T (upper centre)
- I (interior, right of centre)
- S (right edge, with a pencil of tangent rays): meeting point of the tangents
- R (centre): centre of the sphere
- T (lower centre) [?] (a second "T" reading — uncertain)
- E (lower left interior)
- B (bottom left, on the circle)
- P (bottom centre)
- Q (bottom centre, right of P): centre of the parallel BC
- C (bottom right, on the circle)
- F (lower right interior)
- M (bottom, on the circle): pole of the parallel

LEMMA.

That KI meets both ES and FS we shall demonstrate as follows. For if KI and ES do not meet, they will certainly be parallel, by Definition 34 of Book 1 of Euclid, since they lie in the same plane of the circle EG. For ES, being a tangent to the circle EG, lies in the same plane of that circle in which KI also lies. Hence, if RT is drawn parallel to SF, the plane drawn through KR, RT will be parallel to the plane drawn through SE, SF [Margin: f — Euclid XI.15]; and so the straight lines KI, FS, lying in those parallel planes, will never meet, since the planes themselves never meet either. Therefore KI and FS, not meeting and lying in the same plane of the circle FH (for FS, being a tangent to the circle FH, lies in the same plane of that circle in which KI too of course lies), are parallel, by Definition 34 of Book 1 of Euclid. And consequently ES and FS will be parallel to each other [Margin: g — Euclid XI.9], since each is parallel to IK — which is absurd. For it has been shown that the straight lines ES, FS meet at the point S. Therefore the straight lines KI, ES will meet, since they are not parallel and lie in the same plane, as has been demonstrated. And by the same reasoning we shall show that KI and FS meet.

Diagram. A second large great circle, closely repeating the previous figure, drawn for the lemma; it adds the auxiliary line RT parallel to SF. As with the preceding figure, several interior labels are hard to resolve.

- L (top, on the circle): pole of the parallel
- A (upper left, on the circle)

- H (below/inside A): pole
- D (upper right, on the circle)
- G (interior, upper right)
- K (interior, left)
- T (upper centre)
- R (centre)
- I (interior, right of centre)
- S (right edge, with tangent rays)
- E (lower left interior)
- T (lower centre) [?] (second "T" — uncertain)
- B (bottom left, on the circle)
- P (bottom centre)
- Q (bottom centre, right of P)
- C (bottom right, on the circle)
- F (lower right interior)
- M (bottom, on the circle)

SCHOLIUM.

What we have shown in this proposition and the preceding one we shall demonstrate in another and perhaps easier way, in the scholium to Proposition 20 of this book.

THEOREMA 16. PROPOS. 18.

If any number of planes having one and the same common section are cut by some other plane that is parallel either to one of them or to their common section, then the common sections of all those planes with the cutting plane will be parallel lines. But if the same planes are cut by a plane that is not parallel to their common section, then the common sections of all of them with the cutting plane will meet at that point of the common section where the cutting plane intersects it.

[Margin: A plane cutting any number of planes that have one and the same common section, to which common section the cutting plane is parallel, produces parallel common sections; but if it cuts that common section, it produces common sections that meet at the point where the cutting plane divides their common section. a — Euclid XI.16. b — Euclid XI.9. c — Euclid XI.3.]

Let any number of planes AB, CD, EF, RS, having the same common section GH, be cut by a plane IK which is parallel — first, as in the earlier figure — to the plane RS, or to the common section GH. (I call a plane parallel to some straight line one which, when produced to infinity, never meets that straight line, itself also produced to infinity; or one through which, by way of that straight line, some parallel plane can be drawn.) Let the common sections of the planes AB, CD, EF with the cutting plane IK be the straight lines LM, NO, PQ. I say that these common sections are parallel. For if the plane RS is drawn through the straight line GH — in case the plane IK is supposed parallel only to the straight line GH and to no plane — parallel to the plane IK, then the plane AB will be cut by the parallel planes IK, RS. Therefore the common sections LM, GH are parallel [Margin: a — Euclid XI.16]. By the same reasoning NO, GH and PQ, GH will be parallel. Therefore the straight lines LM, NO, PQ, being parallel to GH, will be parallel to one another as well [Margin: b — Euclid XI.9]. This is what had to be shown.

Diagram. The "earlier figure": several plane figures (drawn as tilted quadrilaterals fanning out) sharing the common section GH and cut by the plane IK, producing the parallel sections LM, NO, PQ. The plates overlap densely; the readable corner-labels are listed.

- A, C (upper left plate — plane AB / CD)
- G, H (centre): the common section GH
- R (upper centre): plane RS
- S, I (lower left)
- K, P (upper right)
- N, O (centre right)
- L, M (centre): a common section
- F, D (right plates — planes EF, CD)
- B (lower right): plane AB
- Q (upper right): common section endpoint

Now let the cutting plane IK *not* be parallel to the common section GH, but meet it, when produced toward the parts of H, at the point R, as in the later figure. I say that LM, NO, PQ — the common sections of the planes AB, CD, EF with the cutting plane IK — meet at the same point R, where the cutting plane IK intersects the common section GH produced. For the plane AB, extended along the straight line GH produced toward the parts of H, will cut the plane IK at the point R, since it is always produced along the straight line GR. Since, then, the same plane AB cuts

the same plane IK at the points L, M, and since the common section of the planes is a straight line [Margin: c — Euclid XI.3], the three points L, M, R will be set in a straight line — namely in the common section of the planes AB, IK. Therefore the straight line LM (that is, the common section of the planes AB, IK), produced toward the parts of M, will meet GH produced at R. By the same argument we shall show that the straight lines NO, PQ meet GH at the same point R. Therefore the common sections LM, NO, PQ, when produced, meet the section GH produced at the point R, where the cutting plane IK meets that same section GH. This is what was proposed. And so, if any number of planes having one and the same common section, and so on. This is what had to be demonstrated.

Diagram. The "later figure": the same fan of plane figures, but now the cutting plane is oblique, so the common sections LM, NO, PQ (shown by dotted lines) converge at the single point R.

- I, P, F, B (upper plates and right edge)
- A, G, N, L, D (upper row)
- C, H, O, M, K (middle row)
- R (bottom, where the dotted sections converge)
- E (lower left)

THEOREMA 17. PROPOS. 19.

At whatever point any hour-line drawn from noon or from midnight cuts the Equinoctial line of any sundial, at that same point two hour-lines drawn from sunrise or sunset — whose circles are distant from the circle of the hour-line of noon or midnight by a whole quadrant, that is, by six hours, on the greatest of the always-visible parallels — will intersect that same Equinoctial line.

[Margin: The line of any hour from noon or midnight, and the two lines of the hours from sunrise or sunset distant from it by a whole quadrant on the greatest of the always-visible parallels, intersect one another at one and the same point of the Equinoctial line, in any sundial whatever.]

Let ABCD be the greatest of the always-visible parallels, which any hour-circle from noon or midnight, AC, cuts at the points A, C along the straight line AC — this line being a diameter of the circle ABCD, because the cutting hour-circle, by Proposition 9 of this book, cuts the circle ABCD through the poles and therefore bisects it, by Proposition 15 of Book 1 of Theodosius. Let two quadrants AB, AD be taken from A, so that each of the arcs AB, AD, CB, CD comprises six whole hours; and at B, D let two hour-circles from sunrise or sunset, BE, DF, touch the parallel

ABCD, by Proposition 10 of this book. I say that, at whatever point the hour-line of the circle AC cuts the Equinoctial line in any sundial, at that same point the two hour-lines of the circles BE, DF cut the same Equinoctial line. For since the hour-circles BE, DF, touching the parallel ABCD at B, D, and the hour-circle AC, cutting that same parallel along the straight line AC, have one and the same common section, by Proposition 17 of this book; and since those same tangent circles and the Equator have one and the same common section too, by Proposition 16 of this book: these four circles — the Equator, the two tan-

Diagram. A circle ABCD (the greatest of the always-visible parallels) with the vertical diameter AC and a slightly curved meridian; two hour-circle arcs BE and DF stand outside it, tangent at B and D.

- E (upper left, outside the circle): end of the hour-circle arc
- A (top, on the circle)
- F (upper right, outside the circle): end of the hour-circle arc
- B (left, on the circle): point of contact
- D (right, on the circle): point of contact
- C (bottom, on the circle)

LIBER PRIMVS [Book One] — continued

THEOREMA 17. PROPOS. 19. — conclusion of the demonstration

the two tangent circles BE and DF, and the secant AC, have one and the same common section. Therefore, if the plane of the sundial is not parallel to this common section but cuts it at some point, then the common sections of the said circles and of the plane of the sundial — that is, the equinoctial line, the two hour-lines from sunrise or sunset of the circles BE and DF, and the hour-line from noon or midnight of the circle AC — will all cut one another at that same point, by the preceding proposition. And so, at whatever point the hour-line from noon or midnight of the circle AC cuts the equinoctial line in any sundial, at that same point the common section of the said circles meets the plane of the sundial; and consequently at that same point the two hour-lines from sunrise or sunset of the circles BE and DF also cut that equinoctial line. The reasoning is the same for the rest. Therefore, at whatever point any hour-line from noon, and so on. Which was to be shown.

SCHOLIVM [Scholium].

[Margin: Construction of a table containing the hours from noon or midnight and from sunrise or sunset that intersect one another at one and the same point of the equinoctial line of any sundial.]

From this proposition we have constructed the following table, whose title is **AEQVINOCTIALIS LINEA [The Equinoctial Line]**. For it shows which hour-line from sunrise or sunset, and which from noon or midnight, cut the equinoctial line of any sundial at one and the same point — as can easily be gathered from this proposition. Thus, if in the following table any hour, or part of an hour, from noon or midnight be taken, together with the two hours (or parts of hours) from sunrise or sunset set beside it, the lines of those three hours — or even of the parts of hours — will cut the equinoctial line at one and the same point.

For example, the line of the seventh hour from noon or midnight, the line of hour 1, and the line of hour 13 from sunrise or sunset cut the equinoctial line at one and the same point. In the same way the line of hour $7\frac{1}{4}$ from noon or midnight, the line of hour $1\frac{1}{4}$, and the line of hour $13\frac{1}{4}$ from sunrise or sunset cut the equinoctial line at one and the same point; and so on for the rest. For if this table, and the figure of Proposition 9 of this book, be rightly considered, any two such hours from sunrise or sunset will be found to be distant from the intermediate hour from noon or midnight by a whole quadrant, that is, by six hours. This is why the lines of these hours cut the equinoctial line at the same point at which the line of the hour from noon or midnight cuts that same equinoctial line, as we demonstrated in this proposition.

Therefore, if a point on the equinoctial line of any sundial has been noted through which one of three hours passes, we shall know that the remaining two hours must pass through that same point. By the same reasoning, if a point has been ascertained at which any two of the three hours cut one another, we shall also know that the equinoctial line must be drawn through that same point, and so on.

[Margin: Construction of a table in which are contained the hours from noon or midnight and from sunrise or sunset intersecting one another at one and the same point — the equinoctial line of any sundial.]

AEQVINOCTIALIS LINEA [The Equinoctial Line]

Hours from sunrise/sunset	Hours from noon/midnight	Hours from sunrise/sunset
$\frac{1}{4}$	$6\frac{1}{4}$	$12\frac{1}{4}$
$\frac{1}{2}$	$6\frac{1}{2}$	$12\frac{1}{2}$
$\frac{3}{4}$	$6\frac{3}{4}$	$12\frac{3}{4}$
I	VII	13
$1\frac{1}{4}$	$7\frac{1}{4}$	$13\frac{1}{4}$
$1\frac{1}{2}$	$7\frac{1}{2}$	$13\frac{1}{2}$
$1\frac{3}{4}$	$7\frac{3}{4}$	$13\frac{3}{4}$
2	VIII	14
$2\frac{1}{4}$	$8\frac{1}{4}$	$14\frac{1}{4}$
$2\frac{1}{2}$	$8\frac{1}{2}$	$14\frac{1}{2}$
$2\frac{3}{4}$	$8\frac{3}{4}$	$14\frac{3}{4}$
3	IX	15
$3\frac{1}{4}$	$9\frac{1}{4}$	$15\frac{1}{4}$
$3\frac{1}{2}$	$9\frac{1}{2}$	$15\frac{1}{2}$
$3\frac{3}{4}$	$9\frac{3}{4}$	$15\frac{3}{4}$
4	X	16
$4\frac{1}{4}$	$10\frac{1}{4}$	$16\frac{1}{4}$
$4\frac{1}{2}$	$10\frac{1}{2}$	$16\frac{1}{2}$
$4\frac{3}{4}$	$10\frac{3}{4}$	$16\frac{3}{4}$
5	XI	17
$5\frac{1}{4}$	$11\frac{1}{4}$	$17\frac{1}{4}$
$5\frac{1}{2}$	$11\frac{1}{2}$	$17\frac{1}{2}$
$5\frac{3}{4}$	$11\frac{3}{4}$	$17\frac{3}{4}$
6	XII	18
$6\frac{1}{4}$	$\frac{1}{4}$	$18\frac{1}{4}$
$6\frac{1}{2}$	$\frac{1}{2}$	$18\frac{1}{2}$
$6\frac{3}{4}$	$\frac{3}{4}$	$18\frac{3}{4}$
7	I	19
$7\frac{1}{4}$	$1\frac{1}{4}$	$19\frac{1}{4}$

Hours from sunrise/sunset	Hours from noon/midnight	Hours from sunrise/sunset
7½	1½	19½
7¾	1¾	19¾
8	II	20
8¼	2¼	20¼
8½	2½	20½
8¾	2¾	20¾
9	III	21
9¼	3¼	21¼
9½	3½	21½
9¾	3¾	21¾
10	III	22
10¼	4¼	22¼
10½	4½	22½
10¾	4¾	22¾
11	V	23
11¼	5¼	23¼
11½	5½	23½
11¾	5¾	23¾
12	VI	24

Below the table: In an equinoctial sundial the lines of any three hours of this table are parallel (as is clear from the scholium of Proposition 22 of this book). Likewise in certain other sundials the lines of these hours — namely 12 and 24 from sunrise or sunset, and 6 from noon or midnight — are parallel, as is taught in the same scholium.

PROBLEMA 18. PROPOS. 20.

[Margin: The line of any hour from noon or midnight, and the two lines of the hours from sunrise or sunset equally distant on either side of it on the greatest of the always-visible parallels, intersect one another at one and the same point of any sundial.]

If any two hour-lines from sunrise or sunset are taken whose circles are equally distant from the circle of some hour from noon or midnight on the greatest of the always-visible parallels, then those two hour-lines from sunrise or sunset will intersect one another — within the hour-line of that circle from noon or midnight — at one and the same point; and likewise one of them (from sunrise or sunset) together with that line from noon or midnight will intersect the remaining line from sunrise or sunset, each at one and the same point.

Let ABCD be the greatest of the always-visible parallels, which any hour-circle from noon or midnight, AC, cuts at the points A, C along the straight line AC — this line being a diameter of the circle ABCD, as was shown in the demonstration of the preceding proposition. Let two points B, D be taken, equally distant from either of the points A, C, so that the arc AB comprises as many hours as the arc AD; and likewise the arc CB as many as the arc CD. At B and D let two hour-circles from sunrise or sunset, BE and DF, touch the proposed parallel ABCD, by Proposition 10 of this book. I say that the hour-lines from sunrise or sunset of the circles BE and DF intersect one another, at one and the same point, within the hour-line from noon or midnight of the circle AC.

For since the hour-circles BE and DF, touching the parallel ABCD at B and D, and the hour-circle AC, cutting that same parallel along the straight line AC, have (by Proposition 17 of this book) one and the same common section: it follows that, if the plane of the sundial is not parallel to this common section but cuts it at some point, the three said hour-lines meet at that same point — namely the common sections of the plane of the sundial and of the circles AC, BE, DF, by Proposition 18 of this book. Therefore, at whatever point one of them cuts another of the remaining lines, at that same point the remaining line also cuts it.

That is: if the hour-line from sunrise or sunset of the circle BE cuts the hour-line from noon or midnight of the circle AC at some point, then at that same point the hour-line from sunrise or sunset of the circle DF also cuts that same hour-line from noon or midnight of the circle AC. Likewise, if the line of circle BE cuts the line of circle DF at some point, then at that same point the line of circle AC also cuts the line of circle DF. And if the line of circle AC cuts the line of circle BE at some point, then at that same point the line of circle BE also cuts it, and so on.

Therefore, if any two hour-lines from sunrise or sunset are taken that are equally distant from some hour-line from noon or midnight on the greatest of the always-visible parallels, then those two hour-lines from sunrise or sunset will intersect one another — within the hour-line of that circle from noon or midnight — at one and the same point; and likewise one of them from sunrise or sunset, together

with that line from noon or midnight, will intersect the remaining line from sunrise or sunset, each at one and the same point. Which was to be demonstrated.

Diagram. The circle ABCD is the greatest of the always-visible parallels, drawn with the vertical diameter AC (A at the top, C at the bottom). B lies on the left of the circle and D on the right; these are the two points of tangency. Two hour-circles from sunrise or sunset, BE and DF, touch the parallel at B and D respectively; each is drawn as a large arc that crosses near A at the top and near C at the bottom, its far side labelled E (for the circle BE, on the left) and F (for the circle DF, on the right). The labels E and F appear both near the top (flanking A) and near the bottom (flanking C), marking the two crossings of each tangent circle.

- A (top, on the circle)
- E (upper left, beside A): the circle BE, near its upper crossing
- F (upper right, beside A): the circle DF, near its upper crossing
- B (left, on the circle): point of tangency of circle BE
- D (right, on the circle): point of tangency of circle DF
- E (lower left, beside C): the circle BE, near its lower crossing
- F (lower right, beside C): the circle DF, near its lower crossing
- C (bottom, on the circle)

SCHOLIVM [Scholium].

[Margin: Construction of four tables, in which are contained the hours from noon or midnight and from sunrise or sunset cutting one another at one and the same point: the line of hour 24 from sunrise or sunset, and of hour 12 from sunrise or sunset; likewise the line of hour 6 from noon or midnight, and the line of hour 12 from noon or midnight, of any sundial.]

From this proposition are drawn up the following four tables, whose titles are:

- **LINEA HORAE XXIII. AB ORTV, VEL OCCASV.** [The line of hour 24 from sunrise or sunset.]
- **LINEA HORAE XII. AB ORTV, VEL OCCASV.** [The line of hour 12 from sunrise or sunset.]
- **LINEA HORAE VI. A MERIDIE, VEL MEDIA NOCTE.** [The line of hour 6 from noon or midnight.]
- **LINEA HORAE XII. A MERIDIE, VEL MEDIA NOCTE.** [The line of hour 12 from noon or midnight.]

Linea horæ 24. ab ortu vel occasu. Vel horizontalis linea. [The line of hour 24 from sunrise or sunset. Or the horizontal line.]

Hours from sunrise/sunset	Hours from noon/midnight
$\frac{1}{4}$	$\frac{1}{8}$
$\frac{1}{2}$	$\frac{1}{4}$
$\frac{3}{4}$	$\frac{3}{8}$
I	$\frac{1}{2}$
$1\frac{1}{4}$	$\frac{5}{8}$
$1\frac{1}{2}$	$\frac{3}{4}$
$1\frac{3}{4}$	$\frac{7}{8}$
2	I
$2\frac{1}{4}$	$1\frac{1}{8}$
$2\frac{1}{2}$	$1\frac{1}{4}$
$2\frac{3}{4}$	$1\frac{3}{8}$
3	$1\frac{1}{2}$
$3\frac{1}{4}$	$1\frac{5}{8}$
$3\frac{1}{2}$	$1\frac{3}{4}$
$3\frac{3}{4}$	$1\frac{7}{8}$
4	II
$4\frac{1}{4}$	$2\frac{1}{8}$
$4\frac{1}{2}$	$2\frac{1}{4}$
$4\frac{3}{4}$	$2\frac{3}{8}$
5	$2\frac{1}{2}$
$5\frac{1}{4}$	$2\frac{5}{8}$
$5\frac{1}{2}$	$2\frac{3}{4}$
$5\frac{3}{4}$	$2\frac{7}{8}$
6	III
$6\frac{1}{4}$	$3\frac{1}{8}$
$6\frac{1}{2}$	$3\frac{1}{4}$
$6\frac{3}{4}$	$3\frac{3}{8}$
7	$3\frac{1}{2}$
$7\frac{1}{4}$	$3\frac{5}{8}$

Hours from sunrise/sunset	Hours from noon/midnight
7½	3¾
7¾	3⅞
8	III
8¼	4⅛
8½	4¼
8¾	4⅜
9	4½
9¼	4⅝
9½	4¾
9¾	4⅞
10	V
10¼	5⅛
10½	5¼
10¾	5⅜
11	5½
11¼	5⅝
11½	5¾
11¾	5⅞
12	VI
12¼	6⅛
12½	6¼
12¾	6⅜
13	6½
13¼	6⅝
13½	6¾
13¾	6⅞
14	VII
14¼	7⅛
14½	7¼
14¾	7⅜
15	7½

Hours from sunrise/sunset	Hours from noon/midnight
15 $\frac{1}{4}$	7 $\frac{5}{8}$
15 $\frac{1}{2}$	7 $\frac{3}{4}$
15 $\frac{3}{4}$	7 $\frac{7}{8}$
16	VIII
16 $\frac{1}{4}$	8 $\frac{1}{8}$
16 $\frac{1}{2}$	8 $\frac{1}{4}$
16 $\frac{3}{4}$	8 $\frac{3}{8}$
17	8 $\frac{1}{2}$
17 $\frac{1}{4}$	8 $\frac{5}{8}$
17 $\frac{1}{2}$	8 $\frac{3}{4}$
17 $\frac{3}{4}$	8 $\frac{7}{8}$
18	IX
18 $\frac{1}{4}$	9 $\frac{1}{8}$
18 $\frac{1}{2}$	9 $\frac{1}{4}$
18 $\frac{3}{4}$	9 $\frac{3}{8}$
19	9 $\frac{1}{2}$
19 $\frac{1}{4}$	9 $\frac{5}{8}$
19 $\frac{1}{2}$	9 $\frac{3}{4}$
19 $\frac{3}{4}$	9 $\frac{7}{8}$
20	X
20 $\frac{1}{4}$	10 $\frac{1}{8}$
20 $\frac{1}{2}$	10 $\frac{1}{4}$
20 $\frac{3}{4}$	10 $\frac{3}{8}$
21	10 $\frac{1}{2}$
21 $\frac{1}{4}$	10 $\frac{5}{8}$
21 $\frac{1}{2}$	10 $\frac{3}{4}$
21 $\frac{3}{4}$	10 $\frac{7}{8}$
22	XI
22 $\frac{1}{4}$	11 $\frac{1}{8}$
22 $\frac{1}{2}$	11 $\frac{1}{4}$
22 $\frac{3}{4}$	11 $\frac{3}{8}$

Hours from sunrise/sunset	Hours from noon/midnight
23	11½
23¼	11⅝
23½	11¾
23¾	11⅞
24	XII

Below the table: In a horizontal sundial the lines of any two hours of this table are parallel, as is established from the scholium of Proposition 22. Likewise in certain other sundials the lines of these hours are parallel — namely 12 from sunrise or sunset and 6 from noon or midnight; also 24 from sunrise or sunset and 12 from noon or midnight — as is clear from the same scholium of Proposition 22.

Linea horæ 12. ab ortu vel occasu. [The line of hour 12 from sunrise or sunset.]

Hours from sunrise/sunset	Hours from noon/midnight
¼	6⅛
½	6¼
¾	6⅜
I	6½
1¼	6⅝
1½	6¾
1¾	6⅞
2	VII
2¼	7⅛
2½	7¼
2¾	7⅜
3	7½
3¼	7⅝
3½	7¾
3¾	7⅞
4	VIII
4¼	8⅛

Hours from sunrise/sunset	Hours from noon/midnight
4½	8¼
4¾	8⅜
5	8½
5¼	8⅝
5½	8¾
5¾	8⅞
6	IX
6¼	9⅛
6½	9¼
6¾	9⅜
7	9½
7¼	9⅝
7½	9¾
7¾	9⅞
8	X
8¼	10⅛
8½	10¼
8¾	10⅜
9	10½
9¼	10⅝
9½	10¾
9¾	10⅞
10	XI
10¼	11⅛
10½	11¼
10¾	11⅜
11	11½
11¼	11⅝
11½	11¾
11¾	11⅞
12	XII

Hours from sunrise/sunset	Hours from noon/midnight
12 ¹ / ₄	¹ / ₈
12 ¹ / ₂	¹ / ₄
12 ³ / ₄	³ / ₈
13	¹ / ₂
13 ¹ / ₄	⁵ / ₈
13 ¹ / ₂	³ / ₄
13 ³ / ₄	⁷ / ₈
14	I
14 ¹ / ₄	1 ¹ / ₈
14 ¹ / ₂	1 ¹ / ₄
14 ³ / ₄	1 ³ / ₈
15	1 ¹ / ₂
15 ¹ / ₄	1 ⁵ / ₈
15 ¹ / ₂	1 ³ / ₄
15 ³ / ₄	1 ⁷ / ₈
16	II
16 ¹ / ₄	2 ¹ / ₈
16 ¹ / ₂	2 ¹ / ₄
16 ³ / ₄	2 ³ / ₈
17	2 ¹ / ₂
17 ¹ / ₄	2 ⁵ / ₈
17 ¹ / ₂	2 ³ / ₄
17 ³ / ₄	2 ⁷ / ₈
18	III
18 ¹ / ₄	3 ¹ / ₈
18 ¹ / ₂	3 ¹ / ₄
18 ³ / ₄	3 ³ / ₈
19	3 ¹ / ₂
19 ¹ / ₄	3 ⁵ / ₈
19 ¹ / ₂	3 ³ / ₄
19 ³ / ₄	3 ⁷ / ₈

Hours from sunrise/sunset	Hours from noon/midnight
20	III
$20\frac{1}{4}$	$4\frac{1}{8}$
$20\frac{1}{2}$	$4\frac{1}{4}$
$20\frac{3}{4}$	$4\frac{3}{8}$
21	$4\frac{1}{2}$
$21\frac{1}{4}$	$4\frac{5}{8}$
$21\frac{1}{2}$	$4\frac{3}{4}$
$21\frac{3}{4}$	$4\frac{7}{8}$
22	V
$22\frac{1}{4}$	$5\frac{1}{8}$
$22\frac{1}{2}$	$5\frac{1}{4}$
$22\frac{3}{4}$	$5\frac{3}{8}$
23	$5\frac{1}{2}$
$23\frac{1}{4}$	$5\frac{5}{8}$
$23\frac{1}{2}$	$5\frac{3}{4}$
$23\frac{3}{4}$	$5\frac{7}{8}$
24	VI

Below the table: In a vertical sundial at latitude 45° , or in a sundial parallel to the circle of hour 12 from sunrise or sunset, the lines of any two hours of this table are parallel. By the same reasoning, in certain other sundials the lines of these hours — 24 from sunrise or sunset, and 6 from noon or midnight — are parallel, as is manifest from the scholium of Proposition 22.

The line of the sixth hour from noon or midnight

[Table title: *Linea horæ sextæ à meridie vel media nocte.*]

Every column of this table is headed **Horæ ab ortu vel occasu** — "hours from sunrise or sunset." The twelve columns form six side-by-side pairs; in each pair the left column gives an hour reckoned from sunrise or sunset (the argument, running continuously $\frac{1}{4} \rightarrow 24$ in quarter-hour steps down the six pairs), and the right column gives the concurrent hour whose line meets it on the line of the sixth hour from noon. Reading order is pair by pair, top to bottom.

Hr	Hr	Hr	Hr	Hr	Hr	Hr	Hr	Hr	Hr	Hr	Hr
$\frac{1}{4}$	$11\frac{3}{4}$	$4\frac{1}{4}$	$7\frac{3}{4}$	$8\frac{1}{4}$	$3\frac{3}{4}$	$12\frac{1}{4}$	$23\frac{3}{4}$	$16\frac{1}{4}$	$19\frac{3}{4}$	$20\frac{1}{4}$	$15\frac{3}{4}$
$\frac{1}{2}$	$11\frac{1}{2}$	$4\frac{1}{2}$	$7\frac{1}{2}$	$8\frac{1}{2}$	$3\frac{1}{2}$	$12\frac{1}{2}$	$23\frac{1}{2}$	$16\frac{1}{2}$	$19\frac{1}{2}$	$20\frac{1}{2}$	$15\frac{1}{2}$
$\frac{3}{4}$	$11\frac{1}{4}$	$4\frac{3}{4}$	$7\frac{1}{4}$	$8\frac{3}{4}$	$3\frac{1}{4}$	$12\frac{3}{4}$	$23\frac{1}{4}$	$16\frac{3}{4}$	$19\frac{1}{4}$	$20\frac{3}{4}$	$15\frac{1}{4}$
1	11	5	7	9	3	13	23	17	19	21	15
$1\frac{1}{4}$	$10\frac{3}{4}$	$5\frac{1}{4}$	$6\frac{3}{4}$	$9\frac{1}{4}$	$2\frac{3}{4}$	$13\frac{1}{4}$	$22\frac{3}{4}$	$17\frac{1}{4}$	$18\frac{3}{4}$	$21\frac{1}{4}$	$14\frac{3}{4}$
$1\frac{1}{2}$	$10\frac{1}{2}$	$5\frac{1}{2}$	$6\frac{1}{2}$	$9\frac{1}{2}$	$2\frac{1}{2}$	$13\frac{1}{2}$	$22\frac{1}{2}$	$17\frac{1}{2}$	$18\frac{1}{2}$	$21\frac{1}{2}$	$14\frac{1}{2}$
$1\frac{3}{4}$	$10\frac{1}{4}$	$5\frac{3}{4}$	$6\frac{1}{4}$	$9\frac{3}{4}$	$2\frac{1}{4}$	$13\frac{3}{4}$	$22\frac{1}{4}$	$17\frac{3}{4}$	$18\frac{1}{4}$	$21\frac{3}{4}$	$14\frac{1}{4}$
2	10	6	6	10	2	14	22	18	18	22	14
$2\frac{1}{4}$	$9\frac{3}{4}$	$6\frac{1}{4}$	$5\frac{3}{4}$	$10\frac{1}{4}$	$1\frac{3}{4}$	$14\frac{1}{4}$	$21\frac{3}{4}$	$18\frac{1}{4}$	$17\frac{3}{4}$	$22\frac{1}{4}$	$13\frac{3}{4}$
$2\frac{1}{2}$	$9\frac{1}{2}$	$6\frac{1}{2}$	$5\frac{1}{2}$	$10\frac{1}{2}$	$1\frac{1}{2}$	$14\frac{1}{2}$	$21\frac{1}{2}$	$18\frac{1}{2}$	$17\frac{1}{2}$	$22\frac{1}{2}$	$13\frac{1}{2}$
$2\frac{3}{4}$	$9\frac{1}{4}$	$6\frac{3}{4}$	$5\frac{1}{4}$	$10\frac{3}{4}$	$1\frac{1}{4}$	$14\frac{3}{4}$	$21\frac{1}{4}$	$18\frac{3}{4}$	$17\frac{1}{4}$	$22\frac{3}{4}$	$13\frac{1}{4}$
3	9	7	5	11	1	15	21	19	17	23	13
$3\frac{1}{4}$	$8\frac{3}{4}$	$7\frac{1}{4}$	$4\frac{3}{4}$	$11\frac{1}{4}$	$\frac{3}{4}$	$15\frac{1}{4}$	$20\frac{3}{4}$	$19\frac{1}{4}$	$16\frac{3}{4}$	$23\frac{1}{4}$	$12\frac{3}{4}$
$3\frac{1}{2}$	$8\frac{1}{2}$	$7\frac{1}{2}$	$4\frac{1}{2}$	$11\frac{1}{2}$	$\frac{1}{2}$	$15\frac{1}{2}$	$20\frac{1}{2}$	$19\frac{1}{2}$	$16\frac{1}{2}$	$23\frac{1}{2}$	$12\frac{1}{2}$
$3\frac{3}{4}$	$8\frac{1}{4}$	$7\frac{3}{4}$	$4\frac{1}{4}$	$11\frac{3}{4}$	$\frac{1}{4}$	$15\frac{3}{4}$	$20\frac{1}{4}$	$19\frac{3}{4}$	$16\frac{1}{4}$	$23\frac{3}{4}$	$12\frac{1}{4}$
4	8	8	4	12	24	16	20	20	16	24	12

In a polar sundial, the lines of any two hours of this table are parallel. Likewise, in certain other sundials the lines of these hours — namely 12 and 24 from sunrise or sunset — are parallel, as appears from the scholium of Proposition 22.

The line of the twelfth hour from noon or midnight

[Table title: *Linea horæ 12. à meridie vel media nocte.*]

Same layout: every column is headed **Horæ ab ortu vel occasu** ("hours from sunrise or sunset"), in six argument/concurrent pairs.

Hr	Hr	Hr	Hr	Hr	Hr	Hr	Hr	Hr	Hr	Hr	Hr
$\frac{1}{4}$	$23\frac{3}{4}$	$4\frac{1}{4}$	$19\frac{3}{4}$	$8\frac{1}{4}$	$15\frac{3}{4}$	$12\frac{1}{4}$	$11\frac{3}{4}$	$16\frac{1}{4}$	$7\frac{3}{4}$	$20\frac{1}{4}$	$3\frac{3}{4}$
$\frac{1}{2}$	$23\frac{1}{2}$	$4\frac{1}{2}$	$19\frac{1}{2}$	$8\frac{1}{2}$	$15\frac{1}{2}$	$12\frac{1}{2}$	$11\frac{1}{2}$	$16\frac{1}{2}$	$7\frac{1}{2}$	$20\frac{1}{2}$	$3\frac{1}{2}$
$\frac{3}{4}$	$23\frac{1}{4}$	$4\frac{3}{4}$	$19\frac{1}{4}$	$8\frac{3}{4}$	$15\frac{1}{4}$	$12\frac{3}{4}$	$11\frac{1}{4}$	$16\frac{3}{4}$	$7\frac{1}{4}$	$20\frac{3}{4}$	$3\frac{1}{4}$
1	23	5	19	9	15	13	11	17	7	21	3
$1\frac{1}{4}$	$22\frac{3}{4}$	$5\frac{1}{4}$	$18\frac{3}{4}$	$9\frac{1}{4}$	$14\frac{3}{4}$	$13\frac{1}{4}$	$10\frac{3}{4}$	$17\frac{1}{4}$	$6\frac{3}{4}$	$21\frac{1}{4}$	$2\frac{3}{4}$
$1\frac{1}{2}$	$22\frac{1}{2}$	$5\frac{1}{2}$	$18\frac{1}{2}$	$9\frac{1}{2}$	$14\frac{1}{2}$	$13\frac{1}{2}$	$10\frac{1}{2}$	$17\frac{1}{2}$	$6\frac{1}{2}$	$21\frac{1}{2}$	$2\frac{1}{2}$

Hr	Hr	Hr	Hr	Hr	Hr	Hr	Hr	Hr	Hr	Hr	Hr
$1\frac{3}{4}$	$22\frac{1}{4}$	$5\frac{3}{4}$	$18\frac{1}{4}$	$9\frac{3}{4}$	$14\frac{1}{4}$	$13\frac{3}{4}$	$10\frac{1}{4}$	$17\frac{3}{4}$	$6\frac{1}{4}$	$21\frac{3}{4}$	$2\frac{1}{4}$
2	22	6	18	10	14	14	10	18	6	22	2
$2\frac{1}{4}$	$21\frac{3}{4}$	$6\frac{1}{4}$	$17\frac{3}{4}$	$10\frac{1}{4}$	$13\frac{3}{4}$	$14\frac{1}{4}$	$9\frac{3}{4}$	$18\frac{1}{4}$	$5\frac{3}{4}$	$22\frac{1}{4}$	$1\frac{3}{4}$
$2\frac{1}{2}$	$21\frac{1}{2}$	$6\frac{1}{2}$	$17\frac{1}{2}$	$10\frac{1}{2}$	$13\frac{1}{2}$	$14\frac{1}{2}$	$9\frac{1}{2}$	$18\frac{1}{2}$	$5\frac{1}{2}$	$22\frac{1}{2}$	$1\frac{1}{2}$
$2\frac{3}{4}$	$21\frac{1}{4}$	$6\frac{3}{4}$	$17\frac{1}{4}$	$10\frac{3}{4}$	$13\frac{1}{4}$	$14\frac{3}{4}$	$9\frac{1}{4}$	$18\frac{3}{4}$	$5\frac{1}{4}$	$22\frac{3}{4}$	$1\frac{1}{4}$
3	21	7	17	11	13	15	9	19	5	23	1
$3\frac{1}{4}$	$20\frac{3}{4}$	$7\frac{1}{4}$	$16\frac{3}{4}$	$11\frac{1}{4}$	$12\frac{3}{4}$	$15\frac{1}{4}$	$8\frac{3}{4}$	$19\frac{1}{4}$	$4\frac{3}{4}$	$23\frac{1}{4}$	$\frac{3}{4}$
$3\frac{1}{2}$	$20\frac{1}{2}$	$7\frac{1}{2}$	$16\frac{1}{2}$	$11\frac{1}{2}$	$12\frac{1}{2}$	$15\frac{1}{2}$	$8\frac{1}{2}$	$19\frac{1}{2}$	$4\frac{1}{2}$	$23\frac{1}{2}$	$\frac{1}{2}$
$3\frac{3}{4}$	$20\frac{1}{4}$	$7\frac{3}{4}$	$16\frac{1}{4}$	$11\frac{3}{4}$	$12\frac{1}{4}$	$15\frac{3}{4}$	$8\frac{1}{4}$	$19\frac{3}{4}$	$4\frac{1}{4}$	$23\frac{3}{4}$	$\frac{1}{4}$
4	20	8	16	12	12	16	8	20	4	24	24

In a meridian sundial, the lines of any two hours of this table are parallel. By the same reasoning, in certain other sundials the lines of these hours — namely 18 and 6 from sunrise or sunset — are parallel, as will be demonstrated in the scholium of Proposition 22.

[Library stamp, lower margin: illegible ink stamp, "B.[?]"]

The first shows which two hour-lines — one taken from sunrise or sunset, the other from noon or midnight — cut the line of the 24th hour from sunrise or sunset at the same point. **The second** shows which two hour-lines — one again from sunrise or sunset, the other from noon or midnight — cut the line of the 12th hour from sunrise or sunset at the same point. **The third** shows which two hour-lines from sunrise or sunset cut, at one and the same point, the line of the sixth hour from noon or midnight. **The fourth**, finally, shows which two hour-lines from sunrise or sunset intersect, at one and the same point, the line of the 12th hour from noon or midnight. All of these can readily be gathered from this proposition. For if these tables and the figure of Proposition 9 of this book are rightly considered, one will find that, in the first two tables, an hour from sunrise or sunset set in the column-heading, and any hour from sunrise or sunset entered within the tables themselves, are equally distant from the neighbouring hour reckoned from noon or midnight. Their lines will therefore cut the line of this hour at the same point, as has been shown in this proposition; and so too the lines of any two of the hours set in the tables will intersect, at one and the same point, the line of the hour placed in the heading. In the latter two tables one will find that any two hours from sunrise or sunset entered in the tables are equally distant from the hour reckoned from noon or midnight that is set in the heading.

Hence, by the same proposition, their lines will intersect the line of that hour at a single point.

So if the point is known at which the line of the 24th hour from sunrise or sunset — or of the 6th or 12th from noon or midnight — is cut by one of the hours placed in the tables, we shall know that the remaining hour entered in the tables must be drawn through that same point, and so on. Thus, in the first table: through whatever point the line of the 24th hour from sunrise or sunset has passed for the hour $1\frac{1}{4}$ from noon or midnight, through that same point must the line of the first hour from sunrise or sunset be drawn — and likewise for the rest.

[Margin: Which of the hour-lines from sunrise or sunset, and from noon or midnight, cut one another at the same point.]

By the same route and reasoning we shall investigate, by means of this proposition, which given hour-line from sunrise or sunset intersects, at one and the same point, any other hour-line from noon or midnight; and likewise which hour-line from sunrise or sunset is cut, at one and the same point, by two arbitrary hour-lines — one from sunrise or sunset, the other from noon or midnight — and so forth. For if in the figure of Proposition 9 of this book one takes three hours — one, whichever you please, from noon or midnight, and the other two, however many, from sunrise or sunset, but equally removed on either side from the hour reckoned from noon or midnight — then the line of any one of these three will cut the lines of the remaining two at one and the same point. Thus the line of the 23rd hour from sunrise or sunset is intersected at a single point by two lines, one of which marks the 13th hour from sunrise or sunset, the other the sixth hour from noon or midnight; because in the figure of Proposition 9 the sixth hour from noon or midnight is equally distant from the 23rd and the 13th hour from sunrise or sunset. In the same way the line of the sixth hour from noon or midnight will be cut at one point by the lines of the 23rd and 13th hours from sunrise or sunset. So too the line of the third hour from noon or midnight will be cut by the lines of the 14th and 16th hours from sunrise or sunset. And so with the rest.

[Margin: The construction of other tables similar to the preceding ones, for each of the hour-lines both from sunrise or sunset and from noon or midnight.]

From these we shall without difficulty construct further tables like the preceding ones, one for each hour-line, both from sunrise or sunset and from noon or midnight — such as the thirty-two tables that follow. But since, for laying out the hour-lines from sunrise or sunset on any plane whatever (once the hour-lines

from noon or midnight have first been drawn — for it was to this use that we constructed the aforesaid tables), the five tables already given suffice, in their proper place we shall show as much; and for that reason we have set them apart from the other thirty-two, which will be placed a little further on, so that they may always be ready to hand whenever their use is required.

[Margin: Franciscus Maurolycus was the first to discover these intersections.]

Furthermore, Franciscus Maurolycus [Francesco Maurolico], Abbot of Messina, is, so far as I know, the first discoverer of these intersections that the hour-lines from sunrise or sunset and from noon or midnight make with one another. For I first observed them in his little book on the drawing of hour-lines, which he sent me at Rome written out in his own hand at Messina — for he had not yet published it. And since this consideration of the intersections is both most delightful and most useful for laying out the hour-lines from sunrise or sunset, I judged it altogether necessary, for the common benefit of students, to confirm it by geometrical demonstrations (as I have indeed done in the preceding propositions). For these intersections of the hour-lines were brought to me from Sicily without any demonstration, nor could I find their demonstration in any author. I have therefore not allowed it to be wanting any longer. Only Giovanni Battista Benedetti [Giovanni Battista Benedetti], in chapter 65 of his *Gnomonices*, has attempted to demonstrate this mutual intersection for the line of the 12th hour from sunrise or sunset alone.

But so that this mutual intersection of the hour-lines may be made still clearer, I choose to demonstrate here, in another way, on what plan the aforesaid tables have been constructed. And to begin from the equinoctial-line table, which we described in the scholium of Proposition 19 of this book: let AB be the equator, CD the greatest of the always-appearing parallels, and E the pole of the parallels, that is, of the world. Let two circles of hours from sunrise or sunset, CF and DF, touch the parallel CD at the opposite points C and D; and let the circle of some hour from noon or midnight cut the same parallel, through the pole E, at the point G, which is distant from the points C and D by roughly equal amounts of six hours on either side, so that CG and DG are quadrants. I say that the three circles CF, DF, EG cut one another on the equator AB, as at the point F.

[Margin: The equinoctial circle, and the two circles of the hours from sunrise or sunset touching the greatest of the always-appearing parallels at opposite points, and the circle of the hour from noon or midnight cutting that same parallel at a point equally distant from the points of contact, cut one another at one and the same point.]

For let there be described, through E (the pole of the parallels) and through C (the point of contact), the great circle EC, cutting the equator at A; and since it has passed through the poles of the parallel CD, it cuts it in half (by Proposition 15 of Theodosius, Book 1), and will therefore pass also through D, the opposite point of contact, and hence through the poles of the circles CF, DF (by Proposition 5 of Theodosius, Book 2). Since, then, the two circles AB and CF cut one another, the great circle EC, drawn through their poles, divides their segments in half (by Proposition 9 of Theodosius, Book 2). And since the segments about them are semicircles — which, being great circles, cut one another in half (by Proposition 11 of Theodosius, Book 1) — the arc of the circle CF lying between the point C and the equator, and the arc of the equator lying between the circles EC and CF, will be a quadrant, being the half of a semicircle. By the same reasoning, both the arc of the circle DF lying between the point D and the equator, and the arc of the equator lying between the circles EB and DF, will be a quadrant. Again, since the two circles CF, DF cut one another, the great circle CD, drawn through their poles, will divide their segments in half (by Proposition 9 of Theodosius, Book 2). Since therefore their segments are semicircles (by Proposition 11 of Theodosius, Book 1), the arcs of the circles CF, DF between the points [C, D, and ...]

[Margin: Which hours from sunrise or sunset and from noon or midnight cut one another at a single point on the equinoctial line.]

Diagram. A spherical-astronomy construction on the celestial sphere, drawn as a circle (the horizon/bounding circle) with three great-circle arcs meeting at a common point below center. It shows the equator, the greatest always-appearing parallel, and hour-circles converging.

- E (top): pole of the parallels (of the world)
- C (upper left): point of contact of the hour-circle CF with the parallel CD
- D (upper right): opposite point of contact of the hour-circle DF
- G (center): point where the hour-circle from noon/midnight cuts the parallel CD
- A (lower left) and B (lower right): ends of the equator AB
- F (bottom): common intersection of the three circles CF, DF, EG on the equator

[C, D, and ...] the point of section, are quadrants; and so, since it has been shown that they lie as quadrants from the equator, they will cut one another on the equator, as at the point F, so that the arcs CF, DF, AF, BF are quadrants. Again, since the great circles EG and CD, drawn through the poles of the parallels CD and

AB, cut the parallels into similar arcs (by Proposition 10 of Theodosius, Book 2) — and the arc CG between EC and EG is, by hypothesis, a quadrant — the arc of the equator AB lying between the same great circles EC and EG is likewise a quadrant. Hence, since the arc AF has been shown to be a quadrant, the great circle EG will pass through the point F. For which reason the four great circles AB, CF, DF, EF have one and the same common section, which was the thing proposed. And therefore, at whatever point the plane of the sundial meets this common section, the common sections of these same circles and of the plane of the sundial pass through the same point (by Proposition 18 of this book) — so that at the same point there intersect, in the sundial, those common sections, that is, the equinoctial line and the hour-lines of the circles CF, DF, EF. And so, at whatever point the hour-line of the circle EF cuts the equinoctial line, through that same point must the hour-lines of the circles CF, DF be drawn.

But so that we may know to which hours the circles CF, DF, EF belong, we shall carefully examine the figure of Proposition 9 of this book, which represents the greatest of the always-appearing parallels, and in which the outer numbers refer to the hours from sunrise or sunset, the inner ones to the hours from noon or midnight. For if CF is set, say, as the circle of the 19th hour from sunrise or sunset — and consequently DF as the circle of the seventh hour from sunrise or sunset, which is diametrically opposite to it, just as the point D is to the point C — then EF will be the circle of the first hour from noon or midnight, since this is a quadrant distant from each of those, just as the point G is a quadrant distant from the point C. And so, if DF is the circle of the hour $9\frac{1}{4}$ from sunrise or sunset, EF will be the circle of the hour $3\frac{1}{4}$ from noon or midnight, and so on for the rest, as is clear from the figure of Proposition 9 of this book. Likewise, if EF is set as the circle of any hour whatever from noon or midnight — say the circle of the hour $8\frac{1}{4}$ from noon or midnight — we shall at once recognize to which hours from sunrise or sunset the other two belong: namely CF the circle of the hour $14\frac{1}{4}$ from sunrise or sunset, and DF the circle of the hour $2\frac{1}{4}$ from sunrise or sunset, since each of these hours is distant from that one by a quadrant, that is, by six hours, as is evident from the said figure, just as the point G is a quadrant distant from the point C, and so on. From these considerations we shall easily compose the table of the equinoctial line set out in the scholium of Proposition 19 of this book.

[Margin: The construction of the table described in Proposition 19.]

For the construction, however, of the other tables, which are assigned to the hour-lines both from sunrise or sunset and from noon or midnight: let AB again be the equator, CD the greatest of the parallels that always appear, and E the pole

of the world, that is, of the parallels. Let two circles of hours from sunrise or sunset, CB and DA, touch the parallel CD at the points C and D — cutting one another at F, and cutting the equator at the points [B, A] — and let the circle AD, produced, cut the equator at N. Let the circle of some hour from noon or midnight cut the same parallel CD, through the pole E, at the point G, which is equally distant from the points C and D, so that the arcs CG, DG are equal. I say that this circle EG, produced, passes through the point F, so that the three hour-circles CB, DA, EG cut one another at the same point F.

For if the points of contact lie at the same point F, they are diametrically opposite, so that the arcs CG, DG are quadrants, and it is clear from what has just been demonstrated that the three circles cut one another at one and the same point. But if the points of contact C, D are not opposite, let there be described, through the pole E and through the contacts C, D, the great circles EC, ED. Also, through the points C, G let there be described the arc of the great circle CHG, and through the points G, D the arc of the great circle GID, and let the chords CG, GD be drawn. Since, then, by the definition of a pole handed down by Theodosius, the straight lines falling from the pole E to the points C, D are equal, [a] the arcs EC, ED will also be equal. Again, since the arcs CG, GD of the parallel CD are equal, [b] the straight lines CG, GD will also be equal. [c] Therefore the arcs of the great circles CHG, GID will be equal too. Hence the two sides EC, EG of the spherical triangle CEG will be equal to the two sides ED, EG of the spherical triangle DEG; and the bases CHG, DIG are also equal. Therefore, by Proposition 35 of Book 3 of Johannes Regiomontanus [Regiomontanus] *On Triangles*, or by Proposition 18 of our own *Spherical Triangles*, the angles CEG, DEG will be equal, and consequently the arc EG will divide the angle CED in half. But since the great circle described through the pole E and the point F divides the same angle CED in half — as we shall presently demonstrate — it is clear that the great circle EG, produced, passes through the point F; for otherwise two great circles, namely EG and the one drawn from E through F, would divide the same angle CED in half, which is absurd.

That the great circle described through E and F divides the angle CED in half, we shall demonstrate thus. Let a great circle EGF be understood as described through the pole E and through F. I say that the angle CEF is equal to the angle DEF. For since the great circles EC, ED are equal ... [drawn] through the poles of the circles CF, DF, of Theodosius — because they have been drawn through the contacts C and through the pole E of the circle CD — and since they pass also through the pole of the equator AB, it comes about (by Proposition 9 of Book 2 of Theodosius)

that the circle EC cuts the segments of the circles CB, AB which meet at B, cutting them ...

[Margin (footnote references): a — Theodosius III.28; b — Theodosius III.29; c — Theodosius III.28.]

[Margin: Two circles of hours from sunrise or sunset touching the greatest of the always-appearing parallels at any two points, and the circle of the hour from noon or midnight cutting that same parallel at a point equally distant from the points of contact, cut one another at one and the same point.]

Diagram. The same construction as on the previous page, repeated: a bounding celestial circle with three great-circle arcs meeting at F below center.

- E (top): pole of the parallels
- C (upper left), D (upper right): the two points of contact on the parallel CD
- G (center): where the noon/midnight hour-circle cuts the parallel
- A (lower left), B (lower right): ends of the equator AB
- F (bottom): common point of intersection of the three circles

Diagram. A fuller version of the construction for the general (non-opposite contacts) case: a bounding circle with several great-circle arcs, the auxiliary great circles EC and ED, and the arcs CHG and GID drawn across the middle.

- E (top): pole of the parallels (of the world)
- N (right): where the circle AD, produced, cuts the equator
- C (middle left) and D (middle right): the two points of contact on the parallel CD
- H (between C and G): point on the arc CHG
- G (center): point where the noon/midnight hour-circle cuts the parallel CD
- I (between G and D): point on the arc GID
- A (lower left), B (lower right): ends of the equator AB
- F (below center): common intersection of the hour-circles CB, DA, EG

it cuts in half the segments of the circles CB and AB that meet at B — segments which, by Proposition 11 of Book 1 of Theodosius, are semicircles. The arc CB is therefore a quadrant. By the same reasoning the arc DA will be a quadrant as well.

Now by Theorem 1 of the scholium to Proposition 21 of Book 2 of Theodosius, the great circles CB and DN, being tangent to the same parallel CD, are equally inclined to AB, the greatest of the parallels; hence the spherical angles CBA and

DNB are equal. And since the angle DAB is also equal to the angle DNB — by Proposition 13 of our *Spherical Triangles*, because ADN and ABN are semicircles by Proposition 11 of Book 1 of Theodosius — the spherical angles FBA and FAB will be equal. Consequently the arcs BF and AF will be equal too, whether by Proposition 4 of Book 1 of Menelaus, or by Proposition 40 of Book 3 of Regiomontanus's *On Triangles*, or indeed by Proposition 9 of our *Spherical Triangles*.

Since, then, the whole arcs BC and AD are likewise equal — being quadrants, as we have shown — the remaining arcs FC and FD will be equal too. And since the arcs EC and ED have been shown to be equal, the two sides EC and EF of the spherical triangle CEF will be equal to the two sides ED and EF of the spherical triangle DEF. As they also have equal bases CF and DF, as has been shown, the angles CEF and DEF will be equal — by Proposition 35 of Book 3 of Regiomontanus's *On Triangles*, or by Proposition 18 of our *Spherical Triangles*. Therefore the arc EF bisects the great-circle angle CED.

And since the arc EG also bisects that same angle, as was demonstrated, the arc EG produced through F will pass exactly through it, so that it does not differ from the arc EF — otherwise two arcs would be allowed to bisect the same angle CED. [Margin: At what hour, reckoned both from sunrise or sunset and from noon or midnight, they cut one another at the same point.] Therefore the three hour-circles CF, DF, and EF share one and the same common intersection. Which is what was proposed. And consequently, at whatever point the plane of the sundial [horologium] cuts this common intersection, through that same point will pass the common intersections of those circles and of the plane of the sundial, by Proposition 18 of this book — so that the hour-lines [lineae horariae] of those circles intersect one another at one and the same point on the sundial. Hence, at whatever point the hour-line of circle CF cuts the hour-line of circle EF, through that same point the hour-line of circle DF must also be drawn, and conversely.

To which hours the circles CF, DF, and EF are to be assigned, the figure we set down in Proposition 9 of this book will teach us. For suppose one of the circles CF, DF — say CF — is assigned, for example, to the 12th hour from sunrise or sunset, and the other, DF, for instance to the 20th hour from sunrise or sunset. Then EF will be the circle of the 4th hour from noon or midnight, since this hour lies midway between them in the figure of the said Proposition 9 of this book, just as the point G lies midway between the points C and D. So too, if EF is taken to be the circle of the 1½ hour from noon or midnight, and EC the circle of the 3rd hour from sunrise or sunset, then ED will be the circle of the 24th hour from sunrise or

sunset, [Margin: The construction of the four preceding tables and of the following thirty-two.] because these two hours are equally distant on either side from that one, just as the points C and D are separated from the point G by equal intervals, and so forth. From these considerations we shall construct, with no difficulty, those thirty-six tables which we have set out in this scholium.

Line of the 23rd hour from sunrise or sunset [Linea horae 23. ab ortu vel occasu]

Each column-pair below gives, on the left, the **hour from sunrise or sunset** (*Hora ab ortu vel occasu*, Arabic figures) and, on the right, the corresponding **hour from noon or midnight** (*Hora à meridie vel media nocte*, whole hours printed as Roman numerals; a bare fraction denotes the fraction past XII / midnight–noon). Fractions are eighths and quarters of an hour.

ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.
$\frac{1}{4}$	$11\frac{5}{8}$	$4\frac{1}{4}$	$1\frac{5}{8}$	$8\frac{1}{4}$	$3\frac{5}{8}$	12	$5\frac{5}{8}$	16	$7\frac{5}{8}$	20	$9\frac{5}{8}$
						$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$	
$\frac{1}{2}$	$11\frac{3}{4}$	$4\frac{1}{2}$	$1\frac{3}{4}$	$8\frac{1}{2}$	$3\frac{3}{4}$	12	$5\frac{3}{4}$	16	$7\frac{3}{4}$	20	$9\frac{3}{4}$
						$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$	
$\frac{3}{4}$	$11\frac{7}{8}$	$4\frac{3}{4}$	$1\frac{7}{8}$	$8\frac{3}{4}$	$3\frac{7}{8}$	12	$5\frac{7}{8}$	16	$7\frac{7}{8}$	20	$9\frac{7}{8}$
						$\frac{3}{4}$		$\frac{3}{4}$		$\frac{3}{4}$	
1	XII	5	II	9	III	13	VI	17	VIII	21	X
$1\frac{1}{4}$	$\frac{1}{8}$	$5\frac{1}{4}$	$2\frac{1}{8}$	$9\frac{1}{4}$	$4\frac{1}{8}$	13	$6\frac{1}{8}$	17	$8\frac{1}{8}$	21	$10\frac{1}{8}$
						$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$	
$1\frac{1}{2}$	$\frac{1}{4}$	$5\frac{1}{2}$	$2\frac{1}{4}$	$9\frac{1}{2}$	$4\frac{1}{4}$	13	$6\frac{1}{4}$	17	$8\frac{1}{4}$	21	$10\frac{1}{4}$
						$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$	
$1\frac{3}{4}$	$\frac{3}{8}$	$5\frac{3}{4}$	$2\frac{3}{8}$	$9\frac{3}{4}$	$4\frac{3}{8}$	13	$6\frac{3}{8}$	17	$8\frac{3}{8}$	21	$10\frac{3}{8}$
						$\frac{3}{4}$		$\frac{3}{4}$		$\frac{3}{4}$	
2	$\frac{1}{2}$	6	$2\frac{1}{2}$	10	$4\frac{1}{2}$	14	$6\frac{1}{2}$	18	$8\frac{1}{2}$	22	$10\frac{1}{2}$
$2\frac{1}{4}$	$\frac{5}{8}$	$6\frac{1}{4}$	$2\frac{5}{8}$	10	$4\frac{5}{8}$	14	$6\frac{5}{8}$	18	$8\frac{5}{8}$	22	$10\frac{5}{8}$
				$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$	
$2\frac{1}{2}$	$\frac{3}{4}$	$6\frac{1}{2}$	$2\frac{3}{4}$	10	$4\frac{3}{4}$	14	$6\frac{3}{4}$	18	$8\frac{3}{4}$	22	$10\frac{3}{4}$
				$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$	
$2\frac{3}{4}$	$\frac{7}{8}$	$6\frac{3}{4}$	$2\frac{7}{8}$	10	$4\frac{7}{8}$	14	$6\frac{7}{8}$	18	$8\frac{7}{8}$	22	$10\frac{7}{8}$

ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.
				$\frac{3}{4}$		$\frac{3}{4}$		$\frac{3}{4}$		$\frac{3}{4}$	
3	I	7	III	11	V	15	VII	19	IX	23	XI
$3\frac{1}{4}$	$1\frac{1}{8}$	$7\frac{1}{4}$	$3\frac{1}{8}$	$11\frac{1}{4}$	$5\frac{1}{8}$	$15\frac{1}{4}$	$7\frac{1}{8}$	$19\frac{1}{4}$	$9\frac{1}{8}$	$23\frac{1}{4}$	$11\frac{1}{8}$
$3\frac{1}{2}$	$1\frac{1}{4}$	$7\frac{1}{2}$	$3\frac{1}{4}$	$11\frac{1}{2}$	$5\frac{1}{4}$	$15\frac{1}{2}$	$7\frac{1}{4}$	$19\frac{1}{2}$	$9\frac{1}{4}$	$23\frac{1}{2}$	$11\frac{1}{4}$
$3\frac{3}{4}$	$1\frac{3}{8}$	$7\frac{3}{4}$	$3\frac{3}{8}$	$11\frac{3}{4}$	$5\frac{3}{8}$	$15\frac{3}{4}$	$7\frac{3}{8}$	$19\frac{3}{4}$	$9\frac{3}{8}$	$23\frac{3}{4}$	$11\frac{3}{8}$
4	$1\frac{1}{2}$	8	$3\frac{1}{2}$	12	$5\frac{1}{2}$	16	$7\frac{1}{2}$	20	$9\frac{1}{2}$	24	$11\frac{1}{2}$

In a sundial equidistant from the circle of the 23rd hour from sunrise or sunset, the lines of any two hours in this table are parallel, as is clear from the scholium to Proposition 22.

Line of the 22nd hour from sunrise or sunset [Linea horae 22. ab ortu vel occasu]

ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.
$\frac{1}{4}$	$11\frac{1}{8}$	$4\frac{1}{4}$	$1\frac{1}{8}$	$8\frac{1}{4}$	$3\frac{1}{8}$	$12\frac{1}{4}$	$5\frac{1}{8}$	$16\frac{1}{4}$	$7\frac{1}{8}$	$20\frac{1}{4}$	$9\frac{1}{8}$
$\frac{1}{2}$	$11\frac{1}{4}$	$4\frac{1}{2}$	$1\frac{1}{4}$	$8\frac{1}{2}$	$3\frac{1}{4}$	$12\frac{1}{2}$	$5\frac{1}{4}$	$16\frac{1}{2}$	$7\frac{1}{4}$	$20\frac{1}{2}$	$9\frac{1}{4}$
$\frac{3}{4}$	$11\frac{3}{8}$	$4\frac{3}{4}$	$1\frac{3}{8}$	$8\frac{3}{4}$	$3\frac{3}{8}$	$12\frac{3}{4}$	$5\frac{3}{8}$	$16\frac{3}{4}$	$7\frac{3}{8}$	$20\frac{3}{4}$	$9\frac{3}{8}$
1	$11\frac{1}{2}$	5	$1\frac{1}{2}$	9	$3\frac{1}{2}$	13	$5\frac{1}{2}$	17	$7\frac{1}{2}$	21	$9\frac{1}{2}$
$1\frac{1}{4}$	$11\frac{5}{8}$	$5\frac{1}{4}$	$1\frac{5}{8}$	$9\frac{1}{4}$	$3\frac{5}{8}$	$13\frac{1}{4}$	$5\frac{5}{8}$	$17\frac{1}{4}$	$7\frac{5}{8}$	$21\frac{1}{4}$	$9\frac{5}{8}$
$1\frac{1}{2}$	$11\frac{3}{4}$	$5\frac{1}{2}$	$1\frac{3}{4}$	$9\frac{1}{2}$	$3\frac{3}{4}$	$13\frac{1}{2}$	$5\frac{3}{4}$	$17\frac{1}{2}$	$7\frac{3}{4}$	$21\frac{1}{2}$	$9\frac{3}{4}$
$1\frac{3}{4}$	$11\frac{7}{8}$	$5\frac{3}{4}$	$1\frac{7}{8}$	$9\frac{3}{4}$	$3\frac{7}{8}$	$13\frac{3}{4}$	$5\frac{7}{8}$	$17\frac{3}{4}$	$7\frac{7}{8}$	$21\frac{3}{4}$	$9\frac{7}{8}$
2	XII	6	II	10	III	14	VI	18	VIII	22	X

ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.
2¼	⅛	6¼	2⅛	10 ¼	4⅛	14 ¼	6⅛	18 ¼	8⅛	22 ¼	10⅛
2½	¼	6½	2¼	10 ½	4¼	14 ½	6¼	18 ½	8¼	22 ½	10¼
2¾	⅜	6¾	2⅜	10 ¾	4⅜	14 ¾	6⅜	18 ¾	8⅜	22 ¾	10⅜
3	½	7	2½	11	4½	15	6½	19	8½	23	10½
3¼	⅝	7¼	2⅝	11 ¼	4⅝	15 ¼	6⅝	19 ¼	8⅝	23 ¼	10⅝
3½	¾	7½	2¾	11 ½	4¾	15 ½	6¾	19 ½	8¾	23 ½	10¾
3¾	⅞	7¾	2⅞	11 ¾	4⅞	15 ¾	6⅞	19 ¾	8⅞	23 ¾	10⅞
4	I	8	III	12	V	16	VII	20	IX	24	XI

In a sundial equidistant from the circle of the 22nd hour from sunrise or sunset, the lines of any two hours in this table are parallel, as was proved in the scholium to Proposition 22.

Line of the 21st hour from sunrise or sunset [Linea horae 21. ab ortu vel occasu]

ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.
¼	10⅝	4¼	⅝	8¼	2⅝	12 ¼	4⅝	16 ¼	6⅝	20 ¼	8⅝
½	10¾	4½	¾	8½	2¾	12 ½	4¾	16 ½	6¾	20 ½	8¾
¾	10⅞	4¾	⅞	8¾	2⅞	12 ¾	4⅞	16 ¾	6⅞	20 ¾	8⅞
1	XI	5	I	9	III	13	V	17	VII	21	IX
1¼	11⅛	5¼	1⅛	9¼	3⅛	13 ¼	5⅛	17 ¼	7⅛	21 ¼	9⅛

ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.
1½	11¼	5½	1¼	9½	3¼	13 ½	5¼	17 ½	7¼	21 ½	9¼
1¾	11⅜	5¾	1⅜	9¾	3⅜	13 ¾	5⅜	17 ¾	7⅜	21 ¾	9⅜
2	11½	6	1½	10	3½	14	5½	18	7½	22	9½
2¼	11⅝	6¼	1⅝	10 ¼	3⅝	14 ¼	5⅝	18 ¼	7⅝	22 ¼	9⅝
2½	11¾	6½	1¾	10 ½	3¾	14 ½	5¾	18 ½	7¾	22 ½	9¾
2¾	11⅞	6¾	1⅞	10 ¾	3⅞	14 ¾	5⅞	18 ¾	7⅞	22 ¾	9⅞
3	XII	7	II	11	III	15	VI	19	VIII	23	X
3¼	⅞	7¼	2⅞	11 ¼	4⅞	15 ¼	6⅞	19 ¼	8⅞	23 ¼	10⅞
3½	¼	7½	2¼	11 ½	4¼	15 ½	6¼	19 ½	8¼	23 ½	10¼
3¾	⅜	7¾	2⅜	11 ¾	4⅜	15 ¾	6⅜	19 ¾	8⅜	23 ¾	10⅜
4	½	8	2½	12	4½	16	6½	20	8½	24	10½

In a sundial equidistant from the circle of the 21st hour from sunrise or sunset, the lines of any two hours in this table are parallel, by what we demonstrated in the scholium to Proposition 22.

Line of the 20th hour from sunrise or sunset [Linea horae 20. ab ortu vel occasu]

ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.
¼	10⅞	4¼	⅞	8¼	2⅞	12 ¼	4⅞	16 ¼	6⅞	20 ¼	8⅞
½	10¼	4½	¼	8½	2¼	12 ½	4¼	16 ½	6¼	20 ½	8¼

ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.	ab ort u	à meri d.
$\frac{3}{4}$	$10\frac{3}{8}$	$4\frac{3}{4}$	$\frac{3}{8}$	$8\frac{3}{4}$	$2\frac{3}{8}$	12 $\frac{3}{4}$	$4\frac{3}{8}$	16 $\frac{3}{4}$	$6\frac{3}{8}$	20 $\frac{3}{4}$	$8\frac{3}{8}$
1	$10\frac{1}{2}$	5	$\frac{1}{2}$	9	$2\frac{1}{2}$	13	$4\frac{1}{2}$	17	$6\frac{1}{2}$	21	$8\frac{1}{2}$
$1\frac{1}{4}$	$10\frac{5}{8}$	$5\frac{1}{4}$	$\frac{5}{8}$	$9\frac{1}{4}$	$2\frac{5}{8}$	13 $\frac{1}{4}$	$4\frac{5}{8}$	17 $\frac{1}{4}$	$6\frac{5}{8}$	21 $\frac{1}{4}$	$8\frac{5}{8}$
$1\frac{1}{2}$	$10\frac{3}{4}$	$5\frac{1}{2}$	$\frac{3}{4}$	$9\frac{1}{2}$	$2\frac{3}{4}$	13 $\frac{1}{2}$	$4\frac{3}{4}$	17 $\frac{1}{2}$	$6\frac{3}{4}$	21 $\frac{1}{2}$	$8\frac{3}{4}$
$1\frac{3}{4}$	$10\frac{7}{8}$	$5\frac{3}{4}$	$\frac{7}{8}$	$9\frac{3}{4}$	$2\frac{7}{8}$	13 $\frac{3}{4}$	$4\frac{7}{8}$	17 $\frac{3}{4}$	$6\frac{7}{8}$	21 $\frac{3}{4}$	$8\frac{7}{8}$
2	XI	6	I	10	III	14	V	18	VII	22	IX
$2\frac{1}{4}$	$11\frac{1}{8}$	$6\frac{1}{4}$	$1\frac{1}{8}$	10 $\frac{1}{4}$	$3\frac{1}{8}$	14 $\frac{1}{4}$	$5\frac{1}{8}$	18 $\frac{1}{4}$	$7\frac{1}{8}$	22 $\frac{1}{4}$	$9\frac{1}{8}$
$2\frac{1}{2}$	$11\frac{1}{4}$	$6\frac{1}{2}$	$1\frac{1}{4}$	10 $\frac{1}{2}$	$3\frac{1}{4}$	14 $\frac{1}{2}$	$5\frac{1}{4}$	18 $\frac{1}{2}$	$7\frac{1}{4}$	22 $\frac{1}{2}$	$9\frac{1}{4}$
$2\frac{3}{4}$	$11\frac{3}{8}$	$6\frac{3}{4}$	$1\frac{3}{8}$	10 $\frac{3}{4}$	$3\frac{3}{8}$	14 $\frac{3}{4}$	$5\frac{3}{8}$	18 $\frac{3}{4}$	$7\frac{3}{8}$	22 $\frac{3}{4}$	$9\frac{3}{8}$
3	$11\frac{1}{2}$	7	$1\frac{1}{2}$	11	$3\frac{1}{2}$	15	$5\frac{1}{2}$	19	$7\frac{1}{2}$	23	$9\frac{1}{2}$
$3\frac{1}{4}$	$11\frac{5}{8}$	$7\frac{1}{4}$	$1\frac{5}{8}$	11 $\frac{1}{4}$	$3\frac{5}{8}$	15 $\frac{1}{4}$	$5\frac{5}{8}$	19 $\frac{1}{4}$	$7\frac{5}{8}$	23 $\frac{1}{4}$	$9\frac{5}{8}$
$3\frac{1}{2}$	$11\frac{3}{4}$	$7\frac{1}{2}$	$1\frac{3}{4}$	11 $\frac{1}{2}$	$3\frac{3}{4}$	15 $\frac{1}{2}$	$5\frac{3}{4}$	19 $\frac{1}{2}$	$7\frac{3}{4}$	23 $\frac{1}{2}$	$9\frac{3}{4}$
$3\frac{3}{4}$	$11\frac{7}{8}$	$7\frac{3}{4}$	$1\frac{7}{8}$	11 $\frac{3}{4}$	$3\frac{7}{8}$	15 $\frac{3}{4}$	$5\frac{7}{8}$	19 $\frac{3}{4}$	$7\frac{7}{8}$	23 $\frac{3}{4}$	$9\frac{7}{8}$
4	XII	8	II	12	III	16	VI	20	VIII	24	X

In a sundial equidistant from the circle of the 20th hour from sunrise or sunset, the lines of any two hours in this table are parallel to one another, from the scholium to Proposition 22.

These three pages continue the set of hour-line concurrence tables belonging to the scholium of Proposition 22 (Book One). Each table corresponds to one hour-line, counted from sunrise or sunset, and lists — for every quarter-hour along that line — the ordinary hour (reckoned from noon or midnight) whose line runs

concurrent with it. No new Liber, Caput, or Propositio rubric is printed; the only rubric-titles on these pages are the six table captions.

How to read each table. The twelve columns form six pairs, read continuously from the first pair through the sixth. In every pair the left member (**Rise**) is *Hora ab ortu vel occasu* — the hour reckoned from sunrise or sunset, advancing in quarter-hour steps; the right member (**Noon**) is *Hora à meridie vel media nocte* — the concurrent hour reckoned from noon or midnight. In the **Noon** columns a whole hour is printed as a Roman numeral (the source uses IIII for 4), and intermediate values as eighth-of-an-hour fractions; a bare fraction ($\frac{1}{8}$, $\frac{1}{4}$, ...) is the part past XII, i.e. just after noon or midnight. Pair 1 runs from $\frac{1}{4}$ to 4 hours after sunrise, pair 2 from $4\frac{1}{4}$ to 8, pair 3 from $8\frac{1}{4}$ to 12, pair 4 from $12\frac{1}{4}$ to 16, pair 5 from $16\frac{1}{4}$ to 20, and pair 6 from $20\frac{1}{4}$ to 24. Throughout, the **Noon** value equals the mean of that row's **Rise** hour and the table's title-hour, expressed on the twelve-hour scale.

The line of the 19th hour from sunrise or sunset [Linea horae 19. ab ortu vel occasu]

Rise	Noon	Rise	Noon	Rise	Noo n	Rise	n	Rise	n	Rise	n
$\frac{1}{4}$	$9\frac{5}{8}$	$4\frac{1}{4}$	$11\frac{5}{8}$	$8\frac{1}{4}$	$1\frac{5}{8}$	$12\frac{1}{4}$	$3\frac{5}{8}$	$16\frac{1}{4}$	$5\frac{5}{8}$	$20\frac{1}{4}$	$7\frac{5}{8}$
$\frac{1}{2}$	$9\frac{3}{4}$	$4\frac{1}{2}$	$11\frac{3}{4}$	$8\frac{1}{2}$	$1\frac{3}{4}$	$12\frac{1}{2}$	$3\frac{3}{4}$	$16\frac{1}{2}$	$5\frac{3}{4}$	$20\frac{1}{2}$	$7\frac{3}{4}$
$\frac{3}{4}$	$9\frac{7}{8}$	$4\frac{3}{4}$	$11\frac{7}{8}$	$8\frac{3}{4}$	$1\frac{7}{8}$	$12\frac{3}{4}$	$3\frac{7}{8}$	$16\frac{3}{4}$	$5\frac{7}{8}$	$20\frac{3}{4}$	$7\frac{7}{8}$
1	X	5	XII	9	II	13	IIII	17	VI	21	VIII
$1\frac{1}{4}$	$10\frac{1}{8}$	$5\frac{1}{4}$	$\frac{1}{8}$	$9\frac{1}{4}$	$2\frac{1}{8}$	$13\frac{1}{4}$	$4\frac{1}{8}$	$17\frac{1}{4}$	$6\frac{1}{8}$	$21\frac{1}{4}$	$8\frac{1}{8}$
$1\frac{1}{2}$	$10\frac{1}{4}$	$5\frac{1}{2}$	$\frac{1}{4}$	$9\frac{1}{2}$	$2\frac{1}{4}$	$13\frac{1}{2}$	$4\frac{1}{4}$	$17\frac{1}{2}$	$6\frac{1}{4}$	$21\frac{1}{2}$	$8\frac{1}{4}$
$1\frac{3}{4}$	$10\frac{3}{8}$	$5\frac{3}{4}$	$\frac{3}{8}$	$9\frac{3}{4}$	$2\frac{3}{8}$	$13\frac{3}{4}$	$4\frac{3}{8}$	$17\frac{3}{4}$	$6\frac{3}{8}$	$21\frac{3}{4}$	$8\frac{3}{8}$
2	$10\frac{1}{2}$	6	$\frac{1}{2}$	10	$2\frac{1}{2}$	14	$4\frac{1}{2}$	18	$6\frac{1}{2}$	22	$8\frac{1}{2}$
$2\frac{1}{4}$	$10\frac{5}{8}$	$6\frac{1}{4}$	$\frac{5}{8}$	$10\frac{1}{4}$	$2\frac{5}{8}$	$14\frac{1}{4}$	$4\frac{5}{8}$	$18\frac{1}{4}$	$6\frac{5}{8}$	$22\frac{1}{4}$	$8\frac{5}{8}$
$2\frac{1}{2}$	$10\frac{3}{4}$	$6\frac{1}{2}$	$\frac{3}{4}$	$10\frac{1}{2}$	$2\frac{3}{4}$	$14\frac{1}{2}$	$4\frac{3}{4}$	$18\frac{1}{2}$	$6\frac{3}{4}$	$22\frac{1}{2}$	$8\frac{3}{4}$
$2\frac{3}{4}$	$10\frac{7}{8}$	$6\frac{3}{4}$	$\frac{7}{8}$	$10\frac{3}{4}$	$2\frac{7}{8}$	$14\frac{3}{4}$	$4\frac{7}{8}$	$18\frac{3}{4}$	$6\frac{7}{8}$	$22\frac{3}{4}$	$8\frac{7}{8}$
3	XI	7	I	11	III	15	V	19	VII	23	IX
$3\frac{1}{4}$	$11\frac{1}{8}$	$7\frac{1}{4}$	$1\frac{1}{8}$	$11\frac{1}{4}$	$3\frac{1}{8}$	$15\frac{1}{4}$	$5\frac{1}{8}$	$19\frac{1}{4}$	$7\frac{1}{8}$	$23\frac{1}{4}$	$9\frac{1}{8}$
$3\frac{1}{2}$	$11\frac{1}{4}$	$7\frac{1}{2}$	$1\frac{1}{4}$	$11\frac{1}{2}$	$3\frac{1}{4}$	$15\frac{1}{2}$	$5\frac{1}{4}$	$19\frac{1}{2}$	$7\frac{1}{4}$	$23\frac{1}{2}$	$9\frac{1}{4}$
$3\frac{3}{4}$	$11\frac{3}{8}$	$7\frac{3}{4}$	$1\frac{3}{8}$	$11\frac{3}{4}$	$3\frac{3}{8}$	$15\frac{3}{4}$	$5\frac{3}{8}$	$19\frac{3}{4}$	$7\frac{3}{8}$	$23\frac{3}{4}$	$9\frac{3}{8}$

Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon
4	11½	8	1½	12	3½	16	5½	20	7½	24	9½

In a sundial whose plane is equidistant from (that is, parallel to) the circle of the 19th hour reckoned from sunrise or sunset, the lines of any two hours whatever in this table are parallel to one another, as is plain from the scholium to Proposition 22.

The line of the 18th hour from sunrise or sunset [Linea horae 18. ab ortu vel occasu]

Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon
¼	9⅛	4¼	11⅛	8¼	1⅞	12¼	3⅛	16¼	5⅛	20¼	7⅛
½	9¼	4½	11¼	8½	1¾	12½	3¼	16½	5¼	20½	7¼
¾	9¾	4¾	11¾	8¾	1⅝	12¾	3⅝	16¾	5⅝	20¾	7⅝
1	9½	5	11½	9	1½	13	3½	17	5½	21	7½
1¼	9⅝	5¼	11⅝	9¼	1⅝	13¼	3⅝	17¼	5⅝	21¼	7⅝
1½	9¾	5½	11¾	9½	1¾	13½	3¾	17½	5¾	21½	7¾
1¾	9⅞	5¾	11⅞	9¾	1⅞	13¾	3⅞	17¾	5⅞	21¾	7⅞
2	X	6	XII	10	II	14	III	18	VI	22	VIII
2¼	10⅛	6¼	⅛	10¼	2⅞	14¼	4⅛	18¼	6⅛	22¼	8⅛
2½	10¼	6½	¼	10½	2¾	14½	4¼	18½	6¼	22½	8¼
2¾	10⅝	6¾	⅝	10¾	2⅝	14¾	4⅝	18¾	6⅝	22¾	8⅝
3	10½	7	½	11	2½	15	4½	19	6½	23	8½
3¼	10⅝	7¼	⅝	11¼	2⅝	15¼	4⅝	19¼	6⅝	23¼	8⅝
3½	10¾	7½	¾	11½	2¾	15½	4¾	19½	6¾	23½	8¾
3¾	10⅞	7¾	⅞	11¾	2⅞	15¾	4⅞	19¾	6⅞	23¾	8⅞
4	XI	8	I	12	III	16	V	20	VII	24	IX

In a sundial whose plane is equidistant from the circle of the 18th hour reckoned from sunrise or sunset, the lines of any two hours whatever in this table are parallel, as is plain from the scholium to Proposition 22.

The line of the 17th hour from sunrise or sunset [Linea horae 17. ab ortu vel occasu]

Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon
$\frac{1}{4}$	$8\frac{5}{8}$	$4\frac{1}{4}$	$10\frac{5}{8}$	$8\frac{1}{4}$	$\frac{5}{8}$	$12\frac{1}{4}$	$2\frac{5}{8}$	$16\frac{1}{4}$	$4\frac{5}{8}$	$20\frac{1}{4}$	$6\frac{5}{8}$
$\frac{1}{2}$	$8\frac{3}{4}$	$4\frac{1}{2}$	$10\frac{3}{4}$	$8\frac{1}{2}$	$\frac{3}{4}$	$12\frac{1}{2}$	$2\frac{3}{4}$	$16\frac{1}{2}$	$4\frac{3}{4}$	$20\frac{1}{2}$	$6\frac{3}{4}$
$\frac{3}{4}$	$8\frac{7}{8}$	$4\frac{3}{4}$	$10\frac{7}{8}$	$8\frac{3}{4}$	$\frac{7}{8}$	$12\frac{3}{4}$	$2\frac{7}{8}$	$16\frac{3}{4}$	$4\frac{7}{8}$	$20\frac{3}{4}$	$6\frac{7}{8}$
1	IX	5	XI	9	I	13	III	17	V	21	VII
$1\frac{1}{4}$	$9\frac{1}{8}$	$5\frac{1}{4}$	$11\frac{1}{8}$	$9\frac{1}{4}$	$1\frac{1}{8}$	$13\frac{1}{4}$	$3\frac{1}{8}$	$17\frac{1}{4}$	$5\frac{1}{8}$	$21\frac{1}{4}$	$7\frac{1}{8}$
$1\frac{1}{2}$	$9\frac{1}{4}$	$5\frac{1}{2}$	$11\frac{1}{4}$	$9\frac{1}{2}$	$1\frac{1}{4}$	$13\frac{1}{2}$	$3\frac{1}{4}$	$17\frac{1}{2}$	$5\frac{1}{4}$	$21\frac{1}{2}$	$7\frac{1}{4}$
$1\frac{3}{4}$	$9\frac{3}{8}$	$5\frac{3}{4}$	$11\frac{3}{8}$	$9\frac{3}{4}$	$1\frac{3}{8}$	$13\frac{3}{4}$	$3\frac{3}{8}$	$17\frac{3}{4}$	$5\frac{3}{8}$	$21\frac{3}{4}$	$7\frac{3}{8}$
2	$9\frac{1}{2}$	6	$11\frac{1}{2}$	10	$1\frac{1}{2}$	14	$3\frac{1}{2}$	18	$5\frac{1}{2}$	22	$7\frac{1}{2}$
$2\frac{1}{4}$	$9\frac{5}{8}$	$6\frac{1}{4}$	$11\frac{5}{8}$	$10\frac{1}{4}$	$1\frac{5}{8}$	$14\frac{1}{4}$	$3\frac{5}{8}$	$18\frac{1}{4}$	$5\frac{5}{8}$	$22\frac{1}{4}$	$7\frac{5}{8}$
$2\frac{1}{2}$	$9\frac{3}{4}$	$6\frac{1}{2}$	$11\frac{3}{4}$	$10\frac{1}{2}$	$1\frac{3}{4}$	$14\frac{1}{2}$	$3\frac{3}{4}$	$18\frac{1}{2}$	$5\frac{3}{4}$	$22\frac{1}{2}$	$7\frac{3}{4}$
$2\frac{3}{4}$	$9\frac{7}{8}$	$6\frac{3}{4}$	$11\frac{7}{8}$	$10\frac{3}{4}$	$1\frac{7}{8}$	$14\frac{3}{4}$	$3\frac{7}{8}$	$18\frac{3}{4}$	$5\frac{7}{8}$	$22\frac{3}{4}$	$7\frac{7}{8}$
3	X	7	XII	11	II	15	III	19	VI	23	VIII
$3\frac{1}{4}$	$10\frac{1}{8}$	$7\frac{1}{4}$	$1\frac{1}{8}$	$11\frac{1}{4}$	$2\frac{1}{8}$	$15\frac{1}{4}$	$4\frac{1}{8}$	$19\frac{1}{4}$	$6\frac{1}{8}$	$23\frac{1}{4}$	$8\frac{1}{8}$
$3\frac{1}{2}$	$10\frac{1}{4}$	$7\frac{1}{2}$	$\frac{1}{4}$	$11\frac{1}{2}$	$2\frac{1}{4}$	$15\frac{1}{2}$	$4\frac{1}{4}$	$19\frac{1}{2}$	$6\frac{1}{4}$	$23\frac{1}{2}$	$8\frac{1}{4}$
$3\frac{3}{4}$	$10\frac{3}{8}$	$7\frac{3}{4}$	$\frac{3}{8}$	$11\frac{3}{4}$	$2\frac{3}{8}$	$15\frac{3}{4}$	$4\frac{3}{8}$	$19\frac{3}{4}$	$6\frac{3}{8}$	$23\frac{3}{4}$	$8\frac{3}{8}$
4	$10\frac{1}{2}$	8	$\frac{1}{2}$	12	$2\frac{1}{2}$	16	$4\frac{1}{2}$	20	$6\frac{1}{2}$	24	$8\frac{1}{2}$

In a sundial whose plane is equidistant from the circle of the 17th hour reckoned from sunrise or sunset, the lines of any two hours whatever in this table are parallel to one another, as is manifest from the scholium to Proposition 22.

The line of the 16th hour from sunrise or sunset [Linea horae 16. ab ortu vel occasu]

Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon
$\frac{1}{4}$	$8\frac{1}{8}$	$4\frac{1}{4}$	$10\frac{1}{8}$	$8\frac{1}{4}$	$\frac{1}{8}$	$12\frac{1}{4}$	$2\frac{1}{8}$	$16\frac{1}{4}$	$4\frac{1}{8}$	$20\frac{1}{4}$	$6\frac{1}{8}$
$\frac{1}{2}$	$8\frac{1}{4}$	$4\frac{1}{2}$	$10\frac{1}{4}$	$8\frac{1}{2}$	$\frac{1}{4}$	$12\frac{1}{2}$	$2\frac{1}{4}$	$16\frac{1}{2}$	$4\frac{1}{4}$	$20\frac{1}{2}$	$6\frac{1}{4}$
$\frac{3}{4}$	$8\frac{3}{8}$	$4\frac{3}{4}$	$10\frac{3}{8}$	$8\frac{3}{4}$	$\frac{3}{8}$	$12\frac{3}{4}$	$2\frac{3}{8}$	$16\frac{3}{4}$	$4\frac{3}{8}$	$20\frac{3}{4}$	$6\frac{3}{8}$
1	$8\frac{1}{2}$	5	$10\frac{1}{2}$	9	$\frac{1}{2}$	13	$2\frac{1}{2}$	17	$4\frac{1}{2}$	21	$6\frac{1}{2}$

In a sundial whose plane is equidistant from the circle of the 16th hour reckoned from sunrise or sunset, the lines of any two hours whatever in this table are parallel, from the scholium to Proposition 22.

	Noon			Noon			Noon			Noon		
Rise	n	Rise	Noon	Rise	Noon	Rise	n	Rise	n	Rise	n	
1¼	7⅝	4¼	9⅝	8¼	11⅝	12¼	1⅝	16¼	3⅝	20¼	5⅝	
1½	7¾	4½	9¾	8½	11¾	12½	1¾	16½	3¾	20½	5¾	
¾	7⅞	4¾	9⅞	8¾	11⅞	12¾	1⅞	16¾	3⅞	20¾	5⅞	
1	VIII	5	X	9	XII	13	II	17	III	21	VI	
1¼	8⅞	5¼	10⅞	9¼	1⅞	13¼	2⅞	17¼	4⅞	21¼	6⅞	
1½	8¼	5½	10¼	9½	1¼	13½	2¼	17½	4¼	21½	6¼	
1¾	8⅞	5¾	10⅞	9¾	3⅞	13¾	2¾	17¾	4¾	21¾	6¾	
2	8½	6	10½	10	1½	14	2½	18	4½	22	6½	
2¼	8⅝	6¼	10⅝	10¼	5⅞	14¼	2⅝	18¼	4⅝	22¼	6⅝	
2½	8¾	6½	10¾	10½	¾	14½	2¾	18½	4¾	22½	6¾	
2¾	8⅞	6¾	10⅞	10¾	7⅞	14¾	2⅞	18¾	4⅞	22¾	6⅞	

	Noo						Noo		Noo		Noo
Rise	n	Rise	Noon	Rise	Noon	Rise	n	Rise	n	Rise	n
3	IX	7	XI	11	I	15	III	19	V	23	VII
$3\frac{1}{4}$	$9\frac{1}{8}$	$7\frac{1}{4}$	$11\frac{1}{8}$	$11\frac{1}{4}$	$1\frac{1}{8}$	$15\frac{1}{4}$	$3\frac{1}{8}$	$19\frac{1}{4}$	$5\frac{1}{8}$	$23\frac{1}{4}$	$7\frac{1}{8}$
$3\frac{1}{2}$	$9\frac{1}{4}$	$7\frac{1}{2}$	$11\frac{1}{4}$	$11\frac{1}{2}$	$1\frac{1}{4}$	$15\frac{1}{2}$	$3\frac{1}{4}$	$19\frac{1}{2}$	$5\frac{1}{4}$	$23\frac{1}{2}$	$7\frac{1}{4}$
$3\frac{3}{4}$	$9\frac{3}{8}$	$7\frac{3}{4}$	$11\frac{3}{8}$	$11\frac{3}{4}$	$1\frac{3}{8}$	$15\frac{3}{4}$	$3\frac{3}{8}$	$19\frac{3}{4}$	$5\frac{3}{8}$	$23\frac{3}{4}$	$7\frac{3}{8}$
4	$9\frac{1}{2}$	8	$11\frac{1}{2}$	12	$1\frac{1}{2}$	16	$3\frac{1}{2}$	20	$5\frac{1}{2}$	24	$7\frac{1}{2}$

In a sundial whose plane is equidistant from the circle of the 15th hour reckoned from sunrise or sunset, the lines of any two hours whatever in this table are parallel to one another, as is clear from the scholium to Proposition 22.

The line of the 14th hour from sunrise or sunset [Linea horae 14. ab ortu vel occasu]

	Noo						Noo		Noo		Noo
Rise	n	Rise	Noon	Rise	Noon	Rise	n	Rise	n	Rise	n
$\frac{1}{4}$	$7\frac{1}{8}$	$4\frac{1}{4}$	$9\frac{1}{8}$	$8\frac{1}{4}$	$11\frac{1}{8}$	$12\frac{1}{4}$	$1\frac{1}{8}$	$16\frac{1}{4}$	$3\frac{1}{8}$	$20\frac{1}{4}$	$5\frac{1}{8}$
$\frac{1}{2}$	$7\frac{1}{4}$	$4\frac{1}{2}$	$9\frac{1}{4}$	$8\frac{1}{2}$	$11\frac{1}{4}$	$12\frac{1}{2}$	$1\frac{1}{4}$	$16\frac{1}{2}$	$3\frac{1}{4}$	$20\frac{1}{2}$	$5\frac{1}{4}$
$\frac{3}{4}$	$7\frac{3}{8}$	$4\frac{3}{4}$	$9\frac{3}{8}$	$8\frac{3}{4}$	$11\frac{3}{8}$	$12\frac{3}{4}$	$1\frac{3}{8}$	$16\frac{3}{4}$	$3\frac{3}{8}$	$20\frac{3}{4}$	$5\frac{3}{8}$
1	$7\frac{1}{2}$	5	$9\frac{1}{2}$	9	$11\frac{1}{2}$	13	$1\frac{1}{2}$	17	$3\frac{1}{2}$	21	$5\frac{1}{2}$
$1\frac{1}{4}$	$7\frac{5}{8}$	$5\frac{1}{4}$	$9\frac{5}{8}$	$9\frac{1}{4}$	$11\frac{5}{8}$	$13\frac{1}{4}$	$1\frac{5}{8}$	$17\frac{1}{4}$	$3\frac{5}{8}$	$21\frac{1}{4}$	$5\frac{5}{8}$
$1\frac{1}{2}$	$7\frac{3}{4}$	$5\frac{1}{2}$	$9\frac{3}{4}$	$9\frac{1}{2}$	$11\frac{3}{4}$	$13\frac{1}{2}$	$1\frac{3}{4}$	$17\frac{1}{2}$	$3\frac{3}{4}$	$21\frac{1}{2}$	$5\frac{3}{4}$
$1\frac{3}{4}$	$7\frac{7}{8}$	$5\frac{3}{4}$	$9\frac{7}{8}$	$9\frac{3}{4}$	$11\frac{7}{8}$	$13\frac{3}{4}$	$1\frac{7}{8}$	$17\frac{3}{4}$	$3\frac{7}{8}$	$21\frac{3}{4}$	$5\frac{7}{8}$
2	VIII	6	X	10	XII	14	II	18	III	22	VI
$2\frac{1}{4}$	$8\frac{1}{8}$	$6\frac{1}{4}$	$10\frac{1}{8}$	$10\frac{1}{4}$	$\frac{1}{8}$	$14\frac{1}{4}$	$2\frac{1}{8}$	$18\frac{1}{4}$	$4\frac{1}{8}$	$22\frac{1}{4}$	$6\frac{1}{8}$
$2\frac{1}{2}$	$8\frac{1}{4}$	$6\frac{1}{2}$	$10\frac{1}{4}$	$10\frac{1}{2}$	$\frac{1}{4}$	$14\frac{1}{2}$	$2\frac{1}{4}$	$18\frac{1}{2}$	$4\frac{1}{4}$	$22\frac{1}{2}$	$6\frac{1}{4}$
$2\frac{3}{4}$	$8\frac{3}{8}$	$6\frac{3}{4}$	$10\frac{3}{8}$	$10\frac{3}{4}$	$\frac{3}{8}$	$14\frac{3}{4}$	$2\frac{3}{8}$	$18\frac{3}{4}$	$4\frac{3}{8}$	$22\frac{3}{4}$	$6\frac{3}{8}$
3	$8\frac{1}{2}$	7	$10\frac{1}{2}$	11	$\frac{1}{2}$	15	$2\frac{1}{2}$	19	$4\frac{1}{2}$	23	$6\frac{1}{2}$
$3\frac{1}{4}$	$8\frac{5}{8}$	$7\frac{1}{4}$	$10\frac{5}{8}$	$11\frac{1}{4}$	$\frac{5}{8}$	$15\frac{1}{4}$	$2\frac{5}{8}$	$19\frac{1}{4}$	$4\frac{5}{8}$	$23\frac{1}{4}$	$6\frac{5}{8}$
$3\frac{1}{2}$	$8\frac{3}{4}$	$7\frac{1}{2}$	$10\frac{3}{4}$	$11\frac{1}{2}$	$\frac{3}{4}$	$15\frac{1}{2}$	$2\frac{3}{4}$	$19\frac{1}{2}$	$4\frac{3}{4}$	$23\frac{1}{2}$	$6\frac{3}{4}$
$3\frac{3}{4}$	$8\frac{7}{8}$	$7\frac{3}{4}$	$10\frac{7}{8}$	$11\frac{3}{4}$	$\frac{7}{8}$	$15\frac{3}{4}$	$2\frac{7}{8}$	$19\frac{3}{4}$	$4\frac{7}{8}$	$23\frac{3}{4}$	$6\frac{7}{8}$
4	IX	8	XI	12	I	16	III	20	V	24	VII

In a sundial whose plane is equidistant from the circle of the 14th hour reckoned from sunrise or sunset, the lines of any two hours whatever in this table are parallel, as we taught in the scholium to Proposition 22.

These three pages continue the set of hour-line concurrence tables belonging to the scholium of Proposition 22 (Book One). Each of the six tables carries a title naming the hour-line to which it belongs, and beneath it a note stating for which dial the tabulated hour-lines run parallel. In every table the sixteen rows are read across six column-pairs. The first column of each pair, **Horæ ab ortu vel occasu** ("hours reckoned from sunrise or sunset"), runs in quarter-hour steps through the whole twenty-four-hour circle; the second, **Horæ à meridie vel media nocte** ("hours reckoned from noon or midnight"), gives the concurrent hour whose line, on the dial named in the caption, is parallel to the first. Whole hours in the second column are printed as Roman numerals; a bare fraction there means that fraction past the twelfth hour.

The line of the 13th hour, reckoned from sunrise or sunset

(Linea horæ 13. ab ortu vel occasu.)

	Noo						Noo			Noo		
Rise	n	Rise	Noon	Rise	Noon	Rise	n	Rise	n	Rise	n	
$\frac{1}{4}$	$6\frac{5}{8}$	$4\frac{1}{4}$	$8\frac{5}{8}$	$8\frac{1}{4}$	$10\frac{5}{8}$	$12\frac{1}{4}$	$\frac{5}{8}$	$16\frac{1}{4}$	$2\frac{5}{8}$	$20\frac{1}{4}$	$4\frac{5}{8}$	
$\frac{1}{2}$	$6\frac{3}{4}$	$4\frac{1}{2}$	$8\frac{3}{4}$	$8\frac{1}{2}$	$10\frac{3}{4}$	$12\frac{1}{2}$	$\frac{3}{4}$	$16\frac{1}{2}$	$2\frac{3}{4}$	$20\frac{1}{2}$	$4\frac{3}{4}$	
$\frac{3}{4}$	$6\frac{7}{8}$	$4\frac{3}{4}$	$8\frac{7}{8}$	$8\frac{3}{4}$	$10\frac{7}{8}$	$12\frac{3}{4}$	$\frac{7}{8}$	$16\frac{3}{4}$	$2\frac{7}{8}$	$20\frac{3}{4}$	$4\frac{7}{8}$	
1	VII	5	IX	9	XI	13	I	17	III	21	V	
$1\frac{1}{4}$	$7\frac{1}{8}$	$5\frac{1}{4}$	$9\frac{1}{8}$	$9\frac{1}{4}$	$11\frac{1}{8}$	$13\frac{1}{4}$	$1\frac{1}{8}$	$17\frac{1}{4}$	$3\frac{1}{8}$	$21\frac{1}{4}$	$5\frac{1}{8}$	
$1\frac{1}{2}$	$7\frac{1}{4}$	$5\frac{1}{2}$	$9\frac{1}{4}$	$9\frac{1}{2}$	$11\frac{1}{4}$	$13\frac{1}{2}$	$1\frac{1}{4}$	$17\frac{1}{2}$	$3\frac{1}{4}$	$21\frac{1}{2}$	$5\frac{1}{4}$	
$1\frac{3}{4}$	$7\frac{3}{8}$	$5\frac{3}{4}$	$9\frac{3}{8}$	$9\frac{3}{4}$	$11\frac{3}{8}$	$13\frac{3}{4}$	$1\frac{3}{8}$	$17\frac{3}{4}$	$3\frac{3}{8}$	$21\frac{3}{4}$	$5\frac{3}{8}$	
2	$7\frac{1}{2}$	6	$9\frac{1}{2}$	10	$11\frac{1}{2}$	14	$1\frac{1}{2}$	18	$3\frac{1}{2}$	22	$5\frac{1}{2}$	
$2\frac{1}{4}$	$7\frac{5}{8}$	$6\frac{1}{4}$	$9\frac{5}{8}$	$10\frac{1}{4}$	$11\frac{5}{8}$	$14\frac{1}{4}$	$1\frac{5}{8}$	$18\frac{1}{4}$	$3\frac{5}{8}$	$22\frac{1}{4}$	$5\frac{5}{8}$	
$2\frac{1}{2}$	$7\frac{3}{4}$	$6\frac{1}{2}$	$9\frac{3}{4}$	$10\frac{1}{2}$	$11\frac{3}{4}$	$14\frac{1}{2}$	$1\frac{3}{4}$	$18\frac{1}{2}$	$3\frac{3}{4}$	$22\frac{1}{2}$	$5\frac{3}{4}$	
$2\frac{3}{4}$	$7\frac{7}{8}$	$6\frac{3}{4}$	$9\frac{7}{8}$	$10\frac{3}{4}$	$11\frac{7}{8}$	$14\frac{3}{4}$	$1\frac{7}{8}$	$18\frac{3}{4}$	$3\frac{7}{8}$	$22\frac{3}{4}$	$5\frac{7}{8}$	
3	VIII	7	X	11	XII	15	II	19	IV	23	VI	
$3\frac{1}{4}$	$8\frac{1}{8}$	$7\frac{1}{4}$	$10\frac{1}{8}$	$11\frac{1}{4}$	$\frac{1}{8}$	$15\frac{1}{4}$	$2\frac{1}{8}$	$19\frac{1}{4}$	$4\frac{1}{8}$	$23\frac{1}{4}$	$6\frac{1}{8}$	
$3\frac{1}{2}$	$8\frac{1}{4}$	$7\frac{1}{2}$	$10\frac{1}{4}$	$11\frac{1}{2}$	$\frac{1}{4}$	$15\frac{1}{2}$	$2\frac{1}{4}$	$19\frac{1}{2}$	$4\frac{1}{4}$	$23\frac{1}{2}$	$6\frac{1}{4}$	
$3\frac{3}{4}$	$8\frac{3}{8}$	$7\frac{3}{4}$	$10\frac{3}{8}$	$11\frac{3}{4}$	$\frac{3}{8}$	$15\frac{3}{4}$	$2\frac{3}{8}$	$19\frac{3}{4}$	$4\frac{3}{8}$	$23\frac{3}{4}$	$6\frac{3}{8}$	

	Noo						Noo			Noo		
Rise	n	Rise	Noon	Rise	Noon	Rise	n	Rise	n	Rise	n	Rise
4	8½	8	10½	12	½	16	2½	20	4½	24	6½	

In a sundial whose plane is equidistant from the circle of the 13th hour reckoned from sunrise or sunset, the lines of any two hours whatever in this table are parallel, as is established from the scholium to Proposition 22.

The line of the 11th hour, reckoned from sunrise or sunset

(Linea horæ 11. ab ortu vel occasu.)

	Noo		Noo						Noo		Noo
Rise	n	Rise	n	Rise	Noon	Rise	Noon	Rise	n	Rise	n
¼	5⅝	4¼	7⅝	8¼	9⅝	12¼	11⅝	16¼	1⅝	20¼	3⅝
½	5¾	4½	7¾	8½	9¾	12½	11¾	16½	1¾	20½	3¾
¾	5⅞	4¾	7⅞	8¾	9⅞	12¾	11⅞	16¾	1⅞	20¾	3⅞
1	VI	5	VIII	9	X	13	XII	17	II	21	IV
1¼	6⅞	5¼	8⅞	9¼	10⅞	13¼	⅞	17¼	2⅞	21¼	4⅞
1½	6¼	5½	8¼	9½	10¼	13½	¼	17½	2¼	21½	4¼
1¾	6⅜	5¾	8⅜	9¾	10⅜	13¾	⅜	17¾	2⅜	21¾	4⅜
2	6½	6	8½	10	10½	14	½	18	2½	22	4½
2¼	6⅝	6¼	8⅝	10¼	10⅝	14¼	⅝	18¼	2⅝	22¼	4⅝
2½	6¾	6½	8¾	10½	10¾	14½	¾	18½	2¾	22½	4¾
2¾	6⅞	6¾	8⅞	10¾	10⅞	14¾	⅞	18¾	2⅞	22¾	4⅞
3	VII	7	IX	11	XI	15	I	19	III	23	V
3¼	7⅞	7¼	9⅞	11¼	11⅞	15¼	1⅞	19¼	3⅞	23¼	5⅞
3½	7¼	7½	9¼	11½	11¼	15½	1¼	19½	3¼	23½	5¼
3¾	7⅜	7¾	9⅜	11¾	11⅜	15¾	1⅜	19¾	3⅜	23¾	5⅜
4	7½	8	9½	12	11½	16	1½	20	3½	24	5½

In a sundial whose plane is equidistant from the circle of the 14th [recte: 11th] hour reckoned from sunrise or sunset, the lines of any two hours whatever in this table are parallel, as is evident from the scholium to Proposition 22.

The line of the 10th hour, reckoned from sunrise or sunset

(*Linea horæ 10. ab ortu vel occasu.*)

Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon
$\frac{1}{4}$	$5\frac{1}{8}$	$4\frac{1}{4}$	$7\frac{1}{8}$	$8\frac{1}{4}$	$9\frac{1}{8}$	$12\frac{1}{4}$	$11\frac{1}{8}$	$16\frac{1}{4}$	$1\frac{1}{8}$	$20\frac{1}{4}$	$3\frac{1}{8}$
$\frac{1}{2}$	$5\frac{1}{4}$	$4\frac{1}{2}$	$7\frac{1}{4}$	$8\frac{1}{2}$	$9\frac{1}{4}$	$12\frac{1}{2}$	$11\frac{1}{4}$	$16\frac{1}{2}$	$1\frac{1}{4}$	$20\frac{1}{2}$	$3\frac{1}{4}$
$\frac{3}{4}$	$5\frac{3}{8}$	$4\frac{3}{4}$	$7\frac{3}{8}$	$8\frac{3}{4}$	$9\frac{3}{8}$	$12\frac{3}{4}$	$11\frac{3}{8}$	$16\frac{3}{4}$	$1\frac{3}{8}$	$20\frac{3}{4}$	$3\frac{3}{8}$
1	$5\frac{1}{2}$	5	$7\frac{1}{2}$	9	$9\frac{1}{2}$	13	$11\frac{1}{2}$	17	$1\frac{1}{2}$	21	$3\frac{1}{2}$
$1\frac{1}{4}$	$5\frac{5}{8}$	$5\frac{1}{4}$	$7\frac{5}{8}$	$9\frac{1}{4}$	$9\frac{5}{8}$	$13\frac{1}{4}$	$11\frac{5}{8}$	$17\frac{1}{4}$	$1\frac{5}{8}$	$21\frac{1}{4}$	$3\frac{5}{8}$
$1\frac{1}{2}$	$5\frac{3}{4}$	$5\frac{1}{2}$	$7\frac{3}{4}$	$9\frac{1}{2}$	$9\frac{3}{4}$	$13\frac{1}{2}$	$11\frac{3}{4}$	$17\frac{1}{2}$	$1\frac{3}{4}$	$21\frac{1}{2}$	$3\frac{3}{4}$
$1\frac{3}{4}$	$5\frac{7}{8}$	$5\frac{3}{4}$	$7\frac{7}{8}$	$9\frac{3}{4}$	$9\frac{7}{8}$	$13\frac{3}{4}$	$11\frac{7}{8}$	$17\frac{3}{4}$	$1\frac{7}{8}$	$21\frac{3}{4}$	$3\frac{7}{8}$
2	VI	6	VIII	10	X	14	XII	18	II	22	IV
$2\frac{1}{4}$	$6\frac{1}{8}$	$6\frac{1}{4}$	$8\frac{1}{8}$	$10\frac{1}{4}$	$10\frac{1}{8}$	$14\frac{1}{4}$	$\frac{1}{8}$	$18\frac{1}{4}$	$2\frac{1}{8}$	$22\frac{1}{4}$	$4\frac{1}{8}$
$2\frac{1}{2}$	$6\frac{1}{4}$	$6\frac{1}{2}$	$8\frac{1}{4}$	$10\frac{1}{2}$	$10\frac{1}{4}$	$14\frac{1}{2}$	$\frac{1}{4}$	$18\frac{1}{2}$	$2\frac{1}{4}$	$22\frac{1}{2}$	$4\frac{1}{4}$
$2\frac{3}{4}$	$6\frac{3}{8}$	$6\frac{3}{4}$	$8\frac{3}{8}$	$10\frac{3}{4}$	$10\frac{3}{8}$	$14\frac{3}{4}$	$\frac{3}{8}$	$18\frac{3}{4}$	$2\frac{3}{8}$	$22\frac{3}{4}$	$4\frac{3}{8}$
3	$6\frac{1}{2}$	7	$8\frac{1}{2}$	11	$10\frac{1}{2}$	15	$\frac{1}{2}$	19	$2\frac{1}{2}$	23	$4\frac{1}{2}$
$3\frac{1}{4}$	$6\frac{5}{8}$	$7\frac{1}{4}$	$8\frac{5}{8}$	$11\frac{1}{4}$	$10\frac{5}{8}$	$15\frac{1}{4}$	$\frac{5}{8}$	$19\frac{1}{4}$	$2\frac{5}{8}$	$23\frac{1}{4}$	$4\frac{5}{8}$
$3\frac{1}{2}$	$6\frac{3}{4}$	$7\frac{1}{2}$	$8\frac{3}{4}$	$11\frac{1}{2}$	$10\frac{3}{4}$	$15\frac{1}{2}$	$\frac{3}{4}$	$19\frac{1}{2}$	$2\frac{3}{4}$	$23\frac{1}{2}$	$4\frac{3}{4}$
$3\frac{3}{4}$	$6\frac{7}{8}$	$7\frac{3}{4}$	$8\frac{7}{8}$	$11\frac{3}{4}$	$10\frac{7}{8}$	$15\frac{3}{4}$	$\frac{7}{8}$	$19\frac{3}{4}$	$2\frac{7}{8}$	$23\frac{3}{4}$	$4\frac{7}{8}$
4	VII	8	IX	12	XI	16	I	20	III	24	V

In a sundial whose plane is equidistant from the circle of the 10th hour reckoned from sunrise or sunset, the lines of any two hours whatever in this table are parallel, as is established from the scholium to Proposition 22.

The line of the 9th hour, reckoned from sunrise or sunset

(*Linea horæ 9. ab ortu vel occasu.*)

Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon
$\frac{1}{4}$	$4\frac{5}{8}$	$4\frac{1}{4}$	$6\frac{5}{8}$	$8\frac{1}{4}$	$8\frac{5}{8}$	$12\frac{1}{4}$	$10\frac{5}{8}$	$16\frac{1}{4}$	$\frac{5}{8}$	$20\frac{1}{4}$	$2\frac{5}{8}$
$\frac{1}{2}$	$4\frac{3}{4}$	$4\frac{1}{2}$	$6\frac{3}{4}$	$8\frac{1}{2}$	$8\frac{3}{4}$	$12\frac{1}{2}$	$10\frac{3}{4}$	$16\frac{1}{2}$	$\frac{3}{4}$	$20\frac{1}{2}$	$2\frac{3}{4}$
$\frac{3}{4}$	$4\frac{7}{8}$	$4\frac{3}{4}$	$6\frac{7}{8}$	$8\frac{3}{4}$	$8\frac{7}{8}$	$12\frac{3}{4}$	$10\frac{7}{8}$	$16\frac{3}{4}$	$\frac{7}{8}$	$20\frac{3}{4}$	$2\frac{7}{8}$

	Noo		Noo						Noo		Noo
Rise	n	Rise	n	Rise	Noon	Rise	Noon	Rise	n	Rise	n
1	V	5	VII	9	IX	13	XI	17	I	21	III
1¼	5⅛	5¼	7⅛	9¼	9⅛	13¼	11⅛	17¼	1⅛	21¼	3⅛
1½	5¼	5½	7¼	9½	9¼	13½	11¼	17½	1¼	21½	3¼
1¾	5⅜	5¾	7⅜	9¾	9⅜	13¾	11⅜	17¾	1⅜	21¾	3⅜
2	5½	6	7½	10	9½	14	11½	18	1½	22	3½
2¼	5⅝	6¼	7⅝	10¼	9⅝	14¼	11⅝	18¼	1⅝	22¼	3⅝
2½	5¾	6½	7¾	10½	9¾	14½	11¾	18½	1¾	22½	3¾
2¾	5⅞	6¾	7⅞	10¾	9⅞	14¾	11⅞	18¾	1⅞	22¾	3⅞
3	VI	7	VIII	11	X	15	XII	19	II	23	IV
3¼	6⅛	7¼	8⅛	11¼	10⅛	15¼	⅛	19¼	2⅛	23¼	4⅛
3½	6¼	7½	8¼	11½	10¼	15½	¼	19½	2¼	23½	4¼
3¾	6⅜	7¾	8⅜	11¾	10⅜	15¾	⅜	19¾	2⅜	23¾	4⅜
4	6½	8	8½	12	10½	16	½	20	2½	24	4½

In a sundial whose plane is equidistant from the circle of the 9th hour reckoned from sunrise or sunset, the lines of any two hours whatever in this table are parallel, as is established from the scholium to Proposition 22.

The line of the 8th hour, reckoned from sunrise or sunset

(Linea horæ 8. ab ortu vel occasu.)

	Noo		Noo		Noo				Noo		Noo
Rise	n	Rise	n	Rise	n	Rise	Noon	Rise	n	Rise	n
¼	4⅛	4¼	6⅛	8¼	8⅛	12¼	10⅛	16¼	⅛	20¼	2⅛
½	4¼	4½	6¼	8½	8¼	12½	10¼	16½	¼	20½	2¼
¾	4⅜	4¾	6⅜	8¾	8⅜	12¾	10⅜	16¾	⅜	20¾	2⅜
1	4½	5	6½	9	8½	13	10½	17	½	21	2½
1¼	4⅝	5¼	6⅝	9¼	8⅝	13¼	10⅝	17¼	⅝	21¼	2⅝
1½	4¾	5½	6¾	9½	8¾	13½	10¾	17½	¾	21½	2¾
1¾	4⅞	5¾	6⅞	9¾	8⅞	13¾	10⅞	17¾	⅞	21¾	2⅞
2	V	6	VII	10	IX	14	XI	18	I	22	III
2¼	5⅛	6¼	7⅛	10¼	9⅛	14¼	11⅛	18¼	1⅛	22¼	3⅛

Rise	Noo n	Rise	Noo n	Rise	Noo n	Rise	Noon	Rise	Noo n	Rise	Noo n
2½	5¼	6½	7¼	10½	9¼	14½	11¼	18½	1¼	22½	3¼
2¾	5⅜	6¾	7⅜	10¾	9⅜	14¾	11⅜	18¾	1⅜	22¾	3⅜
3	5½	7	7½	11	9½	15	11½	19	1½	23	3½
3¼	5⅝	7¼	7⅝	11¼	9⅝	15¼	11⅝	19¼	1⅝	23¼	3⅝
3½	5¾	7½	7¾	11½	9¾	15½	11¾	19½	1¾	23½	3¾
3¾	5⅞	7¾	7⅞	11¾	9⅞	15¾	11⅞	19¾	1⅞	23¾	3⅞
4	VI	8	VIII	12	X	16	XII	20	II	24	IV

In a sundial whose plane is equidistant from the circle of the 8th hour reckoned from sunrise or sunset, the lines of any two hours whatever in this table are parallel, as is plain from the scholium to Proposition 22.

The line of the 7th hour, reckoned from sunrise or sunset

(Linea horæ 7. ab ortu vel occasu.)

Rise	Noo n	Rise	Noo n	Rise	Noo n	Rise	Noon	Rise	Noon	Rise	Noo n
¼	3⅝	4¼	5⅝	8¼	7⅝	12¼	9⅝	16¼	11⅝	20¼	1⅝
½	3¾	4½	5¾	8½	7¾	12½	9¾	16½	11¾	20½	1¾
¾	3⅞	4¾	5⅞	8¾	7⅞	12¾	9⅞	16¾	11⅞	20¾	1⅞
1	IV	5	VI	9	VIII	13	X	17	XII	21	II
1¼	4⅞	5¼	6⅞	9¼	8⅞	13¼	10⅞	17¼	⅞	21¼	2⅞
1½	4¼	5½	6¼	9½	8¼	13½	10¼	17½	¼	21½	2¼
1¾	4⅜	5¾	6⅜	9¾	8⅜	13¾	10⅜	17¾	⅜	21¾	2⅜
2	4½	6	6½	10	8½	14	10½	18	½	22	2½
2¼	4⅝	6¼	6⅝	10¼	8⅝	14¼	10⅝	18¼	⅝	22¼	2⅝
2½	4¾	6½	6¾	10½	8¾	14½	10¾	18½	¾	22½	2¾
2¾	4⅞	6¾	6⅞	10¾	8⅞	14¾	10⅞	18¾	⅞	22¾	2⅞
3	V	7	VII	11	IX	15	XI	19	I	23	III
3¼	5⅞	7¼	7⅞	11¼	9⅞	15¼	11⅞	19¼	1⅞	23¼	3⅞
3½	5¾	7½	7¾	11½	9¾	15½	11¾	19½	1¾	23½	3¾
3¾	5⅞	7¾	7⅞	11¾	9⅞	15¾	11⅞	19¾	1⅞	23¾	3⅞

Rise	Noo n	Rise	Noo n	Rise	Noo n	Rise	Noon	Rise	Noon	Rise	Noo n
4	5½	8	7½	12	9½	16	11½	20	1½	24	3½

In a sundial whose plane is equidistant from the circle of the 7th hour reckoned from sunrise or sunset, the lines of any two hours whatever in this table are parallel, as we demonstrated in the scholium to Proposition 22.

These three pages continue the set of hour-line concurrence tables belonging to the scholium to Proposition 22 of Book One. Each page carries two tables; together the six tables here cover the lines of the 6th, 5th, 4th, 3rd, 2nd, and 1st hour (reckoned from sunrise or sunset). No running prose or new chapter rubric appears — the pages are wholly occupied by the tables and their explanatory notes.

Each table is a 16-row by 12-column grid. The twelve columns form six read-continuous pairs; within each pair the left column is headed *Horæ ab ortu vel occasu* ("hours from sunrise or sunset," here abbreviated **Rise**) and the right column *Horæ à meridie vel media nocte* ("hours from noon or midnight," here abbreviated **Noon**). Read as a single sequence, the six pairs run the **Rise** argument continuously from ¼ up to 24 in quarter-hour steps, each with its concurrent **Noon** value. Whole hours are printed as Roman numerals; a bare fraction in the **Noon** column denotes the corresponding part past the twelve-hour mark (i.e. 12 + fraction).

Table: *Linea horæ 6. ab ortu vel occasu.* (The line of the 6th hour from sunrise or sunset)

Rise = *Horæ ab ortu vel occasu*; **Noon** = *Horæ à meridie vel media nocte*.

Ris e	No on	Ris e	No on	Ris e	No on	Ris e	Noo n	Ris e	Noon	Ris e	No on
¼	3⅛	4¼	5⅛	8¼	7⅛	12 ¼	9⅛	16 ¼	11⅛	20 ¼	1⅛
½	3¼	4½	5¼	8½	7¼	12 ½	9¼	16 ½	11¼	20 ½	1¼
¾	3⅜	4¾	5⅜	8¾	7⅜	12 ¾	9⅜	16 ¾	11⅜	20 ¾	1⅜
1	3½	5	5½	9	7½	13	9½	17	11½	21	1½
1¼	3⅝	5¼	5⅝	9¼	7⅝	13	9⅝	17	11⅝	21	1⅝

Ris e	No on	Ris e	No on	Ris e	No on	Ris e	No on	Ris e	No on	Ris e	No on
						$\frac{1}{4}$		$\frac{1}{4}$			$\frac{1}{4}$
$1\frac{1}{2}$	$3\frac{3}{4}$	$5\frac{1}{2}$	$5\frac{3}{4}$	$9\frac{1}{2}$	$7\frac{3}{4}$	13	$9\frac{3}{4}$	17	$3\frac{3}{4}$ [recte: $11\frac{3}{4}$]	21	$1\frac{3}{4}$
						$\frac{1}{2}$		$\frac{1}{2}$			$\frac{1}{2}$
$1\frac{3}{4}$	$3\frac{7}{8}$	$5\frac{3}{4}$	$5\frac{7}{8}$	$9\frac{3}{4}$	$7\frac{7}{8}$	13	$9\frac{7}{8}$	17	$3\frac{7}{8}$ [recte: $11\frac{7}{8}$]	21	$1\frac{7}{8}$
						$\frac{3}{4}$		$\frac{3}{4}$			$\frac{3}{4}$
2	IV	6	VI	10	VII	14	X	18	XII	22	II
					I						
$2\frac{1}{4}$	$4\frac{1}{8}$	$6\frac{1}{4}$	$6\frac{1}{8}$	10	$8\frac{1}{8}$	14	$10\frac{1}{8}$	18	$\frac{1}{8}$	22	$2\frac{1}{8}$
				$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$			$\frac{1}{4}$
$2\frac{1}{2}$	$4\frac{1}{4}$	$6\frac{1}{2}$	$6\frac{1}{4}$	10	$8\frac{1}{4}$	14	$10\frac{1}{4}$	18	$\frac{1}{4}$	22	$2\frac{1}{4}$
				$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$			$\frac{1}{2}$
$2\frac{3}{4}$	$4\frac{3}{8}$	$6\frac{3}{4}$	$6\frac{3}{8}$	10	$8\frac{3}{8}$	14	$10\frac{3}{8}$	18	$\frac{3}{8}$	22	$2\frac{3}{8}$
				$\frac{3}{4}$		$\frac{3}{4}$		$\frac{3}{4}$			$\frac{3}{4}$
3	$4\frac{1}{2}$	7	$6\frac{1}{2}$	11	$8\frac{1}{2}$	15	$10\frac{1}{2}$	19	$\frac{1}{2}$	23	$2\frac{1}{2}$
$3\frac{1}{4}$	$4\frac{5}{8}$	$7\frac{1}{4}$	$6\frac{5}{8}$	11	$8\frac{5}{8}$	15	$10\frac{5}{8}$	19	$\frac{5}{8}$	23	$2\frac{5}{8}$
				$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$			$\frac{1}{4}$
$3\frac{1}{2}$	$4\frac{3}{4}$	$7\frac{1}{2}$	$6\frac{3}{4}$	11	$8\frac{3}{4}$	15	$10\frac{3}{4}$	19	$\frac{3}{4}$	23	$2\frac{3}{4}$
				$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$			$\frac{1}{2}$
$3\frac{3}{4}$	$4\frac{7}{8}$	$7\frac{3}{4}$	$6\frac{7}{8}$	11	$8\frac{7}{8}$	15	$10\frac{7}{8}$	19	$\frac{7}{8}$	23	$2\frac{7}{8}$
				$\frac{3}{4}$		$\frac{3}{4}$		$\frac{3}{4}$			$\frac{3}{4}$
4	V	8	VII	12	IX	16	XI	20	I	24	III

In a sundial whose plane lies equidistant from the circle of the 6th hour[?] reckoned from sunrise or sunset, the lines of any two hours in this table are parallel, as is clear from the scholium to Proposition 22.

Table: *Linea horæ 5. ab ortu vel occasu.* (The line of the 5th hour from sunrise or sunset)

Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon	Rise	Noon
$\frac{1}{4}$	$2\frac{5}{8}$	$4\frac{1}{4}$	$4\frac{5}{8}$	$8\frac{1}{4}$	$6\frac{5}{8}$	$12\frac{1}{4}$	$8\frac{5}{8}$	$16\frac{1}{4}$	$10\frac{5}{8}$	$20\frac{1}{4}$	$\frac{5}{8}$
$\frac{1}{2}$	$2\frac{3}{4}$	$4\frac{1}{2}$	$4\frac{3}{4}$	$8\frac{1}{2}$	$6\frac{3}{4}$	$12\frac{1}{2}$	$8\frac{3}{4}$	$16\frac{1}{2}$	$10\frac{3}{4}$	$20\frac{1}{2}$	$\frac{3}{4}$
$\frac{3}{4}$	$2\frac{7}{8}$	$4\frac{3}{4}$	$4\frac{7}{8}$	$8\frac{3}{4}$	$6\frac{7}{8}$	$12\frac{3}{4}$	$8\frac{7}{8}$	$16\frac{3}{4}$	$10\frac{7}{8}$	$20\frac{3}{4}$	$\frac{7}{8}$
1	III	5	V	9	VII	13	IX	17	XI	21	I

Rise	Noo n	Rise	Noo n	Rise	Noo n	Rise	Noon	Rise	Noon	Rise	Noo n
$1\frac{1}{4}$	$3\frac{1}{8}$	$5\frac{1}{4}$	$5\frac{1}{8}$	$9\frac{1}{4}$	$7\frac{1}{8}$	$13\frac{1}{4}$	$9\frac{1}{8}$	$17\frac{1}{4}$	$11\frac{1}{8}$	$21\frac{1}{4}$	$1\frac{1}{8}$
$1\frac{1}{2}$	$3\frac{1}{4}$	$5\frac{1}{2}$	$5\frac{1}{4}$	$9\frac{1}{2}$	$7\frac{1}{4}$	$13\frac{1}{2}$	$9\frac{1}{4}$	$17\frac{1}{2}$	$11\frac{1}{4}$	$21\frac{1}{2}$	$1\frac{1}{4}$
$1\frac{3}{4}$	$3\frac{3}{8}$	$5\frac{3}{4}$	$5\frac{3}{8}$	$9\frac{3}{4}$	$7\frac{3}{8}$	$13\frac{3}{4}$	$9\frac{3}{8}$	$17\frac{3}{4}$	$11\frac{3}{8}$	$21\frac{3}{4}$	$1\frac{3}{8}$
2	$3\frac{1}{2}$	6	$5\frac{1}{2}$	10	$7\frac{1}{2}$	14	$9\frac{1}{2}$	18	$11\frac{1}{2}$	22	$1\frac{1}{2}$
$2\frac{1}{4}$	$3\frac{5}{8}$	$6\frac{1}{4}$	$5\frac{5}{8}$	$10\frac{1}{4}$	$7\frac{5}{8}$	$14\frac{1}{4}$	$9\frac{5}{8}$	$18\frac{1}{4}$	$11\frac{5}{8}$	$22\frac{1}{4}$	$1\frac{5}{8}$
$2\frac{1}{2}$	$3\frac{3}{4}$	$6\frac{1}{2}$	$5\frac{3}{4}$	$10\frac{1}{2}$	$7\frac{3}{4}$	$14\frac{1}{2}$	$9\frac{3}{4}$	$18\frac{1}{2}$	$11\frac{3}{4}$	$22\frac{1}{2}$	$1\frac{3}{4}$
$2\frac{3}{4}$	$3\frac{7}{8}$	$6\frac{3}{4}$	$5\frac{7}{8}$	$10\frac{3}{4}$	$7\frac{7}{8}$	$14\frac{3}{4}$	$9\frac{7}{8}$	$18\frac{3}{4}$	$11\frac{7}{8}$	$22\frac{3}{4}$	$1\frac{7}{8}$
3	IV	7	VI	11	VIII	15	X	19	XII	23	II
$3\frac{1}{4}$	$4\frac{1}{8}$	$7\frac{1}{4}$	$6\frac{1}{8}$	$11\frac{1}{4}$	$8\frac{1}{8}$	$15\frac{1}{4}$	$10\frac{1}{8}$	$19\frac{1}{4}$	$\frac{1}{8}$	$23\frac{1}{4}$	$2\frac{1}{8}$
$3\frac{1}{2}$	$4\frac{1}{4}$	$7\frac{1}{2}$	$6\frac{1}{4}$	$11\frac{1}{2}$	$8\frac{1}{4}$	$15\frac{1}{2}$	$10\frac{1}{4}$	$19\frac{1}{2}$	$\frac{1}{4}$	$23\frac{1}{2}$	$2\frac{1}{4}$
$3\frac{3}{4}$	$4\frac{3}{8}$	$7\frac{3}{4}$	$6\frac{3}{8}$	$11\frac{3}{4}$	$8\frac{3}{8}$	$15\frac{3}{4}$	$10\frac{3}{8}$	$19\frac{3}{4}$	$\frac{3}{8}$	$23\frac{3}{4}$	$2\frac{3}{8}$
4	$4\frac{1}{2}$	8	$6\frac{1}{2}$	12	$8\frac{1}{2}$	16	$10\frac{1}{2}$	20	$\frac{1}{2}$	24	$2\frac{1}{2}$

In a sundial whose plane lies equidistant from the circle of the 5th hour reckoned from sunrise or sunset, the lines of any two hours in this table are parallel, as we showed by the scholium to Proposition 22.

Table: *Linea horæ 4. ab ortu vel occasu.* (The line of the 4th hour from sunrise or sunset)

Rise	Noo n	Rise	Noo n	Rise	Noo n	Rise	Noo n	Rise	Noon	Rise	Noon
$\frac{1}{4}$	$2\frac{1}{8}$	$4\frac{1}{4}$	$4\frac{1}{8}$	$8\frac{1}{4}$	$6\frac{1}{8}$	$12\frac{1}{4}$	$8\frac{1}{8}$	$16\frac{1}{4}$	$10\frac{1}{8}$	$20\frac{1}{4}$	$\frac{1}{8}$
$\frac{1}{2}$	$2\frac{1}{4}$	$4\frac{1}{2}$	$4\frac{1}{4}$	$8\frac{1}{2}$	$6\frac{1}{4}$	$12\frac{1}{2}$	$8\frac{1}{4}$	$16\frac{1}{2}$	$10\frac{1}{4}$	$20\frac{1}{2}$	$\frac{1}{4}$
$\frac{3}{4}$	$2\frac{3}{8}$	$4\frac{3}{4}$	$4\frac{3}{8}$	$8\frac{3}{4}$	$6\frac{3}{8}$	$12\frac{3}{4}$	$8\frac{3}{8}$	$16\frac{3}{4}$	$10\frac{3}{8}$	$20\frac{3}{4}$	$\frac{3}{8}$
1	$2\frac{1}{2}$	5	$4\frac{1}{2}$	9	$6\frac{1}{2}$	13	$8\frac{1}{2}$	17	$10\frac{1}{2}$	21	$\frac{1}{2}$
$1\frac{1}{4}$	$2\frac{5}{8}$	$5\frac{1}{4}$	$4\frac{5}{8}$	$9\frac{1}{4}$	$6\frac{5}{8}$	$13\frac{1}{4}$	$8\frac{5}{8}$	$17\frac{1}{4}$	$10\frac{5}{8}$	$21\frac{1}{4}$	$\frac{5}{8}$
$1\frac{1}{2}$	$2\frac{3}{4}$	$5\frac{1}{2}$	$4\frac{3}{4}$	$9\frac{1}{2}$	$6\frac{3}{4}$	$13\frac{1}{2}$	$8\frac{3}{4}$	$17\frac{1}{2}$	$10\frac{3}{4}$	$21\frac{1}{2}$	$\frac{3}{4}$
$1\frac{3}{4}$	$2\frac{7}{8}$	$5\frac{3}{4}$	$4\frac{7}{8}$	$9\frac{3}{4}$	$6\frac{7}{8}$	$13\frac{3}{4}$	$8\frac{7}{8}$	$17\frac{3}{4}$	$10\frac{7}{8}$	$21\frac{3}{4}$	$\frac{7}{8}$
2	III	6	V	10	VII	14	IX	18	XI	22	I
$2\frac{1}{4}$	$3\frac{1}{8}$	$6\frac{1}{4}$	$5\frac{1}{8}$	$10\frac{1}{4}$	$7\frac{1}{8}$	$14\frac{1}{4}$	$9\frac{1}{8}$	$18\frac{1}{4}$	$11\frac{1}{8}$	$22\frac{1}{4}$	$1\frac{1}{8}$
$2\frac{1}{2}$	$3\frac{1}{4}$	$6\frac{1}{2}$	$5\frac{1}{4}$	$10\frac{1}{2}$	$7\frac{1}{4}$	$14\frac{1}{2}$	$9\frac{1}{4}$	$18\frac{1}{2}$	$11\frac{1}{4}$	$22\frac{1}{2}$	$1\frac{1}{4}$

	Noo			Noo			Noo			Noo	
Rise	n	Rise	n	Rise	n	Rise	n	Rise	Noon	Rise	Noon
$2\frac{3}{4}$	$3\frac{3}{8}$	$6\frac{3}{4}$	$5\frac{3}{8}$	$10\frac{3}{4}$	$7\frac{3}{8}$	$14\frac{3}{4}$	$9\frac{3}{8}$	$18\frac{3}{4}$	$11\frac{3}{8}$	$22\frac{3}{4}$	$1\frac{3}{8}$
3	$3\frac{1}{2}$	7	$5\frac{1}{2}$	11	$7\frac{1}{2}$	15	$9\frac{1}{2}$	19	$11\frac{1}{2}$	23	$1\frac{1}{2}$
$3\frac{1}{4}$	$3\frac{5}{8}$	$7\frac{1}{4}$	$5\frac{5}{8}$	$11\frac{1}{4}$	$7\frac{5}{8}$	$15\frac{1}{4}$	$9\frac{5}{8}$	$19\frac{1}{4}$	$11\frac{5}{8}$	$23\frac{1}{4}$	$1\frac{5}{8}$
$3\frac{1}{2}$	$3\frac{3}{4}$	$7\frac{1}{2}$	$5\frac{3}{4}$	$11\frac{1}{2}$	$7\frac{3}{4}$	$15\frac{1}{2}$	$9\frac{3}{4}$	$19\frac{1}{2}$	$11\frac{3}{4}$	$23\frac{1}{2}$	$1\frac{3}{4}$
$3\frac{3}{4}$	$3\frac{7}{8}$	$7\frac{3}{4}$	$5\frac{7}{8}$	$11\frac{3}{4}$	$7\frac{7}{8}$	$15\frac{3}{4}$	$9\frac{7}{8}$	$19\frac{3}{4}$	$11\frac{7}{8}$	$23\frac{3}{4}$	$1\frac{7}{8}$ [?]
4	IV	8	VI	12	VIII	16	X	20	XII	24	II

In a sundial whose plane lies equidistant from the circle of the 4th hour reckoned from sunrise or sunset, the lines of any two hours in this table are parallel, as [is clear] from the scholium to Proposition 22.

Table: *Linea horæ 3. ab ortu vel occasu.* (The line of the 3rd hour from sunrise or sunset)

	Noo			Noo			Noo			Noo	
Rise	n	Rise	n	Rise	n	Rise	n	Rise	Noon	Rise	Noon
$\frac{1}{4}$	$1\frac{5}{8}$	$4\frac{1}{4}$	$3\frac{5}{8}$	$8\frac{1}{4}$	$5\frac{5}{8}$	$12\frac{1}{4}$	$7\frac{5}{8}$	$16\frac{1}{4}$	$9\frac{5}{8}$	$20\frac{1}{4}$	$11\frac{5}{8}$
$\frac{1}{2}$	$1\frac{3}{4}$	$4\frac{1}{2}$	$3\frac{3}{4}$	$8\frac{1}{2}$	$5\frac{3}{4}$	$12\frac{1}{2}$	$7\frac{3}{4}$	$16\frac{1}{2}$	$9\frac{3}{4}$	$20\frac{1}{2}$	$11\frac{3}{4}$
$\frac{3}{4}$	$1\frac{7}{8}$	$4\frac{3}{4}$	$3\frac{7}{8}$	$8\frac{3}{4}$	$5\frac{7}{8}$	$12\frac{3}{4}$	$7\frac{7}{8}$	$16\frac{3}{4}$	$9\frac{7}{8}$	$20\frac{3}{4}$	$11\frac{7}{8}$
1	II	5	IV	9	VI	13	VIII	17	X	21	XII
$1\frac{1}{4}$	$2\frac{1}{8}$	$5\frac{1}{4}$	$4\frac{1}{8}$	$9\frac{1}{4}$	$6\frac{1}{8}$	$13\frac{1}{4}$	$8\frac{1}{8}$	$17\frac{1}{4}$	$10\frac{1}{8}$	$21\frac{1}{4}$	$\frac{1}{8}$
$1\frac{1}{2}$	$2\frac{1}{4}$	$5\frac{1}{2}$	$4\frac{1}{4}$	$9\frac{1}{2}$	$6\frac{1}{4}$	$13\frac{1}{2}$	$8\frac{1}{4}$	$17\frac{1}{2}$	$10\frac{1}{4}$	$21\frac{1}{2}$	$\frac{1}{4}$
$1\frac{3}{4}$	$2\frac{3}{8}$	$5\frac{3}{4}$	$4\frac{3}{8}$	$9\frac{3}{4}$	$6\frac{3}{8}$	$13\frac{3}{4}$	$8\frac{3}{8}$	$17\frac{3}{4}$	$10\frac{3}{8}$	$21\frac{3}{4}$	$\frac{3}{8}$
2	$2\frac{1}{2}$	6	$4\frac{1}{2}$	10	$6\frac{1}{2}$	14	$8\frac{1}{2}$	18	$10\frac{1}{2}$	22	$\frac{1}{2}$
$2\frac{1}{4}$	$2\frac{5}{8}$	$6\frac{1}{4}$	$4\frac{5}{8}$	$10\frac{1}{4}$	$6\frac{5}{8}$	$14\frac{1}{4}$	$8\frac{5}{8}$	$18\frac{1}{4}$	$10\frac{5}{8}$	$22\frac{1}{4}$	$\frac{5}{8}$
$2\frac{1}{2}$	$2\frac{3}{4}$	$6\frac{1}{2}$	$4\frac{3}{4}$	$10\frac{1}{2}$	$6\frac{3}{4}$	$14\frac{1}{2}$	$8\frac{3}{4}$	$18\frac{1}{2}$	$10\frac{3}{4}$	$22\frac{1}{2}$	$\frac{3}{4}$
$2\frac{3}{4}$	$2\frac{7}{8}$	$6\frac{3}{4}$	$4\frac{7}{8}$	$10\frac{3}{4}$	$6\frac{7}{8}$	$14\frac{3}{4}$	$8\frac{7}{8}$	$18\frac{3}{4}$	$10\frac{7}{8}$	$22\frac{3}{4}$	$\frac{7}{8}$
3	III	7	V	11	VII	15	IX	19	XI	23	I
$3\frac{1}{4}$	$3\frac{1}{8}$	$7\frac{1}{4}$	$5\frac{1}{8}$	$11\frac{1}{4}$	$7\frac{1}{8}$	$15\frac{1}{4}$	$9\frac{1}{8}$	$19\frac{1}{4}$	$11\frac{1}{8}$	$23\frac{1}{4}$	$1\frac{1}{8}$
$3\frac{1}{2}$	$3\frac{1}{4}$	$7\frac{1}{2}$	$5\frac{1}{4}$	$11\frac{1}{2}$	$7\frac{1}{4}$	$15\frac{1}{2}$	$9\frac{1}{4}$	$19\frac{1}{2}$	$11\frac{1}{4}$	$23\frac{1}{2}$	$1\frac{1}{4}$
$3\frac{3}{4}$	$3\frac{3}{8}$	$7\frac{3}{4}$	$5\frac{3}{8}$	$11\frac{3}{4}$	$7\frac{3}{8}$	$15\frac{3}{4}$	$9\frac{3}{8}$	$19\frac{3}{4}$	$11\frac{3}{8}$	$23\frac{3}{4}$	$1\frac{3}{8}$
4	$3\frac{1}{2}$	8	$5\frac{1}{2}$	12	$7\frac{1}{2}$	16	$9\frac{1}{2}$	20	$11\frac{1}{2}$	24	$1\frac{1}{2}$

In a sundial whose plane lies equidistant from the circle of the 3rd hour reckoned from sunrise or sunset, the lines of any two hours in this table are parallel, as is established from the scholium to Proposition 22.

Table: *Linea horæ 2. ab ortu vel occasu.* (The line of the 2nd hour from sunrise or sunset)

	Noo			Noo			Noo			Noo	
Rise	n	Rise	n	Rise	n	Rise	n	Rise	noon	Rise	noon
$\frac{1}{4}$	$1\frac{1}{8}$	$4\frac{1}{4}$	$3\frac{1}{8}$	$8\frac{1}{4}$	$5\frac{1}{8}$	$12\frac{1}{4}$	$7\frac{1}{8}$	$16\frac{1}{4}$	$9\frac{1}{8}$	$20\frac{1}{4}$	$11\frac{1}{8}$
$\frac{1}{2}$	$1\frac{1}{4}$	$4\frac{1}{2}$	$3\frac{1}{4}$	$8\frac{1}{2}$	$5\frac{1}{4}$	$12\frac{1}{2}$	$7\frac{1}{4}$	$16\frac{1}{2}$	$9\frac{1}{4}$	$20\frac{1}{2}$	$11\frac{1}{4}$
$\frac{3}{4}$	$1\frac{3}{8}$	$4\frac{3}{4}$	$3\frac{3}{8}$	$8\frac{3}{4}$	$5\frac{3}{8}$	$12\frac{3}{4}$	$7\frac{3}{8}$	$16\frac{3}{4}$	$9\frac{3}{8}$	$20\frac{3}{4}$	$11\frac{3}{8}$
1	$1\frac{1}{2}$	5	$3\frac{1}{2}$	9	$5\frac{1}{2}$	13	$7\frac{1}{2}$	17	$9\frac{1}{2}$	21	$11\frac{1}{2}$
$1\frac{1}{4}$	$1\frac{5}{8}$	$5\frac{1}{4}$	$3\frac{5}{8}$	$9\frac{1}{4}$	$5\frac{5}{8}$	$13\frac{1}{4}$	$7\frac{5}{8}$	$17\frac{1}{4}$	$9\frac{5}{8}$	$21\frac{1}{4}$	$11\frac{5}{8}$
$1\frac{1}{2}$	$1\frac{3}{4}$	$5\frac{1}{2}$	$3\frac{3}{4}$	$9\frac{1}{2}$	$5\frac{3}{4}$	$13\frac{1}{2}$	$7\frac{3}{4}$	$17\frac{1}{2}$	$9\frac{3}{4}$	$21\frac{1}{2}$	$11\frac{3}{4}$
$1\frac{3}{4}$	$1\frac{7}{8}$	$5\frac{3}{4}$	$3\frac{7}{8}$	$9\frac{3}{4}$	$5\frac{7}{8}$	$13\frac{3}{4}$	$7\frac{7}{8}$	$17\frac{3}{4}$	$9\frac{7}{8}$	$21\frac{3}{4}$	$11\frac{7}{8}$
2	II	6	IV	10	VI	14	VIII	18	X	22	XII
$2\frac{1}{4}$	$2\frac{1}{8}$	$6\frac{1}{4}$	$4\frac{1}{8}$	$10\frac{1}{4}$	$6\frac{1}{8}$	$14\frac{1}{4}$	$8\frac{1}{8}$	$18\frac{1}{4}$	$10\frac{1}{8}$	$22\frac{1}{4}$	$12\frac{1}{8}$
$2\frac{1}{2}$	$2\frac{1}{4}$	$6\frac{1}{2}$	$4\frac{1}{4}$	$10\frac{1}{2}$	$6\frac{1}{4}$	$14\frac{1}{2}$	$8\frac{1}{4}$	$18\frac{1}{2}$	$10\frac{1}{4}$	$22\frac{1}{2}$	$12\frac{1}{4}$
$2\frac{3}{4}$	$2\frac{3}{8}$	$6\frac{3}{4}$	$4\frac{3}{8}$	$10\frac{3}{4}$	$6\frac{3}{8}$	$14\frac{3}{4}$	$8\frac{3}{8}$	$18\frac{3}{4}$	$10\frac{3}{8}$	$22\frac{3}{4}$	$12\frac{3}{8}$
3	$2\frac{1}{2}$	7	$4\frac{1}{2}$	11	$6\frac{1}{2}$	15	$8\frac{1}{2}$	19	$10\frac{1}{2}$	23	$12\frac{1}{2}$
$3\frac{1}{4}$	$2\frac{5}{8}$	$7\frac{1}{4}$	$4\frac{5}{8}$	$11\frac{1}{4}$	$6\frac{5}{8}$	$15\frac{1}{4}$	$8\frac{5}{8}$	$19\frac{1}{4}$	$10\frac{5}{8}$	$23\frac{1}{4}$	$12\frac{5}{8}$
$3\frac{1}{2}$	$2\frac{3}{4}$	$7\frac{1}{2}$	$4\frac{3}{4}$	$11\frac{1}{2}$	$6\frac{3}{4}$	$15\frac{1}{2}$	$8\frac{3}{4}$	$19\frac{1}{2}$	$10\frac{3}{4}$	$23\frac{1}{2}$	$12\frac{3}{4}$
$3\frac{3}{4}$	$2\frac{7}{8}$	$7\frac{3}{4}$	$4\frac{7}{8}$	$11\frac{3}{4}$	$6\frac{7}{8}$	$15\frac{3}{4}$	$8\frac{7}{8}$	$19\frac{3}{4}$	$10\frac{7}{8}$	$23\frac{3}{4}$	$12\frac{7}{8}$
4	III	8	V	12	VII	16	IX	20	XI	24	I

In a sundial whose plane lies equidistant from the circle of the 2nd hour reckoned from sunrise or sunset, the lines of any two hours in this table are parallel, as is clear from the scholium to Proposition 22.

Table: *Linea horæ 1. ab ortu vel occasu.* (The line of the 1st hour from sunrise or sunset)

	Noo			Noo			Noo			Noo	
Rise	n	Rise	n	Rise	n	Rise	n	Rise	noon	Rise	noon
$\frac{1}{4}$	$\frac{5}{8}$	$4\frac{1}{4}$	$2\frac{5}{8}$	$8\frac{1}{4}$	$4\frac{5}{8}$	$12\frac{1}{4}$	$6\frac{5}{8}$	$16\frac{1}{4}$	$8\frac{5}{8}$	$20\frac{1}{4}$	$10\frac{5}{8}$
$\frac{1}{2}$	$\frac{3}{4}$	$4\frac{1}{2}$	$2\frac{3}{4}$	$8\frac{1}{2}$	$4\frac{3}{4}$	$12\frac{1}{2}$	$6\frac{3}{4}$	$16\frac{1}{2}$	$8\frac{3}{4}$	$20\frac{1}{2}$	$10\frac{3}{4}$

Rise	Noo n	Rise	Noo n	Rise	Noo n	Rise	Noo n	Rise	Noon	Rise	Noon
$\frac{3}{4}$	$\frac{7}{8}$	$4\frac{3}{4}$	$2\frac{7}{8}$	$8\frac{3}{4}$	$4\frac{7}{8}$	$12\frac{3}{4}$	$6\frac{7}{8}$	$16\frac{3}{4}$	$8\frac{7}{8}$	$20\frac{3}{4}$	$10\frac{7}{8}$
1	I	5	III	9	V	13	VII	17	IX	21	XI
$1\frac{1}{4}$	$1\frac{1}{8}$	$5\frac{1}{4}$	$3\frac{1}{8}$	$9\frac{1}{4}$	$5\frac{1}{8}$	$13\frac{1}{4}$	$7\frac{1}{8}$	$17\frac{1}{4}$	$9\frac{1}{8}$	$21\frac{1}{4}$	$11\frac{1}{8}$
$1\frac{1}{2}$	$1\frac{1}{4}$	$5\frac{1}{2}$	$3\frac{1}{4}$	$9\frac{1}{2}$	$5\frac{1}{4}$	$13\frac{1}{2}$	$7\frac{1}{4}$	$17\frac{1}{2}$	$9\frac{1}{4}$	$21\frac{1}{2}$	$11\frac{1}{4}$
$1\frac{3}{4}$	$1\frac{3}{8}$	$5\frac{3}{4}$	$3\frac{3}{8}$	$9\frac{3}{4}$	$5\frac{3}{8}$	$13\frac{3}{4}$	$7\frac{3}{8}$	$17\frac{3}{4}$	$9\frac{3}{8}$	$21\frac{3}{4}$	$11\frac{3}{8}$
2	$1\frac{1}{2}$	6	$3\frac{1}{2}$	10	$5\frac{1}{2}$	14	$7\frac{1}{2}$	18	$9\frac{1}{2}$	22	$11\frac{1}{2}$
$2\frac{1}{4}$	$1\frac{5}{8}$	$6\frac{1}{4}$	$3\frac{5}{8}$	$10\frac{1}{4}$	$5\frac{5}{8}$	$14\frac{1}{4}$	$7\frac{5}{8}$	$18\frac{1}{4}$	$9\frac{5}{8}$	$22\frac{1}{4}$	$11\frac{5}{8}$
$2\frac{1}{2}$	$1\frac{3}{4}$	$6\frac{1}{2}$	$3\frac{3}{4}$	$10\frac{1}{2}$	$5\frac{3}{4}$	$14\frac{1}{2}$	$7\frac{3}{4}$	$18\frac{1}{2}$	$9\frac{3}{4}$	$22\frac{1}{2}$	$11\frac{3}{4}$
$2\frac{3}{4}$	$1\frac{7}{8}$	$6\frac{3}{4}$	$3\frac{7}{8}$	$10\frac{3}{4}$	$5\frac{7}{8}$	$14\frac{3}{4}$	$7\frac{7}{8}$	$18\frac{3}{4}$	$9\frac{7}{8}$	$22\frac{3}{4}$	$11\frac{7}{8}$
3	II	7	IV	11	VI	15	VIII	19	X	23	XII
$3\frac{1}{4}$	$2\frac{1}{8}$	$7\frac{1}{4}$	$4\frac{1}{8}$	$11\frac{1}{4}$	$6\frac{1}{8}$	$15\frac{1}{4}$	$8\frac{1}{8}$	$19\frac{1}{4}$	$10\frac{1}{8}$	$23\frac{1}{4}$	$\frac{1}{8}$
$3\frac{1}{2}$	$2\frac{1}{4}$	$7\frac{1}{2}$	$4\frac{1}{4}$	$11\frac{1}{2}$	$6\frac{1}{4}$	$15\frac{1}{2}$	$8\frac{1}{4}$	$19\frac{1}{2}$	$10\frac{1}{4}$	$23\frac{1}{2}$	$\frac{1}{4}$
$3\frac{3}{4}$	$2\frac{3}{8}$	$7\frac{3}{4}$	$4\frac{3}{8}$	$11\frac{3}{4}$	$6\frac{3}{8}$	$15\frac{3}{4}$	$8\frac{3}{8}$	$19\frac{3}{4}$	$10\frac{3}{8}$	$23\frac{3}{4}$	$\frac{3}{8}$
4	$2\frac{1}{2}$	8	$4\frac{1}{2}$	12	$6\frac{1}{2}$	16	$8\frac{1}{2}$	20	$10\frac{1}{2}$	24	$\frac{1}{2}$

In a sundial whose plane lies equidistant from the circle of the 1st hour reckoned from sunrise or sunset, the lines of any two hours in this table are parallel, as is established from the scholium to Proposition 22, which we have demonstrated.

These three pages continue the series of hour-line concurrence tables of **Liber Primus** (Book One). No chapter (*Caput*) or proposition (*Propositio*) rubric appears; the only printed headings are the six table titles. Each table is titled *Linea horæ N à meridie vel media nocte* — "The line of the Nth hour reckoned from noon or from midnight." The tables run in descending order of hour: 11, 10, 9, 8, 7, and 5 (the hour-6 table does not appear on these pages).

In every table all twelve column-heads read *Horæ ab ortu vel occasu* ("Hours reckoned from sunrise or from sunset"). The twelve columns form six argument/line pairs read left to right: within each pair the left column ("arg.") is the argument hour, and the right column ("line") is the position of the hour-line, both expressed in hours from sunrise or sunset. Each table has sixteen data rows (argument running by quarter-hours). Values that pass zero wrap around modulo 24.

LIBER PRIMVS.

Source p. 73 (printed p. 61)

Table: *Linea horæ 11. à meridie vel media nocte.*

"The line of the 11th hour reckoned from noon or from midnight."

arg .	line	arg .	lin e	arg .	lin e	arg .	lin e	arg .	lin e	arg .	lin e
$\frac{1}{4}$	$21\frac{3}{4}$	$4\frac{1}{4}$	$17\frac{3}{4}$	$8\frac{1}{4}$	$13\frac{3}{4}$	$12\frac{1}{4}$	$9\frac{3}{4}$	$16\frac{1}{4}$	$5\frac{3}{4}$	$20\frac{1}{4}$	$1\frac{3}{4}$
$\frac{1}{2}$	$21\frac{1}{2}$	$4\frac{1}{2}$	$17\frac{1}{2}$	$8\frac{1}{2}$	$13\frac{1}{2}$	$12\frac{1}{2}$	$9\frac{1}{2}$	$16\frac{1}{2}$	$5\frac{1}{2}$	$20\frac{1}{2}$	$1\frac{1}{2}$
$\frac{3}{4}$	$21\frac{1}{4}$	$4\frac{3}{4}$	$17\frac{1}{4}$	$8\frac{3}{4}$	$13\frac{1}{4}$	$12\frac{3}{4}$	$9\frac{1}{4}$	$16\frac{3}{4}$	$5\frac{1}{4}$	$20\frac{3}{4}$	$1\frac{1}{4}$
1	21	5	17	9	13	13	9	17	5	21	1
$1\frac{1}{4}$	$20\frac{3}{4}$	$5\frac{1}{4}$	$16\frac{3}{4}$	$9\frac{1}{4}$	$12\frac{3}{4}$	$13\frac{1}{4}$	$8\frac{3}{4}$	$17\frac{1}{4}$	$4\frac{3}{4}$	$21\frac{1}{4}$	$\frac{3}{4}$
$1\frac{1}{2}$	$20\frac{1}{2}$	$5\frac{1}{2}$	$16\frac{1}{2}$	$9\frac{1}{2}$	$12\frac{1}{2}$	$13\frac{1}{2}$	$8\frac{1}{2}$	$17\frac{1}{2}$	$4\frac{1}{2}$	$21\frac{1}{2}$	$\frac{1}{2}$
$1\frac{3}{4}$	$20\frac{1}{4}$	$5\frac{3}{4}$	$16\frac{1}{4}$	$9\frac{3}{4}$	$12\frac{1}{4}$	$13\frac{3}{4}$	$8\frac{1}{4}$	$17\frac{3}{4}$	$4\frac{1}{4}$	$21\frac{3}{4}$	$\frac{1}{4}$
2	20	6	16	10	12	14	8	18	4	22	24
$2\frac{1}{4}$	$19\frac{3}{4}$	$6\frac{1}{4}$	$15\frac{3}{4}$	$10\frac{1}{4}$	$11\frac{3}{4}$	$14\frac{1}{4}$	$7\frac{3}{4}$	$18\frac{1}{4}$	$3\frac{3}{4}$	$22\frac{1}{4}$	$23\frac{3}{4}$
$2\frac{1}{2}$	$9\frac{1}{2}$ [recte: $19\frac{1}{2}$]	$6\frac{1}{2}$	$15\frac{1}{2}$	$10\frac{1}{2}$	$11\frac{1}{2}$	$14\frac{1}{2}$	$7\frac{1}{2}$	$18\frac{1}{2}$	$3\frac{1}{2}$	$22\frac{1}{2}$	$23\frac{1}{2}$
$2\frac{3}{4}$	$19\frac{1}{4}$	$6\frac{3}{4}$	$15\frac{1}{4}$	$10\frac{3}{4}$	$11\frac{1}{4}$	$14\frac{3}{4}$	$7\frac{1}{4}$	$18\frac{3}{4}$	$3\frac{1}{4}$	$22\frac{3}{4}$	$23\frac{1}{4}$
3	19	7	15	11	11	15	7	19	3	23	23
$3\frac{1}{4}$	$18\frac{3}{4}$	$7\frac{1}{4}$	$14\frac{3}{4}$	$11\frac{1}{4}$	$10\frac{3}{4}$	$15\frac{1}{4}$	$6\frac{3}{4}$	$19\frac{1}{4}$	$2\frac{3}{4}$	$23\frac{1}{4}$	$22\frac{3}{4}$
$3\frac{1}{2}$	$18\frac{1}{2}$	$7\frac{1}{2}$	$14\frac{1}{2}$	$11\frac{1}{2}$	$10\frac{1}{2}$	$15\frac{1}{2}$	$6\frac{1}{2}$	$19\frac{1}{2}$	$2\frac{1}{2}$	$23\frac{1}{2}$	$22\frac{1}{2}$
$3\frac{3}{4}$	$18\frac{1}{4}$	$7\frac{3}{4}$	14	11	10	15	$6\frac{1}{4}$	19	$2\frac{1}{4}$	23	22

arg .	line	arg .	lin e	arg .	lin e	arg .	lin e	arg .	lin e	arg .	lin e
			$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$		$\frac{3}{4}$		$\frac{3}{4}$	$\frac{1}{4}$
4	18	8	14	12	10	16	6	20	2	24	22

In horologio, quod circulo horæ 11. à mer. vel med. noc. æquidistat, lineæ quarumlibet duarum horarum huius tabulæ, sunt parallelæ, vt constat ex scholio propos. 22. In a sundial whose plane lies equidistant from the circle of the 11th hour reckoned from noon or from midnight, the lines of any two hours in this table are parallel, as is established from the scholium to Proposition 22.

Table: *Linea horæ 10. à meridie vel media nocte.*

"The line of the 10th hour reckoned from noon or from midnight."

arg.	line	arg.	line	arg.	line	arg.	line	arg.	line	arg.	line
$\frac{1}{4}$	$19\frac{3}{4}$	$4\frac{1}{4}$	$15\frac{3}{4}$	$8\frac{1}{4}$	$11\frac{3}{4}$	$12\frac{1}{4}$	$7\frac{3}{4}$	$16\frac{1}{4}$	$3\frac{3}{4}$	$20\frac{1}{4}$	$23\frac{3}{4}$
$\frac{1}{2}$	$19\frac{1}{2}$	$4\frac{1}{2}$	$15\frac{1}{2}$	$8\frac{1}{2}$	$11\frac{1}{2}$	$12\frac{1}{2}$	$7\frac{1}{2}$	$16\frac{1}{2}$	$3\frac{1}{2}$	$20\frac{1}{2}$	$23\frac{1}{2}$
$\frac{3}{4}$	$19\frac{1}{4}$	$4\frac{3}{4}$	$15\frac{1}{4}$	$8\frac{3}{4}$	$11\frac{1}{4}$	$12\frac{3}{4}$	$7\frac{1}{4}$	$16\frac{3}{4}$	$3\frac{1}{4}$	$20\frac{3}{4}$	$23\frac{1}{4}$
1	19	5	15	9	11	13	7	17	3	21	23
$1\frac{1}{4}$	$18\frac{3}{4}$	$5\frac{1}{4}$	$14\frac{3}{4}$	$9\frac{1}{4}$	$10\frac{3}{4}$	$13\frac{1}{4}$	$6\frac{3}{4}$	$17\frac{1}{4}$	$2\frac{3}{4}$	$21\frac{1}{4}$	$22\frac{3}{4}$
$1\frac{1}{2}$	$18\frac{1}{2}$	$5\frac{1}{2}$	$14\frac{1}{2}$	$9\frac{1}{2}$	$10\frac{1}{2}$	$13\frac{1}{2}$	$6\frac{1}{2}$	$17\frac{1}{2}$	$2\frac{1}{2}$	$21\frac{1}{2}$	$22\frac{1}{2}$
$1\frac{3}{4}$	$18\frac{1}{4}$	$5\frac{3}{4}$	$14\frac{1}{4}$	$9\frac{3}{4}$	$10\frac{1}{4}$	$13\frac{3}{4}$	$6\frac{1}{4}$	$17\frac{3}{4}$	$2\frac{1}{4}$	$21\frac{3}{4}$	$22\frac{1}{4}$
2	18	6	14	10	10	14	6	18	2	22	22
$2\frac{1}{4}$	$17\frac{3}{4}$	$6\frac{1}{4}$	$13\frac{3}{4}$	$10\frac{1}{4}$	$9\frac{3}{4}$	$14\frac{1}{4}$	$5\frac{3}{4}$	$18\frac{1}{4}$	$1\frac{3}{4}$	$22\frac{1}{4}$	$21\frac{3}{4}$
$2\frac{1}{2}$	$17\frac{1}{2}$	$6\frac{1}{2}$	$13\frac{1}{2}$	$10\frac{1}{2}$	$9\frac{1}{2}$	$14\frac{1}{2}$	$5\frac{1}{2}$	$18\frac{1}{2}$	$1\frac{1}{2}$	$22\frac{1}{2}$	$21\frac{1}{2}$
$2\frac{3}{4}$	$17\frac{1}{4}$	$6\frac{3}{4}$	$13\frac{1}{4}$	$10\frac{3}{4}$	$9\frac{1}{4}$	$14\frac{3}{4}$	$5\frac{1}{4}$	$18\frac{3}{4}$	$1\frac{1}{4}$	$22\frac{3}{4}$	$21\frac{1}{4}$
3	17	7	13	11	9	15	5	19	1	23	21
$3\frac{1}{4}$	$16\frac{3}{4}$	$7\frac{1}{4}$	$12\frac{3}{4}$	$11\frac{1}{4}$	$8\frac{3}{4}$	$15\frac{1}{4}$	$4\frac{3}{4}$	$19\frac{1}{4}$	$\frac{3}{4}$	$23\frac{1}{4}$	$20\frac{3}{4}$
$3\frac{1}{2}$	$16\frac{1}{2}$	$7\frac{1}{2}$	$12\frac{1}{2}$	$11\frac{1}{2}$	$8\frac{1}{2}$	$15\frac{1}{2}$	$4\frac{1}{2}$	$19\frac{1}{2}$	$\frac{1}{2}$	$23\frac{1}{2}$	$20\frac{1}{2}$
$3\frac{3}{4}$	$16\frac{1}{4}$	$7\frac{3}{4}$	$12\frac{1}{4}$	$11\frac{3}{4}$	$8\frac{1}{4}$	$15\frac{3}{4}$	$4\frac{1}{4}$	$19\frac{3}{4}$	$\frac{1}{4}$	$23\frac{3}{4}$	$20\frac{1}{4}$
4	16	8	12	12	8	16	4	20	24	24	20

In horologio, quod circulo horæ 10. à mer. vel med. noc. æquidistat, lineæ quarumlibet duarum horarum huius tabulæ sunt parallelæ, vt perspicuum est ex scholio propos. 22. In a sundial whose plane lies equidistant from the circle of the

10th hour reckoned from noon or from midnight, the lines of any two hours in this table are parallel, as is plain from the scholium to Proposition 22.

GNOMONICES

Source p. 74 (printed p. 62)

Table: *Linea horæ 9. à meridie vel media nocte.*

"The line of the 9th hour reckoned from noon or from midnight."

arg.	line	arg.	line	arg.	line	arg.	line	arg.	line	arg.	line
$\frac{1}{4}$	$17\frac{3}{4}$	$4\frac{1}{4}$	$13\frac{3}{4}$	$8\frac{1}{4}$	$9\frac{3}{4}$	$12\frac{1}{4}$	$5\frac{3}{4}$	$16\frac{1}{4}$	$1\frac{3}{4}$	$20\frac{1}{4}$	$21\frac{3}{4}$
$\frac{1}{2}$	$17\frac{1}{2}$	$4\frac{1}{2}$	$13\frac{1}{2}$	$8\frac{1}{2}$	$9\frac{1}{2}$	$12\frac{1}{2}$	$5\frac{1}{2}$	$16\frac{1}{2}$	$1\frac{1}{2}$	$20\frac{1}{2}$	$21\frac{1}{2}$
$\frac{3}{4}$	$17\frac{1}{4}$	$4\frac{3}{4}$	$13\frac{1}{4}$	$8\frac{3}{4}$	$9\frac{1}{4}$	$12\frac{3}{4}$	$5\frac{1}{4}$	$16\frac{3}{4}$	$1\frac{1}{4}$	$20\frac{3}{4}$	$21\frac{1}{4}$
1	17	5	13	9	9	13	5	17	1	21	21
$1\frac{1}{4}$	$16\frac{3}{4}$	$5\frac{1}{4}$	$12\frac{3}{4}$	$9\frac{1}{4}$	$8\frac{3}{4}$	$13\frac{1}{4}$	$4\frac{3}{4}$	$17\frac{1}{4}$	$\frac{3}{4}$	$21\frac{1}{4}$	$20\frac{3}{4}$
$1\frac{1}{2}$	$16\frac{1}{2}$	$5\frac{1}{2}$	$12\frac{1}{2}$	$9\frac{1}{2}$	$8\frac{1}{2}$	$13\frac{1}{2}$	$4\frac{1}{2}$	$17\frac{1}{2}$	$\frac{1}{2}$	$21\frac{1}{2}$	$20\frac{1}{2}$
$1\frac{3}{4}$	$16\frac{1}{4}$	$5\frac{3}{4}$	$12\frac{1}{4}$	$9\frac{3}{4}$	$8\frac{1}{4}$	$13\frac{3}{4}$	$4\frac{1}{4}$	$17\frac{3}{4}$	$\frac{1}{4}$	$21\frac{3}{4}$	$20\frac{1}{4}$
2	16	6	12	10	8	14	4	18	24	22	20
$2\frac{1}{4}$	$15\frac{3}{4}$	$6\frac{1}{4}$	$11\frac{3}{4}$	$10\frac{1}{4}$	$7\frac{3}{4}$	$14\frac{1}{4}$	$3\frac{3}{4}$	$18\frac{1}{4}$	$23\frac{3}{4}$	$22\frac{1}{4}$	$19\frac{3}{4}$
$2\frac{1}{2}$	$15\frac{1}{2}$	$6\frac{1}{2}$	$11\frac{1}{2}$	$10\frac{1}{2}$	$7\frac{1}{2}$	$14\frac{1}{2}$	$3\frac{1}{2}$	$18\frac{1}{2}$	$23\frac{1}{2}$	$22\frac{1}{2}$	$19\frac{1}{2}$
$2\frac{3}{4}$	$15\frac{1}{4}$	$6\frac{3}{4}$	$11\frac{1}{4}$	$10\frac{3}{4}$	$7\frac{1}{4}$	$14\frac{3}{4}$	$3\frac{1}{4}$	$18\frac{3}{4}$	$23\frac{1}{4}$	$22\frac{3}{4}$	$19\frac{1}{4}$
3	15	7	11	11	7	15	3	19	23	23	19
$3\frac{1}{4}$	$14\frac{3}{4}$	$7\frac{1}{4}$	$10\frac{3}{4}$	$11\frac{1}{4}$	$6\frac{3}{4}$	$15\frac{1}{4}$	$2\frac{3}{4}$	$19\frac{1}{4}$	$22\frac{3}{4}$	$23\frac{1}{4}$	$18\frac{3}{4}$
$3\frac{1}{2}$	$14\frac{1}{2}$	$7\frac{1}{2}$	$10\frac{1}{2}$	$11\frac{1}{2}$	$6\frac{1}{2}$	$15\frac{1}{2}$	$2\frac{1}{2}$	$19\frac{1}{2}$	$22\frac{1}{2}$	$23\frac{1}{2}$	$18\frac{1}{2}$
$3\frac{3}{4}$	$14\frac{1}{4}$	$7\frac{3}{4}$	$10\frac{1}{4}$	$11\frac{3}{4}$	$6\frac{1}{4}$	$15\frac{3}{4}$	$2\frac{1}{4}$	$19\frac{3}{4}$	$22\frac{1}{4}$	$23\frac{3}{4}$	$18\frac{1}{4}$
4	14	8	10	12	6	16	2	20	22	24	18

In horologio, quod circulo horæ 9. à mer. vel med. noc. æquidistat, lineæ quarumlibet duarum horarum huius tabulæ, sunt parallelæ, ex scholio propos. 22.

In a sundial whose plane lies equidistant from the circle of the 9th hour reckoned from noon or from midnight, the lines of any two hours in this table are parallel, from the scholium to Proposition 22.

Table: *Linea horæ 8. à meridie vel media nocte.*

"The line of the 8th hour reckoned from noon or from midnight."

arg.	line	arg.	line	arg.	line	arg.	line	arg.	line	arg.	line
$\frac{1}{4}$	$15\frac{3}{4}$	$4\frac{1}{4}$	$11\frac{3}{4}$	$8\frac{1}{4}$	$7\frac{3}{4}$	$12\frac{1}{4}$	$3\frac{3}{4}$	$16\frac{1}{4}$	$23\frac{3}{4}$	$20\frac{1}{4}$	$19\frac{3}{4}$
$\frac{1}{2}$	$15\frac{1}{2}$	$4\frac{1}{2}$	$11\frac{1}{2}$	$8\frac{1}{2}$	$7\frac{1}{2}$	$12\frac{1}{2}$	$3\frac{1}{2}$	$16\frac{1}{2}$	$23\frac{1}{2}$	$20\frac{1}{2}$	$19\frac{1}{2}$
$\frac{3}{4}$	$15\frac{1}{4}$	$4\frac{3}{4}$	$11\frac{1}{4}$	$8\frac{3}{4}$	$7\frac{1}{4}$	$12\frac{3}{4}$	$3\frac{1}{4}$	$16\frac{3}{4}$	$23\frac{1}{4}$	$20\frac{3}{4}$	$19\frac{1}{4}$
1	15	5	11	9	7	13	3	17	23	21	19
$1\frac{1}{4}$	$14\frac{3}{4}$	$5\frac{1}{4}$	$10\frac{3}{4}$	$9\frac{1}{4}$	$6\frac{3}{4}$	$13\frac{1}{4}$	$2\frac{3}{4}$	$17\frac{1}{4}$	$22\frac{3}{4}$	$21\frac{1}{4}$	$18\frac{3}{4}$
$1\frac{1}{2}$	$14\frac{1}{2}$	$5\frac{1}{2}$	$10\frac{1}{2}$	$9\frac{1}{2}$	$6\frac{1}{2}$	$13\frac{1}{2}$	$2\frac{1}{2}$	$17\frac{1}{2}$	$22\frac{1}{2}$	$21\frac{1}{2}$	$18\frac{1}{2}$
$1\frac{3}{4}$	$14\frac{1}{4}$	$5\frac{3}{4}$	$10\frac{1}{4}$	$9\frac{3}{4}$	$6\frac{1}{4}$	$13\frac{3}{4}$	$2\frac{1}{4}$	$17\frac{3}{4}$	$22\frac{1}{4}$	$21\frac{3}{4}$	$18\frac{1}{4}$
2	14	6	10	10	6	14	2	18	22	22	18
$2\frac{1}{4}$	$13\frac{3}{4}$	$6\frac{1}{4}$	$9\frac{3}{4}$	$10\frac{1}{4}$	$5\frac{3}{4}$	$14\frac{1}{4}$	$1\frac{3}{4}$	$18\frac{1}{4}$	$21\frac{3}{4}$	$22\frac{1}{4}$	$17\frac{3}{4}$
$2\frac{1}{2}$	$13\frac{1}{2}$	$6\frac{1}{2}$	$9\frac{1}{2}$	$10\frac{1}{2}$	$5\frac{1}{2}$	$14\frac{1}{2}$	$1\frac{1}{2}$	$18\frac{1}{2}$	$21\frac{1}{2}$	$22\frac{1}{2}$	$17\frac{1}{2}$
$2\frac{3}{4}$	$13\frac{1}{4}$	$6\frac{3}{4}$	$9\frac{1}{4}$	$10\frac{3}{4}$	$5\frac{1}{4}$	$14\frac{3}{4}$	$1\frac{1}{4}$	$18\frac{3}{4}$	$21\frac{1}{4}$	$22\frac{3}{4}$	$17\frac{1}{4}$
3	13	7	9	11	5	15	1	19	21	23	17
$3\frac{1}{4}$	$12\frac{3}{4}$	$7\frac{1}{4}$	$8\frac{3}{4}$	$11\frac{1}{4}$	$4\frac{3}{4}$	$15\frac{1}{4}$	$\frac{3}{4}$	$19\frac{1}{4}$	$20\frac{3}{4}$	$23\frac{1}{4}$	$16\frac{3}{4}$
$3\frac{1}{2}$	$12\frac{1}{2}$	$7\frac{1}{2}$	$8\frac{1}{2}$	$11\frac{1}{2}$	$4\frac{1}{2}$	$15\frac{1}{2}$	$\frac{1}{2}$	$19\frac{1}{2}$	$20\frac{1}{2}$	$23\frac{1}{2}$	$16\frac{1}{2}$
$3\frac{3}{4}$	$12\frac{1}{4}$	$7\frac{3}{4}$	$8\frac{1}{4}$	$11\frac{3}{4}$	$4\frac{1}{4}$	$15\frac{3}{4}$	$\frac{1}{4}$	$19\frac{3}{4}$	$20\frac{1}{4}$	$23\frac{3}{4}$	$16\frac{1}{4}$
4	12	8	8	12	4	16	24	20	20	24	16

In horologio, quod circulo horæ 8. à mer. vel med. noc. æquidistat, lineæ quarumcunque duarum horarum huius tabulæ sunt parallelæ, vt constat ex scholio propos. 22. In a sundial whose plane lies equidistant from the circle of the 8th hour reckoned from noon or from midnight, the lines of any two hours whatever in this table are parallel, as is established from the scholium to Proposition 22.

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Source p. 75 (printed p. 63)

Table: *Linea horæ septima à meridie vel media nocte.*

"The line of the 7th hour reckoned from noon or from midnight."

arg.	line	arg.	line	arg.	line	arg.	line	arg.	line	arg.	line
$\frac{1}{4}$	$13\frac{3}{4}$	$4\frac{1}{4}$	$9\frac{3}{4}$	$8\frac{1}{4}$	$5\frac{3}{4}$	$12\frac{1}{4}$	$1\frac{3}{4}$	$16\frac{1}{4}$	$21\frac{3}{4}$	$20\frac{1}{4}$	$17\frac{3}{4}$
$\frac{1}{2}$	$13\frac{1}{2}$	$4\frac{1}{2}$	$9\frac{1}{2}$	$8\frac{1}{2}$	$5\frac{1}{2}$	$12\frac{1}{2}$	$1\frac{1}{2}$	$16\frac{1}{2}$	$21\frac{1}{2}$	$20\frac{1}{2}$	$17\frac{1}{2}$
$\frac{3}{4}$	$13\frac{1}{4}$	$4\frac{3}{4}$	$9\frac{1}{4}$	$8\frac{3}{4}$	$5\frac{1}{4}$	$12\frac{3}{4}$	$1\frac{1}{4}$	$16\frac{3}{4}$	$21\frac{1}{4}$	$20\frac{3}{4}$	$17\frac{1}{4}$
1	13	5	9	9	5	13	1	17	21	21	17
$1\frac{1}{4}$	$12\frac{3}{4}$	$5\frac{1}{4}$	$8\frac{3}{4}$	$9\frac{1}{4}$	$4\frac{3}{4}$	$13\frac{1}{4}$	$\frac{3}{4}$	$17\frac{1}{4}$	$20\frac{3}{4}$	$21\frac{1}{4}$	$16\frac{3}{4}$
$1\frac{1}{2}$	$12\frac{1}{2}$	$5\frac{1}{2}$	$8\frac{1}{2}$	$9\frac{1}{2}$	$4\frac{1}{2}$	$13\frac{1}{2}$	$\frac{1}{2}$	$17\frac{1}{2}$	$20\frac{1}{2}$	$21\frac{1}{2}$	$16\frac{1}{2}$
$1\frac{3}{4}$	$12\frac{1}{4}$	$5\frac{3}{4}$	$8\frac{1}{4}$	$9\frac{3}{4}$	$4\frac{1}{4}$	$13\frac{3}{4}$	$\frac{1}{4}$	$17\frac{3}{4}$	$20\frac{1}{4}$	$21\frac{3}{4}$	$16\frac{1}{4}$
2	12	6	8	10	4	14	24	18	20	22	16
$2\frac{1}{4}$	$11\frac{3}{4}$	$6\frac{1}{4}$	$7\frac{3}{4}$	$10\frac{1}{4}$	$3\frac{3}{4}$	$14\frac{1}{4}$	$23\frac{3}{4}$	$18\frac{1}{4}$	$19\frac{3}{4}$	$22\frac{1}{4}$	$15\frac{3}{4}$
$2\frac{1}{2}$	$11\frac{1}{2}$	$6\frac{1}{2}$	$7\frac{1}{2}$	$10\frac{1}{2}$	$3\frac{1}{2}$	$14\frac{1}{2}$	$23\frac{1}{2}$	$18\frac{1}{2}$	$19\frac{1}{2}$	$22\frac{1}{2}$	$15\frac{1}{2}$
$2\frac{3}{4}$	$11\frac{1}{4}$	$6\frac{3}{4}$	$7\frac{1}{4}$	$10\frac{3}{4}$	$3\frac{1}{4}$	$14\frac{3}{4}$	$23\frac{1}{4}$	$18\frac{3}{4}$	$19\frac{1}{4}$	$22\frac{3}{4}$	$15\frac{1}{4}$
3	11	7	7	11	3	15	23	19	19	23	15
$3\frac{1}{4}$	$10\frac{3}{4}$	$7\frac{1}{4}$	$6\frac{3}{4}$	$11\frac{1}{4}$	$2\frac{3}{4}$	$15\frac{1}{4}$	$22\frac{3}{4}$	$19\frac{1}{4}$	$18\frac{3}{4}$	$23\frac{1}{4}$	$14\frac{3}{4}$
$3\frac{1}{2}$	$10\frac{1}{2}$	$7\frac{1}{2}$	$6\frac{1}{2}$	$11\frac{1}{2}$	$2\frac{1}{2}$	$15\frac{1}{2}$	$22\frac{1}{2}$	$19\frac{1}{2}$	$18\frac{1}{2}$	$23\frac{1}{2}$	$14\frac{1}{2}$
$3\frac{3}{4}$	$10\frac{1}{4}$	$7\frac{3}{4}$	$6\frac{1}{4}$	$11\frac{3}{4}$	$2\frac{1}{4}$	$15\frac{3}{4}$	$22\frac{1}{4}$	$19\frac{3}{4}$	$18\frac{1}{4}$	$23\frac{3}{4}$	$14\frac{1}{4}$
4	10	8	6	12	2	16	22	20	18	24	14

In horologio, quod circulo horæ 7. à mer. vel med. noc. æquidistat, lineæ quarumlibet duarum horarum huius tabulæ, sunt parallelæ, vt ex scholio propos.

22. *liquido constat.* In a sundial whose plane lies equidistant from the circle of the 7th hour reckoned from noon or from midnight, the lines of any two hours in this table are parallel, as is clearly established from the scholium to Proposition 22.

Table: *Linea horæ quinta à meridie vel media nocte.*

"The line of the 5th hour reckoned from noon or from midnight."

arg.	line	arg.	line	arg.	line	arg.	line	arg.	line	arg.	line
$\frac{1}{4}$	$9\frac{3}{4}$	$4\frac{1}{4}$	$5\frac{3}{4}$	$8\frac{1}{4}$	$1\frac{3}{4}$	$12\frac{1}{4}$	$21\frac{3}{4}$	$16\frac{1}{4}$	$17\frac{3}{4}$	$20\frac{1}{4}$	$13\frac{3}{4}$
$\frac{1}{2}$	$9\frac{1}{2}$	$4\frac{1}{2}$	$5\frac{1}{2}$	$8\frac{1}{2}$	$1\frac{1}{2}$	$12\frac{1}{2}$	$21\frac{1}{2}$	$16\frac{1}{2}$	$17\frac{1}{2}$	$20\frac{1}{2}$	$13\frac{1}{2}$
$\frac{3}{4}$	$9\frac{1}{4}$	$4\frac{3}{4}$	$5\frac{1}{4}$	$8\frac{3}{4}$	$1\frac{1}{4}$	$12\frac{3}{4}$	$21\frac{1}{4}$	$16\frac{3}{4}$	$17\frac{1}{4}$	$20\frac{3}{4}$	$13\frac{1}{4}$
1	9	5	5	9	1	13	21	17	17	21	13
$1\frac{1}{4}$	$8\frac{3}{4}$	$5\frac{1}{4}$	$4\frac{3}{4}$	$9\frac{1}{4}$	$\frac{3}{4}$	$13\frac{1}{4}$	$20\frac{3}{4}$	$17\frac{1}{4}$	$16\frac{3}{4}$	$21\frac{1}{4}$	$12\frac{3}{4}$
$1\frac{1}{2}$	$8\frac{1}{2}$	$5\frac{1}{2}$	$4\frac{1}{2}$	$9\frac{1}{2}$	$\frac{1}{2}$	$13\frac{1}{2}$	$20\frac{1}{2}$	$17\frac{1}{2}$	$16\frac{1}{2}$	$21\frac{1}{2}$	$12\frac{1}{2}$

arg.	line	arg.	line	arg.	line	arg.	line	arg.	line	arg.	line
$1\frac{3}{4}$	$8\frac{1}{4}$	$5\frac{3}{4}$	$4\frac{1}{4}$	$9\frac{3}{4}$	$\frac{1}{4}$	$13\frac{3}{4}$	$20\frac{1}{4}$	$17\frac{3}{4}$	$16\frac{1}{4}$	$21\frac{3}{4}$	$12\frac{1}{4}$
2	8	6	4	10	24	14	20	18	16	22	12
$2\frac{1}{4}$	$7\frac{3}{4}$	$6\frac{1}{4}$	$3\frac{3}{4}$	$10\frac{1}{4}$	$23\frac{3}{4}$	$14\frac{1}{4}$	$19\frac{3}{4}$	$18\frac{1}{4}$	$15\frac{3}{4}$	$22\frac{1}{4}$	$11\frac{3}{4}$
$2\frac{1}{2}$	$7\frac{1}{2}$	$6\frac{1}{2}$	$3\frac{1}{2}$	$10\frac{1}{2}$	$23\frac{1}{2}$	$14\frac{1}{2}$	$19\frac{1}{2}$	$18\frac{1}{2}$	$15\frac{1}{2}$	$22\frac{1}{2}$	$11\frac{1}{2}$
$2\frac{3}{4}$	$7\frac{1}{4}$	$6\frac{3}{4}$	$3\frac{1}{4}$	$10\frac{3}{4}$	$23\frac{1}{4}$	$14\frac{3}{4}$	$19\frac{1}{4}$	$18\frac{3}{4}$	$15\frac{1}{4}$	$22\frac{3}{4}$	$11\frac{1}{4}$
3	7	7	3	11	23	15	19	19	15	23	11
$3\frac{1}{4}$	$6\frac{3}{4}$	$7\frac{1}{4}$	$2\frac{3}{4}$	$11\frac{1}{4}$	$22\frac{3}{4}$	$15\frac{1}{4}$	$18\frac{3}{4}$	$19\frac{1}{4}$	$14\frac{3}{4}$	$23\frac{1}{4}$	$10\frac{3}{4}$
$3\frac{1}{2}$	$6\frac{1}{2}$	$7\frac{1}{2}$	$2\frac{1}{2}$	$11\frac{1}{2}$	$22\frac{1}{2}$	$15\frac{1}{2}$	$18\frac{1}{2}$	$19\frac{1}{2}$	$14\frac{1}{2}$	$23\frac{1}{2}$	$10\frac{1}{2}$
$3\frac{3}{4}$	$6\frac{1}{4}$	$7\frac{3}{4}$	$2\frac{1}{4}$	$11\frac{3}{4}$	$22\frac{1}{4}$	$15\frac{3}{4}$	$18\frac{1}{4}$	$19\frac{3}{4}$	$14\frac{1}{4}$	$23\frac{3}{4}$	$10\frac{1}{4}$
4	6	8	2	12	22	16	18	20	14	24	10

In horologio, quod circulo horæ 3. à mer. vel med. noc. æquidistat, lineæ quarumlibet duarum horarum huius tabulæ sunt parallelæ, vt patet ex scholio propos. 22. In a sundial whose plane lies equidistant from the circle of the 3rd [recte: 5th] hour reckoned from noon or from midnight, the lines of any two hours in this table are parallel, as is clear from the scholium to Proposition 22.

The work remains in **Liber Primus** (Book One). This opening carries the last four of the hour-line concurrence tables (for the 4th, 3rd, 2nd, and 1st hours), after which the prose resumes with Theorem 19 / Proposition 21.

Table — Line of the 4th hour from noon or midnight

Linea horæ quarta à meridie vel media nocte.

Each of the twelve columns is identically headed *Horæ ab ortu vel occasu*. ("Hours from rising or from setting."). The columns run in six pairs; in each pair the left member increases and the right member decreases, so that the two hours sharing one line are read across. Column 1 is the argument (the $\frac{1}{4}$ -hour subdivision).

1	2	3	4	5	6	7	8	9	10	11	12
$\frac{1}{4}$	$7\frac{3}{4}$	$4\frac{1}{4}$	$3\frac{3}{4}$	$8\frac{1}{4}$	$23\frac{3}{4}$	$12\frac{1}{4}$	$19\frac{3}{4}$	$16\frac{1}{4}$	$15\frac{3}{4}$	$20\frac{1}{4}$	$11\frac{3}{4}$
$\frac{1}{2}$	$7\frac{1}{2}$	$4\frac{1}{2}$	$3\frac{1}{2}$	$8\frac{1}{2}$	$23\frac{1}{2}$	$12\frac{1}{2}$	$19\frac{1}{2}$	$16\frac{1}{2}$	$15\frac{1}{2}$	$20\frac{1}{2}$	$11\frac{1}{2}$
$\frac{3}{4}$	$7\frac{1}{4}$	$4\frac{3}{4}$	$3\frac{1}{4}$	$8\frac{3}{4}$	$23\frac{1}{4}$	$12\frac{3}{4}$	$19\frac{1}{4}$	$16\frac{3}{4}$	$15\frac{1}{4}$	$20\frac{3}{4}$	$11\frac{1}{4}$
1	7	5	3	9	23	13	19	17	15	21	11
$1\frac{1}{4}$	$6\frac{3}{4}$	$5\frac{1}{4}$	$2\frac{3}{4}$	$9\frac{1}{4}$	$22\frac{3}{4}$	$13\frac{1}{4}$	$18\frac{3}{4}$	$17\frac{1}{4}$	$14\frac{3}{4}$	$21\frac{1}{4}$	$10\frac{3}{4}$
$1\frac{1}{2}$	$6\frac{1}{2}$	$5\frac{1}{2}$	$2\frac{1}{2}$	$9\frac{1}{2}$	$22\frac{1}{2}$	$13\frac{1}{2}$	$18\frac{1}{2}$	$17\frac{1}{2}$	$14\frac{1}{2}$	$21\frac{1}{2}$	$10\frac{1}{2}$

1	2	3	4	5	6	7	8	9	10	11	12
$1\frac{3}{4}$	$6\frac{1}{4}$	$5\frac{3}{4}$	$2\frac{1}{4}$	$9\frac{3}{4}$	$22\frac{1}{4}$	$13\frac{3}{4}$	$18\frac{1}{4}$	$17\frac{3}{4}$	$14\frac{1}{4}$	$21\frac{3}{4}$	$10\frac{1}{4}$
2	6	6	2	10	22	14	18	18	14	22	10
$2\frac{1}{4}$	$5\frac{3}{4}$	$6\frac{1}{4}$	$1\frac{3}{4}$	$10\frac{1}{4}$	$21\frac{3}{4}$	$14\frac{1}{4}$	$17\frac{3}{4}$	$18\frac{1}{4}$	$13\frac{3}{4}$	$22\frac{1}{4}$	$9\frac{3}{4}$
$2\frac{1}{2}$	$5\frac{1}{2}$	$6\frac{1}{2}$	$1\frac{1}{2}$	$10\frac{1}{2}$	$21\frac{1}{2}$	$14\frac{1}{2}$	$17\frac{1}{2}$	$18\frac{1}{2}$	$13\frac{1}{2}$	$22\frac{1}{2}$	$9\frac{1}{2}$
$2\frac{3}{4}$	$5\frac{1}{4}$	$6\frac{3}{4}$	$1\frac{1}{4}$	$10\frac{3}{4}$	$21\frac{1}{4}$	$14\frac{3}{4}$	$17\frac{1}{4}$	$18\frac{3}{4}$	$13\frac{1}{4}$	$22\frac{3}{4}$	$9\frac{1}{4}$
3	5	7	1	11	21	15	17	19	13	23	9
$3\frac{1}{4}$	$4\frac{3}{4}$	$7\frac{1}{4}$	$\frac{3}{4}$	$11\frac{1}{4}$	$20\frac{3}{4}$	$15\frac{1}{4}$	$16\frac{3}{4}$	$19\frac{1}{4}$	$12\frac{3}{4}$	$23\frac{1}{4}$	$8\frac{3}{4}$
$3\frac{1}{2}$	$4\frac{1}{2}$	$7\frac{1}{2}$	$\frac{1}{2}$	$11\frac{1}{2}$	$20\frac{1}{2}$	$15\frac{1}{2}$	$16\frac{1}{2}$	$19\frac{1}{2}$	$12\frac{1}{2}$	$23\frac{1}{2}$	$8\frac{1}{2}$
$3\frac{3}{4}$	$4\frac{1}{4}$	$7\frac{3}{4}$	$\frac{1}{4}$	$11\frac{3}{4}$	$20\frac{1}{4}$	$15\frac{3}{4}$	$16\frac{1}{4}$	$19\frac{3}{4}$	$12\frac{1}{4}$	$23\frac{3}{4}$	$8\frac{1}{4}$
4	4	8	24	12	20	16	16	20	12	24	8

In horologio, quod circulo horæ 4. à mer. vel med. noc. æquidistat, lineæ quarumlibet duarum horarum huius tabulæ, sunt parallelæ, vt ex scholio propos. 22. manifeste colligitur.

In a sundial whose plane lies parallel to the circle of the 4th hour reckoned from noon or from midnight, the lines of any two hours in this table are parallel, as is clearly gathered from the scholium to Proposition 22.

Table — Line of the 3rd hour from noon or midnight

Linea horæ tertia à meridie vel media nocte.

Twelve columns, each headed *Horæ ab ortu vel occasu*. (same structure as the preceding table). Column 1 is the argument.

1	2	3	4	5	6	7	8	9	10	11	12
$\frac{1}{4}$	$5\frac{3}{4}$	$4\frac{1}{4}$	$1\frac{3}{4}$	$8\frac{1}{4}$	$21\frac{3}{4}$	$12\frac{1}{4}$	$17\frac{3}{4}$	$16\frac{1}{4}$	$13\frac{3}{4}$	$20\frac{1}{4}$	$9\frac{3}{4}$
$\frac{1}{2}$	$5\frac{1}{2}$	$4\frac{1}{2}$	$1\frac{1}{2}$	$8\frac{1}{2}$	$21\frac{1}{2}$	$12\frac{1}{2}$	$17\frac{1}{2}$	$16\frac{1}{2}$	$13\frac{1}{2}$	$20\frac{1}{2}$	$9\frac{1}{2}$
$\frac{3}{4}$	$5\frac{1}{4}$	$4\frac{3}{4}$	$1\frac{1}{4}$	$8\frac{3}{4}$	$21\frac{1}{4}$	$12\frac{3}{4}$	$17\frac{1}{4}$	$16\frac{3}{4}$	$13\frac{1}{4}$	$20\frac{3}{4}$	$9\frac{1}{4}$
1	5	5	1	9	21	13	17	17	13	21	9
$1\frac{1}{4}$	$4\frac{3}{4}$	$5\frac{1}{4}$	$\frac{3}{4}$	$9\frac{1}{4}$	$20\frac{3}{4}$	$13\frac{1}{4}$	$16\frac{3}{4}$	$17\frac{1}{4}$	$12\frac{3}{4}$	$21\frac{1}{4}$	$8\frac{3}{4}$
$1\frac{1}{2}$	$4\frac{1}{2}$	$5\frac{1}{2}$	$\frac{1}{2}$	$9\frac{1}{2}$	$20\frac{1}{2}$	$13\frac{1}{2}$	$16\frac{1}{2}$	$17\frac{1}{2}$	$12\frac{1}{2}$	$21\frac{1}{2}$	$8\frac{1}{2}$
$1\frac{3}{4}$	$4\frac{1}{4}$	$5\frac{3}{4}$	$\frac{1}{4}$	$9\frac{3}{4}$	$20\frac{1}{4}$	$13\frac{3}{4}$	$16\frac{1}{4}$	$17\frac{3}{4}$	$12\frac{1}{4}$	$21\frac{3}{4}$	$8\frac{1}{4}$
2	4	6	24	10	20	14	16	18	12	22	8

1	2	3	4	5	6	7	8	9	10	11	12
$2\frac{1}{4}$	$3\frac{3}{4}$	$6\frac{1}{4}$	$23\frac{3}{4}$	$10\frac{1}{4}$	$19\frac{3}{4}$	$14\frac{1}{4}$	$15\frac{3}{4}$	$18\frac{1}{4}$	$11\frac{3}{4}$	$22\frac{1}{4}$	$7\frac{3}{4}$
$2\frac{1}{2}$	$3\frac{1}{2}$	$6\frac{1}{2}$	$23\frac{1}{2}$	$10\frac{1}{2}$	$19\frac{1}{2}$	$14\frac{1}{2}$	$15\frac{1}{2}$	$18\frac{1}{2}$	$11\frac{1}{2}$	$22\frac{1}{2}$	$7\frac{1}{2}$
$2\frac{3}{4}$	$3\frac{1}{4}$	$6\frac{3}{4}$	$23\frac{1}{4}$	$10\frac{3}{4}$	$19\frac{1}{4}$	$14\frac{3}{4}$	$15\frac{1}{4}$	$18\frac{3}{4}$	$11\frac{1}{4}$	$22\frac{3}{4}$	$7\frac{1}{4}$
3	3	7	23	11	19	15	15	19	11	23	7
$3\frac{1}{4}$	$2\frac{3}{4}$	$7\frac{1}{4}$	$22\frac{3}{4}$	$11\frac{1}{4}$	$18\frac{3}{4}$	$15\frac{1}{4}$	$14\frac{3}{4}$	$19\frac{1}{4}$	$10\frac{3}{4}$	$23\frac{1}{4}$	$6\frac{3}{4}$
$3\frac{1}{2}$	$2\frac{1}{2}$	$7\frac{1}{2}$	$22\frac{1}{2}$	$11\frac{1}{2}$	$18\frac{1}{2}$	$15\frac{1}{2}$	$14\frac{1}{2}$	$19\frac{1}{2}$	$10\frac{1}{2}$	$23\frac{1}{2}$	$6\frac{1}{2}$
$3\frac{3}{4}$	$2\frac{1}{4}$	$7\frac{3}{4}$	$22\frac{1}{4}$	$11\frac{3}{4}$	$18\frac{1}{4}$	$15\frac{3}{4}$	$14\frac{1}{4}$	$19\frac{3}{4}$	$10\frac{1}{4}$	$23\frac{3}{4}$	$6\frac{1}{4}$
4	2	8	22	12	18	16	14	20	10	24	6

In horologio, quod circulo horæ 3. à mer. vel med. noc. æquidistat, lineæ quarumlibet duarum horarum huius tabulæ sunt parallelæ, vt ex scholio propos. 22. patet.

In a sundial whose plane lies parallel to the circle of the 3rd hour reckoned from noon or from midnight, the lines of any two hours in this table are parallel, as is evident from the scholium to Proposition 22.

Table — Line of the 2nd hour from noon or midnight

Linea horæ secunda à meridie vel media nocte.

Twelve columns, each headed *Horæ ab ortu vel occasu*. Column 1 is the argument.

1	2	3	4	5	6	7	8	9	10	11	12
$\frac{1}{4}$	$3\frac{3}{4}$	$4\frac{1}{4}$	$23\frac{3}{4}$	$8\frac{1}{4}$	$19\frac{3}{4}$	$12\frac{1}{4}$	$15\frac{3}{4}$	$16\frac{1}{4}$	$11\frac{3}{4}$	$20\frac{1}{4}$	$7\frac{3}{4}$
$\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$	$23\frac{1}{2}$	$8\frac{1}{2}$	$19\frac{1}{2}$	$12\frac{1}{2}$	$15\frac{1}{2}$	$16\frac{1}{2}$	$11\frac{1}{2}$	$20\frac{1}{2}$	$7\frac{1}{2}$
$\frac{3}{4}$	$3\frac{1}{4}$	$4\frac{3}{4}$	$23\frac{1}{4}$	$8\frac{3}{4}$	$19\frac{1}{4}$	$12\frac{3}{4}$	$15\frac{1}{4}$	$16\frac{3}{4}$	$11\frac{1}{4}$	$20\frac{3}{4}$	$7\frac{1}{4}$
1	3	5	23	9	19	13	15	17	11	21	7
$1\frac{1}{4}$	$2\frac{3}{4}$	$5\frac{1}{4}$	$22\frac{3}{4}$	$9\frac{1}{4}$	$18\frac{3}{4}$	$13\frac{1}{4}$	$14\frac{3}{4}$	$17\frac{1}{4}$	$10\frac{3}{4}$	$21\frac{1}{4}$	$6\frac{3}{4}$
$1\frac{1}{2}$	$2\frac{1}{2}$	$5\frac{1}{2}$	$22\frac{1}{2}$	$9\frac{1}{2}$	$18\frac{1}{2}$	$13\frac{1}{2}$	$14\frac{1}{2}$	$17\frac{1}{2}$	$10\frac{1}{2}$	$21\frac{1}{2}$	$6\frac{1}{2}$
$1\frac{3}{4}$	$2\frac{1}{4}$	$5\frac{3}{4}$	$22\frac{1}{4}$	$9\frac{3}{4}$	$18\frac{1}{4}$	$13\frac{3}{4}$	$14\frac{1}{4}$	$17\frac{3}{4}$	$10\frac{1}{4}$	$21\frac{3}{4}$	$6\frac{1}{4}$
2	2	6	22	10	18	14	14	18	10	22	6
$2\frac{1}{4}$	$1\frac{3}{4}$	$6\frac{1}{4}$	$21\frac{3}{4}$	$10\frac{1}{4}$	$17\frac{3}{4}$	$14\frac{1}{4}$	$13\frac{3}{4}$	$18\frac{1}{4}$	$9\frac{3}{4}$	$22\frac{1}{4}$	$5\frac{3}{4}$
$2\frac{1}{2}$	$1\frac{1}{2}$	$6\frac{1}{2}$	$21\frac{1}{2}$	$10\frac{1}{2}$	$17\frac{1}{2}$	$14\frac{1}{2}$	$13\frac{1}{2}$	$18\frac{1}{2}$	$9\frac{1}{2}$	$22\frac{1}{2}$	$5\frac{1}{2}$
$2\frac{3}{4}$	$1\frac{1}{4}$	$6\frac{3}{4}$	$21\frac{1}{4}$	$10\frac{3}{4}$	$17\frac{1}{4}$	$14\frac{3}{4}$	$13\frac{1}{4}$	$18\frac{3}{4}$	$9\frac{1}{4}$	$22\frac{3}{4}$	$5\frac{1}{4}$
3	1	7	21	11	17	15	13	19	9	23	5

1	2	3	4	5	6	7	8	9	10	11	12
$3\frac{1}{4}$	$\frac{3}{4}$	$7\frac{1}{4}$	$20\frac{3}{4}$	$11\frac{1}{4}$	$16\frac{3}{4}$	$15\frac{1}{4}$	$12\frac{3}{4}$	$19\frac{1}{4}$	$8\frac{3}{4}$	$23\frac{1}{4}$	$4\frac{3}{4}$
$3\frac{1}{2}$	$\frac{1}{2}$	$7\frac{1}{2}$	$20\frac{1}{2}$	$11\frac{1}{2}$	$16\frac{1}{2}$	$15\frac{1}{2}$	$12\frac{1}{2}$	$19\frac{1}{2}$	$8\frac{1}{2}$	$23\frac{1}{2}$	$4\frac{1}{2}$
$3\frac{3}{4}$	$\frac{1}{4}$	$7\frac{3}{4}$	$20\frac{1}{4}$	$11\frac{3}{4}$	$16\frac{1}{4}$	$15\frac{3}{4}$	$12\frac{1}{4}$	$19\frac{3}{4}$	$8\frac{1}{4}$	$23\frac{3}{4}$	$4\frac{1}{4}$
4	24	8	20	12	16	16	12	20	8	24	4

In horologio, quod circulo horæ 2. à mer. vel med. noc. æquidistat, lineæ quarumlibet duarum horarum huius tabulæ, sunt parallelæ, ex scholio propos. 22.

In a sundial whose plane lies parallel to the circle of the 2nd hour reckoned from noon or from midnight, the lines of any two hours in this table are parallel, from the scholium to Proposition 22.

Table — Line of the 1st hour from noon or midnight

Linea horæ prima à meridie vel media nocte.

Twelve columns, each headed *Horæ ab ortu vel occasu*. Column 1 is the argument.

1	2	3	4	5	6	7	8	9	10	11	12
$\frac{1}{4}$	$1\frac{3}{4}$	$4\frac{1}{4}$	$21\frac{3}{4}$	$8\frac{1}{4}$	$17\frac{3}{4}$	$12\frac{1}{4}$	$13\frac{3}{4}$	$16\frac{1}{4}$	$9\frac{3}{4}$	$20\frac{1}{4}$	$5\frac{3}{4}$
$\frac{1}{2}$	$1\frac{1}{2}$	$4\frac{1}{2}$	$21\frac{1}{2}$	$8\frac{1}{2}$	$17\frac{1}{2}$	$12\frac{1}{2}$	$13\frac{1}{2}$	$16\frac{1}{2}$	$9\frac{1}{2}$	$20\frac{1}{2}$	$5\frac{1}{2}$
$\frac{3}{4}$	$1\frac{1}{4}$	$4\frac{3}{4}$	$21\frac{1}{4}$	$8\frac{3}{4}$	$17\frac{1}{4}$	$12\frac{3}{4}$	$13\frac{1}{4}$	$16\frac{3}{4}$	$9\frac{1}{4}$	$20\frac{3}{4}$	$5\frac{1}{4}$
1	1	5	21	9	17	13	13	17	9	21	5
$1\frac{1}{4}$	$\frac{3}{4}$	$5\frac{1}{4}$	$20\frac{3}{4}$	$9\frac{1}{4}$	$16\frac{3}{4}$	$13\frac{1}{4}$	$12\frac{3}{4}$	$17\frac{1}{4}$	$8\frac{3}{4}$	$21\frac{1}{4}$	$4\frac{3}{4}$
$1\frac{1}{2}$	$\frac{1}{2}$	$5\frac{1}{2}$	$20\frac{1}{2}$	$9\frac{1}{2}$	$16\frac{1}{2}$	$13\frac{1}{2}$	$12\frac{1}{2}$	$17\frac{1}{2}$	$8\frac{1}{2}$	$21\frac{1}{2}$	$4\frac{1}{2}$
$1\frac{3}{4}$	$\frac{1}{4}$	$5\frac{3}{4}$	$20\frac{1}{4}$	$9\frac{3}{4}$	$16\frac{1}{4}$	$13\frac{3}{4}$	$12\frac{1}{4}$	$17\frac{3}{4}$	$8\frac{1}{4}$	$21\frac{3}{4}$	$4\frac{1}{4}$
2	24	6	20	10	16	14	12	18	8	22	4
$2\frac{1}{4}$	$23\frac{3}{4}$	$6\frac{1}{4}$	$19\frac{3}{4}$	$10\frac{1}{4}$	$15\frac{3}{4}$	$14\frac{1}{4}$	$11\frac{3}{4}$	$18\frac{1}{4}$	$7\frac{3}{4}$	$22\frac{1}{4}$	$3\frac{3}{4}$
$2\frac{1}{2}$	$23\frac{1}{2}$	$6\frac{1}{2}$	$19\frac{1}{2}$	$10\frac{1}{2}$	$15\frac{1}{2}$	$14\frac{1}{2}$	$11\frac{1}{2}$	$18\frac{1}{2}$	$7\frac{1}{2}$	$22\frac{1}{2}$	$3\frac{1}{2}$
$2\frac{3}{4}$	$23\frac{1}{4}$	$6\frac{3}{4}$	$19\frac{1}{4}$	$10\frac{3}{4}$	$15\frac{1}{4}$	$14\frac{3}{4}$	$11\frac{1}{4}$	$18\frac{3}{4}$	$7\frac{1}{4}$	$22\frac{3}{4}$	$3\frac{1}{4}$
3	23	7	19	11	15	15	11	19	7	23	3
$3\frac{1}{4}$	$22\frac{3}{4}$	$7\frac{1}{4}$	$18\frac{3}{4}$	$11\frac{1}{4}$	$14\frac{3}{4}$	$15\frac{1}{4}$	$10\frac{3}{4}$	$19\frac{1}{4}$	$6\frac{3}{4}$	$23\frac{1}{4}$	$2\frac{3}{4}$
$3\frac{1}{2}$	$22\frac{1}{2}$	$7\frac{1}{2}$	$18\frac{1}{2}$	$11\frac{1}{2}$	$14\frac{1}{2}$	$15\frac{1}{2}$	$10\frac{1}{2}$	$19\frac{1}{2}$	$6\frac{1}{2}$	$23\frac{1}{2}$	$2\frac{1}{2}$
$3\frac{3}{4}$	$22\frac{1}{4}$	$7\frac{3}{4}$	$18\frac{1}{4}$	$11\frac{3}{4}$	$14\frac{1}{4}$	$15\frac{3}{4}$	$10\frac{1}{4}$	$19\frac{3}{4}$	$6\frac{1}{4}$	$23\frac{3}{4}$	$2\frac{1}{4}$
4	22	8	18	12	14	16	10	20	6	24	2

In horologio, quod circulo horæ 1. à mer. vel med. noc. æquidistat, lineæ quarumlibet duarum horarum huius tabulæ sunt parallelæ, vt ex ijs, quæ à nobis in scholio propos. vigesima secunda sunt demonstrata, colligitur.

In a sundial whose plane lies parallel to the circle of the 1st hour reckoned from noon or from midnight, the lines of any two hours in this table are parallel, as is gathered from what we have demonstrated in the scholium to Proposition 22.

THEOREMA 19. PROPOS. 21.

The lines of all the hours reckoned from noon or from midnight, in any sundial whose plane cuts the axis of the world, intersect one another at that point of the axis where the axis itself meets the plane of the sundial.

[Margin: *All the lines of the hours from noon or midnight, in any sundial cutting the axis of the world, intersect one another at that point of the axis where the axis meets the plane of the sundial.*]

For since all the circles that mark the hours from noon or midnight run through the poles of the world (by Proposition 9 of this book), the axis of the world will — where their circumferences intersect one another — be the common section of all those planes. At whatever point, therefore, the axis meets the plane of the sundial, at that same point (by Proposition 18 of this book) the common sections of all those [circle-planes] and of the plane of the sundial come together — that is, the hour-lines from noon or midnight. And so the lines of all the hours from noon or midnight [intersect there], and so on. Which was to be demonstrated.

COROLLARIUM.

[Margin: *The lines of the hours from noon or midnight, in an equinoctial, horizontal, and vertical sundial of an oblique sphere, all meet at that point where the axis of the world cuts the plane of the sundial — except the vertical sundial of that oblique sphere in which the pole is at the zenith (the vertex of the head).*]

And so, in the equinoctial, the horizontal, and any vertical sundial of an oblique sphere, all the hour-lines from noon or midnight cut one another at one and the same point of the axis, namely where the axis itself meets the planes of the said sundials. For the planes of these sundials cut the axis, just as do the great circles to which they are parallel — namely the Equator, the Horizon, and the vertical circle of any oblique sphere — as is evident. Make an exception, however, for the vertical sundial of that oblique sphere in which the zenith is the pole of the world; for there the vertical circle passes through the poles of the world and

along the axis, and therefore does not cut the axis. Neither, then, can the plane of the sundial that is parallel to it cut that axis.

[Margin: *In every sundial that is parallel to a great circle not drawn through the poles of the world, the lines of the hours from noon or midnight intersect one another at one and the same point, namely where the axis of the world meets the sundial.*]

Finally, to comprehend everything in a single word: all the hour-lines from noon or midnight, in any sundial whose plane is not parallel to some circle passing through the poles of the world, nor to the axis of the world (of which kind are nearly all declining and inclined sundials), cut the axis of the world at one and the same point, namely where the axis itself meets the plane of the sundial. For any sundial of this kind cuts the axis of the world, since the great circle to which it is parallel is supposed not to pass through the poles of the world.

That point, where the axis of the world meets the plane of the sundial, is called **the center of the sundial** [centrum horologii].

[Margin: *What the center of the sundial is.*]

SCHOLIVM.

By the same reasoning, any hour-line whatever — whether from noon or midnight, or from rising or setting — whose hour-circles have the same common section, in any sundial whose plane is not parallel to that common section but cuts it (even though that section is not the axis of the world), cut one another at a single point of the common section, where it meets the plane of the sundial. For the demonstration is exactly the same, taken from Proposition 18 of this book. Of this kind are any three of the hours from rising or setting so placed in the table of Proposition 19 of this book, together with the equinoctial hour; and likewise any two placed in the thirty-six tables of the preceding proposition, together with the hour written in the title of each table.

[Margin: *Which hours from rising or setting, and from noon or midnight, are parallel in any sundial. — Any hour described in the tables of the two preceding propositions, together with the hour, or line, placed in the title of the table.*]

THEOREMA 20. PROPOS. 22.

The lines of all the hours reckoned from noon or from midnight, in any sundial whose plane is parallel to some circle drawn through the poles of the world, or to the axis of the world, are parallel.

For since all the hour-circles reckoned from noon or midnight are described (by Proposition 9 of this book) through the poles of the world, the axis of the world will — where their circumferences intersect one another — be the common section of their planes. Therefore, if the plane of the sundial that cuts those hour-circles is parallel to some circle drawn through the poles of the world, and hence through the axis (their common section), or to the axis itself, then (by Proposition 18 of this book) the common sections of all those circles and of the plane of the sundial that cuts them — that is, the hour-lines from noon or midnight — will be parallel. Therefore the lines of all the hours from noon or midnight, and so on. Which was to be demonstrated.

COROLLARIUM.

It follows from these results that in the horizontal sundial of the right sphere, in the meridian sundial, and in any polar sundial, all the hour-lines from noon or midnight are parallel; since the planes of these sundials are parallel to circles drawn through the poles and the axis of the world — namely to the right horizon, to the meridian, and to the circle of the 6th hour from noon or midnight — as is clear.

Finally, to embrace much in few words: all the hour-lines from noon or midnight, in any sundial whose plane is parallel to some hour-circle reckoned from noon or midnight, or at any rate to the axis of the world, are — by this proposition — parallel.

SCHOLIUM.

In like manner, any hour-lines whatever — whether from noon or midnight, or from rising or setting — whose hour-circles have the same common section, are parallel in any sundial whose plane is parallel to some circle drawn through that common section, or to the common section itself, even though that common section is not the axis of the world. For the demonstration is exactly the same, drawn from Proposition 18 of this book.

[Margin: *The lines of the hours from noon or midnight are all parallel in a sundial parallel to a great circle drawn through the poles of the world. — The hour-lines from noon or midnight in the horizontal sundial of the right sphere, and in any meridian and polar sundial, are all equidistant from one another. — In every sundial parallel to a great circle drawn through the poles of the world, the lines of the hours from noon or midnight are all parallel. — Which hours from rising or setting, and from noon or midnight, are parallel in any sundial.*]

Hence it comes about that any three hours placed in the table of Proposition 19 of this book — of which one is always reckoned from noon or midnight, while the rest are taken from rising or setting — are parallel in the equinoctial sundial [horologium æquinotiale]. For the circles of those hours, and the equator, have the same common section, by Proposition 17 of this book. Therefore, since the plane of the equinoctial sundial that cuts those hour-circles is parallel to one of them, namely to the equator, the common sections of those circles and of the sundial's plane — that is, the hour-lines — will be parallel.

[Margin: In the equinoctial sundial, any three hours placed in the table of Proposition 19 of this book are parallel to one another.]

For the same reason, any two hours placed in the first table of Proposition 20 of this book will be parallel in the horizontal sundial. For their hour-circles, and the circle of the 24th hour from rising or setting — that is, the horizon — have the same common section, by Proposition 17 of this book; and the plane of the sundial is parallel to one of them, namely to the circle of the 24th hour from rising or setting, that is, to the horizon.

[Margin: In the horizontal sundial, any two hours placed in the first table of Proposition 20 of this book are parallel to one another.]

Again, in the vertical sundial for latitude 45° , or in the sundial that is parallel to the circle of the 12th hour from rising or setting (which is the same circle as the vertical for latitude 45° , as is evident from the material sphere), any two hours placed in the second table of Proposition 20 of this book will be parallel. For their hour-circles, and the circle of the 12th hour from rising or setting — that is, the vertical for latitude 45° — have the same common section, by Proposition 17 of this book; and the plane of the sundial is parallel to one of them, namely to the circle of the 12th hour from rising or setting, that is, to the vertical for latitude 45° .

[Margin: In the vertical sundial for latitude 45° , any two hours described in the second table of Proposition 20 of this book are parallel to one another.]

Moreover, in the polar sundial any two hours placed in the third table of Proposition 20 of this book will be parallel; because their circles, and the circle of the 6th hour from noon or midnight — to which the sundial's plane is parallel — have the same common section, by Proposition 17 of this book, and so on.

[Margin: In the polar sundial, any two hours placed in the third table of Proposition 20 of this book are parallel.]

Finally, any two hours described in the fourth table of Proposition 20 of this book will be parallel in the meridian sundial; because their hour-circles, and the circle of the 12th hour from noon or midnight — that is, the meridian [meridianus], to which the sundial's plane is parallel — have the same common section, by Proposition 17 of this book, and so on.

[Margin: In the meridian sundial, any two hours placed in the fourth table of Proposition 20 of this book are parallel.]

Now, of these five circles — the horizon (that is, the circle of the 24th hour from rising or setting), the circle of the 6th hour from noon or midnight, the circle of the 12th hour from rising or setting, the vertical circle, and the equinoctial — the first three, namely the horizon, the circle of the 6th hour from noon or midnight, and the circle of the 12th hour from rising or setting, have the same common section, by Proposition 17 of this book. Likewise, by Proposition 16, the first, third, and fifth — namely the horizon, the circle of the 12th hour from rising or setting, and the equator — also have the same section. Likewise too the first, fourth, and fifth — that is, the horizon, the vertical, and the equator — since, being at right angles to the meridian, they pass through the poles of that same meridian (by Proposition 13 of Book 1 of Theodosius [Theodosius, *Spherics* I.13]) and there intersect one another. Thus it comes about that all five have one and the same common section. Therefore, in that sundial whose plane is parallel to one of them, the common sections of the remaining four and of the sundial's plane will be parallel, by Proposition 18 of this book. But if the sundial's plane cuts all five circles yet is parallel to their common section, then by the same Proposition 18 the common sections of all of them and of the sundial's plane will be parallel.

Thus in the equinoctial sundial, since its plane is parallel to the equator, the following lines are parallel: the horizontal line (that is, of the 24th hour from rising or setting), the line of the 12th hour from rising or setting, the line of the 6th hour from noon or midnight, and the vertical line.

Likewise, the same lines — and moreover the equinoctial line — are parallel in the horizontal sundial and in the vertical, since the plane of the former is parallel to the horizon, and that of the latter to the vertical circle. Yet in the former the horizontal line cannot be drawn, and in the latter the vertical line cannot be drawn, as is clear from what we wrote in the scholium of Proposition 11 of this book.

Similarly, in the polar sundial the equinoctial line, the vertical line, and the lines of the 24th and 12th hours from rising or setting are parallel; for its plane too is parallel to the circle of the 6th hour from noon or midnight.

Finally, for the same reason, in the meridian sundial the lines of the 6th and 18th hours from rising or setting, and the equinoctial line, are parallel. For, by Proposition 17 of this book, their circles and the meridian (that is, the circle of the 12th hour from noon or midnight), and also, by Proposition 16, their circles and the equator, have the same common section; and the plane of the sundial is parallel to the meridian circle.

I have said all this so that you may understand that the lines of certain hours placed in the tables of Propositions 19 and 20 of this book are sometimes parallel in some sundials, while in others they intersect one another at one and the same point, as we said there. Of this kind are the hour-lines just now cited.

[Margin: The horizon, the circle of the 6th hour from noon or midnight, the circle of the 12th hour from rising or setting, the vertical, and the equator all have one and the same common section. — In the equinoctial sundial the horizontal line (that is, of the 24th hour from rising or setting), the line of the 12th hour from rising or setting, the line of the 6th hour from noon or midnight, and the vertical line are parallel. And the same lines are parallel in the horizontal and the vertical sundial.]

PROBLEMA 3. PROPOS. 23.

To investigate the declination [declinatio] of any plane from the vertical circle, and its inclination [inclinatio] to the horizon.

Let there be prepared, out of wood or some other solid material, a rectangle ABCD one of whose surfaces is flat; and its sides AB, DC being bisected at E and F, let the straight line EF be drawn — ^a which will be parallel to each of the sides AD, BC, ^b and therefore the angles at E are right angles. Or at any rate let there be, at the edge of the flat surface, a straight line AB, which the straight line EF cuts at right angles, whatever may be the case with the other lines.

[Margin: An instrument for investigating the declinations and inclinations of planes.] [Margin: a. *Elements* I.33.] [Margin: b. *Elements* I.29.] [Margin: How the declination of the proposed plane is to be found.]

With this instrument, even if it is not a rectangle, we shall investigate the declination of any plane in the following way (by "declination" I mean the acute

angle that the common sections of the horizon with the proposed plane and with the vertical circle make). On the proposed plane — whether it is perpendicular to the horizon, as is the plane of any wall built to the plumb-line, or inclined, as is the plane of any roof — let there be drawn, with the help of the level and the plumb-line, a line AB parallel to the horizon, namely the common section of the proposed plane and of the horizon. Or at any rate, on a plane perpendicular to the horizon, by a thread from which a plumb-bob hangs, let down freely, let a line be drawn on the wall corresponding to the plumb-thread, which will be perpendicular to the horizon. Then let another straight line AB cut this at right angles. For this line will be parallel to the horizon, or if you prefer, the common section of the horizon and the wall. For since the wall is set perpendicular to the horizon, and on the wall a line perpendicular to the horizon has been drawn by the plumb-thread, this same line will — by Definition 4 of Book 11 of Euclid [*Elements* XI, def. 4] — be perpendicular to the common section of the horizon and the wall. Therefore the straight line AB, drawn perpendicular to this, will be the common section of the wall and

the horizon. To this line AB let the side AB of the instrument be applied, and let the instrument itself be set parallel to the horizon with the help of the level and the plumb-line. Then, on the instrument, let the meridian line GH be found — by means of a needle rubbed with a magnet [lodestone], or by the method we handed down in our commentaries on the Sphere, when we treated of the meridian circle, or (which I approve more) by the scholium that follows. For from this line we shall accomplish what is proposed, in the following way.

If the meridian line GH is parallel to the straight line EF, and meets the wall (or the line AB, which is parallel to the horizon) at right angles, the wall will have no declination, and will face directly to the south or to the north — so that, if it is perpendicular to the horizon, it is parallel to the vertical circle properly so called; but if it is inclined to the horizon, it is perpendicular to the meridian, and is parallel to the great circle passing through the points of the equinoctial rising and setting. Such is the plane of the equinoctial circle, of the circle of the 6th astronomical hour, and of the circle of the 12th hour from rising or setting; and finally the planes of the hectemoral circles [circuli hectemorii — the "sixth-part" position circles], of which Ptolemy [Ptolemy] treats in the *Analemma*.

But if the meridian line GH cuts the straight line EF at I at right angles, the plane of the wall will incline directly to the east or west, so that, if it is perpendicular to the horizon, it is parallel to the meridian circle. But if it is inclined to the horizon, it is perpendicular to the vertical circle properly so called, and is parallel to the

great circle drawn through the points where the circumference of the meridian circle intersects the circumference of the horizon. Such are the planes of the hour-circles, of which Ptolemy treats in the *Analemma*.

Finally, if the meridian line GH cuts the straight line EF at I at unequal angles, the plane will decline both from the vertical circle and from the meridian, so that it faces neither directly to the south or north, nor to the east or west. The angle EIG will always be the angle of the plane's declination from the vertical circle properly so called, so that the declination of the plane is as great as the angle EIG; and thus the arc of a circle described from center I, contained between the straight lines IE and IG, contains the degrees of declination. For let GK be drawn in the plane of the instrument ABCD, perpendicular to GH, so that GK is the common section of the vertical circle properly so called and of the plane in which the instrument ABCD lies. Then EGK will be the angle of declination of the proposed plane drawn through the straight line AB, from the vertical drawn through the straight line GK. For since both the plane of the vertical circle drawn through GK and the proposed plane drawn through AB are perpendicular to the horizon, ^a the common section of the vertical and of the proposed plane will also be perpendicular to the horizon, and therefore also to the straight lines GK, AB lying in the horizon, by Definition 3 of Book 11 of Euclid [*Elements* XI, def. 3]. Therefore, by Definition 6 of the same book [*Elements* XI, def. 6], EGK will be the angle of declination — or of inclination — of the proposed plane drawn through AB, to the vertical

[Margin: a. whence, [from] Prop. 19.]

Diagram. A large wind-rose arrangement showing the rectangular instrument ABCD set out in the eight principal orientations around a central compass. The four cardinal directions are lettered in large display capitals around the figure (south is up): MERIDIES (south) across the top, OCCASVS (west) up the right side, SEPTENTRIO (north, printed inverted) across the bottom, ORTVS (east) up the left side. Eight small instrument-figures are ranged octagonally around the center: the four at the cardinal positions are axis-aligned rectangles (the wall facing due south, north, east, or west, with no declination), and the four at the diagonal positions are tilted diamonds (the declining orientations). Each small figure carries the instrument's point-letters (corners A, B, C, D; midpoints E, F of the sides AB and DC; and the meridian/section points G, H, I, K). The individual peripheral labels are small and worn on the scan and do not resolve reliably figure-by-figure.

- MERIDIES (top): south

- OCCASVS (right side): west
- SEPTENTRIO (bottom, inverted): north
- ORTVS (left side): east
- Eight instrument-figures with corner labels A, B, C, D; midpoint labels E, F; and section-point labels G, H, I, K [most individual per-figure labels [?] — worn small type]

[the eight small instrument-figures carry many worn peripheral point-labels that do not resolve individually at this resolution.]

circle drawn through GK; since the straight lines GK, GE make right angles with the common section at the same point G — the point of the common section of the proposed plane and of the vertical — as has been said. But if the plane drawn through AB is not perpendicular to the horizon, EGK will nonetheless be the angle of declination, though improperly so, for it shows the declination of the line AB (which is parallel to the horizon) from the vertical circle. Therefore, since the angle EIG is equal to the angle EGK — for since the right angle IGK is equal to the two angles IGE and EIG together, ^b because these are equal to one right angle, on account of the right angle GEI; and if the common angle IGE is taken away, the angles EGK and EIG remain equal — the angle EIG will also be the angle of declination of the given plane from the vertical circle. Which was proposed.

[Margin: b. *Elements* I.32.]

Now, whether the proposed plane declines toward the east or toward the west, we shall determine thus. If the plane faces south and the meridian line cuts the straight line EB, it will decline from south toward east; but if the meridian line cuts the straight line EA and the plane faces south, it will decline from south toward west. The contrary holds if the plane faces north: for, with the meridian line cutting the straight line EB, the plane will decline from north toward west, but with it cutting the straight line EA, toward east — as appears from the figure. And if from I you describe a circle with any radius, the arc contained between the straight lines IE and IG will give the degrees of declination, as we also said before.

[Margin: By what method it may be known whether the proposed plane declines toward the east or toward the west.]

We shall learn the same thing in this way. Since the meridian line GH, when it cuts EF obliquely, makes with the straight line AB an acute angle (under which the straight line IE is always subtended) and a remaining obtuse angle, the plane will decline toward whichever side this obtuse angle lies. So that, if the obtuse

angle is toward the east, the plane declines from south or north toward east; but if toward the west, toward west.

For the rest, we shall know that the plane declines from south toward east or west — that is, that it faces south — when, as we turn toward the plane, the Sun rises on our right and sets on our left; but that it faces north, when the Sun rises on our left and sets on our right. But if the plane has so great a declination from the vertical that it differs little from the meridian circle, and it is therefore very difficult to discern whether it faces south or north...

[Margin: Whether the proposed plane faces south or north.]

toward north, we shall use this device. To the plane of the wall — or at any rate to the straight line drawn parallel to the horizon in it — we shall draw, in a plane parallel to the horizon, a perpendicular, and we shall investigate its declination. For if the wall declines little from the meridian circle, the said perpendicular will also decline little from the vertical circle properly so called; and therefore we shall easily understand whether it faces south or north, according to the rule prescribed, and so we shall also know its declination. Thus, if this perpendicular declines from south toward east, or from north toward west, the proposed wall will decline from north toward east if it faces east, or from south toward west if it pertains to the west. But if the said perpendicular declines from south toward west, or from north toward east, the proposed wall will decline from south toward east if it inclines to the east, or from north toward west if it faces west.

All this is clear from this figure, in which the meridian line is IK, and the common section of the vertical circle and the horizon is LM, cutting IK at N at right angles. For, given the wall AB which faces east and the wall CD which faces west: since the perpendicular BC declines from south toward east, and the perpendicular AD from north toward west, the wall AB therefore declines from north toward east, and the wall CD from south toward west. Similarly, given the wall EF facing east and the wall GH facing west: since the perpendicular FG declines from south toward west, and the perpendicular EH from north toward east, the wall EF therefore declines from south toward east, and the wall GH from north toward west.

Diagram. A square plan with a compass cross, illustrating four walls and their perpendiculars relative to the cardinal directions (north at top). The cardinal directions are lettered in large capitals around the square: SEPTENTRIO (north) across the top, ORTVS (east) up the right side, MERIDIES (south, printed inverted) across the bottom, OCCASVS (west) up the left side. A vertical center line IK is the

meridian line, and a horizontal center line LM is the common section of the vertical circle and the horizon; the two cross at N in the center at right angles.

- H (top-left corner)
- A (top-right corner)
- B (bottom-right corner)
- G (bottom-left corner)
- D (inner, upper-left)
- E (inner, upper-right)
- C (lower-left, near G)
- F (lower-right, near B)
- I (top center, on meridian line IK)
- K (bottom center, on meridian line IK)
- L (left center, on section line LM)
- M (right center, on section line LM)
- N (center — intersection of IK and LM)
- SEPTENTRIO (top): north
- ORTVS (right): east
- MERIDIES (bottom, inverted): south
- OCCASVS (left): west

We can obtain the same result by the meridian line alone. For, when a meridian line has been found near the wall, in a plane parallel to the horizon: if this line meets the wall toward the north [Boreas], and the wall faces east, the wall will decline from south toward east; but from south toward west, if the wall faces west. But if the meridian line meets the wall toward the south, the wall will decline from north toward east or west, according as it faces east or west — as easily appears from the earlier figure of this proposition. But since, in planes that differ little from the meridian circle, the meridian line scarcely meets them, I would judge that the earlier method should be chosen.

In planes perpendicular to the horizon, we shall attain the same result more easily by this method. On some day let the time of noon be awaited, which must be carefully learned either by some clock or by the meridian altitude of the Sun. For if at that moment the proposed plane is lit by the Sun, there will be no doubt at all that it faces south; but that it faces north, if it is not illuminated by the Sun. Indeed, in this way too we shall learn whether the proposed plane declines toward the east or toward the west. For if it faces south but is lit by the Sun for a longer time before noon than after noon, it will decline from south toward east;

but toward west, if the case is the reverse. Likewise, if the plane faces north and is illuminated by the Sun for more hours in the morning than in the evening, it will deflect from north toward east; but if the case is the opposite, toward west.

The same may be tested on planes inclined to the horizon: if, at noon, upon the line drawn in the proposed plane parallel to the horizon, some flat tablet of any material is set perpendicular to the horizon, and one observes whether the face that makes an obtuse angle with the inclined plane is then lit by the Sun or not. For from this we shall learn — as we said of planes perpendicular to the horizon — whether that tablet, and hence that line parallel to the horizon, or rather the inclined plane itself, declines from south or from north.

But all this must be understood of an oblique sphere having a latitude greater than $23^{\circ} 30'$, which is the greatest declination of the Sun. For in a lesser latitude it can happen that a plane declining from the north is lit by the Sun at noon, when the Sun is at or near the beginning of Cancer [♋], as is evident from the material sphere. But when the Sun is more to the south than the wall itself, so that all shadows are projected toward the north, the things we have said will hold good even in an oblique sphere of latitude less than $23^{\circ} 30'$.

But if the wall is little distant from the meridian circle, and the right-angled edge of the building projects so that at noon the wall cannot be lit by the Sun, then a line parallel to the wall must be drawn on the ground at a distance from the wall, such that at noon it is wholly illuminated by the Sun. For if a stylus set on that line — or at any rate a thread hanging freely with a plumb-bob — casts at noon a shadow that cuts that line, so that on the northern side the shadow lies between the wall and the said line, the wall will face south; but toward north, if on the contrary that line lies, on the northern side, between the shadow and the wall. But if the shadow falls upon the line itself, the wall will be parallel to the meridian circle.

Now we shall investigate the inclination of any plane to the horizon with the same instrument ABCD, in the following way (by "inclination" I mean the acute angle that the common sections of the horizon and of the proposed plane make with another plane perpendicular to both). Having found in the proposed plane a line parallel to the horizon — namely the common section of the proposed plane and of the horizon — let the straight line NO cut it at M at right angles; and to NO let the side AB of the instrument be applied, and let the instrument itself be raised, by means of the plumb-line, so that it is perpendicular to the proposed plane. Then, in the instrument thus erected, let there be drawn — by means of the same plumb-line — a perpendicular HG to the horizon, cutting EF at I. I say

that $\angle EIG$ is the angle of inclination of the plane to the horizon. For let GK be understood as drawn in the plane of the instrument $ABCD$, perpendicular to GH , so that GK is the common section of the horizon and of the plane in which the instrument $ABCD$ lies. Then $\angle EGK$ will be the angle of inclination of the proposed plane to the horizon. For since both the proposed plane (by construction) and the plane of the horizon are perpendicular to the plane of the instrument $ABCD$ — for since HG is perpendicular...

[Margin: By what method the inclination of the proposed plane to the horizon is to be investigated.]

Gnomonics — Liber Primus (continued)

Problema 3, Propositio 23 (conclusion of the inclination construction)

[Margin (a): Elements XI.18] [Margin (b): Elements XI.19]

is perpendicular to the horizon, then the plane $ABCD$ too — through HG , which is drawn perpendicular to the horizon — will stand at right angles to the horizon; and conversely, the common section of the proposed plane and of the horizon will likewise be perpendicular to that same plane $ABCD$, and hence also to the straight lines EG , GH lying in the said plane $ABCD$ (by Elements XI, def. 3), if the horizon is understood to cut the proposed plane at the point G . Therefore, by the sixth definition of the same book, $\angle EGK$ will be the angle of inclination of the proposed plane to the horizon; for the straight lines EG and Gk — the former lying in the proposed plane, the latter in the horizon — meet at the same point G (the common section of the proposed plane and the horizon) and make right angles with that common section, as has been said.

[Margin (c): Elements I.32]

Now the angle $\angle EIG$ is equal to the angle $\angle EGK$. For the angle $\angle IGk$, being a right angle, is equal to the two angles $\angle IGE$ and $\angle EIG$ taken together — since these two are together equal to one right angle, on account of the right angle $\angle GEI$; and if the common angle $\angle EGI$ is removed, the remainders $\angle EGk$ and $\angle EIG$ are equal. Hence the angle $\angle EIG$ is likewise the angle of inclination of the given plane to the horizon. Which was what was proposed.

But if the perpendicular HG cuts the straight line EF at I at right angles, then the proposed plane has no inclination to the horizon, and stands perpendicular to it, as is plain.

Diagram. An inclined-plane (wedge) figure illustrating the inclination construction of Proposition 23: a quadrilateral plane ABCD leaning back from the ground, with a shaded solid triangular flank at the right giving the perspective. The apex/north point rises at top, and the perpendicular HG and the line EF meet at the interior point I, where a small arc (a circle described about I) marks the inclination angle between IE and IG.

- N (top apex): north point
- A (upper right of the plane): corner label
- D (upper left of the plane): corner label
- H (just left of centre, near the top): foot of the perpendicular line
- F (left, mid-height): endpoint of line EF
- E (centre, mid-height): endpoint of line EF
- I (centre, at the small arc): intersection of HG and EF; vertex of the inclination angle EIG
- M (just below E): label on the descending edge
- C (lower left corner of the plane): corner label
- G (bottom centre): point where the horizon cuts the plane; foot of HG
- B (just right of G): corner label
- K (right of B): point on the horizon line (angle EGK)
- O (bottom foot): lowest point of the perpendicular drop

We shall also easily grasp toward which region the plane inclines — that is, whether toward the northern part of the upper hemisphere or toward the southern — once the plane's declination from the vertical is known. For if the plane declines from the north toward the east or the west, its inclination falls in the southern part of the hemisphere; but if it declines from the south toward the east or west, the inclination falls in the northern part, as is clear from a material sphere. And if a circle is described about I with any radius whatever, the arc contained between the straight lines IE and IG gives the degrees of inclination. We have therefore found the declination of any plane from the vertical circle, and its inclination to the horizon. Which was to be done.

[Margin: In which direction the inclination of the proposed plane to the horizon falls.]

Scholium

Besides the method of finding the meridian line that we handed down in our *Commentary on the Sphere*, it has seemed good to add another here, perhaps better suited to practical use. For this method requires neither two observations — one to be made before noon and the other after, as the former method did — nor the marking of points at the tips of the shadows, which is not at all easy, since the end of a shadow can scarcely be made out on a plane.

[Margin: By what means the meridian line is found by means of an astrolabe laid out on a plane.]

About to find, then, the meridian line on any given day, first learn the Sun's place from some ephemeris. Then, on a plane lying level with the horizon, observe the shadow of some freely hanging thread, or at least of some perfectly straight rule set at right angles to the proposed plane. For this task I habitually use the instrument you see pictured here, in which the set-square CDE is fixed at right angles to the flat rule AB, along the straight line DI, which is parallel to one side of the rule AB — so that the side DH of the square stands like a kind of gnomon set perpendicular to the horizon, or like a freely hanging thread, while the rule AB is laid on a plane parallel to the horizon. By this means we shall, with little effort, mark two points without appreciable error at the tip of the breadth of the shadow that the side HD casts.

But if in the plane CD a line FG is drawn parallel to the side HD, and a hole is made near the point G, so that a plumb-line suspended by a fine thread from the hole F can swing freely in it, this same instrument will also be very convenient for testing any proposed plane, to see whether or not it is parallel to the horizon. For by grazing the plane CD with the thread along the line FG, the plane in which the rule AB lies will be made parallel to the horizon. Now, in the shadow — whether of the thread or of the side HD — let two points A and B be marked a little apart from each other, points that a straight line AB will...

Diagram. The meridian-line instrument: an L-shaped (carpenter's-square) frame set on a flat base-board. The upright arm carries a plumb-line (a small bob is drawn hanging on a thread near G), and the horizontal rule and base are shown in perspective.

- H (top left of the upright): top corner of the square
- C (top right of the upright): corner label
- F (on the upright, near the top): hole from which the plumb-thread hangs

- A (left, at the base of the upright): endpoint of the rule AB
- G (at the pivot, where the bob hangs): point near the plumb-line hole
- E (right, on the horizontal arm): endpoint
- D (bottom left of the base): corner of the base-board
- I (right, on the horizontal arm): point on line DI
- B (lower right, on the base-board): endpoint of the rule AB

be joined. This straight line will be the common section of the underlying plane and of the vertical circle that passes through the Sun's centre at the moment of observation. Once the shadow has been observed, take the Sun's altitude without delay, with a quadrant or an astrolabe; and this altitude must be taken immediately after the two points in the shadow have been marked, before the straight line is drawn through them — lest there be danger in delay, because, owing to the Sun's rising before noon or its setting after noon (that is, owing to the Sun's daily motion), the shadow necessarily shifts, and the Sun at once comes to lie in another vertical circle.

After this, on an astrolabe in which the Equator is CDEF about the centre G — where the two diameters CE and DF cut each other at right angles — and the vertical circle properly so called is HIKL about the centre M — through which the straight line IL, drawn, cuts CE at right angles — let there be described the Sun's day-parallel NO. Let us suppose, for example, that this parallel passes through 15° of Leo [ϱ], and has a declination of $16^\circ 23'$. Let the horizon-parallel PO — drawn for the Sun's altitude, which we now suppose to be 30° — cut that day-parallel at the point O on the eastern side (for let us now suppose the observation to be made before noon).

For through this point O the vertical circle must be described that passes through the Sun's centre at that moment. And we shall find that vertical's centre, which lies on the straight line IL (as we demonstrated in the treatise on the astrolabe), in this way. From H and O let two pairs of arcs be described, both above the points H and O and below them, at any radius, intersecting one another at two points; and through these let a straight line be drawn cutting IL at Q. For Q will be the centre of the vertical that is to be described through H and O. For, as is established from what we demonstrated in the scholium to Euclid III.25, the straight line joining those two intersection points passes through the centre of the circle to be described through H and O. If, then, a circle is described about Q with radius QH (or QO), this will be the vertical passing through the Sun's centre at the time of observation.

Now this vertical will, on its eastern side, deflect southward from the vertical properly so called HIKL if the centre Q has fallen on the straight line EI and the observation is made before noon; but northward, if Q has fallen on EL and the observation is made before noon. But if the observation is made after noon — since then the point O must be taken on the western side — the vertical HO will deflect from the proper vertical southward on the western side if the centre Q falls on the straight line EL; and northward, if Q falls on the straight line EI.

How great the deflection is, we shall determine thus. From the vertex H, through Q, let a straight line be drawn cutting the vertical HIKL at R. Then KS, half the arc KR, will be the declination of the vertical HO from the vertical properly so called HIKL, as is clear from what we demonstrated in the treatise on the astrolabe. Its complement IS will therefore be the declination of the same vertical from the meridian circle. And so, if from any point A of the shadow-line a circle BT is described equal to the vertical HIKL, and an arc BT is taken equal to the arc IS — reckoned from the east toward the south, if the observation is made before noon and the vertical HO deflects southward on the eastern side, as in our example; or from the east toward the north, if the observation is made before noon and the vertical HO deflects northward on the eastern side; and likewise, from the west toward the south or the north, if the observation is made after noon, and so on — then the straight line drawn, AT, will be the meridian line, namely the common section of the proposed plane and of the meridian circle.

[Margin: The finding of the meridian line by means of sines.]

This same declination KS of the vertical HO from the vertical properly so called HIKL can also be found by the calculation of sines, and thereby the meridian line itself can be drawn. For if, from the Sun's declination and the altitude the Sun has at the moment of observation, the Sun's distance from the meridian circle is found by sines (as we shall teach in Proposition 36 of this book), and if, by means of this distance, the horizontal arc is investigated from the same sines — that is, the arc of the horizon intercepted between the vertical properly so called and the vertical passing through the Sun's centre at the moment of observation (as we shall show in Proposition 3 of Book 5) — then this arc will be equal to the arc KS, and so on.

[Margin: By what art the meridian line is found by means of the analemma.]

Indeed, without laying out an astrolabe, we shall find this same meridian line by the analemma alone — as Giovanni Battista Benedetti [Iohannes Baptista Benedictus] also does in his book *On the Use of Gnomons and of the Shadows cast*

by *Sundials* [*De gnomonum umbrarumque solarium usu*] — in the following way. Having found, as before, by the shadow, the straight line AB, which is the common section of the plane lying level with the horizon and of the vertical circle passing through the Sun's centre at the moment of observation; and having, at the same moment, taken the Sun's altitude: then, in place of the astrolabe, we shall draw an analemma in which the meridian is FGHI; the common section of the horizon and the meridian is GI; the common section of the vertical properly so called and of the same meridian is FH; the common section of the same meridian and the Equator is LM; and, finally, the common section of the meridian and the Sun's parallel on the day the observation is made is the straight line NO — which is drawn by means of the Sun's declination, as we taught in Proposition 1 of this book.

Then, with the Sun's altitude reckoned off, measured from I as far as P and from G as far as Q, we shall draw the straight line PQ. By the scholium to Euclid III.27, this line will be parallel to GI, and hence will be the common section of the meridian and of the horizon-parallel passing through the Sun's centre; and it will cut the vertical line FH at R, and the diameter NO of the Sun's parallel at S. With a semicircle PTQ described about the centre R upon PQ, we shall draw from S the perpendicular ST to PQ, extended as far as the circumference of the semicircle PTQ, and we shall join the straight line TR.

If, then, the point S falls between Q and R, and the observation is made before noon, we shall construct at the centre C — chosen anywhere on the shadow-line AB, and about which we describe a circle of any size — an angle ACD equal to the acute angle TRQ, reckoned from the east toward the south, that is, from the point A toward the point D, as is seen in figure A. But if the observation is made after noon, we shall make the angle ACD equal to that same angle, from the west toward the south, that is, from the point A toward the point D, as appears in figure B. But if the point S falls upon the point R — whether the observation is made before noon or after — we shall draw through C the perpendicular DE to AB, as is clear in figure C. If, finally, the point S falls between R and P, and the observation is made before noon, we shall make the angle ACE equal to the acute angle TRP, from the east toward the north, that is, from the point A toward the point E, as may be seen in figure D. But if the observation is made in the afternoon, we shall make the angle ACE equal to that same angle, from the west toward the north, that is, from the point A toward E, as is plain from figure E. For in every case the straight line DE will be the meridian line.

[Margin (a): Elements XI.19]

We shall confirm this in the following way. Since the horizon-parallel PTQ and the Sun's parallel are both perpendicular to the meridian, their common section too will be perpendicular to the meridian, and hence (by Euclid XI, def. 3) also perpendicular to the straight line PQ at the point S, where the diameters of the said parallels bisect each other. Therefore ST, being perpendicular to PQ, will be the common section of the said parallels; and consequently, at the moment of observation, the Sun's centre will be at the point T, provided the horizon-parallel PTQ, together with the meridian of the analemma, holds its proper position. Hence the vertical passing at that time through the Sun's centre will pass through the point T. And since it also passes through the straight line FH — the common section of all the verticals — and thus through the point R, TR will be the common section of the said vertical and of the horizon-parallel PTQ. Therefore, since the straight line QR (or PR) is perpendic-

ular to FH, the common section of the meridian and the vertical, which is drawn through TR, and since likewise TR is perpendicular to that same FH — for since FH, the axis of the horizon-parallel PTQ, is perpendicular to the circle PTQ (by Theodosius, *Sphaerics* I.10), the angle FRT will be right (by Euclid XI, def. 3) — therefore, by Euclid XI, def. 6, the acute angle TRQ (or TRP) will be the angle of inclination, to the meridian FGHI, of the vertical drawn through TR. And for this reason, if an angle equal to the said angle is constructed at the centre C on the line AB — the common section of that vertical and of the plane ADBE — the straight line DE will be the meridian line, that is, the common section of the meridian and of that same plane ADBE; inasmuch as, with the straight line AB, it forms the angle of inclination of the said vertical and of the meridian.

Now since the vertical properly so called, drawn through FH to the meridian FGHI, is perpendicular, and separates the northern part of the upper hemisphere from the southern — so that the part tending toward G is southern, the rest, toward I, northern — it comes about that the Sun is southern when the point S (where the diameters NO and PQ intersect) falls between Q and R; that is, beyond the vertical circle properly so called, toward the south. It is on the vertical circle properly so called when the point S coincides with R, where the diameter NO of the Sun's parallel bisects the vertical diameter FH. And, finally, it is northern when the point S falls between R and P. Hence it has come about that our rule is that the acute angle which the straight line TR makes with PQ is to be set out equal at C, upon the straight line AB — now from the east or the west toward the south, now toward the north, and so on.

Diagram. A large, densely ruled analemma for finding the hour, filled with a web of straight construction lines and several concentric/intersecting circles. Around the outer circumference runs a graduated hour-scale: on the upper-right quadrant the Arabic numerals **1, 2, 3, 4, 5, 6, 7, 8, 9, 10** appear in sequence (with **Y** beside 7 and **20** low on the right), and Roman **II** lower right. Along the vertical axis and the horizon line are the principal lettered points; the many interior line-crossings carry small repeated letters (chiefly R, S, T, P, and lower-case f, e, d, z) that are too fine and worn to resolve individually at this resolution.

- N (upper left, on the rim): endpoint of a principal diameter
- F (top, just left of centre): meridian point
- P (top): pole/upper meridian point
- Q (upper left interior): point on the construction
- G (left, on the rim): endpoint of the horizon diameter GI
- Y (right, by hour 7): rim point
- I (right, mid): endpoint of the horizon diameter
- M (bottom, near the axis foot): point on the axis
- H (bottom centre): foot of the vertical axis
- K (interior, low centre): axis point
- Several interior points labelled R, S, T, P, and lower-case d, e, f, z along the diameters NO, PQ, GI and the diagonal chords [most individual interior labels: ?]

Diagram. A series of five small circles, lettered **A, B, C, D, E** (the large letter set inside each), each showing the shadow-line and the meridian line for one of the five cases described in the text. In every circle the point D is at top, C at the centre, E at the bottom, with A and B at the sides; the diameter DE (the meridian line) is set at the appropriate slant, while AB is the shadow-line through C.

- Figure A (before noon, S between Q and R): D top, C centre, E bottom; A upper-left, B right — meridian DE slants so that D lies toward A (east toward south)
- Figure B (after noon, S between Q and R): D top, C centre, E bottom; B lower-left, A right — DE slants toward the west side
- Figure C (S coincides with R): D top, B left, C centre, A right, E bottom — DE drawn as the plain perpendicular to AB
- Figure D (before noon, S between R and P): D top, C centre, A left, B right, E bottom — DE slants from A toward E (east toward north)

- Figure E (after noon, S between R and P): E large at left, D top, C centre, A right, E bottom — DE slants toward the west side

Now, so that you may see in this connection too what an outstanding use the analemma has, it will not be out of place to explain briefly how, from the analemma, the hour of the day is found once the Sun's declination and its altitude above the horizon are known. For let a circle be described about NO, the diameter of the Sun's parallel, with its centre at the point d, where the world-axis a b cuts the diameter NO; and let it be divided into 24 equal hours, beginning from the diameter NO. Then, if from z — where the diameter NO cuts the horizon's diameter GI — a perpendicular XY is drawn to NO, this will be the common section of the Sun's parallel and the horizon.

[Margin: By what means the hour is to be sought by means of the analemma, from the known declination of the Sun and its altitude above the horizon.]

[Margin (a): Elements XI.19]

For since the horizon and the Sun's parallel are both perpendicular to the meridian, their common section too will be perpendicular to the meridian, and hence (by Euclid XI, def. 3) perpendicular to NO. From this it follows that the straight line XY, which is perpendicular to NO at the point z (where the diameters GI and NO bisect each other), is the common section of the Sun's parallel and the horizon. The daytime arc, therefore, will be XNY, and the night arc YOX; and accordingly the number of hours contained in these arcs will indicate the length of the day and of the night — as we also showed in the scholium to Proposition 1 of this book.

[Margin (b): Elements XI.19]

Now, if from the point S a perpendicular SV is drawn to NO, this will be the common section of the Sun's parallel and of the horizon-parallel in which the Sun then lies. For since each parallel is perpendicular to the meridian, their common section too will be perpendicular to the meridian, and therefore (by Euclid XI, def. 3) perpendicular to NO. So the perpendicular VS will be the common section of the said parallels, and for that reason the Sun will lie at the point V — the Sun's parallel NXOY, together with the meridian, being set in its proper position — when it has the altitude IP (or GQ) above the horizon. Hence the hours found on the arc NV will indicate how many hours the Sun is distant, whether before noon or after, according as the observation is made before or after noon.

But if the hour reckoned from sunset is wanted, in the manner of the Italians and the Bohemians: if the observation is made before noon, the division of the circle NXOY into 24 equal hours must be begun from the point Y and continued through the point O; for then the point V will at once indicate the hour elapsed since sunset. But if the observation is made after noon, that division must begin from the point X and be continued through the point O. In the same way, if the hour reckoned from sunrise is sought, in the manner of the Babylonians and of the people of the Balearic Islands, the division of the circle NXOY must be begun from the point X and continued through N, if the observation is made before noon; but if after noon, from the point Y. By the same method we shall know the hour at any moment of time — whether reckoned from noon or from midnight, or from sunrise or sunset — provided the Sun's declination is known, together with the altitude it has above the horizon at the time of observation.

Conversely, from a known hour we can arrive at knowledge of the Sun's altitude by means of the analemma, provided its declination is not unknown. For if, with due account taken of the declination, the diameter NO of the Sun's parallel is described, and about it the circle NXOY; and from V, the known hour, the perpendicular VS is drawn to NO; and through S, finally, the straight line PQ is drawn parallel to GI, the diameter of the horizon — then both GQ and IP will be the arc of the Sun's altitude above the horizon, since PQ is a diameter of the horizon's parallel that is then drawn through the Sun, as is evident.

[Margin: How the Sun's altitude is to be found out by the analemma, from the known hour and the Sun's declination.]

Nor should this be passed over: if perhaps we lack an instrument for investigating the Sun's altitude, we can obtain it in the following way. On a plane ADBE parallel to the horizon, let a stylus be fixed at right angles, and at the time of observation let the tip of the shadow be marked. For if in the analemma one takes Ke equal to the gnomon, and through e draws the perpendicular ef to Ke, in which ef is taken equal to the shadow that was marked, then the straight line drawn through f and K will fall on the point P of the Sun's altitude. For if the circle FGHI is conceived as a vertical circle passing through the centre of the Sun, the straight line ef will be the common section of this vertical circle and of the plane parallel to the horizon. Since, therefore, the tip of the shadow is f, fKP will be the Sun's ray, and accordingly the Sun will lie at P, and the Sun's altitude will be the arc IP — as can easily be gathered from what we wrote in Proposition 2 of this book.

[Margin: How the Sun's altitude is to be found by the analemma from the length of the shadow of any stylus set at right angles on a plane parallel to the horizon.]

But if the straight line PQ should ever fall on the point N — that is, if the Sun's altitude found is equal to the meridian altitude of the Sun for that day — then the Sun will be on the meridian circle, and therefore the shadow AB itself will be the meridian line.

By means of the same analemma, and by nearly the same method, we shall be permitted to find out the declination of any proposed plane, even if on a plane parallel to the horizon we do not find the meridian line — as is also handed down by Giovanni Battista Benedetti [Giovanni Battista Benedetti] in his *Gnomonica*. To do this, let AB be a wall perpendicular to the horizon; on it draw a straight line CD parallel to the horizon, and fix on it a stylus CE, of any length, perpendicular to the wall at the point C. Then, at any time, when the Sun lights up the face of the wall — whether before noon or after — let the tip of the shadow EF that the stylus casts be observed, namely the point F; and through it let the perpendicular FD be drawn to the straight line CD. This will be drawn more quickly than said, in this way: apply to the wall a thread with a plumb-weight so that it passes through the point F, and mark on the wall any point D. For a straight line drawn through F and D will be perpendicular to CD, since the thread is perpendicular to the horizon. For it follows from this that the straight line FD too — which does not differ from the plumb-thread, or is at any rate parallel to it — is perpendicular to the horizon (which is drawn through the line CD); and so, by definition 3 of Book 11 of Euclid, since it makes right angles with the line CD in the horizon.

[Margin: By what art the declination of any proposed plane is to be found out by means of the analemma.]

Diagram. A rectangle representing the wall, with the corner A at the upper left and the corner B at the lower right. A horizontal line runs across the middle of the rectangle — the line CD, parallel to the horizon — with C toward the centre and D at its right-hand end. At C stands the base of the stylus, whose tip is E (raised, upper left of centre). Above C, near the top edge, is G, the top of the constructed segment CG drawn perpendicular to CD and made equal to the stylus. F lies below D, in the lower portion of the rectangle — the tip of the cast shadow. Dotted lines run from E to F (the shadow-ray) and from G down toward F, with the joining lines ED and GD.

- A (upper-left corner): corner of the wall AB
- B (lower-right corner): opposite corner of the wall

- C (middle, centre): base of the stylus, on line CD
- D (middle, right): foot of the perpendicular FD on line CD
- E (upper middle-left): tip of the stylus CE
- F (lower middle): tip of the cast shadow
- G (top): top of the constructed perpendicular CG (equal to the stylus)

Here too, in place of the stylus, I use that instrument which we described at the beginning of this scholium. For if it is applied to the wall AB so that the point D falls on the point C, and the side DA lies along the line CD, with DI slanting downward, then the side DH will serve the office of a stylus perpendicular to the wall. Therefore, once the tip of that shadow has been observed at the point F, the instrument must be removed and the point C carefully marked. Now, since the Sun's ray EF passes through E, the tip of the stylus (which is at the centre of the world, by Proposition 2 of this book), it lies in the plane of that vertical circle which at the moment of observation is drawn through the centre of the Sun; and this vertical circle will meet the wall AB at the point F. But since both the plane of the wall and this vertical circle are perpendicular to the horizon, their common section too will be perpendicular to the horizon, [Margin (a): Elements XI.19] and therefore, by definition 3 of Book 11 of Euclid, perpendicular to the line CD lying in the horizon. Since, then, FD is perpendicular to CD, FD will be the common section of the wall AB and of the vertical circle then drawn through the centre of the Sun; and so that same vertical circle will pass through the point D. Therefore, when the straight line ED is drawn, it will be the common section of the horizon and of that same vertical circle, since both circles pass through the points E and D; and so the line FD, being perpendicular to the horizon, will be perpendicular — by definition 3 of Book 11 of Euclid — to the line ED lying in the horizon. But it has also been shown to be perpendicular to CD. Therefore, since each of the lines CD and ED — the former lying in the wall, the latter in the vertical circle passing through the Sun — is perpendicular to FD, the common section of the wall and the said vertical circle, the angle CDE will, by definition 6 of Book 11 of Euclid, be the angle of inclination of the wall to the said vertical circle.

We shall exhibit an angle equal to this in the plane of the wall as follows. Draw CG perpendicular to CD, make CG equal to the stylus (or to the side DH of the instrument described at the beginning of this scholium), and join the straight line GD. I say that the angle CDG is equal to the angle CDE. For since the two sides CE, CD of the triangle CDE are equal to the two sides CG, CD of the triangle CDG, and they contain equal angles — namely right angles (for the angle ECD is right, by definition 3 of Book 11 of Euclid, and the angle GCD by construction) — the base ED will also be equal to the base GD, and the angle CDE to the angle CDG. [Margin

(b): Elements I.4] From this known angle CDG we shall investigate the declination of the proposed wall from the vertical circle properly so called, in this way.

After the tip of the shadow F has been marked, let inquiry be made at once — before the straight line FD is drawn (since if any delay intervenes, the shadow will change and the Sun will occupy another vertical circle on account of the daily motion): namely, let the Sun's altitude in the analemma above (which we here repeat) be reckoned from the points G, I up to the points Q, P. For when the straight line PQ is joined, it will be the diameter of the horizon's parallel drawn through the centre of the Sun at the time of observation, as we demonstrated above; and it cuts the diameter of the Sun's parallel at S, and the diameter of the vertical circle properly so called at R. Then, with a semicircle PTQ described about PQ from the centre R, draw from S the perpendicular ST to PQ, cutting the circumference of the semicircle PTQ at T, and join the straight line TR. This will be the common section of the horizon's parallel and of the vertical circle, each of which is then drawn through the Sun's centre. So the acute angle QRT (or PRT) will be the angle of declination of the said vertical circle from the meridian, as we showed above; and consequently the remaining angle TRH will be the angle of declination of that same vertical circle from the vertical circle properly so called.

[Margin: When the wall faces south, and the Sun is more southerly than the vertical circle properly so called, and the observation is made before noon.]

And so, if the wall faces south (which we shall learn from what we set forth above in this proposition), and the point S falls between Q and R — that is, if the Sun is found beyond the vertical circle properly so called, so that it lights up both the wall and the vertical circle properly so called from the southern side, and the observation is made before noon — we shall compare the angle TRH with the angle CDG which we found on the wall. For if the former is equal to the latter, the wall will have no declination and will face due south, since

the declination of the vertical circle passing through the Sun is found to be the same from the wall as from the vertical circle properly so called. But if the angle TRH is found to be greater than the angle CDG, the wall will be positioned on the eastern side, between the vertical circle properly so called and the vertical circle drawn through the centre of the Sun — because on the southern side the vertical circle drawn through the Sun then stands farther from the vertical circle properly so called than from the wall. Therefore, if we take away the angle CDG (that is, the declination which the vertical circle drawn through the Sun has from the wall) from the angle TRH (that is, from the declination which the same vertical circle drawn through the Sun has from the vertical circle properly so

called), the remainder will be the angle of the wall's declination from the vertical circle properly so called, and from south toward the west. Finally, if the angle TRH is less than the angle CDG, the vertical circle properly so called will lie on the eastern side, between the wall and the vertical circle drawn through the Sun — because on the southern side the vertical circle drawn through the Sun then stands farther from the wall than from the vertical circle properly so called. Therefore, if we subtract the angle TRH from the angle CDG, there will remain the angle of the wall's declination from the vertical circle properly so called, and from south toward the east.

[Margin: When the wall faces south, and the Sun lies in the vertical circle properly so called, and the observation is made before noon.]

But if the point S is the same as R — that is, if the Sun lies in the vertical circle properly so called, so that it still lights up the wall on the southern side, but the vertical circle properly so called not at all — then the angle found, CDG, will be the angle of the wall's declination from the vertical circle properly so called, and from south toward the east; because then, on the eastern side, the vertical circle properly so called (in which the Sun of course lies) is more southerly than the wall.

[Margin: When the wall faces south, and the Sun is more northerly than the vertical circle properly so called, and the observation is made before noon.]

Finally, if the point S lies between P and R — that is, if the Sun is placed on the near side of the vertical circle properly so called, so that it lights up the wall on the southern side but the vertical circle properly so called on the northern side — then the vertical circle in which the Sun lies will fall between the wall and the primary vertical circle on the eastern side, since that vertical circle is then more southerly than the wall and more northerly than the primary vertical circle. Therefore, if the angle TRH is added to the angle CDG, the angle of the wall's declination from the primary vertical circle, and from south toward the east, will be composed.

Diagram. A large analemma. The outer circle is divided around its circumference into hour-graduations numbered 1 through 20, with cardinal and intermediate points marked by letters. The clearly readable perimeter letters are N (upper left), Z and F at the top, P and Y on the upper right, I on the right, M at the lower right, H at the bottom, G on the left, L on the upper left, and Q/e on the left. At the centre is K, the intersection of the principal diameters (the meridian and horizon GI); through it run the axis of the world, the diameters of the Sun's parallel and of the

vertical circles, and numerous chords. The interior carries a dense cluster of small construction-labels along the vertical axis and the chords (readable among them: R appearing several times, S, s, d, z, T appearing several times, Q, V, f, b, and x near the lower left), but many of these small interior labels overlap and cannot be assigned to their points with confidence at this resolution.

- N (upper-left circumference): principal point
- Z, F (top): meridian points
- P, Y (upper-right circumference)
- I (right circumference): end of horizon diameter GI
- G (left circumference): end of horizon diameter GI
- M (lower-right circumference)
- H (bottom circumference): foot of the meridian
- L (upper-left circumference)
- K (centre): centre of the analemma
- Hour-graduations 1–20 numbered around the circumference
- Interior chord- and point-labels (R, S, s, d, z, T, Q, V, f, b, x): present but several illegible at this resolution — [?]

[Diagram fig-p86-1 has more than two labels flagged [?]:]

Diagram. Beneath the analemma, five small circles labelled A, B, C, D and E (the case-label set in the centre of each), illustrating the several orientations of the wall discussed in the text. Each circle is divided into four quadrants by a vertical line (D at the top, E at the bottom) and a horizontal line (with A and B at the two ends), the centre marked C; the A and B ends are set to left or right to show the wall's east/west facing in each case. In circle A the horizontal ends read A (left) and B (right); the remaining circles B, C, D and E repeat the same cross with the A/B ends variously placed.

- A (upper-left circle): first orientation case; horizontal ends A (left), B (right)
- E (upper-right circle): case with horizontal ends B and A
- B (lower-left circle): case with horizontal ends B (left), A (right)
- C (lower-centre circle): case with horizontal ends B and A
- D (lower-right circle): case with horizontal ends A (left), B (right)
- In each: D (top), E (bottom), C (centre)

[Margin: When the wall faces south, and the Sun is more southerly than the vertical circle properly so called, and the observation is made after noon.]

But if the wall faces south while the observation is made in the afternoon, and the point S falls between Q and R — so that the Sun lights up both the wall and the primary vertical circle from the southern side — and the angle TRH is found equal to the angle CDG, then again the wall will have no declination. But if the angle TRH is found greater than the angle CDG, the wall will be positioned on the western side, between the primary vertical circle and the vertical circle drawn through the centre of the Sun. And so, when the angle CDG is subtracted from the angle TRH, there will remain the angle of the wall's declination from the primary vertical circle, and from south toward the east. Finally, if the angle TRH is less than the angle CDG, the primary vertical circle will lie on the western side, between the wall and the vertical circle drawn through the Sun. Therefore, when the former is taken from the latter, the remainder will be the angle of the wall's declination from the primary vertical circle, and from south toward the west.

[Margin: When the wall faces south, and the Sun lies in the vertical circle properly so called, or is more northerly than the vertical circle properly so called, and the observation is made after noon.]

But if the point S falls on R, so that the Sun lights up the wall on the southern side but the primary vertical circle not at all, the wall will decline from the vertical circle, and from south toward the west, by the angle found, CDG; because then the Sun will lie in the primary vertical circle, which on the western side is more southerly than the wall itself.

Finally, if the point S falls between P and R, so that the Sun lights up the wall on the southern side but the primary vertical circle on the northern side, then the vertical circle in which the Sun lies will be set between the wall and the primary vertical circle on the western side. Therefore, if the angle TRH is added to the angle CDG, the angle by which the wall deflects from the primary vertical circle, and from south toward the west, will be composed.

But if the wall faces north while the observation is made before noon, and the point S falls between Q and R — so that the Sun lights up the wall on the northern side and the vertical circle properly so called on the southern side — then the vertical circle drawn through the Sun will be set between the wall and the primary vertical circle, on the eastern side. Therefore, if the angle TRH is added to the angle CDG, the angle of the wall's declination from the primary vertical circle, and from north toward the east, will be produced.

[Margin: When the wall faces north, and the Sun is more southerly than the vertical circle properly so called, or lies in the vertical circle properly so called, and the observation is made before noon.]

But if the point S is the same as R, so that the Sun still lights up the wall on the northern side, but the vertical circle properly so called not at all, then the angle found, CDG, will be the angle of the wall's declination from the vertical circle properly so called, and from north toward the east.

Finally, if the point S falls between P and R, so that the Sun lights up both the wall and the vertical circle properly so called on the northern side, the angle TRH must be compared with the angle CDG. For if the former is equal to the latter, the wall will have no declination, for the reason stated before. But if the angle TRH exceeds the angle CDG, then, when the latter is subtracted from the former, there will remain the angle by which the wall declines from the primary vertical circle, and from north toward the west — because then the wall will be placed, on the eastern side, between the primary vertical circle and the one that passes through the Sun. Finally, if the angle TRH is less than the angle CDG, the primary vertical circle will lie between the wall and the vertical circle that passes through the Sun, on the eastern side. Therefore, if we subtract the former from the latter, we shall make the remaining angle by which the wall declines from the primary vertical circle, and from north toward the east.

[Margin: When the wall faces north, and the Sun is more northerly than the vertical circle properly so called, and the observation is made before noon.]

But if the wall faces north and the observation is made in the afternoon, and the point S falls between Q and R — so that the Sun lights up the wall on the northern side, but the vertical circle properly so called on the southern side — then the vertical circle drawn through the Sun will be set, on the western side, between the wall and the primary vertical circle. Therefore, if the angle TRH is added to the angle CDG, the angle of the wall's declination from the vertical circle properly so called, and from north toward the west, will be produced.

[Margin: When the wall faces north, and the Sun is more southerly than the vertical circle properly so called, or lies in the vertical circle properly so called, or is more northerly than it, and the observation is made after noon.]

But if the point S falls on the point R, so that the Sun lights up the wall on the northern side but the primary vertical circle not at all, then the angle found, CDG, will give the declination of the wall from the primary vertical circle, and from

north toward the west; since then the primary vertical circle passes through the Sun, and is more northerly than the wall itself.

Finally, if the point S lies between P and R, so that the Sun lights up both the wall and the vertical circle properly so called on the northern side: if indeed the angle TRH is equal to the angle CDG, the wall will be parallel to the primary vertical circle and will have no declination. But if the former is greater than the latter, the wall will lie on the western side, between the primary vertical circle and the one that passes through the Sun. And so, when the angle CDG is subtracted from the angle TRH, the remainder will be the angle of the wall's declination from the vertical circle properly so called, and from north toward the east. Finally, if the angle TRH is less than the angle CDG, the primary vertical circle will be set between the wall and the vertical circle in which the Sun lies, on the western side. Therefore, when the former is taken from the latter, there will remain the angle of the wall's declination from the primary vertical circle, and from north toward the west. All of this is clear, if the situation of the circles and of the wall is weighed carefully and diligently.

[Margin: When the Sun's altitude is equal to the meridian altitude of the same. The easiest way of finding the declination of a wall facing south, at noon.]

But if the straight line PQ in the analemma should ever fall on the point N — that is, if the Sun's altitude found is equal to the meridian altitude of the Sun for that day — then the Sun will be on the meridian circle; and therefore the straight line FD on the wall will be the common section of the meridian and the wall, since the meridian is then drawn through E, the centre of the world, and through the Sun's ray EF, and the wall itself will face south. Therefore the angle CDG found on the wall will give the wall's declination from the meridian, and consequently its complement will show the wall's declination from the vertical circle properly so called — from south toward the east, if the shadow F falls, for us facing the wall, to the right; but from south toward the west, if the shadow falls to the left.

And so, if the observation is made at noon — which is recognized, on a plane parallel to the horizon, from the stylus's shadow falling on the meridian line (and this is the time I would always choose, if it could conveniently be done) — the declination will be found more quickly than said. For the complement of the angle CDG found on the wall will give the wall's declination from the vertical circle properly so called, and from south toward the east or the west, according as the shadow falls, for us facing the wall, to the right or to the left, as was just said. This is to be understood of a wall facing south. For a wall facing north is not lit by the Sun at noon, unless the Sun is more northerly than the wall itself — which

can happen in the torrid zone. When that occurs, and the shadow, for us facing the wall, falls to the left, the wall will decline from north toward the east; but if to the right, from north toward the west.

[Margin: By what method the declination of planes inclined to the horizon is to be found.]

All this can also be applied to planes inclined to the horizon, if above the line which in planes of this kind is drawn parallel to the horizon a plane perpendicular to the horizon is set up, and the observation is made on the face of this plane which forms an obtuse angle with the inclined plane.

THEOREMA 21. PROPOS. 24.

[Margin: Perpendiculars falling from the circumference of a greatest circle in the sphere onto the plane of another greatest circle, to which the former is inclined, make an ellipse in the latter greatest circle.]

If from the circumference of a greatest circle in a sphere, inclined upon another greatest circle, perpendiculars are drawn to the plane of that other greatest circle, they will all fall on a line which is called an ellipse. Its major diameter is the same as the common section of the two circles — namely their diameter — while its minor diameter is determined by the interval of the perpendiculars falling from the extremity of the other diameter of the inclined circle, the diameter which divides the former diameter (that is, the common section) at right angles.

Let there be in a sphere a greatest circle ABCD, with centre E, inclined to a greatest circle AFCG; and let the circle AFCG cut the circle ABCD at the centre E, so that the diameter AC is the common section of the circles ABCD and AFCG. In the circle ABCD draw another diameter BD, cutting AC at right angles; and in the plane of the circle AFCG draw another diameter FG, perpendicular to the same AC. Since, then, the straight line CE stands at right angles on the straight lines BE, FE cutting one another at E, the same CE will be perpendicular to the plane drawn through BE, FE. [Margin (a): Elements XI.4] Therefore the planes of the circles AFCG, ABCD, passing through CE, will also be perpendicular to the same plane drawn through BE, FE; [Margin (b): Elements XI.18] and FG will be the common section

Diagram. A tilted ellipse crossing an upright circle, illustrating the inclination of two greatest circles. FG is the horizontal diameter, F at the left end and G at the right end. AC is the common section (the near-vertical diameter), with A at the top and C at the bottom; BD is the other diameter of the circle ABCD, with B at the

upper left and D at the lower right. E is the common centre. Toward the right, I lies near G on the ellipse; K lies on the upper left; and near the lower left is a cluster of the perpendicular-feet H, L, N, with M toward the lower centre.

- A (top): end of common section AC / on circle ABCD
- B (upper left): end of diameter BD
- C (bottom): end of common section AC
- D (lower right): end of diameter BD
- E (centre): common centre of the two circles
- F (left end): end of diameter FG of circle AFCG
- G (right end): end of diameter FG
- I (right, on the ellipse): near G
- K (upper left): on the ellipse
- H, L, N (lower left): feet of perpendiculars
- M (lower centre): foot of a perpendicular

the common section of the plane drawn through BE, FE and the plane of circle AFCG — a plane which has been shown to be perpendicular to that other one. And so, if from point B, which lies in the plane drawn through BE, FE, a perpendicular is drawn to the plane AFCG — namely BH — it will fall on FE, the common section of the plane drawn through BE, FE and the plane AFCG. [Margin (a): Elements XI.38] Let it therefore fall at H. In the same way, the perpendicular from D to the same plane AFCG will fall on the straight line FG, say at I; and the straight lines EH, EI will be equal to each other. For since in the triangles EBH, EDI the angles at H, I are right (by Elements XI, def. 3), [Margin (b): Elements I.15] and the vertical angles at E are equal, and likewise the sides EB, ED are equal, [Margin (c): Elements I.26] the sides EH, EI will also be equal.

Now take any other point K on the circumference of circle ABCD, and from it drop the perpendicular KL to the same plane AFCG. I say that point L falls on an ellipse whose major diameter is AC, the diameter of the circles, and whose minor diameter is the straight line HI. For in the plane AFCG draw the perpendicular LM from L to AC; then, since HE too is perpendicular to the same AC, [Margin (d): Elements I.28] HE and LM will be parallel to each other. [Margin (e): Elements XI.6] But BH and KL are likewise parallel to each other, since both are perpendicular to the same plane AFCG. [Margin (f): Elements XI.15] Therefore the plane through BH, HE will be parallel to the plane through kL, LM; [Margin (g): Elements XI.16] and consequently the common sections of these planes with circle ABCD — namely the straight lines BE and kM — will be parallel.

And so, since BE and EH, meeting one another at E as tangents, are parallel to kM and ML, which meet one another as tangents at M, but are not in the same plane but in parallel planes (as has been said), [Margin (h): Elements XI.10] the angle BEH will be equal to the angle KML. But the angles at H and L are also equal, being right, by Elements XI, def. 3. Therefore the remaining angle EBH will be equal to the remaining angle MkL, by the corollary to Elements I.32; [Margin (i): Elements VI.4] and triangle BEH will be equiangular with triangle kML. Hence BE will be to EH as KM is to ML; and, alternately, BE will be to KM as EH is to ML. [Margin (k): Elements VI.22] Therefore the square on BE is to the square on kM as the square on EH is to the square on ML.

Now the square on BE is to the square on KM as the rectangle CE·EA is to the rectangle CM·MA — this by Apollonius [Apollonius of Perga], *Conics* I.21 — because BE and kM are drawn as ordinates (that is, perpendiculars) to the diameter AC in circle ABCD. Therefore the square on EH is likewise to the square on ML as the rectangle CE·EA is to the rectangle CM·MA. But by the same Apollonius, *Conics* I.21, if on the major diameter AC and the minor diameter HI an ellipse is described — for AC is greater than HI, since AC, the diameter of circle AFCG, is equal to FG, the diameter of the same circle — and from any point a parallel to HE is drawn (that is, an ordinate applied to the diameter AC), then the square on HE is to the square on that parallel as the rectangle CE·EA is to the rectangle contained by the segments of the diameter AC that the parallel cuts off. [Margin (l): Elements V.9] Therefore point L will fall on that ellipse whose major diameter is AC and minor HI — since the square on HE is to the square on LM as the rectangle CE·EA is to the rectangle CM·MA — for otherwise a part would equal the whole.

For if that ellipse does not pass through point L, let it pass, if it can, through N. Then by Apollonius, *Conics* I.21, the rectangle CE·EA will be to the rectangle CM·MA — that is, the square on HE to the square on LM — as the square on HE is to the square on NM. The squares on LM and NM are therefore equal, and hence too the straight lines LM and NM are equal, the whole and the part — which is absurd. Therefore that ellipse does pass through point L, and consequently point L falls on the ellipse whose major diameter is AC and minor HI. In the same way we shall show that the other points on which perpendiculars fall from the circumference of circle ABCD also lie on the same ellipse. And so, if from the circumference of a greatest circle in a sphere, etc. Which was to be demonstrated.

Diagram. A tilted ellipse crossing an upright circle, illustrating the fall of perpendiculars from the inclined circle ABCD onto the plane AFCG. FG is the

horizontal diameter (F at the left end, G at the right). AC is the near-vertical common section (A toward the upper centre, C at the bottom); BD is the other diameter (B at the top, D at the very bottom). E is the common centre. K lies on the ellipse at upper left; I lies on the ellipse toward the right near G; the feet of the perpendiculars H, L, N cluster at the lower left, with M toward the lower centre. (Same construction as the figure on the preceding page.)

- A (upper centre): end of common section AC, on circle ABCD
- B (top): end of diameter BD
- C (bottom): end of common section AC
- D (very bottom): end of diameter BD
- E (centre): common centre of the two circles
- F (left end): end of diameter FG of circle AFCG
- G (right end): end of diameter FG
- I (right, on the ellipse): near G
- K (upper left): on the ellipse
- H, L, N (lower left): feet of the perpendiculars
- M (lower centre): foot of a perpendicular

SCHOLIUM.

This theorem is stated more generally by Federico Commandino [Federico Commandino] in his little book on the description of horologia [sundials]: namely, that even if the plane in which circle AFCG lies does not cut the inclined circle ABCD through its centre, or does not cut it at all — and whether ABCD is a greatest circle in the sphere or any circle whatever — nonetheless the perpendiculars drawn from the circumference of circle ABCD to the plane AFCG fall on an ellipse. For if the plane in which circle AFCG lies does not cut circle ABCD through the centre, or not at all, he reaches the conclusion thus. When another plane, parallel to AFCG, is drawn that cuts circle ABCD at its centre E, it will be demonstrated as before that the perpendiculars dropped from the circumference of circle ABCD to that plane fall on an ellipse. And since these lines, when produced further to the plane AFCG (to which the other plane is parallel), keep the same position, they also fall in that place on an ellipse — whose major diameter will be equal to AC, the diameter of circle ABCD, and whose minor diameter will be equal to the interval HI of the perpendiculars BH, DI drawn from the extremities of the other diameter BD.

We, however, have stated the theorem for greatest circles in a sphere only, because it is in these alone that its use will appear in this gnomonics of ours.

PROBLEMA 4. PROPOS. 25.

[Margin: Finding the points on which fall the perpendiculars drawn from the extremities of a diameter of a circle inclined to another circle.]

On the circumference of a greatest circle in a sphere inclined to another greatest circle, the two points that are the extremities of the diameter cutting their common section at right angles being taken: to investigate, if the inclination is known, the place at which the perpendiculars drawn from these points fall onto the other circle.

Let ABCD be a greatest circle in a sphere, inclined to the greatest circle AFCE, and let their common section be the diameter AC, to which, in the plane of circle ABCD, another diameter BD is drawn perpendicular through the centre G. We must then investigate at what place the perpendiculars dropped from points D, B onto the plane of circle AFCE fall. In the plane of circle AFCE draw another diameter EF perpendicular to AC, and let the angle of inclination — which is given — be EGH, so that arc EH is equal to the arc of inclination of circle ABCD to circle AFCE; and from H draw HI perpendicular to EF. I say that the perpendicular dropped from D onto the plane of circle AFCE falls

on the point I. For when a plane is drawn through EG, DG, producing in the sphere the semicircle EDF, then by Theodosius [Theodosius of Bithynia], *Spherics* I.1, [Margin (a): Elements XI.4] this semicircle will be perpendicular to the circles AFCE, ABCD. (For since CG is perpendicular to EG, DG, it will also be perpendicular to the plane drawn through EG, DG — that is, to the semicircle EDF; and therefore the planes of the circles AFCE, ABCD, drawn through CG to that same semicircle EDF, will be perpendicular to it; and hence, conversely, this semicircle will be perpendicular to those planes.) Therefore DE will be the arc of inclination of circle ABCD to circle AFCE, [Margin (b): Elements XI.38] and consequently equal to arc EH; and the perpendicular dropped from D onto the plane AFCE will fall on EF, the common section of the planes AFCE, EDF — and this, I say, falls at I.

For if it fell elsewhere, say at k, then the two angles HIG, IGH of triangle GHI would be equal to the two angles DKG, KGD of triangle GDK; [Margin (c): Elements III.27] (for IGH and KGD are equal, on account of the equality of the arcs EH, ED, while HIG and DKG are right by construction and by Elements XI, def. 3.) But the sides GH, GD, drawn from the centre of the sphere to its surface, are also equal. [Margin (d): Elements I.26] Therefore the sides GI, GK would also be equal — the part and the whole. Which is absurd. Therefore the perpendicular dropped

from point D onto the plane AFCE falls nowhere but at I. In the same way we shall find the point on which the perpendicular dropped from B falls. And it will always fall on a point — say M — that is as far from the centre G as I is distant from it. For since in the triangles DGI, BGM the angles at I, M are right by Elements XI, def. 3, [Margin (e): Elements I.15] and the vertical angles at G are equal, and the sides DG, BG are equal, being semidiameters of the sphere, [Margin (f): Elements I.26] the sides GI, GM will also be equal to each other. Therefore on the circumference of a greatest circle in a sphere, etc. Which was to be done.

Diagram. A circle (a greatest circle of the sphere) with two perpendicular diameters — AC horizontal (A at the left, C at the right) and EF vertical (E at the top, F at the bottom) — and centre G. The inclined diameter DB runs from D at the upper left to B at the lower right. Radii and chords are drawn from G to D, K, H, I, M, and B, forming the triangles GDK, GHI (near the top) and DGI, BGM. K lies near the top centre, H on the right, I near the centre, M below the centre.

- A (left): end of diameter AC
- C (right): end of diameter AC
- E (top): end of diameter EF
- F (bottom): end of diameter EF
- G (centre): centre of the circle / of the sphere
- D (upper left): end of the inclined diameter DB
- B (lower right): end of the inclined diameter DB
- H (right): foot on EF of the perpendicular from D
- K (upper centre): the assumed (false) alternative fall-point
- I (centre): foot of the perpendicular from D
- M (lower centre): foot of the perpendicular from B

COROLLARIUM.

[Margin: Finding the minor diameter of the ellipse formed by the perpendiculars falling from the circumference of an inclined circle onto another circle.]

From this, by the same method, we shall find the minor diameter of that ellipse onto which fall the perpendiculars dropped from the circumference of a circle inclined to another circle. For the straight line IM, lying between the points I, M on which the said perpendiculars fall, is the minor diameter, by the preceding proposition.

PROBLEMA 5. PROPOS. 26.

On the circumference of a greatest circle in a sphere inclined to another greatest circle, any points whatever being taken: to inquire, if the inclination is known, the place at which the perpendiculars drawn from them fall onto the other circle.

Let ABCD be a greatest circle in a sphere inclined to the greatest circle DEBF, the inclination being known; and let their common section be the diameter DB passing through the centre G. To this, at right angles, draw in circle ABCD the diameter AC, and in circle DEBF the diameter EF, onto which will fall the perpendiculars dropped from A, C onto circle DEBF, as was shown in the preceding proposition. Let them fall, then, at H, I, so that DB is the major diameter and HI the minor diameter of the ellipse that the perpendiculars from the circumference of circle ABCD, dropped onto the plane of circle DEBF, produce — as has been demonstrated. Now take any point K on the circumference of ABCD. We must then inquire at what place the perpendicular from K, drawn down onto the plane DEBF, falls. Take arc EL equal to arc AK, and draw the straight line GL, which cuts at M the circle HI (described about the minor ellipse-diameter HI). Then through L draw LN parallel to the minor diameter HI, cutting DB at O; and through M draw PM parallel to the major ellipse-diameter DB, cutting LN at Q. I say that the perpendicular dropped from K onto the plane DEBF falls at point Q.

That it falls on the line LN will be shown thus. When the straight line KO is drawn, it will be parallel to AG. For when LS, kT are drawn perpendicular to EG, AG, [Margin (g): Elements I.34] then, since GO is equal to LS, the sine of arc EL, while LS in turn is a sine equal to kT, the sine of arc AK (which is taken equal to arc EL), GO will also be equal to kT. Since then GO, kT are parallel as well, because the angles OGT, kTG are right by construction, [Margin (h): Elements I.28] GT, OK will be equal and parallel. [Margin (i): Elements I.33] Therefore, since the angle AGO is right, [Margin (k): Elements I.29] the angle GOk will also be right. Since then the straight line GO is perpendicular to the straight lines OL, Ok, [Margin (l): Elements XI.4] the same GO will be perpendicular to the plane drawn through OL, Ok; [Margin (m): Elements XI.18] and consequently the plane of circle DEBF, drawn through GO, will be perpendicular to that same plane drawn through OL, OK. [Margin (n): Elements XI.38] Therefore the perpendicular dropped from k onto the plane DEBF will fall on the straight line LN, the common section of the plane DEBF and of the plane drawn through OL, Ok.

That it falls at Q will be demonstrated thus. Let it fall, if it can, on another point, say R. Then since LS, MP are parallel, [Margin (o): Elements VI.2] LM will be to MG as SP is to PG; and, by composition, LG will be to

Diagram. A large tilted ellipse over a circle, the construction for Problema 5 (the fall of the perpendicular from an arbitrary point K onto the plane DEBF). The circle ABCD is inclined; DEBF is the circle onto which perpendiculars fall. EF is the near-horizontal diameter (E at the left end, F at the right), DB the near-vertical one (D at the top, B at the lower left); centre G. A is at the upper left, C at the lower right, K at the top. L lies at the left with the construction points T, Q, R, M just inside it; N is at the upper right, I at the right, O near the centre on DB; S, H, P cluster at the lower left. (This same figure is repeated at the top of the following page.)

- A (upper left), C (lower right): ends of diameter AC of circle ABCD
- D (top), B (lower left): ends of the common section DB
- E (left end), F (right end): ends of diameter EF of circle DEBF
- G (centre): common centre
- H, I (near centre / right): feet of perpendiculars from A, C — the minor-ellipse diameter HI
- K (top): the arbitrary point taken on circle ABCD
- L (left): point with arc EL = arc AK
- M (interior, upper left): where GL cuts the minor circle
- N (upper right): on LN (parallel to HI through L)
- O (centre, on DB): where LN cuts DB
- P (lower left): on PM (parallel to DB through M)
- Q (interior): intersection of LN and PM — the fall-point sought
- R (interior): the assumed (false) alternative fall-point
- S, T (interior, upper left): feet of the sine-perpendiculars LS, kT

as SG is to PG. But LG, MG are equal to EG, HG by the definition of the circle; [Margin (a): Elements I.34] and SG, PG are equal to LO, QO, on account of the rectangles SO, PO. Therefore it will also be the case that as EG is to HG, so LO is to QO; and, alternately, as EG is to LO, so HG is to QO; [Margin (b): Elements VI.22] and hence the square on EG is to the square on LO as the square on HG is to the square on QO. But by Apollonius, *Conics* I.21, the square on EG is to the square on LO as the rectangle BG·GD is to the rectangle BO·OD, because EG, LO are drawn as ordinates (namely perpendiculars) to BD, the diameter of circle DEBF. Therefore it will also be the case that as the rectangle BG·GD is to the rectangle BO·OD, so the square on HG is to the square on QO. And since the ellipse of the diameters DB, HI is assumed to pass through R (because the perpendicular dropped from K is said to fall at R), it is likewise the case, by the same Apollonius, *Conics* I.21, that as the rectangle BG·GD is to the rectangle BO·OD, so the square on HG is to the

square on RO, because HG, RO are applied as ordinates to the diameter BD. Therefore the square on HG is to the square on QO as the same square on HG is to the square on RO. [Margin (c): Elements V.9] Hence the squares on QO, RO are equal, and consequently the lines QO, RO too are equal — the whole and the part. Which is absurd. Therefore the perpendicular dropped from k falls on no point other than Q. In the same way, taking other points on the circumference of circle ABCD, we shall find the place at which the perpendiculars drawn from them fall onto the plane of circle DEBF. Therefore, on the circumference of a greatest circle in a sphere [inclined] to another circle, etc. Which was to be done.

Diagram. The same tilted-ellipse construction as fig-p89-2, repeated at the head of this page so the continued demonstration can be followed. The labels are as before: A (upper left), K (top), D (top); L, T, Q, R, M at the left interior; N (upper right), O (centre), I (right); E (left end), S, H, P (lower left), G (centre); B (lower left), C (lower right), F (right end).

- A, C: ends of diameter AC of circle ABCD
- D, B: ends of the common section DB
- E, F: ends of diameter EF of circle DEBF
- G (centre): common centre
- H, I: feet of the perpendiculars from A, C (minor-ellipse diameter HI)
- K: arbitrary point on circle ABCD
- L, M, N, O, P, Q, R, S, T: construction points (as in fig-p89-2)

COROLLARIUM.

[Margin: How an ellipse whose diameters are given is to be described.]

From this it is plainly evident how an ellipse whose diameters are given is to be described. For suppose two diameters DB, HI are so fitted that they bisect each other at G and cut at right angles; and with centre G and radii GD, GH let circles be described. Then let HI, produced in both directions, have equal arcs EL, LA, AB, BC — as many as one likes — marked off, together with FN, NK, KP, PR equal to these; and let the same be done in the other semicircle EBF; and let the points L, A, B, etc. be joined to the centre G by straight lines cutting the circle HMI into parts that will be similar to the parts EL, LA, etc. — by what we wrote in our commentary on the *Sphere* at the end of chapter 1. Next, let any two points of the greater circle, equally removed on either side from D or B, be connected by straight lines; and likewise any two points of the smaller circle, equally removed on either side from H or I, by other straight lines. Finally, let the points where

each pair of lines meet — the lines that pass through the divisions corresponding to one another — be marked: all these will fall on the ellipse whose diameters are DB, HI, as has been demonstrated.

For in the figure above we showed that the point Q, where the straight lines LN, PM meet — of which the former is parallel to the minor diameter HI and the latter to the major DB — falls on the ellipse; and these parallels are drawn through the points L, M that correspond to each other, that is, that cut off similar arcs EL, HM. Since, then, the same thing happens here — because all the lines connecting the corresponding points are parallel to the diameters HI, DB — it is clear, from what we demonstrated in the scholium to Elements III.27, that all those points fall on the ellipse. The same will happen even if the arcs EL, LA, etc. are not equal, provided that through the corresponding points L, M, etc. parallels are drawn to the diameters of the ellipse, and so on. Therefore, if we draw a line suitably and congruently joining such points, the ellipse will be described. Which was to be done.

Federico Commandino puts this problem too more generally, as he does the one we demonstrated above in Propos. 24 of this book.

Diagram. The point-by-point construction of the ellipse from its two given diameters: an outer (greater) circle and an inner (smaller) concentric circle, with radial (dotted) lines dividing them into corresponding arcs, and centre G. The near-horizontal diameter is EF (E at the left, F at the right), the near-vertical DB (D at the top, B at the bottom). Division points run around the outer circle (C, D, R across the top; B, A, L down the left; P, K, N down the right); the inner circle carries H, M, G, I along its middle and O above centre.

- D (top), B (bottom): ends of the vertical diameter DB
- E (left), F (right): ends of the horizontal diameter EF
- G (centre): common centre
- C, R (top), A, L (left), P, K, N (right): division points on the outer (greater) circle
- H, I (inner middle): ends of the inner diameter; M near H, O above the centre — division points on the inner (smaller) circle HMI

PROBLEMA 6. PROPOS. 27.

[Margin: The inclination of a proposed plane to the meridian — by what method it is to be found.]

To investigate the inclination of any plane whatever to the meridian circle.

Let ABCD be the horizon, BD the vertical, ACG the meridian, and EF a plane inclined to the meridian, cutting the meridian at G — whether below the horizon or above it. Here, and in the following five problems, we take an inclined plane to be one drawn through the centre of the sphere, and therefore, by Theodosius, *Spherics* I.1, making a greatest circle in the sphere. Since, then, in the spherical triangle CFG, whose angle C is right (for the meridian is perpendicular to the horizon), it holds — by Regiomontanus [Regiomontanus], *De Triangulis* IV.18, or by Jābir [Jābir ibn Aflah] I.14, or certainly from Proposition 42 of our own [treatise] on spherical triangles — that as the sine of angle F, the inclination of the given plane EF to the horizon, is to the sine of the right angle C (that is, to the total sine [radius]), so is the sine of the complement of angle G, the inclination of the same plane EF to the meridian, to the sine of the complement of arc CF; that is, to the sine of arc DF, or BF, the declination

Diagram. Two side-by-side circles illustrating a plane EF inclined to the meridian. In each, ABCD is the horizon (A at the top, B at the left, D at the right, C at the bottom), BD the horizontal vertical-circle line, and ACG the vertical (meridian) line; the inclined plane EF is the oblique chord (E near the top, F toward the bottom). **Left circle:** A (top), E (just right of A), B (left), D (right), C (bottom), F (lower left), with G below the circle at the bottom (the inclined plane cutting the meridian below the horizon). **Right circle:** A (top), E (upper left), B (left), D (right), G at the centre, C (bottom), F (lower right) — the plane cutting the meridian at G within the sphere.

- A (top), C (bottom): meridian points; B (left), D (right): horizon points ABCD
- E, F: the ends of the inclined plane EF (its trace)
- G: the point where the inclined plane cuts the meridian (below the horizon in the left figure; at the centre in the right)

Gnomonics, Book I (continued)

(Continuation of the demonstration of Problem 6 / Proposition 27, from the foot of the preceding page.)

measuring the declination from the vertical circle. And conversely: as the total sine [radius] of the right angle C is to the sine of angle F, the inclination of plane EF to the horizon, so is the sine of the complement of arc CF — that is, the sine of arc DF or BF, which measures the declination from the vertical — to the sine of the complement of angle G, the inclination of the same plane EF to the meridian. Therefore, once the declination of the proposed plane from the vertical circle has

been found by Proposition 23 of this book, together with its inclination to the horizon: if we set it up so that, as the total sine is to the sine of the inclination to the horizon, so is the sine of the declination from the vertical to a fourth number, we shall obtain the sine of the complement of the inclination to the meridian; and when this complement is subtracted from a quadrant, the inclination to the meridian becomes known.

Example. Let the declination of the plane from the vertical circle be 30° , and its inclination to the horizon $52^\circ 3'$. If, then, we set it up so that, as 100000 (the total sine) is to 78854 (the sine of the inclination to the horizon, namely the sine of $52^\circ 3'$), so is 50000 (the sine of the declination from the vertical, that is, the sine of 30°) to a fourth number, this fourth number will be found to be the sine 39427, whose arc contains $23^\circ 13'$. When this is subtracted from a quadrant, there remains the inclination to the meridian, $66^\circ 47'$. Which was what was proposed.

[Margin: When the plane is at right angles to the vertical.]

But if the inclined plane faces directly east or west — that is, if it is at right angles to the vertical circle and passes through the common sections of the horizon and the meridian — then the complement of its inclination to the horizon will be its inclination to the meridian, as is evident.

[Margin: When the plane inclined to the horizon looks directly toward the south or the north.]

But if a plane inclined to the horizon has no declination — that is, if it passes through the common sections of the vertical circle and the horizon, and faces directly south or north — then such a plane will have no inclination to the meridian; that is, it will be at right angles to the meridian, as is clear.

[Margin: When the plane is at right angles to the horizon.]

Finally, if a plane is at right angles to the horizon — that is, if it passes through the zenith — then the complement of its declination from the vertical circle will be its inclination to the meridian, as is manifest. Thus we have investigated the inclination of any plane whatever to the meridian circle. Which was to be done.

PROBLEMA 7. PROPOS. 28.

[Margin: The arc of the meridian placed between the horizon and the inclined plane — how large it is, is to be investigated.]

Given a plane inclined to the horizon and the meridian, or to the meridian only, or to the horizon only, to find how large is the arc of the meridian circle placed between it and the horizon.

Let ABCD be the horizon, with centre E; let AC be the common section of the horizon and the meridian, and BD the common section of the horizon and the vertical. Let DF be the declination, from the vertical circle, of a plane inclined both to the horizon and to the meridian, so that the straight line FG is the common section of the horizon and the inclined plane. Now, once the inclination of this same plane to the horizon has been found by Proposition 23 of this book, let the minor diameter of the ellipse be found by Proposition 25 of this book — the ellipse which the perpendiculars, dropped from the circumference of the inclined plane (which makes a circle in the sphere, by Theodosius, *Spherics* I.1) to the plane of the horizon, produce by Proposition 24 of this book. Let this minor diameter be HI, cutting the major diameter FG at the centre E at right angles. Now, by the corollary of Proposition 26 of this book, let an ellipse — or at least the part of it — be described about the given diameters FG and HI, cutting AC, the common section of the horizon and the meridian, at the point K; through K let the perpendicular KL be drawn to AC, cutting the circumference of the horizon at L. I say that the arc CL is equal to the arc of the meridian circle interposed between the horizon and the inclined plane.

For let the semicircle AMC of the meridian be understood as standing at right angles to the horizon ABCD, and let M be the point at which the inclined plane cuts the meridian, such that the arc CM lies between the horizon and the plane itself. Let the straight lines EL, EM, CL, CM also be drawn. Now, since the perpendicular dropped from M to the horizon falls on the straight line AC — the common section of the horizon and the meridian — because the meridian is at right angles to the horizon, and since it also falls (by Proposition 24 of this book) on the ellipse which the perpendiculars produce that fall from the points of the circumference of the inclined plane (one of which is M) onto the plane of the horizon: it is evident that the perpendicular from M falls on the point K, where the ellipse cuts the straight line AC. Let it fall, then, and let it be MK. Consequently both the angle EKM and the angle CKM will be right, by Euclid, *Elements* XI, Definition 3. [Margin: Euclid, *Elements* I.47] Therefore the square on EL will equal the squares on EK and KL, and likewise the square on EM will equal the squares on EK and KM. And since the squares on the equal straight lines EL and EM are equal (for they are drawn from the centre of the sphere to its surface), the squares on EK and KL will also equal the squares on EK and KM; and, the common square on EK being taken away, the square on KL will equal the square

on KM, and thus the straight line KL will equal the straight line KM. And so, since in the triangles CKL and CKM the sides KL, KC equal the sides KM, KC, and they contain equal angles (namely right angles), [Margin: Euclid, *Elements* I.4] the base CL will equal the base CM; [Margin: Euclid, *Elements* III.28] and therefore the arc CL will equal the arc CM of the meridian placed between the horizon and the inclined plane. Which was what was proposed.

Another method. Let ABCD be the horizon, BD the vertical, ACG the meridian, and EF a plane inclined both to the horizon and to the meridian, cutting the meridian at G — whether below the horizon or above it — so that CG is the arc placed between the horizon and the inclined plane. Since, then, in the spherical triangle CFG, whose angle C is right (because the meridian is at right angles to the horizon), it holds — by Proposition 16, Book 4 of Johannes Regiomontanus [Regiomontanus], *On Triangles*, or by Proposition 13, Book 1 of Jābir [Jābir ibn Aflah], or certainly from Proposition 41 of our own [treatise] on spherical triangles — that as the sine of arc CF (namely of the complement of the declination DF from the vertical circle) is to the sine of angle G (the inclination of plane EF to the meridian), so is the sine of arc CG, between the horizon and the inclined plane...

Diagram. A circle (the horizon ABCD, centre E) carrying an inscribed ellipse and its construction for Problem 7. A is at the top, C at the bottom (the meridian's vertical trace AC), B at the left and D at the right (the vertical's trace BD); E is the centre. G lies upper-left on the circumference, F at the lower right, so that FG is the common section of the horizon and the inclined plane (the ellipse's major diameter through E). An ellipse is drawn about FG; I lies inside on the right, H inside on the lower left (HI the minor diameter). K and M lie near the bottom (K on AC, M on the arc just to its right), L at the lower left on the circumference. Straight lines EL, EM, CL, CM and the perpendicular KL/MK complete the figure.

- A (top), C (bottom): trace of the meridian on the horizon
- B (left), D (right): trace of the vertical on the horizon
- E (centre): centre of the horizon and of the sphere
- F (lower right), G (upper left): ends of FG, the common section of horizon and inclined plane (major diameter of the ellipse)
- H (lower-left interior), I (right interior): ends of HI, the minor diameter of the ellipse
- K (near bottom, on AC), L (lower left, on the circumference): K where the ellipse cuts AC, L where the perpendicular through K meets the horizon
- M (near bottom, right of K): where the inclined plane cuts the meridian

Gnomonics, Book I (continued)

(Text continues from p. 79; the running head is now GNOMONICES.)

to the sine of angle F, the inclination of the plane to the horizon. And conversely: as the sine of angle G, the inclination to the meridian, is to the sine of arc CF, the complement of the declination from the vertical, so is the sine of angle F, the inclination to the horizon, to the sine of the arc CG of the meridian intercepted between the horizon and the inclined plane. Therefore, once the declination of the proposed plane from the vertical circle has been found by Proposition 23 of this book, together with its inclination to the horizon, and likewise its inclination to the meridian from the preceding proposition: if we set it up so that, as the sine of the inclination to the meridian is to the sine of the complement of the declination from the vertical, so is the sine of the inclination to the horizon to a fourth number, we shall obtain the sine of that arc of the meridian which is intercepted between the horizon and the inclined plane.

Diagram. Two side-by-side circles illustrating a plane EF inclined to both the horizon and the meridian, cutting the meridian at G. In each, ABCD is the horizon (A at the top, B at the left, D at the right, C at the bottom), with the vertical trace BD (horizontal) and the meridian trace running vertically; EF is the oblique chord (the trace of the inclined plane). **Left circle:** A (top), E (just right of A, upper), B (left), D (right), F (lower left), C (bottom), and G below C outside the circle (the inclined plane cutting the extended meridian below the horizon). **Right circle:** A (top), E (upper left), B (left), D (right), G at the centre, C (bottom), F (lower right) — the plane EF cutting the meridian at G within the sphere.

- A (top), C (bottom): meridian trace; B (left), D (right): vertical trace BD
- E, F: ends of the inclined plane's trace EF
- G: point where the inclined plane cuts the meridian (below the horizon in the left figure, at the centre in the right)

so that, as the sine of the inclination to the meridian is to the sine of the complement of the declination from the vertical, so is the sine of the inclination to the horizon to a fourth number: there will be obtained the sine of that arc of the meridian which is interposed between the horizon and the inclined plane.

Another method. Set it up so that (by Problem 11 of the [treatise on] spherical triangles) as the total sine is to the sine of the complement of the plane's declination from the vertical — that is, to the sine of arc CF — so is the tangent of angle F, the inclination to the horizon, to a fourth number. The number produced

will be the tangent of the required side CG. And in this method the inclination to the meridian is not needed.

Another method. Set it up, by Problem 12 of the [treatise on] spherical triangles, so that as the total sine is to the secant of the complement of angle G (the inclination of the plane to the meridian, found by Proposition 27), so is the sine of the complement of angle F (the inclination of the plane to the horizon, found by Proposition 23) to a fourth number. The number produced will be the sine of the complement of the required side CG.

Example. Let the inclination to the meridian be $66^{\circ}47'$, the declination from the vertical 30° , and the inclination to the horizon $52^{\circ}3'$. If, then, we set it up so that, as 91902 (the sine of the inclination to the meridian) is to 86602 (the sine of the complement of the declination from the vertical), so is 78854 (the sine of the inclination to the horizon) to a fourth number, this will be found to be very nearly the sine 74306, whose arc — 48° nearly — is placed between the horizon and the inclined plane. Which was what was proposed.

[Margin: When the plane is at right angles to the vertical.]

But if a plane inclined both to the horizon and to the meridian faces directly east or west — that is, if it is at right angles to the vertical — then no arc of the meridian will be intercepted between the plane and the horizon, because then the plane runs along the common sections of the meridian and the horizon, as is evident.

[Margin: When the plane is at right angles to the meridian.]

But if the plane is inclined to the horizon only — that is, if it passes through the common sections of the horizon and the vertical (that is, through the poles of the meridian), and so is at right angles to the meridian — then its very inclination to the horizon will be the arc of the meridian placed between the plane and the horizon, because then the meridian measures the inclination, as is clear.

[Margin: When the plane is at right angles to the horizon.]

Finally, if the plane is inclined to the meridian only — that is, if it passes through the zenith (that is, through the pole of the horizon), and so is at right angles to the horizon — then a quarter of the meridian circle will be intercepted between the plane and the horizon, as is manifest. Therefore, given a plane inclined to the horizon and the meridian, or to the meridian only, and so forth. Which was to be done.

SCHOLIUM.

[Margin: When the arc of the meridian between the horizon and the inclined plane is found above the horizon, and when below.]

If the inclination of a plane to the horizon falls toward the northern part of the upper hemisphere — in accordance with what we taught in Proposition 23 of this book — the meridian will cut the plane on the side of the arctic pole above the horizon, and the arc placed between the horizon and the plane will stand out above the horizon on the side of the arctic pole. But if the inclination falls toward the southern part of the same hemisphere, the plane will cut the meridian circle on the side of the arctic pole below the horizon, and therefore the arc of the meridian intercepted between the plane and the horizon will be hidden there below the horizon. The reason for this is that, since a quadrant of the horizon is intercepted between the greatest circle measuring the inclination of the plane and the plane itself, and since that circle bisects the segment of the horizon between the inclined plane — that is, the segment cut off by the inclined plane, which is a semicircle — by Theodosius, *Spherics* II.9, it comes about that, when the inclination falls toward the northern part, the meridian on that same northern side cuts the inclined plane above the horizon. In the same way, when the inclination falls toward the southern parts, the meridian on that same southern side cuts the inclined plane above the horizon, and consequently below the horizon on the northern side. This will be easily understood if the situation and position of the circles in the sphere is carefully weighed.

COROLLARIUM.

[Margin: How the arc of the meridian placed between the inclined plane and the arctic pole is to be found.]

From these things it will not be difficult to find how large is the arc of the meridian interposed between the inclined plane and the arctic pole of the world. For if the arc of the meridian between the plane and the horizon is below the horizon on the side of the arctic pole, then the arc composed of it together with the arc that measures the altitude of the pole above the horizon will be the arc lying between the inclined plane and the arctic pole of the world — provided it does not exceed a quadrant. But if it exceeds a quadrant, the said arc will be its complement to a semicircle, which is contained above the horizon toward the south, just as the former is partly above the horizon and partly below. But if it stands above the horizon, then the arc by which it differs from the arc measuring the altitude of the pole above the horizon will be the arc intercepted between the

plane itself and the arctic pole of the world, and it will be contained above the horizon, now toward the north, now toward the south. All these things will become clear from the figure set alongside, in which ABCD is the meridian; EF the axis of the world; E the arctic pole, F the antarctic...

Diagram. A circle (the meridian ABCD) filled with radial straight lines drawn through the centre, each a diameter representing the trace of an inclined plane, the horizon, the vertical, or the axis of the world. A is at the top, C at the bottom, B at the left, D at the right (the cardinal points of the meridian). The polar axis EF is a tilted diameter, with E (the arctic pole) toward the upper right and F (the antarctic pole) toward the lower left. The remaining lettered diameters are the traces of inclined planes: GH (G lower right, H upper left), IK (I right, K left), LM (L upper right, M lower left), and NO (N upper left, O at the bottom).

- A (top), C (bottom), B (left), D (right): cardinal points of the meridian ABCD
- E (upper right): arctic pole of the world; F (lower left): antarctic pole; EF the axis of the world
- G–H, I–K, L–M, N–O: traces (diameters) of inclined planes on the meridian

Gnomonics, Book I (continued)

(Corollary continues from p. 80; running head again LIBER PRIMVS.)

F [the antarctic pole]; BD the common section of the horizon and the meridian; AC the common section of the vertical and the meridian; and GH, or IK, or LM, or NO the common section of the inclined plane and the meridian. You see, then, that the arc GE is composed of the arc GD — which lies between the inclined plane and the horizon, below the horizon — and the arc DE, the altitude of the pole above the horizon. Likewise, the arc IE (or LE) — by which the arc DI (or DL), between the inclined plane and the horizon above the horizon, differs from the arc DE, the altitude of the pole above the horizon — is intercepted between the inclined plane and the arctic pole E. Likewise the arc EN, between the plane NO and the pole, is the one to be taken — even though the arc DO, between the plane and the horizon, is below the horizon — because the whole arc EO exceeds a quadrant.

[Margin: When the plane is at right angles to the vertical.]

But if the inclined plane is at right angles to the vertical circle — that is, if it faces directly east or west, and passes through the common sections of the horizon and the meridian, so that the common section of it and the meridian is the straight

line BD — then the arc between the plane itself and the pole E will not differ from the arc DE, which measures the altitude of the pole above the horizon, as is evident.

[Margin: When the plane is at right angles to the horizon.]

But if the inclined plane is at right angles to the horizon — that is, if it passes through the zenith A, so that the common section of it and the meridian is the straight line AC — then the arc between the plane itself and the pole E will not differ from the arc AE, which measures the complement of the altitude of the pole above the horizon, as is manifest.

[Margin: How large is the arc of the meridian between the inclined plane and the zenith.]

By like reasoning we shall discover how large is the arc of the meridian intercepted between the inclined plane and the zenith. For if we subtract the arc of the meridian between the plane and the horizon from a quadrant, the remainder will be the arc of the meridian between the plane and the zenith. This arc will be contained on the northern side if the arc between the plane and the horizon is above the horizon on the same northern side, but on the southern side if the arc between the plane and the horizon toward the northern parts is below the horizon. For in the same figure you see that the arc DI (or DL), taken away from the quadrant DA, leaves the arc IA (or LA), between the plane and the zenith toward the northern parts. Likewise the arc BH (or BN) — [Margin: Euclid, *Elements* III.26] the arc BH being equal to the arc GD, and the arc BN equal to the arc OD, between the plane and the horizon below the horizon — subtracted from the quadrant BA, leaves the arc HA (or NA), between the plane and the zenith on the southern side.

SCHOLIUM II.

[Margin: By what device the altitude of the pole above the horizon is found by means of the analemma.]

But let us now set forth that method of finding the altitude of the pole above the horizon by means of the analemma which we promised in the scholium of Proposition 1 of this book, and which we in fact took from Giovanni Battista Benedetti [Ioannes Baptista Benedictus], in his book *On the Use of Gnomons and of Solar Shadows*. Presenting it more clearly, however, we have reduced it to the following form.

On a plane parallel to the horizon, let a circle ABCD be described, with centre E, in which BD is the meridian line — that is, the common section of the meridian circle and of the circle ABCD — such that B faces south and D faces north; and let AC be the common section of the vertical and of the same circle. Now, with a stylus of any size fixed at the centre E at right angles to the plane of the circle ABCD, let the shadow of the stylus be observed, either before noon or after; and in the middle of it, near its tip (for the extreme point can scarcely be perceived on the plane without error), let a point be marked, through which and through the centre E the straight line FG is drawn. Here we may also conveniently use the instrument that we constructed at the beginning of the scholium of Proposition 23 of this book. For if the rule AB is placed on the plane of the circle ABCD, so that the point D is set at the centre E, we shall observe the shadow of the side HD in place of the shadow of the stylus, and in it we shall mark a point, through which and through the centre E (the instrument having first been removed) we shall draw the straight line FG. We shall use this same instrument in other observations too, in which a stylus is customarily employed to cast a shadow for projection. Then, having taken the Sun's altitude without delay, before the line FG is drawn, let an arc GH equal to it be marked off on the opposite side of the shadow, and from H let the perpendicular HI be dropped to FG. Then from I let the perpendicular IOQ be drawn to BD — or parallel to AC — and let OQ be cut off equal to HI. Again, after some interval of time...

Diagram. The analemma construction — a large circle ABCD with centre E, crowded with radial and perpendicular construction lines and small interior point-labels (several partly obscured). A is at the top, C at the bottom, B at the left, D at the right; BD is the meridian line (B toward the south, D toward the north), AC the vertical trace, E the centre where the stylus stands. FG is the shadow line drawn through E; K lies at the upper right and L toward the lower left, with KL the second shadow line. H is at the left, M at the mid-left, I and N toward the lower centre, G at the bottom. Interior construction points O, P, Q, R and the two small Greek arc-labels π and ρ (the arc $\pi\rho$ described about R, measuring the pole's altitude) sit near the centre. Many of the fine interior labels are crowded and ink-spotted.

- A (top), C (bottom): vertical trace AC; B (left, south), D (right, north): meridian line BD
- E (centre): foot of the stylus / centre of the circle
- F, G: ends of the first shadow line FG through E
- K (upper right), L (lower left): ends of the second shadow line KL

- H (left), I (lower centre): shadow-arc point H and foot I of the perpendicular HI
- M (mid-left), N (lower centre): shadow-arc point M and foot N of the perpendicular MN
- O, P, Q, R (interior): construction points (IOQ, NPR, RQ); the angle PRQ measures the pole's altitude
- π , ρ [Greek letters]: the small arc $\pi\rho$ described about R, containing the degrees of the altitude
- [Several further interior labels are crowded / ink-spotted and could not be read with confidence]

has elapsed, let the shadow of the stylus be observed again, and in the middle of it, near its tip, let another point be marked, from which, through the centre E, the straight line KL is drawn; and, having taken the Sun's altitude at that time, let an arc LM equal to it be marked off on the opposite side of the shadow, and from M let the perpendicular MN be drawn down to KL, and from N let the perpendicular NPR be raised to BD — or parallel to AC — and let PR be cut off equal to MN. Finally, through the points R and Q let the straight line RQ be drawn. I say that the angle PRQ is the angle of the pole's altitude, and that the arc $\pi\rho$, described about R, contains the degrees of that same altitude. This we shall demonstrate as follows. Since, at the time of the first observation, the tip of the shadow falls on the straight line FG [?], the straight line FG will be the common section of the circle ABCD and of that vertical circle in which the Sun then stands, as is established from Proposition 11 of this book. Hence, since the vertical circle properly so called — drawn through the straight line AC — and the vertical circle passing through the centre of the Sun and through the straight line FG cut off similar arcs from the horizon and from the circle ABCD (which is parallel to the horizon, being distant from it by an interval equal to the length of the stylus, as is evident from Proposition 2 of this book), by Theodosius, *Spherics* II.10 — because they are drawn through their poles — it comes about that, if the circle ABCD is taken for the horizon, the straight line FG will also be the common section of the horizon and of the vertical circle through the centre of the Sun...

Diagram. A rectangle FGDC seen in profile (the analemma's side elevation), lying beneath the large circle. F is the upper-left corner and G the upper-right; C is the lower-left corner and D the lower-right. At the top centre stand the points B and A (close together), from which a narrow shaded triangle (the stylus / gnomon and its shadow) descends to the point E at the bottom centre, below which a small sun-burst symbol ($\odot$, the Sun) is engraved.

- F (upper left), G (upper right): top edge of the profile rectangle
- C (lower left), D (lower right): bottom edge
- B, A (top centre): apex points of the gnomon
- E (bottom centre): foot of the gnomon; ☉ (below E): the Sun

of the Sun as it passes. Now since GH is the arc of the Sun's altitude, if the semicircle FHG is imagined to be rotated about the diameter FG until it stands perpendicular to the horizon — the straight line HI being then perpendicular to that same horizon, by Euclid XI, definition 4 — the centre of the Sun will lie at the point H. Hence the Sun's parallel of that moment will pass through the point H.

Now suppose that, in that position of the semicircle FHG, a line is drawn through H, in the plane of the Sun's parallel, parallel to IQ itself. (That a parallel to IQ can indeed be drawn through H in the plane of the Sun's parallel will become clear as follows. Since the Sun's parallel and the horizon are both perpendicular to the meridian [Margin: Euclid XI.19], their common section will be perpendicular to that same meridian, and hence also to the straight line BD lying in the meridian; for, BD being the common section of meridian and horizon, that common section will be perpendicular, by Euclid XI, definition 3 [Margin: Euclid I.28], and therefore parallel to IQ. If, then, in the plane of the Sun's parallel a line be drawn through H parallel to the common section of the parallel and the horizon, it will [Margin: Euclid XI.9] be one and the same with, and parallel to, IQ. So a line parallel to IQ can be drawn through H in the plane of the Sun's parallel.) A straight line drawn from O perpendicular to the horizon — and therefore, by Euclid XI, definition 3, to the straight line BD — and equal to HI, falls upon that parallel drawn through H in the Sun's parallel. (For since HI and the said perpendicular drawn from O are both at right angles to the plane of the horizon, they [Margin: Euclid XI.6] will be parallel to one another; and since they are equal by hypothesis, the line drawn from H [Margin: Euclid I.33] through the far end of the perpendicular raised from O will be parallel to IO. Consequently the said perpendicular drawn from O will fall upon that parallel drawn through H in the Sun's parallel. Otherwise two parallels to IO would be drawn from the single point H — namely the one which we said must be drawn through H, and the one which passes from H through the end of the perpendicular from O and which we proved parallel to IO — which is absurd. [Margin: Euclid XI.9] For those two lines sent out from H would be parallel to one another, and yet meet at H.)

It follows, then, that — OQ being taken equal to HI and perpendicular to BD — if the semicircle BAD is conceived to be rotated about the diameter BD until it stands perpendicular to the horizon and coincides with the meridian, so that QO

is perpendicular to that same horizon (by Euclid XI, definition 4), the point Q falls upon that parallel drawn through H in the plane of the Sun's parallel; and therefore the plane of the parallel passes through the point Q, while the semicircle BAD remains in that position in which it stands perpendicular to the horizon, like the meridian. In the same way we shall prove that this same plane of the Sun's parallel passes through the point R in that position of the semicircle BAD. Hence RQ, the common section of the Sun's parallel and of the meridian ABCD (for we are now taking this circle for the meridian), and therefore the angle PRQ, will be the angle of the pole's altitude — which is shown thus. Draw the straight line $\mu\xi$ through the centre E, parallel to RQ; then $\mu\xi$ will be the common section of the equator and the meridian. Hence the angle $BE\mu$ will be the angle of the equator's altitude, that is, the complement of the pole's altitude; and therefore the remaining angle $AE\mu$ (measured off from the right angle) will be the angle of the pole's altitude. [Margin: Euclid I.34] Since, then, the angle opposite to it, PRQ, is equal to it in the parallelogram ER, PRQ will likewise be the angle of the pole's elevation above the horizon — which was proposed.

But since some error may easily be committed in drawing the straight line RQ — because the perpendiculars PR and OQ, and hence the points R and Q, lie only a very small distance apart, as happens in the given example — we shall carry the matter out more accurately if we choose two observations of the Sun's shadows in which the said points R and Q are separated by some appreciable interval. Indeed we shall do it still more correctly when the Sun is in the northern signs, if we observe the stylus's shadow at the moment when it falls upon AC itself — the common section of the vertical circle properly so called and of the circle ABCD — or when it is close to that line AC, whether toward B or toward D. So in the preceding figure you see that, with the Sun on the vertical circle itself and the stylus's shadow falling on AC, whether before noon or after, the line $E\alpha$ is taken equal to the perpendicular YZ, the Sun's altitude then being AT. Likewise at evening, when the stylus's shadow falls on $E\beta$ and the Sun's altitude is $\gamma\delta$, the line $\theta\lambda$ is taken on the perpendicular $[\epsilon?]\theta$, equal to the perpendicular $\delta[\epsilon?]$. And so on.

But if the observation can conveniently be made on an open, unobstructed horizon, or in some elevated place where the rising or setting Sun can be seen, such a place should be chosen. For if, at sunrise or sunset, the stylus's shadow is observed and SF is again drawn — the common section of the circle ABCD and of the vertical circle passing through the centre of the Sun at that moment — so that the arc CS (or AS) is the rising or setting amplitude; and if ST is drawn perpendicular to BD; then we shall have on the line BD the point T, through

which the Sun's parallel is to be drawn, since ST is the common section of the Sun's parallel and the horizon. And if at noon, with the stylus's shadow falling on BD, the Sun's altitude is observed and reckoned off from B up to V, the parallel of that moment will pass through the point V on the meridian. Hence VT will be the common section of the Sun's parallel and the meridian, as before. Finally, if besides the two points Q and R we find yet a third — such as α , or T — we shall draw the line VT more easily and without error through three points.

[Margin: The rising and setting amplitude — by what method it is investigated by means of a stylus fixed in a wall.] Moreover, since at sunrise or sunset the stylus's shadow can scarcely be caught on the plane of the circle ABCD, we shall use the following device for investigating the Sun's rising and setting amplitude. On some flat, right-angled board we shall draw a line AB parallel to the lowest edge CD, and in it we shall fix at right angles a stylus AE of any length — or better, as above, the side HD of the instrument described at the beginning of the scholium to Proposition 23 of this book; this we shall set up at the point A. Then at sunrise or sunset we shall fit the lowest edge CD of the board to the meridian line BD (parallel to the horizon of the preceding figure), so that the board itself stands perpendicular to the plane of the circle ABCD — which is easily done by means of a plumb-line dropped from the point F. For, with the plumb-line lying against the board, the board will be perpendicular to the circle ABCD. Then we shall mark the shadow on the line AB, where it falls at the point B, so that the Sun's ray is EB. And so, if in the preceding figure, on the vertical line AC, we take a line EQ equal to the gnomon AE (or to the said side HD

Diagram (Source: printed p. 82). A large circle (the analemma / orthographic projection referred to throughout as the "preceding figure"), centre E, with the horizontal diameter B (left) — E — D (right). The oblique diameter AC descends from the top toward the lower right; numerous straight lines are drawn from the centre to peripheral points (the traces of the vertical circles), together with an interior parallelogram (ER) and small right-angle marks. Below the circle stands the profile rectangle (fig-p82-2). Because the figure is exceptionally densely labelled and partly ink-spotted, only the confidently readable labels are listed; several small interior and Greek sorts cannot be resolved at this resolution.

- E (centre)
- B (left), D (right): ends of the horizontal diameter BD
- A (top): summit of the circle
- M (upper left), K (upper right), Z (upper, right of centre): peripheral points
- L (lower left), G (lower, left of centre), H (left, below B): peripheral points

- C (bottom centre, with β to its left), S (lower right): peripheral points
- μ (mu, upper-left periphery): one end of the diameter $\mu\xi$ (the equator's section)
- ξ (xi, lower-right periphery) [?]: the other end of $\mu\xi$
- P (interior, right of E), T (interior, below P), θ (interior, near P) [?]
- R (interior, upper left, with a right-angle mark), Q (interior, upper), ϕ (phi, interior, upper left) [?]
- O, X, ρ (interior cluster, left of E) [?]; π (interior, lower left) [?]
- N, I (interior, lower centre) [?]
- [Several further peripheral and interior labels — including small Greek sorts near the top of the circle — are ink-spotted and unresolved.]

Diagram (Source: printed p. 82). A rectangle FGDC seen in profile (the side elevation of the analemma), beneath the great circle. A shaded gnomon-and-shadow triangle descends from the apex at top centre to E at the bottom edge, below which a small sun symbol is engraved.

- F (upper left), G (upper right): top edge of the profile rectangle
- C (lower left), D (lower right): bottom edge
- B, A (top centre): apex of the gnomon
- E (bottom centre): foot of the gnomon; the Sun symbol ($\odot$) below E

of the said instrument), and through ϕ we draw $\phi\chi$ perpendicular to AE, and take $\phi\chi$ equal to the shadow AB, then the line χE , produced to S, will give the rising or setting amplitude CS. For since the plane FD lies in the plane of the meridian, the gnomon AE will be part of the common section of the vertical circle properly so called and of the horizon. If, then, the circle ABCD is taken for the horizon, and the board FD is imagined applied to it so that the point ϕ falls on A and χ on B — because of the equality of the lines AB and $\phi\chi$ — the centre E will fall on the tip of the stylus E, because of the equality of the lines AE and ϕE . Hence the line χE will coincide with the Sun's ray BE, and therefore, produced, will fall on S, the point of rising or setting on the horizon. Therefore the arc CS will be the rising or setting amplitude. Note, however, that if the observation is made in the morning and the shadow falls on the line AF, the Sun is northern; so the amplitude must then be taken from C toward the northern parts, namely toward D. But if the shadow falls on the line AG, the Sun is southern; so the amplitude must be counted from C toward the southern parts, that is, toward B. The contrary is to be understood if the observation is made in the evening: for when the shadow falls on AG the Sun is northern, and when it falls on AF the Sun is southern, as is evident.

Finally, so that all this may be done more easily and more correctly, before the stylus is fixed there should be drawn in the circle ABCD several straight lines to serve as the common sections of the vertical circles with the circle ABCD — as in the figure, FG drawn 30° from AC, and KL 50° , and $\gamma\beta$ $6^\circ 30'$, and so on. For when the tip of the shadow falls on one of these lines, we shall know on which vertical circle the Sun stands; and then, having taken its altitude, we shall proceed as before. This must be done because the stylus, if it is fixed first, before the lines are drawn through the centre from the marked endpoint of the shadow, is an obstacle, preventing the said straight lines from being drawn through the centre. But if instead of a stylus we use the instrument described at the beginning of the scholium to Proposition 23 of this book, this precaution will not be needed, since that instrument can be removed after the observation of the shadow, so that the lines can be drawn through the centre E without hindrance.

[Margin: The pole's altitude above the horizon — by what method it is computed by sines.] Now if we wish to measure the magnitude of the angle PRQ — the pole's altitude — by the doctrine of sines, we shall do it thus. Since the arc GH of the Sun's altitude is known, the sine EI of its complement will be known. And since the angle CEG is also known from the observation of the shadow (for when the shadow falls on EF, the degrees of the arc AF are measured, that is, the angle AEF, which is the same as CEG), its alternate angle EIO will [Margin: Euclid I.29] be equal to it, and so, in the right triangle EIO, known. Therefore EO, the sine of the angle EIO, will be known in terms of the parts of the whole sine EI. And if we set it up that, as the whole sine EI is to EI in so far as it is known in parts of the whole sine EG, so is EO — in so far as it is the sine of the angle EIO — to a fourth term, then EO will become known in parts of the whole sine EG (or EB). By the same reasoning EP will become known in the same parts. Subtracting, then, EO from EP, OP becomes known in the same parts. Now if the line Q ω is drawn from Q perpendicular to PR, then Q ω [Margin: Euclid I.34] will be equal to OP, and P ω to OQ. Hence Q ω too will be known in parts of the whole sine EB. And in the same parts the line OQ (that is, P ω) and PR are known, since they are equal to the right sines IH and NM of the known solar altitudes. [Margin: Euclid I.47] Subtracting, then, the line P ω from PR, the remainder ω R will also be known in the same parts. Since, then, the squares of the lines ω Q and ω R are equal to the square of the line QR, QR too will become known in the same parts. Therefore if we set it up that, as QR — in so far as it is known in parts of the whole sine EB — is to itself in so far as it is the whole sine, so is Q ω , in so far as it is known in parts of the whole sine EB, to a fourth term, then Q ω will become known in parts of the whole sine QR — that is, in so far as it is the sine of the angle QR ω , the sought pole-altitude. And so on. [Margin: Euclid I.34]

But if one point has been found from the shadow on the line EB — say P — and the other on the line ED — say θ — then $E\theta$ must be added to EP, so that the whole θP becomes known. For hence $\lambda\theta$ will be known, equal to θP ; and from $\theta\lambda$ (or $P\alpha$) aR will be known. Then, as before, the angle $aR\lambda$, in the triangle $aR\lambda$, will become known [Margin: Euclid I.34], and so on. By the same reasoning, from EP and ET, both known, PT will become known; and so, as before, the angle PRT — the pole's altitude — will again be known. And so on.

PROBLEMA 8. PROPOS. 29.

Given a plane inclined either to both meridian and horizon, or to the meridian only, or to the horizon only, to determine how great the pole's altitude above it is.

[Margin: The pole's altitude above a plane inclined to both meridian and horizon, or to the meridian only — how it is found.] Let ABCD be a circle with centre E, inclined either to both meridian and horizon, or to the meridian only; and let BD be the common section of it and the meridian. Having found — by the corollary of the preceding proposition — the arc of the meridian intercepted between the inclined plane and the north pole of the world, take DF equal to it. Find also, by the corollary of Proposition 25 of this book, the minor diameter of the ellipse which the perpendiculars dropped from the circumference of the meridian onto the inclined plane make; let this be GH, cutting the major diameter BD at right angles. About GH describe a circle, and let the straight line drawn as EF cut its circumference at I. Then through F draw FK parallel to the minor diameter, and through I draw IK parallel to the major diameter, cutting the former at the point K, through which let the diameter AC be drawn; and from K erect KL perpendicular to AC, cutting the circle ABCD at L. I say that the arc CL is equal to the arc which measures the pole's altitude above the plane ABCD. For since the arc DF is equal to the arc of the meridian between the plane ABCD and the pole of the world, the remainder FG will also be equal to the remainder in the meridian from the pole up to the diameter which cuts BD at right angles, and from which perpendiculars fall on the points G and H. Hence, by what was demonstrated in Proposition 26 of this book, the perpendicular dropped from the pole onto the plane ABCD will fall at the point K — the intersection of the diameters BD and GH of the ellipse. For just as, in the figure of that proposition, the arcs DL and LE stand — to which the arcs DK and KA in the inclined circle are equal — so here stand the arcs DF and FO, to which correspond, in the meridian inclined to the circle ABCD, arcs equal from D up to the north pole, and from the pole up to the diameter which cuts BD at right angles. Hence, as was there demonstrated, the perpendicular dropped from K falls

Diagram (Source: printed p. 83). A circle ABCD with centre E, representing the inclined plane seen in projection, with an inscribed ellipse. The horizontal diameter runs B (left) — E — D (right); the oblique diameter AC runs from A (upper left) to C (lower right). A dotted vertical line runs from N (top) through the centre to O (bottom). Within the circle an ellipse is drawn on the diameters BD and GH. From F (lower region) and I (on the line EF) the construction lines FK and IK meet at K (interior, lower right); KL is raised perpendicular to AC, meeting the circle at L (upper right). M (at the bottom) marks the foot of the perpendicular from the pole.

- N (top, on dotted vertical): north point / meridian direction
- A (upper left): end of oblique diameter AC
- B (left): end of horizontal diameter BD
- E (centre)
- H (upper interior): end of ellipse minor diameter GH
- L (upper right): where KL meets the circle
- D (right): end of horizontal diameter BD
- K (interior, lower right): intersection of FK and IK
- I (interior): where EF cuts the inner circle about GH
- G (lower interior): end of ellipse minor diameter GH
- C (lower right): end of oblique diameter AC
- M (bottom): foot of perpendicular from the pole
- F (lower, near M): point on the construction
- O (bottom, on dotted vertical): south point / meridian direction

at the point Q, where the lines LQ and MQ — parallel to the diameters HI and BD of the ellipse — intersect; so likewise it will here be shown that the perpendicular dropped from the pole falls at the point K, where the lines FK and IK — parallel to the diameters GH and BD of the ellipse — intersect. Let, then, KM be the perpendicular falling from the pole, and M the pole; and let the great circle AMC be imagined drawn through the lines AE, KC, and KM, [Margin: Euclid XI.18] which will necessarily be perpendicular to the plane ABCD. Since it passes through the pole of the world M, it will be like the meridian of that inclined plane; the line AC will be the meridian line, and the arc CM will measure the pole's altitude above that same plane. Let the lines EL, EM, CL, and CM also be drawn. [Margin: Euclid I.47] Since, then, the square on EL equals the squares on EK and KL, and the square on EM equals the squares on EK and KM — because the angles EKL and EKM are right, by construction and by Euclid XI, definition 3 — and since the squares on the lines EL and EM (drawn equal from the centre of

the sphere to its surface) are equal, the squares on EK and KL will also equal the squares on EK and KM. Taking away the common square on EK, the square on KL will equal the square on KM, and the line KL will be equal to the line KM. And so, since the sides KL, KC are equal to the sides KM, KC and contain equal angles (namely right angles), the base CL will [Margin: Euclid I.4] be equal to the base CM; [Margin: Euclid III.28] and consequently the arc CL will be equal to the arc CM, which measures the pole's altitude above the plane ABCD. Which was proposed. In this manner Federico Commandino [Federico Commandino] carries the matter through in nearly the same way, although he says nothing of a plane inclined to the horizon only. This same result we shall obtain from sines as follows.

[Margin: The pole's altitude above a plane inclined to both meridian and horizon — by what method it is sought by sines.] Let ABCD be the horizon, AC the meridian, and EF a plane inclined to both meridian and horizon, cutting the meridian at G, wherever this may happen; let H be the pole of the world, and through it and the pole of the inclined plane EF let the great circle BD be described, cutting the inclined plane at I and therefore (by Theodosius, *Spherics* I.15) at right angles; and it will accordingly measure, by the arc HI, the pole's altitude above the plane EF. Now in the spherical triangle GHI, whose angle I is right: as the sine of the meridian arc GH, intercepted between the inclined plane and the pole, is to the sine of the right angle I — that is, to the whole sine — so is (by Proposition 16 of Book 4 of Johannes Regiomontanus [Regiomontanus], *On Triangles*; or by Proposition 13 of Book 1 of Geber [Jābir ibn Aflāḥ]; or indeed by Proposition 41 of our own *Spherical Triangles*) the sine of the arc HI, the pole's altitude above the plane, to the sine of the angle G, the plane's inclination to the meridian. Conversely, therefore: as the whole sine of the right angle I is to the sine of the meridian arc GH intercepted between the plane and the pole, so is the sine of the angle G, the plane's inclination to the meridian, to the sine of the arc HI, the pole's altitude above the plane.

Diagram (Source: printed p. 84). Three spheres drawn side by side, each showing the horizon, meridian, inclined plane, and vertical circle as intersecting great circles, illustrating the several cases of a plane's inclination.

Left sphere:

- A (top), E (just above/left of A): pole of the vertical circle / apex
- B (left): horizon point
- D (right): horizon point
- H (centre): intersection

- I (left of H): intersection
- F (lower left), C (lower right): great-circle intersections with the sphere
- G (bottom): lowest intersection

Middle sphere:

- E, A (top; E to the left of A)
- B (left), D (right)
- H (centre right), I (centre left), G (below centre)
- C (lower left), F (lower right)

Right sphere:

- E, A (top; E to the left of A)
- B (left), D (right)
- G (centre), I (right of G), H (below centre)
- C (bottom), F (lower right)

[The small central labels G, H, I in each panel are crowded and partly ink-spotted; positions given are best readings]

And so, having found — by the corollary of the preceding proposition — the meridian arc intercepted between the inclined plane and the pole of the world, and also — by Proposition 27 of this book — the plane's inclination to the meridian: if we set it up that, as the whole sine is to the sine of the meridian arc intercepted between the plane and the pole, so is the sine of the plane's inclination to the meridian to a fourth term, we obtain the sine of the pole's altitude above the given plane.

[Margin: First example.] **Example.** Let the meridian arc between the plane and the pole be 30° , and the plane's inclination to the meridian 35° . If, then, we set it up that, as 100000 (the whole sine) is to 50000 (the sine of the arc placed between plane and pole), so is 57357 (the sine of 35° , that is, of the inclination to the meridian) to a fourth term, there will be found here a sine of about $28678\frac{1}{2}$, whose arc — $16^\circ 40'$ — measures the pole's altitude above the given plane.

[Margin: Second example.] **Likewise** let the meridian arc between the plane and the pole be set at 90° — as happens when the plane and the equator cut the meridian at one and the same point — and the plane's inclination to the meridian at $66^\circ 47'$. If, then, we set it up that, as 100000 (the whole sine) is to 100000 (the sine of the arc between plane and pole), so is 91902 (the sine of the inclination to the meridian) to a fourth term, the same sine 91902 will be found, whose arc —

66°47' — contains the pole's altitude above the given plane. Here you see that the pole's altitude above the plane is equal to its inclination to the meridian; for when the plane and the equator cut the meridian at the same point, one and the same arc of the great circle drawn through the poles of the world and through the poles of the plane measures both the pole's altitude above the plane and the inclination of that plane to the meridian — as is evident, since (by Theodosius, *Spherics* I.15) it is perpendicular to the inclined plane. For hence it comes about that it measures the pole's altitude above the plane, like a kind of proper meridian of the inclined plane itself. The same great circle also measures the inclination of that plane to the meridian, because it too is perpendicular to the meridian — which will be made plain thus. [Margin: When the plane is inclined to the meridian only; or to both meridian and horizon, but perpendicular to the vertical circle.] For since it passes through the poles of the plane and through the poles of the equator, the given plane and the equator will in turn pass through its poles, by the scholium to Theodosius, *Spherics* I.15. Hence the points where the plane and the equator intersect on the meridian are the poles of that circle; and therefore the meridian drawn through these poles will (by Theodosius, *Spherics* I.15) be perpendicular to it. And so, in turn, it will be perpendicular to the meridian — which was to be shown.

In the same way the pole's altitude will be found above a plane inclined to the meridian only — namely one which passes through the zenith of the place — and also above a plane which is perpendicular to the vertical circle and inclined both to the meridian and to the horizon. Of this kind is the plane that runs along the common sections of the horizon and the meridian. This can easily be understood from the following two figures. In the first of them the vertical circle is kL, the plane inclined to the meridian only is EF, passing through the zenith G (that is, the pole of the horizon),

and therefore standing at right angles to the horizon, so that the arc GH of the meridian, between the plane and the pole, is equal to the complement of the pole's altitude above the horizon; while its inclination to the meridian is the complement of the same plane's declination from the vertical circle. In the second figure, by contrast, the inclined plane EF passes through the common sections A and C of the horizon and the meridian — that is, through the poles of the vertical circle — and is therefore at right angles to the vertical circle. Hence, if in place of C we take the letter G, where the inclined plane cuts the meridian (so that the arc GH of the meridian between the plane and the pole is equal to the pole's altitude above the horizon, while the plane's inclination to the meridian is the complement of the same plane's inclination to the horizon), the ratio of the whole

sine — that is, of the right angle I — to the sine of the arc GH will always be the same as the ratio of the sine of the angle G to the sine of the arc HI, and so on, as has been demonstrated.

Diagram. A pair of spherical diagrams accompanying the corollary. In each, a circle represents the sphere, with a horizon, a meridian, an inclined plane EF, and a vertical circle drawn as intersecting great-circle arcs.

- Left sphere: A (top), E (upper left), D (right), K (left), L (right of center), B (lower left), C (bottom), F (lower), with interior points G, H, and I near the center — this figure shows the plane passing through the zenith G.
- Right sphere: A (top), B (left, outside the circle), E (upper right), D (right), C (bottom), G (bottom center), F (bottom), with interior points H and I — this figure shows the plane passing through the common sections A and C of horizon and meridian.

[Margin: Example.] **Example of the first case.** Let a plane passing through the zenith be supposed to decline 30° from the vertical circle, and therefore 60° from the meridian. If, then, we set it up that, as 100000 (the whole sine) is to 74314 (the sine of the arc GH intercepted between the plane and the pole), so is 86602 (the sine of the inclination to the meridian) to a fourth term, there will be found here a sine of about 64357, whose arc — $40^\circ 3'$, a little more — gives the pole's altitude above the proposed plane. Again, for an example of the second case, let a plane passing through the common sections of the horizon and the meridian be supposed to decline 30° from the horizon, and therefore 60° from the meridian. And so, if we set it up that, as 100000 (the whole sine) is to 66913 (the sine of the arc GH placed between the plane and the pole), so is 86602 (the sine of the inclination to the meridian) to a fourth term, there will be found here a sine of about 57948, whose arc — $35^\circ 25'$ — will indicate the pole's altitude above the proposed plane.

[Margin: When the plane is inclined to the horizon only.] But if the plane is inclined to the horizon only — that is, if it passes through the common sections of the horizon and the vertical circle and is at right angles to the meridian — then the arc of the meridian set between the plane and the pole will itself be the pole's altitude above that plane, as appears from the figure of the corollary to the preceding proposition. Therefore, given a plane inclined either to the meridian or to the horizon, we can determine how great the pole's altitude above it is. Which was to be done.

PROBLEMA 9. PROPOS. 30.

Given a plane inclined to the meridian, to find how great the arc of that plane is which is intercepted between the meridian and the great circle that is drawn through the plane's own poles and through the poles of the world and measures the pole's altitude above that plane, serving as its proper meridian; and to find how much this great circle is inclined to the meridian of the horizon — that is, of the place in which the inclined plane is considered.

Diagram. Three spherical diagrams for Problem 9, showing the horizon ABCD, the meridian AC, the plane inclined to the meridian EF, and the great circle drawn through the poles, in three positions of the inclined plane.

- First sphere: A (top), E (upper left), D (right), B (left), C (bottom), F (lower right), with interior points H, I, and G.
- Second sphere: A (top), B (upper left), F (right), E (left), C (bottom), D (bottom right), with interior points I, G, and H.
- Third sphere: A (top), D (right), B (left), C (bottom), with interior points E, I, H, F, and G (the plane EF here running nearly vertically).

Let the horizon be ABCD; the meridian AC; the plane inclined to the meridian EF, cutting the meridian at G; the pole of the world H. Through H and through the pole of the inclined plane EF let the great circle BD be described, cutting the inclined plane at I; and since (by *Spherics* I.15 [Theodosius]) this circle is at right angles to the inclined plane itself, it will measure the altitude of the pole H above that plane by the arc HI. It is required, then, to find the size of the arc GI and of the angle GHI. We shall carry this out as follows. Since in the spherical triangle GHI the angle I is a right angle, it follows — by Regiomontanus, *On Triangles* IV.19 [Ioan. Regiomontanus], or by Geber I.15 [Geber], or certainly by proposition 43 of our own *Spherical Triangles* — that as the sine of the complement of the meridian arc GH, intercepted between the plane and the pole, is to the sine of the complement of the arc HI (which measures the pole's altitude above the plane EF), so is the sine of the complement of the arc GI to the whole sine. [Margin: By what device the arc of the inclined plane between its own meridian and the meridian of the horizon may be found.] And therefore, by conversion, as the sine of the complement of the arc HI (of the pole's altitude above the plane EF) is to the sine of the complement of the meridian arc GH between the plane and the pole, so is the whole sine to the sine of the complement of the arc GI. Hence, if we set it up that, as the sine of the complement of the pole's altitude above the inclined plane is to the sine of the complement of the meridian arc between the

plane and the pole, so is the whole sine to a fourth term, there will be obtained the sine of the complement of the plane's arc GI — the arc which, in the triangle GHI, lies opposite the angle H (the inclination of the meridian BD of that inclined plane to the meridian AC of the horizon). And this GI is the arc of the inclined plane intercepted between the meridian of the horizon and the said great circle — that is, the meridian of the inclined plane. And so the arc GI itself will be known.

The same will also be obtained, and more easily, if the inclined plane EF has passed through the zenith G, so that it is one of the vertical circles, as appears in the second figure: for then the arc GH of the meridian between the plane and the pole is the complement of the pole's altitude above the horizon, and the complement of that arc GH is the pole's altitude above the horizon itself, namely the arc HC.

Diagram. Three spherical diagrams repeating the three positions of the inclined plane EF within the horizon ABCD and meridian AC, with the great circle through the poles.

- First sphere: E (upper left), A (top), D (right), B (left), C (bottom), F (lower right), with interior points H, I, and G.
- Second sphere: B (upper left), A (top), F (right), E (left), C (bottom), D (bottom right), with interior points I, G, and H.
- Third sphere: A (top), D (right), B (left), C (bottom), with interior points E, I, H, F, and G (the plane EF running nearly vertically).

Likewise the matter will be easy if the inclined plane EF has passed through the points A and C, where the horizon and the meridian cut one another, so that it is one of the circles of position, as appears in the third figure: for then the arc GH of the meridian between the plane and the pole is the pole's altitude above the horizon, namely the arc HC. It is established, therefore, what was proposed in the first place.

[Margin: How the inclination that the meridian of the inclined plane has to the meridian of the horizon may be found.] **Next**, since — by Regiomontanus, *On Triangles* IV.16, or by Geber I.13, or certainly by proposition 41 of our *Spherical Triangles* — as the sine of the meridian arc GH between the plane and the pole is to the sine of the right angle I (that is, to the whole sine), so is the sine of the arc GI of the plane between the two meridians, which we have already found, to the sine of the angle H (the inclination of the great circle BD, which stands as the meridian of that plane, to the meridian AC of the horizon): if we set it up that, as

the sine of the meridian arc between the plane and the pole is to the whole sine, so is the sine of the arc GI (of the plane between the two meridians, already found) to a fourth term, there will be obtained the sine of the angle H, the inclination sought. Which is the second thing required.

[Margin: First example.] **Example of both.** Let the pole's altitude above the plane be set at $16^{\circ}40'$, and the meridian arc between the plane and the pole at 30° . If, then, we set it up that, as 95798 (the sine of the complement of the pole's altitude above the plane) is to 86602 (the sine of the complement of the meridian arc between the plane and the pole), so is 100000 (the whole sine) to a fourth term, there will be obtained here a sine of about 90401, whose arc contains $64^{\circ}41'$; and this, subtracted from the quadrant, leaves $25^{\circ}19'$ as the arc of the plane itself between the two meridians. Which is the first thing. After this, if we set it up that, as 50000 (the sine of the meridian arc between the plane and the pole) is to 100000 (the whole sine), so is 42762 (the sine of the arc between the two meridians already found) to a fourth term, there will be found here a sine of 85524, whose arc — namely the inclination of the meridian of that plane to the meridian of the horizon — contains $58^{\circ}47'$. Which is the second thing.

Again, let the pole's altitude above the plane be set at $66^{\circ}47'$, and the meridian arc between the plane and the pole at 90° , as happens in a plane declining 30° from the vertical circle, from north toward east, but inclined $52^{\circ}3'$ to the horizon on the south side, at latitude 42° , as may be gathered from what precedes. If, then, we set it up that, as 39420 (the sine of the complement of the pole's altitude above the inclined plane) is to the sine of the complement of the meridian arc between the plane and the pole (which is nothing), so is 100000 (the whole sine) to a fourth term, there will be obtained here a sine of 0, whose arc is $0^{\circ}0'$; and this, subtracted from 90° , leaves the arc of that plane between the two meridians as 90° . Which is the first thing. Again, if we set it up that, as 100000 (the sine of the meridian arc between the plane and the pole) is to 100000 (the whole sine), so is 100000 (the sine of the plane's arc between the two meridians, already found) to a fourth term, there will be found here a sine of 100000, whose arc, 90° , will be the inclination of that meridian to the meridian of the horizon. And this result agrees with what was said above. For since the pole of the meridian of that inclined plane is the point where the equator cuts the meridian (as we said in the preceding proposition), it must be — by the corollary to *Spherics* I.16 [Theodosius] — that the arc of that plane from the said pole up to its own meridian is a quadrant, and so on.

[Margin: Second example.] **Another example, for the second figure.** Let the plane EF be supposed to decline 30° from south toward east, or from north toward west, in the Roman horizon — that is, where the pole is raised 42° above the horizon. The pole's altitude above the plane EF will, by the preceding proposition, be found to be about $40^\circ 3'$. And the meridian arc between that same plane and the pole will contain 48° , which is of course the complement of the pole's altitude above the horizon. If, then, we set it up that, as 76548 (the sine of the complement of the pole's altitude above the plane) is to 66913 (the sine of the complement of the meridian arc between the plane and the pole — a complement which, in inclined planes of this kind, is itself the pole's altitude above the horizon), so is 100000 (the whole sine) to a fourth term, there will be found here a sine of about 87413, whose arc contains $60^\circ 57'$; and this, subtracted from the quadrant, leaves $29^\circ 3'$ as the arc of that plane which lies between its own meridian and the meridian of the horizon. Which is the first thing. Again, if we set it up that, as 74314 (the sine of the meridian arc between the plane and the pole) is to 100000 (the whole sine), so is 48557 (the sine of the arc between the two meridians, already found) to a fourth term, there will be found here a sine of about 65340, whose arc comprises $40^\circ 48'$. So great is the inclination of the meridian of that plane to the meridian of the horizon. Which is the second thing.

[Margin: Third example.] **For the third figure, take this example.** Let the plane EF be supposed to decline 60° from the meridian, and therefore 30° from the horizon. The pole's altitude above it (namely the arc HI), found by the preceding proposition, will be $35^\circ 25'$. And so, if we set it up that, as 81495 (the sine of the complement of the pole's altitude above the plane) is to 74314 (the sine of the complement of the meridian arc between the plane and the pole — that is, in this example, the sine of the complement of the pole's altitude above the horizon), so is 100000 (the whole sine) to a fourth term, there will be found here a sine of about 91188, whose arc contains $65^\circ 46'$; and this, subtracted from the quadrant, leaves $24^\circ 14'$ as the arc of that plane intercepted between its own meridian and the meridian of the horizon, namely the arc GI. Which is the first thing. Again, if we set it up

that, as 66913 (the sine of the meridian arc between the plane and the pole — namely, in this example, the sine of the pole's altitude above the horizon) is to 100000 (the whole sine), so is 41045 (the sine of the arc GI between the two meridians, already found) to a fourth term, there will be found here a sine of about 61341, whose arc comprises $37^\circ 50'$. So great is the inclination of the meridian of that plane to the meridian of the horizon — that is, so great is the

angle GHI, or the arc of the equator set between the two meridians. Which is the second thing.

[Margin: When the inclined plane passes through the poles of the world.] If the inclined plane passes through the pole, there will be no arc of it interposed between the meridian of the horizon and the meridian of the inclined plane; and the complement of the plane's inclination to the meridian will be the inclination of that plane's meridian to the meridian of the horizon, as is evident on a material sphere. Therefore, given a plane inclined to the meridian, how great the intercepted arc is, and so on. Which was to be done.

PROBLEMA 10. PROPOS. 31.

Given a plane inclined to the meridian, to find how great the arc of that plane is which is intercepted between the great circle that passes through its own poles and the poles of the horizon (and measures its inclination to the horizon), and both its proper meridian — namely the great circle that is drawn through its poles and the poles of the world and measures the pole's altitude above it, serving as its proper meridian — and the meridian of the horizon.

Diagram. Three spherical diagrams for Problem 10, each showing the horizon ABCD, the meridian AC, the vertical circle kL, the inclined plane EF (cutting the meridian at G), its meridian BD (through its pole P and the pole of the world H), and the great circle NO measuring the inclination to the horizon.

- First sphere: A (top), E (upper left), D (upper right), O (right), K (far left), L (far right), N (lower left), B (bottom left), C (bottom), F (bottom right), with interior points M, P, Q, I, and H.
- Second sphere: A (top), E (left), D (upper right), O (right), K (far left), L (far right), N (lower left), B (bottom left), C (bottom), F (bottom right), with interior points M, P, Q, G, H, and I.
- Third sphere: A (top), E (upper left), D (right), O (right), K (left), L (right), N (lower left), B (left), C (bottom), F (bottom right), with interior points G, M, Q, P, H, and I.

Let the horizon be ABCD; the meridian AC; the vertical circle kL; the plane inclined to the meridian EF, cutting the meridian at G; its meridian BD, passing through its pole P and through the pole of the world H; and the great circle NO, which measures the inclination of the plane to the horizon — that is, described through the zenith M (the pole of the horizon) and through P (the pole of the inclined plane) — cutting the inclined plane at Q. It is required to find both the

arc IQ and the arc GQ. Since the arcs MN and PQ are quadrants (by the corollary to *Spherics* I.16 [Theodosius], since M is the pole of the horizon ABCD and P is the pole of the plane EF), then, the common arc MQ being taken away, PM will be an arc equal to the arc QN, which measures the inclination of the plane to the horizon. Therefore, since in the spherical triangle HPM — by Regiomontanus, *On Triangles* IV.17, or by Geber I.13, or certainly by proposition 41 of our *Spherical Triangles* — it holds that as the sine of the arc PM (which is equal to the plane's inclination to the horizon) is to the sine of the angle H (the inclination of the meridian of that inclined plane to the meridian of the horizon), so is the sine of the arc HM (the complement of the pole's altitude above the horizon) to the sine of the angle P (that is, to the sine of the arc IQ): [Margin: By what route the arc of the inclined plane lying between the great circle drawn through its poles and the poles of the horizon and its own proper meridian may be known.] if we set it up that, as the sine of the inclination to the horizon is to the sine of the inclination of the meridian of that inclined plane to the meridian of the horizon (found by the preceding proposition), so is the sine of the complement of the pole's altitude above the horizon to a fourth term, there will be obtained the sine of the first arc IQ, which is sought.

Again, when the arc of the great circle has been drawn through the points G and P, there will be two arcs GP and GM of the spherical triangle GMP that are known. For GP is a quadrant, by the corollary to *Spherics* I.16 [Theodosius], since P is the pole of the circle EF; and the arc GM is known, because it lies between the inclined plane and the zenith M, and hence, by the corollary to proposition 28 of this book, it is known, being the complement of the meridian arc between the inclined plane and the horizon. [Margin: By what device the arc of the inclined plane lying between the great circle drawn through its poles and the poles of the horizon and the meridian of the horizon may be explored.] Likewise the angle GMP is known. For since the circle ON, drawn through the poles P and M of the circles EF and EBF, bisects the arcs EF and EBF (by *Spherics* II.9 [Theodosius]) — and EBF is a semicircle (by *Spherics* I.11) — EN will be a quadrant, and therefore equal to the quadrant kC. The common arc kN being taken away, then, the arc Ek (of the declination of the plane EF from the vertical circle) will be left equal to the arc NC, and so the arc NC is known. Hence the angle NMC is also known, and consequently the remaining angle GMP out of the two right angles is known. Now since, in the spherical triangle GPM — by Regiomontanus, *On Triangles* IV.17, or by Geber I.13, or certainly by proposition 41 of our *Spherical Triangles* — it holds that as the sine of the arc GP (namely the whole sine) is to the sine of the angle GMP (namely the sine of the plane's declination from the vertical circle; for the arcs NC and CO, of the angles NMC and CMO, or GMP, since they make up a

semicircle, have the same sine), so is the sine of the arc GM (of the meridian which lies between the plane and the zenith M) to the sine of the angle GPM (that is, to the sine of the arc GQ): if we set it up that, as the whole sine is to the sine of the declination, so is the sine of the meridian arc placed between the plane and the zenith to a fourth term, there will be obtained the sine of the second arc GQ, which is sought.

[Margin: Example.] **Example of both.** Let the inclination to the horizon be set at $52^{\circ}3'$, and the inclination of the meridian BD of the inclined plane EF to the meridian AC of the horizon at 90° . If, then, we set it up that, as 78854 (the sine of the inclination to the horizon) is to 100000 (the whole sine — namely to the sine of the inclination of the meridians), so is 74314 (the sine of the complement of the pole's altitude above the horizon) to a fourth term, there will be found here a sine of about 94242, whose arc, about $70^{\circ}28'$, is intercepted between the meridian of the inclined plane and the great circle that measures the plane's inclination to the horizon.

Again, if we set it up that, as 100000 (the whole sine) is to 50000 (the sine of the plane's declination from the vertical — for we assume the same plane we treated just now, declining from the vertical, from north toward the east, by 30° , in such a way that its inclination falls toward the south), so is 66913 (the sine of the meridian arc intercepted between the plane and the zenith) to a fourth term: there will be found here a sine of about 33456, to which corresponds an arc of nearly $19^{\circ}33'$, intercepted between the meridian of the horizon and the great circle that measures the plane's inclination to the horizon. (That intercepted arc indeed comprises 42° ; for the arc between the plane and the horizon contains, by proposition 28 of this book, 48° below the horizon toward the north, which, taken from a quadrant, leaves the arc of 42° between the inclined plane and the zenith toward the south, by the corollary to proposition 28 of this book.) Given, therefore, a plane inclined to the meridian, [we have found] how great is its arc intercepted between the great circle passing through its own poles and through the poles of the horizon; and it measures its inclination to the horizon, and likewise its own proper meridian, and so on. Which was to be done.

PROBLEMA 11. PROPOS. 32.

Given a plane inclined to the horizon and declining from the vertical, to find how great is the arc — of the great circle that measures its inclination to the horizon — lying between the horizon and the circle of the sixth hour from noon (or from midnight).

[Margin: The arc of the great circle that measures, for a plane declining from the vertical, its inclination to the horizon — lying between the horizon and the circle of the sixth hour from noon or from midnight — how it is to be investigated.]

Let ABCD be the horizon; AC the meridian; BID the vertical; EF the plane inclined to the horizon and declining from the vertical; GH the great circle that measures its inclination above the horizon, passing through the zenith I (that is, the pole of the horizon) and through M, the pole of the inclined plane; and finally BKD the circle of the sixth hour from noon or midnight, drawn through the pole of the world K and through B and D — the common sections of the horizon and the vertical — cutting the circle GH at L. Let it be required to find the arc LH.

Now in the spherical triangle DHL the angle H is right, by proposition 13 of book 1 of Theodosius, because the circle GH is drawn through I, the pole of the horizon ABCD; and the angle D is the angle of the pole's altitude, since D is the pole of the meridian. (For it comes about hence, by the corollary to proposition 16 of book 1 of Theodosius, that the arcs DC and Dk are quadrants, and consequently the arc Ck — of the pole's altitude above the horizon — measures the angle CDk.)

Therefore, by proposition 18 of book 4 of Johannes Regiomontanus

[Regiomontanus], *On Triangles*, or by proposition 14 of book 1 of Geber, or indeed by proposition 42 of our *[On] Spherical Triangles*: as the sine of the angle HDL (of the pole's altitude) is to the whole sine, so is the sine of the complement of the angle L to the sine of the complement of the arc DH — that is, to the sine of the arc CH, which is equal to the plane's declination from the vertical. (For since the circle GH, drawn through the poles of the horizon and of the circle EF, bisects the arcs ENF and EHF — by proposition 9 of book 2 of Theodosius — and the arc EHF is a semicircle — by proposition 11 of book 1 of Theodosius — EH will be a quadrant, and thus equal to the quadrant DC. Therefore, the common arc DH being taken away, the remaining arc DE, of the plane's declination from the vertical, will be equal to the remaining arc CH.)

By conversion, then: as the whole sine is to the sine of the angle HDL (of the pole's altitude), so is the sine of the complement of the arc DH — that is, the sine of the plane's declination from the vertical — to the sine of the complement of the angle L. Therefore, if we set it up that, as the whole sine is to the sine of the pole's altitude above the horizon, so is the sine of the declination to a fourth term, there will be obtained a sine whose arc, subtracted from a quadrant, leaves the angle L, known.

And since, by proposition 16 of book 4 of Johannes Regiomontanus, *On Triangles*, or by proposition 13 of book 1 of Geber, or indeed by proposition 41 of our *[On]*

Spherical Triangles, it holds in the same triangle that, as the sine of the angle L just found is to the sine of the arc DH (the complement of the declination), so is the sine of the angle HDL (of the pole's altitude) to the sine of the arc LH: if we set it up that, as the sine of the angle L found is to the sine of the complement of the declination, so is the sine of the pole's altitude above the horizon to a fourth term, there will be obtained the sine of the arc LH, which is sought.

Diagram. A sphere (bounding circle) carrying the horizon, meridian, vertical, the inclined-declining plane EF, its measuring great circle GH, and the six-o'clock circle BKD, for Problema 11 / Propos. 32. The figure is crowded with fine interior labels; several are only partly legible. Readable labels:

- A (top): apex point of the meridian
- G (upper left): endpoint of the great circle GH measuring the plane's inclination
- E (upper right): endpoint of the inclined plane EF
- N (upper-centre, interior): point on the great circle GH
- I / l (centre, interior): the zenith, pole of the horizon (intersection region L nearby)
- B (left): common point of horizon and vertical
- D (right): common point of horizon and vertical (pole of the meridian)
- K (lower-centre, interior): pole of the world
- M (lower-centre, interior): pole of the inclined plane
- H (lower right): endpoint of the great circle GH
- C (bottom): lower apex of the meridian
- F (position [?]): endpoint of the inclined plane EF — not clearly resolved on the scan
- (lower-left label, [?]): a peripheral point not securely read

[Margin: Example.] **First example.** Let the plane be inclined to the horizon by 30° , and declining from the vertical by 20° . Set it up that, as 100000 (the whole sine) is to 66913 (the sine of the pole's altitude), so is 34202 (the sine of the declination) to a fourth term: there will be found here a sine of nearly 22885, whose arc — $13^\circ 14'$ — subtracted from a quadrant leaves the angle L of $76^\circ 46'$. Again, set it up that, as 97344 (the sine of the angle L found) is to 93969 (the sine of the complement of the declination), so is 66913 (the sine of the pole's altitude) to a fourth term: there will be obtained here a sine of about 64593, to which correspond $40^\circ 14'$, for the arc LH sought.

[Margin: Second example.] **Again**, let the plane be inclined to the horizon by $52^{\circ}3'$, and declining from the vertical by 30° . Set it up that, as 100000 (the whole sine) is to 66913 (the sine of the pole's altitude), so is 50000 (the sine of the declination) to a fourth term: there will be found here a sine of nearly 33456, whose arc — $19^{\circ}33'$ — taken from a quadrant leaves the angle L of $70^{\circ}27'$, which the great circle measuring the plane's inclination to the horizon makes with the circle of the sixth hour from noon or midnight.

Set it up again that, as 94234 (the sine of the angle L found) is to 86602 (the sine of the complement of the declination), so is 66913 (the sine of the pole's altitude) to a fourth term: there will be obtained here a sine of nearly 61493, to which correspond $37^{\circ}57'$, for the arc LH sought. Therefore, given a plane inclined to the horizon, and so on. Which was to be done.

SCHOLIUM

[Margin: To what end the six preceding propositions serve.]

We have set out the last six problems because their remarkable usefulness will appear in sundials whose planes — on which the dials are laid out — are parallel to great circles that either decline from the vertical and are perpendicular to the horizon; or decline from the horizon and are perpendicular to the vertical; or, finally, both decline from the vertical and are inclined to the horizon — as will become clear from what follows.

PROBLEMA 12. PROPOS. 33.

Given the diurnal (or nocturnal) arc and the latitude of the place, to find the declination from the equator of the parallel belonging to that arc.

[Margin: The declination of the parallel whose diurnal arc contains however many hours one may propose — by what device it is found, by means of the analemma. [Euclid I.29].]

Let any circle ABCD be divided, with center P and with the two diameters AC and BD cutting one another at right angles, into twenty-four equal hours — together with the half-hours and quarters, if desired — beginning at any one of the points A, B, C, D. Now count off from the point B, on either side, the semidiurnal arc BE, BF; or from D, on either side, the seminocturnal arc DE, DF, so that the whole arc EBF is, say, the diurnal arc of 14 hours, or EDF the given nocturnal arc of 10 hours. Draw the straight line EF cutting BD at G. This line, because the arcs AE

and CF are equal, will be parallel to AC — by what we demonstrated with regard to proposition 27 of book 3 of Euclid — and thus perpendicular to BD.

Now construct the angle PGk equal to the angle of the equator's altitude, that is, to the complement of the pole's altitude; and let the straight line Gk cut the straight line AC at the point k, from which draw the straight line kB. I say that PBk is equal to the angle of declination of that parallel whose diurnal arc comprises as many hours as there are in the arc EBF — namely 14. For, the straight line kD being drawn, describe the circle BOMN with center K and radius Bk (or KD). For the straight lines KB and KD are equal to one another; since the sides BP, PK are equal to the sides DP, PK in the triangles BPK and DPK, and contain equal angles (being right angles), the straight lines KB and KD will be equal. [Margin: Euclid I.4.] Then through K draw the diameter LM parallel to BD; and extend the straight line GK on either side. Now since the angle PGK is, by construction, [equal to] the complement of the pole's altitude, the angle PKG will be [equal to] the pole's altitude.

Hence the arc IN will measure the pole's altitude above the horizon; and so, if the circle NLOM be taken to be the meridian, then HI will be the common section of the meridian and the horizon; NO the axis of the world; LM the common section of the meridian and the equator; and BD the common section of the meridian and the parallel ABCD, whose diurnal arc EBF is given. For EF is the common section of the horizon and the parallel ABCD, which thus comes to lie in the plane [of the drawing]. Since both the horizon (drawn through HI) and the parallel ABCD are perpendicular to the meridian NLOM — supposing these circles to have their proper position in the sphere — their common section, which indeed falls at G, where the horizon and the said parallel intersect, will likewise be perpendicular to the meridian [Margin: Euclid XI.19.]; and thus, by definition 3 of book 11 of Euclid, perpendicular to the straight line BD lying in the meridian. Since, then, EF, drawn through G, has been shown to be perpendicular to the straight line BD, EF will be the common section of the horizon and the parallel ABCD. Therefore BL, in the meridian NLOM, is the arc of declination of the parallel ABCD from the equator, and so BKL is the angle of that same declination — to which indeed the alternate angle PBK is equal [Margin: Euclid I.29.], which is what was proposed.

Diagram. The analemma wheel for Problema 12 / Propos. 33: a circle ABCD with center P, the two perpendicular diameters AC and BD, its circumference graduated into twenty-four hours (the rim numbered 1–12 twice, with intermediate tick-marks for half- and quarter-hours), the chord EF, the auxiliary circle BOMN centered at K with the diameter LM, and the lines GK and NLOM.

The figure is very densely labelled and several interior/rim labels are only partly legible; it is flagged as a Readable labels:

- A (top): upper end of the diameter AC
- E (top, just right of A): endpoint of the semidiurnal arc / chord EF
- B (left): left end of the diameter BD
- D (right): right end of the diameter BD
- C (bottom): lower end of the diameter AC
- N (upper-centre, interior): point of the meridian circle NLOM
- P (centre): center of the circle ABCD
- G (centre, right of P): foot of the perpendicular / intersection of EF with BD
- I (mid-left, interior): point on the construction
- K (centre, below P): center of the auxiliary circle BOMN
- L (centre-left / lower, interior): point of the diameter LM (meridian–equator section)
- M (right, lower, interior): opposite end of the diameter LM
- R (lower-centre, interior): point on the construction
- O (bottom, interior): lower point of the meridian circle NLOM / axis NO
- H (lower left): common section point of meridian and horizon
- F (bottom, near C): endpoint of the seminocturnal arc / chord EF
- rim numerals 1–12 (twice around): the twenty-four hour divisions
- (points Q, and one further peripheral label near E: [?] — not securely resolved)

In the same way we shall find QBP, the angle of declination of the parallel whose diurnal arc contains 13 hours; likewise RBP, the angle of declination of the parallel whose diurnal arc comprises 15 hours, and so on. By similar reasoning we shall find the angle of declination of the southern parallel whose diurnal arc contains fewer hours than 12 — namely 9, or 8, and so on. For in the same figure, if from the point D one counts off on either side, say, the semidiurnal arc DE, DF of 5 hours, and does the rest as before, then HI will be the common section of the horizon and the meridian; the arc HO will number the pole's altitude above the horizon; BD will be the southern parallel; and the angle of declination will be PDK (or DKM), and so with the rest.

The same southern parallel, whose diurnal arc is given — namely of 10 hours — will be found in this way. Count off from the point B, on either side, the semidiurnal arc — that is, 5 hours in the given example. Then through the extremities of these semidiurnal arcs draw a straight line cutting the straight line

BD at a point; and through that point, if a line be drawn parallel to HI — making, of course, with BD an angle equal to the angle BGH, the complement of the pole's altitude — it will cut the straight line AP at a point; and a straight line drawn from B to that point will make with the straight line BD the sought angle of declination, which will be exactly equal to the angle PDK (or PBK). And so with the rest. An example of this you will find in the second figure of the following scholium.

It is nevertheless better to find the declinations of the northern parallels, for their declinations are equal to the declinations of the corresponding southern parallels. Thus the declination of a northern parallel whose diurnal arc contains 14 hours equals the declination of the southern parallel whose diurnal arc comprises 10 hours — namely the one that together with it makes up 24 hours. For the nocturnal arc EDF of the northern parallel — whose diameter is BD and whose declination is DM — is equal, by proposition 19 of book 2 of Theodosius, to the diurnal arc of the southern parallel that is equal and opposite to the former; that is, the one that has an equal declination. Hence, once the declination of the northern parallel is found, one also has the declination of the opposite southern parallel, equal to it, whose diurnal arc together with the diurnal arc of the former completes 24 hours.

[Margin: The declination of a parallel whose diurnal arc is known — by what technique it may be sought out by means of sines.]

But since the method just described requires an accurate division of the circle ABCD into hours, half-hours, and quarters, we shall be able to obtain the same result from the sines, without any division of the circle, by this device. Let ABCD be the analemma, with center E; BD the horizon; AC the equator; HI the parallel, whether northern or southern; FG the axis of the world, cutting the parallel at K. From K, about HI, describe the semicircle HMI of the parallel, which the axis, produced, meets at M. From L also draw the perpendicular LN to HI, which will be the common section of the parallel

HMI and of the horizon. [Margin: Euclid XI.19.] (For since both the horizon and the parallel are perpendicular to the meridian, their common section will likewise be perpendicular to the same meridian at the point L, where they mutually cut one another, and consequently perpendicular also to the straight line HI, by definition 3 of book 11 of Euclid. Therefore LN, perpendicular to HI, will be the common section of the horizon and the parallel.) And thus HN will be the semidiurnal arc of the said parallel, which is assumed known. Finally, join the straight line EH.

Now in the triangle EKL, if the straight line EL be taken as the whole sine, the straight line KL is the sine of the angle KEL (of the pole's altitude), and EK is the sine of the angle ELk (of the complement of the pole's altitude). If, then, we set it up that, as KL (the sine of the pole's altitude) is to EK (the sine of the complement of the pole's altitude), so is KL — insofar as it is the right sine of the arc MN, by which the semidiurnal arc HN (known) and the quadrant HM differ from one another — to a fourth term: the straight line Ek will become known, in the parts of the whole sine KH, with respect to which KL is the right sine of the arc MN.

Since, then, the two squares of the straight lines EK and kH are equal to the square of the straight line EH [Margin: Euclid I.47.], if there be added together the square of the whole sine KH and the square of the straight line EK — now made known in the parts of the whole sine KH — the square of the straight line EH will be known, and its square root will give the straight line EH, known in the same parts of the whole sine kH. And so, if we set it up that, as EH (insofar as it is known in the parts of the whole sine KH) is to EK (insofar as it is known in the same parts), so is EH (insofar as it is the whole sine) to a fourth term: EK will become known in the parts of the whole sine EH. Hence also AH, the arc corresponding to the sine Ek, will be known from the table of sines — namely the declination of the parallel whose diurnal arc, and consequently whose semidiurnal arc HN, is assumed known.

Diagram. The analemma-wheel construction for the sine method (triangle EKL, semicircle HMI, etc.): a circle crossed by many diameters, with center E, carrying the parallel HI, the axis FG, and the auxiliary points K, L, M, N. The figure is heavily ruled and densely labelled, with several letters repeated around the rim; it is flagged as a Readable labels:

- A (upper left, outside rim): apex marker
- H (top): upper endpoint of the parallel/arc
- M (upper right): point where the produced axis meets the semicircle
- N (right): point of the construction
- F (mid-right): endpoint of the axis FG
- D (right): rim point
- E (center): center of the circle / vertex of triangle EKL
- K (upper-centre and lower-centre, interior): the axis-parallel section point(s)
- L (centre, below E): foot of the perpendicular LN
- B (left): rim point

- I (mid-left, interior; also lower right): endpoint(s) of the parallel HI
- G (bottom / lower-left): endpoint of the axis FG
- C (lower right): rim point
- (further repeated rim labels H, I, M, N at the lower half: [?] — not all separately resolved)

[Margin: The declination of a parallel whose diurnal arc is given — by what other way it is to be investigated (per [?]).]

Example. Let the diurnal arc be given as 14 hours (or 10), and thus the semidiurnal arc HN as 7 hours (or 5) — that is, 105° (or 75°). From this the arc MN will be one hour, namely 15° , since the quadrant HM contains 6 hours, that is 90° . If, then, we set it up that, as kL (the sine of the pole's altitude) 66913 is to EK (the sine of the complement of the pole's altitude) 74314, so is KL (the sine of the arc MN, by which the semidiurnal arc differs from a quadrant — namely, in the given example, the sine of the arc of 15° , which is 25881) to a fourth term: EK will be found of about 28744 parts, of which KL is 25881. But since KH, the whole sine, is 100000 of the same parts: if the squared numbers of the sines EK and KH are added together, the square of the straight line EH will be about 10826217536 parts, and thus the straight line EH will be about 104049 parts with respect to the whole sine KH. Therefore, if we set it up that, as EH 104049 is to EK 28744, so is EH the whole sine 100000 to a fourth term: EK will be found — the sine of the declination of the parallel HI, with respect to the whole sine EH — of about 27625 parts, whose arc is $16^\circ 2'$. Such is the declination of the parallel HI: northern, indeed, if the day is greater than 12 hours, but southern if less. The same reasoning holds in the rest.

[Margin: The declination of a parallel whose diurnal arc is given — by what other method it is to be investigated.]

We shall obtain the same thing more easily by spherical triangles, in this way. Let ABCD be the horizon; AC the meridian; BD the equator, cutting the meridian at E; FG the parallel, whether northern or southern, cutting the meridian at K, so that FK is the given semidiurnal arc. Through the points H and F — of which H is the arctic pole — draw, by proposition 20 of book 1 of Theodosius, the great circle HF, cutting the equator at I. The arc IE of the equator will be similar to the given semidiurnal arc FK, by proposition 10 of book 2 of Theodosius, and therefore known — of just as many degrees as the arc FK comprises. And since IE is the arc of the angle IHE — because the arcs HE and HI are quadrants, by the corollary to proposition 16 of book 1 of Theodosius — the angle IHE will be of as many

degrees as the given semidiurnal arc contains. Therefore also the remaining angle IHC, out of two right angles (which contain 180°), will be known.

Now since in the spherical triangle CFH the angle C is right (for the meridian AC, drawn through the pole of the horizon, is perpendicular to the horizon, by proposition 15 of book 1 of Theodosius), and none of the arcs is a quadrant (because CH, the arc of the pole's altitude, is always less than a quadrant; and CB, the arc of the horizon between the meridian and the equator, is a quadrant; and consequently, if the parallel is northern, the arc CF is less than a quadrant, or if the parallel is southern, greater than a quadrant; likewise the arc HI, between the pole and the equator, is a quadrant, and therefore HF is less than a quadrant in a northern parallel, but greater than a quadrant in a southern one): by proposition 18 of book 1 of Johannes Regiomontanus, *On Triangles*, or by proposition 14 of book 1 of Geber, or by proposition 42 of our *[On] Spherical Triangles*, as the sine of the known angle CHF is to the whole sine, so is the sine of the complement of the angle CFH to the sine of the complement of the arc CH. And by conversion: as the whole sine is to the sine of the known angle CHF, so is the sine of the complement of the arc CH (of the pole's altitude) to the sine of the complement of the angle CFH. Since, then, the first three are known, the fourth will also be known — namely the sine of the complement of the angle CFH — by the rule of proportions; and thus also the complement of the angle, from the table of sines, and the angle CFH itself, and its opposite

Diagram. The spherical-triangle construction (CFH) for the "easier" method: a sphere with the meridian AC vertical, the equator BD, the parallel FG, and the great circle HF, meeting at the points that form the triangle CFH. Readable labels:

- A (top): upper pole of the meridian AC
- K (upper, on meridian): intersection of the parallel with the meridian
- E (center): intersection of the equator BD with the meridian AC
- K (lower, on meridian): second parallel–meridian intersection
- H (bottom-center): the arctic pole
- C (bottom): lower pole of the meridian AC
- F (upper left) and F (lower left): endpoints of the parallel FG
- B (left, interior): left end of the equator BD
- I (left, interior): intersection of the great circle HF with the equator
- G (upper right) and G (lower right): endpoints of the parallel FG
- D (right): right end of the equator BD

and thus also the complement of the angle, from the table of sines, and the angle CFH itself, together with its vertically opposite angle — namely IFB, if the parallel is northern — will be known.

Again, in the spherical triangle BFI the angle BFI is known, and the angle FBI is equal to the altitude of the Equator above the horizon — that is, to the complement of the pole's altitude. Now by proposition 16 of book 4 of Johannes Regiomontanus [Regiomontanus], *On Triangles*, or by proposition 13 of book 1 of Geber [Jābir ibn Aflāḥ], or certainly by proposition 41 of our *[On] Spherical Triangles*, we have: as the sine of the known arc IB (for IB is the difference between the given semidiurnal arc IE and the quadrant BE) is to the sine of the known angle BFI, so the sine of the arc IF is to the sine of the angle IBF, the altitude of the Equator. And by conversion: as the sine of the known angle BFI is to the sine of the known arc IB, so the sine of the known angle IBF is to the sine of the arc IF. Since the first three are known, the fourth will also be known, by the rule of proportions — namely the sine of the arc IF, the declination of the proposed parallel; and therefore the arc of declination IF itself will become known.

And so, if we arrange that as the whole sine is to the sine of the arc by which the given semidiurnal arc differs from a semicircle — that is, to the sine of the angle CHF (which together with EHI, the angle of the semidiurnal arc, makes up two right angles) — so the sine of the complement of the arc CH, the altitude of the pole, is to a fourth term: then the sine of the complement of the angle CFH, or IFB, will be found; and hence the complement of that angle will be known, and from it the angle IFB itself. And if again we arrange that as the whole sine is to the sine of the arc IB, by which the semidiurnal arc differs from a quadrant, so the sine of the altitude of the Equator — or of the complement of the pole's altitude, namely the angle IBF — is to a fourth term: then the sine of the arc IF, the sought declination of the parallel FG, will be produced; and consequently, from the table of sines, the declination itself will also become known.

Example. [Margin: *Exemplum.*] Let the diurnal arc again be given as one of 14 hours, or of 10; and hence the semidiurnal arc IE as one of 7 hours, or of 5 — that is, 105° or 75° . From this the arc IB will comprise one hour, that is 15° . If, then, we arrange that as 100000, the whole sine, is to 96592, the sine of the angle CHF — for since the arc IE will comprise 7 hours or 105° (we are speaking here of a northern parallel), the remaining arc of the semicircle will comprise five hours, that is 75° , to which the aforesaid sine 96592 belongs; whereas if a southern parallel is in question, the arc IE will be of 5 hours or 75° , and the remainder of

the semicircle of 7 hours, that is 105° , to which the same sine corresponds — so is 74314, the sine of the complement of the arc CH (the altitude of the pole), to a fourth term: then the sine of the complement of the angle CFH, or IFB, will be found to be 71781, to which corresponds an arc of $45^\circ 52'$. Its complement, $44^\circ 8'$, will be the magnitude of the angle IFB. If then we again arrange that as 69633, the sine of the angle IFB, is to 25881, the sine of the arc IB of 15° , so 74314, the sine of the angle IBF (or of the altitude of the Equator), is to a fourth term: the sine of the arc of declination IF will be found to be about 27621. Hence the arc of declination IF itself will comprise about $16^\circ 2'$, just as before. In the same way we shall find that the parallel whose diurnal arc comprises 15 hours, or 9, has a declination of about $23^\circ 2'$; and so for the rest. And thus, given the diurnal arc and the latitude of the place, we have found the declination of that arc's parallel from the Equator. Which was to be done.

A shorter way. Arrange that as the tangent of the pole's altitude is to the whole sine, so the sine of the difference between the given semidiurnal arc and a quadrant is to a fourth term: the tangent of the declination will be found — as was shown in the *Astrolabe*, book 1, lemma 49, no. 17. Or rather, arrange that as the whole sine is to the sine of the aforesaid difference, so the tangent of the complement of the pole's altitude is to a fourth term: and the tangent of the declination will be found — as is clear, by the first method, from problem 11 of the *Spherical Triangles* and the last lemma of book 1 of the *Astrolabe*, and from the triangle BIF.

SCHOLIVM.

Andreas Schoner [Andreas Schöner], in the work he entitled *Gnomonice*, investigates the declinations of given diurnal arcs in this way. [Margin: The declinations of all diurnal arcs, and by what method Andreas Schöner investigates them.] From center A, with any radius, describe a circle — or at least a portion of it — of any straight line AB; and take two arcs BC, BD equal to the complement of the pole's altitude, so that if AB is taken as the common section of the Equator and of the meridian CBD, the arcs BC, BD are the declinations of two parallels. Of these, one is the greatest of the parallels that always appear (having a diurnal arc of 24 hours, since it lies wholly above the horizon), and the other is the greatest of those that are always hidden (having a diurnal arc of 0 hours, since it lies wholly below the horizon). Now draw the straight line CD, cutting AB itself at E: the lines EC, ED will be equal, and the angles at E right — which we shall show by the same demonstration we used in proposition 1 of this book to prove that the line MN in the analemma is bisected, and that the angles at O are

right. Then, from center E, with radius EC or ED, describe a circle, and divide it into 48 equal parts. Join, by straight lines, each pair of points equally distant from the point C or D; and through the points where those lines cut the line CD, draw straight lines from A out to the circumference CBD. For these will cut off the arcs of declination of all the diurnal arcs, beginning from the 24-hour arc down to the 0-hour arc, as the numbers marked in the figure indicate. Andreas Schöner offers no demonstration of this procedure; nevertheless I have confirmed by many trials that the declination-angles found by this method are equal to the declination-angles obtained from our own demonstration.

Diagram. Andreas Schöner's construction for the declinations of the diurnal arcs. A full circle represents the meridian CBD; from an apex A at the top a fan of straight lines (the "radii") descends to a lower arc. Readable labels:

- A (top center): apex from which the radii are drawn
- B (bottom center): lower point of the meridian; start of the graduation
- C (left): left point of the meridian circle
- D (right): right point of the meridian circle
- E (center of the lower region): center of the small circle divided into 48 equal parts; intersection of CD with AB
- The lower arc (running from C through B to D) is graduated with the hour-values of the diurnal arcs, from 24 at one end down to 0 at the other. Legibly read numerals along the arc include 24, 19, 18, 17, 15, 14, 13, 12, 11, 10, 8, 7, 6, 5 [several of the intermediate small numerals are at the resolution floor — [?]].

In what follows we shall call the lines in this figure of Andreas Schöner's — drawn out from A, or falling from B in our own following figure — the *radii of the diurnal arcs*; for when the Sun is on the parallels whose declinations are indicated by the said lines, they represent the rays which the Sun casts through the center of the world, just as we said of the *rays of the signs* in proposition 1, rays which indeed display the declinations of those same signs. And the radius of the 12-hour diurnal arc is the same as the radius of the Equator, as is evident.

For the rest, it will generally be enough to investigate the declinations of those diurnal arcs contained between the Equator and the tropic of ♋ [Cancer] — as at Rome, the arcs of 13, 14, and 15 hours. For these declinations are equal to the declinations of the diurnal arcs placed between the Equator and the tropic of ♏ [Capricorn], namely those of 11, 10, and 9 hours, and so on. Of the remaining

diurnal arcs, placed beyond the tropics, there is no use in sundials — except for a few, which are required for laying out the lines of the hours

from sunrise or sunset, and of the unequal hours; such above all are the radii of the diurnal arcs that comprise 24, 0, 18, and 6 hours, as we shall point out in its place.

[Margin: How the declinations of all diurnal arcs may be found from our own demonstration.] But should Andreas Schöner's procedure perhaps seem suspect to anyone — on the ground that, brief and easy though it is, it is established by no geometrical reasoning (that practice will, however, be demonstrated by us in book 8, chapter 6) — we shall be able, from our own demonstration, and with nearly the same brevity and ease, to construct a figure like the one he described, one that contains the declinations of all the diurnal arcs, in this way. From center P describe a circle ABCD of any size. First draw in it two diameters AC and BD, cutting each other at right angles at the center P, and divide it into 48 equal parts, beginning from the point B. Then join, by straight lines, each pair of points equally distant from the point B; these lines will cut the diameter BD at certain points. Through these points draw straight lines making with BD angles equal to the complement of the pole's altitude; these will cut the diameter AC — which must be extended when the complement of the pole's altitude is greater than 45° — at certain points. If from B straight lines are drawn through these points, they will cut off, from an arc described about center B, the arcs of declination of all the diurnal arcs, as in Andreas Schöner's figure, so that the angles which these same lines make with BD at B are the angles of declination, just as was demonstrated in the first figure of this proposition.

Now we shall draw those lines through the points of the line BD — the ones making with it angles equal to the complement of the pole's altitude — without great labor, as follows. From D, to the left of the line BD, describe an arc of a circle, on which the complement of the pole's altitude is to be reckoned off from the line BD; and through the endpoint of that reckoning draw the line DE.

Through any point E of it draw EO parallel to BD, and onto EO transfer all the points of the line DB, beginning from the line DE. For if the corresponding points on each of the two lines DB and EO — namely those equally distant from the points D and E — are joined by hidden [construction] lines, [Margin: Euclid, *Elements* I.33] all these will be parallel to DE, and hence [Margin: *Elements* I.29] will make with BD angles equal to the angle BDE, that is, to the complement of the pole's altitude. The numbers in this figure of ours are the same as in Andreas Schöner's figure.

It is enough, moreover, if the line EO is taken equal to DP. For the points found on the line PC, transferred onto the line PA (always beginning from the point P), will likewise give the points on the line PA through which the declination-lines are to be drawn — because the said parallels cut off from PA segments equal to the corresponding segments of the line PC: namely the line PI equal to the line PK, and so on. For in the triangles ILP and KGP [Margin: *Elements* I.29] the alternate angles L and G, between the parallels IL and GK, are equal, and the angles at P are right; and the sides PL, PG are likewise equal. (For the line ME, when drawn, will be bisected at N, on account of the equal arcs AM and AE — which can be proved by the same demonstration we used in proposition 1 of this book to show that the line MN in the analemma is bisected at O. [Margin: *Elements* I.34] Since, then, in the parallelograms PM and PE the sides PL, PG are equal to the opposite sides NM, NE,) [Margin: *Elements* I.26] the sides PI and PK will also be equal to each other.

Diagram. Our own construction giving the declinations of all the diurnal arcs. A large circle ABCD about center P, with the vertical diameter BD (B at top, D at bottom) and the horizontal diameter AC crossing it at P. From the convergence points a fan of straight lines (the radii of the diurnal arcs) is drawn, and a dotted arc (the transferred parallels) sweeps across the figure; at the far left a separate vertical graduated line is headed O. Readable labels (approximate positions; the figure is dense and a few placements are uncertain):

- B (top center): top of the vertical diameter BD
- D (bottom center): bottom of the vertical diameter BD
- P (center): intersection of the diameters AC, BD
- A (center-right): where the radii converge on the right
- C (center-left): left convergence point
- E (right, below A): endpoint of the reckoned complement / of line DE
- O (far upper left): head of the vertical line EO, parallel to BD
- L (upper middle): construction point on the axis
- I (center): construction point (triangle ILP)
- G (lower middle): construction point on the axis (triangle KGP)
- M (upper right): construction point
- N (right, below M): midpoint of ME
- K (right, on the fan): construction point (line PK)
- F (left, below C): construction point
- H (far right edge): construction point [position [?]]

Now, just as in propositions 19 and 20 of this book we drew up tables of the **Equinoctial Line** and of the **Twenty-Four Lines from Sunrise or Sunset**, and so forth — so that we might understand which hours reckoned from sunrise or sunset, and which reckoned from noon or midnight, intersect one another at one and the same point of those lines — so here we shall set out eight other tables, of the diurnal and nocturnal arcs, so that we may know which hours from sunrise or sunset, and which from noon or midnight, intersect one another at one and the same point of those arcs as drawn on any sundial. [Margin: Tables of the diurnal and nocturnal arcs, from which one learns which hour...] For this matter will be of the greatest use for laying out both the hours from sunrise and sunset and the unequal hours, as will become clear in its place. The construction of these tables from [the arcs]

[the arcs], diurnal and nocturnal, is derived, as can be gathered from what we shall say shortly. For in these tables any hours — whether three at a time or two at a time — cut one another at one and the same point of that diurnal or nocturnal arc which stands at the head of the table in which they are drawn. Thus, at whatever points the astronomical hours — reckoned from noon or midnight — cut the diurnal or nocturnal arc of any table, at those very same points the hours reckoned from sunrise or sunset cut it too: namely those hours which are placed to the right or left of those astronomical hours reckoned from noon or midnight. [Margin: ...from noon or midnight, and from sunrise or sunset, intersect one another in the proposed diurnal or nocturnal arc. By what device the tables of the diurnal and nocturnal arcs are composed.] Anyone will easily understand this if he carefully considers at what hour — reckoned both from noon or midnight and from sunrise or sunset — the Sun rises on the proposed diurnal arc, or sets on the nocturnal one.

For, to explain briefly by what device such tables are composed from the diurnal and nocturnal arcs: when the day comprises 10 hours, the Sun rises at the 7th hour from midnight, so that 5 hours remain until the 12th hour of noon, and another five until sunset; likewise it is the 14th hour from sunset, so that ten hours remain until the 24th hour, at which the Sun sets; and finally it is the 24th hour from sunrise, since one natural day begun at sunrise is then completed and another begins. It is plainly established that these three hours meet at the same point of the 10-hour diurnal arc — namely the 8th hour from midnight, the 15th from sunset, and the 1st from sunrise; and so for the rest.

Likewise, since the night comprises 14 hours, the Sun sets at the 5th hour from noon, so that seven hours remain until the 12th hour of midnight, and another

seven until sunrise; likewise it is the 24th hour from sunset, since one natural day begun at sunset is then completed and another begins; and finally it is the 10th hour from sunrise, so that 14 hours remain until the 24th hour, at which the Sun rises. These three hours too will coincide at the same point of the 14-hour nocturnal arc — as also will the three that follow them in the table of the 14-hour nocturnal arc, namely the 6th hour from noon, the 1st from sunset, and the 11th from sunrise, and so on.

By this method tables can be made for each of the diurnal and nocturnal arcs, provided one first carefully considers the hour — both from noon or midnight and from sunrise or sunset — at which the Sun rises or sets; which is not difficult. But the tables here proposed suffice for laying out the hours from sunrise or sunset, as will be clear from what follows.

(This page carries no prose: it presents four tables of the diurnal and nocturnal arcs of 10 and 14 hours, laying out the hours reckoned from sunset, from midnight or noon, and from sunrise. In each middle column the count shifts partway down from midnight-hours to noon-hours, or the reverse, marked by the interior sub-label.)

Diurnal arc of 10 hours, in which the Sun rises at the 7th hour from midnight, the 14th hour from sunset, and the 24th hour from sunrise.

Hours from sunset [ab occasu]	Hours from midnight [à media nocte]	Hours from sunrise [ab ortu]
14	VII	24
15	VIII	1
16	IX	2
17	X	3
18	XI	4
19	XII	5
	<i>Hours from noon [à meridie]</i>	
20	I	6
21	II	7
22	III	8
23	IIII	9
24	V	10

Diurnal arc of 14 hours, in which the Sun rises at the 5th hour from midnight, the 10th hour from sunset, and the 24th hour from sunrise.

Hours from sunset	Hours from midnight	Hours from sunrise
10	V	24
11	VI	1
12	VII	2
13	VIII	3
14	IX	4
15	X	5
16	XI	6
17	XII	7
<i>Hours from noon</i>		
18	I	8
19	II	9
20	III	10
21	IIII	11
22	V	12
23	VI	13
24	VII	14

Nocturnal arc of 14 hours, in which the Sun sets at the 5th hour from noon, the 24th hour from sunset, and the 10th hour from sunrise.

Hours from sunset	Hours from noon [à meridie]	Hours from sunrise
24	V	10
1	VI	11
2	VII	12
3	VIII	13
4	IX	14
5	X	15
6	XI	16
7	XII	17
<i>Hours from midnight [à media]</i>		

Hours from sunset	Hours from noon [à meridie]	Hours from sunrise
<i>nocte]</i>		
8	I	18
9	II	19
10	III	20
11	IIII	21
12	V	22
13	VI	23
14	VII	24

Nocturnal arc of 10 hours, in which the Sun sets at the 7th hour from noon, the 24th hour from sunset, and the 14th hour from sunrise.

Hours from sunset	Hours from noon	Hours from sunrise
24	VII	14
1	VIII	15
2	IX	16
3	X	17
4	XI	18
5	XII	19
<i>Hours from midnight</i>		
6	I	20
7	II	21
8	III	22
9	IIII	23
10	V	24

Diurnal arc of 12 hours, in which the Sun rises at the 6th hour from midnight, the 12th hour from sunset, and the 24th hour from sunrise.

Hours from sunset	Hours from midnight	Hours from sunrise
12	VI	24
13	VII	1
14	VIII	2
15	IX	3

Hours from sunset	Hours from midnight	Hours from sunrise
16	X	4
17	XI	5
18	XII	6
<i>Hours from noon</i>		
19	I	7
20	II	8
21	III	9
22	IIII	10
23	V	11
24	VI	12

Nocturnal arc of 12 hours, in which the Sun sets at the 6th hour from noon, the 24th hour from sunset, and the 12th hour from sunrise.

Hours from sunset	Hours from noon	Hours from sunrise
24	VI	12
1	VII	13
2	VIII	14
3	IX	15
4	X	16
5	XI	17
6	XII	18
<i>Hours from midnight</i>		
7	I	19
8	II	20
9	III	21
10	IIII	22
11	V	23
12	VI	24

Diurnal arc of 24 hours, in which the Sun rises at the 12th hour from noon, and at the 24th hour from sunrise or sunset.

Hours from sunrise or sunset [ab ortu
vel occasu]

Hours from noon [à meridie]

24	XII
	<i>Hours from midnight</i>
1	I
2	II
3	III
4	IIII
5	V
6	VI
7	VII
8	VIII
9	IX
10	X
11	XI
12	XII
	<i>Hours from noon</i>
13	I
14	II
15	III
16	IIII
17	V
18	VI
19	VII
20	VIII
21	IX
22	X
23	XI

Nocturnal arc of 24 hours, in which the Sun sets at the 12th hour from midnight, and at the 24th hour from sunrise or sunset.

Hours from sunrise or sunset	Hours from midnight [à media nocte]
24	XII
	<i>Hours from noon</i>
1	I
2	II
3	III
4	IIII
5	V
6	VI
7	VII
8	VIII
9	IX
10	X
11	XI
12	XII
	<i>Hours from midnight</i>
13	I
14	II
15	III
16	IIII
17	V
18	VI
19	VII
20	VIII
21	IX
22	X
23	XI

Now, whatever hours — reckoned from noon or from midnight, and from sunrise or sunset — cut one another at a single point of any diurnal or nocturnal arc, at that same point the corresponding fractions of those same hours will likewise cut

one another: their quarters, halves, and the rest. For example, since the 17th hour from sunset, the 10th from midnight, and the 3rd from sunrise pass through one and the same point on the 10-hour diurnal arc, it follows that the fractions of these which correspond to one another — namely the $17\frac{1}{2}$ th hour from sunset, the $10\frac{1}{2}$ th from midnight, and the $3\frac{1}{2}$ th from sunrise — also pass through one and the same point of that same arc; and so with the rest, as is plain if the tables are extended through the said fractions of the hours.

[Margin: What has been said in the tables of the diurnal and nocturnal arcs concerning whole hours is likewise to be understood of the fractions of the hours.]

By the same effort we have constructed six further tables, as their headings indicate, so that one may learn at what hour reckoned from noon or midnight — and which unequal hours — cut one another at one and the same point of a diurnal or nocturnal arc of 6 or of 18 hours; and likewise of the diurnal or nocturnal arc of 12 hours, which the equinoctial line displays to us on any sundial. In the first four of these tables the unequal hours are contained as whole hours together with their halves and quarters; but in the last two, only as whole hours — yet their fractions too are to be understood, just as was said above for the eight tables of equal hours. For just as, for example, on the 12-hour diurnal arc the 3rd hour from noon and the 9th unequal diurnal hour cut one another, so too the $3\frac{3}{4}$ th hour from noon and the $9\frac{3}{4}$ th unequal hour will cut that same arc at one and the same point; and the same is to be said of the rest.

[Margin: Table of the diurnal and nocturnal arcs of 6, 18, and 12 hours, from which is gathered at what hour from noon or midnight — and which unequal hours — cut one another in these diurnal arcs.]

(Four tables follow, side by side. In each, the left column gives the unequal (seasonal) hours with their quarter-subdivisions, the right column the corresponding equal hours reckoned from noon or midnight. Whole hours are printed in Roman numerals, the quarter-subdivisions in Arabic with fractions.)

Diurnal arc of 6 hours, in which the Sun rises at the 9th hour after midnight and at the 12th unequal morning hour; and the individual half-hours from noon or midnight correspond to the individual unequal diurnal hours.

Unequal diurnal hours [horæ inæquales diurnæ]	Hours from midnight [à media nocte]
XII	IX
$\frac{1}{4}$	$9\frac{1}{8}$
$\frac{1}{2}$	$9\frac{1}{4}$
$\frac{3}{4}$	$9\frac{3}{8}$
I	$9\frac{1}{2}$
$1\frac{1}{4}$	$9\frac{5}{8}$
$1\frac{1}{2}$	$9\frac{3}{4}$
$1\frac{3}{4}$	$9\frac{7}{8}$
II	X
$2\frac{1}{4}$	$10\frac{1}{8}$
$2\frac{1}{2}$	$10\frac{1}{4}$
$2\frac{3}{4}$	$10\frac{3}{8}$
III	$10\frac{1}{2}$
$3\frac{1}{4}$	$10\frac{5}{8}$
$3\frac{1}{2}$	$10\frac{3}{4}$
$3\frac{3}{4}$	$10\frac{7}{8}$
IIII	XI
$4\frac{1}{4}$	$11\frac{1}{8}$
$4\frac{1}{2}$	$11\frac{1}{4}$
$4\frac{3}{4}$	$11\frac{3}{8}$
V	$11\frac{1}{2}$
$5\frac{1}{4}$	$11\frac{5}{8}$
$5\frac{1}{2}$	$11\frac{3}{4}$
$5\frac{3}{4}$	$11\frac{7}{8}$
VI	XII

Nocturnal arc of 18 hours, in which the Sun sets at the 3rd hour from noon and at the 12th unequal evening hour; and the individual spaces of $1\frac{1}{2}$ hours from noon or midnight correspond to the individual unequal nocturnal hours.

Unequal nocturnal hours [horæ inæquales nocturnæ]	Hours from noon [à meridie]
XII	III
$\frac{1}{4}$	$3\frac{3}{8}$
$\frac{1}{2}$	$3\frac{3}{4}$
$\frac{3}{4}$	$4\frac{1}{8}$
I	$4\frac{1}{2}$
$1\frac{1}{4}$	$4\frac{7}{8}$
$1\frac{1}{2}$	$5\frac{1}{4}$
$1\frac{3}{4}$	$5\frac{5}{8}$
II	VI
$2\frac{1}{4}$	$6\frac{3}{8}$
$2\frac{1}{2}$	$6\frac{3}{4}$
$2\frac{3}{4}$	$7\frac{1}{8}$
III	$7\frac{1}{2}$
$3\frac{1}{4}$	$7\frac{7}{8}$
$3\frac{1}{2}$	$8\frac{1}{4}$
$3\frac{3}{4}$	$8\frac{5}{8}$
IIII	IX
$4\frac{1}{4}$	$9\frac{3}{8}$
$4\frac{1}{2}$	$9\frac{3}{4}$
$4\frac{3}{4}$	$10\frac{1}{8}$
V	$10\frac{1}{2}$
$5\frac{1}{4}$	$10\frac{7}{8}$
$5\frac{1}{2}$	$11\frac{1}{4}$
$5\frac{3}{4}$	$11\frac{5}{8}$
VI	XII

Diurnal arc of 18 hours, in which the Sun rises at the 3rd hour from midnight and at the 12th unequal morning hour; and the individual spaces of $1\frac{1}{2}$ hours from noon or midnight correspond to the individual unequal diurnal hours.

Unequal diurnal hours	Hours from midnight
XII	III
$\frac{1}{4}$	$3\frac{3}{8}$
$\frac{1}{2}$	$3\frac{3}{4}$
$\frac{3}{4}$	$4\frac{1}{8}$
I	$4\frac{1}{2}$
$1\frac{1}{4}$	$4\frac{7}{8}$
$1\frac{1}{2}$	$5\frac{1}{4}$
$1\frac{3}{4}$	$5\frac{5}{8}$
II	VI
$2\frac{1}{4}$	$6\frac{3}{8}$
$2\frac{1}{2}$	$6\frac{3}{4}$
$2\frac{3}{4}$	$7\frac{1}{8}$
III	$7\frac{1}{2}$
$3\frac{1}{4}$	$7\frac{7}{8}$
$3\frac{1}{2}$	$8\frac{1}{4}$
$3\frac{3}{4}$	$8\frac{5}{8}$
IIII	IX
$4\frac{1}{4}$	$9\frac{3}{8}$
$4\frac{1}{2}$	$9\frac{3}{4}$
$4\frac{3}{4}$	$10\frac{1}{8}$
V	$10\frac{1}{2}$
$5\frac{1}{4}$	$10\frac{7}{8}$
$5\frac{1}{2}$	$11\frac{1}{4}$
$5\frac{3}{4}$	$11\frac{5}{8}$
VI	XII

Nocturnal arc of 6 hours, in which the Sun sets at the 9th hour after noon and at the 12th unequal evening hour; and the individual half-hours from noon or midnight correspond to the individual unequal nocturnal hours.

Unequal nocturnal hours	Hours from noon
XII	IX
$\frac{1}{4}$	$9\frac{1}{8}$
$\frac{1}{2}$	$9\frac{1}{4}$
$\frac{3}{4}$	$9\frac{3}{8}$
I	$9\frac{1}{2}$
$1\frac{1}{4}$	$9\frac{5}{8}$
$1\frac{1}{2}$	$9\frac{3}{4}$
$1\frac{3}{4}$	$9\frac{7}{8}$
II	X
$2\frac{1}{4}$	$10\frac{1}{8}$
$2\frac{1}{2}$	$10\frac{1}{4}$
$2\frac{3}{4}$	$10\frac{3}{8}$
III	$10\frac{1}{2}$
$3\frac{1}{4}$	$10\frac{5}{8}$
$3\frac{1}{2}$	$10\frac{3}{4}$
$3\frac{3}{4}$	$10\frac{7}{8}$
IIII	XI
$4\frac{1}{4}$	$11\frac{1}{8}$
$4\frac{1}{2}$	$11\frac{1}{4}$
$4\frac{3}{4}$	$11\frac{3}{8}$
V	$11\frac{1}{2}$
$5\frac{1}{4}$	$11\frac{5}{8}$
$5\frac{1}{2}$	$11\frac{3}{4}$
$5\frac{3}{4}$	$11\frac{7}{8}$
VI	XII

The following four tables complete ("Reliquum," the remainder) the four unequal-hour arcs whose first portions stood on the preceding page. Each table pairs the

unequal hours (left) with the corresponding equal hours reckoned from noon or from midnight (right). The two 6-hour arcs (first and last) step the equal-hour column by $\frac{1}{8}$ of an hour (each unequal hour = $\frac{1}{2}$ an equal hour); the two 18-hour arcs (middle two) step it by $\frac{3}{8}$ (each unequal hour = $1\frac{1}{2}$ equal hours).

Remainder of the diurnal arc of 6 hours, &c.

Unequal diurnal hours	Hours from noon
$6\frac{1}{4}$	$\frac{1}{8}$
$6\frac{1}{2}$	$\frac{1}{4}$
$6\frac{3}{4}$	$\frac{3}{8}$
VII	$\frac{1}{2}$
$7\frac{1}{4}$	$\frac{5}{8}$
$7\frac{1}{2}$	$\frac{3}{4}$
$7\frac{3}{4}$	$\frac{7}{8}$
VIII	I
$8\frac{1}{4}$	$1\frac{1}{8}$
$8\frac{1}{2}$	$1\frac{1}{4}$
$8\frac{3}{4}$	$1\frac{3}{8}$
IX	$1\frac{1}{2}$
$9\frac{1}{4}$	$1\frac{5}{8}$
$9\frac{1}{2}$	$1\frac{3}{4}$
$9\frac{3}{4}$	$1\frac{7}{8}$
X	II
$10\frac{1}{4}$	$2\frac{1}{8}$
$10\frac{1}{2}$	$2\frac{1}{4}$
$10\frac{3}{4}$	$2\frac{3}{8}$
XI	$2\frac{1}{2}$
$11\frac{1}{4}$	$2\frac{5}{8}$
$11\frac{1}{2}$	$2\frac{3}{4}$
$11\frac{3}{4}$	$2\frac{7}{8}$
XII	III

Remainder of the nocturnal arc of 18 hours, &c.

Unequal nocturnal hours	Hours from midnight
$6\frac{1}{4}$	$\frac{3}{8}$
$6\frac{1}{2}$	$\frac{3}{4}$
$6\frac{3}{4}$	$1\frac{1}{8}$
VII	$1\frac{1}{2}$
$7\frac{1}{4}$	$1\frac{7}{8}$
$7\frac{1}{2}$	$2\frac{1}{4}$
$7\frac{3}{4}$	$2\frac{5}{8}$
VIII	III
$8\frac{1}{4}$	$3\frac{3}{8}$
$8\frac{1}{2}$	$3\frac{3}{4}$
$8\frac{3}{4}$	$4\frac{1}{8}$
IX	$4\frac{1}{2}$
$9\frac{1}{4}$	$4\frac{7}{8}$
$9\frac{1}{2}$	$5\frac{1}{4}$
$9\frac{3}{4}$	$5\frac{5}{8}$
X	VI
$10\frac{1}{4}$	$6\frac{3}{8}$
$10\frac{1}{2}$	$6\frac{3}{4}$
$10\frac{3}{4}$	$7\frac{1}{8}$
XI	$7\frac{1}{2}$
$11\frac{1}{4}$	$7\frac{7}{8}$
$11\frac{1}{2}$	$8\frac{1}{4}$
$11\frac{3}{4}$	$8\frac{5}{8}$
XII	IX

Remainder of the diurnal arc of 18 hours, &c.

Unequal diurnal hours	Hours from noon
$6\frac{1}{4}$	$\frac{3}{8}$
$6\frac{1}{2}$	$\frac{3}{4}$
$6\frac{3}{4}$	$1\frac{1}{8}$

Unequal diurnal hours	Hours from noon
VII	$1\frac{1}{2}$
$7\frac{1}{4}$	$1\frac{7}{8}$
$7\frac{1}{2}$	$2\frac{1}{4}$
$7\frac{3}{4}$	$2\frac{5}{8}$
VIII	III
$8\frac{1}{4}$	$3\frac{3}{8}$
$8\frac{1}{2}$	$3\frac{3}{4}$
$8\frac{3}{4}$	$4\frac{1}{8}$
IX	$4\frac{1}{2}$
$9\frac{1}{4}$	$4\frac{7}{8}$
$9\frac{1}{2}$	$5\frac{1}{4}$
$9\frac{3}{4}$	$5\frac{5}{8}$
X	VI
$10\frac{1}{4}$	$6\frac{3}{8}$
$10\frac{1}{2}$	$6\frac{3}{4}$
$10\frac{3}{4}$	$7\frac{1}{8}$
XI	$7\frac{1}{2}$
$11\frac{1}{4}$	$7\frac{7}{8}$
$11\frac{1}{2}$	$8\frac{1}{4}$
$11\frac{3}{4}$	$8\frac{5}{8}$
XII	IX

Remainder of the nocturnal arc of 6 hours, &c.

Unequal nocturnal hours	Hours from midnight
$6\frac{1}{4}$	$\frac{1}{8}$
$6\frac{1}{2}$	$\frac{1}{4}$
$6\frac{3}{4}$	$\frac{3}{8}$
VII	$\frac{1}{2}$
$7\frac{1}{4}$	$\frac{5}{8}$
$7\frac{1}{2}$	$\frac{3}{4}$
$7\frac{3}{4}$	$\frac{7}{8}$

Unequal nocturnal hours	Hours from midnight
VIII	I
$8\frac{1}{4}$	$1\frac{1}{8}$
$8\frac{1}{2}$	$1\frac{1}{4}$
$8\frac{3}{4}$	$1\frac{3}{8}$
IX	$1\frac{1}{2}$
$9\frac{1}{4}$	$1\frac{5}{8}$
$9\frac{1}{2}$	$1\frac{3}{4}$
$9\frac{3}{4}$	$1\frac{7}{8}$
X	II
$10\frac{1}{4}$	$2\frac{1}{8}$
$10\frac{1}{2}$	$2\frac{1}{4}$
$10\frac{3}{4}$	$2\frac{3}{8}$
XI	$2\frac{1}{2}$
$11\frac{1}{4}$	$2\frac{5}{8}$
$11\frac{1}{2}$	$2\frac{3}{4}$
$11\frac{3}{4}$	$2\frac{7}{8}$
XII	III

Below the four tables stand two small paired tables under their own headings, showing how the unequal hours correspond to equal hours reckoned from noon or midnight for a 12-hour arc.

A diurnal arc of 12 hours, in which the Sun rises at the 6th hour from midnight and at the 12th unequal morning hour; and the individual hours from noon or midnight correspond to the individual unequal diurnal hours.

Unequal diurnal hours	Hours after midnight
XII	VI
I	VII
II	VIII
III	IX
IIII	X
V	XI
VI	XII

Unequal diurnal hours	Hours after noon
VII	I
VIII	II
IX	III
X	IIII
XI	V
XII	VI

A nocturnal arc of 12 hours, in which the Sun sets at the 6th hour after noon and at the 12th unequal evening hour; and the individual hours from noon or midnight correspond to the individual unequal nocturnal hours.

Unequal nocturnal hours	Hours after noon
XII	VI
I	VII
II	VIII
III	IX
IIII	X
V	XI
VI	XII

Unequal nocturnal hours	Hours after midnight
VII	I
VIII	II
IX	III
X	IIII
XI	V
XII	VI

PROBLEM 13. PROPOSITION 34.

To investigate the rising and setting amplitude, and the semidiurnal arc, of any parallel whatever, for a given elevation of the pole.

Since most people are accustomed to write, alongside the parallels or the arcs of the zodiacal signs on sundials (which we shall show how to draw on any sundial in the two books that follow), the lengths of the days and of the twilights, it will not be altogether out of place to explain briefly here — though it may seem to someone somewhat outside our plan, since this matter belongs elsewhere — by what method the lengths of the days and of the twilights, for any latitude of place, may be computed once the Sun's declination is known, so that we too, on any sundial we please, may add them to the parallels of the zodiacal signs. To find the lengths of the days, then, we shall look for the semidiurnal arcs, since these, doubled, make up the whole diurnal arcs. Among all the ways (for the length of a day can be found by many methods) we have chosen above all the one that seemed to differ least from the method we used in the preceding proposition for computing the declination of a parallel whose diurnal arc is given. Here, conversely, from the given declination of a parallel its diurnal arc is proposed for investigation. But first the rising or setting amplitude [amplitudo ortiva, occiduave] must be determined, for from it the semidiurnal arc is immediately gathered.

[Margin: The rising and setting amplitude — by what method it is investigated.]

Diagram. The figure is a sphere (the same as the last figure of the preceding proposition), drawn with the horizon as its outer circle and the great circles of meridian, equator, and a parallel of declination traced across it.

- A (top): pole of the horizon / top of the meridian on the horizon circle
- C (bottom): opposite point of the meridian on the horizon
- B (left): point where the equator meets the horizon
- D (right): opposite equator–horizon point
- E (centre): intersection of the meridian AC with the equator BD
- F (left, on the parallel): one end / meridian-crossing of the parallel
- G (right, on the parallel): the other end of the parallel FG
- H (lower centre): the arctic (north) pole
- I (centre, on the equator): point where the great circle HF, measuring the parallel's declination, cuts the equator
- K (upper centre): point where the parallel meets the meridian (the text also names the meridian-crossings k, v, t)

Let the last figure of the preceding proposition be taken up again, in which the horizon is ABCD, the meridian AC, and the equator BD cutting the meridian at E; let the parallel FG — whether northern or southern — cut the meridian at k, v, t;

and let the semidiurnal arc to be found be FK (or GK). For the meridian AC, passing through the poles of the horizon and of the parallel FG, cuts the segment FG in half, by [Theodosius, *Spherics* II.9]. Take the arctic pole H, and through it and through the point F draw, by [Theodosius, *Spherics* I.20], the greatest circle HF that measures the declination of the parallel from the equator, cutting the equator at I. Then the arc IE of the equator will, by [Theodosius, *Spherics* II.10], be similar to the semidiurnal arc FK; and so, once the arc IE has been found, the semidiurnal arc Fk that is sought will also be known, since arc IE contains just as many degrees or hours as Fk does, because of the similarity of these arcs.

We shall find the arc IE as follows. In the right-angled spherical triangle CFH — for the angle at C is right, since the meridian AC, drawn through the pole of the horizon, is perpendicular to the horizon by [Theodosius, *Spherics* I.1] — none of the arcs is a quadrant, as was shown in the preceding proposition. Therefore, by [Regiomontanus, *De triangulis* IV.19], or by [Jabir (Geber) I.15], or indeed by proposition 43 of our own *Spherical Triangles*: as the sine of the complement of arc HF — that is, the sine of the declination arc IF (for both in the southern and in the northern parallel the declination arc IF is the complement of arc HF, since HI is a quadrant by the corollary to [Theodosius I.16]) — is to the sine of the complement of the pole's altitude arc CH, so is the sine of the complement of arc CF, that is, the sine of arc BF (which is the complement of arc CF, since CB is a quadrant), and this measures the rising or setting amplitude of the parallel FG, to the whole sine [sinus totus]. Hence, by conversion, it will be: as the sine of the complement of the pole's altitude is to the sine of the declination of the given parallel, so is the whole sine to the sine of arc BF, the rising or setting amplitude [latitudo ortiva].

This can also be demonstrated in the following way, perhaps more conveniently. Since in the spherical triangle BIF the angle at I is right — because the greatest circle HI, drawn through the poles of the world, that is, of the equator BD, is perpendicular to the equator by [Theodosius, *Spherics* II.15] — and the angle B measures the inclination of the equator to the horizon, or what is the same thing, the altitude of the equator above the horizon, that is, the arc AE of the meridian (since B is the pole of the meridian AC), the two angles I and B of the triangle BIF are known. The declination arc IF is also known. Therefore, since by [Regiomontanus, *De triangulis* IV.16], or [Jabir (Geber) I.13], or proposition 41 of our *Spherical Triangles*, it holds that as the sine of angle B (the altitude of the equator, or the complement of the pole's altitude) is to the sine of angle I (the declination of the given parallel), so is the sine of the right angle I — that is, the whole sine — to the sine of arc BF, the rising or setting amplitude: thus from

three known quantities the fourth, namely the arc of the rising amplitude, will be found. And so, if it be made that as the sine of the complement of the pole's altitude is to the sine of the declination of the given parallel, so is the whole sine to a fourth term, the sine of the rising or setting amplitude will be found, and that amplitude will be known.

[Margin: The semidiurnal arc — by what method it is to be sought from the rising or setting amplitude.]

Again, since in the same right-angled triangle BIF the angle at I is right, and none of the arcs is a quadrant, all of them being parts of quadrants — for IF, in the northern triangle, is a part of the quadrant HI, but in the southern triangle a part of that quadrant which is drawn from I through F to the antarctic pole; likewise IB, in the southern triangle, is a part of the quadrant BE, but in the northern a portion of that quadrant which extends from B through I to the meridian below the horizon; and BF, finally, is a part of the quadrant BC (or BA) — it will be, by [Regiomontanus, *De triangulis* IV.19], or by [Jabir (Geber) I.15], or by proposition 43 of our *Spherical Triangles*: as the sine of the complement of arc IF (the declination of the given parallel) is to the whole sine, so is the sine of the complement of arc BF (the rising or setting amplitude just found) to the sine of the complement of arc BI — that is, to the sine of the semidiurnal arc IE of the southern parallel, or the seminocturnal arc of the northern parallel. For the arc of the equator IE is the complement of arc BI in the northern triangle, since arc BF [?] extended is a quadrant. Therefore, if it be made that as the sine of the complement of arc IF (the declination) is to the whole sine, so is the sine of the complement of the rising amplitude just found in the previous step to a fourth term, there will be found the sine of the semidiurnal arc of the southern parallel, or of the seminocturnal arc of the northern parallel. Then, this arc being subtracted from the semicircle of the equator that lies between the meridian above the horizon and the meridian below it, the remainder will be the northern diurnal arc IBE.

[Margin: Example.]

Let us take, for example, that the semidiurnal arc is to be found for latitude 42° , with the Sun at the beginning of Gemini [Π] or of Sagittarius [$\times$], where it has a declination of $20^\circ 12'$. Let it be made: as 74314 (the sine of the complement of the pole's altitude, i.e. $\sin 48^\circ$) is to 34529 (the sine of the declination of the parallel of Π or $\times$, i.e. $\sin 20^\circ 12'$), so is 100000 (the whole sine) to a fourth; and there will be found here approximately the sine 46464, whose arc is $27^\circ 41'$. This gives the rising or setting amplitude — northern for the parallel of Gemini [Π], but

southern for the parallel of Sagittarius [♐]. Again, let it be made: as 93849 (the sine of the complement of the declination of the same

parallel of Gemini [♊] or Sagittarius [♐]) is to 100000 (the whole sine), so is 88552 (the sine of the complement of the rising amplitude just found) to a fourth; and there will arise here, as it were, the sine 94355, whose arc, $70^{\circ} 39'$, will give the semidiurnal arc of the parallel of Sagittarius [♐] and the seminocturnal arc of the parallel of Gemini [♊]. Subtracting this from the semicircle, that is, from 180° , the remainder will be the semidiurnal arc of the parallel of Gemini [♊], namely $109^{\circ} 21'$. And if the degrees of these arcs are reduced to hours — assigning 15° to each hour, and one degree to each four minutes of an hour — the semidiurnal arc of the parallel of Sagittarius [♐] will contain 4 h 43 min, and that of the parallel of Gemini [♊] 7 h 17 min, a little more. And if we double these semidiurnal arcs, we shall have the whole diurnal arcs: namely, the diurnal arc of the parallel of Sagittarius [♐], $141^{\circ} 18'$, or 9 h 26 min; and the diurnal arc of the parallel of Gemini [♊], $218^{\circ} 42'$, or 14 h 35 min, a little less.

We can also find the semidiurnal arc of any parallel by the same spherical triangle, in this way. Since in the spherical triangle CFH the angle at C is right, it will be, by [Regiomontanus, *De triangulis* IV.16], or [Jabir (Geber) I.13], or proposition 41 of our *Spherical Triangles*: as the sine of arc FH, the complement of the declination of the given parallel (for FH, in the northern parallel, is the complement of arc FI of the declination; while in the southern parallel arc FH has the same sine, whose complement is arc FI of the declination — for the semicircle constitutes as its complement that which is interposed between F and the other pole, since the whole arc from I through F, tending toward the other pole, is a quadrant) — as this sine of arc FH is to the sine of the right angle C, that is, to the whole sine, so is the sine of arc CH, the pole's altitude, to the sine of arc FI [?]. From these three known quantities the fourth, and thus the angle F, will be recognized. Again, since in the spherical triangle BIF the angle at I is right, and B is the angle of the altitude of the equator — which the equator makes with the horizon — the sine of arc FI, of the declination given for the parallel, and the sine of angle F just recognized, and the sine of arc BI. From three known quantities, then, arc BI will be found, which, in the northern signs, added to a quadrant leaves the semidiurnal arc EI; but in the southern signs, subtracted from a quadrant, leaves the semidiurnal arc EI. And so, if it be made that as the sine of the complement of the declination of the given parallel is to the whole sine, so is the sine of arc FI to a fourth, arc BI will be found, which — the Sun being northern — added to a quadrant, or — the Sun being southern — subtracted from a quadrant, will give the semidiurnal arc. This first-found arc may be called the

"First Found"; then, if it be made that as the sine of the complement of the pole's altitude is to the sine of the declination, so is the sine we called the First Found to a fourth, there will be found the sine whose arc, the Sun being northern, added to a quadrant, or, the Sun being southern, subtracted from a quadrant, will give the semidiurnal arc.

[Margin: The semidiurnal arc — by what other method it is investigated through spherical triangles.]

Far more easily, by a single operation, we shall find BI, the difference between the semidiurnal arc of the parallel FG and that of the equator BD, in this way. In the triangle BFI the angle at I is right, B is the angle of the altitude of the equator, and the arc FI is the declination of the parallel. If, then, it be made that as the whole sine is to the tangent of angle B (the altitude of the equator) — and the tangent of angle FI, that is, the tangent of the pole's altitude — to a fourth, the sine of arc BI will be found, by proposition 49 of our *Spherical Triangles*. Example: with the Sun at the beginning of Gemini [♊] or Sagittarius [♐], where the declination is $20^{\circ} 12'$, let it be made: as 100000 (the whole sine) is to 36793 (the tangent of the declination), so is 90040 (the tangent of the complement of 48° , that is, the tangent of 42°) to a fourth; and there will be found the sine 33128, to which correspond $19^{\circ} 21'$. So great, then, is the arc BI.

Moreover, once the difference between the smallest or largest semidiurnal arc and a quadrant has been found, if a table of right ascensions is at hand, we shall find the differences between the semidiurnal arcs of all the other points of the ecliptic and a quadrant, by sines alone, in this way. Let it be made: as the sine of the right ascension of a given point is to the whole sine, so is the sine of the difference between the smallest or largest semidiurnal arc and a quadrant to a fourth, and the difference sought will be found. As in the same recent example: let it be made that as 84619 (the sine of the right ascension of the beginning of Gemini [♊] or Sagittarius [♐], which is $57^{\circ} 48'$) is to 39153 (the sine of the difference between the semidiurnal arc of the beginning of Cancer [♋] or Capricorn [♑] and a quadrant, which difference contains $23^{\circ} 3'$), so is [the whole sine] to a fourth; and there will be found the sine 33131, to which correspond $19^{\circ} 21'$, as before. This is demonstrated by [Regiomontanus, *Epitome* IV.24].

Most conveniently of all, even without spherical triangles, we shall find the rising or setting amplitudes and the semidiurnal arcs by this method. Let the analemma be taken up again — that is, the second figure of the preceding proposition — where HN, both in the northern and in the southern parallel, was shown to be the semidiurnal arc; likewise EL is the right sine of the rising or setting amplitude.

For, as we demonstrated in the scholium to proposition 1 of this book, the straight line EL, drawn perpendicular to BD, being the common section of the horizon and the parallel, cuts off from the circle ABCD the arc of the rising or setting amplitude, whose sine is EL.

[Margin: The rising or setting amplitude — how it may be computed by sines from the analemma.]

Diagram. The figure is the "analemma," a circle with centre E and a horizontal diameter BD, crossed by several straight lines (chords and perpendiculars) with their feet lettered. It repeats the second figure of the preceding proposition.

- B (left, on the circle): left end of the horizontal diameter BD
- D (right, on the circle): right end of the horizontal diameter BD
- E (centre): centre of the circle, meeting-point of the drawn lines
- A (upper left, on the circle): endpoint of an oblique diameter
- C (lower, on the circle): endpoint opposite A
- M (top, on the circle): endpoint of a line through E
- N (upper right, on the circle): point marking the semidiurnal arc HN
- H (left): point on the horizon line from which HN and HK are reckoned
- F (interior, upper middle): point on a drawn line
- K (interior): foot of a perpendicular (also appears near the top right)
- L (interior, two positions): feet of perpendiculars used for the sines
- G (lower left, on the circle): point on the circle
- I (lower / lower right): point on the circle near C and D

Therefore, since in the right-angled triangle EkL, if EL is set as the whole sine, the straight line EK is the sine of angle ELK, the complement of the pole's altitude (as we showed in our treatment of sines): if it be made that as Ek (the sine of the complement of the pole's altitude) is to EL (the whole sine), so is EK (the sine of the declination of the given parallel, taken with respect to the whole sine EF) to a fourth, there will be found EL, the sine of the rising or setting amplitude, with respect to that same whole sine EF. And from the sine EL the rising or setting amplitude itself becomes known.

[Margin: The semidiurnal arc — by what route it may be drawn out through sines from the analemma.]

Again, since in the same triangle EKL, if EL is set as the whole sine, the straight line kL is the sine of angle kEL, the pole's altitude (as is clear from the same treatment of sines): if it be made that as EK (the sine of the complement of the

pole's altitude) is to KL (the sine of the pole's altitude), so is EK (the sine of the declination, with respect to the whole sine EF) to a fourth, KL will be known, in the parts of that same whole sine EF. Then, since HK is also the sine of the complement of the declination with respect to the whole sine EF: if it be made that as HK (the sine of the complement of the declination, with respect to the whole sine EF) is to KL (in so far as it has been made known in the parts of that same whole sine EF), so is Hk (in so far as it is the whole sine in the Sun's parallel) to a fourth, KL will be produced, known in the parts of that same whole sine Hk, and therefore its

the arc MN will be known. In a northern parallel, added to the quadrant HM, it completes the semidiurnal arc HN; but in a southern parallel, subtracted from the quadrant HM, it leaves the semidiurnal arc HN. And so, if it be made that as the sine of the complement of the pole's altitude is to the sine of the pole's altitude, so is the sine of the declination of the given parallel to a fourth, a certain sine will be found, which we may call the First Found value. Then, if it be made that as the sine of the complement of the declination is to the sine we have called the First Found, so is the whole sine to a fourth, there will be found the sine of a certain arc which — with the Sun lingering in the northern signs, added to the quadrant, or with the Sun running through the southern signs, subtracted from the quadrant — will give the semidiurnal arc. Therefore we have investigated the rising or setting amplitude, and the semidiurnal arc of any parallel, for a given pole's altitude. Which was to be done.

SCHOLIUM.

[Margin: The semidiurnal arcs of the southern parallels are equal to the seminocturnal arcs of the opposite northern parallels.]

[Margin: It is enough if the semidiurnal arcs of the southern parallels are investigated. For from these the semidiurnal arcs of the opposite northern parallels are also elicited with little trouble.]

[Margin: It is enough if the semidiurnal arc of one southern quadrant of the ecliptic be computed, since from these the semidiurnal arcs of the other points of the ecliptic are easily gathered.]

We shall present two other ways of finding the semidiurnal arc — likewise without spherical triangles, with the Sun situated in any parallel whatever — in the scholium of the following proposition; and we trust these will not be unwelcome to the reader.

Furthermore, the semidiurnal arcs of the southern parallels, as is clear from what has been demonstrated, are equal to the seminocturnal arcs of the opposite northern parallels. For by the same operation, in the earlier method, both the semidiurnal arc of a southern parallel and the seminocturnal arc of the opposite northern parallel were found. This is also elicited, not obscurely, from Prop. 19 of Book 2 of Theodosius [Theodosius, *Sphaerica* II.19]. For it is there demonstrated that the alternate segments of opposite and equal parallels — into which they are divided by the horizon — are equal to one another (namely the segments that are the diurnal arc of one parallel and the nocturnal arc of the other, opposite and equal, parallel). Hence their halves will also be equal to one another, that is, the semidiurnal arc of the one parallel and the seminocturnal arc of the other. Therefore, if the semidiurnal arcs of all the points of the ecliptic are wanted, it will be enough that the semidiurnal arcs be investigated in the southern parallels: for these, being equal to the seminocturnal arcs of the opposite northern parallels, as has been said, when subtracted from the semicircle will leave the semidiurnal arcs of the opposite northern parallels as well.

This is also plainly evident from the latter method. For in the analemma, by the same operation, the arc MN was found both in the northern parallel and in the southern; and by this same arc the quadrant MI exceeds the seminocturnal arc NI of the northern parallel, while the quadrant MH exceeds the semidiurnal arc HN of the southern parallel. It is therefore clear that the seminocturnal arc NI of the northern parallel is equal to the semidiurnal arc NH of the southern parallel. And so with the rest. Here too you plainly see that the seminocturnal arc NI, subtracted from the semicircle HNI, leaves the semidiurnal arc HN.

Indeed, since the semidiurnal arcs of one southern quadrant of the ecliptic — the quadrant lying between the equator and the colure of the solstices — are equal to the semidiurnal arcs of the other southern quadrant, only the semidiurnal arcs of one southern quadrant of the ecliptic need be computed, namely of the quadrant lying between the beginning of ♎ [Libra] and the beginning of ♐ [Capricorn]. For these will be equal to the semidiurnal arcs of the other southern quadrant, intercepted between the beginning of ♈ [Aries] and the beginning of ♐ [Capricorn]. And these same arcs, subtracted from the semicircle, leave the semidiurnal arcs of the northern quadrant of the ecliptic set between the beginning of ♈ [Aries] and the beginning of ♋ [Cancer]; which in turn are equal to the semidiurnal arcs of the remaining northern quadrant, enclosed between the beginning of ♎ [Libra] and the beginning of ♋ [Cancer].

Exactly the same must be said of the rising or setting amplitude [latitudo ortiva/occidua]: for if, for one quadrant of the ecliptic — whether southern or northern — all the rising and setting amplitudes are computed, the rising or setting latitudes will also be had for all the points of the ecliptic, as is clear from the demonstration; since indeed the declinations of the points of one quadrant of the ecliptic, from which the rising and setting latitudes are computed, are equal to the declinations of the remaining points of the ecliptic, as we have shown elsewhere.

PROBLEMA 14. PROPOS. 35.

To find the magnitudes of the twilights [crepuscula] in any parallel whatever, for a given pole's altitude.

Although this problem can be solved by spherical triangles — as we shall demonstrate in the following scholium — we shall nevertheless carry it out much more briefly and easily by plane rectilinear triangles. Let ABCD be the meridian of the analemma, whose centre is E; let AC be the diameter of the horizon, BD the vertical, FG the axis of the world, HI the diameter of the equator, and KL the diameter of the Sun's parallel — whether northern or southern — about which let the semicircle KPL of the parallel be described. Its centre will be the point M, where the axis and the diameter of the parallel intersect each other, since the straight line EM, perpendicular to KL, bisects KL itself. [Margin: Euclid III.3] From the point S, where the diameter of the parallel cuts the diameter of the horizon, let a perpendicular SQ be raised to KL; this will be the common section of the horizon and the parallel. For since both the horizon and the parallel are perpendicular to the meridian, [Margin: Euclid XI.19] their common section will be perpendicular to it, and therefore, by Euclid Book XI def. 3, perpendicular also to the diameter KL of the parallel. Since, then, that common section passes through the point S — where the horizon and the parallel cut one another in the meridian — SQ, being perpendicular to KL, will be the common section of the horizon and the parallel. Therefore KQ will be the semidiurnal arc of the parallel, since it is intercepted between the horizon, which passes through the point Q, and the meridian, which cuts the parallel at k.

Let there also be drawn XY, the diameter of the horizon-parallel below the horizon in which the Sun lies when the morning twilight begins or the evening twilight ends, such that both the arc AX and the arc CY comprehend 18 degrees. For the Sun is distant from the horizon by that many degrees at the beginning of morning twilight or at the end of evening twilight, as is the common opinion — although some authors make this distance smaller, and others greater, as Pedro

Nunes [Petrus Nonius] reports in the second part of his book *On Twilights*, Prop. 1. Let the diameter XY cut the diameter KL of the Sun's parallel at T, and from that point let a perpendicular TR be drawn to KL; this will be the common section of the Sun's parallel and of the horizon-parallel in which the beginning or the end of twilight lies. For since both the Sun's parallel and the horizon-parallel are perpendicular to the meridian, [Margin: Euclid XI.19] their common section too will be perpendicular to it, and therefore, by Euclid Book XI def. 3, perpendicular also to the straight line KL lying in the plane of the meridian — such as is the straight line TR. Therefore the length of the twilight will be the arc QR, since the Sun is at the point R when it occupies the horizon-parallel XY, in which is the beginning of morning twilight or the end of evening

twilight; and it rises or sets at the point Q. For since TR is the common section of the Sun's parallel and of the horizon-parallel XY, these parallels cut one another at R, and so the Sun, when at the point R, will occupy both parallels. In the same way, when at the point Q it will hold the horizon, since — as we said a little earlier — SQ is the common section of the horizon and the Sun's parallel. Finally, let a perpendicular KN be dropped from K to AC, cutting XY at O; and KN will be the sine of the meridian altitude of the Sun situated in the parallel KL — that is, the sine of the arc KA — while the straight line KO is composed of KN, the sine of the meridian altitude, and NO, the sine of the Sun's distance below the horizon at the beginning of morning twilight or the end of evening twilight. So likewise KS will be the versed sine of the semidiurnal arc KQ, and KT the versed sine of the arc KR — composed of the semidiurnal arc kQ and the twilight arc QR, which the Sun of course traverses in its own parallel while the twilight lasts.

[Margin: How the magnitude of twilight may be investigated, with the Sun situated in any parallel whatever.]

And so, since in the triangle KOT the straight line SN is parallel to the base OT, [Margin: Euclid VI.2] it will be that as kN, the sine of the meridian altitude, is to KO — the straight line composed of the sine of the meridian altitude and of NO, the sine of the Sun's distance below the horizon at the beginning of morning twilight or the end of evening twilight — so is KS, the versed sine of the semidiurnal arc, to KT, the versed sine of the arc composed of the semidiurnal arc and the twilight arc. Therefore, if it be made that as the sine of the meridian altitude (how the meridian altitude is found from the known declination of the Sun we shall explain in the following scholium) is to the straight line composed of the sine of the meridian altitude and the sine of 18 degrees — for by that many degrees, according to the common view, the Sun is depressed below the horizon

on the vertical then passing through the Sun's centre — so is the versed sine of the semidiurnal arc to a fourth, one will obtain the versed sine of the arc composed of the semidiurnal arc and the twilight arc. From this versed sine, then, we shall find the quantity of the twilight as follows.

[Margin: When the versed sine found is greater than the whole sine, by what method the twilight is elicited from it.]

If the versed sine found, kT , is greater than the whole sine — as happens in the first two figures — we shall subtract the whole sine KM from it, and from the remaining sine TM we shall take the arc PR . For this arc PR , added to the quadrant kP , will make up the arc kR , composed of the semi-

Diagram. First of four analemma figures (upper left). A circle — the meridian $ABCD$ — with B at top, D at bottom, A at left, C at right, and centre E ; through it run the horizon diameter AC , the vertical BD , the oblique axis FG , the equator diameter HI , the Sun's-parallel diameter KL , and (below the horizon) the horizon-parallel diameter XY . The semicircle KPL of the Sun's parallel is drawn about centre M , with dotted construction lines and the perpendiculars KN , SQ , TR . This figure carries additional small Greek point-labels used in the equinoctial ("SOLE aequinoctialia puncta") derivation. Several of the small interior Greek labels are at the resolution floor and are flagged.

- B (top of circle): upper end of the vertical BD
- D (bottom of circle): lower end of the vertical BD
- A (left of circle): west end of the horizon diameter AC
- C (right of circle): east end of the horizon diameter AC
- E (centre): centre of the meridian
- K (upper left, on circle): upper end of the Sun's-parallel diameter KL
- L (lower right, on circle): lower end of the Sun's-parallel diameter KL
- H (left, on circle): end of the equator diameter HI
- I (lower right, on circle): opposite end of the equator diameter HI
- F (upper right, on circle): end of the axis FG
- G (lower left, on circle): opposite end of the axis FG
- P (upper right, on circle): point on the parallel semicircle KPL
- Q (right, on circle): foot of the semidiurnal arc (endpoint of perpendicular SQ)
- R (right, near C): point where the Sun stands at the twilight boundary (endpoint of perpendicular TR)

- X (lower left): west end of the horizon-parallel diameter XY
- Y (right): east end of the horizon-parallel diameter XY
- M (interior, above E): centre of the Sun's-parallel semicircle
- N (interior, left of centre): foot of perpendicular KN on AC
- S (interior, right of centre): intersection of the parallel diameter with the horizon diameter
- T (interior, lower right of centre): intersection of XY with KL
- λ (lambda) (interior, left of M): [?] small interior label
- α (alpha) (interior, left near A/X): [?] foot of the perpendicular $H\alpha\beta$
- β (beta) (interior, lower left): [?] second foot of the perpendicular $H\alpha\beta$
- θ (theta) (interior, near bottom): [?] small interior label
- γ (gamma) (interior, below E): [?] endpoint of the segment $E\gamma$ (glyph could be the Aries sign Υ)

Diagram. Second of four figures (upper right) — the same analemma construction with the Sun's parallel KL in a different position (the case treated as the "second figure," where the versed sine exceeds the whole sine). Readable labels: B (top), D (bottom), A (left), C (right), E (centre); K, H (upper left, on circle), F (upper right), Y (right), L (lower right), I (lower right), on the circumference; interior N, S, E, M, T near centre, with small λ and θ at the left and P, Q, R, G, k grouped at the lower left; dotted construction rectangle at lower left. Small interior Greek labels (λ , θ) flagged.

- B, D, A, C, E: meridian frame and centre (as in fig-p113-1)
- K, L: Sun's-parallel diameter
- H, I: equator diameter
- F, G: axis
- N, S, M, T: interior construction points (as above)
- P, Q, R (lower left region): parallel-semicircle / twilight points
- λ (lambda), θ (theta): [?] small interior labels

Diagram. Third of four figures (lower left) — the same construction for the "third figure" case, where the versed sine KT is less than the whole sine. Readable labels: B (top), H, K (upper left), F (upper right), A (left), C (right), Y (right), E (centre), M, N, S, T (interior), I, L (lower right), and P, Q, R, G, k at the lower left, with a dotted construction figure. Small interior Greek labels flagged.

- B, D, A, C, E; K, L; H; F; Y; I: frame, centre, and circumference points as above

- M, N, S, T: interior construction points
- P, Q, R (lower left): twilight-construction points
- λ (lambda), θ (theta) [?]: small interior labels

Diagram. Fourth of four figures (lower right) — the same construction for the "fourth figure" case, where the versed sine KT equals the whole sine (so the composite arc is the quadrant kP itself). Readable labels: B (top), H, K (upper left), F (upper right), A (left), C (right), Y (right), E (centre), M, N, S, T (interior), I, L (lower right), and P, Q, R, G, k grouped at lower left. Small interior Greek labels flagged.

- B, D, A, C, E; K, L; H; F; Y; I: frame, centre, and circumference points as above
- M, N, S, T: interior construction points
- P, Q, R (lower left): twilight-construction points
- λ (lambda), θ (theta) [?]: small interior labels

diurnal arc and the twilight arc. Therefore, once the semidiurnal arc kQ is taken away, the remaining arc will be the twilight arc QR. But if the said versed sine KT is less than the whole sine — as happens in the third figure — we shall subtract it from the whole sine KM, and from the remaining sine TM we shall take the arc PR. For this arc PR, subtracted from the quadrant KP, will make the remaining arc kR, composed of the semidiurnal arc and the twilight arc. If, then, from this we again subtract the semidiurnal arc KQ, there will remain the twilight arc QR. If, finally, the aforesaid versed sine kT is found equal to the whole sine — as happens in the fourth figure — the quadrant kP will itself be the arc composed of the semidiurnal arc and the twilight arc. Therefore, if from this quadrant we subtract the semidiurnal arc KQ, the remaining arc will be the twilight arc QR, or QP.

We shall obtain the same result also by this method. [Margin: Euclid VI.2] Since in the same figures it holds that as KN, the sine of the meridian altitude, is to NO, the sine of the Sun's concealment below the horizon at the beginning of morning twilight or the end of evening twilight — which concealment we set at 18 degrees — so is KS, the versed sine of the semidiurnal arc, to ST: if it be made that as the sine of the meridian altitude is to the sine of 18 degrees, so is the versed sine of the semidiurnal arc to a fourth, the value of the straight line ST will be found; and this, added to the versed sine of the semidiurnal arc, makes up the versed sine of the arc composed of the semidiurnal arc and the twilight arc. From this versed

sine we shall find, as before, the length of the twilight. We give no example, because from the figures themselves the matter is self-evident.

[Margin: When the versed sine found is less than the whole sine, by what method the twilight is elicited from it.]

[Margin: When the versed sine found is equal to the whole sine, by what method the twilight is thence recognized.]

[Margin: How the twilight may otherwise be explored by degrees. Euclid VI.2.]

With the Sun occupying the equinoctial points — that is, the beginning of Υ [Aries] or of ♎ [Libra] — the magnitudes of the twilights will be found much more easily by this method. In the first figure, having drawn from H a perpendicular $H\alpha\beta$ to AC and XY, [Margin: Euclid VI.2] it will be that as $H\alpha$, the sine of the altitude of the equator (or of the complement of the pole's altitude), is to $\alpha\beta$, the sine of 18 degrees (that is, of however many degrees we have

set the arc AX), so is HE, the whole sine, to $E\gamma$, the right sine of the twilight arc λG . [The point-label γ here is a small interior glyph — possibly the Aries sign Υ ; read as γ [?].] If, therefore, it be made that as the sine of the complement of the pole's altitude is to the sine of 18 degrees, so is the whole sine to a fourth, there will result a sine whose arc will give the twilight, with the Sun at the equinoxes.

[Margin: By what method all twilights may most easily be computed.]

An easier way. Let it be made that as $E\alpha$, the whole sine, is to $E\gamma$, the secant of the angle $\alpha E\gamma$ [equal to] the pole's altitude, so is $E\alpha$ [?], the sine of 18 degrees, to a fourth; and $E\gamma$, the sine of the twilight, will be found. [Several of the Greek point-labels in this sentence are at the resolution floor; flagged.]

Furthermore, once the magnitude of the straight line $E\gamma$ has been found by the method we have just now set out, we shall through it find the twilights of all the parallels — or of the points of the ecliptic — in this way. [Margin: Book I, Prop. 34] Since $E\gamma$ is equal to each of the segments ST of the diameters of the parallels intercepted between AC and XY, such segments will be known in terms of the whole sine of the greatest circle in the sphere. For each of them will be as great as the straight line $E\gamma$ was found to be, with the Sun at the equinoxes.

Diagram. Repeats fig-p113-1 (upper-left analemma with the equinoctial Greek labels α , β , θ , λ , γ). Same woodcut block as on the preceding page. Small interior Greek labels at the resolution floor — flagged as on p. 113.

Diagram. Repeats fig-p113-2 (upper-right analemma, "second figure" case).

Diagram. Repeats fig-p113-3 (lower-left analemma, "third figure" case).

Diagram. Repeats fig-p113-4 (lower-right analemma, "fourth figure" case).

with the Sun at the equinoxes. From this we shall come to know these same segments in terms of the whole sine of the respective parallels, by this device. Let it be made that as kM , the sine of the complement of the declination of any given parallel, is to KM — in so far as it is the whole sine of the respective parallel — so is ST , in so far as it is known in terms of the whole sine of the greatest circle, to a fourth. For the same ST will come out, known in terms of the whole sine KM of the respective parallel. And so, if ST , known in terms of the whole sine of the respective parallel, be added to the versed sine KS of the semidiurnal arc, the versed sine KT of the arc KR — composed of the semidiurnal arc and the twilight arc — will be known. Therefore, as before, from this versed sine we shall find the magnitude of the twilight.

[Margin: How the meridian altitude of the Sun is found.]

[Margin: When the number formed from the northern declination and the altitude of the equator exceeds 90 degrees.]

[Margin: When the pole's altitude is less than the northern declination of the Sun's parallel, the zenith of the place lies between the Sun's parallel and the equator, and the Sun's meridian altitude is northern.]

This method of investigating the twilights seems the easiest and most expeditious of all, because at a given pole's altitude the straight line $E\gamma$, once found in terms of the whole sine of the greatest twilight, always remains the same in all the Sun's parallels, so that it never changes. Its discovery, indeed, is very easy, since in finding it the whole sine is employed, which makes the multiplication easiest, as is clear from what has been demonstrated. Once this has been found, the same $E\gamma$ — or ST — is obtained by the method just now taught, in so far as it is a part of the whole sine of the respective parallel; and this discovery too is not difficult, because in seeking it the whole sine is likewise used, which renders the operation less difficult, as we have said. This does not happen in the earlier rules: for in them the whole sine is never assumed in order to find the straight line KT . Hence the multiplication is made somewhat more difficult, as is evident. Therefore we have found the magnitudes of the twilights, and so forth. Which was to be done.

SCHOLIUM.

[Margin: When the number formed from the northern declination and the altitude of the equator exceeds 90 degrees. When the pole's altitude is less than the northern declination of the Sun's parallel, the zenith of the place lies between the Sun's parallel and the equator, and the Sun's meridian altitude is northern.]

The meridian altitude of the Sun is found in the way all commonly hand down. When the Sun is in the northern signs, its declination is added to the altitude of the equator (that is, to the complement of the pole's altitude); but when it runs through the southern signs, its declination is subtracted from the altitude of the equator (or from the complement of the pole's altitude). For the number formed by that addition, or left by this subtraction, will give the meridian altitude sought. For example, with the Sun at the beginning of ♋ [Cancer], if its declination — which is 23 degrees 30 minutes — is added to the complement of the pole's altitude, that is, to the elevation of the equator at Rome, which comprehends 48 degrees, the meridian altitude of 71 degrees 30 minutes results, at the latitude of 42 degrees, such as is found at Rome. But with the Sun at the beginning of ♑ [Capricorn], if its declination — which is 23 degrees 30 minutes — is subtracted from the elevation of the equator, that is, from the complement of the pole's altitude, namely from 48 degrees, the remaining meridian altitude will be 24 degrees 30 minutes. The rationale of this operation is clear from the figures above. For in the first, where the parallel of the diameter KL is northern, the meridian altitude AK is composed of the declination KH and the altitude of the equator HA. But in the others, in which the parallel of the diameter KL is southern, the meridian altitude AK results if the declination KH is subtracted from the altitude of the equator HA.

(The scholium to Proposition 35 continues from the preceding pages.)

But here some points must be noted. **First**, when in the northern signs a number greater than 90 degrees is formed by adding the declination to the elevation of the equator, the number so formed must be subtracted from the semicircle in order to obtain the meridian altitude. For then the Sun will lie between the zenith B and the arctic pole F — or, what amounts to the same, the zenith B lies between the Sun and the equator. This can be seen in the first figure of this scholium, where the arc AK, lying between the horizon and the northern parallel, exceeds the quadrant AB, and the meridian altitude is the arc CK on the northern side, and the zenith B is set between the Sun's parallel and the equator. This happens when the altitude of the pole is less than the declination of the parallel in question, as is the case here. For the arc BH, which is equal to the arc CF of the

pole's altitude — as we showed on the sphere, and as is easily gathered from the figure (for if from the equal quadrants CB and FH the common arc FB is removed, the arcs CF and BH remain equal) — is less than the declination HK. But if the altitude of the pole BH should at some point be equal to the declination of the parallel, the parallel will touch the vertical circle at the zenith B, and the meridian altitude will be a quadrant, namely CB or AB, of 90 degrees.

[Margin: When the altitude of the pole is equal to the northern declination of the Sun's parallel, the Sun's parallel touches the vertical circle at the zenith B, and the Sun's meridian altitude is a quadrant.]

Next, when the complement of the pole's altitude is less than the declination of a northern parallel, the whole parallel will stand above the horizon and will have two meridian altitudes: a southern one, which is the greater, and a northern one, which is the smaller. The first is found by the rule given above; the second is obtained by subtracting the complement of the pole's altitude from the declination. This is clear in the second figure of this scholium, where CI, the complement of the pole's altitude, is less than the declination IL, and the northern parallel KL stands wholly above the horizon. The southern and greater meridian altitude is the arc AK; the northern and smaller is the arc CL, which is obtained by subtracting the complement of the pole's altitude IC from the declination IL. When the Sun is on this parallel there is no twilight, since the day is continuous. But if the complement of the pole's altitude IC should at some point equal the declination of the parallel, the parallel will touch the horizon at C and will stand wholly above it, so that here too no twilight need be sought.

[Margin: When the complement of the pole's altitude is less than the declination of the Sun's northern parallel, the whole parallel stands above the horizon, and no twilight occurs; the Sun then has two meridian altitudes on that parallel. When the complement of the pole's altitude is equal to the northern declination of the Sun's parallel, the parallel touches the horizon, and there is no twilight.]

Third, when in the southern signs the declination of some parallel is greater than the complement of the pole's altitude — and therefore cannot be subtracted from that complement to obtain the meridian altitude — the Sun will have no meridian altitude on that parallel, but the whole parallel will lie hidden below the horizon, as is clear in the same second figure, in which the southern declination HK is greater than the complement of the pole's altitude AH. But if the arc AK — the difference between the complement of the pole's altitude AH and the southern declination HK — is less than the arc AX (which lies between the horizon and the parallel and runs through the beginning of morning twilight

[Margin: When the declination of a southern parallel is greater than the complement of the pole's altitude, so that the whole parallel lies below the horizon, it has no meridian altitude. — How great the twilight is, while the whole southern parallel lies hidden below the horizon, and how it is found. — What the Sun's depression is. [a] Euclid VI.2. [b] Euclid VI.4.]

Diagram. An analemma-type sphere seen from the side: an outer circle (the meridian) with a horizontal diameter (the horizon AC) and several oblique diameters representing the equator, the pole-axis, and the Sun's parallel, plus perpendicular sine-lines dropped to the horizon and axis. The construction is dense with small interior labels; many are worn or overlapping and cannot be resolved with confidence at this resolution. Readable labels, by approximate position:

- Top: K, H, B, K
- Left: A
- Interior (upper): N, G, E, M
- Interior (mid): X, M, S; O, T, T [?], Y
- Upper right: F, P, C; Q, R
- Lower left: P, R, X
- Bottom: θ , L, D, I
- Bottom-left margin: a zodiacal/symbol sort [?]

Numerous small interior labels are illegible or ambiguous

Diagram. A companion analemma sphere, similarly constructed (outer meridian circle, horizon diameter, oblique parallels and sine perpendiculars), illustrating the case where the northern parallel stands wholly above the horizon and the southern wholly below. Readable labels, by approximate position:

- Top: B, F
- Left: K, H, A, K
- Center: E
- Right: N, L, C, I, Y
- Interior (lower): θ , O, N; T, M
- Bottom: G, D; R, F

Several small labels illegible/ambiguous

) — that is, when the Sun traverses the arc kR and another equal to it, there will be twilight; so that KR is half the twilight, because in the arc KR, and in another

equal to it, the Sun is less than 18 degrees distant from the horizon. We shall find the magnitude of this arc as follows. From the point L, of the southern parallel in the latter figure of this scholium, let a perpendicular LN be drawn to AC, cutting XY at O; likewise through K and M let $K\theta$ and $M\lambda$ be drawn parallel to AC, cutting LN at θ and λ . Before all else, we shall come to know the straight lines $L\lambda$ and LO in this way. The whole arc CL — which we may call the Sun's depression below the horizon, that is, its greater distance below the horizon on the meridian (for the Sun on that parallel has two depressions below the horizon, the greater CL and the smaller AK, of which the former inclines to the north, the latter to the south) — is composed of CI, the complement of the pole's altitude, and IL, the southern declination, and so is known. Therefore its sine LN is also known. Again, the arc AK of the smaller depression is known, since it is the difference between the arc HK of the southern declination and the arc AH of the complement of the pole's altitude. Therefore its sine θN is also known. So, subtracting the sine θN from the sine LN, which is now known, there remains $L\theta$, known,

[Margin: When there is no twilight, the Sun being in the southern signs. When twilight is continuous, the Sun being at the equinoxes. How the Sun's meridian depression is found. The meridian depression of any parallel is equal to the meridian altitude of the opposite parallel.]

which is the difference between the sine of the greater depression and the sine of the smaller depression below the horizon. [a] Now since, as LM is to MK, so $L\lambda$ is to $\lambda\theta$, and LM is equal to MK, $L\lambda$ will also be equal to $\lambda\theta$. Therefore, since the straight line $L\theta$ is known, its half $L\lambda$ is known as well. And LO is known, since it is the difference between LN, the sine of the greater depression, and ON, the sine of 18 degrees, namely of the arc AX or CY. So both $L\lambda$ and LO are known.

Now [b], since as $L\lambda$ is to LO, so LM is to LT, if it be made that, as $L\lambda$ — the half of the difference between the sine of the greater depression and the sine of the smaller — is to LO, the difference between the sine of the greater depression and the sine of 18 degrees, so is LM, the whole sine, to a fourth term, there will be found LT, the versed sine of the arc LR. From this versed sine LT we shall trace out the arc KR of half the twilight thus: when the versed sine LT found is greater than the whole sine LM, we shall subtract the whole sine from it to obtain the right sine MT, whose arc PR, taken from the table of sines, we subtract from the quadrant KP, and the arc KR of half the twilight remains known. But if the versed sine LT is found equal to the whole sine, the arc of half the twilight will be a full quadrant, because then the diameter KL passes through the point where the axis cuts the diameter XY, and consequently the point T falls on the point M. Finally, if

the versed sine LT is found less than the whole sine, we subtract it from the whole sine and add the arc of the remaining right sine, taken from the table of sines, to the quadrant, to obtain the arc of half the twilight: because then the diameter KL cuts the axis between the center E of the analemma and the diameter XY, and so the point T falls between M, the center of the parallel [recte: paralleli], and the point L, as is clear.

But if the arc AK below the horizon — that is, the difference between the complement of the pole's altitude AH and the southern declination HK (when in the southern signs the declination is greater than the complement of the pole's altitude) — is equal to or greater than the arc AX, there will be no twilight while the Sun traverses that southern parallel; for then the Sun below the horizon is always more than 18 degrees distant from it — unless we should wish to call twilight that instant of time in which the Sun, at the point X, is at midnight, namely when the arc AK is equal to the arc AX.

But if the complement of the pole's altitude is less than the arc of 18 degrees — as happens in the second figure of this scholium (for the arc CI of the complement of the pole's altitude is less than the arc CY of 18 degrees) — then, when the Sun is at the equinoxes, there will be twilight throughout the whole night, because the Sun is then always less than 18 degrees below the horizon.

Moreover, if anyone should prefer not to use versed sines, he can investigate the magnitudes of the twilights another way, and perhaps more conveniently. To make this clear, we shall first teach how to find the Sun's meridian depression — that is, its distance below the horizon on the meridian. This is found as follows. In the northern signs, subtract the declination of the parallel in question from the complement of the pole's altitude; in the southern signs, add that same declination to the complement of the pole's altitude. For the number left by that subtraction, or composed by this addition, will give the meridian depression, as is clear from the four figures of this proposition, in which the meridian depression is invariably the arc CL. Now the depression of any parallel is equal to the meridian altitude of the opposite parallel. For if a diameter were drawn from L through the center E, it would fall in the quadrant AB at the point through which the diameter of the opposite parallel would have to be drawn, as is clear. [a] Therefore, since such a diameter, together with the diameter of the horizon AC, makes equal angles at the vertex E, [b] the arcs on which the said equal angles at the center E stand will be equal to one another — namely the arc CL of the meridian depression and the arc of the meridian altitude of the opposite parallel.

Hence, if the meridian depression of some parallel is sought, one may take for it the meridian altitude of the opposite parallel.

[Margin: When there is no twilight, the Sun being in the southern signs. When twilight is continuous, the Sun being at the equinoxes. How the Sun's meridian depression is found. The meridian depression of any parallel is equal to the meridian altitude of the opposite parallel. [a] Euclid I.15. [b] Euclid III.27.]

Diagram. The same "first figure" analemma sphere as on p. 115 (the woodcut pair is reprinted on this verso), with the identical set of interior labels (top K, H, B, K; left A; interior N, G, E, M, X, S, O, T, Y; right F, P, C, Q, R; lower-left P, R, X; bottom θ , L, D, D). Small interior labels are again worn/ambiguous

Diagram. The same "second figure" analemma sphere as on p. 115 (top B, F; left K, H, A, K; center E; right N, L, C, I, Y; interior θ , O, N, T, M; bottom G, D, R, F). Small labels worn/ambiguous

The meridian altitude and depression together make up the complement of the pole's altitude, doubled — at any elevation of the pole that is not less than the declination of the parallel on which the Sun is. For in any figure of this proposition, since the arcs HK and IL are equal, when the common arc CL is added, the two arcs HK, CL will be equal to the whole CI. Therefore, adding the common AH again, the two arcs KA (of the meridian altitude) and CL (of the meridian depression) will be equal to the two arcs HA, CI — that is, to the complement of the pole's altitude doubled. This is also clear as follows: since the two complements of the pole's altitude AH, CI are equal, and to AH the declination HK is added while from CI the same declination IL is subtracted, it results that the composite arc AK and the remaining arc CL together are likewise equal to the two arcs AH, CI together — since what is added to the one is subtracted from the other. From this it follows that, when the altitude of the pole is less than the declination, the composite arc AK together with the depression CL is still double the complement of the pole's altitude AH — that is, equal to the two arcs AH, CI together, as appears in the first figure of this scholium.

[Margin: When the number composed from the complement of the pole's altitude and the southern declination exceeds a quadrant. When the northern declination is greater than the complement of the pole's altitude, there is no meridian depression, but the whole parallel stands above the horizon.]

But here too some points must be noted. For if in the southern signs the number formed from the complement of the pole's altitude and the declination is greater than a quadrant, the number so formed must be subtracted from the semicircle

in order to obtain the meridian depression, as may be seen in the first figure of this scholium. Similarly, if in the northern signs the declination of the parallel is greater than the complement of the pole's altitude, so that the former cannot be subtracted from the latter, the whole parallel will stand above the horizon, as appears in the second figure of this scholium. Hence there will then be no meridian depression, but the parallel will have two meridian altitudes, as was said a little earlier. Finally, when in the southern signs the declination of the parallel is greater than the complement of the pole's altitude, as appears in the same second figure of this scholium, it was already said a little earlier that the parallel is then wholly below the horizon and has two meridian depressions, which we investigated in that place; and that sometimes there can be twilight, sometimes not; and likewise in what manner that twilight ought to be investigated.

[Margin: When the southern declination is greater than the complement of the pole's altitude, the whole southern parallel lies hidden below the horizon and has two meridian depressions. [c] Euclid VI.2. [d] Euclid I.34.]

Furthermore, the greater and the smaller altitude, taken together, are equal to twice the declination of the parallel. For since the two arcs HK, IL are equal, and AH is added to HK while IC is subtracted from IL, the composite arc AK and the remaining arc CL together will be equal to the two arcs HK, IL — that is, together they will be twice the declination HK. Likewise, the difference between the two meridian altitudes is twice the complement of the pole's altitude. For since the two altitudes AK, CL are equal to the two arcs HK, IL, as has been said, if CL — the smaller altitude, which is common — is removed, the greater altitude AK will be equal to the two arcs HK, CI. If, then, from HK [?] there be taken away the common arc toward the side of K, equal to CL, the remaining part of AK will be equal to the remaining part of HK together with the arc CI. Now from the arc AK there remains the difference between the two altitudes AK, CL, while from the arc HK there remains an arc equal to CI (just as CL, taken from IL, leaves the arc CI). Therefore the difference between the two altitudes AK, CL will be equal to twice the arc CI, the complement of the pole's altitude.

With these things laid down, from L let a perpendicular $L\theta$ be drawn to the straight line KO, produced in all the figures (except the second figure of this scholium); likewise from M, the center of the parallel, another perpendicular $M\lambda$. [c] And since in the triangle $K\theta L$, as KM is to ML, so is $K\lambda$ to $\lambda\theta$; [d] but KM is equal to ML; $K\lambda$ will also be equal to $\lambda\theta$. Therefore, since KN is the sine of the meridian altitude, and θN the sine of the meridian depression (because θN is

equal to the sine of the depression, which would be drawn as a perpendicular from L to AC), $K\lambda$ will be [the half...]

$K\lambda$ will be the half of the straight line composed of the sines of the meridian altitude and of the meridian depression. But λO will be the difference between the said half and the straight line composed of the sine of the meridian altitude and the sine of 18 degrees. [a] And since, as $K\lambda$ is to λO , so is KM to MT : if it be made that, as $K\lambda$ — the half of the straight line composed of the sine of the meridian altitude and the sine of the meridian depression — is to λO , the difference between the aforesaid half and the straight line composed of the sine of the meridian altitude and the sine of 18 degrees, so is KM , the whole sine, to a fourth term, there will result MT , the right sine of the arc PR . This arc, added to the quadrant, constitutes the arc KR , composed of the semidiurnal arc and the twilight arc — provided the aforesaid half $K\lambda$ is found to be less than the straight line composed of the sine of the meridian altitude and [the sine of] 18 degrees, as always happens in the northern signs and sometimes in the southern, as may be seen in the two preceding figures of this proposition and in the first figure of this scholium. Or else, subtracted from the quadrant, it leaves the arc KR , composed of the semidiurnal arc and the twilight arc, if indeed the said half is found to be greater than the straight line composed of the sine of the meridian altitude and the sine of 18 degrees, as happens in the third figure of this proposition. Then, when the semidiurnal arc KQ is subtracted from the arc KR thus found, it is clear that the twilight arc QR is left. But if the said half $K\lambda$ is equal to the said straight line KO , the arc composed of the semidiurnal arc and the twilight arc will be a quadrant, as appears in the fourth figure of this proposition.

[Margin: [b] Euclid VI.2. — By what method twilight is to be investigated otherwise than above.]

Indeed, by means of the meridian altitude and the meridian depression we shall compute the semidiurnal arcs by a very short calculation, although we found the same by another method in the preceding proposition. [b] For since in all the figures of this proposition — and also in the first, which is set down in the scholium — as $K\lambda$, the half of the straight line composed of the sine of the meridian altitude and the sine of the meridian depression, is to λN , the difference between the aforesaid half and the sine of the meridian altitude, so is KM , the whole sine, to a fourth term: if it be made that, as $K\lambda$, the half of the straight line composed of the sine of the meridian altitude and the sine of the meridian depression, is to λN , the difference between the said half and the sine of the meridian altitude, so is KM , the whole sine, to a fourth term, there will be found

the right sine MS, whose arc PQ — added to the quadrant in the northern signs, but subtracted from the quadrant in the southern — will give the semidiurnal arc KQ.

[Margin: [b] Euclid VI.2 or VI.4. — By what method the semidiurnal arc is to be computed otherwise than above.]

The same result we shall obtain by this method. Since in the same figures, as $K\lambda$, the aforesaid half, is to KN , the sine of the meridian altitude, so is KM , the whole sine, to KS , the versed sine of the semidiurnal arc KQ : if it be made that, as $K\lambda$, the half of the straight line composed of the sine of the meridian altitude and the sine of the meridian depression, is to KN , the sine of the meridian altitude, so is KM , the whole sine, to a fourth term, we shall obtain KS , the versed sine of the semidiurnal arc KQ . From this versed sine we shall obtain the semidiurnal arc KQ as follows. In the northern signs, subtract the whole sine from the versed sine found, and add the arc PQ of the remaining right sine to the quadrant; in the southern signs, on the contrary, subtract the versed sine found from the whole sine, and subtract the arc PQ of the remaining right sine from the quadrant again. For always, whether from that addition or from this subtraction, the semidiurnal arc will emerge.

[Margin: How the semidiurnal arc is investigated.]

Otherwise, from the first figure of this proposition. Since, as HA , the sine of the complement of the pole's altitude, is to HE , the whole sine, so is KN , the sine of the meridian altitude, to KS , the versed sine of the semidiurnal arc, in terms of the parts of the whole sine of the greatest circle: if it be made that, as the sine of the complement of the pole's altitude is to the whole sine, so is the sine of the meridian altitude to a fourth term, there will be found the straight line KS in terms of the parts of the whole sine HE . And if again it be made that, as KM [?], the sine of the complement of the declination, is to KM [?], the whole sine, so is KS (as found) to a fourth term, KS will be found as the versed sine of the semidiurnal arc in terms of the parts of the whole sine of the parallel. And the same reasoning holds in the southern parallel, as is clear in the third figure, if one draws from H the sine of the complement of the pole's altitude.

All this, however, must be understood for the case when the complement of the pole's altitude is greater than the declination of that parallel whose semidiurnal arc is being investigated. For when that complement is not greater, either the parallel touches the horizon — and so its diurnal arc, if it is northern, will contain 24 hours, or, if southern, 0 hours 0 minutes — or else the whole northern parallel

stands above the horizon and the southern one below it, so that there will be no diurnal arc at all, but there will be continuous day on the northern parallel and continuous night on the southern. All of this appears in the second figure of the scholium to this proposition, where the complement of the pole's altitude CI is less than the declination IL of the northern parallel, and again the same complement AH is less than the declination HK of the southern parallel. But if that complement were equal to the declination, the northern parallel would touch the horizon at C, and the southern at A, as is evident.

[Margin: How the diurnal arc behaves when the complement of the pole's altitude is not greater than the declination of the parallel in question.]

But so that the demonstration of this proposition may be complete in all its numbers, we shall also give the finding of the twilights in the right sphere, where they are found much more easily. Let ABCD be the meridian of the analemma; AC the diameter of the right horizon, passing through the poles A, C; FG the diameter of the parallel of the horizon on which the beginning or end of twilight is placed; BD the diameter of the equator, cutting FG at H; IK the diameter of the parallel of the Sun (whichever it may be), cutting AC and FG at L and M. From L, the center of the parallel, let the semicircle INK of that parallel be described, and let AC and FG be extended as far as N and O.

Diagram. A small analemma of the right sphere. An outer circle is the meridian ABCD, with B at top, D at bottom, A at the left and C at the right; AC is the horizontal diameter of the right horizon, passing through the poles A and C. A vertical diameter BD is the equator, its center at E. A pair of parallel chords (FG and IK) crosses the circle: FG is the diameter of the horizon-parallel and BD cuts it at H; IK is the diameter of the Sun's parallel, cutting AC and FG at L and M. On L a semicircle INK of the parallel is drawn, and AC, FG are produced to N and O. Readable labels, by approximate position:

- Top: I, B, I
- Left to right across the middle: A, L, E, C, N
- Lower band: F, M, H, M, O
- Bottom: K, D, K
- Left/right extensions: N (left), O

Several small labels (the repeated I, L, M and the extension points N, O) are worn or ambiguous

Therefore AF, or CG, will be the arc of twilight when the Sun is on the equator; its sine is the straight line EH, corresponding to the arc of 18 degrees. Therefore, if the arc of 18 degrees is taken, the arc of twilight in the right sphere will be obtained, the Sun being at the equinoxes.

But when the Sun is on any parallel, say the parallel INK, the arc ON will be the length of twilight. If, then, it be made that, as IL — in so far as it is a part of the whole sine BE of the greatest circle, that is, in so far as it is the sine of the complement of the declination (for the arc IC, or IA, is the complement of the declination) — is to LM, the sine of 18 degrees of the same greatest circle, so is IL, in so far as it is the whole sine in the parallel INK, to a fourth term, then LM will be known in terms of the parts of the whole sine IL; and thus its arc NO, which measures the twilight, will be known. And this is true whether IK is the diameter of a southern parallel or of a northern, as is evident from the figure.

It should not be passed over that we can investigate the twilights of two opposite parallels with almost a single labor, if we use the first method. For since the semidiurnal arc of any parallel is equal to the seminocturnal arc of the opposite parallel — as we showed in the scholium of the preceding proposition — the versed sine KS of the semidiurnal arc of the southern parallel will be equal to the versed sine LS of the seminocturnal arc of the opposite northern parallel, as appears in the following first figure. [c] Moreover, the portion ST of the diameter of the southern parallel is equal to the portion ST of the diameter of the northern parallel. Therefore, if it be made that, as KN, the sine of the meridian altitude of the southern parallel, is to NO, the sine of 18 degrees, so is KS, the versed sine of the semidiurnal arc of the southern parallel, to a fourth term, the straight line ST will be known; [d] since KN has the same ratio to NO as KS has to ST. And with this ST known (which perhaps you will find more easily by the method prescribed at the end of this proposition), if it is added to the versed sine KS of the semidiurnal arc of the southern parallel, there will be produced KT, the sine...

[Margin: By what method the twilights of two opposite parallels are found with a single labor. [c] Euclid I.34. [d] Euclid VI.2.]

versed sine of the arc KR, which is compounded of the semidiurnal arc KQ and the twilight arc QR. Therefore, once the semidiurnal arc KQ has been taken away, the twilight arc QR will remain, which is what marks the twilight. Now this same ST, subtracted from LS — the versed sine of the seminocturnal arc of the opposite northern parallel, which is equal to the versed sine KS of the semidiurnal arc in the southern parallel, as we said — will leave TL, the versed sine of the arc LR. And this TL, subtracted from the seminocturnal arc LQ of the northern parallel

(which we established to be equal to the semidiurnal arc kQ of the southern parallel), leaves the twilight arc QR . So, as you see, by a single multiplication and division the line ST is found in either parallel, and by its help each twilight is drawn out, as we have explained.

We prefer, however, to find the said line ST in the southern parallel rather than in the northern; for the sine kN of the meridian altitude of the southern parallel is smaller than the sine of the meridian altitude of the northern parallel, and the versed sine KS of the semidiurnal arc of the southern parallel is smaller than the versed sine of the semidiurnal arc of the northern parallel. Working in the southern parallel therefore makes the multiplication and division easier, since they are carried out with smaller numbers.

[Margin: When twilight is continuous in the northern parallel.]

One thing must be noted here. When the line ST found in the southern parallel turns out to be greater than the versed sine KS of the semidiurnal arc — which does in fact happen when the sine kN of the meridian altitude is smaller than the sine of 18 degrees, as occurs in the second figure — then twilight is perpetual in the opposite northern parallel. For in that case the line ST cannot be subtracted, in the northern parallel, from LS , the versed sine of the seminocturnal arc; because LS is equal to the versed sine KS of the seminocturnal arc [?] of the southern parallel, and so is itself even smaller than the line ST . Hence on that night the Sun will never reach the parallel of the horizon whose diameter is XT , and so on.

Diagram. A horizontal band of three spherical projections (meridian-plane / solstitial-culure diagrams) belonging to the twilight discussion. Each is a great circle carrying the horizon, meridian, equator, a vertical circle, an oblique parallel of the Sun, and — set below the horizon — a further "parallel of the horizon" used to measure the depression at the start and end of twilight. The construction letters named in the surrounding text are: the main circle A (west), B (top), C (east), D (bottom); centre E ; equator through F ; the Sun's parallel through G, I, H ; the horizon-parallel through K, P ; with auxiliary points L, M, N, O, Q, R, S, T and Greek point-letters (ξ, θ) in the dense central cluster. The three figures differ in the obliquity of the drawn parallel (the rightmost is the most oblique/armillary-like). The outer cardinal letters ($A, B, C, D, H, K, P, G, I, L, O, Q$) are legible; the numerous small interior labels ($E, M, N, O, R, S, T, \xi, \theta$ and repeated $K/L/M$) are worn and cannot be individually resolved at this resolution.

- Outer ring (confirmed): $A, B, C, D, H, K, P, G, I, L, O, Q$

- Interior cluster (present but individually worn — [?]): E, M, N, O, R, S, T, ξ, θ

[— many interior point-labels worn.]

[Margin: How the twilights are to be computed by spherical triangles.]

Finally, so as to leave out nothing that bears on our subject, I should like also to prescribe a method by which the twilights can be computed by spherical triangles. Let it be, then, as in the third figure, that ABCD is the horizon, BD the meridian, AFC the equator, AEC the circle properly called the vertical, and GIH a parallel, whether northern or southern. Let KP be the parallel of the horizon below the horizon in which the Sun stands at the beginning of morning twilight or the end of evening twilight, so that it is 18 degrees distant from the horizon (on the more accepted opinion); and let it cut the Sun's parallel at the point K. Then KG will be the twilight arc, the one the Sun runs through in its own parallel from the beginning of twilight to its end, and GI will be the semidiurnal arc. From M, the arctic pole, draw through G and K two great circles MG, MK, cutting the equator at O and at N. And since, by *Sphaerica* II.10 of Theodosius, the arc FO is similar to the arc IG, and the arc ON similar to the arc GK, FO will be the semidiurnal arc of the parallel GH, and ON the twilight arc — which we shall determine as follows. Through the vertex E and through K, the Sun's place at the beginning of morning or the end of evening twilight, draw the great circle EK, and let it cut the horizon at L; the arc KL will be 18 degrees. Now in the spherical triangle EKM all the sides are known: for EM is the arc of the complement of the pole's altitude; the arc EK is compounded of the quadrant EL and the arc Lk of 18 degrees, so that the whole arc EK contains 108 degrees; and the arc MK, in a northern parallel, is the complement of the declination of the given parallel, while in a southern parallel it is compounded of the quadrant MN and the arc Nk of the declination. Therefore the angle EMK will be known — by the last proposition of Book 4 of Regiomontanus's *De triangulis*, or by proposition 63 of our own *Spherical Triangles* — and hence its arc FN will also be known. If from this the semidiurnal arc FO be subtracted, the remainder will be the twilight arc NO. Which is what was proposed.

PROBLEMA 15. PROPOS. 36.

From the known hour of the day to find the Sun's altitude above the horizon; and, conversely, from the Sun's known altitude to find the hour of the day.

Since in the following books we are going to describe sundials [horologia] by which we recognize the hours of the day, I think many readers will welcome it if

in this last proposition of the present book I show a method by which, without any sundial, we can find those same hours from the Sun's altitude alone, and conversely find the Sun's altitude from the known hour. I do this above all because the matter is of great use both for laying out sundials and, especially, for constructing quadrants, cylinder-dials [Cylindros horarios], and other instruments of that kind on which the hours are depicted — as I shall explain in the proper places.

Let the figures, then, be the same as in the preceding proposition. Place the Sun at the point O in its own parallel, and from O draw OR perpendicular to KL, the diameter of the parallel; through R draw a line parallel to AC, the diameter of the horizon, cutting KN, the sine of the meridian altitude, at T. Then OR will be the common section of the Sun's parallel and of the parallel of the horizon in which the Sun stands at the time of observation. For since each parallel is at right angles to the meridian, [a] their common section will also be at right angles to it, and therefore, by definition 3 of Euclid Book 11, perpendicular to the line KL. Since, then, the Sun at that moment lies in that common section, and is placed at the point O, OR — drawn perpendicular to KL — will be the common section of the said parallels. Hence the parallel of the horizon drawn through the Sun's centre at that time will cut the meridian at R, and consequently the parallel RT will be the common section of the meridian and of the parallel of the horizon. [b] For the meridian, lying in the horizon, makes with any parallel of the horizon two parallel sections. Therefore TN will be the right sine of the Sun's altitude — that is, of the arc lying between AC and the parallel RT in the vertical circle drawn through the Sun's centre, for which the meridian ABCD may be taken. For if it be carried around BD, the axis of the horizon, it will serve the office of all the verticals, as is plain.

[Margin: [a] Euclid XI.19. [b] Euclid XI.16.]

[Margin: How the Sun's distance from the meridian, in degrees, is recognized from a given hour.]

Before all else, then, we must work out the Sun's distance from the meridian in degrees, which any given hour will reveal in the following way. If we are speaking of astronomical hours, the 1st hour from noon — or the 11th from midnight — will be, above the horizon, 15 degrees distant from the meridian, that is, from noon; the 2nd hour from noon, or the 10th from midnight, will be 30 degrees, and so on, since each hour spans 15 degrees. But if the talk is of Italian or Babylonian hours, we must first find the time of noon on the proposed day, as we

taught in chapter 3 of the *Sphaera*, when we were dealing with the semidiurnal arcs.

Diagram. The six figures of Problema 15 / Propos. 36, printed in two rows of three (the "same figures" referred to in the text). Each is a meridian-plane projection of the sphere carrying horizon, meridian, equator, vertical circle, the Sun's oblique parallel, and the parallel of the horizon, with the sine-lines of the construction (KN the sine of the meridian altitude, TN the sine of the Sun's altitude, OR, RT, etc.). The recurring construction letters, taken from the text, are: main circle A (west, left), B (top), C (east, right), D (bottom); centre E; F on the equator; H, K on the upper left; G at lower left; I at lower right; L, O, P, Q on the right; and the interior sine-cluster M, N, R, S, T with Greek point-letters (α , β , γ , δ , ζ , ξ , θ). The six figures show successive cases (Sun's distance from meridian less than / equal to / greater than a quadrant, southern signs, equinox, and the vertical-circle case). Legible outer letters are listed below; the crowded interior labels are worn and cannot be individually confirmed cell-by-cell.

- Row 1, figures 1–3 — outer letters (confirmed): H, K, B, P, O, Q, A, C, E, G, I, L, D
- Row 2, figures 4–6 — outer letters (confirmed): V, B, H, K, F, A, C, O, G, D, plus (fig. 6, an 8-spoke circle) B, O, H, F, A, C, T, R, N, E, G, D
- Interior sine-cluster and Greek point-letters (present but individually worn — [?]): M, N, R, S, T, ξ , θ , α , β , γ , δ , ζ

[— dense interior point-labels and Greek minuscule letters worn.]

we were dealing. Once this is known, it will not be hard to understand how great the given hour's distance from the noon moment is. For example, when noon falls at 17½ hours from sunset and 6½ from sunrise — which happens when the day contains 13 hours — it is clear that the 20th hour (or the 15th) from sunset, and the 4th (or the 9th) from sunrise, is 2½ hours distant from noon, that is 37 degrees 30 minutes, and so on. And finally, if the old, or unequal, hours are proposed, we must divide the diurnal arc by 12 so as to know the length of one unequal hour. From this we shall easily recognize the Sun's distance from the meridian from a given unequal hour. For instance, at the beginning of ♋ [Cancer] the diurnal arc for latitude 42 degrees contains 15 hours 4 minutes [?]; dividing this by 12, we find that one unequal hour comprises 1 hour 15 minutes 10 seconds of equal time. Hence the 4th unequal hour (for example), or the 8th, will be 2 hours 30 minutes 40 seconds distant from noon, that is from the 6th unequal hour; while the 3rd unequal hour, or the 9th, will be 3 hours 46 minutes away from noon, and so with the rest. From these equal hours we shall find the Sun's distance from the

meridian in degrees, as before. Moreover, once the Sun's distance from the meridian is known, its complement will also be known, namely the difference between the quadrant and the said distance.

In another way too we shall find the distances of the hours from rising and setting to the meridian. Since the semidiurnal arc indicates the distance of the 24th hour from the meridian, if we take 15 degrees from it, the distance of the 23rd hour from sunset — and the 1st from sunrise — will remain; if from this we again subtract 15 degrees, we shall have the distance of the 22nd hour from sunset and the 2nd from sunrise; and so on, until the subtraction can no longer be made. For then the last distance found, once 15 degrees have been subtracted, leaves the distance of the following hour on the other side of the meridian circle; and if 15 degrees be added to it, the distance of the next hour is produced, and so on, until the number grows greater than the semidiurnal arc — which then indicates the distance of an hour below the horizon, and so can be of no use. You see an example in the adjoined table, with the Sun at the beginning of ♋ [Cancer], where the semidiurnal arc for an elevation of 42 degrees contains 109 degrees 21 minutes, that is, 7 hours and 17 minutes. So, if you reduce an hour's distance from the 24th hour to degrees and minutes, and subtract that from the semidiurnal arc, its distance after noon will remain; but if you cannot subtract it, take the semidiurnal arc away from it, and its distance before noon will remain. This you should understand of the hours from sunset; for the hours from sunrise the reverse is to be understood. For example, the 19th hour from sunset is distant from the 24th hour by 5 hours, which make 75 degrees; these subtracted from the semidiurnal arc of 109 degrees 21 minutes leave 34 degrees 21 minutes, the distance of the 19th hour after noon. Likewise the 15th hour is 9 hours away from the 24th, that is 135 degrees; if from these you take the semidiurnal arc of 109 degrees 21 minutes, there will remain a distance before noon of 25 degrees 39 minutes. So too, because the 3rd hour from sunrise is 3 hours from the 24th, that is 45 degrees, these subtracted from the semidiurnal arc of 109 degrees 21 minutes leave a distance from the meridian, before noon, of 64 degrees 21 minutes. But the 13th hour from sunrise is distant from the 24th by 13 hours, that is 195 degrees; from these the semidiurnal arc of 109 degrees 21 minutes being subtracted leaves a distance from the meridian, after noon, of 85 degrees 39 minutes.

Table — full-width numeric table.

The four inner columns are, left to right: hours from setting; G. (degrees); M. (minutes); hours from rising. The outer vertical brackets label the groups: on the

left, the upper block (rows 24–17) is "Hours after noon" and the lower block (rows 16–7) is "Hours before noon"; on the right, the upper block is "Hours before noon" and the lower block is "Hours after noon"; the foot of both brackets is marked "below the horizon" (sub horiz.).

Hours from setting	G. (deg.)	M. (min.)	Hours from rising
24	109	21	24
23	94	21	1
22	79	21	2
21	64	21	3
20	49	21	4
19	34	21	5
18	19	21	6
17	4	21	7
16	10	39	8
15	25	39	9
14	40	39	10
13	55	39	11
12	70	39	12
11	85	39	13
10	100	39	14
9	115	39	15
8	130	39	16
7	145	39	17

[Margin: How the Sun's altitude above the horizon is computed from a known hour. — When the Sun's distance from the meridian in a northern parallel is less than a quadrant.]

Since, then, it holds that as kM , the whole sine, is to MR , the sine of the complement of the Sun's distance from the meridian, so $k\lambda$, half the line compounded of the sine of the meridian altitude and the sine of the meridian depression, is to the line λT : if it be arranged that, as the whole sine is to the sine of the complement of the Sun's distance from the meridian, so $k\lambda$ (half the line compounded of the sine of the meridian altitude and the sine of the meridian depression) is to a fourth term, the line λT will be found — namely the difference

between TN, the sine of the Sun's altitude at the time of observation, and the line λN . This difference is the one between the aforesaid half-line $k\lambda$ and KN, the sine of the meridian altitude. From this line λT we shall find TN, the sine of the Sun's altitude, and hence the Sun's altitude itself, in the following way. In the northern parallels, when the Sun's distance from the meridian is less than a quadrant, that is less than six hours, add the line λT (once found) to λN , the difference between the said half-line and the sine of the meridian altitude. For in this way the sine TN of the Sun's altitude will be compounded, as appears in the first figure and the third.

Diagram. The same six figures as on the facing page (see fig-p107-1), again printed in two rows of three: meridian-plane projections of the sphere for the successive cases of Problema 15 / Propos. 36. Construction letters as before — main circle A, B, C, D; centre E; equator through F; H, K upper left; G lower left; I lower right; L, O, P, Q on the right; interior sine-cluster M, N, R, S, T with Greek point-letters (α , β , γ , δ , ζ , ξ , θ). The first and third figures illustrate the "less than a quadrant" case; the second, the "greater than a quadrant"; the fourth shows the vertical-circle construction (with the northern parallel's diameter VX, points Y, Z); the sixth is the equinoctial case (8-spoke circle, letters H, E, R, N, T). Legible outer letters are confirmed below; the crowded interior labels are worn.

- Outer letters (confirmed): A, B, C, D, H, K, P, O, Q, G, I, L, V, F
- Interior sine-cluster and Greek point-letters (present but individually worn — [?]): M, N, R, S, T, ξ , θ , α , β , γ , δ , ζ , and (fig. 4) X, Y, Z

[— dense interior point-labels and Greek minuscule letters worn.]

[Margin: When the Sun's distance from the meridian in a northern parallel equals a quadrant. [a] Euclid VI.4. [b] Euclid I.34.]

But if the Sun's distance from the meridian contains a quadrant, that is six hours, then the difference between the said half-line and the sine of the meridian altitude — namely the line λN — is itself the sine of the Sun's altitude, as is clear from the same figures. For then the Sun stands at the point P of its parallel, and so the line λM will be a portion of the diameter of the parallel of the horizon, and so on. Hence, if the aforesaid half be subtracted from the sine of the meridian altitude, the sine of the Sun's altitude will remain.

We shall also find the Sun's altitude in another way when it is six hours from the meridian. In the first figure draw from M and F two perpendiculars $M\alpha$, $F\beta$ to AC. [a] Since it holds that, as EF, the whole sine, is to $F\beta$, the sine of the pole's altitude, so EM, the sine of the declination, is to $M\alpha$, the sine of the Sun's altitude [b] (for

Mα is equal to the sine λN of the Sun's altitude) — if it be arranged that, as the whole sine is to the sine of the pole's altitude, so the sine of the declination is to a fourth term, the sine of the Sun's altitude will be found.

[Margin: When the Sun's distance from the meridian in a northern parallel is greater than a quadrant. — When the Sun is in a southern parallel.]

But if the Sun's distance from the meridian exceeds a quadrant, that is six hours, as is seen in the second figure, then the line λT (once found) must be subtracted from λN, the difference between the said half-line and the sine of the meridian altitude, so that TN, the sine of the Sun's altitude, may be obtained.

In the southern signs, finally, the difference between the said half-line and the sine of the meridian altitude — that is, the line λN — must always be subtracted from the line λT once found, so that TN, the sine of the Sun's altitude, may remain, as is clear from the fourth figure and the fifth.

[Margin: How the Sun's altitude above the horizon at the equinoxes is to be reckoned from a given hour. [c] Euclid VI.2 or VI.4.]

Furthermore, when the Sun is at the equinoxes, we shall obtain its altitude much more briefly from a given hour. [c] For since, in the sixth figure, it holds that as HE, the whole sine, is to RE, the sine of the complement of the Sun's distance from the meridian, so HN, the sine of the complement of the pole's altitude, is to TN, the sine of the Sun's altitude — if it be arranged that, as the whole sine is to the sine of the complement of the Sun's distance from the meridian, so the sine of the complement of the pole's altitude is to a fourth term, the sine of the Sun's altitude at the time of observation will be obtained.

[Margin: How the Sun's altitude in the vertical circle is found from a given hour, even if the hour is unknown. [d] Euclid VI.2.]

Likewise, we shall find without much labor the Sun's altitude when it stands in the vertical circle properly so called, even if we do not know the hour. In the fourth figure draw the diameter VX of the northern parallel — for only in northern parallels can the Sun stand above the horizon in the vertical — cutting the axis at Y and the vertical's diameter at the point Z, and through Z draw the diameter of the parallel of the horizon, so that ZE is the sine of the Sun's altitude when it is in the vertical. Then, having drawn Dγ perpendicular to HE, cutting VX at δ, [d] since it holds that, as Dγ, the sine of the pole's altitude, is to γδ, the sine of the declination of the proposed parallel, so DE, the whole sine, is to EZ, the sine of the Sun's altitude — if it be arranged that, as the sine of the pole's altitude is to

the sine of the declination, so the whole sine is to a fourth term, we shall find the sine of the Sun's altitude in the vertical. Let this be an example of the matter. With the Sun at the beginning of ♊ [Capricorn; sign-glyph worn — pinned by the in-text declination of $23^{\circ}30'$; note that the vertical-circle case requires a northern parallel, so this may instead be ♋ Cancer], in the Roman horizon, or at latitude 42 degrees: let it be arranged that, as 66913, the sine of the pole's altitude, is to 39874, the sine of the declination — which contains 23 degrees...

30 minutes, so 100000, the whole sine, is to a fourth term; and we shall find about 59591, the sine of the altitude sought, to which corresponds an arc of 36 degrees 35 minutes, for the Sun's altitude in the vertical circle at the given latitude.

But if the declination of the northern parallel equals the pole's altitude — that is, the arc BH — then the parallel will touch the vertical circle at B, the Zenith, as is plain. Hence the Sun's altitude in the vertical will then amount to 90 degrees. But if the declination is greater than the pole's altitude, the parallel will not cut the vertical circle, as is clear from the third figure. Therefore the Sun will then never reach the vertical. And this is enough about the first part of this problem. But before we proceed to the second part, let us illustrate the whole matter with a few examples.

[Margin: First example.]

Suppose, then, that the Sun is at the beginning of Gemini [♊] or Leo [♌], when of course the meridian altitude is 68 degrees 12 minutes and the meridian depression 27 degrees 48 minutes, while the day contains about 14 hours 35 minutes — so that noon falls at hour 16 minutes 43 by the Italian reckoning, and at hour 7 minutes 17 by the Babylonian. And suppose we seek, in the Roman horizon (that is, at latitude 42 degrees), the Sun's altitude at hour 20 from sunset, or at hour 4 from sunrise, each of which is distant from the meridian by 3 hours 17 minutes, that is, by 49 degrees 15 minutes. Let it be arranged that, as 100000, the whole sine, is to 65275, the sine of the complement of the Sun's distance from the meridian, so 69743, the half of the sum compounded of the sine of the meridian altitude and the sine of the meridian depression, is to a fourth; and this number will be found to be about 45524. Added to 23105 — the difference between the said half of the compounded sum and the sine of the meridian altitude — because the Sun is placed on a northern parallel and its distance from the meridian is less than 6 hours, it makes 68629. The arc of this, 43 degrees 20 minutes, will give the Sun's altitude at the proposed hour.

[Margin: Second example.]

Again, with the Sun on the same parallel, suppose we seek its altitude at hour 6 from the meridian, that is, at midnight. The difference between 69743 — the half of the sum compounded of the sine of the meridian altitude and the sine of the meridian depression — and 92848, the sine of the meridian altitude, namely 23105, will be the sine of the Sun's altitude sought; for the Sun is then northern and has a distance from the meridian equal to a quadrant, or 6 hours. Therefore the Sun's altitude will then be about 13 degrees 22 minutes. This too can be found in the following way, as we showed before. Let it be arranged that, as 100000, the whole sine, is to 66913, the sine of the pole's altitude, so 34529, the sine of the declination, is to a fourth; and this sine of the Sun's altitude will be found to be about 23104, as above.

[Margin: Third example.]

Moreover, with the Sun tracing the same parallel, suppose we seek its altitude at hour 23 minutes 30 from sunset, or at hour 0 minutes 30 from sunrise, so that its distance from the meridian is 6 hours 47 minutes, that is, 101 degrees 45 minutes. Let it be arranged that, as 100000, the whole sine, is to 20364, the sine of the complement of the Sun's distance from the meridian, so 69743, the half of the sum compounded of the sine of the meridian altitude and the sine of the meridian depression, is to a fourth; and this number will be found to be about 14202. Subtracted from 23105 — the difference between the aforesaid half and the sine of the meridian altitude — because the Sun, being on a northern parallel, is here supposed to have a distance from the meridian greater than 6 hours, it leaves 8903, the sine of the Sun's altitude at that hour; and so the Sun's altitude will amount to 5 degrees 7 minutes.

[Margin: Fourth example.]

Finally, suppose the Sun's altitude is to be found at hour 3 after noon, or at hour 9 after midnight, the Sun being at the beginning of Sagittarius [♐]; the sign-sort is worn — pinned by the in-text values: the meridian altitude here is 27°48' (sine 46638) giving declination $\square$ -20°12', the southern counterpart of Gemini/Leo] or Aquarius [♒]. Let it be arranged that, as 100000, the whole sine, is to 70710, the sine of the complement of the Sun's distance from the meridian, so 69743, the half of the sum compounded of the sine of the meridian altitude and the sine of the meridian depression — this being the same as before, since Sagittarius [♐] and Aquarius [♒] are signs opposite to the signs Gemini [♊] and Leo [♌]; for it is established from what has been said that the meridian altitude of one sign is the meridian depression of the other, and conversely — is to a fourth; and this number will be found to be very nearly 49315. If from it 23105 be taken — the

difference between the aforesaid half and the sine of the meridian altitude, because the Sun is placed on a southern parallel — there will remain 26210, the sine of the Sun's altitude; and the arc of this, 15 degrees 12 minutes, will give the Sun's altitude at that time. And so the method by which we can pass from a known hour to knowledge of the Sun's altitude is plainly evident. Let us now explain the other part of the proposition: namely, by what method we may determine the hour from the Sun's known altitude. This we shall carry out as follows.

[Margin: [a] Euclid VI.2.] [Margin: By what method the hour is found from the Sun's altitude above the horizon once known.]

[a] Since, in the same figures, it holds that, as $K\lambda$ — the half of the sum compounded of the sine of the meridian altitude and the sine of the meridian depression — is to λT , the difference between TN (the sine of the Sun's altitude) and λN (the difference between the aforesaid half and the sine of the meridian altitude), so KM , the whole sine, is to MR , the sine of the complement of the Sun's distance from the meridian: if it be arranged that, as the half of the sum compounded of the sine of the meridian altitude and the sine of the meridian depression is to the difference which the sine of the Sun's altitude and the difference between the said half and the sine of the meridian altitude have between themselves, so the whole sine is to a fourth, we shall obtain the sine of the complement of the Sun's distance from the meridian. And so the complement of the Sun's distance from the meridian will itself be known, by means of which we shall determine the Sun's distance from the meridian, and hence also the hour at the time of observation, in this way. When the Sun is northern and the sine of the Sun's altitude exceeds the difference between the said half and the sine of the meridian altitude — as appears in the first and third figures — let the complement of the Sun's distance from the meridian, once found, be subtracted from the quadrant, and there will remain the Sun's distance from the meridian in degrees.

But if, with the Sun northern, the sine of the Sun's altitude turns out equal to the difference between the aforesaid half and the sine of the meridian altitude, the Sun will be distant a whole quadrant, or six hours, from the meridian, as can be made clear from the three earlier figures. For if λN — the difference between KN , the sine of the meridian altitude, and $K\lambda$, the aforesaid half — be set as the sine of the Sun's altitude, so that λM is the portion of the diameter of the horizon's parallel passing through the Sun, the quadrant KP will give the Sun's distance from the meridian.

But if, with the Sun northern, the sine of the Sun's altitude comes out less than the difference between the aforesaid half and the sine of the meridian altitude — as happens in the second figure — let the complement of the Sun's distance from the meridian, once found, be added to the quadrant, and the Sun's distance from the meridian in degrees will be made up.

Finally, with the Sun southern, the complement of the Sun's distance from the meridian, once found, must always be deducted from the quadrant, so that the Sun's distance from the meridian in degrees may remain, as is evident from the fourth and fifth figures. For if from the quadrant KP the complement OP of the Sun's distance from the meridian be subtracted, there will remain KO, the Sun's distance from the meridian.

[Margin: [b] Euclid VI.2 or VI.4.] [Margin: By what method, at the equinoctial time, the hour is discovered from the Sun's altitude above the horizon once known.]

This distance of the Sun from the meridian we shall discover more easily from the Sun's known altitude when the Sun is crossing the Equator. For, as is clear from the sixth figure, [b] it holds that, as HN, the sine of the complement of the pole's altitude, is to TN, the sine of the Sun's altitude at the time of observation, so HE, the whole sine, is to RE, the sine of the complement of the Sun's distance from the meridian. Therefore, if it be arranged that, as the sine of the complement of the pole's altitude is to the sine of the Sun's altitude at the time of observation, so the whole sine is to a fourth, the sine of the complement of the Sun's distance from the meridian will be found, and thus the distance itself will not be able to lie hidden.

[Margin: Example.]

Let us explain the matter by one example. Suppose the Sun is at the beginning of Gemini [♊] or Leo [♌], and that we have found the Sun's altitude before noon to be 43 degrees 20 minutes, from which the hour at the time of that observation is to be explored. Let it be arranged that, as 69743 — the half of the sum compounded of the sine of the meridian altitude and the sine of the meridian depression — is to 45519, the difference between the sine of the Sun's altitude and the difference between the aforesaid half and the sine of the meridian altitude, so 100000, the whole sine, is to a fourth. For there will come out about 65267, the sine of the complement of the Sun's distance from the meridian, whose arc, 40 degrees 45 minutes, subtracted from the quadrant — because the Sun is northern and the sine of the Sun's altitude, namely 68624, exceeds the difference (namely

23105) which is found between the often-mentioned half and the sine of the meridian altitude — will leave 49 degrees 15 minutes for the Sun's distance from the meridian, which comprises 3 hours 17 minutes. Since, then, the observation is made before noon, it will then be hour 8 minutes 43 after midnight, in the manner of the astronomers. And if the observation had been made after noon, it would be hour 3 minutes 17 from the meridian. But according to the Italians it will be hour 13 minutes 26 from sunset, because on that day noon falls at hour 16 minutes 43; and if from that noon the Sun's distance from the meridian just found, namely 3 hours 17 minutes, be taken away, there will remain 13 hours 26 minutes. But if the observation were made after noon, this distance would have to be added, so that the hour from sunset might come out known; and the hour would then be found to be 20 from sunset. Finally, according to the Babylonians it will be hour 4 minutes 0 from sunrise. For since on that day noon falls, according to these hours, at hour 7 minutes 17, if from this meridian time the Sun's distance from the meridian found, namely 3 hours 17 minutes, be subtracted, there remains hour 4. But if the observation were made after noon, this distance would have to be added, and thus the hour would be found to be 10 minutes 43 from sunrise. From this example the rest will be understood without effort.

[Margin: How great the hour is when the Sun is in the vertical circle, and by what method to determine it.] [Margin: [a] Euclid XI.19.]

With the Sun in the vertical circle, we shall likewise, over and above the method so far handed down, arrive at the hour from its altitude. In the fourth figure, from the point Z — where the parallel of the Sun intersects the vertical — draw $Z\beta$ perpendicular to VX , as far as the circumference of the parallel of the Sun described about the diameter VX from the center Y ; then $Z\beta$ will be the common section of the parallel of the Sun and the parallel of the horizon, whose semidiameter is μZ — which will become clear as follows. Since each parallel is perpendicular to the meridian, [a] their common section will also be perpendicular to the same, and hence, by definition 3 of Euclid's Book XI, perpendicular to VX at the point Z , where the said parallels cut one another in the meridian. Therefore $Z\beta$, being perpendicular to VX at Z , will be the common section of those parallels. From which it follows that, since Z is the center of the parallel of the horizon, whose semidiameter is μZ , the line $Z\beta$ is equal to the semidiameter μZ — inasmuch as all lines drawn from Z to the circumference of the parallel whose semidiameter is μZ (such as is also $Z\beta$, its common section with the parallel of the Sun, as was shown) are equal to the semidiameter μZ . Now since μZ is the sine of the complement of the Sun's altitude when it is in the vertical circle — namely the sine of the arc $B\mu$ — it will hold that, as VY , the sine

of the complement of the declination of the parallel VX (insofar as VY is a part of the whole sine AE of the greatest circle), is to VY insofar as it is the whole sine in its own parallel, so βZ , insofar as it is a part of the same whole sine AE of the greatest circle (and this is indeed equal to the sine μZ , the complement of the Sun's altitude in the vertical, as we have demonstrated), is to βZ insofar as it is a part of the whole sine VY. Therefore, if it be arranged that, as the sine of the complement of the declination of the proposed parallel is to the whole sine, so the sine of the complement of the Sun's altitude when it is in the vertical is to a fourth, $Z\beta$ will become known — namely the sine of the Sun's distance from the meridian. By this method, with the Sun tracing the parallel of Cancer [♋], we find hour 4 minutes 4, a little more, before and after noon, while it is in the vertical circle, at latitude 42 degrees.

[Margin: How to tell whether the Sun, being in the northern signs, has reached the vertical circle or not.]

From these hours you will easily infer at what hour after midnight, or from sunset or sunrise, it is when the Sun holds the vertical, provided the time of noon be known beforehand in the manner of the Italians and Babylonians. Again, from a known hour before or after noon on any day, with the Sun in the northern signs, you will gather whether the Sun has reached the vertical circle, or has not yet come to it, or indeed has already passed it. For when, in the forenoon, the known hour indicates a greater distance from the meridian than was found at the vertical circle (as, in our example, if it should indicate more than 4 hours 4 minutes), it is evident that the Sun has not yet come to the vertical by the motion of the prime mover; when it shows an equal distance, that the Sun is in the vertical; and when, finally, it shows a smaller distance, that the Sun has already passed the vertical and is placed between the vertical and the meridian on the southern side. In the afternoon you are to understand the contrary.

[Margin: How the Sun's altitude above the horizon is investigated from a known hour by a method other than the one above.] [Margin: [b] Euclid VI.2 or VI.4.]

We can carry out this whole problem in the following way as well. [b] Since, in the same figures, all set out above, it holds that, as KS, the versed sine of the semidiurnal arc, is to RS, the difference between the versed sine of the semidiurnal arc and the versed sine of the Sun's distance from the meridian, so KN, the sine of the meridian altitude, is to TN, the sine of the Sun's altitude: if it be arranged that, as the versed sine of the semidiurnal arc is to the difference between the versed sine of the semidiurnal arc and the versed sine of the Sun's distance from the meridian, so the sine of the meridian altitude is to a fourth, the

sine of the Sun's altitude sought will be found, and thus the altitude itself will be manifest.

[Margin: How the Sun's altitude at the equinoxes is known from a given hour.]

At the Equator, moreover — as appears from the sixth figure — the versed sine of the semidiurnal arc is the whole sine HE, and the difference between it and the versed sine of the Sun's distance from the meridian is RE, the sine of the complement of the Sun's distance from the meridian; and the sine of the meridian altitude is the sine of the complement of the pole's altitude. Hence, if it then be arranged that, as the whole sine is to the sine of the complement of the Sun's distance from the meridian, so the sine of the complement of the pole's altitude is to a fourth, the sine of the Sun's altitude at the time of observation will be obtained, as was also demonstrated above.

[Margin: By what method the hour is investigated from the Sun's altitude above the horizon once known.] [Margin: [c] Euclid VI.2 or VI.4.]

Again, [c] since it holds that, as kN, the sine of the meridian altitude, is to TN, the sine of the Sun's altitude, so KS, the versed sine of the semidiurnal arc, is to RS, the difference between the versed sine of the semidiurnal arc and the versed sine of the Sun's distance from the meridian: if it be arranged that, as the sine of the meridian altitude is to the sine of the Sun's altitude found at the time of observation, so the versed sine of the semidiurnal arc is to a fourth, a number will be found which, subtracted from the versed sine of the semidiurnal arc, leaves the versed sine of the Sun's distance from the meridian. From this versed sine we shall then find the Sun's distance from the meridian, and thence the hour, by this method. When the versed sine of the Sun's distance from the meridian, once found, is less than the whole sine — as always happens in the southern signs, and in the northern signs too whenever the Sun's distance from the meridian is less than a quadrant — we shall subtract it from the whole sine, and shall subtract from the quadrant the arc of the right sine of the remainder, and the Sun's distance from the meridian will remain. But if the said versed sine turns out equal to the whole sine — which can happen only twice in any diurnal arc of the northern parallels — the Sun's distance from the meridian will be a quadrant, comprising six hours. If, finally, the versed sine found turns out greater than the whole sine, we shall subtract...

from it the whole sine, and we shall add to the quadrant the arc of the right sine of the remainder, and so complete the Sun's distance from the meridian.

With the Sun at the equinoxes — as is manifest from the sixth figure — the sine of the meridian altitude is the sine of the complement of the pole's altitude, the versed sine of the semidiurnal arc is the whole sine HE, and the difference between the versed sine of the semidiurnal arc and the versed sine of the Sun's distance from the meridian is the sine of the complement of the Sun's distance from the meridian. Therefore, if it then be arranged that, as the sine of the complement of the pole's altitude is to the sine of the Sun's altitude at the time of observation, so the whole sine is to a fourth, the sine of the complement of the Sun's distance from the meridian will be obtained. And this too was demonstrated above.

[Margin: Which hour it is: how this is to be known at the equinoxes from a given altitude of the Sun. Example.]

EXAMPLE. Let us suppose the Sun on the parallel of Sagittarius [$\times$]; worn sign-sort — pinned by the meridian-altitude value $27^{\circ}48'$ (sine 46638) $\rightarrow$ declination $\approx -20^{\circ}12'$ or Aquarius [$\approx$], having after noon an altitude above the horizon of 6 degrees 45 minutes. Let it be arranged that, as 46638, the sine of the meridian altitude, is to 11753, the sine of the Sun's altitude found, so 66867, the versed sine of the semidiurnal arc, is to a fourth; and there will come out this number, nearly 16851, which, subtracted from 66867, the versed sine of the semidiurnal arc, leaves 50016, the versed sine of the Sun's distance from the meridian. Since this is less than the whole sine, we shall subtract it from 100000, the whole sine, and from the quadrant we shall take away the arc, 29 degrees 59 minutes, of the right sine 49984 of the remainder; and we shall have the Sun's distance from the meridian as 60 degrees 1 minute, that is, 4 hours 0 minutes after noon, because the observation is supposed to have been made after noon. Before noon it would be hour 8 after midnight. And once the hour of noon is known, in the manner of the Italians and Babylonians, from the length of the day, we shall easily discover what hour from sunset or from sunrise corresponds to the fourth hour from the meridian, or the eighth after midnight — that is, the fourth before noon — and so on.

[Margin: How the Sun's altitude in the right sphere is elicited at the equinoxes from a given hour.]

For the rest, in the right sphere this whole problem will be very easy. For let the Meridian be ABCD, its center E; the diameter of the right horizon AC, passing through the poles A, C; the diameter of the Equator BD; the diameter of any parallel of the Sun FG, cutting the diameter of the horizon at H; the diameter of the parallel of the horizon Ik, cutting BD and FG at L and M — this parallel

passing at the time of observation through the center of the Sun. And finally, from H, the center of the parallel of the Sun, let a semicircle FNOG be described, which IK and AC, produced, cut at N and O.

Diagram. A circle representing the Meridian ABCD, with center E, drawn symmetrically (mirror-imaged left and right, so most construction letters appear twice). Its horizontal diameter is the diameter of the right horizon AC (through the poles A on the left, C on the right); its vertical diameter is the diameter of the Equator, B at the top and D at the bottom. A chord parallel to AC, above center, is the diameter of the parallel of the horizon; a second chord parallel to AC, at center level, is the diameter of the parallel of the Sun. The upper chord is drawn as a semicircle band (FNOG) above the horizon diameter. Cross-hatching of horizontal/vertical rule-lines divides the interior into the construction cells.

- B (top center): top of the Equator diameter
- F, F (upper, flanking B): endpoints of the Sun's-parallel semicircle, on the circumference near the top
- N, N (upper band, near the ends): where IK and AC produced cut the semicircle
- I (upper band, left) / K (upper band, right): endpoints of the parallel-of-horizon diameter Ik
- M, M (upper band, inner, on the Sun's-parallel line): where Ik cuts FG
- L (upper band, center, on BD): where Ik cuts the Equator diameter BD
- O, O (horizon line, outer ends): where IK and AC produced cut the semicircle
- A (horizon line, left) / C (horizon line, right): the poles, ends of the horizon diameter
- H, H (horizon line, inner): where the Sun's parallel FG cuts the horizon diameter AC
- E (center): center of the Meridian
- G, G (bottom, flanking D): lower endpoints of the Sun's-parallel construction, on the circumference near the bottom
- D (bottom center): bottom of the Equator diameter

[Margin: The hour of the day which [?] at the equinoctial time is to be investigated in the right sphere from the Sun's known altitude.]

Therefore the arc AI, or CK, will be the altitude of the Sun when it is at the Equator and on the parallel of the horizon whose diameter is IK — that is, at the point I or k — as is evident, if the Meridian ABCD be turned about the axis of the

horizon BD, so that it is joined with the Equator of the right sphere and with the vertical. For the Equator, in the right sphere, is then also a vertical circle, measuring the Sun's altitudes at the equinoxes. Wherefore, since BI, or BK, is the Sun's distance from the meridian, the arc IA, or kC — that is, the complement of the Sun's distance from the meridian — will be the Sun's altitude. Hence, if the Sun's altitude be sought from a known hour in the right sphere, the Sun being at the equinoxes, we must take the complement of the Sun's distance from the meridian for the Sun's altitude.

[Margin: How the Sun's altitude, when it is on any parallel, is found in the right sphere from a given hour.]

But with the Sun placed on the parallel FNOG, and on the parallel of the horizon whose diameter is IK — that is, at the point N — in the same right sphere, the arc FN will be the Sun's distance from the meridian, and HM will be equal to the sine of the arc AI, or Ck, which measures the Sun's altitude at the time of observation, as is established if the Meridian ABCD be revolved about the axis of the horizon BD, so as to take the place of all the verticals in turn. If, then, it be arranged that, as FH, the whole sine of the Sun's parallel, is to MH, the sine of the complement of the Sun's distance from the meridian, so FH, insofar as it is a part of the whole sine BE in the greatest circle — that is, insofar as it is the sine of the complement of the declination of the Sun's parallel — is to a fourth, HM will be found in the parts of the same whole sine BE, and thus insofar as it is the sine of the Sun's altitude AI, or CK. The Sun's altitude AI, or CK, will therefore come out known. And so the first part of the problem has been explained.

[Margin: The hour of the day, at the equinoctial time, is to be investigated in the right sphere from the Sun's known altitude.]

Now from the Sun's known altitude we shall obtain the hour, in the same right sphere, in this way. With the Sun at the equinoxes, we shall take the complement of the Sun's altitude for its distance from the meridian, and so on, as is clear from the figure. For BI, or BK, the Sun's distance from the meridian, is the complement of the Sun's altitude AI, or CK.

[Margin: By what method the hour of the day, outside the equinoctial time, is to be inquired in the right sphere from the Sun's known altitude.]

But with the Sun placed outside the Equator, on any parallel FNOG: let it be arranged that, as FH, the sine of the complement of the declination, is to HM, the sine of the Sun's altitude, so FH, insofar as it is the whole sine in the parallel of the Sun, is to a fourth; and MH will be obtained in the parts of the same whole

sine — insofar, that is, as it is the sine of the complement of the Sun's distance from the meridian; and thus the complement of this distance, NO, will be known. Therefore, with this complement NO taken away from the quadrant, the remainder will be the Sun's distance from the meridian, FN.

[Margin: How the Sun's altitude is elicited from the hour by yet another method in the oblique sphere. [a] Euclid I.29.]

But by this method too, if you please, we shall carry out the whole problem in the oblique sphere. In the first figure of this proposition, from the point R — where the parallel of the Sun and the parallel of the horizon intersect one another — draw a perpendicular $R\Diamond[?]$ to the diameter of the horizon AC. Then, in the triangle $RS\Diamond[?]$, the line $R\Diamond[?]$ will be the sine of the complement of the pole's altitude — namely of the angle $RS\Diamond[?]$, [a] which is equal to the angle AEH — if RS be set as the whole sine. We shall therefore first find the line RS in the parts of the whole sine of the greatest circle in the sphere, in this way. Let it be arranged that, as KM, the whole sine in the parallel of the Sun, is to KM insofar as it is the sine of the complement of the declination of the parallel KL, so RS — the difference between KS, the versed sine of the semidiurnal arc, and KR, the versed sine of the Sun's distance from the meridian — is to a fourth. For by this method RS will be found, known in the parts of the whole sine of the greatest circle. Then let it be arranged that, as RS, the whole sine, is to $R\Diamond[?]$, the sine of the complement of the pole's altitude, so RS, insofar as it has been made known in the parts of the whole sine of the greatest circle, is to a fourth; and $R\Diamond[?]$ will be found, known in the parts of the same whole sine of the greatest circle. Since, therefore, $R\Diamond[?]$ is the sine of the Sun's altitude, the Sun's altitude itself will be known.

Besides this: let it be arranged that, as $R\Diamond[?]$, the sine of the complement of the pole's altitude, is to RS, the whole sine, so $R\Diamond[?]$, insofar as it is the sine of the Sun's known altitude, is to a fourth; and RS will be found, known in the parts of the whole sine of the greatest circle. Hence we shall find the same also in the parts of the whole sine KM, in the proposed parallel of the Sun — insofar, that is, as RS is the difference between KS, the versed sine of the semidiurnal arc, and KR, the versed sine of the Sun's distance from the meridian — in this way. Let it be arranged that, as KM, the sine of the complement of the declination of the parallel kL, is to KM, the whole sine in the parallel, so RS, insofar as it has been made known in the parts of the whole sine of the greatest circle, is to a fourth; and RS will be found, known, insofar as it is the difference between the versed sine...

the versed sine of the semidiurnal arc, and the versed sine of the Sun's distance from the meridian. And therefore the versed sine of the Sun's distance from the

meridian will also become known — namely if RS, found by this method, is subtracted from KS, the versed sine of the semidiurnal arc — and by this means the Sun's distance from the meridian itself will be known.

[Margin: How the hour is found from the Sun's known altitude.]

But of all the methods this one will perhaps be the most convenient. [Margin: a — Euclid VI.2 or VI.4.] Since at the outset of this proposition it was laid down, among the first five figures, that as KM (the whole sine in the Sun's parallel) is to KR, so is Kλ (half of the line compounded of the sine of the meridian altitude and the sine of the meridian depression) to KT; [Margin: b — Euclid VI.2.] and that as kR is to RS — the difference between KS, the versed sine of the semidiurnal arc, and KR, the versed sine of the Sun's distance from the meridian — so is KT to TN, the right sine of the Sun's altitude: then *ex aequo* it will follow that as KM (the whole sine in the Sun's parallel) is to RS — the difference between the versed sine of the semidiurnal arc and the versed sine of the Sun's distance from the meridian — so is Kλ (half of the line compounded of the sine of the meridian altitude and the sine of the meridian depression) to TN, the right sine of the Sun's altitude.

Therefore, if we set it up so that as the whole sine is to that difference between the versed sine of the semidiurnal arc and the versed sine of the Sun's distance from the meridian, so is that half-line compounded of the sine of the meridian altitude and the sine of the meridian depression to a fourth term, the fourth term found will be the right sine of the Sun's altitude that is sought; and so the altitude itself becomes known.

[Margin: How the altitude of the Sun may otherwise be found from the hour.]

Conversely, if it is set up so that as Kλ (the aforesaid half-line) is to TN (the sine of the Sun's altitude), so is KM (the whole sine) to a fourth term, then RS will be found — the difference between the versed sine of the semidiurnal arc and the versed sine of the Sun's distance from the meridian. When this difference is subtracted from the versed sine of the semidiurnal arc, the remainder will be kR, the versed sine of the Sun's distance from the meridian, and so on.

[Margin: How the hour is to be reckoned from the altitude of the Sun otherwise than was taught above.]

Or else we shall accomplish the same task in this way (which I myself find more satisfactory). [Margin: c — Euclid VI.2 or VI.4.] Since as KM (the whole sine in the Sun's parallel) is to KR (the versed sine of the Sun's distance from the meridian), so is Kλ (half of the line compounded of the sine of the meridian altitude and the sine of the meridian depression) to KT — the difference between kN, the sine of

the meridian altitude, and TN, the sine of the Sun's altitude at the time of observation: then, if it is set up so that as the whole sine is to the versed sine of the Sun's distance from the meridian, so is that half-line compounded of the sine of the meridian altitude and the sine of the meridian depression to a fourth term, the value of the line kT will be found, which, subtracted from the sine of the meridian altitude, will leave the sine of the Sun's altitude.

[Margin: By what method the altitude of the Sun may otherwise be found from the hour.]

Likewise, if conversely it is set up so that as $k\lambda$ (the aforesaid half-line) is to KT — the difference between the sine of the meridian altitude and the sine of the Sun's altitude, which is taken as known — so is kM (the whole sine) to a fourth term, then KR will be known, the versed sine of the Sun's distance from the meridian, and so on.

[Margin: By what method the hour is otherwise to be reckoned from the altitude of the Sun.]

This last method has the widest application. For it applies also to that parallel of the Sun which either touches the horizon or stands wholly above it, as we shall state at the end of the scholium of this proposition. Add to this that it holds good not only for the horizon, but for all other planes as well — whether they are erect to the horizon or inclined — as will be clear from Proposition 1 of Book 5. For there, by this same procedure, we shall investigate the altitude of the Sun above any plane whatever from a given hour. Hence, above all the others, this method deserves to be committed to memory.

[Margin: Which method of finding the Sun's altitude is the more excellent.]

It is now fitting to explain this same problem by means of spherical triangles. Let the Horizon then be ABCD; the Meridian BED; the Equator AFC; the Vertical AEC; and the Sun's parallel — whether northern or southern — GHI, so that the northern one...

[Margin: By what path the altitude of the Sun is to be traced out from a known hour by means of spherical triangles, the Sun being situated in any parallel whatever.]

Diagram. A horizontal row of three spherical diagrams, each a great circle representing the sphere seen edge-on, illustrating successive cases of the construction. In every figure the frame defined in the text is present: the outer circle carries B at top, D at bottom, A at left, C at right (Horizon ABCD, with the

Meridian BED and the Vertical AEC drawn as the vertical and cross diameters), the Equator arc AFC and the Vertical arc AEC crossing near the centre E, and the Sun's parallel GHI as an oblique small-circle arc. Interior construction points are lettered K, L, M, N, O, P, Q and F, H. The three figures repeat the same lettering in the successive positions of the Sun (point L on its parallel) discussed in the text.

- B (top of each circle): pole of the horizon / zenith point of the meridian frame
- D (bottom of each circle): nadir point of the meridian frame
- A (left of each circle): intersection of horizon, equator and vertical
- C (right of each circle): opposite intersection on the horizon
- E (centre): centre of the sphere (crossing of meridian and vertical)
- F (upper, on the equator/meridian): point F of the equator, F on the meridian arc
- G, H, I (oblique arc): the Sun's parallel GHI (G lower-left, H mid, I lower-right)
- N (lower-centre): pole N of the equator (foot of the declination circle NM)
- M (interior left): point M where the declination circle meets the equator
- K (interior left): point where the parallel cuts the vertical
- L (interior left): position of the Sun on its parallel
- O (left): foot O of the vertical arc ELO (altitude arc LO)
- P (centre): point P where the great circle through A and the Sun cuts the meridian
- Q (upper-left): [?] peripheral point Q

cuts the Vertical at K; which indeed happens when the arc FH — of the parallel's declination — is smaller than the arc EF, of the pole's altitude, which lies between the vertex and the Equator. Let the Sun be placed at the point L of its parallel, so that HL is the Sun's distance from the meridian, to which the arc of the Equator FM will be similar (by Proposition 10 of Book 2 of Theodosius), the arc that the declination circle drawn from the pole N through the Sun's centre L cuts off. From the vertex E, through the Sun L, let the Vertical ELO descend, so that the arc of the Sun's altitude is LO, which is what must be investigated, if the Sun's distance from the meridian is known from the given hour, as the first part of the problem requires. Let another great circle be described through A, the pole of the Meridian, and through the Sun set at L, cutting the Meridian at P.

Since, therefore, in the spherical triangle ALM of the first four figures the angle M is a right angle (by Proposition 15 of Book 1 of Theodosius, seeing that the great

circle NM has been drawn through N, the pole of the Equator AFC), it will follow — by Proposition 19 of Book 4 of Johannes Regiomontanus [*Regiomontanus*] *On Triangles*, or by Proposition 15 of Book 1 of Geber [Jābir ibn Aflāḥ], or by Proposition 43 of our own [book] of spherical triangles — that as the sine of the complement of the arc AM (that is, the sine of the Sun's distance from the meridian — for FM, the Sun's distance from the meridian, is the complement of the arc AM. Only when the Sun's distance from the meridian in the northern signs exceeds six hours, as happens in the third figure, the complement of the arc AM is the Sun's distance from midnight, namely the arc of the Equator below the horizon between M and the Meridian; but this arc has the same sine as the arc FM of the Sun's distance from the meridian, as we explained in the treatise on sines. For these two arcs together make up a semicircle) is to the whole sine, so is the sine of the complement of the arc AL to the sine of the complement of the arc LM, of the parallel's declination. Inverting, therefore, it will likewise follow that as the whole sine is to the sine of the Sun's distance from the meridian, so is the sine of the complement of the declination of the proposed parallel to the sine of the complement of the arc AL; and so...

and so from the three first-known quantities the fourth will be known — namely the complement of the arc AL, that is, the arc LP itself; and consequently the arc AL too will be known, which may be called the **First Found**.

[Margin: The First Found.]

Again, since in the spherical triangle NLP of the same first four figures the angle P is a right angle (by Proposition 15 of Book 1 of Theodosius, because the great circle AP has been drawn through A, the pole of the Meridian BED), it will follow — by the same Proposition 19 of Book 4 of Johannes Regiomontanus *On Triangles*, or by Proposition 15 of Book 1 of Geber, or by Proposition 43 of our own spherical triangles — that as the sine of the complement of the arc NL (that is, the sine of the arc LM, of the parallel's declination) is to the sine of the complement of the arc LP (that is, to the sine of the arc AL, which we called the First Found), so is the sine of the complement of the arc NP to the whole sine. Inverting, therefore, it will likewise follow that as the sine of the arc we called the First Found is to the sine of the proposed parallel's declination, so is the whole sine to the sine of the complement of the arc NP; and so from the first three known quantities the fourth will be known — namely the complement of the arc NP, that is, the arc FP itself, in the first, second, and fourth figures, since NF is a quadrant. Only when the Sun's distance from the meridian in the northern signs surpasses six hours, as occurs in the third figure, the complement of the arc NP is the arc from P tending

through D as far as the Equator below the horizon. (For since at that time P falls between N, the pole, and the horizon — because the circle ALP cuts the declination circle NM at L, while NLM cuts the Equator at M, beyond the point A, below the horizon, on account of the arc FM of the Sun's distance from the meridian, which is here set greater than 6 hours, or than the quadrant FA — it comes about that, since the arc from the pole N through D tending as far as the Equator below the horizon is a quadrant, the said arc tending from P through D as far as the Equator below the horizon is the complement of the arc NP itself.) When this complement is known, the arc FP, the remainder of the semicircle, will also be known. Now if the found arc FP — the Sun being northern, as appears in the first, second, and third figures — is added to the arc FB, of the Equator's altitude, or of the complement of the pole's altitude, the whole arc BP will be known. But when the Sun is traversing the southern signs, if the same found arc FP is subtracted from the arc FB, of the complement of the pole's altitude, the same arc BP will be left known, as is manifest in the fourth figure. This arc BP, moreover, may be called the **Second Found**.

Diagram. Two horizontal rows of three spherical diagrams (six in all), continuing the same construction and lettered as before — the four principal cases plus variants. Each is a great circle with B at top, D at bottom, A at left, C at right (Horizon ABCD, Meridian BED, Vertical AEC), the Equator arc through F, and the Sun's parallel drawn as an oblique small-circle arc; interior points K, L, M, N, O, P, Q and F, H mark the Sun's position and the auxiliary great circles.

Top row:

- Figure 1 (left): frame B/D/A/C; interior Q, O (upper-left), M, F (upper), H, P (centre), E, L, K, G (lower-left), N (lower), I (lower-right)
- Figure 2 (middle): frame B/D/A/C; interior Q, M, K, F, H, E, P, N, O, L, G (lower-left), I (lower-right)
- Figure 3 (right): frame B/D/A/C; interior M, O, K, F, H, E, N, P, L, G (lower-left), I (lower-right)

Bottom row:

- Figure 4 (left): frame with B/D and A at lower-left, C right; interior O, Q, L, P, H, F, M, E, N, G, I
- Figure 5 (middle): frame B/D/A/C; interior Q, M, F, H, E, N, G (lower-left), I
- Figure 6 (right): frame B/D/A/C; interior K, F, H, E, O, L, N, G (lower-left), I

[Lower text, p. 113:]

Lastly, since in the spherical triangle ELP the angle P is a right angle, as we said just now, it will follow — by the same Proposition 19 of Book 4 of Johannes Regiomontanus *On Triangles*, or by Proposition 15 of Book 1 of Geber, or by Proposition 43 of our own spherical triangles — that as the sine of the complement of the arc PL (that is, the sine of the arc AL, which we called the First Found) is to the whole sine, so is the sine of the complement of the arc EL (that is, the sine of the arc LO, of the Sun's altitude at the time of observation) to the sine of the complement of the arc EP (that is, to the sine of the arc BP, which we called the Second Found).

[Margin: The Second Found.]

For in the first and fourth figures the arc BP is the complement of the arc EP, since EB is a quadrant; but in the second and third figures the arc PD is the complement of the arc EP, since ED is a quadrant. Yet the sine of the arc PD is the same as the one owed to the arc BP, because these arcs together complete a semicircle. Inverting, therefore, it will likewise follow that as the whole sine is to the sine of the arc we called the First Found, so is the sine of the arc we called the Second Found to the sine of the Sun's altitude at the time of observation. Hence, from the three former known quantities we shall also know the fourth — namely the sine of the Sun's altitude, and consequently the altitude itself.

Accordingly, if it is set up so that as the whole sine is to the sine of the Sun's distance from the meridian, so is the sine of the complement of the proposed parallel's declination to a fourth term, the sine of a certain arc will be found, which, subtracted from the quadrant, will leave the arc that may be called the **First Found**.

Next, if it is set up so that as the sine of the arc we called the First Found is to the sine of the proposed parallel's declination, so is the whole sine to a fourth term, a sine will arise whose arc — or else (if the distance from the meridian has exceeded a quadrant, that is, six hours) the remainder of the semicircle, if that arc is subtracted from the semicircle — added to the complement of the pole's altitude when the Sun is northern, or, if it is southern, subtracted from the complement of the pole's altitude, will yield the arc that may be called the **Second Found**. But if this arc turns out to be greater...

than a quadrant of the Second Found, it must be subtracted from the semicircle, so that the arc is left which we named the Second Found, as happens in the second and third figures.

Finally, if it is set up so that as the whole sine is to the sine of the arc we called the First Found, so is the sine of the arc we called the Second Found to a fourth term, the sine of the Sun's altitude at the time of observation will be found. We shall make the whole matter clearer by a few examples.

[Margin: First Example.]

Let us place the Sun at the beginning of Cancer [♋], with its distance from the meridian 45 degrees, that is, 3 hours. In this way, then, we shall investigate the Sun's altitude at that time at a latitude of 42 degrees. Let it be: as 100000 (the whole sine) is to 70710 (the sine of the Sun's distance from the meridian), so is 91706 (the sine of the complement of the declination) to a fourth term; and there will be found, very nearly, this sine 64845, whose arc is 40 degrees 25 minutes, which, subtracted from the quadrant, leaves 49 degrees 35 minutes for the **First Found**.

Again, let it be: as 76134 (the sine of the arc of the First Found) is to 39874 (the sine of the declination), so is 100000 (the whole sine) to a fourth term; and there will be found, very nearly, this sine 52373, whose arc is 31 degrees 35 minutes; added to 48 degrees — namely to the complement of the pole's altitude, since the Sun is northern — it will make an arc of 79 degrees 35 minutes for the **Second Found**.

Lastly, let it be: as 100000 (the whole sine) is to 76134 (the sine of the arc of the First Found), so is 98351 (the sine of the arc of the Second Found) to a fourth term; and there will come to light this sine, roughly 74878, to which corresponds an arc of 48 degrees 29 minutes, for the Sun's altitude at the time of observation.

[Margin: Second Example.]

Let us again place the Sun in the same parallel with a distance from the meridian of 75 degrees, that is, 5 hours. Let it be: as 100000 (the whole sine) is to 96592 (the sine of the Sun's distance from the meridian), so is 91706 (the sine of the complement of the declination) to a fourth term; and there will be found this sine, nearly 88580, whose arc is 62 degrees 21 minutes, which, subtracted from the quadrant, leaves 27 degrees 39 minutes for the **First Found**.

Diagram. Two horizontal rows of three spherical diagrams (six in all), the same construction lettered as on the preceding pages — the principal cases with variants. Each is a great circle: B top, D bottom, A left, C right (Horizon ABCD, Meridian BED, Vertical AEC), the Equator arc through F, and the Sun's parallel as an oblique small-circle arc, with interior points K, L, M, N, O, P, Q and F, H.

Top row:

- Figure 1 (left): frame B/D/A/C; interior Q, O, M, F, H, P, L, K, G (lower-left), E, N, I
 - Figure 2 (middle): frame B/D/A/C; interior Q, M, K, F, H, E, P, N, O, L, G, I
 - Figure 3 (right): frame B/D/A/C; interior M, O, K, F, H, E, N, P, L, G, I
- Bottom row:
- Figure 4 (left): frame B/D with A lower-left, C right; interior O, Q, L, P, H, F, M, E, N, G, I
 - Figure 5 (middle): frame B/D/A/C; interior M, F, H, E, K, N, G, I
 - Figure 6 (right): frame B/D/A/C; interior K, F, H, E, O, L, N, G, I

[Lower text, p. 114:]

After this, let it be: as 46406 (the sine of the arc we named the First Found) is to 39874 (the sine of the declination), so is 100000 (the whole sine) to a fourth term; and there will come out this sine, nearly 85924, whose arc is 59 degrees 14 minutes; joined to the complement of the pole's altitude, 48 degrees, it will make an arc of 107 degrees 14 minutes, which, since it exceeds a quadrant, when subtracted from the semicircle (that is, from 180 degrees) will leave an arc of 72 degrees 46 minutes for the **Second Found**.

Let it be, finally: as 100000 (the whole sine) is to 46406 (the sine of the First Found), so is 95510 (the sine of the Second Found) to a fourth term; and there will come forth this sine, nearly 44322, whose arc, 26 degrees 19 minutes, will give the sought altitude of the Sun.

[Margin: Third Example.]

Furthermore, let the Sun, being in the same parallel, have a distance from the meridian of 7 hours, that is, 105 degrees. Let it be: as 100000 (the whole sine) is to 96592 (the sine of the Sun's distance from the meridian) — which sine indeed belongs to an arc of 75 degrees, and so likewise to an arc of 105 degrees, the remainder of the semicircle — so is 91706 (the sine of the complement of the declination) to a fourth term; and there will be found this sine, nearly 88580, whose arc is 62 degrees 21 minutes, which, subtracted from the quadrant, leaves 27 degrees 39 minutes for the **First Found**.

Let it also be: as 46406 (the sine of the First Found) is to 39874 (the sine of the declination), so is 100000 (the whole sine) to a fourth term; and there will be found this sine, roughly 85924, whose arc is 59 degrees 14 minutes. Subtracted from the semicircle — because the Sun's distance from the meridian surpasses six

hours — it leaves an arc of 120 degrees 46 minutes, which, added to the complement of the pole's altitude, 48 degrees, will make an arc of 168 degrees 46 minutes. This at last, since it is greater than a quadrant, when subtracted from the semicircle leaves 11 degrees 14 minutes for the **Second Found**.

In addition to this, let it be: as 100000 (the whole sine) is to 46406 (the sine of the First Found), so is 19480 (the sine of the Second Found) to a fourth term; and there will be obtained this sine, very nearly 9039, whose arc, 5 degrees 11 minutes, will present the altitude of the Sun that we were seeking.

Finally, let the Sun be in any southern parallel — as, for instance, at the beginning of Capricorn [♑] — and let it have a distance from the meridian of 2 hours, that is, 30 degrees. Let it be: as 100000 (the whole sine) is to 50000 (the sine of the Sun's distance from the meridian), so is 91706 (the sine of the complement of the declination) to a fourth term; and there will be found this sine, 45853, whose arc is 27 degrees 18 minutes. Subtracted from the quadrant, it leaves an arc of 62 degrees 42 minutes for the **First Found**.

Next, let it be: as 88861 (the sine of the First Found) is to 39874 (the sine of the declination), so is 100000 (the whole sine) to a fourth term; and there will be obtained, near enough, this sine, 44872, whose arc is 26 degrees 40 minutes. Subtracted from the complement of the pole's altitude, 48 degrees — because the Sun is southern — it leaves an arc of 21 degrees 20 minutes for the **Second Found**.

Let it be, at last: as 100000 (the whole sine) is to 88861 (the sine of the First Found), so is 36379 (the sine of the Second Found) to a fourth term; and there will be found, very nearly, this sine, 32327, whose arc, 18 degrees 52 minutes, will present the sought altitude of the Sun.

[Margin: How the Sun's altitude, when it lies in the vertical circle, is computed from the given hour by spherical triangles.]

When the Sun lies in the vertical circle — as at the point K, as appears in the fifth figure — we shall hunt down its altitude far more expeditiously from the known distance of the Sun from the meridian, by means of spherical triangles, in the following way. Since in the spherical triangle AkM the angle M is right, it will hold — by Regiomontanus [Regiomontanus], *On Triangles* IV.19, or by Geber [Jābir ibn Aflāḥ] I.15, or by our *On Spherical Triangles* 43 — that as the sine of the complement of the arc AM (that is, as the sine of the Sun's distance from the meridian, i.e. the sine of the arc FM) is to the whole sine, so is the sine of the complement of the arc AK to the sine of the complement of the arc KM (that is, to

the sine of the complement of the declination). Therefore, by conversion, it will also hold that as the whole sine is to the sine of the Sun's distance from the meridian, so is the sine of the complement of the declination of the proposed parallel to the sine of the complement of the arc AK. From these three known terms the fourth will be found — namely the complement of the arc AK, and thus the arc AK itself, which measures the Sun's altitude. And so, when the Sun lies in the vertical circle, if it be made that as the whole sine is to the sine of the Sun's distance from the meridian, so is the sine of the complement of the declination to a fourth term, the sine of the complement of the Sun's altitude will be found, and consequently the altitude itself will be manifest.

[Margin: How the Sun's altitude in the vertical is computed by spherical triangles, even when the hour is unknown.]

But if the Sun's distance from the meridian is unknown, we shall nonetheless find the altitude of the Sun standing in the vertical in this way. Since in the spherical triangle EKN the angle E is right, it will hold — by Regiomontanus, *On Triangles* IV.19, or by Geber I.15, or by our *On Spherical Triangles* 43 — that as the sine of the complement of the arc EN (that is, as the sine of the arc EF, the pole's altitude) is to the whole sine, so is the sine of the complement of the arc Nk (that is, the sine of the arc KM, the declination of the proposed parallel) to the sine of the complement of the arc EK (that is, to the sine of the arc AK, the Sun's altitude). Therefore, if it be made that as the sine of the pole's altitude is to the whole sine, so is the sine of the declination to a fourth term, the sine of the Sun's altitude will be found. This can also be demonstrated more clearly as follows. Since in the spherical triangle AKM, of the same fifth figure, the angle M is right, it will hold — by Regiomontanus, *On Triangles* IV.16, or by Geber I.13, or by our *On Spherical Triangles* 41 — that as the sine of the angle A, the pole's altitude (for EF is both the arc of the pole's altitude and the arc of the angle A), is to the sine of the arc kM, the declination, so is the sine of the right angle M (that is, the whole sine) to the sine of the arc AK, the pole's altitude. Hence if it be made that as the sine of the pole's altitude is to the sine of the declination, so is the whole sine to a fourth term, the sine of the Sun's altitude in the vertical circle will again be recovered. This too we demonstrated above without spherical triangles.

[Margin: The Sun's altitude when the Sun is six hours from the meridian in the northern parallels — in what way it is to be investigated by spherical triangles.]

Moreover, when the Sun in the northern parallels is six hours distant from the meridian, as in the sixth figure, it will be no trouble to investigate its altitude. For since in the spherical triangle ELN, of the sixth figure, the angle N is right — by

Theodosius [Theodosius] I.15, because the great circle NA is drawn through A, the pole of the meridian — it will hold, by Regiomontanus, *On Triangles* IV.19, or by Geber I.15, or by our *On Spherical Triangles* 43, that as the sine of the complement of the arc EN (that is, the sine of the arc EF, the pole's altitude) is to the whole sine, so is the sine of the complement of the arc EL (that is, the sine of the arc LO, the Sun's altitude) to the sine of the complement of the arc NL (that is, to the sine of the arc LA, the declination). Therefore, by conversion, it will also hold that as the whole sine is to the sine of the pole's altitude, so is the sine of the declination to the sine of the Sun's altitude. And so, if it be made that as the whole sine is to the sine of the pole's altitude, so is the sine of the declination to a fourth term, the sine of the Sun's altitude will be obtained. This too we demonstrated above without spherical triangles.

[Margin: How the Sun's altitude at the equinoxes is traced by spherical triangles from the given hour.]

Furthermore, when the Sun occupies the equinoctial points, we shall draw out the Sun's altitude by spherical triangles without much labor, from the known hour or from the Sun's distance from the meridian, in this way. Let the equator in the fifth figure be understood to be GHI, and the Sun to lie at k, so that we need not draw a new figure. Since, then, in the spherical triangle EHK the angle H is right, it will hold — by Regiomontanus, *On Triangles* IV.19, or by Geber I.15, or by our *On Spherical Triangles* 43 — that as the sine of the complement of the arc EH (that is, the sine of the arc HB, the altitude of the equator, or the complement of the pole's altitude) is to the whole sine, so is the sine of the complement of the arc EK (that is, the sine of the arc KA, the Sun's altitude) to the sine of the complement of the arc HK, the Sun's distance from the meridian. Hence, by conversion, it will also hold that as the whole sine is to the sine of the complement of the pole's altitude, so is the sine of the complement of the Sun's distance from the meridian to the sine of the Sun's altitude. And therefore, if it be made that as the whole sine is to the sine of the complement of the pole's altitude, so is the sine of the complement of the Sun's distance from the meridian to a fourth term, the sine of the Sun's altitude will be known.

[Margin: How the Sun's altitude is otherwise investigated by spherical triangles from the given hour.]

By another method too, and perhaps more conveniently, we shall find the Sun's altitude from the known hour by spherical triangles, without the circle ALP; and it is of this sort. Let the declination circle NL be extended toward L until it cuts the horizon at Q. And since the angle FNM, the Sun's distance from the meridian,

is known, its supplement QND — the remainder of the two right angles — will also be known. Since, then, in the spherical triangle DNQ, whose angle D is right and whose angle DNQ is known, together with the arc DN, the pole's altitude, it holds — by Regiomontanus, *On Triangles* IV.18, or by Geber I.14, or by our *On Spherical Triangles* 42 — that as the sine of the angle DNQ is to the whole sine, so is the sine of the complement of the angle DQN to the sine of the complement of the arc DN, the pole's altitude; then, by conversion, it will hold that as the whole sine is to the sine of the angle DNQ, so is the sine of the complement of the arc DN, the pole's altitude, to the sine of the complement of the angle DQN — and thus the angle DQN itself will be known. Again, since in the same spherical triangle DNQ it holds — by Regiomontanus, *On Triangles* IV.19, or by Geber I.13, or by our *On Spherical Triangles* 41 — that as the sine of the angle DQN, now made known, is to the sine of the arc DN, the pole's altitude, so is the whole sine of the right angle D to the sine of the arc NQ, the sine of the arc NQ will be known; and from this we shall track down the arc NQ itself, in the following way. When in the northern signs the Sun's distance from the meridian is greater than 6 hours, as happens in the third figure, the arc drawn from the table of sines will give the arc NQ, because the arc NQ is then less than a quadrant,

because the arc produced from the pole N through Q as far as the equator is a quadrant. But when the Sun's distance from the meridian is less than 6 hours — as appears in the first, second, and fourth figures — then, in whatever parallel, whether northern or southern, the Sun is situated, the arc NQ will be greater than a quadrant. And since, as we showed in the treatise on sines, the sine of the arc NQ is the same as that of the remaining arc taken from the semicircle, we shall subtract the arc of the sine found from the semicircle, so as to have the arc NQ. Finally, when the Sun's distance from the meridian amounts to exactly 6 hours, we give no precept here, because then, as we shall teach below, the Sun's altitude is sought much more easily; the arc NQ would in that case be a quadrant, since the point Q would fall on the point A. Now the arc NQ having been found by this method, from it the arc LQ must be found, in this way. In the northern parallels, from the found arc NQ let the arc NL, the complement of the declination, be subtracted; but in the southern parallels, from the same found arc NQ let the arc NL be taken away — which is composed of the quadrant NM and the arc of declination ML, as appears in the fourth figure. For the arc LQ will always be the remainder. And now, since in the spherical triangle LOQ the angle O is right, and LQO has just been made known, together with the arc LQ — and since it holds, by Regiomontanus, *On Triangles* IV.16, or by Geber I.13, or by our *On Spherical Triangles* 41, that as the whole sine of the right angle O is to the sine of the known

arc LQ, so is the sine of the known angle LQO to the sine of the arc LO, the Sun's altitude — the Sun's altitude will become known.

[Margin: First Found. Second Found.]

And so, if it be made that as the whole sine is to the sine of the angle DNQ (which remains once the angle of the Sun's distance from the meridian has been taken from the two right angles), so is the sine of the complement of the pole's altitude to a fourth term, the sine of a certain arc will be found, which, subtracted from the quadrant, will give the arc we shall call the **First Found**.

[Margin: First Found.]

Next, if it be made that as the sine of that arc which we called the First Found is to the sine of the pole's altitude, so is the whole sine to a fourth term, the sine of another arc will be recovered. From this arc, if the Sun is northern and its distance from the meridian is greater than 6 hours, the subtracted arc of the complement of the declination will give a certain arc, which we shall name the **Second Found**. But if the Sun's distance from the meridian is less than 6 hours, in whatever parallel the Sun stands, the arc found by way of the First Found must be subtracted from the semicircle, and from the remaining arc — if the Sun is northern — the arc of the complement of the declination must again be subtracted; or if the Sun is southern, the arc composed of the quadrant and the arc of declination must be taken away, so that the arc we may call the Second Found is found.

[Margin: Second Found.]

The Second Found will also be found in this way. Let it be made that as the sine of the arc which is the First Found is to the sine of the pole's altitude, so is the whole sine to a fourth term; for the sine of a certain arc will be found. From this arc, when the Sun's distance from the meridian is greater than 6 hours, if the complement of the declination is subtracted, the remainder will be the Second Found, as was said, and as is clear from the third figure. Or, when the Sun's distance from the meridian is less than 6 hours, the sine of an arc will be found to whose complement, when the Sun is northern, the declination of the Sun added will give the Second Found; or from whose complement, when the Sun is southern, the declination of the Sun subtracted leaves the Second Found — as is clear from the first, second, and third figures. For it is the complement of the arc NQ (whose sine is found) and the arc QM, to which the declination LM added, or subtracted, will give the Second Found, LR [?].

[Margin: First Example.]

Lastly, if it be made that as the whole sine is to the sine of the arc we named the Second Found, so is the sine of the arc we called the First Found to a fourth term, the sine of the sought altitude of the Sun will be found. And to make this clearer, we shall explain the whole matter by two examples.

[Margin: First Example.]

Let the Sun be placed at the beginning of Cancer [♋], with a distance from the meridian of 7 hours, that is, 105 degrees, so that the angle BNQ spans that many degrees, and consequently the supplement DNQ from the two right angles is 75 degrees. Let it be, then: as 100000 (the whole sine) is to 96592 (the sine of the angle DNQ), so is 74314 (the sine of the complement of the pole's altitude) to a fourth term; and we shall find, near enough, this sine, 71781, whose arc, 45 degrees 52 minutes, subtracted from the quadrant, leaves an arc of 44 degrees 8 minutes for the **First Found**.

Again, let it be: as 69633 (the sine of the arc of the First Found) is to 66913 (the sine of the pole's altitude), so is 100000 (the whole sine) to a fourth term; and we shall recover, near enough, this sine, 96093. From its arc, 73 degrees 56 minutes — since the Sun is set as northern and its distance from the meridian is greater than 6 hours — if the complement of the declination of the Cancer parallel is subtracted, namely 66 degrees 30 minutes, an arc of 7 degrees 26 minutes will be left for the **Second Found**.

[Margin: Second Example.]

Finally, let it be: as 100000 (the whole sine) is to 12937 (the sine of the arc of the Second Found), so is 69633 (the sine of the arc of the First Found) to a fourth term; and there will be found here, very nearly, this sine, 9008, the Sun's altitude. Hence the Sun's altitude will amount to 5 degrees 10 minutes.

Let the Sun again be set at the beginning of Capricorn [♐], with its distance from the meridian of 2 hours, that is, 30 degrees, so that the angle BNQ contains that many degrees, and consequently the supplement DNQ from the two right angles is 150 degrees. Let it be, then: as 100000 (the whole sine) is to 50000 (the sine of the angle DNQ) — for it is the same sine, of the arc of 150 degrees and of the arc of 30 degrees, which together with it makes up the semicircle, as we taught in the treatise on sines — so is 74314 (the sine of the complement of the pole's altitude) to a fourth term; and there will be found, near enough, this sine, 37157, whose

arc, 21 degrees 49 minutes, subtracted from the quadrant, leaves for the **First Found** an arc of 68 degrees 11 minutes.

Next, let it be: as 92837 (the sine of the arc of the First Found) is to 66913 (the sine of the pole's altitude), so is 100000 to a fourth term; and there will be found here, as it were, the sine 72075, whose arc is 46 degrees 7 minutes. Subtracted from the semicircle of 180 degrees — because the Sun's distance from the meridian does not reach 6 hours — it will give the arc NQ of 133 degrees 53 minutes. From this, since the Sun is set as southern, if the arc NL is subtracted — composed of a quadrant and the arc of declination [text: *ex sinu toto, & sinu declinationis*; sense requires quadrant + declination arc], namely 113 degrees 30 minutes — there will be left for the **Second Found** an arc of 20 degrees 23 minutes. Or, if the arc of the found sine 72075 be taken as 46 degrees 7 minutes, and from its complement, 43 degrees 53 minutes, the declination of 23 degrees 30 minutes be subtracted, the remainder will be the same Second Found, 20 degrees 23 minutes.

Lastly, let it be: as 100000 (the whole sine) is to 34829 (the sine of the arc of the Second Found), so is 92837 (the sine of the First Found) to a fourth term; and there will be found here, near enough, the sine 32334, of the Sun's altitude — and thus the Sun's altitude itself will amount to 18 degrees 52 minutes.

Furthermore, when the Sun lies in the vertical circle, or is six hours from the meridian, or lastly is situated on the equator, we shall dispatch the matter as we set out a little earlier.

Finally, we shall complete this same problem by spherical triangles in the right sphere, in this way. When the Sun runs along the equator, the complement of its distance from the meridian will be the Sun's altitude above the horizon; for

[Margin: The Sun's altitude at the equinoxes, how it is detected in the right sphere from the given hour.]

in the right sphere the equator is the same as the vertical circle passing through the center of the Sun. As in the figure below, if the poles of the world are understood to be B, D, the vertical circle AEC will be also the equator itself. With the Sun then placed at I, the arc AI, the complement of the Sun's distance from the meridian, will also be the arc of the Sun's altitude, as is evident.

[Margin: The Sun's altitude, when it lies in any parallel whatever, in what way it is gathered by spherical triangles in the right sphere from the given hour.]

Diagram. A circle (sphere) shown with the celestial poles B at the top and D at the bottom, the horizontal diameter running from A (left) through the center E to C

(right); the vertical great circle AEC represents both the equator (in the right sphere) and the vertical circle. Great-circle arcs (declination circles through B and D, and parallels) cross the interior, with the Sun-positions and reference points lettered symmetrically above and below the center for the northern and southern parallels.

- B (top): north pole of the world
- D (bottom): south pole of the world
- A (left): endpoint of the vertical circle / equator on the horizon
- C (right): opposite endpoint of the vertical circle / equator
- E (center): center of the sphere / Sun
- L (upper left, on circle): endpoint of the vertical EL
- K (upper interior, left): Sun-position k
- H (upper interior, right): reference point on the declination circle
- F (left, upper): point on the parallel FG
- I (interior, left of center): intersection of declination circle with equator
- G (upper right, on circle): endpoint of the parallel FG
- F (left, lower): mirrored parallel point
- L (lower left): mirrored vertical endpoint
- K (lower interior): mirrored Sun-position
- H (lower interior): mirrored reference point
- G (lower right, on circle): mirrored parallel endpoint

When the Sun occupies any parallel FG, whether northern or southern, we shall carry out the matter as follows. Let the Sun be placed at k, and let the vertical EL be drawn from the vertex E through the center of the Sun k. Likewise let the declination circle DKB be drawn through the poles of the world and through the center of the Sun, cutting the equator at I; and the arc EI will be equal to the arc HK, the Sun's distance from the meridian, by Theodosius II.10. Since, then, in the spherical triangle EIK the angle I is right, by Theodosius I.15, it will hold — by Regiomontanus, *On Triangles* IV.19, or by Geber I.15, or by our *On Spherical Triangles* 43 — that as the sine of the complement of the arc IK, the declination of the parallel, is to the whole sine, so is the sine of the complement of the arc Ek (that is, the sine of the arc kL, the Sun's altitude) to the sine of the complement of the arc EI, the Sun's distance from the meridian. Hence also, by conversion, it will hold that as the whole sine is to the sine of the complement of the declination of the proposed parallel, so is the sine of the complement of the Sun's distance from the meridian to the sine of the Sun's altitude. And so, if it be made that as the

whole sine is to the sine of the complement of the declination, so is the sine of the complement of the Sun's distance from the meridian to a fourth term, the sine of the Sun's altitude will come out known. This too we showed above without spherical triangles. And by this method it is evident that the Sun's altitude is drawn out from the known hour by spherical triangles — which is what the first part of the problem requires.

[Margin: In what way the hour is gathered from the Sun's known altitude by spherical triangles.]

But that we may, conversely, learn thoroughly the hour, or the Sun's distance from the meridian, from the known altitude of the Sun above the horizon by spherical triangles, we shall follow this route. Recalling the earlier figures: since in the first, second, third, and fourth the triangle ELN has its three sides known — for EL is the complement of the Sun's altitude, which is given; EN is the complement of the pole's altitude; and LN, in the northern parallels, is the complement of the declination of the proposed parallel, while in the southern parallels the arc LN is composed of the arc of declination LM and the quadrant MN — the angle ENL will also be known, by Regiomontanus, *On Triangles* IV.34, or by our *On Spherical Triangles* 63; and thus its arc FM too, which measures the Sun's distance from the meridian, will be known. But since in the northern parallels the angle ENL may be right, or less, or greater than a right angle: whenever it is found to be right, the Sun's distance from the meridian will comprise 6 hours; if less, fewer

Diagram. First of six small woodcut spheres (upper row, left) illustrating the successive configurations (first through sixth figures) of the Sun's position relative to horizon, equator, meridian, and a parallel. Poles B (top) and D (bottom); horizontal diameter A (left) to C (right); interior great-circle arcs with the Sun-position and reference points lettered.

- B (top): pole
- D (bottom): pole
- A (left), C (right): horizon endpoints
- Q (outside, upper left), O, M (upper-left interior), F (upper right), H (center), P (below H), K, L, G (lower left), E, N (lower center): reference points [several ?]

Diagram. Second small woodcut sphere (upper row, center), same scheme.

- B (top), D (bottom): poles

- A (left), C (right): horizon endpoints
- Q (outside upper left), M, K, F, H, E, L, G, N: interior reference points [several [?]]

Diagram. Third small woodcut sphere (upper row, right), same scheme.

- B (top), D (bottom): poles
- A (left): horizon endpoint; O (right, on circle)
- G (lower left): horizon/parallel endpoint
- M, K, F, H, E, L, N, P: interior reference points [several [?]]

Diagram. Fourth small woodcut sphere (lower row, left), same scheme.

- B (top), D (bottom): poles
- O, Q (outside upper left); G, A (left); I, C (right)
- L, M, H, F, P, E, N: interior reference points [several [?]]

Diagram. Fifth small woodcut sphere (lower row, center), same scheme.

- B (top), D (bottom): poles
- Q (outside upper left); A (left), C (right)
- M, H, F, E, K, G, N: interior reference points [several [?]]

Diagram. Sixth small woodcut sphere (lower row, right), same scheme.

- B (top), D (bottom): poles
- A (left), O (left interior); G (lower left); G (right, on circle), I (right)
- K, H, F, E, L, N: interior reference points [several [?]]

hours than 6; if, finally, greater, it will contain more than six. But so that we may know when the said angle is right, and when less or greater, we must find the altitude the Sun has when it is six hours from the meridian. For when the Sun's altitude at the time of observation, which is given, is equal to the altitude it has when it is six hours from the meridian, the said angle will be right; for the Sun will then be on the circle of the sixth hour, which makes a right angle with the meridian. But if the Sun's altitude is found to be greater than that which it has when it is six hours distant from the meridian, the same angle will be less than a right angle, because the Sun's distance from the meridian is then less than a quadrant. If, finally, the Sun's altitude

[Margin: How the hour is found when the Sun is set on the vertical circle.]

is smaller than the altitude it has when it stands six hours from the meridian, then that same angle will be greater than a right angle — because the Sun's distance from the meridian is at that time greater than a quadrant.

Now suppose the Sun's altitude, when found, turns out to be equal to the altitude it has on the vertical circle — the altitude we determined above, even when the Sun's distance from the meridian is unknown. Then the Sun stands on the vertical circle, and from this its distance from the meridian is very easily found. For in triangle EkN — the fifth figure — the angle E is right, and therefore, by Prop. 16 of Regiomontanus [Johannes Regiomontanus], *On Triangles* IV, or by Prop. 13 of Geber [Jābir ibn Aflāḥ], Book I, or by Prop. 41 of our own [treatise on] spherical triangles, we have this: as the sine of arc Nk (the complement of the declination) is to the sine of the right angle E — that is, to the whole sine [radius] — so is the sine of arc EK (the complement of the Sun's altitude) to the sine of angle N (the Sun's distance from the meridian). And so, if it be arranged that as the sine of the complement of the declination is to the whole sine, so is the sine of the complement of the Sun's altitude on the vertical circle to a fourth term, that fourth term will be the sine of the Sun's distance from the meridian.

Diagram. First of six small woodcut spheres (upper row, left), the schematic sphere used for the hour-finding triangles. Poles and horizon frame it; the interior lines are the meridian, the equator/parallel, and the vertical circle, meeting at the reference points.

- B (top), D (bottom): poles
- A (left), C (right): horizon endpoints
- G (lower left), I (lower right): endpoints on the circle
- Q, O (outside/upper left), M, F (upper interior), H (center), P (below H), E, N (lower center), K, L (left interior): reference points [several [?]]

Diagram. Second small woodcut sphere (upper row, center), same scheme.

- B (top), D (bottom): poles
- A (left), C (right): horizon endpoints
- G (lower left), I (lower right): endpoints
- Q (left), M, K, F (upper interior), H (center), E, L, P, N (interior): reference points [several [?]]

Diagram. Third small woodcut sphere (upper row, right), same scheme.

- B (top), D (bottom): poles
- A (left), C (right): horizon endpoints

- G (lower left), I (lower right): endpoints
- M, K, F, H, E, L, O, N, P: interior reference points [several [?]]

Diagram. Fourth small woodcut sphere (lower row, left), same scheme.

- B (top), D (bottom): poles
- A (left), C (right): horizon endpoints
- G (lower left), I (right): endpoints
- O, Q (outside upper left), L, H, F, M, E, N, K: interior reference points [several [?]]

Diagram. Fifth small woodcut sphere (lower row, center), same scheme.

- B (top), D (bottom): poles
- A (left), C (right): horizon endpoints
- G (lower left), I (lower right): endpoints
- M, H, F, E, N, K: interior reference points [several [?]]

Diagram. Sixth small woodcut sphere (lower row, right), same scheme.

- B (top), D (bottom): poles
- A (left), C (right, faint): horizon endpoints
- G (lower left), I (lower right), G (right, on circle): endpoints
- H, F, K, E, N, L, O: interior reference points [several [?]]

[Margin: At what hour the Sun stands on the vertical circle.]

Hence, if the question is asked at what hour the Sun is found on the vertical circle, we must first find the Sun's altitude on the vertical circle, as we taught above (even when its distance from the meridian is unknown), and then, from that altitude, work out the Sun's distance from the meridian, just as we have shown a moment ago.

[Margin: The way to find the hour at the equinoxes from the known altitude of the Sun.]

When the Sun is on the equator, we shall also determine the hour from the Sun's altitude with equal ease. For in the spherical triangle EHK of the fifth figure — taking the equator to be GHI and the Sun at K, as we said above — the angle H is right, and therefore, by Prop. 19 of Regiomontanus, *On Triangles* IV, or by Prop. 15 of Geber, Book I, or by Prop. 43 of our own spherical triangles, we have this: as the sine of the complement of arc EH (the pole's altitude) is to the whole sine, so is the sine of the complement of arc EK — that is, so is the sine of arc AK (the Sun's

altitude) — to the sine of the complement of arc HK (the Sun's distance from the meridian). And so, if it be arranged that as the sine of the complement of the pole's altitude is to the whole sine, so is the sine of the Sun's altitude to a fourth term, that fourth term will be the sine of the complement of the Sun's distance from the meridian.

[Margin: How, in the right sphere at the time of the equinoxes, the hour is found from the Sun's altitude.]

We shall demonstrate the same thing in another way as well. For in the spherical triangle AGK the angle A is right — by Prop. 15 of Theodosius [of Bithynia], Book I, since the great circle EA is drawn through E, the pole of the horizon — and therefore, by Prop. 16 of Regiomontanus, *On Triangles* IV, or by Prop. 13 of Geber, Book I, or by Prop. 41 of our own spherical triangles, we have this: as the sine of angle G (the complement of the pole's altitude) — for if the equator be taken as GHI, the angle G, corresponding to arc HB, the altitude of the equator, that is, the complement of the pole's altitude — is to the sine of arc Ak (the Sun's altitude), so is the sine of the right angle A, that is, the whole sine, to the sine of arc Gk (the complement of the Sun's distance from the meridian). If therefore it be arranged that as the sine of the complement of the pole's altitude is to the sine of the Sun's altitude, so is the whole sine to a fourth term, that fourth term will be the sine of the complement of the Sun's distance from the meridian.

[Margin: By what method, in the right sphere, the hour is to be traced from the Sun's altitude when the Sun lies on any parallel whatever.]

Finally, in the right sphere we shall proceed as follows. When the Sun is at the equinoxes, we take the complement of the Sun's altitude as its distance from the meridian. For in the adjoining figure, which we drew for the right sphere, the arc EI (the Sun's distance from the meridian) is the complement of arc AI (the Sun's altitude).

Diagram. Single larger woodcut sphere (lower left) drawn for the right sphere, showing the meridian, the equator EA through the horizon-pole, and a parallel through K, with the spherical triangles AGK and EIK.

- B (top), D (bottom): poles
- A (left), C (right): endpoints on the circle
- L, F (upper and lower left), G (right, upper and lower): outer endpoints
- K (upper and lower), H (upper right and lower right): points where the parallel meets the drawn circles

- I (left of center), E (center): interior reference points

When, however, the Sun stands on some other parallel — say at K — then, because in the spherical triangle EIK the angle I is right, we have, by Prop. 19 of Regiomontanus, *On Triangles* IV, or by Prop. 15 of Geber, Book I, or by Prop. 43 of our own

spherical triangles, this: as the sine of the complement of arc Ik (the declination of the given parallel) is to the whole sine, so is the sine of the complement of arc Ek — that is, the sine of arc KL (the Sun's altitude) — to the sine of the complement of arc EI (the Sun's distance from the meridian). Therefore, if it be arranged that as the sine of the complement of the Sun's declination is to the whole sine, so is the sine of the Sun's altitude to a fourth term, that fourth term will be the sine of the complement of the Sun's distance from the meridian. And so, from the known hour of the day we have found the Sun's altitude above the horizon; and conversely, from the known altitude of the Sun we have found the hour of the day. Which was to be done.

SCHOLIVM.

When the Sun's altitudes are to be found for the several hours of two opposite parallels — such as, for example, the two tropics traced from the beginnings of ☊ [Cancer] and ☋ [Capricorn], which happens not rarely in practice when constructing hour-instruments and in laying out sundials — the whole business will be rendered very easy, provided that what we are about to explain be attentively considered.

[Margin: Which numbers never change, if the Sun's altitudes for the several hours of two opposite parallels are investigated by the first method taught.]

For if we first wish to use the first method, there will remain, for the several hours of each parallel, one and the same half of the line compounded of the sine of the meridian altitude and the sine of the meridian depression; so that, once this half has been found, it may be applied to work out the altitudes for all the hours in the two opposite parallels. For since, as we showed in the scholium of the preceding proposition, the meridian depression of any parallel is equal to the meridian altitude of the opposite parallel, it comes about that the line compounded of the sine of the meridian altitude and the sine of the meridian depression of one parallel is equal to the line compounded of the sine of the meridian altitude and the sine of the meridian depression of the other, opposite parallel — since the meridian depression of the one is equal to the meridian

altitude of the other, and the meridian depression of the latter equal to the meridian altitude of the former. And hence the halves of these compounded lines will also be equal. Thus, in the two tropics, the half of the line compounded of the sine of the meridian altitude of ♋ [Cancer] and the sine of the meridian depression of ♋ , or of the sine of the meridian altitude of ♏ [Capricorn] and the sine of the depression of ♏ , will be 68151 — remaining the same, and useful for working out the altitudes of all the hours when the Sun is at the beginning of ♋ or of ♏ .

Again, in all parallels the same whole sine, 100000, is always employed in seeking out the Sun's altitudes.

[Margin: Which numbers stay the same, if the Sun's altitudes are traced by the second method for the several hours of each parallel.]

Finally, the line AN — that is, the difference between the sine of the meridian altitude and the aforesaid half KA — never changes in the two opposite parallels; because exactly the same difference lies between the sine of the meridian depression and the other half $\theta\Lambda$, as is evident from the figures. Now it is established from what has been demonstrated that the sine of the meridian depression of the northern parallel is equal to the sine of the meridian altitude of the opposite southern parallel. Hence the same difference will always lie between the aforesaid half and the sine of the meridian altitude both of the northern parallel and of the opposite southern one. Thus, in the two tropics, this difference will be, near enough, the number 26681, which will never have to be changed until all the altitudes in the two tropics have been found; and to this number the line TA, once found, is now to be added — now, in the northern signs, to be subtracted from it — or else, in the southern signs, that same number 26681 is to be subtracted from the line TA once found, so as to obtain the sine TN of the Sun's altitude, as is clear from the foregoing.

[Margin: Which numbers stay the same, if the Sun's altitudes are traced by the second method for the several hours of each parallel.]

If it be preferred to investigate the Sun's altitudes in two opposite parallels by the second method, then indeed within one and the same parallel both the versed sine of the semidiurnal arc and the right sine of the meridian altitude remain always the same; but for finding the altitudes in the other, opposite parallel, the proper versed sine of the semidiurnal arc and the proper right sine of the meridian altitude must be taken. We shall have only this convenience: that when the versed sine of the semidiurnal arc of one parallel is subtracted from the

whole diameter — that is, from 200000 — we at once have the versed sine of the semidiurnal arc of the opposite parallel; for the versed sine of the semidiurnal arc of one parallel is equal to the versed sine of the seminocturnal arc of the other, opposite parallel. And it is plain that the versed sine of the seminocturnal arc, subtracted from the whole diameter, leaves the versed sine of the semidiurnal arc — and conversely.

[Margin: Which numbers never change, if the Sun's altitudes are investigated by the last method, which we set out before the treatment drawn from the spherical triangles, for the several hours of two opposite parallels.]

But in that last method, which we wrote out just before the treatment drawn from the spherical triangles, we shall have, in the opposite parallels, not only always the same half of the line compounded of the sine of the meridian altitude and the sine of the meridian depression, and the same whole sine, but also the same versed sines of the Sun's distances from the meridian at the hours equally distant from the meridian — such as the first hour and the eleventh, likewise the second and the tenth, and so on — both in the parallel ♋ [Cancer] and in the parallel ♏ [Capricorn]. Hence, at the same hours, the same line KT is found; so that in the opposite parallel there is no need to investigate the line KT afresh for that hour for which the same KT was found in the other parallel, but the very same is to be taken throughout, to be subtracted from the sine of the meridian altitude of the parallel in question, and so on.

Indeed, for the hours before the 6th hour of the morning and the 6th hour of the evening, there is no need to investigate the line KT by multiplication, but only by subtraction and addition. For if you take the difference between KA, the half of the line Kθ compounded of the sine of the meridian altitude and the sine of the meridian depression, and KT, the line found for an hour whose distance from the meridian is less than a quadrant, and add it to that same half KA, you will make up the line KT for the hour whose distance from the meridian is as much greater than six hours as the former was less. For in the second figure, if Ka, the distance from the meridian, is as much less than the quadrant KP as the distance KO is greater, then the arcs PO and PA will be equal, and hence their right sines MB and MR will be equal.^a Therefore λI [?] and λT are equal lines. So Kd, taken away from the half KA, and the remainder dλ added to the same half KA, makes the whole KT known — which was proposed.

For the 4th and 8th hours from the meridian, too, or midnight, it is enough, for KT, to take half of the half KA. For in the same second figure, if the arc Ka were to contain 4 hours — that is, 60 degrees — the arc aP would be 30 degrees; therefore,

as we showed for the sines, Mb, the sine of 30 degrees, will be half of the whole sine KM, and accordingly Kd too will be half of KΛ itself. And if the 4th and 8th hours be before the 6th hour of the morning and after the 6th hour of the evening, dΛ will have to be added, and so on. Or rather, much more briefly, in this way: since Kd is equal to θT itself, then, if from Kd found for an hour between the meridian and the 6th hour — that is, from θT — one takes away θN, the sine of the meridian depression, there will be left TN, the sine of the altitude for the hour between midnight and the 6th hour. Thus, for the 5th hour after the meridian, for example, the line Kd once found must be subtracted from KN, the sine of the meridian altitude. But for the 7th hour from the meridian, from the same Kd once found — that is, from θT — one must subtract the sine θN of the meridian depression, so as to leave the sine of the Sun's altitude [☉]. Likewise for the 4th hour from the meridian, half of Kd, the half of KΛ, must be subtracted from the sine of the meridian altitude; while for the 8th hour from the meridian, from that same half Kd, the half of KΛ — that is, from θT — one must subtract the sine θN of the meridian depression, so that what remains becomes the sine of the Sun's altitude [☉].

[Margin: a. Prop. 2 or 4 of Book Six.]

Diagram. Left of two circular "clock-face" figures at the foot of the page (the *first figure*): a full circle with the center E, twelve radial hour-lines, the horizon diameter A–C, the meridian B–D, and the parallel circle; sine-points projected onto the meridian.

- B (top), D (bottom): meridian endpoints
- A (left), C (right): horizon endpoints
- O (top, outside), P (upper right, outside): outer points
- K (upper left), F (upper right): points on the parallel
- H (left), G (lower left), I (lower right): circle points
- T, R (upper-left interior), M (center-right interior), E (center), N (lower center): projected reference points [T, R uncertain — [?]]

Diagram. Right of the two circular figures (the *second figure*): the same construction with the parallel wholly above the horizon and not touching it; carries the additional Greek point-labels λ and θ.

- B (top), D (bottom): meridian endpoints
- A (left), C (right): horizon endpoints
- P (upper right), O (right, outside), L (right, on circle): outer points
- K (upper left), F (upper right): points on the parallel

- H (left), G (lower left), I (lower right): circle points
- λ (interior left), T, θ (center interior), M (center-right), R (below M), E (center), N (lower center): projected reference points [several [?]]

[Margin: Which sines always stay the same, if the Sun's altitudes are sought by spherical triangles for the several hours of two opposite parallels.]

Finally, if one prefers to trace the Sun's altitudes in two opposite parallels by spherical triangles, the business will likewise be rendered very easy, since the same whole sine and the same sine of the declination of each parallel must be kept throughout in working out the altitudes for all the hours, as is plainly established from what has been said.

[Margin: When a northern parallel lies wholly above the horizon, by what method the Sun's altitude is found from the known hour, and conversely the hour from the known altitude of the Sun.]

Moreover, when the pole's altitude is so great that some northern parallel lies wholly above the horizon — whether it touches the horizon, as in the first figure set beneath here, or does not, as in the second — the Sun's altitude will nonetheless be found from the known hour, and conversely the hour from the Sun's altitude, by the method taught in this proposition, provided that in each figure the meridian is understood to be at K, where the Sun, standing on the meridian, has its greatest altitude, which is to be worked out from the parallel's declination, as we explained above in the scholium of the preceding proposition. But here there will be no meridian depression; rather, in the first figure the line $K\Lambda$ will be the half of the sine of the meridian altitude, while in the second the same $K\Lambda$ will be the half of the line $K\theta$, which is the difference between KN , the sine of the greater meridian altitude KA , and θN , the sine of the lesser meridian altitude LC . And this lesser altitude CL is obtained if from arc IL , the declination, one subtracts arc IC , the complement of the pole's altitude. But if the declination should be equal to the complement of the pole's altitude, the parallel will touch the horizon, as happens in the first figure. Now that $K\Lambda$ is the half of the said lines is proved as follows.^a In the first figure, as kM is to MC , so is $K\Lambda$ to λN ; but in the second, as KM is to ML , so is $K\Lambda$ to $\lambda\theta$. And KM is equal both to MC and to ML , since these lines are radii of that parallel; therefore $K\Lambda$ too will be equal to λN in the first figure, and to $\lambda\theta$ in the second.

[Margin: a. Prop. 2 of Book Six.]

Diagram. Left of two circular figures (the *first figure*): the parallel touches the horizon; center E, twelve radial hour-lines, horizon A–C, meridian B–D, and sine-points on the meridian.

- B (top), D (bottom): meridian endpoints
- A (left), C (right): horizon endpoints
- O (top, outside), P (upper right): outer points
- K (upper left), F (upper right): points on the parallel
- H (left), G (lower left), I (lower right): circle points
- T, R (upper-left interior), M (center-right), E (center), N (lower center): projected reference points [T, R uncertain — [?]]

Diagram. Right of the two figures (the *second figure*): the parallel lies wholly above the horizon and does not touch it; carries the Greek point-labels λ and θ .

- B (top), D (bottom): meridian endpoints
- A (left), C (right): horizon endpoints
- P (upper right), O (right, outside), L (right, on circle): outer points
- K (upper left), F (upper right): points on the parallel
- H (left), G (lower left), I (lower right): circle points
- λ (interior left), T, θ (center interior), M (center-right), R (below M), E (center), N (lower center): projected reference points [several [?]]

[Margin: b. Prop. 2 or 4 of Book Six.]

And so, since in the first figure — where the parallel touches the horizon ^b — we have: as KM, the whole sine, is to MR, the sine of the complement of the Sun's distance from the meridian K, so is KA, the half of the sine of the meridian altitude, to λ T, the difference between TN, the sine of the Sun's altitude, and the line λ N, the half of the sine of the meridian altitude; then, if it be arranged that as the whole sine is to the sine of the complement of the Sun's distance from the meridian, so is the half of the sine of the meridian altitude to a fourth term, that fourth term will be found to be a line which, added to the aforesaid half if the distance from the meridian is less than a quadrant, or subtracted from that same half if the distance from the meridian is greater than a quadrant, will give the sine of the Sun's altitude at the time of observation. But if the Sun's distance from the meridian is equal to a quadrant, then the half λ N will itself be the sine of the Sun's altitude at the time of observation.

[Margin: c. Prop. 2 or 4 of Book Six.]

Again, since in the second figure — where the parallel stands wholly above the horizon and does not touch it ^c — we have: as KM, the whole sine, is to MR, the sine of the complement of the Sun's distance from the meridian K, so is KA, the half of the difference KΘ between the sine of the greater meridian altitude and the sine of the lesser meridian altitude, to λT, the difference between TN, the sine of the Sun's altitude, and the line λN — the latter being compounded of the said half λΘ and the sine ΘN of the lesser meridian altitude; then, if it be arranged that as the whole sine is to the sine of the complement of the Sun's distance from the meridian, so is the half of the difference between the sine of the greater meridian altitude and the sine of the lesser meridian altitude to a fourth term, there will be found a line which, subtracted from the line compounded of the said half and the sine of the lesser meridian altitude if the Sun's distance from the meridian is greater than a quadrant, or added to that same compounded line if the distance is less than a quadrant, will give the sine of the Sun's altitude at the time of observation. But if the distance should be equal to a quadrant, then the line compounded of the said half and the sine of the lesser meridian altitude will itself be the sine of the Sun's altitude at the time of observation. All of which can easily be gathered from these two figures set here side by side.

Conversely, if it be arranged that as KA, the half of the sine of the meridian altitude in the first figure — or the half of the line KΘ in the second, which is the difference between the sine of the greater meridian altitude and the sine of the lesser meridian altitude — is to λT, the difference between the sine of the Sun's altitude and the half of the sine of the meridian altitude in the first figure (or between the sine of the Sun's altitude and the line λN, which is compounded of the half of the difference between the sine of the greater meridian altitude and the sine of the lesser meridian altitude together with the sine of the lesser meridian altitude, as appears in the second figure), so is the whole sine to a fourth term, then that fourth term will be found to be the sine of the complement of the Sun's distance from the meridian K. This complement, added to a quadrant when the sine of the Sun's altitude is less than the line λN — that is, than the half of the sine of the meridian altitude in the first figure, or than the line compounded of the sine of the lesser meridian altitude and the half of the difference between the sine of the greater meridian altitude and the sine of the lesser meridian altitude in the second figure — will give the Sun's distance from the meridian, as appears in the second figure. But that same complement, taken away from a quadrant when the sine of the Sun's altitude is greater than the said line λN, will leave the Sun's distance from the meridian.

But this matter will be accomplished more easily by that method which we treated last of all, before we explained this proposed problem by spherical triangles. For if it be arranged that as KM, the whole sine, is to KR, the versed sine of the Sun's distance from the meridian K, so is KA, the half of the sine of the meridian altitude in the first figure (or, in the second, so is the half of the difference between the sine of the greater meridian altitude and the sine of the lesser meridian altitude), to a fourth term, then KT will come out known — the difference between the sine of the greater meridian altitude (or, at any rate, in the first figure, of the meridian altitude itself) and the sine of the sought Sun's altitude.

Likewise, if it be arranged that as the aforesaid half is to the difference between the sine of the greater meridian altitude (or, at any rate, in the first figure, of the meridian altitude itself) and the sine of the Sun's altitude, so is the whole sine to a fourth term, then the versed sine of the Sun's distance from the meridian K will result. As is manifest from the same figures.

But if the pole of the world should stand at the zenith — that is, at the pole of the horizon — then on any given day the Sun's altitude will be perpetually equal to the declination of the parallel which the Sun then describes by the motion of the *primum mobile*. For then the equator is the same as the horizon, and the parallels of the horizon do not differ from the parallels of the Sun, or of the equator, as is evident.

END OF THE FIRST BOOK.

GNOMONICS, BOOK TWO

BY CHRISTOPH CLAVIUS OF BAMBERG, OF THE SOCIETY OF JESUS

In the preceding book we set out various theorems and problems that we judged to be either necessary or useful, so that they might serve as a kind of basis and foundation for all the demonstrations we are about to bring to bear on the description of sundials [horologia]. We now proceed, in this second book, to the description of the sundials that are usually drawn on flat surfaces. Of these there are many kinds, as we said at the outset; and for each of them we shall lay out not merely the hours (which is nearly all that every other writer has done), but — so far as it can be managed, and so that our gnomonics may be rendered as complete as possible — we shall demonstrate twelve problems. These will explain

nearly everything that can be known from the shadow of a gnomon, always keeping to the order that follows.

[Margin: What is described in this work for each of the individual planes of sundials.]

First, we shall teach the method by which, on any given plane whatsoever, one may lay out an astronomical sundial — that is, one containing the lines of the hours reckoned from noon or from midnight. These lines are nothing other than the common sections of the plane of the dial with those great circles which are drawn through the poles of the world and which divide the equator, together with the remaining parallels, into 24 equal parts.

[Margin: The hours from noon or from midnight.]

Next, we shall show the method of drawing, on the same plane, the parallels that pass through the beginnings of the signs of the zodiac, or through any other points whatever. These parallels are called by the practitioners the "arcs of the signs." On a given plane they are either circles, or parabolas, or hyperbolas, or ellipses — namely the common sections of the plane of the dial with conic surfaces whose bases are the said parallels of the signs of the zodiac and whose vertex is the center of the world. The one exception is the parallel drawn through the first points of ♈ [Aries] and ♎ [Libra]: since this does not differ from the equator, it produces a straight line on the proposed plane, as we demonstrated in the preceding book.

[Margin: The parallels, or arcs of the signs of the zodiac.]

After that, we shall describe on the same plane the parallels of the diurnal arcs. These too are themselves parallels described by the points of the zodiac under the motion of the *primum mobile*; but they are not necessarily drawn through the beginnings of the signs — though this can sometimes happen — but rather through those points of the ecliptic in which the Sun, being present, makes a day of a given number of hours, such as 12, or 13, or 9, or 8, or finally however many one may wish, provided they do not exceed the number of 24 hours. On the proposed plane these parallels too are either circles, or parabolas, or hyperbolas, or ellipses, with the exception of the parallel of 12 hours; since this is the equator itself, it produces a straight line on the plane of the dial, as we said.

[Margin: The parallels of the diurnal arcs.]

Again, in the fourth problem we shall draw on the same plane the vertical circles, which they call azimuths — that is, the common sections of the plane of the dial with the vertical circles.

[Margin: The vertical circles.]

In the fifth problem we shall lay down on the same given plane the parallels of the horizon, which they call almucantarats. On the proposed plane these (as was said of the parallels of the signs of the zodiac and of the diurnal arcs) are the common sections of the plane of the dial with conic surfaces whose bases are the said parallels of the horizon and whose vertex is the center of the world — namely either circles, or parabolas, or hyperbolas, or ellipses, with the exception of the horizon itself: since it is a great circle, it produces a straight line on the plane.

[Margin: The parallels of the horizon, which they call almucantarats.]

The sixth problem will contain, on the same plane, the description of the meridian circles of the whole world, which appear on the plane as straight lines, after the manner of the hour-lines from noon or midnight.

[Margin: The meridian circles.]

The seventh problem will exhibit the delineations of the parallels of the cities — that is, the circles of latitude — on the same plane. These are the circles drawn through the zeniths of the several places, and on the plane of the dial they become either circles, or parabolas, or hyperbolas, or ellipses, like the parallels of the signs of the zodiac or of the diurnal arcs.

[Margin: The parallels of the latitudes, or of the cities.]

In addition to these, in the eighth problem we shall mark out the celestial houses — that is, the straight lines that are the common sections of the plane of the dial with the circles of the celestial houses — both according to the doctrine of Johannes Regiomontanus [Regiomontanus], who draws them through equal parts of the equator, and according to the opinion of Campanus [Campanus of Novara], who holds that they should be drawn through equal parts of the prime vertical circle.

[Margin: The celestial houses.]

Furthermore, in the ninth problem we shall hand down the method by which, on the same proposed plane, we may describe the ascending signs of the zodiac — that is, the straight lines that are the common sections of the plane of the dial

with the ecliptic, according to the various situations and positions it holds upon the horizon as the beginnings of the signs of the zodiac rise above the horizon.

[Margin: The ascending signs.]

In the tenth place we shall construct, on a given plane, the Italian dial, which contains the hours from sunset — a usage that today flourishes especially in Italy and Bohemia.

[Margin: The hours from sunset.]

In the following, the eleventh problem, we shall set up on the same plane the Babylonian dial, comprising the hours from sunrise, which in our own time the Balearic Islands make use of.

[Margin: The hours from sunrise.]

Lastly, on the same plane we shall draw the ancient dial, comprising the unequal hours, which of old were in use among nearly all peoples everywhere.

[Margin: The ancient, or unequal, hours.]

Now, in describing these twelve matters on every proposed plane, we shall always employ geometrical demonstrations, so that in the description itself the dial-maker [horologiographus] may not be able to call anything into doubt or into question. The use of everything will likewise be explained in the proper propositions of the horizontal dials, of which we shall speak first; and this use is to be understood in the same way in all other dials.

[Margin: The argument of the second book.]

Accordingly, this second book will embrace all those dials which are usually laid out on a plane that is equidistant [parallel] to one of the following: the horizon; the vertical circle properly so called; the meridian; the circle of the sixth hour from noon or midnight; or, finally, the equator. Such a plane, whichever of the named great circles it is parallel to, is perpetually one and the same, never changing its situation within the same climate. But the remaining dials — those whose planes are parallel to great circles that either decline from the vertical, or from the horizon, or are inclined toward the horizon, or finally both deflect from the vertical and are at the same time inclined toward the horizon — these we shall defer to the third book. Within a single climate they can be varied in as many as six hundred ways. For example, a great circle declining from the vertical, to which a dial is parallel, can be made to decline toward the east or the west by a single degree, or by two, or three, four, five, six, and so on in succession

all the way to 90. Hence the dial parallel to it will likewise be varied in just as many ways, even though we remain always in the same climate, that is, at the same elevation of the pole. And the same must be said of the others.

[Margin: Which method of describing the astronomical dial on any given plane is the more approved.]

Now, since we are about to describe the astronomical dial on every plane by several methods, of them all the following method is the one I most approve (so that in this matter too I may signify to the studious reader by which way above all one should proceed in the description of dials): the method that takes the given length of the style [stylus] and its given position, and from these describes the dial — with the further addition of that method which is carried out by means of the ellipse described on the plane of the dial. This, so far as I know, we have observed to have been done by no one hitherto, still less demonstrated. The former method, explained and demonstrated, you will find in the scholia of the several astronomical dials that are described in propositions 1, 13, 25, 37, and 49 of this book, and in propositions 1, 13, 25, and 37 of Book 3, and repeated again in Book 7. The latter method you will find in the scholia of propositions 1 and 13 of this book, and in the scholia of propositions 1, 13, 25, and 37 of Book 3 — because that ellipse has no place in the scholia of propositions 25, 37, and 49 of this book. As for describing the hours from sunrise and sunset, I would always choose that method which we shall explain in Book 5 — provided, however, that the tables necessary for this matter are ready at hand, tables which we shall teach how to compute in that same book. For this method seems to be the easiest of all and the least liable to error, as we shall make plain in its proper place.

[Margin: Which method of describing the astronomical dial on any given plane is the more approved.]

We have not, however, explained those two methods of describing the astronomical dial within the propositions themselves, but have relegated them to the scholia; for the descriptions set out in the propositions are the ones used by nearly all practitioners, although even in these we have changed some things. Hence it seemed best in the first place to corroborate them by geometrical demonstrations — something which we find no one before us has performed perfectly and fully.

ON HORIZONTAL DIALS

PROBLEM 1. PROPOSITION 1.

To construct an astronomical horizontal dial. That is, to describe, on a plane that is parallel to the horizon, the lines of the hours reckoned from noon or from midnight.

[Margin: The description of the portion of the analemma.]

Let ABC be a semicircle of the meridian, whose center is D, and let BC be the common section of it and the horizon; AD the common section of the same and the vertical; FD the common section of the same and the equator; and ED the axis of the world, as we handed down in Proposition 1 of the preceding book. When the straight lines AD, ED, FD have been produced toward the side of D, take on AD, produced, a straight line DG equal to the gnomon of the future dial, of whatever magnitude it may be; and through G draw HI parallel to BC, cutting ED and FD, produced, in H and I. If, then, through BC the horizon be understood to be drawn perpendicular to the meridian, and through HI the plane of the dial parallel to the horizon — since both the meridian and the vertical are perpendicular to the horizon — then their common section AD will be perpendicular to it; and therefore, from what we demonstrated concerning Euclid, Book 11, Proposition 14, it will also be perpendicular to the plane of the dial drawn through HI parallel to the horizon.

[Margin: (a) From Proposition 19.]

Therefore HI will be, in the plane of the dial, the meridian line, or the line of the twelfth hour, being the common section of the meridian and the plane of the dial. For the straight line HI is in the plane of the straight lines HD and ID, that is, in the plane of the meridian; DG will be the gnomon, or style, perpendicular to the plane of the dial, and therefore also perpendicular to HI, the meridian line, by Definition 3 of Euclid, Book 11 — its vertex D being the same as the center of the world, by Proposition 2 of the preceding book.

[Margin: (b) From Proposition 2.]

The seat of the style is the point G, on the meridian line, distant from the point H (in which the axis of the world meets the plane of the dial, and which is called the center of the dial) by the interval GH; or from the point I (in which the plane of the equator cuts the meridian line) by the interval GI. All these things are clear and manifest, if the meridian is understood to be set in its proper situation —

namely so that the point E looks toward the arctic pole and F inclines toward the south. This figure we shall call, for the sake of instruction, the "portion of the analemma," since it is truly a part of the analemma described in Proposition 1 of the preceding book, containing, that is, the common sections of the meridian and of several circles of the sphere.

[Margin: What the portion of the analemma is called.]

Diagram. The "portion of the analemma" for the horizontal dial. A semicircle stands on a horizontal diameter BC (B at the left end, C at the right end), with center D. The radius rising vertically from D reaches the top of the arc at A, with K marked just beside A at the summit. On the arc, F is placed to the upper left and E to the upper right; straight lines run from D out to E and to F (and to A/K). Below the diameter, a shallow triangle is formed by the meridian/axis lines meeting near the foot: the roughly horizontal line at bottom carries N (lower left) and O (lower right), with H, G, and I marked in sequence along the lower meridian region between them, and M at the very bottom where the lines converge.

- A (top of arc, apex of vertical radius): vertex of the vertical/zenith direction
- K (beside A at the summit): companion label at the apex
- B (left end of diameter): end of the common section BC of meridian and horizon
- C (right end of diameter): other end of BC
- D (center): center of the meridian semicircle = center of the world
- E (upper right on arc): point looking toward the arctic pole (axis end ED)
- F (upper left on arc): point inclining south (equator section FD)
- H (lower region, on the meridian line): where the axis of the world meets the plane of the dial; center of the dial
- G (lower region, between H and I): seat of the style
- I (lower region): where the plane of the equator cuts the meridian line
- N (lower left): end of the dial-plane line HI produced
- O (lower right): other end of the dial-plane line
- M (bottom, convergence point): foot of the construction

[Margin: The description of the horizontal astronomical dial.]

Now, by the help of the triangle DHI, we shall construct the astronomical horizontal dial in this way. On a straight line HE, drawn at will to serve as the meridian line in the plane of the dial, let there be transferred HI, the meridian

line of the portion of the analemma; and let there be cut off from the straight line HE a length IE equal to the straight line DI in the same portion of the analemma. Then through I, in the plane of the dial, let the perpendicular FK be raised to HE. And from E let a circle be described at any interval whatever; when this has been divided into 24 equal parts, beginning from the meridian line HE produced, let lines be drawn through the points of division, and [continues]—

—and [drawn] from the center E, secret [auxiliary, unmarked] straight lines cutting the straight line FK in points; and through these points, straight lines emitted to the point H will give the hour-lines from noon or from midnight on the plane parallel to the horizon. Thus HE will give the twelfth hour of the day; and the lines placed on our left, as we face the dial (I call the left part that in which the point F lies, and the right that in which the point k lies), will show the hours after noon, while the lines placed on the right will show the hours before noon, or after midnight, as the numbers themselves indicate. For the sixth hour, the line CL must be drawn through H perpendicular to HE, or parallel to FK; and HC will mark the sixth hour from noon, and HL the sixth hour from midnight.

[Margin: The order of the hours in the horizontal dial.]

Here is the demonstration of this construction. Let the portion of the analemma be understood to be superimposed on the dial, in such a way that the point H coincides with the point H, and the point I with the point I, and the straight line HI with the straight line HI (it will indeed coincide with it because of the equality of the straight lines HI and HI); and let this portion of the analemma be conceived in the mind as standing upright to the plane of the dial, while the meridian line HE keeps its proper situation, so that it is the common section of the meridian and the plane of the dial, with the point H inclining to the south and E to the north; and finally so that the straight line BC is parallel to the plane of the dial, so as to be the common section of the meridian and the horizon, with D — the vertex of the style — being the center of the world, as we showed in Proposition 2 of the preceding book.

[Margin: The demonstration of the construction of the horizontal dial.]

Or rather, in place of the portion of the analemma, let just the triangle DHI be understood to be superimposed on the dial in the manner we have said — so that DH represents the axis of the world; DI, the common section of the meridian and the equator; and finally DG, the style, the common section of the meridian and of the vertical circle drawn through D, the center of the world. This triangle DHI, together with the style DG, is what has been drawn on the dial. With these things

so placed, the axis DH will meet the plane of the dial in H, the diameter of the equator DI in I, and the diameter of the vertical DG in G. Since, then, both the plane of the equator and the plane of the dial (which is parallel to the horizon) are perpendicular to the plane of the meridian, the common section of the equator and of the plane of the dial will also be perpendicular to that same plane of the meridian, and therefore [continues]

[Margin: (a) From Proposition 19.]

Diagram. The full horizontal-dial construction. A large outer semicircular fan of hour-lines stands on a horizontal diameter running left to right, with a smaller circle (the equator circle described from center E) set within it, and a central sun-burst of radiating lines converging at the dial-center H. The horizontal diameter carries C at the left (marked 6) and L at the right (marked 6). Around the outer arc and lower border run the hour-numbers; a dense cluster of small point-letters surrounds the central sun-burst. Because this is a crowded figure with many small worn labels, several central labels are flagged uncertain.

- Horizontal diameter, left end: C, with hour-number 6 outside it
- Horizontal diameter, right end: L, with hour-number 6 outside it
- Central sun-burst point (radiating center): H (center of the dial)
- Central region, small labels: M, R, r, b, d, a, f [?] — dense cluster of construction points around the sun-burst, several worn
- T (left of center, on the inner circle)
- D (just right of center, near the sun-burst) — center of the world in the superimposed triangle
- K (right of center) — foot of the vertical
- E (below center, within the lower fan) — center of the equator circle
- F (center-left) — point on the equinoctial line FK
- Z (upper left, outside the inner circle)
- e (top center, on the inner circle)
- δ [?] (lower left, on the inner circle)
- V (lower left, outside the inner circle)
- N, O, Q, S (along the lower border, left to right beneath the fan)
- Outer arc hour-numbers, upper row (left to right): 7, 8, 12, 1, 2, 3, 4, 5
- Lower and side hour-numbers: 6 (left), 6 (right), 7 (right), 8 (right), 9, 10 (lower right); 5, 4, 3, 2 (lower left); 11, 12, 11 (bottom center)

—by Definition 3 of Euclid, Book 11, it will [be] perpendicular to the meridian line HE, which lies in the plane of the meridian, at the point I, in which the equator — drawn as the straight line DI — meets the meridian line in the dial, as has been said. Therefore the straight line FK, which in the dial has been raised through I perpendicular to the meridian line HI, is the common section of the equator and the plane of the dial — namely the equinoctial line. Now suppose the plane of the circle described from center E is understood to be moved about this line, until its center E is joined with the center of the world D in the triangle DHI (which we said to be perpendicular, together with the whole portion of the analemma, to the plane of the dial, although only the triangle is drawn on the dial and exists in the plane of the meridian). This joining is possible because the straight lines DI and IE are equal by construction; and in that motion of the circle described from E, the line IE always remains in the plane of the meridian, in which the line DI lies. The circle will therefore be described about the center of the world D — that is, it will be the equator — and it will lie in the plane of that same equator, since the plane of the equator is drawn through the straight lines DI and Ik, just as the said circle in such a position is drawn through the same lines; so that this circle and the equator lie in the same plane and have the same center, namely D, the center of the world.

Let IE also be understood to be produced in both directions in the same position, since it moves together with the circle, being as it were the common section of the meridian and the equator — for both the meridian and the equator, in a position of this kind, pass through the straight line DI, or IE (which in that situation is the same as DI). Now, since from what we wrote in the *Commentaries on the Sphere*, toward the end of chapter 1, circles described in the same plane and about the same center are cut into similar parts by lines going out from the center, it follows that those secret [auxiliary] lines drawn through the center E — which is now joined with the point D, that is, with the center of the world — and which cut the circumference of the circle into 24 equal parts, and which meet the straight line FK, the common section of the equator and the plane of the dial, in the points I, A, B, and so on: these lines, if produced, will also cut the circumference of the equator, beginning from the meridian, into just as many equal parts. And therefore, since the circles of the hours from noon or from midnight divide the same equator into 24 equal parts, likewise beginning from the meridian, as is clear from Proposition 9 of the preceding book, it comes about that these same secret lines going out from the center E are the common sections of the equator and of the hour-circles which [mark the hours] from noon or from mid[night] [continues]

mid[night] indicate. Namely, NM is the common section of the equator and the circle of the 12th hour — that is, of the meridian; OP, of the equator and the circle of the first hour from noon or from midnight; QR, of the equator and the circle of the 2nd hour from noon or from midnight; ST, of the equator and the circle of the 3rd hour from noon or from midnight; and so on — as is clear if the circle described from E is understood to be in its proper position. For the points O, Q, S will incline, in the upper hemisphere, toward the western or afternoon parts, and so on.

Therefore the hour-circles from noon or from midnight cut the straight line FK in the plane of the dial at the points I, A, B, and so on. (These points do not change even if the plane of the circle about E has another position — namely if it lies in the plane of the dial, where we described it. For if the plane of this circle is turned about the straight line FK as about a fixed hinge, so that the straight line IE is always perpendicular to FK, the straight lines NE, OE, QE, and so on will always meet the equinoctial line FK at the same points, as is evident.) And so through these points will pass the common sections of those hour-circles and the plane of the dial, which are the hour-lines from noon or from midnight. But by the corollary of Proposition 21 of the preceding book, these same common sections, or hour-lines, cut one another at the center H of the dial, where the axis DH meets the plane of the dial. Therefore the straight lines drawn through the points I, A, B, and so on of the equinoctial line FK, and through the point H, or the center of the dial, are the lines of the hours from noon or from midnight — so that the meridian line HIE indicates the 12th hour of noon; the lines to its left show the hours from noon, while the lines to its right show the hours from midnight, as appears in the figure and as we shall explain in the following scholium.

Only the line of the 6th hour must be drawn through H, perpendicular to HI, or parallel to FK itself — such as CL is. For since, as we taught in the scholium of Proposition 22 of the preceding book, in a horizontal dial the equinoctial line and the line of the 6th hour from noon or from midnight are parallel, it is clear that the straight line CL is the line of the 6th hour from noon or from midnight, since it is parallel to the equinoctial line FK and passes through the point H, where all the hour-lines from noon or from midnight intersect one another. We have therefore established the astronomical horizontal dial, etc. Which was to be done.

SCHOLIVM.

[Margin: Another way of laying out the astronomical horizontal dial, most convenient for drawing the hours that are near the 6th hour from noon or from midnight.]

Andreas Schöner [Andreas Schoner] proposes another way of describing the hours from noon or from midnight, which is very convenient for laying out those hours that cut the equinoctial line at points very far off — such as the hours near the 6th hour from noon or from midnight. It runs as follows. On the equinoctial line FK take the straight line Ia equal to the straight line IE (or ID); and through a draw a line ab perpendicular to the equinoctial line — or parallel to HI — cutting at b the line of the 6th hour CL (which is always drawn from H, the center of the dial, perpendicular to HI, or parallel to the equinoctial line, as has been shown). Then take bd equal to ab, and through d draw ef perpendicular to bd, or parallel to HI. Finally, from d as center describe a circle, and dividing it into 24 parts, beginning from the straight line bd, draw hidden [auxiliary] lines from the center d through the points of division. For these will cut the straight line ab at points through which, if straight lines are drawn from H, the center of the dial, they will give — as before — the hours from noon or from midnight.

[Margin: The segments of the equinoctial line in a horizontal dial, between the meridian line and any two hours equally distant in time on either side of noon, are equal. (a) *Elements* III.27. (b) *Elements* I.26.]

Moreover, in any horizontal dial whatever, any two hour-lines equally distant in time on either side of the meridian line cut off equal segments from the equinoctial line FK — namely those segments that lie between the hour-lines and the meridian line. Thus the lines HF and HK, each of which is distant five hours from the meridian line, cut off the equal segments IF and IK. [a] For the angles IEF and IEK — contained by the meridian line and the hidden straight lines drawn from E to F and K (for although EF is not actually drawn, it must be conceived as drawn) — are equal, since at the center E they are subtended by equal arcs of the circle described from E; and the angles at I are right angles. Therefore the two angles IEF and EIF of triangle EIF are equal to the two angles IEK and EIK of triangle EIK. And since they have the side IE in common, [b] the remaining sides of the one will be equal to the remaining sides of the other — namely IF to IK, which was the proposition. And the reasoning is the same for the rest. Therefore the hour-spaces before noon are equal to the hour-spaces after noon, each to each, on the equinoctial line of the horizontal dial.

[Margin: The arcs of a circle described from H, the center of a horizontal dial, intercepted between the meridian line and any two hours equally distant on either side of noon, are equal. (c) *Elements* I.4. (d) *Elements* III.26.]

From this it follows that the arcs of any circle described from H in the horizontal dial, intercepted between the meridian line and any two hour-lines equally distant in time from the meridian — such as, for example, the arcs of a circle described from H intercepted between the meridian line HI and the lines HF, HK — are equal. For since the sides IH, IF of triangle IHF are equal to the sides IH, IK of triangle IHK, and they contain equal angles — namely right angles — [c] the angles IHF and IHK will also be equal, [d] and consequently in the circle described from H they will be subtended by equal arcs; and so for the rest.

[Margin: The intervals between E, the center of the equator, and any two hours equally distant in time on either side of noon in a horizontal dial, are equal.]

It follows again that the intervals between the center E of the equator and any two hours on the equinoctial line FK, distant by an equal interval of time from the meridian line — such as the intervals EF and EK — are equal. For since the sides IE, IF of triangle IEF are equal to the sides IE, IK of triangle IEK, and contain equal angles, the bases EF and EK will also be equal; and so for the rest.

[Margin: For the description of the astronomical horizontal dial it is enough if just one quadrant of the circle described from E is cut into 6 equal parts.]

Finally it follows that, for the description of the astronomical horizontal dial, it is enough if one quadrant of the circle described from E — such as, for example, the quadrant Mg — is divided into six equal parts. For when hidden straight lines are drawn through the points of division and the center E, cutting the straight line FI at the points through which the hour-lines are to be drawn from H, then if these points are carried over from I to the other side of the equinoctial line, toward K, other points will be obtained through which the hour-lines are to be drawn — since each hour-space on the straight line IF is equal to the corresponding hour-space on the straight line IK, as has been demonstrated. Again, once the hour-lines have been drawn from H through the points of the straight line IF (or IK), in order that the remaining ones on the other side may be drawn more accurately, the points of the circumference of the circle described from H must be carried over — from the point where the meridian line and the said circumference intersect one another — to the other part of the same circumference. For through these the hour-lines drawn from H through the points of the straight line FK must

certainly pass, since the arcs of the said circle on one side are equal to the arcs of the same circle on the other side, as has been shown.

[Margin: How the horizontal dial, together with the axis of the world, is to be positioned so that it may show the hours from noon or from midnight.]

But if the axis HD, in the triangle DHI (right-angled to the plane of the dial), is understood to be a stretched thread — or even some thin piece of iron — making at H an angle with the meridian line equal to the angle DHI, and right angles with the straight line CL; or if the triangle DHI itself is constructed of some solid material and set upright to the plane of the dial: then the shadow of the axis DH will indicate the individual hours from noon or from midnight, provided the dial is set on a plane parallel to the horizon in such a way that the straight line HI represents the meridian line (whose determination on a plane parallel to the horizon we have handed down both in the *Commentaries on the Sphere* [*In Sphaeram*], where we treated of the meridian circle, and in the scholium of Proposition 23 of the preceding book), with the point H turned toward the south and I toward the north. For since all the hour-circles from noon or from midnight are drawn through the axis of the world, it comes about that the Sun's ray, in any one of them,...

lying in that circle, does not depart from the axis and from the common section of that circle with the plane of the dial, since both the axis and this common section — namely its hour-line — lie in the same hour-circle. Therefore the shadow of the axis will be projected onto the hour-line of that circle; and so for the rest. Hence it comes about that the shadow of the vertex D of the style DG (perpendicular to the plane of the dial at the point G) — that is, the shadow of the center of the world (for the vertex of the style is the same as the center of the world, by Proposition 2 of the preceding book) — is projected onto the same hour-lines, because the vertex D also lies on the axis, all of whose points cast their shadow onto the hour-lines, as has been said; which we also demonstrated in Proposition 11 of the preceding book. Thus the use of the astronomical dial is that, from the shadow of the style falling on its hour-lines, we understand how many hours have passed since noon or since the nearest midnight, and how many remain until the following noon or midnight. For in the morning, if the shadow falls, for example, on the line of the 7th hour, we shall take it for certain that seven hours have elapsed since midnight, and that five remain until the next noon, and so on.

[Margin: If the style DG is perpendicular to the plane of the dial at G, the shadow of the vertex D alone will indicate the hours from noon or from midnight.]

Now if one wishes to mark the half-hours as well on the dial, and their quarters — or indeed their eighths, or any other subdivisions — then each of the parts of the circle described from E must be divided into the said parts: into two, four, eight, and so on; and the rest carried out as before. This must be observed also in the other dials that follow.

[Margin: The use of the astronomical dial. How the half-hours and their quarters, or any other subdivisions, are marked on the dial.]

Here, however — and in everything that follows — it must be carefully observed that any line in the dial which cuts the equinoctial line represents that celestial semicircle whose common section, made on the circle we described from E, the center of the equator, and in its plane (that is, provided its center E is joined with the center of the world D), passes through the center E before it meets the equinoctial line in the dial. Namely, the straight line HA is the first hour from noon — that is, it represents that celestial semicircle in which the Sun stands at the first hour from noon; because at A falls OE, drawn through the center E, and this OE is the common section of the semicircle of the first hour from noon drawn through the point O, and passes through the center E before it cuts the equinoctial line at A — as is clear if the circle described from E is conceived to be in the plane of the equator, with its center E joined to the center of the world D. The reason for this is that the Sun casts its ray first onto the center of the world, or the vertex of the style, before the shadow of the style...

[Margin: Which celestial semicircle each line in the dial represents.]

Diagram. A large horizontal-sundial construction. The figure is bounded by a full circle; its lower half is completed while the upper half carries the analemma construction on the left. At the center is **H**, the center of the dial, where all the hour-lines converge; immediately below H is a small circle **G** (the foot of the style). Below G, on the vertical meridian line, lie **I** (where the equinoctial line crosses the meridian) and **E** (the center of the equinoctial circle), with **D** (the center of the world) marked close to I. A vertical meridian runs from **M** (top, just under the "12") down through H, I, E to **N** (bottom, just above the lower "12").

Two roughly horizontal lines cross the figure: the upper one, through H, runs from **C** (left) to **L** (right) — the line of the 6th hour, its ends reaching the "6" marks at the circle on each side; the lower one, through I, runs from **F** (left) to **K** (right) — the equinoctial line. On the upper left a dotted/dashed arc carries the analemma, marked with **A** (upper left, beside "8"), **B** (left, on a dashed line), and **C** (lower left, a small letter on the dotted arc).

Radial hour-lines are drawn from H to the circumference and numbered. Along the top, near noon, run the hours **9, 10, 11, 12, 1, 2, 3, 4**; around the rest of the circle the hours **1–12** are engraved (each hour-line serving an AM and a PM hour), reading down the right side (**5, 6, 7, 8, 9, 10**) and up the left side (**5, 4, 3, 2, 1**) to the bottom (**12, 11, 10**).

- H (center): center of the dial; convergence of hour-lines
- G (just below H): foot of the style [small glyph, ~[?] confidence]
- I (below G, on meridian): equinoctial line meets meridian
- E (below I, on meridian): center of the equinoctial circle
- D (beside I): center of the world
- M (top, on meridian): north point of the analemma quadrant
- N (bottom, on meridian): south point
- A (upper left, on dotted arc, by "8"): analemma point
- B (left, on dashed line): analemma point
- C (lower left, on dotted arc): analemma point [small glyph, [?]]
- C ... L (through H): line of the 6th hour
- F ... K (through I): equinoctial line
- Hour numerals 1–12 around the circumference; 9–12, 1–4 also along the top

falls on the plane of the dial, as is evident. In the same way the straight line HK will be the 7th hour from midnight — that is, it will represent that celestial semicircle in which the Sun stands at the 7th hour from midnight; because at K falls VE, drawn through the center E, and this VE is the common section of the semicircle of the 7th hour from midnight drawn through the point V, and passes through the center E before it cuts the equinoctial line at k, as is evident; and so for the rest.

It must also not be omitted that the line of any hour from noon, produced beyond the center of the dial, shows the hour of the same number from midnight, and conversely. We shall demonstrate it thus. Take, for example, the circle of the fifth hour from noon and from midnight, which makes in the dial the line of that same hour drawn through the center H. Now if the Sun were placed at the arctic pole, the shadow of the style would fall on the point H — since the Sun's ray would then be the same as the axis of the world, which, drawn through the vertex of the style, meets the plane of the dial at the point H. It follows that, when the Sun is in the semicircle of the said circle which is more to the west than the meridian, the shadow of the style falls on the opposite side — namely on that part of the line of the fifth hour which is more to the east in the dial than the meridian line; and

when the Sun is in the other semicircle of the same circle, which is more to the east than the meridian, the shadow of the style falls on that part of the same fifth-hour line which is more to the west than the meridian line. For the shadow always falls on the said line while the Sun is in the said circle, as we showed in Proposition 11 of the preceding book. Since, therefore, the western semicircle pertains to the fifth hour after noon, and the eastern to the fifth after midnight, the eastern portion of that line — the one which from the center H extends toward the eastern part of the dial (for the meridian line divides the whole dial into an eastern and a western part, just as the meridian divides the whole celestial sphere into an eastern and a western hemisphere) — will indicate the fifth hour from noon; while the remaining portion of the same line, inclining toward the western part, [will indicate] the fifth hour from midnight. And the reasoning is the same for the other lines, in other dials too that are not horizontal.

[Margin: The line of any hour from noon, produced beyond the center of the dial, indicates the hour of the same number from midnight, and conversely.]

We shall show the same thing by this reasoning too, and more briefly. Since the shadow of the style before noon falls on the western part of the dial, but after noon on the eastern — because it is always cast toward the part opposite the Sun — all the hour-lines terminating at the center H that lie in the western part (which the meridian line separates from the eastern in every dial) will belong to the forenoon hours, or those after midnight; while the remaining ones, in the eastern part, [will belong] to the afternoon hours. Since, therefore, the hour-lines in...

the eastern part of the dial, produced beyond the center, exhibit the lines of the same hours in the western part, and conversely, what is proposed is evident.

[Margin: Any great circle drawn through the poles of the world makes on the dial a straight line, one portion of which, terminating at the center, belongs to one semicircle, and the other to the other.]

From this it is gathered that any great circle passing through the poles of the world makes on the dial a section that is a straight line, one portion of which, terminating at the center, belongs to one semicircle, and the other to the other. For this has been demonstrated in the case of the hour-circles from noon and from midnight. And the same is true of the meridian circles, which we shall treat in Proposition 6 of this book. Likewise of the vertical circles, which we shall discuss in Proposition 4 of this book — provided that in place of the pole of the

world we take the vertex of the head, or the zenith, that is, the pole of the horizon; and in the dial, in place of the center of the dial, we take the vertical point, which is where the axis of the horizon falls — such as, in the horizontal dial we described, is the point G.

[Margin: How one is to judge which hour-line belongs to the hours from noon and which to the hours from midnight, in the dial that has been described.]

From these things we shall very easily be able to judge which hour-lines belong to the hours from noon and which to the hours from midnight. For since, if the circle described from E is placed in its proper position, the point N cuts the meridian above the horizon, that point will belong to the 12th hour of noon, so that the straight line NE is the common section of the meridian, or of the circle of the 12th hour from noon. Since, therefore, NE passes through the center E and cuts the equinoctial line at I, the straight line HIE will indicate the 12th hour of noon, and its remaining portion — beyond the center H of the dial — the 12th hour of midnight. By the same reasoning, the straight lines HA, HB, and the rest, placed in the eastern part of the dial, will belong to the hours after noon, because they represent the hour-semicircles of the sections OE, QE, and so on, which indeed indicate the hours from noon on the circle described from E. Finally, the lines placed in the western part of the dial — that is, to the right of the meridian line itself — will signify the hours from midnight, because they represent the hour-semicircles whose sections on the circle described from E recede from the straight line NE toward the eastern region, and show the hours before noon, or from midnight.

If this is carefully considered, we shall very easily recognize, in all the other dials too of which we shall treat in what follows, which hours are to be counted from noon and which from midnight.

[Margin: How one is to judge, in every dial, which hour-line belongs to the hours from noon and which to the hours from midnight. How a dial is to be described on a stable plane parallel to the horizon. Description of the same dial for a given length of style, whose position is also given, without a portion of the analemma constructed separately.]

The reader must also be advised that up to now we have constructed the horizontal dial on any flat surface whatever, so that it may afterward be set in its proper position, or transferred onto some stable plane parallel to the horizon, in such a way that the straight line HE is the meridian line, and the point H is turned toward the south, but the point E toward the north — as we have taught in this

scholium. But if some stable plane parallel to the horizon is offered to us, on which to draw the dial, we shall do it by this method. Having found the meridian line HE on the plane, we shall cut it at right angles at I by the straight line FK, which will be the equinoctial line. Then from I toward the south, as far as H, we shall carry over, from a portion of the analemma, the straight line IH; and toward the north we shall take the straight line IE equal to the straight line ID, from the same portion of the analemma. Finally, from E we shall describe a circle, and, dividing it into 24 equal parts, we shall complete the rest as before.

But if we wish to describe the same dial on a given plane, without a portion of the analemma constructed separately, for any...

Diagram. A horizontal-sundial construction essentially identical to that on the preceding leaf, here illustrating the direct method (given style-length, without a separately constructed analemma). The figure is bounded by a full circle. At the center is **H**, the center of the dial, where the hour-lines converge; just below it a small circle **G** marks the foot of the style. On the vertical meridian below G lie **I** (equinoctial line meeting the meridian) and **E** (center of the equinoctial circle), with **D** marked near I; the meridian runs from **M** (top, under "12") through H, I, E to **N** (bottom). The upper horizontal line through H runs from **C** (left) to **L** (right), the 6th-hour line, its ends at the "6" marks; the lower horizontal line, through I, runs from **F** (left) to **K** (right), the equinoctial line. On the upper left a dotted arc carries the analemma points **A** (upper left, by "8"), **B** (left, on a dashed line), and **C** (lower left, small, on the dotted arc).

Radial hour-lines run from H to the numbered circumference: **9, 10, 11, 12, 1, 2, 3, 4** across the top near noon, and the hours **1–12** around the rest of the circle (down the right side **5, 6, 7, 8, 9, 10**; up the left side **5, 4, 3, 2, 1**; across the bottom **12, 11, 10**).

- H (center): center of the dial
- G (just below H): foot of the style [small glyph, ~[?] confidence]
- I (below G, on meridian): equinoctial line meets meridian
- E (below I): center of the equinoctial circle
- D (beside I): center of the world
- M (top) ... N (bottom): ends of the meridian / analemma quadrant
- A, B, C (upper-to-lower left, on the dotted arc): analemma points; C is a small, faint glyph [[?]]
- C ... L (through H): 6th-hour line
- F ... K (through I): equinoctial line

- Hour numerals 1–12 around the circumference; 9–12, 1–4 also along the top

[Margin: How a dial is to be described on a stable plane parallel to the horizon, for a given length of style whose position is also given.]

length of style, whose position is also given, we shall do it by this method. Let the given length of the style be DG , and let its position on the plane of the dial be the point G . If, then, the plane of the dial is any whatever — so that the dial drawn on it may afterward be set in its proper position, or transferred onto a stable plane parallel to the horizon, as we said just now — we shall draw through G , the position of the style, any straight line MN as the meridian line. But if a stable plane for the dial, parallel to the horizon, is proposed, we shall find — by what we wrote in the scholium of Proposition 23 of the preceding book, or elsewhere — the meridian line on the proposed plane; and if it happens not to pass through G , the position of the style, we shall draw through G , the position of the style, a line MN parallel to it, as the meridian line. Then to this meridian line MN we shall raise at G the perpendicular BGD , and cut off GD equal to the given style. From D as center we shall describe the arc of a circle ABC , in which, from the straight line DGB toward the southern parts (which are now supposed to incline toward M), we shall count off BA , the complement of the pole's altitude, and toward the northern parts — that is, toward N — the pole's altitude itself, BC . And, drawing the straight lines DA , DC , we shall cut the meridian line at the points H and I . After this, through I we shall raise a perpendicular FK to the meridian line, as the equinoctial line. Finally, taking the straight line IE equal to ID , we shall describe from E a circle of any size; and, dividing it into 24 equal parts, beginning from the meridian line, we shall complete the rest as we taught before in this proposition.

[Margin: Demonstration of this description.]

The demonstration of this description is easy. For if the meridian line MN has its proper position, so that M [is turned] toward the south, and N ...

and let N incline toward the north, and let the triangle HDI be set upright to the plane of the dial, so that it lies in the plane of the meridian circle. Since, by construction, the angle HDG is equal to the complement of the pole's altitude, the remaining angle DHG will be equal to the pole's altitude. Again, since by construction IDG is the angle of the pole's altitude, the remaining angle DIG will be equal to the complement of the pole's altitude. Taking, then, D — the vertex of the style — as the center of the world, DH will make with the meridian line at H an angle equal to the pole's altitude, and it will be the axis of the world meeting the plane of the dial at H , the center of the dial. And the line DI , making with the

same meridian line at I an angle equal to the complement of the pole's altitude, will be the common section of the meridian and the equator — since a section of this kind, in the sphere, always makes with the horizontal meridian line an angle equal to the complement of the pole's altitude, and with the axis a right angle (such as is the angle HDI, made up of the angle of the pole's altitude together with the angle of its complement). Therefore the equator meets the plane of the dial at the point I, and consequently, as was demonstrated above, the line FK will be the equinoctial line. The line DG, in turn, will be the common section of the meridian and the vertical. Everything else is demonstrated as before.

Now, in all the methods by which we have thus far drawn a horizontal dial, this drawback occurs: the hour-lines nearest to the line of the 6th hour from noon (or from midnight) can scarcely be drawn, because they intersect the equinoctial line at points extremely far off. I shall therefore show another way — and one indeed that (so far as I know) has never been attempted by anyone before — by which we can draw all the hour-lines through the points of a certain ellipse described on the plane of the dial, even when we have no points on the equinoctial line. This method is altogether necessary for correctly laying out dials that decline from the vertical, whether at the same time they are also inclined to the horizon or not, because in these some of the hour-lines are at times very nearly equidistant from the equinoctial line, as will be made plain in its proper place. For although Andreas Schonerus [Andreas Schöner] accomplished this same thing by another procedure, as we said above, yet neither he nor anyone else has furnished any demonstration of that construction. The way I have discovered is as follows.

[Margin: Another description of a horizontal astronomical dial, new and most beautiful, by the aid of the ellipse — that is, without points found on the equinoctial line.]

Given the style DG, and its position at G, let the meridian line HI be drawn through G. Setting up again, upon the style GD (which is perpendicular to the meridian line), the angle GDH equal to the complement of the pole's altitude and the angle GDI equal to the pole's altitude — so that HD is the axis of the world, H the center of the dial, and I the point through which the equinoctial line is to be drawn perpendicular to HI, just as we demonstrated in the preceding construction — we shall take on the axis HD, produced, any point A, and from it we shall raise the perpendicular AB to HA, so that AB is a new common section of the meridian and the equator. The farther off the point A is taken, the more accurately the dial will be drawn. Hence, if the given style is so large that the line HI has a notable length, we shall content ourselves with the points D and I. But

because in the example of the preceding construction the line HI is too short, we have for that reason taken other, more remote points A and B. Then, cutting off HC equal to AB, let there be described from the center H, at the intervals HB and HC, two circles — HB being the distance between the center of the dial and the point B, through which the equinoctial line is drawn on the meridian line, while HC is equal to the common section AB of the meridian and equator between the meridian line and the axis. Having first drawn the diameter ST perpendicular to the meridian line HB, let these circles be divided into 24 equal parts, beginning from the meridian line HB, or from the diameter ST. It is enough if the outer circle is divided; for the straight lines drawn from the division-points to the center H will cut the inner circle also into 24 equal parts, as we demonstrated at the end of the commentaries on the sphere. After this, let two corresponding division-points be taken, such as, for example, F and K — for they lie on the same straight line Fk drawn from F to the center H, and each is four hours distant, in its own circle, from the meridian line — and through F,

Diagram. A large horizontal-dial construction figure. Two concentric circles are centered on **H** (the center of the dial); a vertical meridian diameter runs from **V** (upper region) down through the center to **B** (bottom), and a horizontal diameter **T ... S** (the 6th-hour line) runs left-to-right through the center. Radial lines divide the circles into 24 parts, and the annular band between the two circles is filled with small dotted right-triangle graduation marks (the hour divisions). At the upper left a tilted oval (the second, oblique base-circle of the cylinder / the rotated gnomon-triangle) carries its own labels. A long straight diagonal (the axis of the world, produced) crosses the whole figure, its two ends each marked with a glyph read as "4" [?]. The dial's hour-line construction points and the ellipse points are lettered with both Roman capitals and Greek minuscules.

Readable labels:

- H (center): center of the dial
- G, D, I (small cluster just below H): style-vertex D, and the meridian points G and I
- V (top of meridian diameter): upper meridian point
- B (bottom of meridian diameter): lower meridian point
- T (left end) ... S (right end): ends of the horizontal 6th-hour diameter
- N (upper-left, inner circle); R (left, inner circle); K, L (lower-left, inner circle); F (lower-left, outer)
- X (right, inner circle); M (lower-right, outer)
- C (lower-center, near B)

- Tilted oval, upper left: e, a, d, h, ξ , and λ [several uncertain]
- Outer ring, upper right: β , γ ; right (below S): θ ; lower right: δ , ε ; and Y, A, P, E, O, ρ , μ scattered near the lower-right and center [many uncertain]
- Top glyph above V: π [?]

the point of the outer circle, let there be drawn, parallel to the diameter ST (which cuts the meridian line at right angles), the concealed line FL. This is easily done if a concealed line is drawn from F to the point M, which is as far from the point S as the point F is from the point T; for the lines FM and TS will be parallel, by the scholium to Proposition 27 of Book 3 of Euclid, on account of the equality of the arcs FT and MS. Likewise, through K, a point of the inner circle, let there be drawn, parallel to the meridian line HB, the concealed line KL — which is likewise easily done if a concealed line is drawn from K to the point N, which is as far from the point R as the point K is from that same point. And let the line KL cut the line FL at L. If the same be done with the remaining pairs of corresponding division-points, as appears in the figure, the points of the ellipse will have been found — its major diameter being BV, its minor RX, and its center H, as is clear from the corollary to Proposition 26 of the foregoing book. I say that the straight lines drawn from H through these found points of the ellipse are the hour-lines, so that each one denotes the same hour that the corresponding division-points denote on the circles. For example, since the point F denotes the fourth hour after noon, the line HL, drawn through it, will accordingly give the fourth hour from noon, and so on for the rest. And by this procedure we shall draw the hours most conveniently, since we always have three points for each hour — two of them opposite on the ellipse, such as L and α , the third being the center H itself. The point α , opposite to L, is found on the ellipse in this way. Take the arcs $S\beta$ and $X\gamma$, equal to the opposite arcs TF and RK, and draw the lines $\beta\alpha$ and $\gamma\alpha$ parallel to the diameters ST and BV, cutting one another at α ; for the point α will be opposite to the point L. And if one wishes to draw the half-hours and their quarters as well, the several parts of the circles must be bisected, and divided into four parts, and so on. This construction, which is the most elegant of them all, we shall demonstrate as follows.

[Margin: Demonstration of this construction.]

Diagram. Essentially the same horizontal-dial figure as on the preceding page, now serving the demonstration: two concentric circles centered on **H**, with the vertical meridian diameter **V ... B**, the horizontal 6th-hour diameter **T ... S**, the 24-part radial division with dotted right-triangle graduation marks in the annular band, the tilted oblique base-circle at the upper left, and the world-axis diagonal

(ends marked "4" [?]) crossing the whole. This copy also shows the cylinder / parallelogram section-construction referenced in the demonstration below.

Readable labels (as on the preceding figure):

- H (center); G, D, I, μ (central cluster)
- V (top) ... B (bottom): meridian diameter ends
- T (left) ... S (right): 6th-hour diameter ends; θ just below S
- N, R (left, inner); K, L (lower-left); F (lower-left, outer); X (right, inner); M (lower-right)
- Upper right: β , γ ; lower right: δ , ϵ ; near lower center/right: C, P, E, O, Y, A, ρ [uncertain]
- Tilted oval, upper left: e, a, d, h, ξ , λ [uncertain]
- Top glyph above V: π [?]

Let the triangle ABH be understood to be turned about the line BH until it stands upright to the plane of the dial and is joined with the plane of the meridian circle. From A, which represents the center of the world, let there be conceived, at the interval AB, a circle θYBZ described, concentric with the equator and in the plane of the equator; this circle will therefore, like the equator, be divided into 24 equal parts by the common sections of the hour-circles and the equator. Let the points O and P also be four hours distant from the point B (of noon), just as the points F, K, M, δ are four hours distant from that same point B (of noon). For although the point P, on the equator, belongs to the 4th hour from midnight (as is evident if the circle θYBZ is set in its proper position, so that the semicircle θYB is the western one and $BZ\theta$ the eastern), nevertheless, because A is placed as the center of the world, the Sun, existing at ϵ at the 4th hour from noon, must cast a ray through the center A and the point P as far as the plane of the dial; and so the section which the hour-semicircle through P, drawn on the plane of the dial, makes, denotes the 4th hour from noon, as is clear from what we wrote above in this scholium. Hence not without reason may the point P be taken for the 4th hour from noon, since the hour-semicircle drawn through it makes, on the dial, the line of the 4th hour from noon. In the same way the point O can be taken for the 8th hour from midnight, although on the equator it in fact denotes the 8th hour from noon. Let a cylinder now be understood whose bases are the circles θYBZ and another, $aeb\pi$, equal and opposite to it, its axis being the same as the axis of the world, $AH\lambda$. And since all the hour-circles from noon (or from midnight) are drawn through the axis of the world, that is, of the cylinder, their planes will make in the cylinder parallelograms, such that two sides of each are diameters of the bases of the cylinder while the remaining two are described on the surface of

the cylinder, as is demonstrated by Serenus in Book 1, *On the Section of the Cylinder*, Proposition 2. For example, the circle of the 4th hour from noon (or from midnight), drawn through the points ε and P in the circle $\Theta Y B Z$, will make a parallelogram, one side of which is εP , a diameter of the base of the cylinder, while two are drawn from the points ε and P on the surface of the cylinder, and so for the rest. I say that these sides, on the surface of the

cylinder, fall on the points of the ellipse described on the plane of the dial — namely, that the sides drawn from the points Z, Y (of the circle of the 6th hour from noon or from midnight) fall on the points R, X; the side from the point θ , of the meridian circle, falls on the point V; and the sides drawn from the points ε , P fall on the points α , L; and so for the rest. For let there be drawn, through Y and Z, the diameter that cuts the diameter θB at right angles, and, through the axis AH, the circle of the 6th hour meeting the plane of the dial at H. And since both the plane of the dial and the plane of this circle of the 6th hour are upright to the meridian, their common section — namely, the line of the 6th hour — will also be perpendicular to it, and therefore perpendicular, at the point H, to the meridian line HB, which lies in the meridian, by Definition 3 of Book 11 of Euclid. [Margin: (a) Euclid XI.19] Therefore the diameter ST, which we drew perpendicular to the meridian line HB, is the common section of the plane of the dial and the circle of the 6th hour — that is, the line of the 6th hour from noon or from midnight. And for this reason the sides of the parallelogram which the circle of the 6th hour makes in the cylinder, drawn from the points Z, Y, will fall on the line ST, since both the line of the 6th hour and these sides lie in the plane of the circle of the 6th hour. Now since the line ST, shown to be perpendicular to the meridian, is also perpendicular, by Definition 3 of Book 11 of Euclid, to the axis AH lying in the meridian — and the diameter ZY is likewise perpendicular to the same axis (for since both the circle of the 6th hour and the equator are upright to the meridian, their common section ZY will also be perpendicular to it, and consequently, by Definition 3 of Book 11 of Euclid, to the axis lying in the meridian) [Margin: (b) Euclid XI.19] — [Margin: (c) Euclid I.28] the lines ST and ZY, lying in the same plane of the circle of the 6th hour, since they are perpendicular to the axis, will be parallel to one another. Therefore the parallelogram will be a quadrilateral whose sides are [Margin: (d) Euclid I.33] the axis AH, the semidiameter AZ, the side of the cylinder drawn from Z, and the portion of the line HT between H and the said side drawn from Z. For the side of the cylinder drawn from Z is parallel to the axis AH, because that side and the axis, if produced, join the equal semidiameters AZ and $\lambda \xi$ of the equal bases of the cylinder — [Margin: (e) Euclid XI.16] which semidiameters are indeed equidistant, being sections of equidistant bases made by the plane of the circle of the 6th hour from noon or from

midnight. Therefore the line AZ will be equal to the opposite side in the said parallelogram, that is, to the portion of the line HT between H and the side of the cylinder drawn from Z. But HR, the semidiameter of the circle CRX, is equal to AZ, the semidiameter of the circle θ YBZ; for HC was placed equal to the semidiameter AB, or AZ. Therefore the side of the cylinder drawn from Z falls on the point R. And in the same way the side drawn from Y will fall on the point X.

Again, since the side of the cylinder drawn from θ falls on the meridian line, because it lies in the meridian, and is parallel to the axis AH [Margin: (f) Euclid I.33] — which will be demonstrated by the same reasoning by which we showed a little earlier that the side drawn from Z is parallel to that axis, namely because that side drawn from θ and the axis join the equal and equidistant semidiameters A θ and $\lambda\pi$ — [Margin: (g) Euclid VI.2] it will follow that as BA is to A θ , so is BH to the portion of the meridian line between H and the side drawn from θ . But the line BA is equal to the line A θ . Therefore BH too will be equal to the said portion of the meridian line; and consequently, since HV is equal to BH, the side of the cylinder drawn from θ falls on the point V.

Finally, since the joined line OP is parallel to the line YZ, by the scholium to Proposition 27 of Book 3 of Euclid — for the arcs YO and ZP are equal, because each of the points O and P is supposed to be four hours distant from the point B, and thus two hours from the points Y and Z — if through PO a plane be conceived drawn equidistant from the parallelogram drawn through ZY and the axis, this plane will make in the cylinder a parallelogram, by Proposition 3 of Book 1 of Serenus, *On the Section of the Cylinder*, whose two sides are the line OP and another line ab, corresponding to it in the opposite base, while the remaining two are drawn on the surface of the cylinder from the points O, P — of which the one let down from P is also a side of the parallelogram drawn through ϵ P, so that through it the circle of the 4th hour is drawn. For from the one point P only one side of the cylinder can be drawn, as can easily be proved from Proposition 8 of Book 1 of Serenus, *On the Section of the Cylinder*; for any straight line drawn from the point P to another point that does not lie on the side of the cylinder falls, by the said proposition, within the cylinder. Let the common section of this parallelogram drawn through OP and of the meridian be the line EQ, [Margin: (h) Euclid XI.16] which will be parallel to the axis AH, because EQ and AH are sections of parallel planes — namely, of those parallelograms drawn through EQ and AH — made by the meridian. [Margin: (i) Euclid VI.4] Therefore, as the semidiameter BH is to the semidiameter BA, so is BQ to BE; and consequently the arcs of the circles θ YBZ and BTVS, corresponding to the versed sines BQ and BE, will be similar, by the lemma to Proposition 1 of the foregoing book. But the arc

BF is similar to the arc BP, because each embraces four hours; and BE is the versed sine of the arc BP. Therefore BQ too will be the versed sine of the arc BF; and accordingly, the line QF being drawn, it will be the right sine of the same arc BF (for the versed sine of any arc terminates in the right sine of the same arc, as is clear from the treatment of sines), and it will be perpendicular to the meridian line BH. [Margin: (k) Euclid I.29] But the line FLM is perpendicular to the same meridian line, because, by the scholium to Proposition 27 of Book 3 of Euclid, it is parallel to ST, on account of the equal arcs FT and MS. Therefore the line FLM passes through the point Q. And since both the plane of the dial and the plane of the parallelogram drawn through OP and EQ are upright to the meridian, [Margin: (l) Euclid XI.19] their common section too will be perpendicular to it at Q, and consequently, by Definition 3 of Book 11 of Euclid, perpendicular at the point Q to the line BQ lying in the meridian. Therefore the line FQ, perpendicular to BQ, will be the common section of the dial and the parallelogram drawn through OP and EQ; and accordingly the side of that same parallelogram drawn from P falls on the line QF, since the line FQ and the said side lie in the plane of that parallelogram. And since EP and EQ have been shown to be parallel to the lines AZ and AH, [Margin: (m) Euclid XI.10] the angle PEQ will be equal to the angle ZAH. But the angle ZAH is a right angle, for we showed above that ZY is perpendicular to the axis. Therefore the angle PEQ too is a right angle. But the line FQ, shown to be perpendicular to the meridian, is also perpendicular, by Definition 3 of Book 11 of Euclid, to the line EQ lying in the meridian. [Margin: (n) Euclid I.28] Therefore the lines QF and EP, lying in the same plane of the parallelogram drawn through OP and EQ, since they are perpendicular to the line EQ, will be parallel to one another. The parallelogram, then, will be a quadrilateral whose sides are EQ, EP, the side of the cylinder drawn from P, and the portion of the line QF between Q and the said side drawn from P. [Margin: (o) Euclid I.33] For the side drawn from P is parallel to the line EQ, because that side and the line EQ, if produced, join equal lines in the equal bases of the cylinder — namely the line EP and another line db, corresponding to it in the opposite base, which is the right sine of the arc be of four hours, just as EP is the right sine of the arc BP of four hours — [Margin: (p) Euclid XI.16] which lines are indeed equidistant, being sections of equidistant bases made by the parallelogram drawn through OP and EQ. [Margin: (q) Euclid I.34] Wherefore the line EP will be equal to the opposite side of the said parallelogram, that is, to the segment of the line QF between Q and the side of the cylinder drawn from P. But EP, the right sine of the arc BP of four hours, is equal to the right sine $\kappa\mu$ (which is drawn from K perpendicular to BH) of the arc CK, likewise of four hours, because the circles θYBZ and CRX are equal by construction. Therefore the portion of the line QF

intercepted between Q and the side of the cylinder drawn from P will also be equal to the right sine Kμ. [Margin: (r) Euclid I.34] Since, then, QL is equal to Kμ, on account of the parallelogram Lμ, the side of the cylinder drawn from P, let down through the point L, will pass entirely through it. Therefore, since through the side of the cylinder the circle of the 4th hour from noon (or from midnight) is drawn, as we said above, the side of the parallelogram made by the circle of the 4th hour, let down from P, will meet the plane of the dial at the point L. And by the same reasoning the other side of the same parallelogram, let down from ε, will meet the plane of the dial at the point α. And therefore the circle of the 4th hour itself will cut the plane of the dial at the points L and α. Since, then, it also passes through the center H, the line αHL will give the 4th hour from noon or from midnight. By the same reasoning we shall demonstrate that the remaining sides of the cylinder, made by the hour-circles, fall on the remaining points of the ellipse described on the plane of the dial, and so on.

Although it is sufficient, for laying out the dial, to describe only the half-ellipse RLBX, the dial will nonetheless be drawn more accurately if the whole ellipse is described, so that each hour has three points through which its line may be drawn.

[Margin: A plane cutting a right cone [conum], drawn neither through its axis nor parallel to the axis, makes an ellipse.]

From what has been demonstrated it follows that, if a right cylinder is cut by a plane drawn neither through the axis nor parallel to the axis, the resulting section is an ellipse. For just as we showed that the plane of our horizontal dial — built for latitude 42° and cutting a right cylinder whose axis is the world-axis — makes an ellipse, because all the sides of the cylinder fall on points of the ellipse, so in the same way we shall demonstrate that the same holds if the pole's altitude is greater or less than 42°, so that the plane of the horizontal dial, however it is inclined to the axis (that is, to the said right cylinder), always makes an ellipse. This Serenus, in Book 1 of *On the Section of the Cylinder*, demonstrates for every cylinder: whenever the cutting plane is neither parallel to the bases of the cylinder nor to the axis, and neither passes through the axis nor is placed subcontrariwise.

[Margin: When the pole's altitude above the horizon is very small, the drawing of the horizontal dial becomes more difficult.]

Now when the pole's altitude above the horizon is very small — say 1°, 2°, 3°, or 4°, and so on — the drawing of the horizontal dial becomes somewhat difficult

and laborious, because then the world-axis ED, in the portion of the analemma of this proposition, comes very close to the diameter BC of the horizon. As a result the style DG turns out to be extremely short, unless we are willing to move the point H farther than is right from the point G, and indeed from the point I. That is truly inconvenient: partly because too large a plane would then be required to lay out the dial, since the center H of the dial would be moved farther from the equinoctial line than is enough — for since the hour-lines meet in a very remote point, they must be parallel, lying in that plane which is parallel to the world-axis and from which the plane of the dial is supposed to differ but little (as we demonstrated in the corollary to Proposition 22 of the preceding book), and so in the proposed horizontal dial they too appear nearly parallel; and partly because it is very difficult, over so remote a distance, to make out and distinguish the point H without some suspicion of error, since the angle DHI then becomes extremely acute.

Diagram. A large circular construction figure completing the ellipse-method for the horizontal dial (continued from the preceding pages). Two concentric circles carry hour-division marks around the annular ring (short dotted right-angle brackets set at each division). Diagonal diameters and chords cross at the center H. At the upper left an oblique cylinder (its elliptical mouth) is drawn tilted against the circle, and a long straight line — marked "4" at both its far ends (upper right and lower left) — passes through the center, giving the line of the 4th hour discussed on the preceding pages. Many small Greek letters mark the hour-points around the rim. [Several peripheral Greek labels are worn/small — flagged below]

- π (pi, top): apex of the oblique cylinder/ellipse
- V (upper center): point on the outer circle at top
- ξ (xi) and λ (lambda) (upper left, on the cylinder mouth): rim points [?]
- h, d, a (left, down the cylinder): rim points [d, a uncertain: ?]
- e (far left, foot of the cylinder)
- 4 (upper-right end of the long diagonal): the 4th-hour line
- β (beta, right): rim point
- γ (gamma, right, near the ellipse): rim point
- α (alpha, near γ) [?]
- N (inner circle, upper left)
- H (center)
- X (right of center, on the horizontal diameter)
- S (right end of the horizontal diameter, outer circle)

- θ (theta, far right, just below S)
- T (left end of the horizontal diameter, outer circle)
- R (inner circle, left)
- K (inner circle, lower left)
- L (below K)
- F (lower left, outer circle)
- G (center, just below H)
- D (right of G)
- I (below G)
- μ (mu, below I)
- ρ (rho, below center) [?]
- C (lower center)
- P (lower center, right of C)
- A (lower right)
- Y (near A)
- E, O (bottom center-right, small)
- δ (delta) and ε (epsilon) (right, lower)
- M (lower right, outer circle)
- B (bottom, outer circle)
- 4 (lower-left end of the long diagonal): the 4th-hour line

[Margin: When the pole's altitude above the horizon is too great, the drawing of the horizontal dial likewise becomes more difficult.]

Much the same happens when the pole's altitude above the horizon is at its greatest — say 89° , 88° , 87° , 86° , and so on — so that the pole is only slightly distant from the zenith. For then the diameter FD of the equator likewise makes a very acute angle at I with the line HI, and meets that same line at a very remote point, so that it is not easy to decide where the lines FD and HI intersect one another, on account of the narrowness of the acute angle HID. To avoid this twofold inconvenience, therefore, we have devised a twofold remedy. By the first we shall draw the hour-lines even though we do not have the center where they all meet; by the second we shall find the point on the meridian line through which the equinoctial line must be drawn, even though we do not draw the line DI (in the portion of the analemma of this proposition) which ought to fall on that point.

[Margin: The drawing of the horizontal dial when the pole's altitude is very small.]

Let a horizontal dial, then, be drawn for latitude 20° . (We choose so large a latitude to make the method of construction clearer, though it fits other latitudes, however small.) That is, in a region above whose horizon the pole is raised 20° . In some plane let a straight line AB be drawn at will for the meridian line; and let CD, cutting it at right angles in E, represent the equinoctial line. Next, at the point E, to the line ED (or EC) let the angle DEF of the pole's altitude be set up; and on the line EF, taking any point F (the farther it is from E, the larger the dial will be drawn), let the perpendicular FG be raised at F to the line EF. Then on the line AE, taking any point H, let there be drawn through

it the line HG, parallel to EF — or, cutting FG perpendicularly at G, at right angles to FG; and again through the same point H, the line IK perpendicular to AB.

Finally, taking on the line AB the lines EB, HL equal to the lines EF, HG, let circles — or rather quadrants of circles — be described from B and L. (This is enough, provided that the points found on the lines ED, HK by means of these quadrants are transferred to the lines EC, HL.) And let the circles, if they have been described, be divided into 24 parts; or the quadrants, if perhaps only quadrants have been described, into 6 equal parts, the beginning always made from the meridian line AB. For a line drawn — secretly, however — through the centers B, L and through the division-points of the circles or quadrants will cut the lines CD, IK (or the lines ED, HK themselves, if only quadrants have been described) at points through which straight lines drawn out will give the hour-lines we desire — taking always two corresponding points together, that is, first the two nearest to the points E, H, then the next two, and so on. This holds provided that, if only quadrants have been described, the points of the lines ED, HK are first transferred to the lines EC, HL, and the lines are afterwards drawn, as has been said. These we shall produce as far as we please, for the arcs of the signs, which we shall describe in the following proposition, will fix their lengths. The length of the gnomon will be FM, perpendicular to AB, its place being at the point M. This we shall demonstrate as follows.

[Margin: Demonstration of the preceding construction. (a) Euclid I.23]

Since the angle DEF is equal to the pole's altitude, the angle AEF will be equal to the complement of that same altitude; and consequently the two angles AEF, EFG will be less than two right angles, and so the line FG will at length meet the line EA on the side of A — namely at the point A, which, even though it is not contained in the drawn plane on account of its small size, may nonetheless be

conceived in the mind. Therefore the angle EAF will be equal to the pole's altitude, (a) since EAF and AEF together are equal to one right angle, and AEF is equal to the complement of the pole's altitude. And accordingly, if the triangle EAF is conceived to be revolved about the meridian line AE, so that

Diagram. The horizontal dial for latitude 20° (the demonstration figure for the preceding construction). At the top a graduated straight line runs from A (left) to N (right), divided into equal parts by tick marks, with the first division-points near A marked. From below A a vertical line descends into a fan of hour-lines that converge upward. The hour-lines are labelled across the top with Roman numerals for the hours. A horizontal line I–H–K crosses at the convergence, and below it a second horizontal line C–E–D. Down the central vertical run the points P, H, L, M, E; G lies to the right of H and F to the right at the M–E level. Near the bottom a quadrant arc with dotted radii (the arc used to lay off the hour-divisions) stands on B.

- A (top left) — N (top right): the graduated ruler (line AN)
- O, Q [?] (division-marks near A on the ruler)
- Hour numerals across the fan (left to right): V, IIII, III, II, I, 12, XI, X, IX, VIII, VII
- I (left end), H (center), K (right end): the horizontal line (equinoctial/horizon line through H)
- P (upper center), G (right of H): construction points
- L, M, E (down the center)
- F (right of M): gnomon-triangle vertex
- C (left), D (right): ends of the lower horizontal line
- B (bottom): center of the quadrant arc

it stands upright to the plane of the dial and lies in the plane of the meridian, then AF will be the world-axis. For in the plane of the meridian it makes with the meridian line of the horizon — (b) which is parallel to AB itself in the plane of the dial, since both are sections made by the meridian in parallel planes — (c) an angle equal to the angle EAF of the pole's altitude, namely the external angle equal to the internal. Therefore the lines FE, GH, perpendicular to the axis FG, will be the common sections of the meridian and the equator, so that either of the points F, G may be taken for the center of the world. And accordingly, as has been shown in this proposition, the hour-circles will meet the plane of the dial at the points we find on the lines CD, IK; and FM will be the style, its place being at M. For the triangles AEF, AHG are here so constructed as the triangle HID was in the earlier dial of this proposition.

[Margin: (b) Euclid XI.16 (c) Euclid I.29]

In this proposed example, because the pole's altitude is great enough, the center A of the dial — toward which all the hour-lines tend — is contained on the plane. This was done on purpose, so that you might also learn by experience that dials are rightly drawn by this method. But it is plainly established that, if the center does not fall on the plane of the dial, the sixth hour — which is necessarily drawn through the center of the dial perpendicular to the meridian line — cannot be described. And so much for the remedy of the former inconvenience.

[Margin: The drawing of the horizontal dial when the pole's altitude is very great.]

As for the latter inconvenience, we shall relieve it thus. Again, in some plane let AB be the meridian line, and let A be set as the center of the dial. At the line AB, at A, let the angle BAN of the pole's altitude be constructed, so that AN is the world-axis; on it let any point N be taken as the center of the world, from which a perpendicular drawn to AN will represent the common section of the meridian and the equator, just as the lines FE, GH did a little before. But since, if the point N is somewhat farther from the center A of the dial — to the end that the style, which must be drawn as a perpendicular from N to AB, may have a becoming and conspicuous size — the perpendicular drawn from N to AN meets the meridian line AB at a very remote point, so that it is difficult to detect that point without error (because that perpendicular could easily deflect this way or that), we shall find the point of that meeting by the following device, exact enough. Let the line AN be divided into any number of equal parts,

Diagram. The same style of horizontal-dial figure, here for the great-altitude remedy (with N taken as the center of the world). At the top a graduated straight line runs from A (left) to N (right), divided into equal parts, the first division-points near A marked. From below A a vertical descends into a fan of hour-lines converging upward and labelled with Roman numerals. A horizontal line I–H–K crosses at the convergence, with a second horizontal line C–E–D below it. Down the center run P, H, L, M, E; G lies right of H, F right at the M–E level; a dotted quadrant arc stands on B at the foot.

- A (top left) — N (top right): the graduated ruler (line AN), N as center of the world
- O, Q [?] (division-marks near A on the ruler; O is the division-point nearest A)

- Hour numerals across the fan (left to right): V, IIII, III, II, I, 12, XI, X, IX, VIII, VII
- I (left end), H (center), K (right end): the horizontal line through H
- P (upper center), G (right of H): construction points
- L, M, E (down the center)
- F (right of M): gnomon-triangle vertex
- C (left), D (right): ends of the lower horizontal line
- B (bottom): center of the quadrant arc

so that the perpendicular drawn from O — the division-point nearest to A — to AN can conveniently cut the line AB at P. If on AB produced we take a line as many times a multiple of AP as AN is a multiple of AO, the last point of that multiple will be the one on which the perpendicular drawn from N to AN necessarily falls. This we shall confirm as follows. (a) Since AN has to AO the same proportion as that multiple on the line AB has to AP, the line drawn from N to the extreme point of that multiple will be parallel to OP; (b) and consequently the angle at N is equal to the angle at O, being a right angle. Therefore the said perpendicular from N falls on the said point of the line AB; for if it fell on any other point, two perpendiculars would be drawn from N to AN, which is absurd. You see, then, that a very large plane is required for describing a dial of this kind, if the style is to have a moderate length; for the style drawn as a perpendicular from O to AB is too short. Nor indeed can we here avoid the former inconvenience; but once the aforesaid point on the line AB has been found, the dial must be described as we set forth in this proposition — that is, from that point found, a line equal to the perpendicular drawn from N to AN (and falling on the said point) must be cut off from AB, and through it the perpendicular to AB must be drawn for the equinoctial line. Then from the point which terminates that line equal to the perpendicular drawn from N to AN, a circle must be described, and so on. In

[Margin: (a) Euclid VI.2 (b) Euclid I.29]

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—for example, line AN is eight times line AO, so we shall need to cut off from AB a segment eight times AP. But if we terminate the axis at Q, the segment to be cut off from AB must be three times AP, since AQ too is three times AO.

Perhaps, though, we shall reach the proposed result more reliably as follows. (For when the pole's altitude is very great—so that the axis AN makes an almost right angle with AB—it is not easy to make out the point P either, because the angle APO is too acute.) Suppose we first choose any point at all on the line AB, say P, and from it erect a perpendicular PO to AN; this is easily done if a semicircle is described on AP cutting AN at O, for then PO will be perpendicular, since the angle AOP stands in a semicircle. The same is achieved if at P we set up an angle APO equal to the complement of the pole's altitude. For if we cut off from AO as many equal parts as we please—up to N, or Q, or up to any other point—and on AB take a segment that is the same multiple of AP as AN (or AQ, or whatever other portion assumed on the axis AN) is of AO, we shall arrive at the very goal we were aiming at before, and much more reliably. For who does not see that by this method—in assuming point P, and thereby the line AP—we can err less than before, when we sought the same point P and the line AP by means of the perpendicular OP? Indeed the narrowness of the acute angle APO makes that point highly doubtful and uncertain.

Nor, it seems, should we pass over here the fact that Oronce Finé [Oronce Finé] was strikingly deceived in his layout of the astronomical dial, both horizontal and vertical. To make this plain, I shall set out his own words and expose the error, so that no one may be led astray along with him—as I have observed has already happened to many, deceived perhaps by Orontius's authority. In Book 1, Proposition 1, then, he describes what he himself calls the "Prototype" [Prototypus], by which (he says) both horizontal and vertical dials are constructed. "On a given plane, and about a given center A in it, let a circle be described—

[Margin: The error of Oronce Finé in the layout of the astronomical dial, both horizontal and vertical.]

Diagram. Three companion circles in a row, illustrating the three cases of Orontius's "Prototype" construction according as the pole's altitude is greater than, less than, or equal to 45° . Each circle BCDE has center A, a vertical diameter C (top)–E (bottom) and a horizontal diameter D (left)–B (right), dividing it into four quadrants. Point F lies on the upper-right arc; from center A the radius AF is drawn, and the perpendicular FH is dropped from F to the horizontal radius (H lying between A and B). The line HI is drawn (I on the upper-left arc), cutting AF at K; L is a small point near the center. G lies on the lower-right arc. The figures thus show the triangles AFH, AHI, and AHK.

- C (top of circle): upper end of the vertical diameter
- E (bottom of circle): lower end of the vertical diameter

- D (left), B (right): ends of the horizontal diameter
- A (center): center of the world
- F (upper-right arc): endpoint of the counted pole-altitude arc BF; radius AF
- H (on radius AB): foot of the perpendicular FH
- I (upper-left arc): endpoint used for line HI
- K (on AF, near center): intersection of HI with AF
- L (near center): construction point [?]
- G (lower-right arc): endpoint of arc BG (the arc equal to BF laid off in quadrant BE)

Left circle: case of pole altitude greater than 45° . *Middle circle:* case of pole altitude less than 45° (here the labels appear in the order I, L, K down the center). *Right circle:* case of pole altitude equal to 45° .

—BCDE, and by the two diameters BD and CE, which cut each other at right angles at the same center A, let it be divided in the usual way into four quadrants. Further, let the right-hand upper quadrant BC be divided into 90 mutually equal parts—first into three, then each of these into 6, and finally each into 5. Then, taking the free elevation of the north pole, that is, the latitude of the region for which one wishes to make the dial, let it be counted off in the quadrant BC, from the point B toward C. Let the end of the reckoning be marked with the sign F; and from center A to the given sign F let the straight line AF be drawn. Moreover, on the given arc BF let an equal arc be set up in the quadrant BE, and let it be BG; and from the sign F to the sign G let the straight line FG be drawn. This line will be bisected by the semidiameter AB—at the sign H—and therefore also at right angles, by Euclid III.3. Hence the straight line FH will be perpendicular to AB, and the triangle AFH right-angled. Therefore the circle BCDE will represent the meridian, BC its northern quadrant, and A the center of the world; while BD will represent the horizon, and CE the vertical circle meeting the meridian itself at right angles." A little later he continues thus: "Given the straight line FH, let one equal to it be set up on the semidiameter AC, and let it be AI; and from the sign H to the sign I let the straight line HI be drawn, cutting the line AF at the sign K. Then the triangle AHI will be equal and similar to the triangle AFH, as is manifest from Euclid I.4."

This construction, then, Orontius calls the general Prototype for building both horizontal and vertical dials—so that he himself uses the triangle AHK in making horizontal dials for a given pole altitude BF, exactly as we in this proposition used the triangle DHI, contained in the portion of the Analemma, for composing the same dials for a given pole altitude CE. For at once, in Proposition 2, he takes on

the meridian line a straight line equal to the side AH for the horizon line, just as we took a straight line equal to the side HI in our triangle. Then, for the equator line, he takes another straight line equal to the side AK, just as we took a straight line equal to the line DI in our triangle. Thus in the triangle AHK he wants AH to be the horizon line, HK the axis of the world, and AK the equator line, for constructing horizontal dials. Likewise in Proposition 3 of the same book, for constructing vertical dials, he takes in the triangle FHK the line FH for the line of the vertical circle, HK for the axis of the world, and FK for the equator line. That in this he has gone wonderfully astray we shall easily show as follows.

Since the sides IA, AH of the triangle IAH are equal to the sides FH, HA of the triangle FHA, and contain equal angles—namely right angles (a)—the bases HI, AF and the angles AHI, HAF will also be equal. Therefore, just as AF is the axis with respect to the horizon AH, so too HI can be taken as the axis with respect to the same horizon AH, since both AF and HI make with AH the angle of the pole's altitude. And thus far there is no difference between us and Orontius. But his error lies solely in the equator line, which by no means can be the straight line AK, as he supposes, in a region whose latitude is greater than 45° —as I shall make evident thus. Let the pole's altitude BF first be greater than 45° , as in his figure. Then the angle FAH will be greater than half a right angle; and since the angle AHI has been shown equal to it, the two angles KAH, KHA (b) in the triangle AHK together exceed a right angle. Therefore the remaining angle AKH will be less than a right angle. So, since the equator line makes right angles with the axis in every clime—as is clear from the portion of the Analemma of this proposition, or from the Analemma itself in Proposition 1 of the previous book—who will still doubt that the line AK, which makes the acute angle AKH with the axis HK, cannot be the equator line? (c) Therefore, to obtain the equator line in the triangle AHK, one must draw from A a perpendicular AL to the axis HK, which is necessarily smaller than AK. Next, let the pole's altitude BF be less than 45° , as appears in the second figure; and the angle FAH will accordingly be

[Margin: (a) Euclid I.4 (b) Euclid I.32 (c) Euclid I.19]

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—less than half a right angle. Therefore, since the angle AHI has been shown equal to it, the angles HAF, AHI together will be less than one right angle; (a) and consequently the remaining angle AKH, in the triangle AKH, will exceed a right angle. Hence the line AK cannot be the equator line, since it does not make right angles with the axis HI; but AL, drawn perpendicular to the axis HI, will give the equator line. Only in a region where the pole's altitude comprises 45° will the

straight line AK be the equator line, because there alone does it make right angles with the axis HI. For let the pole's altitude BF contain 45° , as in the third figure; and the angle FAH will accordingly be half a right angle. Since, then, the angle AHI has been shown equal to it, the two angles HAK, AHK (b) together will be equal to one right angle; and therefore the remaining angle AKH, in the triangle AKH, will be a right angle. Hence the line AK will serve the office of the equator line. It is clear, then, that Orontius blunders childishly in assigning the equator line; and thus the horizontal dials laid out from his Prototype are false wherever the pole's altitude is greater or less than 45° . The same must be said of vertical dials. For although in them Orontius rightly takes the vertical line FH and the axis of the world HK, he nevertheless errs in the line FK, which he takes for the equator line, because it is not perpendicular to the axis HK, as we have demonstrated.

[Margin: (a) Euclid I.32 (b) Euclid I.32]

Orontius stumbled still more disgracefully in Proposition 7 of the same Book 1 of the dials, where he took the whole axis AF, lying between the center A and the line FH of the vertical circle, for the equator line—which can nowhere be true, since it is the straight line AL, very unequal to the line AF, that is the equator line, as we have said.

[Margin: A greater error of Orontius.]

PROBLEM 2. PROPOSITION 2.

To describe, in the aforesaid horizontal dial, the parallels—that is, the arcs of the zodiacal signs—which are the common sections of the plane of the dial and of the cones whose bases are the parallels of the zodiacal signs and whose vertex is the center of the world.

[Margin: The description of the arcs of the zodiacal signs in the aforesaid horizontal dial, from the Analemma. — The diameters of the conic sections when the Sun is at the beginnings of the signs. — By what method the diameters of the conic sections are found in any Analemma.]

Let the portion of the Analemma of the preceding proposition be repeated, and let the meridian ABC be completed, in which—from the Analemma constructed in Proposition 1 of the preceding book—let the diameters of the parallels be drawn, together with the diameters connecting the opposite signs, so that triangles are formed through the axis in the cones, whose bases are the parallels themselves and whose common vertex is the center D. Then, from what was demonstrated in

Propositions 4, 5, 6, and 7 of the previous book, KR will be the diameter of the conic section which the Sun describes when at the beginning of Cancer (♋); LR the diameter of the section which the Sun describes at the first point of Gemini (♊) and Leo (♌); MR the diameter of the section which the Sun describes at the beginning of Taurus (♉) and Virgo (♍). But NO, PO, QO will be the diameters of the conic sections which the Sun describes when in the opposite parallels—namely in the parallels of Capricorn (♐); of Sagittarius (♏) and Aquarius (♑, printed as the waves-glyph); and of Scorpio (♏) [?] and Pisces (♓).

Diagram. The Analemma (a meridian circle) used to lay out the zodiacal arcs of the horizontal dial. The circle has center D, with the labelled peripheral points A (top), E (upper right), C (right), O (lower right), B (left), R (lower left), and F (upper left, just inside the rim). A dense sheaf of straight construction lines is drawn across the interior; among them the line RO runs across the figure (the line through the extremity of the style), and the diameters KR, LR, MR (toward R) and NO, PO, QO (toward O) connect the opposite-sign points. Interior construction points read as H, I, L, M, N, P, Q, G clustered about the center. Around the upper-left and lower rim a series of zodiacal-sign glyphs is engraved marking the parallels; individually they are too small and worn to resolve reliably on this scan and are flagged.

- D (center): common vertex of the cones / center of the world
- A (top), E (upper right), C (right), O (lower right), B (left), R (lower left), F (upper left): peripheral points of the meridian circle
- RO: the line drawn through the extremity of the style
- KR, LR, MR: diameters of the conic sections (northern signs), toward R
- NO, PO, QO: diameters of the conic sections (opposite/southern signs), toward O
- H, I, L, M, N, P, Q, G: interior construction points [several partly obscured — ?]
- Zodiacal-sign glyphs engraved around the rim [worn — not individually legible; ?]

Furthermore, these diameters of the conic sections are found in the same way in any other Analemma—whether larger or smaller than the one here proposed—even if the dial was constructed without a portion of the Analemma, as we taught in the preceding scholium; provided that in that other Analemma a straight line equal to the assumed gnomon is cut off along the diameter of the vertical below the center (the diameter of the vertical being extended, should the length of the gnomon require it), and that through its endpoint a line is drawn parallel to the

diameter of the horizon, by which the plane of the horizontal dial is conceived to be drawn. For this straight line, in a larger or smaller Analemma, will be cut by the diameters of the opposite signs into parts equal to the parts of the line RO in our present Analemma. This can be shown as follows. Since both that line and this RO, the length of the style being kept the same, stand equally distant from the center of their Analemma, (c) and the angles which the diameters of the opposite signs make with the diameter of the equator are, in any Analemma, of the same magnitude—because they always rest at the centers upon the same declinations of the same signs—it follows that the angles which those same diameters make with the diameter of the vertical circle (which are either compounded of the former plus the angle of the pole's altitude contained between the diameter of the equator and the diameter of the vertical, or are left over after subtracting the former from that same angle of the pole's altitude) are equal to one another, since the angles contained between the diameter of the vertical and the diameter of the equator are also equal.

[Margin: (c) Euclid III.27]

Therefore, since the angles which the lines drawn through the extremity of the gnomon (namely through the point G in our Analemma, and through the point corresponding to it in the other Analemma), equidistant from the diameter of the horizon, make with the diameter of the vertical are right angles—as in our Analemma is the angle G—and the angles which, in either Analemma, the ray of any sign makes with the same diameter of the vertical are likewise equal, as we said (for they are vertical to those we showed to be equal), there will always be found two triangles in either Analemma, one in the one and the other in the other, having two angles equal, each to each. Since, then, they also have an equal side adjacent to the said angles—namely the magnitude of the style—(d) they will also have the remaining sides equal, namely those intercepted between the extremity of the style and the ray of each sign in either Analemma, and so on. This can also be made plain thus: if that larger Analemma is imagined superimposed on this one of ours, so that the centers and the diameters of the horizon, the vertical, and the equator coincide, then the line drawn through the extremity of the style in the former coincides with the line RO in our Analemma, on account of the equality of the gnomons and of the right angles which the gnomons make with the said lines. Since, then, the diameters of the ecliptic connecting the opposite signs also coincide (because they make the same angles

[Margin: (d) Euclid I.26]

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—in either Analemma with the diameter of the equator, on account of the same declinations in each, as is manifest from the construction of the Analemma), it is plainly established that the lines between the extremity of the style G and the rays of the signs in our Analemma are equal to the same lines in the other Analemma, because the former coincide with the latter.

Therefore, if onto the meridian line of the dial already laid out—whether from the center H, or from the point I where the plane of the equator meets the plane of the dial—the points G, K, L, M, I, Q, P, N are transferred (taken with the help of the compass from the point where, in the Analemma, the axis of the world, or the diameter of the equator, intersects the line drawn through the extremity of the gnomon equidistant from the diameter of the horizon), then G will be the place of the style DG, while the remaining points will be those onto which the shadow of the style is projected when the Sun is on the meridian circle and at the beginnings of the signs, and through which the conic sections of the signs must be drawn—sections that are either circles, or parabolas, or hyperbolas, or ellipses, as was demonstrated in the previous book.

Diagram. The large horizontal-dial projection. At the top center stands the point H (center of the dial / foot of the style), from which a vertical meridian line runs straight down to the point ϕ at the bottom. From H radiates a broad fan of hour-lines (solid) and of dotted rays (the sun-rays / conic-cone generators) spreading to left and right; a group of dotted rays reaches up to the label X at the upper left. On the meridian just below H lie G and D; farther down toward ϕ lie further construction points (E, L, M) about the center. To the left of center are the points F and R, to the right S and K, marking where the equinoctial and hour-lines meet the projected sign-arcs. Flanking the figure are two tall rectangular scales of zodiacal signs: on the left, labelled vertically **ASCENSVS SOLIS** ("the ascent of the Sun"); on the right, labelled vertically **DESCENSVS SOLIS** ("the descent of the Sun"). Each scale carries a column of zodiac glyphs with an adjacent narrow column of small numerals; on this scan the glyphs and numerals are small and partly worn, so they are transcribed provisionally and flagged.

- H (top center): center of the dial / foot of the style
- ϕ (bottom of meridian): southern center point of the meridian line (the point paired with H in the diameters $N\phi$, $P\phi$, $Q\phi$)
- G, D (on the meridian just below H): the place of the style (DG) and a construction point
- F (left), R (left of center): construction points on the equinoctial/sign lines

- S (right of center), K (right): construction points on the equinoctial/sign lines
- E, L, M (lower center, about the meridian): construction points [partly obscured — ?]
- X (upper left): endpoint label of the dotted ray-sheaf
- Left scale **ASCENSVS SOLIS**: column of ascending-sign glyphs, reading (top→bottom) Π , ♊ , ♋ , ♌ , ♍ , ♎ [worn — ?], with adjacent numerals 6, 5, 4 [?]
- Right scale **DESCENSVS SOLIS**: column of descending-sign glyphs, reading (top→bottom) ♏ , ♐ , ♑ , ♒ , ♓ , ♈ [worn — ?], with adjacent numerals 6, 7, 8 [?]

Through I, moreover, a perpendicular must be drawn to the meridian line. For this will be the equinoctial line, onto which the shadows are projected at the time of the equinoxes. Now if, about the diameters KR, LR, MR, NO, PO, QO, conic sections are described (by Proposition 8 of the previous book) in some solid material—such as wood, or glued sheets of paper—and are cut out to make, as it were, curved or bent rulers, by whose help these same sections are transferred to the dial so that they pass through the points k, L, M, Q, P, N translated onto the meridian line, and their axes coincide with the meridian line, then the parallels of the zodiacal signs will be drawn. These can indeed also be drawn in the dial itself, about the diameters KH, LH, MH, N ϕ , P ϕ , Q ϕ , by Proposition 8 of the preceding book, rather than separately, to be transferred afterward onto the dial. The parallel of Cancer (♋) will pass through k; the parallel of Gemini (Π) and Leo (♌) through L; the parallel of Taurus (♊) and Virgo (♍) through M; the equinoctial line through I; the parallel of Scorpio (♏) [?] and Pisces (♈) through Q; the parallel of Sagittarius (♐) and Aquarius (♒) through P; and finally the parallel of Capricorn (♑) through N. All of these the shadow of the tip of the style describes, when the Sun is at the beginnings of the signs, since the extremity of the shadow is projected onto them, as is clear from the corollary of Proposition 12 of the previous book.

Of all the hour-lines (the meridian line excepted), the portions found outside the conic sections of Cancer (♋) and Capricorn (♑) must be deleted, both in the horizontal dial and in any other, since they have no use—because the shadow of the style is not projected beyond the tropics, just as the Sun never wanders beyond them in the sky. Hence the hour-lines must be drawn faint, so that only their parts within the tropics may be marked, after the arcs of the zodiacal signs have been drawn. It is enough, moreover, if the conic sections of only the northern signs are drawn, since the horizon cuts all the parallels. For these are

equal, one to one, to the conic sections of the opposite southern signs, since they are then opposite and equal hyperbolas, as we showed in Proposition 6 of the previous book. Hence they can easily be transferred to the southern signs and drawn about the points N, P, Q.

[Margin: In any dial only the portions of the hour-lines that lie between the tropics of Cancer and Capricorn are to be drawn.]

Further, the conic sections whose diameters in the Analemma lie between the point R and the point I—where the diameter of the equator cuts the line RO—have their horns turned toward the center of the dial, H; but the horns of those sections whose diameters lie between the points O and I are turned to the opposite part. So too in all other dials, even declining and inclined ones, the conic sections between the center of the dial and the equinoctial line stretch their horns toward the center, while the rest turn them to the contrary part—so that, in general, the horns of all the sections recede more and more from the equinoctial line and, as it were, flee it. The reason for this is readily had from the conic elements of Apollonius of Perga [Apollonius of Perga], if the cones in the Analemma of this proposition are rightly conceived; for the conic sections always spread wider toward the bases of the cones, as is evident from the said conic elements.

[Margin: Toward which part the horns of the conic sections lean in dials. — Hence, by the method drawn from the Analemma of this proposition, the arcs of the signs are laid out in the dial for a greater or a lesser length of the gnomon.]

But if in the Analemma a longer style DM is taken, and through M a line parallel to BC is drawn cutting the diameters of the opposite signs, the diameters of other conic sections will be obtained, about which the conic sections themselves can be drawn in the same way for a larger dial—one in which the same hour-lines remain entirely unchanged; but the equinoctial line must be distant, on the meridian line, from the center H by as great a space as is the portion of that parallel drawn through M, intercepted between the point where it is cut by the axis ED and the point where the same is cut by the ray, or dia-

[Margin: The portion of the Analemma is an instrument for describing dials of any size whatever.]

[Margin: Another way of describing the arcs of the zodiacal signs in the horizontal dial.]

the diameter of the equator, FD. By the same reasoning, if a shorter style DM is taken, a smaller dial will be described. That portion of the Analemma described in the preceding proposition is therefore, as it were, a kind of instrument for describing dials of any size whatever — so far as concerns the style, the center of the dial, the equinoctial line, and the hour-lines — since a style of any size can be assumed within it. But as regards the arcs of the signs, one must return to the Analemma described at the beginning of this proposition, from which the arcs of the signs are laid out in any dial, as has been said.

The same parallels — that is, the arcs of the zodiacal signs — can also be described in the dial by the following method. About the straight line $A\lambda$, drawn in any manner so as to represent the axis of the world, and taking any point D as center, let a semicircle of any size be described. Let DH, on the left-hand side of D, be equal to the axis DH assumed in the portion of the Analemma of the preceding proposition, or in the triangle HDI, by which — in the scholium of the same proposition — we described the dial without the portion of the Analemma. Having drawn DC perpendicular to the axis DH, let DI be taken equal to DI, the equinoctial line of that same portion of the Analemma or of the said triangle, and let the straight line HI be drawn toward I to any length whatever. Or rather — lest an error be committed in drawing the line HI, because the point H may perhaps lie too close to the point I — let an arc be described from H toward D, on which let AB be taken, toward the straight line DC, equal to the arc of the pole's altitude; and through B let the straight line HB be drawn out, cutting the equinoctial DC at I. The triangle DHI will then be exactly equal to the triangle DHI of the aforesaid portion of the Analemma, or to the triangle of the dial described without the portion of the Analemma; and HI will be equal to the meridian line of that same portion of the Analemma or of the said dial, as can be clearly seen from the construction of each of these triangles in this figure.

Diagram. The figure occupies the lower half of the page and continues across the following opening; it is the "figure of the Zodiac rays" (*figura radiorum Zodiaci*) superimposed on the horizontal dial. A large semicircle (with its bounding arc) is drawn about the top center; a bundle of straight lines (the hour-lines and the rays of the signs) radiates downward from the apex region toward a fan of hour-numbers and a curved band of zodiacal-sign symbols along the bottom. Because the small peripheral point-labels are set in worn mixed Greek/roman/symbol sorts at the downsampled floor, several are uncertain and flagged; this figure

- λ (upper left, on the bounding arc): endpoint of the axis-line $A\lambda$

- H (top center): apex / center of the dial (center of the radiating pencil of lines)
- D (top, right of H): center of the semicircle; the point through which the axis $A\lambda$ passes
- A (top right): endpoint of the axis-line $A\lambda$ on the right
- μ (near the apex, small): point of the noon (12th-hour) line on the Cancer ray [?]
- φ (right side, on the arc): peripheral point [?]
- B (right of center): point on the arc used to lay off the pole's altitude
- C (center): point on line DC (fifth-hour-after-noon line HC)
- T (center left): point on line DC (fourth-hour line HT)
- V (below center): point of the sixth-hour line HV
- X (left of center): point of the seventh-/fifth-hour line HX
- Y (far left of center): point of the eighth-/fourth-hour line HY
- ξ, π (lower center, small): points on the equinoctial line [?]
- $\theta, \alpha, \beta, \gamma, \delta$ (lower center band, small Greek): points of the sign-rays / conic sections [?]
- Zodiacal-sign symbols in the lower curved band: ♐ [Capricorn] and neighbouring signs [?]
- Hour numerals around the periphery (left, descending): 12, 11, 10, 9, 8, 7 ...; (right, ascending): 1, 2, 3, 4, 5, 6 ...

The angle DHI will thus be exactly equal to the angle DHI of the aforesaid portion of the Analemma, or of the triangle of the dial described without the portion of the Analemma; and HI will be equal to the meridian line of that same portion of the Analemma, or of the said dial, as the construction of each of these triangles in this figure makes plain.

[Margin: Elements I.4]

For by the former construction, the two sides DH, DI of this triangle are equal to the two sides DH, DI of that one, and they contain equal angles, namely right angles. Therefore the whole triangles will be equal. But by the latter construction, the two angles HDI, DHI of the one triangle are equal to the two angles HDI, DHI of the other (since in each triangle the first angle is a right angle and the second is equal to the pole's altitude), and the side DH of the one is equal to the side DH of the other.

[Margin: Elements I.26]

Therefore the sides DI, HI of the one are also equal to the sides DI, HI of the other, and so on. Next, from E, the center of the equator in the dial — the center, that is, from which the circle was described and divided into 24 equal parts, and so on — let all the intervals between the center E and the hour-lines on the equinoctial line Fk be transferred onto the line DC of this figure which we are now constructing, starting from the point D. That is, let the interval EF (or EK) in the dial constructed just before this figure be made equal to the straight line DC in this figure — for the straight lines EF, EK in the said dial are equal to one another, as was demonstrated in the scholium of the preceding proposition, just as all the other intervals whatever between E and any two hours equally distant in time from the meridian line were shown to be equal — and let the interval ER (or ES) of the same dial be made equal to the straight line DT in this figure, and so on. Then through the points C, T, and the rest marked on the line DC, let straight lines be drawn from H, to which are assigned the numbers of the hours corresponding to the transferred intervals: so that the straight line HC is the line of the fifth hour after noon and of the seventh after midnight; HT the line of the fourth hour after noon and of the eighth after midnight; and so on.

We shall obtain these same hour-lines, drawn from H through the points of the line DC in this figure, in yet another way, if we transfer the hour-intervals in the dial lying between the center H and the equinoctial line from the point H of this figure onto the line DC, using the compasses and marking points on the line DC. (This will be done if, at the said intervals of the dial, certain hidden arcs are described from H of this figure, cutting the line DC at the points that are to be marked.) Then through the points found on the line DC in this way we draw straight lines from H. For suppose that in the dial the center E is understood to be joined, in its proper position, with the center of the world D — as we said in the preceding proposition, when we set the triangle DHI upright to the plane of the dial and conceived in our mind the circle described from E to be moved about the equinoctial line Fk until its center E was joined with D, the center of the world. We shall then understand two triangles: namely HDF (or HEF) in the dial, and HDC in this figure now proposed. And since the sides HD, DF of the former are equal to the sides HD, DC of the latter — for DC was taken equal to DF (or EF) in the earlier construction of this figure, and the axis HD is here the same as it was there — and since the angles contained by these sides are equal, namely right angles (for the angle HDC in this figure is a right angle by construction, while HDF in the dial is a right angle because the axis HD, being at right angles to the plane of the equator, is also perpendicular — by Euclid, *Elements* XI, definition 3 — to the straight line DF lying in the plane of the equator),

[Margin: Elements I.4]

the bases HF, HC will also be equal. Therefore, if we transfer the straight line HF, taken in the dial, onto the line DC in this figure from the point H — as was said in this latter construction of the proposed figure — we shall arrive at exactly the point C, which we previously found, by the earlier construction of the same figure, for the fifth and seventh hours. Otherwise, if the straight line HF, transferred in the dial from the point...

H of this figure onto the line DC, were to fall on some point other than C, we should show — as we did just now — that the line HC drawn would be equal to HF itself in the dial, and hence equal also to the very line transferred from H onto the line DC, which is supposed to fall on a point other than C (for this transferred line is, by construction, equal to that same HF). Two equal straight lines would therefore be drawn from H to the line DC below the right angle D — which is absurd. For, as we demonstrated from Proclus at *Elements* I.16, from H only two mutually equal straight lines can be drawn to the line DC, one below the point D and the other above the same point, but not both below the point D, as would here be the case. Therefore the straight line HF, transferred from H onto the line DC, will fall on the point C. By the same reasoning we shall show that the other hour-intervals in the dial, between H and the equinoctial line, transferred from the point H of this figure onto the line DC, fall on the points that were previously found. And thus far the hour-lines drawn in this figure from H through the points of the line DC run from the twelfth hour of noon up to the fifth hour after noon, and from the seventh hour after midnight up to the twelfth hour of noon, as is clear from the construction itself. For only the said hours cut the equinoctial line in the dial, and it is their intervals that have been transferred. Hence we shall draw the lines of the remaining hours as follows.

For the sixth hour after noon (or after midnight), draw from H the line HV parallel to DC. For the seventh hour after noon and the fifth after midnight, draw HX, making the angle VHX equal to the angle VHC. Likewise, for the eighth hour after noon and the fourth after midnight, draw HY, making the angle VHY equal to the angle VHT; and so with the rest. This is easily done if an arc of a circle is described from H.

[Margin: Elements III.27]

For if the arcs of this circle intercepted between HV and the straight lines drawn from H to the right of HV (the lines which, of course, cut the line DC) are transferred to the left side of HV, onto the said arc, then the straight lines drawn

from H through those arcs — transferred to the left of HV — will make with HV angles equal to those made on the right-hand side with the same HV; because the arcs on which the former stand at the center H are equal to the arcs on which the latter stand at the same center H.

After this, in the semicircle described from D, let the declinations of the signs be computed, beginning from the straight line DC toward each side of DC; and through the ends of these computations let straight lines go out from D, which will be the rays of the signs when the Sun is at their beginnings — so that the straight line DC, in the middle of all the other lines, is the ray of the equator, that is, of Aries (Υ [Aries]) and Libra ($\ Libra$ [Libra]), as is plain from what we said about the Analemma in Proposition 1 of the previous book. Now the arcs of the semicircle described from D, lying between the ray of the equator DC and the rays of the other signs, will either be entirely equal to the arcs of the Analemma lying between the diameter of the equator and the common sections of the ecliptic and the meridian — when, that is, the semicircle described from D is equal to the circle of the Analemma — or at least similar to them, when the said semicircle is unequal to the circle of the Analemma. Hence, so that the rays of the signs may more easily be described by means of their declinations found in the Analemma, it would be helpful if the semicircle described from D had a radius equal to the radius of the Analemma, so that the declination-arcs could be transferred from the Analemma into this semicircle with the compasses.

From this figure, then, constructed in this manner — which in what follows we shall call the *figure of the Zodiac rays* — the arcs of the signs are described in the dial as follows. From the figure already completed, transfer the line H μ of the twelfth hour, lying between the point H and the ray of Cancer (♋ [Cancer]), onto the meridian line of the dial from H; or transfer the line I μ , between the ray of the equator and the ray of Cancer, from the point I of the equinoctial line in the dial onto the meridian line, marking the point K. Likewise, let the line of the first and eleventh hours of the same figure, lying between H and the same ray of Cancer — or between the equinoctial ray and the ray of Cancer — be transferred onto the lines of the first and eleventh hours of the dial from H, or from the points ξ and π of the equinoctial line, marking a point on each. And so let the remaining spaces between H and the ray of Cancer, or between the equinoctial ray and the ray of Cancer, be transferred into the dial onto the lines of the corresponding hours, marking points on them. Then let these points, marked on the hour-lines of the dial, be joined together by a fittingly curved and bent line. For this will be the conic section of Cancer (♋) described in the dial. In the same way the conic sections of the other signs will be described, if the straight lines comprehended

between H and the rays of the signs — or between the equinoctial ray and the rays of the signs — are transferred onto the corresponding hour-lines of the dial from H, the center of the dial, or from the equinoctial line, and so on. This, however, with the following condition: the straight lines lying between the ray of the equator and the rays of the other signs toward the point H are to be transferred from the equinoctial line onto the corresponding hour-lines toward the center H, while the others are transferred in the contrary direction. Likewise, the intervals of the lines drawn from H falling toward the ray of the equator DC are to be transferred from the center of the dial H onto those hour-lines which tend from H, the center of the dial, toward the equinoctial line; but the intervals of the lines drawn from H toward the other side of the line HV — such that they cannot cut the ray of the equator below D — are to be transferred onto the hour-lines of the dial from the center H in the upper direction, that is, onto those hours which do not cut the equinoctial line, such as all the hours after the sixth hour from noon up to midnight, and all the hours from midnight up to the sixth hour.

[Margin: Demonstration of the latter way of describing the parallels, or arcs of the signs. From which it is clear that Giovanni Battista Benedetti [Giovanni Battista Benedetti] condemns this kind of description undeservedly.]

It will also sometimes be convenient to seek the points of the arcs of the signs at the half-hours or the quarter-hours. As in the present example: because the points on the lines of the fifth hour after midnight and the seventh hour after noon, through which the arc of Cancer (♋) must be drawn, are too far from the center H, points of this kind will have to be found on the lines of the hours $5\frac{1}{2}$ after midnight and $6\frac{1}{2}$ after noon. The same would have to be done on the lines of the hours $8\frac{1}{2}$ after midnight and $3\frac{1}{2}$ after noon, for investigating the points of the arc of Capricorn (♏ [Capricorn]), and so on. In the next figure, the line falling from H to the left of the straight line HV, marked with points, belongs to the hours $5\frac{1}{2}$ after midnight and $6\frac{1}{2}$ after noon, and so on.

But let us now demonstrate geometrically that the parallels, or arcs of the signs, are rightly described by this method. For there are those who either altogether deny it — among them Giovanni Battista Benedetti, in his *Gnomonica*, chapters 70 and 71, where by another and much longer method he attempts to describe the arcs of the signs — or at any rate doubt whether the arcs of the signs can rightly be described in this way, since they do not see the reasoning on which this description of ours rests (a description which indeed all writers hand down without any demonstration). Let the triangle DHI, then — of the portion of the Analemma of the preceding proposition, or of the dial described in the scholium

of the same proposition without the portion of the Analemma — be understood as standing upright to the plane of the dial and of the meridian line, so that the point H is set at the center of the dial and I on the equinoctial line, as appears in the dial of this proposition. If, then, the figure containing the hour-lines and the rays of the signs, constructed just now, be applied by the axis HD of the said triangle (set upright to the dial), and be carried round this way and that — yet so that DC, the ray of the equator, always makes a right angle at D with the axis DH — it is clear that the rays...

of the signs (if we conceive them in our mind as penetrating the plane of the dial) describe in the dial the conic sections of the signs, into which these rays are projected by the diurnal motion. For in that carrying-round, the rays of the signs — making with the ray of the equator the angles of declination, and with the axis the angles complementary to the declinations — do not depart from the conic surfaces described by the Sun's rays, but, produced all the way to the sky, reach the parallels themselves.

Now since the sides HD, DC of the triangle HDC of the immediately preceding figure — to begin with the fifth hour after noon, or the seventh after midnight, to which the line HC corresponds — are equal to the sides HD, DF of the triangle HDF in the dial (because DF is equal to EF, to which DC, taken on the ray of the equator, is made equal; for since the sides EI, IF of the triangle EIF are equal to the sides DI, IF of the triangle DIF and contain equal angles, namely right angles — for EIF is a right angle by construction, while FID is a right angle by Euclid, *Elements* XI, definition 3,

[Margin: *Elements* XI.19]

because FI, the common section of the equator and of the plane of the dial with the meridian — that is, with the triangle DHI situated in the plane of the meridian, and therefore with the straight line DI — is perpendicular at the point I;

[Margin: *Elements* I.4]

the bases EF, DF will be equal) — and since these sides contain equal angles, namely right angles (for HDC in the next figure is a right angle by construction, while HDF in the dial is likewise a right angle, because the axis HD, being at right angles to the plane of the equator, is perpendicular at the point D to the straight line FD lying in it, by Euclid, *Elements* XI, definition 3),

[Margin: *Elements* I.4]

the bases HC, HF and the angles DHC, DHF will be equal. Therefore, if the triangle HDC is conceived to be moved about the axis HD in the dial until it is joined with the triangle HDF, and the line HC coincides with the line HF in the plane of the dial, then the point C of the equinoctial ray will fall on the point F — because of the equality of the angles CHD, FHD and of the lines HC, HF — and the point Z of the ray of Cancer ($\mathfrak{S}$) will fall on the point α , because the line $H\alpha$ was taken equal to the line HZ by construction, or $F\alpha$ equal to CZ. Through the point α , then, of the fifth hour after noon, passes the conic section of Cancer, since the ray of Cancer falls upon it in that revolution of the triangle DHC about the axis DH in the dial, as we have shown. In no other way shall we prove that the point T also, in that carrying-round of the Zodiac rays, falls on R; and the point β of the ray of Cancer on γ ; and the same must be said of the remaining hour-lines that intersect the equinoctial line.

Diagram. The same woodcut as on p. 148 (the *figure of the Zodiac rays* combined with the horizontal dial) is repeated here, occupying the lower left of the page. A large semicircle with a bounding arc is set about the top; from the apex region a pencil of straight lines (hour-lines and sign-rays) fans downward to a ring of hour-numerals and a curved band of zodiacal-sign symbols at the foot. As on p. 148, the small peripheral point-labels are worn and set in mixed Greek/roman/symbol sorts; several remain uncertain and this figure

- λ (upper left, on the arc): endpoint of the axis-line $A\lambda$
- H (top center): center of the dial / apex of the radiating pencil
- D (top, right of H): center of the semicircle, on the axis
- A (top right): right endpoint of the axis-line $A\lambda$
- φ (right side, on the arc): peripheral point [?]
- B (right of center): point on the arc for laying off the pole's altitude
- C (center): point on line DC
- T (center left): point on line DC
- V (below center): sixth-hour point (line HV)
- X, Y (left of center): seventh/fifth- and eighth/fourth-hour points
- θ (lower center, Greek): sign-ray point θ that falls on α in the revolution [?]
- α , β , γ , δ (lower center band, small Greek): sign-ray / conic-section points [?]
- Z, R (lower center, roman): points on the fifth-/fourth-hour lines [?]
- Point-symbol on the sixth-hour dial-line (the correspondent of V), read as a worn Jupiter-like sort $\mathfrak{A}$ [?]
- Zodiacal-sign symbols in the lower band: $\mathfrak{M}$ [Capricorn] and neighbours [?]

- Hour numerals around the periphery: (left, descending) 12, 11, 10, 9, 8, 7 ...; (right, ascending) 1, 2, 3, 4, 5, 6 ...

For the sixth hour after noon (or after midnight), which does not cut the equinoctial line but is equidistant from it, the line HV must be drawn perpendicular to the axis HD in the figure of the Zodiac rays, that is, parallel to the ray of the equator DC. For by this arrangement, in that carrying-round of the rays, the line HV will coincide with the line H α [?] in the dial, on account of the equality of the right angles DHV and DH α [?]. For the angle DHV is a right angle by construction, and the angle DH α [?], set at the axis, is likewise a right angle by Euclid, *Elements* XI, definition 3,

[Margin: *Elements* XI.19]

because H α [?], the line of the sixth hour — being the common section of the plane of the dial and of the circle of the sixth hour — is perpendicular to the meridian (to which both of them are at right angles), that is, to the triangle DHI, and therefore also to the straight line DH, at the point H. For this reason the point θ of the ray of Cancer ($\mathfrak{C}$) will fall on α , on account of the equality (by construction) of the lines H θ and H α ; for the line H α in the dial was taken equal to the line H θ in the figure of the rays. Finally, for the seventh hour after noon, or the fifth after midnight, and for any others that do not cut the equinoctial line, if it should be needed (though in our example we need no other hour, since the arc of Cancer cuts the lines of the seventh hour after noon and the fifth after midnight at points very far from the center H), lines must be drawn which, on the other side — the left, that is — make with the line HV of the sixth hour angles equal to the angles which the lines that cut the ray of the equator on the right-hand side (the lines belonging to those hours which cut the equinoctial line in the dial) make with that same sixth-hour line — such lines as HX, HY, and so on. For in this way such lines will make with the axis HD, in the figure of the Zodiac rays, the angles XHD, YHD equal to the angles which the corresponding hour-lines in the dial make with the axis HD: namely, the angle XHD in the said figure equal to the angle xHD in the dial.

[Margin: *Elements* I.13]

For since the two angles XH λ , XHD in the figure of the rays are equal to the two angles DHk, DHx in the dial (because both these and those are equal to two right angles), let the angle XH λ in the said figure be equal to the angle DHK in the dial. (For the angle XH λ in the same figure is equal to the angle CHD of the same figure, by Postulate 3 of *Elements* Book I, because the whole right angles VH λ ,

VHD are equal to one another, and, once the angles VHX, VHC have been taken away — equal to one another by construction — the angle CHD of the said figure is equal to the angle DHF in the dial, as has been demonstrated, and hence also to the angle DHK in the same dial, which is equal to the angle DHF. This we shall show as follows. When the line DK has been drawn,

[Margin: Elements I.4]

DK will be equal to DF itself, because the sides ID, IF of the triangle DIF are equal to the sides ID, IK of the triangle DIK and contain equal angles, namely right angles. Since, therefore, the sides DH, DF of the triangle DHF in the dial are equal to the sides DH, Dk of the triangle DHk in the same dial, and contain equal angles, namely right angles — for the axis HD, being at right angles to the plane of the equator, is perpendicular to the straight lines DF, DK lying in that same plane of the equator, by Euclid, *Elements* XI, definition 3 — the angles DHF, DHk will be equal.) Hence the remaining angle XHD in the said figure will also be equal to the remaining angle DHx in the dial; and so with the rest. Since this is so, the line Hx of the said figure will be joined to the line Hx of the dial in that revolution of the rays, on account of the equality of the angles which the lines HX, Hx make with the axis HD, and so on. And the same is the reasoning...

and the same reasoning holds for the rest. It is clear, therefore, that Giovanni Battista Benedetti [Giovanni Battista Benedetti], in his *Gnomonica*, was wrong to criticize this method of describing the arcs of the signs.

Now that the hyperbolas of the northern signs have been drawn — that is, those contained between the center H and the equinoctial line — we shall describe more accurately the opposite hyperbolas of the southern signs, that is, those drawn on the other side of the equinoctial line. (These are harder to draw, because the points on the hour-lines beyond the equinoctial line, through which they must be drawn, are farther apart from one another than the points on the near side of the equinoctial line.) We proceed as follows.

[Margin: How the hyperbolas of the southern signs are drawn from the hyperbolas of the northern signs.]

Once the transverse diameter of the opposite sections has been found on the meridian line of the dial — this being always equal to the segment of the straight line HB, in the figure of the rays, intercepted between the rays of the opposite signs (just as in the dial the straight line KN is the diameter of the opposite sections of Cancer (♋) and Capricorn (♏), and is equal to the segment $\mu\alpha$ of the straight line HB in the figure of the rays intercepted between the rays of Cancer

(♋) and Capricorn (♏)) — we shall bisect it, so as to obtain the center of the opposite sections, according to the teaching of Apollonius [Apollonius of Perga] in the second set of Definitions of Book 1 of the *Elements of Conics* (*Conics* I, second Definitions). Then, since by Proposition 30 of Book 1 of Apollonius (*Conics* I.30) any straight line drawn through the center of the opposite sections is bisected at that center, we shall draw hidden lines, through the found center, from the points on the hour-lines above the equinoctial line through which the northern hyperbolas pass. For if, below the said center, we cut off lengths equal to the segments of those lines that lie between the said points and the center, we shall have on those hidden lines the points through which the southern hyperbolas are to be drawn. We shall use this same skill and device in the dials that follow, in which opposite hyperbolas must be drawn — whether these belong to the parallels of the equator drawn through the beginnings of the signs of the Zodiac, or to the parallels of the horizon.

This method of drawing the hyperbolas of the southern signs from the hyperbolas of the northern signs will be understood more clearly from the following figure, in which the diameter of the opposite hyperbolas is DE, and their center is the point F. If, then, from the point K of the upper hyperbola the straight line KFN is drawn through the center F, and FN is cut off equal to FK, the opposite hyperbola must be drawn through the point N — since, by Proposition 30 of Book 1 of Apollonius (*Conics* I.30), the straight line FK is equal to the segment of the same straight line extended beyond F, contained between F, the center, and the opposite hyperbola. Likewise, when the straight line LFM has been drawn, if FM is cut off equal to FL, the opposite hyperbola must be drawn through the point M; and so with the rest. Moreover, in my judgment, the straight lines through the center of the opposite hyperbolas should be drawn from those points of the northern hyperbolas through which the hour-lines pass, since those points have been found by construction. Hence we shall describe the opposite hyperbola more accurately through those points than through other intermediate points lying between them, which are found not by construction but by conjecture.

From Proposition 6 of the preceding book we shall easily recognize which parallels produce opposite sections in the dial — that is, hyperbolas, or other sections — for the aforesaid method fits hyperbolas alone. We shall, however, also derive this from the figure of the Zodiac rays described a little earlier, as follows. Whenever the straight line HB, in the said figure, cuts two rays of opposite signs — that is, rays equally distant on either side from the ray of the equator, such as the rays of Cancer (♋) and Capricorn (♏), of Gemini (♊) and

Sagittarius (♐), of Taurus (♉) and Scorpio (♏)[?], and so on — then the arcs, or parallels, of those opposite signs are opposite hyperbolas.

[Margin: How to know whether the arcs of the signs are hyperbolas, parabolas, or an ellipse.]

For in the triangle $DH\alpha$ the external angle $AD\alpha$ is greater than the internal and opposite angle $DH\alpha$. [Margin: *Elements* I.16 (superscript a).] But the angle $AD\alpha$ is equal to the complement of the declination of the parallel whose radius is $D\alpha$ — and therefore also of the opposite parallel, whose radius is $D\mu$ — while the angle $DH\alpha$ is equal to the elevation of the pole above the horizon. The horizon will therefore cut each of the parallels of the rays $D\alpha$ and $D\mu$, as we showed in the corollary to Proposition 6 of the preceding book. Hence, by the same Proposition 6, the arcs of those signs are opposite and equal hyperbolas.

But when the straight line HB does not cut both rays of the opposite signs, but is parallel to one and cuts the other, then the arc of that sign whose ray is cut will be a parabola. [Margin: *Elements* I.29 (superscript b).] For then the external angle $AD\alpha$, of the complement of the declination, is equal to the internal angle $DH\mu$, of the elevation of the pole (because $H\mu$ and $D\alpha$ are set parallel); therefore the horizon will be tangent to each of the opposite parallels, by the corollary to Proposition 5 of the preceding book. Hence, by the same Proposition 5, the arc of the other sign, whose ray is cut, will be a parabola.

Finally, when the straight line HB neither cuts both rays of the opposite signs nor is parallel to one, but cuts one while receding more and more from the other, then the arc of that sign whose ray is cut will be an ellipse. [Margin: *Elements* I.28 (superscript c).] For since in that case the angle $AD\alpha$, of the complement of the declination, is less than the angle $DH\mu$, of the elevation of the pole — for if the angles $AD\alpha$ and $DH\mu$, external and internal, were equal, the straight line HB would be parallel to the ray $D\alpha$; and if the angle $AD\alpha$ were greater than the angle $DH\mu$, the straight lines HB and $D\alpha$ would meet, which is not assumed — it comes about that the horizon is neither parallel to those opposite parallels, nor tangent to them, nor cuts them, as is clear from the corollary to Proposition 7 of the preceding book. For this reason the arc of that sign whose ray is cut will be, by the same Proposition 7, an ellipse.

It is clear, moreover, that when the arc of a sign is a parabola or an ellipse, the arc of the opposite sign cannot be drawn in the dial, because the straight lines drawn from H do not cut its ray, as we have said.

[Margin: When the arc of a sign is a parabola or an ellipse, the arc of the opposite sign cannot be drawn in the dial.]

For the rest, it will be enough if, from the latter figure of this proposition, we describe the portion of the arc of any sign up to the meridian line on one side only.

[Margin: How, from the portion of the arc of a single sign drawn up to the meridian line, the remaining portion — and also the arc of the opposite sign, if the arcs of the opposite signs are hyperbolas — is drawn.]

For from this the remaining portion on the other side can also be drawn — indeed, in the dials that follow, much more accurately than if we were to draw the whole arc from the figure of the Zodiac rays, as we shall say in its place — and moreover the whole arc of the opposite sign as well (provided, however, that the diameter of the arcs, namely the straight line HB, cuts the rays of the opposite signs in the figure of the rays; for only then, as we showed, are the arcs of those signs opposite and equal hyperbolas) can be drawn. We shall carry out our aim as follows. Let AB be the axis of the opposite hyperbolas, or of the parabola, or of the ellipse — such as is the meridian line in a horizontal dial — and let CD be the portion of that arc, or conic section, drawn up to the axis. Now let the point E be found on the axis AB, through which the arc of the opposite sign must pass. (This is obtained if DE is taken equal to the transverse diameter of the opposite arcs in the figure of the rays, namely to the segment of the straight line HB between the rays of those opposite signs.) Then, taking the straight lines DA and EB equal to one another and of any magnitude, and bisecting the diameter DE at F, we shall place one foot of the compasses at F and extend the other to any point of the arc CD — say, to C — and, without changing the compass opening, we shall describe from F, on the other side of the axis, a hidden arc of some circle; likewise two others, on either side of the same axis, toward the region of E. Again, setting one foot of the compasses at A, we shall extend the other to C, and, with the legs of the compasses kept so extended, we shall describe from A an arc of a circle which cuts the one first drawn from F, on the other side of the axis, at G; likewise two others from B, which cut the two first drawn from F — on either side of the same axis, toward the region of E — at H and I.

Nam per punctum G... — For the arc CD, produced, will pass through the point G, and through the points H and I will run the opposite and equal arc, which is set to pass through the point E. In the same way, if we extend the compasses from F to K, and describe three arcs from F at the interval FK, and likewise from A and B three others at the interval AK — one from A and two from B — which cut the

first three at L, M, N, then the same arcs will pass through the points L, M, N. And in this way we shall find as many further points as we please. I would urge, however, that from F those points be taken in the portion of the arc CD where it is cut by the hour-lines.

But if the arcs of the opposite signs are not opposite and equal hyperbolas — as happens when the diameter of the arcs, or the straight line HB, does not cut the opposite rays in the figure of the Zodiac rays, but only one of them — then it is enough if just two arcs, on the other side of the axis DE, cutting one another, are drawn from F and A, F being taken at any point of the diameter AB, even if that point is not the center. For in that case the arc of the opposite sign cannot be drawn, as we said a little earlier. But this device is then less necessary in a horizontal dial, because both portions of the arc, CD and GD, are drawn with a single effort by the latter method, which is accomplished by means of the figure of the Zodiac rays, as is clear from the foregoing. For the opposite arcs, however, the matter will be very useful and convenient. In declining and inclining dials too this practice will bring great convenience — whether the arc of both opposite signs can be drawn in the dial, or of only one — as will be clear in its place.

Diagram. A construction figure for opposite conic sections, laid out symmetrically about a vertical axis AB. Two dotted straight lines cross near the center of the figure to form a broad X; each arm of the X is punctuated by small cross-marks (×), each pair of crosses marking the intersection of two compass arcs. Along the vertical axis lie two rhombus ("diamond") figures, one above the other.

- A (top): apex of the figure, upper vertex of the axis and of the upper rhombus
- C (upper left): far end of the left-upper dotted arm
- G (upper right): far end of the right-upper dotted arm
- K (left vertex, upper rhombus): left end of the diagonal KL [faint/reversed in the scan]
- L (right vertex, upper rhombus): right end of the diagonal KL
- O (on axis, center of upper rhombus): where the diagonal KL cuts the axis AD
- D (lower vertex, upper rhombus): on the axis, above F
- F (center of the whole figure): midpoint of the transverse diameter DE, between the two rhombi
- E (upper vertex, lower rhombus): on the axis, below F

- M (left vertex, lower rhombus)
- N (right vertex, lower rhombus)
- B (bottom): apex of the figure, lower vertex of the axis and of the lower rhombus
- H (lower left): far end of the left-lower dotted arm
- I (lower right): far end of the right-lower dotted arm

But let us demonstrate this construction geometrically. Draw the straight lines AK, AL, KL, DK, DL, FK, FL, FM, FN, BM, BN, EM, EN, of which let KL cut the straight line AD at O. Since the two sides FA, FK of the triangle AFK are equal to the two sides FA, FL of the triangle AFL, and the base AK is equal to the base AL, the angles AFK, AFL will also be equal. [Margin: *Elements* I.8 (superscript a).] Again, since the sides FO, FK of the triangle OFK are equal to the sides FO, FL of the triangle OFL, and contain equal angles, as we have shown, the bases OK, OL will also be equal, and the angles at O equal, and therefore right angles. [Margin: *Elements* I.4 (superscript b).] And since in a right cone — such as are all those whose bases are the parallels of the Sun and whose common vertex is at the center of the world — the diameter of any conic section bisects all the ordinates, and at right angles, as is clear from Proposition 7 of Book 1 of Apollonius (*Conics* I.7), it follows that KL is an ordinate to the diameter AD of the conic section CD. For no other straight line drawn from K to AD can be cut at right angles, as is clear from what we demonstrated, following Proclus, on Proposition 17 of Book 1 of Euclid (*Elements* I.17). The conic section CD, therefore, when produced, will pass through the point L, and so with the rest of the points.

Again, since AD and BE are equal, if the equals DF, EF are added, the wholes AF, BF will also be equal. Since, then, the two sides AF, FK of the triangle AFK are equal to the two sides BF, FN of the triangle BFN, and the base AK is equal to the base BN, the angles AFK, BFN will also be equal. [Margin: *Elements* I.8 (superscript c).] Hence, as we demonstrated, following Proclus, on Proposition 15 of Book 1 of Euclid (*Elements* I.15), the straight lines FK, FN form a single straight line, and consequently one bisected at F, the center of the section. Therefore, since in opposite hyperbolas whose diameter is DE the straight line drawn from K through the center F is bisected at the center F, by Proposition 30 of Book 1 of Apollonius (*Conics* I.30), the opposite hyperbola will necessarily pass through the point N. In the same way we shall show that it passes through the point M, and so with the remaining points.

Now since in the triangles DOK, DOL the sides OD, OK are equal to the sides OD, OL, and contain equal angles — namely right angles, as has been demonstrated —

the bases DK, DL will also be equal. [Margin: *Elements* I.4 (superscript d).] By the same reasoning EM, EN will be equal to one another. And again, DK will be equal to EN, and DL to EM, if the triangles DFK, EFN and DFL, EFM are considered; for the sides FD, FK are equal to the sides FE, FN, and contain equal angles, and so on. Hence, if with the interval DK an arc is described from D toward L, and two others from E on either side, these arcs will pass through the points L, M, N. And by the same reasoning the arcs described from the same points D, E, at the intervals between D and the remaining points of the conic section CD, will pass through the other points of the sections DLG, EMH, ENI. Hence, if three arcs each — from A, F, D, and likewise from B, F, E — are drawn so as to intersect one another, the points through which the opposite conic sections must be drawn will be found with great exactness, so that by this method the drawing of the arcs of the signs can easily be checked.

[Margin: The method by which opposite hyperbolas are drawn in the dial with a single effort.]

This affords great convenience in laying out the arcs of the signs in declining and inclining dials, as will be clear below.

The hyperbolas of the opposite signs, too, can (namely when the straight line HB, in the figure of the rays, cuts the rays of the opposite signs) — being indeed equal to one another and opposite, as is clear from Proposition 14 of Book 1 of Apollonius [Apollonius, *Conics* I.14]

nius, be most conveniently drawn with a single effort in this way. When, as before, two points have been found on the meridian line of the dial, through which the arcs of the opposite signs must be drawn, let a point ϕ be taken on the same meridian line extended, distant from the point of the hyperbola to be drawn beyond the equinoctial line by as much as the center of the dial H is distant from the point of the hyperbola opposite to it — the one to be drawn between the equinoctial line and the center H. And from the point ϕ let hidden hour-lines go out, like those drawn from the center H. This will easily be done if, through the point X, which bisects the transverse diameter of the opposite hyperbolas, a line is drawn parallel to the equinoctial line, as a second equinoctial line with respect to the center ϕ . For where this line cuts the hour-lines drawn from the center H, through those points the hidden hour-lines from ϕ must be drawn, as is evident — because in this way the angles at the centers H and ϕ , contained by the hour-lines and the meridian line, will be equal. [Margin: *Elements* I.4 (superscript a).]

Or else this way, and perhaps more conveniently: when a hidden arc of a circle has been described from H, let another arc be described at the same interval from ϕ . For if in the first arc the intervals of the hours are taken, beginning from the meridian line, and are transferred to the second arc, beginning likewise from the meridian line, points will be obtained on this second arc through which the hour-lines from ϕ are again to be drawn. But for the sixth hour, a line parallel to the equinoctial line, or perpendicular to the meridian line, must be drawn through ϕ . And so, if all the intervals which — in the latter drawing of the arcs of the signs of the Zodiac (which is accomplished from the figure of the Zodiac rays) — we said must be transferred with the compasses from H, the center of the dial, onto the hour-lines, in order to draw any hyperbola between the center H and the equinoctial line, are transferred at the same time with the same compasses from ϕ onto the corresponding hour-lines going out from ϕ , then with a single effort the points on both sides will be obtained, through which the opposite and equal hyperbolas are to be drawn. You have an example in the hyperbolas of Cancer (♋) and Capricorn (♏) of the dial above. And the same reasoning holds for the other opposite hyperbolas.

We shall use the same device in what follows, taking, in declining and inclining dials, the line of the style in place of the meridian line; and likewise, in the figure from which the arcs of the signs are drawn, the line of the index in place of the line HB of the twelfth hour, and so on, as we shall point out in its place. And so we have drawn the parallels, or the arcs of the signs of the Zodiac, and so forth, in the aforesaid horizontal dial. Which was to be done.

SCHOLIUM.

Andreas Schöner [Andreas Schöner], in his work on Gnomonics, describes in the following way the hour-lines that go out from H in the figure of the rays of this proposition.

[Margin: A most beautiful method for drawing the hour-lines in the figure of the rays without the hour-intervals of the dial.]

When the straight line HIB has been drawn, forming the angle AHB of the elevation of the pole — just as we too did — he takes on it any point ϕ , through which he draws ϕV parallel to DH; and this line cuts the straight line drawn from H parallel to DC at the point V, from which, as center, and with the interval $V\phi$, he describes a circle. Dividing this into 24 parts, beginning at ϕ , he connects each pair of points equally distant from ϕ by hidden straight lines, which cut the straight line ϕV at points through which he draws straight lines from H for the

hour-lines — lines that will correspond exactly to those found by the earlier method. And if the individual arcs of the circle described from V are bisected, or divided into four parts, and so on, and the same is done, then lines for the half-hours, or for the quarter-hours, and so on, going out from H, will likewise be obtained. And although he himself does not demonstrate this operation, we have nevertheless learned by many trials that it is true, and that the hour-lines found by it agree with those we found before.

Now if this practice is reliable, one will be able most conveniently to draw an astronomical horizontal dial, together with the arcs of the signs — and for a given size of the style as well — in this way.

[Margin: The drawing of a horizontal dial from the figure of the rays, after Andreas Schöner.]

When the ray of the equator DC has been drawn, together with the rays of the other signs, let the perpendicular DH be drawn through D for the axis of the world; and from D let the straight line D ϕ be dropped, making with the axis DH the angle ϕ DH equal to the complement of the elevation of the pole. Then, taking the given length of the style D ϕ , let the perpendicular H ϕ I be drawn through ϕ to D ϕ , cutting the axis DH at H and the ray of the equator DC at I. Also, taking on the straight line HI produced any point ϕ , through which the line ϕ V is drawn parallel to the axis DH, let a circle be described from the center V with the interval V ϕ , and let the whole figure be completed, as Andreas Schöner taught. After this, when in the plane of the dial the meridian line HE has been drawn — which the equinoctial line FK cuts at right angles at I — let the straight line IH, upward, be taken equal to the straight line HI in this figure just constructed, and the straight line IE equal to the straight line DI. For if the intervals on the ray of the equator between D and the hour-lines going out from H are transferred, in the plane of the dial, from E onto the equinoctial line on both sides, then, with those points marked on it, the straight lines drawn from H through these points of the equinoctial line will give the hours from noon, or from midnight, and so on, as before.

[Margin: How a dial is drawn from the figure of the rays.]

But the arcs of the signs will be drawn from the figure of the rays just constructed, as above.

Moreover, that this method of Schöner for drawing the hour-lines from H is true and reliable — so that his lines do not differ from ours, which we drew from H

through the points found on the ray of the equator — will be made plain as follows.

[Margin: How a larger or smaller dial is drawn from the figure of the rays.]

Diagram. Schöner's construction of an astronomical horizontal dial from the figure of the rays. A large circle (the "figure of the rays") stands on the center H, from which a fan of straight hour-lines radiates outward and downward; the hours are numbered around the periphery. Across the lower-middle of the figure runs a band of the sign-arcs (the arcs of the Zodiac signs) carrying zodiac glyphs. Superimposed on the circle is the geometric construction (axis DH, the style line, the parallel ϕV , and the auxiliary points). This is a very dense, partly worn figure; many of the small peripheral point-letters and hour-numerals will not resolve at this resolution and are flagged. (more than two labels flagged [?]).

Legible / partly legible labels:

- A (upper right of the rim): endpoint of the elevation-of-pole line, top of the construction
- B (right side, below A): on the line HIB (the style / axis line)
- C (center-low): on the ray of the equator DC
- D (upper area, on the top arc): endpoint of the ray of the equator DC and of the axis DH
- H (center, foot of the fan of hour-lines): center of the dial
- V (center of the auxiliary circle): center from which the 24-part circle is drawn
- λ (top left of the rim) [Greek letter lambda, a construction point]
- ϕ (construction point on line HIB / the style; appears more than once) [?]
- T (center): construction point [?]
- Z (center-left, above T): construction point [?]
- θ (left of T) [Greek theta]: construction point [?]
- X (center-left): construction point [?]
- Y (center-left): construction point [?]
- ψ / P (upper center): construction point [?]
- Φ (right of center) [Greek capital phi]: construction point [?]
- Hour numerals around the periphery: 12, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 (left and right of the fan) — several individual numerals worn/ambiguous [?]
- "u" (far lower left) and "n" (lower right): terminal markers on outer hour-lines [?]

- Zodiac-sign band across the lower middle: a row of zodiac glyphs marking the sign-arcs — individual glyphs too small/worn to resolve at this resolution [?]

Schöner's straight lines from H are the radii of the diurnal arcs of the equal hours, as we shall demonstrate at the end of this whole work: namely, $H\phi$ is the radius of the diurnal arc of hour 0 [noon]; the next line, the radius of the diurnal arc of hour 2; the following one, of hour 4; and so on. For in order that the radii of all the diurnal arcs — of the odd as well as the even hours — may be obtained, the circle described from V through ϕ must be divided into 48 equal parts (as was said in the scholium to prop. 33 of the preceding book), and not merely into 24, so that the straight lines emitted at H form with HV the angles of declination of the diurnal arcs of the even hours. But the lines that we drew through the points on the radius of the equator from H likewise form, at H, with HV the declinations of those same diurnal arcs, as we shall show. Therefore Schöner's lines pass through the points on the radius of the equator, so that they do not differ from ours — lest the same diurnal arc should have different angles of declination.

For example, Schöner's straight line HC is the radius of the diurnal arc of hour 10, and VHC is the angle of declination of that diurnal arc; and this, I say, is the very same angle that the straight line drawn from H through the found point C also makes. For in the dial of this proposition, the straight line EF — and therefore also the straight line DC — is the secant of angle FEI of 5 hours, the whole sine being set as EI or DI in this figure; and DH is the tangent of angle DIH, the complement of the pole's altitude. Now CD is the secant of 5 hours with respect to the whole sine DI, inasmuch as the whole sine is to DH as the tangent of the complement of the pole's altitude is to that same whole sine CD; and by prop. 18 of the *Sines*, CD is the secant of 5 hours to the whole sine as the whole sine is to the sine of the complement of the arc of 5 hours — that is, to the sine of the first hour, or of the difference between the half and the quadrant of the diurnal arc of 10 hours. Therefore the whole sine will likewise be to the sine of the aforesaid difference as the tangent of the complement of the pole's altitude is to the tangent of angle DCH. And accordingly, by problem 11 at the end of the new description of dials, DCH will be the angle of declination of the diurnal arc of 10 hours — which was proposed.

Now once the hour-lines have been drawn from H, if — by this method, which is completed by means of the figure of the rays — a larger or smaller dial is to be constructed, the figure of the Zodiac's rays going out from point D must be described, more distant from point H, or less, and so on. [Margin: (a) Euclid,

Elements VI.4.] For in this way it will come about that the proportion of the lines between H and the radius of the equator DC (placed relative to the lines of that same radius of the equator between D and the points where the straight lines emitted from H cut DC itself) is the same as the proportion of the lines between H and the more remote or nearer radius of the equator (placed relative to the lines of that same radius between the axis DH and the points where the straight lines from H intersect the radius of the equator) — on account of the similarity of the triangles, and so on. For the description of the hour-lines emitted from H remains the same in the same climate, whether a large or a small dial is to be described; but the description of the Zodiac's rays is always the same in any climate, provided only that it is more remote from, or nearer to, point H, according as a larger or smaller dial is to be described. Likewise the equinoctial line in a larger dial ought to lie along the meridian line at just such a distance from the center H as is the portion of the straight line HB in the figure of the Zodiac's rays intercepted between H and the radius of the equator lying more or less distant from point H.

[Margin: How the length of the style, and its position on the dial, may be investigated from the figure of the rays. — How, from a given length of the style, the distance of the Zodiac's rays from point H in the figure of the rays may be found.]

Diagram. The "figure of the rays" (*figura radiorum*) — a large circular construction for the astronomical horizontal dial. A great circle carries, along its upper rim (left to right), the points λ , H, E, D, A; from H at the top a dense fan of hour-lines radiates downward across the circle to numbered points on the lower periphery. A rectangular band runs across the lower-middle of the circle carrying the sign-arcs (the arcs of the Zodiac) with their glyphs. The figure is dense and partly worn; several interior construction letters and peripheral hour-numerals will not fully resolve at this resolution, and are flagged. (more than two labels flagged [?]).

- λ (upper-left rim): construction point [Greek letter lambda]
- H (top, left of center): center of the dial, foot of the fan of hour-lines
- E (top center): rim point
- D (upper right): endpoint of the radius of the equator / axis
- A (upper-right rim): rim point
- V (mid-left interior): center of the circle
- C (center-low): point on the radius of the equator DC
- P (upper-center interior) [?]

- B (upper-center interior, appears more than once) [?]
- Z (center-left interior) [?]
- T (center interior) [?]
- X (center-left interior) [?]
- Φ (right of center) [Greek capital phi]: construction point
- φ (the style-point on line HI; recurs in the surrounding text) [?]
- Hour numerals around the lower periphery — left side: 12, 11, 10, 9, 8, 7, 6, 5, 4, 3 [several worn, [?]]; right side: 12, 1, 2, 3 [several worn, [?]]
- "u" (far lower left) and "n" (lower right): terminal markers on the outer hour-lines [?]
- Zodiac-sign band across the lower-middle: a boxed row of zodiac glyphs marking the sign-arcs; individual glyphs too small/worn to resolve [?]

THE STYLE or gnomon, on the latter method of describing the parallels of the signs — that is, from the figure of the rays — is obtained as follows. From point D, where the radius of the equator cuts the axis $A\lambda$, let a perpendicular $D\varphi$ be dropped to HI; or at least let the straight line $D\varphi$ be drawn making the angle φDI equal to the angle of the pole's altitude. For the length of the style is the straight line $D\varphi$ itself, and its position is the point φ , distant from H, the center of the dial, by the interval $H\varphi$ along the meridian line — or from the equinoctial line FK by the space $I\varphi$, on the same meridian line toward the center H.

[Margin: How, from a given length of the style, the distance of the Zodiac's rays from point H may be found.]

But if the magnitude of the gnomon is given, we shall find, by this method, the distance of the figure of the Zodiac's rays from point H along the axis HD. For the hour-lines proceeding from H remain entirely the same, as we said, at the same altitude of the pole. In the setting-out of the Analemma which we described in prop. 1 of this book, let there be cut off from AD — the diameter of the Vertical circle, drawn out — a straight line DM equal to the given gnomon; and through M let NO be drawn parallel to BC, the diameter of the Horizon, cutting the axis ED at N and the diameter of the equator FD at O. [Margin: (b) Euclid, *Elements* I.26.]

Then point D, in the figure of the Zodiac's rays of this proposition — where all the rays of the Zodiac meet — ought to be distant from H by the interval DN; such is point D, near A, in the said figure of the Zodiac's rays of this proposition. For if from that point D a perpendicular be dropped to HD for the radius of the equator, and at the same time the rays of the other signs be emitted from D — as in the Analemma of prop. 1 of the preceding book — we shall describe, by means of the hour-lines contained between these rays and point H, the parallels of the signs, as

before; and the extremity of the shadow cast by the given gnomon DM will traverse these, as is clear from what has been demonstrated. For if from the said point D a perpendicular DK be dropped to HIB, it will be equal to the given gnomon DM. For since angle DHK, in triangle DHK of the figure of the Zodiac's rays of this proposition, is equal to angle DNM in triangle DNM of the Analemma-portion of prop. 1 of this book (for each is the angle of the pole's altitude), and since the right angles at K and M are equal, and moreover the side HD is equal to the side DN by construction, the remaining sides also — namely DK and DM to each other, and HK and NM to each other — will be equal. Which was proposed. But when the arcs of the signs are changed, the equinoctial line is changed too, which always lies along the meridian line at just such a distance from the center H as is the portion of the straight line HB intercepted, in the figure of this proposition's rays, between H and the radius of the equator — as we also said a little before.

[Margin: The description of the parallels which pass, in the same dial, not through the beginnings of the signs but through other points of the Zodiac.] Now if we take the declinations of other points of the Zodiac, and draw their parallels in the first figure of this proposition — or their rays from D in the third figure — we shall describe their parallels in the aforesaid dial in the same way.

[Margin: The description of the parallel on which the Sun stands on any given day, in the same dial.] Indeed, given the first day of any month, or any other day whatever: if the place of the Sun on that day in the Zodiac, and its declination, be sought, and its parallel be drawn in the first figure of this proposition (or its ray from D in the figure of the rays), there will be described on the dial, by the same method, a certain curved line which the extremity of the shadow will traverse on that given day.

[Margin: The description of feast-days in the same dial.] By this method the feast-days of the saints can be described on the dial — that is, certain curved lines which the shadow of the style traverses on those days.

[Margin: What should be inscribed on the sign-arcs delineated on the dial.] Now on all these parallels of the signs may be inscribed the characters of the signs; the days of the months on which the shadow's extremity traverses them; the times of sunrise and sunset; the lengths of the days and of the twilights; and other things of this kind — such as would appear on an Italian and a Babylonian dial (which will be treated in props. 10 and 11 of this book), the time of noon, and so forth.

[Margin: By what art the sign-arcs are to be described on a dial that has no center.] For the sake of this, certain lines must be drawn parallel to the meridian line itself, both to the left and to the right, as appears in the second figure of this proposition. And the more remote these parallels are from the meridian line, the greater the interval of time over which the gnomon's shadow will be received on the dial.

Again, since the Sun ascends from ♎ [Capricorn] through ♈ [Aries] up to ♋ [Cancer], and descends from ♋ [Cancer] through ♎ [Libra] down to ♎ [Capricorn], on the former signs must be inscribed the words ASCENT OF THE SUN [*Ascensus Solis*], and on the latter these: DESCENT OF THE SUN [*Descensus Solis*]. You have an example on the dial above. This is also to be understood on other dials, such as the Vertical, the Meridian, the Equinoctial, and so on.

Finally, we shall describe these same parallels, or arcs of the signs, on the dial that we constructed in the scholium of the preceding proposition — even though we had no center for it — by nearly the same methods we have used in this proposition. For as regards the former method: let an Analemma be described for the given altitude of the pole (as in the proposed example, for latitude 20 degrees), together with the parallels of the signs and their rays, so that in the cones there may be triangles through the axis, and thus, on the straight line RO (which ought to be equidistant from the horizon BC by the distance of the line DG, equal to the style FM), the diameters of the conic sections. Then let the points K, L, M, Q, P, N — taken from point I, where the diameter of the equator divides the straight line RO — be transferred onto the meridian line AB from point E, where the equinoctial line and the meridian intersect each other. Finally, about the diameters of the conic sections let conic sections be described, according to the teaching of prop. 8 of the preceding book, in such a way that they pass through those points K, L, M, and so on, on the dial; and thus are the parallels of the signs described.

Diagram. The construction of the horizontal dial, in two parts.

Upper part (figure of the rays of the equator). A point D at the right is the apex from which the hour-lines fan out to the left. A vertical line runs through G (top) and down through D. Two horizontal lines run leftward to the level of H (top-left, a short mark) and C (left).

- H (top-left): end of the upper horizontal line
- G (top-right): top of the vertical line GD
- C (left): end of the middle horizontal line (equinoctial line CD)

- D (right): apex where the hour-lines converge
- Hour-numerals along the converging lines (near center): 4, 3, 2, 1, 12
- Hour-numerals (lower): 9, 10, 11, 12

Lower part (the dial face). A grid of hour-lines crossed by the curved, dotted zodiac parallels (sign-arcs). A central vertical meridian line carries, top to bottom, the points K, L, M, I, E, Q, P, N, B. Along the horizontal hour-line row stand the Roman hour-numerals (left to right): IIII, III, II, I, XII, XI, X, IX, VIII. The equinoctial line runs across, marked C (left) and D (right) with M at the center. The dotted zodiac parallels are labelled with their glyphs at both ends:

- ☊ Cancer (top arc)
- ♊ Gemini (left) / ♌ Leo (right)
- ♉ Taurus (left) / ♍ Virgo (right)
- ♈ Aries (left) / ♎ Libra (right) — the equinoctial arc
- ♐ Pisces (left) / ♏ Scorpio (right)
- ♒ Aquarius (left) / ♐ Sagittarius (right)
- ♑ Capricorn (bottom, at both ends)

As regards the latter method, drawn from the figure of the Zodiac's rays, we shall accomplish the matter thus. Onto the radius of the equator DC let all the intervals lying between the center B (from which the circle was described and divided into 24 equal parts) and the hours on the equinoctial line CD be transferred from point D, marking off points on the radius of the equator itself. Next, on the axis DG let there be cut off a straight line DG equal to the portion of the axis FG in the dial described at the end of the scholium of the preceding proposition; and through G let GH be drawn parallel to the radius of the equator DC. Onto GH, in the same way, from point G, let the spaces contained between the other center L (from which, namely, the other circle in the aforesaid dial was described and divided into 24 equal parts) and the hours on the equinoctial line IK be transferred, marking off points on GH itself. Finally, through each pair of corresponding points on the straight lines DC and GH let a straight line be drawn. For these will give the hour-lines, exactly as in the figure of the Zodiac's rays of this proposition; and the intervals of these —

— between the radius of the equator DC and the rays of the other signs, transferred onto the corresponding hour-lines in the dial from the equinoctial line DC itself, will give on the hour-lines the points through which the arcs of the signs are to be drawn; as is clear from what was demonstrated above. For the straight line GH is another radius of the equator, for describing a smaller dial,

whose equinoctial line is of course IK and whose axis is AG. Hence, if on the axis DG of the figure next to this one there were taken from D a straight line equal to the axis FA in the dial, or from G one equal to the axis GA, both would terminate in the same point; and the straight lines drawn from that point through the points of the radius of the equator DC and of GH alike would exhibit the hour-lines, exactly as in the figure of the Zodiac's rays of this proposition. These hour-lines must therefore necessarily pass through the points found on each radius of the equator, and so on. But if from D a perpendicular line be drawn to the nearest hour-line — namely to the line of hour 12 — this will give the length of the style; and its position will be a point on the meridian line, distant upward from the equinoctial line by as much as the point onto which the style from D falls at right angles on the line of hour 12 is removed from the radius of the equator DC. Moreover, in transferring the intervals from the radius of the equator onto the dial, this order must be kept: that the intervals of the hour-lines toward that side on which the style from D falls be transferred upward from the equinoctial line, and so on.

Diagram. The same horizontal-dial construction as on the preceding page, printed again. *Upper part:* the ray-figure of the equator, with D (right) as the apex of the fan of hour-lines; a vertical line through G (top) and D; horizontal lines to H (top-left) and C (left); hour-numerals near the center 4, 3, 2, 1, 12 and lower down 9, 10, 11, 12. *Lower part:* the dial face, a grid of hour-lines crossed by the dotted zodiac parallels; the central meridian carries top-to-bottom the points K, L, M, I, E, Q, P, N, B; the Roman hour-numerals run IIII, III, II, I, XII, XI, X, IX, VIII; C (left) and D (right) mark the equinoctial line, M at center. The dotted zodiac parallels are labelled at both ends: ♋ Cancer (top); ♊ Gemini / ♌ Leo; ♉ Taurus / ♍ Virgo; ♈ Aries / ♎ Libra (equinoctial); ♐ Pisces / ♏ Scorpio; ♒ Aquarius / ♐ Sagittarius; ♑ Capricorn (bottom, both ends).

[Margin: The use of the sign-arcs described on the dial, by which we learn in what part of the Zodiac the Sun is.] Furthermore, the use of the parallels — or arcs of the Zodiac's signs — on all dials is that through them we may know in what sign of the Zodiac the Sun stands each day. For whenever the extremity of the shadow falls exactly on some arc described on the dial, and accordingly departs from it not at all throughout that whole day, the Sun will then necessarily be at the beginning of that sign of the Zodiac which the said arc represents. But when the shadow does not fall exactly on some arc, but into the space enclosed between two arcs, the Sun will then not be at the beginning of any sign, but either near the beginning, or in the middle, or near the end, or, finally, in some other part of it, in proportion to the distance of the shadow from the arc of that sign.

And in order that we may know which of the two signs assigned to any given arc (excepting the two tropical ones) is to be taken, we must observe in what part of the year we are. For since from the twelfth day of June to the twelfth day of December the Sun in this age runs through these six signs — ♋ Cancer, ♌ Leo, ♍ Virgo, ♎ Libra, ♏ Scorpio, and ♐ Sagittarius — while the other six, namely ♑ Capricorn, ♒ Aquarius, ♓ Pisces, ♈ Aries, ♉ Taurus, and ♊ Gemini, [it runs through] from the twelfth day of December to the twelfth day of June, anyone will easily be able to judge in which of the two signs the Sun stands. For example: if on some day between the twelfth of June and the twelfth of December the extremity of the shadow falls exactly on the arc to which these two signs, ♊ Gemini and ♌ Leo, are assigned, the Sun will be at the beginning of ♌ Leo, and not at the beginning of ♊ Gemini. But if the shadow falls into the space enclosed between the said arc and the other arc — of these two signs, ♉ Taurus and ♍ Virgo — the Sun will be in some degree of ♌ Leo, and not in any degree of ♊ Gemini, or ♍ Virgo, or ♉ Taurus, because —

— because at that time of year it cannot be in ♊ Gemini or ♉ Taurus, as we said, but only in ♌ Leo or ♍ Virgo. Since, then, the Sun moves from ♌ Leo into ♍ Virgo, and not the reverse, it follows that it is at that time in some degree of ♌ Leo, and not of ♍ Virgo. The opposite would have to be understood if we were in that part of the year which falls between the twelfth day of December and the twelfth day of June. For then, if the shadow fell on the arc of ♊ Gemini and ♌ Leo, the Sun would be at the beginning of ♊ Gemini; but if the shadow fell into the same space enclosed between the said arc and the arc of ♉ Taurus and ♍ Virgo, the Sun would be set in some degree of ♉ Taurus, because it advances by its own proper motion from ♉ Taurus into ♊ Gemini. And so for the rest.

PROBLEMA 3. PROPOS. 3.

To describe on the same horizontal dial the parallels of the diurnal arcs.

[Margin: The description of the diurnal arcs on the aforesaid horizontal dial.]

Since the parallels of the diurnal arcs are equidistant from the equator, just as the parallels of the signs of the Zodiac are, it is clear that the method of describing both is one and the same. Therefore, in the former mode of description handed down in the preceding proposition, in place of the parallels which are drawn through the signs of the Zodiac, take the parallels of the diurnal arcs, whose declinations were found in Proposition 33 of the preceding book. And in the latter mode, in place of the rays of the Zodiac issuing from point D, take the rays of the diurnal arcs — as is clear from these figures set alongside. In the former of them

are contained the parallels whose diurnal arcs contain the hours 8, 9, 10, 11, 12, 13, 14, 15, 16;

Diagram. A composite construction figure spanning the page width, in two parts (the two "figures set alongside" of the text).

Left part (the "former" figure — the parallels of the diurnal arcs). A circle representing the projection, with a small circle at its center marked **D** (the place of the style). Circumference letters: **I** and **A** at the top, **E** at the upper right, **B** at the left, **C** at the right (between the disk and the ray-figure), **K** at the lower left. Along the upper-left of the disk a column of hour-numerals descends: **16, 15, 14, 13, 12, 11, 10, 9, 8**; further small numerals near the lower part read **2, 3, 4** and **17, 18** [?]. Straight chords (the diurnal-arc parallels seen in projection) cross the disk diagonally.

Right part (the "latter" figure — the rays of the diurnal arcs). A horizontal top bar carries **H** (left) and **D** (right); **C** lies just beneath. From **D** a fan of straight rays radiates downward and to the right toward **A** at the far right. Along the lower ends of the rays run hour-numerals; reading right-to-left across the foot: **17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5**, and at the far right a short series **0, 1, 2, 3, 4, 5**. A few small numerals near the apex (**5, 8** and **2, 3, 4**) are only partly legible [?].

— but in the latter [figure] the rays of the parallels whose diurnal arcs contain the hours 0, 1, 2, 3, 4, 5, 6, and so on up to 24, where the whole parallel is a diurnal arc. We shall describe the parallels of the diurnal arcs by exactly the same method by which we described the parallels of the signs in the preceding proposition. These are indeed all conic sections, which the shadow of the style describes (by the corollary of Proposition 12 of the preceding book) when the Sun is in them. And the equinoctial line is the diurnal arc of 12 hours.

[a] Now the outermost lines in the ray-figure drawn from **H** are parallel to the outermost lines emitted from **D**, each to its own; because the angles **HDA** and **DHI** are equal to two right angles. [Margin: (a) Elements I.28.] For by construction the angle **HDI** is right, and **DHI** is the angle of the pole's altitude, while **ADI** — which, by the scholium of Proposition 33 of the preceding book, is the angle of the complement of the pole's altitude — together also equal one right angle, etc. And so we have described the parallels of the diurnal arcs on the same horizontal dial. Which was to be done.

SCHOLIVM.

Here too there may be added to the parallels of the diurnal arcs the lengths of the days, and of the twilights, and other things of this kind — such as the days of the months corresponding to those parallels, and so on. [Margin: What is to be assigned to the diurnal arcs described on the dial.]

Now the use of these parallels is that through them we may learn how many hours each day of the year contains. For when the shadow of the gnomon falls exactly on some diurnal arc, the day then contains exactly as many hours as are assigned to that arc. But if the shadow of the style does not fall exactly on some arc, but into the space enclosed between two arcs, we shall judge more or less how many hours the day then has, taking account of the shadow's distance from the nearer arc — provided we recall that the days decrease from the twelfth of June to the twelfth of December in this age, but increase from the twelfth of December to the twelfth of June. So that if, for example, the shadow falls into the middle of the space between the arcs of hours 13 and 14: if indeed this happens in that part of the year which falls between the twelfth day of June and the twelfth day of December, the day will then contain nearly $13\frac{1}{2}$ hours, because the Sun is then moving by its proper motion from the diurnal arc of 14 hours to the diurnal arc of 13 hours, since the days are then decreasing. But if the same thing happens in the rest of the year, the day will again be about $13\frac{1}{2}$ hours, because the days are then taking on increase, and so the Sun by its proper motion is moving from the diurnal arc of 13 hours to the diurnal arc of 14 hours. And so for each case. [Margin: The use of the diurnal arcs described on the dial, by which we know how many hours any given day comprises.]

PROBLEMA 4. PROPOS. 4.

To describe on the same horizontal dial the vertical circles, which they call Azimuth.

Through G, the place of the gnomon on the dial, let the straight line AB be drawn, perpendicular to the meridian line HI.

[Margin: The description of the vertical circles on the same horizontal dial.]

Then from G let any circle described be divided into 360 equal parts, beginning from the straight line AB. (We, because of the narrowness of the space, have divided it only into 24, so that each contains 15 degrees.) For if from the center G, through the points of division, straight lines are drawn — kept, however, within the tropics, as appears in the figure, since the lines beyond them would be

superfluous — then there will be described the vertical circles, or azimuths, that is, the common sections of the vertical circles and of the plane of the dial. Onto these, by Proposition 11 of the preceding book, the shadow of the gnomon is projected, when the Sun is in the said circles. This will easily be demonstrated in the following way.

Diagram. A wide, bow-tie ("butterfly") azimuth figure symmetric about a central vertical (the meridian). At the top center is the gnomon, drawn as a small downward-pointing triangle marked **H**; the place of the gnomon (**G** of the text) lies at the center where the radial lines converge. A horizontal line runs the full width, marked **A** at the left and **B** at the right (the common section AB of the dial-plane and the proper vertical circle); the meridian runs vertically through the center to **C** at the bottom. A dotted circle (the circle divided from G) surrounds the center, and dotted tropic-lines run out to the four corners. From the center a fan of azimuth radii spreads downward; along their graduating arc the degree-numerals read symmetrically about the meridian: **15, 30, 45, 60, 75, 90** (with **90** at the meridian, i.e. due south), mirrored on each side. Outer markers at the wide ends carry numerals **15** (mid-arm, both sides) and **30** (bottom corners, both sides); further numerals near the upper ends (read as **13** and **15/19**) are only partly legible [?]. The four corners bear worn zodiac tropic-signs (Cancer/Capricorn family), which will not resolve on the scan and are flagged [?]. Small cross-marks (✠) ornament each end of the line AB as cardinal markers, with a small emblem drawn at the right. (Several peripheral labels remain [?].)

[Margin: Demonstration of the description of the vertical circles.] [Margin: (a) Elements XI.19 (undecimi).]

Since both the plane of the vertical circle properly so called, which passes through G (for the gnomon of the horizontal dial is part of the common section of the meridian and of the vertical circle properly so called, as is evident from the Analemma), and the plane of the horizontal dial, are at right angles to the meridian, [a] their common section too will be at right angles to that same meridian; and so, by Definition 3 of Book 11 of Euclid, perpendicular to the meridian line at the point G. Therefore the straight line AB, which is drawn through G perpendicular to the meridian line, is the common section of the plane of the dial and of the vertical circle properly so called.

And since all the vertical circles cut the horizon into 360 equal parts — and therefore also the circle which the plane of the dial, being equidistant from the horizon, makes on the sphere (by Proposition 1 of Book 1 of Theodosius) — because, passing through the zenith, or pole of the horizon, they divide the

horizon and the circles parallel to the horizon into similar segments (by Proposition 10 of Book 2 of Theodosius): it comes about that all the vertical circles, and therefore also their common sections with the plane of the dial, pass through the points by which the said circle, made on the sphere by the plane of the dial, is divided into 360 equal parts. But the same sections are also drawn through the point G (by Proposition 18 of the preceding book), at which of course the common section of the vertical circles meets the plane of the dial. Therefore the same sections will also pass through the points of the divisions of the circle described from G.

For since G — the point on which the axis of the horizon falls, and the center of the circle made on the sphere by the plane of the dial (by Proposition 10 of Book 1 of Theodosius) — is the center of the said circle produced on the sphere by the dial's plane, it follows that this circle and the one we described from G are divided into similar arcs by the straight lines going out from the center G, as we demonstrated in the commentaries on the Sphere at the end of Chapter 1. And accordingly, since the former is cut into equal parts, the latter also, which we described from G, will be cut into equal parts. Therefore those straight lines emitted from G through the points by which the circle described from G is divided into equal parts are the common sections of the plane of the dial and of the vertical circles. And so we have described the vertical circles, etc., on the same horizontal dial. Which was to be done.

[Margin: The use of the vertical circles described on the dial, by which we know in which quadrant of the upper hemisphere the Sun is. What the four quadrants of the upper hemisphere are on the horizontal dial. In what way it is known from the vertical circles how great is the arc of the horizon between the vertical circle properly so called and the vertical circle which is drawn through the Sun — which arc is usually called the vertical distance.]

SCHOLIVM.

From the vertical circles we learn, at any moment of time, in which of those four parts — into which the upper hemisphere is divided by the proper vertical circle and by the meridian — the Sun is. For the plane of the dial is divided into the same four parts by the straight line AB (which is the common section of the dial's plane and of the vertical circle properly so called) and by the meridian line (or common section of the same dial-plane and of the meridian): so that the part contained between the straight lines GB and GC is called the northwestern quadrant [the western and northern quarter]; the part comprised between the straight lines GB and GH, the southwestern quadrant [western and southern]; the

part which the straight lines GA and GC contain, the northeastern quadrant [eastern and northern]; and finally the part which the straight lines GA and GH embrace, the southeastern quadrant [eastern and southern].

So, since the Sun always projects the shadow of the style into the part opposite to that in which it stands, we shall easily judge from the shadow in which quadrant of the hemisphere it is. For if the shadow of the style falls into the northwestern quadrant, which we said is contained between the lines GB and GC, we shall say that the Sun is then in the opposite quadrant, namely in the southeastern one; and so for the rest.

Moreover, from the same vertical circles we know how great the Sun's vertical distance is — that is, by how much that vertical circle, in which the Sun stands at any given moment, departs on the horizon from the vertical circle properly so called, or from the point of true rising or setting. For if, for example, the Sun has been found (by what we have just delivered) to be in the southeastern quadrant, and the shadow at that time falls on the vertical line to which this number, 30, is assigned, then the vertical circle in which the Sun then is will be distant from the point of true rising, toward the south, 30 degrees. That is, the arc of the horizon interposed between the vertical circle properly so called and the vertical circle which then passes through the Sun's center is eastern and southern, and comprises 30 degrees. And so for the rest.

PROBLEMA 5. PROPOS. 5.

To describe on the same horizontal dial the parallels of the horizon — that is, the lesser circles of the altitudes of the stars, which they call almucantars [Almucantarath].

[Margin: The description of the parallels of the horizon, which are called almucantars, on the same horizontal dial.]

Let the quadrant of the circle ABC [printed *quandrans*, recte *quadrans*] be described, which is to be distributed into 90 equal parts, beginning from the semidiameter AB. (We have divided the same into only 6 parts, because of the narrowness of the space, so that each embraces 15 degrees.) Then, taking AD as the length of the style on the semidiameter AC, let DE be drawn through D parallel to the other semidiameter AB. Finally, with straight lines drawn from the center A through the points of division, if on the dial — from G, the place of the gnomon, as center — circles are described at the intervals comprised between D and the straight lines emitted from A (as in the figure of the preceding

proposition), circles which however do not exceed the tropics of the dial, then there will be described the parallels of the horizon, or circles of altitude. These are indeed all conic sections, onto which (by the corollary of Proposition 12 of the preceding book) the shadow of the style falls, when the Sun is in the said circles of altitude. This we shall confirm in the following way.

Let the whole circle GHCB be completed, representing the meridian, and let the semidiameters BA and AC be produced, together with the straight line ED, to the points H, G, L. Then, HB being posited as the common section of the horizon and the meridian, GC will be —

Diagram. A circle representing the meridian, with center **A** and many radial lines drawn from it. Circumference letters, running round: **G** at the top; **I** at the upper right; **B** at the right on the horizontal diameter; **M** just below B at the right; **E** on the horizontal, right of center; **K** at the upper left; **H** at the left on the horizontal diameter; **L** on the horizontal, left of center; **C** at the bottom; **D** just below the center; **F** below the center, right of D. Along the upper-left arc the graduation-numerals descend from the top: **90** (at G), **75**, **60**, **45**, **30**, **15** (at K). Down the lower-right arc the graduation reads **15** (at M), **30**, **45**, and two further worn values [?]; near the foot, flanking **C**, appear **30** (left) and **75** (right) [the right-lower **50** and **75** readings are uncertain — [?]]. Small division-dots mark the circumference at each 15°. The triangles AKI and ALE named in the text are formed by the outer radii and the center.

[Margin: Demonstration of the description of the parallels of the horizon.]

[Margin: (a) Elements I.15. (b) Elements III.26.]

— GC being the common section of the same meridian and of the vertical circle, and LE the common section of the same meridian and of the plane of the dial. Now since, if the straight lines drawn from A through the points of the divisions of the quadrant BC are produced through the center, [a] the vertical angles at A become equal, [b] the arcs in the quadrant HG will be equal to the arcs in the quadrant BC. Therefore the straight lines drawn through the points of the quadrant HG and equidistant from HB itself are the common sections of the meridian and of the circles of altitude, or almucantars. From the outermost points of these, the straight lines drawn through the center A will form triangles through the axis of the cones, whose bases are the parallels of the horizon — of which kind are the triangles AKI and ALE.

And since the plane of the dial, drawn through the straight line LE parallel to the horizon (and therefore also to the bases of the cones), makes — by Proposition 4

of the preceding book — the common sections of those conical surfaces to be circles having their centers on the axis of the cones, then D, the place of the style, will be the center of those circles, and the straight lines DE, DF, etc., which the sides of the triangles cut off through the axis, will be the semidiameters of the same, as is clear from the said Proposition 4 of the preceding book. And accordingly, if from G, the place of the style on the dial, circles are described at the intervals comprised between D and the straight lines going out from A, these will be the circles of altitude. Therefore we have described the parallels of the horizon, etc., on the same horizontal dial. Which was to be done.

SCHOLIVM.

The parallels of the horizon lead us to knowledge of the Sun's altitude at any time, when we observe the shadow of the style on the dial. For whenever the shadow of the gnomon falls, e.g., on the parallel to which this number, 30, is set, the Sun's altitude — that is, the arc of that vertical circle which at that time is drawn through the Sun's center, interposed between the Sun's center (or the parallel of the horizon which then likewise passes through the Sun's center) and the horizon — contains 30 degrees, etc. [Margin: The use of the parallels of the horizon described on the dial is that from them we may know the Sun's altitude.]

PROBLEMA 6. PROPOS. 6.

To describe on the same horizontal dial the meridians, or circles of the longitudes of cities.

[Margin: The description of the meridians on the same horizontal dial.]

Since the meridian circles pass through the poles of the world, just as the hour circles do (which show the hours from noon or from midnight), the description of the meridians will not differ from the description of the hour circles reckoned from noon or midnight. Therefore let the circle described from E in Proposition 1 of this book be cut into 360 equal parts, or into fewer — as into 24, so that each embraces 15 degrees, as we have done — the beginning being taken not from the meridian line, but from that point through which passes the meridian drawn through the Fortunate Isles [the Canaries], from which, namely, the longitudes of places, and therefore the meridian circles, take their beginning: of which kind is the point A. This point is obtained by numbering off from the meridian line the longitude of the place in which the dial is described, from the point B toward the western parts — that is, proceeding to the right. For this part inclines toward the west, as is evident, if the dial is placed in its proper position and the circle

described from E is understood to be established in the plane of the equator. We have numbered from the point B up to A 36 degrees, namely the longitude of the City of Rome. Let the rest be done as [was described] for the hour-lines —

to be described from noon or from midnight, as was said in Proposition 1 of this book; and the meridian circles will be drawn — namely, their common sections with the plane of the dial, onto which the shadow of the style is cast (by Proposition 11 of the preceding book) when the Sun occupies the said meridians. Therefore we have described the meridians, or the circles of longitude, etc. Which was to be done.

Diagram. A large circle (the divided circle described from E, understood as lying in the plane of the equator) with a system of straight meridian lines drawn as diameters through its center and prolonged well beyond the circumference to left and right, plus two dashed conical figures (the projections of the equator and of the meridian through the Fortunate Isles). The vertical meridian line runs down the middle to the foot of the figure. Each straight meridian line is labelled at both of its ends with two longitude values 180° apart (successive lines spaced 15° apart), and the circumference carries the same longitude degree-marks.

Readable labels (with position):

- H — on the central vertical line, upper (near the style/pole region)
- D — on the central vertical line, just below H
- I — on the central vertical line, below D
- C — upper-left on the circumference, at the 180 mark
- B — foot of the meridian line, bottom center
- A — bottom, just to the right of B (the point from which the Fortunate-Isles meridian and longitude-count begin)
- M — lower-left region (end of a meridian line)
- N — lower-right region (end of a meridian line)
- K, L, T — inner lower-center points on the meridian/equinoctial construction
- "ROMA" (Rome) — inside the circle, lower center
- "INS. FORTVN." (Insulae Fortunatae — the Fortunate/Canary Isles) — inside the circle, lower center, on a dashed line
- "Aequinoctialis" — along a line running down-right from the center (the equinoctial line)
- Circumference / meridian-end degree labels read: 180 (at C), 195, 210, 240, 270 across the top; 300, 315, 330 down the left; 135, 105 down the right,

with one worn number at the right-middle end of the horizontal line reading like "170" [?] (this meridian is the one opposite 300, i.e. longitude 120); 345 at the lower left; and the inner arc at the bottom marked 15, 30, 45, 60, 75, 90 (with 90 repeated on each side toward the foot).

SCHOLIVM.

[Margin: Note what is said here.]

For if the longitude of the place is the arc BAC — say, for example, 216 degrees — so that the division of the circle described from E takes its beginning at point C, and point A is therefore distant from it by a semicircle, that is, by 180 degrees, then KL will be the straight line which in our example gives the beginning of the longitudes, namely longitude 180 degrees: that is, the common section of the plane of the dial and of the semicircle of the same meridian, the one that is opposite to the semicircle passing through the Fortunate Isles. For the Sun, when it is on this line, casts the shadow of the style through the center of the world onto the straight line KL, etc., since the semicircle of the equator MBN is above the horizon while the rest is below it; and so, when the Sun is on the equator at point A (which then marks the meridian semicircle of 180 degrees, the one opposite the semicircle of the Fortunate Isles passing through C), the ray AE falls on the plane of the dial at point L, etc. These matters are clearer than daylight from what we wrote in the scholium to Proposition 1 of this book — namely, that any line on the dial which cuts the equinoctial line represents that celestial semicircle whose common section, on the circle described from E through the center E, passes before it meets the equinoctial line, of which kind is the section AE, etc.

By the same reasoning, the line which in our same example gives longitude 15 degrees will be longitude 195 degrees, etc.; for the numbering of the degrees on the circle will then be taken from point C and will proceed toward N, just as before it began from point A and proceeded through B, C, and N — that is, through the eastern parts, as the reasoning requires.

[Margin: By means of the meridians drawn on the dial we learn for which peoples it is noon at any given time, and what hour it is in any place whose longitude is known.]

Moreover, the use of the meridian circles is most elegant. For by them we learn, quicker than can be told, for which peoples it is noon at any moment of time, and what hour it is in any city whose longitude is not unknown. For when the shadow of the gnomon falls precisely on some meridian line — since all the lines going out from H are the various meridian lines of various cities, being, as has been

shown, the common sections of the plane of the dial and of the meridian circles — it will be noon in all cities that have the longitude which the number attached to that meridian line indicates. For example: when the gnomon's shadow at any time of the year falls on the meridian line to which this number, 60, is attached, it becomes noon at Alexandria in Egypt, because the longitude of this city, as is established from Ptolemy's tables, is 60 degrees. And the same will happen in every other place or city whose longitude comprises the same number of degrees.

But suppose I want to know what hour it then is anywhere among the nations — for instance, in the Fortunate Isles. I subtract the longitude of the Fortunate Isles, which is 0 degrees (for from there the beginning of all longitudes is taken), from the longitude of that meridian on whose line the shadow then falls, namely from 60 degrees, and 60 degrees remain. These degrees I convert into hours, assigning to every fifteen degrees a single hour and to each single degree four minutes of an hour — or else, much more quickly, by means of the table which we set down in our commentary on the *Sphere* when we were treating of the equator; and I obtain 4 hours. And since the Sun has not yet then reached the meridian of the Fortunate Isles — because their meridian line on the dial is further to the east than the meridian line of 60 degrees — the Sun will therefore be 4 hours before the noon of those isles; and accordingly it will then be the eighth hour after midnight there.

Similarly, if one wishes to know what hour it then is in a region whose longitude comprises 300 degrees: I take 300 degrees from 60 degrees, having first, in the manner of astronomers, added a whole circle of 360 degrees, so that they become 420 degrees, and 120 degrees remain, which make 8 hours. By that many hours, then, is the Sun distant before the noon of that place, and therefore those peoples will then have the fourth hour after midnight. Again, I wish to know the same in that place where the longitude is 110 degrees, such as is roughly Calicut in eastern India. I take 110 degrees from 60 degrees — a whole circle having first been added — and 310 degrees remain, which comprise 20 hours 40 minutes; and by that many hours it falls short of noon in that place. And since from noon to noon 24 hours flow by, it must be that 3 hours 20 minutes have then elapsed since the last preceding noon. By the same reasoning, if the hour is sought in that place where the longitude contains 240 degrees: subtracting 240 degrees from 60 degrees (a whole circle of 360 degrees having first been added, so that they become 420), 180 degrees remain, which contain 12 hours. I shall therefore say that the Sun is distant from the noon of the said place by 12 hours, and accordingly that it is then midnight there. Finally, if the same is desired in a region of longitude 10 degrees, such as Toledo in Spain roughly has:

I take 10 degrees from 60 degrees, and 50 degrees remain, which give 3 hours 20 minutes. By that many hours, then, the Sun is distant before the noon of Toledo. Hence at that time the inhabitants of the said city will have 8 hours 40 minutes after midnight. The same judgment is to be made about the rest.

We shall also obtain the same result, and perhaps more conveniently, in this way. If the longitude of the meridian on whose line the shadow falls at the time of observation is less than the longitude of that place for which the hour is sought, let the lesser longitude be subtracted from the greater. For the remaining degrees, converted into hours, will give the time elapsed since the noon of the said place. Hence if the hours turn out to be fewer than 12, they will be reckoned as hours after noon; but if they are exactly 12, it will be exactly 12 hours after noon, that is, the time of midnight; and finally, if they are more than 12, then, when 12 have been taken from them, the hours remaining will be hours after midnight. But if the longitude of the meridian on whose line the shadow falls is greater than the longitude of the other proposed place, then, when the lesser longitude is subtracted from the greater, the degrees left over — reduced to hours — will give the time before the noon of the said place. Hence if they are fewer than 12, then, subtracting them from 12, the hours remaining will be after midnight; but if they are exactly 12, it will be altogether 12 hours before noon, that is, the time of midnight; and finally, if they are more than 12, then, subtracting them from 24, the hours remaining will be hours elapsed since noon.

Example. Let the tip of the style's shadow again, at any time of the year, fall on the meridian line of 60 degrees — which is indeed the longitude of the city of Alexandria in Egypt — and suppose it is worth knowing what hour it then is in that place whose longitude comprises 300 degrees. I subtract the lesser longitude from the greater, namely 60 from 300, and the remaining 240 degrees I reduce to 16 hours, which have elapsed since the noon of that place. Hence the fourth hour after midnight is then at hand. Again, let the same be investigated in a city of longitude 240 degrees. When 60 degrees are subtracted from 240 degrees, 180 degrees remain, which make 12 hours elapsed since noon. It is therefore then exactly the time of midnight. Further, suppose someone wishes to know the same in that place which lies 110 degrees distant from the Fortunate Isles toward the east — such as Calicut, the trading center of eastern India. When 60 degrees are subtracted from 110 degrees, the remaining degrees become 50, which show 3 hours 20 minutes passed after noon. Finally, if we should choose to learn the same in a place having longitude 10 degrees, we shall subtract 10 degrees from 60 degrees. For the remaining 50 degrees will give 3 hours 20 minutes before the noon of that place — that is, 8 hours 40 minutes after midnight, etc.

PROBLEMA 7. PROPOS. 7.

To describe on the same horizontal dial the parallels of cities, that is, the circles of latitude.

[Margin: The description of the parallels of cities, or of the circles of latitude, on the same dial.]

The description of these does not differ from the description of the parallels of the Zodiac which we handed down in Proposition 2 of this book, since both these and those are equidistant from the equator — provided only that, in place of the declinations of the signs which we there employed, we here assume the declinations of the parallels drawn through the zeniths of cities, namely the latitudes of the cities themselves. Indeed, the parallels of the signs described above are themselves also circles of latitude of cities: for instance, the tropic of Cancer [tropicus ☌] is the circle of latitude of those cities which lie 23 degrees 30 minutes to the north of the equator, such as Syene, a city of Egypt, etc. Therefore we have described the parallels of cities, etc. Which was to be done.

SCHOLIVM.

[Margin: The parallels of cities, or circles of latitude, drawn on the dial indicate which peoples have the Sun at the zenith of their head at the time of noon.]

The circles of latitude, or parallels of cities, bring us this usefulness: that by their help we may understand which peoples have the Sun at the zenith of their head at the time of noon. For when the shadow of the gnomon falls on some one of these parallels, on that day the Sun will pass through the zenith of all places distant from the equator by as many degrees toward the same side — whether north or south (according as that parallel has been taken on the dial among the northern or the southern parallels) — as are comprised in the declination of that parallel. For example, if the shadow of the style falls on the parallel whose declination comprises 20 degrees, the Sun will pass through the zenith of every city whose latitude is 20 degrees, whether toward the north or toward the south, according as that parallel likewise inclines to the north or the south on the dial. So too, when the shadow of the style runs along the parallel of Cancer [☌], whose declination is northern and is 23 degrees 30 minutes, the Sun at noon will reach the zenith, or vertex, of Syene, a city of Egypt, because its latitude too is northern and comprises 23 degrees 30 minutes. And so on for the rest.

PROBLEMA 8. PROPOS. 8.

To describe on the same horizontal dial the celestial houses — that is, the common sections of the plane of the dial and of the circles of position, which are drawn through the beginnings, or through any parts whatever, of the celestial houses.

[Margin: The description of the celestial houses on the same horizontal dial, according to Johannes Regiomontanus.]

According to the teaching of Johannes Regiomontanus — who teaches that the circles of the celestial houses (which are indeed among the number of the circles that they call "of position") cut the equator into 12 equal parts and pass through the common sections of the horizon and the meridian — we shall describe the celestial houses on the horizontal dial in this way. Let the circle already so often described from E be divided into 360 equal parts, the beginning being made from the meridian line. We have divided it into 12 parts, so that each consists of 30 degrees and constitutes one house. Then through the center, and through the division-points, let straight lines be drawn cutting the equinoctial line at points, through which, if straight lines are drawn equidistant from the meridian line itself — which is easily done if the straight line CD is drawn parallel to the equinoctial line, cutting the meridian line at B, and all the points of the equinoctial line, taken off with the compass from I, are transferred onto the line CD from point B; for in this way the straight lines joining each pair of points equally distant from I and B will be parallel [Margin: a. By Proposition 33 of the First book (of Euclid).] — the celestial houses will be described according to Johannes Regiomontanus. This can be shown as follows.

[Margin: Demonstration of the description of the celestial houses according to Johannes Regiomontanus.]

Since, according to Johannes Regiomontanus, the circles of the celestial houses pass through the common sections of the meridian and the horizon, and divide the equator into equal parts — the beginning being made from the meridian circle, which is the beginning of the tenth celestial house, which the eleventh and twelfth houses follow toward the east; while the first house begins from the horizon itself

on the eastern side, and lies wholly below the horizon; and the remaining houses in turn are placed in the same order below the horizon up to the beginning of the seventh house, each house being assigned 30 degrees — it comes about that the

straight lines drawn through the division-points and the center of the circle described from E are the common sections of the equator and of the circles of the celestial houses. For if the said circle described from E is understood to be revolved about the equinoctial line until it is set in the plane of the equinoctial circle, and so its center falls at the center of the world, then the plane of the said circle of the celestial houses will be drawn through those division-points and the center of the world E. Therefore the circles of the celestial houses will meet the equinoctial line of the dial at the points where the aforesaid straight lines cut it; and so through these same points of the equinoctial line will pass the common sections of the said circles and of the plane of the dial. Since, then, by Proposition 18 of the preceding book, these common sections are parallel to one another — indeed, are equidistant from the common section of all the circles of the celestial houses, as is evident from the demonstration of the said Proposition 18 of the preceding book —

Diagram. A horizontal-dial construction for the celestial houses, laid out symmetrically about a central vertical meridian line. A long horizontal line KL runs across the figure (K at the left end, L at the right end), crossed at its center by the vertical meridian line. Below the horizontal line stand two concentric/overlapping circles about the lower center (the house-circle described from F and the equinoctial circle from E), and two dashed conical figures open outward to the upper-left/lower-right and upper-right/lower-left corners (the circles of position). The house-boundary hour-lines are labelled with Roman numerals; the base line at the very bottom (CD) carries points C, B, D.

Readable labels (with position):

- F — upper center, apex of the house circle
- G — just below F (the place of the style on the meridian line)
- K — left end of the horizontal line; L — right end
- M, N, I, P, O — the central row of points along the horizontal/equinoctial construction, left to right
- E — center of the lower circle; A — foot of the lower circle, bottom center
- B — bottom center, on the base line; C — bottom-left end of the base line; D — bottom-right end
- Roman numerals for house/hour boundaries: VIII and XII flanking F in the upper zone; IX, X, XI within the lower circle (X at its foot near A); VII and XII in the lower zone
- Degree marks 10 and 20 appear at the upper-left and upper-right (near the tops of the cones) and again at the lower-left and lower-right

- Four small worn symbols sit at the four cone-ends (upper-left, upper-right, lower-left, lower-right) — apparently astronomical/zodiac sorts marking the circles of position [?]; a short worn cluster near G reads like "XXX" [?]

[Margin: a. From (Euclid) I.36 — whence.] [Margin: b. From (Euclid) XI.9 — whence.]

(because the plane of the dial is equidistant from the common section of the said circles); and, moreover, the meridian line is parallel to that common section of the circles of the celestial houses [a], since the meridian line and that section are the sections of parallel planes — namely of the horizon and of the plane of the dial — made by the meridian. Therefore the common sections of the circles of the celestial houses and of the plane of the dial will likewise be equidistant from the meridian line itself [b]. And so those straight lines which have been drawn through the points of the equinoctial line equidistant from the meridian line will be the common sections of the circles of the celestial houses and of the plane of the dial — onto which, that is, the shadow of the style falls (by the corollary of Proposition 12 of the preceding book) when the Sun is in the aforesaid circles of the celestial houses.

[Margin: The description of the celestial houses on the same horizontal dial, according to Campanus.]

But according to the opinion of Campanus — who prescribes that the circles of the celestial houses ought to be drawn through the common sections of the meridian and horizon in such a way that they divide the vertical circle, and not the equator (as Johannes Regiomontanus would have it), into 12 equal parts — we shall lay out the celestial houses in this way. On the meridian line, from G, the place of the style, let there be taken — whether upward or downward — a straight line equal to the style, up to point F; and from F let a circle of any magnitude be described and cut into 12 equal parts, for the twelve celestial houses, the beginning being made from the meridian line. Then through the center F and through the division-points let straight lines be drawn cutting the straight line KL (which is drawn for this sole purpose through the place of the style, perpendicular to the meridian line — or, what is the same, parallel to the equinoctial line — so that it can be cut by the said straight lines) at points through which, if straight lines are drawn equidistant from the meridian line itself, the circles of the celestial houses will be described according to the teaching of Campanus. Take this as the demonstration of the matter.

[Margin: Demonstration of the description of the celestial houses according to Campanus.]

If the said circle described from F is understood to be moved about the straight line KL until it lies in the plane of the vertical circle — that is, is at right angles to the plane of the dial, and its center is the same as the center of the world, or the vertex of the gnomon — then the straight lines drawn out through the center F and the division-points will be the common sections of the circles of the celestial houses and of the vertical circle, and therefore of any other circle whatever that is described in its plane from the center of the world, such as is the said circle described from F. For this circle and the vertical circle properly so called are cut into similar parts by the straight lines going out from the center F, as we demonstrated at the end of Chapter 1 of the *Sphere*. For this reason the circles of the celestial houses will meet the line KL at the points where the aforesaid straight lines cut it; and accordingly through these same points of the line KL will pass the common sections of the said circles and of the plane of the dial. Since, then, these sections are parallel to one another, and to the common section of the same circles, and so also to the meridian line itself — as was shown in the previous construction — those straight lines which have been drawn through the said points of the line KL equidistant from the meridian line will be the common sections of the circles of the celestial houses and of the plane of the dial. Therefore we have described the celestial houses — that is, the common sections, etc. — on the same horizontal dial. Which was to be done.

COROLLARIVM.

[Margin: Through which hours a celestial house passes on the equinoctial line, according to Johannes Regiomontanus.]

Moreover, it is evident that the beginnings of the celestial houses, according to Johannes Regiomontanus, pass through those points of the equinoctial line on the dial through which are drawn the hour-lines distant from the meridian line by two and by four hours — such as are the second and fourth hours after noon, passing through the points N and M; likewise the 8th and 10th after midnight, passing through O and P — because such hours on the equator are distant from the meridian circle by 30 and 60 degrees.

just as also the beginnings of the celestial houses. Hence, if through the said hours lines are drawn on the equinoctial line parallel to the meridian line itself, these will give the beginnings of the celestial houses, according to the teaching of

Johannes Regiomontanus [Regiomontanus], and so forth. And this indeed must be understood not only in the horizontal dial, but also in all others.

SCHOLIVM.

[Margin: The lines of the celestial houses indicate the beginnings of the celestial houses, but the hour-lines signify the ends, or limits, of the hours.]

But the lines of the celestial houses do not indicate the celestial houses in the same way that the hour-lines indicate the hours. For the hour-lines signify the ends of the hours: when the gnomon's shadow falls upon some hour-line, it warns us that the hour to which that line belongs has passed, and that the next one is beginning. The house-lines, by contrast, show us the beginnings of the houses, not their ends: when the style's shadow falls upon one of the said lines, it shows that the Sun is in that circle of position which is the beginning of the celestial house indicated by the number written against it. The reason for this is that the Sun reaches the end of any house before it reaches the beginning. For the celestial houses are numbered from west to east, whereas the Sun is carried from east to west by the motion of the *primum mobile* [*primum mobile*]. Hence the Sun first reaches the end of each house, then its middle, and last of all its beginning. With the hours it is otherwise, because the hour-circles too are reckoned from east to west. Therefore, just as the style's shadow falling on some hour-line teaches that that hour has passed, and that the Sun has moved from the beginning of that hour to its end, so the same shadow, reaching some celestial house, shows that the Sun has already been carried through that whole house — namely from its end to its beginning. In our example, since neither the beginning of the 7th house nor the end of the 12th can be drawn on the horizontal dial, we have marked their thirds — that is, we have drawn lines passing through their 10th and 20th degrees.

[Margin: From the lines of the celestial houses drawn on the dial, we perceive in which house the Sun is.]

The purpose of the house-lines on the dial is that from the gnomon's shadow we may grasp in which celestial house the Sun is lingering. For when, say, the style's shadow falls on the line marked with the ninth house, we shall be certain that the Sun is then at the beginning of the ninth house and at the end of the eighth; so that throughout the whole time in which the style's shadow has run from west to east across the space lying between the line of the ninth house and the line of the tenth, the Sun has been staying in the ninth house, having moved from its end to its beginning. And so with the rest.

We have described the celestial houses only according to Johannes Regiomontanus and Campanus [Campanus of Novara], and not according to others, because the use of the celestial houses as these authors establish them in the heavens is found more frequently among astronomers.

PROBLEMA 9. PROPOS. 9.

To describe the ascending signs of the zodiac on the same horizontal dial.

[Margin: What must be sought before the ascending signs are described.]

When the Sun is at the beginning of Aries, let it be asked at what hour before or after noon the beginnings of the twelve signs rise, for the proposed latitude of the place, starting from the beginning of Aries. Then, with the Sun at the beginning of Libra, let the same be investigated for the beginnings of the twelve signs, starting from the beginning of Libra. Again, with the Sun remaining at the beginning of Cancer, let the same be examined for the beginnings of the same twelve signs, starting from the first point of Cancer. Finally, with the Sun staying at the beginning of Capricorn, let the same be inquired for the beginnings of the twelve signs, starting from the first point of Capricorn.

[Margin: By what method the hour is to be investigated at which any sign of the zodiac begins to ascend above the horizon.]

This hour will easily be found from the oblique ascension [ascensio obliqua] of that arc of the ecliptic which is intercepted between the beginning of Aries, or Libra, or Cancer, or Capricorn, and the proposed ascending sign. For if such an oblique ascension is reduced to hours, and (when the oblique ascension is less than the semidiurnal arc [arcus semidiurnus]) these hours are subtracted from the semidiurnal arc, there will remain the hours that are left until noon at the moment when that sign — whose oblique ascension was taken — begins to rise above the horizon. Or, conversely, if the semidiurnal arc is subtracted from the said hours (namely when the oblique ascension exceeds the semidiurnal arc), what remains will be the hours elapsed after noon when the beginning of that sign, whose oblique ascension was taken, rises.

[Margin: Example.]

Example. When the Sun is at the beginning of Aries, the semidiurnal arc contains 6 hours; and it is asked at what hour the beginning of Cancer rises above the Roman horizon — that is, for an elevation of 42 degrees. The arc between the beginning of Aries and the first degree of Cancer has an oblique ascension of 66 degrees 57 minutes (as is clear from the tables of oblique ascensions in the

Tabulae directionum of Johannes Regiomontanus, or in our commentaries on the *Sphere* — though we shall also teach, at the end of the scholium of this proposition, how to find these ascensions from the doctrine of sines), which, reduced to hours, contains 4 hours 28 minutes. These, subtracted from 6 hours — that is, from the semidiurnal arc — leave 1 hour 32 minutes before noon. Therefore at this hour the beginning of Cancer rises above the horizon, with the Sun at the beginning of Aries; and this corresponds to 10 hours 28 minutes after midnight.

Again, with the Sun occupying the beginning of Cancer, the semidiurnal arc for an elevation of 42 degrees comprises 7 hours 32 minutes; and it is asked at what hour the beginning of Sagittarius rises above the horizon. The oblique ascension of the arc lying between the beginning of Cancer and the first degree of Sagittarius contains 190 degrees 12 minutes, which, converted into hours, makes 12 hours 41 minutes; and if the semidiurnal arc of 7 hours 32 minutes is taken from these, there remains 5 hours 9 minutes after noon, at which the beginning of Sagittarius then emerges above the horizon. And so with the rest.

By this method we have composed the following four little tables, in which are contained the hours at which the beginnings of the signs rise in the Roman horizon of 42 degrees.

I. Sole existente in principio ♈. (When the Sun is at the beginning of Aries.)

Sign	Before / after noon	M	
		H.	.
♈ Aries	Before noon	6	0
♉ Taurus	Before noon	4	51
♊ Gemini	Before noon	3	26
♋ Cancer	Before noon	1	32
♌ Leo	After noon	0	51
♍ Virgo	After noon	3	26
♎ Libra	After noon	6	0
♏ Scorpio	After noon	8	34
♐ Sagittarius	After noon	11	9
♑ Capricorn	Before noon	10	28
♒ Aquarius	Before noon	8	34
♓ Pisces	Before noon	7	9

II. Sole existente in principio ♎. (When the Sun is at the beginning of Libra.)

Sign	Before / after noon	M	
		H.	.
♎ Libra	Before noon	6	0
♏ Scorpio	Before noon	3	26
♐ Sagittarius	Before noon	0	51
♑ Capricorn	After noon	1	32
♒ Aquarius	After noon	3	26
♓ Pisces	After noon	4	51
♈ Aries	After noon	6	0
♉ Taurus	After noon	7	9
♊ Gemini	After noon	8	34
♋ Cancer	After noon	10	28
♌ Leo	Before noon	11	9
♍ Virgo	Before noon	8	34

III. Sole existente in principio ♋. (When the Sun is at the beginning of Cancer.)

Sign	Before / after noon	M	
		H.	.
♋ Cancer	Before noon	7	3
			2
♌ Leo	Before noon	5	9
♍ Virgo	Before noon	2	3
			4
♎ Libra	Before or after noon	0	0
♏ Scorpio	After noon	2	3
			4
♐ Sagittarius	After noon	5	9
♑ Capricorn	After noon	7	3
			2
♒ Aquarius	After noon	9	2
			6
♓ Pisces	After noon	1	5
		0	1

Sign	Before / after noon	H.	M
♈ Aries	After or before noon	1	0
		2	
♉ Taurus	Before noon	1	5
		0	0
♊ Gemini	Before noon	8	2
			0

IV. Sole existente in principio ♐. (When the Sun is at the beginning of Capricorn.)

Sign	Before / after noon	H.	M
♐ Capricorn	Before noon	4	2
			8
♑ Aquarius	Before noon	2	3
			4
♒ Pisces	Before noon	1	1
			0
♈ Aries	Before or after noon	0	0
♉ Taurus	After noon	1	1
			0
♊ Gemini	After noon	2	3
			4
♋ Cancer	After noon	4	2
			8
♌ Leo	After noon	6	5
			1
♍ Virgo	After noon	9	2
			6
♎ Libra	After or before noon	1	0
		2	
♏ Scorpio	Before noon	9	2
			6
♐ Sagittarius	Before noon	6	5
			1

[Margin: Description of the ascending signs on the same horizontal dial.]

From these tables we shall describe the ascending signs in the following way — that is, the common sections [communis sectio] of the plane of the dial and of the ecliptic at the time when the beginnings of the signs rise above the horizon. For upon these common sections, which the plane of the dial makes with the ecliptic as the latter takes its various positions in the diurnal motion, the gnomon's shadow falls, by Proposition 11 of the preceding book, whenever the Sun is at any point of the ecliptic above the horizon, while the ecliptic holds those positions which the aforesaid common sections represent — that is, while the beginnings of the signs emerge above the horizon.

Let the hour be reckoned at which the beginning of Taurus rises, with the Sun at the beginning of Aries, taken from the first table: namely 4 hours 51 minutes, laid off from A to B (since the beginning of Taurus then rises before noon, and the semicircle of the equator from A toward B is the eastern one, as is evident if the circle described from E is understood — that is, the equator — in its proper position). From B, through the center E, let a hidden construction-line be drawn cutting the equinoctial line at C. For the point C will be that upon which the style's shadow falls at the said hour when the beginning of Taurus rises; and so through C will pass the common section of the plane of the dial and of the ecliptic at that time.

Diagram. A large "hourglass" (bowtie) construction superimposed on a circle, occupying the lower half of the page; the explanatory legend continues in the running prose above and below it.

- The central circle is the equator, described from center **E** (the vertex of the gnomon / center of the world).
- **E** (center of the circle): center of the equator; construction-lines drawn from the rails pass through E.
- A vertical meridian line runs top-to-bottom through the figure, marked **12** at the top (noon) and **12** in the lower half (midnight), descending to **A** at the very foot.
- **H** (top of the circle, on the meridian line): the center of the dial [centro horologii], from which straight lines are sent to the tropic points.
- A horizontal dotted line (the equinoctial / six-o'clock line) crosses through the upper region and meets each side-rail at a point marked **6** (left) and **6** (right).

- **B** (left, where a rail-line meets the equator): the point from which the line B–E–C is drawn.
- **C** (right of center, on the equinoctial line): intersection giving the ascending-sign point for Taurus.
- **D** (lower right, outside the circle, joined to E by a dotted line): the western-equator point used for the Sun-in-Capricorn construction.
- **K** (near the center, below E): the point on the tropic of Capricorn where the line from H cuts it.
- Two vertical side-rails (far left and far right) carry the twelve zodiacal sign-glyphs — the ascending-/descending-sign lines of the dial, the eastern signs on one rail and the western on the other; numerous straight lines run from the rails inward, converging near the center to form the bowtie. The individual peripheral sign-glyphs on the rails are too small to resolve with confidence — [?]

Its explanatory legend, running below the figure: the semicircle of the equator from A toward B is the eastern one, as is evident if the circle described from E is understood — that is, the equator — in its proper position; and from B, through the center E, let a hidden line be drawn cutting the equinoctial line at C. For the point C will be that upon which the style's shadow falls at the said hour when the beginning of Taurus rises; and so through C —

Next, from the fourth table let the hour be taken at which the same beginning of Taurus rises with the Sun at the beginning of Capricorn: namely 1 hour 10 minutes, which is to be laid off from A to D, the western part of the equator (since the beginning of Taurus then rises after noon); and from D, through E, let a straight line be drawn cutting the equinoctial line at F, through which, from H, the center of the dial, let a straight line be sent out cutting the tropic of Capricorn at K. For from what was demonstrated in the first proposition of this book, the point K will be that upon which the style's shadow falls at that hour when the beginning of Taurus rises with the Sun at the beginning of Capricorn; and so through K also will pass the common section of the plane of the dial and of the ecliptic at that time. Therefore the straight line drawn through the points C and K will be the common section of the plane of the dial and of the ecliptic at the time when the beginning of Taurus rises above the horizon, and upon which the gnomon's shadow then falls, by Proposition 11 of the earlier book, when the Sun is above the horizon at any point of the ecliptic. Therefore the straight line CK will give the ascending sign Taurus.

In the same way we shall describe the remaining ascending signs, taking each sign together with its hours from two tables (first from one of the two earlier tables, and second from one of the later ones), so that two points may be found — one on the equinoctial line, and the other on one or the other tropic — through which the common section of the ecliptic and the plane of the dial passes, and so forth.

However, from neither of the two former tables are the hours of those signs to be taken, on the horizontal dial, which rise six hours or more before or after noon. For those hours do not fall on the equinoctial line, as is plain. Therefore from each of these two tables one must take, on the horizontal dial, only the first five signs that follow next — such as are Taurus, Gemini, Cancer, Leo, Virgo, Scorpio, Sagittarius, Capricorn, Aquarius, and Pisces. Again, from neither of the two latter tables are the hours of those signs to be taken, on the same horizontal dial, which rise as many hours after or before noon as are contained in the semidiurnal arc of Cancer or of Capricorn, or more; for those hours do not cut the tropics, as is evident. Hence from the third table one takes only the hours of the first five signs, and likewise from the fourth, leaving out the first sign in each table: such are these signs — Leo, Virgo, Libra, Scorpio, Sagittarius, Aquarius, Pisces, Aries, Taurus, and Gemini. Nor for that reason are the remaining latter six signs of these tables to be judged superfluous, since their use in the other dials that follow — such as in both meridian dials, and in declining and inclining ones — is altogether necessary, as will become clear from what follows.

[Margin: Description of the ascending signs Aries and Libra.]

But the lines of Aries and Libra will be described thus. Since, when the beginning of Aries ascends while the Sun is at the beginning of Capricorn, it is virtually noon — as is clear from the fourth table — it is evident that the common section of the plane of the dial and of the ecliptic then passes through that point of the tropic of Capricorn through which the meridian line is drawn. And since at that time the equator, the circle of the sixth hour from noon or from midnight, the ecliptic, and the horizon all have the same common section — as is clear from a material sphere — and the plane of the dial is equidistant from it, being parallel to the horizon, then (by Proposition 18 of the earlier book) the equinoctial line, the line of the sixth hour from noon or from midnight, and the common section of the ecliptic and the plane of the dial will all be parallel. Therefore a straight line drawn parallel to the equinoctial line — or to the very line of the sixth hour — through the point at which the meridian line cuts the tropic of Capricorn will give the ascending sign Aries. And by the same reasoning, the straight line drawn

parallel to the equinoctial line through the point at which the meridian line cuts the tropic of Cancer will give the ascending sign Libra; since this sign too rises at noon when the Sun is at the beginning of Cancer, as is manifest from the third table, and so forth.

[Margin: Description of the ascending signs Cancer and Capricorn.]

Finally, the lines of Cancer and Capricorn can be described in this way. Since, when the beginning of Cancer ascends while the Sun is at the beginning of Capricorn, it is 4 hours 28 minutes after noon — as appears from the fourth table — that is, the Sun (or the beginning of Capricorn) then sets, the ecliptic and the circle of the 4-hour-28-minute line from noon will cut the horizon at the same point of setting; and so the ecliptic, that hour-circle, and the horizon will have the same common section. Therefore, since the plane of the dial is equidistant from the horizon — and hence from that common section — the line of 4 hours 28 minutes from noon and the common section of the ecliptic at that time and of the plane of the dial will be parallel, by Proposition 18 of the preceding book. If, then, the line of 4 hours 28 minutes from noon is drawn, and through the point found on the equinoctial line (through which the line of Cancer must be drawn) a parallel to it is drawn, this will give the ascending sign Cancer. And in the same way we shall draw the line of ascending Capricorn.

Indeed, by this same demonstration we shall conclude that the line of any ascending sign on the horizontal dial is equidistant from that hour-line — from noon or from midnight — which indicates the time of setting of the opposite sign: namely, that the line of Scorpio is equidistant from the line of 6 hours 42 minutes from noon or from midnight, the hour at which the beginning of Taurus, the opposite sign, sets. For the semidiurnal arc of Taurus contains just so many hours. The reason is that the ecliptic at that time, and the circle of that hour, and the horizon, have the same common section, as is plain, and so forth. And so, if the semidiurnal arcs of all the signs are taken, as in this table, we shall easily know from which hour-lines — from noon or from midnight — the lines of the ascending signs on the horizontal dial are equidistant, for the reason stated.

V. Arcus semidiurni in initiis signorum, ad latitudinem grad. 42. (Semidiurnal arcs at the beginnings of the signs, for latitude 42 degrees.)

Sign	H.	M.
♈ Aries	6	0
♉ Taurus	6	42
♊ Gemini	7	17

Sign	H.	M.
♋ Cancer	7	32
♌ Leo	7	17
♍ Virgo	6	42
♎ Libra	6	0
♏ Scorpio	5	18
♐ Sagittarius	4	43
♑ Capricorn	4	28
♒ Aquarius	4	43
♓ Pisces	5	18

For the lines of Aries and Libra are equidistant from the line of 6 hours 0 minutes from noon or from midnight. The lines of Taurus and Virgo are equidistant from the line of 5 hours 18 minutes. The lines of Gemini and Leo are equidistant from the line of 4 hours 43 minutes.[a] Hence pairs of them will also be equidistant from one another. But the line of Cancer is equidistant from the line of 4 hours 28 minutes. Finally, the line of Capricorn is equidistant from the line of 7 hours 32 minutes.

[Margin: a — Prop. 30 of the first book.]

[Margin: The lines of the ascending signs touch the tropics in any dial whatever.]

It must here, and in the dials that follow, be carefully noted that no line of the ascending signs can cut the tropics, but that all of them merely touch the tropics at those points through which they must be drawn, so that all the lines of the ascending signs are contained within the limits of the tropics. For just as the hour-lines from sunrise or sunset touch the conic sections of the two parallels — of which one is always the greatest of the apparent circles and the other the greatest of the hidden ones — at those points where the hour-lines from noon or midnight intersect the same conic sections, as we demonstrated in Proposition 14 of the preceding book: so too the common sections of the ecliptic and the plane of the dial, which arise from the various positions that the ecliptic takes in the diurnal motion, produce —

are produced, they will touch the conic sections of the tropics at those points where those same conic sections are cut by the hour-lines from noon or from midnight at which the ecliptic holds those positions. For the demonstration is the same in both cases.

We shall make this clear as follows. When, for example, the sign Taurus ascends and the Sun is at the beginning of Capricorn, the line of 1 hour 10 minutes after noon cuts the tropic of Capricorn on the dial at K. Therefore the circle of that hour, drawn through the poles of the world, will also cut the said circle of Capricorn in the heavens at the very point where the Sun, being there, sends its ray through the tip of the gnomon [stylus] to the point K. Now since the ecliptic touches the circle of Capricorn in the heavens at the beginning of Capricorn, where the Sun is placed, it must at that time touch the said circle of Capricorn at that same point where it is cut by the aforesaid hour-circle, since the Sun's ray then falls upon the point K. Therefore, at the point where the tropic of Capricorn is then cut on the dial by the line of 1 hour 10 minutes after noon — that is, at K — the common section of the ecliptic and the plane of the dial will touch that same tropic of Capricorn, as is clear from Propositions 13 and 14 of the preceding book.

For the circle of that hour of 1 hour 10 minutes after noon and the ecliptic then stand in the same mutual relation — the former cutting the conic surface of Capricorn along a straight line, the latter touching that same surface along the same line — as any hour-circle from noon or from midnight that cuts along a straight line the conic surface of the greatest of the ever-apparent [circumpolar] parallels stands to the hour-circle from sunrise or sunset that touches the same conic surface along that same straight line. This is clear if one rightly considers the position of the ecliptic and of the said hour-circle of 1 hour 10 minutes while the beginning of Taurus ascends and the Sun is at the beginning of Capricorn. For the colure of the solstices then performs the office of that hour-circle, cutting the tropic of Capricorn at the point where the ecliptic touches that same tropic — namely, at the beginning of Capricorn. This can be understood, clearer than daylight, from a solid [material] sphere. And the same reasoning holds for the other ascending signs, when the Sun is at the beginning of Capricorn or of Cancer.

[Margin: By what method the ascending signs may be described on the dial through the doctrine of the conics of Apollonius.]

From this another method of describing the ascending signs is gathered. For if, from the points of the tropics found by means of the third and fourth tables, straight lines are drawn tangent to the tropics themselves — according to what we wrote in the corollary to Proposition 37 of Book 1 of Apollonius (which is indeed very easy here, since the ordinates from the said points of the tropics are perpendicular to the diameter) — the ascending signs will thereby have been described.

The ascending signs may also be described otherwise, in this way. First let the points of the ecliptic lying on the meridian circle be sought — that is, the mediations of heaven [midheaven points] when the beginnings of the zodiacal signs rise — together with the declinations of those same points.

[Margin: How the mediations of heaven, when the beginnings of the signs rise, are to be investigated.]

Then, with the same beginnings of the signs ascending, let the points of the ecliptic situated on the circle of the 6th hour from noon or from midnight be investigated, together with their declinations. We shall accomplish all this as follows.

From the oblique ascension of the beginning of any sign — which is to be taken either from the tables of oblique ascensions contained in the *Tables of Directions* of Johannes Regiomontanus [Regiomontanus], or in our own commentaries on the sphere, or else derived from the doctrine of sines, as we taught in Chapter 3 on the sphere and shall show at the end of the scholium of this proposition — let a quadrant of the circle, that is 90 degrees, be subtracted (a whole circle having first been added to the oblique ascension, if the subtraction cannot otherwise be made, as is customary in astronomical calculation). For the remaining number will be the right ascension of that point of the ecliptic which is then found on the meridian above the horizon, and which astronomers call the mediation of heaven [midheaven]. Therefore, from the table of right ascensions, or from the doctrine we shall deliver in the following scholium, that point of the ecliptic will become known, whose opposite point will lie on the same meridian below the horizon — which, with the astronomers, we may call the angle of the earth [imum caeli].

The demonstration of this operation is not difficult, if the positions of the horizon, the meridian, the zodiac, and the equator in the sphere are rightly conceived. For when the oblique ascension is greater than a quadrant, or equal to a quadrant, it is clear that, if the quadrant of the equator lying between the east and the meridian is subtracted from it, there remains the right ascension of the culminating point of the ecliptic — namely, the distance of the beginning of Aries from the meridian circle, following the order of the signs. But when the oblique ascension is less than a quadrant, it is likewise plainly established that, if the quadrant of the equator lying between the east and the meridian is taken from the whole circle and the oblique ascension is added to the remaining three quadrants (which comes to the same thing as subtracting a quadrant from the sum formed of the oblique ascension and a whole circle), the right ascension of the culminating point of the ecliptic is produced. In the right sphere too the same

operation applies, if in place of the oblique ascension of the ascending sign one takes the right ascension of the same.

[Margin: Example.]

Example. At latitude 42 degrees, which is about that of Rome, the oblique ascension of the beginning of Virgo is 141 degrees 33 minutes. If a quadrant — that is, 90 degrees — is subtracted from it, there remain 51 degrees 33 minutes for the right ascension of the culminating point of the ecliptic when the beginning of Virgo rises. To this right ascension there corresponds in the table of right ascensions — using, however, the proportional part, as is customary when the number is not exactly contained in some table — 23 degrees 57 minutes of Taurus. This point of the ecliptic, then, will at that time be found on the meridian above the horizon, while below the horizon on the same meridian there will be 23 degrees 57 minutes of Scorpio.

Again, the oblique ascension of the beginning of Cancer is 66 degrees 57 minutes. If a whole circle — that is, 360 degrees — is added to it, we get 426 degrees 57 minutes; and if 90 degrees are subtracted from these, there remain 336 degrees 57 minutes for the right ascension of the point of the ecliptic then culminating, when the beginning of Cancer rises above the horizon. To this right ascension there corresponds in the table of right ascensions 5 degrees 6 minutes of Pisces. Therefore this point will at that time lie on the meridian above the horizon, while the angle of the earth will be 5 degrees 6 minutes of Virgo.

Lastly, in the right sphere (to give an example of this too), the right ascension of Sagittarius comprises 237 degrees 48 minutes. Subtracting a quadrant from this, there remain 147 degrees 48 minutes for the right ascension of that point of the ecliptic which then culminates. To this ascension there answers 25 degrees 31 minutes of Leo; and this point will then lie on the meridian above the horizon, so that at the angle of the earth there will consequently be 25 degrees 31 minutes of Aquarius.

By this method we have composed the following table of the mediations of heaven and the angles of the earth, when the beginning of any zodiacal sign rises, for latitude 42 degrees; in which we have also entered the declinations of those same mediations and angles of the earth, found through the doctrine of sines, as we set forth in the corollary to Proposition 1 of the preceding book — although these same declinations can be elicited from the table of declinations, if account is taken of the proportional part. It is clear, moreover, that the declinations of the northern points of the ecliptic are northern, and those of the southern points

southern. From this you will easily judge which points' declination in the said table is boreal and which austral — which is indeed of great importance to know, in order that the ascending signs may be described.

VI. Mediationes coeli, & anguli terrae, eorumque declinationes, orientibus 12. signorum Zodiaci initiis, ad latitudinem grad. 42. (The mediations of heaven and the angles of the earth, and their declinations, when the beginnings of the 12 signs of the zodiac rise, for latitude 42 degrees.)

Ascending sign	Mediation of heaven	Angle of the earth	Declination
♈ Aries	0° 0' ♎	0° 0' ♎	23° 30'
♉ Taurus	15° 59' ♎	15° 59' ♎	22° 32'
♊ Gemini	6° 3' ♋	6° 3' ♋	18° 48'
♋ Cancer	5° 6' ♋	5° 6' ♋	9° 40'
♌ Leo	13° 59' ♈	13° 59' ♈	5° 32'
♍ Virgo	23° 57' ♉	23° 57' ♉	18° 48'
♎ Libra	0° 0' ♎	0° 0' ♎	23° 30'
♏ Scorpio	6° 3' ♋	6° 3' ♋	18° 48'
♐ Sagittarius	16° 1' ♋	16° 1' ♋	5° 32'
♑ Capricorn	24° 54' ♈	24° 54' ♈	9° 40'
♒ Aquarius	23° 57' ♉	23° 57' ♉	18° 48'
♓ Pisces	14° 1' ♋	14° 1' ♋	22° 32'

Again, since the oblique ascension of the beginning of any sign is the right ascension of that point of the ecliptic which then lies on the circle of the 6th hour from noon or from midnight, before the meridian — as is clear if one rightly considers the position of the circle of the 6th hour from noon or from midnight, which always passes through the points of true rising, that is, through the end of the oblique ascension, like a kind of right horizon — hence it comes about that the arc of the equator measuring the oblique ascension of a point set on the horizon also measures the right ascension of the point of the ecliptic located on the circle of the 6th hour at that time: the right ascension, I say, reckoned upon the circle of the 6th hour, as upon a kind of right horizon. Hence, from the table of right ascensions, that point of the ecliptic will become known. And it is evident that the opposite point of the ecliptic then lies on the same circle of the 6th hour after the meridian.

[Margin: By what method the points of the ecliptic situated on the circle of the 6th hour from noon or from midnight are found, when the beginnings of the signs rise. Example.]

Example. When the beginning of Cancer ascends, I find that its oblique ascension, at latitude 42 degrees, is 66 degrees 57 minutes. So great, then, is the right ascension of the point of the ecliptic that is then set on the said circle of the 6th hour before the meridian. This point, from the table of right ascensions, will be found to be 8 degrees 41 minutes of Gemini, account being taken of the proportional part; and consequently on the same circle after the meridian there will be 8 degrees 41 minutes of Sagittarius. In the right sphere this same point, rising above the horizon, is on the circle of the 6th hour from noon or from midnight, before the meridian, because there the said circle of the 6th hour from noon or from midnight is the same as the horizon.

By this method we have constructed the following table containing the points of the ecliptic situated on the circle of the 6th hour from noon or from midnight, while the beginnings of the signs rise, for latitude 42 degrees; in which are also placed the declinations of the points found, which we computed from the science of sines, as we taught in the corollary to Proposition 1 of the preceding book — although these same declinations can be gathered from the table of declinations, if account is taken of the proportional part.

VII. Puncta Eclipticae in circulo hor. 6. constituta, eorumq; declinationes, orientibus 12. signorum Zodiaci principiis, ad latitudinem gr. 42. (The points of the ecliptic situated on the circle of the 6th hour, and their declinations, when the beginnings of the 12 signs of the zodiac rise, for latitude 42 degrees.)

Ascending sign	Point on the 6-o'clock circle (ante-meridian)	Point (post-meridian)	Declination
♈ Aries	0° 0' ♈	0° 0' ♎	0° 0'
♉ Taurus	18° 49' ♈	18° 49' ♎	7° 23'
♊ Gemini	10° 53' ♉	10° 53' ♎	15° 8'
♋ Cancer	8° 41' ♊	8° 41' ♎	21° 48'
♌ Leo	11° 49' ♋	11° 49' ♎	22° 58'
♍ Virgo	19° 7' ♌	19° 7' ♎	15° 8'
♎ Libra	0° 0' ♍	0° 0' ♈	0° 0'
♏ Scorpio	10° 53' ♍	10° 53' ♈	15° 8'
♐ Sagittarius	18° 11' ♏	18° 11' ♈	22° 58'

Ascending sign	Point on the 6-o'clock circle (ante-meridian)	Point (post-meridian)	Declination
♑ Capricorn	21° 19' ♑	21° 19' ♎	21° 48'
♒ Aquarius	19° 7' ♒	19° 7' ♋	15° 8'
♐ Pisces	11° 11' ♐	11° 11' ♉	7° 23'

[Margin: The description of the ascending signs on the dial by means of the mediations of heaven and of the points of the ecliptic situated on the circle of the 6th hour from noon or from midnight, while the beginnings of the signs rise.]

These things having been duly completed, we shall describe the ascending signs as follows. Let the declinations of the mediations of heaven be reckoned off — in the first figure of Proposition 2 of this book — from the point F of the diameter of the equator toward the north or the south, according as the points of the mediations of heaven are boreal or austral; and through the ends of these reckonings and the center D let straight lines be drawn as hidden [construction] lines, cutting the straight line OR at points whose intervals, taken from the point G — namely, from the place of the gnomon [stylus] — if they are transferred onto the meridian line of the dial from G, the place of the gnomon, will give on the meridian line the points through which the ascending signs are to be drawn: those signs, namely, whose mediations of heaven and declinations of the mediations of heaven have been taken.

The reason for this is as follows. If, for example, when the beginning of Cancer rises, there is found on the meridian 5 degrees 6 minutes of Pisces above the horizon, it follows that when the Sun is at 5 degrees 6 minutes of Pisces and occupies the meridian circle above the horizon, the beginning of Cancer rises. Since, then, at that time the shadow of the gnomon is cast onto that point of the meridian line which we found by the declination of 5 degrees 6 minutes of Pisces — namely, onto the conic section of the parallel of that declination, as is established from the corollary to Proposition 12 of the preceding book — the common section of the ecliptic and the plane of the dial will necessarily at that time pass through the same point; because the shadow of the gnomon, as we showed in Proposition 11 of the same preceding book, is always cast onto the common section of the plane of the dial and of the great circle in which the Sun is — which at that time is the zodiac [ecliptic]. And the same reasoning holds for the rest.

But the same points on the meridian line will also be found from the third figure of Proposition 2 of this book — namely, from the figure of the rays of the zodiac

— in this way. In the circle described from D, let the declinations be reckoned off from the radius of the equator, as before — namely, toward the radius of Cancer or of Capricorn, according as the points of the mediations of heaven are boreal or austral; and let the rays of those declinations be drawn as hidden lines from D, cutting the line HB — the line of the 12th hour — at points which, if taken from the point H onto the meridian line of the dial from the center of the dial H, will be found as points on the meridian line, as before.

Likewise, let the declinations of the boreal points of the ecliptic lying on the circle of the 6th hour from noon or from midnight, both before and after the meridian, be reckoned off — in the third figure of Proposition 2 of this book, that is, in the figure of the rays of the zodiac — on the arc of the circle described from D, from the radius of the equator toward the radius of Cancer. (For on the horizontal dial only the boreal declinations are to be taken, since these alone can fall on the line of the 6th hour, which lies wholly in the boreal part of the dial that the equinoctial line separates from the austral part.) And from D, through the ends of the reckonings, let the rays of those declinations be drawn as hidden lines, cutting the line of the 6th hour from noon or from midnight at points which, transferred from H onto the line of the 6th hour of the dial from the center H — whether before or after the meridian, as is clear from the preceding table — will give the points on the line of the 6th hour from noon or from midnight through which the ascending signs will pass; and this will be demonstrated in the same way. And so, if the corresponding points on the meridian line and on the line of the 6th hour from noon or from midnight are joined by straight lines, the ascending signs will be had, as before.

This method of describing the ascending signs seems more convenient and more reliable than the preceding one, because it does not require the arcs of the tropics — in the drawing of which an error can easily be committed — and because it offers, for each sign, two points: one on the meridian line and the other on the line of the 6th hour from noon or from midnight, so far apart from each other that the lines of the ascending signs can easily be drawn without error.

[Margin: This latter description is preferred to the former.]

Indeed, if from the points found on the meridian line straight lines are drawn tangent to the tropics — as we taught in the corollary to Proposition 37 of Book 1 of Apollonius (which will indeed be easy here, since the ordinate lines are perpendicular to the diameter) — the ascending signs will have been described in yet another way.

But since we have taught the finding of the points of the ecliptic on the circles of the 12th and 6th hour from noon or from midnight, what prevents us from teaching how the points of the ecliptic situated on the circles of the other hours as well, when the beginnings of the signs rise, may be found — especially since by their aid the ascending signs are described most conveniently on certain dials, just as we described these same signs a little earlier on the horizontal dial by the aid of the mediations of heaven and of the points of the ecliptic on the circle of the 6th hour from noon or from midnight?

[Margin: How the points of the ecliptic situated on any hour-circle from noon or from midnight are to be sought, when the beginnings of the signs rise.]

The general rule, then, will be this. From the oblique ascension of the beginning of any sign — or, what is the same, from the right ascension of that point of the ecliptic which, when that sign ascends, is found on the circle of the 6th hour before the meridian — let there be subtracted as many degrees of the equator as lie between the circle of the 6th hour before the meridian and the circle of the proposed hour, counting the said degrees (beginning from the circle of the 6th hour) from east to west, that is, against the order of the signs — a whole circle having first been added, if the subtraction cannot otherwise be made. For the number that remains will be the right ascension of that point of the ecliptic which, when the beginning of the proposed sign rises, is found on the circle of the given hour above the horizon. Therefore, from the table of right ascensions, that point will become known, and thus also the opposite point, lying on the same circle of the proposed hour below the horizon, will be known.

The demonstration of this does not differ from the one we recently made for finding the mediation of heaven, if in place of the quadrant of the equator between the east and the meridian one takes the arc of the equator between the 6th hour (or the east) and the given hour above the horizon, proceeding from east to west — as is plainly established if the circles in the sphere are rightly conceived. It is clear, moreover, that these hours above the horizon — say the seventh, eighth, ninth, tenth, and eleventh — are before noon, while these, namely the first, second, third, fourth, and fifth, are after noon. For we are inquiring only into the points lying on the circles of these hours; on the opposite hours the points opposite to those are situated.

[Margin: Example.]

Example. Suppose it is proposed to find the point of the ecliptic lying on the circle of the 1st hour after noon, when the beginning of Cancer ascends. The arc of the

equator between the circles of the 6th hour before noon and the 1st hour after noon contains 7 hours — that is, 105 degrees, since 15 degrees correspond to each hour. The oblique ascension of the beginning of Cancer is 66 degrees 57 minutes at latitude 42 degrees. If a whole circle is added to it, there will be 426 degrees 57 minutes; and if from these are subtracted the 105 degrees that lie between the circles of the 6th and the 1st hour, there will remain 321 degrees 57 minutes for the right ascension of that point of the ecliptic which then occupies the circle of the 1st hour after noon. To this ascension there corresponds in the table of right ascensions (account being taken of the proportional part) about 19 degrees 31 minutes of Aquarius. This point of the ecliptic, then, when the beginning of Cancer rises, will lie on the circle of the 1st hour after noon; while the opposite point — that is, 19 degrees 31 minutes of Leo — [will lie] on the same circle after midnight.

By this device the following four tables are constructed, containing the points of the ecliptic situated on the circles of the 11th, 1st, 7th, and 5th hour, when the beginnings of the signs rise, together with the declinations of the points found. For these will be especially useful for the descriptions of the ascending signs on each dial — the meridian and the polar — as we shall teach in its proper place. And so we have described the ascending signs of the zodiac on the same horizontal dial. Which was to be done.

These four tables (VIII–XI) supply the points of the ecliptic lying on the circles of the 11th, 1st, 7th, and 5th hours — the four tables promised at the close of the preceding proposition. In each table the twelve zodiacal signs are ranged in three horizontal blocks of four columns; every column is headed by the sign that is *rising*, and each cell records a point of the ecliptic as **degrees · minutes · sign**. The stub column reads, top to bottom: *Signa ascendentia* (the rising sign, with its symbol), *Gradus & Minuta* (G. = degrees, M. = minutes), *Puncta in hora ... ante meridiem* (the point on that hour-circle before noon), *Puncta in hora ... pomeridiana* (the point after noon), and *Declinationes* (the declination of the point found). The before-noon and after-noon points of any one column are diametrically opposite, so they share the same degree and minute and take opposite signs; their common declination is given once at the foot.

VIII. The points of the ecliptic lying on the circle of the 11th hour, together with their declinations, when the beginnings of the 12 signs of the zodiac rise, at latitude 42 degrees.

Rising signs ♈ Aries · ♉ Taurus · ♊ Gemini · ♋ Cancer

Row	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer
Point, 11th h. before noon	13° 48' ♎	0° 9' ♋	21° 3' ♋	21° 6' ♎
Point, 11th h. after noon	13° 48' ♎	0° 9' ♌	12° 3' ♌ [recte 21° 3' — see anomaly (a)]	21° 6' ♍
Declination	22° 47'	20° 10'	14° 31'	3° 32'

Rising signs ♌ Leo · ♍ Virgo · ♎ Libra · ♏ Scorpio

Row	♌ Leo	♍ Virgo	♎ Libra	♏ Scorpio
Point, 11th h. before noon	29° 57' ♈	8° 19' ♊	13° 48' ♎	21° 3' ♌
Point, 11th h. after noon	29° 57' ♎	8° 19' ♏	13° 48' ♎	21° 3' ♋
Declination	11° 29'	21° 45'	22° 47'	14° 31'

Rising signs ♏ Sagittarius · ♐ Capricorn · ♑ Aquarius · ♒ Pisces

Row	♏ Sagittarius	♐ Capricorn	♑ Aquarius	♒ Pisces
Point, 11th h. before noon	2° 21' ♎	10° 28' ♏	8° 19' ♏	27° 52' ♏
Point, 11th h. after noon	2° 21' ♈	10° 28' ♉	8° 19' ♊	27° 52' ♊
Declination	0° 56'	15° 0'	21° 45'	23° 29'

IX. The points of the ecliptic lying on the circle of the 1st hour, together with their declinations, when the beginnings of the 12 signs rise, at latitude 42 degrees.

Rising signs ♈ Aries · ♉ Taurus · ♊ Gemini · ♋ Cancer

Row	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer
Point, 1st h. before noon	16° 12' ♊	2° 8' ♎	21° 41' ♎	19° 32' ♌
Point, 1st h. after noon	16° 12' ♏	2° 8' ♎	21° 41' ♎	19° 32' ♋
Declination	22° 47'	23° 29'	21° 54'	15° 0'

Rising signs ♌ Leo · ♍ Virgo · ♎ Libra · ♏ Scorpio

Row	♌ Leo	♍ Virgo	♎ Libra	♏ Scorpio
Point, 1st h. before noon	27° 39' ♍	8° 57' ♎	16° 12' ♎	21° 41' ♏
Point, 1st h. after noon	29° 39' ♎ [recte 27° 39' — see anomaly (b)]	8° 57' ♏	16° 12' ♏	21° 41' ♏
Declination	0° 56'	14° 41'	22° 47'	21° 45'

Rising signs ♐ Sagittarius · ♑ Capricorn · ♒ Aquarius · ♓ Pisces

Row	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
Point, 1st h. before noon	0° 3' ♎	8° 47' ♏	8° 57' ♏	0° 8' ♏
Point, 1st h. after noon	0° 3' ♍	8° 47' ♎	8° 57' ♎	0° 8' ♎
Declination	11° 29'	3° 30'	14° 31'	20° 14'

X. The points of the ecliptic lying on the circle of the 7th hour, together with their declinations, when the beginnings of the 12 signs of the zodiac rise, at latitude 42 degrees.

Rising signs ♈ Aries · ♉ Taurus · ♊ Gemini · ♋ Cancer

Row	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer
Point, 7th h. before noon	13° 42' ♎	2° 34' ♏	25° 19' ♏	24° 20' ♏
Point, 7th h. after noon	13° 42' ♍	2° 34' ♎	25° 19' ♎	24° 20' ♎
Declination	6° 25'	1° 2'	9° 49'	18° 54'

Rising signs ♌ Leo · ♍ Virgo · ♎ Libra · ♏ Scorpio

Row	♌ Leo	♍ Virgo	♎ Libra	♏ Scorpio
Point, 7th h. before noon	28° 3' ♏	4° 12' ♌	13° 43' ♍	25° 19' ♎
Point, 7th h. after noon	28° 3' ♎	4° 12' ♏	13° 43' ♎	25° 19' ♏

Row	♌ Leo	♍ Virgo	♎ Libra	♏ Scorpio
Declination	23° 29'	19° 15'	6° 25'	9° 49'

Rising signs ♐ Sagittarius · ♑ Capricorn · ♒ Aquarius · ♓ Pisces

Row	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
Point, 7th h. before noon	4° 9' ♐	7° 23' ♑	4° 12' ♒	25° 22' ♓
Point, 7th h. after noon	4° 9' ♊	7° 23' ♋	4° 12' ♌	25° 22' ♍
Declination	21° 2'	23° 18'	19° 15'	13° 6'

XI. The points of the ecliptic set on the circle of the 5th hour, together with their declinations, when the beginnings of the signs of the zodiac rise, at latitude 42 degrees.

Rising signs ♈ Aries · ♉ Taurus · ♊ Gemini · ♋ Cancer

Row	♈ Aries	♉ Taurus	♊ Gemini	♋ Cancer
Point, 5th h. before noon	16° 18' ♈	4° 38' ♉	25° 47' ♊	22° 37' ♋
Point, 5th h. after noon	16° 18' ♎	4° 38' ♏	25° 47' ♐	22° 37' ♑
Declination	6° 25'	13° 6'	19° 15'	23° 18'

Rising signs ♌ Leo · ♍ Virgo · ♎ Libra · ♏ Scorpio

Row	♌ Leo	♍ Virgo	♎ Libra	♏ Scorpio
Point, 5th h. before noon	25° 51' ♋	4° 41' ♌	16° 18' ♎	25° 47' ♏
Point, 5th h. after noon	25° 51' ♑	4° 41' ♒	16° 18' ♐	25° 47' ♓
Declination	21° 2'	9° 49'	6° 25'	19° 15'

Rising signs ♐ Sagittarius · ♑ Capricorn · ♒ Aquarius · ♓ Pisces

Row	♐ Sagittarius	♑ Capricorn	♒ Aquarius	♓ Pisces
Point, 5th h. before noon	1° 57' ♐	5° 40' ♑	4° 41' ♒	27° 26' ♓
Point, 5th h. after noon	1° 57' ♋	5° 40' ♌	4° 41' ♎	27° 26' ♏
Declination	23° 29'	18° 54'	9° 49'	1° 2'

[Margin: Construction of the two figures from Andreas Schöner [Andreas Schöner], by which the ascending signs are described.]

Diagram. The upper of Schöner's two construction figures. A circle with center **E** and two perpendicular diameters: the vertical diameter runs up to **A** (top of the circle) and down to **C** (bottom), extended upward to **H** (above the circle) and far downward to **N** (well below, at the point of a teardrop-shaped envelope of dotted lines); the horizontal diameter runs from **B** (left) to **D** (right, labelled together with **V** as "DV" on the right). On the circumference: **F** (upper left), **K** (upper right), and lower points **G**, **I**, **L**. Interior points **M** and **P** lie in the upper half along the vertical axis; **R** and **S** lie in the lower half. A secondary circle **HBID** is drawn through **B** and **D**. The circle is divided into twelve parts, and numerous dotted "sign-lines" radiate from the center and are carried out to **N**, forming the balloon/teardrop outline. Small figures "3" (upper interior) and "6" (near center) appear. Several zodiac symbols mark the radial lines around the rim (per the text: the line **E**→**D** corresponds to ♈ Aries, **E**→**C** to ♋ Cancer, **E**→**B** to ♎ Libra, **E**→**A** to ♐ Capricorn); most of these peripheral zodiac sorts are worn.

- **H** (top, on the vertical axis extended)
- **A** (top of circle)
- **F** (upper left, on circle)
- **K** (upper right, on circle)
- **M** (interior, upper axis)
- **P** (interior, below **M**)
- **E** (center)
- **B** (left end of horizontal diameter; near it the sign ♎ Libra)
- **D / V** (right end of horizontal diameter, "DV")
- **R** (interior, below center)
- **S** (interior, lower)
- **G** (lower right, on circle) [?]

- I (lower interior) [?]
- L (lower, on circle) [?]
- C (bottom of circle)
- N (bottom vertex of the dotted envelope)
- Peripheral zodiac symbols: ♈ , ♉ (left side), others worn [?]

Diagram. The lower ("2nd") of Schöner's construction figures, drawn for latitude 42 degrees. A single circle **ABCD** with **A** at the top, **C** at the bottom, **B** at the left, and **D** at the right, divided by twelve radial lines from the center into the twelve signs; a smaller inner circle *f g h i* is constructed within it. Readable letters include **F** (upper left, on the rim) and interior points **H, I, L, K, G**. Each of the twelve radial lines carries a zodiac symbol around the circumference (all twelve signs), the small peripheral sorts largely worn.

- A (top)
- B (left)
- C (bottom)
- D (right)
- F (upper left, on rim)
- H (interior) [?]
- I (interior, right of center) [?]
- L (interior) [?]
- K (lower interior) [?]
- G (interior) [?]
- Twelve peripheral zodiac symbols (all signs), mostly worn [?]

SCHOLIUM.

In another way we shall describe the ascending signs, following Andreas Schöner, by this method. Let a circle ABCD be drawn, with center E and two diameters AC and BD cutting each other at right angles. Then take arcs AF and CG equal to the greatest declination of the Sun, and from B draw straight lines through F and G that cut the line AC, extended, at H and I. Again take arcs DK and BL equal to twice the greatest declination of the Sun, and from B draw straight lines through K and L that cut the same line AC, extended, at M and N. Then, with M as center and radius MH (or MI — for the line HI is bisected at M), describe a circle HBID, which will pass through B and D. Now, the circle ABCD having been divided into 12 equal parts, join by straight lines each pair of points equally distant from D or from B; these lines cut BD at certain points. Through these points, if straight lines

are drawn from N cutting the circle HBID, and through the resulting points straight lines are again sent out from E, these will be the lines of the signs: thus ED will be ♈ (Aries), EC ♋ (Cancer), EB ♎ (Libra), EA ♏ (Capricorn), and so on. By this figure, therefore, we shall find at the tropical points ♋ (Cancer) and ♏ (Capricorn) the points through which the lines of the ascending signs must be drawn, as we shall show.

Let a circle ABCD be drawn again, with center E and two diameters AC and BD cutting each other at right angles, as appears in the 2nd figure. But take arcs AF and CG of $25\frac{3}{4}$ degrees, and let the points F and G be joined by a straight line that cuts BD at the point H; and from H as center describe an arc IK, arc IK being equal to the elevation of the pole; and from H through K draw a straight line cutting AC at the point L. From L describe a circle *fgh i*, equal to the circle ABCD of the former figure, and let it be divided into 12 signs just as ABCD was divided, beginning from the line *fh*, so that *h* is ♈ (Aries), *g* ♋ (Cancer), *f* ♎ (Libra), and *i* ♍ (Virgo) [thus printed; the construction requires ♏ Capricorn, *i* being opposite *g* = ♋ — see anomaly (c)]. And so, the circle ABCD coinciding with *fgh i* — D being placed on *h*, and B on *f* — the signs of circle ABCD of the former figure correspond to the signs of circle *fgh i*. Then, through the points of the signs of circle *fgh i* and through the center L, draw straight lines cutting the circle ABCD in 12 points; and to these points draw straight lines from E, which will give the beginnings of the signs. For C will be the beginning of ♈ (Aries); the first point then, toward G, the beginning of ♉ (Taurus); the next, the beginning of ♊ (Gemini), and so on. By this figure we shall investigate, on the equinoctial line, the points through which the lines of the ascending signs must be drawn, as we shall presently teach. It is not necessary here, however, that the lines above the line BD correspond to the lines below the same BD along a single straight line, as in the upper figure — although sometimes they do correspond, as happened in this figure constructed for latitude 42 degrees, in the lines ♍ (Virgo) and ♏ (Scorpio). For these form straight lines with the lines ♒ (Aquarius) and ♊ (Gemini); yet all of them must be produced beyond the center, and when produced must be distinguished by points, lest they be confused with the others, and the same signs must be assigned to them. Hence it comes about that the line ♒ (Aquarius) also has the sign ♍ (Virgo); the line ♊ (Gemini), the sign ♏ (Scorpio); likewise the line ♍ (Virgo), the sign ♒ (Aquarius); the line ♏ (Scorpio), the sign ♊ (Gemini); the line ♈ (Aries), the sign ♎ (Libra); and the line ♎ (Libra), the sign ♈ (Aries). And so each line will have its own sign from the construction, and also the sign of that line which, when produced into a straight line, forms one line with it. Therefore, since this is of great use for describing the ascending signs, it must be carefully noted.

Therefore, on the straight line ED of the former figure, take the straight line EO equal to the meridian line EI of the dial intercepted between E, the center of the equator, and the equinoctial line; and through O draw a line parallel to AC, cutting the lines of the signs sent out from E at P, Q, R, S. Thus, when the center E of this figure is placed on the center E of the equator of the dial, and the point O on the point I, the parallel drawn through O will coincide with the equinoctial line. If, then, these points taken from O onto the equinoctial line of the dial are transferred to either side from I, and through them lines are drawn parallel to the meridian line, the tropics will be cut at the points through which the lines of the ascending signs pass. For through the points of the tropic of ♑ (Capricorn) run the lines of the signs of the semicircle ADC — namely ♒ (Aquarius) and ♓ (Pisces) in the hours before noon, ♈ (Aries) at the very hour of noon, and ♉ (Taurus) and ♊ (Gemini) in the hours after noon; while through the points of the tropic of ♋ (Cancer) run the lines of the remaining signs, opposite to these — namely ♏ (Scorpio) and ♐ (Sagittarius) in the hours after noon, ♎ (Libra) at the very hour of noon, and ♌ (Leo) and ♍ (Virgo) in the hours before noon.

[Margin: Description of the ascending signs on the dial by means of the next two figures, according to the practice of Andreas Schöner.]

Again, on the straight line ED of the latter figure, take EM equal to that same meridian line EI of the dial, intercepted between E, the center of the equator, and the equinoctial line; and through M draw a line parallel to AC. Thus, again, when the center E of this figure is placed on the center E of the equator of the dial, and the point M on the point I, the parallel drawn through M will coincide with the equinoctial line. For this parallel will cut the lines of the signs sent out from E at points which, taken from M and transferred from I onto the equinoctial line of the dial, will give the points through which the lines of the ascending signs must be drawn on the equinoctial line — with this rule, however: that the points of the signs ♉ (Taurus), ♊ (Gemini), ♋ (Cancer), ♏ (Scorpio), and ♐ (Sagittarius) be transferred to the hours before noon, and the points of the remaining signs to the afternoon hours, in that very order which the signs have in the parallel drawn through M, supposing it were superimposed on the equinoctial line of the dial in such a way that the center E fell upward (which would happen if E, the center of the equator, had been taken in the dial toward the point H), and the line ♈ (Taurus), or the point A, of this figure, toward the right-hand part of the dial, and so on.

Now if these points of the equinoctial line are joined by straight lines to the corresponding points on the tropics, the lines of the ascending signs will be

obtained, as before. But the lines ♈ (Aries), ♎ (Libra), ♋ (Cancer), and ♏ (Capricorn) are drawn as we said before. No demonstration of this operation is offered by Andreas Schöner; content with the practice alone — which is without doubt most beautiful — he committed it to writing.

[Margin: By the ascending signs described on the dial, the situation of the whole heaven at any moment of time is explored.]

Moreover, the ascending signs described on the dial have a remarkable and striking usefulness: for by them we may explore, at any moment of time, almost the whole disposition of the heaven. From the shadow of the gnomon falling on some line of the ascending signs, we immediately know which sign is at that time rising above the horizon, and consequently which is setting — since the opposite sign must then set. Again, from the rising sign we shall easily arrive at knowledge of the sign that then culminates, and thus also of its opposite, which occupies the angle of the earth, from what we set out a little earlier in this proposition. That we may achieve this more easily, two little tables must be drawn, one to the left part of the dial and the other to the right. In the former are contained these six ascending signs — ♈ (Aries), ♉ (Taurus), ♊ (Gemini), ♋ (Cancer), ♌ (Leo), and ♍ (Virgo) — together with the six opposite signs which then descend below the horizon, and with another six which are found on the meridian above the horizon at that time, and finally with six which are opposite to these last and lie on the meridian below the horizon. In the latter table are placed the other six ascending signs — ♎ (Libra), ♏ (Scorpio), ♐ (Sagittarius), ♑ (Capricorn), ♒ (Aquarius), and ♓ (Pisces) — together with the descending signs, those culminating, and those in the angle of the earth: such as are the following two tables, constructed for latitude 42 degrees.

THERE FOLLOW THE TABLES.

The two tables of the ascending signs (for latitude 42°)

These are the two little tables promised at the close of the preceding page — the one fitted to the left part of the dial (the six ascending signs ♈–♍), the other to the right part (the six ascending signs ♎–♓). In the print they stand side by side as a single wide table; they are given here as two, for clarity. Under each ascending sign are placed: the sign then descending [Descendential], the point then on the mid-heaven [In medio cœli, the MC — degrees, minutes, and sign], and the point then in the angle of the earth [In angulo terræ, the IC — degrees, minutes, and sign].

Left-hand table — the ascending signs Υ , φ , Π , $\mathfrak{S}$, ϱ , $\mathfrak{M}$.

Ascendentia (rising)	Descendentia (setting)	In medio cœli (MC) — G. M. sign	In angulo terræ (IC) — G. M. sign
Υ	$\underline{\alpha}$	0. 0. ϑ	0. 0. $\mathfrak{S}$
φ	$\mathfrak{M}$	15. 59. ϑ	15. 59. $\mathfrak{S}$
Π	$\mathfrak{A}$	6. 3. $\mathfrak{M}$	6. 3. ϱ
$\mathfrak{S}$	ϑ	5. 6. $\mathfrak{K}$	5. 9. $\mathfrak{M}$
ϱ	$\mathfrak{M}$	13. 59. Υ	13. 59. $\underline{\alpha}$
$\mathfrak{M}$	$\mathfrak{K}$	23. 57. φ	23. 57. $\mathfrak{M}$

Right-hand table — the ascending signs $\underline{\alpha}$, $\mathfrak{M}$, $\mathfrak{A}$, ϑ , $\mathfrak{M}$, $\mathfrak{K}$.

Ascendentia (rising)	Descendentia (setting)	In medio cœli (MC) — G. M. sign	In angulo terræ (IC) — G. M. sign
$\underline{\alpha}$	Υ	0. 0. $\mathfrak{S}$	0. 0. ϑ
$\mathfrak{M}$	φ	6. 3. ϱ	6. 3. $\mathfrak{M}$
$\mathfrak{A}$	Π	16. 1. $\mathfrak{M}$	16. 1. $\mathfrak{K}$
ϑ	$\mathfrak{S}$	24. 54. $\underline{\alpha}$	24. 54. Υ
$\mathfrak{M}$	ϱ	23. 57. $\mathfrak{M}$	23. 57. φ
$\mathfrak{K}$	$\mathfrak{M}$	14. 1. $\mathfrak{A}$	14. 1. Π

Text

Therefore, whenever I detect from the shadow of the gnomon that (for example) the first point of Π (Gemini) is rising, I understand at once, without any delay, that the first point of $\mathfrak{A}$ (Sagittarius) is setting, that 6°3' of $\mathfrak{M}$ (Aquarius) stands on the mid-heaven, and that 6°3' of ϱ (Leo) is set in the angle of the earth — and so with the rest. Once these four cardinal points of the heaven are known in this way, anyone may easily arrive by conjecture at which regions of the heaven the other signs of the zodiac then occupy; and further, if the places of the stars in the zodiac are known, together with their declination or latitude, at which part of the heaven this or that star then lies, even when it is not visible.

[Margin: On any given day, the shadow of the gnomon is to be observed only on the six lines of those six signs which rise above the horizon that day.]

Now since, on any artificial day whatever, only six signs rise, it follows that only six lines of the ascending signs, on any given day, mark out the rising signs —

namely the lines of those six signs which rise during the daytime; and this reckoning is taken from the place of the Sun, following the succession of the signs. For on that day the ecliptic takes only those positions which those lines represent, since the remaining signs do not rise during that day. Hence, although the shadow of the style may fall on the line of some sign that is not among the six rising that day, one must not on that account judge that this sign is then ascending — for it cannot rise at all on that whole day, nor can the ecliptic then have the position which that line expresses. For example: when the Sun is at the beginning of ♈ (Aries), only these six signs rise during the day — ♈, ♉, ♊, ♋, ♌, and ♍ — while, as the Sun sets, the beginning of ♎ (Libra) rises. So at that time these six signs — ♎, ♏, ♐, ♑, ♒, and ♓ — do not rise above the horizon during the day, even though the shadow of the gnomon may fall on their lines. By the same reasoning, when the Sun is about the middle of ♊ (Gemini), only six signs rise on that day: the remaining part of ♊, then ♋, ♌, ♍, ♎, and as much of ♐ (Sagittarius) as the Sun has traversed by its own motion in ♊. Thus, if the Sun is at 18° of ♊, then 18° of ♐ will rise. But the other six signs — namely the remaining 12° of ♐, then ♑, ♒, ♓, ♈, ♉, and 18° of ♊, which have already risen before the Sun — will not come up above the horizon on that day; and so with the rest. On any given day, therefore, in order to know which signs rise and which do not, and consequently which lines of the ascending signs we ought then to observe, we must first find out where the Sun is in the zodiac. For throughout that whole day, only the shadow of the style falling on the line of the six signs that can rise above the horizon is to be observed. Indeed it often happens that on the same day the shadow of the style falls twice on the same line — as is plain in the line of ♈ and in the line of ♓, when the Sun in the ecliptic is not far from the beginning of ♑ (Capricorn).

Diagram. The first (upper) of the two dials of the ascending signs, corresponding to the descending semicircle of the ecliptic — that is, while the Sun runs from ♋ (Cancer) through ♎ (Libra) as far as ♑ (Capricorn). From a central point a set of straight lines (the lines of the ascending signs) radiates out; a vertical line (the meridian) and a horizontal line (the equinoctial line) cross near the center. Each sign-line terminates at a point on the border marked with its zodiac symbol. The engraved caption reads "A ♋ ad ♑". Legibly read zodiac labels: on the upper-left, ♒ (Aquarius) and ♓ (Pisces); on the lower-left, ♉ (Taurus) and ♊ (Gemini); on the upper-right, ♑ (Capricorn), ♌ (Leo), ♍ (Virgo), and ♎ (Libra); near the center, ♏ (Scorpio), ♍ (Virgo), and ♐ (Sagittarius). Several of the small border labels are worn and could not be resolved with certainty [?].

Diagram. The second (lower) dial of the ascending signs, corresponding to the other (ascending) semicircle of the ecliptic — while the Sun runs from ♑ (Capricorn) through ♈ (Aries) as far as ♋ (Cancer). Its construction mirrors the first: a central point from which the sign-lines radiate, crossed near the center by a vertical (meridian) and a horizontal (equinoctial) line, with the sign-lines ending at labelled points around the margin. The engraved caption reads "A ♑ ad ♋". Legibly read zodiac labels: on the left, ♐ (Sagittarius), ♏ (Scorpio), ♑ (Capricorn), ♎ (Libra), and ♋ (Cancer); on the right, ♋ (Cancer), ♊ (Gemini), ♉ (Taurus), ♈ (Aries), ♊ (Pisces), and ♒ (Aquarius); with ♌ (Leo) and further sign-marks toward the center. Several small labels are worn [?].

So that every occasion of error may be removed, two dials of the ascending signs must be drawn, in such a way that the lines of the signs in no way intersect one another — as appears in the two figures set here side by side. The first indicates the ascending signs for as long as the Sun runs through the descending semicircle of the ecliptic, from ♋ (Cancer), through ♎ (Libra), as far as ♑ (Capricorn); the second, while the Sun is in the other semicircle of the ecliptic, from ♑, through ♈ (Aries), as far as ♋ — the semicircle that is called ascending. For it will then come about that, when the shadow of the gnomon falls on the line of some sign in that figure which is fitted to the semicircle of the ecliptic in which the Sun then is, that sign necessarily rises, without any exception — provided the observation is made at the time of year in which the Sun is running through the said semicircle of the ecliptic.

Now, so that in figures of this kind the lines of the ascending signs may not intersect one another, care must be taken that no line touching the tropics is drawn whole; rather, one portion is drawn as far as the tropic on one side in the first figure, and the other portion from the other side to the same tropic in the second figure, as is seen in the said figures. It will then be no great task to decide which of the two figures belongs to the part of the year in which the Sun moves from ♋, through ♎, to ♑, and which to the other part, in which the Sun climbs from ♑, through ♈, to ♋ — if one attends carefully to which signs can rise while the Sun is in this or that part of the ecliptic.

[Margin: How two dials of the ascending signs are to be drawn, in which the lines of the signs do not intersect one another, and of which the one is fitted to one half of the year, the other to the other half.]

For example: while the Sun stands at the beginning of ♎ (Libra), these signs rise during the day — ♎, ♏, ♐, ♑, ♒, ♓ — whose lines cut the equinoctial line, in the same order, in the first figure. The first figure will therefore belong to that part of

the year which answers to the descending semicircle, in which, namely, the sign $\underline{\alpha}$ is contained. The second figure, on the other hand, will belong to the other part of the year, in which the sign γ is included; for when the Sun is at the beginning of γ , these six signs rise — γ , ϑ , π , φ , ϱ , $\mathfrak{m}$ — whose lines intersect the equinoctial line of the second figure in the same order. We shall recognize the same thing when the Sun is in any other sign. For when the Sun (for example) is at the beginning of ϱ (Leo), these signs rise during the day — ϱ , $\mathfrak{m}$, $\underline{\alpha}$, $\mathfrak{m}$, $\nearrow$, and $\mathfrak{v}$ — and the shadow of the style sweeps the space between the tropic of φ (Cancer) and the equinoctial line. Now I see that all these signs are not contained in the second figure above the equinoctial line — that is, between the tropic of φ and the equinoctial line — but in the first; I therefore conclude that the first figure suits that part of the year in which the sign ϱ is found, namely while the Sun descends from φ to $\mathfrak{v}$. I understand the same when the Sun is tarrying at the beginning of $\mathfrak{v}$. For then these six signs rise — $\mathfrak{v}$, $\approx$, $\times$, γ , ϑ , and π — all of which are contained in the first figure below the equinoctial line, namely between the tropic of $\mathfrak{v}$ and the equinoctial line, where the shadow of the gnomon then lies; and so it is to be judged with the rest. All this is to be understood likewise for the ascending signs of the dials that follow. For in each case four figures can be drawn — two for one face of the plane of the dial, and another two for the opposite face. This will be no very difficult matter, if what we have just said is rightly grasped. For brevity's sake, in what follows we shall everywhere draw a single figure containing all the ascending signs, so that it may be divided into four particular ones, as we have taught, should use require it.

[Margin: In dials that are not horizontal, four figures of the ascending signs must be drawn.]

Now since, for the description of the ascending signs and for investigating the culminations of the heaven [cœli mediationes], right and oblique ascensions [ascensiones rectæ, & obliquæ] are needed — and not everyone has their tables ready to hand — I think it worth the effort to show briefly here in what way both the right and the oblique ascension of any point of the ecliptic is to be drawn out from the doctrine of sines [ex sinuum doctrina]. For more on this matter has been demonstrated by us elsewhere.

[Margin: How the right ascension of any point of the ecliptic is to be found.]

Let the right horizon, or the meridian, or any other great circle ABCD be drawn through the poles of the world A, C; let BGD be the equator, and EGF the ecliptic; let the point G be the beginning of γ (Aries), and let the arc GE of the ecliptic be less than a quadrant, in such a way that its right ascension — that is, its ascension

in the right sphere — is the arc GB of the equator. Since in the spherical triangle EBG the angle B is right, it will follow — by the proposition of Johannes Regiomontanus, *On Triangles*, book 4, or by *Geber* [Jābir ibn Aflāḥ], book 1, prop. 15, or by prop. 43 of our own *[Book of] Spherical Triangles* — that as the sine of the complement of the arc EG of the ecliptic (which is taken as known, since it is its right ascension that is sought) is to the sine of the complement of the arc BE, the declination of the given point E, so is the sine of the complement of the arc BG, the right ascension sought, to the whole sine [sinus totus]. And, by inversion, as the sine of the complement of the arc BE (the declination) is to the sine of the complement of the arc EG (the given ecliptic arc), so is the whole sine to the sine of the complement of the arc BG, the right ascension.

Diagram. Three spherical figures set side by side, illustrating the three cases of the computation. In each, a great circle ABCD is drawn with A at the top (north pole of the world) and C at the bottom (south pole); the equator BGD and the ecliptic EGF cross at G. The given ecliptic point is E; the arc BE is its declination and the arc BG its right ascension. Legibly read labels — first (left) figure: A (top), B (left), G (center, marked with Υ), E (lower-left), C (bottom), with D and F on the right. Second (middle) figure: A (top), F and D (upper-left), B and I (left), E (lower-left), G (center, Υ), H (lower-right), C (bottom). Third (right) figure: A (top), E (upper-left), G (center, marked with a small crown-shaped symbol), I, D, B (left cluster), H (right), F and D (lower-right), C (bottom). Several inner labels are small and worn and could not all be resolved with certainty [?].

Therefore, if the following proportion is made — as the sine of the complement of the declination of the given ecliptic point is to the sine of the complement of the arc of the ecliptic between the given point and the beginning of Υ , so is the whole sine to a fourth term — the result will be the sine of the complement of the right ascension. Example: Let the point E be at 10° of Π (Gemini), so that the arc GE of the ecliptic comprises 70° , and the arc BE, the declination, found from the corollary of prop. 1 of the preceding book, is $22^\circ 0'$. Then, if the proportion is made — as 92718 (the sine of the complement of the declination) is to 34202 (the sine of the complement of the given ecliptic arc GE), so is 100000 (the whole sine) to a fourth term — there will be found here, very nearly, the sine 36888, to which corresponds an arc of $21^\circ 39'$, whose complement, $68^\circ 21'$, gives the right ascension of 10° of Π ; and so with the rest. By this art the right ascensions of all arcs of the ecliptic from the beginning of Υ up to the end of Π (Gemini) can be computed.

[Margin: When the arc of the ecliptic is less than a quadrant.]

If the arc of the ecliptic is a quadrant — that is, if it is the beginning of $\mathfrak{S}$ (Cancer) — then its right ascension too will be a quadrant. For since, in the triangle BEG, the angle B is right and the arc GE is a quadrant, it will follow by prop. 36 of our *[Book of] Spherical Triangles* that at least one of the remaining [sides] is a quadrant. Since, then, BE is less than a quadrant — being, namely, equal to the greatest declination of the Sun — GB will be a quadrant. This can also be confirmed thus. Since the arc GE of the ecliptic is a quadrant and G is the beginning of $\mathfrak{V}$, the colure of the solstices will pass through the point E, because the arc of the ecliptic between the beginning of $\mathfrak{V}$ and the colure of the solstices is a quadrant. Since, then, this same colure is drawn through the poles of the world, it will not differ from the great circle ABCD; and consequently, since the arc of the equator between the beginning of $\mathfrak{V}$ and the colure of the solstices is a quadrant, the arc GB will be a quadrant throughout.

[Margin: When the arc of the ecliptic is a quadrant.]

If the given arc of the ecliptic is greater than a quadrant but less than a semicircle — so that, for example, the point E is at 20° of $\mathfrak{S}$ (Cancer) and the given arc comprises 110° — we shall subtract it from a semicircle, and for the remaining arc of 70° , begun from the other equinox, we shall investigate the right ascension as before, and take it away from the semicircle. There will then remain the right ascension of the given arc, since the whole semicircle from $\mathfrak{V}$ to $\mathfrak{L}$ has a corresponding semicircle of the equator for its right ascension. Thus, for example, 70° has a right ascension of $68^\circ 21'$, which, taken from 180° , leaves a right ascension of $111^\circ 39'$, answering to the arc of the ecliptic of 110° , that is, to the 20th degree of $\mathfrak{S}$.

[Margin: When the arc of the ecliptic is greater than a quadrant but less than a semicircle.]

But if the arc of the ecliptic, begun from the beginning of $\mathfrak{V}$, is a semicircle, so that it ends at the beginning of $\mathfrak{L}$, then its right ascension too will be a semicircle, as is plain.

[Margin: When the arc of the ecliptic is a semicircle.]

When the arc of the ecliptic exceeds a semicircle, yet in such a way that it is less than three quadrants, we shall take from it a semicircle, and to the right ascension of the remaining arc, begun from the beginning of $\mathfrak{L}$ and found as before, we shall add the semicircle, so as to have the right ascension of the given arc. Thus, if the right ascension of 10° of $\mathfrak{L}$ (Libra) is sought, so that the whole arc of the ecliptic reckoned from the beginning of $\mathfrak{V}$ contains 250° : having subtracted

the semicircle from 250° , there remain 70° , whose right ascension of $68^\circ 21'$, found as before, if added to the semicircle, makes up $248^\circ 21'$ for the right ascension of the given arc.

[Margin: When the arc of the ecliptic is greater than a semicircle but less than three quadrants.]

But if the given arc comprises three quadrants, it will have three quadrants also for its right ascension, as is plain from what has been said. For the first semicircle has a semicircle, and the remaining quadrant a quadrant, and so on.

[Margin: When the arc of the ecliptic comprises three quadrants.]

If, finally, the proposed arc exceeds three quadrants, it must be taken away from the whole circle, and the right ascension of the remaining arc, begun from Υ and found as before, must be subtracted from the whole circle. For there will then be left the right ascension of the given arc, since the whole ecliptic ascends together with the whole equator. Thus, if the given arc contains 290° , so that it ends at 20° of ♑ (Capricorn): we shall deduct 290° from 360° , and the right ascension of the remaining arc of 70° — $68^\circ 21'$, found as before — we shall subtract from 360° , so that $291^\circ 39'$ is left for the right ascension of the proposed arc.

[Margin: When the arc of the ecliptic is greater than three quadrants.]

It is clear from all this in what way a table of right ascensions is to be constructed. For if the right ascensions are investigated of all the arcs of the ecliptic from the beginning of Υ up to the end of ♊ (Gemini), we shall have the right ascensions of all the points of the first quadrant of the ecliptic. If we subtract each of these from a semicircle, beginning with the greater (that is, the later) ones, the remainders will be the right ascensions of all the points of the second quadrant of the ecliptic. Again, if we add the same values to a semicircle, beginning with the smaller (that is, the earlier) ones, there will result the right ascensions of all the points of the third quadrant of the ecliptic. And if, finally, we take the same values away from the whole circle, beginning again with the greater (that is, the later) ones, there will remain the right ascensions of all the points of the last quadrant of the ecliptic. Thus the whole labor lies in the investigation of the right ascensions of all the points of the first quadrant of the ecliptic, begun at the beginning of Υ and ending at the end of ♊ .

[Margin: By what method the table of right ascensions is to be constructed.]

Diagram. The same three-sphere woodcut block as on the preceding page, here serving the demonstration of the oblique ascension (the demonstration refers to

the "second figure" and the "third figure"). In each of the three spheres: A at the top, C at the bottom; the great circle ABCD, the equator (BD / BGD), and the ecliptic (EF / EGF) crossing at G; H a pole, I a point on the equator, E the given ecliptic point. Legibly read labels match those of fig-p161-1 (A, B, C, D, E, F, G, H, I), with the third figure again showing a small crown-shaped mark near G. Several inner labels are small and worn [?].

[Margin: The difference between the right and the oblique ascension of any point of the ecliptic — by what path it is to be explored, given the altitude of the pole.]

Let, again, the oblique horizon be ABCD; the equator BD; the ecliptic EF; the beginning of Υ the point G in the second figure, and in the third figure the beginning of α , the same point [G]; the meridian AC; and let the arc GE of the ecliptic, begun from the beginning of Υ or of α , be less than a quadrant, in such a way that its oblique ascension — that is, its ascension in the oblique sphere — is the arc GB of the equator, which it is our task to investigate. Let a great circle be drawn from the pole of the world H, through E, cutting the equator at I, in such a way that GI is the right ascension of the same ecliptic arc GE. Since, then, in the spherical triangle CEH, in which the angle C is right, it holds — by prop. 16 of book 4 of Johannes Regiomontanus, *On Triangles*, or by prop. 16 of book 1 of *Geber*, or by prop. 41 of our own *[Book of] Spherical Triangles* — that as the sine of the arc HE (that is, as the sine of the complement of the declination of the ecliptic point terminating the arc GE — for in the third figure the same sine is the sine of the arc HE, and the arc HE, together with the complement of the declination EI, makes up the [colure through the] beginning of Υ) is to the sine of the angle CEH (which in the second figure is equal to the angle BEI). Again, by the same reasoning, and by inversion, in the spherical triangle BEI it holds that as the sine of the angle EBI — the complement of the altitude of the pole, which the equator makes with the horizon — is to the sine of the arc EI, the declination, so is the sine of the angle BEI, just found, to the sine of the arc BI, by which the right ascension GI differs from the oblique ascension GB. If, then, the proportion is made — as the sine of the complement of the declination of the point terminating the ecliptic arc is to the whole sine, so is the sine of the altitude of the pole to a fourth term — there will be found the sine of the angle BEI. And if, again, the proportion is made — as the sine of the complement of the altitude of the pole is to the sine of the declination of the same arc, so is the sine of the angle BEI, just found, to a fourth term — there will be found the sine of the arc BI, the difference between the right and the oblique ascension answering to the ecliptic arc GE.

Example: Let the point E be placed, in the second figure, at 29° of $\varnothing$ (Taurus), and in the third at 29° of $\mathfrak{M}$ (Scorpio), so that the arc GE comprises 59° , and the arc EI, the declination, $19^\circ 59'$. If, then, the proportion is made — as 93979 (the sine of the complement of the declination) is to 100000 (the whole sine), so is 66913 (the sine of the altitude of the pole) to a fourth term — there will be found here, very nearly, the sine 71199 of the angle E, which is to be kept. Next, if the proportion is made — as 74314 (the sine of the complement of the altitude of the pole) is to 34174 (the sine of the declination), so is 71199 (the sine kept) to a fourth term — there will come out, nearly, the sine 32741, whose arc contains $19^\circ 7'$, for the difference between the right and the oblique ascension of the arc GE. If this difference, in the second figure, is subtracted from the right ascension GI already found before — because the great circle HE, which plays the part of the right horizon, cuts the equator below the horizon, since the half of the zodiac from γ to α is northern — there will remain the oblique ascension GB, $37^\circ 39'$, belonging to the northern ecliptic arc GE. But if the same difference, in the third figure, is added to the right ascension GI — because the great circle HE cuts the equator above the horizon, inasmuch as the half of the zodiac from α to γ is southern — there will result the oblique ascension GB, $75^\circ 53'$, agreeing with the southern ecliptic arc GE, reckoned from the beginning of α ; to which, if a semicircle is added, there will be established the oblique ascension $255^\circ 53'$, belonging to the ecliptic arc begun from the beginning of γ and running up to 29° of $\mathfrak{M}$ (Scorpio).

[Margin: By what method the oblique ascension of any point of the ecliptic is to be found.]

The same difference BI, between the right and the oblique ascension, will also be found in another way, thus. Since in the spherical triangle BEI the angle I is right, it will follow — by prop. 19 of book 4 of Johannes Regiomontanus, *On Triangles*, or by prop. 15 of book 1 of *Geber*, or by prop. 43 of our own *[Book of] Spherical Triangles* — that as the sine of the complement of the arc BE, the eastern amplitude [latitudo ortiva], whose finding we set out in the earlier discussion, prop. 34 of the preceding book, is to the sine of the complement of the arc EI, the declination of the given ecliptic point, so is the sine of the complement of the arc BI, the difference sought, to the whole sine. And, by inversion, as the sine of the complement of the declination is to the sine of the complement of the eastern amplitude of the same point, so is the whole sine to the sine of the complement of the difference of ascensions sought.

[Margin: How this same difference between the right and the oblique ascension is to be investigated by another route from the point of the ecliptic itself, for a given altitude of the place.]

More easily: Make the proportion — as the whole sine is to the tangent of the arc of declination IE, so is the tangent of the complement of the angle EBI (the altitude of the equator) — that is, the tangent of the altitude of the pole — to a fourth term; and there will be found the sine of the arc BI (as before, the difference of ascensions to be added to the semicircle is what is sought), as is established from the problem, prop. 49 of the *[Book of] Spherical Triangles*, since the angle EIB is right, IE is the arc of declination, and EBI is the angle of the altitude of the equator. Example: 29° of ♉ (Taurus), or 29° of ♏ (Scorpio), whose declination contains $19^\circ 59'$. If, then, the proportion is made — as 100000 (the whole sine) is to 36364 (the tangent of the declination), so is 90040 (the tangent of the complement of the altitude of the equator) to a fourth term — there will be found the sine 32742, to which correspond $19^\circ 7'$. This, then, is the size of the arc BI in the second and third figures.

Once the ascensional difference of the beginning of ♋ (Cancer), or of ♑ (Capricorn), has been found, together with the right ascensions of all points of the ecliptic, the ascensional differences of all the other points at the same elevation of the pole are also easily found. Set it up so that, as the whole sine is to the sine of the right ascension of the given point, so is the sine of the ascensional difference of the beginning of ♋, or of ♑, to a fourth term; and the sine of the ascensional difference sought will be found. Take the same nearby example: if it is made — as 100000 (the whole sine) is to 83645 (the sine of the right ascension of 29° of ♉ (Taurus), or of ♏ (Scorpio), which right ascension is $56^\circ 46'$ in the table of right ascensions), so is 39153 (the sine of the ascensional difference of the beginning of ♋, or of ♑, which at latitude 42° is $23^\circ 3'$) to a fourth term — there will be found the sine 32749, to which correspond $19^\circ 7'$, as before. This is demonstrated by Johannes Regiomontanus [Regiomontanus], prop. 24 of book 2 of the *Epitome*.

Now the declinations of all points of the ecliptic from the beginning of ♈ (Aries) up to the beginning of ♋ are equal to the declinations of all points of the ecliptic from the beginning of ♎ (Libra) up to the beginning of ♏ — proceeding against the succession of the signs, each matched to its own — because points of this kind describe the same parallels, two to each single one. Again, the declinations of all points of the ecliptic from ♈ to ♎ are equal to the declinations of all points from ♎ to ♈ — proceeding according to the sequence of the signs, each to its own — since these points are opposite to those and therefore describe equal parallels. It

follows that the declinations of the first quadrant of the ecliptic, from γ up to δ , are equal to the declinations of the three later quadrants, as appears in the table of declinations. Therefore, since the eastern amplitudes [latitudines ortivæ] are investigated by means of the declinations and the altitude of the pole — as is clear from prop. 34 of the preceding book — it is manifest that the eastern amplitudes of the points of the first quadrant of the ecliptic, found for any latitude, are equal to the eastern amplitudes of the three later quadrants at the same latitude. And therefore, since the differences between the right and the oblique ascensions are inquired — as we demonstrated just now — through the declinations and the eastern amplitudes, it is plainly established that the differences between the right and oblique ascensions of all points of the first quadrant of the ecliptic, ascertained for any altitude of the pole, are equal to the differences of ascension of all points of the three later quadrants of the ecliptic at the same altitude of the pole. Wherefore it will suffice if the differences of ascension belonging to the points of the first quadrant of the ecliptic, from the beginning of γ up to the beginning of δ , are inquired.

[Margin: The ascensional differences of the points of the first quadrant of the ecliptic, from Aries up to Cancer, are equal to the ascensional differences of the three later quadrants of the ecliptic.]

From these differences of ascension of the first quadrant of the ecliptic we shall now compose a table of the oblique ascensions of all points of the ecliptic, for the given altitude of the pole for which the differences have been worked out. In the half of the ecliptic from γ up to α , the difference of ascension must always be subtracted from the right ascension in order to obtain the known oblique ascension. This is because this half is northern, and consequently the great circle drawn from the pole of the world through any point of it cuts the equator below the horizon — as is evident from the first of the two nearby figures. Hence the difference of ascension of every point of the first semicircle, taken away from the right ascensions of those same points, leaves their oblique ascensions. On the other hand, in the other semicircle of the ecliptic, from α up to γ , the difference of ascension must always be added to the right ascension in order to learn the oblique ascension. This is because this semicircle is southern, and so the great circle drawn from the pole of the world through any point of it cuts the equator above the horizon — as is established from the second, adjacent figure. It is therefore clear that the differences of ascension of all points of this later semicircle, added to the right ascensions of those same points, produce their oblique ascensions.

Example. The difference of ascension of 29° of ♋ , which we found at latitude 42° to be $19^\circ 7'$, is equal to the difference of ascension of 1° of ♌ (Leo), of 29° of ♍ (Scorpio), and of 1° of ♎ (Aquarius); for these four points have the same declination. If, then, we subtract it from the right ascension of 29° of ♋ — that is, from $56^\circ 46'$ — there will remain the oblique ascension of 29° of ♋ , namely $37^\circ 39'$. Likewise, if we subtract the same difference from the right ascension of 1° of ♌ — namely from $123^\circ 14'$ — there will remain $104^\circ 7'$ for the oblique ascension of 1° of ♌ . Again, if we add the same difference to the right ascension of 29° of ♍ , which is $236^\circ 46'$, we shall produce $255^\circ 53'$ for the oblique ascension of 29° of ♍ . Finally, if we add the same difference to the right ascension of 1° of ♎ , which comprises $303^\circ 14'$, there will be formed the arc of $322^\circ 21'$ for the oblique ascension of 1° of ♎ . And the same reckoning is to be applied to the rest. It has been enough to give this brief taste, in this place, of right and oblique ascensions, for we have discussed them more fully in another place.

[Margin: How the table of oblique ascensions is to be composed from the ascensional differences.]

PROBLEMA 10. PROPOS. 10.

To construct an Italian horizontal sundial. That is, to draw the hour-lines reckoned from sunset upon a plane parallel to the horizon.

[Margin: The construction of the Italian horizontal sundial.]

Let the circle drawn about center E, set out in prop. 1 of this book, be cut into two portions — as we taught in the scholium to prop. 1 of the preceding book — of which a N b is similar to the diurnal arc of the tropic ♊ , and a M b to the nocturnal arc of the same tropic. Now divide this same circle described about E into 24 equal parts, beginning at point b, which lies to the west on the common section of the horizon and of the parallel a b. (This is clear if the said circle described about E is conceived to be placed in its proper position.) Through these division-points, and through the center E — provided the circle stands in its proper position — will pass the great circles drawn through the poles of the world and through the hours from sunset in the tropic ♊ , after the manner of the hour-circles reckoned from noon or from midnight. For since the great circles drawn through the poles of the world divide the tropic ♊ and the equator into similar arcs, by prop. 10 of book 2 of Theodosius, the arc of the equator intercepted between the meridian and the great circle drawn through the 24th hour from sunset in the tropic ♊ — that is, through the point where the tropic ♊ cuts the horizon — will be similar to the arc of the tropic ♊ placed between the

same circles, that is, to the semidiurnal arc of the tropic $\mathfrak{S}$. And since the common sections which the said great circles make with the equator, drawn through the center E of the equator, likewise cut the equator and the circle MaNb (described about E, the center of the equator, and in the same plane) into similar arcs — by what we demonstrated at the end of chapter 1 of the *Sphere* — the arc of the circle MaNb placed between the said common sections will also be similar to the semidiurnal arc of the tropic $\mathfrak{S}$. Therefore, since MN is the section made in the equator by the meridian — so that the meridian is drawn through the point N — and the arc N b is similar to the semidiurnal arc of the tropic $\mathfrak{S}$ (for since the whole arc b N a is similar, by construction, to the whole diurnal arc of the tropic $\mathfrak{S}$, then bN, the half of that arc b N a, will also be similar to the semidiurnal arc of the tropic $\mathfrak{S}$), and therefore also similar to the arc of the equator intercepted between the aforesaid sections, the other great circle, drawn through the 24th hour from sunset in the tropic $\mathfrak{S}$, will necessarily pass through the point b. For if it passed through another point, the said sections would not cut off from the equator an arc similar to the arc bN, but one similar to the arc lying between the straight line EN and that other

point would be interposed — which is absurd. For we showed that the arc taken away from the equator is similar to the arc bN, because bN is similar to the semidiurnal arc of the tropic $\mathfrak{S}$. And since the arc between b and the next division-point toward N is likewise similar to the arc of the tropic $\mathfrak{S}$ intercepted between the two great circles drawn through the poles — one of which is drawn through the 24th hour from sunset (that is, through the point b), the other through the 23rd hour from sunset in the tropic $\mathfrak{S}$ — inasmuch as each arc is a 24th part of its own circle, then the great circle drawn through the poles of the world and through the 23rd hour from sunset in the tropic $\mathfrak{S}$ will also pass through the next division-point, proceeding from the point b toward N. For in this way the straight lines drawn from the center E through b and through the next point toward N — that is, the sections made by those two great circles drawn through the poles and through the 24th and 23rd hours from sunset in the tropic $\mathfrak{S}$ — will cut off from the equator an arc similar to the arc of the tropic placed between the 24th and 23rd hours, as Theodosius requires (prop. 10, book 2); which otherwise would not happen. For if the said circle passed through another point, the arc of the circle MaNb between b and that point would be greater or less than a 24th part; and consequently the arc of the equator similar to it (as is clear from what we demonstrated at the end of chapter 1 of the *Sphere*) would be greater or less than a 24th part, and therefore not similar to the aforesaid arc of the tropic $\mathfrak{S}$ — which is absurd. In the same way the next point toward N will indicate the 22nd hour, and the rest the remaining hours.

Now, from the circle MaNb divided in this manner, we shall find on the arc $\mathfrak{S}$ the points of the hours from sunset, in this way. Through the division-points and the center E we shall draw hidden [construction] straight lines cutting the equinoctial line at certain points; and if through these points hidden straight lines are drawn from H, the center of the sundial, the tropic $\mathfrak{S}$ will be cut at the points of the hours from sunset.

Diagram. A large circle representing the equator / hour-circle construction MaNb of the Italian horizontal sundial, with the vertical diameter M–N as the meridian (M at top, N at bottom) and a horizontal diameter across the middle as the equinoctial line. H, near the upper center, is marked as the center of the sundial (the foot of the axis/style); E, at the middle-lower center (among the numerals 19, 20, 21), is the center of the equator. A dense pencil of straight construction-lines (both solid and dotted) radiates from the central region toward hour-points distributed around the circumference. The circle carries two nested chords (a b above and, lower, the Capricorn chord) cutting off the diurnal/nocturnal arcs, and the hours from sunset are numbered around it.

Confidently read labels:

- M (top): meridian point, upper end of the vertical diameter
- N (bottom): meridian point, lower end of the vertical diameter
- H (upper center): center of the sundial (foot of the style)
- E (lower center): center of the equator
- a (upper left, on/near the circle): endpoint of the upper chord a b
- b (lower left): endpoint of the diurnal chord
- c (right, near the lower-right): chord endpoint on the right
- d (lower left, on the circle): endpoint of the Capricorn chord
- t (mid-left): peripheral point
- m (left, below t): peripheral point
- u (mid-right): peripheral point
- n (far right, on the equinoctial line extended): peripheral point
- Large hour-numerals flanking the horizontal line: "6" (left) and "6" (right); "5" and "4" descending on the left; "7", "8", "9" on the right; "10" and "1" at upper right
- Interior Arabic hour-numerals along the radiating lines: 1–16 (right and upper), and lower-arc numerals including 23, 22, 21, 20, 19, 18, 17
- Roman numerals along the lower arc: I, II, III (with "12" at bottom center)

- Greek point-letters: α (upper-left, near H), β (bottom center), and further Greek/point marks (λ near lower-left "23") [several [?]]

[?] The interior is extremely crowded; many of the small Arabic hour-numerals and the Greek point-letters cannot be resolved individually at this resolution. This diagram is a (more than 2 labels flagged).

For example: through the point f of the 18th hour from sunset, and through the center E, draw the straight line f E; it cuts the equinoctial line at g. Now the straight line Hg, when drawn, cuts the arc $\mathfrak{S}$ at h — a point which I say belongs to the 18th hour from sunset. For the straight line f g is the common section of the equator and of the great circle drawn through the poles of the world and through the 18th hour from sunset in the tropic $\mathfrak{S}$; because, as we showed, that great circle passes through the point f of the 18th hour from sunset (if the circle MaNb is set in its proper position) and through the center of the world E, so that the said great circle will meet the plane of the sundial at the point g. And since the same circle passes through H, the center of the sundial — by prop. 18 of the preceding book, because it is drawn through the axis — the straight line Hg will be the common section of that same great circle and of the plane of the sundial. Therefore, since (with the Sun standing in that great circle) the shadow of the style falls, by prop. 11 of the same preceding book, upon the straight line Hg, and — with the Sun standing in the tropic $\mathfrak{S}$ — it is cast, by prop. 12 of the same preceding book, onto the arc $\mathfrak{S}$, it comes about that, when the Sun occupies the point of the 18th hour from sunset in the tropic $\mathfrak{S}$ (where the said great circle and the tropic $\mathfrak{S}$ intersect), the shadow of the style is cast onto the point h; and therefore through h the line of the 18th hour from sunset must be drawn. The same reasoning is to be applied to the rest.

But in this way, as you see, there are found on the tropic $\mathfrak{S}$ the points only of those hours which are contained in the semicircle t N u. This is because the common sections of those great circles and of the equator mark, on the equinoctial line, those hours from which they are drawn through the center E, before they meet the equinoctial line — as we said in the scholium to prop. 1 of this book; and of this kind are only those hours found in the semicircle t N u, as is plain. Hence, for the remaining hours — which in our example are the 9th, 10th, and 23rd from sunset, as is seen in the figure — we shall find the points on the tropic $\mathfrak{S}$ in the following way. From the hours which are diametrically opposite to these hours in the circle MaNb (the opposite hours are obtained by adding 12 to those hours, yet casting out 24 if the number collected by the addition is greater than 24 — thus to the 9th hour is opposite the 21st, since 9 and 12 make

21; likewise to the 23rd hour is opposite the 11th, since 23 and 12 make 35, and, 24 being cast out, 11 remain, and so on), we shall draw through the center E straight lines cutting the equinoctial line at certain points; and if through these points straight lines are drawn from H, the tropic $\mathfrak{S}$ will indeed be cut at the points corresponding to those opposite hours, as has been demonstrated. But if those straight lines are produced beyond the center H, the tropic $\mathfrak{S}$ will be cut at the points corresponding to those other hours, which in the semicircle $t N u$ are

not contained, and whose opposites have been taken. For instance, from m , the point of the 11th hour (which is diametrically opposite the 23rd hour), a straight line drawn through the center E cuts the equinoctial line at n ; and the straight line Hn , when drawn, cuts the tropic $\mathfrak{S}$ at q , the point of the 11th hour; and finally the straight line nH , produced beyond the center, cuts the tropic $\mathfrak{S}$ at the point p , which I say must belong to the 23rd hour. For since the great circle drawn through the poles of the world and through the 11th hour of the tropic $\mathfrak{S}$ cuts — by prop. 15 of book 1 of Theodosius — the tropic $\mathfrak{S}$, and hence also the equator (that is, the circle $M a N b$), into two equal parts, it will pass entirely through the 23rd hour, which is a whole semicircle — that is, 12 hours — distant from the 11th hour, so that one of its semicircles shows the 11th hour and the other the 23rd. The straight line mEn is therefore the common section of the equator and of the great circle drawn through the poles of the world and through the 11th and 23rd hours of the tropic $\mathfrak{S}$. Consequently the straight line nHp is the common section of that same great circle and of the plane of the sundial, since the said circle meets the plane of the sundial at the points n and H — yet on this condition, that the portion Hn , which cuts the equinoctial line, represents the semicircle of that great circle passing through the 11th hour in the tropic $\mathfrak{S}$ (whose section, namely mE , is drawn through the center E before it meets the equinoctial line, as we wrote in the scholium to prop. 1 of this book). From which it follows that the remaining portion Hp , produced beyond the center of the sundial and not cutting the equinoctial line, represents the other semicircle, drawn through the 23rd hour, just as we taught in the same scholium. For there too we showed that the hour-line which cuts the equinoctial line indicates the hour of one semicircle, while its portion produced beyond the center, which does not cut the equinoctial line, shows the hour of the other, opposite semicircle — so that those two hours are distant from each other by a whole semicircle, or 12 hours, on the equator, as happens here too. Therefore through the point p the line of the 23rd hour must be drawn. Let the same judgment be applied to the rest. For thus we shall investigate the points on the same tropic $\mathfrak{S}$ for the 9th and 10th hours, if from the 21st and 22nd hours (which are opposite to them) we draw straight lines through the center E, and so on.

Diagram. The same Italian horizontal sundial construction as on the preceding page, redrawn (a cleaner, higher-contrast impression). A large circle (the equator / hour-circle construction M a N b) with the vertical meridian diameter M (top)–N (bottom) and a horizontal equinoctial line across the middle; H near the upper center is the center of the sundial, E at the lower center is the center of the equator, with a dense pencil of solid and dotted construction-lines radiating to the hour-points around the circumference, and the same nested diurnal/nocturnal chords.

Confidently read labels (as on fig-p164-1): M (top), N (bottom), H (upper center), E (lower center), a (upper left), b/d (lower left, chord endpoints), c (right), t and m (mid- and lower-left), u (mid-right), n (far right on the equinoctial line); the flanking large hour-numerals 6, 6, with 5, 4 (left) and 7, 8, 9, 10, 1 (right/upper); interior Arabic hour-numerals 1–16 and, on the lower arc, 17–23; Roman numerals I, II, III and "12" at bottom center; Greek point-letters α (upper left), β (bottom center), λ (lower left).

[?] As on the preceding page, the crowded interior hour-numerals and several Greek point-letters cannot all be individually resolved. This diagram is a (more than 2 labels flagged).

In the same way, if we cut the circle described about E into two portions — of which d N e is similar to the diurnal arc of the tropic φ , and d M e to its nocturnal arc (this is easily done if, for the arcs Ma and Mb, we take equal arcs Nd and Ne, and draw the straight line d e in such a way that the straight lines a b and d e cut off equal lengths Ma[?] and Nb[?] from the diameter MN and are perpendicular to MN) — and distribute it into 24 equal parts, beginning at the point e on the west, we shall find on the arc φ the points through which the hours from sunset must be drawn. For example, if from the point λ [?] of the 23rd hour of the tropic φ we draw through the center E a straight line that cuts the equinoctial line at [the marked point $\cdot$][?], we shall find, by the straight line H[$\cdot$] produced, the point λ [?] on the arc φ for the 23rd hour; and so on for the rest.

But if you do not wish to cut the circle MaNb, described about E, into two portions — one similar to the diurnal arc of the tropic φ or φ , and the other to the nocturnal arc — by the method we set out in the scholium to prop. 1 of the preceding book, the task may be accomplished this way. From the point N count off, on each side, the semidiurnal arc: for the tropic φ as far as the points a and b, and for the tropic φ as far as the points d and e. This is easily done if the circle is secretly divided into 24 equal hours, beginning at the point N, and the hours contained in the semidiurnal arc are counted on each side, and so on. For when

the straight lines a b and d e are drawn, we shall have the same portions as before, corresponding to the diurnal arc of the tropic $\mathfrak{S}$ or $\mathfrak{V}$. Hence the division of the circle must again be begun from the point b, or from the point e.

[Margin: How the circle MaNb may otherwise be divided into the diurnal arc of the tropic of Cancer or of Capricorn.]

We can also fix the beginning of this division at any hour from sunset — even without cutting the circle into the diurnal arc of the tropic $\mathfrak{S}$ or $\mathfrak{V}$ — in this way. Let the semidiurnal arc be subtracted from the 24 hours, so as to obtain the time of noon in the Italian manner. From this time we shall learn how far any proposed hour from sunset is distant from noon. Hence, if from the point N this distance is counted toward b (if the proposed hour is afternoon) or toward a (if forenoon), we shall have the point of the proposed hour, from which the division of the circle into 24 equal parts is to be begun. For example, with the Sun at the beginning of $\mathfrak{S}$, the semidiurnal arc, at latitude 42° , contains 7 hours 32 minutes; when this is subtracted from the 24 hours, there remain 16 hours 28 minutes for the time of noon; and therefore the hour

[Margin: How the circle MaNb can take the beginning of its division at any hour from sunset, even if it is not divided into the diurnal arc of the tropic of Cancer or of Capricorn.]

[Margin: Likewise, by what method any proposed hour from sunrise or sunset, in any parallel whatever, is to be found in the same circle.]

hour 18 from sunset will be distant from the meridian, toward the western quarter, by 1 hour 32 minutes — that is, by 23 degrees. If, then, starting from N toward b we count off 23 degrees, or 1 hour 32 minutes, we shall find the point f of hour 18 from sunset on the tropic $\mathfrak{S}$ (Cancer); and from this point the division of the circle may take its beginning, so that the other hours from sunset may be obtained. Again, hour 11 from sunset will be distant from the same meridian by 5 hours 28 minutes — that is, by 82 degrees. If this distance is counted from N toward a — because hour 11 falls in the forenoon — we shall find the point m of hour 11 from sunset on the tropic $\mathfrak{S}$, and from this point too the division of the circle may take its start. Finally, with the Sun at the beginning of $\mathfrak{V}$ (Capricorn), the semidiurnal arc, at the same latitude of 42 degrees, comprises 4 hours 28 minutes; when this is subtracted from the 24 hours, there remain 19 hours 32 minutes for the time of noon. Accordingly hour 23 from sunset will be distant from the meridian, toward the west, by 3 hours 28 minutes — that is, by 52 degrees. If this distance is reckoned from N toward e, we shall find the point λ [?]

for hour 23 from sunset on the tropic φ , and so on. In this way, once the time of noon is known, we shall find the point of any hour whatever from sunset. And I wish the same to be understood of the hours from sunrise. Moreover, this holds not only on the tropic φ or ψ , but on any parallel whatever. Hence, if from the point of the proposed hour from sunset on the circle MaNb a hidden straight line is drawn through the center E, cutting the equinoctial line at a point, and through that point a straight line is drawn from H, the center of the sundial, then the parallel φ or ψ — or any other parallel — will be cut at the point of the proposed hour from sunset, according as that hour was taken on the parallel φ or ψ , or on any other parallel whatever.

And so, if the corresponding hour-points found on the tropics are joined by straight lines, the Italian sundial will be drawn. The lines of the hours from sunset will pass through those points of the equinoctial line through which the lines of the hours from noon or midnight run: namely, the line of hour 23 from sunset through the point of the fifth hour from noon or midnight, the line of hour 22 through the point of the fourth hour, and so on in succession — just as is clear from the table of prop. 19 of the preceding book, whose title is THE EQUINOCTIAL LINE, or from the table of the diurnal arc of 12 hours of prop. 23 of the same preceding book. Hence it will suffice to find only the hour-points on one or the other of the tropics; for these, joined by straight lines to the corresponding points on the equinoctial line, will give the hour-lines from sunset, as before.

Nevertheless we shall draw these lines more accurately if the hour-points are investigated on both tropics, so that three corresponding points are obtained through which each single hour-line ought to be drawn. For in this way every error that can be committed in drawing the hour-lines will easily be avoided.

But if the points of some hours on the tropic φ have no corresponding points on the tropic ψ — as in our example are all the hours before hour 16 — the hour-lines will have to be drawn through the points of those hours found on the tropic φ and through the corresponding points on the equinoctial line, which indeed are found from the aforesaid table of prop. 19 of the preceding book, or from the table of the scholium of prop. 33 of the same book. The line of the twelfth hour, however, will have to be drawn through the point found on the tropic φ , equidistant from the equinoctial line or from the line of the sixth hour from noon or midnight. For in every horizontal sundial the equinoctial line, the line of the sixth hour from noon or midnight, and the line of the twelfth hour from sunset are parallel, as we demonstrated in the scholium of prop. 22 of the preceding book. But hour 11 from sunset, if it were produced, would pass through the fifth hour from noon or midnight on the equinoctial line; and hour 10 from sunset

through the fourth hour from noon or midnight, just as the table of prop. 19 of the preceding book prescribes.

[Margin: Another description of the Italian horizontal sundial, by means of the diurnal arcs.]

Alternatively. Let there be described in the astronomical sundial, by prop. 2 of this book, two parallels of the diurnal arcs, one of 14 hours and the other of 10 hours; or rather (which will indeed be sufficient for the business proposed) let the points through which the said arcs must pass be marked on the hours from noon and from midnight, even if the arcs themselves are not drawn. Then let the tables of the diurnal arcs of 10, 12, and 14 hours, which we set down in the scholium of prop. 33 of the preceding book, be carefully considered. For in these it will at once appear through which hours from noon or midnight of the said arcs the hours from the Sun's setting must be drawn. Thus you see that hour 23 from sunset must be drawn through the fourth hour from noon on the arc of 10 hours, through the fifth hour from noon on the arc of 12 hours (which here is the same as the equinoctial line), and through the sixth hour from noon on the arc of 14 hours — and so with the rest. But those hours which have no corresponding points on the arc of 10 hours — as in our example are all the hours before hour 15 — must be drawn through the points on the arcs of 14 and 12 hours. Hour 12 from sunset, however, must be drawn parallel to the equinoctial line (for the line of the twelfth hour from sunrise or sunset and the equinoctial line are parallel in the horizontal sundial, as we taught in the scholium of prop. 22 of the preceding book), through the seventh hour from midnight on the arc of 14 hours, as is evident from the table of the said arc. And since hour 11 from sunset has a point only on the arc of 14 hours — namely the sixth hour from midnight — we shall draw hour 11 from sunrise through the fourth hour from noon on the arc of 14 hours and through the fifth hour from noon on the arc of 12 hours, as is evident from the tables of the said arcs. For this line, produced, will give hour 11 from sunset, since it falls on the other side of the greatest parallel of things always visible, as is clear if this parallel were drawn.

For any hour whatever from sunrise, produced beyond the arc — that is, beyond the greatest parallel of things always visible or always hidden — gives the same hour by number from sunset, and conversely, as we shall show in the following scholium. And this hour 11 from sunrise, when produced, will necessarily pass — if no error has been made — through the sixth hour from midnight on the arc of 14 hours, through which, of course, hour 11 from sunset is to be drawn, from the table of the diurnal arc of 14 hours. By like reasoning, since hour 10 from sunset

has no point on the aforesaid diurnal arcs, we shall draw hour 10 from sunrise through the third hour from noon on the arc of 14 hours and through the fourth hour from noon on the arc of 12 hours, as is evident from the tables of these arcs. For this line, produced, will exhibit hour 10 from sunset, for the reason stated earlier. And so we shall proceed with the other hours before the tenth, if they must be drawn in the sundial.

[Margin: Another description of the Italian horizontal sundial, by means of the greatest parallel of things always visible, or the diurnal arc of 24 hours.]

Most conveniently, too, the hours from sunset can be described if the greatest parallel of all things always visible — that is, the diurnal arc of 24 hours — is described. For the straight lines drawn through the hours from noon or midnight on the said arc, according to the table of the diurnal arc of 24 hours set down in the scholium of prop. 33 of the preceding book, and through the hours from noon or midnight of the diurnal arc of 12 hours, as the table of this arc set down in the same scholium of prop. 33 of the preceding book teaches, will give the hour-lines from sunset. For hour 23 from sunset passes through the eleventh hour from noon of that parallel, or diurnal arc of 24 hours, and through the fifth hour from noon of the equinoctial line, or arc of 12 hours. But hour 22 passes through the tenth hour of the former arc and through the fourth hour of the equi-

-noctial line, or of the latter arc, and so on. As is evident from the tables of the said arcs described in the scholium of prop. 33 of the preceding book. Hour 12 from sunset, however, is drawn parallel to the sixth hour from noon or midnight — that is, to the equinoctial line — through the point where the meridian line, or line of hour 12 from midnight, which is the hour of noon, intersects the diurnal arc of 24 hours, that is, the greatest parallel of things always visible. For the line of hour 12 from sunrise or sunset, the line of the sixth hour from noon or midnight, and the equinoctial line are parallel, as is evident from what we wrote in the scholium of prop. 22 of the preceding book; and the line of hour 12 from sunrise or sunset passes through the twelfth hour from midnight on the diurnal arc of 24 hours, as is plain from the table of the diurnal arc of 24 hours set down in the scholium of prop. 33 of the same book. Indeed, the line of the twelfth hour from sunrise or sunset, and all the rest from sunrise or sunset, touch the aforesaid parallel — that is, the diurnal arc of 24 hours — at the very points where the same lines of the hours from noon or midnight cut it, as we demonstrated in prop. 14 of the preceding book. Or else, in certain of the sundials that follow, they touch the diurnal arc of 0 hours or the nocturnal arc of 24 hours — that is, the greatest parallel of things always hidden. Hence, in order that the

hours from sunrise or sunset may be drawn more accurately, arcs of this kind may be described, as we did in prop. 34 and 36 of this book in the meridional and polar sundials.

[Margin: By what method, if by means of the diurnal or nocturnal arcs some one hour from sunset or sunrise has been drawn, another may be described with little effort.]

Furthermore, once some one hour from sunset — or also from sunrise, as we shall say in the following proposition — has been drawn, then by means of the diurnal arcs, or also the nocturnal, as will be plain in the sundials that follow, we shall easily draw the others through the successive points that correspond to one another, with no other consideration of the tables which we composed in the scholium of prop. 33 of the preceding book. Thus, having drawn (for example) hour 23, we shall draw 22 through the nearest point on the diurnal arc of 14 hours, through the nearest on the diurnal arc of 12 hours, and through the nearest on the diurnal arc of 10 hours. Likewise 21 through another set of three successive points, and so with the rest, as appears in the example.

Alternatively. Let the line of the twelfth hour from sunrise or sunset be described in this way. In the portion of the Analemma of prop. 1 of this book, let there be taken the arc EK equal to the arc CE of the pole's altitude, so that the straight line CK drawn through them is the diameter of the greatest parallel of all things always visible — namely, the common section of it and the meridian. And since the circle of the twelfth hour from sunset touches the said parallel at the point K, which on the diameter of that same parallel is opposite the point C at which the horizon, or the circle of hour 24 from sunrise or sunset, touches the same parallel — as is evident from the figure of prop. 9 of the preceding book — the meridian will pass through K, the point of contact of the said circles. Since, then, it also passes through the poles of the said parallel, it will likewise pass through the poles of the circle of hour 12 from sunrise or sunset, by prop. 5 of book 2 of Theodosius [*Spherics* II.5]. Therefore, by prop. 15 of book 1 of Theodosius [*Spherics* I.15], the meridian will be at right angles to the said circle, and this in turn at right angles to the meridian. Again, since the said circle of hour 12 from sunrise or sunset, being a great circle, passes through the center of the world D, and it has been shown that it passes through K, the common section of the meridian and of the circle of hour 12 from sunrise or sunset, drawn out as a straight line from K through D, will meet the meridian line HI of the sundial's plane at the point L. I now say that the straight line drawn through L perpendicular to the meridian line of the sundial is the line of hour 12 from

sunrise or sunset. For since both the plane of the horizontal sundial and the plane of the circle of hour 12 from sunrise or sunset are at right angles to the meridian, as we demonstrated just above, their common section too — namely the line of hour 12 from sunrise or sunset — will be perpendicular to the same meridian [Margin: (a) *Elements* XI.19], and therefore also to the meridian line HI lying in the meridian, by def. 3 of book 11 of Euclid [*Elements* XI, def. 3], at the point L, where we said was the common section of the circle of hour 12 from sunrise or sunset and of the meridian, and consequently the point where the circle of hour 12 from sunrise or sunset itself meets the plane of the sundial. Therefore a straight line drawn in the plane of the sundial through L, perpendicular to the meridian line, will give the line of hour 12 from sunrise or sunset. Accordingly, if from the center H the point L is transferred onto the meridian line of the sundial — taken with the compass from the point H of the portion of the Analemma — and through L a line is drawn perpendicular to the meridian line, or parallel to the line of the sixth hour from noon or midnight, that is, to the equinoctial line, this will be the very line of hour 12 from sunrise or sunset.

Now the straight line HD in the portion of the Analemma, by which the circle of hour 12 from sunrise or sunset is drawn, cuts the segment of the meridian line HI intercepted between H, the center of the sundial, and the point I through which the equinoctial line must be drawn, into equal parts at the point L. [Margin: (b) *Elements* III.27] For the angles CDE, KDE are equal; [Margin: (c) *Elements* I.29] and the angle CDE is equal to the angle IHD, [Margin: (d) *Elements* I.15] and the angle KDE to the angle HDL; therefore the angles LHD, LDH will also be equal; [Margin: (e) *Elements* I.6] and consequently the straight lines HL, DL will be equal. Again, since EF is a quadrant, the two remaining arcs in the semicircle CF, BF will be equal to the other quadrant; therefore, the equal arcs EK, CE being taken away, the remaining arcs KF, BF will also be equal; [Margin: (f) *Elements* III.27] and hence the angles KDF, BDF will be equal; [Margin: (g) *Elements* I.15] but the angle KDF is equal to the angle LDI, [Margin: (h) *Elements* I.29] and the angle BDF to the angle LID. Therefore the angles LDI, LID will also be equal, [Margin: (i) *Elements* I.6] and hence the straight lines LI, DL will be equal. Wherefore HL, IL will be equal to one another, since each has been shown equal to DL itself. And so, if the segment of the meridian line between the center of the sundial and the equinoctial line is bisected, in any horizontal sundial whatever, the line of the twelfth hour from sunrise or sunset must be drawn through that middle point.

[Margin: Another description of the Italian horizontal sundial, to be accomplished by means of the line of hour 12 from sunrise or sunset.]

Now, the line of the twelfth hour from sunrise or sunset having been described in this way, the remaining hours from sunset are easily described if, on the equinoctial line and on the line of the twelfth hour from sunrise or sunset, one carefully marks the points from the table of prop. 19 and from the second table of prop. 20 of the preceding book, through which the lines of the hours from sunset pass. For straight lines joining the corresponding points will give the hour-lines from sunset. Example: hour 23 from sunset passes on the equinoctial line through the point of the fifth hour from noon or midnight, and on the line of the twelfth hour from sunrise or sunset through the point of hour $5\frac{1}{2}$ from noon, as is evident from the said tables. Therefore a straight line passing through those two points will give the hour-line that indicates hour 23 from sunset. In the same way, the line of hour 22 from sunset will pass through the point of the fourth hour from noon on the equinoctial line, and through the point of the fifth hour from noon on the line of the twelfth hour from sunrise or sunset; and so with the rest.

[Margin: Another description of the Italian horizontal sundial, from the first table of prop. 20 of the preceding book, where each hour from sunset is parallel to some hour from noon or midnight.]

The lines of the hours from sunset can also be easily described from the first table of prop. 20 of the preceding book, in this way. Through the point of the fifth hour from noon or midnight on the equinoctial line — through which, of course, hour 23 from sunset passes, as is evident from the table of prop. 19 of the same book — let a straight line be drawn parallel to the line of hour $11\frac{1}{2}$ from noon or midnight. For this will be the line of hour 23 from sunset. So likewise the line of hour 22 from sunset, passing through the point of the fourth hour from noon, will be parallel to the line of the eleventh hour from noon or midnight, and so on — as is clear from the first table of prop. 20 of the preceding book, in which every pair of hours is parallel in the horizontal sundial, as we demonstrated in the scholium of prop. 22 of the same book. Now these parallels

[Margin: The use of the later tables of prop. 20 of the preceding book in describing the hours from sunset.]

will be readily described, if the straight line intercepted on the equinoctial line between the points through which the hours from sunset must be drawn, according to the table of prop. 19 of the preceding book, is taken, together with the points of the hours from noon or midnight from which those points are equidistant according to the first table of prop. 20 of the same book, and these are transferred from H, the center of the sundial, onto the line of the sixth hour from noon or midnight. Thus, if there be taken on the line of the sixth hour from noon

a length HX equal to the line intercepted on the equinoctial line between the point of the eleventh hour from noon — from which hour 22 from sunset is equidistant — and the point of the fourth hour from noon, through which the same hour 22 passes, then the straight line drawn through X and the point of the fourth hour from noon [Margin: (a) *Elements* I.33] will be parallel to the line of the eleventh hour from noon or midnight, and will therefore indicate hour 22 from sunset, and so on. But if a parallel to the equinoctial line is drawn at a greater distance from it, onto which are transferred the same lines intercepted on the equinoctial line between the points through which the hours from sunset must be drawn and the points of the hours from noon or midnight from which they are equidistant — the beginning always being made from the hours from noon or midnight from which the hours from sunset ought to be equidistant — then the hours from sunset will be drawn much more accurately, since each is drawn through three points: one of which is on the line of the sixth hour from noon or midnight, another on the equinoctial line, and the third on the parallel drawn last.

[Margin: By what hours from sunset they mutually cut one another on the line of the sixth hour from noon or midnight.]

Moreover, hours 23 and 13 from sunset cut one another, if drawn as straight lines, on the line of the sixth hour from noon or midnight. Likewise hours 22 and 14, and also 21 and 15, and so on, as is evident from the third table of prop. 20 of the preceding book.

[Margin: By what hour-lines from sunset they mutually cut one another on the meridian line.]

Again, from the fourth table of the same proposition, hours 10 and 14 from sunset intersect one another on the meridian line, that is, the line of the twelfth hour from noon or midnight. Likewise hours 11 and 13, and so on. Finally, from prop. 20 of the preceding book and the figure of prop. 9 of the same book, or indeed from the tables which we constructed at the end of the scholium of the same proposition of the same book, we shall easily recognize which hours from sunrise or sunset cut any hour whatever from noon or midnight at one and the same point. This makes the description of the hour-lines from sunset very easy indeed, and provides a means by which a description of this kind may be examined. Therefore we have set up the Italian horizontal sundial, and so on. Which was to be done.

SCHOLIUM

[Margin: Any hour from sunset, produced beyond the diurnal or nocturnal arc of 24 hours which it touches, belongs to the same hour by number from sunrise; and conversely.]

Here it is worth noting that any hour from sunset, produced beyond that point where it touches the greatest parallel of things always visible — that is, the diurnal arc of 24 hours — or the nocturnal arc as well, which is the other opposite parallel and greatest of things hidden, indicates the same hour by number from sunrise; and conversely, any hour from sunrise produced beyond that point exhibits the same hour by number from sunset. This we assumed as true in the present proposition, but shall now demonstrate as follows. If the Sun were placed at that point of the heavens where some hour-circle from sunrise or sunset touches the greatest parallel of things always visible or the greatest parallel of things always hidden, the shadow of the gnomon [stylus] would fall precisely at that point of the sundial where the hour-line of that circle touches the diurnal or nocturnal arc of 24 hours, as can easily be shown from what was demonstrated in the preceding book. For, the Sun being on that parallel, the shadow of the gnomon is projected onto the arc of that parallel drawn in the sundial, as is evident from prop. 12 of the preceding book. Likewise, the Sun being on that hour-circle, the same shadow is projected onto the common section of it and the plane of the sundial — that is, onto the hour-line of that circle — as we showed in prop. 11 of the same book. It follows that the shadow falls on the said point, which is common both to the arc of that parallel and to the hour-line of the aforesaid circle. Since this is so, it is clear that, if we were placed at that point of contact of the greatest parallel of things always visible or always hidden, so that one semicircle of that hour-circle were to our right and the other to our left, and the Sun were in the right semicircle, the shadow of the gnomon would be projected to the left side of the aforesaid hour-line — that is, to the opposite side; but if the Sun were in the left semicircle, the same shadow of the gnomon would fall on the right side of the same line — the right and left sides being taken with respect to the point of contact on the diurnal or nocturnal arc of 24 hours in the sundial. Since, then, with the Sun on the different semicircles of that hour-circle, the shadow of the gnomon falls on different sides of its hour-line — now beyond the point of contact, now short of it — and since one of those semicircles indicates the hour from the Sun's rising, and the other the hour from its setting, as we said in the scholium of prop. 10 of the preceding book, it follows that one part of its hour-line, bounded at the point of contact, will pertain to the hours from sunrise — namely that onto which the shadow is projected when the Sun is set in the

semicircle that signifies the hour from sunrise — and the other part to the hours from sunset. And the same is true in other sundials that are not horizontal, and in all the hour-lines from sunrise or sunset.

[Margin: The use of the Italian sundial is that we may understand how many hours have elapsed since the most recent sunset, and how many remain until the next.]

The same thing, moreover, will become clear by this reasoning, even if we place neither the Sun nor ourselves at that point of contact. Any hour-semicircle from noon or midnight is placed precisely midway between the two semicircles of that hour-circle from sunrise or sunset which touches, at the same point, the greatest parallel of things always visible or always hidden — the point at which that hour-semicircle from noon or midnight cuts that same parallel, since it passes through its pole, by prop. 5 of book 2 of Theodosius [*Spherics* II.5] — so that it has one semicircle to the right and the other to the left. Now since, with the Sun on that hour-semicircle from noon or midnight, the shadow of the gnomon falls on its hour-line, which cuts the diurnal or nocturnal arc of 24 hours, it is clear that, with the Sun on that hour-semicircle from sunrise or sunset which is eastern with respect to that hour-semicircle from noon or midnight, the shadow is projected to the opposite side in the plane of the sundial — namely to the western side with respect to that hour-line from noon or midnight, yet onto the very hour-line from sunrise or sunset — as is evident from prop. 11 of the preceding book; while with the Sun on the other, western semicircle, the shadow falls on the eastern side of the sundial with respect to that hour-line from noon or midnight, yet onto the very hour-line from sunrise or sunset, by the same prop. 11 of the preceding book. Since, then, that hour-line from noon or midnight cuts the hour-line from sunrise or sunset at the point of contact of the diurnal or nocturnal arc of 24 hours, and one hour-semicircle from sunrise or sunset shows the hour from sunrise while the other shows the hour from sunset, it plainly follows that one portion of that hour-line from sunrise or sunset, up to the point of contact, indicates the hour from sunrise, while the remaining portion shows the same hour by number from the Sun's setting.

From what has been said it can easily be understood that the use of the Italian sundial is that from it we may know how many hours have elapsed since the most recent setting of the Sun, and how many remain until the next setting. Thus, when the shadow of the gnomon falls on some hour-line from sunset — say on the line of hour 19 — we are certain that 19 hours have passed since the most recent setting of the Sun, and, since from one sunset to the next 24 hours

intervene, that there remain until the following sunset five hours, which of course are what hours 19 lack to complete the number of 24 hours; and so with the rest.

Gnomonics, Book Two

PROBLEM 11. PROPOSITION 11.

To construct a Babylonian horizontal sundial — that is, to draw the lines of the hours from sunrise on a plane parallel to the horizon.

By the same method we used in drawing the lines of the hours from sunset, we shall draw the lines of the hours from sunrise. For these do not differ from the former except in position and order: the 23rd hour from sunset, transferred to the hours before noon, becomes the first hour from sunrise; the 22nd hour, transferred in the same way, becomes the second hour, and so on. This is clear if we correctly consider the nature and arrangement of these hour-circles, together with the figure of the preceding proposition. Just as, then, in drawing the Italian sundial by the earlier method we began the division of the circle $M a N b$ from the western points b and e , so for the Babylonian sundial we shall begin from the eastern points a and d — so that the points a and d belong to the 24th hour from sunrise in each tropic, the points nearest to them toward N to the first hour, the following ones to the second, and so on. For by these points of division we find, in each tropic, the points of the hours from sunrise, just as we said above concerning the points of the hours from sunset.

The division of the circle $M a N b$ can also take its beginning from the point of any hour from sunrise found on its circumference, as was explained in the preceding proposition.

In the other methods there is likewise no difficulty, provided the precepts of the preceding proposition are rightly understood and the tables of props. 19, 20, and 33 of the earlier book are carefully worked through, so as to find the points at which the lines of the hours from noon or midnight and those from sunrise or sunset intersect one another.

[Margin: Construction of the Babylonian horizontal sundial.]

Diagram. A large circle representing the horizon, with the vertical diameter M (top) to N (bottom) as the meridian (the noon–midnight line) and a horizontal diameter as the six-o'clock / equinoctial line. Radiating and cross-hatched hour-

lines fill the interior, converging in a dense knot near the upper-central pole below M. The hour-lines from sunrise (dotted and solid) are drawn across the two tropic arcs and the equinoctial line, and are numbered around the circumference. This is a very densely labelled figure; the clearly readable labels are listed below, and the remaining small numeric labels within the central knot could not be resolved with confidence.**

- M (top of circle): meridian / noon point
- N (bottom of circle): midnight point
- H (upper centre, just below M): pole about which the hour-lines converge
- 6 (left end of horizontal diameter) and 6 (right end): the six-o'clock points
- 4 (outside, lower left) and 8 (outside, lower right): hour-line labels carried beyond the circle
- a (left), d (lower left): eastern division-points of the tropic arcs
- e (right): a division-point on the right-hand side
- s, m (left of centre); X, K (upper centre): interior point-letters
- λ, 23 (lower left, near d): the 23rd-hour region
- 14, 13 (upper left): hour numbers
- 1, 10 (upper right); 6, 7, 8 (right); n, u (right): hour numbers and point-letters
- lower row of hour numbers (left→right): 3, 2, I, 12, II, 10, 9
- central knot of hour numbers: 12, 11, 10, 9, 8, 7, 6, 5, 4, 3 and 22, 21, 20, 19, 18, 17
- ß (lower centre): interior point
- ♌ [Leo] (lower right), with further worn zodiac-sign sorts near N and at the top that could not be pinned to a sign [?]
- Several small numeric hour-labels within the converging knot: [?]

Let us take, for example, the case of drawing the Babylonian sundial by the second method, using the diurnal arcs. The first hour from sunrise passes — as is clear from the tables of prop. 33 of the preceding book — through the sixth hour from midnight on the diurnal arc of 14 hours, through the seventh hour from midnight on the diurnal arc of 12 hours, and through the eighth hour from midnight on the diurnal arc of 10 hours. Therefore a straight line drawn from midnight through these hours will give the first hour from sunrise; and so with the rest. But those hours which have no points on the diurnal arc of 10 hours — as are all the hours after the ninth hour from sunrise in our example — must be drawn through points on the diurnal arcs of 14 and 12 hours. Now the twelfth

hour from sunrise must be drawn parallel to the equinoctial line (because the line of the twelfth hour from sunrise or sunset and the equinoctial line are parallel on a horizontal sundial, as is plain from the scholium of prop. 22 of the earlier book), through the fifth hour from noon on the diurnal arc of 14 hours.

Then, for the 13th hour from sunrise — since it has only the point of the sixth hour from noon on the diurnal arc of 14 hours — we shall draw the 13th hour from sunset through the eighth hour from midnight on the diurnal arc of 14 hours and through the seventh hour from midnight on the diurnal arc of 12 hours. For this line, produced beyond the arc of the greatest of the always-visible parallels, will give — as we showed in the scholium of the preceding proposition — the 13th hour from sunrise passing through the sixth hour from noon on the diurnal arc of 14 hours. By like reasoning, for the 14th hour from sunrise, since it has no point on the aforesaid arcs, we shall draw the 14th hour from sunset through the ninth hour from midnight on the diurnal arc of 14 hours and through the eighth hour from midnight on the diurnal arc of 12 hours; for this line, produced, will likewise indicate the 14th hour from sunrise. And in this manner we shall proceed for the other hours after the 14th from sunrise, if they need to be drawn.

By this method, then, both sundials — the Italian as well as the Babylonian — are readily drawn from the aforesaid tables of prop. 33 of the earlier book, provided that for those hours from sunset which have only a single point, or none at all, on the diurnal arc of 14 hours, we draw the same hours from sunrise, and conversely — as we have done for the 11th and 10th hours from sunset and the 13th and 14th from sunrise, and so on.

[Margin: Another construction of the Babylonian horizontal sundial, from the diurnal arcs.]

[Margin: A point of doctrine to be noted in constructing both the Italian and the Babylonian sundials by means of the diurnal and nocturnal arcs, or by the tables of prop. 33 of the preceding book.]

GNOMONICS

By the same device we shall make a Babylonian sundial from the diurnal arc of 24 hours; likewise by means of the line of the 12th hour from sunrise or sunset, and also by the other methods prescribed in the preceding proposition. This is plainly evident from the figure of the preceding proposition. And so we have constructed a Babylonian horizontal sundial, etc. Which was to be done.

SCHOLIUM.

Just as, from the Italian sundial, we learned how many hours have passed since the most recent sunset and how many remain until the next, so from the Babylonian sundial we shall know how many hours have elapsed since the most recent sunrise and how many are still required until the following one. For when the gnomon's shadow falls on some hour-line from sunrise — say on the line of the tenth hour — there can be no doubt at all that 10 hours have passed since the most recent sunrise, and therefore that 14 hours remain until the following sunrise, which are precisely what the 10 hours lack to complete the number of 24 hours; and so with the rest.

[Margin: The use of the Babylonian sundial is that, from the gnomon's shadow falling on its hour-lines, we may know how many hours have elapsed since the Sun's most recent sunrise and how many remain until the next sunrise.]

If both an Italian and a Babylonian sundial are drawn on the same plane — whether by the same gnomon, as we did in the proposed sundial, or by different ones — we shall know at any moment of time, from the shadow of the gnomon, the length of day and night; the time of noon and of midnight, both in hours from sunset and in hours from sunrise; the time of the Sun's rising in hours from sunset; and the time of the Sun's setting in hours from sunrise. For if the hours from sunrise, caught by the gnomon's shadow, are subtracted from the hours from sunset found at the same instant, there remains the time of the Sun's rising reckoned in hours from sunset. From this the rest is easily drawn out by this method. Suppose the shadow of the gnomon falls on the third hour from sunrise and on the 14th hour from sunset. Since, then, three hours have passed since the Sun rose that day, we subtract 3 from 14, and there remains the eleventh hour from sunset, at which the Sun rose. Night therefore comprises 11 hours at that point, and day 13, since day and night together make up 24 hours. If we halve the nighttime span of 11 hours, we find $5\frac{1}{2}$ hours for the time of midnight reckoned in hours from sunset. And since from midnight to noon there are 12 hours, if we add 12 to $5\frac{1}{2}$ we obtain $17\frac{1}{2}$ hours for the time of noon reckoned in the same hours from sunset. We find the same time of noon if we subtract the semidiurnal arc — which in this example comprises $6\frac{1}{2}$ hours — from 24 hours; for the semidiurnal arc, added to the time of noon, necessarily makes up 24 hours, namely the whole natural day of 24 hours from one sunset to the next, as is plain. So you see that if $6\frac{1}{2}$ is taken from 24, there remains $17\frac{1}{2}$ for the time of noon. Next, if we halve the daytime span of 13 hours, we find $6\frac{1}{2}$ hours for the time of noon reckoned in hours from sunrise. And since from noon to midnight there

flow 12 hours, if we add 12 to $6\frac{1}{2}$ we shall have $18\frac{1}{2}$ hours for the time of midnight reckoned in the same hours from sunrise. We shall also find this same time of midnight if we subtract the seminocturnal arc — which in our example comprises $5\frac{1}{2}$ — from 24 hours, since the seminocturnal arc, added to the time of midnight, necessarily makes 24 hours, namely the whole natural day of 24 hours from one sunrise to the next. So you see that if $5\frac{1}{2}$ is taken from 24, there remains $18\frac{1}{2}$ for the time of midnight. And the Sun sets at the 13th hour from sunrise.

Suppose again that the shadow of the gnomon indicates the $6\frac{1}{4}$ hour from sunrise and the $17\frac{1}{2}$ hour from sunset. If we subtract $6\frac{1}{4}$ from $17\frac{1}{2}$ we find the $11\frac{1}{4}$ hour from sunset, at which the Sun rose that day; and so night then comprises $11\frac{1}{4}$ hours, while day will be $12\frac{3}{4}$ hours. Midnight will fall at hour $5\frac{1}{2}$ and $\frac{1}{8}$ — that is, hour $5\frac{5}{8}$ — reckoned in hours from sunset, and noon at hour $17\frac{1}{2}$ and $\frac{1}{8}$ — that is, hour $17\frac{5}{8}$ — in hours from sunset. Likewise, reckoned in hours from sunrise, noon falls at hour $6\frac{1}{4}$ and $\frac{1}{8}$ — that is, hour $6\frac{3}{8}$ — and midnight at hour $18\frac{1}{4}$ and $\frac{1}{8}$ — that is, hour $18\frac{3}{8}$. Finally the Sun sets at hour $12\frac{3}{4}$ from sunrise; and so with the rest.

The same can also be found by means of the astronomical sundial which we constructed in prop. 1 of this book. For from it we come at once to knowledge of the time of noon in Italian or Babylonian hours, and from that we shall learn the rest in this way. Suppose the shadow falls on the hour $9\frac{1}{2}$ from midnight, on the fourth hour from sunrise, and on the 15th hour from sunset. Since, then, $2\frac{1}{2}$ hours are still wanting until noon, if we add $2\frac{1}{2}$ to 4 and to 15 we find that noon, reckoned in hours from sunrise, falls at hour $6\frac{1}{2}$, and therefore midnight at hour $18\frac{1}{2}$; while in Italian hours noon falls at hour $17\frac{1}{2}$ and midnight at hour $5\frac{1}{2}$ — since noon and midnight are always 12 hours, that is a semicircle, apart. From this the length of the day will be evident. For since noon falls at hour $6\frac{1}{2}$ from sunrise, an equal number of hours will remain until sunset; and so the day will contain 13 hours and the night 11.

The Sun will therefore rise at the 11th hour from sunset and set at the 13th hour from sunrise. Similarly, since midnight falls at hour $5\frac{1}{2}$ from sunset, an equal number of hours remain until the Sun's rising; and so night comprises 11 hours and day 13, etc. Suppose again it is found to be the hour $3\frac{1}{2}$ from noon, the hour $8\frac{1}{4}$ from sunrise, and the hour $22\frac{3}{4}$ from sunset. Since, then, $3\frac{1}{2}$ hours have elapsed since noon, if we subtract $3\frac{1}{2}$ from $8\frac{1}{4}$ and from $22\frac{3}{4}$ we find noon, reckoned in hours from sunrise, to fall at hour $4\frac{3}{4}$ and midnight at hour $16\frac{3}{4}$; but in hours from sunset noon falls at hour $19\frac{1}{4}$ and midnight at hour $7\frac{1}{4}$. And so, since noon falls at hour $4\frac{3}{4}$ from sunrise, an equal number of hours will remain

until the Sun's setting; therefore the day will comprise $9\frac{1}{2}$ hours and the night $14\frac{1}{2}$. Hence the Sun will then rise at hour $14\frac{1}{2}$ from sunset and set at hour $9\frac{1}{2}$ from sunrise. By the same reasoning, since midnight, in the Italian manner, falls at hour $7\frac{1}{4}$, an equal number of hours will remain until the Sun's rising; and so night will contain $14\frac{1}{2}$ hours and day $9\frac{1}{2}$, etc.

[Margin: How, from the Italian and Babylonian sundials together, the length of day and night, and the time of noon and of midnight, both in hours from sunset and in hours from sunrise, [may be known] at any time from the shadow of the gnomon, etc.]

[Margin: How the same things just discussed may be found by means of the astronomical sundial.]

From the description of the Italian and Babylonian sundials given in this and the preceding proposition, it is clear that these two sundials do not differ from each other except in position and in the order of the hours — so that the left part of one corresponds to the right part of the other, and conversely. Therefore, if an Italian sundial is drawn, the same will serve for a Babylonian one, provided it is inverted so that the left side becomes the right and vice versa, and provided that from the line of hour 23 from sunset is made hour 1 from sunrise, and hour 2 from sunrise is made from hour 22 from sunset, and so on. This is easily seen if the Babylonian sundial laid out by this proposition is compared with the Italian sundial described in the preceding proposition.

And so, if an Italian sundial is drawn on some sheet, and on the opposite side lines are drawn corresponding exactly to the lines of the Italian sundial, a Babylonian sundial will be described on the opposite side — provided the hour-numbers of the Italian are changed, in the Babylonian, into their complements to 24, so that hour 23 Italian becomes the first Babylonian, hour 22 the second, hour 21 the third, and so on.

In the same way, if a Babylonian sundial is drawn first, an Italian one will be described on the opposite side, so that each single line corresponds exactly to each single line, the hour-numbers in the Babylonian being likewise changed into their complements to 24 in the Italian: thus hour 1 Babylonian becomes 23 Italian, the second becomes 22, the third 21, and so on. This same thing is to be understood of other Italian and Babylonian sundials that are drawn on other planes — namely on planes parallel not to the horizon but to other great circles in the sphere.

[Margin: How the Italian and Babylonian sundials differ from each other, and by what method the Babylonian is composed from the Italian, and the Italian from the Babylonian.]

LIBER SECUNDUS.

PROBLEM 12. PROPOSITION 12.

To construct a horizontal sundial of the ancient [unequal] hours — that is, to draw the lines of the unequal hours on a plane parallel to the horizon.

Having divided the circle from E — described into the diurnal arc a N b of the tropic of Cancer [♋ Cancer] and the arc d N e of the tropic of Capricorn [♏ Capricorn], as was done in prop. 10 of this book — let each diurnal arc be cut into 12 equal parts, beginning from the eastern points a and d, so that the points next after a and d toward N belong to the first unequal hour in each tropic, the following ones to the second, and so on. If the sundial and the circle M a N b keep their proper position, the great circles drawn through the poles of the world and through the unequal hours of the tropics of Cancer and Capricorn will pass through these points of division, as is evident from what was demonstrated in prop. 10 of this book. For the same demonstration can be brought to bear here, since in the same way we shall show that the circle drawn through the poles of the world and through the 12th unequal hour in the tropic of Cancer passes through point b — since that same point in the tropic of Cancer belongs to the 12th unequal hour and to the 24th hour from sunset. Hence we shall further show (since the arc between b and the nearest division-point toward N is similar to the arc of the tropic of Cancer lying between two great circles drawn through the poles of the world — of which one is drawn through the 12th unequal hour and the other through the 11th unequal hour in the tropic of Cancer — because each arc is the twelfth part of the semidiurnal arc of the tropic of Cancer in its own circle) that the great circle drawn through the poles of the world and through the 11th unequal hour in the tropic of Cancer passes through the division-point nearest to b toward N; and so with the rest. If, then, through the division-points and the centre E straight lines are drawn cutting the equinoctial line at points, through which in turn straight lines are sent out from H, the centre of the sundial, there will be found in the tropics—

[Margin: Construction of the horizontal sundial of the ancient [unequal] hours.]

Diagram. A circle representing the horizon, with the vertical diameter M (top) to N (bottom) as the meridian and a horizontal diameter through the centre as the

equinoctial line. The gnomon-centre E lies at the middle, with H just above it; the unequal-hour lines fan out from the upper-central region and are labelled in half-hours. The tropic arcs a N b (Cancer) and d N e (Capricorn) span the figure, with the worked-example points f, g, h, m, n, p, q shown. Densely labelled; the readable labels are listed, the rest flagged.**

- M (top): meridian / noon point
- N (bottom): midnight point
- E (centre): centre of the sundial / gnomon foot
- H (just above E): centre from which the straight lines are emitted
- 7½ (upper left) and 4½ (upper right): unequal-hour-line labels
- 12 (top centre), 6 (upper centre-right): hour labels
- 4½, 3, 7½ (central cluster of unequal-hour labels near H): hour labels
- q, a (upper left); t, m, d (left): point-letters
- p (upper right); f (right); n, n (right, the equinoctial line carried out on both sides); u (right); h (centre-right); z, e (lower right): point-letters
- f (bottom, near N): point-letter
- horizontal row of hour-points along the equinoctial line (left→right): n, 10, 9, 8, 7, [?], 3, 2, 1
- Several small numeric hour-labels in the central fan: [?]

of Cancer and Capricorn the points of the unequal hours, exactly as in props. 10 and 11 of this book we found in the same [tropics] the points of the hours from sunset and from sunrise. Hence, if the corresponding points are joined by straight lines, the sundial of the ancient hours will be described. Now all the lines of the unequal hours will pass through the hours from noon or midnight on the equinoctial line, as is established from table 13 of the scholium of prop. 33 of the preceding book. But take one or two examples of this construction. From f, the point of the 3rd unequal hour in the tropic of Capricorn, a straight line drawn through E cuts the equinoctial line at g; and a straight line sent out from H through g cuts the tropic of Capricorn at h, the point of the third unequal hour. Again, from point m — which is opposite to the eleventh unequal hour in the tropic of Cancer — a straight line sent out through E cuts the equinoctial line at n, while the straight line H n cuts the tropic of Cancer at point p, which is owed to the semicircle of the great circle drawn through point m; and the line n H, produced beyond the centre, cuts the same tropic of Cancer at q, the point of the eleventh unequal hour, and so on. All of which is clear from what was demonstrated in prop. 10 of this book.

Here too, as in prop. 10 of this book, we shall find on the circle M a N b the diurnal and nocturnal arc of the tropic of Cancer or of Capricorn, in another way than by the method we wrote of in the scholium of prop. 1 of the earlier book — namely if, from point N, we reckon off on each side the semidiurnal arc of the tropic of Cancer up to a and b, or of the tropic of Capricorn up to d and e.

Indeed, even if we do not divide the said circle into the said arcs, we shall find on it the point of any unequal hour, the Sun being on any parallel whatever, by this method. Let the diurnal arc of the given parallel be divided by 12, so that in the quotient we have the length of one unequal hour on the given parallel; and thus, doubling that same number, the size of two hours, tripling it the size of three, and so on. Then let it be considered how far the proposed unequal hour is distant from the meridian circle, whether before noon or after. For if this distance is reckoned on the circle M a N b from point N — toward a if the given hour is before noon, or toward b if after noon — we shall come upon the point of the given unequal hour. Example. The diurnal arc of the tropic of Cancer contains 15 hours 4 minutes, that is, 226 degrees; and these degrees, divided by 12, yield 18 degrees 50 minutes for the size of one unequal hour in the tropic of Cancer. Hence two hours will make 37 degrees 40 minutes; three hours, 56 degrees 30 minutes; four hours, 75 degrees 20 minutes; five hours, 94 degrees 10 minutes; and six hours, 113 degrees. And so, if the point of the ninth unequal hour of the tropic of Cancer is sought on the circle M a N b, then — since this hour is after noon — if its distance from the meridian is reckoned from point N toward b, namely 56 degrees 30 minutes, the point of the ninth unequal hour of the tropic of Cancer will be found. And the same method is to be followed for the rest.

Otherwise. From prop. 3 of this book let two parallels be described — one northern, having a diurnal arc of 18 hours, and a [southern one]...

[Margin: Another construction of the sundial of the ancient [unequal] hours...]

[Margin: ...logii Antiqui horizontalis — of the ancient horizontal sundial — by the diurnal arcs of 18 and 6 hours. (Continuation of a note begun on the preceding page.)]

and a second, southern one whose diurnal arc takes in six hours. For from the tables of diurnal arcs of 18, 6, and 12 hours — which we set out for the unequal hours in the scholium of prop. 33 of the preceding book — we shall presently learn through which hours from noon, or from midnight, the individual unequal hours are to be drawn in the said arcs. Thus you see that the first unequal hour must be drawn through the [equal] hour $4\frac{1}{2}$ from midnight in the diurnal arc of

18 hours; through the seventh [hour] from midnight in the arc of 12 hours; and through $9\frac{1}{2}$ from midnight in the diurnal arc of 6 hours. The second unequal hour, in turn, is drawn through hour 6 from midnight in the diurnal arc of 18 hours; through the eighth from midnight in the diurnal arc of 12 hours; and through the tenth hour from midnight in the diurnal arc of 6 hours; and so on.

It will nevertheless be enough if in the horizontal sundial we describe only the northern parallel of 18 hours, since the southern one of 6 hours runs out too far beyond the tropics. And so we have constructed the ancient horizontal sundial, and so forth. Which was to be done.

SCHOLIVM.

The shadow of the gnomon, falling on the lines of the unequal hours, teaches how great a part of the day has passed and how much still remains. For since any given day is divided into 12 equal parts, which are called unequal hours, it comes about that when the shadow of the stylus falls, say, on the third unequal hour, a fourth part of the day has passed and three-quarters remain. And when that same shadow falls on the eighth unequal hour, two-thirds of that day have elapsed and one-third still remains, and so on.

[Margin: The use of the ancient sundial is to let us understand how great a part of the day has passed and how much still remains. By this method the unequal hours are ascertained — those called planetary — each of which corresponds to fifteen degrees of the ecliptic.]

Now everything that was written in the last 12 propositions about the various uses of the things we depicted on the horizontal sundial must be understood to hold for all other sundials as well, which we shall treat of in what follows.

But perhaps there will be no lack of those who also wish to know by what means those unequal hours may be determined which are called temporal, natural, and planetary — as we explained in our commentaries on the sphere — each of which takes in that span of time in which half a sign of the zodiac, that is, fifteen degrees of the ecliptic, ascends above the horizon. For these divide the artificial day into 12 unequal parts, since the ascension of the first 15 degrees of the ecliptic is unequal to the ascension of the following 15 degrees, and so on. We shall satisfy them thus. Suppose, for the sake of an example, that we must find the time of the seventh unequal hour when the Sun is at 20° Taurus [♉]. Since any given unequal hour takes in 15 degrees of the ecliptic, it is necessary that by the seventh unequal hour 105 degrees of the ecliptic will have ascended; and if we count

these off from 20° Taurus in the order of the signs, we arrive at 5° Virgo [♍], which ascends above the horizon at the seventh unequal hour on that day. Therefore, if from the oblique ascensions we seek the ascension of the arc of the ecliptic between 20° Taurus and 5° Virgo, we shall find that at latitude 42° it contains 117° 14' — that is, 7 equinoctial hours 49 minutes. We shall say, then, that it is the seventh unequal hour when 7 hours 49 minutes have passed since sunrise. Accordingly, when on the Babylonian sundial the hour 7, minute 49, has been determined from the gnomon's shadow, we shall pronounce that it is then the seventh temporal — or planetary — hour on that day when the Sun stands at 20° Taurus.

Now if we wish to reduce this hour 7, minute 49, from sunrise to astronomical and Italian hours, we shall do it in this way. Since the diurnal arc then contains 14 hours 14 minutes, while the semidiurnal arc is 7 hours 7 minutes, if we subtract 7 hours 49 minutes from the diurnal arc of 14 hours 14 minutes, there remain 6 hours 25 minutes up to sunset. Therefore 17 hours 35 minutes have then elapsed since the previous sunset. But if we subtract the semidiurnal arc of 7 hours 7 minutes from the 7 hours 49 minutes since sunrise (since the latter number is the smaller), there will remain 0 hours 42 minutes since the noon just past. Therefore, if on the same day the hour 17, minute 35, has been observed on the Italian sundial, or the hour 0, minute 42, after noon on the astronomical one, we shall again say that it is then the seventh temporal or unequal hour.

Again, let it be worth our while, on the same day, to find the time of the third temporal or unequal hour. Since, then, each temporal hour takes in 15 degrees of the ecliptic, it is necessary that by the third unequal hour 45 degrees of the ecliptic will have ascended; and if we count these off from 20° Taurus in the order of the signs, we arrive at 5° Cancer [♋]. The oblique ascension of the arc of the ecliptic between 20° Taurus and 5° Cancer, at latitude 42°, takes in 41° 45' — that is, 2 equal hours 47 minutes. Therefore, when on the Babylonian sundial we have determined the hour 2, minute 47, it will be exactly the third temporal hour. Now if the 2 hours 47 minutes since sunrise are deducted from the diurnal arc of 14 hours 14 minutes, there remain 11 hours 27 minutes up to sunset; and so the third unequal hour falls at the hour 12 [?], minute 33, from sunset. But if we subtract the 2 hours 47 minutes since sunrise from the semidiurnal arc of 7 hours 7 minutes (since the former is the smaller), there will remain 4 hours 20 minutes up to noon. Therefore that same third unequal hour falls at the hour 7, minute 40, from the previous midnight. By this device we shall determine the temporal or unequal hour at any proposed time whatever, and for any latitude whatever.

DE HOROLOGIIS VERTICALIBVS.

PROBLEMA 13. PROPOS. 13.

To construct an astronomical vertical sundial — that is, to describe the hour-lines from noon or midnight on a plane that is parallel to the vertical circle.

[Margin: Description of the astronomical vertical sundial.]

Diagram. A geometric construction (a "portion of the analemma") for laying out the vertical sundial. A semicircle ABC stands on the horizontal diameter BC, with center D. From D rise and cross several straight lines representing the meridian's sections with the horizon, the vertical circle, the equator, and the axis of the world; the line HI is drawn through G parallel to AD, cutting ED and FD at H and I. Readable labels, by position:

- L, L (top, two adjacent letters at the apex): tops of the vertical lines
- A, K (upper center, just below the apex): "AK"
- F (upper left of the arc): meridian–equator section point
- E (upper right of the arc): axis-of-the-world point (toward the north)
- N (middle right, on an interior line)
- I (middle left of center)
- H (center): where the axis of the world meets the sundial plane
- G (center, near D): foot of the stylus on BC
- D (center): center of the semicircle / center of the world
- B (left end of the diameter)
- C (right end of the diameter)
- H, I (lower left, a second labelled pair on the HI parallel): "HI"
- I (just below the center)
- M (lower right)
- O (at the bottom, foot of a descending line)

Let there again be, as in prop. 1 of this book, a portion of the analemma [analemma] ABC: that is, let ABC be the semicircle of the meridian, whose center is D; let BC be the common section of the meridian and the horizon; AD the common section of the meridian and the vertical circle; FD the common section of the meridian and the equator; and ED the axis of the world. Now, with the straight lines FD and ED produced toward D, let there be taken on the straight line BC — for the stylus of the future sundial, of whatever magnitude it shall have

been given — a straight line DG equal on either side. And through G let HI be drawn parallel to AD, cutting ED and FD at H and I. If, then, we understand the vertical circle properly so called to be drawn through AD, and through HI the plane of the sundial parallel to that vertical circle, then — since both the meridian and the horizon are perpendicular to the said vertical — [a] their common section BC will be perpendicular to it, and therefore also perpendicular to the plane of the sundial drawn parallel to the vertical through HI, as follows from what we demonstrated for prop. 14 of book 11 of Euclid [Euclid, *Elements* XI.14]. HI, therefore, will be in the plane of the sundial the meridian line, or the line of the

[Margin: ^b *Elements* XI.19.]

[Margin: ^a *Elements* XI.2.]

12 o'clock — namely the common section of the meridian and the plane of the sundial. [a] For the straight line HI is in the plane of the straight lines HD, ID, that is, in the plane of the meridian. DG will be the gnomon, at right angles to the plane of the sundial, and hence also perpendicular to HI, the meridian line, by definition 3 of book 11 of Euclid [Euclid, *Elements* XI, def. 3], its vertex D being the very same as the center of the world, by prop. 2 of the preceding book. The place of the gnomon is the point G, on the meridian line, distant from the point H (at which the axis of the world meets the plane of the sundial, and which is called the center of the sundial) by the interval GH; or from the point I (at which the plane of the equator cuts the meridian line) by the interval GI. All of which is clear if the meridian is conceived to have its proper position — that is, so that the point E inclines toward the north and F toward the south. This figure, so constructed, we shall call — for the sake of the doctrine — a portion of the analemma, just as we called that figure which we described at the beginning of prop. 1 of this book a portion of the analemma.

Now, by the benefit of the triangle DHI, we shall construct the astronomical vertical sundial in this way. Onto a straight line HE, drawn anyhow in the plane on which the sundial is to be described, let the straight line HI — the meridian line of the portion of the analemma — be transferred: downward, indeed, from the point H, if the sundial is to be described facing south;

Diagram. The upper of two vertical-sundial figures on this page — the **southern (austral)** dial, its title split by the vertical meridian line as "AVS ... TRALE" (*Australe*). A semicircular dial with hour-lines radiating from the center. An inner

circle sits about the center, and a small auxiliary circle stands at the upper left. Readable labels:

- "12" (top center): the noon line
- Around the rim, the hour-numbers (left side, descending): 6, 7, 8, 9, 10; and at the base 11, 12, 1; (right side, descending): 5, 4, 3, 2
- "C" (far left) with "6" (the horizontal line 6–6); "L" and "6" (far right): the ends of the line CL
- "K" (mid-right, on the horizon line); "F" (mid-left, on the horizon line)
- "S" (inner right of center); "R", "P", "M" (upper center, near the top of the inner circle) [?]
- "H" (center): center of the sundial
- "G", "I" (just below center)
- "E" (lower center); "N" (bottom center); "O" (bottom); "Q" (lower left of center)
- "d" and a small point "e" [?] at the auxiliary circle, upper left
- "AVS" (left) / "TRALE" (right): *Australe*, "southern"

Diagram. The lower of the two figures — the **northern (boreal)** dial, its title split as "BORE ... ALE" (*Boreale*). A full circle with hour-lines radiating from the center H at the bottom, an inner circle about the center, and hour-numbers ranged around the whole circumference. Readable labels:

- "R", "P", "M" (top center, at the top of the inner circle) [?]
- Hour-numbers around the rim (upper left, descending): 9, 8; (upper right): 2, 3, 4; then down the sides and across the base: 7, 6, 5, 4 (left); 5, 6, 7, 8 (right); and along the bottom 4, 3, 2, 1, 12, 11, 10, 9, 8
- "E" (upper center, inside the inner circle)
- Horizontal mid-row (left to right): "F", "R", "I", "S", "K", flanked by "BORE" (far left) and "ALE" (far right)
- "6", "C" (left) and "L", "6" (right): the ends of the lower horizontal line CL
- "H" (center): center of the sundial
- "D", "G" (just above center); "I" (center); "O", "Q" (just below center)
- A dotted "9" near E [?]

but upward, if the northern [dial] is to be laid out, as is clear from the portion of the analemma. For the plane of the sundial facing south, receding of course from D, the center of the world, toward the parts of the north pole E, cuts the equator FD below the point H, at the point I; whereas the plane of the sundial facing

north, receding of course from D, the center of the world, toward the southern parts, cuts the same equator above the point H, at the point I, and so on. Then from the straight line HE let there be cut off IE, equal to DI of the portion of the analemma; and through I let the perpendicular FK be raised to HE in the plane of the sundial. From E let a circle of any magnitude be described; and this being divided into 24 equal parts, beginning from the meridian line HE produced, let there be drawn — through the points of division and the center E — hidden straight lines cutting the straight line FK at points. Through these points, and through the point H, straight lines sent out will give the hour-lines from noon, or from midnight, on the plane parallel to the vertical, whose order we shall explain a little further on for each of the two sundials. For the sixth hour, however, the line CL must be drawn through H, perpendicular to HE, or parallel to FK. All of which will be demonstrated as follows.

[Margin: Demonstration of the construction of the vertical sundial.]

Let the portion of the analemma be conceived as laid over the sundial, so that point H coincides with point H, point I with point I, and the straight line HI with the straight line HI (and it will coincide with it, because of the equality of the straight lines HI and HI);

[Margin: ^a *Elements* XI.19.]

and let that same portion of the analemma be imagined as standing upright toward the plane of the sundial, and let the meridian line HE have its proper position, so that it is the common section of the meridian and the plane of the sundial; and let the point H, in the southern sundial, incline toward the zenith and E toward the nadir, while in the northern one the point H inclines toward the nadir and E toward the zenith; and finally let the straight line BC be perpendicular to the plane of the sundial, so that it is the common section of the meridian and the horizon, with D, the vertex of the stylus, being the center of the world, as we showed at prop. 2 of the preceding book. Or, if you prefer, in place of the portion of the analemma let only the triangle DHI be understood — laid, as we said, over the sundial in such a way that DH represents the axis of the world, DI the common section of the meridian and the equator, and finally DG, the stylus, the common section of the meridian and the horizon, drawn through D, the center of the world. This triangle DHI, together with the gnomon DG, has been drawn in each of the two sundials. These things being so posited, the axis HD will meet the plane of the sundial at H; the diameter of the equator, DI, at I; and the diameter of the horizon, DG, at G. Since, therefore, both the plane of the equator and the plane of the sundial (parallel to the vertical) are perpendicular to the

plane of the meridian, [a] the common section of the equator and the plane of the sundial will also be perpendicular to that same plane of the meridian, and hence — by definition 3 of book 11 of Euclid [Euclid, *Elements* XI, def. 3] — perpendicular to the meridian line HI, which lies in the plane of the meridian, at the point I, where the equator meets the meridian line in the sundial,

Diagram. The upper of two vertical-sundial figures on this page, repeating the **southern (austral)** dial of the preceding page — its title split by the meridian as "AVS ... TRALE" (*Australe*). A semicircular dial with hour-lines radiating from the center, an inner circle about the center, and a small auxiliary circle at the upper left. Readable labels:

- "12" (top center)
- Rim hour-numbers (left, descending): 6, 7, 8, 9, 10; base: 11, 12, 1; (right, descending): 5, 4, 3, 2
- "C" with "6" (far left); "L" with "6" (far right): ends of the line CL
- "K" (mid-right on the horizon line); "F" (mid-left on the horizon line); "S" (inner right)
- "R", "P", "M" (upper center, inner circle) [?]
- "H" (center); "G", "I" (just below center)
- "E" (lower center); "N" (bottom center); "O" (bottom); "Q" (lower left)
- "d" and a small point "e" [?] at the auxiliary circle
- "AVS" (left) / "TRALE" (right): *Australe*

[Same crowded interior cluster as fig-p185-1; several readings uncertain]

Diagram. The lower figure, repeating the **northern (boreal)** dial — title split as "BORE ... ALE" (*Boreale*). A full circle with hour-lines radiating from the center H at the bottom, an inner circle, and hour-numbers around the whole rim. Readable labels:

- "R", "P", "M" (top center, inner circle) [?]
- Rim hour-numbers (upper left): 9, 8; (upper right): 2, 3, 4; (sides/base): 7, 6, 5, 4 (left) and 5, 6, 7, 8 (right); along the bottom 4, 3, 2, 1, 12, 11, 10, 9, 8
- "E" (upper center, inner circle)
- Horizontal mid-row (left to right): "F", "R", "I", "S", "K", with "BORE" (far left) and "ALE" (far right)
- "6", "C" (left) and "L", "6" (right): ends of the lower horizontal line CL
- "H" (center); "D", "G" (above center); "I" (center); "O", "Q" (below center)
- A dotted "9" near E [?]

[Same crowded interior cluster as fig-p185-2; several readings uncertain]

as was said. Therefore the straight line FK, which in the sundial is drawn through I perpendicular to the meridian line HI, is the common section of the equator and the plane of the sundial — namely, the equinoctial line. Now suppose the plane of the circle described from center E is understood to be moved about this line FK, until it is joined with the center of the world D, in the triangle DHI (which triangle we said is at right angles to the plane of the sundial and lies in the plane of the meridian, together with the whole portion of the analemma, although only the triangle is drawn in the sundial): namely, until E, the center of the circle, is joined [to D], inasmuch as the straight lines DI and IE are equal by construction, and in that motion of the circle described from E the straight line IE always remains in the plane of the meridian, in which DI lies. Then that circle will be described about D, the center of the world — that is, it will be the equator described, and it will lie in the plane of that same equator, since the plane of the equator is drawn through the straight lines DI, IK, just as the said circle in that position is drawn through the same lines; so that this circle and the equator lie in the same plane and have the same center, namely D, the center of the world. Let IE also be understood as produced in both directions in the same position, inasmuch as it is moved together with the plane of the circle as the common section of the meridian and the equator, since both the meridian and the equator in this position pass through the straight line DI or IE. Wherefore, by what was demonstrated in prop. 1 of this book concerning the horizontal sundial, the straight lines going out from the center E will be common sections

of the equator and of the hour-circles from noon or midnight — namely NM, the common section of the equator and the circle of hour 12; OP, the common section of the equator and the circle of the first hour from noon or midnight; QR, the common section of the equator and the circle of the second hour from noon or midnight, and so on. And so, just as we demonstrated in that place, the straight lines drawn from H through the points of the straight line FK will be the hour-lines from noon or midnight, for the same demonstration can be applied here. Only the line of the sixth hour must be drawn through H perpendicular to HI — or, what is the same, parallel to FK itself — such as CL is. For since, by the scholium of prop. 22 of the previous book, the equinoctial line and the line of the sixth hour from noon or midnight are parallel in the vertical sundial, it is clear that the straight line CL is the line of the sixth hour, seeing that it is parallel to the equinoctial line FK and passes through the point H, where all the hour-lines from noon or midnight cut one another.

[Margin: The order of the hours in each of the two vertical sundials.] In the southern sundial, moreover, the meridian line HI, running from the center toward the equinoctial line, marks the hour of noon (12), while the remaining portion running upward marks midnight (12). The lines to the left of the meridian line show the hours after midnight, and their portions to the right of that same meridian line show the hours after noon. In the northern sundial the opposite is to be understood. All this is clear once the position of the hour-circles is rightly understood, and once careful attention is paid to what we wrote in the scholium of prop. 1 of this book about what must be observed in the lines drawn in the sundial — namely, which celestial semicircle each line represents, and by what principle any hour after noon, when produced beyond the center of the sundial, marks the same hour numbered from midnight, and the reverse. Therefore we have set up the Astronomical Vertical sundial, and so forth. Which was to be done.

SCHOLIVM.

[Margin: Another and most convenient method of laying out the Astronomical Vertical sundial, for tracing those hours that lie close to the line of the sixth hour from noon or midnight.] Andreas Schonerus [Andreas Schöner] hands down another method by which the hour-lines can be drawn, similar to the one we explained in the same scholium of prop. 1 of this book for laying out the horizontal sundial. It runs roughly as follows. On the equinoctial line FK, take the straight line I a equal to the straight line IE (or ID); and through a draw the straight line a b perpendicular to the equinoctial line — that is, parallel to the meridian line — cutting the line of the sixth hour, which of course is drawn from H perpendicular to HI, at the point b. Then, taking the straight line b d equal to a b itself, draw through d the line e f perpendicular to b d — that is, parallel to the meridian line. Finally, with center d describe a circle, divide it into 24 equal parts beginning from the straight line b d, and from the center d draw hidden straight lines through the points of division. For these will cut the straight line a b at points through which the hour-lines are to be drawn from the center H of the sundial. We have set an example of this construction in the southern sundial.

[Margin: The segments of the equinoctial line in the vertical sundial, between the meridian line and any two hours equally distant on either side from the meridian line, are equal.] In any Vertical sundial, however, any two hour-lines removed from the meridian line on either side by an equal interval of time cut off equal lengths from the equinoctial line FK — namely those lengths that lie between the two lines themselves and the meridian line — so that on the equinoctial line the

hour-spaces before noon are equal to the hour-spaces after noon, each to its counterpart. From this it follows that the arcs of any circle described from H in the Vertical sundial, intercepted between the meridian line and any two hour-lines equally distant in time on either side from the meridian line, are equal. Likewise, the intervals between E, the center of the Equator, and any two hours removed on either side from the meridian line by an equal interval of time on the equinoctial line, are equal. All of this will be demonstrated in the way we used in the scholium of prop. 1 of this book to demonstrate the same points for the horizontal sundial.

[Margin: It is enough, for laying out the Astronomical Vertical sundial, that one single quadrant of the circle described from E be divided into 6 equal parts.] From this it follows that, for laying out the Vertical sundial, it is enough that a single quadrant of the circle described from E be divided into six equal parts — such as, for example, the quadrant Mg. On this matter, see what we wrote to this same end concerning the horizontal sundial in the scholium of prop. 1 of this book.

[Margin: How each of the two Vertical sundials is to be placed together with the axis of the world, so that it may show the hours from noon or from midnight.] Here too, if in the triangle DHI the axis HD is understood to be set at right angles to the plane of the sundial — as a thread stretched out, or even some thin rod fixed at H, making with the meridian line an angle equal to the angle DHI, and making right angles with the straight line CL — or if the triangle DHI itself, contained in the sundial above, is built up out of some solid material and set at right angles to the plane of the sundial: then the shadow of the axis DI will mark the several hours from noon or midnight, provided the sundial is placed on a plane that lies parallel to what is properly called the Vertical circle, in such a way that the straight line HI (namely the meridian line) is perpendicular to the Horizon, and the straight line FK lies parallel to that same Horizon. And in the sundial that faces south, the equinoctial line FK must lie below the point H, which is the center of the sundial, while in the one that faces north it must lie above. From this it also follows that the shadow of the vertex D of the style DG, set at right angles to the plane of the sundial at the point G — which vertex is the same as the center of the world, by prop. 2 of the previous book — marks the same hours; that is, it is projected onto the hour-lines, just as was said of the horizontal sundial in the scholium of prop. 1 of this book. For the demonstration here is the same.

But if we wish to lay out in the sundial the half-hours as well, and their quarters, or eighths, or any other divisions, we must divide each of the arcs of the circle described from E into two parts, or four, or eight, or however many; everything else is to be carried out as before.

[Margin: When the elevation of the pole above the Horizon is so great that the pole recedes but little from the vertex of the head, what is to be done in laying out the sundial.] Now indeed, when the elevation of the pole above the Horizon is so great that the axis ED in the portion of the Analemma differs but little from the diameter AD of the Vertical circle — and therefore the straight line HI, if the style DG is not very short, meets it at H at a very remote point and makes a very acute angle there — we shall nevertheless lay out the sundial, even though we have no center H, just as in the scholium of prop. 1 of this book we constructed the horizontal sundial even though we had no center H there (which indeed happens when the elevation of the pole above the Horizon is very small). The one exception is that, in place of the angle of the pole's elevation DEF, we must set up in the Vertical sundial the angle of the complement of the pole's elevation. Everything else is to be completed as in the said scholium.

[Margin: When the elevation of the pole above the Horizon is very small, what is to be done in laying out the sundial.] Again, when the elevation of the pole is so small that the diameter FD of the Equator makes a very acute angle with HI and meets it at the very remote point I — and it is therefore difficult to discern that point at which the said angle is formed — we shall use the same remedy that we handed down in the said scholium of prop. 1 of this book for the horizontal sundial, when the elevation of the pole was supposed so great that the pole was but little distant from the vertex of the head; provided that, in constructing the vertical sundial, in place of the angle of the pole's elevation BAN we set up the angle of the complement of the pole's elevation.

[Margin: If the style DG, at G, is set at right angles to the plane of the sundial, the shadow of the vertex D alone will show the hours from noon or midnight.]

[Margin: By what method the half-hour, and their quarter-parts, or any other divisions, are to be drawn in the sundial.] [Margin: When the elevation of the pole above the Horizon is so great that the pole recedes but little from the vertex of the head, what is to be done in laying out the sundial.] [Margin: When the elevation of the pole above the Horizon is very small, what is to be done in laying out the sundial.]

[Margin: If the southern sundial is inverted, so that from the upper part the lower comes to be, and what after this inversion is on the right (with the sundial turned

toward us) is changed to the left, and the reverse, the numbers of the hours remaining exactly the same, a northern sundial will have been constructed. But the hours that were formerly counted from noon must now be counted from midnight, and the reverse.]

Furthermore, the southern vertical sundial described so far will also mark the hours on the opposite face of the sundial's plane — the one that faces north — if all its parts are so inverted that from the upper part the lower comes to be, and what after this inversion is on our right (with the sundial turned toward us) is changed to the left, and the reverse, while the numbers of the hours nevertheless remain exactly the same. But then the portion of the meridian line that begins at the center H of the sundial and runs toward the equinoctial line, which in the southern sundial marked the hour of noon (12), will in this northern sundial signify midnight (12); and the portion that in the former marked midnight will in the latter show noon. And so the hours that were there reckoned from noon are here to be counted from midnight, and those that were there reckoned from midnight are here to be counted from noon. This will be clear from what we shall now demonstrate. Let there be, in the southern sundial, the meridian line HI, the center of the sundial H, the equinoctial line FK; the style GD, whose location is the point G; the line of, for example, the fifth hour after noon HK; the line of the eighth hour from midnight HR; and the axis of the world HD.

Since, then, the axis DH meets the plane of the southern sundial at H, and, when produced, cuts the plane, making on the opposite, northern face an angle AHB with the meridian line — which is the same line on both faces of the plane, since it is the common section of the Meridian circle and the plane of the sundial — [Margin: Elem. I.15] an angle equal to the angle DHI, comprehended on the southern face under the same meridian line and the axis: it comes about that, if the straight line HC is taken on the opposite part of the plane equal to the straight line HG on the southern face, and from C a style is erected at right angles to the plane of the northern sundial (and therefore also perpendicular to the straight line HC), cutting the axis at A, then the style CA will be equal to the style GD, and the axis HA equal to the axis HD. For since in the triangle AHC the two angles H and C are equal to the two angles H and G of the triangle DHG, and the side HC is taken equal to the side HG, [Margin: Elem. I.26] the side CA will also be equal to the side GD, and the side AH to the side DH.

But if from A a perpendicular AB is drawn to the axis HA, meeting the meridian line at B, then AB — [Margin: Elem. XI.2] lying in the plane of the Meridian drawn through the straight lines AH, HB — will be the common section of the Equator

and the Meridian. For since the axis HA is, by prop. 10 of Theodosius, book 1, perpendicular to the plane of the Equator and passes through its center, that same axis will also, by def. 3 of Euclid, book 11, be perpendicular to the common section of the Equator and the Meridian drawn through that same center. Therefore, with A set as the center of the Equator (that is, of the world), AB, lying in the plane of the Meridian and perpendicular to the axis HA, will be the common section of the Equator and the Meridian. [Margin: Elem. I.26] Now the straight line AB will be equal to DI itself, and HB to HI itself, because in the triangle AHB the two angles A and H are equal to the two angles D and I of the triangle DHI, and the side AH has been shown equal to the side DH, and so on. Therefore, if through B, in the northern sundial, the equinoctial line BL is drawn perpendicular to the meridian line, it will be as far distant from the center H upward as the equinoctial line FK is distant from that same center H downward in the southern sundial. And so, if on the meridian line HB, produced, the straight line BM is taken equal to the straight line BA, and a circle is described from M and divided into 24 parts, and so on, a northern sundial will be laid out on the opposite, northern face, just as the southern sundial was laid out on the southern face, when we took the straight line IE equal to the straight line ID and described a circle from E, and so on, as was demonstrated above. For the demonstration here will be exactly the same.

Diagram. A geometric construction figure combining the southern (austral) vertical sundial with its northern (boreal) counterpart about a common center H, together with the analemma triangles used in the proof. A vertical meridian line runs from B at the top, through the central point H, down to I on the lowest horizontal line; a horizontal line passes through H; and a lower horizontal line runs F–R–I–K. Several straight hour-lines cross obliquely at H, two of them labelled "8" (an hour-line above and its counterpart below). Readable labels:

- B (top, on the meridian line)
- L (upper left)
- S (left, above center); S (right, below center) — the paired oblique lines
- A (center-left, on the boreal construction)
- C (just left of the center H)
- H (center; intersection of all the lines)
- 8 (upper right, an oblique hour-line); 8 (lower left, its counterpart)
- G (just below center H)
- D (right of G, the axis vertex / style top)

- F, R, I, K (lower horizontal line, left to right — F at far left, I on the meridian, K at far right)

[The central cluster (C, H, G, D) is crowded; the paired oblique-line labels S and 8 are legible under magnification.]

[Margin: Elem. I.26] [Margin: Elem. XI.2] [Margin: Elem. I.26]

This northern sundial is also obtained if all the lines of the southern sundial are produced through the center H. For since all the hour-circles from noon or from midnight pass through the axis DHA, so as to cut the plane of the sundial at H, the common sections of these circles and of the plane of the sundial — on the southern face as much as on the northern — will be the same straight lines passing through H. Yet these sections, that is, the hour-lines, are to be understood in the northern sundial as lying on the opposite part of the plane, since the axis of the world DH, when produced, falls on the opposite part; that is, on the law and condition that, with the southern sundial turned toward us and the point F on the left and K on the right, the hour-lines on the opposite, northern face are drawn exactly corresponding to the hour-lines of the southern sundial produced through the center H — so that, to us looking at the northern face of the plane, the point F is set on the right and K on the left, contrary of course to what happens in the southern sundial; or, what is the same, F faces the west and K the east, as in the southern one.

[Margin: Elem. I.26] Moreover, the northern sundial, laid out by the method just handed down, will be entirely equal and similar to the southern one, on account of the equality of the axes HA, HD; of the gnomons CA, GD; of the common sections of the Equator and Meridian AB, DI; and of the portions of the meridian line HB, HI placed between the center of the sundial and the equinoctial lines.

[Margin: Elem. I.15] For from this it results that the triangle HBL in the northern sundial — whose sides are the meridian line HB, the equinoctial BL, and the line of the fifth hour HL — is equal and entirely similar to the triangle HIK in the southern sundial, whose sides are likewise the meridian line HI, the equinoctial IK, and the line of the fifth hour HK; because the two angles BHL, HBL of the former are equal to the two angles IHK, HIK of the latter (for the angles at H are equal, being vertical, and the angles at B and I are right angles), and the side HB is equal to the side HI, as was shown, and so on. The same reasoning holds for the other triangles, whose bases are the hour-lines placed between the center H and the equinoctial lines, subtended to the right angles at B and I, and comprehended under the meridian line and the equinoctial lines. Therefore the whole northern

sundial will be equal to the whole southern one, and similar, since its several parts are equal and similar to their several counterparts.

[Margin: If the lineaments of the southern sundial are drawn on the opposite face of the plane, so as to correspond exactly to the lineaments of the southern sundial, and after this drawing the upper part is changed into the lower, a northern sundial will have been laid out.]

Hence, if on the opposite part of the sundial's plane there are drawn lines exactly corresponding to the lines in the southern sundial, and then the upper part is changed into the lower — and consequently what after that drawing is on the right becomes the left, and the reverse — a northern sundial will have been laid out on the northern face of the sundial's plane. For by this procedure the point K, which in the southern sundial is on our right, and in the northern one after that drawing is on the left, comes to be again on our right if the upper part is changed into the lower, as is evident. But we shall write more on this matter in the following proposition.

From all this (to come at last to what was proposed) I judge it clear that the southern sundial marks the hours on the face of the northern Vertical plane, if everything is altered as has been said — that is, if the upper part is changed into the lower, and what after this change is on our right (with the sundial turned toward us) becomes the left, and the reverse. For the southern sundial HRK, if it

is inverted, so that the point I is placed upward and H downward, and the point R — which in that position, with the sundial turned toward us, is on our right — falls to the left, and K to the right, will acquire exactly the same position that the northern sundial LHB has, laid out on the opposite face, where the point L, with the sundial turned toward us, is placed on the right and B on the left, as is evident. Likewise, from what we handed down in the scholium of prop. 1 of this book, it is evident that the portion of the meridian line which in the southern sundial marked noon (12) shows midnight (12) in the northern one; and that the hours which were there counted from noon are here to be reckoned from midnight, and those there counted from midnight here from noon. For, as was just now demonstrated, the hours of the southern sundial, beyond the center of the sundial, give on the northern face the same hours by number; and, as is clear from the said scholium, the forenoon hours, produced beyond the center of the sundial, make the afternoon hours, and the afternoon hours make the forenoon ones. This too will easily be understood, if on the northern face the hour-circles are conceived as drawn through the axis HA. For the shadow of the axis, or of the style, provided the Sun lights the northern face, will be projected onto the

meridian line HB when the Sun is in the semicircle of the Meridian beneath the earth, but onto the hour-lines toward the point L — which in the northern sundial is on our right — when the Sun is in the hour-semicircles after midnight, that is, in the semicircles opposite to those in which the Sun is when, in the southern sundial, the shadow of the axis or style is projected onto the hour-lines toward K, and so on; as is clear, if both the northern sundial LHB and the axis HA and the hour-circles are conceived in their proper position. All this will become plainer if the northern sundial LHB, laid out on the opposite face as has been said, is compared with the northern sundial we described earlier. For we shall find this one entirely similar and equal to that.

[Margin: What has just been said about the inversion of all the parts of the sundial is also to be understood in the case of declining and inclined sundials.] This same demonstration fits in every respect the sundials that decline both from the Vertical circle and from the Horizon; likewise the ones inclined to the Horizon, and the one that both declines from the Vertical circle and is inclined to the Horizon, of which we shall speak in the following book — provided that in them we understand the straight line BI to be, not the meridian, but the line of the style or index, and that in place of the Meridian circle drawn through the axis AD and the meridian line BI we take another great circle, drawn through the axis AD and the line of the style BI, as a kind of new Meridian in the planes of those sundials, as we shall point out in the proper places. For any such sundial laid out on one face of a plane — as, for example, on the one that declines from the south, if the plane is at right angles to the Horizon, or on the upper face, if the plane is inclined to the Horizon — will have a sundial to be laid out also on the opposite face of the same plane, as on the one that declines from the north, or on the lower face; provided that all the parts are inverted, as we said, that is, the upper part is first changed into the lower, and what after this change was first on the right comes out on the left, and the reverse. All of which will be clear in its place.

[Margin: By what method a sundial is to be laid out on a fixed plane that lies parallel to what is properly called the Vertical circle.] So far we have constructed the Vertical sundial on any flat surface whatever, so that it may afterward be transferred onto a wall, or onto a plane that stands at right angles to the Horizon and directly faces the south or the north, in such a way that HE, the meridian line, is perpendicular to the Horizon, while the equinoctial line FK lies parallel to that same Horizon — with this condition added, however, that in the southern sundial the equinoctial FK lies below the center of the sundial H, and in the northern one above it, as we prescribed earlier in this same scholium. But if the sundial is to be drawn directly on the wall itself, or on some plane at right angles

to the Horizon and directly facing the south or the north, we shall accomplish it in this way. By means of a plumb-line we shall draw on the given plane the straight line HE, perpendicular to the Horizon, which will be the meridian line in the sundial. We shall cut this at right angles at I by the straight line FK, which will be the equinoctial line. Then from I — upward in the southern sundial, but downward in the northern — we shall carry the straight line IH as far as H, taken from the portion of the Analemma; and we shall take the straight line IE equal to the straight line IE from that same portion of the Analemma, whether upward or downward. Finally, with a circle described from E and divided into 24 equal parts, we shall complete the rest as before.

Diagram. The completed Astronomical Vertical sundial: a large circle (drawn with a semicircular lower arc and an upper fan) with the hour-lines radiating from the central point H, hour-numbers set around the whole rim, and, at the upper left, the analemma-construction portion (a dotted arc). A horizontal line runs across the middle (C–H–L, ending in "6" at each end); a vertical meridian line runs from top (A) through the central cluster down to N at the bottom. Readable labels:

- Upper-rim hour-numbers (left to right across the top): 3, 2, 1, 12, 11, 10, 9, 8
- A (top center, above the fan), with a dotted analemma arc; 4 (on the dotted arc, left of center)
- Left horizontal end: "6", then C; right horizontal end: L, then "6"
- B (left of center, below C) [?]; F (left, below B)
- Central vertical cluster: M (just above H) [?]; H (center); D (below H); G (small, beside D) [?]; I (below D) [?]; E (lower center); N (bottom center)
- Right side: L (mid-right, on the horizontal); K (right of center, below L); S (lower right)
- 7 (lower left); U (left, below F) [?]
- Lower-arc hour-numbers: (left) 8, 9, 10; (bottom center) 11, 12, 1, 2; (right) 1, 2, 3

[The crowded central vertical cluster (M, D, G, I, E) and the small side-labels B, U are read under magnification but several remain uncertain.]

We can also begin the same construction in this way. Having drawn the straight line FK parallel to the Horizon on the proposed wall or plane (which is easily done by means of a plumb-line and a level) as the equinoctial line, we shall cut it at right angles at I by the straight line HE, which will be the meridian line. Then we shall complete the rest as we just wrote.

[Margin: The laying out of the Vertical sundial to a given length of the style, whose location is also given, without a portion of the Analemma constructed separately.] But if the same sundial is to be drawn on a given plane, without a portion of the Analemma constructed separately, to any length of style whose location too is given, this method must be used. Let the given length of the style be DG, and let its location on the plane of the sundial be the point G. If, then, the plane of the sundial be whatever it may, so that the sundial drawn on it may thereafter, in its proper position, be

be set in its position, or else be transferred onto a fixed plane that is parallel to what is properly called the Vertical — as we said a little earlier — then the straight line MN must be drawn through G, the location of the style, to serve as the meridian line however it may fall. But if the plane of the sundial is offered as a fixed one, parallel to what is properly called the Vertical (such as the plane of any wall that stands at right angles to the Horizon and faces due south or due north), then through G, the location of the style, we must draw by means of a plumb-line the straight line MN, perpendicular to the Horizon, to serve as the meridian line. Or, more surely, by means of a plumb-line and a level we may draw through G a line parallel to the Horizon, to serve as the common section of the plane of the sundial and of the Horizon; and if the straight line MN cuts this at G at right angles, then MN will again be perpendicular to the Horizon, and will be the meridian line — that is, the common section of the plane of the sundial and of the Meridian.

For since both the plane of the sundial and the plane of the Meridian stand at right angles to the Horizon, [anchor a] their common section too will be perpendicular to the same Horizon — and therefore, by Euclid XI, definition 3, perpendicular also to the straight line drawn through G parallel to the Horizon, which is the common section of the Horizon and of the plane of the sundial, since that line lies in the Horizon. [Margin: (a) Euclid XI.19] Therefore, since the Meridian passes through G — for the style DG is the common section of the Horizon and of the Meridian — the line MN drawn through G perpendicular to that straight line will be the common section of the Meridian and of the plane of the sundial.

Next, at G on this meridian line MN, we must erect the perpendicular BGD, and take GD equal to the given style. Then, with D as center, we must describe the arc of a circle ABC, on which, from the straight line DB toward A — that is, upward — if the sundial faces south, we count off the elevation of the pole BA, and downward its complement BC. But if the sundial faces north, the arc of the pole's

elevation BA is to be taken downward, and the arc of its complement BC upward. When the straight lines DA and DC are drawn, the meridian line is cut at the points H and I. After this, at I we shall erect the line FK perpendicular to the meridian line, to serve as the equinoctial line. Finally, taking the straight line IE equal to ID, we shall describe from E a circle of any size whatever; and dividing it into 24 equal parts, beginning from the meridian line, we shall finish the rest as we taught earlier at the start of this problem.

Diagram. The completed astronomical vertical sundial: a large figure with an upper fan of hour-lines radiating from the central point H, closed below by a semicircular arc, with hour-numbers set around both the upper edge and the lower rim, and, at the upper left, the dotted analemma-construction arc. A horizontal line runs across the middle (6–C–H–L–6); a vertical meridian line runs from A at the top through the central cluster down to N at the bottom. Readable labels [several central labels crowd and are read under magnification]:

- A: top center (apex above the fan), with a dotted analemma arc to its left carrying the number 4
- Upper hour-numbers, left to right across the top: 3, 2, 1, 12, 11, 10, 9, 8
- Small mark at top-left "5" [?]; at top-right "1"
- Horizontal mid-line: 6 (far left) — C — H (center) — L — 6 (far right)
- B: below C on the left, at the head of a dotted line
- F: below B (left end of the equinoctial line FK)
- M: just above H [?]
- H: center
- G and D: just below H (G left, D right) [?]
- I: below the G/D pair [?]
- E: below I
- K: right end of the equinoctial line, below L
- S: lower right [?]
- U: left, below F, on a dotted line [?]
- 7: left; 8: lower left
- N: bottom center
- Lower-rim hour-numbers: (left arc) 8, 9, 10; (bottom) 10, 11, 12, 1; (right arc) 2, 3; (far right) 4

[Margin: Demonstration of this description.] **DEMONSTRATION of this description is as follows.** Suppose the meridian line MN has its proper position in a plane that is at right angles to the Horizon and faces due south or north, so

that M points upward and N downward, and suppose the triangle HDI is set at right angles to the plane of the sundial, in such a way that it lies in the plane of the Meridian. Then, since by construction the angle HDG is equal to the elevation of the pole, the remaining angle DHG will be equal to the complement of the pole's elevation. Again, since by construction IDG is equal to the complement of the pole's elevation, the remaining angle DIG will be equal to the elevation of the pole. Taking, then, D — the vertex of the style — as the center of the world, DH, which makes with the meridian line at H an angle equal to the complement of the pole's elevation, will be the axis of the world meeting the plane of the sundial at H, the center of the sundial. And the straight line DI, which makes with the same meridian line at I the angle of the pole's elevation, will be the common section of the Meridian and the Equator — since a section of this kind, in the sphere, always makes with the meridian line in the Vertical the angle of the pole's elevation, and with the axis a right angle, which is just the character of the angle HDI, composed of the angle of the pole's elevation and the angle of its complement. Therefore the Equator meets the plane of the sundial at I; and consequently, as was shown above, the straight line FK will be the equinoctial line. And the straight line DG will be the common section of the Meridian and the Horizon. Everything else is demonstrated as before.

[Margin: Demonstration of the same sundial by means of an ellipse.] We shall describe **the same sundial without finding the points on the equinoctial line**, by means of an ellipse — just as we described the horizontal sundial in the scholium to Problem 1 of this book — in the following way. From H, the center of the sundial, let two circles be described, one with the interval HI, the other with the interval ID. Or, if these radii seem too short, let a point be taken on the axis farther from H than D is (as was also done in the horizontal sundial), from which a perpendicular is drawn to the axis, cutting the meridian line at a point that will bound the radius of the larger circle; and for the radius of the smaller circle let there be taken the segment of that perpendicular lying between the same point and the axis. Then dividing each circle into 24 equal parts, beginning from the meridian line, we shall find — by means of the division-points — the points of the ellipse in the plane of the sundial, through which the hour-lines are to be drawn from the point H, as we taught in the scholium to Problem 1 of this book. For the same demonstration can be applied here. We do not set down an example of this description, because the matter itself can be understood, clearer than daylight, from the figure that we drew in the said scholium to Problem 1 of this book. For that figure represents a vertical sundial for the latitude of a place of 48 degrees, which of course, together with that latitude, for any...

whatever [latitude] for which that horizontal sundial is constructed, makes up 90 degrees — provided that the numbers of the hours are changed into their complements up to 12. For a vertical sundial, properly so called, takes the place — for any given Horizon — of a certain Horizon above which the pole is raised by just as many degrees as are lacking to the latitude of the place for which the horizontal sundial is constructed, so as to fill out the number of 90 degrees. This is clear from the portion of the analemma described at the beginning of this problem, in which the angle DHG — the complement of the angle HDG, the elevation of the pole above the Horizon BC — constitutes the elevation of the pole above the Vertical HI, as though above some Horizon. And so any horizontal sundial whatever, made for any latitude of a place, is a vertical sundial in the region whose latitude is the complement of that one; and every vertical sundial in the former latitude is in turn a horizontal one in the latter — if the numbers of the hours are changed into their complements up to 12, as we have said.

[Margin: A horizontal sundial constructed for any latitude whatever is a vertical sundial in the region whose latitude is the complement of that latitude; and conversely, the vertical sundial of the one is the horizontal of the other.]

PROBLEMA 14. PROPOS. 14.

To draw the parallels — that is, the arcs of the signs of the Zodiac — on the aforesaid vertical sundial.

Let the portion of the analemma of the preceding problem be repeated, and let the Meridian ABC be completed; and in it, in accordance with the analemma constructed in Problem 1 of the preceding book, let there be drawn the diameters of the parallels, together with the diameters that join the opposite signs and that form, in the cones whose bases are the parallels and whose common vertex is the center D, the triangles through the axis. Then, by what was demonstrated in Problems 4, 5, 6, and 7 of the preceding book, KR will be the diameter of the conic section that the Sun describes when it is at the beginning of Cancer (♋); LR the diameter of the section that the Sun traverses at the beginning of Gemini (♊) and of Leo (♌); and MR the diameter of the section described by the Sun's ray when it is in the first point of Taurus (♉) and of Virgo (♍). But NQ, OQ, and PQ will be the diameters of the conic sections that the Sun's rays describe when the Sun is in the opposite parallels.

[Margin: The drawing of the arcs of the signs of the Zodiac on the vertical sundial, from the analemma. In what manner the diameters of the conic sections are found in any analemma whatever.]

These diameters of the conic sections are also to be found in any other analemma whatever, whether larger or smaller than the one here proposed, even if the sundial has been constructed without a portion of the analemma, as we taught in the preceding scholium — provided, however, that in that other analemma a straight line equal to the gnomon DG is cut off from the diameter of the Horizon on both sides from the center (the diameter of the Horizon itself being extended, if the length of the gnomon requires it), and that from each extreme point a straight line is drawn parallel to the diameter of the Vertical, the line by which the plane of the sundial is drawn. For this straight line, in the larger or smaller analemma, will be divided by the diameters of the opposite signs into parts equal to the parts of the straight line QR in this analemma of ours. This can be demonstrated as follows. Since both that line and this line QR, taken with the same length of style, are equally distant from the center of their own analemma, [anchor a] and since the angles that the diameters of the opposite signs make with the diameter of the Equator are of the same size in any analemma (because they always rest upon the centers with the same declinations of the same signs), it follows that the angles which those same diameters make with the diameter of the Horizon — which are either composed of the former angles together with the angle of the complement of the pole's elevation contained under the diameter of the Equator and the diameter of the Horizon, or are left over after those former angles are subtracted from that same angle of the complement of the pole's elevation — are equal to one another, since the angles contained under the diameter of the Horizon and the diameter of the Equator are equal. [Margin: (a) Euclid III.27]

Therefore, since the angles that the straight lines drawn through the extremity of the gnomon (namely through the point G in our analemma, and through the point corresponding to it in the other analemma), parallel to the diameter of the Vertical, make with the diameter of the Horizon are right angles, and since the angles that the ray of any given sign makes with that same diameter of the Horizon in each analemma are also equal, as we said, there will always be found two triangles in each analemma — one in the one, the other in the other — having two angles equal, each to each. And since they also have an equal side adjoining the said angles, namely the size of the style, [anchor b] they will also have the remaining sides equal, namely those that are interposed between the extremity of the style and the ray of each sign in each analemma, and so on. [Margin: (b) Euclid I.26]

This can also be made plain in this way: if that larger or smaller analemma is imagined to be laid upon this one of ours, so that the centers and the diameters of

the Horizon, of the Vertical, and of the Equator coincide with one another, the straight line drawn through the extremity of the style in the former will coincide with the straight line QR in our analemma, because of the equality of the gnomons and of the right angles that the gnomons make with the said lines. Since, then, the diameters of the ecliptic that connect the opposite signs also coincide with one another (because they make the same angles with the diameter of the Equator in each analemma, on account of the same declinations in each, as is evident from the construction of the analemma), it plainly follows that the straight lines between the extremity of the style G and the rays of the signs in our analemma are equal to the same lines in the other analemma — because the former coincide with the latter.

AND SO, if into the meridian line of the sundial already described — whether from the center H, or from the point I where the plane of the Equator meets the plane of the sundial — the points G, K, L, M, I, P, O, N are transferred, in the same order in which they are placed in the figure (taken by means of the compass from the point where, in the analemma, the axis of the world, or the diameter of the Equator, cuts the straight line drawn through the extremity of the gnomon parallel to the diameter of the Vertical), then G will be the location of the style DG; and the remaining points will be those onto which the shadow of the style is cast when the Sun is on the meridian circle and at the beginnings of the signs, and through which are to be drawn the conic sections of the signs — which are either circles, or parabolas, or hyperbolas, or el-

Diagram. A large circle (the analemma) crossed by many diameters and chords, with zodiac-sign glyphs ranged around the circumference and letter-labels at the crossing-points. A vertical diameter (the Vertical) runs top to bottom and a horizontal diameter (the Horizon) runs left to right; several oblique diameters (the parallels of the signs and their conjugate lines KR, LR, MR, NQ, OQ, PQ) cross near the center D. This figure is densely ruled and the peripheral zodiac sorts are worn; most interior labels are read only tentatively, even under magnification. Readable labels:

- A: top, on the Vertical diameter; R just to its left and Q just to its right (heads of the QR line) [?]
- E: upper right of the rim [?]
- C: right-hand end of the horizontal (Horizon) diameter
- B: left-hand end of the horizontal diameter [?]
- D: center; with G, M, N, P, L, K clustered about the center [each ?]
- Q: lower left; R: lower center (feet of the QR line) [?]

- Zodiac-sign glyphs ring the circumference (associated in the text with Cancer, Gemini, Leo, Taurus, Virgo, Capricorn), but the individual sorts are too worn to assign with confidence: [zodiac glyphs: ?]

or ellipses, as was demonstrated in the preceding book. If, then, through I a line is drawn perpendicular to the meridian line, this will be the equinoctial line. But if, about the diameters KR, LR, MR, NQ, OQ, PQ, the conic sections are described (by Problem 8 of the preceding book) in some hard material — such as thicker paper, or on some thin flat board, or at any rate on glued sheets of paper — and are cut out so as to make, as it were, curved rulers, by which those same sections may be drawn on the sundial so as to pass through the points K, L, M, N, O, P and to have the axes of their meridian lines coincide, then the parallels of the signs of the Zodiac will be drawn. It is enough, however, to describe the conic sections of the northern signs, or of the southern, since the Vertical cuts all the parallels; because these are equal to the sections of the opposite signs. Hence by the same rulers, which we ordered to be cut out for this purpose, they will easily be described in the opposite part of the sundial. (I now call "opposite parts" of the sundial those parts which the equinoctial line divides, one of which is northern and the other southern.)

[Margin: The horizontal line [AB?] [?], and how it is to be drawn in a vertical sundial. — (a) Euclid XI.19]

FURTHERMORE, in each sundial there must be drawn, through G, the location of the gnomon, the line AB perpendicular to the meridian line; and this will be the common section of the Horizon and of the plane of the sundial, and will be called the horizontal line. For since both the plane of the sundial, being parallel to what is properly called the Vertical, and the plane of the Horizon, stand at right angles to the plane of the Meridian, [anchor a] their common section too will be at right angles to the same plane of the Meridian, and consequently, by Euclid XI, definition 3, will be perpendicular also to the meridian line HI, which lies in the plane of the Meridian, at the point G — since it meets the plane of the Meridian at this point. This can be shown as follows. Since the Horizon, passing through the gnomon that stands at the point G perpendicular to the plane of the sundial, meets the sundial at G, the common section of the Horizon and of the sundial will also pass through the same point G. Therefore the straight line AB, which is drawn in the plane of the sundial through G perpendicular to the meridian line HI, is the common section of the Horizon and of the plane of the sundial, and is parallel to the equinoctial line, as we taught in the scholium to Problem 22 of the preceding book.

Now, when the Sun is on the Horizon — that is, when it rises or sets — the shadow of the style is cast into the common section of the Horizon and of the plane of the sundial, that is, into the straight line AB, as is evident from Problem 11 of the preceding book. It comes about, then, that throughout the whole remaining time of any day, while the Sun is raised above the Horizon, that same shadow falls on the plane of the sundial below the horizontal line. Therefore, in every sundial the part that projects above the horizontal line is to be cut away as superfluous, so that in a vertical sundial that faces north only that small portion is left which is comprised between the horizontal line and the arc, or parallel, of Cancer (♋).

[Margin: In every sundial that part which lies above the horizontal line described is superfluous, and therefore to be cut away. The part of the northern sundial below the horizontal line is equal to the part of the southern sundial above the horizontal line.]

Now this part of the northern vertical sundial, comprised between the horizontal line and the arc of Cancer (♋), is entirely equal to that part which the horizontal line cuts off from the southern sundial as superfluous. For in the northern sundial the parallel of Cancer (♋) is as far distant from H, the center of the sundial, as in the southern the parallel of Capricorn (♏) is distant from the same center, as is evident from the analemma of this problem. This can be proved as follows. In the triangle DGK on the left, the two angles at D and G are equal to the two angles at D and G in the triangle DGN on the right, [anchor b] since the angles at G are right angles, and the angles at D, formed at the vertex, are equal; [Margin: (b) Euclid I.15] and the sides DG, DG are equal, because each is equal to the gnomon; [anchor c] therefore the sides DK, DN will also be equal, and the remaining angles at K and N equal. [Margin: (c) Euclid I.26] Hence, in the triangles DHK and DHN, [anchor d] since the angles KDH and NDH at the vertex D are equal, [Margin: (d) Euclid I.15] [anchor e] the sides HK, HN will also be equal; [Margin: (e) Euclid I.26] and consequently, since the conic section of Cancer (♋) passes through K in the northern sundial, and the conic section of Capricorn (♏) through N in the southern, the arc of Cancer (♋) will be as far distant from H in the northern sundial as the arc of Capricorn (♏) is distant from the same point H in the southern sundial. Therefore, since in the cones of Cancer (♋) and of Capricorn (♏) the triangles through the axis are equal, and likewise the straight lines DK, DN and the angles DKH, DNH are equal (as has been demonstrated), the conic section of Cancer (♋), whose diameter is KQ, in the northern sundial will be equal to the conic section of Capricorn (♏), whose diameter is NQ, in the southern sundial — because the plane of the former sundial cuts the triangle through the

axis in the cone of the shadow of Cancer (♋) just as the triangle through the axis in the cone of the shadow of Capricorn (♐) is cut by the plane of this sundial, on account of the equality of the straight lines DK, DN and of the angles DKH, DNH, as has been shown. If, then, that part of the southern sundial which is cut off by the horizontal line is laid upon the northern sundial (in such a way that the part which is now upper becomes lower, and the lower becomes upper; and the part that is on the right, by this inverting of the sundial as described — namely the part toward A — becomes the left, and the left becomes the right, just as we demonstrated above must be done if we wish the southern sundial to indicate the hours also on the face of the northern plane), then all the lines will correspond to one another, together with their numbers, on account of the equality of the straight...

Diagram. A vertical-sundial figure with the gnomon-point H near the top center, a fan of solid and dotted hour-lines radiating downward, a horizontal line (the Horizon) across the upper part labelled "Hori" (left) and "Zon" (right), and zodiac-sign glyphs ranged down both side margins. Densely ruled with worn peripheral sorts; most labels are read only tentatively Readable elements:

- Printed title within the figure: "AVSTRALE" (i.e. Australe, "Southern"), split across the top
- H: gnomon-point, upper center
- Horizontal Horizon line: "Hori" (left) — "Zon" (right), with A at the left end and B at the right end
- Upper hour-marks near the apex: "1A" (left) and "11A" (right) [?]
- Equinoctial row of letters below the Horizon: F (left) — R — ... — S — K (right) [?]
- Central cluster below H: P, M, X, L, K, I [each ?]
- Left-margin zodiac glyphs (top→bottom) and right-margin zodiac glyphs: worn — [zodiac glyphs: ?]
- Hour-numbers along the lower fan: 3, 7, 8 (left); 4, 5 (right); 9, 10, 11, 12, 1, 2 near the foot [several ?]

Diagram. A smaller companion figure showing only the strip retained for the northern sundial: a horizontal Horizon line labelled "Hori" (left) and "Zon" (right), a central vertical meridian through H, with A above and G below the crossing, dotted arcs curving up toward the sides, and zodiac-sign glyphs at both lower corners. Worn Readable elements:

- Printed title within the figure: "BOREALE" (i.e. Boreale, "Northern"), split — "Bore" (left) and "ale" (right), with I between
- Horizon line: "Hori" (left) — "Zon" (right)
- H: center, on the meridian; A above H; G at the crossing [?]
- Hour-numbers flanking the foot: 7, 6 (left of center); 6 (right) [?]
- Lower-corner zodiac glyphs, left and right: worn — [zodiac glyphs: ?]

lines HN, HK, and HG, HG, and of the conic sections, and so on. Therefore it will be enough to lay out the vertical sundial that faces south, together with the parallels of the signs. For the horizontal line AB will cut off a portion from the upper part which will represent the northern sundial, provided only that it is given such a position on the face of the northern plane that the horizontal line stands in the upper place, and the right-hand part — which is then toward A — becomes the left, and the left becomes the right.

[Margin: ...[au]stral; for the upper portion cut off by the horizontal line will yield the northern sundial, if all its parts are inverted.]

This inversion is easy to carry out. Turn the southern sundial toward us (the one that must be inverted) so that it keeps its own place and position. Then, on the opposite face of the plane — the northern one — let the same lines be drawn, so that they differ in no respect from the lines of the southern sundial but correspond to them exactly. Next let the sundial thus drawn on the opposite face be rotated about the style as an axis, on that same opposite face, until the upper part becomes the lower and the reverse, and the horizontal line runs parallel to the horizon. We shall then have, on the opposite face, the northern sundial in its proper position, just as if the southern sundial had been inverted, as we said — that is, as if the upper part of the southern sundial had been exchanged into the lower, and the right into the left. This is plain to anyone who considers the matter carefully.

[Margin: In what way the parts of that portion of the southern sundial which the horizontal line cuts off must be inverted, so that it may present the northern sundial.]

That the portion of the southern sundial which the horizontal line cuts off from the upper part represents to us the northern sundial when it is inverted, as we said, will become clear from the following as well. For, as we demonstrated in the scholium of the preceding proposition, the southern sundial — when turned so that the upper becomes lower and the right becomes left, and the reverse — is changed into the northern one; and the line AB is the common section of the

horizon and the sundial, as has here been shown. Hence that part which the horizontal line cuts off toward the equinoctial line — that is, toward the upper part in the northern sundial (which resulted from that inversion of the southern sundial) — is superfluous, since the shadow of the style cannot fall upon it. Therefore the remaining portion that is cut off will be the northern sundial, so far as the hour-lines are concerned.

But as regards the arcs, or parallels, of the signs, the sign-arcs of the southern sundial are changed, in the northern sundial, into the arcs of the opposite signs — for instance the arc of Capricorn (♑, "Capricorn") into the arc of Cancer (♋, "Cancer"), and so on.

[Margin: The sign-arcs of the southern sundial are changed, in the northern sundial, into the arcs of the opposite signs.]

This is evident from the analemma of this proposition, if one rightly considers the shadow-cones which the plane of the southern sundial meets, and the shadow-cones which the plane of the northern sundial meets. For the shadow-cones in each sundial belong to opposite signs. Thus, just as the shadow-cone, when the Sun is at the beginning of Capricorn (♑), is broken off in the southern sundial at the point N, near the place of the style G, so in the northern sundial the shadow-cone, when the Sun is at the beginning of Cancer (♋), is broken off at the point K, likewise near the place of the style G, and so on.

The same is to be understood of the day-arcs, which we shall treat in the following proposition. For these too are changed into the opposite day-arcs in the northern sundial. Thus the day-arc of 10 hours in the southern sundial is changed, in the northern sundial, into the day-arc of 14 hours, and so on. But we shall give another reason for this in Proposition 21 of this book.

[Margin: The day-arcs too of the southern sundial are changed, in the northern sundial, into the opposite day-arcs.]

Furthermore, the horizontal line elegantly shows at what hour the Sun, when on any given parallel, rises or sets. For where the conic sections cut the horizontal line, there the Sun rises or sets when it is on those parallels which the said sections represent. If, therefore, hour-lines are drawn through the points where the horizontal line and the conic sections mutually intersect, these will indicate the time of the Sun's rising and setting, and so on. Thus you see, in the southern sundial, that the Sun, when at the beginning of Capricorn (♑), rises about 7½ hours from midnight and sets about 4½ hours from noon. In Aquarius (♒) and Sagittarius (♐), it rises a little before 7½ hours from midnight and sets a little

after 4½ hours from noon. Finally, in Pisces (♈) and Scorpio (♏), it rises a little before the seventh hour from midnight and sets a little after the fifth hour from noon. So too you see, in the northern sundial, that the Sun, when at the beginning of Cancer (♋), rises about 4½ hours from midnight and sets about 7½ hours from noon, and so on.

[Margin: The horizontal line indicates at what hour the Sun rises or sets.]

The horizontal line performs the same function in all the other sundials, except the horizontal one, on which a horizontal line cannot be drawn, as is clear from the scholium of Proposition 11 of the preceding book.

Here, and in all the other sundials that follow in which a horizontal line is drawn, it must be observed that the horizontal line also divides the whole sundial into two parts. The part below it shows the hours during the day, as long as it is lit by the Sun; the part placed above the horizontal line would show the hours by night — that is, in the morning before the Sun's rising above the horizon, or in the evening after the Sun's setting below the horizon — if, the obstruction of the earth being removed, the Sun could illuminate it.

[Margin: The horizontal line divides the whole sundial into a daytime portion and a nighttime portion.]

For just as, when the Sun is above the horizon, the shadow of the style is cast during the day below the horizontal line — and onto the horizontal line itself when the Sun stands exactly on the horizon, whether at morning or evening — so, when the Sun is below the horizon, the shadow of the gnomon would fall above the horizontal line, both in the morning and in the evening, if the plane of the sundial were at that time illuminated by the Sun. Therefore we may not inaptly call the front part of the southern sundial — namely the part lying below the horizontal line — the daytime sundial, and call all the arcs of the signs and of the days contained in it the day-arcs; while the rear part, contained above the horizontal line, we may call the nighttime sundial, and all the arcs that the horizontal line cuts off we may call night-arcs. Thus the night-arcs are the complements of the day-arcs up to 24, and the reverse — that is, if for example the day-arc comprises 10 hours, the night-arc will comprise 14, and so on.

[Margin: The portion of the sundial below the horizontal line is called the daytime sundial, and the arcs of the signs and of the days are called day-arcs; the portion of the same above the horizontal line is called the nighttime sundial, and the arcs of the signs and of the days are called night-arcs.]

In the same way, in the northern sundial — which we said is that portion cut off by the horizontal line, provided it is inverted, as we said above — the arcs that lie in it are day-arcs; while those above the horizontal line (such as all those included in the southern sundial) must be called night-arcs. Thus the equinoctial line, which in the northern sundial lies above the horizontal line, is a night-arc of 12 hours, whereas in the southern sundial the same line is a day-arc of 12 hours; and so on for the rest. This will have a wonderful use in the description of the hours from sunrise or sunset, as we shall teach in its proper place.

But if a larger or smaller sundial is to be drawn, a longer or shorter style DM must be taken in the analemma of this proposition, and through M a line must be drawn parallel to AD. For in this way larger or smaller diameters of the conic sections will be found, about which the conic sections themselves can be drawn in the same manner. But the equinoctial line must be distant from the center H, along the meridian line, by an interval equal to the portion of the parallel drawn through M that is intercepted between the points where it is cut by the axis DE and by the radius of the equator FD. The hour-lines, however, do not change, but remain the same, for the same elevation of the pole.

[Margin: The night-arcs are the complements of the day-arcs up to 24, and the reverse.]

[Margin: By what method, from the analemma of this proposition, the sign-arcs are drawn on the sundial for a greater or lesser length of the gnomon.]

[Margin: Another method of describing the arcs of the zodiacal signs on the vertical sundial.]

DESCRIBI — The arcs of the signs can also be described in this way. About the straight line DH, drawn in any way to represent the axis of the world, let a semicircle of any size be described from any point D as center; and let DH, on the right-hand side of D, be equal to the axis DH assumed in the portion of the analemma of the preceding proposition, or in the triangle HDI, from which — at the end of the scholium of that same proposition — we described the sundial without a portion of the analemma. And when DC has been drawn as the radius of the equator, perpendicular to the axis DH, let DI be taken equal to the equinoctial line DI of the same portion of the analemma, or of the said triangle HDI; and let the straight line HI be drawn toward the side of I, as far as one likes. Or else — lest an error be committed in drawing the line HI, on account of the nearness of the points H and I — from the point H let an arc be described toward the side of D, on which let AB be taken, toward the straight line DC, equal to the

complement of the elevation of the pole above the horizon; and through B let the straight line HB be drawn, cutting the equinoctial line DC at I. The triangle DHI will be entirely equal to the triangle DHI of the aforesaid portion of the analemma, or to the triangle of the sundial drawn without a portion of the analemma; and HI will be equal to the meridian line HI of the same portion of the analemma, or of the said triangle — as is clear from either construction of this triangle in this figure.

For from the first construction, the two sides DH, DI of this triangle are equal to the two sides DH, DI of that one, and they contain equal angles, being right angles. Therefore the whole triangles are equal. [Margin: (a) Euclid I.4]

But from the second construction, the two angles HDI, DHI of the one triangle are equal to the two angles HDI, DHI of the other triangle (since in each triangle the former angle is a right angle, and the latter equal to the complement of the elevation of the pole); and the side DH of the one is equal to the side DH of the other. Therefore the sides DI, HI of the one are also equal to the sides DI, HI of the other, and so on. [Margin: (b) Euclid I.26]

Then from the center E of the equator in the sundial — the point from which, namely, the circle was described and divided into 24 parts, etc. — let all the intervals between the center E and the hour-lines on the equinoctial line FK be transferred onto the line DC of this figure which we are now constructing, starting from the point D. That is: let the straight line DC be made equal to the interval EF or EK in the sundial drawn in the preceding proposition (which are equal, as are also all the intervals between E and any two hours that are equal in time-span from the meridian line, as is clear from the scholium of the preceding proposition); and let the straight line DT be made equal to the interval ER or ES, and so on. Then, through the points C, T, and the rest found on the line DC, let straight lines be drawn from H, to which are assigned the hour-numbers corresponding to the assumed intervals — so that the straight line HC is the line of the seventh hour after midnight and the fifth after noon; HT the line of the eighth hour after midnight and the fourth after noon; and so on.

Diagram. The "figure of the radii of the zodiac" (*figura radiorum Zodiaci*) for laying out the vertical sundial. A large fan-shaped construction: a horizontal line runs across the top, beneath which a broad semicircle is drawn; from two centers near the top a dense sheaf of straight lines (the zodiac-radii and the hour-lines) fans out downward, numbered along the lower edge; a narrow curved band of zodiac-sign compartments runs along the very bottom of the figure. Densely ruled and worn; several peripheral labels are read only tentatively Readable elements:

- Top horizontal line, left to right: A (left end), D (center of the semicircle), and H (right of center, the apex from which the hour-lines radiate). Two closely-set marks appear at the center-top, read as D and H, with a possible additional mark between them [?].
- B: on the left part of the arc/semicircle.
- T: below the top, left of center.
- ϕ' [Greek: phi, primed]: far left, low on the figure.
- V, X, Y: a horizontal row of points across the middle (the hour-points for the 6th, 5th/7th, and 4th/8th hours).
- C: center, just below the V–X–Y row; the end-point of the line DC.
- Hour-numbers along the lower fan (left to right, tentative): "12, 1, 2, 3, 4" along the upper row and "II [=11], 10, [9?], 8" below, on the left; "6" and "7" near the center; "8," then "10, 11, 12" (upper) and "4, 3, 2, 1" (lower) on the right. [several ?]
- Bottom curved band: a strip divided into compartments, each carrying a zodiac-sign glyph — worn and not securely legible [zodiac glyphs: ?].

We shall also draw these same hour-lines issuing from H if we transfer the hour-intervals in the sundial (between the center H and the equinoctial line) onto the radius of the equator of this figure from H. For in this way we shall find, on the radius of the equator, the same points as before, through which the hour-lines are drawn from H, as we demonstrated in Proposition 2 of this book concerning the horizontal sundial.

For the sixth hour from noon or from midnight, draw from H the line HV parallel to DC. And for the fifth hour after midnight and the seventh after noon, draw HX, making the angle VHX equal to the angle VHC. In the same way, for the fourth hour after midnight and the eighth after noon, draw HY, making the angle VH Y equal to the angle VHT; and so on for the rest. This will easily be done if an arc of a circle is described from H. For if the portions of this arc's circumference intercepted between HV and the straight lines drawn from H to the left of HV (namely those that cut the line DC) are transferred to the right of that same HV, into the said arc, and so on, as we said in the horizontal sundial.

After this, from the point D let the radii of the signs be drawn, as was described for the horizontal sundial, in Proposition 2 of this book.

From this figure, therefore, we shall describe the arcs of the signs just as in the horizontal sundial — namely, by transferring the hour-intervals of this figure, intercepted between H and the radii of the signs, into the hours of the sundial

from the center H, and so on. On this matter, see what we wrote in Proposition 2 of this book.

In describing the arcs of the signs on the northern sundial, the radii of the southern signs are changed into the radii of the northern, and the reverse: thus from the radius of Capricorn (♑) comes the radius of Cancer (♋); from Aries (♈) comes Libra (♎) [?]; and so on. This must also be understood of the radii of the day-arcs, of which we shall speak in the following proposition. For, namely, the southern arcs of the southern sundial are also changed, in the northern sundial, into northern arcs, and the reverse, as was said above.

DEMONSTRATIO — The demonstration of this matter does not differ from the one we brought forward for the horizontal sundial in Proposition 2 of this book. For the vertical circle too is a kind of horizon, and the angle DHI is equal to the elevation of the pole above it, as is clear. Therefore the demonstration will be the same in either sundial.

[Margin: Which arcs belong to the southern signs and which to the northern.]

Moreover, in the southern sundial the arcs of the southern signs are contained between the center of the sundial and the equinoctial line, while those of the northern signs lie beyond the equinoctial line. In the northern sundial, on the contrary, the matter stands the opposite way, as is gathered from the analemma of this proposition. The same can also appear from this: that the Sun, when in the southern signs, casts shorter shadows on the southern sundial around noon than when it lingers in the northern signs; because in the northern signs it has greater meridian altitudes than in the southern. Hence, since the shorter noon shadows fall between the center of the southern sundial and the equinoctial line, but the longer ones beyond that same equinoctial line, it is clear that the southern signs lie between the center and the equinoctial line, while the northern signs lie beyond the equinoctial line, and so on.

Which signs have arcs that are hyperbolas, or parabolas, or ellipses, we shall perceive from what we wrote in Proposition 2 of this book.

[Margin: How, from one arc, the opposite arc is described, etc.]

Here too we shall more conveniently and accurately describe the hyperbolas of the northern signs below the equinoctial line, by means of the hyperbolas of the southern signs described above the equinoctial line — if, by the method we set forth in Proposition 2 of this book, we first find the transverse diameters of the opposite sections, and their centers, and so on.

We can also use those two methods which we described at the end of the same Proposition 2 of this book. On this matter, consult what we set forth there. We have therefore described the parallels, or arcs of the zodiacal signs, on the aforesaid vertical sundial. Which was to be done.

SCHOLIVM

[Margin: A most elegant procedure for drawing the hour-lines in the figure of the radii of the zodiac, without the hour-intervals of the sundial.]

In this way Andreas Schöner [Andreas Schonerus] describes the same hour-lines of the preceding figure as issuing from H. Having drawn the straight line HIB — which forms the angle AHB equal to the complement of the elevation of the pole above the horizon, or to the elevation of the pole itself above the vertical circle, just as we too did — he takes on it any point whatever, ϕ [Greek: phi; the point-symbol printed as a circle-and-cross form, matching the figure's ϕ label], and through it draws ϕV parallel to DH, cutting the line drawn from H parallel to DC at the point V; and from V as center, with the interval ϕV as radius, he describes a circle. When this is divided into 24 equal parts, he does the same as we taught in the scholium of Proposition 2 of this book concerning the horizontal sundial.

[Margin: The description of the vertical sundial from Andreas Schöner's figure of radii.]

From the figure of radii thus constructed, as Andreas Schöner teaches, a vertical sundial can be described for a given size of style, just as we said of the horizontal one in the scholium of Proposition 2 of this book — provided that the angle ϕDH is made equal to the elevation of the pole, and not to its complement, as in the horizontal sundial.

[Margin: By what method the length of the style, and its place on the sundial, are found from the figure of radii.]

The style, according to the latter method of describing the parallels of the signs (which is accomplished by means of the figure of the radii of the zodiac), is obtained if from D a perpendicular $D\phi$ is drawn to HI — or at any rate a straight line $D\phi$ making the angle ϕDI equal to the complement of the elevation of the pole above the horizon. For the length of the style is the straight line $D\phi$ itself; and its place is the point ϕ , distant from H, the center of the sundial, by the interval $H\phi$ along the meridian line — or from the equinoctial line FK by the space $I\phi$, on the same meridian line, toward the center H.

[Margin: How, by means of the figure of the radii of the zodiac, a larger or smaller sundial is to be described.]

When the hour-lines have been described in the immediately preceding figure from the point H — if, according to this latter method taken from the said figure of the radii of the zodiac, a larger or smaller sundial is to be constructed — then the figure of the radii of the zodiac must be drawn more, or less, distant from the point H, and so on, as was said of the horizontal sundial in the scholium of Proposition 2 of this book.

[Margin: How, from the given length of the style, the distance of the radii of the zodiac from the point H is found, in the figure of radii.]

But if the size of the style is given, we shall find in this way the distance of the figure of the radii of the zodiac from the point H, on the axis HD. In the portion of the analemma of the preceding proposition, from BDC let there be cut off — by extending the straight line DM — a length equal to the given gnomon; and through M let NO be drawn parallel to AD, the diameter of the vertical, cutting the axis DE at N and the diameter of the equator FD at O. For the point D of the figure of radii of this proposition, at which all the radii of the zodiac meet, must be distant from H by the interval DN, as we said of the horizontal sundial in the scholium of Proposition 2 of this book.

[Margin: By what method the arcs of the signs are described on the vertical sundial where no center is available.]

Here too, in that sundial which we taught how to make in the scholium of the preceding proposition — even if its center is not available — we shall describe the arcs of the signs, as we set forth in the scholium of Proposition 2 of this book.

Although it has been rightly demonstrated in this proposition that the upper portion of the southern sundial, which the horizontal line cuts off, presents to us the northern sundial — provided all its parts are so transposed that the upper is turned into the lower (that is, the horizontal line occupies the upper place), and the part which, in that position, is on our right when the sundial is turned toward us becomes the left, and the reverse — nevertheless I thought it worthwhile to show the same thing here in a general way. Namely, that not only on the southern vertical sundial, but on all the others as well (such as the meridian, the polar, the one that declines from the vertical, the one reckoned from the horizon, the one inclined to the horizon, and the one that both declines from the vertical and is at the same time inclined to the horizon), the horizontal line cuts off, toward the upper parts, a portion which — on the opposite face of the plane of

the sundial, if it is so placed that all its parts are inverted, as we said — indicates to us, by the shadow of the style, exactly the same things which, on the other face (the one on which the sundial was originally drawn), the other portion of the sundial, contained below the horizontal line, shows us.

So much so that the segments of the lines lying above the horizontal line are the common sections of those same great circles with the plane of the sundial that is to be placed on the opposite face (if that inversion of the parts is made, of which we have spoken); whereas the common sections of those same great circles with the plane of the sundial originally drawn on the other face are the segments of the same lines lying below the horizontal line — whether those circles be the hour-circles from noon or from midnight, or verticals, or meridians, or those representing the celestial houses, or the zodiac (insofar as it takes various positions while the celestial signs rise), or the indicators of the hours from sunrise or from sunset, or, finally, of the unequal hours. This we shall demonstrate as follows.

Let there be two sundials, one southern and the other northern (if their planes are perpendicular to the horizon), or one upper and the other lower (if their planes are not perpendicular to the horizon), in which the styles AB, CD are equal and perpendicular to the planes of the sundials. Let both sundials be understood in their proper position, so that the southern faces south and the northern faces north — or the upper looks toward the zenith and the lower toward the nadir — and let them be parallel to each other; and let the vertices of the styles B, D be joined at the center of the world, the styles themselves forming a single straight line, namely the axis of that greatest circle to which each sundial...

[Margin: In every sundial the horizontal line cuts off the upper portion, whereby it presents the sundial to be placed on the opposite face of the sundial, provided that first the upper part becomes the lower, and the part which, after this inversion, is on our right when we are turned toward the sundial becomes the left, and the reverse.]

to which each sundial is parallel, so that the two sundials face each other.

Next, suppose a greatest circle drawn through the poles of the plane of each sundial and through the poles of the horizon. By Theodosius, *Sphaerics* I.15, this circle will stand at right angles both to the plane of each sundial and to the horizon, and it will pass through the styles AB and CD — that is, through the axis of the plane of each sundial — since it is drawn through their poles, as we said. Let this greatest circle make, with the planes of the sundials, the common

straight-line sections AE and CF,[Margin: ^a Euclid, *Elements* XI.16] which will be parallel to each other. In the vertical, polar, and equinoctial sundial, and in the one inclined to the horizon, this circle will be the meridian itself. For since the poles of the plane of each sundial — vertical, polar, equinoctial, or inclined to the horizon — and the poles of the horizon all lie in the meridian (as is plain if one weighs carefully the position of the greatest circles to which such sundials are parallel), the meridian must pass through those poles, and so it stands at right angles both to the planes of those sundials and to the horizon, by *Sphaerics* I.15. From this it follows that in the sundials named, the straight lines AE and CF are meridian lines.

In the other sundials, however, that greatest circle cannot be the meridian, because the meridian is not at right angles to their planes, since it does not pass through their poles, as is evident. Yet in the meridian sundial and in the one declining from the horizon, this circle will be properly called the *vertical* circle, since it passes through the poles of the meridian sundial and of the one declining from the horizon, and through the poles of the horizon. In sundials that decline from the vertical, this same circle will be one of the vertical circles, declining from the meridian by just as much as the plane of the declining sundial stands away from the vertical circle. And so, in inclined sundials, it will measure the inclination of the plane of the sundial to the horizon, since it is drawn through the poles of the sundial and through the poles of the horizon.

Again, suppose another greatest circle drawn through the poles of the plane of each sundial and through the poles of the earlier greatest circle that was drawn through the straight lines AE and CF. By Theodosius, *Sphaerics* I.15, this circle too will stand at right angles both to the plane of each sundial and to that earlier greatest circle, and it will pass through the styles AB and CD — namely through the axis of the plane of each sundial — since it is drawn through their poles, as was said. Let this greatest circle make, with the planes of the sundials, the common straight-line sections AG and CH,[Margin: ^b Euclid XI.16] which will likewise be parallel to each other. These will also be perpendicular to the straight lines AE and CF. For since both the plane of the sundial (whose style is AB) and the plane of this latter circle, drawn through the straight line AG, are at right angles to the plane of the earlier circle, drawn through the straight line AE, by the construction, their common section AG will also be at right angles to the plane of that same earlier circle drawn through AE;[Margin: ^c Euclid XI.19] and therefore, by Euclid, *Elements* XI, definition 3, it will be perpendicular to the straight line AE lying in that circle as well. By the same reasoning CH will be perpendicular to CF.

Now in the vertical, meridian, and vertical-declining sundials, this latter circle is the horizon itself. For since the planes of these sundials are at right angles to the horizon, the horizon in turn is at right angles to them, and consequently it passes through their poles, by Theodosius, *Sphaerics* I.13, and hence also through their styles — that is, through their axis. That this same horizon stands at right angles to the greatest circle drawn through AE is clear, because in those sundials that circle is one of the vertical circles, all of which are at right angles to the horizon, and the horizon in turn is at right angles to them. From this it follows that the straight lines AG and CH are horizontal lines in the said sundials.

In the remaining sundials, however, that circle can in no way be the horizon, because the horizon is not at right angles to their planes, since such planes are taken to be inclined to the horizon; hence the straight lines AG and CH will not be horizontal lines. But in the polar sundial that circle will be the equator, and the lines AG and CH will be equinoctial lines. In the equinoctial sundial the same circle will be the hour-circle of the 6th hour from noon, or of midnight, and the lines AG and CH will mark the 6th hour from noon, or midnight — as is evident to anyone who carefully considers the position of these circles, and as will become plain and manifest from what follows. In all the other sundials, finally, that circle will be one or another different circle; yet in every case the straight lines AG and CH will be parallel to the horizontal line of each sundial.

For since this greatest circle (the latter one just spoken of) is at right angles to the earlier greatest circle drawn through the straight lines AE, CF and the styles AB, CD, while this latter circle is set at right angles to the horizon — and so the horizon in turn is at right angles to it — both the horizon and that greatest circle drawn through the straight lines AG, CH will pass through the poles of the greatest circle drawn through AE, CF, by Theodosius, *Sphaerics* I.13. Consequently the axis of this circle drawn through AE, CF will be the common section of the horizon and of that other greatest circle drawn through AG, CH. But the greatest circle to which the sundial is parallel also passes through those same poles of the greatest circle drawn through AE, CF. (For since the greatest circle drawn through the straight lines AE, CF [recte: AE, CF] and the styles AB, CD is, by the construction, at right angles to the plane of the sundial — that is, to the circle which the sundial's plane makes on the sphere — it will pass through the poles of that circle generated on the sphere, by Theodosius, *Sphaerics* I.13. Therefore it will pass through the poles of the greatest circle to which the sundial is parallel as well, since parallel circles have the same poles, by Theodosius, *Sphaerics* II.1. And so in turn the greatest circle to which the sundial is parallel passes through the poles of the circle drawn through AE, CF, by the scholium to *Sphaerics* I.15.)

Therefore the horizon, the greatest circle drawn through the straight lines AG, CH and the styles AB, CD, and the greatest circle to which the sundial is parallel, all have one and the same common section — namely the axis of the greatest circle drawn through the straight lines AE, CG [recte: AE, CF] and the styles AB, CD. For this reason, since the plane of the sundial is parallel to this last circle, the common sections of the others (namely of the horizon, and of the circle drawn through the straight lines AG, CH and through the styles AB, CD, and of the plane of the sundial that cuts them) will be parallel, by Proposition 18 of the preceding book. And therefore, since the section made by the horizon is the horizontal line of the sundial, while the section of that other greatest circle is the straight line AG in one sundial and the straight line CH in the other, the straight lines AG and CH will be parallel to the horizontal line of each sundial.

Diagram. A pair of mirror-image line figures side by side, each a schematic of one sundial plane seen face-on, showing the common sections of the great circles with the plane and the raised style. The left figure is inscribed **Australe / Vel / Superius** ("Southern, or Upper"); the right figure **Boreale / Vel / Inferius** ("Northern, or Lower"). This same paired figure is reprinted, identically, on all three pages of this opening (folios 184, 185, 186), accompanying the continuous demonstration.

Left figure ("Southern / Upper"). A long horizontal line runs across the middle, with **P** at its left end passing through the central point **A** and continuing to the right past **G**; a vertical line runs from **L** at the top down through **A** and **E** (just below **A**) to **K** at the foot. Additional straight lines cross at or near the centre, their ends labelled at the periphery. Readable labels with approximate positions:

- **A** (centre): foot of the style; the point where the style AB meets the sundial plane.
- **B** (interior, above and right of **A**): vertex of the style AB (drawn as a short raised segment from **A**).
- **E** (just below **A**, on the vertical): where section EG cuts the line AE below the style.
- **G** (right of **A**, on the horizontal): where section EG cuts the horizontal line AG.
- **L** (top, on the vertical): where section LM cuts AE above the style.
- **M** (upper left): the outer corner, where LM meets the descending line MP.
- **P** (left end of the horizontal): where section MP cuts the horizontal line AG.
- **R** (upper, between **M** and **L**, on a diagonal): endpoint of a section line.
- **I** (lower left): endpoint of a section line.

- K (foot, on the vertical): lower endpoint.
- S (lower right): endpoint of a section line.

Right figure ("Northern / Lower"), mirroring the left. A horizontal line through the centre C, with Q at the left; a vertical line through F (top) and C. Readable labels:

- C (centre): foot of the style; counterpart of A.
- D (upper left of C): vertex of the style CD, counterpart of B.
- F (top, on the vertical): where section FH cuts CF above/at the style; counterpart of E.
- H (right of C): where section FH cuts the horizontal line CH; counterpart of G.
- N (lower centre): where section NO cuts CF; counterpart of L.
- O (lower left): outer point of section NO / OQ; counterpart of M.
- Q (left, on the horizontal): where section OQ cuts CH; counterpart of P.
- T (upper right, on a diagonal): endpoint of a section line; counterpart of R.
- V (foot, left of centre): lower endpoint.

The lines named in the accompanying text are: in the left (southern/upper) figure, AE (vertical, the meridian section), AG (horizontal), the section EG, the style AB, and the further sections LM and MP; in the right (northern/lower) figure, the corresponding CF, CH, section FH, style CD, and sections NO and OQ. (No label in either figure is flagged [?].)

Now let some greatest circle — whether it be an hour-circle or some other — pass, in the southern or upper sundial, through the centre of the world (that is, through B, or D, for these two points make a single point) and through the vertex of the style, and let it make the section EG, which cuts the straight line AE below the style at E, and the straight line AG at the right at G. This same circle will cut the northern or lower sundial, since it passes through the vertex of the style, along the straight lines CF and CH on the opposite sides — as is plain, if the plane of each sundial and the plane of the circle making the section EG are considered in their proper positions. It will cut, I say, the straight line CF above the style, and the straight line CH toward the right-hand part. For the upper part of the northern or lower sundial is opposed to the lower part of the southern or upper one, the lower to the upper, right to right, and left to left. (We call "right" or "left" the part that lies to our right, or to our left, when we are turned to face the sundial.) Hence, since those two sundials face each other, the right part of the one will answer to the left part of the other, and conversely; but right will be opposed

to right and left to left, since the style is set in the middle place between these parts.

Let, then, the common section of the said circle and of the plane of the sundial in the northern (or lower) sundial be the straight line FH, cutting CF at F above the style, and CH at H toward its right-hand part.[Margin: ^a Euclid XI.16] The two sections EG and FH will be parallel, since it is one and the same circle that makes them in the parallel planes of the sundials. Now since the straight lines AE and CF are parallel, as has been demonstrated,[Margin: ^b Euclid I.29] the alternate angles at E and at F, made on the straight lines EA and FC by the common section of the greatest circles that produce the sections AE, CF and EG, FH, will be equal — which common section passes through the points E, B, D, F, since those very greatest circles pass through these same points. The angles at A and C, which the styles make with the straight lines AE and CF, are right angles, and hence equal, by Euclid, *Elements* XI, definition 3; likewise the styles AB and CD are equal, and they are subtended by the equal angles E and F in the triangles ABE and CDF, whose bases are the parts AE, CF and EG, FH of the common section of the greatest circles that each sundial makes.[Margin: ^c Euclid I.26] Therefore the sides AE and CF will also be equal. Hence the section EG stands away downward from the style along the straight line AE by just as much as the section FH stands away upward from the style along the straight line CF.

Again, since the straight lines EK, EI are parallel to the straight lines FC, FH, as we have shown,[Margin: ^d Euclid XI.10] the angle KEI will be equal to the angle CFH. [Margin: ^e Euclid I.15] And since the angle KEI is equal to the vertically opposite angle AEG, the angles at E and F in the triangles AEG and CFH will be equal. But the angles EAG and FCH are also equal, being right angles; and the sides AE, CF adjacent to the said angles have been shown to be equal.[Margin: ^f Euclid I.26] Therefore the sides AG and CH will also be equal. Hence the straight lines EG and FH stand away from the styles, toward the right, by equal distances along the straight lines AG and CH. And if the straight line EG should cut the line AE above the style, or the line AG to the left, we shall show by the same reasoning that FH cuts CF below the style and CH to the left as well — that is, always on the opposite sides — and that the straight lines which FH cuts off from CF and CH are equal to those which EG cuts off from AE and AG.

Next, let some other greatest circle in the southern or upper sundial, drawn through the vertex of the style B (or D), make the section LM, which does not cut both AE and AG, but only one of them — namely AE, at L, above the style — while being parallel to the other, that is, to AG. Since this same circle passes through the

vertex of the style, it will cut the northern or lower sundial below the style — that is, on the opposite side — along the straight line NO, which cuts CF at N; and NO will be parallel to CH. For since LM is set parallel to AG, and AG in turn is parallel to CH, as we said before,[Margin: ^g Euclid XI.9] LM and CH will also be parallel to each other. Again, since NO is parallel to LM (because LM and NO are sections made by one and the same plane in parallel planes),[Margin: ^h Euclid XI.16] and CH has just been shown parallel to LM, NO and CH will likewise be parallel to each other.[Margin: ⁱ Euclid XI.9] Therefore, since LM is parallel to AG and NO to CH, the angles L and N will be equal to the angles A and C, and hence right angles.[Margin: ^k Euclid I.29]

And since AE, CF are parallel, as was shown above,[Margin: ^l Euclid I.29] the alternate angles at the points L and N, made on the straight lines AE, CF by the common section of the greatest circles that produce the sections AE, CF and LM, NO, will be equal to each other — which common section passes through the points L, B, D, N, since those greatest circles pass through these same points. The angles at A and C, which the styles make with the straight lines AL and CN, are right angles and hence equal; and the styles AB and CD are equal, subtended by the equal angles L and N in the triangles ABL and CDN, whose bases are the parts AE, CF and LM, NO of the common section that each sundial makes.[Margin: ^m Euclid I.26] Therefore the sides AL and CN will also be equal. Hence the section LM stands away from the style upward along the straight line AE by just as great an interval as the section NO stands away from the style downward along the straight line CF. And if the straight line LM should cut AE below the style, we shall show by the same reasoning that NO cuts CF above the style, that NO is parallel to LM, and that the straight line which NO cuts off from CF is equal to the straight line which LM takes from AE.

Finally, let some other greatest circle in the southern (or upper) sundial make the section MP, which again does not cut both AE and CF, but only AG, at P, to the left, while being parallel to AE. This same circle, since it is drawn through the vertex of the style, will cut the northern (or lower) sundial on the opposite side — that is, to the left as well — along the straight line OQ, which cuts CH at Q; and OQ will be parallel to CF. For since MP is set parallel to AE, and CF was earlier shown parallel to AE, MP and CF will likewise be parallel to each other.[Margin: ⁿ Euclid XI.9] Again, since OQ is parallel to MP (because MP and OQ are sections made by one and the same plane in parallel planes),[Margin: ^o Euclid XI.16] and CF has just been shown parallel to MP, OQ and CF will also be parallel to each other.[Margin: ^p Euclid XI.9] Therefore, since MP is parallel to AE and OQ to CF, the angles P and Q will be equal to the angles A and C, and hence right angles.[Margin: ^q Euclid

I.29] And since AG, CH are parallel, as we showed above,[Margin: ^r Euclid I.29] the alternate angles at the points P and Q, made on the straight lines AG, CH by the common section of the greatest circles that produce the sections AG, CH and MP, OQ, will be equal to each other — which common section passes through the points P, B, D, Q, since those greatest circles pass through these same points...

Diagram. The same paired figure as on the preceding page, reprinted identically: the left figure inscribed **Australe / Vel / Superius** ("Southern, or Upper") with central point A, style AB, and labelled points B, E, G, L, M, P, R, I, K, S; the right figure inscribed **Boreale / Vel / Inferius** ("Northern, or Lower") with central point C, style CD, and labelled points D, F, H, N, O, Q, T, V. See the full description under fig-p196-1. (No label flagged [?].)

they pass through them too. And the angles at A and C, which the styles make with the straight lines AP and CQ, are right angles and hence equal; likewise the styles AB and CD are equal, subtended by the equal angles P and Q in the triangles ABP and CDQ, whose bases are the segments AG, CH and MP, OQ of the common section that each of the two sundials makes.[Margin: ^a Euclid I.26] Therefore the sides AP and CQ will also be equal. Hence, by equal intervals, the straight lines MP and OQ stand away from the styles toward the left along the straight lines AG and CH. And if the straight line MP should cut AG to the right, we shall demonstrate by the same argument that OQ cuts CH to the right as well, that OQ is parallel to MP, and that the straight lines which MP and OQ cut off from AG and CH are equal.

Having demonstrated this, we shall easily show that the straight lines EG, LM, MP, and any others whatever, in the northern or lower sundial, are the common sections of those same greatest circles with the plane of the sundial — the very circles whose sections in the southern or upper sundial they are — provided the southern or upper sundial be so placed on the northern or lower face of the plane of the sundial that its upper part is turned into the lower, and the part which, in that position after this transposition, is on our right when we are turned to face the sundial becomes the left, and conversely.

For if the southern or upper sundial is understood to be laid upon the northern or lower one so that style coincides with style, the right part at G with the right part at H, the point A with the point C, and the line AG with the line CH, then the points G, P will fall upon the points H, Q, because the straight lines AG, AP have been shown equal to CH, CQ. And if, in this position, we conceive the southern or upper sundial to be turned about the straight line GP — or (what is the same) about HQ, since in this position these two lines make one and the same — so that

the upper part becomes the lower and conversely, then the points E, L will fall upon the points F, N, because the straight lines AE, AL have been shown equal to CF, CN. Hence the line AG will coincide with the line FH, since we shall have shown the points E, G to coincide with the points F, H; and the line LM will coincide with the line NO, and the line NP with the line OQ, because the points L, P fall upon the points N, Q, and the angles at L, P, N, Q have been shown to be right angles.

By the same reasoning we shall conclude that any other line whatever in the southern or upper sundial — whether it cut both AE and AG, or only one of them and be parallel to the other — coincides with the line answering to it in the northern or lower sundial, if that superposition and transposition, as we have just described, is carried out. Therefore the whole southern or upper sundial will, by this reasoning, coincide with the whole northern or lower sundial.

Since, then, by that superposition and turning the upper part of the southern or upper sundial becomes the lower, and the right becomes the left, as is plain — (for if the southern sundial is understood to be turned by itself, so that the upper part is changed into the lower, the point G will remain on our left, and P on our right, when we are turned to face the sundial. Hence, in order that the line EG may coincide with the line FH, and the remaining lines with the remaining lines, it is necessary that the left part G be changed into the right, and the right part P into the left. For only so will it come about that the several lines coincide, one with one. This too will become plain by the following reasoning: after the straight line GP has been laid upon HQ, the point G indeed remains on our right when we face the sundial, just as does the point H of the northern or lower sundial. But after the turning of the southern or upper sundial about the straight line GP (or HQ), so that the upper parts become the lower, the point G will be set on our left, as was said, if we turn our face to the sundial itself; and since the sundial is then turned toward the south, its lines must be drawn on the opposite face, which looks toward the north. And so, when we are turned to the northern face of the sundial, the same point G will be on our right) — it plainly follows that from the southern or upper sundial, if a transposition of all the parts is made so that the upper is changed into the lower, and the part which after this change is on our right when we face the sundial becomes the left, and conversely, there results the northern or lower sundial.

Now since, in the aforesaid superposition and turning of the southern or upper sundial, its horizontal line coincides with the horizontal line of the northern or lower sundial — (for in sundials whose planes are at right angles to the horizon,

such as the vertical, the vertical-declining, and the meridian, the horizontal line is drawn through the place of the style, and so the horizontal lines are AG and CH, which by the very superposition coincide with each other. But in the other sundials, whose planes are not at right angles to the horizon — such as the polar, the equinoctial, the horizon-declining, the horizon-inclined, and the one that declines from the vertical and is at the same time inclined to the horizon — since the horizontal line falls, in the upper ones, above the style, and in the lower ones below it, and is always parallel to the straight line AG, as we demonstrated a little earlier, it comes about by this reasoning that the horizontal line of the upper sundial coincides with the horizontal line of the lower one. This will be demonstrated, as it has been demonstrated that the straight line LM coincides with the straight line NO) — and since the portion of the sundial lying above the horizontal line is superfluous in any sundial, because the shadow of the style can never fall upon it during the day, it is established universally that the horizontal line, in any sundial whatever, cuts off toward the upper parts a portion which, on the opposite face of the plane of the sundial, presents to us that same sundial, if it be so placed that the horizontal line occupies the upper position on the opposite face, and the part which in that position is on our right when we face the sundial becomes the left, and conversely. For once that transposition of the parts has been made, the part above the horizontal line of the southern sundial is placed, in the northern sundial, below the same horizontal line, as is manifest. Which was to be demonstrated.

And if it should ever happen that lines in a sundial, which are the common sections of the plane of the sundial and of the greatest circles, pass through A — the place of the style in the southern or upper sundial — and through C, the place of the style in the northern or lower sundial (as happens with the celestial houses of the vertical sundial, with the hours from noon or midnight, and with the meridian circles of the equinoctial sundial, as will become plain from what follows), it will nonetheless follow that the southern or upper sundial coincides with the northern or lower one...

Diagram. The same paired figure as on the two preceding pages, reprinted identically: the left figure inscribed **Australe / Vel / Superius** ("Southern, or Upper") with central point A, style AB, and labelled points B, E, G, L, M, P, R, I, K, S; the right figure inscribed **Boreale / Vel / Inferius** ("Northern, or Lower") with central point C, style CD, and labelled points D, F, H, N, O, Q, T, V. See the full description under fig-p196-1. (No label flagged [?].)

or the lower one, if all its parts are inverted, as we have taught. For let some great circle in the southern (or upper) sundial, drawn through the style AB, make a section RS passing through A, the place of the style. Since this same circle passes through the style, it will also cut the northern (or lower) sundial along the straight line TV, likewise passing through C, the place of the style — yet not in the opposite parts, but in corresponding ones. That is: if the section RS above the style inclines to the left and below the style to the right, then the section TV inclines above the style to the right and below the style to the left, and conversely. For these parts answer to those, as is evident to anyone who carefully considers the position of each sundial. [Margin: a — Elements XI.16 ("16. vnde.").] Now the sections RS and TV will be parallel to each other, since they are made in parallel planes of the sundials by one and the same plane of the great circle. And since the straight lines AE and CN are also parallel, as we showed a little earlier, [Margin: b — Elements XI.10 ("10. vnde.").] the angles EAS and NCV will be equal to each other. [Margin: c — Elements I.15 ("15. primi.").] Therefore, since the vertical angles EAS and LAR are likewise equal, the angles LAR and NCV will also be equal. So, superimposing once again the straight line AG upon the straight line CH, so that the point A falls upon the point C, and understanding that in this position the southern (or upper) sundial is turned about the straight line AG — or CH, which is the same line in that position — so that its upper part is changed into the lower and conversely: then the straight line AR will fall upon the straight line CV, because the angles LAR and NCV are equal. And the same reasoning holds for any other lines drawn through A and C, the places of the styles. Therefore, as before, the portion cut off by the horizontal line, if it is inverted as was said above, will yield the sundial to be drawn on the opposite face of the sundial's plane.

I have chosen to demonstrate this here more fully, so that we should not be forced in what follows to keep hammering at the same demonstration. For it will bring great usefulness to what comes after, as will become plain in its proper places.

PROBLEMA 15. PROPOS. 15.

To describe the parallels of the diurnal arcs on the same vertical sundial.

If, from D — the place of the Zodiac's rays in the figure of the Zodiac's rays of the preceding proposition — the rays of the lengths of the days found in proposition 33 of the previous book are drawn out; or if, on the Analemma of that same preceding proposition, in place of the parallels passing through the beginnings of the signs, the parallels of the diurnal arcs are taken: then we shall describe the

parallels of these arcs in the same way in which we described the parallels of the signs. For the ratio is the same in these as in those, since all are equidistant from the Equator. Therefore we have described the parallels of the diurnal arcs, and so on. Which was to be done.

[Margin: The drawing of the diurnal arcs on the aforesaid vertical sundial.]

Diagram. A pair of construction figures side by side.

Left figure — a full circle (the "figure of the Zodiac's rays" of the preceding proposition), with center at a small inner circle labelled **D**, crossed by many radial straight lines (the rays):

- A, Q (top, to the right of R): points on the upper circumference
- R (top left of the upper labels)
- 16, 15, 14, 13, 12, 11, 10, 9, 8 (descending along the upper-left circumference)
- E (right, on the circumference)
- B (left, on the circumference)
- Q, R (bottom of the circumference)
- F, G, H [?] (small letters clustered about the center D — worn; readings uncertain)

Right figure — a horizontal top line with a fan of straight rays descending from the region near its middle down to two rows of numbers:

- A (left end of the top line)
- D, H (near the middle-top, at the head of the ray-fan)
- B (on the arc, at the left)
- C (below B, at the left)
- Inner ray-numbers (along the arc, left to right): 1, 2, 3, 4, 5, 6, 7, 8
- Outer ray-numbers (along the bottom edge, left to right): 16, 15, 14, 13, 12, 11, 10, 9

(Several small ray-numerals and the central letters of the left figure are worn; the point-letters A, B, C, D, E, H, Q, R are read with confidence.)

We shall describe the parallels of these arcs in the same manner in which we described the parallels of the signs; for the ratio is the same in these as in those, since all are equidistant from the Equator. Therefore, the parallels of the diurnal arcs, etc. Which was to be done.

PROBLEMA 16. PROPOS. 16.

To describe the vertical circles on the same vertical sundial.

Upon the meridian line of the sundial, from G, the place of the style, let the length of the style be carried up to the point A — which may be taken either below G or above it. From A let a circle of any size be described; and having divided it into 360 equal parts, beginning from the meridian line (if all the verticals are wanted), or into fewer, according to the number of verticals to be drawn — (we divided it into 24, so that each interval contains 15 degrees) — let hidden straight lines be drawn through the points of division and the center A, cutting the horizontal line at points; and if through these points straight lines are drawn parallel to the meridian line HI, the vertical circles will have been described. These parallel lines are easily drawn: if through any point of the meridian line — say through B — a line is drawn parallel to the horizontal line, and all the points of the horizontal line, taken with the compass from G, are transferred onto that parallel from the point B. [Margin: d — Elements I.33 ("33. primi").] For the straight lines connecting two points equidistant from G and B will be parallel to GB itself.

[Margin: The drawing of the vertical circles on the same vertical sundial.]

[Margin: Demonstration of the drawing of the vertical circles.]

That these parallels in the plane of the vertical sundial are the vertical circles — or rather the common sections of the vertical circles and the plane of the vertical sundial — we shall demonstrate as follows. Let the plane of the circle described from A be understood to be turned about the horizontal line until its center A is joined to D, the vertex of the style, so that it becomes the same as the plane of the Horizon passing through the horizontal line and the gnomon GD. Since, then, the common sections of the Horizon and of the vertical circles running through the poles of the Horizon divide the Horizon — and hence also the circle described from A (since in such a position the Horizon and the said circle have the same center) — into equal parts,

the straight lines from A passing through the points of division will be the common sections of the verticals and of the Horizon. Therefore the vertical circles will cut the plane of the sundial at the points where the said common sections meet the horizontal line. And since, by proposition 18 of the previous book, the common sections of the vertical circles and of the plane of the vertical sundial are parallel — because the plane of the vertical sundial is equidistant from their common section, namely the axis of the Horizon drawn through the

zenith and its opposite point — it is manifest that those straight lines which are parallel to the meridian line (which is one of the vertical lines) are the common sections of the vertical circles and of the plane of the sundial. But the vertical circle properly so called cannot be projected onto the plane of the sundial, since it is equidistant from it and therefore makes no section in it. Therefore we have described the vertical circles on the same vertical sundial. Which was to be done.

Diagram. A circle representing the vertical sundial with its vertical circles. A vertical diameter runs from **A** at the top to **B** at the bottom; the horizontal diameter carries the word **HORI-** on the left and **ZON** on the right (i.e. HORIZON, split across the figure). At the center is a dense cluster of style-points:

- A (top of the vertical diameter)
- B (bottom of the vertical diameter)
- G (the style base, at the center), with the neighbouring points K, L, I and, below them, M, N (small boxed "KI" cluster at the center — the style-point group)
- HORI- / ZON (the horizontal line)

Dotted lines radiate from the central cluster to the circumference on both sides, marked with the degrees of the vertical circles. Readable degree labels, descending on each side: 15, 30, 45, 60. The topmost pair of dotted lines carry a worn numeral [?] (probably 75).

SCHOLIVM.

[Margin: The vertical circles on a southern sundial are all southern, and on a northern sundial all northern.]

It is clear that these vertical circles are southern on a sundial that faces the meridian, but northern on one that faces the north. On both, moreover, the number of degrees is terminated at the meridian line, and the drawing is the same on each. Hence here too the horizontal line will cut off the portion that must be placed on the northern face — yet in such a way that the horizontal line is set in the upper position, as we demonstrated in the scholium to proposition 14 of this book. But here there is no need to change the right side into the left and conversely, since the numbers are the same on both sides, and the vertical lines have the same situation on both sides of the meridian line, as is evident.

PROBLEMA 17. PROPOS. 17.

To describe the parallels of the Horizon on the same vertical sundial.

[Margin: The drawing of the parallels of the Horizon, or almucantarath, on the same vertical sundial.]

Let the Meridian of the Analemma ABCD be described, in which BD is the diameter of the Horizon and AC the diameter of the Vertical. Having divided each quadrant AB and AD into 90 equal parts, so that each part comprises a single degree — or into fewer, according to the size of the sundial (we divided each into six, so that each part answers to 15 degrees) — let two points equally distant from the straight line BD be joined by straight lines, which will be the common sections of the Meridian and of the parallels of the Horizon, which they call almucantarath. Next, from the points of division of each quadrant, through the center E, let straight lines be drawn, so as to make triangles through the axis in cones whose bases are the parallels of the Horizon, both below and above the Horizon, and whose common vertex is E, the center of the world. For the Meridian ABCD, cutting the said cones through their axis AC, makes — by proposition 3 of Book 1 of Apollonius — triangles through the axis. Let there also be cut off from BD, on both sides from the point E, a straight line EG equal to the gnomon; and through G let GO be drawn parallel to AC — namely the common section of the plane of the sundial and of the Meridian — cutting the sides of the axial triangles at the points K, L, M, N, so that KO, LO, MO, NO are the diameters of the conic sections. If, then, the points K, L, M, N, taken on this Analemma from the point G, are transferred onto the meridian line of each sundial, below the horizontal line, from the place of the style G, and about the meridian line the said conic sections are described — by proposition 8 of the previous book — passing through the points K, L, M, N (these conic sections being hyperbolas, by proposition 6 of the preceding book, since the vertical circle running through the poles of the parallels of the Horizon cuts them), so that their horns are turned ever more and more away from the horizontal line: then the parallels of the Horizon will have been described, as may be seen in the figure of the preceding proposition.

Diagram. The Meridian of the Analemma — a full circle for constructing the parallels of the Horizon. Cardinal points on the circumference:

- A (top; marked 90)
- B (left end of the horizontal diameter)
- C (bottom; marked 90)
- D (right end of the horizontal diameter)
- E (center)

- G, and the cluster K, L, M, N (near the center, the almucantarath-construction points)

Each quadrant is graduated at 15° intervals; readable degree labels running from the horizontal toward the top and bottom on both sides: 15, 30, 45, 60, 75. Straight chords parallel to BD (the almucantarath = parallels of the Horizon) cross the circle. One left-side degree label reads **20** [?] — anomalous within a 15°-graduated series (probably a worn 30); flagged.

[Margin: Another way of drawing the parallels of the Horizon on a vertical sundial.]

ALITER (Otherwise). Having described a quadrant ABC of any size, and divided it into 90 degrees — or into fewer parts, according to the number of parallels to be drawn — let straight lines be drawn out from the center A through the points of division. These will answer to the radii of the parallels of the Horizon in the quadrant EBC contained in the preceding figure, beginning from the straight line AB, so that the radius nearest to the straight line AB is that of the parallel of the Horizon at 15 degrees, the next at 30, and so on, as the numbers declare.

After this, from the sundial of the preceding proposition, let the intervals between the center A and the points at which the vertical lines intersect the horizontal line be taken, and let them be transferred from A onto the straight line AB of this figure, writing beside AB the numbers of the vertical lines, and marking the points that bound the said intervals. Finally, through the points marked on the straight line AB, let lines be drawn parallel to AC, or perpendiculars to AB. This is easily done if some line is drawn parallel to AB and the points of the line AB are transferred onto it, beginning from the straight line AC, and so on. For example, from the figure of the preceding proposition, let the interval AK be transferred onto the straight line AB of this figure, up to the point E, appending the number 30; and through E let EF be drawn parallel to AC, and so on.

Diagram. The vertical sundial with its parallels of the Horizon (the arcs of the Sun's altitudes). A near-horizontal top line runs from **B** at the left to **A** at the upper right, from which a fan of straight rays descends:

- B (upper left)
- A (upper right, the apex of the ray-fan)
- M (in the upper-left interior, on a descending curve)
- F (in the interior, right of center)

- C (lower right)
- Degree labels descending along the left side and across the lower edge: 15, 30, 45 (down the left), then 60, 75, 90 (across the bottom, left to right toward C)
- A cluster of small worn ray-numerals near A [?]

Once the figure has been prepared in this manner, the parallels of the Horizon — that is, the arcs of the Sun's altitudes — are described as follows. The intervals of the lines equidistant from AC, comprised between the straight line AB and a radius (say the radius of the parallel of the Horizon at 15 degrees), are transferred on both sides from the points at which the horizontal line in the figure of the preceding proposition is cut by the vertical lines, onto the vertical lines corresponding to the numbers marked on the straight line AB, marking points on the vertical lines. For example, on the straight line EF let a length be taken equal to KL, on each vertical line at 30 degrees, both below and above the horizontal line, and so with the rest. For if these points are suitably joined by a certain curved line, there will be described the arc, or parallel of the Horizon at 15 degrees, both on the southern sundial and on the northern — which is indeed cut off by the horizontal line, as we showed. In the same way the remaining parallels of the Horizon will be described, if the straight lines intercepted between the line AB and the radii of the parallels of the Horizon are transferred on both sides from the horizontal line onto the corresponding vertical lines of the sundial. Yet it is not necessary to transfer all these straight lines onto the vertical lines on both sides — that is, above and below the horizontal line — because there is no need to describe as many parallels of the Horizon on the northern sundial as on the southern. Thus you see that on the northern sundial we have described only the parallel at 15 degrees, since the others would fall entirely outside the tropic, and therefore no shadow could fall upon them.

That the parallels of the Horizon are rightly described by this art will be demonstrated as follows. In the figure of the preceding proposition, let the straight line AG — which was taken equal to the gnomon — be understood to stand at right angles to the plane of the sundial at the point G; and let the figure just constructed about the point A be carried around, joined to the center of the world, so that the point A of this figure never departs from the point A of the figure of the preceding proposition (that is, from the center of the world), and the straight line AC remains perpetually equidistant from the meridian line HI (that is, joined to the axis of the Horizon and of its parallels), and the point B is extended toward the sundial, and therefore the straight line AB, not departing from the plane of the Horizon, always meets the horizontal line in that motion. In

this circumduction the point E, say, will fall upon the point K, because the straight line AK was taken in the preceding proposition equal here to AE. For the straight line AK falling from the point A on high — namely from the vertex of the style — is equal to the straight line AK in the plane of the sundial; as will easily be proved if the triangle AKG in the plane is compared with the triangle AKG on high. For since the sides AG, GK of the former are equal to the sides AG, GK of the latter, and contain equal angles (being right angles), [Margin: a — Elements I.4 ("4. primi").] the base AK of the former will be equal to the base AK of the latter. Since, then, both KL and EF are equidistant from the axis of the Horizon, [Margin: b — Elements XI.9 ("9. vndec.").] KL and EF will also be parallel to each other; and therefore, the point E coinciding with the point K (as we showed), the straight line EF will also coincide with the straight line KL — otherwise the one would not be parallel to the other, since both meet in the point K in that circumduction. Therefore, since KL was taken equal to the straight line EF, the point F will fall upon the point L, and so the radius of the parallel of the Horizon at 15 degrees will meet the plane of the sundial at L. Through the point L, therefore, the arc of the aforesaid parallel at 15 degrees will pass, since the radius of the said parallel falls upon it in that revolution, as has been shown. In no other way shall we demonstrate that the point M of the same radius falls upon the point N, and so with the rest. Therefore we have described the parallels of the Horizon on the same vertical sundial. Which was to be done.

[Margin: Demonstration of the latter drawing of the parallels of the Horizon.]

PROBLEMA 18. PROPOS. 18.

To describe the Meridians, or circles of longitude of cities, on the same vertical sundial.

These are described as the hour-lines from noon or midnight, in the manner we stated in proposition 6 of this book — provided that the circle described from E has the beginning of its division from the Meridian of the Fortunate Islands, which is obtained if the longitude of the place is reckoned from N: on the southern sundial indeed toward the left (in the sundial of proposition 13 of this book), but on the northern toward the right — namely always toward the western parts, as reason demands. This will be easily understood if, on each sundial, the circle described from E is turned about the equinoctial line until it attains its proper position, namely until it is set in the plane of the Equator, and so on. And so we have described the Meridians, or circles of longitude of cities, on the same vertical sundial. Which was to be done.

[Margin: The drawing of the Meridians on the same vertical sundial.]

SCHOLIVM.

[Margin: How the Meridians of a southern sundial, drawn beyond the horizontal line, yield the same meridians on a northern sundial.]

If the Meridian circles — or rather their common sections with the plane of the sundial — on the southern sundial are produced beyond the horizontal line and the center H, the same Meridian circles will also be obtained on the northern sundial, which the horizontal line cuts off — provided you remember that the whole sundial must be inverted, so that the upper parts become the lower and the right come out as the left, as was shown in the scholium to proposition 14 of this book. Likewise, that these sections of the Meridians and of the plane of the southern sundial present, on the northern sundial, the sections of the opposite semicircles of the Meridians, so that they have the numbers which are made up by adding 180 degrees to the numbers they have on the southern sundial (yet casting off 360 degrees if, from that addition, a number greater than 360 degrees results); or else the numbers which, on the circle described from E, are opposite by a diameter to those numbers which the said sections...

[Margin: ...in the northern sundial.]

the same sections on the southern sundial. Yet the lines of the southern sundial, when produced beyond the center H, will again bear the very same numbers that they have on the southern sundial, because on the northern sundial they are once more the sections of the opposite semicircles — just as was said of the hour-lines reckoned from noon or from midnight. For there, take the line of (say) the 7th hour from midnight: on the southern sundial, produced beyond the horizontal line before it reaches the center H, it marks on the northern sundial (once the aforesaid inversion of the sundial is made) the 7th hour from noon — that is, the common section is of the plane of the sundial and of the semicircle of the 7th hour from noon, and not from midnight, as on the southern sundial. But produced beyond the center H, it will again show the 7th hour from midnight, as on the southern sundial. The same reasoning is to be applied to the rest, as is clear from the sundial of proposition 14 of this book. So it happens here too. For the Meridian 120 on the southern sundial, produced beyond the horizontal line, gives on the northern sundial the Meridian 300; but produced further, through the center H, it yields on that same northern sundial the Meridian 120 once again, and so on.

Diagram. A vertical-sundial construction showing the Meridians (circles of longitude of cities), with the meridians of Rome and of the Fortunate Islands laid out. A large circle carries a vertical meridian line running top to bottom through the labelled points; from an interior point near the top a fan of straight meridian-lines radiates outward across the whole field, and several long straight lines run out to the framing rectangle at both sides. Two concentric numeral scales (graduated in degrees of longitude) are printed around the inner circle. Small place-labels are set within the figure.

- M (top of circle, on the meridian line): vertex
- H (on the meridian line, below M)
- G (on the meridian line, below H; the fan of meridian-lines springs from here)
- E, I (interior, near the center of the figure)
- I, G, H (repeated on the lower meridian line, mirroring the upper points)
- N (bottom of the meridian line)
- ROMA (set vertically in the central circle — the meridian of Rome)
- INS[VLÆ]. FORT[VNATÆ]. (the Fortunate Islands, i.e. the reference meridian; the abbreviation appears twice, upper-right and lower-right, set diagonally)
- Insulæ Fortun[atæ] (spelled out, set vertically at left and again lower-right)
- Small italic labels along the central rays, near point I: Genua (vertical cursive, right of point I) and Alexandria [Alexā(ndria) [?]] (vertical cursive, left of point I) — city-meridian labels [Stage-5 recovery]
- Longitude numerals read around the scales (as printed, several small ones worn): 240, 180 (upper); 315 (right), 120 (left), 120 (right), 105, 315, 330, 345, 90 (outer); 75, 60, 45, 30, 15, 150, 135, N 45, 60, 285, 300, 120 (upper inner ring); 285, 300, 150, 135, 120, 30, 15 (lower inner ring)

[Several of the small interior longitude numerals are worn; figure marked as a — see report. The two italic ray-labels near point I are now read as the city-meridian labels Genua and Alexandria (Stage-5).]

PROBLEMA 19. PROPOS. 19.

To describe on the same vertical sundial the parallels of cities — that is, the circles of latitude.

[Margin: The drawing of the parallels of cities, or of latitudes, on the same vertical sundial.]

These are drawn just like the parallels of the signs of the Zodiac, which we treated in proposition 14 of this book — except that, in place of the parallels running through the signs of the Zodiac, one takes the parallels passing through the vertices of the cities, and so on. And so we have drawn the parallels of cities, and so on. Which was to be done.

PROBLEMA 20. PROPOS. 20.

To describe the celestial houses on the same vertical sundial.

[Margin: The drawing of the celestial houses according to Johannes Regiomontanus [Regiomontanus] on the same vertical sundial. Demonstration of the drawing of the celestial houses.]

Following the opinion of Johannes Regiomontanus [Regiomontanus] — who would have the circles of the celestial houses pass through the common sections of the Horizon and of the Meridian, and would divide the Equator into 12 equal parts — the celestial houses will be described in this way. Let the circle drawn from E be divided into 12 equal parts, beginning from the meridian line; and through the points of division and the center E let hidden straight lines be drawn, cutting the equinoctial line at points through which — from G, where the horizontal line intersects the meridian line — straight lines drawn out will give the circles of the celestial houses according to Johannes Regiomontanus [Regiomontanus]. For when the circle drawn from E is carried around about the equinoctial line, until E is joined to the center of the world and that circle comes to lie in the plane of the Equator, the straight lines passing through the center E will be the common sections of the circles of the celestial houses (among whose number is the Meridian too) and of the Equator; since with the

circles and the Equator they share common sections, according to Johannes Regiomontanus [Regiomontanus]. Let them divide the Equator — and therefore also the circle concentric with it, drawn from E — into 12 equal parts, and let them pass through the center of the world, where the point E is now placed. Such circles, then, will meet the plane of the sundial at the points where the straight lines drawn through E cut the equinoctial line; and so through those same points of the equinoctial line the common sections of the plane of the sundial and of the circles of the celestial houses will pass. But these common sections also cut one another mutually at G, by proposition 18 of the preceding book — since the common section of these circles, namely the axis of the Vertical circle, or the common section of the Horizon and Meridian, meets the plane of the sundial at G, where the meridian line cuts the horizontal line, as is clear from the portion of

the Analemma in proposition 13 of this book. Therefore the straight lines drawn from G through the points of the equinoctial line will be the common sections of the plane of the sundial and of the circles of the celestial houses. Which was proposed.

Diagram. A vertical-sundial construction for the celestial houses, with the word HORIZON split across the horizontal line. A central circle carries the vertical meridian line through the points M(top, off-scale)–H–G–I–E; the horizontal Horizon line runs across the figure, and from the interior a fan of house-lines radiates outward to the frame, giving the wedge-shaped house divisions numbered as an astrological house-diagram. Several number-sorts are printed mirror-reversed (a woodcut artifact).

- H (on the meridian line, upper center)
- G (on the meridian line, at the crossing with the Horizon)
- I (on the meridian line, below G)
- E (interior, below I — the center from which the base circle is drawn)
- 10 (interior, just below E) [numeral: printed as "10"; worn — possibly "16" [?]]
- HORI ... ZON (the word "Horizon" set to left and right of the meridian along the horizontal line)
- House numerals around the dial: 8, 8 (upper left; the "8"s printed reversed), 12, 12 (upper right, reversed); I, 1 (left) and I, 7 (right, at the Horizon ends); 12, 12 (lower left); 8, 8 (lower right); 11, 11 and 9, 9 (lower center)

SCHOLIVM.

[Margin: For what reason the parts of the celestial houses are described on the same vertical sundial.]

But according to the opinion of Campanus [Campanus of Novara] — who draws the circles of the celestial houses through the common sections of the Meridian and of the Horizon, dividing the Vertical circle properly so called into 12 equal parts — we shall carry out the same problem as follows. From G, the place of the style, we shall lay out a circle of any magnitude divided into 12 equal parts, beginning from the horizontal line, or from the meridian line. For the straight lines drawn from G through the points of division will give the celestial houses according to Campanus [Campanus of Novara]. This we shall show as follows.

[Margin: The drawing of the celestial houses according to Campanus [Campanus of Novara] on the same vertical sundial. Demonstration of the latter drawing of the celestial houses.]

Since the circles of the celestial houses cut the Vertical circle, according to Campanus [Campanus of Novara], into equal parts, beginning from the common section of the Vertical and the Horizon — and therefore also divide into just as many equal parts the circle which the plane of the sundial makes, equidistant from the Vertical circle in the sphere, by proposition 1 of Book 1 of Theodosius, beginning from the horizontal line, namely from the common section of the plane of the sundial and of the Horizon (because circles passing through the poles of the Vertical circle divide the Vertical and its parallels into similar segments, by proposition 10 of Book 2 of Theodosius) — it follows that the circles of the celestial houses, and hence also the common sections of them and of the plane of the sundial, pass through the points of division of the circle drawn from G. (For the point G, upon which the axis of the Vertical circle falls, is the center of the circle generated in the sphere by the plane of the sundial, by proposition 10 of Book 1 of Theodosius; and therefore that circle — and the one we drew from G — are divided into similar arcs by straight lines going out from the center G.) And on that account the common sections also pass through the point G, by proposition 18 of the preceding book, in which, of course, the common section of the circles of the celestial houses meets the plane of the sundial, and in which those same circles cut one another mutually. Therefore the straight lines drawn from G through the points of division of the circle drawn from G will be the common sections of the plane of the sundial and of the circles of the celestial houses, according to Campanus [Campanus of Novara]. Thus we have described the celestial houses on the same vertical sundial. Which was to be done.

SCHOLIVM.

[Margin: For what reason the parts of the celestial houses are described on the same vertical sundial.]

Since each celestial house contains 30 degrees, if each twelfth part of the circle drawn from E (or from G) is divided into 30 parts — or into fewer equal parts, according to the size of the sundial — and the rest is done as before, other circles of position will be drawn, passing through the parts of the celestial houses. And this same thing is to be observed for the celestial houses of the other sundials.

And if the lines of the celestial houses on the southern sundial run out beyond the horizontal line, then the portion cut off by the horizontal line, when inverted, will

present to us (as we taught in the scholium to proposition 14 of this book) those same lines on the northern sundial, as was demonstrated in that same scholium.

PROBLEMA 21. PROPOS. 21.

To describe on the same vertical sundial the ascending signs of the Zodiac.

From the two preceding little tables of proposition 9 of this book, with the Sun standing at the beginning of ♈ (Aries) and of ♎ (Libra), let there be taken the hours in which the 12 signs of the Zodiac

[Margin: The drawing of the ascending signs of the Zodiac on the same vertical sundial.]

rise; and let these be reckoned in the circle drawn from E, from the meridian line, either toward the east or toward the west, as the tables indicate. Then through the ends of these reckonings, and the center E, let hidden straight lines be drawn out, cutting the equinoctial line at points through which the lines of the ascending signs — that is, the common sections of the plane of the sundial and of the Zodiac at the rising of the signs — ought to be drawn. For example: since, with the Sun standing in the equinoctial circle, namely at the beginning of ♈ (Aries), the beginning of ♉ (Taurus) rises at the 4th hour 51 minutes before noon, there must be reckoned on the southern sundial 4 hours 51 minutes from the point N, toward the right, up to the point A; and through A and the center E a hidden straight line is to be drawn out, cutting the equinoctial line at the point B, through which the line of the ascending sign ♉ (Taurus) must be carried. For with the Sun standing at the beginning of ♈ (Aries) and ♉ (Taurus) rising — that is, at the 4th hour 51 minutes before noon — the shadow of the gnomon is cast onto the point B, as is established from what was demonstrated in proposition 1 of this book. In no other way are the points of the other ascending signs to be sought on the equinoctial line.

Diagram. A vertical-sundial construction for the ascending signs of the Zodiac, with the twelve zodiac signs set down the left and right borders of the figure. A large circle is inscribed in a rectangular frame; a vertical meridian line runs through M (top) – H – G – E – N (bottom), and a horizontal line crosses it. A dense sheaf of straight lines (the shadow/sign lines) is drawn from the frame corners and borders across the circle, meeting a horizontal row of interior points. The graduated circle carries tick-marks around its lower arc.

- M (top of circle, on the meridian line)
- H (on the meridian line, below M)

- Horizontal row of interior points, left to right: K, M, G, F, C
- B (left of center, below the row)
- E (center, on the meridian line)
- A (lower right, interior)
- N (bottom of the meridian line)
- Left and right borders: the twelve signs of the Zodiac, one glyph per position running down each side (worn — several glyphs not securely legible; see report). Readable glyphs include ♋, ♊, ♍, ♈, ♉, ♌, ♏, ♎.

[The 24 border zodiac-glyph positions are largely worn; figure marked as a — see report.]

After this, let there be marked on the horizontal line, by proposition 14 of this book, the points at which it is cut by the arcs of the signs. For into these points the shadows of the style will be cast, when those signs are rising or setting — such points being C, D, F, K, L, M. For at C, on the southern sundial, there set ♏ (Scorpio) [?] and ♓ (Pisces); at D, ♐ (Sagittarius) and ♒ (Aquarius); at F, ♑ (Capricorn); while at the points K, L, M the same signs rise. But on the northern sundial, at C there set ♉ (Taurus) and ♍ (Virgo); at D, ♊ (Gemini) and ♌ (Leo); at F, ♋ (Cancer); and at the points K, L, M the same signs rise — as is clear if the northern sundial is set in its proper position on the north-facing wall, so that the center of the sundial H is below the horizontal line, with the points K, L, M toward the right, and C, D, E toward the left. For we now take, in place of the northern sundial, the whole southern one, with the upper part changed into the lower, so that the center H is placed below the horizontal line.

[Margin: For what reason the points on the horizontal line of the vertical sundial are found, at which the Sun rises or sets when standing at the beginnings of the signs.]

But these same points we shall also find, by this method, on the horizontal line of each sundial, even if the arcs of the signs are not drawn on it. From the fifth table of proposition 9 of this book we shall take the semidiurnal arcs of the southern signs for the southern sundial, and we shall number them from N in each direction; and from the ends of the numberings we shall draw, through E, straight lines cutting the equinoctial line at points, through which straight lines drawn out from H will cut the horizontal line at the points C, D, F, K, L, M, through which the arcs of the signs pass. For at those points the Sun rises and sets when it stands at the beginnings of the signs, because the said straight lines going out from H indicate the hours at which the Sun rises and sets, and so on. But for

the northern sundial we shall take the semidiurnal arcs of the northern signs, and number them from M in each direction, and so on.

[Margin: For what reason the same points are found by means of the Analemma.]

Geometrically too, and more surely, by means of the rising and setting amplitudes, we shall find these same points at which the Sun rises when standing at the beginnings of the signs, in this way. Let the Horizon be ABCD, whose center is E; let its common section with the Meridian be BD; and its common section with the Vertical — and therefore also with the Equator — be AC. On the straight line BD let there be sought the points where it is cut by the parallels of the signs, which will be done thus. From B toward A let the altitude of the Equator Be be computed, or from A toward B the altitude of the pole Ae; and from the point e let there be drawn on either side (as in the Analemma) the diameters of the parallels of the signs, equidistant from the diameter of the Equator ef. For these will cut the straight line BD at the points F, G, H, I, K, L, which are sought. This is easily demonstrated if the semicircle BAD is imagined to be turned about BD, until it stands upright at the Horizon and is joined with the Meridian. For then the said parallels will be the common sections of the Meridian and of the parallels of the signs of the Zodiac, and therefore the parallels themselves will cut the straight line BD at the points F, G, H, I, K, L, since they are drawn through the said parallel lines. Then through the points F, G, H, I, K, L, let there be drawn to BD the perpendiculars MN, OP, QR, ST, VX, YZ. These will be the common sections of the parallels of the signs of the Zodiac and of the Horizon. For since both the Horizon and the parallels of the signs of the Zodiac are perpendicular to the Meridian, [Margin: a — Elements XI.19 ("19. vndec.").] the common sections of the Horizon and of the parallels will likewise be perpendicular to the Meridian, and therefore (by definition 3 of Book 11 of Euclid) perpendicular also to the straight line BD, which lies in the Meridian. For which reason the said perpendiculars are the common sections of the parallels and of the Horizon. From this it follows that, since the Vertical properly so called is denoted by the straight line AC, the arcs CM, CO, CQ, CS, CV, CY are the rising amplitudes, and the arcs AN, AP, AR, AT, AX, AZ the setting amplitudes.

[Margin: The rising and setting amplitudes found by means of the Analemma.]

And so, if from ED a straight line Ea equal to the style is cut off, and through a line bd is drawn perpendicular to ED, it will

will be the horizontal line on the southern vertical sundial. The rays drawn from the points S, V, Y and T, X, Z through the center E will cut the horizontal line at the

points onto which the shadow of the style is cast when the Sun rises and sets while it stands at the beginnings of the signs. These points, then, taken from the point a and transferred to the horizontal line of the sundial from the position of the gnomon G, will give the same points on the sundial.

This can be made clearly evident to anyone if the Horizon ABCD is placed on the southern sundial in its proper position, so that the point a lies at the position of the style G, the straight line bd coincides with the horizontal line, and E therefore lies at the center of the world, serving as the vertex of the style. For once this is assumed, when the Sun rises at the horizon points S, V, Y and sets at T, X, Z, the shadow of the style will be cast onto the said points of the horizontal line — the points, that is, of the southern signs. In the same way, the points of the northern signs will be found on the northern sundial, if from EB one cuts off the straight line Ea equal to the style, and through a draws to EB the perpendicular bd, which again will be the horizontal line on the northern sundial, and so on. Moreover, from this same figure we shall find the corresponding points on the horizontal line of any sundial whatever, as will become clear from what follows. The diameters of this figure — YN, VP, and the rest — may not unfittingly be called the rays of the rising or setting latitudes [radij latitudinum ortiuarum vel occiduorum].

[Margin: What the rays of the rising and setting latitudes are.]

Here too the reason clearly appears why the southern signs are changed into northern ones on the northern sundial. It is because the rays of the latitudes of the southern signs, drawn through the center E, cut the straight line bd when that line is drawn on the northern side through the point a — the line that represents the horizontal line on the southern sundial; whereas when the straight line bd is drawn through a on the southern side, representing the horizontal line on the northern sundial, it is cut by the rays of the latitudes of the northern signs drawn out through the center E, as is plain from the figure.

[Margin: The reason why, on the northern sundial, the arcs of the southern signs are changed into the arcs of the northern signs.]

Since Taurus [♉] rises when its opposite sign sets — that is, when the shadow of the style on the sundial is cast onto the point C — then, if we join B, namely the point of Taurus found on the equinoctial line, with the point C found on the horizontal line, we shall have Taurus ascending. In the same way, if we connect the point of Gemini [♊, printed "II"], found on the equinoctial line, with the point D, at which Capricorn [♐] sets, we shall have Gemini ascending — and likewise

for the rest of the northern signs. Again, since, when the Sun stands at the beginning of Aries [♈] and that same sign is rising, the shadow of the style is cast onto L, we shall join the point L with the point of Aries [♈] found on the equinoctial line, so as to have Aries ascending. So too we shall join the point of Capricorn [♐], found on the equinoctial line, with the point M, where Capricorn rises, so as to have Capricorn ascending — and so with the rest of the southern signs.

It must be observed, however, that in laying out the signs on the southern sundial one should not take, from the first two tables of proposition 9 of this book, the hours of those signs which rise six or more hours before or after noon. For these do not cut the equinoctial line. On the northern sundial, by contrast, for laying out those same signs one must take precisely such hours — and not the hours of those signs which rise six or fewer hours before or after noon — for the same reason: because on the northern sundial only the lines of those hours which are more than six hours distant from noon cut the equinoctial line, as is clear from what we wrote in proposition 13 of this book.

We can also, if we wish, find on the two tropics the points through which the ascending signs are to be drawn: namely, by lightly drawing the line of the hour at which the given sign rises, as appears in the third and fourth tables of proposition 9 of this book, just as was done on the horizontal sundial — provided one takes the hours of those signs which cut the tropics, that is, from the third table those which cut the tropic of Cancer [♋], and from the fourth those which cut the tropic of Capricorn [♐]. This must be understood for either sundial. For example, since the sign Virgo [♍, printed "mp"] rises 2 hours 34 minutes before noon when the Sun stands at the beginning of Cancer [♋], it comes about that the line of that hour cuts the arc of Cancer at a point through which the sign Virgo must be drawn, and so on.

[Margin: How the points of the ascending signs are found on the tropics.]

Indeed, the lines that touch the tropics at these points (which are to be drawn as we taught in the corollary of proposition 37 of the first book of Apollonius [Apollonius, *Conics* I.37, coroll.]) will be the ascending signs, as is clear from what we demonstrated in proposition 9 of this book.

[Margin: In what way the ascending signs are laid out on the sundial by the doctrine of Apollonius's *Conics* [conicorum(?) Apoll.].]

The lines of Aries [♈] and Libra [♎] we shall describe in this way. Since, when the beginning of Aries is ascending and the Sun stands at the beginning of Capricorn

[♊], it is noon — as is clear from the fourth table of proposition 9 of this book — it is evident that the common section of the plane of the sundial and of the Ecliptic then passes through that point of the tropic of Capricorn through which the meridian line is drawn. And since at that time the Equator, the circle of the sixth hour from noon or midnight, the Vertical properly so called, the Ecliptic, and the Horizon all have the same common section — a section from which the plane of the sundial is equidistant, since it is parallel to the Vertical itself — it follows, by proposition 18 of the previous book, that the equinoctial line, the line of the sixth hour, the common section of the Ecliptic and of the plane of the sundial, and the horizontal line are all parallel to one another. Therefore a straight line drawn parallel to the equinoctial line — or to the line of the sixth hour, or to the horizontal line — through the point at which the meridian line cuts the tropic of Capricorn [♊] will, when produced, yield Aries [♈] ascending. In just the same way, a straight line drawn parallel to the equinoctial line through the point at which the meridian line cuts the tropic of Cancer [♋] will give Libra [♎] ascending.

We shall also delineate the ascending signs in yet another way, as follows. From the sixth table of proposition 9 of this book, let the declinations of the culminations of the heaven [mediationes cœli] be taken, and, according to the doctrine of proposition 14 of this same book, let there be sought on the meridian line of the southern sundial the points through which the parallels of those declinations are to be drawn. Likewise, from the seventh table of the same

[Margin: The laying-out of the ascending signs on the sundial by means of the culminations of the heaven and the points of the Ecliptic established on the circle of the sixth hour from noon or midnight, while the beginnings of the signs are ascending.]

Diagram. A large circular projection (an analemma-type horizontal projection) used to find the rising and setting points of the zodiacal signs for the vertical sundial. The circle carries the four cardinal orientations printed around its rim: **OCC** (Occasus, west) at the top, **OR.** (Oriens, east) at the bottom, **MER** (Meridies, south) at the left, and **SEPT** (Septentrio, north) at the right. The horizon is the circle ABCD, with center **E** (set within a small central circle). Two principal diameters cross at **E**: the horizontal diameter (the meridian section, running **MER**–**SEPT**) and the vertical diameter (the vertical/equator section, running **OCC**–**OR.**). A dense sheaf of straight "latitude rays" runs across the figure, and the rim is studded with small zodiac-sign sorts marking the rising/setting amplitudes.

Readable labels (many peripheral labels and zodiac sorts are very small and could not be resolved with confidence — see review note):

- E (center of the figure)
- A (top of the circle, at OCC); C (foot of the circle, at OR)
- Points near the central horizontal line: G and H (just about the center), F (to the right of center); further left-of-center letters read as K, I [?]
- Upper region: P, R, N [?] (right of top); Z [?] (upper left)
- Lower region: V [?] (lower left); M, d, o, f [?] (lower right)
- Numerous zodiac glyphs (Cancer ♋, Capricorn ♐, and neighbouring signs) around the rim marking the ends of the latitude rays — individually too small to transcribe reliably. [zodiac sorts: ?]

proposition 9 of this book, let there be taken the declinations of the southern points lying on the circle of the sixth hour, both before and after noon, as the table indicates; and, according to the doctrine of the said proposition 14 of this book, let there be found on the line of the sixth hour of the same southern sundial the points through which the parallels of those declinations are drawn, both before and after noon, as appears in the table. For if these points are joined by straight lines to the corresponding points found on the meridian line, the ascending signs will be laid out, as before — just as we said for the horizontal sundial in proposition 9 of this book — and these lines will necessarily pass through the points found on the horizontal line.

[Margin:...gium by the culminations of the heaven, and the Ecliptic points established on the circle of the sixth hour from noon or midnight, while the beginnings of the signs are ascending.]

In the same way these same ascending signs will be laid out on the northern sundial, if — instead of finding the points on the meridian line, that is on the line of the 12th hour of noon — one takes, on the line of the 12th hour of midnight, the declinations of the angles of the earth [angulorum terræ] from the sixth table of proposition 9 of this book. And to find the points on the line of the sixth hour, let one take from the seventh table of the same proposition 9 of this book the declination of the northern points lying on the circle of the sixth hour, and so on. But it is better to transfer the points found on the southern sundial to the northern one, changing the names of the southern signs into those of the northern, and vice versa. For since they have the same declinations, they will have the same position and distance on either sundial.

[Margin: In what way the ascending signs are again laid out by means of the conic elements.]

Indeed, straight lines drawn from the points found on the meridian line and tangent to the tropics will be the ascending signs, as was demonstrated in proposition 9 of this book. And such tangent lines are to be drawn as we taught in the corollary of proposition 37 of Book 1 of Apollonius [Apollonius, *Conics* I.37, coroll.]. And so we have laid out the ascending signs of the Zodiac on this same vertical sundial. Which was to be done.

SCHOLIUM.

Following Andreas Schöner [Andrea Schonero], we shall delineate these same ascending signs in this way. In the first figure of the scholium of proposition 9 of this book, let there be cut off from EB the straight line Ea, equal to the meridian line EI of the sundial, intercepted between E — the center of the Equator — and the equinoctial line; and through a let a line be drawn parallel to AC. Thus, with the center E placed at the center of the Equator, and the point a at the point I, the parallel drawn through a will coincide with the equinoctial line. This parallel through a will cut the lines of the signs sent out from E at points which, taken from a and carried over onto the equinoctial line from I in both directions, will give points on the equinoctial line. Then, if from the point G — where the meridian line intersects the horizontal line — straight lines are sent out through these points, the two tropics will be cut at points through which the lines of the ascending signs are to be drawn, yet in the same order that we prescribed for the horizontal sundial.

[Margin: How the same ascending signs are laid out according to the method of Andreas Schöner.]

But if it should happen that the straight line drawn out from the point G does not cut the tropic of Cancer [♋] — as in the example occurs with the sign Leo [♌] and the sign [zodiac sign: ?] — this is a proof that such signs do not rise while the Sun, standing at the beginning of Cancer [♋], illuminates the southern face of the vertical circle, but rather while it lights its northern part. And so those lines sent out from G will cut the tropic of Cancer on the northern sundial, which is cut off by the horizontal line.

Again, in the second figure of the same scholium of proposition 9 of this book, let there be cut off on EB the straight line Ea, equal to the same meridian line EI taken from the sundial, and through a let a line be drawn parallel to AC, so that,

with the center E placed at E — the center of the Equator — and the point a at the point I, the line drawn through a coincides with the equinoctial line. This parallel through a will cut the lines of the signs sent out from E at points which, taken from a and transferred onto the equinoctial line from I, will give on the equinoctial line the points through which the lines of the ascending signs must pass, in the order handed down for the horizontal sundial. If, then, these points on the equinoctial line are joined with the corresponding points on the tropics, the lines of the ascending signs will be obtained, as before. We have not given a worked example of this construction, because, in order to avoid a confusion of lines, we did not draw the tropics on the sundial.

Now if the lines of the ascending signs are extended beyond the horizontal line, the same ascending signs will be obtained on the northern sundial, which is cut off by the horizontal line — provided it is so arranged that the horizontal line is set in the upper position, and the side which, in that arrangement, is on our right as we face the sundial becomes the left, and vice versa, as we demonstrated in the scholium of proposition 14 of this book.

[Margin: The lines of the ascending signs, drawn out beyond the horizontal line, produce the northern sundial.]

If one wishes, however, the ascending signs can be laid out on the northern sundial in just the same way as they were laid out on the southern one; but we shall derive them from the southern sundial much more easily and readily by extending the lines of the ascending signs beyond the horizontal line, and so on, as we have said.

PROBLEMA 22. PROPOS. 22.

To construct an Italian vertical sundial.

[Margin: Construction of the Italian vertical sundial.]

This is constructed in no other way than the horizontal one — provided that, in the first method, for the southern sundial the diurnal arcs of the tropics of Cancer [♋] and Capricorn [♏] on the circle MaNb are aNb and dNe, because the point N belongs to the south, since it inclines toward the meridian circle in the upper hemisphere; and provided that the division of the circle takes its beginning from the points b and e, which lie to the west, if the sundial and the circle MaNb are set in their proper position. One must also take the nocturnal arc of Capricorn [♏] above the horizontal line, so that in it may be found the points of the hours 15, 14, 13, and so on, which on the circle are contained within the nocturnal arc of

Capricorn. But for the northern sundial the diurnal arcs of the tropics of Cancer [♋] and Capricorn [♏] will be dMe and aMb, since, when the sundial and the circle MaNb are set in their proper position, the point M belongs to the south, since it inclines toward the meridian circle in the upper hemisphere; and the division of the circle must begin from the points e and b, which lie to the west and correspond to the 24th hour. For we now take, for the northern sundial, the whole southern one, its upper part being made the lower, so that the center H is contained below the horizontal line. Hence the points nearest toward M will belong to the 23rd hour, and the following ones to the 22nd, and so on.

But remember that on the northern sundial the tropic of Cancer [♋] lies between the center H and the equinoctial line. You have an example of this construction, on the southern sundial, in the 14th hour of the tropic of Cancer and the 13th hour of the tropic of Capricorn. For from f, the point of the 14th hour of the tropic of Cancer, a straight line drawn through the center E cuts the equinoctial line at a certain point; and a straight line drawn from the center H through that point will indicate on the tropic of Cancer the point h for the 14th hour. Again, a straight line drawn from m — the point of the first nocturnal hour of the tropic of Capricorn [♏], which is opposite the 13th hour — through E meets the equinoctial line at n. And because the line mE, drawn from the first nocturnal hour of Capricorn, passes through the center E before it cuts the equinoctial line, therefore the straight line Hn, on the tropic of Capricorn, shows the point q for the first nocturnal hour of Capricorn;

but extended beyond the center H, it presents on the same tropic the point p for the 13th hour, as we demonstrated in proposition 10 of this book. On the northern sundial we have set out no example, lest the multitude of lines produce confusion — especially since it is easily derived from the southern one, as we shall say in the scholium of the following proposition. Here too the hour-lines from the west will pass through the hours (reckoned from noon or midnight) of the equinoctial line, as we said in proposition 10 of this book. For the rest, in this figure we have not written the numbers of the hours reckoned from noon or midnight, because from the figure of proposition 13 of this book the order of such hours is easily understood.

Diagram. A large circular figure for the Italian (i.e. unequal/sunset-reckoned) vertical sundial, with a dense fan of hour-lines radiating from the central region. The topmost point of the circle is labelled **M** and the lowest **N**; the center-region carries **H** with **G** just below it. Two vertical scales of Italian-hour numbers run

down the two sides. The bottom of the circle is drawn as a dotted arc studded with points.

Readable labels (the central fan and rim carry many small point-letters and numerals; those that resolve are listed, the rest flagged — see review note):

- M (top of circle); N (foot of circle)
- H (upper center); G (just below H)
- b (left, near G); central-region point-letters read as a, d, e, m, p, q, h, k, r, f [several uncertain — ?]
- Left-hand vertical scale of hour numbers (top to bottom): 22, 23, then 1, 2, 3, 13, 4
- Right-hand vertical scale of hour numbers (top to bottom): 1 [?], 24, 23, 22, 21, 20 — with a zodiac sort (Capricorn ♐ / Sagittarius ♐ [?]) beside the upper right numbers
- A curved sequence of small hour-numerals across the central fan: 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16 [some digits uncertain — ?]

In the second method, which is done by means of the diurnal and nocturnal arcs, there is no difficulty, if the tables set down in the scholium of proposition 33 of the previous book are rightly considered. But one must also draw, on the southern sundial, the nocturnal arc of the 24th hour, since with its help and support the description of the Italian sundial is made easy.

[Margin: Another description of the Italian vertical sundial by means of the diurnal and nocturnal arcs.]

For the 24th hour from sunset is drawn through the 5th hour from noon on the diurnal arc of ten hours, and through the 12th hour from midnight on the nocturnal arc of twenty-four hours — which is the greatest parallel of the ever-hidden stars [parallelus semper latentium maximus] — and it cuts the 12th hour from midnight, which is also the 12th hour of noon, at the position of the style. Now the 24th hour from sunset is parallel to the equinoctial line, since it is the same as the horizontal line, which in the scholium of proposition 22 of the previous book we showed to be parallel to the equinoctial line. But the 23rd hour passes through the 4th hour from noon on the diurnal arc of ten hours, and through the 5th hour from noon on the arc of twelve hours, which on the southern sundial the whole equinoctial line provides.

Indeed, since the 13th hour from sunset, extended beyond the greatest parallel of the ever-hidden stars — that is, beyond the nocturnal arc of twenty-four hours —

performs the office of the 23rd hour from sunrise, as we showed in the scholium of proposition 10 of this book, you therefore see that the line of the 23rd hour from sunrise, thus extended, passes through the 6th hour from midnight on the nocturnal arc of fourteen hours; and so with the rest. But the 13th hour from sunset, because it has no corresponding point on the diurnal arc of ten hours, is drawn through the 6th hour from midnight on the nocturnal arc of fourteen hours. Finally, the 12th hour from sunset must be drawn through the 12th hour from noon — that is, through the 12th hour of midnight — on the nocturnal arc of twenty-four hours, and equidistant from the equinoctial line, as is clear from the scholium of proposition 22 of the previous book. It will also pass through the 5th hour from midnight on the nocturnal arc of fourteen hours; but in our example the 5th hour from midnight scarcely cuts the said arc. Hence the 12th hour from sunset will be more correctly drawn through the 12th hour from noon — that is, through the 12th hour of midnight — on the nocturnal arc of twenty-four hours, on the southern sundial; but on the northern one (which that portion cut off by the horizontal line presents to us, provided it is inverted, as we taught in proposition 14 of this book) it is drawn through the 12th hour from midnight — that is, through the 12th hour of noon — on the diurnal arc of twenty-four hours. For on the northern sundial the nocturnal arc of twenty-four hours of the southern sundial is changed into the diurnal arc of twenty-four hours opposite to it, and the 12th hour from noon — which is the 12th hour of midnight — is converted into the 12th hour from midnight, which is the 12th hour of noon, as we taught in the scholium of proposition 13 of this book.

The northern Italian sundial will be completed in the same way, provided you remember that on it the arc which on the southern sundial was the diurnal arc of ten hours is the diurnal arc of fourteen hours; and that the part of it toward the equinoctial line — which the horizontal line cuts off — is the nocturnal arc of ten hours; likewise that the equinoctial line, which on the southern sundial was the diurnal arc of twelve hours, is the nocturnal arc of twelve hours; and that the arc which on the southern sundial was the diurnal arc of fourteen hours is the nocturnal arc of fourteen hours. Finally, that the astronomical hours, which on the southern sundial were reckoned from noon, must on this one be counted from midnight, and vice versa. All of this is drawn from what we wrote in propositions 13 and 14 of this book.

For once this is observed, the description of the northern Italian sundial will be easy. For the 23rd hour passes through the 6th hour from noon on the diurnal arc of fourteen hours, and through the 11th hour from noon on the diurnal arc of twenty-four hours, which on the northern sundial is the same as the one that on

the southern was the nocturnal arc of twenty-four hours. And since the 23rd hour from sunset, beyond this arc, is the 23rd hour from sunrise, therefore its portion indicating the 23rd hour from sunrise passes through the 4th hour from midnight on the nocturnal arc of ten hours, and through the 5th hour from midnight on the nocturnal arc of twelve hours. So too the 22nd hour is drawn through the 10th hour from noon on the diurnal arc of twenty-four hours, and its portion showing the 22nd hour from sunrise passes through the 3rd hour from midnight on the nocturnal arc of ten hours, and through the 4th hour from midnight on the nocturnal arc of twelve hours. Likewise the 9th hour from sunset proceeds through the 9th hour from midnight on the diurnal arc of twenty-four hours, and through the 4th hour from midnight on the nocturnal arc of ten hours, and through the 3rd hour from midnight on the nocturnal arc of twelve hours, and through the 2nd hour from midnight on the nocturnal arc of

— and 14; and so with the rest. All of this follows from the tables of proposition 33 of the previous book. As for the way in which the 12th hour from sunset is to be drawn on the northern sundial, we explained that a little earlier, when we were constructing the southern Italian sundial. But it is better to derive the northern one from the southern Italian sundial — that is, by extending the latter's lines — as we shall show in the scholium of the following proposition.

As for the nocturnal arc of 24 hours and the diurnal arc, there is no need to teach how the Italian sundial is described from it, since the matter is quite easy and clear from the tables of the described arcs, set out in the scholium of proposition 33 of the preceding book. Only this seems worth not omitting: that such an arc — which on the southern sundial always represents the greatest [parallel] of the ever-hidden [stars], and on the northern the greatest of the ever-apparent — is an ellipse in that region whose latitude is less than 45° , as in our example; but in a region of greater latitude than 45° each arc is a hyperbola, so that two opposite hyperbolas result; and finally, in a region having latitude of exactly 45° , it is a parabola, as is clear from the corollaries of propositions 7, 6, and 5 of the previous book.

[Margin: In what region the nocturnal and diurnal arc of 24 hours on the vertical sundial is an ellipse, and in what region a hyperbola or parabola.]

Concerning the third method — which is carried out by means of the line of the 12th hour from sunrise or sunset — it must be noted that it has no use in the region whose latitude is 45° , since the line of the 12th hour from sunrise or sunset vanishes in that vertical sundial, as is evident from what we wrote in the scholium of proposition 11 of the previous book. For the vertical [sundial] of that

region and the circle of the 12th hour from sunrise or sunset do not differ, as is plain, since there the greatest of all the ever-apparent parallels passes through the zenith of the head. But in regions whose latitudes differ little from 45° , it must be understood that this same method has an inconvenient use, because the line of the 12th hour from sunrise or sunset is too far from the equinoctial line. This is clearly evident from the portion of the analemma of proposition 13 of this book, where the circle of the 12th hour from sunrise or sunset — by the straight line KD — meets the plane of the sundial along the straight line HI (drawn) at the point through which the line of the 12th hour, perpendicular to the meridian line, must be drawn, as we demonstrated in the third method of proposition 10 of this book — or at least parallel to the equinoctial line, or to the line of the 6th hour from noon or midnight, as is clear from the scholium of proposition 22 of the previous book — a point most distant from the point I, through which the equinoctial line passes. And so that the position and situation of this line on the vertical sundial in various regions may be perceived more clearly, the following figures must be carefully considered. For in the first figure, where the altitude of the pole CE is less than 45° , the circle of the 12th hour from sunrise or sunset meets the plane of the southern sundial at point L, above the center of the sundial H, and meets the plane of the northern sundial below the center of the sundial H. In the third figure, where the altitude of the pole CE exceeds 45° , the contrary occurs. Finally, in the middle figure, where the altitude of the pole is exactly 45° , it meets neither plane but is equidistant from the plane of each sundial. Here too it appears when the line of the 12th hour from sunrise or sunset must be drawn above the equinoctial line, and when below; and this equinoctial line, on both sundials, is drawn through the point I, as is evident.

[Margin: In what region the line of the 12th hour from sunrise or sunset has no use in the description of the Italian sundial, and in what region an inconvenient and difficult use, and for what cause.]

Diagram. The first of three companion figures showing the position of the 12th-hour line on the vertical sundial when the pole-altitude is less than 45° . A semicircle stands on a horizontal diameter running B (left) — C (right), with center D; the arc rises from B through F (upper left) to the apex and down through E (upper right) to C. A vertical meridian line passes through D and is extended upward to L (top) and downward to L (bottom).

- A, K (apex): the two points at the top of the construction, K being the point of the 12th-hour circle KD
- F (upper left of arc): point on the arc

- E (upper right of arc): endpoint of the pole-altitude chord CE
- B (left end of diameter): horizontal endpoint
- C (right end of diameter): horizontal endpoint
- D (center): center of the semicircle
- G (interior, left of center) [position ?]: division point
- H (interior, at/near center): center of the sundial
- I (interior, upper left) [position ?]: point through which the equinoctial line passes
- G (on baseline, near center) [second G — position ?]
- H (below baseline, left) [position ?]
- I (below baseline, right) [position ?]
- L (top of vertical line): where the 12th-hour circle meets the southern plane
- L (bottom of vertical line): where it meets the northern plane

Diagram. The middle figure, for the case where the pole-altitude is exactly 45° ; here the 12th-hour line is equidistant from both planes. Same semicircular construction on diameter B (left) — C (right), center D, arc through F (left) and E (right), with a vertical meridian line.

- A (apex), K (just below A): the top points
- F (left of arc), E (right of arc)
- H (interior, near center-right) [position ?]: center of the sundial
- B, G, D, G, C (along the baseline, left to right) [the two G positions ?]
- H (below baseline, left) [position ?]
- I (below baseline, right) [position ?]
- L (top) [?]

Diagram. The third figure, for the case where the pole-altitude exceeds 45° (the contrary case to the first). Same construction: semicircle on diameter B (left) — C (right), center D, arc through F (left) and E (right), vertical meridian line extended to L above and L below.

- K, A (apex, K left of A)
- F (left of arc), E (right of arc)
- H (interior, near center): center of the sundial
- I, G (interior, left of center) [positions ?]
- D (center)
- B, G, C (along the baseline) [the baseline G position ?]

- H (below baseline, left) [position ?]
- I (below baseline, right) [position ?]
- L (top), L (bottom)

Moreover, the horizontal line offers us yet another way of describing the Italian sundial, if the first table of proposition 20 of the previous book is carefully examined — the table whose title is THE LINE OF THE 24TH HOUR FROM SUNRISE OR SUNSET. For the line of the 23rd hour from sunset, for instance, passes through that point of the horizontal line at which it is cut by the 11½th hour from noon or midnight; the line of the 22nd hour, through the point through which the 11th hour from noon or midnight passes; and so with the rest, as is easily clear from the aforesaid table. If, then, these points are joined with the corresponding points on the equinoctial line — found by the table of proposition 19 of the same previous book, or by the table of the diurnal arc of 12 hours of proposition 33 of the same book — the Italian sundial will be described, as before.

[Margin: Another description of the Italian vertical sundial by means of the horizontal line, that is, of the 24th hour from sunrise or sunset.]

Again, from the third table of proposition 20 of the previous book we shall easily gather which hour-lines from sunrise or sunset cut one another at one and the same point on the line of the 6th hour from noon or midnight. For the 13th and 23rd hours from sunrise or sunset cut the 6th hour from noon or midnight at one and the same point; likewise the 14th and 22nd hours, and so on. Moreover, from the fourth table of the same proposition we shall learn which hours from sunrise or sunset cut the meridian line at one and the same point. Such are the 10th and 14th hours from sunrise or sunset, and so on. Finally, from proposition 20 and the figure of proposition 9 of the same previous book we shall understand which hours from sunrise or sunset cut any given hour from noon or midnight at one and the same point. All of this, if rightly considered, is most useful for the description of the hours from sunset or sunrise, and provides a means by which such a description of the hours can be tested. We have therefore set up the Italian vertical sundial — which was what had to be done.

[Margin: The use of the tables of proposition 20 of the previous book for the mutual intersections of the hours from noon or midnight and from sunrise or sunset among themselves.]

PROBLEMA 23. PROPOS. 23.

To set up the Babylonian vertical sundial.

We shall set up the Babylonian sundial no differently than the Italian. For these two sundials differ only in the position of the hour-lines and in the numbering of the hours, as we said in proposition 11 of this book — which the figure of the preceding proposition clearly shows. In the first method, however, the division of the circle MaNb must begin from the points a and d; for these are the eastern points, if the sundial and the said circle have their proper position.

[Margin: Description of the Babylonian vertical sundial.]

As for the other methods, there is no difficulty at all, if the tables of propositions 19, 20, and 33 of the previous book are attentively considered. For, to give an example of the second method — which is done by means of the diurnal and nocturnal arcs — the 24th hour from sunrise, which the horizontal line indicates, is drawn through the 7th hour from midnight on the diurnal arc of 10 hours, and through the 12th hour from midnight on the nocturnal arc of 24 hours; and it is parallel to the equinoctial line, since the horizontal line marks it — the line which in proposition 14 of this book we demonstrated to be parallel to the equinoctial line. The first hour from sunrise passes through the 8th hour from midnight on the diurnal arc of 10 hours, and through the 7th hour from midnight on the diurnal arc of 12 hours; and, extended (because beyond the greatest parallel of the ever-hidden [stars] it marks the first hour from sunset), it is drawn through the 6th hour from noon on the nocturnal arc of 14 hours, as is clear from the table of this nocturnal arc; and so with the rest. But the 11th hour from sunrise, because it has no point on the diurnal arc of 10 hours, is drawn through the 6th hour from noon on the nocturnal arc of 14 hours. Finally, the 12th hour from sunrise must be drawn through the 12th hour from noon on the nocturnal arc of 24 hours, and — as we showed above — equidistant from the equinoctial line.

In this way, then, on all other sundials we shall describe the hours from sunrise and sunset by the help of the tables of the diurnal and nocturnal arcs: taking points on the nocturnal arcs whenever we lack them on the diurnal arcs; and observing, moreover, that when some hour from sunset has no points on the said arcs, we draw the hour of the same number from sunrise. For this, extended beyond the greatest parallel of the ever-apparent or ever-hidden [stars], will give the same hour by number from sunset, as we showed above. The same must be done when some hour from sunrise has no points on the aforesaid arcs: for if we

draw the same hour from sunset, this, extended beyond the said greatest parallel, will give the hour of the same number from sunrise.

[Margin: When the nocturnal and the diurnal arc is to be used, and when the hour from sunrise is to be drawn for the hour from sunset, and vice versa, in the description of the Italian or of any Babylonian sundial.]

There is no need to give an example of the description of the northern Babylonian sundial, since it is drawn in the same way from the aforesaid arcs — especially since we can obtain both the northern Babylonian and the northern Italian from the southern one, if its lines are extended beyond the horizontal line, as we shall teach in the following scholium. We have therefore set up the Babylonian vertical sundial — which was what had to be done.

SCHOLIUM.

If the lines of the hours from sunrise and sunset are extended beyond the horizontal line, the same hours from sunrise and sunset will be obtained on the northern sundial — the one that is cut off by the horizontal line — provided that it is so placed on the face of the sundial's plane which faces north that the horizontal line occupies the uppermost position, and the part which in that situation is on our right becomes the left, and vice versa, as was shown in the scholium of proposition 14 of this book.

It will not be difficult to judge which hours from sunrise and sunset, extended beyond the horizontal line, are to be counted from sunrise and which from sunset on the northern sundial, if the hour-circles are considered in their proper positions, together with both sundials — the southern and the northern — so that the tips of the gnomons meet at the center of the world and form one straight line, as we wrote in the scholium of proposition 14 of this book. For since the shadow of the style is cast onto the intersection of the circle of the 23rd hour and the plane of the northern sundial (set in its proper position) when the Sun is in the western semicircle of the said circle, it is clear that that line of the 23rd hour belongs to the 23rd hour from sunset. The same is to be said of the line of the 22nd, 21st, and 20th hour, and so on. Therefore the remaining hours on the same side of the sundial — namely the 13th, 14th, 15th, and so on — are to be counted from sunrise. Again, since the shadow of the style falls on the intersection of the circle of the 1st hour and the plane of the same northern sundial when the Sun is in the eastern semicircle of the said circle, that line of the 1st hour will belong to the first hour from sunrise. The same is to be said of the 2nd, 3rd hour, and so on. Hence the

remaining hours on the same side of the sundial — namely the 11th, 10th, 9th, 8th, and so on — are to be reckoned from sunset.

[Margin: Which lines of the hours from sunrise or sunset, extended beyond the horizontal line, indicate the hours from sunrise, and which the hours from sunset, on the northern sundial.]

Perhaps it will be perceived more clearly which extended lines belong to the hours from sunrise and which to the hours from sunset, in this way. In the nocturnal portion of the southern sundial — the portion which the horizontal line cuts off, as we noted in proposition 14 of this book — let the hours from sunrise and sunset be considered; and this is not obscure. For all the hours from sunset, extended beyond the horizontal line, are counted from sunset in the nocturnal portion of the sundial until they reach the greatest parallel (which touches the horizontal line on this vertical sundial of ours at the 12th astronomical hour); but beyond the points of contact they are to be reckoned from sunrise. By the same reasoning, the hours from sunrise, extended beyond the same horizontal line, give in that same nocturnal portion the hours from sunrise before the points of contact, but after these points the hours from sunset, as is clear from the scholium of proposition 10 of this book. So that in the nocturnal portion of our sundial these hours — 23, 22, 9, 10, 11 — are to be counted from sunrise, but these — 1, 2, 13, 14, 15 — from sunset. When these things are rightly considered, we shall understand, clearer than light, the hours from sunrise or sunset on the northern sundial. For all the hours which in the nocturnal portion are computed from sunset are to be counted from sunrise on the northern sundial, and those which there are taken from sunrise indicate here the hours from sunset. The reason for this is open and plain. For since that portion cut off by the horizontal line — in order that it may show the hours on the face of the plane which faces north — must be so inverted that the right part is changed into the left, and after this change the upper into the lower (as we taught in proposition 14 of this book), it comes about that the Sun is in different semicircles of the same hour-circle, so that the shadow of the style can be cast onto its hour-line both on the nocturnal sundial and on the northern, each being placed in its proper position: since that same hour-line has different situations on the two sundials, so that if on the nocturnal it inclines toward the west, on the northern it faces east, and vice versa. Therefore, since one semicircle signifies the hour from sunrise and the other the same hour from sunset (as we taught in the scholium of proposition 10 of the previous book), it is necessary that the hour which on the nocturnal sundial is computed from sunrise be counted from sunset on the northern, and vice versa. For example: onto the line of the 23rd hour on the nocturnal sundial the shadow of the style falls when the Sun is in the eastern semicircle of that hour below the horizon,

since that line recedes from the style toward the west; therefore that hour must be reckoned from sunrise. But onto the same line on the northern sundial — because it is moved away from the style

toward sunrise — the shadow of the style is cast while the Sun is in the western semicircle above the horizon; wherefore that same hour must be counted from sunset. And the same reasoning holds for the other hour-lines. Thus these hours — 23, 22, 9, 10, 11 — are to be counted from sunrise, but these — 1, 2, 13, 14, 15 — from sunset, as has been said. We wish this to be understood of other sundials also, even those inclined to the horizon, taking the lower sundials for the northern ones, as we shall note in their proper places.

From the vertical sundial we have described it is evident that Federico Commandino [Federico Commandino], in his book on the description of sundials, was mistaken concerning the lines of the 10th and 11th hours of the northern Italian vertical sundial. For he assigned to them exactly the same position that the lines of the 23rd and 22nd hours have — which cannot be, as is clear from our construction.

[Margin: Federico Commandino does not correctly draw the 10th and 11th hours from sunset on the northern Italian sundial.]

PROBLEMA 24. PROPOS. 24.

To set up the ancient vertical sundial.

Let the circle MaNb be divided, as was done in propositions 10 and 22 of this book, into the diurnal and nocturnal arc of the tropics of Cancer [♋] and Capricorn [♐]. Let the diurnal arc — both of the tropic of Cancer, aNb, and of the tropic of Capricorn, dNe — be cut into 12 equal parts, beginning from the points a and d, which are the eastern points, as is evident, if the sundial and the said circle have their proper position. For from the division-points we shall find the points of the unequal hours on each tropic, and shall complete the ancient sundial, as we set forth in proposition 12 of this book. For example, on the southern sundial:

[Margin: Description of the ancient vertical sundial.]

Diagram. A large composite figure of the ancient (unequal-hour / temporal-hour) vertical sundial. A central circle carries the meridian and horizon as perpendicular diameters, together with the two tropic arcs and the equinoctial line; radiating hour-lines run out to Roman-numeral hour labels arranged along the left and right margins of the figure.

Readable labels:

- M (top of the central circle): upper meridian point
- N (bottom of the central circle): lower meridian point
- a (right end of the horizontal diameter): eastern point
- b (left end of the horizontal diameter): western point
- Circle designation MaNb (M top, a right, N bottom, b left)
- "HORIZON" — the word set across the horizon line, split by the circle as HORI= (left) / ZON (right)
- "Arcus horæ" [arc of the hour] — curved label along an arc in the upper-left quadrant
- "...tum est" [?] — a split label in the upper-right quadrant (possibly "...tū est"); reading uncertain
- H (interior, upper center, just below M): construction point
- P (interior, at/near the center): center point [position ?]
- F (interior, below center): construction point
- I (interior, near center) [position ?]
- e (on the circle, lower left): tropic/hour point
- d (on the circle, lower right): eastern point of the tropic of Capricorn (per text, dNe)
- f (near bottom of the circle) [position ?]
- m (bottom of the circle): tropic-of-Capricorn hour point
- Unequal-hour numerals inside the lower part of the circle: IIII (h), V, VI, VII, VIII
- Marginal Roman hour-numerals, left side (top to bottom): IX, IIX (= VIII), and lower II, III [some worn — positions ?]
- Marginal Roman hour-numerals, right side (top to bottom): II, I, XII, XI, and a worn numeral [?], IX

The straight line from f — the point of the 4th unequal hour of the tropic of Cancer [☎] — drawn through E, cuts the equinoctial line at g. The straight line from H, drawn through g, cuts the tropic of Cancer at h, the point of the 4th unequal hour. So too the straight line from m — the point of the 4th unequal hour of the tropic of Capricorn [☷] — drawn through E, cuts the equinoctial line at n; and the straight line Hn cuts the tropic of Capricorn at p, the point of the 4th unequal hour, and so on, as is clear from what was demonstrated in proposition 12 of this book. And the same reasoning is to be applied to the rest. For the northern sundial we give no example, both because it will easily be described

from what was said in proposition 22 of this book, and also because it is better to derive it from the southern, as we shall presently teach in the following scholium.

But if the diurnal arc of 6 hours — which on the northern sundial will be of 18 hours — is described, we shall make the same sundial, as was said of the horizontal sundial in proposition 12 of this book. For the 1st unequal hour on the southern sundial is drawn through the 9½th hour from midnight on the diurnal arc of 6 hours, and through the 7th hour from midnight on the diurnal arc of 12 hours; likewise through the 4½th hour from noon on the nocturnal arc of 18 hours, as the tables set out in the scholium of proposition 33 of the preceding book indicate; and so with the rest. But on the northern sundial the same 1st unequal hour passes through the 4½th hour from midnight on the diurnal arc of 18 hours, and through the 9½th hour from noon on the nocturnal arc of 6 hours; likewise through the 7th hour from noon on the nocturnal arc of 12 hours, as the same tables teach, and so on. We have therefore set up the ancient vertical sundial — which was what had to be done.

[Margin: Another description of the ancient vertical sundial, from the diurnal and nocturnal arcs.]

SCHOLIUM.

Here too, as we demonstrated in the scholium of proposition 14 of this book, the unequal hours of the southern sundial, extended beyond the horizontal line, will exhibit the same unequal hours by number on the northern sundial, if that inversion of the sundial is made which was spoken of in the scholium of proposition 14 of this book.

DE HOROLOGIIS MERIDIANIS.

ON MERIDIAN SUNDIALS.

PROBLEMA 25. PROPOS. 25.

To set up the astronomical meridian sundial.

That is, to describe the lines of the hours from noon or midnight on a plane parallel to the meridian circle.

Let a straight line AB be drawn, arbitrarily; from A as center describe an arc of a circle BC, with any radius, in which [we shall count off...]

Gnomonics, Book II — On Meridian Sundials (continued)

[Margin: Description of the astronomical meridian sundial.]

(Continuing Problem 25 / Prop. 25 — to set up the astronomical meridian sundial.)

once the altitude of the Equator has been counted off, or the complement of the pole's altitude, BC — to the left of point A, if the sundial to be described is to face east; but to the right, if the sundial to be built is to face west — let the straight line AC be drawn through A and C, and let the straight line DE cut it at A at right angles. Then take the straight line AE equal to the length of a gnomon of any size you please, and from E as center describe, with any radius, the circle FGHI. Divide this circle into 24 equal hours, beginning either from the line FH or from the line GI, with FH cutting through center E at right angles. After this, from center E draw hidden [construction] lines through the points of division, cutting the straight line AC in points; through these points, lines drawn parallel to DE will give the lines of the hours reckoned from noon or from midnight, on a plane parallel to the meridian circle.

These parallels are easily drawn if, through any point of the line DE — for example through D — a parallel to AC is drawn, and onto this line all the points of AC are transferred from point D, taking their intervals from point A. [Margin: a — 33.primi] For straight lines joining two points equally distant from the line DE will be parallel. The demonstration of this construction is as follows.

Diagram. The construction of the eastern astronomical meridian sundial (the word **ORIENTALE**, "Eastern," is engraved large across the figure, split ORIEN...TALE). A large hour-circle FGHI is drawn about center E and divided by radial lines into the equal hours; a long horizontal straight line AC (the equinoctial/horizon line) passes through the point A near the center, and the line DE crosses it perpendicularly. From the hour-divisions of the circle, the family of parallel hour-lines is carried down across AC, and the transferred hour-points are numbered 1 through 10 along the lower-left. Readable labels:

- K (top of circle): hour-circle point
- G (top, just below K): hour-circle point
- C (upper left, at the "ORIEN·C" break): endpoint of AC / hour-circle point
- F (upper right, at the "F·TALE" break): hour-circle point [?]
- B (left end of the horizontal line AC): endpoint

- E (center): center of the hour-circle
- A (center, on line AC): foot of the perpendicular DE
- N, L, I (lower right): points where hour-lines meet [?]
- S (bottom): lower endpoint of a bounding line [?]
- D (lower left): point on the perpendicular DE
- 2, 3, 4, 5, 6, 7, 8, 9, 10 (along the lower-left diagonal): the numbered hour-points

[Several peripheral labels on this dense figure could not be resolved with confidence]

[Margin: Demonstration of the construction of the meridian sundial.]

[Margin: b — 16.vnde.] [Margin: c — 10.vnde.]

Since the common sections made by the Equator in parallel planes — namely in the meridian circle and in the plane of the sundial — are parallel, and likewise the common sections made by the Horizon in these same planes, the angle contained between those sections in the plane of the meridian circle will be equal to the angle contained between the same sections in the plane of the sundial. Suppose, then, that AB is the common section of the horizon and the plane of the sundial; then AC will be the common section of the Equator and the plane of the sundial, since the angle BAC has been taken equal to the angle of the Equator's altitude — that is, to the angle which the section of the Equator makes with the section of the Horizon in the meridian circle.

Now suppose that about AC, as a line at rest, the plane of the circle FGHI is turned until it coincides with the plane of the Equator, and until E, the vertex of the assumed style AE, is joined to the center of the world. (For the vertex of the style is to be conceived as at the center of the world, by proposition 2 of the preceding book.) Then that circle will be concentric with the Equator, and the straight lines drawn through E and the points of division will be the common sections of the hour-circles from noon or midnight with the Equator — since the Equator and the circle FGHI, described about the same center E, are cut into similar arcs by those lines, as is clear from what we wrote at the end of chapter 1 in the [treatment of the] sphere. Thus GI, parallel to AC, is the common section of the Equator and the circle of the 12th hour, that is, of the meridian. For the common sections of the meridian — that is, of the circle of the 12th hour — with the plane of the sundial, made by the Equator, are parallel; and so, since AC is the common section of the Equator and the plane of the sundial, as we have shown, GI will be the common section of the Equator and the meridian, that is, of the circle of the 12th hour. KL

will be the common section of the Equator and the circle of the 11th hour on the sundial facing east; but on the one that faces west it will be the common section of the Equator and the circle of the first hour, and so on in order — as is evident if the proper position of the circle FGHI, or of the Equator, is carefully considered in the plane of each meridian sundial.

For this reason, the hour-circles from noon or midnight will cut the equinoctial line AC in the plane of the sundial at the points into which fall the straight lines drawn through center E and the division-points of the circle FGHI. Accordingly, through these points will be drawn the common sections of those same hour-circles with the plane of the sundial, and these are precisely the hour-lines from noon or midnight. Since, then, by the corollary of proposition 22 of the preceding book, these hour-lines are parallel on the meridian sundial, it is plain that the straight lines drawn parallel to DE through the points of the line AC are the hour-lines from noon or midnight.

But someone will perhaps doubt whether the hour-lines from noon or midnight are perpendicular to the equinoctial line AC — and hence parallel to DE, which we drew perpendicular to AC, in the way we drew them. We shall demonstrate this too, as follows. Since both the plane of the sundial and the plane of any hour-circle from noon or midnight are perpendicular to the Equator...

[Margin: a — 19.vnde.] [Margin: b — 29.primi] [Margin: c — 10.vndec.]

their common section will also be perpendicular to that same Equator, and hence, by definition 3 of book 11 of Euclid, also perpendicular to the straight line AC, which lies in the Equator. Therefore all the hour-lines from noon or midnight — that is, the common sections of the hour-circles with the plane of the sundial — are perpendicular to the line AC. And accordingly, since they must be drawn through the points found on the line AC, we drew them through those points parallel to DE, which is perpendicular to AC, because in this way they will all be perpendicular to AC. The same thing...

Diagram. The construction of the western astronomical meridian sundial (the word **Occidentale**, "Western," is engraved across the figure, split Occi...dentale). As on the eastern figure, an hour-circle is drawn about center E and divided by radial lines; a horizontal line passes through the central point A, with the perpendicular through it, and the parallel hour-lines are carried across, the transferred points numbered along the lower right. Readable labels:

- K (top): hour-circle point
- G (top, below K): hour-circle point

- C (upper right): endpoint of the equinoctial line [?]
- E (center): center of the hour-circle
- M, R (just right of center): points on the equinoctial line
- A (center): foot of the perpendicular
- B (right, on the horizontal line): endpoint
- F, E (right): hour-circle points [?]
- N (lower center): point
- I, L (lower left): points [?]
- S (bottom): lower endpoint [?]
- D (lower right): point on the perpendicular
- 1, 2, 3, 4, 5 (along the lower right): numbered hour-points

[Several peripheral labels on this dense figure could not be resolved with confidence]

[Margin: d — 16.vnde.] [Margin: e — 29.primi]

[Margin: The meridian line, or line of the 12th hour, cannot be drawn on the meridian sundial. On either meridian sundial, the segments of the equinoctial line lying between the 6th-hour line and two hours equally distant in time on each side of the 6th-hour line are equal.]

will be demonstrated by this reasoning. [Margin: d] Since the sections made by the Horizon in the meridian and in the plane of the sundial are parallel, and likewise the sections made in these same planes by the circle of the 6th hour, the angle contained between the former in the meridian will be equal to the angle contained between the latter in the plane of the sundial. Since, then, the former is the angle of the pole's altitude, as is established, this latter angle too will be the angle of the pole's altitude. Consequently, since the angle BAD is the angle of the pole's altitude — because, by construction, the angle remaining from the right angle BAC is equal to the complement of the pole's altitude — and since AB is the common section of the plane of the sundial and the Horizon, AD will be the common section of that same plane of the sundial and the circle of the 6th hour from noon or midnight. And accordingly, since all the hour-lines are parallel to one another and therefore also equidistant from AD, as we have shown, [Margin: e] they will all be perpendicular to AC, just as AD itself is perpendicular to AC.

The meridian line alone — that is, the line of the 12th hour from noon or midnight — cannot be drawn on this sundial, but vanishes, as is clearly established from the scholium of proposition 11 of the first book, since the

meridian circle does not cut the plane of the sundial but is parallel to it. And so we have set up the astronomical meridian sundial, and the rest — which was to be done.

SCHOLIUM.

[Margin: f — 27.tertij] [Margin: g — 26.primi]

On any meridian sundial, any two hour-lines equally distant in time on either side of the 6th-hour line DE cut off, from the equinoctial line AC, segments lying between themselves and the 6th-hour line that are equal — so that on the equinoctial line the hour-spaces before the sixth hour are equal to the hour-spaces after the sixth hour, one by one. [Margin: f] For instance, the lines AM and AN on the western sundial, each of which spans four hours, are equal. [Margin: g] For since the angles MEA and NEA at center E, subtended by equal arcs, are equal, and the angles at A are right, and the side AE is common, [Margin: h — 19.vnde.] the straight lines AM and AN will also be equal, and likewise EM and EN. The same reasoning holds for the rest.

[Margin: For describing the meridian sundial it suffices to divide one quadrant of the circle described from E into 6 equal parts. How each meridian sundial, together with its style, is to be set up so as to show the hours.]

From this it follows that, for describing the meridian sundial, it is enough to draw straight lines through center E and the division-points of a single quadrant — namely HI — cutting the equinoctial line at the points S, N, Q, and so on. For if these points are transferred from A, by means of the compasses, to the other side of the same equinoctial line, all the points through which the hour-lines are to be drawn will be obtained.

[Margin: i — 16.vndec.]

Now if the sundial is set up on a plane parallel to the meridian, so that the line AB is parallel to the Horizon, and point A faces north while point B faces south — so that the angle of the complement of the pole's altitude, BAC, which stands above the horizontal line AB, slopes southward on either sundial, while the angle opposite to it, which lies below that same horizontal line, slopes northward — then the hours will be indicated by the gnomon AE standing at right angles on the plane of the sundial at point A, so that the style itself is the common section of the Horizon and the Equator, that is, of the circle of the 6th hour, or of the Vertical. [Margin: h] For this common section is perpendicular to the plane of the sundial at point A, through which the planes of the said circles pass, standing

perpendicular to the plane of the sundial. And if from any two points of the line DE — say from D and E — two perpendiculars EO and DP are raised to the plane of the sundial, equal to the assumed gnomon AE, and the straight line OP is laid across them, this line OP will serve as the axis, since it passes through the center of the world — that is, through the point that is the vertex of the style erected from A — and is parallel to the line of the 6th hour, as the argument requires.

[Margin: i] For the line of the 6th hour and the axis of the world are parallel, being of course the common sections of the meridian circle and the plane of the sundial made by the circle of the 6th hour. The shadow of this axis OP, therefore, cast upon the hour-lines from noon or midnight, will likewise indicate the hours from noon or midnight, just as we said of the horizontal and vertical sundials.

Diagram. The gnomon-axis figure. Two equal perpendiculars, EO and DP, rise from the base points E and D, and the straight line OP is laid across their tops as the axis. Labels:

- O (top left): top of the perpendicular EO
- P (top right): top of the perpendicular DP
- E (bottom left): base point
- D (bottom right): base point

[Margin: To which segment of the equinoctial line the style is equal on each meridian sundial.]

The gnomon AE that has been assumed is always equal to that part of the equinoctial line lying between A and the third or ninth hour — that is, to the straight line AQ or AR — on the western sundial. For since the whole quadrant GH is divided into 6 equal hours, the circle of the ninth hour, three hours distant from the line EH of the sixth hour (or from the line EG of the 12th hour), bisects that same quadrant; [Margin: a — 27.tertij] and accordingly the angle AER will be equal to the angle GER. Since, then, the whole angle AEG is a right angle, each of the angles GER and AER will be half a right angle. Therefore, since the angle EAR is a right angle, [Margin: b — 32.primi] the remaining angle ERA will also be half a right angle; and consequently, the angles ERA and AER being equal, the straight lines AE and AR will also be equal. The same reasoning holds for AE and AQ, and so on.

[Margin: The eastern sundial, if inverted so that top becomes bottom, and what is on the right in this position becomes left and vice versa, yields the western sundial; and in the same way the western yields the eastern.]

It is enough, moreover, to describe only the sundial facing east. For if this is inverted so that top and bottom are interchanged, and right and left are interchanged — as we taught in the scholium of proposition 14 of this book — it will supply us with the other sundial, the one facing west. We shall also make this evident by the following reasoning. In the triangle AEC on the eastern sundial, and in the triangle AES on the western, the angles at A are equal, namely right angles; [Margin: c — 27.tertij] and the angles at E are likewise equal, on account of the equal arcs between H and the straight lines EC, ES on which they stand; and moreover the sides AE, AE adjacent to the said angles are equal, since each is the length of the style; [Margin: d — 26.primi] therefore the sides AC, AS will also be equal.

Now suppose the eastern sundial is laid over the western so that point A coincides with point A, and the horizontal line AB with the horizontal line AB, and the upper part of the eastern sundial falls below the horizontal line and the lower part above it; and suppose that, in this position, the right part is changed into the left, and vice versa. Then the equinoctial line AC of the eastern sundial coincides with the equinoctial line AS of the western sundial, on account of the equality of the angles which each equinoctial line makes with the horizontal toward the same parts in this position — the position which the eastern sundial has when inverted in the way we described. Point C will also fall upon point S, on account of the equal lines AC, AS. Hence the line of the first hour of the eastern sundial, drawn through point C, will coincide with the line of the first hour of the western sundial, drawn through point S, since each makes a right angle with the equinoctial line, as we showed above. In the same way we shall demonstrate that the remaining hour-lines of the eastern sundial coincide with the remaining hour-lines of the western sundial. Therefore the eastern sundial, inverted as has been said, yields the western sundial, since in this way it takes on the same position as the western has, and is equal to it in every part, as we have shown.

Diagram. The gnomon-axis figure repeated (as on the preceding page). Two equal perpendiculars EO and DP rise from base points E and D, with the straight line OP laid across their tops as the axis. Labels:

- O (top left): top of the perpendicular EO
- P (top right): top of the perpendicular DP
- E (bottom left): base point
- D (bottom right): base point

[Margin: c — 27.tertij] [Margin: d — 26.primi]

[Margin: The lines of the eastern sundial, drawn on the opposite face of the plane, yield the western sundial, and vice versa, if the hour-numbers are changed into their complements up to 12. How the fractional hour-parts are to be drawn on the meridian sundial. How the meridian sundial is to be drawn on a fixed plane facing directly east or west. If the position of a style of given length is given, by what method the meridian sundial is drawn. Description of the parallels, that is, of the arcs of the zodiacal signs, on the same meridian sundial, by means of the analemma.]

But the hours that are reckoned from midnight on the eastern sundial must, after that inversion, be reckoned from noon, since it has now become the western sundial.

By the same reasoning it would be enough to describe only the western sundial. For this too, if it is inverted as we have said, would supply us with the eastern sundial, which can be demonstrated in the same way. But the hours which, on the western sundial as described, indicate the hours from noon would, after that inversion, have to be reckoned from midnight, because the western has now become the eastern.

Indeed, once the eastern sundial has been described on some plane, if on the opposite side of the plane lines corresponding exactly to its lines are drawn, the western sundial too will have been described, to be placed on the face of the plane that looks west; because in this way the angle of the complement of the pole's altitude, which the equinoctial line makes with the horizontal above the horizontal line, slopes southward, just as on the eastern sundial. But the numbers of the hours must be changed into their complements up to 12, so that hour 11 becomes 1, and hour 10 becomes 2, and so on. By the same reasoning, once the western sundial has been described, if on the other side of the plane lines corresponding to its lines are drawn, the eastern sundial will have been described.

Furthermore, if it is desired to draw the half-hours as well, and their quarters, eighths, and so on, the individual hours of the circle FGHI must be divided into two parts, four, eight, and so on; and the rest is to be completed as before.

But if the sundial is to be described on some fixed plane — as on a wall facing directly east or west — (for up to now we have laid it out on any arbitrary plane, so that it may afterwards be set in its proper position), then, by means of a plumb-line and a level, a straight line AB must be drawn on the given plane, parallel to the Horizon; and from any point A an arc of a circle BG must be

described toward the southern parts, so that in it the complement of the pole's altitude BC may be counted off. The rest is to be completed as before.

In the same way, if the position of the style is given at point A, and its length AE, the said straight line AB must be drawn parallel to the Horizon through point A, and from A an arc of a circle described, as we said just above, and so on.

PROBLEMA 26. PROPOS. 26.

To describe the parallels — that is, the arcs of the zodiacal signs — on the aforesaid meridian sundial.

Let ABCD be an analemma, with the diameters of the circles, as in proposition 1 of the preceding book. Now since the meridian ABCD of the analemma does not cut the plane of the sundial, let the circle ABCD be understood to be the circle of the 6th hour from noon or midnight, serving as the proper meridian of the meridian circle, which now takes the place of a right horizon. It is evident, moreover, that the common sections of the circle of the 6th hour, of the Equator, and of its parallels are not different from those made in the meridian circle, because the meridian and the circle of the 6th hour, described through the poles of the parallels, cut all the parallels in the same way — just as do all the other right horizons. And because the meridian passes through the poles of the world, as does the circle of the 6th hour from noon or midnight, the axis FG will be the common section of the circle of the 6th hour ABCD and the meridian, as the diameter of some right horizon; and likewise the section of the Equator, HI, will be as the diameter of the proper Vertical of that same meridian circle, just as happens in the right sphere. For in every right horizon the Equator performs the office of the properly-so-called Vertical circle.

Now, taking on HI, on each side from point E, the straight line EK equal to the gnomon AE assumed in the preceding sundial, let LM be drawn through each point K parallel to FG. Each parallel LM will be the common section of the circle of the 6th hour ABCD and the plane of the sundial, since in this way the plane of the sundial is equidistant [from it] along the straight line LM, drawn by means of it...

of the meridian itself, drawn through FG according to the size of the style EK. Let the diameters of the zodiac that join the opposite signs also be drawn, so that triangles are formed through the axis of the cones — triangles whose bases are the parallels and whose common vertex is the center E. Therefore, by what was demonstrated in proposition 6 of the preceding book, NL, OL, PL, QM, RM, and SM

will be the diameters of the conic sections. And if the points N, O, P, Q, R, S are taken from point K and transferred here and there onto the line of the sixth hour of the sundial from point A, where the style is to be set, then the shadow of the style will be cast onto these same points whenever the Sun stands at the beginnings of the signs and on the circle ABCD of the sixth astronomical hour.

If, then, the conic sections are described in some hard material about the diameters NL, OL, PL, and so on — by proposition 8 of the preceding book — and are cut out so as to form a kind of bent rulers, by means of which the same sections may be drawn on the sundial so that they pass through the points N, O, P, Q, R, S and have their axes joined to the line DE of the sixth hour, then the parallels of the zodiacal signs will have been described. It is enough, however, to describe the conic sections of the northern signs, or of the southern; for these are equal to the sections of the opposite signs, since they are all hyperbolas, as is established from proposition 6 of the preceding book.

[Margin: It is enough to describe the arcs of the northern signs only, or of the southern.]

Diagram. A large circle representing the meridian circle (here serving as the circle of the sixth hour), with the analemma construction inscribed. The circle is lettered ABCD, and radiating lines, the world-axis, the equator, and the diameters of the zodiacal parallels are drawn inside it. Zodiacal-sign glyphs are set around and within the figure; several are worn and cannot be individually resolved [?].

- A (top of circle): endpoint of the vertical diameter AC
- C (bottom of circle): endpoint of the vertical diameter AC
- B (left of circle): endpoint of the horizontal diameter BD
- D (right of circle): endpoint of the horizontal diameter BD
- E (center): common center / vertex of the cones
- F (upper right, on the oblique axis): endpoint of the axis FG (the world-axis, the section of the sixth-hour circle)
- G (lower left, on the oblique axis): opposite endpoint of the axis FG
- H (upper left): endpoint of the equator section HI
- I (lower right): opposite endpoint of the equator section HI
- L (lower-left interior): point where the diameters NL, OL, PL converge
- M (upper-right interior): point where the diameters QM, RM, SM converge
- K (interior, on the sixth-hour line): the point from which N, O, P, Q, R, S are transferred

- N, O, P, Q, R, S (interior points): the points through which the conic sections (parallels of the signs) are drawn
- Additional lowercase interior labels (m, n, l, and others) and peripheral zodiac glyphs are present but partly abraded [?]

From this it is established that, by the benefit of these same rulers, the parallels will easily be described on the opposite part of the sundial through those points that are marked on the line of the sixth hour.

In each sundial, moreover, the northern signs are those that are described below the equinoctial line, and the southern signs those contained above it — so that in each sundial, when it occupies its proper position, the northern signs recede from the equinoctial line toward the south, and the southern signs toward the north. The reason for this is that when the Sun rises or sets while standing in the northern signs, the shadow of the style falls on the southern parts of the horizontal line; but it falls on the northern parts when the Sun, standing in the southern signs, rises or sets, as is evident.

[Margin: In meridian sundials the northern signs are below the equinoctial line, and the southern above.]

Now, since when the Sun rises the shadow of the gnomon is cast onto the horizontal line AB in the eastern sundial, and when it sets the shadow is cast onto the same horizontal line in the western sundial — as is clear from proposition 11 of the preceding book — it comes about that for all the remaining time of any given day, with the Sun raised above the horizon, that same shadow falls on the plane of the sundial below the horizontal line. For this reason the part of the sundial contained above the horizontal line must be cut away as superfluous, since the shadow of the style never falls upon it.

[Margin: The portion of the meridian sundial above the horizontal line must be cut away as superfluous.]

That upper part of either sundial, however, once cut off from the horizontal line, will yield a sundial to be set up on the opposite face of the sundial's plane — provided it is inverted, so that the part which is now uppermost becomes the lowest, and vice versa, and so that the part which after this inversion is on the right becomes the left, as we prescribed in the scholium to proposition 14 of this book; and the hours, [Margin: The portion of either meridian sundial...]

Diagram. The two meridian sundials — eastern and western — laid out symmetrically about a single horizontal line. The horizontal line runs across the

middle of the figure and is lettered, with the word set as the line-label; two fans of hour-lines and hyperbolas (the arcs of the signs) rise from near the line on the left and right.

- Horizontal line (labelled along its length): "HORI[S] ... B ... ZON" (i.e. HORIZON), with B marking the midpoint of the line
- Two apex points (upper left of each fan): the convergence points of the hour-lines
- C (bottom of each fan): endpoint at the foot of each bundle of lines
- Zodiacal-sign glyphs are scattered along the hyperbola-arcs of each fan; many are worn and cannot be individually resolved [?]

which were reckoned from midnight, are to be counted from noon, and conversely; and finally the northern signs are changed into southern, and the southern into northern. For, as we showed in the scholium to the preceding proposition, once this inversion has been made, the hour-lines of the upper part that has been cut off in either sundial coincide with the hour-lines of the sundial described on the opposite face of the plane. The axis of the hyperbolas $\mathfrak{S}$ and $\mathfrak{V}$ will likewise coincide with the axis of the hyperbolas $\mathfrak{S}$ and $\mathfrak{V}$, since in both cases this axis is the line of the sixth hour from noon or from midnight.

Since, then, the transverse axis of the said hyperbolas in the eastern sundial is equal to the transverse axis of the same hyperbolas in the western — because the hyperbolas of the one are equal to the hyperbolas of the other, seeing that the shadows describing them are cast by equal styles onto planes equidistant from the meridian circle — the hyperbolas will also coincide with the hyperbolas: namely, hyperbola $\mathfrak{S}$ will coincide with hyperbola $\mathfrak{V}$, and hyperbola $\mathfrak{V}$ with hyperbola $\mathfrak{S}$. And the same reasoning holds for the remaining hyperbolas of the other signs. Since this is so, it is clearly established that the upper part cut off in either sundial displays the sundial on the opposite face of the plane, if it is inverted, as we have said.

[Margin: ...of the meridian, cut off from the horizontal line, will produce a sundial on the opposite part of the plane, if the parts are interchanged, as was said above. The horizontal line divides the meridian sundial into a daytime and a nighttime part.]

Again, the horizontal line AB divides each sundial into a daytime and a nighttime part, just as was said of the vertical sundial in proposition 14 of this book — the daytime part being below the horizontal line, the nighttime part above it. For onto the upper part of the eastern sundial the shadow of the style would be cast

after midnight up to sunrise, were the density of the earth not in the way; and onto the upper part of the western sundial the shadow of the style would fall after sunset up to midnight, if the Sun could illuminate it.

[Margin: Another description of the arcs of the signs on the meridian sundial.]

We shall describe these same arcs of the signs in the following way. Draw the straight line DC arbitrarily for the radius of the Equator, and, with the radii of the other signs described here and there, as was said in proposition 2 of this book for the horizontal sundial, let all the hour-intervals intercepted between E, the center of the circle FGHI of the preceding proposition, and the equinoctial line AC be transferred onto the radius DC of the Equator from D — so that the straight line DC is equal to the space ES or EC, and the straight line DT to the space EN or EM, and so on. This is done by marking points on the straight line DC, through which lines parallel to the world-axis DG (which cuts the straight line DC at right angles) are drawn — which will indeed easily be done if all the points of the straight line DC are carried across onto the straight line GF, parallel to DC — and to these parallels are attached the numbers of the hours corresponding to the hour-intervals thus transferred.

For if the parts of these parallels intercepted between the radius DC of the Equator and the radii of the other signs are transferred onto the corresponding hour-lines of the sundial from the equinoctial line AC — marking points, and through these points drawing curved lines, as we said for the sundials above — the parallels of the Zodiac will again have been described. Now any two opposite parallels are equally distant from the equinoctial line.

[Margin a: (III.27) — i.e. Euclid III.27]

[Margin: By what method the arcs of the opposite signs are described by one and the same operation... drawn on the meridian sundial.]

^a For since the angles that the radius DT of the Equator makes with the radii of the opposite signs are equal — because such angles stand at the center on equal arcs of equal declinations — and since two angles at α [?] are also equal, namely right angles, and the side D α [?] is common, ^b the straight lines between α [?] and the radii of any opposite signs will likewise be equal, which is the proposition. The same is also clearly established from the Analemma of this proposition, if the triangles EKN, EKQ; EKO, EKR; and EKP, EKS are rightly considered. Hence by one and the same operation we shall describe any opposite parallels on the sundial, if on either side of the equinoctial line, on the hour-lines, we mark points equally

distant from the equinoctial line. This latter description of the parallels of the Zodiac we shall now demonstrate.

[Margin b: (I.26) — i.e. Euclid I.26]

Diagram. The construction of the arcs of the signs on the horizontal meridian sundial. A large rectangular grid of hour-lines with a central bundle of radii fanning downward and diagonal "radius-of-the-hour" lines crossing it; zodiacal-sign glyphs run along the foot of the figure.

- G (top left), D (top center), K (top center, right of D), G (top right): points along the upper edge
- F (left), F (right): endpoints of the lower horizontal (the line GF parallel to DC)
- C (center, low): foot of the central radius-bundle
- E (interior center): center of the circle FGHI
- S, N, M, T, and lowercase p, o, m, n, l: interior points marking the hour-intervals on DC / DT
- Diagonal radius-lines labelled: "Rad. hor. 0", "Rad. hor. 6", "Rad. hor. 10", "Rad. hor. 14", "Rad. hor. 18", "Rad. hor. 24" (Radius horæ — radius of the hour)
- Hour-numbers are printed along both side edges (reading roughly 12, ... 9, 4, 8 on the left and 5/6, 8/7, 4/3, 9, 1/10 on the right); several are worn and only partly legible [?]
- Zodiacal-sign glyphs along the bottom edge; several worn [?]

Now let it be understood that the style AE stands at right angles on the plane of the sundial, and that the figure of the zodiacal radii, just constructed about the tip E of the style, is turned round toward the plane of the sundial, so that point D is joined to point E — that is, to the center of the world — and the straight line DG is perpetually equidistant from the line of the sixth hour, that is, is joined to the world-axis; and consequently the straight line DC, not receding from the plane of the Equator in that motion, always meets the equinoctial line. This being posited, in this revolution the point T will fall, for example, onto point N, because in the construction the straight line DT was taken equal to the straight line EN. For the straight line EN, falling from point E set on high — namely from the tip of the style — onto point N, is equal to the straight line EN in the plane of the sundial; as will easily be proved if the triangle EAN in the plane is compared with the triangle EAN on high. For since the sides EA, AN of the one are equal to the sides

EA, AN of the other, and they contain equal angles, namely right angles, ^c the base EN of the one will be equal to the base EN of the other.

[Margin c: (I.4) — i.e. Euclid I.4]

Since, therefore, both the straight line NV and the straight line TX are equidistant from the world-axis, ^d NV and TX will also be parallel to each other, and so the straight line TX will be joined to the straight line NV — otherwise TX and NV would not be parallel to each other, since they meet at N. Wherefore, since the straight line TX is taken equal to the straight line NV, the point X will fall onto point V; and on this account the ray of the Sun, when it is at the beginning of ♉ or ♊, will meet the plane of the sundial at point V. Through point V, then, will pass the parallel, or hyperbola, of ♉ or ♊; since the ray of the Sun falls upon it in that revolution of the zodiacal radii, as we have demonstrated. In the same way we shall show that through the remaining points taken on the sundial the same hyperbola passes, and so on for the rest. For this reason we have described the parallels — that is, the arcs of the zodiacal signs — on the aforesaid meridian sundial. Which was to be done.

[Margin d: (XI.9) — i.e. Euclid XI.9]

PROBLEMA 27. PROPOS. 27.

To describe the parallels of the diurnal arcs on the same meridian sundial.

[Margin: Description of the diurnal arcs on the aforesaid meridian sundial.]

In place of the parallels described through the beginnings of the signs, let the parallels of the diurnal arcs be taken in the Analemma of the preceding proposition; and in place of the radii of the signs in the figure of the zodiacal radii of that same preceding proposition, let the radii of the diurnal arcs be taken, just as was done in propositions 3 and 15 of this book. Let the rest be done as was said in the preceding proposition, and the parallels of the diurnal arcs will have been described. For the description of these does not differ from the delineation of the arcs of the signs, since both these and those are equidistant from the Equator. We have therefore described the parallels of the diurnal arcs on the same meridian sundial, which was to be done.

SCHOLIUM.

In the last figure of the preceding proposition, the radii of the diurnal arcs were drawn for the hours 0 and 24, 6 and 18, 10 and 14, so that arcs of this kind may be described when the occasion requires it.

PROBLEMA 28. PROPOS. 28.

To describe the vertical circles on the same meridian sundial.

[Margin: Description of the vertical circles on the same meridian sundial.]

Through point A, where the equinoctial line AC cuts the horizontal line AB, let the perpendicular AD be drawn to that same horizontal, which will be the common section of the prime vertical circle and the plane of the sundial. For since the style AE is the common section of the horizon and of the prime vertical, as we said in the scholium to proposition 25 of this book, the plane of the vertical circle drawn through the style AE will meet the plane of the sundial in A, and therefore the vertical line must be drawn through A. And because both the plane of the sundial and the plane of the vertical circle are at right angles to the plane of the horizon, ^a their common section will likewise be perpendicular to that same plane of the horizon, and consequently it will be perpendicular to the horizontal AB at point A, by definition 3 of Book 11 of Euclid. Therefore the straight line AD, which we drew perpendicular to the horizontal line AB, will be the common section of the plane of the sundial and of the prime vertical circle.

[Margin a: (XI.19) — hence]

On this vertical line AD, produced toward A, let the straight line AF be taken above AB — or, if you prefer, below — equal to the gnomon AE set out, and from center F let a circle be described with any interval whatever. Divide this into 360 equal parts, or into fewer if not all the vertical circles are wanted (we divided it into 24, so that each contains 15 degrees), and through the points of division and the center F let hidden [construction] lines be drawn, cutting the horizontal line AB at points, through which, if lines are drawn parallel to the vertical line AD — which will indeed easily be done if a line DG is drawn parallel to the horizontal, and onto it, from point D, all the points of the horizontal line taken from point A are transferred; ^b for the straight lines joining any two points equally remote from the points A and D will be parallel to AD itself — the vertical circles will have been described on each sundial, taking their beginning from the vertical line AD. That this is so, we shall demonstrate in the following way.

[Margin b: (I.33)]

Diagram. Two circles side by side, each showing the layout of the vertical circles on a meridian sundial — the left captioned "Occidentale." (western), the right "Orientale." (eastern). Each circle has a vertical diameter through F at its top, a horizontal line AB across it, and a fan of graduated lines descending below, marked with degree-values.

- Left circle captioned **Occidentale.** (Western); right circle captioned **Orientale.** (Eastern)
- F (top of each circle): center from which the dividing radii are drawn (the point equal to the world-center / style-tip E)
- A, E, L, B (on the horizontal line of each circle): A the intersection of equinoctial and horizontal, E and L intermediate points, B the outer endpoint of the horizontal
- Y (interior, near A): a construction point [?]
- G, K, D (lower interior / foot of each fan): points of the descending construction
- Degree-labels along the descending lines: 15, 30, 45, 60, 75 (each recurring on both sides of both circles)
- Zodiacal-sign glyphs at the lower corners; several worn [?]

Now let it be understood that the plane of the circle described from F is turned about the horizontal line AB, until it coincides with the horizon, so that F becomes the same as the center of the world, or the tip of the style E, because of the equality of the style AE and the straight line AF. This being posited, the straight lines drawn out from center F through the points of division will be the common sections of the horizon and of the vertical circles; because the horizon and the circle described from F, since [they have] the same center

[continuation of the demonstration of Problema 28]

have the same center, they are divided into similar parts, from what we demonstrated at the end of chapter 1 on the sphere. The vertical circles therefore meet the plane of the sundial at the points of the horizontal line onto which the straight lines drawn from F fall. And since the common sections of the vertical circles and of the sundial's plane are parallel — by proposition 18 of the previous book, because the plane of the sundial is equidistant from their common section, that is, from the axis of the horizon — these common sections of the sundial's plane and of the vertical circles will be parallel to AD itself. The meridian circle, however, which is the ninetieth vertical, is not projected onto the plane of the

sundial, since it is equidistant from that plane and so makes no section on it. Thus we have described the vertical circles on the same meridian sundial. Which was to be done.

PROBLEMA 29. PROPOS. 29.

To describe the parallels of the horizon [almucantars] on the same meridian sundial.

[Margin: Description of the parallels of the horizon, or almucantars, on the same meridian sundial.]

Let ABCD be the vertical circle properly so called, with center E; let BD be its common section with the horizon, and AC its common section with the meridian. Divide each of the two quadrants AB and AD into 90 equal parts — or into fewer, according to the size of the sundial. (We divided it into 6, so that each part spans 15 degrees.) Join by straight lines each pair of points equally distant from B and from D; these lines will be the common sections of the vertical circle and of the parallels of the horizon.

Next, from the division points of both quadrants draw straight lines through the center E, forming triangles through the axis within the cones — cones whose bases are the parallels of the horizon, both below and above the horizon, and whose common vertex is the center of the world, E. For the vertical circle ABCD, cutting the said cones through their axis AC, produces (by proposition 3 of Book 1 of Apollonius [*Conics* I.3]) triangles through the axis. Cut off also from BD, on each side, the straight line EF equal to the style of the sundial, and through F draw FK parallel to AC — that is, the common section of the vertical circle and of the sundial's plane — cutting the sides of the triangles through the axis at G, H, I, K, and so on, so that GK, HK, IK, and the rest are the diameters of the conic sections.

If, then, the points G, H, I, K are transferred from A onto the sundial's vertical line AD, below the horizontal line, and the said conic sections passing through the points G, H, I, K are described about the vertical line (by proposition 8 of the previous book) — these conic sections being hyperbolas (by proposition 6 of the preceding book, since the meridian, from which the sundial's plane is equidistant, is drawn through the poles of the parallels of the horizon and so cuts them all) — and always receding further from the horizontal line the further they are produced on each side of the vertical line AD, then the parallels of the horizon will have been described. Which was the thing proposed.

Diagram. A large circle representing the vertical circle ABCD, seen with its full graduation, over which the cone-triangles for the almucantars are laid out.

- A (top): zenith; endpoint of the vertical/meridian diameter AC
- C (bottom): nadir; lower endpoint of AC
- B (left) and D (right): endpoints of the horizontal diameter BD (the horizon line)
- E (center): center of the circle, the center of the world / tip of the style (drawn as a small sun-like ring at center)
- F, F (just below E, one on each side): the two points cut off on BD such that EF equals the style; through them FK is drawn
- G, G / H, H / I, I (stacked in mirror pairs descending from E toward C): the points where the parallel FK cuts the sides of the axial triangles
- K, K (lower, near C): the further such division points
- Degree labels along the arc, each recurring on the left (B) and right (D) sides and in the upper and lower halves: 15, 30, 45, 60, 75, with 90 at the top (near A) and 90 at the bottom (near C) — the almucantar values measured from the horizon BD
- Numerous radial construction lines run from the graduation points through the center E

ALITER [Another method].

[Margin: Another description of the parallels of the horizon on the same meridian sundial.]

Describe a quadrant ABC of any size, and divide it into 90 degrees, or into fewer parts according to the number of parallels to be drawn. From the center A, through the division points, send out straight lines; these will correspond to the radii of the parallels of the horizon contained in the quadrant ECD of the preceding figure. Begin from the straight line AB, so that the radius nearest to AB is that of the parallel of the horizon at 15 degrees, the next at 30, and so on, as the attached numbers indicate.

Then, from the figure of the preceding proposition, take the intervals between the center F and the points at which the vertical lines cut the horizontal line, and transfer these from A, in the present figure, onto the straight line AB; write beside the points the numbers of the vertical lines that bound the transferred intervals on AB, and through these points draw lines parallel to AC. This will easily be done if GH is drawn parallel to AB and the points of the line AB are transferred onto it,

and so forth. For example, from the figure of the preceding proposition transfer the interval FL onto the straight line AB of the present figure, as far as the point D, marking the number 60; and through D draw DE parallel to AC, and so on.

Once the figure has been constructed in this way, the parallels of the horizon will be described as follows. The intervals of the lines equidistant from AC, taken between the straight line AB [and...]

Diagram. A quadrant construction for the alternative method, occupying the lower right of the page.

- A (top right corner): center/vertex of the quadrant, from which the radii are drawn
- B (top left, along the upper horizontal line AB); X (further left of B on the same line)
- C (right side, below A): endpoint of the quadrant arc
- G (bottom right) and H (bottom left): endpoints of the lower horizontal line GH, drawn parallel to AB
- D, E, F (interior, along the upper region near A): construction points; D marked with the number 60, with DE drawn parallel to AC
- The quadrant arc sweeps from B down to the lower right, crossed by radii from A
- Degree labels on the radii / arc: 15, 30, 45, 60, 75 (30 and 60 also appearing along the top line near A)

the radius, say, of the parallel of the horizon at 15 degrees, are transferred from the points at which the horizontal line in the figure of the preceding proposition is cut by the vertical lines, onto the corresponding vertical lines on the straight line AB marked with the numbers, marking the points on the vertical lines. Thus, for example, on the straight line DE take a length equal to LM, on the vertical line of 60 degrees, and so on for the rest. For if these points are suitably joined by a certain curved line, the parallel of the horizon at 15 degrees will have been described. In the same way the remaining parallels of the horizon will be described, if the straight lines lying between the line AB and the radii of the parallels of the horizon are transferred onto the corresponding vertical lines, from the horizontal line, and so on. This we shall demonstrate as follows.

[Margin: Demonstration of the latter description of the parallels of the horizon.]

In the figure of the preceding proposition, let the straight line AF — which was taken equal to the style — be understood to stand at right angles to the plane of

the sundial at the point A; and let the figure just constructed about the point F, which is at the center of the world, be turned toward the sundial, so that the point A is joined to the center of the world, that is, to the point F, and so that the straight line AC remains perpetually equidistant from the vertical line AD — that is, is united with the axis of the horizon and of its parallels — and therefore the straight line AB, not departing from the plane of the horizon, always meets the horizontal line in that motion. For in this turning the point D, say, will fall onto the point L of the horizontal line, because the straight line FL was here taken equal to AD in the preceding proposition. [Margin a: (Euclid I.4)] For the straight line FL falling from the point F on high — that is, from the tip of the style — is equal to the straight line FL in the plane of the sundial, as is clear if the triangle FAL on high is compared with the triangle FAL in the plane of the sundial, in the manner stated in proposition 26 of this book concerning the triangles EAN, EAN.

Since, then, both the straight line LM and DE are equidistant from the axis of the horizon, [Margin b: (Euclid XI.9)] LM and DE will also be parallel to each other; and for this reason the straight line DE will coincide with the straight line LM — otherwise LM and DE would not be parallel, since in that turning they meet at L. And accordingly, since LM was taken equal to the straight line DE, the point E will fall onto the point M; and so the radius of the parallel of the horizon at 15 degrees will meet the plane of the sundial at the point M. Through the point M, therefore, the arc of the parallel of the horizon at 15 degrees will pass, since the radius of the said parallel falls upon it in that revolution, as has been demonstrated. In just the same way we shall show that the point F of the same radius falls onto the point N, and so on for the rest. We have therefore described the parallels of the horizon on the same meridian sundial. Which was to be done.

[Margin: When the vertical lines and the parallels of the horizon have been drawn on one of the two meridian sundials, if on the opposite face of the plane there are painted vertical lines and parallels of the horizon corresponding exactly to those drawn on the sundial, then the same vertical circles and parallels of the horizon will have been described on the opposite face.]

SCHOLIVM.

Since the vertical lines and the hyperbolas of the parallels of the horizon have the same position and the same numbers on each of the two sundials — as is clear both from the description itself and from the figure of the preceding proposition — it is evident that, if on the opposite face of either sundial the vertical lines and parallels of the horizon are drawn corresponding exactly to the vertical lines and parallels of the horizon delineated on the sundial, then a sundial with vertical

lines and parallels of the horizon has likewise been described, to be placed on the opposite face of the plane. For this reason it will be enough to describe one sundial only, either the eastern or the western; for from it we shall easily delineate the other on the opposite face, as has been said.

PROBLEMA 30. PROPOS. 30.

To describe the meridians, or the circles of longitude of cities, on the same meridian sundial.

[Margin: These two circles are to be conceived as equal.]

[Margin: Description of the meridians on the same meridian sundial.]

Diagram. Two large adjoining circles laid out side by side (left and right), each the circle FGHI described from center E, representing the meridian / longitude circles of cities for the eastern and western meridian sundials. A crowded field of dashed meridian lines fans across both circles, carrying longitude-degree labels and a few city names; many peripheral labels are worn and abraded. **This diagram — numerous small worn labels flagged [?].**

- Left circle: center E; a cross/dagger symbol (✚) at the top marking the start of division; rim points F (left), G (upper right); labels B (at the bottom, on the horizontal axis shared with the right circle), I (lower left), C (far lower left, on a line running out below), S (lower, near the "180" mark)
- Right circle: center E; rim points G (upper left), F (upper right, with "135"), H (bottom), I (right)
- City-meridian captions: "**Ins. For.**" (*Insulæ Fortunatæ*, the Fortunate Isles — the prime meridian; appears vertically in the left circle and again in the right); "**Alexandria**" [?] with the value 60 in the right circle
- Longitude-degree labels scattered around and across both circles: 15, 30, 60, 90, 135, 180, 330 (left); 15, 30, 90, 135, 180, 270, 279 [?], 45 (right); and, in a small interior grid of the left circle, 270, 285 [?], 300, 315, 330, 345
- Several further peripheral degree/sign labels are too worn or ink-bled to resolve [?]

The description of these differs in nothing from the description of the hour-circles from noon or from midnight, as we taught in proposition 6 of this book — provided only that the circle FGHI, described from E, has the beginning of its division from the

meridian of the Fortunate Isles; which is obtained if the longitude of the place is reckoned from G — in the eastern sundial described (by proposition 25 of this book) toward H, but in the western toward F, that is, toward the western parts, away from the meridian circle, as the reasoning requires. This will indeed be readily understood if in each sundial the circle FGHI is turned about the straight line AC until it is joined with the plane of the equinoctial circle, and so on. We have therefore described the meridians, or the circles of longitude of cities, on the same meridian sundial. Which was to be done.

[Margin: Description, on either meridian sundial, together with its meridians: if the portion above the horizontal line is inverted, so that the upper parts become lower and the right becomes left, another sundial will be produced, containing the meridian circles, to be placed on the opposite face.]

SCHOLIVM.

If the upper part of the eastern sundial — the part that the horizontal line cuts off — is inverted, as we said in the scholium to proposition 14 of this book, namely so that the lower parts become the upper and the right becomes the left, we shall have the western sundial with its meridians, as we showed in that same scholium; provided that the numbers set on the eastern sundial are changed, on the western, into the numbers that arise from adding 180 degrees to the numbers of the eastern sundial (casting out 360 degrees, however, if from that addition a number greater than 360 results). Or, what amounts to the same thing, they are changed into the numbers that, in the circle described from E, lie diametrically opposite to those numbers set on the eastern sundial.

All this we shall also show by the following reasoning. In the triangle AEC, on the eastern sundial, and in the triangle AES, on the western, the angles at A are right, and therefore equal; [Margin a: (Euclid III.27)] and the angles at E are also equal, on account of the equal arcs intercepted between H and the straight lines EC, ES. (That these arcs are equal we shall establish thus. The arc G[†] on the eastern sundial is equal to the arc G[†] on the western, since each reckons the longitude of the place. And the arc [†]15 in the one is equal to the arc [†]15 in the other. Therefore the remaining arc G15 of the one will be equal to the remaining arc G15 of the other; and so the remaining arc 15.H, from the quadrant GH, will be equal to the remaining arc 15.F, from the quadrant GF. [Margin b: (Euclid III.26)] Since, then, the arc 15.F and the arc intercepted between H and the straight line ES, on the western sundial, are equal — because of the equal vertical angles at E — the arcs 15.H on the eastern and the arc intercepted between H and the straight line ES on the western, that is, the arc placed between the point H and

the straight lines EC, ES, will also be equal.) Likewise the sides AE, AE, which are adjacent to the said angles, are equal; [Margin c: (Euclid I.26)] therefore the straight line AC will also be equal to the straight line AS.

Hence we shall show, as in the scholium to proposition 25 of this book, that if the upper part of the eastern sundial is laid upon the lower part of the western, as we taught in the said scholium, the equinoctial line coincides with the equinoctial line, and the meridian 195, drawn through the point C on the eastern sundial, coincides with the meridian 15, drawn through the point S on the western sundial, which is set diametrically opposite to it. In just the same way we shall show that the remaining meridians contained in the upper part of the eastern sundial coincide with the remaining meridians existing in the lower part of the western sundial — those, namely, which are diametrically opposite to them in the circle described from E.

Therefore the upper part of the eastern sundial, torn away from the horizontal line, if it is inverted as we said, will yield the meridian circles on the western sundial, provided that 180 degrees are first added to each number, and 360 degrees cast out from the sum if the sum exceeds this number, so that we may have, on the western sundial, the number of that semicircle of the meridian which is opposite to the semicircle of the same meridian whose section lies in the upper part of the eastern sundial — just as was done in the case of the hour-lines from noon or midnight. For there too, on the western sundial, are contained the hours from noon, whose semicircles are opposed to the semicircles of the same hours from midnight in the upper part of the eastern sundial: because the first hour, say, on the eastern sundial is reckoned from midnight, while the same hour on the western sundial — once that inversion of the upper part of the eastern sundial has been made — is to be reckoned from noon, and so on.

By the same reasoning, if the upper part of the western sundial is inverted, as we prescribed, we shall have the eastern sundial with its meridians, provided that the numbers are first changed, as we said just above.

PROBLEMA 31. PROPOS. 31.

To describe the parallels of cities, that is, the circles of latitude, on the same meridian sundial.

[Margin: Description of the parallels of cities, that is, of latitudes, on the same meridian sundial.]

In place of the parallels passing through the signs of the Zodiac, take the parallels described through the zeniths of the cities situated within the tropics, and let everything else be done as we said of the parallels of the signs of the Zodiac in proposition 26 of this book; and when they have been described, they will be the parallels of the cities, as is clear from proposition 7 of this book. We have therefore described the parallels of the cities, and so on. Which was to be done.

PROBLEMA 32. PROPOS. 32.

To describe the celestial houses on the same meridian sundial.

[Margin: Description of the celestial houses according to Johannes Regiomontanus, on the same meridian sundial.]

According to the opinion of Johannes Regiomontanus [Regiomontanus], the celestial houses, dividing the equator into 12 equal parts, will be described as follows. Let the circle described from E be cut into 12 equal parts, beginning from the straight line GI, or FH; and let hidden straight lines, sent out through the division points and through the center E, cut the equinoctial line AC at points; the lines drawn parallel to the horizontal line AB through these points will be the lines of the celestial houses.

For when the circle described from E is turned about the equinoctial line until its center E is joined with the center of the world and the circle itself is set in the plane of the equator, the straight lines passing through the center E will be the common sections of the circles of the celestial houses — among whose number is also the meridian, passing through the straight line GI, and the horizon, running through FH — and of the equator. This is so because the common sections of these circles and of the equator, from what has been demonstrated by us for the sphere, divide the equator, and a circle concentric with it described from E, into 12 equal parts, and pass through the center of the world. The circles of the celestial houses will therefore meet the plane of the sundial at the points where the straight lines drawn through E cut the equinoctial line; and so through these same points of the equinoctial line will pass the common sections of those circles and of the plane of the sundial. Since, then, these common sections are parallel (by proposition 18 of the previous book), because the plane of the sundial is equidistant from the common section of those circles — namely, from the axis of the vertical circle — they will be straight lines parallel to the horizontal line [par-]

[Margin: Description of the celestial houses according to Johannes Regiomontanus, on the same meridian sundial. Demonstration of the description of the celestial houses according to Johannes Regiomontanus.]

parallel, and — drawn through the points of the equinoctial line — they are the common sections of the circles of the celestial houses and of the plane of the sundial. Which is what was proposed.

[Margin: Description of the celestial houses according to Campanus, on the same meridian sundial.]

Now, according to the opinion of Campanus, the celestial houses — which divide the vertical circle into 12 equal parts — will be laid out in this way. Through A, drop the perpendicular AD onto the horizontal line AB; this will be the common section of the vertical circle and of the plane of the sundial, as we demonstrated in proposition 28 of this book. Then, on the horizontal line, taking the straight line AB, with the style AE equal to it, describe from B, at any interval, a circle, and cut it into 12 equal parts, beginning from the horizontal line. Through the center B and the division points let hidden straight lines be drawn, cutting the line AD at points; the lines drawn through these points parallel to the horizontal line will be the lines of the celestial houses.

Diagram. A pair of circles set side by side — the celestial-house constructions for the eastern and the western meridian sundial. Both are ruled by a horizontal diameter and by a fan of straight house-lines carrying Roman house-numbers, with small "15" marks for the half-house (15°) subdivisions. Many peripheral tick-marks divide each circle's circumference.

Left circle:

- ♈ (?) (far left on the horizontal diameter): a zodiac glyph, apparently Aries
- B (on the horizontal diameter, left of center): center of the Campanus circle
- A (on the horizontal diameter, at center)
- VII (right end of the horizontal diameter)
- XII, VIII, XI, IX (on the radiating house-lines below the diameter)
- 15 (several, marking the mid-house divisions) [?]
- D (at the foot, center)
- C (at the lower left)

Right circle:

- G (upper left), F (upper right)

- E (upper center, at the crossing of two diagonals)
- I (left of center on the horizontal), A (center), VII, VIII (right of center)
- H (lower left)
- XII, XI, IX (on the radiating house-lines)
- I (right side)
- C (lower right)
- 15 (several, marking the mid-house divisions) [?]

[Margin: Demonstration of the description of the celestial houses according to Campanus.]

For when the circle from B, described about the line AD, is turned until it stands perpendicular to the plane of the sundial, and its center B is joined with the center of the world — so that AB becomes the same as the line of the style, and the circle itself is set in the plane of the vertical circle — then the straight lines passing through the center B will be the common sections of the circles of the celestial houses (among whose number is also the horizon, running along the line AB) and of the vertical circle. For the common sections of these circles with the vertical circle divide the vertical circle, and therefore — from what we have demonstrated for the sphere — also divide the circle concentric with it described from B into 12 equal parts, and they are drawn through the center of the world. The said circles will therefore meet the plane of the sundial at the points where the straight lines emitted from B cut the line AD; and thus through those same points of the line AD will pass the common sections of these circles and of the plane of the sundial. Since, then, these common sections are parallel — by proposition 18 of the previous book, because the plane of the sundial is equidistant from the common section of those circles, namely from the axis of the vertical circle — it is clearly established that the straight lines drawn through the points of the line AD parallel to the horizontal line are the common sections of the circles of the celestial houses and of the plane of the sundial. We have therefore described the celestial houses on the same meridian sundial. Which was to be done.

SCHOLIVM.

[Margin: Through which hours the celestial houses are drawn on the equinoctial line according to Johannes Regiomontanus [Regiomontanus].]

It is clear, moreover, that the lines of the celestial houses according to the former method are drawn through those points of the equinoctial line through which

pass the hour-lines that are distant two and four hours from the horizontal line — such as, in the eastern sundial, the hours 8 and 10 after midnight, and in the western, 2 and 4 after noon. And since each house comprises 30 degrees, if each twelfth part of the circle described from E or B is divided into 30 parts — or into fewer equal parts, according to the size of the sundial — and everything else is done as before, the lines of the other position-circles passing through the parts of the celestial houses will be described. In the example, the individual houses are each divided in two, so that between any two lines there fall 15 degrees, as the figures indicate.

[Margin: The lines of the celestial houses have the same position in both meridian sundials. Hence it is enough that only the eastern or the western sundial be described.]

For the rest, since the lines of the celestial houses are described in the same way in both the eastern and the western sundial — both according to Johannes Regiomontanus and according to the opinion of Campanus — we have accordingly drawn them, in the prior figure, on the eastern sundial. For the same lines, described on the western face and corresponding to those, will give the celestial houses on the western sundial, as the numbers indicate: of these, the ones attached to the tropic of Cancer ☊ belong to the eastern sundial, while the others, assigned to the tropic of Capricorn ☋, pertain to the western. In the latter figure, however, we have drawn the lines of the houses on the western sundial, because the same lines on the eastern face, corresponding to them, give the celestial houses on the eastern sundial. For here the numbers assigned to the tropic of Cancer ☊ refer to the western sundial, while the others, attached to the tropic of Capricorn ☋, look to the eastern.

PROBLEMA 33. PROPOS. 33.

To describe the ascending signs of the Zodiac on the same meridian sundial.

Let there be taken, from the two prior tables of proposition 9 of this book — with the Sun at the beginning of Aries ♈ and of Libra ♎ — the hours

[Margin: Description of the ascending signs on the same meridian sundial.]

at which the 12 signs of the Zodiac rise (in the eastern sundial before noon, in the western after noon). Let these hours be counted, on the circle described from E, from the line GI, either toward the east or toward the west — that is, either toward the forenoon side or toward the afternoon side — as the tables indicate. In the eastern sundial the counting must always be done toward the right, in the

western toward the left — that is, always toward F — as is clear if the circle described from E is considered in its proper position in each sundial.

[Margin: Finding of the points on the horizontal line of each meridian sundial, onto which the shadow of the style falls when the signs rise or set.]

Then, through the ends of the countings and through the center E, let hidden straight lines be drawn, cutting the equinoctial line (whether below the horizontal line or above it) at the points through which the lines of the ascending signs are to be drawn. Afterward, from proposition 26 of this book, mark on the horizontal line the points at which the arcs of the signs cut it. For onto these points the shadows of the gnomon will be cast — in the eastern sundial when the signs rise, and in the western sundial when they set.

[Margin: The reason why, on the same meridian sundial, the shadow of the style falls both when the signs rise and when they set.]

These points we shall also find by this method, even if the arcs of all the signs are not drawn on the sundial. From the fifth table of proposition 9 of this book we shall take the semidiurnal arcs of the signs, and reckon them in each sundial from G toward F; and from the ends of the reckonings, through the center E, we shall draw straight lines cutting the equinoctial line at points, through which points lines drawn perpendicular to the equinoctial line will cut the horizontal line at the points where the arcs of the signs divide it. For when the Sun is at the beginnings of the signs, its style's shadow is cast onto those points as it rises or sets — because the said perpendicular lines indicate the hours of the rising and setting of the Sun when it stands at the beginnings of the signs, as is established from what we demonstrated in proposition 25 of this book. Since, then, at the Sun's rising and setting the shadow of the style is cast onto the horizontal line, it is clear that the Sun rises and sets at those points where the said perpendicular lines cut the horizontal line.

[Margin: How the points on the horizontal line may be found otherwise than before.]

We shall find these same points in the following way. From the straight line EA of the last figure of proposition 21 of this book, subtract a straight line equal to the style of the meridian sundial, and through its extreme point draw a perpendicular to EA; this will be the horizontal line in the eastern sundial. If, then, from the extreme point of the line cut off from EA equal to the style, we take the intervals of all the points at which the said perpendicular is cut by the rays drawn through the center E from M, O, Q, S, V, Y, and transfer them onto the

horizontal line of the eastern sundial from A, in the place of the style, we shall obtain the points that are sought — as is clear if the horizon ABCD in the eastern sundial is conceived in its proper position, so that the extreme point of the straight line taken on EA lies at A, in the place of the style, and the perpendicular drawn through that point becomes the same as the horizontal line in the sundial, and accordingly E is set at the center of the world, as the apex of the gnomon. For thus it will come about that, when the Sun rises at the points M, O, Q, S, V, Y, the shadow of the style is cast onto the said points of the horizontal line in the eastern sundial, as is clear. In the same way, in the western sundial the same points will be found, if from EC of the same figure of proposition 21 of this book a straight line equal to the style is cut off, and through its extreme point a perpendicular to EC is drawn to serve as the horizontal line of the western sundial, and so on.

And so, if the points found on the equinoctial line are joined by straight lines with the points found on the horizontal line — those of the corresponding signs in the eastern sundial, but those of the opposite signs in the western — the ascending signs will be described. For when in the eastern sundial the shadow is cast onto some point of the horizontal line, the sign in which the Sun stands rises; but when the shadow in the western sundial falls onto some point of the horizontal line, the opposite sign rises, since the sign in which the Sun then stands is setting.

[Margin: How the points of the ascending signs are investigated in the tropics.]

We can also find, in the two tropics, the points — from the third and fourth tables of proposition 9 of this book — through which the ascending signs are to be drawn: namely, by secretly drawing the line of the hour at which each of the signs rises. For where this line cuts the tropic, through that point the line of that ascending sign must be drawn, as was done in the horizontal sundial. It is very useful, indeed almost necessary, to find in the tropics the points of those signs whose points found on the equinoctial line lie so close to the horizontal line that the line of the ascending sign can scarcely be drawn through those two neighboring points without some error, unless a third point found on the tropic is added. But if a point is to be found on the tropic above the horizontal line, we shall find it by this method even if the tropic is not drawn above the horizontal line.

Suppose, for example, that a point is to be found for the sign Virgo ♍. Since, with the Sun at the beginning of Virgo ♍, the sign Virgo rises at hour 9 min. 26 after noon — as is clear from the fourth table of proposition 9 of this book — we shall count in the western sundial, from G toward F, 9 hours 26 minutes; or (what is

the same) from point I, since the circle in the western sundial is not drawn complete, [count] toward F the complement of those hours up to 12, that is, 2 hours 34 minutes, as far as point O. Through O we shall draw a straight line passing secretly through the center E, which cuts the equinoctial line at the point L; through L, toward the arc of Capricorn ♑, we shall draw the perpendicular LK to the equinoctial line, which — if the tropic of Capricorn ♑ were drawn — would cut it at the point through which the ascending sign Virgo ♍ would have to be drawn. But because the tropic of Capricorn ♑ is not drawn beyond the horizontal line in the western sundial, we shall cut off from the equinoctial line, below the horizontal line, a straight line AM equal to AL, and from M we shall draw the perpendicular MN to the equinoctial line, which cuts the tropic of Capricorn ♑ at N; and from the line MN we shall cut off a portion equal to LK. The point K, therefore, will be that through which both the tropic of Capricorn ♑ and the ascending sign Virgo ♍ must be drawn, as is clear. The same method is to be observed in the rest.

Nor should it seem strange to you that we have taken some points, both on the equinoctial line and on the tropics, above the horizontal line — even though the part of the sundial that stands above the horizontal line is never illuminated by the Sun. For if the density of the earth did not stand in the way, the plane of the eastern sundial would always be lit by the Sun's rays from midnight until noon, for a full 12 hours, one part of which is contained below the horizontal line and the other above it. The same you will say of the plane of the western sundial, from noon to midnight. Hence at that time those hours would indicate which signs of the Zodiac touch the horizon.

[Margin: By what method the ascending signs may be described through the doctrine of the Conics of Apollonius [Apollonius].]

But if lines tangent to the tropics are drawn through the points found on the tropics — which will be drawn as we taught in the corollary of proposition 37 of book 1 of Apollonius — the ascending signs will again be described, as is established from what was demonstrated in proposition 9 of this book.

[Margin: How the ascending signs of Aries and Libra are described.]

Furthermore, we shall describe the lines of Aries ♈ and of Libra ♎ — which necessarily pass through the point A, where the horizontal line, the equinoctial line, and the line of hour 6 from noon or midnight mutually intersect — in the following way. (They pass through A because, when the Sun is at the beginning of Aries ♈, the beginning of Aries rises above the horizon at hour 6 before noon,

and the beginning of Libra ♎ at hour 6 after noon; whereas when the Sun is at the beginning of Libra ♎, the beginning of Aries rises at hour 6 after noon, and the beginning of Libra at hour 6 before noon, as is clear.) For the ecliptic cuts the equator at the beginning of

Aries ♈ and Libra ♎, and makes with it two angles of 23 degrees 30 minutes — which is indeed the greatest declination of the Sun — if to both sides of the equinoctial line at the point A we set two angles of 23 degrees 30 minutes. (This will be done if from A a circumference of a circle is described, from which two arcs of 23 degrees 30 minutes are taken on either side from the equinoctial line, and from A straight lines are drawn through the ends of the arcs.) The lines of Aries ♈ and Libra ♎ will thus be described; of these, the one that inclines toward the tropic of Cancer ♋ will give the beginning of Libra ♎, and the other the beginning of Aries ♈. This is because in the celestial sphere, when the beginning of Libra ♎ is placed in the east, the ecliptic deflects from the equator northward, toward the parallel of Cancer ♋; but southward, toward the tropic of Capricorn ♏, when the beginning of Aries ♈ is set in the east — as is evident from the material sphere.

Hence it comes about that, when the beginning of Libra ♎ is set in the east, all the points of the northern semicircle of the ecliptic that stand above the horizon — in some one of which the Sun must be, since the beginning of Libra rises in the daytime — cast their shadows into the opposite, namely southern, parts, toward which the tropic of Capricorn ♏ also inclines. But when the beginning of Aries ♈ is set in the east, all the points of the southern semicircle of the ecliptic that stand above the horizon — in some one of which the Sun necessarily is, since the beginning of Aries rises in the daytime — cast their shadows into the opposite, namely northern, parts, toward which the tropic of Cancer ♋ declines. [The two tropic-signs here read against their glyphs on the scan: southern shadows are paired with Cancer's sort and northern shadows with Capricorn's sort; I have set each to the tropic its own hemisphere requires (southern → Capricorn, northern → Cancer) and flagged both — the zodiac sorts here are worn.] That the lines of Aries ♈ and Libra ♎ are correctly described in this way will become clear as follows.

[Margin: Another description of the ascending signs on the meridian sundial, through the points of the ecliptic lying on the circles of hour 1 from noon and hour 11 from midnight, when the beginnings of the signs ascend.]

Because, when the beginning of Aries ♈ or of Libra ♎ is placed at the rising point, the style of the sundial is the common section of the plane of the ecliptic and of

the equator, each circle will meet the plane of the sundial at the point A. [Margin: (a) — from proposition 16, whence:] But because the common sections that the ecliptic and the equator make on the plane of the sundial are parallel to the common sections they make on the meridian circle, [Margin: (b) — from proposition 10, whence:] the angle contained in the plane of the sundial between the equinoctial line and the line of Aries ♈ or of Libra ♎ will be equal to the angle enclosed by the same lines on the plane of the meridian circle. Since, then, on the plane of the meridian circle the said angle comprises 23 degrees 30 minutes, the proposition is established.

Diagram. A large figure combining the eastern and western meridian sundials for the ascending signs. Two large circles overlap near the center; a horizontal line and the diagonal equator-line (labelled ÆQVATOR) run across them, and a dense fan of straight lines radiates from the crossing points — these are the hour-lines and the sign-lines, their outer ends marked with the twelve zodiac glyphs (♈ ♉ ♊ ♋ ♌ ♍ ♎ ♏ ♐ ♑ ♒ ♓). Four of the lines carry engraved captions naming their hours.

Labelled points and lines I can read:

- F (top left of the left circle)
- G (at the top, near where the two circles meet)
- P (top right of the right circle)
- O (far left, on the horizontal line)
- E (center of each circle — two instances)
- H (two instances: lower left, and center-bottom)
- Near the left circle's center, a cluster of small labels: K, I, N, M (and an L in the same cluster) [?]
- ÆQVATOR / ÆQVA= — labelling the equator (equinoctial) line
- Line captions:
- "Hora 1^a (prima) antemeridiana, quæ est post mediam noctem." ["1st hour before noon, which is after midnight"] — upper right
- "Hora 1^a (prima) pomeridiana." ["1st hour of the afternoon"] — lower left
- "Hora ij^a (secunda) antemeridiana, quæ est... post mediam noctem." ["2nd hour before noon, which is after midnight"] — lower center [Stage-5 corrected from "Hora 12 [?]" ; the reported 4th "illegible" caption does not exist — there are exactly three]

Peripheral zodiac glyphs mark the outer ends of the sign-lines around both circles; several are worn and cannot be individually resolved [?].

ALITER. The ascending signs will be laid out otherwise, in this manner. From the eighth table of proposition 9 of this book, take the declinations of all the points on the circle of hour 11 before noon — which hour is the eleventh after midnight — and, according to the doctrine of proposition 26 of this book, find on the line of hour 11 in the eastern sundial the points through which the parallels of those declinations are to be drawn. For these points, joined with the corresponding points found on the horizontal line, will give the ascending signs. Or indeed, from the ninth table of the same proposition 9 of this book, take the declinations of the points on the circle of the first hour before noon — which is the first hour after midnight — and, according to the doctrine of proposition 26 of this book, find the points on the line of hour 1 from midnight, that is, the first before noon (which is drawn above the horizontal line, distant from it by just as much as the line of hour 11 is distant below it), through which the parallels of those declinations are to be drawn. For these, joined with the corresponding points found at hour 11, will give the ascending signs, as before, which will necessarily pass through the points of the horizontal line.

Again, from the ninth table of the same proposition 9 of this book, take the declinations of the points on the circle of hour 1 after noon, and, according to the doctrine of proposition 26 of this book, investigate on the line of hour 1 in the western sundial the points through which the parallels of those declinations are to be drawn. For these points, [joined] by straight

joined by straight lines to the corresponding points of the horizontal line, will give the ascending signs.

Or else, from the eighth table of proposition 9 of this book, take the declinations of the points on the circle of hour 11 after noon, and, by the doctrine of proposition 26 of this book, find the points on the line of hour 11 after noon — which line is described above the horizontal line, at just the distance from it that the line of hour 1 from noon is distant from the same line — through which the parallels of those declinations are to be drawn. For these points, joined by straight lines to the corresponding points found at hour 1 from noon, will give the ascending signs, as before; and they will necessarily pass through the points found on the horizontal line.

[Margin: Another description of the ascending signs, from the doctrine of the conics of Apollonius.]

And if from the seventh table of proposition 9 of this book one finds on the line of hour 6 the points through which the parallels of the declinations of the points of

the ecliptic existing at hour 6 pass, and if through these one draws — by the doctrine of the corollary of proposition 37 of Book 1 of Apollonius — the straight lines tangent to the tropics, the signs will be obtained as before, as is clear from what we demonstrated in proposition 9 of this book. We have therefore described the ascending signs of the zodiac on the same meridian dial. Which was to be done.

SCHOLIUM.

[Margin: Yet another description of the ascending signs, according to the practice of Andreas Schöner.]

By the doctrine of Andreas Schöner [Andreas Schöner] the ascending signs may be described as follows. In the first figure of the scholium of proposition 9 of this book, cut off from the straight line EC the straight line E a, equal to the gnomon of the dial, and through a draw a line parallel to BD — so that, with the center E placed at the center E of the dial and the point a at the point A, the parallel drawn through a coincides with the equinoctial line. The parallel drawn through a will then cut the lines of the signs issuing from E in points; and if the distances of these points from a are taken with the compass and transferred onto the equinoctial line of the dial, beginning from A, both above the horizontal line and below it, then through the points marked on the equinoctial line let there be drawn hidden straight lines parallel to the horizontal line itself, cutting both tropics — including the one described above the horizontal line — in the points through which the ascending signs are to be laid out; observing, however, this order.

[Margin: The order of the ascending signs on the tropics.]

In the eastern dial, the point of the tropic of Cancer [♋], where it cuts the horizontal line, belongs to Cancer [♋]; the point next below it, to Leo [♌]; the one following, to Virgo [♍]; while the nearest point above the horizontal line looks to Gemini [♊], and the one following, to Taurus [♉]. Likewise the point of the tropic of Capricorn [♏], where it cuts the horizontal line, looks to Capricorn [♏]; the point next below, to Aquarius [♑]; the one following, to Pisces [♓]; while the nearest point above the horizontal line belongs to Sagittarius [♐], and the one following, to Scorpio [♏ — glyph worn; pinned by the zodiacal sequence] [?]. But in the western dial, the point of the tropic of Cancer, where it intersects the horizontal line, belongs to Capricorn [♏]; the point next below, to Sagittarius [♐]; the one following, to Scorpio [♏] [?]; while the nearest point above the horizontal line looks to Aquarius [♑], and the one following, to Pisces [♓]. Likewise the point

of the tropic of Capricorn, where it intersects the horizontal line, looks to Cancer [♋]; the point next below, to Gemini [♊]; the one following, to Taurus [♉]; while the nearest point above the horizontal line belongs to Leo [♌], and the one following, to Virgo [♍].

[Margin: Another description of the same, by the equinoctial line.]

Again, in the second figure of the scholium of proposition 9 of this book, cut off from the straight line EC the segment E a, equal to the gnomon, and through a draw a line parallel to BD — so that once more, with the center E placed at E, the center of the dial, and the point a at the point A, the parallel drawn through a coincides with the equinoctial line. This parallel drawn through a will cut the lines of the signs issuing from E — including those signs which lie above the line BD, provided their line be produced below the center E — in points; and if the distances of these points from a are taken with the compass and transferred onto the equinoctial line of the dial, beginning from A, both above the horizontal line and below it, there will be obtained on the equinoctial line the points through which the lines of the ascending signs will pass, in this order. In the eastern dial the points of the equinoctial line marked below the horizontal line are owed to the signs that lie to the left of the straight line EC — counting also the signs above the line BD which lie below the center E and are produced beyond the center E, such as Taurus [♉], Gemini [♊], Cancer [♋], [?], and Scorpio [♏] [?]. The points of the equinoctial line above the horizontal line, however, are to be assigned to those signs found to the right of the straight line EC, of which sort are Pisces [♐], [?], [?], Leo [♌], and Virgo [♍]. But in the western dial, understand the whole matter reversed. For to the signs that lie to the left of the straight line EC correspond the points of the equinoctial line above the horizontal line; whereas to the signs that lie to the right of the same straight line EC belong the points of the equinoctial line below the horizontal line. When all this is duly carried out, if the points of the corresponding signs found on the equinoctial line and on the tropics are joined by straight lines, the ascending signs will be described. The lines of Aries [♈] and Libra [♎], however, we shall describe as before.

Now the lines of the ascending signs of either dial, produced beyond the horizontal line, exhibit the lines of those same signs on the opposite face of the plane of the dial, provided that the upper part cut off by the horizontal line is inverted, as we demonstrated in the scholium of proposition 14 of this book. It will therefore be enough to describe the ascending signs on the eastern dial, or on the western; for all its lineaments produced beyond the horizontal line will give the same ascending signs on the opposite dial, if that upper part be inverted, and

so on. In the figure we have here, for the most part, described the ascending signs from the eighth and ninth tables of proposition 9 of this book — namely, by the points found on the lines of hour 11 from midnight and hour 1 from midnight, and likewise hour 1 from noon and hour 11 from noon.

PROBLEMA 34. PROPOS. 34.

HOROLOGIVM Italicum Meridianum constituere. [To construct an Italian meridian sundial.]

[Margin: The description of the Italian meridian dial.]

Let the circle FGHI be cut into the diurnal and nocturnal arc of the tropic of Cancer [♋], as we taught in the scholium of proposition 1 of the preceding book, and let the diurnal arc be BGC and the nocturnal CIB; these arcs will also be found, as we wrote in proposition 10 of this book, if from the point G of noon there be reckoned on either side the semidiurnal arc of the tropic of Cancer, as far as B and C — namely 7 hours 32 minutes, where the pole has an altitude of 42 degrees, as it is at Rome. Next let the circle FGHI be divided into 24 equal hours, taking the beginning from the western terminus of the diurnal arc — which in the eastern dial is the point C, and in the western the point B, as is evident if the circle FGHI is understood to be moved to its proper position about the equinoctial line, until it is set in the plane of the Equator. Then, in the eastern dial from the hours between the points B and G, of sunrise and of noon, and in the western between the points B and G, of sunset and of noon, let there be drawn through the center of the circle FGHI straight lines which, as we showed in proposition 10 of this book, are the common sections of the Equator and of the great circles which are drawn through the poles of the world and parallel to the Italian hours of Cancer [♋], in the manner of hour-circles reckoned from noon or from midnight. These lines will cut the equinoctial line in points; and through these points, perpendicular hidden lines drawn to the equinoctial line will cut the tropic of Cancer [♋] in the Italian hours, or the hours from sunset — since these perpendiculars are the hour-lines from sunset, in the manner of the hour-lines from noon or from midnight, the Sun being at the beginning of Cancer [♋], as is clear from what we demonstrated in proposition 25 of this book. In the same way, if to the arcs IB, IC there be taken equal arcs GD, GE, so that DGE is the diurnal arc of the tropic of Capricorn [♋],

and again the circle FGHI be cut into 24 equal parts, beginning from the western terminus of the diurnal arc of the tropic of Capricorn [♋] — so that from the point E in the eastern dial, and from D in the western, we shall find the Italian

hours on the tropic of Capricorn [♊]. And so, if the mutually corresponding hours on the tropics are joined by straight lines, the Italian dial will be described. These hour-lines will necessarily pass through the points of the hours from noon, or from midnight, on the equinoctial line — as the 9th Italian hour through the 3rd from midnight, the 10th through the 4th from midnight, and the 20th through the 2nd from noon, and so on, as is clear from the table of proposition 19 of the preceding book, whose title is *ÆQVINOCTIALIS LINEA* [the Equinoctial Line]. And if some hours from sunset do not, in either of the tropics, have corresponding points on the other tropic, they must be drawn through the corresponding hours on the equinoctial line, as is gathered from the said table of proposition 19 of the preceding book. Thus in the eastern dial hour 16 and hour 17 are drawn through the points found on the tropic of Capricorn [♊], and through hour 10 and hour 11 from midnight on the equinoctial line, because on the tropic of Cancer [♋] they have no corresponding points. The line of hour 18, moreover, must be drawn through the

Diagram. A large combined construction figure spanning the width of the page, showing the eastern and western Italian meridian sundials side by side. Two large circles (each the circle FGHI, one for the eastern dial at left, one for the western at right) overlap near the center; a horizontal line runs across both through the points K...K, and a dense fan of straight and dotted lines radiates from the crossing points, forming the hour-lines and sign-lines. The circumference and interior carry numeric hour-labels 1–24.

Labelled points and lines I can read:

- Left circle: M (upper left), D, L, F, B ranged down the upper-left; K on the horizontal line at left; G near the upper center; H and C toward the lower center; E near the center; I on the lower left.
- Right circle (mirror image): L, D, M (upper right), F, B ranged down the upper-right; K on the horizontal line at right; G near the upper center; H and C toward the lower center; E near the center; I on the lower right.
- "12" appears near the top of each circle; "6" near the upper center between the circles.
- Numeric hour-labels around the lower portions: on the left circle 24, 23, 22, 21, 20, 18, 16, 10, 8, 7, 5, 4, 3, and "12" at the very bottom of a long line descending below the circle; on the right circle 24, 18, 17, 16, 6, 5, 4, 3, and "12" at the foot of a descending line. Several of the interior numerals are faint or crossed by other lines and cannot be individually resolved [?].

- Dotted lines mark the nocturnal-arc constructions descending below each circle to the "12" termini.

hour...

point found on the tropic of Capricorn [γ], parallel to the equinoctial line, as is evident. For since the circles of hour 6 and hour 18 from sunrise, or from sunset, have with the Equator one and the same common section (by proposition 16 of the preceding book), and likewise one and the same with the Meridian (by proposition 17 of the same preceding book), and since those two circles have but one common section, it follows that those two circles, together with the Equator and the Meridian, have one and the same common section — namely that which is also the common section of the Meridian and the Equator. Since, then, the plane of the meridian dial is equidistant from this common section — that is, from the Meridian circle itself — the sections of the remaining circles with the plane of the dial (such as the line of hour 6, the line of hour 18 from sunrise or from sunset, and the equinoctial line) will, by proposition 18 of the preceding book, be parallel to one another.

[Margin: Another description of the same Italian meridian dial, by the diurnal and nocturnal arcs of the hours of 14 and 10; likewise of 24.]

By the two diurnal and nocturnal arcs of the hours of 14 and 10, the same Italian dial will be described, as we explained in the horizontal and vertical dial. The same will be done by means of two parallels, one of which is always the greatest of the ever-apparent — that is, the diurnal arc of 24 hours — while the other is the greatest of the ever-hidden — that is, the nocturnal arc of 24 hours — if their hyperbolas be described on the dial. For the hour-lines from sunset or from sunrise touch the said hyperbolas at those points where the same are cut by the hour-lines from noon or from midnight, by proposition 14 of the preceding book — excepting the lines of hours 12 and 24, for these do not meet the said hyperbolas, as is clear from the corollary of proposition 15 of the same preceding book.

Furthermore, for the eastern Italian dial there are to be taken, out of the diurnal and nocturnal arcs, only those hours from sunset which have corresponding hours from midnight, since the hours from noon are not contained in it. For the western dial, however, there are to be taken those hours to which the hours from noon correspond, because these alone are found described in it. Let the same be observed in the diurnal and nocturnal arc of 24 hours. Thus we shall describe these hours 9, 10, 11, 12, 13, 14 from sunset on the eastern dial by the hours 2, 3,

4, 5, 6, 7 from midnight in the nocturnal arc of 14 hours. Likewise these hours 9, 10, 11, 12 from sunset by the hours 3, 4, 5, 6 from midnight in the nocturnal arc of 12 hours. But these 13, 14, 15, 16, 17 from sunset by 7, 8, 9, 10, 11 from midnight in the diurnal arc of 12 hours. Moreover hours 9, 10 from sunset by the hours 4, 5 from midnight in the nocturnal arc of 10 hours. And these 10, 11, 12, 13, 14, 15, 16 from sunset by 5, 6, 7,

8, 9, 10, 11 from midnight in the diurnal arc of 14 hours. And these 14, 15, 16, 17, 18 from sunset by the hours 7, 8, 9, 10, 11 from midnight in the diurnal arc of 10 hours. By the same reasoning, in the western dial, from the said tables we shall find the hours from noon by which the hours from sunset are to be drawn. But since hour 18 from sunset has only a single point in each dial — namely in the eastern hour 11 from midnight in the diurnal arc of 10 hours, and in the western hour 1 from noon in the diurnal arc of 14 hours — we shall draw it in the eastern dial through hour 11 from midnight in the diurnal arc of 10 hours, and through hour 6 from midnight in the nocturnal arc of 24 hours; but in the western dial through hour 1 from noon in the diurnal arc of 14 hours, and through hour 6 from noon in the diurnal arc of 24 hours. In each dial, however, the line of hour 18 from sunset is parallel to the equinoctial line, as is clear from the scholium of proposition 22 of the preceding book.

[Margin: Another description of the Italian meridian dial by the line of hour 12 from sunrise and sunset, and by the line of hour 24 from sunrise or sunset.]

Finally, the same Italian dial will be completed by means of the line of hour 12 from sunrise and from sunset, as is evident from the second table of proposition 20 of the preceding book, whose title is LINEA HORAE XII. AB ORTV, VEL OCCASV [the Line of Hour 12 from Sunrise or Sunset]. Likewise by means of the line of hour 24 from sunrise or from sunset (which is the same as the horizontal line), as is gathered from the first table of the same proposition, whose title is LINEA HORAE XXIII. AB ORTV, VEL OCCASV [the Line of Hour 24 from Sunrise or Sunset]. Indeed it will be described more accurately by means of the line of hour 12 from sunrise or sunset, and the line of hour 24 from sunrise or sunset, together with the equinoctial line — because in the eastern dial the hour-lines from sunset which are but little distant from the line of hour 12 from sunrise or sunset barely intersect the line of hour 12 itself (the same being seen in the hour-lines from sunset which are but little distant from the horizontal line in the western dial, if they be compared with the horizontal line, for it is scarcely cut by them). Hence in describing the former we shall use the horizontal line and the

equinoctial line; but in the latter, the line of hour 12 from sunrise or sunset and the equinoctial line.

[Margin: The description of the line of hour 12 from sunrise or sunset on the meridian dial.]

Furthermore, we shall trace the line of hour 12 from sunrise or sunset by this method. From the point A we shall describe a circle KL of any magnitude whatever, which cuts the line of hour 6 from noon, or from midnight, produced, in M; and the arc KM (if no error has been made in the description of the line of hour 6) will be the altitude of the pole above the Horizon, on account of the angle KAM — because the angle contained by the line of hour 6 and the equinoctial line is right, by construction, and the remaining angle out of this right angle, which the equinoctial line makes below the horizontal with the horizontal line at A, is the angle of the altitude of the Equator above the Horizon, by construction, as was said in proposition 25 of this book. Then to the arc KM we shall take an equal arc ML, and through L and A we shall produce the straight line LA. This, I say, is the line of hour 12 from sunrise or sunset.

[Margin: a — 16, hence.] For since the common sections of the circle of hour 12 from sunrise or sunset, made in parallel planes — namely in the plane of the Meridian and in the plane of the dial — are parallel; and likewise the common sections of the Horizon made in the same planes; [Margin: b — 10, hence.] the angle contained beneath those sections in the plane of the Meridian will be equal to the angle contained beneath the same sections in the plane of the dial. Since, then, AK is the common section of the Horizon and of the plane of the dial, and the angle KAL is equal to that which the common sections made by the circle of hour 12 from sunrise or sunset and by the Horizon constitute in the plane of the Meridian — because the arc KL is similar to the arc of the Meridian intercepted between the Horizon and the circle of hour 12, as is evident from the figure of proposition 9 of the preceding book, since the diameter of the parallel of the ever-apparent, the greatest of all, is bounded by the points K, L — the straight line LA will be the common section of the circle of hour 12 from sunrise or sunset and of the plane of the dial; which was proposed. Moreover, the equinoctial line will always bisect the angle contained beneath the line of hour 12 from sunrise or sunset and the horizontal line, in each dial. For since the angle MAL is equal to the altitude of the pole above the Horizon, the remaining angle out of the right angle, contained beneath the line LA and the equinoctial line, will be equal to the altitude of the Equator above the Horizon. Wherefore, since the acute angle contained beneath the horizontal line and the equinoctial line is also equal, by

construction, to the same altitude of the Equator, the angles which the equinoctial line makes with the line of hour 12 and with the horizontal line will be equal to one another. Hence the line of hour 12 will also be easily drawn, if at the equinoctial line there be constituted an angle equal to the altitude of the Equator, toward the obtuse angle which the equinoctial line makes with the horizontal line.

[Margin: When the line of hour 12 from sunrise or sunset is right, and when acute or obtuse.]

It is also manifest that the line of hour 12 from sunrise or sunset makes with the horizontal line a right angle KAL when the altitude of the pole KM contains 45 degrees, because then the whole arc KL is a quadrant; an acute angle when the altitude of the pole is less than 45 degrees, as in our example, since then the whole arc KL is less than a quadrant; and finally an obtuse angle when the altitude of the pole exceeds 45 degrees, because then the whole arc KL is greater than a quadrant. This is clearly gathered from the description of the line of hour 12 from sunrise or sunset here given.

For the rest, neither the horizontal line nor the line of hour 12 from sunrise or sunset can here be described in the manner which we prescribed for the vertical dial in proposition 22 of this book — namely by the arcs of the parallels of the ever-apparent and of the greatest of the ever-hidden cutting the meridian line — because in this dial the meridian line cannot be drawn, as we showed in proposition 25 of this book; and the aforesaid lines, namely the horizontal line and the line of hour 12 from sunrise or sunset, neither cut nor touch the arcs of the said parallels, but never meet them, as was demonstrated by us in proposition 15 of the preceding book. And so we have constructed the Italian meridian dial. Which was to be done.

PROBLEMA 35. PROPOS. 35.

HOROLOGIVM Babylonicum Meridianum constituere. [To construct a Babylonian meridian sundial.]

[Margin: The description of the Babylonian meridian dial.]

We shall describe the Babylonian dial by nearly the same methods by which we constructed the Italian — provided that, in the first method, with the circle FGHI divided into 24 equal hours, the beginning is taken from the eastern terminus of the diurnal arc, both of the tropic of Cancer [♋] and of the tropic of Capricorn [♐] — that is, both from the point B and from the point D in the eastern dial, and in

the western both from the point C and from the point E. The line of hour 6 from sunrise, moreover, must be drawn parallel to the equinoctial line, as we showed in the preceding proposition: in the eastern dial through the point of the said hour found on the tropic of Cancer [☿], and in the western through the same point found on the tropic of Capricorn [♊].

[Margin: Another description of the Babylonian meridian dial by the diurnal and nocturnal arcs.]

In the second method, which is accomplished by the diurnal and nocturnal arcs, the same is to be observed here as we observed in the Italian dial: that is, for the eastern Babylonian dial there are to be taken, out of the diurnal and nocturnal arcs, those hours from sunrise to which the hours from midnight correspond; while for the western, those to which the hours
the hours from noon [correspond].

Now the line of hour 6 from sunrise — which is also parallel to the equinoctial line in both dials, as was shown in the scholium of proposition 22 of the preceding book — must likewise be drawn: in the eastern dial through hour 6 from midnight in the diurnal arc of 24 hours, and in the western through hour 6 from noon in the nocturnal arc of 24 hours.

[Margin: Another description of the Babylonian meridian dial by the lines of hours 12 and 24 from sunrise or sunset.]

The third method, drawn from the table LINE OF HOUR XII FROM SUNRISE OR SUNSET and from the table LINE OF HOUR XXIII FROM SUNRISE OR SUNSET, presents no difficulty, provided only this is noted: in the western dial the hours from sunrise are more easily drawn by the horizontal line (that is, the line of hour 24) and by the equinoctial line than by the line of hour 12 from sunrise or sunset, because that line scarcely, or only with difficulty, cuts the line of hour 12 from sunrise or sunset in the western dial. In the eastern dial, by contrast, these same hours are more easily drawn by the line of hour 12 from sunrise or sunset together with the equinoctial line, because they scarcely, or only with difficulty, intersect the horizontal line, as is clear from the figure.

[Margin: Which line in the figure indicates the hours from noon or midnight, and which the hours from sunset or from sunrise.]

An example of the Babylonian dial is set out in the figure of the preceding proposition, for it contains the hours from noon and midnight as well as those from sunrise and sunset. The lines that are parallel to one another and

perpendicular to the equinoctial line — beside which, along the equinoctial line, the hour-numbers are placed — pertain to the hours from noon and midnight. The other lines, which in the eastern dial cut the horizontal line and in the western the line of hour 12 from sunrise or sunset, show the hours from sunset. Finally, the lines which in the eastern dial cut the line of hour 12 from sunrise or sunset, and in the western the horizontal line, signify the hours from sunrise. And so we have described the Babylonian meridian dial. Which was to be done.

SCHOLIVM.

[Margin: The Italian eastern dial, drawn on the opposite face, gives the Babylonian western; and the Italian western dial drawn on the opposite face exhibits the Babylonian eastern, and vice versa — provided that the hour-numbers are changed into their complements up to 24.]

From the figure of the preceding proposition it is clear that the hour-lines from sunset in the eastern dial have exactly the same position that the hour-lines from sunrise keep in the western dial. The same holds of the hours from sunrise in the eastern dial and of the hours from sunset in the western. Hence the Italian eastern dial, drawn on the opposite face of the dial's plane so that line answers to line, will be the Babylonian western; and likewise the Italian western will be the Babylonian eastern, and conversely — provided only that each hour-number is changed into its complement up to 24: thus 9 into 15, and 3 into 21, and so on.

PROBLEMA 36. PROPOS. 36.

HOROLOGIVM Antiquum Meridianum constituere. [To construct an ancient meridian dial.]

[Margin: Description of the ancient meridian dial.]

Let the diurnal arc both of the tropic of Cancer [♋], BGC, and of the tropic of Capricorn [♐], DGE, in the circle FGHI, be divided into 12 equal parts. For by their aid we shall find, in each tropic, the points through which the unequal hours are to be drawn — in the same manner in which, in the two preceding propositions, we investigated the points of the hours from sunrise and sunset in these same tropics.

Diagram. Two large circles set side by side, representing the eastern and western ancient (unequal-hour) meridian dials, drawn for Problema 36. Each circle carries a network of straight and dotted construction lines radiating from a dense cluster of small numerals near its center, together with the two divided tropic

arcs and the hour-lines. Because the interior of each circle is crowded with small hour-numerals that do not resolve cleanly even under magnification, the numeric labels near the centers are given as read where legible and flagged elsewhere.

Left (eastern) circle:

- D (top): circle point
- F (upper left, on circumference): circle point
- G (near top, short dashed segment): circle point
- B (left): circle point
- A (center): stylus foot / center of the hour-line pencil
- E (center-right, where the two circles meet): common point
- C (bottom): circle point
- H (lower right): circle point
- I (lower left): circle point
- Peripheral hour-numerals read: 10 (right), 8 and 9 (upper middle), 8 and 2 (lower middle), 7 (far lower left, outside the circle)
- interior numeral cluster around A: [numeral: ?]

Right (western) circle:

- D (top): circle point
- G (upper left, dashed): circle point
- F (upper right, on circumference): circle point
- B (right): circle point
- E (center-left, where the two circles meet): common point
- C (bottom): circle point
- H (lower left): circle point
- I (lower right): circle point
- Peripheral hour-numerals read: 2 (upper left), 3 and 4 (center), 13 (center), 10 (lower middle), 11 (bottom), S (lower right, outside the circle)
- interior numeral cluster: [numeral: ?]

The unequal hours, moreover, will pass through the hours from noon or midnight on the equinoctial line, as we taught in the horizontal and vertical dials, and as is clear from the tables of proposition 33 of the preceding book. If again two diurnal arcs are drawn, one comprising 18 hours and the other 6, we shall describe the same dial — just as was done in the horizontal and vertical dials —

from the tables of the scholium of proposition 33 of the preceding book, which are devoted to the unequal hours.

[Margin: Another description of the ancient meridian dial by the diurnal arcs of hours 6 and 18.]

An example of each description appears in the figure set before us. We have therefore constructed the ancient meridian dial. Which was to be done.

SCHOLIVM.

From the figure and description of each dial it appears that the lines of the unequal hours have the same position and situation in both dials. Therefore, once one dial has been drawn, if on the opposite face the same lines are laid down, corresponding in every way to the earlier ones, the other dial too will be obtained. The hour-numbers, however, must be changed into their complements up to 12: thus 1 into 11, and 2 into 10, and so on.

[Margin: The ancient eastern dial, on the opposite face of the plane, gives the ancient western, and conversely.]

DE HOROLOGIIS POLARIBVS. [ON POLAR DIALS.]

PROBLEMA 37. PROPOS. 37.

HOROLOGIVM Astronomicum polare delineare. [To delineate an astronomical polar dial.]

That is: to describe the lines of the hours from noon or midnight on a plane that is parallel to the circle of the sixth hour from noon or midnight — the circle which is drawn through the poles of the world and stands perpendicular to the meridian.

[Margin: How the astronomical polar dial differs from the meridian dial.]

This dial does not differ from the meridian dial except in the number of the hours and their order. For here, on each side of the straight line DE, all the hours are to be taken that can fall upon the plane of the dial; and the straight line DE no longer shows hour 6 from noon or midnight, as it did there, but hour 12. The lines that recede from it toward the east indicate the hours from noon, while those

placed toward the west show the hours from midnight, as appears in the opposite dial.

For suppose the dial is so placed that the straight line CS is parallel to the horizon, the straight line DE lies in the plane of the meridian circle, the point E inclines toward the south and D toward the north, and finally the point D is raised on the northern side by the altitude of the pole — that is, so that the straight line DE, together with the meridian line in a plane parallel to the horizon, makes on the side of point D an angle equal to the altitude of the pole. Then the stylus AE, standing at right angles to the plane of the dial at A (so as to be the common section of the equator and meridian, or at any rate the straight line OP, as we taught in the scholium of proposition 25 of this book), will indicate the hours from noon and midnight — such that the lines more toward the east than DE, of the sort that run toward the point S, give the hours from noon, while those more toward the west, such as the lines running toward the point C, pertain to the hours from midnight, as appears in the figure.

Diagram. The astronomical polar dial of Problema 37, drawn in the manner of an analemma. A horizontal straight line CS (the equinoctial line, parallel to the horizon) runs across the figure; a vertical straight line DE (the meridian line, the line of hour 12) crosses it, with D toward the top (north) and E toward the bottom (south). At the foot lies a circle (described from E and divided into 24 equal parts) carrying the numeral 12 within it; from its divisions the dashed hour-lines and sign-arcs radiate outward to left and right, ending at the symbols of the zodiacal signs ranged down both margins of the figure. Two arcs cross in an X above the center.

- C (left end of the horizontal line): equinoctial-line terminus toward the west
- S (right end of the horizontal line): equinoctial-line terminus toward the east
- D (upper center): meridian-line terminus toward the north
- E (lower center): stylus vertex and center of the divided circle
- 12: numeral within the lower circle
- interior points near the center (labels resembling N, O, P, S): not legibly resolved — [?]
- zodiacal sign-symbols down the left and right margins: mostly worn / not legibly resolved — [?]

Therefore, if an astronomical polar dial is to be drawn, two straight lines CS and DE must be laid down on some plane, cutting one another at right angles at A; and on the straight line DE the segment AE, equal to the proposed stylus, must be taken. Then, describing from E a circle of any size and dividing it into 24 equal parts, the rest is to be completed as in the meridian dial, and so on.

That the meridian dial becomes polar if that exchange of the hour-numbers is made, as we said, can easily be understood in this way. Both the circle of hour 6 from noon or midnight — to which the plane of the polar dial is parallel — and the meridian — to which the meridian dial is parallel — are drawn through the poles of the world. It therefore comes about that the circles of the hours from noon or midnight cut the plane of each dial in the same way, since they stand in the same relation to each, only the order and number of the hours being changed.

For as the circle of hour 6 stands to the meridian — cutting it, namely, at right angles — so in turn the meridian stands to the circle of hour 6; and therefore the line which in the meridian dial gives hour 6 will give hour 12 in the polar dial, and so on. But we shall also demonstrate geometrically the construction of the polar dial which we have just now set out. Since the planes of the meridian and of the circle of hour 6 are perpendicular to the plane of the equator (by proposition 15 of Book 1 of Theodosius [Theodosius, *Sphaerica* I.15]), inasmuch as they pass through its poles, [a] their common section will be perpendicular to that same plane of the equator. [b] Since, then, the section which the meridian makes in the plane of the polar dial — namely the meridian line — is parallel to this common section, [c] the meridian line of the dial will also be perpendicular to the plane of the equator; and therefore, by definition 3 of Book 11 of Euclid, it will also be perpendicular to the common section of the plane of the dial and of the equator — namely to the equinoctial line of the dial, which lies in the plane of the equator.

[Margin: Demonstration of the construction of the astronomical polar dial. a: Euclid XI.19; b: Euclid XI.16; c: Euclid XI.8.]

Wherefore, if the straight line CS is set as the equinoctial line, the straight line DE will be the meridian line in the polar dial, since DE is by construction perpendicular to CS. Therefore, if the plane of the circle described from E is imagined to turn about the stationary line CS until it coincides with the plane of the equator, and E, the vertex of the assumed stylus AE, is joined to the center of the world, then that circle will be concentric with the equator; and the straight lines drawn out through E and the points of the circle's divisions will be the common sections of the hour-circles from noon or midnight with the equator. For

the equator and the circle described from E, being described about the same center E, are cut into similar arcs by the said straight lines, as we demonstrated at the end of chapter 1 on the sphere — that is, the diameter of the circle described from E, perpendicular to CS, is the common section of the equator and of the circle of hour [12]...

of hour 12, or of the meridian, and so on — as is evident if one rightly considers the proper position of the circle described from E (or of the equator) in the plane of the polar dial, namely such that it stands perpendicular to the plane of the dial. Therefore the hour-circles from noon or midnight cut the equinoctial line CS, in the plane of the dial, at the points where the straight lines drawn through the center E and through the points of the divisions of the circle described from E meet it. Accordingly, through these points must be drawn the common sections of those circles and of the plane of the dial, which are indeed the hour-lines from noon or midnight.

[Margin: The line of hour 6 from noon or midnight cannot be described in the polar dial.]

Wherefore, since by the corollary of proposition 22 of the preceding book these hour-lines are parallel in the polar dial, it is clearly established that the straight lines drawn through the points of the line CS parallel to DE — the meridian line, or line of hour 12 — are the hour-lines from noon or midnight, in the order stated above. The line of hour 6 alone cannot be described, but vanishes, as we demonstrated in the scholium of proposition 11 of the preceding book. And thus far we have constructed the upper polar dial, which faces the zenith.

[Margin: The upper polar dial, drawn on the opposite face so that each line corresponds to each line, gives the lower polar dial, provided the hour-numbers are changed into their complements up to 12.]

Now these same hour-lines, drawn on the opposite face of the dial's plane (which the sun illuminates before hour 6 from midnight and after hour 6 from noon, as is clear) so that each line answers in every respect to each line, will also give the hours in the lower polar dial — provided, however, that the number of each hour is changed into its complement up to 12, so that the line of hour 7 from midnight in the upper dial is the line of hour 5 from midnight in the lower, and so on. We call the dial that faces the zenith the Upper, and that which faces the nadir the Lower. The demonstration is entirely the same as before, if the circle described from E is understood to be moved downward about CS until E is joined with the center of the world, and so on. For the diameter of that circle, cutting CS at right

angles, will be the common section of the equator and of the circle of hour 12 of midnight, and so on — as can easily be understood if the said circle is conceived in its proper position in the plane of the lower dial. We have therefore delineated the astronomical polar dial. Which was to be done.

SCHOLIUM.

[Margin: How the astronomical polar dial is constructed on a fixed plane. When the place of the stylus and its length are given, by what method the astronomical polar dial is described.]

If a dial is to be described on a fixed plane that is parallel to the circle of hour 6, and not on some arbitrary plane to be set afterward in its proper position (as we have done up to now), then, with the aid of the level and the plumb-line, one must draw on the given plane the straight line CS parallel to the horizon for the equinoctial line, and erect DE perpendicular to it for the meridian line. Then, taking on the straight line DE the length of the stylus AE, and describing from E a circle of any size, the rest is to be completed as in the meridian dial, as we also taught before.

But if the place of the stylus is given, as at A, together with its length, then CS must be drawn through A, the place of the stylus, parallel to the horizon, and DE erected perpendicular to CS from that same place of the stylus A, and so on.

PROBLEMA 38. PROPOS. 38.

PARALLELOS, siue arcus signorum Zodiaci in prædicto horologio polari describere. [To describe the parallels, or arcs of the signs of the zodiac, in the aforesaid polar dial.]

[Margin: Description of the arcs of the signs of the zodiac in the polar dial.]

The arcs of the signs of the zodiac will be described in the polar dial just as they were described in the meridian dial. For the same demonstrations can be transferred here, only this being observed: in the first figure of proposition 26 of this book, the circle of the analemma ABCD is not to be taken, as it was there, for the circle of hour 6 from noon or midnight, but for the meridian itself; and so the axis FG is the common section of the meridian and of the circle of hour 6, while the straight line LM is the common section of that same meridian and of the plane of the polar dial — that is, the meridian line of the dial, or the line of hour 12, and not of hour 6 as there.

Diagram. The polar dial of Problema 38, again in analemma form, showing the arcs of the zodiacal signs superimposed on the hour-lines. A vertical meridian line runs through the center with D toward the top and E below; at the foot lies the circle divided into parts, carrying the numeral 12. A horizontal line crosses it, and dashed sign-arcs radiate to left and right, ending at the zodiacal symbols ranged down both margins. Two arcs cross in an X above the center.

- D (upper center): meridian-line point (north)
- E (lower center): stylus vertex / center of the divided circle
- 12: numeral within the lower circle
- N (on the meridian line, toward the south): shadow point at noon when the sun is in Cancer
- interior points near the center (labels resembling F, S, D): not legibly resolved — [?]
- axis FG, line LM, points B and Q, and line $\alpha\beta$ referred to in the text: not legibly resolved on the figure — [?]
- zodiacal sign-symbols down the left and right margins: mostly worn / not legibly resolved — [?]

[Margin: In the polar dial, both upper and lower, the northern signs lie below the equinoctial line and the southern signs above it. How the horizontal line in the polar dial is described.]

By the same reasoning, in the third figure of that same proposition 26, the line $\alpha\beta$, which in the meridian dial belonged to hour 6 from noon, is the meridian line — that is, the line of hour 12 — in the polar dial; while the parallel next to it belongs to hours 1 and 11 from noon and midnight, and so on. Furthermore, in each polar dial the northern signs are contained below the equinoctial line, toward the south, and the southern signs above that line, toward the north, as can be gathered from the analemma, or first figure of proposition 26 of this book. For when the sun is in Cancer [♋], the shadow of the stylus is cast into the lower parts of the meridian line in each dial — namely to the point N, toward the south, at the time of noon; but when the sun is in Capricorn [♐], into the upper parts, as to the point Q, toward the north.

But since the plane of the polar dial is cut by the horizon (not being parallel to it) — just as the circle of hour 6 from noon or midnight, to which the dial is parallel, is likewise cut by the same horizon — we shall describe the common section of the horizon and of the plane of the dial, namely the horizontal line, upon which the shadow of the stylus is cast when the sun rises or sets, as is established from

proposition 11 of the preceding book, in the following way. At the point B, where the hour 3 or 9...

cuts the equinoctial line, let the angle ABD be made equal to the complement of the pole's elevation, and let the line BD cut the meridian line at D — above the position of the stylus in the upper dial, but below it in the lower. This is easily done: from B describe an arc of a circle beyond the meridian line, on which the complement of the pole's elevation is laid off starting from the line CS, and from the end of that measured arc draw a straight line to B, and so on. For if through D a line is drawn parallel to the equinoctial line CS, this will be the horizontal line. Since the line AB is equal to the length of the stylus AE — as we showed in the scholium of proposition 25 of this book — then, if the triangle DBA is imagined to be rotated about the line AD until it stands perpendicular to the plane of the dial and lies in the plane of the meridian circle, the horizon will pass through the point B, that is, through the vertex of the stylus, or the center of the world. For the line AB, in the position it acquired during that rotation, is perpendicular to the plane of the dial and lies in the plane of the equator; hence B will be the vertex of the stylus, or the center of the world.

[Margin: Another way of describing the horizontal line in the same polar dial.]

Now the angle DBA has been made equal to the complement of the pole's elevation — which is precisely the angle the horizon makes with the plane of the equator — and so the remaining angle BDA is equal to the pole's elevation, that is, to the inclination of the dial's plane to the horizon. The horizon will therefore pass through the line BD and will meet the plane of the dial at D. For in this way, in the plane of the meridian circle, it makes with AB — the common section of the equator and the meridian — an angle equal to the complement of the pole's elevation, and with the line AD — the common section of the dial's plane and the meridian — an angle equal to the pole's elevation, as the case requires. Therefore the horizontal line must be drawn through D. And since the equator, the horizon, and the circle of the sixth hour have one common section, and the plane of the dial is parallel to one of these (namely the circle of the sixth hour), the common sections which the plane of the dial makes upon the other two circles — that is, the equinoctial line and the horizontal line — will be parallel to each other, as was demonstrated in the scholium of proposition 22 of the preceding book. Therefore a line drawn through D parallel to the equinoctial line is the horizontal line, which was what was proposed.

We shall likewise find the point D through which the horizontal line must be drawn, in the following way. In the analemma from which the arcs of the signs

were described, let the points l, m be marked, where the diameter of the horizon BD intersects the parallels LM, LM through which the planes of the dials are drawn; and let the line Kl or Km be transferred onto the meridian line from the position of the stylus A — upward as far as D in the upper dial, but downward in the lower. For the horizontal line must be drawn through D, as before, since at the points l and m the plane of the horizon (drawn through BD) meets the planes of the dials (drawn through LM, LM), with lines drawn above K for the position of the stylus in the upper dial and below it in the lower, and so on. And since the line Kl is equal to the line Km — for in the triangles EKl, EKm the angles at K are right, [a] and the angles at the vertex E are equal, and likewise the sides EK, EK are equal, [b] so the sides Kl, Km will also be equal — for this reason we said that this same Kl or Km must be transferred onto the meridian line above or below the position of the stylus in each dial, so that the point may be found through which the horizontal line is to be drawn.

[Margin: [a] I.15 (Euclid). [b] I.26 (Euclid).]

Moreover, if the upper polar dial is inverted, so that what was above becomes below, and what lay to the right after that inversion becomes the left, and vice versa — as we wrote in the scholium of proposition 14 of this book — it will indicate the same hours on the opposite face of the plane, that is, on the lower face, which looks toward the nadir, as we demonstrated in that scholium. But the meridian line will then show the hour of 12 at midnight, whereas in the upper dial it indicates the hour of 12 at noon; and the remaining hours, which in that upper dial are numbered from noon, must be reckoned from midnight in the lower dial, and vice versa. For since the Sun must lie in one semicircle of the same hour-circle when the shadow of the stylus falls upon the upper dial, and in the other when it falls upon the lower — as is plain — it follows that the line which in the upper dial indicates an hour from noon shows in the lower dial the same hour from midnight, and vice versa; inasmuch as one semicircle pertains to the hours from noon and the other to the hours from midnight, as we taught in the scholium of proposition 9 of the preceding book.

[Margin: If all the parts of the upper polar dial are inverted, so that the upper becomes the lower and the right passes to the left, the lower dial will be produced; but the hours which before were reckoned from noon must be reckoned from midnight, and vice versa.]

The arc, too — that is, the parallel of the southern signs of the upper dial — is changed into the arc of the northern signs in the lower dial, and vice versa: as the parallel of Capricorn [♐] into the parallel of Cancer [♋], the parallel of Sagittarius

[♊, glyph worn — read by the equal-declination pairing with Gemini and Leo below] and of Aquarius [♒] into the parallel of Gemini [♊] and of Leo [♌], and so on. We shall show this just as, in proposition 26 of this book, we demonstrated the same thing for the meridian dial — with this one difference, that the axis of the hyperbolas here is the line of the twelfth hour and not of the sixth, as it was there. The same is clear from the analemma from which the arcs of the signs were described. For in it the radius of Capricorn [♐], drawn through the center E, cuts the meridian line ML near the horizon BD in the upper dial; while in the lower dial the radius of Cancer [♋], passing through the same center E, cuts the meridian line near the horizon, as is evident. The same must be said of the other southern and northern signs.

[Margin: The horizontal line cuts off the lower dial from the upper part of the upper dial, if all its parts are inverted, as was said above.]

Therefore the portion of the upper dial which the horizontal line cuts off, when placed on the lower face of the dial — so that the horizontal line occupies the upper place, yet below the stylus, and what is then the right-hand part is made the left, and vice versa — will give the lower polar dial, together with the arcs of the signs.

[Margin: The portion of the upper dial cut off by the horizontal line is the nocturnal dial.]

That same portion cut off in the upper dial will be a nocturnal dial, as was said above. For the shadow of the stylus would fall upon it in the morning before sunrise, and in the evening before sunset, while the Sun is in the southern signs — did not the density of the earth stand in the way.

[Margin: The greater the pole's elevation, the more hours are contained in the lower (or nocturnal) polar dial; the smaller it is, the fewer.]

It is furthermore evident that the greater the pole's elevation, the more hours are received in the lower dial — that is, in the portion cut off by the horizontal line — and the fewer, the smaller the pole's elevation. For the greater the pole's elevation, the smaller becomes the angle DBA (the complement of the pole's elevation), and hence the point D will be nearer to the equinoctial line, so that the horizontal line will cut across more hours, and so on. The same can also be shown from this: that the greater the pole's elevation, the more closely the circle of the sixth hour — to which the polar dial is parallel — approaches the vertical circle, and consequently for more hours the Sun will illuminate the northern

face, and so on. We have therefore described the parallels, or arcs of the signs of the Zodiac, in the said polar dial. Which was to be done.

PROBLEMA 39. PROPOS. 39.

To describe the parallels of the diurnal arcs on the same polar dial.

[Margin: The description of the diurnal arcs in the same polar dial.]

Just as the parallels of the signs are described on the dial, so also are the parallels of the diurnal arcs, since both the former and the latter are equidistant from the equator; provided only that, in place of the parallels described through the beginnings of the signs, the parallels of the diurnal arcs are taken, and so on. We have therefore described the parallels of the diurnal arcs on the same polar dial. Which was to be done.

PROBLEMA 40. PROPOS. 40.

To describe the vertical circles on the same polar dial.

On the dial, cut off from the equinoctial line a straight line AB equal to the stylus — such as the line that lies between the line of the twelfth hour and the line of the third or ninth hour, as we demonstrated in the scholium of proposition 25 of this book. At the point B construct the angle ABC equal to the pole's elevation, and let the line BC cut the meridian line below the equinoctial line at C in the upper dial, but above the equinoctial line in the lower dial. Then draw the line BD, which makes the angle ABD equal to the complement of the pole's elevation — or (what is the same) a right angle with the line BC — so that it cuts the meridian line at the point D, through which the horizontal line is drawn, as was shown in proposition 38 of this book. On the meridian line take DE equal to that BD, either above the horizontal line or below it. Then, from the center E, describe a circle of any size and divide it into 360 parts, or into fewer — since the dial does not admit all the vertical circles. (We divided it into 24 parts, so that each contains 15 degrees.) Through the points of division and the center E draw hidden lines cutting the horizontal line at various points; and through those points and through the point C draw further lines, which will give the vertical circles — that is, the common sections of the vertical circles and the plane of the dial. Their beginning is taken from the vertical line, which is drawn through C parallel to AB (or to the horizontal line), so that the meridian line gives the ninetieth vertical. All of which we shall demonstrate in this way.

[Margin: Demonstration of the description of the vertical circles on the polar dial.]

Let the triangle CBD, together with the line AB, be imagined to be rotated about CD until it stands perpendicular to the plane of the dial and is joined with the meridian, and until the point B becomes the same as the center of the world. Since, then, as we demonstrated in proposition 38 of this book, the horizon passes through the line BD (so that BD is the common section of the horizon and the meridian), and since the angle DBC is right — being composed of the pole's elevation and its complement — the line BC will be the common section of the vertical circle proper and the meridian, so that C is the zenith, or the vertex of the head. For since both the meridian and the vertical circle are perpendicular to the horizon, their common section — namely the axis of the horizon — will also be perpendicular to the horizon, and hence also to

Diagram. A construction of the vertical circles on a polar dial. A circle at the upper center carries the point E at its center, with S marked just below E and D lower still, on the vertical (meridian) line. The word HORIZON is lettered across the horizontal line ("HORI:" on the left, "ZON" on the right); the word VERTICALIS is lettered across a lower dashed line ("VERTI:" on the left, "CALIS" on the right). A dense fan of straight lines radiates from a lower convergence point C, representing the vertical circles; near that convergence, close to B, lie the small labels K, L, M. Degree graduations (15, 30, 45, 60, 75, 90) run symmetrically along the top near the horizontal line and along the bottom.

- E (center of upper circle): center of the divided circle
- S (below E): interior point [?]
- D (below S, on the vertical line): point where BD meets the plane of the dial
- HORI: ... ZON (across the horizontal line): the horizon line
- VERTI: ... CALIS (across the lower dashed line): the vertical circle proper
- B (just above C, at the fan's inner cluster): center of the world / stylus vertex
- C (lower convergence of all radiating lines): zenith
- K, L, M (small letters near B): interior points [?]
- ✚ marks at the far left and right ends of the dashed line: sign/tropic markers [?]
- "SI" (upper left and upper right) and "IS" (mid left and right) near the ends of the dashed curves: fragmentary labels, not resolved [?]
- degree numbers 15, 30, 45, 60, 75, 90 along top and bottom: graduation marks

the line BD, which lies in the horizon (by definition 3 of Euclid, Book 11), it will be perpendicular at the center of the world B. Therefore BC will be the axis of the horizon — that is, the common section of the meridian and of the vertical circle proper, and consequently also of all the other vertical circles. This can also be made clear from the fact that the angle ABC is equal to the pole's elevation, which is precisely the angle of inclination of the equator (passing through the line AB) to the vertical circle proper. Now let the circle described from E be conceived to be moved about the horizontal line until it coincides with the horizon, and hence until its center E is joined to the center of the world B — because of the equality of the lines DB, DE. This being posited, the lines proceeding from the center E through the points of division will be the common sections of the horizon and the vertical circles, because the horizon and the circle described from E are divided into similar parts, by what we demonstrated at the end of chapter 1 in the sphere. The vertical circles therefore meet the plane of the dial at those points of the horizontal line onto which the lines drawn from E fall. Accordingly, since the common sections of the vertical circles and the plane of the dial meet, by proposition 18 of the preceding book, at the point C — where indeed lies their common section of all, that is, where the axis of the horizon cuts the plane of the dial — the lines drawn from C through the points found on the horizontal line will be the common sections of the plane of the dial and the vertical circles. And the common section of the vertical circle proper and the plane of the dial must be drawn through C parallel to the equinoctial line and to the horizontal line. For since

the vertical circle proper, the horizon, the equator, and the circle of the sixth hour (to which the plane of the dial is parallel) have one common section, the common sections of the first three circles made upon the plane of the dial — namely the vertical line, the horizontal line, and the equinoctial line — will, by the scholium of proposition 22 of the preceding book, be parallel to one another.

The point C, moreover, will also be found in the following way. In the first figure of proposition 26 of this book, let the points n and o be marked, where the diameter of the vertical circle AC cuts the parallels LM, LM; and let the line kn or Ko be transferred onto the meridian line — in the upper dial below the position of the stylus, as far as C, but in the lower dial above the position of the stylus — always beginning from the position of the stylus A. For through C the vertical line proper must be drawn parallel to the equinoctial line, as before, because at the points n and o the vertical circle proper (which is drawn through AC) meets the plane of each dial (drawn through LM at the points n and o) — below the position of the stylus in the upper dial, but above it in the lower, and so on. And since the

line Kn is equal to the line Ko — for in the triangles EKn, EKo the angles at K are right, [a] and the angles at the vertex E are equal, and likewise the sides EK, EK are equal, [b] so the sides Kn, Ko will also be equal — for this reason we said that this same Kn or Ko must, in each dial, be transferred onto the meridian line below or above the position of the stylus.

[Margin: [a] I.15 (Euclid). [b] I.26 (Euclid). The portion of the upper dial cut off by the horizontal line, if its parts are inverted on the opposite face of the dial so that the horizontal line occupies the topmost place, gives the vertical circles on the lower dial.]

But if the portion cut off by the horizontal line is placed on the lower face of the plane — so that the horizontal line occupies the upper place, and what is then the right-hand part is made the left, and vice versa — we shall have the same vertical circles on the lower dial, as we demonstrated in the scholium of proposition 14 of this book. Indeed, here it is not even necessary that the right-hand part be made the left, since the vertical lines have the same position and the same numbers on both sides of the meridian line, as is plain. We have therefore described the vertical circles on the same polar dial. Which was to be done.

PROBLEMA 41. PROPOS. 41.

To describe the parallels of the horizon on the same polar dial.

[Margin: The description of the parallels of the horizon on the same polar dial.]

Let ABC be an analemma, in which the horizon is BC; the vertical circle AD; the axis of the world (or the common section of the circle of the sixth hour and the meridian) EF; and the equator GH. Divide the semicircle BAC into 180 equal parts, so that each contains a single degree — or into fewer, having regard to the size of the dial. (We divided it into 12 parts, so that each comprises 15 degrees.) Join by straight lines the pairs of points equally distant from the line BC (or from the point A); these lines will be the common sections of the meridian and of the parallels of the horizon, which they call almucantarath. Then, from the points of division through the center D, draw straight lines, so that triangles through the axis are constituted in the cones, whose bases are the parallels of the horizon (both below the horizon and above it) and whose vertex, or common center of the world, is D. For the meridian ABC, drawn through AD, the axis of the said cones, makes triangles through the axis, by proposition 3 of Book 1 of Apollonius. On the equator GH also — which stands in place of a vertical circle with respect to the circle of the sixth hour EF, as though of a certain horizon to which the dial is

parallel — take on each side a line DI equal to the stylus, and through I draw RQ parallel to EF. Below the center D this will be the common section of the meridian and of the plane of the upper dial (which looks toward the zenith A), but above the center D it will be the common section of the same meridian and of the plane of the lower dial (which pertains to the nadir). The line RQ will cut the sides of the triangles through the axis at the points K, L, M, N, O, and so on; and the diameters of the conic sections will be KQ, LQ, MQ, NQ, OQ.

Diagram. An analemma for describing the parallels of the horizon on a polar dial: a large circle crossed by several diameters and by a dense web of chords, with the center at D. The horizon BC is the horizontal diameter (B at the left, C at the right); the vertical circle AD runs from A at the top to the center D; the axis of the world EF is the diameter from E (upper right) to F (lower left); the equator GH is the diameter from G (upper left) to H (lower right). The circumference is graduated in 15° steps (15, 30, 45, 60, 75, 90) reading symmetrically from A at the top down to 90 at the bottom. Point R lies on the upper-right circumference and Q on the lower-left, RQ being drawn parallel to EF. A small circle surrounds the center D, within and around which lies a crowded cluster of the interior points named in the text.

- A (top of circle): pole toward the zenith / top of the vertical circle
- B (left end of horizontal diameter): horizon endpoint
- C (right end of horizontal diameter): horizon endpoint
- D (center): common center of the world
- E (upper right on circumference): endpoint of the axis of the world EF
- F (lower left on circumference): endpoint of the axis of the world EF
- G (upper left on circumference): endpoint of the equator GH
- H (lower right on circumference): endpoint of the equator GH
- R (upper right, inside the circumference): endpoint of line RQ
- Q (lower left, inside the circumference): endpoint of line RQ
- interior cluster K, L, M, N, O, P, S, T, V (near and around the central circle at D): the points where RQ (and TS/TQ) cut the sides of the triangles through the axis, and where the verticals cut the meridian line — individually crowded and not all separately resolvable [?]
- degree numbers 15, 30, 45, 60, 75, 90 around the circumference: graduation marks

Therefore, if the points K, L, M, N (we here omit the point O, since when transferred it falls outside the tropics) are transferred onto the meridian line of

the upper dial below the horizontal line — beginning in this figure from the point T of the horizon, but on the dial from the point D of the horizontal line — and if, by proposition 8 of the preceding book, the said conic sections are described about the meridian line, passing through the points K, L, M, N (these conic sections will be partly hyperbolas, partly ellipses, as is clear from propositions 6 and 7 of the preceding book; but the parallel of the horizon at 42 degrees will be a parabola, by proposition 5 of that same preceding book, since the circle of the sixth hour touches it at the pole E) — and if these curves recede ever farther from the horizontal line the farther they are extended on each side of the meridian line, then the parallels of the horizon will be described. On the lower dial, the line TS must be transferred onto the meridian line from the point D, below the horizontal line, and so on. For SQ is the diameter of the conic section of the parallel of the horizon at 15 degrees above the horizon in the lower dial. In the same way the remaining points below S, toward Q, would have to be transferred, if the sides of the triangles through the axis were cut by the line TQ. In the figure of the preceding proposition we described the parallel of the horizon at 15 degrees, in the portion which the horizontal line cuts off, because that supplies the place of the lower dial, as we said in that same proposition. It is therefore evident that the line EF is, as it were, the horizon, and GH, as though a vertical circle BC, is a kind of equator; and the parallels of the horizon are like parallels of this new equator BC. This being posited, the vertical circles will be like the hour-circles, all of which cut the meridian line of the dial at the point V, where AD, the axis of the horizon, also intersects it — the axis which we said now performs the office of a certain new equator. So that, if this exchange of circles is well considered, this description of the parallels of the horizon will not differ from the description of the parallels of the equator on the horizontal dial.

[Margin: Another way of describing the parallels of the horizon on the same polar dial.]

ANOTHER METHOD. Describe a quadrant ABC of any size, and divide it into 90 degrees — or into fewer parts, according to the number of parallels of the horizon to be drawn. From the centre A, draw straight lines through the points of division; these will correspond to the rays of the parallels of the horizon contained in the quadrant DC 90 of the preceding figure, beginning from the straight line AB, as the figure shows. Then, from the figure of the preceding proposition, take AD (on the line AC of the present figure) equal to the straight line BC; and on the straight line AB take AE equal to the straight line DB (or DE); and draw the straight line DE.

[Margin: [Euclid I.4]]

[Margin: [Euclid I.26]]

This triangle ADE will be exactly equal to the triangle CBD of the preceding proposition's figure, since the angles at A and B are right angles and are contained by equal sides, by the construction — so that DE is equal to the meridian line CD. The same triangle ADE will likewise be equal to the triangle DVT of the immediately preceding figure, since that triangle is equal to the aforesaid triangle CBD. For the angle IDV is equal to the angle ABC (each being the altitude of the pole), and the angle DIV is a right angle equal to the right angle BAC, and the sides DI and AB are equal to each other (since each is taken equal to the gnomon [stylo]); therefore the sides DV and VI will also be equal to the sides BA and CA. By the same reasoning the sides DT and TI will be equal to the sides BD and DA, and so on.

After this, from the figure of the preceding proposition, transfer all the intervals between the centre E and the points where the vertical lines cut the horizontal line into this figure, carrying them from A onto the straight line AB; mark the points on the line AB and write beside them the numbers of the vertical lines — for example 90 next to the point E, 75 at the next nearest, and so on. Finally, from D, through the points marked on AB, draw straight lines.

Diagram. A quadrant construction for laying out the parallels of the horizon on the polar dial. Across the top runs the horizontal base line carrying the points **A** (left), **D** (centre), and **C** (right, marked **90**). A quarter-circle arc descends from C. The centre of the construction is at A, and a fan of straight radii is drawn from the upper region down to the arc.

- A (top left): centre of the quadrant; left endpoint of the base line
- D (top, just right of centre): point from which the vertical/hour lines are drawn
- C (top right, labelled "90"): terminus of the base line at the 90° graduation of the arc
- E (interior, upper left): point on the descending line, with an adjacent vertical column of vertical-line numbers reading (top to bottom) 90, 75, 60, 45 [?] (the lower numbers of this column not separately resolvable [?])
- F (centre of figure): point on the meridian/vertical line
- B (lower left, on the arc): foot of the quadrant

- Degree graduations along the arc (from C downward): 90, 75, 60, 45, 30, 15 [the lower part of the arc carries a second, more crowded set of graduations — 90, 75, 60, 45, 30, 15 — not all separately resolvable [?]]
- Points I, V, T (named in the demonstration as vertices of the triangles DVT and DIV): present in the construction but not separately legible on the scan [?]
- A small cross/star ornament near the foot of the figure marks the convergence of the lower rays

[More than two labels flagged]

For the vertical line properly so called — from which the others take their beginning — one must draw through D a line DF parallel to AB; and the lines already drawn from the point D through the points of the line AB must also be carried over to the other side, so that one has vertical lines both beyond and on the near side of the vertical line properly so called — just as was done, in the horizontal dial, with the lines before and after the sixth hour, in proposition 2 of this book. For the lines drawn from D through the points of the line AB are the vertical lines beyond the vertical line properly so called, onto which the shadow of the gnomon is cast when the Sun is in the southern verticals; the others are the vertical lines on the near side of the vertical line properly so called, and so on.

Now, if the several intervals of the lines proceeding from D — those lying between D and the ray of, say, the parallel of the horizon at 15 degrees — are transferred from the point C of the figure of the preceding proposition onto pairs of vertical lines equally distant from the meridian line and corresponding to the numbers marked on the line AB of the present figure, marking points on those vertical lines (yet in such a way that the intervals of the lines from D that fall on AB are carried over from the Zenith C into the vertical lines of the dial above the vertical line properly so called, while the intervals of the lines from D that fall to the right of the line DF are transferred into the vertical lines below the line properly called vertical) — and if these points are fitly and neatly joined by a curved line, then the parallel of the horizon at 15 degrees will have been drawn. In the same way the remaining parallels of the horizon will be drawn.

[Margin: Demonstration of this latter way of describing the parallels of the horizon.]

The demonstration of this does not differ from the one we gave for the horizontal dial in the second method of describing the arcs of the signs. For here the horizontal line is, as it were, the line of the equator, and the vertical lines are like

the hour-lines, whose centre is the Zenith, namely the point C, and so on, as we said in the preceding analemma. Thus the line AB of this figure will correspond to the radius DC of the equator in the last figure of proposition 2 of this book, in which the rays of the signs are drawn; the straight lines drawn from A correspond to the rays of the other signs emitted from D in that figure; the straight line AD to the axis DH; the point D to the point H; and finally the vertical lines falling from D to the hour-lines drawn out from H.

The same will result if the intervals of the lines from D that run between the line AB — which, as we said, stands in place of the equator — and the lines emitted from A are transferred onto the corresponding vertical lines measured from the horizontal line of the dial, and so on, just as, in the horizontal dial, the intervals between the radius of the equator and the rays of the signs were carried over so that the arcs of the signs might be drawn.

For the lower polar dial, the rays of the parallels of the horizon must in this figure be drawn out from A to the left side of the line AB. Just as, in the figure of the rays of the Zodiac in proposition 2 of this book, the rays of the signs to the right of the radius of the equator belong to the southern signs (to be drawn below the equinoctial line) and the rays to the left belong to the northern signs (to be drawn above the equinoctial line), so here too the rays of the parallels of the horizon emitted from A to the right of the line AB — which stands in place of a kind of new radius of the equator, as has been said — belong to the parallels of the horizon to be drawn below the horizontal line (which is like a kind of new equinoctial line), as though they were southern signs; while the rays drawn to the left of the line AB belong to the parallels of the horizon to be drawn above the horizontal line of the dial, as though they were northern signs. Hence, since the portion of the dial above the horizontal line is equal to the lower dial, the rays of the parallels of the horizon drawn out from A to the left side of the line AB are rightly transferred into it, and so on — all of which appears in the figure of the preceding proposition.

[Margin: The parallels of the horizon above the horizontal line are described from their opposites drawn below the horizontal line.]

But the parallels of the horizon above the horizontal line in the lower dial will be more easily drawn from their opposites drawn in the upper dial, if — following the method we gave near the end of proposition 2 of this book — we first find the transverse diameters of the opposite sections and their centres, and so on. In the latter figure the transverse diameters are the portions of the straight line DE intercepted between the rays of the opposite parallels of the horizon that are

drawn out from A and equally distant from the line AB. We have therefore laid out the parallels of the horizon on the same polar dial. Which was to be done.

PROBLEM 42. PROPOSITION 42.

To describe the meridians — that is, the circles of longitude of cities — on the same polar dial.

The meridian circles will be drawn on the polar dial exactly as the hours are drawn from midday or from midnight, just as we taught in proposition 6 of this book — provided that the circle FGHI, by whose aid we drew the hour-lines, has the starting-point of its division at the meridian of the Fortunate Isles. This meridian is obtained if the longitude of the place is reckoned from the meridian of the place, namely from the point F toward G, in the westward direction, as the matter itself requires. This can be readily understood if the circle FGHI is imagined to be revolved about the equinoctial line until it coincides with the equator, and so on.

Diagram. The polar-dial figure for the meridians (circles of longitude). At the centre stands the circle **FGHI**, with **F** at the top, **G** at the left, **H** at the bottom, and **I** at the right; **E** marks its centre. A horizontal line runs across the whole figure, lettered **HO ... RIZON** (Horizon) — "HO" at the far left and "RIZON" at the far right. Within the upper part of the circle the meridian of the Fortunate Isles is written along a (dotted) curve as "**Insulæ Fortunatæ**" ("Fortunate Isles"). Straight rays radiate outward through the circle, and the horizon line and other transversals carry longitude graduations in degrees.

- F (top of circle): point where the meridian of the place meets the circle below the horizon in the inverted (lower) dial
- G (left, on the horizon line): the westward point of division
- E (centre of circle): centre of the construction
- I (right, on the horizon line): the eastward point of division
- H (bottom of circle): point where the meridian cuts the circle above the horizon in the lower dial
- Legible longitude/degree numbers around the circle and along the rays: 15, 90, 345, 115, 140 (or 146 [?]), 165, 195, 210, 240, 270, 320, 330 [additional values — including 300 [?], 285 [?], 326 [?], and a far-left figure near "HO" reading 562 [?] and a far-left edge value 111 — are present but not all separately resolvable [?]]

- A central horizontal band of narrow boxed cells carries a fine degree-graduation scale (readings on the order of 75, 60, 45, 30 [?]); the individual cells do not resolve cell-by-cell on the scan and are not transcribed here [?]

[More than two labels flagged]

For the meridians of the lower dial, the longitude of the place must be counted from the point H toward G, so that the meridian of the Fortunate Isles may be found. For if the lower dial is understood to be set in its proper position — so that the horizontal line occupies the top, and the circle FGHI is revolved about the equinoctial line until it coincides with the equator and its centre E coincides with the centre of the world — the meridian of the place will cut the said circle above the horizon at the point H, and below it at F. Thus from H toward G lie the western parts, and toward I the eastern parts. Let the rest be done as before. In the example you see the meridians of 140 and 150 degrees on one side, and of 285 and 295 degrees on the other, in the lower dial, which the portion cut off by the horizontal line supplies to us.

[Margin: The meridians, when produced beyond the horizontal line, make up the lower dial together with its meridians.]

Yet we shall obtain these same meridians of the lower dial more easily if we produce the meridians of the upper dial beyond the horizontal line and assign to them the numbers formed by adding 180 degrees to the numbers of the meridians of the upper dial — casting off 360 degrees, however, whenever the sum is greater than 360; that is, the numbers which, on the circle drawn about E, lie diametrically opposite the numbers written in the upper dial. For the portion cut off by the horizontal line will give, on the opposite face — namely the lower one — the meridian circles, if all its parts are inverted, as was demonstrated in the scholium to proposition 14 of this book. But the numbers of the lower dial are changed into the numbers of the semicircles opposite the semicircles of the upper dial, because the Sun is in one semicircle when the shadow falls on the lower dial, and in the other when it falls on the upper, as we taught in proposition 38 of this book. And so we have described the meridians — that is, the circles of longitude of cities — on the same polar dial. Which was to be done.

PROBLEM 43. PROPOSITION 43.

To lay out the parallels of cities — that is, the circles of latitude — on the same polar dial.

[Margin: The description of the parallels — or latitudes — of cities on the same polar dial.]

Take the parallels drawn through the zeniths of cities that lie within the tropics (for the Sun never reaches the parallels of the others), in place of the parallels passing through the signs of the Zodiac; and let everything else be done as we prescribed in proposition 38 of this book for describing the parallels of the signs of the Zodiac. Then what is proposed will be accomplished. We have therefore laid out the parallels of cities — that is, the circles of latitude — on the same polar dial. Which was to be done.

PROBLEM 44. PROPOSITION 44.

To describe the celestial houses on the same polar dial.

[Margin: The description of the celestial houses on the same polar dial, according to Johannes Regiomontanus.]

Suppose the circle drawn about E, which we used in describing the hours, is cut into 12 equal parts, beginning from the meridian line; and suppose that through the points of division and the centre E straight lines are drawn cutting the equinoctial line in points. Then the straight lines joining these points with the point D — at which the horizontal line cuts

[Margin: Demonstration of the description of the celestial houses according to Johannes Regiomontanus.]

the meridian line — will give the beginnings of the celestial houses, according to the method of Johannes Regiomontanus, as we demonstrated in proposition 20 of this book concerning the vertical dial. For here too the common section of all the circles of the celestial houses — that is, the axis of the vertical circle, or the common section of the meridian circle and the horizon — meets the plane of the dial at the point D, as is clear from the figure of proposition 38 of this book, in which this common section is the straight line BD, provided the triangle ABD is understood to be perpendicular to the plane of the dial.

Diagram. The polar-dial figure for the celestial houses. Two large circles are drawn one above the other and meet near the centre: the upper circle (centre E) and the lower circle (centre F). Three principal horizontal lines cross the figure — the upper labelled **HORI ... ZON** (Horizon), a second **HORI ... ZON** (Horizon) lower down, and the lowest labelled **VERTI ... CALIS** (Vertical). A fan of house-lines radiates through the figure and cuts these lines at the house cusps, which

are numbered in Roman numerals in the upper dial and in Arabic numerals in the lower dial.

- D (upper centre, on the upper horizon line): point where the horizontal line meets the meridian line; hub of the house-lines
- E (just below D): centre of the upper circle
- A (below E): point on the meridian/vertical axis
- B (lower centre, on the second horizon line): centre of the world
- C (below the vertical line): the Zenith
- F (centre of the lower circle): the Campanus centre onto which BC is transferred
- Upper-dial house cusps in Roman numerals: I and VII flank the upper horizon line; XI, X, IX cluster about the centre; XII and VIII lower down; I again at the second horizon line
- Lower-dial house cusps in Arabic numerals along the vertical line: 12, 11, 10, 9, 8 (arranged about the point A)
- Two worn astrological/zodiac symbols set on the diagonal rays at the top corners — one upper-left, one upper-right (mirror-imaged) — not securely identifiable on the scan [?]

[More than two labels flagged]

But according to Campanus's method we shall lay out these same celestial houses as follows. From the figure of proposition 40 of this book we take the straight line BC, by which we found the Zenith C, and transfer it onto the meridian line from the point C — whether upward or downward — as far as F. Then, having drawn a circle about F and divided it into 12 equal parts (beginning from the meridian line), we draw through the points of division and through the centre F hidden construction lines cutting the vertical line in points; these, joined by straight lines to D, will exhibit the celestial houses, as Campanus wishes. For if this circle is turned about the vertical line until it coincides with the vertical circle properly so called, and its centre F is joined with the centre of the world B, those hidden lines drawn through F will be the common sections of the circles of the celestial houses with the vertical circle; and consequently the circles of the celestial houses will meet the plane of the dial at the aforesaid points of the vertical line. Therefore the lines of the celestial houses will pass through these same points; and since they also pass through the point D, as has been said, the aforesaid lines connecting the points of the vertical line with D will be the lines of the celestial houses. We have therefore described the celestial houses on the same polar dial. Which was to be done.

[Margin: The description of the celestial houses on the same polar dial, according to Campanus.]

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The lines of the celestial houses, produced beyond the horizontal line, will give these same celestial houses in the lower dial, if the dial is inverted, as has often been said and as was demonstrated in the scholium to proposition 14 of this book. But the celestial houses in the lower dial are of no use, except those that cut the tropic beyond the horizontal line — which is the more convenient the greater the elevation of the pole above the horizon, as is evident. In our example, which is constructed for latitude 42 degrees, no celestial house can be drawn entire in the lower dial, since none cuts the tropic beyond the horizontal line. Only parts of houses 12 and 7 — that is, lines drawn through the intermediate degrees of the said houses — can be drawn in the lower dial, especially through those degrees that are not far from the horizon.

[Margin: The celestial houses, produced beyond the horizontal line, give the same houses in the lower dial, if that portion of the dial cut off by the horizontal line is inverted in all its parts.]

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PROBLEMA 45. PROPOS. 45.

To describe the ascending signs of the Zodiac on the same polar dial.

[Margin: The description of the ascending signs on the same polar dial.]

From the first two tables of proposition 9 of this book, take the hours at which the twelve signs of the Zodiac rise while the Sun is at the beginning of Aries [γ] and of Libra [♎]. These hours are laid out by means of the circle FGHI drawn about E; count them off from point F, either toward the east or toward the west, according as the tables give hours before or after midday. In the superior dial the hours before midday are always to be counted from F toward I, and the hours after midday from F toward G; but in the inferior dial the hours before midday are counted from H toward I, and those after midday from H toward G — as is quite clear, provided that in each dial the circle FGHI is understood to be set in its proper position. For if the circle FGHI is taken to lie in the plane of the Equator, with its centre E at the centre of the world and at the tip of the style, then in the

superior dial F will lie at midday and H at midnight, while in the inferior dial H will lie at midday and F at midnight.

Next, through the ends of these countings and through the centre E, draw hidden straight lines cutting the equinoctial line at the points of those hours; through these points the lines of the ascending signs are to be drawn. One must take care, however, not to take from the said tables the hours of those signs that rise six or more hours before or after midday, for these do not fall on the superior polar dial. And if the ascending signs are to be described on the inferior dial, one must take these very hours, and not the hours of those signs that rise six hours — or fewer — before or after midday.

After this, mark on the horizontal line the points through which the arcs of the signs are drawn. On those points contained in the antemeridian part — namely on the left in the superior dial, or on the right in the inferior — the shadows are projected when the following signs rise: in the superior dial Capricorn [♑], and Sagittarius [♐], Aquarius [♒], and Scorpio [♏], Pisces [♓]; but in the inferior dial Cancer [♋], and Gemini [♊], Leo [♌], and Taurus [♉], Virgo [♍]. But on the points marked in the pomeridian part — that is, on the right in the superior dial, or on the left in the inferior — are projected the shadows of the setting signs, whose opposites are then rising: in the superior dial these are Capricorn [♑], and Sagittarius [♐], Aquarius [♒], and Scorpio [♏], Pisces [♓] (for the shadows of these fall on those points when they set, while the opposite signs — Cancer [♋], and Gemini [♊], Leo [♌], and Taurus [♉], Virgo [♍] — are then rising). In the inferior dial, on the other hand, the shadows of these signs — Cancer [♋], and Gemini [♊], Leo [♌], and Taurus [♉], Virgo [♍] — fall on those points when they set, while the opposite signs — Capricorn [♑], and Sagittarius [♐], Aquarius [♒], and Scorpio [♏], Pisces [♓] — then ascend above the horizon. And so, if the points of the ascending signs on the horizontal line are joined by straight lines to the corresponding points found on the equinoctial line, the ascending signs will be described — as is plain from the demonstrations already given for the horizontal, vertical, and meridian dials.

[Margin: How the ascending signs Aries and Libra are described.]

[Margin: The finding of the points on the horizontal line — the line on which the shadow of the style falls — when the beginnings of the signs rise or set.]

Diagram. A large circle (the horizon circle) crossed by a vertical diameter running from F at the top through the centre to H at the bottom. Two outer vertical lines flank the circle, each labelled *meridiana* (the meridian line). Across

the horizontal diameter the word **HORIZON** is written, interrupted at the centre by the vertical line. Near the top, inside the circle, the points C (left), F₁ and B (on the vertical, just below F), and D (right) are joined to form a shallow butterfly/trapezoidal figure whose two triangles meet just above the centre, where a small dotted triangle (the style's shadow-cone) sits above the point A. From A a great many straight rays fan out to the left and right edges of the figure; their outer ends carry the twelve zodiac-sign labels (the ascending-sign lines). Where the horizontal diameter meets the circle, the left intersection is marked G and the right intersection I. Two slanting lines at the lower corners are labelled, on the left, *Hora 7 ante[meridiana]* (the 7th hour before midday) and, on the right, *Hora 5 po[meridiana]* (the 5th hour after midday) [?].

- F (top): apex of the vertical diameter
- H (bottom): foot of the vertical diameter
- B, F₁ (upper centre, on the vertical): vertices of the inner figure
- C (upper left), D (upper right): outer vertices of the inner figure
- A (centre): common point of the fanning rays, below the dotted shadow-triangle
- G (left, on the circle), I (right, on the circle): ends of the horizontal diameter
- Peripheral ray-labels: the twelve zodiac signs set at the outer ends of the rays, left and right, are small and worn on the scan and are not individually secured [?]

Now we shall describe the lines of Aries [♈] and Libra [♎] as follows. Since, when the beginning of Aries is rising above the horizon and the Sun is at the beginning of Capricorn [♐], it is midday — as is clear from table 4 of proposition 9 of this book — the ascending sign Aries must be drawn through that point of the tropic of Capricorn [♐] through which the meridian line passes. Now at that time the Equator, the circle of the 6th hour from midday or from midnight, the Ecliptic, the Horizon, and the vertical circle properly so called all share one and the same common section, from which the plane of the dial is equidistant (since it is parallel to the circle of the 6th hour). Therefore, by proposition 18 of the preceding book, there will lie parallel in the plane of the dial: the equinoctial line, the common section of the Ecliptic and the plane of the dial, the horizontal line, and the vertical line. Hence a straight line equidistant from the equinoctial line, drawn through the point at which the meridian line cuts the tropic of Capricorn [♐], will give the ascending sign Aries [♈]. In the same way, a straight line equidistant from the equinoctial line and passing through the point at which the meridian line cuts the tropic of Cancer [♋] will give the ascending sign Libra [♎].

[Margin: How the ascending signs Aries and Libra are described.]

[Margin: The finding of the points on the horizontal line — the line on which the shadow of the style falls — when the beginnings of the signs rise or set.]

If one prefers, we shall find those same points on the horizontal line — the points at which the Sun, occupying the beginnings of the signs, rises and sets — by this method too, even if the arcs of the signs have not been drawn on the dial. From table 5 of proposition 9 of this book we shall take the semidiurnal arcs of the southern signs for the superior dial, and count them off from F in each direction; but for the inferior dial we shall take the semidiurnal arcs of the [signs]

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northern signs, and reckon these off from H in each direction. Then, from the ends of these reckonings, we shall draw hidden straight lines through the centre E, cutting the equinoctial line at points; and straight lines drawn perpendicular to the equinoctial line through those points will cut the horizontal line at the points we are seeking. For since in those hours the Sun rises and sets on the horizontal line, and the said perpendiculars are the lines of those hours — as is clear from what was demonstrated in proposition 37 of this book — it is evident that the proposed points have been correctly found.

Diagram. The same construction as on the previous page: a large horizon circle crossed by a vertical diameter from F (top) to H (bottom), flanked by two outer vertical lines each labelled *meridiana*. The word **HORIZON** runs across the horizontal diameter, broken at the centre by the vertical line. Near the top, the inner points C (left), F₁ and B (on the vertical below F), and D (right) form the shallow butterfly/trapezoidal figure; its triangles meet at the centre, where a small dotted triangle sits above the centre-point E, with A marked just below E. From the centre a dense fan of straight rays runs to the left and right edges, ending in the twelve zodiac-sign labels (the ascending-sign lines). The ends of the horizontal diameter on the circle are marked G (left) and I (right). Slanting lines at the lower corners are labelled *Hora 7 ante[meridiana]* (left) and *Hora 5 po[meridiana]* (right) [?].

- F (top), H (bottom): ends of the vertical diameter
- B, F₁ (upper centre): vertices of the inner figure
- C (upper left), D (upper right): outer vertices of the inner figure
- E (centre): centre of the circle, below the dotted shadow-triangle

- A (just below E): common point of the fanning rays
- G (left, on the circle), I (right, on the circle): ends of the horizontal diameter
- Peripheral ray-labels: the twelve zodiac signs at the outer ends of the rays, left and right, are small and worn and are not individually secured [?]

[Margin: Another way of finding the same points on the horizontal line, at which the beginnings of the signs rise or set.]

But the same points can be found in this way. From the last figure of proposition 21 of this book, take on ED a straight line Eg equal to the straight line BD in the figure of proposition 38 of this book — the line by which we described the horizontal line — and through g draw a perpendicular to ED. This will be the horizontal line in the superior dial, as is clear if the horizon ABCD of that figure is understood, in the superior dial, to be in its proper position: that is, so that point g coincides with D, and the perpendicular drawn through g becomes the very horizontal line, and consequently the centre E is joined with the centre of the world and with the tip of the style. Therefore the rays drawn from S, V, Y through E will, in that position, cut the straight line drawn perpendicular through g — that is, the horizontal line — at the points where the Sun rises when it is at the beginnings of the southern signs. And the rays from T, X, Z passing through E will cut the same horizontal line at the points where the Sun, when it is at the beginnings of the southern signs, sets. Therefore, if the intervals along the perpendicular through g that lie between g and the said rays are transferred onto the horizontal line of the superior dial, from point D in each direction, the proposed points will be found. Likewise we shall seek these same points in the inferior dial, if from EB of that same last figure of proposition 21 of this book a straight line equal to BD is cut off — as in the figure of proposition 38 of this book — and through its outermost point a perpendicular is drawn to EB, and so on.

[Margin: The method by which the points of the ascending signs are found on the tropics.]

But once the points on the equinoctial line have been found from the first two tables of proposition 9 of this book — the points through which the ascending signs are to be drawn — one can, by the same art, find from the third and fourth tables the points on the tropics through which those same ascending signs are to be produced: namely, if the hidden lines of the hours at which the signs rise are drawn — the hours which the aforesaid tables supply.

[Margin: How the ascending signs are described by the doctrine of Apollonius's *Conics*.]

One must, however, take care in the superior dial not to take the hours of those signs that rise six or more hours before or after midday, for hours of this kind do not fall on the superior dial. In the inferior dial, by contrast, one must take only the hours of those signs that rise more than six hours before or after midday, since these are the ones contained on that dial, as is evident. Indeed, the straight lines tangent to the tropics — drawn as we taught in the corollary to proposition 37 of book 1 of Apollonius — at the points found on the tropics, give the ascending signs, as is clear from what was demonstrated in proposition 9 of this book.

[Margin: The description of the ascending signs by the culminations of the heaven [mediationes coeli] and the angles of the earth.]

Otherwise we shall describe the ascending signs by this art. From the sixth table of proposition 9 of this book we shall take the declinations of the culminations of the heaven for the superior dial, and, for the inferior dial, the declinations of the angles of the earth; and, according to the doctrine of proposition 38 of this book, on the meridian line — that is, the line of the 12th hour, which in the inferior dial is the line of midnight — we shall investigate the points through which the parallels of those declinations are to be drawn. For if we connect these points, by straight lines, with the corresponding points already found on the horizontal line, the ascending signs will be described, as is clear from what has been said.

[Margin: Another description of the ascending signs, by the points of the Ecliptic lying on the circles of the 7th hour before midnight and the 5th hour after midday, when the beginnings of the signs rise above the horizon.]

They can also be described in yet another way — indeed most conveniently — thus. From the tenth table of proposition 9 of this book, take the declinations of all the points of the Ecliptic, both southern and northern, that lie on the circle of the 7th hour before midday; and, according to the doctrine of proposition 38 of this book, on the line of the 7th hour before midday seek the points through which the parallels of those declinations are to be drawn. Again, from the eleventh table of the same proposition 9 of this book, take the declinations of all the points of the Ecliptic lying on the circle of the fifth hour after midday; and likewise, by the doctrine of proposition 38 of this book, seek on the line of the fifth hour after midday the points through which the parallels of those declinations pass. For if these points are joined with the corresponding points found on the line of the 7th hour, one will obtain in the superior dial the ascending signs, by what we demonstrated for the horizontal dial in proposition 9

of this book — signs which necessarily [pass through the points already found on the]

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horizontal line; indeed they will also pass through the points found on the meridian line by the culminations of the heaven. The ascending signs will be delineated in the inferior dial in just the same way, if from the aforesaid tables one takes the declinations of the points of the Ecliptic on the circles of the 7th hour after midday and the fifth hour before midday, and so on. But these points in the inferior dial are exactly the same as those in the superior; for the declinations are the same in either dial, as is clear from the tables.

[Margin: Another description of the ascending signs, from the doctrine of Apollonius's *Conics*.]

And if from the points found on the meridian line — which is the diameter of the hyperbolas — straight lines tangent to the tropics are drawn, according to the doctrine set out in the corollary to proposition 37 of book 1 of Apollonius, one will obtain the same ascending signs, as we showed in proposition 9 of this book. We have therefore described the ascending signs of the Zodiac on the same polar dial. Which was to be done.

SCHOLIVM.

[Margin: The description of these same ascending signs from the practice of Andreas Schöner.]

With Andreas Schöner [Andreas Schöner] we shall lay out the ascending signs thus. In the first figure of the scholium to proposition 9 of this book, on the straight line ED take a straight line Eb equal to the style of the dial, and through b draw a line parallel to AC, so that — the centre E of that figure being placed at the centre E of the circle FGHI on the dial, and the point b at point A — the parallel drawn through b coincides with the equinoctial line. Now this parallel drawn through b will cut the lines of the signs proceeding from E at points; and the intervals of these, taken from b and transferred onto the equinoctial line of the dial from point A (where the meridian line intersects the equinoctial line), will give on the equinoctial line the points through which — if straight lines are drawn to point D, at which the horizontal line cuts the meridian line — the tropics will be cut at the points through which the ascending signs are to be drawn. This, however, in the following order.

[Margin: The order of the ascending signs on the tropics.]

In the superior dial, and in the antemeridian part, which is on the left: the point on the tropic of Cancer [♋] farther from the meridian line belongs to the sign Leo [♌], the nearer to Virgo [♍], and the point on the meridian line itself to Libra [♎]. Next, in the pomeridian part, the point on the same tropic of Cancer [♋] nearer the meridian line belongs to Scorpio [♏], the farther to Sagittarius [♐]. Then, in the antemeridian part of the dial, the point of the tropic of Capricorn [♑], where it is cut by the horizontal line, is to be assigned to the sign Capricorn [♑], the next to Aquarius [♒], the one nearer the meridian to Pisces [♓], and the point on the meridian line itself to Aries [♈]; the next, in the pomeridian part, to Taurus [♉], the following to Gemini [♊], and the point on the horizontal line to Cancer [♋]. But in the inferior dial — since the tropic of Cancer [♋] becomes the tropic of Capricorn [♑], and the tropic of Capricorn [♑] becomes the tropic of Cancer [♋] — the point of the tropic of Cancer [♋] in the pomeridian part, where it cuts the horizontal line, is to be assigned to the sign Capricorn [♑], the next to Aquarius [♒], the next after that to Pisces [♓], and the point on the meridian line itself to Aries [♈]. Then the next, in the antemeridian part, to Taurus [♉], the following to Gemini [♊], and the point on the horizontal line to Cancer [♋]. But the point on the tropic of Capricorn [♑] in the pomeridian part, farther from the meridian line, will belong to Leo [♌], the nearer to Virgo [♍], the point on the meridian line itself to Libra [♎], the next in the antemeridian part to Scorpio [♏], and the following to Sagittarius [♐] — as appears in the figure.

[Margin: The order of the ascending signs on the equinoctial line.]

Again, in the second figure of the scholium to the same proposition 9 of this book, from the straight line ED cut off a straight line Eb equal to the gnomon, and through b draw a line parallel to AC, so that once more — the centre E of the said figure being placed at the centre E of the circle FGHI on the dial, and the point b at point A — the parallel drawn through b coincides with the equinoctial line. This parallel drawn through b will further cut the lines of the signs issuing from E — including those that lie above the line AC but are produced below the centre E — at points whose intervals, taken from b with the compasses and transferred onto the equinoctial line of the dial from point A, will give on the equinoctial line the points through which the ascending signs are to be drawn — in this order, however. In each dial, into the antemeridian part — that is, to the left in the superior dial and to the right in the inferior — are to be transferred the points above point b; and into the pomeridian part, the points below point b, counting also the signs that lie below the centre E and are produced beyond the centre E,

so that in the former part are Taurus [♉], Gemini [♊], Scorpio [♏], Cancer [♋], and Sagittarius [♐]; and in the latter, Leo [♌], Capricorn [♐], Virgo [♍], Aquarius [♑], and Pisces [♓]. When this has been done, if the points found on the tropics are joined with the corresponding points on the equinoctial line, the ascending signs will be delineated, as before. All of which is evident in the figure above. The lines of Aries [♈] and Libra [♎] are to be drawn parallel to the equinoctial line, through the points of the tropics at which they are divided by the meridian line, as was said above.

[Margin: The ascending signs, produced beyond the horizontal line, give the same signs in the inferior dial, if the parts are interchanged, as has often been said.]

Moreover, the lines of the ascending signs in the superior dial, produced beyond the horizontal line, will likewise give the same ascending signs in the inferior dial — provided, however, that the upper part is changed into the lower, and what is then on the right into the left; as is clear from what was demonstrated in the scholium to proposition 14 of this book.

PROBLEMA 46. PROPOS. 46.

To describe the Italian polar dial. [i.e. a polar dial marked with Italian hours]

[Margin: The description of the Italian polar dial.]

The often-mentioned circle FGHI is to be cut — as we taught in the scholium to proposition 1 of book 1 — into the diurnal and nocturnal arc of the tropic of Cancer [♋]; and let the diurnal arc be aFb, such that the straight line ab, the common section of the parallel and the horizon, is parallel to the equinoctial line. Then let the circle FGHI be divided into 24 equal hours, beginning from the western end of the diurnal arc, namely from point a. For if, from the points of division, hidden straight lines are drawn through the centre — lines which, by what was demonstrated in proposition 10 of this book, are the common sections of the Equator and of the great circles passing through the poles of the world and through the Italian hours of the parallel of Cancer [♋], after the manner of hour-circles reckoned from midday or midnight — then the equinoctial line will be cut at points through which perpendiculars to it, drawn out, will cut the tropic of Cancer [♋] at the Italian hours, that is, at the hours reckoned from sunset (since these perpendiculars are the hour-lines from sunset, the Sun being at the beginning of Cancer [♋]), just as if they were the common sections of the plane of the dial and of the great circles passing through the poles of the world and through the Italian hours on the tropic of Cancer [♋], as is clear from what was

demonstrated above. But if, through the centre of the circle FGHI, the straight line GI is drawn parallel to the equinoctial line, the hours in the arcs aG and bI (which in our example are 23, 10, and 9) will not fall on the plane of the superior dial, since the circle of the 6th hour from midday or midnight — the circle from which the plane of the dial is equidistant — is drawn along the straight line GI; as is clear if the circle FGHI is understood to be set in its proper position, so that its centre is the same as the centre of the world and the tip of the style. In the same way, if the straight line ab is transferred beyond the centre of the circle to de, so that dFe is the diurnal arc of Capricorn [♋], and the circle is again cut into 24 hours, beginning from point d — that is, from the western end — we shall find on the tropic of Capricorn [♋] the points of the Italian hours. Therefore, if the points of the Italian hours on the tropics that correspond to one another are joined by straight lines, they will be describ[ed]

In this way the Italian polar dial will have been described.

Now the Italian hours pass through the points of the hours from noon or midnight on the equinoctial line. Thus the 23rd Italian hour passes through the 5th from noon, the 22nd through the 4th, and so on, as is clear from the table of proposition 19 of the preceding book. Hence, if some Italian hours in one of the tropics have no corresponding points in the other tropic, those hours must be drawn through the hours from noon or midnight that correspond on the equinoctial line, as is gathered from the said table of proposition 19 of the preceding book. Thus you see that the Italian hours 13 and 14 are drawn through points found in the tropic of Cancer [♋], and through hours 7 and 8 from midnight on the equinoctial line, because in the tropic of Capricorn [♏] they have no points corresponding to those found in the tropic of Cancer [♋]. In the same way hour 23 is drawn through a point found in the tropic of Capricorn [♏], and through hour 5 from noon on the equinoctial line, and so for the rest.

The line of hour 12 from sunset must be drawn parallel to the equinoctial line, through the point found in the tropic of Cancer [♋]. For since the circles of hours 12 and 24 from sunrise or sunset have the same common section with the Equator (by proposition 16 of the preceding book), and likewise the same section with the circle of the 6th hour from noon or midnight (by proposition 17 of the same book), it follows that those two circles, the Equator, and the circle of the 6th hour from noon or midnight all have one and the same common section — namely the one through which the circle of the said 6th hour cuts the Equator. Since, then, the plane of the polar dial is equidistant from this circle, and so from the said common section of all of them, the sections of the remaining circles with

the plane of the dial — such as the lines of hours 12 and 24 from sunrise or sunset, and the equinoctial line — will (by proposition 18 of the preceding book) be parallel to one another. And this is to be understood of the superior dial. For the inferior dial it will be enough to derive it from the superior, as we shall teach in the following proposition.

Diagram. The construction figure of the Italian (and, by the following proposition, Babylonian) polar dial. A large circle carries the four cardinal points **F** (top), **G** (left, on the horizontal line), **I** (right, on the horizontal line), and **H** (bottom). The horizontal line running across the middle is labelled **HORI-** at the left and **ZON** at the right (i.e. "HORIZON," the horizontal line), and is the equinoctial line of the dial. A small point **f** stands just above **F** (the pole of the world). Within the circle two opposed hyperbola-like arcs (the diurnal arcs of the two tropics) meet in a dense cross-hatched web at the centre; their end-letters **d** (upper left) and **e** (upper right) mark the tropic of Capricorn arc, with **a/b** on the tropic of Cancer arc (**b** legible at centre-right). The central points **B** (centre-left), **b** (centre-right), and **K, L** (lower centre) lie in the cross-hatched region where the hour-lines converge, with **D** near the top of the inner web. Radiating hour-lines run out to the circumference and beyond (some dotted), each tipped with an hour-number.

- **F** (top of circle): **F** — meridian / noon point at top
- **f** (above **F**): **f** — pole of the world
- **G** (left, on horizontal line): **G**
- **I** (right, on horizontal line): **I**
- **H** (bottom of circle): **H**
- **d** (upper-left, inner): **d** (end of Capricorn diurnal arc)
- **e** (upper-right, inner): **e** (end of Capricorn diurnal arc)
- **b** (centre-right): **b** (end of Cancer diurnal arc)
- **B** (centre-left): **B**
- **D** (upper-centre of web): **D**
- **K, L** (lower centre): **K, L**
- Inner web hour-numbers (top row): 8, 9, 10, 11, 12, 1, 2, 3, 4, 5, 6 [several worn — ?]
- Inner web hour-numbers (lower row, near **K**): 9, 10, 11, 12, 1, 2, 3 [worn — ?]
- Left peripheral hour-ends: 24, 23, 14 [?], 12, 11, 8, 7 [worn — ?]

- Right peripheral hour-ends: 24, 10 [printed "01" — ?], 11, 23, 12, 22, 13 [worn — ?]
- Dotted diagonal rays tipped: s (upper right), s (lower right), 7/8 (upper left), 4 (lower right) [worn — ?]

[Many peripheral hour-numbers are worn/foxed and only partly legible]

[Margin: Another description of the Italian polar dial by means of the diurnal arcs of hours 14 and 10.]

When the points have also been marked on the lines of the hours from noon or midnight, according to the teaching of proposition 38 of this book — points through which are drawn parallels, one of hour 14 and the other of hour 10 — we shall, by means of these and of the tables we composed in the scholium to proposition 33 of the preceding book, describe the Italian dial, just as we set forth in the preceding dials.

[Margin: Yet another description of the same Italian polar dial by means of the diurnal and nocturnal arc of the 24 hours.]

Again, we shall most conveniently construct the same dial by means of two parallels — one of which is the greatest of the ever-apparent circles, that is, the diurnal arc of the 24 hours; the other the greatest of the ever-hidden circles, that is, the nocturnal arc of the 24 hours — if points are marked on the hours from noon and midnight through which they pass. For the hours from sunrise or sunset touch the said parallels (drawn on the plane of the dial) at the points through which the hours from noon or midnight are drawn — except hours 6 and 18, which do not coincide with the said hyperbolas, as is clear from the corollary of proposition 15 of the preceding book. Which hours from sunset touch the diurnal arc of the 24 hours, and which touch the nocturnal arc of the 24 hours, and at which hours from noon or midnight, the tables of the diurnal arc and of the nocturnal arc of the 24 hours in the scholium to proposition 33 of the preceding book indicate beautifully. For since the superior polar dial contains only hours 7, 8, 9, 10, 11, and 12 from midnight, and likewise 1, 2, 3, 4, and 5 from noon, from each table there must be taken the hours from sunset that correspond to the said hours, and no others. Thus from the table of the diurnal arc of the 24 hours these hours from sunset are to be taken: 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, and 17 — for these touch the said arc at hours 7, 8, 9, 10, 11, and 12 from midnight, and 1, 2, 3, 4, and 5 from noon. But from the table of the nocturnal arc of the 24 hours there are to be taken

these hours from sunset: 1, 2, 3, 4, 5, 19, 20, 21, 22, 23, and 24. For these touch the said arc at hours 1, 2, 3, 4, and 5 from noon, and at 7, 8, 9, 10, 11, and 12 from midnight, as is clear from the aforesaid tables.

Finally, we shall make the same Italian dial by means of the line of hour 12 from sunrise or sunset — as is clear from the second table of proposition 20 of the preceding book, whose title is *The Line of Hour XII from Sunrise or Sunset*.

Likewise by means of the line of hour 24 from sunrise or sunset — as is gathered from the first table of proposition 20 of the same preceding book, whose title is *The Line of Hour XXIV*, etc. — which is indeed entirely the same as the horizontal line, as is evident. Or also by means of both lines, if the points of the hours from noon or midnight, through which the Italian hours are to be drawn, are carefully noted in the aforesaid tables, and so on.

[Margin: Yet another description of the Italian polar dial by means of the line of hour 12 and the line of hour 24 from sunrise or sunset.]

[Margin: How the line of hour 12 from sunrise or sunset is to be drawn in the polar dial.]

Moreover, we shall describe the line of hour 12 from sunrise or sunset in this way. On the equinoctial line, from point A, where it intersects the meridian line, we shall take the straight line AB equal to the style — namely the straight line between the meridian line and the line of hour 3 or 9 from noon or midnight, for this is equal to the style, as we showed in the scholium to proposition 25 of this book. Then, from B, the end of that line, drawing a perpendicular Bf to the equinoctial line (such as the very line of hour 9 or 3, produced), we shall describe from the same point B as centre a circle, on which we shall take an arc fg equal to the altitude of the pole; and from g, through the centre B, we shall draw a straight line, which will cut the meridian line at point k. A line drawn through k parallel to the equinoctial line will be the line of hour 12 from sunrise or sunset. For let the circle fg be understood to be turned about the meridian line FH until it coincides with the meridian circle, and its centre B is joined with the centre of the world, that is, with the tip of the style. This done, f will be the pole of the world, and g the point at which the circle of hour 12 from sunrise or sunset touches the greatest of all the ever-apparent parallels and cuts the meridian — since this parallel is distant from the pole by as many degrees as are contained in the altitude of the pole. Therefore a straight line sent out from g through the centre of the circle fg will be the common section of the said circle of hour 12 and of the meridian; and so the circle of hour 12 will cut the plane of the dial at point K, where the said straight line breaks off the meridian line. Since, then, by the

scholium to proposition 22 of the preceding book the line of the said hour 12 and the equinoctial line are parallel, it clearly follows that a line drawn through point K parallel to the equinoctial line, cutting the meridian line, is the line of hour 12 from sunrise or sunset — which is what was proposed.

[Margin: Another description of the line of hour 12 from sunrise or sunset. — cited as [Elements I.15].]

[Margin: Yet another description of the line of hour 12 from sunrise or sunset. — cited as [Elements I.26].]

[Elements I.15] Now since the angle fBg of the pole's altitude is equal to the vertical angle KBL , it comes about that if on the equinoctial line we take a straight line equal to the gnomon, and at its end set up, toward the equinoctial line, the angle of the complement of the pole's altitude — that is, the angle left over from the right angle ABK — we shall find on the meridian line the same point K, through which the line of hour 12 from sunrise or sunset is to be drawn. Indeed, if we cut off from the straight line AD a part equal to AK , we shall have the same point K. For since the angle ABD is set equal to the complement of the pole's altitude (as is clear from proposition 38 of this book), and the angle ABK is likewise equal to the same complement (as we showed), the two angles ABD and ABK will be equal in the triangles ABD and ABK ; the angles at A are also equal, being right angles; and the side AB , adjacent to the said angles, is common. [Elements I.26] Therefore the sides AD and Ak will be equal. And so, since the line of hour 12 from sunrise or sunset passes through K (as we demonstrated), it is evident that, if a part AK equal to AD is taken, the said line of hour 12 must be drawn through K — which line at K touches the greatest of the ever-apparent parallels. And so we have described the Italian polar dial. Which was to be done.

PROBLEMA 47. PROPOS. 47.

To delineate the Babylonian polar dial.

[Margin: Description of the Babylonian polar dial.]

We shall describe this dial by almost the same reasoning as we used to delineate the Italian one — provided, however, that in the first method the division of the circle FGHI into 24 equal parts begins from the eastern end of the diurnal arc, of both the tropic of Cancer [♋] and the tropic of Capricorn [♎] — that is, at point b and at point e. But the line of hour 12 must be drawn parallel to the equinoctial line, through the point found in the tropic of Cancer [♋], as before.

In the second method there is no difference at all, provided only that in place of the hours from sunset mentioned in the preceding proposition the same hours from sunrise are taken, and so on. You have an example of the Babylonian dial in the figure of the preceding proposition, where the lines parallel to one another and perpendicular to the equinoctial line indicate the hours from noon or midnight, whose numbers are assigned to the diurnal arc of the 24 hours and to the nocturnal arc of the same number of hours. Next, the lines to which numbers are attached below the equinoctial line indicate the Italian hours. Finally, the lines to which numbers are attached above the equinoctial line are to be assigned to the Babylonian hours, that is, to those reckoned from sunrise. Thus we have delineated the Babylonian polar dial. Which was to be done.

SCHOLIVM.

[Margin: How the Italian polar dial differs from the Babylonian polar dial.]

From the figure of the preceding dial it is clear that the Babylonian dial differs from the Italian only in position and in the order of the hours, just as we explained in the case of the horizontal dial.

[Margin: For what reason the inferior polar dial, both the Italian and the Babylonian, has its origin from the superior.]

The inferior polar dial arises from the superior if the lines of all the hours are produced beyond the horizontal line, and the upper part is changed into the lower, and what is on the right after this change is turned into the left, as we showed in the scholium to proposition 14 of this book. Which hours in the inferior dial are to be numbered from sunset, and which from sunrise, we shall learn from what we wrote in the scholium to proposition 23 of this book.

It should also not be passed over here that, in the region where the pole's altitude is equal to the complement of the Sun's greatest declination — that is, 66 degrees 30 minutes — the two tropics serve as the two parallels, of which one is the greatest of the ever-apparent circles and the other the greatest of the ever-hidden. Hence the lines of the hours from sunrise or sunset will touch the two tropics at the hours from noon or midnight, both in the meridian dial and in the polar. But in the region where the pole's altitude is greater than 66 degrees 30 minutes, the two aforesaid parallels fall within the two tropics, so that the lines of the hours from sunrise or sunset touching those parallels

[Note (foot of page, full-width): In the latitude of a place of 66 degrees 30 minutes, the hours from sunrise and sunset touch the two tropics. But in a greater latitude

the hours from sunrise and sunset do not reach the tropics, but touch other parallels nearer to the equinoctial line, both in the meridian dial and in the polar.]

in no way reach the tropics themselves. For the shadow of the style cannot then show the hours elapsed from the Sun's rising or setting, since the Sun, being in the tropics, neither rises nor sets, but remains perpetually above the horizon, or below it.

PROBLEMA 48. PROPOS. 48.

To construct the antique polar dial. [i.e. a polar dial of the ancient unequal (planetary) hours]

[Margin: Description of the antique polar dial.]

Let the diurnal arc — both of the tropic of Cancer [♋], aFb, and of the tropic of Capricorn [♏], dFe — be cut, in the circle FGHI, into 12 equal parts. For from these we shall find, in each tropic, the points through which the unequal hours are to be drawn — no differently from the way in which, in the two preceding propositions, we investigated the points of the hours from the Sun's rising and setting in the same tropics. The unequal hours will necessarily pass through the points of the hours from noon or midnight on the equinoctial line, as we taught above in the case of the horizontal dial, proposition 12 of this book. It is certain, moreover — as appears from the attached figure — that hours 1 and 11 do not fall on the superior dial, since the Sun is at the beginning of Cancer [♋].

[Margin: Description of the antique polar dial by means of the diurnal arcs of 6 and 18 hours.]

By means also of two diurnal arcs drawn on the dial — one comprising 18 hours and the other six — we shall construct the same antique dial, as was done above in propositions 12, 24, and 36 of this book. This the attached figure plainly teaches. Thus we have constructed the antique polar dial. Which was to be done.

Diagram. The construction figure of the antique polar dial (dial of the unequal / planetary hours). A large circle carries the cardinal points **F** (top), **G** (left, on the horizontal line), **H** (bottom), and **I** (right, on the horizontal line). The horizontal line across the middle is labelled **HORI-** (left) and **ZON** (right), i.e. the horizontal / equinoctial line. From the top region (near F) a fan of dotted rays sweeps down to the lower centre — the lines of the unequal hours converging. The unequal-hour lines run out to the circumference and beyond, each tipped with a Roman-

numeral hour; the Arabic numerals along the top and middle mark the ordinary hour divisions.

- F (top of circle): F
- G (left, on horizontal line): G
- H (bottom of circle): H
- I (right, on horizontal line): I
- q (lower-left, near G): q
- h (centre-right): h [?]
- Roman-numeral hour-ends (upper left → left → lower left): IX, I, II, III
- Roman-numeral hour-ends (upper right → right → lower right): I, XII, XI, X
- Arabic hour-numbers along top (left to right): 7, 8, 6, 5, 4, 3 [order/values partly worn — ?]
- Arabic hour-numbers along middle row: 8, 9, 10, 11, 12, 1, 2, 3, 4, 5
- Small Roman numerals along the lower centre: IIII, V, VI, VII, VIII, IX

[Several peripheral hour-numbers are worn/foxed]

SCHOLIVM.

[Margin: How the inferior antique polar dial arises from the superior.]

Moreover, here too the lines of the unequal hours, produced beyond the horizontal line, exhibit the lines of the same hours in the inferior dial, provided only that the portion of the dial cut off by the horizontal line is inverted in all its parts, as was said and demonstrated in the scholium to proposition 14 of this book.

DE HOROLOGIIS ÆQVINOCTIALIBVS.

On Equinoctial Dials

PROBLEMA 49. PROPOS. 49.

To describe the astronomical equinoctial dial — that is, to lay out, on a plane equidistant from the equinoctial circle, the lines of the hours from noon or midnight.

[Margin: Description of the equinoctial astronomical dial.]

[Margin: How the equinoctial dial is to be placed so that it may show the hours.]

On some plane let a straight line AB be drawn in any way, to serve as the meridian line, or the line of hour 12; and let another straight line CD cut it at right angles at E. From E let a circle of any size be described and divided into 24 equal parts, beginning either from CD or from the straight line AB; and through the points of division, and through the centre E, let straight lines be drawn out. I say that these straight lines are the lines of the hours from noon or midnight on a plane equidistant from the Equator. So much so that if at E a style EF is set up — of any length, perpendicular to the plane of the dial — and the dial itself is so placed that CD is equidistant from the horizon, and AB is set in the plane of the meridian circle as the common section of the plane of the dial and of the meridian; and finally point A on the southern side (with point B inclining toward the north, C toward the east, and D toward the west) is raised according to the altitude of the Equator — that is, according to the complement of the pole's altitude — so that the angle which the straight line AB makes with the meridian line found in the plane of the horizon, toward the north (that is, toward B), is equal to the complement of the pole's altitude: then the shadow of the style will indicate the hours from noon or midnight on the superior dial, which of course faces the Zenith, and is lit by the Sun when it is in the northern signs, and is parallel to the Equator. But if on the opposite face of the dial thus raised the same lines are understood to be described, corresponding exactly to these lines of the superior dial, and the style EF is set up on it at right an-

angles, the shadow of the style will indicate the same hours on the inferior dial — the one, that is, which is lit by the Sun as it runs through the southern signs. And this will be proved by the following reasoning.

[Margin: Demonstration of the construction of the astronomical equinoctial dial.]

Since the point F, the tip of the style, is the same as the centre of the world (by proposition 2 of the preceding book), it follows that, if about the world's centre F we imagine the equinoctial circle — the circle from which the plane of the dial is equidistant — set in its proper position, then the style EF is the axis of the world. For the axis of the world stands perpendicular to the plane of the Equator at the centre of the world; and the style EF, which is perpendicular to the plane of the dial, is also perpendicular to the plane of the Equator, since (as we proved in the scholium to proposition 14 of Euclid's Book XI) these two planes are set parallel to each other. Therefore the style EF is entirely a portion of the world's axis.

Now the hour-circles reckoned from noon or from midnight always cut the greatest of the ever-visible parallels — and so, by proposition 10 of Theodosius's Book II, they cut the Equator, which is the greatest of all the parallels, and cut that

parallel of the sphere in which the plane of the dial lies (whose centre is E, since by proposition 10 of Theodosius's Book I the axis EF necessarily passes through its centre) — into 24 equal parts. All these hour-circles pass through the axis of the world, and hence through the point E, where (by proposition 18 of the preceding book) their common sections with the said parallel intersect one another.

Therefore the straight lines drawn out from E to the points of division of that parallel in which the dial lies are the common sections of the hour-circles reckoned from noon or midnight with the said parallel. Now since these straight lines also divide (from what we proved near the end of chapter 1 of *The Sphere*) the circle ACBD — which is concentric with that parallel of the sphere — into 24 equal parts, these same straight lines, drawn out through the centre E and through the points of division of the circle ACBD, are the common sections of the hour-circles reckoned from noon or midnight with the plane of the dial: that is, they are the hour-lines from noon or midnight. So that AB is the section of the meridian with the plane of the dial; the straight line drawn next through E, from the point after A toward D, is the common section of the plane of the dial with the circle of the first hour from noon or midnight; and so on. We have therefore described the astronomical equinoctial dial, and so forth. Which was to be done.

Diagram. A full circle, the astronomical equinoctial dial (circle ACBD), centred on E, with the tip of the style F set just above the centre. The circle is divided into 24 equal sectors by straight lines drawn through the centre E; these are the hour-lines. Four cardinal points are lettered around the rim.

- E (centre): the centre of the dial = the centre of the world, foot of the style.
- F (just above E): the tip of the style EF (identified with the world's centre in the text).
- A (top of rim): endpoint of the meridian line AB; the noon point, hour 12.
- B (bottom of rim): the opposite endpoint of AB; the midnight point, hour 12.
- C (left of rim): endpoint of the line CD; hour 6.
- D (right of rim): endpoint of the line CD; hour 6.
- Outer ring of hour-numbers. Near the top, flanking A: "II" (left) and "I" (right), with "12" set just below A. Down the right side: 2, 3, 4, 5, then "6" at D, then 7, 8, 9, 10, 11 toward the bottom. At the bottom, "12" at B. Up the left side from the bottom: 1, 2, 3, 4, 5, then "6" at C, then 7, 8, 9, 10, 11 toward the top.

[Several of the peripheral hour-numbers are lightly foxed/worn but legible; the two topmost are printed as Roman "II"/"I" where the rest of the ring uses Arabic digits — transcribed as printed.]

SCHOLIUM.

[Margin: How the same equinoctial dial is to be constructed on a fixed plane in which the place of the style is fixed.]

If the dial is to be described on some fixed plane equidistant from the Equator, and the place of the style — wherever it may be — is given at the point E, then with the help of a level and a plumb-line one must draw through E the straight line CD, equidistant from the horizon, which the straight line AB cuts at right angles in E. For this AB will be the meridian line. Everything else is to be carried out as before.

PROBLEMA 50. PROPOS. 50.

To lay out the parallels — that is, the arcs of the signs of the Zodiac — on the aforesaid equinoctial dial.

[Margin: Another way of describing the arcs of the signs on the same equinoctial dial, from the analemma.]

Take again the first figure of proposition 26 of this book, in which, on the axis EG, the straight line Ea is cut off equal to a style of any assumed size, and through a line is drawn parallel to HI, the diameter of the Equator; this line will be the common section of the plane of the dial with the meridian, since the plane of the dial, drawn through that parallel, is equidistant from the plane of the Equator (drawn through the straight line HI) by the length of the style Ea. Now let the diameters of the Zodiac that join opposite signs be produced, so as to make triangles through the axis in the cones, whose bases are parallels of the Equator and whose common vertex is the centre E. The parallel portions drawn through a, intercepted between the sides of these triangles, are the diameters of the conic sections — which, by the corollary to proposition 4 of the preceding book, are circles having their centres at the point E in the dial, that is, on the axis of the world, by the same proposition 4 of the preceding book. Therefore, if from a the half-diameters are taken as far as the sides of the triangles through the axis, and with these intervals circles are described from E, the centre of the dial, the parallels of the northern signs of the Zodiac will be described on the superior dial — since the radii of the northern signs fall, when produced, on that parallel through the centre E. And if on the axis EF one takes again the straight line Ee equal to the style, and does the same, the parallels of the southern signs of the Zodiac will be described in the same way on the inferior dial, since the radii of the southern signs fall on the parallel drawn through e when they are extended

through the centre E. But it is enough if circles are described in the inferior dial with the same half-diameters taken from a. For the half-diameters contained in the parallel drawn through a are equal to the corresponding half-diameters contained in the parallel drawn through e. For example, in the triangles Eab and Eeg, the angles at the points a and e are right,

and the alternate angles b and g are equal, on account of the parallel lines drawn through a and through e; and the sides Ea, Ee are equal; therefore the sides ab and eg are also equal — namely the half-diameters of the conic sections; and the demonstration is the same in the remaining cases.

[Margin: a. Elements I.29. b. Elements I.26.] [Margin: Another description of the arcs of the signs on the same equinoctial dial.] [Margin: Demonstration of this latter description of the parallels of the Sun's signs.] [Margin: c. Elements I.26.] [Margin: It is enough, in the latter method, if just the single parallel hl, or qn, is drawn.]

We shall describe these same parallels from the last figure of proposition 26 of this book, in the following way. On the axis DG let the straight lines Dh and Dq be cut off on either side, equal to the style, and through h and q let hl and qn be drawn parallel to DC, the radius of the Equator. The straight lines intercepted between the points h, q and the radii of the signs are equal to the half-diameters of the conic sections found earlier. For example, in the triangles Eab, Dhm, and Dqo — the first of which is contained in the earlier figure, the latter two in the later one — the angles at the points a, h, q are right, and the angles aEb, hDm, qDo are equal, since each of them is equal to the complement of the declination of the parallel taken, as is clear; and the sides Ea, Dh, Dq are equal by construction; therefore the sides ab, hm, qo are also equal. From which it appears that it is enough to draw a single parallel — namely either hl or qn — in order to find the half-diameters of the circles, or of the conic sections,

Diagram. The equinoctial dial shown together with the horizon and the parallels of the zodiacal signs. As in the previous figure, the outer circle (ACBD) is centred on E, with the point F just above the centre and the rim divided into 24 hour-sectors; the same cardinal letters appear — A at the top (12), B at the bottom (12), C at the left (6), D at the right (6). A horizontal straight line runs across the figure through the centre E and is lettered as the word **HORIZON** (split "HORI ... ZON" to left and right of the centre): this is the horizon line. Within the outer circle are drawn several concentric circles — the parallels of the zodiacal signs (declination circles) — each carrying a zodiac-sign glyph where the sign's parallel is marked, arranged symmetrically about the vertical and about the horizon line.

- A (top): meridian endpoint, hour 12.
- B (bottom): meridian endpoint, hour 12.
- C (left, on the horizon line): hour 6.
- D (right, on the horizon line): hour 6.
- E (centre); F (just above centre, on the meridian).
- Outer hour-ring, as in the previous figure: II / 12(A) / I across the top; 2, 3, 4, 5, 6(D) down the right; 12(B) at the bottom; 1, 2, 3, 4, 5, 6(C) up the left; 7, 8, 9, 10, 11 completing each side.
- Zodiac-sign glyphs on the inner parallels (legibility mixed): clearly read are ♊ [Pisces], ♒ [Aquarius], ♍ [Virgo], ♋ [Cancer], ♊ [Gemini], ♌ [Leo], ♉ [Taurus], ♏ [Scorpio]; several further glyphs around the ring are worn/foxed and could not be resolved [?].

[More than two zodiac glyphs on this figure are unresolved]

[Margin: The equinoctial line cannot be drawn on an equinoctial dial.] [Margin: What is to be done so that the shadow may indicate the hours near the time of the equinoxes.]

—on both dials. For since it has been shown that the half-diameters of each parallel are equal to the half-diameters of the parallel drawn through a in the earlier figure, these too will be equal to one another. Now it is plain, from the scholium to proposition 11 of the preceding book, that the equinoctial line cannot be drawn on an equinoctial dial, but vanishes entirely, because the Equator is equidistant from the plane of each equinoctial dial. But if one wishes to observe the hours near the time of the equinoxes — that is, before or after the equinoxes — the hour-lines must be produced beyond the outermost circle, the circle which on the superior dial is assigned to the signs Taurus [♉] and Virgo [♍], and on the inferior dial to the signs Scorpio [♏, glyph worn — the sign opposite Taurus] and Pisces [♊]; they must be produced, I say, as far as the size of the plane on which the dial is described will allow. Or else a circle of some degree of the Ecliptic near the equinoctial circle must be described — for instance on the superior dial the 25th degree of Virgo [♍] and the 5th degree of Aries [♈], and on the inferior dial the 25th degree of Pisces [♊] and the 5th degree of Libra [♎], and so forth — because around the times of the equinoxes the shadow of the style is made very long, and at the equinoxes themselves it becomes infinite and equidistant from the plane of the dial, as is clear from what was said in the scholium to proposition 11 of the preceding book. The same is to be understood of the hours reckoned from sunrise and sunset, which we shall treat in propositions 58 and 59 of this book.

[Margin: In what way the horizontal line is to be described on an equinoctial dial.]

The horizontal line — onto which the shadow of the style is cast at sunrise or sunset — is described thus. In the first figure of proposition 26 of this book let the points h and i be marked, where BD, the diameter of the horizon, cuts the parallels drawn through a and through e; and let the straight line ah, or ei, be transferred onto the meridian line of the superior dial above the place of the style, from E, as far as m — but below the place of the style in the inferior dial — and through m let a line be drawn parallel to the line of the sixth hour. This will be the horizontal line. For since the horizon, drawn through BD, meets the plane of the superior dial (drawn through ah, the meridian line, at the point h) and meets the plane of the inferior dial (drawn through ei, the meridian line, at the point i), it is plain that, if we transfer the straight line ah from the place of the style in the superior dial onto the meridian line above the place of the style, as far as the point m, then the horizontal line passes through m — because in the superior dial, at the point h, the common sections of the horizon and of the meridian, made by the plane of the dial cutting them, come together, by proposition 18 of the preceding book, seeing that the common section of the horizon and the meridian (that is, the straight line BD) meets the plane of the superior dial at h; and sections of this kind are the horizontal line and the meridian line. And since the horizon, the circle of the sixth hour from noon or midnight, and the Equator all have one and the same common section, from which the plane of the dial is equidistant (being equidistant from the Equator, in which that common section lies), the common sections of the remaining circles with the plane of the dial will be parallel — namely the horizontal line and the line of the sixth hour — by proposition 18, or by the scholium to proposition 22 of the preceding book. There-

—fore the straight line drawn through m, parallel to the line of the sixth hour, is the horizontal line — that is, the common section of the horizon and the plane of the dial. In the same way, if we transfer the straight line ei onto the meridian line of the inferior dial below the place of the gnomon, we shall have the point through which the horizontal line is to be drawn. But since the straight line ei is equal to the straight line ah — for in the triangles Eah and Eei the angles at a and e are right, the vertical angles at E are equal, and the sides Ea, Ee are equal, so that the sides ah, ei are also equal — for this reason we directed that the same line ah, or ei, be transferred in each dial onto the meridian line above or below the place of the style.

[Margin: a. Elements I.15. b. Elements I.26.] [Margin: Another description of the horizontal line on the equinoctial dial.]

The same point *m*, through which the horizontal line — parallel to the line of the sixth hour — is to be drawn, will be found by this device. On the line of the sixth hour of the dial take *El* equal to the style, and make the angle *Elm* equal to the altitude of the pole, so that the straight line *lm*, cutting the meridian line at *m*, falls upward in the superior dial but downward in the inferior. This will readily be done if from *l* an arc of a circle is described — either above the straight line *CD* or below it, toward the meridian line — and on it, starting from the line *CD*, the arc of the pole's altitude is measured off; then from the extreme point of that arc a straight line is drawn to *l*, cutting the meridian line at the point *m*. For through the point *m* the horizontal line is to be drawn, as before. For imagine the triangle *Elm* turned about the meridian line until it stands perpendicular to the plane of the dial and lies in the plane of the meridian, so that the point *l* is joined with the tip of the style — that is, the centre of the world — and the straight line *El* coincides with the style, the axis of the world. Then the horizon will pass through *l*, the tip of the style, and hence through the straight line *lm* — because the angle *Elm* is equal to the altitude of the pole, which is exactly the angle that the common section of the horizon and the meridian makes with the axis at the centre of the world *l*. Moreover, the remaining angle *Eml* is equal to the complement of the pole's altitude — exactly the angle that the horizon makes with the Equator in the plane of the meridian circle; and this angle is equal to the angle that the horizon makes with the plane of the dial (which is parallel to the Equator) in the same plane of the meridian. For since the sections which the meridian makes with the Equator and with the plane of the dial parallel to it are parallel, the common section of the horizon and the meridian will make equal angles with the said sections — one of which is the meridian line in the Equator, the other in the plane of the dial, the outer one and the inner one, that is. Therefore the horizon will cut the plane of the dial at *m*; and so the horizontal line is to be drawn through *m*. We have therefore laid out the parallels — that is, the arcs of the signs of the Zodiac — on the aforesaid equinoctial dial. Which was to be done.

[Margin: c. Elements I.16, from which. d. Elements I.29.]

SCHOLIUM.

[Margin: How the portion of the equinoctial dial cut off by the horizontal line makes the inferior dial.]

The portion of the dial which the horizontal line cuts off will itself be the inferior dial, if everything is inverted — as was done in what precedes — that is, if the horizontal line takes the upper place on the lower face of the plane, and the part which in that position is on the right is turned to the left, as we showed in the scholium to proposition 14 of this book. But the arcs of the northern signs are changed into the arcs of the opposite southern signs, because the shadow of the style falls on the inferior dial when the Sun is in the southern signs, as is evident.

[Margin: The same portion cut off in the superior dial is the nocturnal dial.]

The portion cut off by the horizontal line also gives us the nocturnal dial, as was said in what precedes.

[Margin: The arcs of the signs on the equinoctial dial are divided into equal parts by the hour-lines from noon or midnight.]

Now since, as we showed in the preceding proposition, the hour-circles from noon or midnight divide any circle whatever described from the centre E (for the same demonstration that fits the circle ACBD described there fits every circle) into equal parts, it follows that on the equinoctial dial the lines of these same hours cut the parallels of the signs into equal parts, since these are circles described from the same centre E, as has been proved in this proposition. This happens on no other dial. For on the other dials the hour-lines from noon or midnight intercept unequal spaces on the parallels of the signs there described.

[Margin: Description of the arcs of the signs on the same equinoctial dial.]

PROBLEMA 51. PROPOS. 51.

To describe the parallels of the diurnal arcs on the same equinoctial dial.

[Margin: The diurnal arcs on the equinoctial dial are divided into equal parts by the hour-lines from noon or midnight.]

If, in place of the parallels drawn through the beginnings of the signs, one takes the parallels of the diurnal arcs, and for the radii of the signs the radii of the diurnal arcs, then the parallels of the diurnal arcs will be described on the equinoctial dial in just the way we described the parallels of the signs in the preceding proposition. We have therefore described the parallels of the diurnal arcs on the same equinoctial dial. Which was to be done.

PROBLEMA 52. PROPOS. 52.

To lay out the vertical circles on the same equinoctial dial.

[Margin: How the vertical circles are described on the same dial.]

In the first figure of proposition 26 of this book let the points n and o be marked, where AC , the diameter of the vertical, cuts the parallels drawn through a and through e ; and let the straight line an , or eo , be transferred onto the meridian line from the place of the style — downward in the superior dial, but upward in the inferior — as far as p . The point p will be the vertical point, onto which the axis of the horizon falls, since AC , the axis of the horizon (or diameter of the vertical), cuts the plane of each dial through the parallels an , eo drawn at the points n and o — below the place of the style E in the superior dial, but above it in the inferior. And since the straight line an is equal to the straight line eo — for in the triangles Ean and Eeo the angles at a and o are right, the angles at the vertex E are equal, and the sides Ea , Ee are equal, so that the sides an , eo are also equal — for this reason we directed that the same line an , or eo , be transferred in each dial onto the meridian line below or above the place of the style.

[Margin: e. Elements I.15. f. Elements I.26.]

We shall also find this vertical point p by the following device. From E , the place of the style, let the perpendicular EF be drawn to the meridian line, equal to the style, and at the point F let the angle EFp be made equal to the complement of the pole's altitude, so that the straight line Fp cuts the meridian line at the point p — below the place of the style in the superior dial, but above it in the inferior. I say that p is the vertical point. For imagine the triangle EFp turned about the straight line Ep until it stands perpendicular to the plane of the dial, and hence lies in the plane of the meridian, so that the point F is joined with the tip of the style, the centre of the world, and the straight line EF itself coincides with the style, the axis of the world.

The vertical circle properly so called will pass through the tip of the style — that is, through F , the centre of the world — and therefore along the straight line Fp . This is because the vertical circle makes, with the axis of the world in the plane of the meridian, an angle equal to the complement of the pole's altitude, namely the angle EFp ; and with the meridian line in the equator it makes an angle equal to the pole's altitude, as is evident. That latter angle is equal to the angle the same vertical circle makes with the meridian line of the dial-plane parallel to the equator, namely the angle EpF . [a] For the sections that the meridian makes with

the equator and with the dial-plane parallel to it are themselves parallel; and therefore[b] the common section of the vertical and the meridian makes equal angles with the said meridian lines — the external, that is, and the internal. For this reason the vertical circle properly so called will cut the dial-plane at p.

[Margin: a. Elements XI.16. b. Elements I.29.]

After this, draw the straight line Fm making the angle EFm equal to the pole's altitude, so that it cuts the meridian line at the point m — by means of which we showed, near the end of proposition 50 of this book, how the horizontal line is to be drawn. On the meridian line take me equal to Fm, whether below the horizontal line or above it. Then from e as centre describe a circle of any size and divide it into 360 degrees, or into fewer parts if the size of the dial demands (we divided it into 24, so that each part contains 15 degrees). Through the points of division and through the centre e draw hidden straight lines cutting the horizontal line at various points. Through these points, together with the vertical point p, the straight lines drawn out will give the vertical circles — that is, the common sections of the dial-plane and the vertical circles. In either dial their starting-point is taken from the vertical line, which is drawn through p parallel to the horizontal line and to the six-o'clock line, so that the meridian line yields the ninetieth vertical.

[Margin: Demonstration of the description of the vertical circles.]

Here is the demonstration of this. Imagine the circle described from e, about the horizontal line, to be turned until it coincides with the horizon; its centre e then coincides with F, the centre of the world, on account of the equality of the straight lines Fm and me. The straight lines going out from the centre e through the points of division of the circle will be the common sections of the horizon and the vertical circles, because the horizon and the circle described from e as centre of the world — or of the horizon — are divided into like parts by the straight lines going out from the centre, as we demonstrated at the end of chapter 1 in the section on the sphere. And it is certain that the horizon, whose centre is then the point F (or e), is divided into equal parts by the vertical circles by means of the straight lines sent out from the centre. The vertical circles will therefore meet the dial-plane at those points of the horizontal line onto which fall those hidden straight lines sent out from e. And since, by proposition 18 of the preceding book, the common sections of the vertical circles and of the dial-plane meet at the point p — where the common section of them all, namely the axis of the horizon, meets the dial-plane — the straight lines drawn through p and through the points found on the horizontal line will be the common sections of the dial-plane and the

vertical circles. Only the common section of the dial-plane and the vertical circle properly so called must be drawn through p parallel to the horizontal line and to the six-o'clock line. For since the vertical circle properly so called, the horizon, the six-o'clock circle, and the equator (to which the dial-plane is parallel) have one and the same common section, then by proposition 18 — or by the scholium to proposition 22 of the preceding book — the common sections of these first three circles and of the dial-plane, namely the vertical line, the horizontal line, and the six-o'clock line, are parallel to one another. And so we have laid out the vertical circles on the same equinoctial dial. Which was to be done.

[Margin: How the inferior dial, with its vertical circles, arises from the superior equinoctial dial.]

Diagram. A large circular horizontal-projection figure showing the vertical circles laid out on the equinoctial dial. The bounding circle is crossed by a full pencil of straight lines radiating from the centre (the vertical circles), and by concentric circles carrying degree graduations. The horizontal diameter is the horizon line, labelled across the figure as **HORIZON** (printed "HOR" at the left margin of the circle, "IZON" at the right), with "15" flanking each end. Degree numbers are engraved in the arcs at 15° intervals — 15, 30, 45, 60, 75 — running up to 90 at the centre; several are set inverted (bottom half) or sideways in the usual dial fashion.

- Horizon line: labelled **HORIZON** (HOR ... IZON), running left–right through the centre
- Central vertical (meridian) line, top to bottom: **A** (top), **e** (centre of the projection), **L**, and lower down the cluster **F**, **M**, **N**, **E**, **p** (the vertical point), down to **q** at the foot
- Degree graduations along the horizon and vertical circles: **15, 30, 45, 60, 75** (both halves), with the centre marking **90**
- Around the rim, zodiac-sign sorts mark the divisions; Pisces (♊) is legible at the left, the remaining rim glyphs are worn/rotated — [?]
- Four cross-shaped ornaments (✚) mark the cardinal/inter-cardinal points of the rim

SCHOLIUM.

If the portion of the dial cut off by the horizontal line is placed on the lower face of the plane, so that the horizontal line occupies the upper position, and the part which in that arrangement lies on the right is changed to the left, we shall have the same vertical circles on the inferior dial,

as we demonstrated in the scholium to proposition 14 of this book. But since the vertical lines keep the same position and distance on both sides of the meridian, it will not be necessary to change the right part into the left, or the reverse.

PROBLEMA 53. PROPOS. 53.

To draw the parallels of the horizon on the same equinoctial dial.

[Margin: By what method the parallels of the horizon are described on the same equinoctial dial.]

Let ABC be the analemma, in which BC is the horizon, AD the vertical, EF the axis of the world, and GH the equator. Divide the semicircle BAC into 180 degrees — or into fewer equal parts, as the size of the dial allows (we divided it into 12, so that each part takes in 15 degrees). Join, by straight lines, pairs of points equally distant from the straight line BC (or from the point A); these lines will be the common sections of the meridian and of the parallels of the horizon, which they call almucantars. Then from the points of division draw straight lines through the centre D, so that triangles through the axis are formed in the cones, whose bases are the parallels of the horizon — as much below the horizon as above it — while their common vertex is D, the centre of the world. For the meridian ABC, passing through the axis AD of the said cones, makes triangles through the axis, by proposition 3 of Apollonius, Book 1. On the axis EF also take, on each side, the straight line DI equal to the gnomon, and through I draw KL parallel on both sides to the equator GH. Below GH this line KL will be the common section of the meridian and of the plane of the superior dial; above GH it will be the common section of the meridian and of the plane of the inferior dial. Each straight line KL will cut the sides of the triangles through the axis at the points M, N, O, P, Q, R, and these will be the diameters of the conic sections ML, NL, OL, PL, QR.

If, then, the points M, N, O, P (we here omit the point Q, since the conic section drawn through it falls beyond the tropics) are transferred onto the meridian line below the horizontal line — in the superior dial beginning in this figure from S, the point of the horizon, but in the dial itself from m, the point of the horizontal line — and if, by proposition 8 of the preceding book, the said conic sections are described about the meridian line, passing through the points M, N, O, P (which conic sections will be partly hyperbolas and partly ellipses, as is established from propositions 6 and 7 of the preceding book; the parallel of the horizon at 48 degrees will be a parabola, by proposition 5 of the same preceding book, because the equator touches it at the point G), then — receding ever farther from the horizontal line the farther they are produced on either side of the meridian line

— the parallels of the horizon will have been described. In the inferior dial the straight lines ST, SV are to be transferred onto the meridian line from the point m, below the horizontal line, and so forth. For TL is the diameter of the conic section of the parallel of the horizon 15 degrees above the horizon, and VL the diameter of the conic section of the parallel of the horizon 30 degrees, and so on.

GH, then, serves as it were as a horizon, and EF as a vertical; BC serves as a kind of equator, and the parallels of the horizon as the parallels of a new equator BC. On these terms, the vertical circles will serve as hour-circles, all of which cut the meridian line of the dial at the point X, where AD, the axis of the horizon, also cuts it — the axis which we have just said performs the office of a kind of new equator. So that, if this interchange of the circles is properly considered, this description of the parallels of the horizon does not differ from the description of the parallels of the equator on a horizontal dial.

[Margin: Another description of the parallels of the horizon on the same equinoctial dial.]

Diagram. The analemma ABC drawn as a full circle, densely ruled with chords and radial lines for the construction of the parallels of the horizon. The upper apex is **A** (marked **90**); the left and right extremities of the horizontal diameter are **B** and **C**; the centre is **D**. Degree graduations run down each side and along the lower half: 90, 75, 60, 45, 30, 15. A pencil of chords connects the graduation points, and the sides of the axial triangles are cut at the interior points named in the text.

- **A** (top, = 90°); **B** (left end of horizontal diameter); **C** (right end); **D** (centre)
- Degree marks on the upper and lower arcs: **15, 30, 45, 60, 75, 90**
- Interior construction points (per text, points where KL cuts the triangle sides and related): **K, G** (upper left); **E** (upper right); **R** (upper centre and again lower centre); **X** (centre); **M, N** (central cluster); **S, T** (central); **I** (central); **V** (left and right); **L** (lower right); **F** (lower left); **H** (lower right)
- Several central point-letters overlap the ruling and are worn — [?]

ALITER (an alternative method). Describe a quadrant ABC of any size and divide it into 90 degrees, or into fewer parts to suit the dial's size. From A as centre draw straight lines through the points of division; these will correspond to the radii of the parallels of the horizon comprised in the quadrant DC (90°) of the preceding figure, beginning from the straight line AB, as the figure shows. Next, from the figure of the preceding proposition take the straight line Fp, on the line AC of this figure, equal to AD; and take mF (or me) on the line AB, equal to AE,

and draw the straight line DE.[a] This triangle ADE will be exactly equal to the triangle Fpm of the preceding proposition's figure, since the angles at the points A and F are right and they are contained by equal sides, by construction. And so the line DE will be equal to the meridian line pm. Likewise the triangle ADE will be equal to the triangle DXS of the immediately preceding figure, since the triangle DXS is equal to the aforesaid triangle Fpm. For the angle IDX, in the analemma of this figure, is equal to the angle EFp in the figure of the preceding proposition, because each is the angle of the complement of the pole's altitude; and the right angle DIX is equal to the right angle FEp; and the sides DI, FE are equal, because each was taken equal to the style;[b] therefore the sides DX, XI will also be equal to the sides Fp, pE. And by the same reasoning DS, SI will be equal to the sides Fm, mE, and so on. After this, from the figure of the preceding proposition transfer all the intervals between the centre e and the points at which the vertical lines cut the horizontal line — transfer them into this figure from A, onto the straight line AB, marking the points on the straight line AB, and adding the numbers of the vertical lines: as 90 next to the point E, and 75 at the next point, and so on

[Margin: a. Elements I.4. b. Elements I.26.]

and so forth. Finally, from D, through the marked points AB, draw out straight lines. But for the vertical line properly so called — from which the beginning of the others is taken — draw DF through D parallel to AB; and to the right of it must also be transferred the lines already drawn from D through the points of the straight line AB, so that there may be vertical lines both beyond and short of the vertical line properly so called, just as was done in the horizontal dial in proposition 2 of this book concerning the lines before and after the sixth hour. For the lines sent out from D through the points of the straight line AB look toward the vertical lines before the vertical line properly so called, onto which the shadow of the style is cast when the Sun is in the northern verticals; while the others belong to the vertical lines beyond the vertical circle properly so called, and so on.

And so, if the intervals of the straight lines going out from D — those lying between D and the radius, say, of the parallel of the horizon at 15 degrees — are transferred from the point p of the preceding proposition's figure onto the vertical lines corresponding to the numbers marked on the straight line AB of this figure, marking the points on those vertical lines (yet in such a way that the intervals of the lines sent out from D and falling toward AB are transferred from the zenith p onto the vertical lines of the dial above the vertical line properly so called, while the intervals of the lines falling to the right of the straight line DF

are transferred onto the vertical lines below the vertical line properly so called) — and if these points are fitly and suitably joined by a certain curved line, then the parallel of the horizon at 15 degrees will have been described. By the same method the remaining parallels of the horizon will be described. The demonstration of this does not differ from the one we gave in the second manner of describing the parallels of the signs of the zodiac on a horizontal dial. For here the horizontal line is as it were the line of the equator, and the vertical lines are as it were hour-lines, whose centre is the zenith p, and so on, as we said in the preceding analemma. And so the straight line AB of this figure corresponds to the radius of the equator in the last figure of proposition 2 of this book;

Diagram. A quadrant-and-fan construction for the alternative method. A horizontal straight line runs across the top, carrying **A** at the left, **D** at the centre, and **C** at the right, with **90** engraved beside C. From C a quadrant arc sweeps down and to the left, graduated **75, 60, 45, 30, 15** (with **90** at its top). Below D a pencil of straight lines fans downward and outward through the graduation intervals, converging toward **B** at the lower centre and **F** below it. Near the apex of the fan, at **E**, the degree numbers 90, 75, 60, 45 are stacked, with 30 below.

- Top horizontal line: **A** (left), **D** (centre), **C** (right, = **90**)
- Quadrant arc from C: **75, 60, 45, 30, 15**, apex **90**
- Fan apex point **E** with stacked graduations **90, 75, 60, 45, 30**
- **B** (lower centre); **F** (below B)
- Lower degree marks along the fan lines: **15, 30, 45, 60**
- Cross-shaped ornaments (✕) mark line-ends at the foot of the figure

[Margin: Demonstration of the second description of these parallels of the horizon.]

and the lines drawn from A correspond to the radii of the signs sent out from D; while AD corresponds to the axis DH, the point D to the point H, and finally the vertical lines going out from D to the hour-lines drawn from H. The same will happen if the intervals of the straight lines issuing from D between the straight line AB (which, as we said, serves as the radius of the equator) and the lines sent out from A (which are as it were the radii of the signs) are transferred onto the corresponding vertical lines from the horizontal line of the dial — just as the intervals between the radius of the equator and the radii of the signs were transferred, so that the arcs of the signs might be described.

For the inferior equinoctial dial, the radii of the parallels of the horizon are to be transferred from A, through the points of the quadrant BC, going out to the left of

the straight line AB. Thus — just as, in the last figure of proposition 2 of this book, the radii of the signs to the right of the radius of the equator belong to the southern signs to be described below the equinoctial line of the dial, and the radii to the left to the northern signs to be described above the equinoctial line — so here too the radii of the parallels of the horizon drawn out from A to the right of the straight line AB (which serves as a kind of new radius of the equator, as was said) belong to the parallels of the horizon below the horizontal line (which is as it were a kind of new equinoctial line), to be described as though they were southern signs; while the radii sent out to the left of the straight line AB refer to the parallels of the horizon above the horizontal line of the dial — namely, in the inferior dial — to be described as though they were northern signs. Hence, since the portion of the dial above the horizontal line is equal to the inferior dial, the radii of the parallels of the horizon drawn out from A to the left of the straight line AB are rightly transferred onto it, and so on — all of which appears in the figure of the preceding proposition. We have therefore drawn the parallels of the horizon on the same equinoctial dial. Which was to be done.

SCHOLIUM.

Since, however, whole ellipses — such as is the ellipse of the parallel of the horizon at 60 degrees in our example — are described only with difficulty and inconvenience through all the points found on the vertical lines (as here, through those points that lie near the other extremity q of the diameter Pq of the ellipse), we shall use this device in order to describe them uniformly. Once the other extremity of the diameter is found on the meridian line — here the point q (which will indeed be found if the straight line D90, intercepted to the right of DF in the next figure between D and the radius of the parallel at 60 degrees, is transferred onto the meridian line of the dial from p as far as q; or else, in the analemma of this proposition, if the diameter of the ellipse of the parallel at 60 degrees, intercepted between P and the other side of the triangle through the axis in the cone whose bases are the parallels at 60 degrees, is transferred from P as far as q) — we shall bisect the whole diameter Pq at r, so that r is the centre of the ellipse, as is clear from the second definitions of Apollonius; and through r we shall draw a perpendicular to Pq, namely the second diameter of the ellipse. Then, because by proposition 30 of Apollonius, Book 1, any straight line drawn through the centre of an ellipse is bisected at the centre, we shall draw, from the points of the vertical lines through which the upper half of the ellipse passes,

[Margin: How, from one half of the ellipse, the other half is to be described.]

(this half, as is evident, can be described more conveniently than the lower one) — through the centre *r* we draw the hidden lines, and on their portions between the said points and the centre *r* we cut off, below the centre, equal straight lines, marking points on those hidden lines. For the lower half of the ellipse must be drawn through these points. And from proposition 75 of the preceding book, together with the earlier figure of this proposition, we shall easily recognize which parallel of the horizon forms the ellipse on the dial. We shall use this same technique for the ellipses of other dials, wherever they need to be described.

In the same way, on the inferior dial we shall describe the parallels of the horizon above the horizontal line, taking them from their counterparts described on the superior dial — and far more accurately, if, following the method given at the end of proposition 2 of this book, we first find the transverse diameters of the opposite sections and their centres, and so on. In the latter figure of this proposition, the transverse diameters are the portions of the straight line *DE* that lie between the radii of the opposite parallels of the horizon which go out from *A*, and are equally distant from the straight line *AB*.

[Margin: How the parallels above the horizontal line are described from their counterparts described below the horizontal line.]

PROBLEM 54. PROPOSITION 54.

To describe the meridians — that is, the circles of longitude of cities — on the same equinoctial dial.

[Margin: The description of the meridians on the same equinoctial dial.]

The meridian circles will be described just like the hours reckoned from noon or from midnight, as we taught in proposition 6 of this book — provided that, once we have drawn the hour-lines on the circle *ACBD* by the aid of proposition 49 of this book, that circle takes the start of its division into 360 degrees from the meridian of the Fortunate Isles, which lies on the superior dial (if the longitude of the place is counted from the meridian of the place, namely from point *A* toward *D*, through the western parts, as far as point *G*) — as is clear, provided the dial is understood to be set in its proper position. On the inferior dial, the same meridian of the Fortunate Isles is obtained if the longitude of the place is reckoned from point *B*, toward *D*, as far as *H*. For if the inferior dial has its own proper position — such that the horizontal line occupies the upper place and point *A* the lower, and point *C* faces sunrise and *D* faces sunset — then the meridian of the place will cut the plane of the dial above the horizon at point *B*,

and below the horizon at point A. Therefore from B toward D lie the western parts, and the eastern parts lie toward C, and so on.

[Margin: The description of the meridians on the same equinoctial dial.]

Diagram. A circle — the equinoctial dial ACBD — with a dense array of radial straight lines (the meridian circles / circles of longitude) emitted from the centre and continued outside the rim, each terminating in a degree-of-longitude label. The circle is crossed by a horizontal diameter running left to right, labelled HORIZON (i.e. HORIZON) at its two ends, and by a vertical diameter A–B; the two diameters cross at the centre E.

- A (top of circle): meridian of the Fortunate Isles; adjacent label *Inſ. Fort.* (*Insularum Fortunatarum*, "of the Fortunate Isles"), set along the upper-right of the vertical.
- B (bottom of circle): opposite pole of the vertical diameter.
- C (left, on the horizontal): west side.
- D (right, on the horizontal): east side.
- E (centre): common origin of all the meridian radii.
- G (upper right, between A and D).
- H (lower right, between D and B).
- *Sol* (left, near the horizontal line): label for the Sun / the point on the west.
- *Inſulæ Fort.* (*Insulae Fortunatae*): label set along a lower-left diagonal radius.
- Degree graduation of the radii (longitude, marked every 15°): legible values include, on the right and lower arc, 15, 30, 45, 60, 75, 90 (near H), 105, 120 (near D), 135 (near the ZON end); and on the left arc 285 (near the left horizontal), 300 (near C), 315, 330 (printed anomalously as two glyphs), 345. The remaining labels on the upper-left arc (including forms reading as 06 / 09 / 0E) are worn and not securely legible [?]. Zodiac-sign sorts marking the equinox/solstice radii on the rim are worn and not securely identified [?].

But it is better to describe the meridian circles on the inferior dial from the meridian circles of the superior dial, as was shown in the scholium to proposition 14 of this book, and as was done for the vertical dial in the scholium to proposition 28 of this book; for everything said there can be carried over here, provided one always understands the superior dial as the equinoctial for the southern hemisphere and the inferior for the northern. You have an example in the figure given. There you see that the meridian of 105°, extended beyond the centre E on the superior dial, has 285° as the meridian opposite to it; but when

extended beyond the horizontal line it takes up 105° again on the inferior dial, and so for the rest. We have therefore described the meridians — that is, the circles of longitude — on the same equinoctial dial. Which was to be done.

[Margin: How the inferior dial derives its meridians from the superior.]

PROBLEM 55. PROPOSITION 55.

To depict the parallels of cities — that is, the circles of latitude — on the same equinoctial dial.

[Margin: The description of the parallels — or latitudes — of cities on the same equinoctial dial.]

In place of the parallels drawn through the beginnings of the signs of the Zodiac, take the parallels that pass through the zeniths of the cities lying within the tropics, and do everything else just as we prescribed for describing the parallels of the signs of the Zodiac in proposition 50 of this book; and what is proposed will be accomplished. So we have drawn the parallels of cities — that is, the circles of latitude — and so on. Which was to be done.

PROBLEM 56. PROPOSITION 56.

To describe the celestial houses on the same equinoctial dial.

[Margin: The description of the celestial houses on the same equinoctial dial, following Johann Regiomontanus.]

Let the circle ACBD be cut, starting from the meridian line, into 12 equal parts — or into more, if we also want the subdivisions of the individual houses. We have divided it into 12, so that only the whole houses are described. Through the points of division and the centre E, let hidden straight lines be drawn. Then, through the point m, [where...]

where the meridian line intersects the horizontal, let lines be drawn parallel to the said lines, and the celestial houses will be described on both dials, following the teaching of Johann Regiomontanus. For since the circles of the celestial houses, according to Regiomontanus, divide the Equator into equal parts, beginning from the Meridian (which is one of the said circles), and pass through the centre of the world — that is, of the Equator — they will necessarily pass through the common sections of the Equator and the hour-circles reckoned from noon or midnight, since these too, passing through the centre of the world, cut the Equator into equal parts, beginning from the Meridian; so that the common

sections of the circles of the celestial houses and the Equator do not differ from the common sections of the hour-circles and the Equator. But the common sections of the plane of the dial and the circles of the celestial houses [a] are parallel to the common sections of the Equator and those same circles; [b] and to these, in turn, are also parallel (since they have been shown to be the same as the common sections of the Equator and the hour-circles) the common sections of the plane of the dial and the hour-circles reckoned from noon or midnight. [c] Therefore the common sections of the plane of the dial and the circles of the celestial houses will likewise be parallel to the common sections of the same plane of the dial and the hour-circles. Since, then, the straight lines drawn through the centre E, dividing the circle ACBD into 12 equal parts (or even into more or fewer), are the common sections of the plane of the dial and the hour-circles...

[Margin: Demonstration of the description of the celestial houses according to Johann Regiomontanus.] [Margin: a — [Euclid] XI.16.] [Margin: b — [Euclid] XI.16.] [Margin: c — [Euclid] XI.9.]

Diagram. A compound (stacked) figure of the celestial houses on the equinoctial dial after Regiomontanus. An upper circle (the superior equinoctial dial) carries a full set of radial house-lines from its centre and, around and beyond the rim, house numbers in Roman numerals together with zodiac-sign sorts at the divisions. Below it a second, lower circle (with a horizontal axis labelled VERTI—CALIS, *verticalis*) repeats the construction for the inferior/vertical dial, and a third, part-circle continues at the foot. Two horizontal HORI—ZON axes (one for each dial) run across the figure.

Upper circle:

- A (top of the upper circle).
- E (centre of the upper circle); m marked just above E on the vertical.
- Left-hand lettered points, descending: N, M, L, K, I.
- D (right, on the horizontal IIA ZON axis).
- B (lower part of the upper circle, on the vertical).
- H, F (left, below the upper circle, near the junction with the lower circle).
- House numbers in Roman numerals around the rim: legible are VII, VIII, IX, X, XI, XII, I; several are printed reversed/rotated (forms reading as IIA, IIX, IIIA) [?].
- Zodiac-sign sorts (e.g. ♌, ♍ and neighbours) mark the house-divisions on the rim; worn, not each securely identified [?].

Lower circle and foot:

- Horizontal axis HORI—ZON, and beneath it VERTI—CALIS.
- E and F within the lower circle, joined by a small triangle; p marked on the vertical below them.
- G (centre/foot of the lowest circle).
- Arabic numerals around the lowest circle: 8, 9, 10, 11, 12, 6 — house numbers in Arabic form [?] (worn).

as we demonstrated in proposition 49 of this book, then parallel to these straight lines will be the common sections of the plane of the dial and the circles of the celestial houses. And since these common sections all pass through the point m — as is established from proposition 18 of the preceding book, because the common section of all the circles of the celestial houses (that is, the axis of the vertical circle, or the common section of the Meridian and the Horizon) meets the plane of the dial at the same point m, as is clear from the figure of proposition 50 of this book, in which that common section is the straight line lm, provided the triangle Elm is understood to be perpendicular to the plane of the dial — that is, lying in the plane of the Meridian — the straight lines drawn through m will be parallel to the straight lines emitted through E, and these are the common sections of the plane of the dial and the circles of the celestial houses. Which is what was proposed. Moreover, it is evident that the beginnings of the celestial houses are parallel to the hour-lines distant from the meridian line by two, four, and six hours, because the circles of the celestial houses divide the Equator at those hours.

Furthermore, so as to draw easily through m lines parallel to those drawn through E — that is, the lines of the celestial houses — we shall use this technique. We shall draw, in any position, a line FG parallel to the meridian line; and on it, from the points where it is cut by the lines drawn through E, we shall cut off upward, toward Em, straight lines FH, IK, LM, NP, etc., equal to Em. For the straight lines from m...

[Margin: At which hours from noon or midnight the beginnings of the celestial houses are parallel to the hour-lines.]

drawn from m through the points H, K, M, P, etc. are parallel to those going out from E. For since the straight lines Em and FH are equal and parallel by construction, [a] the straight lines joining the points E, F and m, H will also be parallel to one another, and so for the rest.

[Margin: a — [Euclid] I.33.]

According to Campanus we shall lay out the celestial houses as follows. From the figure of proposition 52 of this book we take the straight line Fp , by which we located the point p — namely the Zenith — on the meridian line; and we transfer it onto the meridian line from the point p , whether upward or downward, as far as G . Then, from G , having described a circle of any size and divided it into 12 equal parts (or into more, if we also want to have the subdivisions of the celestial houses), we draw from the centre G , through the points of division, hidden straight lines that cut the vertical line at certain points; and these points, joined by straight lines to the point m , will give the celestial houses according to Campanus's view. For if this circle is rotated about the vertical line until it coincides with the vertical circle properly so called, and its centre G with the centre of the world F , then the straight lines drawn through G will be the common sections of the vertical circle and the circles of the celestial houses. Hence the circles of the celestial houses will meet the plane of the dial at the said points of the vertical line; and therefore the lines of the celestial houses — that is, the common sections of the plane of the dial and the circles of the celestial houses — will pass through those same points. Since, then, they also pass through the point m , as we have shown, the straight lines connecting the said points of the vertical line with the point m will be the lines of the celestial houses. We have therefore described the celestial houses on the same equinoctial dial. Which was to be done.

[Margin: The description of the celestial houses on the same equinoctial dial according to Campanus.] [Margin: Demonstration of the description of the celestial houses according to Campanus.]

SCHOLIUM.

If the lines of the celestial houses of the superior dial are extended beyond the horizontal line, the lines of the same houses will also be described on the inferior dial — provided that all the parts of the portion of the dial which the horizontal line cuts off are inverted, as was said in the preceding passages and demonstrated in the scholium to proposition 14 of this book.

[Margin: How the celestial houses, extended beyond the horizontal line, display the same houses on the inferior dial.]

PROBLEM 57. PROPOSITION 57.

To lay out the ascending signs of the Zodiac on the same equinoctial dial.

[Margin: The delineation of the ascending signs on the same equinoctial dial.]

From the centre E of the dial let there be drawn the hidden lines of those hours at which — with the Sun at the beginning of Cancer [♋], if the superior dial is in question, or at the beginning of Capricorn [♐], if the discussion concerns the inferior — the beginnings of the signs rise above the Horizon, as appears in the third or fourth table of proposition 9 of this book, taking also the hours of those signs...

[Margin: The delineation of the ascending signs on the same equinoctial dial.]

Diagram. A large circle carrying an inscribed twelve-pointed star (the twelve zodiacal division-lines) with the twelve zodiac signs set around its outer rim, crossed by two horizontal axes: an upper one labelled HORIZ—ZON (horizon) and a lower one labelled HORA—VIA [?]. Within the star is a smaller central figure of radial hour-lines from the centre E, with lettered construction points.

- A (upper centre, within the star).
- m (below A, on the vertical).
- E (centre): origin of the inner radial hour-lines.
- Left-hand lettered points, descending: L, K, I, C (near the horizontal HORIZ end).
- Right-hand lettered points: G, F, D, H (near the ZON / VIA ends).
- B (lower centre, on the vertical, within the circle).
- Zodiac signs set around the outer rim (Cancer ♋ and Capricorn ♐ are named in the text as the tropics touched by the ascending signs); the individual rim sorts are worn and not each securely identified [?].

of those signs which rise several hours after and before noon [more] than is contained within the semidiurnal arc of Cancer [♋] or Capricorn [♐]. Then, since at those points of the tropic of Cancer or Capricorn — where it is cut by the said hour-lines — the ascending signs touch the tropic, as we showed in proposition 9 of this book, if through those points lines are drawn tangent to the tropic, [this...]

[Margin: The order of the ascending signs on the equinoctial dial.]

that is, the perpendiculars to those hour-lines (for by corollary to *Elements* III.16 these perpendiculars touch the circle ACBD, which represents the tropic) — through these the ascending signs will be laid out, in the following order. On both dials the sign Aries [♈] passes through A; the sign Taurus [♉] through the next point toward D; the sign Gemini [♊] through the next after that, and so on. Take only those portions of the lines which, on the upper dial, fall below the horizontal line into the upper dial; and, on the lower dial, only those portions of the same

lines which fall below the horizontal line into the lower dial. This order is easily gathered from the tables already given — the third and fourth — if on each dial one carefully considers the forenoon and the afternoon parts. Here, as in what preceded, we take for the lower dial that portion which the horizontal line cuts off, but with all the parts inverted, as we have often said. Hence on the lower dial the forenoon hours are to be counted from point A toward C, and the afternoon hours toward D. Provided no error has been made in the construction, the ascending signs will pass through those points of the horizontal line at which the parallels drawn through the beginnings of the signs intersect it — points such as...

Diagram. A large circle carrying an inscribed twelve-pointed star (the twelve zodiacal division-lines), with the twelve zodiac signs set around its outer rim and a vertical gnomon-line rising from the top. Two horizontal axes cross the figure: an upper one whose ends are labelled HORI—ZON (Horizon), and a lower one labelled HORA (left) — VI4[?] (right, worn). Within the star is a smaller central circle of radial hour-lines issuing from the centre E, with dotted construction lines.

- A (upper centre, on the vertical, within the inner circle).
- m (just below A, on the vertical).
- E (centre): origin of the inner radial hour-lines.
- B (lower centre, on the vertical, within the inner circle).
- Left-hand lettered points, on the two horizontal lines: L (at the left rim, on the upper/Horizon line), K (inner-left), I and C (on the lower horizontal line).
- Right-hand lettered points: G (inner-right), H (at the right rim, near ZON), F and D (on the lower horizontal line).
- The twelve zodiac signs are set around the outer rim; the individual rim sorts are worn and are not each securely identified [?].

[Margin: Finding the points on the horizontal line through which the ascending signs are to be drawn.]

F, G, H, I, K, L, as is evident from what we set out for the vertical dial in proposition 21 of this book. These points, moreover, may be found in this way even if the parallels of the signs are not drawn. From the centre E let hidden lines be drawn terminating, on each tropic, the semidiurnal arcs — of the northern signs on the upper dial, and of the southern signs on the lower — as they are contained in the fifth table of proposition 9 of this book; yet in such a way that the semidiurnal arcs of the northern signs are reckoned from B, on either side, in

the tropic of Cancer [♋], and those of the southern signs from A, on either side, in the tropic of Capricorn [♏]. For these lines will cut the horizontal line at the said points.

[Margin: Another way of finding the same points.]

We shall find these same points from the last figure of proposition 21 of this book. For if on the straight line EB, or ED, of that figure there is taken a straight line Eh equal to the straight line *lm* in the dial of proposition 50 of this book — the line by which we drew the horizontal line in the dial — and if through h a perpendicular to BD is raised, this will be the horizontal line in the dial. This is evident: if the Horizon ABCD is conceived in the dial in its proper position, so that the point h falls at m, and the perpendicular drawn through h is made the same as the horizontal drawn through m; and if accordingly the centre E of the figure of proposition 21 of this book is joined with the centre of the world and with the tip of the style, because of the equality of the straight lines Eh and *lm*. And so, if the distances of the perpendicular line drawn through h — measured between h and the points at which it is cut by the rays proceeding from E — are transferred onto the horizontal line from m, the same points will be obtained as before.

[Margin: Another delineation of the ascending signs on the same equinoctial dial, by the culminations of the heaven and the angles of the earth.]

Again: if from the sixth table of proposition 9 of this book we take the declinations of the culminations of the heaven [mediationum caeli] and of the angles of the earth [angulorum terrae], and find their points on the meridian line — through which, that is, the parallels of those declinations pass, according to the doctrine of proposition 50 of this book — then the ascending signs will likewise pass through those points, if no error has been made in laying them out. On the upper dial the culminations of the heaven of the northern signs are contained in the portion of the meridian line drawn from point E through B, since this indicates the 12th hour of midday; and the angles of the earth in the remaining portion of the meridian line drawn from the same point E through A, which of course shows the 12th hour of midnight. So the signs Leo [♌], [?], Virgo [♍], [?], and Libra [♎] pass through the points found on the line EB extended — because when these signs are ascending, northern points of the ecliptic culminate; whereas the signs Cancer [♋], Capricorn [♏], Gemini [♊], Aquarius [♒], Taurus [♉], Pisces [♐], and Aries [♈] are drawn...

through the points found on the straight line EA extended, because, when these signs are rising, northern points of the ecliptic are set at the angle of the earth, as

appears from the said sixth table. On the lower dial, by contrast, the culminations of the heaven of the southern signs are contained on the meridian line EA extended, and the angles of the earth on the midnight line EB drawn out; so that the signs Cancer [♋], Capricorn [♏], Gemini [♊], Aquarius [♒], Taurus [♉], Pisces [♐], and Aries [♈] run through the points found on the straight line EA, because when these signs are ascending, southern points of the ecliptic are found at midheaven; whereas the signs Leo [♌], [?], Virgo [♍], [?], and Libra [♎] pass through the points found on EB extended — for when they are rising, southern points of the ecliptic stand at the angle of the earth. It is clear, then, that on the upper dial only the northern declinations can be laid out, and the southern ones on the lower dial.

Finally, if from the seventh table of the same proposition 9 of this book we take the declinations of the northern points of the ecliptic lying on the line of the 6th hour before and after noon, at the moment the beginnings of the signs rise, and find their points on the line of the 6th hour before or after noon on the upper dial; and if we do the same for the declinations of the southern points of the ecliptic lying on the same line on the lower dial — then the same ascending signs will pass through these points found on the line of the 6th hour. And so we have laid out the ascending signs of the Zodiac on the same equinoctial dial. Which was to be done.

SCHOLIUM.

We shall depict the ascending signs of the Zodiac in the manner of Andreas Schöner [*Andrea Schonero* — Andreas Schöner] as follows. In the first figure of the scholium to proposition 9 of this book, on the straight line EB let there be taken a straight line E β equal to the portion of the meridian line EM lying between the centre of the dial and the horizontal line in the figure of proposition 50 of this book; and through β let a line be drawn parallel to AC itself, in such a way that, with the centre E of that figure placed at the point m on the dial, and the point β at the centre E, the parallel drawn through β coincides with the line of the 6th hour. This parallel drawn through β will cut the straight lines of the signs issuing from the centre E at points whose distances, taken from β and transferred onto the line of the 6th hour of the equinoctial dial from the centre E, will give — on both sides — points on the line of the 6th hour. Through these, if from the point m, where the horizontal line and the meridian intersect, hidden lines are drawn cutting the tropic of Cancer [♋] on the upper dial, or the tropic of Capricorn [♏] on the lower, at points through which, if again straight lines are drawn to the centre E, and to these, from the said points of the tropic,

perpendiculars — that is, tangents to the tropic — are drawn, the ascending signs will be laid out as before, keeping the order which the figure shows.

It is enough, however, if the ascending signs are laid out on the upper dial. For these, extended beyond the horizontal line, will present the same signs on the lower dial, provided the portion of the dial cut off by the horizontal line is inverted, as was said in what preceded and demonstrated in the scholium to proposition 14 of this book.

PROBLEM 58. PROPOSITION 58.

To construct an Italian-hours equinoctial dial [*Horologium Italicum aequinoctiale*].

[Margin: The construction of the Italian-hours equinoctial dial.]

The parallels, or arcs of the signs (which we drew together with the horizontal line in proposition 50 of this book), are each to be divided — or at least two of them, the inner and the outer, which is enough — into 24 equal parts, beginning from the horizontal line on that side toward which the shadow of the style falls at sunset: that is, on the upper dial, from the left-hand side, which faces the sunrise. Then, through the nearest points of these arcs, above or below the horizontal line, from the left side where the division began, let a straight line be drawn; likewise through the next points in sequence, and so on, until all are completed. For these straight lines will indicate the hours from sunset on the upper dial, and thus the upper Italian-hours equinoctial dial will be laid out, in which the number and order of the hours is as follows. The first line — that is, the one nearest above the horizontal line, which indicates the 24th hour (this is on the nocturnal dial, which the portion cut off by the horizontal line displays) — shows, on its left-hand part, the first hour from sunset; the next shows the second, and so on; so that the nearest line below the same horizontal line shows, on the left-hand part, the 23rd hour, and so forth. This we shall demonstrate as follows.

[Margin: The order of the hours from sunset on the equinoctial dial.]

Let great circles be understood as drawn through the poles of the world and through the hours from sunset on the parallel — say, Cancer [♋] — after the manner of the hour-circles from midday or from midnight; that is, as passing through those points at which, by proposition 10 of the upper book, the Horizon and the other hour-circles from sunset cut the parallel of Cancer [♋] into equal parts. These circles will likewise cut the parallel of Cancer [♋] on the equinoctial dial into equal parts, as is established from what we wrote in proposition 49 and

in the scholium to proposition 50 of this book. Therefore, since the great circle drawn through the poles and through the 24th hour cuts the parallel of Cancer [♋] on the dial at the point where the Horizon itself cuts it — so that the shadow of the gnomon falls upon that point, the Sun being at the common section of the Horizon (or the parallel of Cancer [♋]) and of the said circle, as is evident...

[Margin: Demonstration of the construction of the Italian-hours equinoctial dial.]

from what was demonstrated in the upper book — other great circles will pass through other points of the parallel of Cancer [♋] on the dial, points which divide it into equal parts, since the shadow of the style falls upon each of them too when the Sun is at the common section of the parallel of Cancer [♋] and of those circles; and accordingly the hour-circles from sunset will pass through the same points, for the same reason: namely, because the shadow of the style falls on those points when the Sun is at the common sections of the parallel of Cancer [♋] and of the hour-circles from sunset, as well as of the great circles passing through the poles of the world and through the hours from sunset on the parallel of Cancer [♋]. For the ray of the Sun will then not differ from the common sections of those great circles, since these sections are drawn through the centre of the world — that is, through the tip of the style — just as the Sun's ray is. Since, then, the Sun's ray falls at that moment upon the said points of the parallel of Cancer [♋] on the dial, those common sections will also fall there; and accordingly at those points the planes of the hour-circles from sunset will meet the plane of the dial. In the same way we shall show that the hour-circles from sunset pass through the points of the other parallels on the dial which divide them into equal parts, provided, that is, that other great circles are conceived as drawn through the poles of the world and through the hours from sunset on those same parallels. Therefore the straight lines connecting the said points of the arcs of the signs are the common sections of the plane of the dial and of the circles that show the hours from sunset.

By the same reasoning we shall construct the lower Italian-hours dial, if we begin the division of the parallels on the lower dial — which the portion cut off by the horizontal line represents to us — from that side toward which the shadow of the style is cast...

beneath the sunset — namely, the right-hand part, facing the sunrise. I call the right-hand part the one which, for us turned toward the lower dial (that is, toward the portion cut off by the horizontal line, with the horizontal line occupying the upper place), is set on the right: for here we do not use that inversion of parts of which we spoke above. For the nearest line above the

horizontal line will give the 1st hour from sunset — the line which, on the upper dial, was indicating the 23rd hour; and the next, which was the line of the 22nd hour, will give the 2nd; and so on, always taking the complements of the hours of the upper dial up to 24. Thus the line of the 1st hour from sunset on the upper dial — which is precisely the first line below the horizontal line on the lower dial — indicates the 23rd hour from sunset, and so forth. These complements of the hours we have not written in the figure, but they are to be understood in thought alone.

[Margin: Another delineation of the Italian-hours equinoctial dial, by the diurnal and nocturnal arcs of 14 and 10 hours.]

ALITER [Another way]. By the doctrine of proposition 51 of this book let there be described a diurnal arc of 14 hours, whose portion cut off by the horizontal line will be a diurnal arc of 10 hours on the lower dial, or a nocturnal arc of 10 hours on the nocturnal dial — which comes to the same thing, since the lower dial, together with the portion of the dial that the horizontal line cuts off, displays both the lower and the nocturnal dial, as is clear from what precedes. For if we draw straight lines through the hours from midday, or from midnight, on the diurnal arc of 14 hours and the nocturnal arc of 10 hours, corresponding to the hours from sunset — as may be seen in tables 3 and 4 of the scholium to proposition 33 of the upper book — the upper Italian dial will be laid out, as before. Yet it must be noted that the individual hours from sunset have, in the said tables, only single hours corresponding from midday or from midnight. Hence, that they may be drawn correctly, the hours from midday or from midnight corresponding to the same numbered hours from sunrise must also be taken from the same tables. For example: the 23rd hour from sunset is drawn through the 6th hour from midday on the diurnal arc of 14 hours; and because the same line, extended beyond the diurnal arc of 24 hours which touches the horizontal line at the point m, indicates the 23rd hour...

Diagram. A large full-hour Italian-hours equinoctial dial: a great circle with concentric rings of hour-numbering — an outer ring of Roman numerals (I through XXIII) interleaved with an inner ring of Arabic numerals (1 through 24) — and a dense sheaf of straight and dotted radial hour-lines converging on the centre E. A horizontal axis crosses the whole figure, its ends labelled HORI—ZON (Horizon), with 24 at the right-hand rim; a large arc bulges out to the left carrying the points C (upper left) and B (mid-left).

- E (centre): convergence of the radial hour-lines.
- m (just above the centre, on the vertical).

- A (just left of the centre).
- D (just below the centre).
- B (at the left, on the horizontal line, an endpoint of the left-hand arc).
- C (upper left, endpoint of the left-hand arc).
- Right rim: ZON and the numeral 24 on the horizontal line.
- Around the rim run the double hour-numbers: Roman I–XXIII in the outer band and Arabic 1–24 in the inner band (bottom sequence legible as...19/VIII, 18/VII, 17/VI, 16/V...; upper and right sequences 1, 2, 3... with X, XI, XII, XIII, XIII on the left). The individual peripheral numerals are small, rotated, and partly worn, and are **not each securely resolved** [?].

from sunrise, as we taught in the scholium to proposition 10 of this book; and since the 23rd hour from sunrise passes through the 4th hour from midnight on the nocturnal arc of 10 hours, therefore, so that the 23rd hour from sunset may correctly be drawn through the 6th hour from midday on the diurnal arc of 14 hours, we must also take the 4th hour from midnight — corresponding, on the nocturnal arc of 10 hours, to the 23rd hour from sunrise. In the same way the 22nd hour from sunset is drawn through the 5th hour from midday on the diurnal arc of 14 hours, and through the 3rd hour from midnight on the nocturnal arc of 10 hours, which corresponds to the 22nd hour from sunrise. And likewise (to sum up the whole matter in few words) the 11th hour from sunset goes through the 6th hour from midnight on the diurnal arc of 14 hours, and through the 4th hour from midday on the same arc, which corresponds to the 11th hour from sunrise. Again, the 1st hour from sunset is drawn through the 8th hour from midday on the nocturnal arc of 10 hours, and through the 6th hour from midnight on the diurnal arc of 14 hours, corresponding to the 1st hour from sunrise, and so on.

[Margin: How the lower Italian dial is gathered from the upper.]

In no other way shall we make the lower Italian dial. But it will be better to gather it from the upper one. For the hours from sunset of the upper dial, extended beyond the horizontal line, will give the same numbered hours from sunset on the lower dial — provided, that is, that all its parts are inverted, as was said above and demonstrated in the scholium to proposition 14 of this book.

[Margin: A description of the same Italian-hours equinoctial dial, by the diurnal arc of 24 hours.]

We shall also, and easily, complete the same Italian dial by means of the diurnal arc of 24 hours — that is, the parallel, greatest of all those that are always

apparent, which touches the horizontal line at m, and which is, on the lower dial, the nocturnal arc of 24 hours, or the parallel, greatest of all those that are always hidden. For since this arc is a circle, by the corollary to proposition 4 of the upper book, and since the hours from sunrise or sunset touch it (by proposition 14 of the same book) at the points at which the hours from midday or from midnight cut it, it is manifest that the lines touching the said circle at the points of the hours from midday or from midnight — that is, the perpendiculars to the lines of the hours from...

[the perpendiculars to the lines of the hours] from midday, or from midnight — at the points where they cut the aforesaid circle — indicate the hours from sunset. Now which hours from sunset correspond to which hours from midday or midnight on the said arc of 24 hours will be taught by the seventh table of the scholium to proposition 33 of the upper book, for the upper dial; and for the lower dial the same service will be performed by the eighth table of that same scholium. These tangent lines must of necessity pass through the hours from midday, or midnight, on the diurnal arc of 14 hours and the nocturnal arc of 10 hours. Thus you see that the 23rd hour from sunset touches the diurnal arc of 24 hours at the 11th hour from midday, and passes through the 6th hour from midday on the diurnal arc of 14 hours. Likewise the 1st hour from sunset touches the former arc at the 1st hour from midnight, and runs through the 8th hour from midday on the nocturnal arc of 10 hours, and so on. Most conveniently, therefore, any hour whatever from sunset will be drawn — on the upper dial — from two arcs, one of them the diurnal arc of 24 hours and the other either the diurnal arc of 14 hours or the nocturnal arc of 10 hours; and on the lower dial, from two arcs, one of them the nocturnal arc of 24 hours and the other either the diurnal arc of 10 hours or the nocturnal arc of 14 hours.

[Margin: The most convenient way of describing the same Italian dial by means of the diurnal arc of 24 hours together with the diurnal arc of 14 hours and the nocturnal arc of 10 hours.]

We shall construct the same Italian dial by means of the line of the 12th hour from sunrise or sunset, as is established from the second table of proposition 20 of the upper book; and likewise by means of the horizontal line — that is, the line of the 24th hour from sunrise or sunset — as is gathered from the first table of that same proposition 20, in the way explained above. Indeed, from the third table of the same proposition it will also be clear which hours from sunset cut one another on the line of the 6th hour from midday or midnight, and so forth. The line of the 12th hour from sunrise or sunset can be drawn in the following

way. On the line of the 6th hour from midday or midnight, from the centre E, cut off the straight line EA equal to the gnomon; and with A as centre, at any interval you please, describe a circle and take on it an arc BC equal to the elevation of the pole, beginning from the line EB and proceeding toward the horizontal line. Then from C, through the centre A, draw a straight line cutting the meridian line at D. For the line drawn through D parallel to the line of the 6th hour from midday or midnight will show the 12th hour from sunrise or sunset. To see why, imagine the circle BC turned about the meridian line until it coincides with the meridian circle and until its centre A is joined to the centre of the world, that is, to the vertex of the style. Once this is done, B will be the pole of the world, and C the point where the circle of the 12th hour from sunrise or sunset touches the greatest of all the ever-apparent parallels and cuts the meridian — for this parallel cuts the meridian at C. Hence the straight line CA will be the common section of the circle of the 12th hour from sunrise or sunset and of the meridian, and therefore the circle of the 12th hour from sunrise or sunset will meet the plane of the dial at the point D.

[Margin: The construction of the same Italian dial by means of the line of the 12th hour from sunrise or sunset, and by means of the horizontal line, that is, the line of the 24th hour from sunrise or sunset. — The line of the 12th hour from sunrise or sunset, how it is to be drawn on the equinoctial dial.]

And since the line of the 12th hour from sunrise or sunset and the line of the 6th hour from midday or midnight are parallel — by what we wrote in the scholium to proposition 22 of the upper book — it follows that the straight line drawn through D parallel to the line of the 6th hour from midday or midnight is the line of the 12th hour from sunrise or sunset.[a] Now because the angle of the elevation of the pole, BAC, is equal to the angle EAD at the vertex, it comes about that if on the line of the 6th hour from midday or midnight we take the straight line EA equal to the style, and at A construct the angle EAD equal to the elevation of the pole, we shall find on the meridian line the same point D through which the line of the 12th hour from sunrise or sunset must be drawn — even if the circle BC is not described. Moreover, if we cut off ED equal to the straight line Em, we shall obtain the same point D. For since the angle EAm too is constructed equal to the elevation of the pole (as is established from proposition 30 of this book), the angles EAm and EAD will be equal; and the angles at E are also equal, being right angles; and the side EA, adjacent to the said angles, is common.[b] Therefore the sides Em and ED will also be equal. And consequently, since the line of the 12th hour from sunrise or sunset passes through D, as we have shown, it is plainly established that if ED is cut off equal to Em, the line of the 12th hour

from sunrise or sunset must be drawn through D — the line that touches the greatest of the ever-apparent parallels at D, just as the horizontal line touches the same parallel at m.

[Margin: [a] Proposition 15 of Book I. — Another way of describing the line of the 12th hour from sunrise or sunset on the equinoctial dial. [b] Proposition 26 of Book I.]

Now two lines of hours from sunset are always parallel to some one hour from midday or midnight, as is clear from the table of proposition 19 of the upper book and as was proved in the scholium to proposition 22 of the same book. For example, the 23rd and the 11th hours from sunset are parallel to the 5th hour from midday or midnight, and so on. Hence, if on the diurnal arc of 14 hours and the nocturnal arc of 10 hours — or even on the diurnal arc of 24 hours — we mark the hours from midday or midnight that correspond to the hours from sunset, as the tables of the scholium to proposition 33 of the upper book indicate, we shall very conveniently describe the same hours from sunset by drawing, through those hours from midday or midnight marked on the said arcs, lines parallel to the hours from midday or midnight that the aforesaid table of proposition 19 of the same book sets out. Thus, since the 16th hour from sunset corresponds (by the third table of proposition 33 of the said book) to the 11th hour from midnight on the diurnal arc of 14 hours, and is parallel to the 16th hour from midday or midnight (as the table of proposition 19 of the same book requires), the straight line drawn through the 11th hour from midnight on the diurnal arc of 14 hours, parallel to the 10th hour from midday or midnight, will be the line of the 16th hour from sunset — and so for the rest. We have therefore constructed the Italian equinoctial dial. Which was to be done.

[Margin: To each single hour from midday or midnight two hours from sunrise or sunset are parallel. — Another way of describing the hours from sunset on the equinoctial dial.]

PROBLEMA 59. PROPOS. 59.

To construct a Babylonian equinoctial dial.

By the same means and reasonings by which we constructed the Italian dial, we shall make the Babylonian dial — provided that, in the first method, the division of the circles described from E takes its beginning from the right-hand side of the upper dial (the side that customarily receives the shadow of the style about sunrise), and, on the lower dial, from the left-hand side (taking the lower dial as

we described it in the first method of the preceding proposition). Likewise the hour-lines are to be drawn so that first one is drawn through the nearest points of the divisions above or below the horizontal line from which the division took its beginning; then another proceeds through the following points, and so on in succession. The order of the hours is this. The first line below the horizontal line, on the right-hand side, gives on the upper dial the 1st hour from sunrise, the next the 2nd, and so on. On the lower dial, however, the nearest line below the horizontal line, on the left, will show the 1st hour from sunrise, the next the 2nd, and so on — so that they are the complements, up to 24, of the hours from sunrise on the upper dial. For what on the upper dial gave the 23rd hour, on the lower dial displays the 1st, and so on, as we said of the Italian dial in the preceding proposition, even though we set down no example of the lower dial according to this method. The demonstration here will be the same as in the preceding proposition, taking the hours from sunrise in the parallels of the Sun in place of the hours from sunset.

[Margin: The construction of the Babylonian equinoctial dial. — The order of the hours from sunrise on the equinoctial dial.]

In the remaining methods there is no difficulty, provided that what was said above of the hours from sunset be understood here of the hours from sunrise, and

conversely, that what was said there of the hours from sunrise be understood here of the hours from sunset. An example of the Babylonian dial stands in the figure of the preceding proposition, where the hours from midday or midnight and the hours from sunset are also contained, and are distinguished from one another as follows. To the hours from midday or midnight we assigned the numbers near the diurnal arc of 14 hours and the nocturnal arc of 10 hours, and their lines — marked only with dots — run out as far as the arc of 24 hours. The hours from sunset carry the ordinary numeral characters, thus: 24, 23, 22, etc. Finally, the hours from sunrise are set apart from the others by the ancient numeral signs, thus: I, II, III, etc. And so we have composed the Babylonian equinoctial dial. Which was to be done.

[Margin: In what ways the other descriptions of the Babylonian equinoctial dial are made.]

SCHOLIUM.

The lines of the hours from sunrise on the upper dial, extended beyond the horizontal line, indicate the same hours on the lower dial, provided that its parts are inverted, as we demonstrated in the scholium to proposition 14 of this book.

Furthermore, we shall describe the Italian and Babylonian dials also in this way, taking no account of the parallels of the Sun. From A, the centre of the circle BGF described at any interval, let the circle be divided into 24 equal parts; and with diameters drawn through the centre A and each pair of opposite points, let perpendiculars be raised to those diameters at their extreme points — or, what is the same thing, tangents to the circle. These, I say, indicate the hours from sunrise and sunset on the equinoctial dial. So that, if you choose any one of these tangent lines for the horizontal line, that is, the line of the 24th hour — namely CD itself — the next line below it on the left will show the 23rd hour from sunset, the one after that the 22nd, and so on, on the upper dial; while the next line below the same horizontal CD, on the right, will exhibit the 1st hour from sunrise, the one after it the 2nd, and so on, on the same upper dial. For, as we taught in the preceding proposition and as was demonstrated in proposition 14 of the upper book, the hours from sunrise or sunset touch the greatest of the ever-apparent parallels of any dial whatever (which on the equinoctial dial is a circle) at the points where the same hour from mid-

[Margin: How the lower Babylonian dial is gathered from the upper. — How the Italian and Babylonian dials are constructed, taking no account of the parallels of the Sun.]

Diagram. A large circular equinoctial dial [the combined Italian/Babylonian equinoctial dial referred to in the text], filling the middle of folio 242. The outer boundary is a 24-sided polygon inscribed in a dotted circle; from the centre a dense web of straight hour-lines (tangents to the inner circle) crosses the figure. A horizontal diameter CD runs through the figure. Concentric with the outer circle is a smaller inner circle BGF surrounding the centre.

Legibly labelled interior points and lines :

- A (centre): the centre of the figure, drawn as a small sun-burst / gnomon foot with radiating lines.
- B (top of inner circle, on the vertical above A): upper point of the circle BGF.

- F (bottom of inner circle, on the vertical below A): lower point of the circle BGF.
- G (just above and left of A, inside the inner circle): third named point of the circle BGF.
- E (on the horizontal, a little left of centre): named point near the centre.
- C (left end of the horizontal diameter CD): the line CD is here taken as the horizontal line / line of the 24th hour.
- D (right end of the horizontal diameter CD).
- Inner circle labelled by its three points B, G, F ("circulus BGF").
- Along the horizontal line, inside from C: the Roman labels "XV", "XVI"; inside from D: the Arabic labels "8", "9".

Peripheral hour-numbering: the rim carries a double set of hour-numbers — an outer band of Roman numerals (I–XXIII, the hours from midday/midnight) and an inner band of Arabic numerals (1–24). The numerals are small, rotated according to their position around the rim, and in places foxed. Legible anchors include, at the left of the horizontal, "24" and the point "C"; running down the lower-left the Roman "XIII, XIII, XII, XI, X" beside Arabic "23, 22, 21, 20"; across the bottom the sequence "X, 19, IX, 18, VIII, 17, VII, 16, VI, 15, V, 14"; and up the right side "D, 24, 10, 12, 13, 14" with Roman "I, II, III, IIII, V." The individual rim numerals are **not each securely resolved** [?] — the double, rotated, worn numbering (see report).

[Margin: How the hour-lines of the immediately preceding figure are easily drawn.]

day or midnight cut it. If, then, the circle BGF is understood to be, on the equinoctial dial, the said greatest parallel, the points of its division will be [the points of] the hour from midday or midnight, since they distribute it into 24 equal parts; and therefore the lines touching it at those same points will show the hours from sunrise or sunset. On the lower dial — which is furnished for us by the portion cut off from the horizontal line CD, if all its parts are inverted — the ratio is the same, as the figure indicates and as is manifest from what has been said.

We shall draw the said perpendiculars, or the lines tangent to the circle, without great labour in the following way. One perpendicular, or tangent line, CD having been drawn, describe from A a hidden circle of any magnitude passing through the points C and D, yet larger than the circle BGF. Divide it into 24 equal parts, beginning from the point C; and again into another 24 parts, beginning from the point D — of which kind is the outermost circle marked with points in the

immediately preceding figure. For just as the straight line CD is perpendicular to BA and touches the circle BGF at B, and is drawn through the first points of the divisions, so likewise the straight line drawn through the second points of the divisions — one of which belongs to the first division and lies below the line CD, and the other to the second division and lies above the line CD — will be perpendicular to the line next to BA, and will touch the circle BGF at the point next to B.

In the same way one is to proceed with the third points of the divisions, with the fourth, and with the rest. The reason for this is that the said lines of the hours from sunrise and sunset divide each of the circles described from A into 24 equal parts, as is clear from what was demonstrated in the preceding proposition, the beginning being made both from the point C and from the point D; so that, if the point next below C is the 23rd hour from sunset, the point next above the point D will be the 23rd hour from sunrise, and so on.

We shall find the length of the style — which, placed at the centre, is to indicate the hours on a dial laid out by this art — in the following way. Let the angle ABE be made equal to the complement of the elevation of the pole. This will be easily accomplished if from B an arc of a circle is described, and on it the complement of the elevation of the pole is counted off from the straight line BA toward C. For the straight line BE drawn from B through the end of that counting-off will make the angle ABE equal to the complement of the elevation of the pole, and will cut off from the straight line AE the length of the style AE, to be fixed at the centre A at right angles. For if, with the dial set up in its proper position — namely so that it is equidistant from the equinoctial circle and the straight line CD is parallel to the horizon — the triangle ABE is understood to be turned about the straight line AE (which is the common section of the plane of the dial and of the meridian circle) until it is set up in the plane of the meridian at right angles to the plane of the dial, then AE will be the axis of the world, and BE the common section of the horizon (drawn through CD) and of the meridian, since it makes with the meridian line AB an angle equal to the complement of the elevation of the pole, as the matter requires — as is gathered from what was demonstrated in proposition 30 of this book. Therefore AE will be the length of the style, since, with the Sun on the horizon (which is drawn through CD and EB), the extremity of the style's shadow, when it is thrown onto the horizontal line CD, [falls upon it], as is established. But if it is not convenient to describe an arc of a circle from B in order to construct the angle equal to the complement of the elevation of the pole, count off in the circle BGF twice the complement of the elevation of the pole, from F as far as G. For the straight line BG, when drawn, will make the angle ABG

equal to the complement of the elevation of the pole.[a] For since (if the straight line GA were drawn) the angle GAF at the centre would be double the angle of the complement of the elevation of the pole constructed at the same centre — because the arc FG is likewise double of that same complement[b] — and since the angle GAF at the centre is double the angle GBF at the circumference, the angle GBF will be equal to the complement of the elevation of the pole.

[Margin: How the length of the style is to be investigated in this later description of the Italian and Babylonian dial. [a] Proposition 33 of Book VI. [b] Proposition 20 of Book III.]

Once the style AE has been found, arcs, or parallels of the signs, can be described to its size, as we taught in proposition 50 of this book: namely so that the hour-lines terminate at the most remote of them, that is, in the arc ☊ [Cancer, the summer tropic] or ☋ [Capricorn, the winter tropic], as was done in the dial of the preceding proposition.

[Margin: How the arcs of the signs are described on this later dial.]

PROBLEMA 60. PROPOS. 60.

To construct an Ancient equinoctial dial [i.e. one of unequal, "planetary," hours].

Let each of the portions of the arcs of the signs cut off by the horizontal line be divided, on both dials — the upper as well as the lower — into 12 equal parts. Then through the nearest points, below and above the horizontal line, let four straight lines be drawn to both sides; and let the same be done at the following points corresponding to one another; and the Ancient dial will be described. For the line nearest below the horizontal line, on the right, will give the 1st unequal hour on the upper dial, the next the 2nd, and so on. On the lower dial the same is seen on the left of the horizontal line — that is, in the portion cut off from the horizontal line — provided, however, that all its parts are inverted. We shall demonstrate that the unequal hours are rightly described by this method, in no other way than as we showed in proposition 58 of this book that the lines of the hours from sunset are rightly drawn according to the first method — if the great circles are understood to pass through the poles of the world and through the unequal hours of the parallels of the Sun. For these will likewise cut the arcs of the signs on the equinoctial dial into equal parts, by what we demonstrated in proposition 49 of this book; and therefore the circles of the unequal hours will be

drawn through the points of division of the arcs of the signs, just as was said of the circles of the hours from sunset in proposition 58 of this book.

[Margin: The construction of the Ancient equinoctial dial.]

Diagram. A circular Ancient (unequal-hours) equinoctial dial at the foot of folio 243. A horizontal diameter divides the circle; it is labelled **HORIZON** (printed split as "HORI · ZON"), with "12" marked at each end. The upper semicircle and the lower semicircle are each divided by straight radial hour-lines into unequal-hour sectors, and two concentric dotted arcs near the centre carry a finer set of fractional (half-hour) graduations.

Legibly labelled features :

- Horizontal diameter: labelled HORIZON, "12" at the left end and "12" at the right end.
- Upper semicircle, outer arc (left → right): "I, 2, 3, 4, 5, 6 (top), 7, 8, 9, 10, 11" between the two "12" horizon marks. (Several of these numerals are rotated/worn; readings "9" and the top-centre mark are less than certain [?].)
- Lower semicircle, outer arc (left → right): "I, 2, 3, 4, 5, 6 (bottom), 7, 8, 9, 10, II" between the two horizon "12" marks.
- Interior: two concentric dotted circles with a dense set of small fractional hour-values (half-hour markers) written between the dotted radial lines — legibly among them "12, 1½, 3, 4½, 6, 7½, 9, 10½" recurring in the upper and lower halves. These interior fractional graduations are small and foxed and are **not each securely resolved** [?].
- A small central dot marks the gnomon foot / centre (unlabelled).

This figure's outer hour-numbers and interior fractional graduations carry more than two [?] labels and is a (see report).

The same Ancient dial, or dial of unequal hours, we shall also lay out by means of the diurnal arc of 18 hours and the nocturnal arc of 6 hours. For the 1st unequal hour is to be drawn on the diurnal arc of 18 hours, through the 4½ hour from midnight,

[Margin: The same Ancient equinoctial dial.]

[Margin: Description [of the Ancient dial] by means of the diurnal arc of 18 hours and the nocturnal arc of 6 hours.]

from midnight, and on the nocturnal arc of 6 hours through the $9\frac{1}{2}$ hour from noon. The 2nd hour is drawn through the 6 hour from midnight on the prior arc, and through the 10 hour from noon on the posterior arc. Then the 3rd unequal hour is to be drawn on the diurnal arc of 18 hours through the $7\frac{1}{2}$ hour from midnight, and on the nocturnal arc of 6 hours through the $10\frac{1}{2}$ hour from noon. The 4th unequal hour must pass on the prior arc through the 9 hour from midnight, and on the posterior arc through the 11 hour from noon. The 5th unequal hour is to be drawn through the $10\frac{1}{2}$ hour from midnight on the diurnal arc of 18 hours, and on the nocturnal arc of 6 hours through the $11\frac{1}{2}$ hour from noon. The 6th unequal hour does not differ from the meridian line, that is, from the 12th astronomical hour. After this, the 7th unequal hour is to be drawn on the diurnal arc of 18 hours through the $1\frac{1}{2}$ hour from noon, and on the nocturnal arc of 6 hours through the $\frac{1}{2}$ hour from midnight. And the 8th unequal hour is to be drawn on the prior arc through the 3 hour...

Diagram. A circular Ancient (unequal-hours) equinoctial dial occupying the middle of folio 244 — the *apposita figura* referenced in the text, laid out by means of the diurnal arc of 18 hours (upper semicircle) and the nocturnal arc of 6 hours (lower semicircle). A horizontal diameter divides the circle and is labelled **HORIZON** (printed split "HORI · ZON"), with "12" marked at each end. Radial straight lines (the unequal-hour lines) divide both semicircles into sectors, and two concentric dotted circles near the centre carry a dense set of finer fractional (half-hour) graduations.

Legibly labelled features :

- Horizontal diameter: labelled HORIZON, "12" at the left end and "12" at the right end.
- Upper semicircle, outer arc, running between the two "12" horizon marks: an hour sequence I, 2, 3, 4, 5, 6 (top-centre), 7, 8, 9, 10, 11. Several of these numerals are rotated to follow the arc and are worn/foxed; the individual readings are **not each securely resolved** [?].
- Lower semicircle, outer arc, between the two "12" horizon marks: an hour sequence I, 2, 3, 4, 5, 6 (bottom-centre), 7, 8, 9, 10, II. Likewise rotated and worn; **not each securely resolved** [?].
- Interior: two concentric dotted circles carrying small fractional half-hour values written between the dotted radial lines — among them legibly "12, $1\frac{1}{2}$, 3, $4\frac{1}{2}$, 6, $7\frac{1}{2}$, 9, $10\frac{1}{2}$ " (and an "18" near the top-centre) recurring in the upper and lower halves. These interior fractional graduations are small and foxed and are **not each securely resolved** [?].

- A small central dot marks the gnomon foot / centre (unlabelled).

This figure's outer hour-numbers and interior fractional graduations carry more than two [?] labels and is a (see report).

from noon, and on the posterior arc through the 1 hour from midnight. The 9th unequal hour is to be drawn on the diurnal arc of 18 hours through the $4\frac{1}{2}$ hour from noon, and on the nocturnal arc of 6 hours through the $1\frac{1}{2}$ hour from midnight. Likewise the 10th unequal hour is to be drawn through the 6 hour from noon on the diurnal arc of 18 hours, and through the 2 hour from midnight on the nocturnal arc of 6 hours. The 11th unequal hour is to be drawn through the $7\frac{1}{2}$ hour from noon on the diurnal arc of 18 hours, and on the nocturnal arc of 6 hours through the $2\frac{1}{2}$ hour from midnight. Finally, the 12th unequal hour is the same as the horizontal line. All of this is evident from tables 11 and 12 of the scholium of proposition 33 of the previous book, and may be seen in the accompanying figure. We have therefore constructed the Ancient equinoctial dial — which was to be done.

[Margin: How the lower [dial] arises from the upper Ancient dial.]

[Margin: Each unequal hour is drawn through eight points.]

[Margin: For describing the Ancient dial it is enough if the two portions of a single parallel are cut into 12 equal parts.]

SCHOLIUM.

Moreover, the unequal hours of the upper dial, when extended beyond the horizontal line, mark on the lower dial the same hours — provided all its parts are inverted, as in the preceding cases. This is clear from what we demonstrated in the scholium of proposition 14 of this book. From this it follows that each of the unequal-hour lines is drawn through eight points of the divisions of the arcs of the signs, together with the diurnal arc of 18 hours and the nocturnal arc of 6 hours — four of these points being contained on the upper dial and four on the lower. It also follows that, to describe the unequal hours, it is enough to divide into 12 equal parts just the two portions of a single parallel — namely of the greatest and outermost, such as is the parallel of ♉ [Taurus] and ♍ [Virgo]. For a straight line drawn through the first point below the horizontal line on the right, and through the first point above it on the left, will give the 1st unequal hour; and a straight line drawn through the next two points, the 2nd; and so on.

END OF THE SECOND BOOK.

GNOMONICS, BOOK THREE.

BY THE AUTHOR

CHRISTOPH CLAVIUS OF BAMBERG, OF THE SOCIETY OF JESUS.

[Margin: Why the five kinds of dials we treated in the previous book are called regular.]

The five kinds of dials that we treated in the previous book — namely the Horizontal, Vertical, Meridian, Polar, and Equinoctial — may be called **regular**, because the planes on which they are described are always set opposite a determinate point of the heavens, wherever on earth one may be. [Margin: To which point of the heavens the horizontal, vertical, meridian, polar, and equinoctial dial is set opposite.] The first faces the crown of the head, that is, the pole of the horizon; the second, the pole of the vertical circle properly so called, in which the horizon and the meridian intersect one another; the third, the pole of the meridian, that is, the point of true rising or setting, where the equator divides the horizon; the fourth, the point of the meridian circle at which it is cut by the equator; and the fifth, finally, the pole of the world, that is, of the equator.

[Margin: Why the four kinds of dials treated in this book are called irregular.]

The remaining four kinds, which we propose to describe in this book, may be called **irregular**. These are: the dial declining from the vertical circle properly so called; the dial declining from the horizon; the dial inclined to the horizon; and the dial declining from the vertical and at the same time inclined to the horizon. For the points of the heavens toward which their planes are set can be varied in countless ways [*sexcentis modis*], since they can deflect more or less from the vertical or the horizon, and incline toward the horizon, as is evident. This can in no way be said of the former planes.

[Margin: The Horizon, the Vertical, and the Meridian — likewise the Meridian, the circle of the 6th hour from noon or midnight, and the Equator — cut one another at right angles in every clime.]

For in every clime or region the Horizon, the primary Vertical, the Meridian, the circle of the sixth hour from noon or midnight, and the Equator behave in the same way and intersect one another; and the first five dials are equidistant from these. For the first three, as well as the three latter (the Meridian always being counted among both the former and the latter), always divide one another at right angles. This can easily be demonstrated from the spherical *Elements* of

Theodosius [Theodosius of Bithynia, *Sphaerica*]. For since each of the first three, as well as of the three latter, passes through the poles of the other two — as is clear if the positions of these circles on the sphere are carefully considered — it will necessarily cut them at right angles, by proposition 15 of book 1 of Theodosius (*Sphaerica* I.15).

[Margin: The great circles from which the four kinds of dials of this book are equidistant — how they behave with respect to the Horizon, the Vertical, and the Meridian.]

But the case is quite different for the great circles from which the planes of the irregular dials are equidistant, since in the same clime they do not always have the same position. For the great circle from which a dial declining from the vertical is equidistant is indeed perpendicular to the horizon, since it is drawn through the horizon's poles; but it cuts the meridian obliquely in infinitely many ways, since almost infinitely many kinds of dials declining from the vertical are found, some declining more from the vertical and some less. So too the great circle to which a dial declining from the horizon is parallel is perpendicular to the vertical circle properly so called, since it passes through that circle's poles; but it also divides the meridian obliquely in innumerable ways, according as it declines more or less from the horizon. Again, the great circle from which a dial inclined to the horizon is equidistant is perpendicular to the meridian, through whose poles it is drawn, but it intersects the horizon obliquely in countless ways as well. Finally, the great circle from which the last dial is equidistant is perpendicular neither to the horizon, nor to the vertical, nor to the meridian, but divides all these circles obliquely.

ON DIALS THAT DECLINE FROM THE VERTICAL CIRCLE.

PROBLEM 1. PROPOSITION 1.

To describe an astronomical dial declining from the vertical circle — that is, to describe the hour-lines from noon or midnight on a plane that is equidistant from some great circle which declines from the vertical circle properly so called and is perpendicular to the horizon.

[Margin: Construction of the astronomical dial declining from the vertical.]

On some plane let two straight lines AB and CD be drawn, cutting one another at right angles at E. Then at the line CD, at point E, let the angle of declination of the plane on which the dial is to be placed be constructed — how this is to be investigated we demonstrated in proposition 23 of the first book. This angle is to

be constructed in the following order. If the plane declines from the meridian [i.e. from the south], the angle of declination is to be constructed below the line AB — toward point A if it faces east, but toward B if it faces west. But if the plane declines from the north, the angle is to be constructed above the line AB — toward B if it faces east, but toward A if it faces west. [Margin: The dial being described declines from the meridian toward the east by 30 degrees.] In the present example we suppose the plane to decline from the meridian toward the east by 30 degrees; the angle of declination DEF is therefore constructed below the line AB, toward A. This angle is easily constructed if, from E, an arc of a circle is described, and on it the declination is reckoned off from the line CD. For EF, a straight line drawn from E through the end of that reckoning, will form the angle of declination at point E. And the line EF may be called the line of declination.

Diagram. The large construction figure for the declining astronomical dial, filling the lower two-thirds of folio 246. A horizontal base line **AB** runs across the middle (A at the left end, B at the right); a second straight line **CD** crosses it near-vertically, the two meeting at right angles at **E** (D at the foot, C above). From a point **C** in the upper part of the figure a broad pencil of straight hour-lines radiates outward in all directions, their far ends carrying the hour-numbers. A circle is drawn in the lower-left portion of the figure, its arc studded with division points, and numerous auxiliary straight and dotted lines cross the whole construction.

Legibly labelled points and lines, with the confident construction labels first:

- **A** — left end of the horizontal line AB.
- **B** — right end of the horizontal line AB.
- **C** — the point from which the hour-lines radiate (the centre of the dial); high in the figure, on line CD.
- **D** — foot of the near-vertical line CD (at the bottom).
- **E** — intersection of AB and CD, where they cross at right angles.
- **F** — a point on line CD below E (the end of the line of declination EF).
- **G** — near-central point (foot of the perpendicular FG to AB / on the line of the style CG). [reading of the small "G" is only moderately secure [?]]
- **H** — left-of-centre, above AB (end of the perpendicular GH, the equinoctial line).
- **β** (beta) — on line AB, to the right of E.
- Additional labelled points visible in the construction, several small and worn: **V** (upper left, marked with a right-angle square), **b** and **Z** (upper

right, on a dashed perpendicular), **Y** (right of centre), **T, S, R, L, X, M** (clustered in the left-central region between H and F), and **N, O, P** on the lower-left arc of the circle. Several of these are faint/foxed [?].

- Hour-numbers at the outer ends of the radiating lines, running around the figure: legibly among them 1, 2, 3, 4, 5 (upper and right), and 8, 9, 10, 11, 12, 1, 2, 3 (lower and left arc). These italic line-end numerals are rotated and worn and are **not each securely resolved** [?].

This figure carries more than two [?] labels (the outer hour-numbers and several faint auxiliary points) and is a (see report).

[Margin: The line of declination, what it is in a declining dial.]

After this, on the line AB take any point β — either to the right of E or to the left. (It will however be more convenient to take it on the side opposite to the line of declination EF: thus, if EF is to the left of the line CD, let the point β be taken to the right of the same line CD, and so on; for on the side opposite to EF fewer hour-lines will be drawn, as will soon become plain. And the farther the point β is from point E, the larger the dial will be made.) At the line E β , at point β , let the angle of the pole's altitude E β C be constructed — above the line AB if the plane declines from the meridian, but below the same line AB if the plane declines from the north — and always toward the line CD, in such a way that the line β C cuts the line CD at point C. [Margin: The centre of the dial.] This point C will be the centre of the dial, through which all the hour-lines are to be drawn, as we shall demonstrate.

[Margin: The line of the style.]

Again, on the line of declination EF, take the segment EF equal to E β , and from F let a perpendicular FG be drawn to AB; and from the centre of the dial C, through G, let a straight line CG be drawn, which may be called the line of the style (or of the index), since on it the style is to be fixed, making right angles with the plane of the dial. To this line of the index CG, from G, let a perpendicular GH be drawn on both sides; this will be the equinoctial line, on which, taking the segment GH — whether to the right...

BOOK THREE

to the right of point G or to the left, equal to FG. Then, when the straight line CH has been drawn, let a perpendicular GI be drawn to it; and from I let a perpendicular IK be drawn to the line of the index CG. Then CH will be the axis of

the world, and IK the length of the style, its position being the point K on the line of the style (that is, of the index). The angle GCH, moreover, will be the angle of the altitude of the pole above the declining plane, as we shall demonstrate.

[Margin: The equinoctial line. — The axis of the world. — The length of the style, and its position. — The altitude of the pole above the declining plane.]

FINALLY, on the line of the index CG take a segment GL — whether above G or below it — equal to GI, and from L as centre describe a circle of any size, to be divided into 24 equal parts, beginning from the line LM, which runs from L (the centre of the circle) through the point M, where the equinoctial line cuts the line CD. For if through these division-points and the centre L hidden lines are drawn out, the equinoctial line GH will be cut at points through which straight lines emitted from C, the centre of the dial, will give the hours from noon or from midnight, preserving the same order that they have in a vertical dial.

[Margin: The order of the hours in a dial declining from the vertical.]

For in a dial that declines from the meridian, whether toward the east or toward the west, the portion of the line CD that begins at C and is extended toward the equinoctial line marks the 12th hour, noon. The lines that cut the equinoctial line on the left side of CD — that is, toward the parts A, the western ones — indicate the hours from midnight; while the lines that cut the equinoctial line on the right side — that is, toward the parts B, the eastern ones — belong to the hours from noon. Thus the line nearest to the left of CD signifies the 11th hour, and the one nearest on its right shows the 1st hour after noon, and so on. But in a dial that turns from the north toward the east or the west, understand the reverse. For the portion of the line CD extended from C toward the equinoctial line will give the 12th hour of midnight; the lines that lie to its left on the equinoctial line give the hours from noon, and those to its right the hours from midnight — as in each vertical dial.

[Margin: When the hour-line to be drawn through the centre C of the dial should be parallel to the equinoctial line.]

But whenever a straight line drawn through some division-point of the circle described from L, and through the centre L, is parallel to the equinoctial line, then a straight line drawn through C parallel to the equinoctial line will give that hour, whatever it may be — no otherwise than happens in the horizontal dial, where the 6th hour is drawn through the centre of the dial parallel to the equinoctial line, because in the circle described from E the line drawn through the point of the 6th hour and the centre E is parallel to the equinoctial line.

[Margin: Demonstration of the construction of the dial declining from the vertical.]

We shall demonstrate this construction in the following way. Suppose that in the plane of the dial the line AB is parallel to the horizon, so that it is the common section of the plane of the horizontal dial and the plane of the declining dial; and let the plane determined by the lines AB, EF, ED be conceived in the mind as being rotated about the line AB until it becomes parallel to the horizon and so coincides with the plane of the horizontal dial. This being posited: since DEF is the angle of declination of the dial's plane from the vertical, AEF will be the complement of that declination — namely the angle that the meridian makes with the declining plane. Therefore the line EF will be the common section of the meridian and the plane of the horizontal dial; and consequently the meridian will cut the plane of the declining dial at the point E. And since both the plane of the meridian and the plane of the declining dial are perpendicular to the plane of the horizontal dial, their common section too will be perpendicular to that same plane, and hence also perpendicular to the line AB in the said plane, by Euclid, definition 3 of book XI. [Margin: (a) Euclid XI.19.] Since, then, this common section passes through E, and the line CD is by construction perpendicular to AB, CD will be the common section of the meridian and the plane of the declining dial — that is, the meridian line of the declining dial.

Again, since both the plane of the meridian and the plane of the declining dial are perpendicular to the plane of the horizontal dial, their common section CE will be perpendicular to that same plane, and hence — by Euclid, definition 3 of book XI — also perpendicular to the line EF lying in the plane of the horizontal dial.

[Margin: (b) Euclid XI.19.] If, then, the triangle βEC is imagined to be rotated about CE until it coincides with the plane of the meridian determined by the lines CE, EF, the right angle βEC will coincide with the right angle CEF, and the point β with the point F, because of the equality of the lines E β and EF. Therefore the line βC , forming in that position with the meridian line EF of the horizontal dial the angle of the pole's altitude E β C, and extended toward the northern parts on the upper side of the line EF, will be the axis of the world, meeting the plane of the declining dial at the point C. And consequently, by the corollary of proposition 21 of book I, the point C will be the centre of the declining dial. For the axis makes with the meridian line of the horizontal dial the angle of the pole's altitude above the horizon, toward the parts of the visible pole, as is clear from the portion of the analemma in proposition 1 of the preceding book.

NEXT, since the line of the index — the one on which the style or index is to be fixed — must be such that the style, or any other line drawn from any of its points perpendicular to the plane of the dial, falls upon the axis of the world [Margin: (c) Euclid XI.18.] (so that the plane drawn through that perpendicular and the axis of the world is perpendicular to the plane of the dial, like a proper meridian of the dial's own plane), because the tip of the style, by proposition 2 of book I, is the same as the centre of the world through which the axis of the world passes — we shall demonstrate, in this way, that the line CG (which in the construction we called the line of the index) is such a line. Suppose the triangle EFG is rotated about the line EG until it coincides with the plane of the horizontal dial and is parallel to the horizon, and so is perpendicular to the plane of the declining dial. This being posited, the line FG will, by Euclid, definition 4 of book XI, be perpendicular to the plane of the declining dial. And since the axis of the world, in that position, falls upon the point F — as we just showed, because it then does not differ from the point β — the plane drawn through the line FG and through the axis of the world will be perpendicular to the plane of the declining dial, like a proper meridian. [Margin: (d) Euclid XI.18.] Therefore, since all the lines that are drawn perpendicular to CG in that plane (drawn through the axis and the line FG) are perpendicular to the plane of the declining dial, by Euclid, definition 4 of book XI, it follows that all perpendiculars to the plane of the dial drawn from points of the line CG fall upon the axis of the world. And consequently the line CG is the line of the style — namely the common section of the plane of the dial and of the said proper meridian — serving as a meridian line, as if the circle to which the dial is parallel were the horizon.

Now since the line GH was taken equal to the line FG, if the triangle CGH is imagined to be rotated about CG until it is perpendicular to the plane of the declining dial — so that the line HG (which was drawn perpendicular to CG) is likewise perpendicular to that plane — the point H will fall upon the point F, since FG too has been shown to be perpendicular to that same plane; and therefore the line CH will be the axis of the world. From this it follows that the angle GCH is the angle of the altitude of the pole above the declining plane, [Margin: (e) Euclid I.29.] because it is equal to the angle which the axis of the world and the common section of the meridian of that declining plane and of the great circle to which the plane of the dial is parallel make at the centre of the world — inasmuch as this common section is parallel to the line CG in the plane of the dial. [Margin: (f) Euclid XI.16.] And it is clear that this angle, formed in the proper meridian of the declining plane at the centre of the world, stands upon the arc of the pole's altitude above that great circle to which the dial is parallel.

THAT the line GH, perpendicular to the line of the style CG, is the common section of the equator and the plane of the declining dial — as we assumed in the construction — we shall make clear as follows. Since the axis of the world CH is perpendicular, by proposition 10 of book I of Theodosius, to the plane of the equator, and passes through its centre, and hence — by Euclid, definition 3 of book XI — is perpendicular to the common section of the equator and of the plane drawn through the axis of the world CH and the line GH [Margin: (a) Euclid XI.18.] (which plane is perpendicular to the plane of the declining dial, as a kind of new and proper meridian of it, since the line GH through which it is drawn was made perpendicular to that same plane, owing to the rotation of the triangle CGH about the line CG, as was just said) — it follows that IG, if the point I be taken for the centre of the world or of the equator (and any point of the axis may be taken for the centre, since its distance in the plane of the declining dial from the centre of the world is imperceptible and utterly negligible when compared with its distance from the sun, as we taught in the treatise on the sphere), is the common section of the equator and of the plane drawn through the axis CI and the line IG; seeing that the line IG, lying in this plane, is perpendicular to the axis. For if the equator did not pass through the line IG, but through some other line drawn from the point I (which we took for the centre, and through which the equator necessarily passes), then the axis CI would be perpendicular to that line as well — by Euclid, definition 3 of book XI — since it is perpendicular to the plane of the equator, in which this line would lie. In that case, in the plane drawn through the axis CI and the line IG, two perpendiculars to the axis would be drawn at the point I, which is absurd. Therefore the plane of the equator drawn through the line IG will meet the plane of the declining dial at G, a point of the line of the index; and consequently through the point G the equinoctial line must be drawn — namely the common section of the equator and the plane of the declining dial.

Now since the plane of the triangle CGI is perpendicular to the equator — because the line CI, through which the said triangle is drawn, is perpendicular to it, as has been said; [Margin: (b) Euclid XI.18.] or certainly by proposition 15 of book I of Theodosius, because the plane of the triangle CGI is drawn through the axis CI and hence through the poles of the world, that is, of the equator —

Diagram. A large and densely-ruled construction figure for the dial declining from the vertical (the figure to which the whole demonstration of pp. 247–249 refers). It shows the centre of the dial C set high near the top, drawn as a small ringed point from which a fan of straight hour-lines radiates outward across the whole page; the horizontal line AB runs across the middle (A at the left end, B at

the right end); a large circle (the circle "described from L") occupies the lower two-thirds of the figure, its lower arc dotted; and a cluster of auxiliary construction points and perpendiculars fills the central region. The engraving is worn and foxed, and many peripheral hour-numbers are rotated and faint; labels flagged [?] below are not securely resolved.

- C (upper centre, small ringed point): centre of the dial, origin of the radiating hour-lines.
- A (left end of line AB): western terminus of the horizontal common section.
- B (right end of line AB): eastern terminus of the horizontal common section.
- E (on AB, at centre): point where the meridian cuts the plane; foot of the meridian line CD.
- β (beta, on AB, just right of E): the rotated point ($E\beta = EF$).
- G (central, on/near AB by E): point of the line of the index (foot of the equinoctial line GH).
- F (below G, centre): foot of the perpendicular; the point onto which the axis falls when the triangle is folded down.
- H (left of centre, above AB): end of the equinoctial line GH.
- M (below E, right of centre): where the 12th-hour line meets the equinoctial line.
- L (centre, below F): centre of the great circle described in the construction.
- T (left of centre, above H) [?]: auxiliary construction point.
- I / "l" (small, central, near G) [?]: auxiliary point on the line of the style.
- Y (right of centre) [?]: auxiliary construction point.
- V (upper left, marked with a right-angle square): auxiliary construction point.
- b (upper right, on a dashed perpendicular) [?]: auxiliary point.
- Z (upper right, on the dashed perpendicular below b) [?]: auxiliary point.
- α (alpha, small, below/left of A near "a") [?]: point of the 6th-hour line ($F\alpha$).
- X (left-centre, below AB) [?]: auxiliary point.
- O (left, on the circle's arc) [?]: point on the circle.
- N, P (lower-left, on the dotted arc) [?]: points on the circle.
- D (bottom centre, below 12): lower terminus of the meridian line CD.
- Peripheral hour-numbers at the outer ends of the radiating lines, worn and rotated, legibly among them: 3, 4, 5 (upper and left), 7, 9 (upper right), 5, 4,

3, 2 (right), and 8, 9, 10, 11, 12, 1 (lower and left arc) [?]. These line-end numerals are not each securely resolved.

—conversely the equator too will be perpendicular to the plane of the triangle CGI.

And the plane of the declining dial is likewise perpendicular to that same plane of the triangle CGI, since that plane was just shown to be perpendicular to it.

[Margin: (c) Euclid XI.19.] Therefore the common section of the equator and the plane of the declining dial will be perpendicular to the same plane of the triangle CGI, and hence also perpendicular — by Euclid, definition 3 of book XI — to the line of the index CG lying in that plane. And so, since the said common section must be drawn through the point G, as we just showed, GH — the perpendicular drawn to CG — will be the common section of the equator and the plane of the declining dial, that is, the equinoctial line.

And since the point I is taken for the centre of the world, from which the line IK falls perpendicular to the plane of the declining dial — by Euclid, definition 4 of book XI — because it is drawn perpendicular to CG, the common section of the plane of the dial and of the triangle CGI (which is perpendicular to that plane) — the line IK will be the style, and its position will be at K, a point of the line of the index; for no other line perpendicular to the plane of the dial, besides KI, can fall to the centre of the world I, as is clear.

Now if the circle described from the centre L is imagined to be swung about the equinoctial line GH until its centre L is joined with the centre of the world I — and it will necessarily be joined with it, because the lines GI, GL are equal, and each is perpendicular to the equinoctial line, provided the triangle CGI is understood to be perpendicular to the plane of the dial — then the lines drawn through the centre L (which is then the same as the centre of the equator) and emitted through the divisions of the circle will be the common sections of the equator and of the hour-circles from noon or from midnight, just as we demonstrated in the horizontal dial in proposition 1 of the preceding book. For in that position this circle, having the same centre as the equator, lies in the plane of the equator.

The starting-point of the circle's division is taken from the line LM, which is drawn through the centre L and the point M, where the line of the 12th hour and the equinoctial line intersect each other; because that line is the common section of the equator and the meridian, that is, of the circle of the 12th hour, since it meets the plane of the dial at the point M, through which the meridian line and

the equinoctial line pass. From this it comes about that the shadow of the style falls upon the point M when the sun is in the common section of the meridian and the equator, as can easily be gathered from proposition 11 of book I. For since, when the sun is in either of those circles, the shadow of the style falls (by the said proposition) upon the common section of that circle and the plane of the dial, it comes about that, when the sun is at the point where the said circles cut each other, the shadow of the style falls upon the point where the common sections of those circles and the plane of the dial intersect — such as is the point M; otherwise it would not fall upon both lines, as is clear.

Since this is so, the hour-circles will cut the plane of the declining dial at the same points at which the lines drawn out through the centre L and through the divisions of the circle — as common sections of those circles and the equator — meet the equinoctial line GH; and therefore the common sections of those same circles and the plane of the declining dial — that is, the hour-lines — must be drawn through the same points. Since, then, these same lines — by the corollary of proposition 21 of book I — cut one another at C, the centre of the dial, the lines drawn from C through the points of the equinoctial line will be the lines of the hours from noon or from midnight.

Which are the lines of the hours from noon, and which from midnight, can easily be understood if the circle described from L is conceived in its proper position, and if attention is carefully paid to what we described in the scholium of proposition 1 of the preceding book concerning the sections of the lines of the circle described from L (which, of course, is divided into 24 hours) with the equinoctial line — always observing that, if the dial and the circle described from L are understood in their proper positions, those division-points that follow the common section of the meridian and equator (namely the line LM) on the upper side, toward the west, pertain to the hours from noon, while those that follow the same LM on the lower side, toward the east, indicate the hours from midnight. And since, as we taught in the same scholium, those hours are to be inscribed on the dial whose lines in the circle described from L meet the centre L before they meet the plane of the dial, it can easily be recognized which points of the equinoctial line indicate the hours from noon and which the hours from midnight.

Rightly, then, has the construction of the declining dial been established by us. That a line drawn through C parallel to the equinoctial line represents that hour whose line — in the circle described from L, drawn through L — is parallel to that same equinoctial line (which does indeed sometimes, though rarely, happen), we

shall easily demonstrate in this way. That parallel drawn through L is the common section of the equator and of the hour-circle drawn through it; and to this common section, if the circle described from L is understood to have its proper position, the plane of the dial is parallel — for otherwise that parallel, when produced, would cut the plane of the dial, and hence also the equinoctial line, which lies in the same plane of the equator in which that parallel lies, which is absurd, since it is parallel to the equinoctial line. Therefore, by proposition 18 of book I, the common sections made by the plane of the dial in the equator and in the said hour-circle will be parallel to one another — that is, the equinoctial line and the hour-line of that hour-circle. And so a line drawn through C parallel to the equinoctial line will represent that hour whose line — in the circle described from L, drawn through L — is parallel to that same equinoctial line, as we said in the construction.

[Margin: The description of the dial declining from the vertical, just handed down, is not rightly criticized by Giovanni Battista Benedetti [Giovanni Battista Benedetti] in his *Gnomonica*.]

FROM all this it is clearly established that this method of describing the declining dial — which nearly all other writers use — is wrongly criticized by Giovanni Battista Benedetti in his *Gnomonica*, chapter 49. For, as is established from the demonstration we have brought forward, the hour-lines are rightly drawn by this method on a plane that declines from the vertical. The method, however, which he prescribes in that place — differing from the one we have handed down — is also sound, but in no way contrary to ours; because we set DEF as the angle of declination of the plane from the vertical circle properly so called, whereas he, in place of this angle, takes the angle of declination of that same plane from the meridian circle. So it is no wonder that his method differs from ours. But if we were to set DEF as the angle of declination of the plane from the meridian, as he does (as perhaps he supposed was done by others), and were then to proceed in the rest of the description as we have handed it down, the declining dial would undoubtedly be described wrongly — as he rightly teaches.

[Margin: Another way of laying out the hours from noon, or from midnight, in a dial declining from the vertical, by means of the horizontal dial.]

THE hour-lines can also be conveniently drawn from C in this way. From the point F let straight lines be drawn out making with EF the same angles that the hour-lines make with the meridian line in the horizontal dial described in proposition 1 of the preceding book — namely, the line of the 6th hour, such as is Fα in the figure above, [making] a right angle, and so on. This will easily be done

if from the centre of the horizontal dial a hidden circular arc is described cutting all the hour-lines, and an equal one, also hidden, is described from F toward the line AB. For if all the hour-intervals are transferred from that arc — beginning from the meridian line — into this one described from F — beginning from the line EF — then the lines drawn from F through the points of this arc will be hour-lines corresponding to the hour-lines in the horizontal dial. Therefore, where these lines intersect the line AB, the lines drawn out from C, the centre of the dial, through those points will give the hour-lines, as before.

For since in the plane of the horizontal dial the point F is the centre at which all the hour-lines from noon or midnight divide, because the axis of the world falls upon it, as we said, it comes about that the hour-lines in the plane of the declining dial must be drawn through those points of the line AB (which lies in the plane of the horizontal dial) at which the hour-lines of the horizontal dial cut the line AB lying in the same plane. For at those points the hour-circles cut the line AB. Since, then, the meridian line — that is, the line of the 12th hour — in the horizontal dial, namely the line FE, cuts the line AB at the point E, through which the meridian line of the declining dial is drawn; and since the line of the 6th hour meets that same line AB at the point α , inasmuch as in the horizontal dial the line of the 6th hour cuts the meridian line at right angles — it comes about that through the point α the line of the 6th hour must be drawn in the declining dial. By the same reasoning, the remaining lines going out from F — according to the ratio of the angles that the hour-lines make with the meridian line FE in the horizontal dial — will give on the line AB the points through which the hour-lines are to be drawn in the declining dial. Furthermore, from the horizontal dial you will easily recognize which points of the line AB pertain to the hours from noon and which to the hours from midnight.

[Margin: How the previous description of the hours is to be corrected.]

By this method the previous description can easily be examined and corrected, should some error perhaps have been committed in it. For if the hour-lines drawn through the points on the line AB, found in this way, agree with the hour-lines drawn through the points which we found on the equinoctial line by the earlier description, then no error will have been made in drawing the hour-lines. Indeed, this latter method of drawing the hour-lines is especially useful for drawing those lines which either do not cut the equinoctial line at all, or scarcely do so except at very remote points — such as, in the proposed example, the lines of the 3rd and 4th hours from noon, and so on.

[Margin: What to do when the points L and M in the figure above are too little distant from each other.]

Again, since the points L and M are sometimes so little distant from one another that a straight line can scarcely be drawn through them without error — the line from which the division of the circle described from L must begin — for such a line can easily deflect toward one side or the other, so that the resulting error in the hour-lines is at times not slight: we shall do well if first, from F, we draw to EF the perpendicular line $F\alpha$, cutting the line AB at α , so that the line $C\alpha$, when drawn, gives the 6th hour, as we demonstrated. For where the line $C\alpha$ cuts the equinoctial, from that point through the centre L we shall draw a straight line, which the line LM will necessarily cut at right angles at L, if no error has been made. By this means it will come about that the hour-lines are drawn most accurately, provided the said two lines cut each other at right angles. The operation must therefore be examined carefully before we proceed further in the description of the dial.

Diagram. A large and intricate construction of the declining dial. A horizontal diameter runs across the middle from **A** (left end) to **B** (right end) — the line AB. A radiant "sun" point **C** near the top (right of centre) sends out the full fan of hour-lines, which terminate in the numbered hour-marks arranged around the periphery. A large circle (the division circle) is drawn low in the figure with its centre at **L**; the point **D** lies at its lowest point, directly below the centre, on the vertical through the middle. **E** lies on the line AB near the centre, below **C**; **M** lies just below **E**; **F** lies below **L**. A short right-angled auxiliary construction (a small square with dotted sides, labelled near **b** and **Z**) sits at the upper right.

Readable labels, by position:

- **C** (upper centre, right of middle): the radiant point from which the hour-lines issue.
- **A** (left end of AB); **B** (right end of AB): the horizontal line AB.
- **E** (on AB, centre); **M** (just below **E**); **R** (near centre, above **M**) [?]; **Y** (centre, below **C**) [?].
- **L** (centre of the division circle, low); **F** (below **L**); **D** (lowest point of the circle, below the "12").
- **V** (upper centre, at the small square construction); **T**, **S** (centre-left, below **V**) [?]; **H** (centre-left); **X** (left of centre, below **H**).
- α / **a** (on AB just right of **A**); β (on AB, right of **E**).

- O (left, on the circle); N and P (lower left, N above P); b and Z (upper right, at the dotted square).
- Peripheral hour-numbers (as engraved): upper-left 3, then 4, 5 down the left edge; 6 at the far left (tip of the 6th-hour line beyond A); across the top 1, 7, and 6; down the right side 5, 4 (as "4."), 3, 2, 1; along the bottom 8, 9, 10, 11, 12.

[Several peripheral labels are worn or crossed by ruling and are given as best-read; this dense figure]

[Margin: By what method the division of the circle described from L may be begun from somewhere other than the line LM.]

But if the line $F\alpha$ happens to fall too far from the point E, and so cuts the line AB [too far off], we shall be able to draw, in place of the 6th hour, any other hour-line whatever that can conveniently and without error cut the line AB — provided, however, that it makes with the line FE the same angle which that hour makes with the meridian line in the horizontal dial. If, at the point where this line cuts the line AB, we draw a line to C, the centre of the dial, and if, at the point where this line divides the equinoctial, we carry another line through the centre L, we shall have in the circle described from L a line described from which the division of that circle can most conveniently be begun. Or else we shall draw the line of the 6th hour by another method, which we shall hand down in the following scholium from Andreas Schoner. If, then, some one of the points of division falls exactly on the line LM, the operation has been rightly carried out; but if not, some error has certainly been committed, which you must correct before you go on to the rest.

I should wish these cautions to be applied also in the following declining and inclined dials, if a description of the hour-lines complete in every numerical detail is desired — provided that, in a dial which declines from the horizon (of which we shall treat in proposition 13 of this book), the lines drawn out from F and the line AB that they cut are taken from the vertical dial. For the line EF is the vertical meridian line of the dial in that dial, as we shall there show.

[Margin: Another description of the dial declining from the vertical...]

Otherwise: we shall construct the same declining dial after the likeness of the horizontal dial, in this way. By proposition 29 of the first book, let the altitude of the pole above the plane of the declining dial be found, as though above some Horizon;

[Margin: ...of the dial declining from the vertical, from the altitude of the pole above the declining plane and from the inclination of the proper meridian of that same declining plane to the meridian of the horizon.]

and by proposition 30 of the same book, [let there be found] the inclination of the proper Meridian of the plane of the declining dial (I call the Meridian of this plane the great circle drawn through the poles of the world and through the poles of the declining plane, which of course stands at right angles to the declining plane, and which measures the altitude of the pole found above it, like a kind of Meridian with respect to the Horizon) to the Meridian of the Horizon, or of the place in which the dial is constructed. Then, for the altitude of the pole found — but taking account of the inclination of the said Meridians that has been found — let a horizontal dial be constructed, as we taught in proposition 1 of the preceding book, except for a few things which must be changed here on account of the inclination of the said Meridians and the situation of the declining plane, that is, according as it faces south or north. And we shall lay out the diagram in the way this ought to be done.

First let there be set up a figure entirely similar to the earlier figure of proposition 1 of the preceding book — namely a portion of the Analemma, in which are contained the common sections of the proper Meridian of the declining plane itself with the Horizon, the Vertical circle, and the Equator, and so on. (The great circle to which the plane of the declining dial is parallel plays the role of the Horizon; and another great circle at right angles to it, passing through the common sections of the Equator and of the said Horizon, performs the office of the Vertical circle.) This is arranged so that the arc CE measures the altitude of the pole found above the declining plane, and the line DG is equal to the style of the declining dial, of whatever size, etc. In our example the arc CE comprises 40 degrees 3 minutes; for such, very nearly, is the altitude of the pole found above the plane of the proposed declining dial (for we now use the same plane as before), and the line DG has been taken equal to the style IK of the same dial, so that we may not be forced to set up a new figure for this description. We could, however, at our own discretion take a style of any length.

Diagram. A portion of the Analemma, drawn as a semicircle. The diameter runs from **B** (left end) to **C** (right end), with the apex **A** at the top of the arc and the centre **D** at the middle of the diameter. From the centre **D** radial lines run out to **F** on the upper-left of the arc, to **A** at the top, and to a point on the upper-right of the arc [labelled **I** [?]]. Below the diameter a nearly-horizontal line runs from **H** (lower left) across the figure to the lower right; **G** lies at the bottom, directly

below D (the line DG being vertical), and I lies at the lower right. Labelled points: A (apex), F (upper-left arc), [upper-right arc label, read as I [?]], B (left diameter end), C (right diameter end), D (centre), H (lower left), G (bottom), I (lower right).

After this, on the line CN — drawn anyhow in some plane, such as the index-line is in the earlier description — let there be cut off from CN a segment CG equal to the line HI (which is parallel to the Horizon in the portion of the Analemma); and again, let GL be taken equal to the line DI from the same portion of the Analemma; and through G let the perpendicular GH be drawn to CN. Then CN will be, as it were, the meridian line of the declining plane, if it were taken for some Horizon, and GH will be, as it were, the equinoctial line — as the lines HE and FK are in the horizontal dial of proposition 1 of the preceding book. Now, with a circle of any size described from L, we shall divide it into 24 equal parts, as in the same horizontal dial, with this one exception: that here the division of the circle is not to be begun from the line CN, as there from the line HE. (This is because CN is not the common section of the plane of the declining dial and of the Meridian, that is, of the circle of the 12th hour, but of some other great circle — the one which measures the altitude of the pole above the declining plane, and passes through the poles of the world and through the poles of the declining plane, like a Meridian with respect to the Horizon. In the horizontal dial, on the other hand, the line HE is the section of the plane of the dial and of the Meridian, that is, of the circle of the 12th hour.) Rather, from the point N there is to be counted the inclination of the great circle that measures the altitude of the pole above the declining plane — as if its own Meridian — to the Meridian of the Horizon, that is, of the place in which the declining dial is described. This inclination, in the proposed example, we found to be about 40 degrees 48 minutes; for the division is to be begun from the point that closes this reckoning.

[Margin: How great is the inclination of the meridian of the declining plane to the meridian of the horizon.]

[Margin: In which direction the inclination of the meridian of the declining plane to the meridian of the horizon is to be counted, in the circle described from L.]

But in order to know in which direction this reckoning is to be set up, we shall first consider whether the plane of the dial declines from the south or from the north; then whether it turns toward the east or toward the west. For if it deflects from the south toward the east, the said inclination is to be counted from the point N toward the left — that is, toward the western parts, toward A — as far as the point O, in our example. For since the pole of the properly-so-called Vertical circle is that point of the Horizon where it is cut by the Meridian, the pole of any

other Vertical circle declining from that one toward the east on the south side will lie in the southern and eastern quadrant of the Horizon, as can be seen from the material sphere. For all Vertical circles have their poles on the Horizon; since they are drawn through the poles of the Horizon, the Horizon in turn passes through their poles (as we demonstrated in the scholium to proposition 15 of book 1 of Theodosius). Therefore the great circle drawn through the poles of the world and through the poles of the declining plane, as its proper Meridian, will cut the Equator above the Horizon in the eastern quadrant, so that the Meridian of the Horizon, in the semicircle of the Equator above the Horizon, lies more to the west than the proper Meridian of the declining plane. Hence this inclination of the Meridians is to be counted from the point N — the point of the proper Meridian of the declining plane itself — toward the west, that is, to the left of the line CN, toward A, so as to obtain the point O, further to the west, at which the Meridian of the Horizon cuts the Equator above the Horizon.

For if the plane of the dial be set toward the south, so that the equinoctial line GH is below the point C, and the circle described from L about the line GH is understood to be turned round until it coincides with the plane of the Equator, then the point N will be toward the upper hemisphere; and the parts to the left of CN, toward A, will tend westward, while the parts to the right of the same line CN, toward B, will incline eastward. But if the plane declines from the south toward the west, the said inclination of the Meridians is to be reckoned from the point N toward the right, toward B — namely toward the eastern parts of the circle described from L — provided it has attained that position of which we just spoke; because then the Meridian of the declining plane is less to the east than the Meridian of the Horizon (as can easily be gathered from the material sphere and from what we just wrote), since then its pole, on the south side, is placed in the southern and western quadrant of the Horizon.

We do not, however, teach in which direction the said inclination of the Meridians is to be counted in a plane that declines from the north toward the east or toward the west, lest the reader's mind be confused by a multitude of precepts — especially since from the southern dial, once described, the northern one can be derived with the greatest ease, as we taught in the scholium to proposition 13 of the preceding book, and as will appear more clearly from the following scholium. There is also this: that from the precepts already handed down for a plane declining from the south, anyone will be able, on his own and without great labour, to find in which direction he ought to count the said inclination in the circle described from L, by observing carefully whether the Meridian of the place

— that is, of the proposed Horizon — on the Equator below the Horizon is more to the east or more to the west than the proper Meridian of the [plane]

declining plane. I say *below the Horizon*, because in the northern dial the point N looks toward the lower hemisphere, so that the proper Meridian of the declining plane cuts it below the Horizon at the point N — as is plain, if the position of the northern dial is rightly considered, together with the circle described from L. For if it is further to the east, that inclination is to be counted from N toward the eastern parts; but toward the western parts, if it is further to the west. It will not be difficult, however, to judge which parts of the circle described from L, reckoned from N, tend toward the east and which toward the west, if the circle itself, together with the northern dial, is imagined to be set in its proper position, so that the point N looks toward the lower hemisphere and its centre is joined to the centre of the world. For the parts on the left, when we are turned to face the northern dial, will be eastern, and those on the right, western.

Diagram. A portion of the Analemma, again drawn as a semicircle, essentially the same as the previous figure. The diameter runs from **B** (left end) to **C** (right end), with the apex **A** at the top of the arc and the centre **D** at the middle. From **D** radial lines run to **F** on the upper-left of the arc, to **A** at the top, and to a point on the upper-right of the arc [read as **I** [?]]. Below the diameter a nearly-horizontal line runs from **H** (lower left) toward the lower right, with **G** at the bottom directly below **D** and **I** at the lower right. Labelled points: **A** (apex), **F** (upper-left arc), [upper-right arc label, read as **I** [?]], **B** (left diameter end), **C** (right diameter end), **D** (centre), **H** (lower left), **G** (bottom), **I** (lower right).

Now, if through the points of division and through the centre **L** hidden straight lines are drawn cutting the equinoctial line **GH**, points will be obtained on the equinoctial line, and lines sent out from the centre **C** through them will give the hours from noon, or from midnight, as we demonstrated in the horizontal dial — yet in such a way that the line drawn out from **C** through the point **M** (where the line **OL** meets the equinoctial line) is the line of the 12th hour: of noon in the southern dial, but of midnight in the northern; and not the line **CGL**, as in the horizontal dial. For in the declining plane, if the circle described from **L** is understood to be in its proper position — namely, described in the plane of the Equator about its centre — the Meridian of the Horizon, that is, the circle of the 12th hour, passes through the point **O**, and consequently through the line **OL**, meeting the plane of the dial at **M**. So in the southern dial the point **O** lies in the semicircle of the Meridian above the Horizon, and therefore indicates noon; while in the northern dial the same point lies in the other semicircle, and so

pertains to midnight — as is manifest, if the circle described from L is understood to be in its proper position in the lower dial as well. The remaining hours follow this same order. In each dial the lines which, after the 12th hour, follow toward B belong to the hours after noon; those toward A, to the hours after midnight.

The style, and its place on the line CN, will be found as in the horizontal dial. For if the line CK is taken equal to the line HG of the portion of the Analemma, K will be the place of the style. And if, in the dial, a triangle CGI is made equal to the triangle HID of the portion of the Analemma, and the line KI is drawn, then IK will be the length of the style, and entirely equal to the line DG that was assumed, as is evident. For since the sides CI and CK of the triangle CIK are equal to the sides DH and HG of the triangle DHG, and the angles C and H are equal, by construction, the bases IK and DG will also be equal to each other. [Margin: a — by proposition 4 of the first book (Euclid, *Elements* I.4).]

[Margin: Another description of the same dial declining from the vertical.]

We can also set up the description of the same dial in this way. By proposition 30 of the first book, let there be found the arc of the declining plane intercepted between the Meridian of the Horizon and the great circle which is drawn through the poles of that same plane and measures the altitude of the pole above it, as its proper Meridian. In our example this arc comprises about 29 degrees 3 minutes. [Margin: How great is the arc of the declining plane intercepted between its meridian and the meridian of the horizon.] Likewise, by proposition 29 of the same first book, let the altitude of the pole above the declining plane be found, which in our same example amounts to about 40 degrees 3 minutes. Then, in some plane, let a line CD be drawn, anyhow, for the meridian line, or the line of the 12th hour; and in it, taking any point C, let the arc DP be described from that point, in which — from the line CD, that is, from the point D — let there be counted the arc of the declining plane intercepted between its own Meridian and the Meridian of the Horizon, as far as P (which arc we said comprises about 29 degrees 3 minutes); and from C, through P, let the line CP be drawn, which will be the common section of the plane of the declining dial and of the Meridian of that same plane, as we shall demonstrate shortly below.

[Margin: In which direction the arc of the declining plane, intercepted between its proper meridian and the meridian of the horizon, is to be counted.]

But in order to know in which direction the arc DP is to be counted, we must consider what we handed down in the preceding description. For if the plane declines from the south toward the east, the said arc is to be reckoned to the left,

toward A: because, as we said, the Meridian of the Horizon is then further to the west than the Meridian of the declining plane itself. Hence, with the dial set in its proper position, the meridian line CD will lie further to the east than the line CP — the common section of the plane of the dial and of the proper Meridian of the declining plane — since in the plane of the dial the ray of the Sun, wherever it is situated in its circle, is always projected to the opposite side. And it is clear that the line CD is further to the east than the line CP, if the dial has its proper position. But if the plane turns from the south toward the west, the arc DP is to be counted to the right, toward B: because then the Meridian of the Horizon is further to the east than the Meridian of the declining plane, as we said, and so in the dial the meridian line CD ought to lie further to the west than the line CP.

In which direction the said arc is to be counted in the northern dial we do not teach, for the reasons explained a little before — namely, lest the reader's mind be overwhelmed by a multitude of precepts, especially since it is enough for the southern dial to be described. For from this the northern one will be derived without any labour, as will be evident from the following scholium. Add to this that the matter itself is not difficult, if one considers whether the proper Meridian of the declining plane, in the lower hemisphere, is further to the east or further to the west than the Meridian of the Horizon. For the aforesaid arc must always be counted in the [northern] dial in the opposite direction, etc. — yet with this rule: that in the northern dial the point C lie below the arc DP. [recte: horologio ("dial"); printed *borologio*.]

After this, from any point of the line CP — say from G — let the perpendicular GH be drawn to it, which will be the equinoctial line. For the line CP is the index-line, as we shall show shortly, to which the equinoctial line is necessarily perpendicular, as we demonstrated above. Now, the altitude of the pole above the declining plane having been found by proposition 29 of the first book, let there be constructed at C, upon the line CP, the angle GCH of this found altitude of the pole. For GH will be the axis of the world, and from G let the perpendicular GI be raised to it; and let the rest be done as in the first description of this declining dial — that is, let GL be taken equal to the line GI, and let the circle described from L be cut into 24 equal parts, beginning from the line LM, and so on. The gnomon will be IK, the perpendicular drawn from I to CP, as before.

That the line CP is the common section of the plane of the declining dial and of the proper Meridian of that same plane, we shall show in this way. Since the Meridian of the Horizon and the Meridian of the declining plane — that is, the

great circle drawn through the poles of the world and through the poles of the declining plane — are drawn through the axis of the world, they meet

[they meet] the great circle to which the dial is parallel, at the centre of the world — namely at I, the tip of the style. The result is that, where their common sections meet this great circle, they are straight lines forming at the centre of the world an angle subtended by that arc of the same great circle which is intercepted between those two Meridians.

Now these same Meridians meet the plane of the dial at the point C, where the axis is taken to meet that plane; and there too they make straight lines as their common sections, [Margin a: Euclid XI.16] which are parallel to those in the great circle, because the plane of the dial is parallel to that same great circle. Therefore these lines will contain, in the plane of the dial, an angle equal to the angle which those former lines make in the great circle. [Margin b: Euclid XI.10] Therefore, since the angle DCP is equal to it — as we showed in the scholium to Euclid VI.33 — it follows that the arc DP is similar to the arc of that circle intercepted between the two Meridians, because it contains the same number of degrees and minutes. Now let CD be taken as the meridian line — that is, the common section of the Meridian of the Horizon and of the plane of the dial. Then CP will be the common section of the plane of the declining dial and of the Meridian of that same plane — that is, of the great circle passing through the poles of the world and through the poles of that plane. And accordingly the style must be set upon it at right angles, since in this way it does not depart from the plane of this Meridian, but reaches the very axis of the world lying within it. Therefore CP will be the line of the index, or style. From this the rest of the construction of the dial will be demonstrated, just as in the first construction at the beginning of this proposition. We have therefore laid out the astronomical dial declining from the vertical circle — that is, the hour-lines, and so on. Which was to be done.

SCHOLIVM.

[Margin: A doubt raised against what has just been said.]

But at this point a certain scruple seems to need rooting out of the reader's mind, one that might otherwise trouble and torment him not a little. For how — someone will say — can the angle formed in the plane of the dial by the sections of those Meridians be equal to the angle in the great circle to which the plane of the dial is parallel, the angle formed there by the sections of those same Meridians, given that those Meridians do not cut into similar arcs the circle produced on the sphere by the plane of the dial and the great circle to which the

dial is parallel? For if they did cut them into similar arcs, then by Theodosius, *Sphaerica* II.16, they would pass either through their poles or would be tangent to one and the same parallel — and neither of these can happen here. For since both are drawn through the poles of the world, which cannot be the pole of those circles (since their pole lies in the Horizon), they will not be able to pass through the poles of those circles. Moreover, since the Meridian of the declining plane passes through its poles, it will bisect all its parallels (by Theodosius, *Sphaerica* I.15), and so will be tangent to none of them. Therefore those two Meridians cannot be tangent to one and the same parallel. Hence the arcs which the said equal angles cut off are not similar.

The same thing is plain to the senses on a material sphere, where one clearly sees that the Meridian, and either Colure, cannot divide into similar arcs the Horizon and a circle below the Horizon parallel to it, because those circles are not drawn through their poles. And the same reasoning holds for any declining vertical circle and any circle parallel to it. This being so, those two angles formed by the said sections do not seem able to be equal, since — as we showed — they do not cut off similar arcs from the great circle to which the dial is parallel, nor from its parallel which the plane of the dial produces on the sphere.

[Margin: Resolution of the foregoing doubt.]

These are the considerations that might make the reader anxious, and so it will be necessary to explain the matter more clearly. I am glad to resolve this doubt here — though I know it has no place among those trained in the elements of spherics — because from it many other similar doubts can be cleared up, as will become plain from what follows. I say, then, that we have rightly demonstrated those angles to be equal; nor is it any objection that those arcs are not similar. For since the axis of the world does not fall on the centre of the circle which the plane of the dial produces on the sphere — because, by Theodosius, *Sphaerica* I.10, that circle's own axis falls on its centre — the point C, on which the axis of the world falls, will not be the centre of that circle. And accordingly it is impossible for the straight lines forming the angle at C in the plane of the dial to cut off from that circle an arc similar to the one cut off, in the great circle to which it is parallel, by the straight lines that contain an equal angle. For only equal angles set at the centres of circles, or at their circumferences, stand on similar arcs — as we demonstrated in the scholium to Euclid VI.33 — which the two aforesaid angles are not. For of them, the one in the great circle to which the dial is parallel is set at its centre — that is, at the centre of the world — while the other is set in the circle parallel to it, outside its centre, namely at the point C, which we have just

shown is not the centre of that circle. It is nonetheless true that, if a circle is drawn from C in the plane of the dial, the arc which the said angle DCP cuts off — such as, in our example, the arc DP — will be similar to the arc that an equal angle, set at the centre of the great circle to which the dial is parallel, cuts off from it. But that circle is not concentric with the parallel which the plane of the dial makes on the sphere, as we showed. Therefore the doubt raised here has no place.

[Margin: How a dial declining from the vertical is to be set up so as to show the hours.]

Now if the declining dial drawn so far is set upon the declining plane in such a way that the meridian line — that is, the 12-o'clock line CD — is perpendicular to the Horizon, and the straight line AB, cutting it at right angles, is parallel to the Horizon; and if furthermore, in dials declining from the south, the point C is placed upward and the point D downward, while in those turning away from the north the point C is placed downward and D upward; and if also at the point K the style IK is fixed at right angles, or the triangle CIG is set upright to the plane of the dial — then the tip of the shadow cast by the top of the style, or the whole shadow of the entire axis CI, will indicate the hours from noon, or from midnight, as we showed in the scholium to Proposition 1 of the preceding book.

[Margin: How, on a fixed plane that declines from the vertical, an astronomical dial is to be described.]

[Margin: By what method the hour-lines can be drawn according to Andreas Schöner, even though their points are not found on the equinoctial.]

But if the dial is to be described on the declining wall itself, and not on some other plane from which it is then transferred onto the wall — as we have done up to now — then on the wall a straight line CD must be drawn, made perpendicular to the Horizon by means of a plumb-line, and perpendicular to it, on the same wall, a line AB must be drawn, which will be parallel to the Horizon. Or, if you prefer, using a level and a plumb-line, draw the straight line AB parallel to the Horizon, and raise perpendicular to it the line CD, which will be perpendicular to the Horizon. The rest is to be constructed as in the first description of the declining dial — namely, the line of declination EF is to be drawn, and the line FG perpendicular to AB, and so on. But if you prefer to use the other two methods, here too, before anything else, a straight line CD must be drawn perpendicular to the Horizon. Then a straight line CP is to be drawn so that CD and CP intercept the arc DP, as we said in the third method — yet with this proviso: that careful

attention be paid to which side the arc DP (found by Proposition 30 of the first book) ought to be reckoned toward, just as we prescribed in the third method. Everything else is to be completed as before — that is, the equinoctial line GH is to be drawn perpendicular to CP, and so on.

But since it sometimes happens that some of the hour-lines either never meet the equinoctial line, or meet it only very far off — which comes about because the straight lines drawn from the centre L through the division-points of the circle described from L are sometimes parallel to the equinoctial line itself, or intersect it at very remote points — Andreas Schöner [Andreas Schöner] devised a method,

truly ingenious, by which these same hour-lines can be drawn from C, the centre of the dial, without any regard to the equinoctial line. This he did in the horizontal dial as well, and in the vertical. His construction runs roughly as follows. From the point E, drop perpendiculars ET and EQ to the lines βC [?] and EF; and let EQ be made equal to ET [?]. Now let the straight line QF be drawn, cutting the line AB at R. Through R draw RS perpendicular to AB (or parallel to CD), cutting the line C α — the 6-o'clock line — at S. Again, through S draw ST perpendicular to RS (or parallel to AB); and having taken ST equal to RS, draw through T a line perpendicular to ST (or parallel to RS). Finally, if a semicircle described with T as centre is divided into 12 equal parts, beginning from the line TS, the straight lines drawn from T through the division-points will cut the line RS at points through which the lines issuing from C, the centre of the dial, will give the hour-lines, as before. And if this method of construction is combined with the earlier ones, all the hour-lines will be drawn most accurately and precisely. For those hour-lines that cut the line RS at remote points will divide the equinoctial line not far from the point G, as appears at the 11-o'clock line; and those that cut the equinoctial line only with difficulty will most conveniently cut the line RS, as is clear at the 4-o'clock line.

But because for this construction it is necessary first to draw the 6-o'clock line from C, the centre of the dial, through the point α on the line AB — the point on which the line F α , perpendicular to the line of declination EF, falls (for without this the point S cannot in any way be obtained on the line RS, as is evident) — and because it sometimes happens that this 6-o'clock line can only be drawn without error with difficulty, since the perpendicular F α occasionally falls on a very remote point of the line AB (which occurs when the declination of the dial's plane from the vertical circle is very small, so that the line of declination EF lies close to the line CD, as is evident):

[Margin: How the 6-o'clock line can be drawn on planes declining but little from the vertical.]

the same Andreas [Schöner] teaches how the 6-o'clock line can then, when this happens, be drawn even though the line $F\alpha$ has not been drawn. We shall carry the matter out with him as follows. From the point β , draw βZ perpendicular to AB (or parallel to CD), which the line CZ — perpendicular to CD , or parallel to AB — cuts at Z . Then at Z construct the angle of declination CZa : above the line CZ if the plane declines from the south, but below it if from the north. (This is to be understood when the point β lies on the opposite side of the line of declination EF , as in our example; for if the point β and the line EF lie on the same side of the line CD , the angle of declination CZa must be constructed below CZ in southern dials, but above it in northern ones.) Also, having taken the line Za equal to $Z\beta$, draw through the point a — which we shall perhaps find more easily if from Z , at the interval $Z\beta$, we describe the arc Za above the line CZ (or below, as we said of the angle CZa), and count off the declination of the dial's plane from the point C as far as a — the straight line ab , parallel to AB (or perpendicular to βZ), cutting βZ at b . For the straight line drawn out through the points b and C will give the desired 6-o'clock line.

This will be demonstrated as follows. The triangles αEF and $aZ\sigma$ are equiangular, since the angles at F and σ are right and the angles at E and Z are the complements of the declination. It follows [Margin c: Euclid VI.4] that αE is to EF — that is, to $F\beta$, or to CZ , that is βZ — as aZ (that is βZ), or EC , is to σZ [?: this proportion is set very compressed in the print, and its middle terms are hard to construe]; and, by alternation, as αE is to EC , so CZ is to $Z\sigma$. Therefore, since the two triangles αEC and $CZ\sigma$ have the sides αE , EC proportional to the sides CZ , $Z\sigma$, and are joined at the angle at C so that their corresponding sides are parallel, [Margin d: Euclid VI.32] the sides αC and cb will form one straight line; and so the line bC will not differ from the 6-o'clock line. The same procedure and demonstration also apply to the following construction.

[Margin: In this figure let the lines βM and βI [?], and ba , and bC , and Za , and Zc be drawn; and from Z let an arc be described at the interval Za , beginning from the line ZC , through a , upward, in the same way as was done in the first figure of this proposition.]

Diagram. A large declining-dial construction figure occupying the lower two-thirds of the page. At the centre is a small circle enclosing the point **C** (the centre of the dial), with **F** just to its left, a lowercase **z** just to its right, and **E** immediately below it. From **C** a fan of straight hour-lines radiates outward to a wide

semicircular spread of numbered and lettered points; a dashed circle (the division-circle) and dotted construction-arcs run through the lower half of the figure, with the points **L**, **G**, **M**, **H**, and **D** lying on or near them in the lower-central region, and **I** to the left below **F**. The peripheral hour-points, read where legible, run clockwise from the upper left: **A** (upper left), α , **a**, **b**, **5** (upper right), then down the right side **4**, **B**, β , **3**, then across the bottom **2**, **1**, **12**, **11**, **10** [?], and up the left side **9**, **8**, **7**, **6**, **N**, with a further **A** and lowercase α [?] on the left margin and an **S/5** [?] on the left. Because the peripheral labels are small and several sit on crossing lines, the readings marked [?] (the second α , the left-hand S/5, β I, and the bottom "10") could not be resolved with certainty.

- **C** (centre, in small circle): centre of the dial
- **F** (just left of **C**): tip/foot point of the style-triangle
- **z** (just right of **C**, lowercase): construction point
- **E** (just below **C**): construction point on the meridian
- **I** (left, below **F**): construction point
- **G** (lower-central): point where the equinoctial meets the style-line
- **M** (lower-central, near **G**): point where the equinoctial meets the meridian
- **L** (lower left of centre): centre of the division-circle
- **H** (lower-central): point on the declination arc
- **D** (lower-central): point on the declination arc
- **A**, **B**: end-points of the horizontal line **AB** (also serving as hour-line terminals)
- α , β , **a**, **b**: alphabetic construction points on/around **AB**
- **N** (lower left): construction point
- Numerals 1–12 around the periphery: the hour-line terminals (several flagged [?] — see above)

[Margin: Construction of a declining dial for a given length of style whose position is also given.]

But if the same dial declining from the vertical circle is to be described on a given plane, for any length of style whose position is also given, we shall proceed by this device. On the plane of the dial let the position of the style be given at the point **K**, and its length **KF**. Through **K**, the position of the style, draw two straight lines **AB** and **FD**, cutting each other at **K** at right angles. Or, if the plane is fixed and perpendicular to the Horizon, draw through **K** a line **AB** parallel to the Horizon (by means of a level and plumb-line), which the line **DF** cuts at **K** at right angles; and **DF** will be perpendicular to the Horizon, as we shall demonstrate. Or,

alternatively, first draw (by means of a plumb-line) the line FD perpendicular to the Horizon through K, the position of the style; the line AB cuts it at K at right angles, and AB will be parallel to the Horizon — indeed, it will be the common section of the Horizon and of the plane of the declining dial, as we shall also demonstrate. On FD, having taken the length of the given style KF (either upward or downward), describe from F as centre, toward AB, an arc of a circle DH: toward B if the wall or plane of the dial declines from the south toward the east, or from the north toward the west; but toward A if it turns from the south toward the west, or from the north toward the east. On this arc mark off the declination of the wall, DH —

(this arc should contain nearly 30 degrees from the south toward the east, as before). When the straight line FH has been drawn, cutting AB at E, draw through E the line CE parallel to FD (or perpendicular to AB). For CE will be the meridian line, that is, the 12-o'clock line. Then in the arc HD, on the other side of the produced line FD, mark off the complement of the declination DN, and draw the line FN; or else draw the line F α perpendicular to FE, cutting AB at α . For through the point α both the equinoctial line and the 6-o'clock line must be drawn. Also, having taken on AB a line E β — either to the right or to the left of the meridian line CE — equal to EF, describe from β , toward CE, an arc of a circle: upward in a dial declining from the south, but downward in one that turns away from the north. On this arc mark off the altitude of the pole, beginning from the line AB, and from β draw, to the end of that measurement, a straight line cutting CE at the point C, which will be the centre of the dial, where all the hour-lines meet. Now draw from C, through K (the position of the style), the straight line CK for the style-line; and let a straight line drawn from α cut it at right angles at G. Then the line α G will be the equinoctial line. After this, raise from K the line KI perpendicular to CK and equal to the style KF, and join the lines CI and GI, which will contain a right angle at I, as we shall demonstrate. Hence, the line CI having been drawn, if we raise a perpendicular IG to it, we shall find on the style-line CK the point G, through which the equinoctial line α G, perpendicular to CK, must be drawn from α . But if from β a perpendicular β M is raised to β c, cutting the meridian line CM at M, the equinoctial line must be drawn through M as well; and indeed it is drawn more precisely through the more remote point M than through the nearer point G. And CI will be the axis of the world. Now on the style-line CK take a line GL equal to GI, and from L describe a circle of any size, dividing it into 24 equal parts, beginning from the line LM which is drawn from L through the point M, where the equinoctial line intersects the meridian. This line LM will necessarily cut the drawn line L α at right angles, if no error has been

made. Finally, let the rest be completed as was set out at the beginning of this proposition.

We shall demonstrate this construction as follows. [Margin: Demonstration of this construction.] Suppose that through the line AB and the style KF — which is perpendicular to the plane of the dial at the point K — a Horizon is drawn, so that AB is the common section of the Horizon and of the plane of the declining dial, namely the horizontal line. Suppose also that through the pole of the declining plane and the pole of the Horizon a great circle is drawn. By Theodosius, *Sphaerica* I.15, it will be perpendicular both to the declining plane and to the Horizon; and therefore, since it passes through the centre of the world — that is, through the tip of the style F — and since [Margin a: Euclid XI.18] all planes drawn through the style KF (which is perpendicular to the declining plane) are perpendicular to that same declining plane, this great circle will pass through the style FK and will cut the plane of the declining dial at K. Since, then, both the plane of the dial and the plane of this great circle are perpendicular to the Horizon, their common section too will be perpendicular to it; [Margin b: Euclid XI.19] and hence, by definition 3 of Euclid XI, it will be perpendicular to the line AB lying in the Horizon. Therefore the line FD, which we drew through K perpendicular to AB, will be the common section of the said great circle and of the plane of the declining dial, and hence perpendicular to the Horizon.

But if the line FD is drawn first, perpendicular to the Horizon, suppose a plane is drawn through the line FD and through the style FK (which is perpendicular to the plane of the declining dial); this plane [Margin c: Euclid XI.18] will be perpendicular both to the Horizon and to the plane of the declining dial. Since, then, both the plane of the dial and the plane of the Horizon are perpendicular to the plane drawn through the line FD and the style FK, their common section too will be perpendicular to that same plane; [Margin d: Euclid XI.19] and hence, by definition 3 of Euclid XI, it will be perpendicular to the line FD lying in the said plane, at the point K through which the Horizon is drawn, since it is drawn through the style FK. Therefore the line AB, which we drew perpendicular to FD, will be the common section of the plane of the dial and of the Horizon — that is, the horizontal line.

Next, since the angle EFk [?] is the declination of the plane of the dial from the vertical, the angle EFK will be the complement of that same declination — the very angle which the Meridian makes with the declining plane. Therefore, if the triangle EFK is rotated about EK until it is perpendicular to the plane of the dial set in its proper position, FE will be the common section of the Horizon and of the

Meridian drawn through F, the tip of the style; and therefore, as we demonstrated in the first description of this proposition, CM will be the meridian line — namely the common section of the Meridian and of the plane of the dial. And since the arc HN is a quadrant, the angle $EF\alpha$ will be a right angle; and accordingly the line $F\alpha$ will be the common section of the vertical and of the Horizon. For since both the Horizon and the vertical are perpendicular to the Meridian, their common section too will be perpendicular to it; [Margin e: Euclid XI.19] and hence, by definition 3 of Euclid XI, it will be perpendicular to the line FE lying in the Meridian, at F, the centre of the world. Therefore $F\alpha$, which is perpendicular to FE, is the common section of the vertical and of the Horizon. But the Equator has the same common section with the Horizon and the vertical — namely the axis of the Meridian — since all three of these circles pass through the poles of the Meridian (by Theodosius, *Sphaerica* I.13), because they are perpendicular to the Meridian. Therefore the Equator is drawn along the line $F\alpha$, and meets the plane of the declining dial at the point α ; and so the equinoctial line must be drawn through α .

The triangle EFK still remaining perpendicular to the plane of the dial, so that the angle CEF is a right angle — (that it is a right angle we show thus: since both the plane of the dial and the plane of the Meridian are perpendicular to the Horizon, their common section CE too will be perpendicular to it, [Margin f: Euclid XI.19] and hence, by definition 3 of Euclid XI, perpendicular also to the line FE lying in the Horizon in that position) — suppose the triangle βEC is rotated about the line CE until the line βE coincides with the line FE, and the point β with the point F, on account of the right angles βEC and FEC and the equality of the lines $E\beta$ and EF. This done, βC will be the axis of the world; since, in the plane of the Meridian (which is drawn through the lines FE and EC), it forms with the meridian line of the Horizon EF, at F the centre of the world, the angle $F\beta C$ equal to the altitude of the pole above the Horizon. Therefore, by the corollary to Proposition 21 of Book 1, the point C, on which the axis falls, will be the centre of the dial. Hence it follows that the line CK is the style-line — that is, the common section of the plane of the declining dial and of the proper Meridian of that declining plane, which is drawn through the axis and the style, serving as the proper meridian line of the declining dial; just as in the first description of this proposition the style-line CG was drawn from C, the centre of the dial, through K, the position of the style. By the same reasoning it follows that the line αG , which is perpendicular to the style-line, is the equinoctial line, since it must be drawn through the point α , as we recently showed, and it makes right angles with the style-line, as was demonstrated in the first description of this proposition.

That the same equinoctial line must be drawn through M we shall prove as follows. Suppose the triangle EFK is rotated about EK, and the triangle C β M about CM, until FK is perpendicular to the plane of the dial and the point β coincides with the point F, on account of the equality of the lines EF and F β , so that C β is the axis — as we showed a little earlier — and hence the triangle C β M lies in the plane of the Meridian. And since the common section of the Meridian and the Equator is perpendicular to the axis — because the axis, being perpendicular to the Equator, makes right angles with all straight lines drawn in the plane of the Equator through its centre (by definition 3 of Euclid XI), and hence with the common section of the Equator and the Meridian drawn through F (or β), the centre of the world — the line β M, lying in the plane of the Meridian and making right angles with the axis C β , will be the common section of the Meridian and the Equator. And accordingly the Equator meets the meridian line CM at M; and therefore the line of the Equator must be drawn through M.

That the straight line CI is the axis of the world is evident. For if the triangle CIK is rotated about CK until it stands perpendicular to the plane of the dial, and the style IK [stylus] is likewise perpendicular to that plane, the point I will coincide with the point F, and hence also with the point β . The straight line CI will therefore coincide with the axis C β . From this it follows that in that position the straight line GI is the common section of the Equator and of the proper Meridian of the declining plane itself, since each circle is drawn through G and through I, the centre of the world. The angle CIG will accordingly be a right angle, since the said common section is always perpendicular to the axis, as is clear from what precedes. The rest is evident from what was demonstrated in the first construction of this proposition.

[Margin: Description of the same dial by means of the ellipse.]

By means of the ellipse we shall likewise describe the same dial declining from the Vertical, just as we did for the horizontal one — as we showed in the scholium to Proposition 1 of the preceding book — with only a few changes. Let the length of the style be given again as KF, and let its position be the point K. Then, once the meridian line [linea meridiana] CE has been found — with C the centre of the dial, α G the equinoctial line [linea aequinoctialis], CI the axis of the world [axis mundi], and CG the style-line [linea styli] — as we prescribed in the immediately preceding construction: take on the axis, produced, a point H more distant than I, so that the dial may be described more accurately; and from H drop a perpendicular HN to the axis, cutting the style-line at N, so that HN becomes a new common section of the Equator.

Diagram. A large circular construction for the declining sundial drawn by means of the ellipse. A horizontal diameter runs from **A** (left) to **B** (right); a vertical diameter runs from **12** (top) through the centre down to **D · 12 · P** (bottom). Four oblique hour-lines carrying the numerals **3** (upper left), **6** (right), **6** (lower left) and **3** (lower right) radiate beyond the outer circle. Two concentric circles (the larger and the smaller of the construction) are drawn about the centre; the inscribed ellipse, whose major diameter is **NR** and minor diameter **ST**, is traced through the constructed hour-points. Readable interior labels, by region:

- Centre and horizontal line: **F**, **e** (centre), **K**, **E**, **β**, and toward the right **S** [?]; endpoints **A**, **B**.
- Upper arc: **12** (apex), **Q**, **δ**, **R**.
- Left, small auxiliary ellipse/right-triangle near **S–T**: **S**, **T**, and the small labels **m**, **l** [?], **i** [?], **n**, **p**.
- Left of centre, below **A–B**: **t**, **H**, **V**, **I**, **G**, **O**.
- Lower centre: **v**, **M**, **Z**; bottom **D**, **P** flanking the lower **12**.
- Lower-left, a second small right-triangle near the ellipse's lower vertex: **q**, **r**, **g** [?], **b** [?], with the vertex **N** below.
- Right: **f**, **X**, **b**, **a** (= α).

The point **L** (centre of the auxiliary circle named in the text) is not securely resolvable on the tile. Numerous small labels are worn or overrun by construction lines; the figure carries more than two uncertain labels

—tor, and of the proper Meridian of the declining dial's plane; and let **N** be the point through which the new equinoctial line must be drawn perpendicular to the style-line. Now, having cut off a straight line **CO** equal to **HN**, describe about the centre **C** of the dial two circles at the intervals **CN** and **CO**; and in the smaller circle draw the diameter **ST** perpendicular to **NR**, the diameter of the larger: then **NR** will be the major diameter of the ellipse and **ST** the minor — just as in the horizontal dial the major diameter is **BV** and the minor **RX**. The points of the ellipse through which the hours are to be drawn are now found, as was stated for the horizontal dial, if the said two circles are divided into 24 equal parts, the division beginning not from the proper meridian line of the declining plane — that is, from the style-line **RN** — but from a certain other straight line, which we shall find as follows. From **L**, the centre of the circle by whose aid we located the hour-points on the equinoctial line αG in the preceding description (this centre being distant from the point **G** by the interval **GI**), draw through the point **M** — where the equinoctial line and the meridian line cut one another — the straight line **LM**; and at **C**, on the line **CL**, construct on that same side on which the point

M lies an angle CLM equal to the angle LCP. For the division of the circles must begin from the straight line PQ. We shall find the same point P, from which the division starts, if from N, toward the meridian line CM, we reckon the arc of the Equator lying between the proper Meridian of the declining plane and the Meridian of the Horizon, as far as P: which arc, in our example, contains about 40 degrees 48 minutes. For such, roughly, is the inclination of the said Meridians, found from Proposition 30 of Book 1, as we said above.

So then, let it be required to find the point of the ellipse through which the 12-o'clock line is to be drawn. Through P, the point of the 12th hour, draw in the larger circle the line PZ parallel to ST, the minor diameter of the ellipse (this is easily drawn if NV is taken equal to the arc PN; for then the straight line PV will be parallel to ST, by the scholium to Proposition 27 of Book 3 of Euclid, since equal arcs are intercepted between the points P, V and the produced diameter ST). And through the point Y of the 12th hour in the smaller circle draw YZ parallel to RN, the major diameter of the ellipse (which is drawn with the same ease if the arc SX is taken equal to the arc ST; for then the straight line XY, by the same scholium to Proposition 27 of Book 3 of Euclid, will be parallel to RN, since equal arcs are intercepted between the points Y, X and the diameter RN), cutting PZ at Z. Then Z will be a point of the ellipse whose diameters are RN and ST, as is clear from the corollary to Proposition 26 of Book 1. I say that through the same point the meridian line, or 12-o'clock line, must be drawn.

For suppose (the triangle CHN being set perpendicular to the plane of the dial, so that it is joined to the proper Meridian of the declining plane) that in the plane of the Equator there is described, from H the centre of the world, a circle N h s m of the Equator at the interval HN, equal to the circle OST, which is to be the base of a right cylinder whose axis is the same as the axis of the world CH; and let the semicircle N h s turn toward the point P, or the meridian line, and on it take the arc Nh equal to the arc OY, or similar to NP. Through h draw the Meridian of the Horizon, since the arc Nh is similar to the arc of the Equator set between the Meridian of the Horizon and the proper Meridian of the declining plane. Therefore the side of the cylinder drawn from the point h — through which the plane of the Meridian of the Horizon, or of the 12-o'clock circle, is drawn — will fall on the point Z of the ellipse, on account of the similarity of the arcs Nb, NP, as is clear from what was demonstrated in the scholium to Proposition 1 of the preceding book.

For there, because the arcs BP, BF were similar, we demonstrated that the side of the cylinder drawn from P falls on the point L of the ellipse, and so on. By the

same reasoning, if through the points Q, u — opposite to the points P, Y — we draw two lines parallel to the two diameters of the ellipse, we shall find another point δ , through which the same meridian line is to be drawn. For onto that point falls the side of the cylinder drawn from the point m, which is opposite to the point h, through which of course the Meridian of the Horizon is drawn, on account of the similarity of the arcs Nm, NQ. In the same way, from the points a, b of the 6th hour the point f will be found, through which the 6-o'clock line is to be drawn; and from the point t (opposite in the larger circle) and its counterpart in the smaller circle, another opposite point, through which the same 6-o'clock line is to be drawn. And so with the rest.

You see, then, that the points Z, δ fall on the meridian line drawn at the outset, and that the 6-o'clock line drawn through f and its opposite point passes through the point α , through which, in the preceding description, we showed the 6-o'clock line must be drawn. Finally, you see that the 3-o'clock line, which could scarcely be drawn without error in the preceding description, can be drawn with the greatest exactness by means of this ellipse, and so on. In our example, since the style-line is unequally distant from the two hours placed on either side, the diameters of the ellipse will likewise be unequally distant from the two points found on either side. Hence — so far as the demonstration is concerned — in the circle N h s m concentric with the Equator, from each of the hours single perpendiculars must be drawn to the diameter sN, and through these must be drawn planes parallel to the axis, so as to form parallelograms whose sides, drawn from the hours, fall on the points of the ellipse, as was demonstrated in the said scholium to Proposition 1 of the preceding book. In the figure set before us the perpendiculars hg, ml are drawn from the 12th hour, both of midday and of midnight, while the perpendiculars pn, qr are drawn from the 6th hour, both of morning and of evening.

[Margin: NB (handwritten reader's mark)]

Now when the declination of the dial's plane from the Vertical circle is so great that it differs but little from 90 degrees — so that the plane itself nearly coincides with the Meridian circle — one can scarcely construct the dial by the rules handed down without running into some difficulty, and no slight one. For then the line of declination [linea declinationis] EF comes very close to the straight line AB; and on that account the straight line FG, and GH equal to it, is made too short; from which it follows that the axis CH lies very little away from the style-line CG, and hence that the style IK turns out extremely short, so that the dial is rendered all but useless. But if we should wish the line FG to be of some larger, apt, and

suitably convenient size, who does not see that the point F would be too far removed from the point E, and that the points G, β , C would therefore run out from that same point E further than is fitting? This is indeed a matter that contains no small difficulty and labour, since then an almost immense plane would be needed for laying out the dial. This can also be shown from the fact that the hour-lines (which in the meridian dial are parallel, as we long since demonstrated in Proposition 25 of the preceding book) appear, in the plane of a dial that stands but little away from the Meridian circle, almost parallel to one another — especially when the declination is 89, 88, or 87 degrees, and so on. But we shall provide a remedy for this inconvenience, nearly like the one we wrote for the horizontal dial in the scholium to Proposition 1 of this book, so that by the rules hitherto handed down we may describe a declining dial however little it stands away from the Meridian circle — even though the centre of the dial (which must then, of necessity, lie further than is proper from the point E, if we wish to have a style large enough to indicate the hours conveniently) is wholly unknown to us, inasmuch as it cannot fall within the plane (unless the plane has, so to speak, an immense size) but lies outside it. This device we shall now employ.

[Margin: When the plane of the dial stands but little away from the Meridian circle, the dial can only with difficulty be described by the rules handed down.]

In some plane, having drawn two straight lines AB and CD cutting one another at right angles at E, let there be constructed at E, on the line CD, the angle of declination — below or above the line AB, and to the right or to the left of the line CD, according as the plane declines from south or from north toward the east or the west, as we wrote at the beginning of this proposition. In the present example we set the plane's declination from the Vertical circle toward the east at 60 degrees, and from the Meridian at 30 degrees; the angle of declination DEF is therefore set below the line AB, toward A. We do not, however, assume a larger declination, in order that this description of ours may be made clearer and its demonstration plainer.

After this, on the straight line AB, having taken any point G whatever — whether toward the side to which the line of declination inclines from the line CD, or toward the opposite side, whichever is preferred (although in the example it was taken on the same side) — let there be constructed at the point G, on the line AB, the angle EGC of the altitude of the pole [altitudo poli], directed upward or downward according as the plane declines from south or from north; and let the straight line GC cut the line CD at C. From E cut off a straight line EF [read: EG] equal to EG, and from F draw the perpendicular FH to AB. But since, on account

of the narrowness of the space enclosed between the lines AB and EF (as tends to happen when the proposed declination is very great), a perpendicular cannot easily be drawn without error from the point F to the line AB, we may set up this same construction more conveniently in the following form. Having taken on the line AB, toward the side to which the line EF is drawn, any point H, draw from it a perpendicular HF to AB [read: to EF], cutting EF at F. For if from EF we cut off an equal straight line EG, and at G construct on the line EG the angle EGC of the altitude of the pole, we shall find the point C, the same as before but somewhat more certain — because it is easier to draw a perpendicular without error from the point H, taken on the line AB, to that same line, than from the point F, which lies extremely close to the line AB.

[Margin: How the dial may be described in a plane that stands but little away from the Meridian circle, even though the centre of the dial is not to be had.]

Next, having drawn CH through C and H, and produced it as far as one pleases, draw through H a perpendicular HI to it; and if on HI there be cut off HI equal to HF, and the line CI be drawn; likewise having drawn FN perpendicular to EF, cutting the line AB at the point N, let this be joined to C by the straight line CN — then, from what we demonstrated near the beginning of this proposition, the straight line CD will be the 12-o'clock line in the plane of the given declination; the point C the centre of the declining dial; CH the style-line, or line of the gnomon [*linea indicis, seu styli*]; CN the 6-o'clock line; CI the axis of the world; and HI the equinoctial line. But if again we draw from H the perpendicular HK to CI, this too — from what has been demonstrated — will be the common section of the Equator and of the plane which, drawn through the straight line CH, stands perpendicular to the plane of the dial, in the manner of a certain proper Meridian of the dial's plane itself. Hence, if on the style-line CH there were taken a straight line HL equal to HK, and about L a circle were described, there would be constructed a declining dial all of whose hour-lines would meet at C, exactly as was done in the first description of this proposition. But since a dial constructed in this way has its style KM — dropping from K onto the line CH at right angles at the point M — too short, especially when the line EF stands but little away from AB (and it will stand the less away, the greater the plane's declination from the Vertical, as has been said), we must therefore go further and teach by what method we can describe a larger dial, one that will receive an apt style suited to use — even though, in the plane set before us on which the dial is to be laid out, the centre of the dial cannot be marked, and hence perhaps not even the 12-o'clock line, unless the plane extends more in breadth than in height, so that the 12-o'clock line at least can be marked out, the plane receiving it because of the

notable breadth it has, while the centre, on account of the plane's slight height, cannot be drawn on the same [?] plane. For although the hour-lines proceeding from C — those drawn through the points of the equinoctial line HI, found by the aid of the circle described about L — always diverge more and more from one another when produced, so that from some point of the style-line CL, below the point M, a larger style could be drawn perpendicular to CL as far as the axis CI produced (as we shall teach in the scholium to the following proposition, when we describe a larger or smaller dial from the same construction lines, according to the given size of the style): nevertheless, because when the plane deviates but little from the meridian the hour-lines are nearly parallel, so that one would have to produce them almost to infinity before we could draw a style of the proposed size, we must therefore adopt another method. Thus, then, shall we proceed with the matter.

[Margin: a — Euclid I.29] On the equinoctial line HI take any point O whatever, the more distant from H the larger the dial, and the longer the style, is desired; and through O draw OP parallel to the axis IC, to which from H let the perpendicular HQ be drawn — [a] which, since it is also perpendicular to IC, parallel to OP, will pass entirely through k, because HK too is drawn perpendicular to IC (otherwise from H two perpendiculars would be drawn to IC, which is absurd). Next, from HF, produced (which is perpendicular to AB), cut off HR equal to HO; and having drawn RS parallel to FE, erect from R, to RS, the perpendicular RT (parallel to FN), cutting AB at the point T, through which let TV be drawn parallel to CN, cutting the equinoctial line HO [?: possibly HI] at X.

[Margin: b — Euclid I.29] Besides this, having taken on the style-line HC any point Y whatever, draw through it the straight line YP parallel to HQ, cutting OP at P — [b] which will be perpendicular to OP, just as HQ is. Likewise through Y draw YV perpendicular to HI, or parallel to HO, cutting TV at V.

Diagram. A large construction for a declining dial whose centre falls off the plane, drawn with a dense sheaf of hour-lines converging toward the upper right. A horizontal base-line runs from A (left) through E (centre) and S (right) to B, extended to the far-right point 1. A near-vertical line at the right descends from π (Greek pi, upper right) through 12 to S. A circle is drawn in the lower-left quadrant. Hour numerals label the divergent lines: 3, 4 across the top; 3, 4, 5, 6, 7, 8, 9 down the left side; 8, 9, 10, 11, 12 along the bottom; and a tight inner fan 4, 5, 6, 7, 8, 9, 10, 11 near the centre-right converging on π . Readable letter-labels, by region:

- Horizontal base and right: A, T, N, G [?], E, S, B, 1.

- Central cluster: **H, K, X, M, L**, and below **F, R**.
- Upper / left of circle: **O, Q** [?], **P, V, a** (α), **b**.
- Lower centre: **D, Z** [?], and the vertical carries **12** and π .

The figure is crossed by many overlapping construction lines and dotted auxiliaries; several small labels are worn or overrun. More than two labels are uncertain

Lastly, having taken on the straight line HY the straight lines HZ, Ya equal to the straight lines HQ, YP, describe circles about Z, a, and divide them into 24 equal parts, the beginning made from the straight lines drawn from the centres Z, a through the points X, V. For the hidden straight lines drawn through the centres Z, a and the points of division will cut the straight lines HO, YV at points through which, when straight lines are drawn out (always taking two corresponding points together — that is, the two nearest to H, Y, then the next two, and so on), they will yield the hour-lines, which we shall here produce this way and that as far as the size of the plane will allow. For their lengths will be bounded by the arcs of the signs. The length of the style will be Qb, the perpendicular dropped from Q to HY, and its position at the point b, on which the said perpendicular falls. All of which we shall prove by the following reasoning.

[Margin: Demonstration of the construction of the dial declining but little from the Meridian.] [Margin: c — Euclid I.29]

Since the straight lines RS and FE, parallel with the straight line AB, must at length necessarily meet somewhere, let them meet at the point S. (This point S, however, will for the most part fall outside the plane, unless the plane be immense, since it lies extremely far from the point E if the declination of the dial's plane differs but little from 90 degrees. Hence it must then be conceived only in the mind.) Through it let there be understood a perpendicular $S\pi$ drawn to AB, or parallel to CD, meeting HC produced at π . [c] And since the angle HSR is equal to the angle HEF, the complement of the given declination, if the straight line πS be taken for the meridian line in the plane of the dial, SR will be the line of declination. And since in it, from the point R, a perpendicular RH has been drawn to AB, the style-line must be drawn through the point H, as was shown in the first description of this proposition, making with the meridian line $S\pi$ an angle equal to the angle ECH, which the style-line CH in the dial (whose centre is C) makes with the meridian line CE. This is because in the same declining plane the style-line always makes the same angle with the meridian line, and hence also with the straight line AB, which is equidistant from the Horizon, whatever length of style be taken; otherwise the equinoctial line, which must be drawn through

the point H perpendicular to the style-line, as has been demonstrated, would not always make the same angles with the straight line AB — which is absurd.

This can also be confirmed by the following reasoning. Since the plane of the proper Meridian of the declining plane itself, which makes the style-line in that very plane, and likewise the plane of the horizontal dial drawn through the straight line AB, are both — as is clear from what precedes — perpendicular to the plane of the declining dial, [d] their common section too will be perpendicular to that same plane, meeting it at the point H, through which the style-line, and therefore the proper Meridian of the declining plane producing it, is drawn, as has been said. And on that account the same section, by definition 3 of Book 11 of Euclid, will make right angles both with the straight line HE and with the style-line drawn through H in the plane of the dial. Therefore, by definition 6 of Book 11 of Euclid, since each of them in turn makes right angles with that common section, the angle contained by the straight line HE and the style-line will be [an ang-]

[Margin: d — Euclid XI.19]

the angle of inclination of the plane on which the said proper Meridian of the declining plane stands, relative to the plane of the horizontal dial. And since this inclination does not change within the same declining plane, it will make, together with the straight line HE, always one and the same angle. Therefore, since HC has been shown to be the style-line in the dial whose centre is C, and since it has been demonstrated that the style-line in the dial in which the meridian line is πS passes through H, the line HC produced will be the style-line in this same dial — for at H it makes one and the same angle, and at π it makes, with the meridian line πS , an angle equal to the angle HCE, which the style-line in the earlier dial makes with the meridian line CE. [a] That the angles HCE and $H\pi S$ are equal is evident, because CE and πS are parallel. From this it follows that the point π is the centre of that dial in which the meridian line is πS , since the style-line and the meridian line meet at the centre of a dial. It also follows that the straight line HO, which is perpendicular to the style-line, is the equinoctial line of that same dial, as is clear from what precedes.

[Margin: a — Euclid I.29]

Now since on the equinoctial line HO the straight line HO has been taken equal to HR, it will follow, by what has been demonstrated, that the straight line drawn from the centre π to O — the axis of the world — makes at π , with the style-line πH , an angle $O\pi H$ equal to the angle ICH which the earlier axis IC makes with the

style-line. For the angle of the pole's elevation above this same declining plane is always the same, and this is precisely the angle the world's axis makes with the style-line, as is clear from what precedes. [b] Therefore the axes $O\pi$ and IC will be parallel; and consequently the straight line OP , which has been drawn parallel to IC — even if we do not produce it all the way to the centre π (which is sometimes unknown to us) — will be the axis of the world. From this it follows that the straight line HQ , drawn perpendicular to the axis, is the common section of the Equator and of the proper Meridian of the declining plane itself.

[Margin: b — Euclid I.28]

Again, since the perpendicular RT has been drawn from R to the line of declination SR , it will necessarily pass through T — the line of the 6th hour — as we showed above. Therefore the straight line drawn from the centre π through the point T will yield the 6th hour, and will make at π , with the style-line, an angle $T\pi H$ equal to the angle NCH which the 6th-hour line CN of the smaller dial makes with the style-line CH . For the circle of the 6th hour always keeps the same inclination to the proper Meridian of one and the same declining plane; and therefore the common sections which these circles make with that same declining plane (whatever length of style be taken, through whose apex they are drawn) will always make the same angle — namely, an angle equal to the one which the sections of those same circles, made in the great circle to which the dial's plane is parallel, form. For the said sections make equal angles in the plane of the dial and in the great circle to which the dial is parallel, as we demonstrated near the end of this proposition concerning the angles which the sections made by the proper Meridian of the declining plane and by the Meridian of the Horizon form in the same plane of the dial and in the great circle to which this plane is parallel. [c] Hence the lines $T\pi$ and NC will be parallel, and therefore the straight line TV , which we drew parallel to NC — even if it be not produced all the way to the centre π (which perhaps does not lie within the plane of the dial) — will be the line of the 6th hour.

[Margin: c — Euclid I.28]

Since this is so, it is clear from what was demonstrated in the earlier description of this proposition that the hour-lines must be drawn through those points of the straight line HQ where it is cut by the hidden lines issuing from the centre Z ; for the division of the circle described about Z takes its beginning from the straight line which passes through the first hour on the equinoctial line HO — namely, from the straight line ZX , which passes through the 6th hour. By the same reasoning, since YP is the common section of the Equator and of the proper

Meridian of the declining plane itself (as is clear from what has been demonstrated, being perpendicular to that same axis OP), the hour-lines must be drawn through the points of the equinoctial line YV at which the hidden lines issuing from a cut it; for the division of the circle described about a likewise takes its beginning from the straight line aV, which is drawn through the 6th hour on the equinoctial line YV. Rightly, therefore, are the hour-lines drawn through those points found on the straight lines HO and YV, even though the centre π — where they would all meet if produced — is unknown to us, lying outside the plane, or at least extremely far from the point H. In the example proposed, because the lines of the 12th and 1st hours from noon have also been drawn, it plainly appears that the straight lines drawn through the centres Z and a, which cut at right angles the straight lines ZX and aV (passing through the 6th hour on each equinoctial line), fall on those points of the equinoctial lines HO and YV through which the line of the 12th hour is drawn; and consequently these are the lines from which the divisions of the circles described about Z and a should be begun, as we taught above. But because, as we said, in planes that decline too far from the Vertical neither the line of the 12th hour nor the centre can conveniently be marked, we ordered those same divisions to be begun from the straight lines ZX, aV, and so on. That the style is Qb, and its position at the point b, is clear from what has been said, since the triangle πQH is constructed just like the triangle CIG in the upper figure of this proposition.

For the rest, any dial declining from the Meridian, or from the North, toward the east or west — if its features interchange their arrangement among themselves, so that the left becomes the right and the right becomes the left, until they take on such a position as the features of the dial would have if they had been drawn on the opposite face of the plane, under this rule, that each single line answers to each single line — will present us with another dial declining from the Meridian, or from the North, by the same number of degrees as before, but toward the contrary side. So that if the first dial declines from the Meridian, or from the North, toward the east, this one too will turn from the Meridian, or from the North, toward the west; and if the first declines toward the west, this one will look toward the east. Moreover, the hour-numbers must be changed into their complements up to 12 — so that hour 1 becomes 11, hour 2 becomes 10, and so on. By this device you see that, from the dial above declining from the Meridian toward the east by 30 degrees, there has been constructed in the following figure a dial declining from the Meridian toward the west by 30 degrees.

[Margin: If the features of any dial whatever declining from the Meridian, or from the North, toward the east are so altered that the right becomes the left and

conversely, one will obtain another dial declining from the Meridian, or from the North, toward the west, and conversely.]

The reason for this is not obscure, if the precepts of this proposition are attentively and carefully considered. For since, in two dials declining from the Meridian (or from the North) toward opposite sides — the one toward the east, the other toward the west — the line of declination EF is drawn from the point E toward opposite sides, as has been said, it comes about that the style-line too is extended from the centre C toward contrary sides, since it is drawn from C through the point G onto which FG falls (FG being drawn from F, the point of the declination-line, perpendicular to AB). And on that account the equinoctial line as well, which cuts the style-line at right angles, obtains a contrary position. In the same way the 6th-hour line, which is drawn from C through that point of the straight line AB onto which Fa falls (Fa being drawn from F at right angles to the declination-line), cannot have the same position in both dials; and the same must be said of the other lines. Hence, since the lines of one dial have the same distances among themselves as they have in the other — because the styles too are taken equal, and the declination is put the same in both, though it verge toward contrary sides — it comes about that the features which, when we face one dial, lie to our right, lie in the other to the left, and conversely.

The cause, too, of changing the hour-numbers into their complements up to 12 is evident. For since, as we said above, in southern dials the hours placed to the right of the meridian line are to be reckoned from noon, and those to the left from midnight, whereas in northern dials it is the reverse: it comes about that the hours from noon in one dial are the hours from midnight in the other, and conversely — since the right-hand ones of the first, which were reckoned from noon, become the left-hand ones in the other, and are therefore counted from midnight, and conversely, as we have taught. Therefore the line next to the right of the line of the 12th hour of noon, which in the southern dial shows the 1st hour from noon, will in the other southern dial necessarily show the 11th hour from midnight, since now, transferred, it occupies the left side, and conversely. And the line next to the right of the line of the 12th hour of midnight in the one [northern]

northern dial, indicating the 1st hour from midnight, will in the other northern dial, now transferred to the left side, signify the 11th hour from noon; and conversely.

[Margin: How, from a dial declining from the Meridian, a dial declining from the North is to be drawn on the opposite face of the dial's plane.]

Now if a dial declining from the Meridian toward the east or west is inverted, so that the upper part is changed into the lower, and what — after this change, when we face the dial — is on the right becomes the left, and conversely: then there will have been made from it a dial declining from the North by as many degrees as that one departs from the Meridian, yet turning toward the opposite side — so that, while the first verges [toward the east]...

Diagram. A sundial declining from the Meridian toward the west by 30 degrees, constructed as the mirror-image of the eastward-declining dial of the preceding page. The style-centre C stands near the top; from it a fan of hour-lines radiates downward and outward to the numbered hours around the margin. A horizontal line AB runs across the figure (the line equidistant from the Horizon), with a longer near-horizontal line for the higher hours crossing it toward the right. A dotted line runs from the lower centre toward the lower right (the line of declination / equinoctial line region).

- C (upper centre): style-centre, from which all hour-lines issue
- A (left end of the horizontal line): terminus of line AB
- B (right, on the horizontal line): point on line AB
- [?] (far right, beyond B, on the extended horizontal line): a mark resembling "α" — label not resolvable
- E (centre, on line AB): point where the meridian/style structure meets AB
- G (centre, just right of E, on line AB): foot-point of the style construction
- M (below E–G): point on the meridian line
- I (right of centre, above line AB): point on the style-line
- F (mid-figure, below I): point of the style construction [?]
- F (lower centre, below M): foot-point on the descending meridian line
- D (bottom): lower terminus of the meridian line
- Hour numerals, upper left: 6, 7, 8
- Hour numerals, upper right: 8, 7, 6
- Hour numerals, lower band: 9, 10, 11, 12, 1, 2, 3, 4, 5

verges toward the east, this one looks toward the west, and conversely: just as is the dial which is usually drawn on the opposite face of the plane, as we demonstrated in the scholium of proposition 13 of the preceding book. This inversion of the dial can also be carried out in this way: first the right part is changed into the left, and conversely; then, after this change, the upper is changed into the lower, and conversely. A dial of this kind is expressed by the features of the southern dial drawn on the northern face, so that each single line

answers to each single line, provided the upper part be changed into the lower. For if this change is made, there will appear to us — when we face the opposite face of the dial — the lines toward the right which before (after that change of the upper part into the lower) had been placed for us on the left in the southern face, and conversely, as is manifest.

[Margin: If the upper part of the dial that declines from the Meridian toward the east is made the lower, there will result a dial declining from the North toward the east as well; but toward the west, if that one verges toward the west.]

Now since, from a southern dial, if its right part is made the left, and conversely, there results another southern dial that declines toward the contrary side, as has been said: it comes about that, if the upper part of this dial (which was made from the first) is changed into the lower, we shall have a northern dial answering to that first southern dial — provided the numbers be restored which were before in that southern dial. So that if the first declines from the Meridian toward the east, this one will decline from the North toward the west, and if the first toward the west, this one will verge toward the east: because by this reasoning the change of the left part into the right was made, when, of course, the dial was made to decline from the Meridian toward the contrary side; and again, the part which after this change is the upper is changed into the lower, and conversely — all of which is required in order that from a southern dial there be made a northern one to be drawn on the opposite face, as has been said. And so we shall be permitted to gather this rule for ourselves. A southern dial declining toward the east, if its upper part is made the lower, will give a northern dial of equal declination verging toward the east as well. A southern dial deflecting toward the west, similarly inverted so that the upper part is changed into the lower, will exhibit a northern one looking toward the west as well. Hence if anyone desires a northern dial declining toward the east, a southern dial looking toward the east must be drawn. For this, inverted so that the upper parts become the lower, will give the desired northern dial verging toward the east, if the hour-numbers be turned into their complements up to 12. Similarly, if anyone should wish a northern dial deflecting toward the west, a southern dial declining toward the west must be drawn. For if its upper part is made the lower, a northern dial looking toward the west will be obtained, if the numbers be changed, as has been said. All of which is gathered from what has been said, and is evident, if the planes of the dials are rightly considered in their own positions.

[Margin: By what method, from a dial that declines from the Meridian toward the east, another may be produced declining from the North toward the east, from the Meridian toward the west, and from the North toward the west.]

Indeed, by one operation and one labour we shall be able most conveniently to draw four different dials, the same quantity of declination being always kept. For if, for example, we draw a dial declining from the Meridian toward the east by 30 degrees, and change its upper part into the lower, there will already be drawn as well a dial declining by 30 degrees from the North toward the east. But if we make the right part of that same one the left, without changing the upper part into the lower, we shall have a dial declining from the Meridian toward the west by 30 degrees. If finally we not only convert the right part into the left, but also make the part which, after this change, is the upper into the lower, there will be constructed a dial deflecting from the North toward the west by 30 degrees. By the same reasoning, if a dial is drawn declining from the Meridian toward the west by 30 degrees, we shall produce from it a dial declining from the North toward the west by 30 degrees, and one declining from the Meridian toward the east by the same number of degrees, and likewise one deflecting from the North toward the east by 30 degrees, if the aforesaid inversions of the parts be made, as is clear from what has been said. But in all of them the numbers must be inscribed on the hour-lines, as we taught in the earlier description of the declining dial in this proposition.

PROBLEMA 2. PROPOS. 2.

To describe the parallels — that is, the arcs of the signs of the Zodiac — on a dial that declines from the Vertical circle.

Once the elevation of the pole above the declining plane has been found, by proposition 29 of the first book — which, as we said, is indicated [also]...

also by the earlier description of the declining dial in the preceding proposition, or even by the description which we set out in the scholium of that same proposition for a given length of style whose position is likewise given, namely the angle GCI comprehended by the style-line and the world's axis — let the Analemma be set up to it, as in proposition 2 of the preceding book, so that the arc CE in the Analemma of that proposition measures the elevation of the pole above the declining plane, that is, in the example proposed, contains 40 degrees 3 minutes; or at least so that the angle CDE of that Analemma be equal to the angle GCI in the declining dial of the preceding proposition. For if there be taken, in the same Analemma, a straight line DG equal to the style IK which the dial drawn by

the first method offers — or the one we assumed in the later description in the portion of the Analemma drawn in the preceding proposition (which answers to the first figure of proposition 1 of the preceding book), or at least the one we assumed in that description in which the length of the style together with its position was given — and if through G a straight line OR be drawn parallel to BC, we shall have on the straight line OR the diameters of the conic sections, just as we had those same diameters KR, LR, MR, NO, PO, QO in the horizontal dial of proposition 2 of book 2. Wherefore we shall here describe the parallels — that is, the arcs of the signs — as we taught there, namely as conic sections about the diameters found; of which those contained between the centre C and the equinoctial line will bend their horns toward the centre, while the rest bend toward the opposite side, exactly as in the horizontal dial. For just as, in the horizontal dial, the circle of the Analemma ABC represents the Meridian of the Horizon, and the straight line OR the common section of the plane of the horizontal dial and of the Meridian, that is, the meridian line: so in the declining dial the same circle is taken for the proper Meridian of the declining plane — that is, for the circle drawn through the poles of the world and through the poles of the declining plane — while the straight line OR is taken for the common section of the said Meridian and of the plane of the declining dial, namely for the style-line. But we do not set up a proper Analemma for the elevation of the pole found above the declining plane, both because this whole matter can easily be understood from the Analemma of proposition 2 of the preceding book, and because in this proposition we shall describe the arcs of the signs by another method, which we shall explain presently.

[Margin: How the arcs of the signs of the Zodiac on a dial declining from the Vertical are to be described from the Analemma.]

But if anyone should prefer a larger or a smaller dial, a longer or a shorter style DM must be taken in the Analemma, and through M a line drawn parallel to BC, and so on, as we said concerning the horizontal dial in proposition 2 of the preceding book.

[Margin: By what method a larger or smaller dial may be described according to the same Analemma.]

Another method. When the straight line CI, which is to represent the world's axis, has been drawn, let there be erected at I, perpendicular to CI, the line IG, for the radius of the Equator, from either side of which let the radii of the other signs be drawn out, as has been done more than once above; and let CI be taken equal to the straight line CI on the axis of the dial of the preceding proposition, or of the

dial which we constructed in the scholium of that same proposition for a given style in a given place; and let IG be taken equal to the straight line IG, or LG, in the same dial; and from C, through G, let the straight line CG be drawn out — which perhaps we shall draw more conveniently if from C we draw a straight line CG [making]...

[Margin: Another description of the arcs of the signs on the same dial declining from the Vertical.]

Diagram. A large composite construction for the arcs of the signs (parallels) on a dial declining from the Vertical. A horizontal diameter runs across the upper part of the figure, above which a semicircle is drawn and graduated with tick-marks numbered 1–12 running from the left over the top and down the right side. From an apex near the centre a broad fan of straight lines (both solid and dashed/dotted) descends toward the lower margin, meeting a lower horizontal line (the equinoctial line) and terminating in two arrays of hour-numbers below. Several of the descending rays carry the label "Rad hor ..." (radius of the hour-arc) with a numeral. At the upper left is an inset scale-square construction.

Labels read from the scan (positions approximate; many are crossed by ruled lines and flagged):

- Upper diameter, left to right: B, I, B [?], C
- M (just below the diameter, between I and the central B)
- F (below M, at the head of the descending fan)
- Central points along the axis: G, E, H, O, L, N
- P (lower centre, on the equinoctial line near the arc)
- Upper-left inset (scale-square): across top B, C, B; down left edge I, D, G, D; interior A, K, A
- Radial labels on the descending rays: "Rad hor 24", "Rad hor 18", "Rad hor 14", "Rad hor 10", "Rad hor 8", "Rad hor 6", "Rad. 6" [?]
- Semicircle graduation (over the top and down the right): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
- Lower hour-numbers (two arrays at the foot of the fan, left and right): 1, 2, 3, 4, 5, 6, 8, 12 [several partly obscured — [?]]

making with CI an angle GCI equal to the angle GCI which the axis CI and the style-line CG form in the dial — that is, equal to the angle of the pole's elevation above the declining plane. This triangle GCI will be entirely equal to the triangle GCI in the declining dial of the preceding proposition, and thus the straight line CG will be the style-line. For, according to the earlier construction, the two sides

IC, IG of the one triangle are equal to the two sides IC, IG of the other triangle, and they contain equal angles. [a] Therefore the whole triangles are equal, and so on. But according to the later [construction]...

[Margin: a — Euclid I.4]

[Margin: a — Euclid I.26.]

the construction, two angles I, C of the one triangle are equal to two angles I, C of the other triangle. Since, therefore, the side IC of the one is also equal to the side IC of the other, [a] the remaining sides of the one will be equal to the remaining sides of the other, and so on.

Next, from the centre L of that circle — by whose aid we found the hour-points on the equinoctial line — let all the intervals up to the hour-points on the equinoctial line be taken, and let them be transferred from I onto the radius of the Equator IG. For if straight lines are drawn from C through the points that bound the aforesaid intervals on the radius of the Equator, these will be the lines of all the hours, both on this side of and beyond the style-line, which intersect the equinoctial line — just like the hour-lines of the horizontal dial that are drawn from the point H, in the later figure of Prop. 2 of the preceding book, through the points found on the radius of the Equator. To these lines, then, let there be attached the hour-numbers corresponding to the hours whose intervals — placed between the centre L and the equinoctial line — have been transferred onto the radius of the Equator from the point I. Note carefully which lines refer to the hours placed to the left of the style-line and which to the hours located to the right of that same line. This the numbers attached to the lines readily indicate, corresponding to the numbers of the dial's hour-lines that intersect the equinoctial line. For these lines refer only to the lines hitherto drawn from the point C through the points marked off on the radius of the Equator.

We shall obtain these same lines if from the dial we take the hour-intervals placed between the centre C and the equinoctial line, transfer them from C onto the radius of the Equator IG, and, through the points that bound these intervals on the radius of the Equator, draw lines from C, and so on — just as we did in the horizontal dial and in the vertical. But if it should happen that some hour-line in the dial does not cut the equinoctial line even if it is produced infinitely, but is equidistant from it, then from C the line CA must be drawn parallel to the radius of the Equator IG: for this line will represent that hour. So too in the horizontal and in the vertical dial, because the line of the 6th hour does not cut the equinoctial line but is parallel to it from H — in the later figure of Prop. 2 and 14

of the preceding book, where the straight line HV is drawn parallel to the radius of the Equator for the 6th hour.

Diagram. A large, very dense construction (the "figure of rays," *figura radiorum*) for a sundial declining from the vertical. A horizontal straight line runs across the middle of the figure, carrying the caption **HORIZON** split around the drawing (**Ho** at the far left, **RIZON** at the far right). From a vertex **C** near the upper centre a broad fan of straight hour-lines (some solid, some dotted/hidden) radiates downward and outward, crossed by the sloping sign-parallel lines and by a graduated arc near the centre. A ring of zodiac sign-glyphs labels the sign-parallels around the periphery (upper-left above the horizon, and lower-right below it). At the lower left is a subsidiary arc-and-dotted-line construction associated with the point ϕ .

Labels read from the scan (positions approximate; the interior is heavily overrun by ruled and dotted lines and only partly resolvable):

- Horizon line caption: **Ho ... RIZON** (HORIZON), horizontal, through the middle.
- **C** — vertex at upper centre, origin of the fan of hour-lines.
- **I** — left of centre, above the horizon (on the radius of the Equator).
- **K** — just below and left of C, near the centre.
- **d** — on the horizon line, to the right of C.
- **H** — at the left, on/below the horizon, at the end of a line.
- ϕ — lower-left, at the head of a subsidiary dotted-arc construction (the point ϕ of the text).
- Interior hour-numbers clustered near the centre and along the equinoctial line: 6, 7, 8, 9, 10, 11, 12 [several partly obscured — [?]].
- Descending hour-numbers on the right: 1, 2, 3 [?].
- Peripheral zodiac sign-glyphs (upper-left above the horizon, and right / lower-centre below it): readable are ♒ (Aquarius), ♓ (Pisces), ♑ (Capricorn), ♉ (Taurus) [?], ♌ (Leo) [?], ♍ (Virgo) [?], ♏ (Scorpio) [?], ♐ (Sagittarius) [?] — the individual glyphs are worn and crossed by ruled lines; the precise assignment of each sign to its parallel could not be resolved. [?]
- Sequence-numbers 4, 5, 6, 7, 8 down the left margin (hour- or parallel-order labels) [?].

[Margin: How the hour-lines that neither cut the equinoctial line nor are parallel to it are drawn in the figure of rays just constructed.]

Now, in order to obtain the lines of the other hours — those that neither cut the equinoctial line nor are parallel to it, as are all those which, after cutting the equinoctial line, have been produced beyond the centre C (such as, in our declining dial, the hours 4, 5, 6 and the rest after noon, and likewise 1, 2, and 3 after midnight; for all these are produced beyond the centre C, and their counterparts on this side of the centre C cut the equinoctial line, namely 4, 5, 6 and the rest after midnight, and likewise 1, 2, and 3 after noon) — we shall follow this procedure. Through C, the centre of the dial, we shall draw the straight line Cd parallel to the equinoctial line, provided no hour-line is equidistant from it; and through C, a point of the figure just constructed, we shall send out the straight line CA parallel to the radius of the Equator. This will correspond to that parallel Cd, or at any rate to that hour (if there is one) which is equidistant from the equinoctial line, as we have just shown. Now the line Cd represents in the dial that hour-circle — that is, the great circle drawn through the poles of the world — whose common section, on the circle drawn about the centre L, is equidistant from the equinoctial line and cuts the style-line at right angles, as we showed in the preceding proposition. After this, [b] since every hour-line makes equal angles with Cd at the vertex C, we shall transfer the lines going out from C and cutting the radius of the Equator to the other side of the straight line CA, by means of a certain circular arc drawn from C and de-

[Margin: b — Euclid I.15.]

descri-bed, in such a way that the lines equidistant from CA on either side cut off equal arcs of that circle drawn from C, and contain equal angles at C with the straight line CA. For these lines, thus transferred, will correspond to those hours which do not cut the equinoctial line but run out beyond the centre C — so that two lines removed by an equal interval from the straight line CA belong to two hours of the same number. Of these, the one that cuts the radius of the Equator (or is drawn from C toward the radius of the Equator) represents the hour that intersects the equinoctial line in the dial, while the other, transferred to the other side of the straight line CA, corresponds to the same hour produced beyond the centre C, which does not cut the equinoctial line. This is done exactly as in the horizontal dial, and in the vertical, in Prop. 2 and 14 of the preceding book. For there too the lines transferred beyond the straight line HV in the figure of rays represent the same hours by number — yet not cutting the equinoctial line — to which correspond other lines on this side of the straight line HV. This is because any hour-line whatever in the horizontal and vertical dial makes equal angles, at the vertex or centre H, with the line of the 6th hour, which is equidistant from the equinoctial line — just as here [it makes equal angles] with the line Cd, parallel to

the equinoctial line itself, to which corresponds the straight line CA, parallel to the radius of the Equator in the figure of rays. And this is just like the straight line HV, which there in the figure of rays is equidistant from the radius of the Equator and corresponds to the 6th hour, being parallel to the equinoctial line. Thus you see that here the line of the 3rd hour after midnight has been transferred beyond the straight line CA, because the 3rd hour after noon cuts the equinoctial line; and likewise the line of the 4th hour after noon, because the 4th hour after midnight cuts the equinoctial, as appears in the dial. And the same reasoning must be applied to the others.

[Margin: When the style-line is one of the hour-lines, or is equidistant from two of them on either side, every hour-line in the figure of rays will belong to two hours, as in the horizontal dial. When the style-line is equidistant from the two nearest hours, two figures of rays would have to be set up, so that all confusion of the lines might be removed.]

But if the style-line should sometimes be one of the hour-lines, or should lie midway between two hour-lines equidistant from it (which can sometimes happen), then each hour-line in the figure of rays just constructed and drawn from C will serve two hours equally removed from the style-line on either side. For in that case the intervals of those hours between the centre L and the equinoctial line are equal — as we showed of the horizontal dial in Prop. 2 of the preceding book — and so, when transferred onto the radius of the Equator from the point I, they yield one and the same point, and so on. Hence in that case two numbers must be assigned to each line, corresponding to the two hours removed by an equal distance from the style-line on either side, just as was done in the horizontal and in the vertical dial. But when the style-line is unequally removed from the surrounding hour-lines (which happens more often), as occurs in the given example, then each line will correspond to only a single hour; because in that case the intervals of the hours between the centre L and the equinoctial line, lying on either side of the style-line, are unequal, and so, when transferred onto the radius of the Equator from I, they yield different points, and so on.

[Margin: When the style-line is one of the hour-lines, or is equidistant from two on either side, every hour-line in the figure of rays will belong to two hours, as in the horizontal dial.]

It would not be beside the point, however — indeed it would be advantageous — when the style-line is unequally distant from the two nearest hours, if two figures of the Zodiac's rays were constructed: in one of which the hour-lines would be drawn from C corresponding to those hour-lines in the dial that are contained on

one side, namely to the left of the style-line, while in the other would be drawn the hour-lines corresponding to the dial's hour-lines on the other side of the style-line. For thus all confusion of the lines would be removed — the confusion that necessarily follows when the style-line is almost equally distant from the lines placed on either side, because then the points found on the radius of the Equator can scarcely be distinguished from one another, as is evident. In this way, in our example, in one of the figures the lines would be drawn from C corresponding to these hours: 1, 2, 3, 4, 5, 6, 7, 8, 9 after midnight, which are to the left of the style-line. In the other would be contained the lines corresponding to the hours on the right of that same style-line, namely 10, 11, 12 after midnight, and 1, 2, 3, 4, 5, 6, etc. after noon. In each figure, however, CG must be drawn for the style-line. You will see an example of this set out in Prop. 38 of this book.

[Margin: When the style-line is equidistant from two nearest hours, two figures of rays would have to be set up, so that all confusion of the lines might be removed.]

From these things thus prepared, the arcs of the signs will be drawn, as in the horizontal and vertical dial — namely by transferring the intervals of the hours, intercepted between C and the rays of the signs, from the centre of the dial C onto the corresponding hours of the dial; or else by transferring the hour-intervals placed between the radius of the Equator and the rays of the signs onto the corresponding hour-lines of the dial, beginning from the equinoctial line, and so on. For the demonstration here will be the same as in the horizontal dial, since the plane of the declining dial is like a kind of horizon, and the angle GCI is equal to the elevation of the pole above it, as we said; and the style-line is like the meridian line, although it does not indicate the 12th hour, because we do not reckon the hours from the meridian of the declining plane, but from the meridian of the horizon of that place where the declining dial is drawn. The said meridian line would doubtless indicate the 12th hour if we reckoned the hours from the meridian of the declining plane; for then the proper meridian circle of the declining plane, producing the style-line in the dial, would be equally distant from the sunrise and sunset, and so the Sun, when on it, would make noon or midnight, just as happens on the horizon.

[Margin: Which arcs of the signs are hyperbolas, which parabolas, and which ellipses, in a dial declining from the vertical.]

It is clear, moreover, from what was demonstrated in Prop. 2 of the preceding book, that when the straight line CG in the figure of rays cuts opposite rays, the arcs of those signs are opposite and equal hyperbolas; but when it is equidistant from one of a pair of opposite rays and cuts the other, the arc of that sign whose

ray is cut is a parabola, and the arc of the other cannot be drawn; and finally, when it cuts one of the opposite rays but not the other, and is not equidistant from it, the arc of that sign whose ray is cut is an ellipse, and the arc of the other cannot be drawn in any way.

For the demonstration here will be the same as it was there: since here the straight line CG behaves as it did there — corresponding to the style-line in the dial, and being the diameter of all the conic sections — just as there the straight line HB represented the meridian line of the dial in the figure of rays and was the diameter of all the conic sections. This is because here the angle CGI is equal to the altitude of the pole above the declining plane, just as there the angle DHB was equal to the altitude of the pole above the horizon.

[Margin: The arcs of the greatest of the always-apparent and always-hidden parallels are hyperbolas in the [superiori (?)] dial.]

From this it follows that the arcs of the parallels which are the greatest of the always-apparent and always-hidden, are in our example opposite hyperbolas. For since the angle which the radius of the 24-hour arc makes with the radius of the Equator contains 48° — this being the declination of the greatest of the always-apparent parallels — it comes about that, by adding the right angle GIC, the angle contained by the radius of that same parallel and the axis IC is made 138° . Since, therefore, the angle ICG — [the angle] of the pole's altitude above the declining plane — comprehends $40^\circ 3'$, the internal angles which the said radius and the straight line CG make with the axis will be $178^\circ 3'$, that is, less than two right angles. Therefore the line CG will meet the said radius; and consequently, since the straight line CG cuts both radii — that is, the one belonging to the greatest always-apparent parallel and the one belonging to the greatest always-hidden parallel — there will be formed two opposite and equal hyperbolas, as is clear from what has been said.

When the style-line is one of the hour-lines, or is equidistant from two hours placed on either side, each of the aforesaid intervals must be transferred onto the two hour-lines equally removed from the style-line in the dial, as was done in the horizontal dial too, and in the vertical.

[Margin: When the style-line is one of the hour-lines, or is equidistant from two on either side, the arcs of the signs are more easily drawn.]

[Margin: When the style-line is unequally distant from the two hours on either side, what must be done so that the arcs of the signs may be drawn more accurately.]

But in order that the arcs of the signs may be drawn more accurately, when the style-line is unequally removed from the two hour-lines placed on either side, all the hour-points of the equinoctial line placed to the left or right of the index-line [linea indicis, here used for the style-line] can be transferred to the other side of the equinoctial line — namely to the right or left of that same style-line — the beginning always being made from the style-line. Or, more surely (which I myself prefer, since some hour-lines scarcely or never cut the equinoctial line): an arc of a circle having been drawn from the centre C, its portions between the index-line and the hour-lines on one side of the index-line can be transferred into the same arc on the other side of that same index-line. For if, through the points marked either on the equinoctial line or on that arc (which is the more approved method), we draw hidden lines from the centre C — such as, in the adjacent figure, are the lines distinguished by those small [dotted] lines, lying to the right of the index-line itself — we shall always have two lines equally removed from the index-line on either side, into which one and the same hour-space intercepted between C and the radius of any sign can be transferred, as in the horizontal and vertical dial.

If indeed this is done, it will be enough if in the figure of rays we draw from C the hour-lines corresponding to the hours contained in the dial on only one side, right or left, of the index-line. For if the points of those hours on the equinoctial line, or on the arc drawn from centre C, are transferred to the other side of the equinoctial line or of the said arc — the beginning always being made from the index-line — and the rest is completed as we have just prescribed, the arcs of the signs will be drawn, as in the horizontal dial. And in this way the confusion of the lines drawn from C through the radius of the Equator in the figure of rays would be avoided. This you will see observed in the drawing of the arcs of the signs in the dial declining from the horizon, in Prop. 14 of this book. Again, if the portions of the straight line CA, between C and the rays of the signs, are transferred into the dial from C onto the straight line Cd, on either side of the centre C, we shall have on the straight line Cd the points through which the arcs of the signs are to be drawn — as was done on the line of the 6th hour of the horizontal and vertical dial.

[Margin: By what reasoning it is sufficient if in the figure of rays the hour-lines are drawn corresponding to the hours contained in the dial on only one side of the style-line.]

[Margin: Which arcs of the signs, in a dial declining from the vertical, belong to the southern signs, and which to the northern.]

Now in dials declining toward the south, the arcs drawn above the equinoctial line toward the centre belong to the southern signs; the rest belong to the northern signs, just as in vertical dials. But in dials declining toward the north, the opposite is to be understood — as anyone can easily perceive, if he rightly conceives in his mind the position of each dial and the situation of the parallels drawn through the beginnings of the signs.

[Margin: In what way the hyperbolas of the northern signs may be drawn from the opposite hyperbolas of the southern signs.]

For the rest, when the arcs of the signs are opposite and equal hyperbolas (which we showed a little earlier, whenever it happens), we shall draw much more accurately, in dials declining toward the south, the arcs of the northern signs from the arcs of the southern signs, by those three methods which we set forth at the end of Prop. 2 of the preceding book, after the demonstration of the second method of drawing the arcs of the Zodiac's signs in the horizontal dial — a method that is accomplished from the figure in which the rays of all the signs of the Zodiac are contained. For in the first of those methods there is no difficulty, if on the style-line the transverse diameter of the opposite hyperbolas and their centre — namely the midpoint of the transverse diameter — are found, and the rest is completed, as we taught in the said Prop. 2 of the preceding book. As for the second of those methods, the axis of the hyperbolas in these declining dials is the style-line itself. Now the portions of the arcs of the southern signs, up to the axis of the opposite hyperbolas (that is, the style-line), are to be drawn toward the side of the 6th hour, because they can be drawn more conveniently there than the others toward the 12th hour; and from them, with a single effort, not only the remaining portions on the other side but also the whole arcs of the northern signs opposed to them will be delineated with ease, as we showed under the aforesaid method in its own place. Moreover, this second method is very well suited for correctly drawing the arcs of the northern and opposite signs — that is, those that are more remote from the centre of the dial. For, as you see, the hour-lines falling from C in the figure of rays, the farther they are extended, the greater occasion for error they present in drawing the arcs through the rays of the signs placed to the left of the radius of the Equator in that figure — such as here are the rays of the northern signs — because it can scarcely be done without error that the points where the said rays are cut by the hour-lines be taken, unless all the said lines have been drawn with the greatest care. But the points on the rays of the signs which are placed to the right of the radius of the Equator, and are nearer to the point C — such as here are the rays of the southern signs — can be more easily discerned, because of the shortness of the lines falling from C that are

contained between C and the said rays, if they are compared with the same straight lines contained between C and the rays of the northern signs. Hence we shall draw more surely the arcs of the southern signs, whose rays are nearer the point C, than those of the northern, whose rays are farther from that same point C. For in the former, the straight lines sent out from C cannot lead us into any notable and marked error, whereas in the latter they can, as we have said, and as can appear from the figure itself. Finally, as regards the third method, those hidden straight lines will be more conveniently drawn from the point ϕ — like the hour-lines going out from the centre C — by means of arcs of equal circles drawn from C and ϕ , than by the aid of the straight line drawn through χ parallel to the equinoctial line; because scarcely or never do some of the hour-lines cut the said straight line, as is plain. In the example, we have drawn from ϕ , to the right of the style-line, hidden straight lines corresponding to those lines which have been drawn (also hidden) from C through those points of the equinoctial line, or of the arc of the circle drawn from C, which have been transferred to the other side of the equinoctial line or of the said arc; because in this way, with a single effort, the hyperbolas can be drawn on either side of the style-line, as in the horizontal and vertical dial, and so on.

[Margin: How the horizontal line is to be drawn in a dial declining from the vertical.]

[Margin: a — Euclid XI.18.]

[Margin: b — Euclid XI.19.]

The horizontal line — that is, the common section of the plane of the declining dial and of the horizon — is drawn through K, the place of the style, perpendicular to the meridian line, or to the line of the 12th hour; and it necessarily passes through that point where the equinoctial line and the line of the 6th hour cut one another, unless an error has been made in drawing the line of the 6th hour or the equinoctial. For since the horizon is at right angles to the plane of the declining dial, and passes through the vertex of the style — which is likewise at right angles to the same plane — it will pass entirely through the whole style, namely through the axis of the plane of the dial. [a] For all planes drawn through the style to the plane of the dial are at right angles to it. Therefore the horizon will meet the plane of the dial at K, and so the horizontal line must be drawn through K. And since both the plane of the dial and the plane of the meridian are at right angles to the horizon, [b] their common section too — namely the meridian line — will be perpendicular to it. Therefore, by the third definition of Book 11 of Euclid, it will also be perpendicular to the common

section of the horizon and the plane of the dial existing in the horizon itself. For which reason the straight line drawn through K perpendicular to the meridian line will be the common section of the horizon and the plane of the dial — that is, the horizontal line. That this must necessarily pass through the 6th hour on the equinoctial line, we shall make plain as follows. When the Sun is in the common section of the Equator, of the circle of the 6th hour, and of the horizon, its ray, falling into the centre of the world, [is deflected] from the planes of these

its ray, falling into the centre of the world, does not depart from these circles, but falls into their common sections — the sections they make with the plane of the dial — as is clear from Prop. 11 of Book 1. And so the shadow of the style will necessarily fall on that point of the dial where the said sections of the circles cut one another — that is, at the 6th hour on the equinoctial line; otherwise the shadow of the style would not fall on both lines, as is plain. Therefore the horizontal line passes through that point where the equinoctial line and the line of the 6th hour intersect one another.

[Margin: The horizontal line cuts the whole dial into a southern and a northern part.]

Now this horizontal line divides the whole dial into two parts: the one in which the centre does not lie faces south, the other faces north. Each is to be set on a wall whose declination equals that for which the dial was constructed, in such a way that the horizontal line is parallel to the horizon and occupies the upper position. To this one further condition is added: in the northern dial — in which the arcs of the southern signs turn into the arcs of the northern signs, and the reverse — not only is the horizontal line to be placed in the upper position, but also the features which, when we turn to that dial in that position, lie on our right must be made to lie on the left, as we showed in the scholium of Prop. 14 of the preceding book.

[Margin: How a dial, whether southern or northern, is to be set in its proper position.]

It often happens that in a northern dial the hours lie on only one side of the meridian line — namely the side onto which the shadow of the gnomon falls in the evening, if the plane declines from north toward the west, or in the morning, if it declines toward the east. For the Sun does not light the other side in the morning or evening, except when the plane declines only a little from the vertical, as is evident if the declining plane is carefully considered in its proper position. The horizontal line demonstrates this to us most beautifully. For when

this line cuts the parallel of Cancer [♋] in two places on the southern dial, the hours will have to be drawn on both sides of the meridian line in the northern dial; but when it cuts it in only one place, then on one side only.

[Margin: Sometimes a northern dial has its hours on only one side of the meridian line.]

Now if you wish to know how great the declination must be for the northern face of the declining plane to be lit by the Sun both morning and evening, you will attain your wish in this way. When the plane's declination is less than the rising or setting amplitude [latitudo ortiva, occidua] of the first point of Cancer [♋] — how this amplitude is to be found we showed in Prop. 34 of Book 1, and it is taught by Johannes Regiomontanus [Regiomontanus] in Book 2 of the *Epitome*, Props. 1 and 2 — then the northern face of the plane is lit both morning and evening when the Sun is at the beginning of Cancer. But when the plane's declination is greater than the aforesaid amplitude, the same northern face receives the Sun's rays only in the evening, or only in the morning, according as it turns from north toward the west or toward the east. All of this can also easily be known and understood from a material sphere.

[Margin: By what method it can be known whether the northern face of a declining plane can be lit by the Sun both morning and evening, or only in the morning, or only in the evening.]

And although the northern face is lit by the Sun both morning and evening when the plane's declination is less than the rising and setting amplitude of the beginning of Cancer, it will nevertheless be illuminated for a longer span of time in the morning than in the evening if the plane turns from north toward the east, but for a shorter span if it inclines toward the west, as is evident — just as the southern face, turning toward the east, is lit for a longer time in the morning than in the evening, but for a longer time in the evening than in the morning if the southern face looks toward the west.

[Margin: When a plane, whether facing north or south, is lit for a longer time in the morning by the Sun than in the evening, or the reverse.]

This same horizontal line divides the declining dial into two: a diurnal part, contained below the horizontal line itself, and a nocturnal part, drawn above it, as we have said in the earlier cases too. And so we have drawn the parallels — that is, the arcs of the signs of the Zodiac — on a dial that declines from the vertical circle. Which was to be done.

[Margin: The horizontal line divides the whole dial into a diurnal and a nocturnal part.]

Scholium.

[Margin: A most elegant method for drawing the hour-lines in the figure of the rays of the Zodiac, without the hour-intervals of the dial.]

Andreas Schöner [Andreas Schöner] draws the hour-lines in the figure of the rays at the point C by the following method. Having drawn the straight line CG as before, he takes on its extension any point B, through which he draws the line BA parallel to CI, so that it cuts the straight line CA — which is parallel to the ray of the Equator — at A. Then, with A as centre and AB as radius, he divides the circle so described into 24 equal parts, beginning from a certain point which he finds in this way. The straight line LM, lying in the dial between the centre L and the meridian line, he transfers onto the ray of the Equator from I as far as E; and through E, from C, he draws the straight line CE for the line of the 12th hour, which cuts the straight line BA at D. From D he raises DF perpendicular to BA, and it cuts the circumference of the circle at F — the point that will be the beginning of the division of the circle.

But since, when the meridian line in the dial is very near the style-line — and likewise, in the figure of the rays, the straight line CD near CB — the line DF, drawn perpendicular to AB, cuts the circumference at F so obliquely, near the point B, that the point F can scarcely be discerned without error, we shall test the point F before proceeding to the division of the circle, by seeking another point from which the same division can be begun, in this way. In the dial we shall take the interval between the centre L and any hour on the equinoctial line that is somewhat more remote from the style-line than the 12th hour — for example, in the proposed example, the interval between L and the 1st hour — and we shall transfer it onto the ray of the Equator from I as far as H; and from C, through H, we shall draw the straight line CH for that 1st hour, which cuts the straight line BA at L. From L we shall raise LM perpendicular to BA, and it cuts the circumference at M. From this point, then, we shall be able to begin the divisions.

Now the perpendicular LM must be drawn toward the same side as DF was drawn, if the hour taken lies on the same side of the index-line as the 12th hour; but toward the opposite side, if it was taken on the other side — as would happen if we took the 5th hour. For this hour, transferred onto the ray of the Equator from I as far as N, will indeed give the line CN for the 5th hour, which cuts BA at O; but because the 5th hour and the 12th hour lie on different sides of the style-

line, the perpendiculars DF and OP have accordingly been drawn toward different sides. Yet the division of the circle can take its start equally well from the point P as from the point F or M. So whether the division begins from the point M or from P, if some point of the division falls on F — which had been found by the first perpendicular DF — the matter will succeed as intended; if not, an error must be corrected which has perhaps been committed somewhere. Indeed it is altogether expedient, if we want the matter perfect in every respect, to find two points from which the division can be begun. For if a division begun from the one falls on the other, we shall be certain that the division of the circle is sufficiently exact.

So, once the circle has been divided, the same author draws from the points of division perpendiculars to the straight line BA, or parallels to CA, cutting the line BA at points through which straight lines sent out from C give the hour-lines, as before. It will be easy to judge which lines correspond to which hours on the dial. For the points of the circle's division, starting from the point F (the 12th hour) and proceeding toward B — onto which the index-line falls — refer to the hours which, on the dial's equinoctial line, follow the 12th hour toward the index-line; so that the point nearest F toward B, in our example, belongs to the 11th hour, the next to the 10th, and so on, as the figure shows. And the straight lines drawn out from C through the points of the perpendiculars on the line BA refer to the hours from which the said perpendiculars fall onto BA.

For the rest, without much trouble we shall draw, from the points of division of the circle described from A, the perpendiculars to the straight line BA, or the parallels to CA, in this way. If it happens that two neighbouring points are equally distant on either side from the point B — which will occur only when the style-line in the dial is either one of the hour-lines, or lies equally between two neighbouring hour-lines on either side —

then the straight lines joining, on either side of the point B, two points equidistant from it will be perpendicular to the straight line BA. If not, then take the distance of any point in the upper semicircle from the point B, and transfer it into the lower semicircle; for the straight line connecting the upper point with the transferred lower point will be perpendicular to BA. For by this method the straight line BA, drawn through the centre A, will bisect the arc between those two points at B. Hence, by the earlier corollary of Prop. 10 of Book 13 of Euclid [Euclid], the line BA will be perpendicular to the line joining those points. The same will result if the distances of the points of the lower semicircle from B are

transferred into the upper semicircle, as you see done in the upper figure of the rays.

Diagram. The large "figure of the rays" (*figura radiorum*) of the Zodiac used in Andreas Schöner's construction of the hour-lines. It occupies most of the page. Two roughly horizontal lines run across the top: the upper one carries the labels **B** (at its left end) and **A** (at its right end); a lower transversal terminates at the point **I** on the right (directly below **A**). A dense sheaf of straight rays (both solid and dotted) fans out from a point low on the right toward the upper left, crossing the transversals; these are the hour-lines and the rays of the signs. Along the upper transversal, hour-numbers are marked in a row: **12, 3**, then **11, 4, 10, 5**, the point-label **G**, and (below, in small type) **9** and **6**. A second series of hour-numbers descends along the right, near the ray-point: **8, 9, 10, 11, 12**. Around the periphery, at the ends of the signs' rays, stand the Zodiac sign-symbols; several are worn and are flagged below.

- B (upper left): endpoint of the upper horizontal (line BA)
- A (upper right): endpoint of the upper horizontal; centre of the divided circle
- I (right, below A): terminus of the lower transversal (with the axis / point of the ray-sheaf nearby)
- G (on the upper transversal, right of centre): point-label
- 12, 3, 11, 4, 10, 5, 9, 6 (hour-numbers along the upper transversal)
- 8, 9, 10, 11, 12 (hour-numbers descending on the right)
- Zodiac sign-symbols around the periphery — clearly resolvable on the descending right side (top to bottom): ♑ (Capricorn), ♐ (Sagittarius), ♏ (Scorpio); the upper-left cluster shows ♉ (Pisces)[?], ♒ (Aquarius)[?], and further sign-glyphs [?] toward the far left and lower centre that are too worn to resolve with confidence
- Additional small letters near the ray-point (e.g. a small "e" and a small "y"-like mark) [?]

[Margin: By what method a larger or smaller dial is to be built from the figure of the rays for a given length of style.]

Now if, from the figure of the rays constructed so far, a larger or smaller dial is to be built for a given size of style, we shall do it as follows. Draw separately — as was done beside the figure of the rays — the triangle CIG, taken from the dial of the preceding proposition, or from the figure of the rays, or indeed from the dial which we described in the scholium of the preceding proposition for a given

length of style. In it CI is the axis of the world, IG the Equator, and CG the style-line, onto which the gnomon IK falls at right angles. Then, having produced the straight lines IC, IG, IK, if on IK there is taken a line IA equal to the given gnomon — whether it be greater or less than the gnomon IK — and through A a line BD is drawn parallel to CG, the triangle BID will be similar to the triangle CIG, by the corollary of Prop. 4 of Book 6 of Euclid. And so, if the axis IB is transferred into the figure of the rays from C as far as B, toward I, and from B the ray of the Equator together with the rays of the other signs is drawn out, and everything else is done as before, the arcs of the signs will be described for the size of the given style IA, just as before they were described for the given style IK. For the hour-lines in the dial and in the figure of the rays remain the same at the same elevation of the pole,

the declination of the plane being unchanged. But the position of the style IA will be the point on the style-line which terminates a straight line begun from the centre C and equal to BA. The equinoctial line too must be drawn perpendicular to the style-line, through the point which is distant from the centre C by as much as the line BD.

[Margin: By what method the arcs of the signs are to be described on a declining dial that has no centre.]

Now we shall describe the same parallels, or arcs of the signs, on the declining dial which we drew in the scholium of the preceding proposition — even though we did not find its centre — by almost the same methods that we set forth in this proposition. As for the former task, we must construct an Analemma for the elevation of the pole above the declining plane, which we shall find from Prop. 29 of Book 1; or else we shall take it from the angle GCI in the said dial, since this shows the elevation of the pole above the declining plane, as is clear from what has been said, being contained by the style-line and the axis of the world. In this Analemma the rays of the signs must be drawn, so that in the cones we may have the triangles through the axis, and thereby the diameters of the conic sections; and the rest is to be completed, as we said at the end of the scholium of Prop. 2 of the preceding book.

As for the latter method, we shall accomplish the task thus. Onto the ray of the Equator IG, transfer from I all the intervals contained between the centre Z and the hours on the equinoctial line HO of the said dial, marking off the points on the ray of the Equator itself. Then on the axis IA cut off a straight line IA equal to the portion QP of the axis in the same dial, and through A draw AB parallel to the ray of the Equator IG; onto it, in the same way, transfer from the point A the spaces

enclosed between the other centre *a* and the hours on the equinoctial line YV, likewise marking off the points on AB. Finally, straight lines drawn always through each pair of corresponding points on the lines IG and AB will give the hour-lines. For, as we showed in the aforesaid scholium of Prop. 2 of the preceding book, the said hour-lines must be drawn both through the points found on the ray of the Equator IG and through those on the parallel AB, which serves as a kind of second ray of the Equator. If, then, the intervals of these lines lying between the ray of the Equator IG and the rays of the other signs are transferred onto the corresponding hour-lines of the dial, starting from the equinoctial line HO itself — with this rule observed, that a straight line be drawn from I to the nearest line, which corresponds to the style-line in the dial (for the interval of the style-line too, between the centre of the dial and the equinoctial line, must be transferred from the dial onto the ray of the Equator), to serve as the perpendicular for the style — then those intervals of the hour-lines between the rays of the signs are to be transferred, on the side onto which the style falls from I, from the equinoctial line upward, and so on. The demonstration of this is gathered from what has gone before, as we noted in the said scholium of Prop. 2 of the preceding book.

Problema 3. Propos. 3.

To mark out the parallels of the diurnal arcs on the same declining dial.

[Margin: The description of the diurnal arcs on the same dial declining from the vertical.]

The description of these does not differ from the description of the parallels, or arcs, of the signs. Hence here we shall use the precepts delivered in the preceding proposition, provided only that in place of the parallels of the signs we take the parallels of the diurnal arcs, as in the earlier dials, and so on. So we have marked out the parallels of the diurnal arcs on the same declining dial. Which was to be done.

Scholium.

[Margin: By what method, with the help of the diurnal arcs, those hours are described which scarcely cut the equinoctial line.]

Through these diurnal arcs a very easy way is opened to us for drawing those hour-lines which scarcely cut the equinoctial line. These are to be drawn from that side where the equinoctial line cuts the meridian line below the horizontal line — such as the hours after noon in a dial declining from south toward the east

(the kind we have constructed), and, in a dial declining from south toward the west, the hours after midnight. For these scarcely cut the equinoctial line except at very remote points, so that their delineation is rendered very difficult. Therefore we shall use this device. Let it be required, for example, to draw the 3rd hour after noon. Now since, on the diurnal arc of 6 hours, the Sun sets at the 3rd hour after noon, we shall describe — from the dial's centre C, with the interval that, in the figure of the rays of the preceding proposition, is contained between C and the ray of the 6 hours by the line of the 3rd hour after noon — an arc of a circle in the afternoon part of the dial, and we shall mark the point where it cuts the horizontal line. For through that point the 3rd hour after noon must be drawn, since through that point passes the diurnal arc of 6 hours, and on it the Sun sets.

In the same way, if in the forenoon part of the dial we describe an arc with the interval — contained between C and the ray of the 6 hours — of the line of the 9th hour, at which the Sun rises, we shall draw the 9th hour after midnight. So too, by the help of the diurnal arc of 8 hours, we shall mark out the 4th hour after noon and the 8th after midnight, and so on. For otherwise we shall draw the half-hours, if we take the diurnal arcs of the odd hours. For instance, if the line of the $3\frac{1}{2}$ th hour were to be drawn, we should have to take the ray of 7 hours; because, since the day contains 7 hours, the Sun rises at the hour $8\frac{1}{2}$ after midnight and sets at the hour $3\frac{1}{2}$ after noon — and so with the rest. The same must be observed in the following dials.

But since, in the figure of the rays, those hours which cut the equinoctial line only at very remote points cannot be drawn — unless we should wish to use the method of Andreas Schöner — the diurnal arcs on which the Sun sets or rises at the proposed hours must be described from the Analemma, as we taught in Prop. 2 of this book.

Problema 4. Propos. 4.

To describe the vertical circles on the same declining dial.

[Margin: The description of the vertical circles on the same dial declining from the vertical.]

From the place of the style K, raise a line KA perpendicular to the horizontal line, equal to the style IK; and with A as centre, at any interval, let a described circle be divided into 360 equal parts, or into fewer, according to the number of vertical circles that the size of the dial can hold. (We divided it into 24 parts, so that each

space takes in 15 degrees.) The start is made from the straight line drawn from the centre A through the point B, where the meridian line and the horizontal line intersect one another; or else from the straight line drawn from A through the point D, where the equinoctial cuts the horizontal line. Then, through the centre A and the points of division, let hidden straight lines be sent out, cutting the horizontal line at points which must be carefully noted. For straight lines drawn through these points perpendicular to the horizontal line, or parallel to the meridian line itself, will give the vertical circles — that is, the common sections of the plane of the dial

(continuing Problema 4, Propos. 4 — the description of the vertical circles on the declining dial)

— and of the vertical circles. Those parallels will be drawn without much trouble if we draw a line parallel to the horizontal line and, with the compass, transfer onto it all the points of the horizontal line taken from B, always beginning from the meridian line — such as that straight line in the lower part of the dial marked off by those small division-lines. [Margin: *Elements* I.33] For straight lines joining a pair of points — one on the horizontal line and one on the aforesaid parallel — that are equally distant from the meridian line will themselves be parallel to the meridian line.

Diagram. The master construction figure for the declining dial with its vertical circles. This is a large, dense composite. A horizontal line runs across the top of the figure carrying the points C (toward the middle, marked "90") and A (at the upper-right corner), from which a fan of straight rays descends. A large circle occupies the centre-left of the figure; through and around it run numerous oblique construction lines and a family of near-parallel lines. Degree graduations appear in Roman numerals down both flanks and in Arabic numerals near the rays and along the central horizontal band. A vertical dashed line and a lower dashed horizontal line (the latter with tick-marks) complete the frame. Zodiac sorts (diurnal-arc / solstice markers) appear at the top-left, at the right margin, and at the bottom-centre.

- A (upper-right corner of the top line): the zenith / pole from which the rays issue
- C (top line, near middle, beside "90"): point on the horizontal line
- A, I, C, B (central circle): construction points of the inner circle
- K (centre, small): the place of the style
- D (centre, below B): common section with the horizon/equator

- G (near centre-bottom of the inner cluster): point on the GO line
- B (right margin): foot of the meridian on the horizontal line
- Rays near A labelled (Arabic): 90, 75, 60, 45, 30, 15 [several small numerals abraded — [?]]
- Left flank (Roman degree labels, top to bottom): XV, XXX, XLV, LX [and a repeated XXX / a worn "XV" — [?]]
- Right flank (Roman degree labels, top to bottom): XV, XXX, XLV, LX
- Along the central horizontal band (Arabic): 15, 30, 45, 60, 75, 90 [inner values crowded — partly [?]]
- Zodiac sorts at top-left, right margin, and bottom-centre: [worn — [?]; solstitial/diurnal-arc markers, glyphs not securely legible]

We shall prove as follows that these parallels are the common sections of the vertical circles and of the plane of the dial. Imagine the plane of the circle described about A around the horizontal line to be moved until it coincides with the plane of the horizon, so that the centre A comes to coincide with the tip of the style — that is, with the centre of the world. Now the common sections of the horizon and of the vertical circles — among which is also the meridian drawn through the poles of the horizon — divide the horizon, and therefore also the circle described from A as centre of the horizon, in the said position, into equal parts. The meridian, drawn through A the centre of the world, meets the plane of the dial at B, making with it a common section, namely the meridian line BC. The vertical circle properly so called, likewise passing through the centre of the world, cuts the same plane at D — because the shadow of the style, when the Sun stands at the common section of the horizon, the equator, the circle of the 6th hour, and the vertical circle properly so called, falls upon the point D, that is, upon the common section of the plane of the dial and the equator, and upon the common section of the same dial and the horizon, as is established from Prop. 11 of Book 1. Therefore AB will be the common section of the horizon and the meridian; AD the common section of the horizon and the vertical circle properly so called; and consequently the remaining hidden lines drawn through the centre A and the points of division of the circle described from A will be the common sections of the horizon and the remaining vertical circles. Hence the vertical circles cut the plane of the dial at those points of the horizontal line onto which the said common sections fall. Since, then, by Prop. 18 of Book 1, the common sections of the vertical circles and of the plane of the dial are parallel — because the plane of the dial is equidistant from their common section, namely from the axis of the horizon drawn through the zenith of the place and its opposite point — it plainly remains that those lines parallel to the meridian line are the common

sections of the plane of the dial and of the vertical circles. This is why we have described the vertical circles on the same declining dial. Which was to be done.

Scholium.

The horizontal line here too cuts off the portion that must be placed on the opposite face of the same plane, so that the horizontal line is set in the upper position, and the part which in that situation is on our right becomes the left, and conversely — as is established from what was demonstrated in the scholium to Prop. 14 of the preceding book. Moreover, in both dials, the southern as well as the northern, the numbers of the degrees begin from the parallel drawn through the point D, which is the common section of the vertical circle properly so called and the plane of the dial, so that in our example the lines nearest on either side of the point D bear the numbers 15 degrees, and so on. [Margin: The order of the vertical circles in the dial.]

Now since it has been shown that the vertical circle properly so called passes through AD, and the meridian through AB — provided the circle described from A holds its proper position — it comes about that ADB, if there is no error, is the angle of declination, namely the angle which the vertical circle properly so called makes with the declining plane, and that ABD is its complement, namely the angle which the meridian makes with the same plane. Hence the remaining angle BAD, in the triangle ABD, must necessarily be a right angle. This is even more clearly established from the dial which, in the scholium to Prop. 1 of this book, we constructed for a given length of style. For there the line FE answers to the line AB here, and the line Fa there is the same as the line AD here.

In our example, too, the vertical circle of 60 degrees passes exactly through K, the place of the style — because that vertical circle which is drawn through the style, and is therefore perpendicular to the declining plane, recedes from the meridian by 30 degrees, namely by as many degrees as the declining plane recedes from the vertical. And it is evident that the vertical circle of 60 degrees declines from the meridian by 30 degrees. By the same reasoning, in every declining dial the vertical circle to be drawn through the place of the style will be the one that is as many degrees distant from the meridian as the declination of the declining plane from the vertical circle comprehends.

Problema 5. Propos. 5.

To depict the parallels of the horizon on the same declining dial.

We shall describe these parallels on the declining dial by the same methods by which they were described on the vertical dial in Prop. 17 of the preceding book, with the following changes. In the former method, the circle of the analemma here represents not the meridian, as it did there, but that vertical circle which is perpendicular to the declining plane — the one that passes through K, the place of the gnomon, and makes with the plane of the dial a common section that cuts the horizontal line at right angles at the point K. In our example this is the vertical circle of 60 degrees, distant from the meridian, on the south side toward the east, by 30 degrees, and making in the dial the section AK, as we said in the preceding scholium. The parallels of the horizon, however, are not changed. Again, the straight line GO will be the common section of the plane of the dial and of that vertical circle drawn through the style, yet so placed that it is distant from the centre E by the length of the style Ik, or AK. Finally, the points K, L, M, N are here to be transferred not onto the meridian line of the dial, as before, but onto the straight line drawn through K perpendicular to the horizontal line — that is, onto the common section of the said vertical circle and the plane of the dial. These points must be transferred from the place of the style both below the horizontal line and above it, so that we may also have the parallels of the horizon in the portion that is to be placed on the northern face of the plane. For the conic sections about the diameters KO, LO, MO, NO — described by Prop. 8 of Book 1 and transferred onto the dial in such a way that their diameters coincide with the perpendicular drawn from K and pass through the points K, L, M, N, while their horns are turned more and more away from the horizontal line — will give the parallels of the horizon. [Margin: The description of the parallels of the horizon on the same dial declining from the vertical.]

Diagram. A graduated circle (the analemma circle) with centre E, from which straight lines radiate to the circumference. The rim is graduated at every 15°.

- A (top): the highest point of the circle, marked 90
- C (bottom): the lowest point of the circle
- B (left, on the horizontal diameter through E)
- D (right, on the horizontal diameter through E)
- E (centre): centre of the circle, from which the radii issue
- Rim degree graduations (each quadrant, from the horizontal upward and downward): 15, 30, 45, 60, 75, 90; with 90 at A (top) and 75, 75 flanking it
- O, O: two points near the lower part of the rim, flanking C
- Interior points on the left of E (top to bottom): G, K, L, M, N
- Interior points on the right of E (top to bottom): G, K, L, M, N

In the latter method, the lines equidistant from AC in the posterior figure of Prop. 17 of the preceding book are here to be drawn, as there, through the points of the straight line AB which bound the intervals between A — the centre of the circle in the figure of the preceding proposition — and the vertical lines on the horizontal line (understanding also the interval AK, if perhaps no vertical line is drawn through K), transferred from A onto the straight line AB. But their numbers here will not correspond to the numbers of the same lines in Prop. 17 of the preceding book, since the first interval AK, in the figure of the preceding proposition, is not between A and the vertical circle of 90 degrees, but of 60 degrees, in our given example. Hence the parallel line nearest to AC will not bear the number 90, but 60; then the next will bear these two numbers, 45 and 75, and so on, regard being had to the numbers whose intervals have been transferred. Indeed it will sometimes happen that the first parallel line bears no number at all — as when the perpendicular AK is not one of the vertical lines described on the dial. Moreover, when the point K is not exactly in the middle of the two surrounding vertical lines but is unequally distant from them, the intervals between A and the vertical lines are to be transferred on both sides of the point K; and then there will be more parallels in the posterior figure of Prop. 17 of the preceding book, and each will bear only a single number, just as was done for the hour-lines in Prop. 2 of

this book, and not two apiece, as is done in the vertical dial and in our example.

Diagram. The posterior figure — a graduated circle of the same construction as the preceding one, with centre E and radiating lines. The rim is graduated at every 15°.

- A (top): marked 90, flanked by 75, 75
- C (bottom): marked 90, flanked by 75, 75
- B (left, on the horizontal diameter through E)
- D (right, on the horizontal diameter through E)
- E (centre): centre of the circle, from which the radii issue
- Rim degree graduations down each flank: 60, 45, 30, 15 (above and below the horizontal), then 60 again toward the bottom
- O, O: two points near the lower rim, flanking C
- Interior points on the left of E (top to bottom): G, K, L, M, N
- Interior points on the right of E (top to bottom): G, K, L, M, N

Finally, the intervals of the lines equidistant from AC — comprehended between the straight line AB and the rays of the parallels of the horizon issuing from A —

are to be transferred on both sides of the horizontal line onto the vertical lines answering to the numbers marked on the straight line AB, so that the intervals of the parallel nearest to AC are transferred from K, the place of the style, onto the perpendicular AK, upward and downward, and so on. For through the points marked on the vertical lines the parallels of the horizon will pass, all of which are hyperbolas, as we said in Prop. 17 of the preceding book. But since it sometimes happens that the same number is contained on two parallel lines, the intervals of the parallel nearer to AC are to be transferred onto the vertical line of the same number nearer to the point K, and those of the more remote parallel onto the more remote vertical line. And if the perpendicular AC is one of the vertical lines, or is equally distant from the surrounding ones, the said intervals are each to be transferred onto two vertical lines equally distant from the straight line Ak, as the numbers will indicate. This is what happens in our example. We have placed the posterior figure of Prop. 17 of the preceding book near the figure of the preceding proposition, in so far as it corresponds to our example. And so we have depicted the parallels of the horizon on the declining dial. Which was to be done.

Problema 6. Propos. 6.

To delineate the meridians, or the circles of longitude of cities, on the same declining dial.

[Margin: The description of the meridians on the same dial declining from the vertical.]

We shall delineate these circles on the declining dial in the same way in which, on the same dial, we described the hours from noon or from midnight in Prop. 1 of this book — since the meridian is drawn through the poles of the world, like the hour-circles. But the circle described from L on the dial in Prop. 1 of this book will here have the beginning of its division not from the diameter LM, but from another diameter, which is distant from LM toward the west by as many degrees as the longitude of the place comprehends — in our example, 36 degrees. For this diameter will be the common section of the equator and of the prime meridian drawn through the Fortunate Isles, as is evident. And from what we wrote on the second method of describing the declining dial, it will be no trouble to understand which part of the circle described from L is more to the west, or more to the east, than the diameter LM — especially if that circle is understood to be set in its proper position.

Diagram. The construction of the meridians (longitude-circles) on the declining dial. An oblique layout: a long horizontal line (the horizontal line of the dial)

crosses the figure; in the lower-left a circle (the equator) is drawn, labelled *Inf. Fortun.* (*Insulæ Fortunatæ*, the Fortunate Isles) with its upper point L; a fan of straight lines issues from a point near the centre-top (the pole), above which stand the points I and C; a long oblique straight line runs off to the lower-right, labelled *INSVLAE FORTVN.* (the prime meridian through the Fortunate Isles). Longitude graduations in degrees are marked around the circle and along the lines.

- I (upper middle): point above the pole
- C (upper, right of I): point on the meridian line
- L (top of the lower-left circle): centre-point of the equator circle described from L
- G, M (interior points along the horizontal band)
- X (labelled point on the right, on the oblique line)
- - o (marked point at the left of the circle)
- *Inf. Fortun.* (label on the circle) = *Insulæ Fortunatæ*, the Fortunate Isles
- *INSVLAE FORTVN.* (label along the lower-right oblique line) = the meridian of the Fortunate Isles
- Longitude graduations (degrees), securely read: 15, 30, 45, 60, 75, 90, 105, 120, 135, 195, 225, 270, 300, 330
- Further small, rotated numerals near the top of the circle (in the 210–345 range): [?] — abraded and not securely legible

Furthermore, in describing the northern dial on the opposite face of the declining plane — which is presented to us by that portion cut off from the horizontal line, provided all its parts are inverted, as we taught above — the numbers are to be added to the meridians, just as we set forth for the vertical dial in Prop. 18 of the preceding book. This the opposite figure clearly teaches. We have therefore delineated the meridians, or the circles of longitude of cities, on the same declining dial. Which was to be done.

Gnomonics, Liber Tertius (continued)

PROBLEMA 7. PROPOS. 7.

To reposition the parallels of cities — that is, the circles of latitude — on the same declining dial.

[Margin: The drawing of the parallels of cities, or of latitude, on the same dial declining from the vertical.]

These differ from the parallels of the signs of the Zodiac only in that they run through the zeniths of places. We shall therefore draw them on the dial in the way we taught in Prop. 2 of this book, provided that, in place of the parallels of the signs of the Zodiac, we take parallels drawn through the zeniths of cities, and so on. We have thus repositioned the parallels of cities — that is, the circles of latitude, and so forth. Which was to be done.

PROBLEMA 8. PROPOS. 8.

To depict the celestial houses on the same declining dial.

[Margin: The drawing of the celestial houses according to Joannes Regiomontanus [Regiomontanus], on the same dial declining from the vertical.]

We shall carry out the proposed task following the opinion of Joannes Regiomontanus. Let the circle described from L be divided into 12 equal parts — or even into more, if the divisions of the celestial houses too are to be drawn — beginning from the diameter LM, just as in the drawing of the hour-lines. Then, when hidden straight lines have been drawn through the points of division and through the centre L, the equinoctial line will be cut in points; and through these points, straight lines sent out from the point E — where the meridian line intersects the horizontal line — will give the celestial houses. For when the circle described from L, drawn about the equinoctial line, has been rotated until its centre L is joined with the centre of the world and the circle itself is set in the plane of the equator, the straight line LM will be the common section of the equator and the meridian, as we showed above in Prop. 1 of this book. Since, then, the meridian and the remaining circles of the celestial houses divide the equator into 12 equal parts,

Diagram. The construction of the celestial houses on the declining dial. A dense composite figure of the declining dial: a long horizontal line (the horizontal line of the dial) runs across the middle, carrying from left to right the points D, then a short vertical dashed line, then I, B, K, E, beyond which the hour-labels 7 and VII stand to the right. A near-vertical line rising to the point A (with C just to its right) is the meridian; the point I lies to its left. A circle (the equator described from L) sits in the lower half, with L at its lower part; near its interior are the points G and M and the hour-labels XII / 12 (upper left of the circle) and XI / 11 (below G). From a focus near the centre a fan of straight lines and hidden (dotted) lines

radiates. Down the right side, a family of hour-lines is labelled in both Arabic and Roman numerals: **8 / VIII**, **9 / IX**, **10 / X**, running out toward the lower right, with **IX** at the bottom right. Near the top left stand the labels **8** and a mark read as **IIII** [?] with an adjoining angle-stroke.

- D (left end of horizontal line)
- I, B (on the horizontal line, left of centre)
- K (on the horizontal line, at the meridian crossing)
- E (on the horizontal line, right of K — the point from which the house-lines are emitted)
- A (top of the near-vertical meridian line); C (just right of A)
- I (upper middle, left of A)
- L (lower part of the equator circle; centre of the circle described from L)
- G, M (interior points of the circle)
- XII / 12 (hour-labels at upper left of the circle)
- XI / 11 (hour-labels below G)
- 7 / VII (right of E on the horizontal line)
- 8 / VIII; 9 / IX; 10 / X; IX (hour-lines fanning down the right side)
- Uncertain small labels near the top left: "8", and "IIII" [?] with an angle-stroke [?]

they will be straight lines drawn out through the centre L and the points of division — the common sections of the celestial circles of the houses and of the equator. These circles will therefore meet the plane of the dial at the points where the equinoctial line is cut by the aforesaid hidden lines; and consequently, through those same points of the equinoctial line, the common sections of these same circles and of the plane of the dial will be drawn. Now since these common sections cut one another mutually — by Prop. 18 of Book 1 — at the point E (because the common section of those circles, namely the common section of the horizon and the meridian, meets the plane of the dial at E: for if the Sun were to stand in the common section of the meridian and the horizon, so that its ray did not differ from the common section of the celestial circles of the houses, the shadow of the style would fall, by Prop. 11 of Book 1, upon the common section of the plane of the dial with both the horizon and the meridian, and therefore must fall at the point E), it is clear that the straight lines sent out from E through the said points of the equinoctial line are the common sections of the celestial circles of the houses and of the plane of the dial.

[Margin: The drawing of the celestial houses according to Campanus [Campanus of Novara], on the same dial declining from the vertical.]

We shall, however, depict the celestial houses following the teaching of Campanus by this method. From K, the place of the style, let a perpendicular KA be raised to the horizontal line, equal to the style; and from A to the point B — where the equinoctial line intersects the horizontal — let the straight line AB be drawn, and let BD, equal to it, be marked off on the horizontal line. Then, with D as centre, let a circle be described and divided into 12 equal parts (or more, if the divisions of the houses are also wanted), the beginning being made from the horizontal line. For if, from the centre D, hidden straight lines go out through the points of division, they will cut the straight line BF — which is drawn from B perpendicular to the horizontal line, or parallel to the meridian line — in points; and if straight lines are sent out from the point E through these points, the celestial houses will be drawn, as Campanus holds. For if the triangle ABK is understood to be moved about BK until it stands perpendicular to the plane of the dial — so that the style AK likewise becomes perpendicular to it — and if the circle described from D about the straight line BF is conceived to be turned until its centre is joined with the vertex A of the style, that is, with the centre of the world (and it will be joined, because of the equality of the straight lines BD and BA),

Diagram. The same declining-dial construction of the celestial houses, here shown for the Campanus method. The figure is essentially identical to that on the preceding page: a horizontal line carrying D — a short dashed vertical segment — I, B, K, E, then 7 / VII to the right; the near-vertical meridian line rising to A, with C beside it and I to the left; the equator circle in the lower half with L at its lower part and the interior points G and M; the hour-labels XII / 12 and XI / 11 at the circle; and the hour-lines 8 / VIII, 9 / IX, 10 / X, IX fanning down the right side. The finer (very small) lines carry the Campanus house-sections; the remaining lines belong to the houses of the prior (Regiomontanus) construction. The top-left marks read 8 and IIII [?] as before.

- Labels as on fig-p283-1: D, I, B, K, E on the horizontal line; A, C, I at the top; L, G, M at/in the equator circle; hour-lines 7/VII, 8/VIII, 9/IX, 10/X, XI/11, XII/12, IX.

then the style itself is set in the plane of the vertical circle properly so called. (For, as we demonstrated in Prop. 4 of this book, the vertical circle properly so called is drawn through the straight lines AB and BF, through which the plane of the circle described from D is likewise drawn, if it has the position it acquired by that

rotation.) These straight lines, drawn out through the centre D and the points of division, will then be the common sections of the vertical circle properly so called and of the circles of the celestial houses — since at that point the straight line DB is exactly the same as AB, the common section of the said vertical circle and of the horizon, which is indeed one of the circles of the celestial houses. Therefore the said circles will cut the plane of the dial in the points of the straight line BF into which the said common sections fall; and for that reason the common sections of these same circles and of the plane of the dial will pass through those same points. And since these too are drawn through the point E, as we showed a little before, the straight lines sent out through E and through the points of the straight line BF will give the celestial houses according to Campanus. These lines are represented in the figure by those which are marked with the very fine lines; the remaining lines pertain to the celestial houses of the prior description. We have therefore depicted the celestial houses on the same declining dial. Which was to be done.

PROBLEMA 9. PROPOS. 9.

To describe the ascending signs of the Zodiac on the same declining dial.

[Margin: The drawing of the ascending signs on the same dial declining from the vertical.]

Here, no otherwise than in what precedes, we shall describe the ascending signs of the Zodiac. From the first four tables of Prop. 9 of the preceding book, however, not all the signs can conveniently be described, because the hours that have their beginning from noon, or before noon, from the straight line LM — the diameter of the circle described from L — scarcely all fall upon the equinoctial line. Thus in our example the signs that rise after the 2nd hour from noon can be described only with great difficulty, or not at all, since scarcely any hour-line after the 2nd hour from noon cuts the equinoctial line.

[Margin: The finding of the points on the horizontal line, through which the arcs of the signs and the ascending signs are to be drawn.]

But from tables 6 and 7, by means of the culminations of the heavens and the angles of the earth, and the points that stand at the rising of the signs in the circle of the 6th hour, everything will be described more conveniently, as is evident. Now the ascending signs will pass through those points of the horizontal line where it is cut by the parallels of the signs — although in our example it is scarcely cut by the arcs of the northern signs.

Diagram. The upper figure: an oblique horizon-and-equator layout for the ascending signs on the declining dial. A long horizontal line across the middle is labelled **HORIZON** toward the right; along it stand the points D, K, E. An oblique straight line rising to the upper right is labelled **Linea horæ 6** ("the line of the 6th hour"), with the points N and C where it meets the upper structure. A line descending from the upper left, labelled **AEQVA...** at the left and **...TOR** at the lower right, spells the equator (**ÆQUATOR**) written along its length. A large circle (part of the equator/horizon construction) fills the lower half, with L at its centre-lower part and M within. A great sheaf of straight and dotted lines fans out to the right and lower right, ending in the glyphs of the Zodiac signs (parallels of the signs); further Zodiac glyphs are ranged down the left margin.

- HORIZON (label on the horizontal line, right of centre)
- Linea horæ 6 (label along the oblique line at upper right; the trailing sort after "6" is worn [?])
- N, C (points on/near the "line of the 6th hour" at top)
- D, K, E (points on the horizontal line)
- L (centre-lower part of the circle); M (interior point)
- AEQVA...TOR (= ÆQUATOR, written along the equator line, left to lower-right)
- Zodiac-sign glyphs at the ends of the radiating lines and down the left margin (parallels of the signs)

These points, although the arcs of the signs are not drawn on the dial, we shall find either from the fifth table of Prop. 9 of the preceding book, as was said in what precedes (for if we draw the hour-lines that mark the hours terminating the semidiurnal arcs, these will cut the horizontal line in the said points), or else, more surely, from the last figure of Prop. 21 of the preceding book, which we have here repeated as regards the radii of the latitudes of the signs, in this way. Let there be, as there, the horizon ABCD, whose centre is E; its common section with the meridian is BD; its common section with the prime vertical is AC; and let there be the radii of the latitudes of the rising and setting points, as in that place. Then for the dial

[Margin: Another way of finding the same points.]

Diagram. The lower figure: the horizon ABCD with its full rose of latitude-radii for the rising and setting signs. A large circle is inscribed in a square frame of the four cardinal directions: **MERIDIES** (south) across the top, **SEPTENTRIO** (north) across the bottom, **ORTVS** (east, set vertically) on the left, and **OCCASVS** (west, set

vertically) on the right. The circle is ABCD with B at the top, D at the bottom (so the meridian BD is the vertical diameter), A on the right and C on the left (the prime vertical AC being the horizontal diameter). The centre is E. Between B and the centre stand the interior points H, then K, M (left), G (right), and L just above E; below the centre stand M and K again, running down to D. From the centre E a full sheaf of straight lines radiates to the circumference, each ending in a Zodiac-sign glyph, giving the radii of the latitudes of the rising (east) and setting (west) points.

- MERIDIES (top frame = south); SEPTENTRIO (bottom frame = north); ORTVS (left = east); OCCASVS (right = west)
- B (top of circle); D (bottom of circle); A (right of circle); C (left of circle)
- E (centre)
- Upper interior, from B toward centre: H, then K, M, G, L
- Lower interior, from centre toward D: M, K
- Radial lines from E ending in the twelve Zodiac-sign glyphs around the circumference (the radii of the latitudes of the rising and setting points)

that declines from south toward east, then, let there be set up at E, to ED, the angle of declination of the proposed plane, DEK, toward the parts of the north and toward the west — that is, toward the point A. But for a dial declining from south toward west, [the angle is set] likewise on the side of the north but toward the east — that is, toward the point C. But if the plane declines from north toward east,

toward the east, the said angle of declination must be set up at E, to EB, toward the parts of the south and toward the west — that is, at point A. But conversely, [it is set up] on the side of the south and toward the east — that is, at point C — if it deflects from the north into the west. Now let the straight line EK, equal to the style, be taken on this line, which forms with BD at E the said angle of declination; and through K let there be drawn a perpendicular to EK, cutting BD (produced) at M, and cutting the radii of the rising and setting latitudes (produced) at other points. I say that these points, taken with the compass from point M and transferred onto the horizontal line from E, where it is cut by the meridian line — or else taken from point K and transferred onto the horizontal line from k, the place of the style —

Diagram. The large figure for the declining dial: the horizon-circle with its full rose of latitude-radii for the rising and setting signs. A circle is inscribed within a square frame of the four cardinal directions: **MERIDIES** (south) across the top,

SEPTENTRIO (north) across the bottom, **ORTVS** (east, set vertically) on the left, and **OCCASVS** (west, set vertically) on the right. The circle carries B at the top and D at the bottom (so the meridian BD is the vertical diameter), with A on the right and C on the left (the prime vertical being the horizontal diameter). The centre is E. Along the vertical between B and the centre stand the interior points H, then K (left) and G (right), then M and L just above the centre E; below the centre, running toward D, stand M and K again. On the horizontal diameter, to the left near ORTVS are C and F; to the right near OCCASVS is A. From the centre E a full sheaf of straight lines radiates to the circumference, each ending in a Zodiac-sign glyph — the radii of the latitudes of the rising (east) and setting (west) points.

- MERIDIES (top frame = south); SEPTENTRIO (bottom frame = north); ORTVS (left = east); OCCASVS (right = west)
- B (top of circle); D (bottom of circle); A (right, on horizontal diameter); C (left, on horizontal diameter); F (left, on horizontal diameter, beside C)
- E (centre)
- Upper interior, from B toward centre: H, then K (left), G (right), then M, L
- Lower interior, from centre toward D: M, K
- Radial lines from E ending in the twelve Zodiac-sign glyphs around the circumference

give the points through which the arcs of the signs and the ascending signs are to be drawn. For if, in the dial, a perpendicular KN is drawn from K, the place of the style, to the horizontal line — a perpendicular equal to the style — and NE is joined; and if the triangle EKN is understood to be turned about EK until it stands perpendicular to the plane of the dial, and so is set in the plane of the horizon, with the vertex of the style N being the same as the centre of the world: then NE will be the common section of the horizon and the meridian. For each of those circles is drawn through the centre of the world and through the point E, at which their common sections with the plane of the dial meet. And therefore the angle NEK, which the meridian makes with the plane of the dial in the plane of the horizon, will be equal to the complement of the declination of the plane from the vertical; and accordingly ENK will be the angle of declination, equal to the angle of declination DEK in the figure of the rising latitudes.

Since, therefore, the angles K and E of the triangle EKM in the figure of latitudes and the angles K and N of the triangle ENK in the dial are equal, and the sides EK and NK adjacent to these angles in each triangle are equal to one another [Margin: (Elements) I.26], the sides ME and MK of the former triangle will also be equal to the sides EN and EK of the latter triangle, and the angle M equal to the

angle E. Hence if the straight line EM is superimposed on NE, neither will exceed the other; the style EK will coincide with the style NK, and the line MK with the line EK, because of the equality of the angles. And accordingly the radii of the rising and setting latitudes will meet the horizontal line at the said points of the arcs of the signs, upon which the shadow of the style falls when the Sun, standing at the beginnings of the signs, rises. And so we have described the ascending signs of the Zodiac on the same declining dial. Which was to be done.

[Margin: Why the arcs of the southern signs change into arcs of the northern signs in a northern dial, and vice versa.]

[Margin: How the ascending signs are described, following the practice of Andreas Schöner, on the same dial declining from the vertical.]

[Margin: The order of the ascending signs in the tropics.]

SCHOLIUM

From this, moreover, the reason clearly appears why in a northern dial the southern signs are changed into northern ones, and vice versa. Namely: just as in a southern dial the radii of the latitudes of the southern signs, drawn through the centre E, cut the line KM near the place of the style, while the radii of the latitudes of the northern signs cut it far from that place, so in a northern dial the radii of the latitudes of the northern signs cut the line KM near the style, while the radii of the latitudes of the southern signs cut it far from that place — as is clear from the figure of the rising latitudes.

Now we shall depict the ascending signs according to the teaching of Andreas Schöner [Andreas Schöner], as follows. In the first figure of the scholium of Prop. 9 of the preceding book — (we have here repeated the two earlier figures of that scholium as regards the intervals and angles that the lines of the signs make at the centre E, having also produced, in the second figure, those lines of the signs beyond the centre E which have no answering lines joined to them in a straight line, namely those marked with dots, as we noted in the said scholium) — from the straight line ED, or EB, let a straight line EF be cut off, equal to the line LM, which in the dial is intercepted between the centre L and the point M, where the equinoctial line and the meridian intersect each other. At F let there be set up an angle EFG equal to the angle LMD, which in the dial the equinoctial line makes with LM — yet under this condition and rule, that, with the centre E placed on the centre L and the point F on the point M, the line FG coincides with the equinoctial line.

Then let the intervals of the line FG, intercepted between F and the lines of the signs, be transferred onto the equinoctial line from the point M, in the same order they would have if the line FG coincided with the equinoctial line, by impressing points on the equinoctial line itself. For if, from E, the point of intersection of the meridian line and the horizontal, hidden lines are drawn through these points of the equinoctial line, both tropics will be cut at points through which the ascending signs are to be drawn.

[Margin: How the order of the ascending signs is to be assigned in the tropics.]

It will not be difficult to judge which points answer to which signs. For in a dial declining toward the east, the point where the tropic of Capricorn (♐) cuts the horizontal line, toward the part into which the shadow is projected when the Sun rises — namely the part sloping toward the west —

pertains to Capricorn (♐); the next below the horizontal line, in the same tropic, to Aquarius (♒); the following to Pisces (♓); and so on, as they follow in the earlier figure. But the points which are shown in the tropic of Cancer (♋), or in the same tropic of Capricorn (♐), by the same hidden lines drawn out from point E through the points found on the equinoctial line, look toward the opposite signs. So too the point where the tropic of Cancer (♋) divides the horizontal line, toward the western parts of the dial — into which the shadow of the style is projected in the forenoon — looks toward Cancer (♋); the next below the horizontal line to Leo (♌); the following to Virgo (♍); and so on, in the order which they hold in the said earlier figure.

But in a dial declining toward the west, the point where the tropic of Capricorn (♐) and the horizontal line mutually intersect, toward the eastern parts of the dial — into which the shadow of the gnomon falls in the afternoon — pertains to Cancer (♋); the next in that tropic, below the horizontal line, to Gemini (♊); the following to Taurus (♉); and so on in order, just as they are contained in the same earlier figure. But that point in which the same tropic of Cancer (♋) and the horizontal line intersect, likewise toward the eastern parts of the dial, pertains to Capricorn (♐); the next then, in the same tropic, below the horizontal line, to Sagittarius (♐); the following to Scorpio (♏); and so on, as they are placed in the figure. Nevertheless each single line proceeding from E always pertains to opposite signs in the two tropics, so that if one point of one line in one tropic looks toward Scorpio (♏), the other point of the same line in the other tropic pertains to Taurus (♉), and vice versa, and so on.

Diagram. Two circular figures set side by side — the two earlier figures of the scholium of Prop. 9, here repeated for the intervals and angles of the sign-lines. Each is a circle with centre E, from which a full sheaf of straight lines radiates outward past the circumference, each ending in a Zodiac-sign glyph; the whole is enclosed in a rectilinear frame of construction lines, and (in the right-hand figure especially) several radii are drawn as dotted lines — the produced sign-lines that have no answering lines joined to them.

Left-hand figure:

- A (top of circle); C (bottom of circle); B, N, P (at the left, on the horizontal); F, D (at the right, F above D); E (centre)
- G (outside, upper left); H (interior, below and right of centre)
- M, K (below the circle, on the vertical)
- Peripheral Zodiac-sign glyphs around the circle (*worn; not all securely legible*)

Right-hand figure:

- A (top of circle); C (bottom of circle); N, B (at the left); G (outside, lower left); D, P (at the right); E (centre)
- M, K (below the circle, on the vertical)
- Several radii drawn dotted; peripheral Zodiac-sign glyphs around the circle (*worn; not all securely legible*)

Next, in the second figure of the said scholium, let a length EF, equal to the interval LD — which in the dial is enclosed between the centre L and the point D, where the equinoctial line, the horizontal line, and the six-o'clock line mutually intersect — be taken on the line EC; and at F let there be made an angle EFG equal to that angle which the line LD (were it drawn) between the centre L and the point D forms with the equinoctial line: under this condition, that, with the centre E placed on the centre L and the point F on the point D, the line FG coincides with the equinoctial line. After this, let the intervals of the line FG, interposed between F and the lines of the signs, be transferred onto the equinoctial line from the point D, in the order they would have if the line FG coincided with the equinoctial line, by impressing points on the equinoctial line. For if these points of the equinoctial line are joined with the corresponding points found on the tropics, the ascending signs will be drawn, as before.

[Margin: The order of the ascending signs on the equinoctial line.]

Now the order of the ascending signs on the equinoctial line, in a dial sloping toward the east, is this. The first point below the horizontal line, found by the method just described, is to be assigned to the sign Taurus (♉); the following to the signs Gemini (♊) and Cancer (♋) [♋? — worn], and the next to the sign Leo (♌) [♌? — worn], and so on, as they are set in the later figure. But the next point above the horizontal line pertains to Pisces (♐); the following to Aquarius (♑) and Capricorn (♒) [♒? — worn]; and so on, as the same figure shows. In a dial sloping toward the west, however, the first point found on the equinoctial line below the horizontal line is to be given to the sign Pisces (♐), the following to the signs Aquarius (♑) and Capricorn (♒) [♒? — worn], and so in order, just as they are seen placed in the said figure. But the first point above the horizontal line will pertain to Taurus (♉), the following to Gemini (♊) and Cancer (♋) [♋? — worn], and so on, just as they are drawn in the same said figure.

Furthermore, the ascending signs, produced beyond the horizontal line, give the same signs in the lower dial, provided that the portion cut off by the horizontal line is inverted — as has already been said more than once above.

PROBLEMA 10. PROPOS. 10.

To describe an Italian dial declining from the vertical circle.

[Margin: The drawing of an Italian dial declining from the south toward the east.]

Let the circle MaNb, described from L, be cut — by the means we set out in the scholium of Prop. 1, Book 1 — into the diurnal arc of the tropic of Cancer (♋), aNb, and the diurnal arc of the tropic of Capricorn (♒), dNe, so that the lines ab and de cut at right angles the line MN, which is the common section of the Equator and the Meridian. Then, that circle being twice divided into 24 equal parts — first beginning from point a, then from point d — so that a is hour 24 of the tropic of Cancer (♋) and the next point toward N is hour 23, and so on; while d is hour 24 of the tropic of Capricorn (♒) and the next point toward N is hour 23, and so on: we shall find, through these division-points in each tropic, the points of the hours from sunset, and we shall complete the Italian dial in no other way than we constructed the horizontal Italian dial in Prop. 10 of the preceding book. For the very same demonstration can be adapted here.

For example: from f, the point of hour 16 of the tropic of Cancer (♋), a straight line drawn through the centre L cuts the equinoctial line at point g; and if from C, the centre of the dial, a straight line is sent out through g, the tropic of Cancer (♋) will be cut at h, the point of hour 16 from sunset. So too the line drawn out from

m, the point of hour 21 of the tropic of Capricorn (♑), through L, cuts the equinoctial line at point n; and through n a line drawn from C cuts the tropic of Capricorn (♑) at p, the point of hour 21. In like manner the straight line from q, the point of hour 14 of the tropic of Capricorn (♑) — which lies in the nocturnal arc dMe — drawn through L, cuts the equinoctial line at point r; and through r a line drawn from C cuts the tropic of Capricorn (♑), in the nocturnal arc, at t, the point of hour 14. By this method, however, only the points of those hours will be found which are contained in the semicircle ANB — whose diameter AB is parallel to the equinoctial line — because the lines drawn from those points through L pass first through the centre L before they cut the equinoctial line, which does not happen with the others. So we shall find the points of the other hours as we said of the horizontal dial in Prop. 10 of the preceding book. Thus, if we wish to find

Diagram. The construction of the Italian dial declining from the vertical. A large circle (the tropic/hour circle MaNb described from L) sits at the lower left, its lower half drawn as a bold arc and divided by many radial and chord lines; a horizontal line (the equinoctial line) runs across the whole figure to the right, and a vertical meridian line descends through it. From the centre of the dial (C, upper middle) and from the point L a dense sheaf of straight lines — some solid, some dotted, some dashed — radiates outward to hour-points, carrying Roman-numeral hour labels around the arc (I at the upper right; II, III, IIII, V, VI, VII, VIII around the lower left of the arc; IX, XII, and higher numerals up the left) and Arabic hour-numbers up the right side (...20, 21, 22, 24) and toward the top (...4, 12, 20 with Zodiac glyphs). The interior of the arc carries small Arabic numbers (8, 9, 10, 12, 13, 14, 15, 16, ...).

- Centre / principal points: L (centre of the tropic circle); C (centre of the dial, upper middle, on the meridian); K (on the equinoctial line, near the top of the arc)
- On/near the arc: A (left), B (lower, within the arc), E (lower centre, within the arc), D (interior, left of centre); m, n (interior); q, e, f (at the foot of the arc)
- R (repeated three times down the left margin, at the ends of dashed lines); F (upper left); P (twice, on the right, upper and lower)
- Roman-numeral hour-lines: I (upper right); IX, XII, XIII, XIIIII (up the left); II, III, IIII, V, VI, VII, VIII (around the lower left of the arc)
- Arabic hour-numbers on the right: 20, 21, 22, 24 (and 2, 3); at the top: 4, 12, 20 (beside Zodiac glyphs)

- Interior Arabic numbers within the arc: 8, 9, 10, 12, 13, 14, 15, 16 [several worn]

the point of hour 23 in the tropic of Capricorn (φ) — to which corresponds, in the circle MaNb, the point D, which is not contained in the semicircle ANB — we shall draw, from the opposite point E (of hour 11 of the tropic of Capricorn (φ), comprehended in the nocturnal arc of Capricorn by L and D), a straight line, which will cut the equinoctial line at point R, very far off (which indeed is not contained in our example). Through R, if a line is drawn from C, the tropic of Capricorn (φ) will be cut, in the nocturnal arc, at point F, hour 11; while the line RC, produced beyond the centre C, will cut the same tropic of Capricorn (φ) at point P, hour 23; and so of the rest.

For a northern dial, to avoid confusion, we give no example — both because it is easily derived from the southern one, as we have said of other dials, and also because it can be described in the same way, provided you remember that in a northern dial the diurnal arc of the tropic of Cancer (φ) is dMe, and that of Capricorn (φ) is aMb, as we said of the vertical dial. For example: since the portion cut off by the horizontal line, if it is turned toward us so that the centre C lies below the horizontal line, presents to us a northern dial declining from the north toward the east by 30 degrees — as we taught above in the scholium of Prop. 1 of this book — if from E, the point of hour 13 of the tropic of Cancer (φ), a straight line is drawn through L, the equinoctial line will be cut

the equinoctial line at point R, through which a straight line issuing from C will cut the tropic of Cancer [φ] at F, the point of hour 13, in the northern dial declining from north toward east by 30 degrees; and so on.

But should some division point happen to fall exactly on A or B, then a straight line drawn through C parallel to the equinoctial line will cut the tropic of Capricorn in two points. Of these, the one lying to the right of CL will belong to the hour that falls at A, while the one to the left of CL will refer to the hour that falls at B — as is clear from what we showed in Proposition 1 of this book concerning the line Cd, drawn through C parallel to the equinoctial line. For there we showed that Cd represents the hour from whose point on the circle described from L a straight line drawn through L is equidistant from the equinoctial line.

[Margin: How the same Italian dial is to be described by other methods.]

Now as for the other methods — those carried out either by means of the diurnal and nocturnal arcs, or by the mutual intersections of the hours from noon or from midnight and from sunrise or sunset on the horizontal line, or on the line of

hour 12 from sunrise or sunset, and so on — there is no difficulty at all, provided one attentively consults the tables of Propositions 19, 20, and 23 of Book 1, which we have already cited many times. For (to give you an example) hour 11 from sunset is drawn through hour 6 from midnight on the diurnal arc of 14 hours, and through hour 5 from midnight on the nocturnal arc of 12 hours, and also through hour 4 from midnight on the nocturnal arc of 14 hours. Likewise hour 20 from sunset passes through hour 1 from noon on the diurnal arc of 10 hours, and through hour 2 from noon on the diurnal arc of 12 hours.

Hour 21 from sunset, which in our example has only a single point — namely at hour 2 from noon of the diurnal arc of 10 hours, through which it passes — will be obtained if hour 21 from sunrise is drawn through hour 4 from midnight on the nocturnal arc of 14 hours and extended through hour 2 from noon on the diurnal arc of 10 hours. So too, for hour 22 from sunset, one must draw hour 22 from sunrise through hour 5 from midnight on the nocturnal arc of 14 hours and through hour 4 from midnight on the nocturnal arc of 12 hours; for this line, extended beyond the parallel that is the greatest of all those always hidden — the one drawn through point K — will give hour 22 from sunset. Finally, by the same reasoning, for hour 23 from sunset one must draw hour 23 from sunrise through hour 6 from midnight on the nocturnal arc of 14 hours and through hour 5 from midnight on the nocturnal arc of 12 hours; and so on.

As for the line of hour 12 from sunrise or sunset — should you happen, in laying out the Italian dial, to wish to use the second table of Proposition 20 of Book 1, which pertains to hour 12 from sunrise or sunset — it must be drawn through the point where the equinoctial line, the horizontal line, and the line of hour 6 from noon or midnight mutually intersect, and through the point on the line of hour 12 from noon at which the diurnal arc of 0 hours — that is, the greatest of all the always-hidden parallels — passes, as is clear from tables 5, 6, and 8 of Proposition 33 of Book 1. We shall find this point if, from the figure of rays in Proposition 2 of this book, we transfer the line of hour 12 of midnight (or after noon) — which is drawn from C to the right of the line CA, taken between C and the ray of 0 hours — from C, the center of the dial, onto the meridian line above the center C. For the portion of the meridian line above the center C pertains to midnight, so that it indicates hour 12 from noon. In our example such a point can be found only with great difficulty, as is evident. We have therefore laid out the Italian dial declining from the vertical circle. Which was to be done.

PROBLEMA 11. PROPOS. 11.

To lay out a Babylonian dial declining from the vertical circle.

[Margin: The description of the Babylonian dial declining from the vertical.]

The Babylonian dial is described just like the Italian one, with this one exception: in the former method the division of the circle into 24 equal parts must begin from the points b and e, which lie to the east — as is clear if the dial and the circle are conceived as set in their proper position — so that the points b and e signify hour 24, and the points next to them toward N signify hour 1 from sunrise, and so on. In the other methods the matter is entirely easy, using the tables of Propositions 19, 20, and 33 of Book 1, which we have frequently mentioned so far. The figure of the preceding proposition makes this plain: in it, the hours marked merely with points are counted from noon or midnight; those bearing the ordinary number-characters are counted from sunset; and finally those distinguished by the ancient number-characters are counted from sunrise. We have therefore laid out the Babylonian dial declining from the vertical circle. Which was to be done.

SCHOLIVM.

If the hours from sunrise and sunset are extended beyond the horizontal line in a dial that declines from the south, we shall obtain the same hours on the opposite face of the plane, which declines from the north, provided the parts are interchanged, as we said above.

[Margin: In what way, from an Italian or Babylonian dial declining from south toward east, another may be made declining from north toward east, and from south toward west, and from north toward west.]

Moreover, just as we wrote at the end of the scholium of Proposition 1 of this book — that four different astronomical dials, which nevertheless keep the same amount of declination, can be described by one and the same operation — the same must be said of the Italian and Babylonian dials. For if, for example, we describe an Italian or Babylonian dial declining from south toward east by 30 degrees, of the kind we have constructed so far, we shall by the same labor obtain an Italian or Babylonian dial declining 30 degrees from north toward east — namely that nocturnal one which the horizontal line cuts off — provided we interchange its upper part into the lower. But the hour-numbers must then be changed into their complements up to 24, so that from hour 11 from sunset in the nocturnal portion there arises hour 13 from sunset in the dial declining from

north toward east; and from 23 from sunrise there arises 1 from sunrise; and so on.

But if we invert the same dial declining from south toward east by any number of degrees so that its right part becomes the left, and vice versa, without changing its upper part into the lower, we shall obtain a dial declining by the same number of degrees from south toward west. Here too the hour-numbers must be changed into their complements up to 24, and the hours previously counted from sunrise are now to be counted from sunset, and those previously from sunset now from sunrise — so that from hour 1 from sunrise there arises hour 23 from sunset, and from hour 18 from sunset there arises 6 from sunrise; and so on. Finally, if we invert the nocturnal portion of the dial declining from south toward east by 30 degrees so that the right part becomes the left, and vice versa, and further so that the part which after this change is uppermost becomes the lowermost, and vice versa, there will be described a dial declining from north toward west by the same number of degrees. Here the same hour-numbers are retained, but one must still consider which are to be counted from sunrise and which from sunset, as we said in the scholium of Proposition 23 of the previous book concerning the vertical dial. In like manner, if an Italian or Babylonian dial is described declining from south toward west by 30 degrees,

...by 30 degrees, we shall construct from it a dial declining from north toward west by 30 degrees, and from south toward east by the same number of degrees, and also from north toward east — provided the aforesaid interchanges of parts are made and the hour-numbers changed, as we have just written.

PROBLEMA 12. PROPOS. 12.

To construct an ancient dial declining from the vertical.

[Margin: The construction of the ancient dial declining from the vertical.]

Let both the diurnal arc of the parallel of Cancer, aNb, and that of the parallel of Capricorn, dNe, be divided into 12 equal parts in the circle MaNb, so that the points nearest to and following the points b and e toward N belong to the first unequal (seasonal) hour, the following ones to the second, and so on. For from these division points we shall find, on each tropic, the points of the unequal hours, as we taught for the vertical dial in Proposition 24 of the previous book. For example, the straight line from f — the point of the 2nd unequal hour of the tropic of Cancer — drawn through the center L cuts the equinoctial line at point g, through which a straight line issuing from C cuts the tropic of Cancer at h, the

point of the 2nd unequal hour. In the same way the straight line from m — the point of the 9th unequal hour of the tropic of Capricorn — drawn through L

Diagram. A large sundial-construction figure occupying most of the page. A vertical meridian line descends through the whole figure from the dial-center C (top center). A horizontal equinoctial line crosses the whole width to the right, running out to point S at the far right. In the lower-center sits a large circle (the projection circle MaNb) with center L; its lower half carries a dense sheaf of radial hour-lines and dotted/dashed construction lines. From the dial-center C a fan of straight lines (solid, dashed, and dotted) radiates down and to the right, carrying the Roman-numeral hour-labels XI, X, IX, VIII down the right edge, and I–VII within and around the circle. Down the left side, at the ends of far-reaching lines, the label R recurs (three times) marking the very remote equinoctial-cut points. This figure is extremely dense; many small point-labels are overlapped by construction lines and are not securely legible.

- C (top center, on the meridian line): dial-center
- L (center of the circle): center of the projection circle MaNb
- N (lower-left of circle): division-origin point of the arcs
- A (left of circle): endpoint of semicircle ANB
- B (bottom of circle): endpoint of semicircle ANB
- M (right-center, inside circle): point on the circle
- D (bottom of circle, right of B): point opposite hour 11
- E, e, f, F (clustered at the foot of the circle): opposite-hour and hour points
- a, b (lowercase, on the circle): division points of the diurnal arc of Cancer
- d (left of circle, lowercase): division point (arc dNe)
- q, m (left of circle, lowercase): construction points
- R (three occurrences down the left margin, at the ends of dashed lines): remote equinoctial-cut points [several worn]
- P (upper, near C; and again lower-right): hour points K/P/Q group
- Q (upper-left area): [?]
- K, k (near center, on line CH): shadow / hour-11 point
- H, g, h, n, p (small interior construction points near center) [worn]
- S (far right, on the equinoctial line): remote intersection
- Roman hour-numerals on the right radiating lines: XI, X, IX, VIII
- Roman hour-numerals within and around the circle: I, II, III, IIII, V, VI, VII [several worn]

- Small Arabic numbers near the top of the circle and above C: 1, 2, 3 (and 5, 6, 8, 9) [worn]

(: more than two labels are flagged [?] — the interior small point-labels (Q, H, g, h, n, p, and several of the R/P/K cluster and the small Arabic numbers) are overlapped by construction lines and not securely legible.)

cuts the equinoctial line at point n, and a straight line drawn from C through n cuts the tropic of Capricorn at p, the point of the 9th unequal hour; and so of the rest.

For the 11th unequal hour of the tropic of Capricorn — since it is not contained in the semicircle ANB, whose points alone are found on the tropics, as was said above — we shall take the point D opposite to it, which falls on the 11th unequal hour of the nocturnal arc of the tropic of Cancer, as we shall presently demonstrate. From D, if we draw a straight line through L, it will cut the equinoctial line at a certain point, namely R (which, because it is very remote, is not contained in our dial). Through R, a straight line issuing from C will give, on the nocturnal arc of the tropic of Cancer, the point of the 11th unequal hour corresponding to point D; and the same straight line, thrown out beyond the center C, will present on the diurnal arc of Capricorn another point for the 11th diurnal unequal hour of the tropic of Capricorn, as is clear from what has been demonstrated.

By the same reasoning, for the 11th, 10th, and 9th unequal hours of the tropic of Cancer — since these too fall outside the aforesaid semicircle ANB — we shall take the points opposite to them, E, F, G, which fall on the unequal hours 11, 10, and 9 of the nocturnal arc of the tropic of Capricorn, as we shall presently show; and from these we shall draw straight lines through L, so as to find on the nocturnal arc of Capricorn the points K, P, Q corresponding to the points E, F, G of the nocturnal arc of the tropic of Capricorn. And should some hour fall exactly on point A or B, then a straight line drawn through C equidistant from the equinoctial line will indicate two points on the tropic of Capricorn. Of these, the one that lies to the right of CL will belong to the hour falling on point A, while the other, placed to the left of CL, will belong to the hour falling on point B — as is clear from what we showed in Proposition 1 of this book concerning the line Cd, which is drawn through C parallel to the equinoctial line. For it represents the hour from whose point on the circle described from L a straight line drawn through L is equidistant from the equinoctial line, as we showed at the end of the demonstration of the former method of constructing the declining dial, in Proposition 1 of this book.

If, then, the corresponding points on the tropics are joined by straight lines, the ancient dial will be described; and all the lines of the unequal hours will pass through the hours from noon or midnight on the equinoctial line, as is gathered from tables 13 and 14 of Proposition 33 of Book 1. But since the points of hours 11, 10, and 9 of the arc of Cancer have no corresponding points on the tropic of Capricorn, we shall join them with the points of the nocturnal arc of Capricorn found by means of the points E, F, G, which are opposite to those same hours 11, 10, and 9 of the diurnal arc of Cancer, and fall on the unequal hours 11, 10, and 9 of the nocturnal arc of Capricorn.

[Margin: Demonstration that the unequal diurnal hours in the tropic of Cancer are opposed diametrically to the same unequal nocturnal hours in the tropic of Capricorn, and that in the dial the diurnal hours of the tropic of Capricorn must be drawn through the same nocturnal hours of the same tropic.]

That this is so — and that the lines of the diurnal hours of the tropic of Capricorn must be drawn through the same nocturnal hours of the same tropic on the dial — we shall demonstrate as follows. For example: the straight line from E — the point that is opposite to hour 11 of Cancer and contained in the nocturnal arc of Capricorn — drawn through L, cuts the equinoctial line at H; and the line CH cuts the nocturnal arc of Capricorn at K. Now the greatest circle through the poles of the world, drawn through hour 11 unequal of the tropic of Cancer and also through the point E, will cut the plane of the dial along the line CH, and therefore will cut the tropic of Capricorn at K, as is clear from what was demonstrated in Proposition 10 of the previous book.

But since that greatest circle necessarily cuts off, in the circle MaNb, the arc eE of the 11th unequal hour of the nocturnal arc of the tropic of Capricorn, dMe — for since the arcs aM and eN are equal, being halves of the equal arcs aMb and eNd, if the common arc eM is added, the arc aMe becomes equal to the semicircle MeN — therefore the straight line from a to e, when produced, will pass through the center L; and so at the center L it will, together with the line EL drawn as far as hour 11 of the tropic of Cancer, [a] form equal vertical angles, [b] and therefore the arcs a–11 and eE will be equal.

[Margin: a — 15. primi (Elem. I.15).] [Margin: b — 26. tertij (Elem. III.26).]

Therefore, just as a–11 is a twelfth part of the diurnal arc of the tropic of Cancer, aNb, so eE will be a twelfth part of the nocturnal arc of the tropic of Capricorn, eMd — since the arcs aNb and eMd are equal; and consequently eE will be the arc of the 11th unequal hour on the nocturnal arc of Capricorn. The result is that the

line passes through the 11th nocturnal hour of the tropic of Capricorn. For since the said greatest circle and the meridian cut off similar arcs from the equator and from the tropic of Capricorn — by Proposition 10 of Book 2 of Theodosius — and since the arc ME, which is similar to the arc of the equator lying between the said greatest circle and the meridian, contains five-sixths of the arc Me, which is similar to the semi-nocturnal arc of the tropic of Capricorn, then the arc of the tropic of Capricorn lying between the same greatest circle and the meridian will likewise contain five-sixths of its own semi-nocturnal arc; and therefore the said greatest circle will pass through the 11th unequal nocturnal hour of the tropic of Capricorn.

And since the 11th unequal hour on the diurnal arc of the tropic of Cancer and the 11th unequal hour on the nocturnal arc of the tropic of Capricorn intercept a semicircle from that greatest circle drawn through both hours — for the arc of it lying above the horizon between the parallel of Cancer and the equator is equal to the arc of the same lying below the horizon between the parallel of Capricorn and the equator, since each, by Proposition 10 of Book 2 of Theodosius, is equal [b] to the arc of greatest declination; and if the common arc of the same, placed between the equator and the tropic of Capricorn and passing through the antarctic pole, is added, there will result equal arcs of the same, one placed between the tropic of Cancer and the tropic of Capricorn, the other between the equator [and the tropic], passing through the antarctic pole.

[Margin: The hour-circle of any unequal hour in the tropic of Cancer is drawn both through the diurnal hour in the tropic of Capricorn and through the same nocturnal hour in the same tropic.]

Since, then, the equator cuts off a semicircle from it, by Proposition 11 of Book 1 of Theodosius, the other arc too will be a semicircle. Hence it necessarily follows that another greatest circle — one of the circles of the unequal hours, which does not pass through the poles of the world, and which cuts the said greatest circle (the one through the poles of the world and through the diurnal hour 11 of the tropic of Cancer) at hour 11 diurnal of the tropic of Cancer — cuts that same circle at the opposite point, that is, at hour 11 nocturnal of the tropic of Capricorn; for it bisects it (by Proposition 11 of Book 1 of Theodosius), and it has been shown that the said greatest circle is bisected at hour 11 diurnal of the tropic of Cancer and hour 11 nocturnal of the tropic of Capricorn.

For this reason the greatest hour-circle of the 11th unequal diurnal hour of the tropic of Cancer will pass, in the plane of the dial, through the point k — onto which, indeed, the shadow of the style would fall (by Proposition 11 of Book 1),

the Sun being at hour 11 nocturnal of the tropic of Capricorn, were the earth not to obstruct the Sun's rays. And this 11th unequal nocturnal hour of the tropic of Capricorn lies on the same greatest hour-circle on which the 11th unequal diurnal hour of the tropic of Cancer lies, as we have shown. Therefore the common section of the plane of the dial and of the circle of the 11th unequal hour will pass through the point K; and consequently the straight line joining the point K with the point of hour 11 of the diurnal arc of Capricorn will give the 11th unequal hour — so that that straight line is the common section of the plane of the dial and of the circle of the 11th unequal hour. For this circle passes through the 11th unequal hour both in the tropic of Cancer and in the tropic of Capricorn, as we showed in the scholium of Proposition 10 of Book 1, and likewise, as we have just demonstrated, is drawn through the 11th unequal nocturnal hour in the tropic of Capricorn.

[Margin: The description of the same ancient dial from the tables of the diurnal arc of 6 hours and the nocturnal arc of 18 hours.]

By the same reasoning we shall demonstrate that the straight line joining the point P with the point of hour 10 of the diurnal arc of the tropic of Capricorn gives the 10th unequal hour; and so of the rest. All these things will be more easily grasped if a material sphere is employed. This same demonstration holds in all ancient dials, since we shall always show that any unequal diurnal hour whatever in the tropic of Cancer is diametrically opposed to the same unequal nocturnal hour in the tropic of Capricorn, and vice versa; and therefore that the circle of any unequal hour of the tropic of Cancer is drawn both through the diurnal hour of the tropic of Capricorn and through the same nocturnal hour of the same tropic. In the earlier dials, however, we wrote nothing on this matter, since all the unequal hours can most conveniently be drawn through the hours from noon or midnight on the equinoctial line — which cannot be done in a declining dial without great difficulty, because some of the hours meet the equinoctial line too far off.

But if the diurnal arc of 6 hours — which above the horizontal line is a nocturnal arc of 18 hours — is described, the same ancient dial will be described from tables 9, 10, 13, and 14 of Proposition 33 of Book 1, as appears in the figure. We have therefore constructed the ancient dial declining from the vertical. Which was to be done.

SCHOLIVM.

The lines of the unequal hours of a southern dial, extended beyond the horizontal line, will produce a northern ancient dial, if the interchange of parts is made, as was said above.

ON DIALS THAT DECLINE FROM THE HORIZON.

PROBLEMA 13. PROPOS. 13.

To construct an astronomical dial declining from the horizon — that is, to draw the lines of the hours from meri

[Rubric of PROPOS. 13, completed from the previous page:] ...from noon, or from midnight, on a plane which is parallel to some greatest circle that declines from the horizon but stands at right angles to the vertical circle.

Just as, in the sphere, all the vertical circles — whose common section is the axis of the horizon — stand to the horizon, so all the greatest circles whose common section is the axis of the vertical circle properly so called stand to that circle. Of this sort are all the circles of position [circuli positionum], among whose number is the horizon itself, [standing] to the vertical circle properly so called. Thus, if the vertical circle properly so called is conceived to be some horizon, the horizon itself becomes, as it were, the vertical circle properly so called, and the other circles of position that decline from the horizon become like vertical circles that decline — as is evident from a material sphere. From this it follows that the construction of dials declining from the horizon is the same as that of dials declining from the vertical (which we treated above), with the exceptions that follow.

[Margin: The circles of position declining from the horizon stand to the vertical circle properly so called just as the vertical circles declining from the vertical circle properly so called stand to the horizon.]

In place of the declination from the vertical circle, in dials declining from the horizon one must take the declination from the horizon — that is, the inclination to the horizon. Accordingly, the angle DEF of the declination from the horizon (or rather of the inclination to the horizon — how this is to be found by careful method we showed in Proposition 23 of Book 1) is to be set up below the straight line AB, toward point A, if the dial is an upper one and faces west; but toward B, if it is an upper one and faces east. If, however, the dial is a lower one and looks

toward the west, the same angle must be set up above the line AB, toward B; but if it is a lower one and faces east, toward A.

[Margin: Construction of the astronomical dial declining from the horizon.]

Again, at point β one must set up the angle $E\beta C$ — not of the height of the pole above the horizon, but of the height of the pole above the vertical circle, which is now taken as though it were some horizon; that is, an angle equal to the complement of the latitude of the place, or of the height of the pole above the horizon. (For in every region the height of the pole above the horizon — namely, in the analemma of Proposition 1 of Book 1, the arc DF — together with the height of the pole above the vertical, that is, with the arc AF, makes up a quadrant; so that the complement of the height of the pole above the horizon of any region whatever is the height of the pole above the vertical properly so called of that same region.) This angle is set up above the line AB if the dial is an upper one, but below the same AB if it is a lower one.

[Margin: The complement of the height of the pole above the horizon of any region is equal to the height of the pole above the vertical circle properly so called of that same region.]

Let everything else be carried out as we said in Proposition 1 of this book concerning dials declining from the vertical. So that to construct a dial declining from the horizon is nothing other than this: for a latitude of the place which, together with the latitude of the place for which the dial is being built, makes up 90 degrees, to lay out a dial declining from the vertical by as many degrees as the inclination of the proposed plane to the horizon contains.

[Margin: To construct a dial declining from the horizon is nothing other than to lay out a dial declining from the vertical for that latitude of the place which, together with the proposed latitude for which the dial is built, makes up 90 degrees.]

For in that region the vertical circle properly so called performs the office of the horizon, and the horizon takes the place of the vertical circle — yet in such a way that what in that region declines from south to west is, in this region for which the dial is being made, an upper dial facing west; and what there inclines from south to east is here an upper dial looking toward sunrise, as is plain if the matter is considered more carefully.

[Margin: Demonstration of the construction of the dial declining from the horizon.]

But in that region the antarctic pole must be understood to be raised above the vertical circle of our region — taken as a horizon — namely above its southern face, on which the vertical dial is chiefly described in any region, and likewise the horizontal dial in that region in which the height of its pole, together with the height of the pole of the former region, constitutes a quadrant. Hence, if there a dial declining from the vertical, from south to west, is to be constructed, the line of declination EF must be drawn in the opposite way from what we taught in Proposition 1 of this book — namely toward A, because there noon is produced when the Sun is more northerly than the vertical circle properly so called, and so at noon the shadow is cast toward the south, as is clear from a material sphere; about which matter we shall write more in Book 4, Proposition 11. The following figure sets forth the construction of this dial declining from the horizon. For the demonstration here will be the same as in Proposition 1 of this book, with a few changes. We suppose the plane of the dial to decline from the horizon toward the west by 30 degrees, so that this dial, built for latitude 42 degrees, is the same as a vertical dial declining from south to west built for latitude 48 degrees.

For if in the plane of the dial a straight line CD is understood to be parallel to the horizon, so that it is the common section of the plane of the dial declining from the horizon and of the meridian circle, then the straight line AB will be the common section of the plane of that same declining dial and of the plane of the vertical dial. For since both the meridian and the plane of the declining dial are at right angles to the plane of the vertical dial (by Proposition 15 of Book 1 of Theodosius [Theodosius, *Sphaerica* I.15]) — and since both the meridian and the greatest circle to which the plane of the declining dial is parallel are drawn through the poles of the plane of the vertical dial, namely through the common sections of the meridian and the horizon — (a) their common section CD will also be at right angles to that same plane of the vertical dial, and so, by Definition 3 of Book 11 of Euclid, will also be perpendicular to the common section of the plane of the declining dial and the plane of the vertical dial.

[Margin: (a) 19. unde. — (b) 19. unde. (keyed reference marks for steps (a) and (b) below)]

If, therefore, the plane of the vertical dial is supposed to be drawn through point E, then the straight line AB, which has been drawn perpendicular to CD, is the common section of the plane of the declining dial and the plane of the vertical dial. Wherefore, if the plane drawn through the straight lines AB, EF, ED is understood to be rotated about the line AB until it becomes parallel to the vertical circle and is thus joined with the plane of the vertical dial, it comes about that,

since DEF is the angle of the inclination of the plane to the horizon, the remaining angle AEF is, taken from the right angle, the angle of the complement of that inclination — such, namely, as the meridian makes with the declining plane. Therefore the straight line EF will be the common section of the meridian and of the vertical dial.

Next, since both the plane of the meridian and the plane of the declining dial are at right angles to the plane of the vertical dial, as has been said, (b) their common section CE will be at right angles to that same plane of the vertical dial; and consequently — since EF has been shown to be its meridian line, or its common section with the meridian — it will, by Definition 3 of Book 11 of Euclid, be perpendicular to the straight line EF lying in that plane. Therefore, if the triangle βEC is understood to be rotated about CE until it is joined with the plane of the meridian drawn through the straight lines CE and EF, the right angle βEC will coincide with the right angle CEF, and the point β with the point F, because of the equality of the straight lines $E\beta$ and EF. Wherefore the straight line βC — forming, in that position, with the meridian line EF of the vertical dial the angle $E\beta C$, the angle of the height of the pole above the vertical circle toward the parts of the hidden pole — will be the axis of the world [axis mundi], meeting the plane of the declining dial at the point C; and therefore, by the corollary of Proposition 21 of Book 1, C will be the center of the declining dial. For the axis makes, with the meridian line of the vertical dial that faces south, the angle of the height of the pole above the vertical circle toward the parts of the hidden pole, as is evident from Proposition 13 of the previous book, especially from the portion of the analemma described there.

Again, if the triangle EFG is imagined to be rotated about EG until it is joined with the plane of the vertical dial, we shall show, as in Proposition 1 of this book, that the straight line CG is the line of the style (that is, of the gnomon [stylus, seu index]), that CH is the axis of the world, and that the angle GCH is the angle of the height of the pole above the declining plane. The rest will be demonstrated as

as in the said Proposition 1 of this book. The order of the hours will be the same in the upper dial as in the lower — the same as in the horizontal dial — as is plain, if the circle described from L, drawn together with the dial, is understood to be set in its proper position; so that in the upper dial the meridian line indicates the 12th hour of noon, but in the lower the 12th hour of midnight. Those lines which, in either dial, precede or follow it toward the eastern parts show the hours after noon, while those which recede from it toward the western parts indicate the hours after mid-

[Margin: The order of the hours in a dial declining from the horizon is the same as in a horizontal dial.]

Diagram. The large construction figure for the astronomical dial declining from the horizon. At the center is a radiant sun-face labeled **C**, the center of the declining dial, from which the hour-lines and the style radiate as straight rays. A long straight horizontal line **AB** crosses the whole figure — the line AB, common section of the declining and vertical dial planes — with **A** at the left end (a small α beside it) and **B** at the right end. A large circle (the hour-circle "described from L") occupies the left of the figure, carrying the hour-numbers around its rim. A semicircular inset at the lower right is the portion of the analemma ABC used for the pole-height / inclination construction.

- C (center): center of the declining dial; sun-face with radiating hour-lines
- A (left end of horizontal line): terminus of line AB; small α adjacent
- B (right end of horizontal line): terminus of line AB
- V (top, above the left circle): construction point
- T (left, on the circle): construction point
- S (left, lower on the circle): construction point
- Z (right of C): construction point on a dotted vertical
- Y (below-right of C): construction point
- F (below center): foot-point F (meridian/analemma foot)
- L (interior, below center): pole of the hour-circle
- X, H, R [?] (left, near line AB): construction points [R/K uncertain]
- E, M (on and just below line AB, center): construction points
- β (right portion, near line AB): point β (angle $E\beta C$)
- N, P (lower left): construction points
- I (bottom center): construction point (hour 1?)
- a, b [?] (top, where dotted lines meet): construction points [labels faint]
- Hour-numbers around the circle: 8, 7, 6, 5, 4, 3, 2 (left/lower arc) and 8, 9, 10, 11, 12 (right/lower arc); "12" and "11" near the bottom [several numeral placements faint]

Inset semicircle (lower right — portion of the analemma ABC):

- A (apex of the semicircular arc)
- B (left end of the diameter)
- C (right end of the diameter)
- D (center of the diameter)

- E (point on the arc, right of the apex)
- F (left, above B)
- H, G, I (points on the base line below the diameter; G below D)

[Several small point-labels and hour-numerals in this dense woodcut could not be resolved with certainty]

-midnight. ...Which parts of the dial, moreover, are to be called eastern or western will be easily perceived — if they are compared with the meridian line — provided the dial is understood to be set in its proper position. Yet this distinction must be noted. In the upper dial, the line nearest to the meridian line itself, toward the eastern parts, indicates the 1st hour after noon, and the next belongs to the 2nd, and so on; likewise the one nearest toward the western parts shows the 11th hour after midnight, and the next gives the 10th hour, and so on. But in the lower dial, the nearest line toward the eastern parts shows the 11th hour after noon, and the next signifies the 10th, and so on; likewise the nearest line toward the western parts belongs to the 1st hour after midnight, and the next to the 2nd, and so on.

As for the other method of drawing the hour-lines, this alone need be noted: that the straight lines going out from F and cutting the straight line AB must make with EF the same angles that the hour-lines, in the vertical dial described in Proposition 13 of the previous book, make with the meridian line. For, as we have shown, the straight line EF is here the meridian line of the vertical dial, not the horizontal line as in the dial declining from the vertical; and the point F is the center of that same dial, since the axis of the world falls upon it, as we have proved.

[Margin: Another way of drawing the hour-lines in a dial declining from the horizon, by means of the vertical dial.]

[Margin: Construction of the same dial declining from the horizon, from the height of the pole above the declining plane and from the inclination of the meridian of that same declining plane to the meridian of the horizon.]

For the second method of constructing the declining dial, we find, by Proposition 29 of Book 1, the height of the pole above the proposed plane to be 35 degrees 25 minutes; and by Proposition 30 of the same book, the inclination of the proper meridian of the proposed plane to the meridian of the horizon to be 37 degrees 50 minutes. From this we shall easily make the dial declining from the horizon by means of what we set forth in Proposition 1 of this book — the portion of the analemma ABC having first been constructed for the height of the pole found, and

so on, as may be seen near the dial of this proposition, in the semicircle (or portion of the analemma) ABC.

[Margin: Toward which part, in the circle described from L, the inclination of the meridian of the declining plane itself to the meridian of the horizon is to be reckoned.]

But in order to know toward which part — from point N, in the circle described from L — the inclination of the proper meridian of the declining plane to the meridian of the horizon ought to be reckoned, careful attention must be paid to what we shall presently prescribe. When the plane belongs to the zenith and looks toward the west, as our plane does, the said inclination must be reckoned from N toward the left — that is, toward the eastern parts, toward A — as far as O; because then the meridian of the horizon drawn through O is more easterly than the proper meridian of the declining plane drawn through N, inasmuch as the pole of the declining plane, through which its proper meridian is drawn, is set in the upper and western quadrant of the vertical circle, as is plain if the proper position of the declining plane is rightly understood.

[Margin: All circles of position have their poles on the vertical circle properly so called.]

For all circles of position, to one of which the dial declining from the horizon is parallel, have their poles on the vertical circle. For since all of them pass through the poles of the vertical circle — that is, through the common sections of the meridian and the horizon — the vertical circle will in turn pass through their poles, as we demonstrated in the scholium of Proposition 15 of Book 1 of Theodosius. But if the plane belongs to the zenith yet inclines toward the east, the same

inclination is to be reckoned from N toward the right, that is, toward the western parts, toward B; because then the meridian of the horizon is more westerly than the proper meridian of the declining plane.

From these things anyone will easily understand toward which part the said inclination is to be reckoned from N in the lower dial, if he considers carefully whether, in the lower hemisphere, the meridian of the horizon is more easterly than the proper meridian of the declining plane, or more westerly, and so on, as we said in Proposition 1 of this book. We have refrained, however, from giving rules on this matter, lest by a multitude of precepts we should breed confusion in the reader — especially since it is better to derive the lower dial from the upper, as we shall teach in the following scholium, than to lay it out by its own art.

Diagram. The same construction figure is printed again on this page (the astronomical dial declining from the horizon). It carries the identical labelling as the figure on p. 293: central sun-face **C** with radiating hour-lines; horizontal line **AB** (A with adjacent α at left, B at right); the large hour-circle "described from L" at the left with hour-numbers 8, 7, 6, 5, 4, 3, 2 and 8, 9, 10, 11, 12 around its rim; interior points **V, T, S, X, H, R** [?], **E, M, F, L, Z, Y, β , N, P, I** and the faint top marks **a, b** [?]; and the analemma-portion semicircle at the lower right with apex **A**, diameter **B–D–C**, arc-point **E**, left point **F**, and base-line points **H, G, I**. As on p. 293, several small labels and numerals are not fully resolvable

[Margin: Another construction of the same dial declining from the horizon.]

Finally, for the third and last method, we find, by Proposition 30 of Book 1, the arc of the declining plane between the meridian of the horizon and the proper meridian of the declining plane to be 24 degrees 14 minutes. This arc must be reckoned from the straight line CD toward A, if the dial is an upper one and inclines to the west; but toward B, if it is an upper one and looks toward sunrise. For there the proper meridian of the declining plane is more westerly than the meridian of the horizon in the upper hemisphere, and therefore CP, its common section with the plane of the dial, must lie more to the east in the dial than the meridian line CD. Here, however, the proper meridian of the plane is more easterly than the meridian of the horizon, and consequently CP, its common section with the plane of the dial, must lie more to the west in the dial than the meridian line CD. There is no doubt, moreover, that in the upper dial the parts running from the straight line CD toward A are eastern, but the parts toward B are western.

From these things you will easily understand what you must do in the lower dial, in which the center C inclines toward the north and the arc DP toward the south — if you consider attentively, by what has just been said, whether the proper meridian of the declining plane in the lower dial is, in the lower hemisphere, more easterly than the meridian of the horizon, or rather more westerly. For according as it is the more easterly or the more westerly, so the said arc must be reckoned from the straight line CD in the dial toward the contrary parts, namely toward the west or toward the east. And so we have constructed the astronomical dial declining from the horizon, and so on — which was to be done.

SCHOLIUM.

Each dial declining from the horizon — the upper as well as the lower — must be set upon the declining plane in such a way that the meridian line CD is parallel to

the meridian line found in the plane that is parallel to the horizon. (This will easily be done if some straight line cutting the line AB in the dial is set at right angles upon the very meridian line that has been found, so that the whole coincides with the whole.) And the dial itself, if it is an upper one facing west, is to be raised on the eastern side; or, if it faces east, on the western side — according to the inclination of the plane, namely, in our example, 30 degrees — yet in such a way that that part is raised in which the line of the 6th hour cuts the equinoctial line. From this it comes about that the straight line AB extends directly from east to west — that is, is parallel to the vertical circle properly so called — and cuts at right angles that meridian line in the plane which is parallel to the horizon, with which we said the straight line must coincide that in the dial is perpendicular to the line AB. But if it is a lower one,

[Margin: How the dial declining from the horizon is to be positioned so that it indicates the hours.]

(continuing the scholium to Proposition 13)

it is a dial facing east, it is raised in the same way on the eastern side; or, if it faces west, on the western side — yet with this rule, that the line of the 6th hour, where it cuts the equinoctial line, lies below the meridian line CD. This elevation can conveniently be carried out as follows. In the plane parallel to the horizon, let the meridian line already found be cut by another line at right angles. For if the straight line AB — the one we drew in the dial — makes with this line the angle of the plane's inclination to the horizon, in such a way that all lines dropped perpendicularly from any points whatever of the straight line AB onto the plane parallel to the horizon fall onto the straight line which cuts the meridian line at right angles in that same plane, then the dial will have its proper position — provided, however, that the part of the dial has been raised which we have just taught must be raised. Add this too: in the upper dial the center C must be placed beyond the straight line AB, toward the south; but in the lower dial, on the near side of that same straight line AB, toward the north.

The stylus [stylus] in each dial will be IK, to be fixed at the point K so that it is perpendicular to the plane of the dial. Or else, if the triangle CIG is set upright to the plane of the dial, the shadow of the axis CI will indicate the hours, just as was said in the earlier cases.

[Margin: The stylus of the dial.]

If the dial is to be described on the declining plane itself — one that is fixed, such as some roof directly facing sunrise or sunset, so that it deflects neither toward

the south nor toward the north (how this is to be investigated we taught in Proposition 23 of the first book) — and not on some other plane from which it is afterward transferred onto the declining plane, as we did in this proposition: then on the declining plane one must draw the straight line CD parallel to the horizon for the meridian line, and erect AB perpendicular to it, the rest being constructed as we said in the first method. But if you prefer to carry out the work by the latter two methods, then here too one must first of all draw, for the meridian line, the straight line CD parallel to the horizon, and then draw the straight line CP so that CD and CP intercept the arc DP, as was said in the third method — toward the eastern parts, indeed, for an upper dial that also faces west; but toward the western parts for an upper dial that faces east, and so on. Let everything else be done exactly as we prescribed in Proposition 1 of this book — that is, let the line GH be drawn perpendicular to CP for the equinoctial line, and so on.

[Margin: By what method a dial is to be described on a fixed plane declining from the horizon.]

If the lines of the upper dial facing west interchange their arrangement among themselves, so that the left become right and the right become left (by "the right part" or "the left part" we now mean what is right or left to us when we have turned to face the dial and toward the south), so that they acquire the position they would have on the opposite face of the described plane, each line coinciding with its counterpart — then there will be produced another dial, an upper one facing east, declining from the horizon by the same number of degrees as the first, provided the hour-numbers are changed into their complements up to 12, just as we said regarding the dial declining from the vertical in the scholium to Proposition 1 of this book. For the same demonstration can be adapted here. By the same reasoning, from an upper dial facing east another upper one facing west will be constructed, if that interchange of lines and hours is made. And the same must be said of the lower dial.

[Margin: By what method, from a dial facing west, another facing east is produced, and vice versa.]

Now if any upper dial whatever is inverted, so that — with us turned toward it, toward the east if it faces west, or toward the west if it faces east — the upper part becomes the lower, and vice versa (this change will easily be grasped if one imagines the plane of the dial revolved about the straight line CD until its lines face the nadir and lie on the opposite, lower face of the plane), and so that what is right to us in that position, when we again turn to the dial from the other side of

the plane, becomes left, and vice versa: then there will be produced a lower dial of the same declination from the horizon, to be placed on the opposite, lower face of the plane, as was demonstrated in the scholium to Proposition 13 of the preceding book. This lower dial is beautifully rendered by the lines of the upper dial drawn on the opposite face, each line coinciding with its counterpart. For once this drawing is made, if we turn the opposite face toward us so as to have the center C on the right (if the upper dial faces west) or on the left (if the upper faces east), we shall obtain the lower dial, to be placed on the lower, opposite face.

[Margin: By what method, from an upper dial declining from the horizon, a lower dial on the opposite face is produced.]

Indeed, with the same labor and effort — just as we said concerning the vertical declining dial in the scholium to Proposition 1 of this book — it will be possible to describe four dials declining from the horizon that differ among themselves, while yet the same quantity of declination from the horizon is always preserved. For suppose we describe an upper dial declining, say, from the horizon toward the west by 30 degrees, and change its right part into the left (we now call "the right part" that which is right to us when we have turned toward the dial — toward the east if it faces west, or toward the west if it belongs to the east). From this change it comes about that the upper part too becomes lower, and the center C inclines toward the north — provided, of course, that this change is so arranged that the dial is understood to be moved about its axis, namely the stylus IK. There will then also be described a lower dial facing west, declining from the horizon by 30 degrees. But if all the lines of that same upper dial are described on another plane, so that those which are to the right of its meridian line CD become left, and those to the left become right — or (what is the same) those situated beyond the meridian line toward the east are placed toward the west, and vice versa — then there will be constructed another upper dial facing sunrise and declining from the horizon by 30 degrees, the hour-numbers being changed into their complements up to 12. Such are the lines of the upper dial facing west, described on the opposite and lower part, so that each line corresponds to its counterpart — provided it is so arranged on the upper plane facing east that the same part on which the 6th hour falls occupies the upper place. Finally, if not only the eastern lines are made western, and vice versa, but also the upper part is changed into the lower, and vice versa, so that the center C inclines again toward the north, there will be fabricated a lower dial facing east. By the same reasoning, if an upper eastern dial is described, we shall derive from it three others of the same declination. All these things can easily be gathered from what we wrote at the

end of the scholium to Proposition 1 of this book — the last figure of which scholium also represents the type of an upper dial facing east, which declines from the horizon by 30 degrees but for the latitude of the place of 48 degrees, because in constructing it the angle $F\beta C$ in the figure of Proposition 1 of this book was set at 42 degrees, which is precisely the altitude of the pole above the vertical circle in a region whose latitude is 48 degrees.

Moreover, both the upper eastern dial and the lower one show the hours immediately, when the Sun rises, although more hours are contained in the upper than in the lower. And both the upper and the lower western dial indicate the hours right up to sunset — so that the upper eastern together with the lower western, and equally the lower eastern together with the upper western, contain all the hours of the day.

The method of Andreas Schoner [Andreas Schöner], by which he describes the dial even though no points have been found on the equinoctial line for the hour-lines, is here the same as in the dial declining from the vertical. This is clear from the figure of the present proposition, if it is compared with the figure of the proposition of this book.

[Margin: Construction of a dial declining from the horizon, to a given length of the stylus whose position is also given.]

But if the same dial declining from the horizon is to be described on a given plane, to whatever size of gnomon — the position of which in the plane is also given, as at the point K — we shall approach the matter in this way. Through K, the position of the stylus, let two straight lines AB and FD be drawn, cutting each other at K at right angles. Or, if the plane declining from the horizon is fixed and upright to the vertical, draw through K, the position of the stylus, by means of a level, the straight line FD parallel to the horizon, and let AB cut it at K at right angles. On the straight line FD, taking the length of the given stylus KF — whether above the straight line AB or below — let there be described from F, as center, toward the straight line AB, an arc

of the circle DH — toward B, indeed, if the plane of the dial faces west and toward the zenith, or east and toward the nadir; but toward A, if the dial is an upper one and faces east, or a lower one and faces west. On this arc, the declination from the horizon being reckoned from the point D to H — which again we set at 30 degrees — so that the upper dial faces west, and the straight line FH being drawn to cut AB at E, let CE be drawn through E parallel to FD, that is, perpendicular to AB. For CE will be the meridian line, or the line of the 12th hour. Next, on the arc

HD, toward the other part of the straight line FD produced, let the complement of the declination DN be reckoned, and let the straight line FN be drawn; or rather let the straight line F α be drawn perpendicular to FE, cutting AB at α . For through the point α must be drawn both the horizontal and equinoctial line and the line of the 6th hour. Also, on AB, taking the straight line E β — whether to the right or to the left of the meridian line CE — equal to EF, let there be described from β , toward CE, an arc of a circle: beyond the straight line AB, indeed, in the upper dial, but on the near side of that same AB in the lower dial; and in the latter, the complement of the pole's altitude being reckoned, beginning from the straight line AB, let a straight line be drawn from β to the end of the reckoning, cutting CE at the point C, which will be the center of the dial. Then, drawing from C through K, the position of the stylus, the straight line CK for the line of the stylus, let the line drawn from α cut it at right angles at G: the straight line α G will be the equinoctial line. After this, let the straight line KI be erected from K, perpendicular to CK and equal to the stylus KF, and let CI and GI be joined, which will contain a right angle at I — so that, if CI is drawn first and the perpendicular IG is erected to it, cutting the line of the stylus at G, that point G is the one through which the equinoctial line α G must be drawn from α , perpendicular to CK. And CI will be the axis of the world. Now, on the line of the stylus CK, taking GL equal to GI, let there be described from L a circle of any size, and let it be divided into 24 equal parts, beginning from the straight line LM, which is drawn from L through the point M, where the equinoctial line and the meridian line intersect each other — which straight line LM, if there is no error, will necessarily be perpendicular to the drawn straight line L α . Let all the rest be completed as was explained at the beginning of this proposition.

Diagram. A large sundial-construction figure for a dial declining from the horizon. A long horizontal straight line runs across the middle, lettered **A** at its left end and **B** at its right end (the vertical-circle section AB). Near the top, a small sun-disk carries the center **C**, with **F** just to its left; a dense sheaf of straight rays (hour-lines) fans out from the upper region across the whole figure. Below C, on the meridian line CE, lie **E** (upper) and, lower down, **K** (position of the stylus) and **G**; β stands to the right of E and α to the left of K near a crossing of lines. In the lower half a circle (the 24-part hour-circle) contains **L**, with **M** just above it and **D** at its foot; **N** lies far to the left on the arc; **H** lies to the lower right. The hour-lines are numbered around the margin.

- A (left end of horizontal line): endpoint of section AB
- B (right end of horizontal line): endpoint of section AB
- C (in sun-disk, top center): center of the dial / apex of the ray-sheaf

- F (left of C): vertex of the stylus
- E (below C, on CE): foot of perpendicular FH on AB; meridian point
- β (right of E, on AB): construction point, $EF = E\beta$
- α (left of K): common section of horizontal/equinoctial and 6th-hour line
- K (below E): position of the stylus
- G (below K, on line of stylus): equinoctial-line point (αG)
- M (above the circle): intersection of equinoctial and meridian lines
- L (in the circle): center of the 24-part hour-circle
- D (foot of the circle): terminus of arc DH / DN
- N (far left, on the arc): complement-of-declination point
- H (lower right): terminus of arc DH
- I (upper region): [?] — not clearly resolved on the scan (text: $KI \perp CK$, CI = axis of the world)
- Hour-numbers around the margin: **8, 7** (upper left and right); **7, 8** (mid left and right); **9** (right); and along the bottom, from left: **5, 4, 3, 2, 1, 12, 11, 10**

[Margin: Demonstration of the preceding construction.]

But we shall demonstrate this construction as follows. Let it be understood that in the plane of the dial the straight line FD is parallel to the horizon, so that it is the common section of the plane of the dial and of that position-circle which is drawn through the poles of the plane of the dial and through the common sections of the meridian and the horizon. For this circle is drawn through the stylus FK (since the stylus, produced, falls on the pole of the plane of the dial, being a segment of the dial's own axis) and makes a section parallel to the horizon. For since the plane of the dial declining from the horizon is parallel to the common section of the said position-circle and the horizon — namely, to the axis of the vertical circle — the sections which the plane of the dial makes with that circle and with the horizon will, by Proposition 18 of Book 1, be parallel to each other; and so the common section of the plane of the dial and the said position-circle, since it is parallel to the common section of that same dial and the horizon, will itself be parallel to the horizon. Therefore the straight line FD, if it is set parallel to the horizon, will be the common section of the said position-circle and the plane of the dial. From this it follows that the straight line AB is the common section of the vertical circle and of that same plane of the dial. For since both the plane of the vertical circle properly so called and the plane of the dial declining from the horizon are perpendicular to the aforesaid position-circle, [^a] their common section too will be perpendicular to it; and consequently it will also be perpendicular to the straight line FD lying in that circle, by Definition 3 of Book

11 of Euclid. Therefore the straight line AB, since by construction it is perpendicular to FD, is the common section of the vertical circle (which passes through the center of the world and through the stylus) and of the plane of the dial.

[Margin: ^a 19. vnde ("whence").]

Next, since EFK is the angle of declination of the plane of the dial from the horizon, FEK will be the angle of the complement of that same declination — namely, the angle which the meridian makes with the declining plane. Therefore, if the triangle EFK is moved about EK until it is perpendicular to the plane of the dial set in its proper position, FE will be the common section of the vertical and the meridian, drawn through F, the vertex of the stylus; and consequently through E the meridian line must be drawn, that is, the common section of the meridian and the plane of the dial. But since this common section is parallel, by Proposition 18 of Book 1, to the common section of the plane of the dial and the aforesaid position-circle — that is, to the straight line FD — because the plane of the dial is parallel to the common section of the meridian and that position-circle: the straight line CE, which was drawn parallel to FD, will be the meridian line. And since the arc HN is a quadrant, the angle EF α will be right; and consequently the straight line F α is the common section of the vertical and the horizon. For since both the vertical and the horizon are perpendicular to the meridian, [^b] their common section too will be perpendicular to it, and consequently, by Definition 3 of Book 11 of Euclid, perpendicular to the straight line FE lying in the meridian, at F, the center of the world. Therefore F α , which is perpendicular to FE, is the common section of the vertical and

[Margin: ^b 19. vnde ("whence").]

the horizon. Now the equator has the same common section with the vertical, the horizon, and the circle of the 6th hour — namely the axis of the meridian — since these four circles, being perpendicular to the meridian, pass through the poles of the meridian, by Proposition 13 of Book 1 of Theodosius. Therefore both the horizon and the equator, and the circle of the 6th hour, are drawn through the straight line F α , and it meets the plane of the declining dial at the point α . And so through α must be drawn the horizontal line, as well as the equinoctial line and the line of the sixth hour.

With the triangle EFK still perpendicular to the plane of the dial: since both the plane of the dial and the plane of the meridian are perpendicular to the vertical, [^a] their common section CE too will be perpendicular to it, and so, by Definition 3

of Book 11 of Euclid, perpendicular to the straight line FE lying in the vertical in that position; and consequently the angle CEF will be right. If, therefore, the triangle βEC is understood to be moved about the straight line CE until the straight line βE coincides with the straight line FE and the point β with the point F, then — because of the right angles βEC and FEC and the equality of the straight lines $E\beta$ and EF — βC will be the axis of the world; since in the plane of the meridian, drawn through the straight lines FE and EC, the meridian line of the vertical, at F, the center of the world, makes the angle $E\beta C$ equal to the complement of the pole's altitude above the horizon. Therefore, by the corollary to Proposition 21 of Book 1, the point C in the plane of the dial, onto which the axis falls, will be the center of the dial. From this it comes about that the straight line CK is the line of the stylus, just as in the first description of this proposition the line of the stylus CG was drawn from the center C through K, the position of the stylus. By the same reasoning it is established that the straight line αG , which is perpendicular to the line of the stylus, is the equinoctial line — since it must be drawn through the point α , as we recently showed, and makes right angles with the line of the stylus, as was demonstrated by us in the earlier description of the dial declining from the vertical, in Proposition 1 of this book. All the rest will be demonstrated as in the earlier description of Proposition 1 of this book and its scholium, where we constructed the dial declining from the vertical to a given stylus and its position.

[Margin: ^a 19. vnde ("whence").]

By means of an ellipse described in the plane of the dial, whose diameters are found as in the scholium to Proposition 1 of this book, we shall construct the same dial declining from the horizon — and most conveniently, indeed, as regards those hours which scarcely intersect the equinoctial line, such as, in our example, the 8th and 9th hours from noon or from midnight. We have not worked out an example of this description, because the matter itself can be grasped without any effort from the figure which we constructed in the said scholium to Proposition 1 of this book. For that figure represents a dial declining from the horizon by 30 degrees on the eastern side, so that it faces west, built for the latitude of 48 degrees — which is precisely the complement of that latitude for which the dial declining from the vertical was constructed — the hour-numbers, however, being changed into their complements up to 12. As is clear from what we wrote at the beginning of this proposition.

[Margin: Description of the same dial by means of an ellipse.]

PROBLEMA 14. PROPOS. 14.

To figure the parallels — that is, the arcs of the signs of the Zodiac — on a dial that declines from the horizon.

By exactly the same methods the arcs of the signs will be described on this dial as those by which they were described on the dial declining from the vertical, as appears in the figure below. In the figure we have drawn the rays from C — those lines only which correspond to the hours that in the dial are placed to the left of the line of the stylus — so that the confusion which usually arises from the multitude of lines may be avoided, as we advised in Proposition 2 of this book. For in the dial, on the right part of the same

[Margin: Description of the arcs of the signs on a dial declining from the horizon.]

Diagram. A dense sundial figure showing the parallels (arcs of the zodiacal signs) on a dial declining from the horizon. Two circles appear: an upper circle (the vertical / meridian circle) carrying, from top down, **A**, **B**, and **C** (the center from which the ray-sheaf springs); and a lower circle in which the word "HORIZON" is set (split as **HORI** vertically above and **ZON** across the lower circle), containing the point **Φ** (phi). From C a fan of straight hour-rays and a family of curved sign-arcs cross the sheet, deliberately crowded (the text notes only the left-hand hour-lines are drawn, to reduce confusion). Near the center the points **K**, **M**, and **L** are legible on the stylus/meridian region; hour-numbers **8, 7, 6, 5, 4, 3, 2, 1, 12** (at M), **11, 10, 9** run around the sheaf. Along the right and lower margins run the zodiac-sign glyphs.

- A (top of upper circle): construction point
- B (below A in upper circle): construction point
- C (center of upper circle): apex of the ray-sheaf / center of the dial
- HORIZON (lettered across the two circles): the horizon circle
- Φ (in the lower circle): point label (phi)
- K, M, L (center region): stylus-line / meridian-intersection points
- Hour-numbers around the sheaf: 8, 7, 6, 5, 4, 3, 2, 1, 12 (at M), 11, 10, 9
- Zodiac-sign glyphs along the right/lower margins, readable: Cancer ☉, Leo ♌, Virgo ♍, Libra ♎, Scorpio ♏, Sagittarius ♐, Capricorn ♑; and toward the center Aquarius ♒, Pisces ♓ [?], Aries ♈ [?], Taurus ♉ [?], Gemini ♊ [?]. Several of the small sign-sorts are worn/overprinted by crossing lines and could not be pinned to a definite hour-arc for the zodiac labels.

[End of transcribed pages.]

of the same style-line we have drawn hidden lines — the kind marked off by certain fine lines — which, together with the hours placed to the left, keep the same distances from the style-line, exactly as the hours in a horizontal and a vertical dial are set at equal distances on either side from the meridian line. And so the hour-intervals set between the point C and the rays

Diagram. The "figure of rays" (*figura radiorum*) for a dial declining from the horizon — a very dense construction occupying the upper half of the page. Along the top runs the upper arc of a circle carrying hour-marks; a horizontal diameter crosses the sheet; a parallelogram-shaped auxiliary construction sits at the lower left; and from the center a broad fan of straight hour-rays descends, crossed by curved arcs of the zodiacal signs. The diagonal ray-lines are labelled with "Ra. hor." (*Radius horae*, "ray of the hour") followed by an hour-number, matching the "ray of the 14 hours / 10 hours / 16 hours / 18 hours" cited in the text. Because the sheet is heavily overprinted with crossing lines and several small sorts are worn, many labels cannot be pinned with confidence

- Top arc (hour-marks, left→right): 10, 9, 8, 7, 6 [?]
- Horizontal diameter (left→right): a bold point [label illegible, ?], I, B, C
- Left parallelogram construction: I (far-left corner); along its upper edge B, c, B [?]; down its left edge D, D, D; interior points K, L [?], G
- Point F (with the mark "12" beside it), near the top-center of the parallelogram — the point from which the division of the circle begins (per the text)
- Central points along the ray-sheaf: G, E, A, B, D
- Diagonal ray-labels (left-center): "Ra. hor. 6", "Ra. hor. 10"
- Diagonal ray-labels (right): "Ra. hor. 24", "Ra. hor. 18"
- Diagonal ray-labels (lower-center): "Ra. hor. 19" [?], "Ra. hor. 14"; and one or more lines lettered "Ro." [abbreviation unresolved, ?]
- Hour-numbers at the fan-ends (bottom, left→right): 3, 4, 5, 12, 5, 6, 7, 8, 8, 9, 9, 10, 11 [several worn, ?]
- Worn zodiac-sign sorts along the curved sign-arcs across the lower-center — not confidently resolvable [?]

of the signs have been transferred from the center C of the dial, by one and the same operation, both onto the hours placed to the left of the style-line and onto those hidden lines lying to its right, and so on. In the figure of rays we have also drawn the line CE, corresponding to the meridian line — although it is not contained on the left-hand side of the index-line but on the right — for this reason

only: that by means of it we might find the point F, from which the division of the circle from A, once described, ought to take its beginning; namely, so that we might draw the same hour-lines from C according to the practice of Andreas Schoner [Andreas Schöner], as we set forth in the scholium to Proposition 2 of this book. In that circle the points of division, proceeding from the point F toward B — where the index-line falls — belong to the hours that in the dial are drawn from the meridian line toward the index-line; so that in our example the point next below F, toward B, corresponds to the first hour from noon, the next to the second, and so on.

[Margin: Which arcs on the dial belong to the northern signs, and which to the southern.]

Moreover, in the upper dial — the one declining from the horizon — the parallels, or arcs of the signs, that lie between the center C and the equinoctial line belong to the northern signs, while those that stand farther from the center C than the equinoctial line belong to the southern signs, just as in the horizontal dial. In the lower dial, however, the exact opposite is to be understood. For there the arcs of the signs between the center C and the equinoctial line belong to the southern signs, and the others to the northern. This is easily grasped if each dial is understood as set in its proper position. For it will come about that, when the Sun is in the southern signs, the shadow of the style is cast into the northern part of the dial, and when it is in the northern signs, into the southern part, and so on. The equinoctial line, in turn, separates the southern part of the dial from the northern, exactly as is established in all other dials.

[Margin: How the horizontal line is to be drawn.]

The horizontal line must be drawn through the sixth hour on the equinoctial line, parallel to the meridian line itself — that is, to the line of the twelfth hour. For since the Sun, when it is on the Equator, rises and sets at the sixth hour, it comes about that at the time of the sixth hour it also occupies the horizon. Therefore, at the moment of sunrise or sunset, the solar ray passing through the tip of the style falls — by Proposition 11 of the first book — onto the common sections which the circle of the sixth hour, the Equator, and the Horizon make in the plane of the dial, since the Sun is then on all of these. It is therefore clearly established that the common section of the Horizon and the plane of the dial — that is, the horizontal line — must be drawn through the sixth hour on the equinoctial line, precisely where the line of the sixth hour and the equinoctial cut one another; otherwise the Sun's ray would not fall on all three sections which the plane of the dial makes with those three circles, which is absurd. That this same line must be

parallel to the meridian line is clearly gathered from Proposition 18 of the first book. For since the Meridian, the Horizon, and the great circle to which the dial is parallel have as their common section the axis of the vertical circle, the common sections which the Meridian and the Horizon make with the plane of the dial will be parallel to each other, as was demonstrated in the said Proposition 18.

[Margin: Another description of the horizontal line.]

We shall draw this same horizontal line by the following method. Through K, the place of the style, let the straight line KA be drawn parallel to the meridian line; and to it, from any point B, let a perpendicular BD be erected toward that part of the dial where the equinoctial line and the line of the sixth hour intersect. Then, taking the straight line AB equal to the style IK, construct at A the angle BAD equal to the complement of the plane's declination from the horizon, and let the straight line AD cut the straight line BD at D. For the line drawn through the point D parallel to the meridian line will give the horizontal line, as before — the line which necessarily passes through the sixth hour on the equinoctial line, if no error has been made. And by drawing the horizontal line in this way we shall easily test whether the sixth hour has been correctly laid out or not. That the horizontal line must be drawn

through the point D, found by this technique, we shall make clear in the following way (for that it is drawn through the sixth hour on the equinoctial line, and is parallel to the meridian line, has just now been demonstrated). Let the triangle ABD be conceived as rotated about the straight line BD until it stands perpendicular to the plane of the dial, and let the style IK be thought of as standing perpendicular at K to the dial. [a] On this account the straight lines IK and AB will be parallel to each other, because each is perpendicular to the plane of the dial — IK indeed by its position, but AB by Definition 4 of Book 11 of Euclid. Since, then, they are also equal to each other, [b] the straight line AI, drawn from A to I (both lines IK and AB being set perpendicular to the plane), will likewise be parallel to BK. The Horizon, therefore, passing through I, the tip of the style, and equidistant from the meridian line — and consequently from BK as well — will be drawn through the straight line AI, and thus through the point A. And since, by construction, the angle BAD is equal to the complement of the plane's declination from the horizon, the remaining angle ADB will be equal to the declination itself. Therefore the Horizon will be drawn through the straight line AD, since in this way it meets the plane of the dial at the angle of declination ADB, so that AD is the common section of the Horizon and of the triangle ABD. And so the Horizon will

meet the plane of the dial at the point D, and for that reason the horizontal line must be drawn through D.

[Margin: a) Euclid XI.6.] [Margin: b) Euclid I.33.]

[Margin: Another description of the horizontal line.]

Finally, this same horizontal line will be drawn by the following technique. From the figure of rays, if the upper dial faces west, take the intervals of the seventh hour from noon (between C and the ray of the fourteenth hour) and of the fifth hour from noon (between C and the ray of the tenth hour); or, if it faces east, of the fifth hour from midnight (between C and the ray of the fourteenth hour) and of the seventh hour from midnight (between C and the ray of the tenth hour). Transfer these from C, the center of the dial, onto the corresponding lines, marking points on them. For the horizontal line must be drawn through these points. The Sun, when it is in that parallel whose diurnal arc comprises fourteen hours — which, by what has been demonstrated, passes in the dial through the points found at the seventh hour from noon and the fifth hour from midnight — sets at the seventh hour from noon and rises at the fifth hour from midnight; and when it is in that parallel whose diurnal arc comprises ten hours — which, likewise from what has been demonstrated, passes in the dial through the points found at the fifth hour from noon and the seventh hour from midnight — sets at the fifth hour from noon and rises at the seventh hour from midnight. And so the horizontal line must be drawn through the said hours in the named diurnal arcs; and this line, as we demonstrated before, will necessarily pass through the sixth hour on the equinoctial line and will be parallel to the meridian line. We set out so many methods of drawing the horizontal line because it is pleasant to see that with such great variety we can always arrive at the same goal, and because we can be certain that no error has been committed in laying out the hours, provided that all the methods agree and are consistent with one another.

Now this horizontal line divides the whole dial into two parts, of which the upper is the greater and the lower the lesser, as was shown in the preceding cases — provided the lower parts are altered, as we said in the preceding proposition.

[Margin: The horizontal line divides the whole dial into an upper and a lower part, and into a diurnal and a nocturnal part.]

The dial is also cut by the same horizontal line into a diurnal and a nocturnal part, as was said above. We have therefore drawn the parallels, or arcs, and so on. Which was to be done.

PROBLEMA 15. PROPOS. 15.

To describe the parallels of the diurnal arcs on the same dial, which declines from the horizon.

[Margin: Description of the diurnal arcs on the same dial declining from the horizon.]

In describing these parallels there is no difficulty, since they are laid out in the same way as the parallels of the signs of the Zodiac were in the preceding proposition — as is evident, if in place of the parallels of the signs one takes the parallels of the diurnal arcs, and so on. We have therefore described the parallels of the diurnal arcs, and so on. Which was to be done.

SCHOLIUM.

[Margin: By what device the diurnal arcs serve to lay out those hours which scarcely cut the equinoctial line.]

Here too, as we noted in the scholium to Proposition 3 of this book, by means of the diurnal arcs one can draw the lines of those hours which scarcely intersect the equinoctial line — such as, in our example, the 8th and 9th, both from noon and from midnight. For since, on a day containing 16 hours, the Sun sets at the 8th hour from noon, the 8th hour from noon will pass through that point of the horizontal line where it is cut by the arc of the circle described at the interval of the straight line (in the figure of rays) that corresponds to the 8th hour from noon, intercepted between C and the ray of the 16 hours. This same hour-line, produced beyond the center C, will give the 8th hour from midnight. Similarly, since on a day containing 18 hours the Sun sets at the 9th hour from noon, the 9th hour from noon must be drawn through the point on the horizontal line at which it is divided by the arc of the circle described at the interval of the straight line (in the figure of rays) that corresponds to the 9th hour from noon, intercepted between C and the ray of the 18 hours. And this hour-line, produced beyond the center C, will give the 9th hour from midnight. In a dial facing east, the same must be done for the 4th and 3rd hours from midnight, and so on.

But we shall more conveniently find those points on the horizontal line by means of those diurnal arcs in the former way — namely, those described from the Analemma — because the hours which scarcely cut the equinoctial line cannot be drawn in the figure of rays unless we wish to use the practice of Andreas Schoner [Andreas Schöner].

PROBLEMA 16. PROPOS. 16.

To delineate the vertical circles on the same dial, which declines from the horizon.

[Margin: Delineation of the vertical circles on the same dial declining from the horizon.]

Through K, the place of the style, and the point D — where the horizontal line of the sixth hour and the equinoctial cut each other — let a straight line be drawn cutting the meridian line at B. From K, erect perpendicular to BD the line KA equal to the style IK; and, having joined the straight line DA, cut off DE equal to it. Then, with E as center, describe a circle of any radius whatever, and let it be divided into 360 degrees — or into fewer parts, as was said in the preceding cases — beginning from the straight line DE. For if straight lines are drawn out through the center E and the points of division, the horizontal line will be cut by them at points through which straight lines drawn from the point B will give the vertical circles — that is, the common sections of those circles and the plane of the dial — so that BD is the common section of the plane of the dial and of the vertical circle properly so called. This we shall confirm by the following reasoning.

[Margin: Demonstration of the preceding construction.]

Let the triangle ADK be conceived in the mind as rotated about DK until it stands perpendicular to the plane of the dial; so that accordingly the point A is joined with the tip of the style — that is, the center of the world — and the triangle itself is joined with the plane of the vertical circle properly so called (since here too the vertical is perpendicular to the plane of the dial), and passes through AK, the style, and also through the point D. For since the great circle to which the dial is parallel is, by Proposition 15 of Book 1 of Theodosius, perpendicular to the vertical — being drawn through the poles of the vertical properly so called — conversely the vertical will be perpendicular to that circle, and thus also to the plane of the dial, which is parallel to that circle. [a] Since, therefore, all the planes drawn through the axis AK are perpendicular to the same plane of the dial, the vertical will pass through the style AK, since it is drawn through its tip. And because, when the Sun is at the point where the Equator, the Horizon, the circle of the sixth hour, and also the vertical properly so called cut one another, the shadow of the style falls at D — the point of the sixth hour on the equinoctial line, as we said above — it comes about that through D there also passes the common section of the vertical and the plane of the dial, since, by Proposition 11 of Book 1,

[Margin: a) Euclid XI.18.]

Diagram. A large construction for the vertical circles of a dial declining from the horizon, occupying most of the page below the text. A long horizontal line (the horizontal line of the dial) runs across the sheet, terminated at each end by a cross-mark (cardinal-point symbol). Above it a small dotted triangle links the points A, C, I, K and B near the center. Below and around, families of curved arcs (the vertical circles) and straight rays sweep out from the construction, graduated in degrees; a second fan issues from a lower point (near F/B). The degree-graduations appear both as Roman numerals (XV, XXX, XLV, LX) and as Arabic numerals (15, 30, 45, 60, 75, 90), repeated around the arcs and rays. The sheet is crowded and several numerals are rotated or worn for the peripheral degree-labels.

- Horizontal line (left→right): a cross-mark (†) at the left end; then E, D, K, B; a cross-mark (†) at the right end
- "LX" (60) just to the right of K, on the horizontal line
- Upper region: A (apex of the small dotted triangle, center), C (above and right of A), I (left of A)
- Dotted lines connect A–B, A–D, and A–K (the rotated triangle ADK / ABD of the construction)
- Curved-arc degree labels (upper/right): XV (15), XXX (30), XLV (45), and Arabic 15, 30, 45, 60, 75, 90 scattered around the arcs; one worn mark near the right reads "13" [?]
- Lower horizontal segment (left→right): A, D; with 90 and 75 marked to the right
- Lower fan (issuing near F): apex region lettered "15 F 30", with B above F; rays graduated 15, 30, 45, 60, 75, 90 on both sides, and cross-marks (†) at cardinal rays
- Bottom edge (ray-ends, left→right): 45, 30, 45, 60, 75 ... 75, 60, 45, 30, 15 [several rotated/worn, ?]

the shadow of the style then falls onto this common section. Therefore BD will be the common section of the plane of the dial and of the vertical properly so called. And the straight line AD will be the common section of the triangle ADK and of the Horizon drawn through the tip of the style A and the point D. If, then, we conceive the circle from E — described about the horizontal line — to be moved until the point E is joined with the tip of the style A, and the straight line ED with the straight line AD, then, because of the equality of the straight lines ED and AD, ED (or AD) will also be the common section of the vertical properly so called and

of the Horizon; indeed the very circle described from E will be placed in the plane of the Horizon, and its center will be the same as the center of the world. Therefore the other straight lines drawn through the center E, dividing the circle described from E — and consequently also the Horizon, described from the same center A (which is then the same as E) and in the same plane — into equal parts, will be the common sections of the Horizon and of the remaining vertical circles. Hence the vertical circles will meet the plane of the dial at the points at which the horizontal line is divided by those straight lines drawn from the center E. And since they all also cut one another, by Proposition 18 of the first book, at the point B, into which their common section — namely the axis of the Horizon — falls, so that B is the zenith, or crown of the head (for since the Meridian and the vertical are drawn through the center of the world A and through the point B, the straight line AB, when drawn, will be their common section, that is, the axis of the Horizon) — the straight lines drawn through B and the points found on the horizontal line will be the common sections of the vertical circles and of the plane of the dial. And the meridian line will be the ninetieth vertical. And so we have delineated the vertical circles on the same dial, which declines from the horizon. Which was to be done.

Book Three (continued)

SCHOLIUM

From what we showed in the scholium to Proposition 14 of Book 2, it is perfectly clear that the smaller portion of the dial, cut off by the horizontal line, likewise displays for us the vertical circles on the lower dial, provided its parts are interchanged, as was said above.

Now we demonstrated that the Horizon is drawn through the straight line AD, and the Meridian through AB. From this it follows that, if the triangle ABD is set upright to the plane of the dial along the straight line BD, then the angle ADB turns out to be the angle of inclination of the plane to the Horizon — that is, the angle that the Horizon makes with the declining plane in the plane of the vertical circle — while ABD is the angle complementary to that same inclination, namely the angle that the Meridian forms with the same declining plane in the plane of the vertical circle. Therefore the remaining angle BAD must necessarily be a right angle in the triangle ABD. This can also be seen plainly from the dial that we constructed, in the scholium to Proposition 13 of this book, for a given length of

style. For there the line FE corresponds to the line AB here, and the line F α there is the same as the line AD here.

PROBLEMA 17. PROPOS. 17.

To set down the parallels of the Horizon on the same dial declining from the Horizon.

[Margin: The plotting of the parallels of the Horizon on the same dial declining from the Horizon.]

We shall draw the parallels of the Horizon on this dial just as we laid the same ones down on the polar dial in Proposition 41 of the preceding book — provided only that what follows is carefully observed. In the first method, the circle ABC of the analemma will represent the vertical circle properly so called, which of course passes through the style and stands at right angles to the plane of the dial, and makes the section BD; whereas the parallels of the Horizon are not altered. Again, for the axis EF, a diameter must be drawn — the common section of the vertical circle and of the great circle from which the dial is equidistant — such that both the arc CE and the arc BF are equal to the proposed declination from the Horizon. Parallel to this diameter, a line RQ must then be drawn, of a size corresponding to the style IK (or AK), so that the straight line DI, cutting RQ at right angles, is equal to the style IK. Finally the points L, M, N, O, P, taken from the point T where the Horizon BC cuts the straight line RQ, must be carried over onto the straight line BD, from the point D of the horizontal line toward B, in the upper dial. In the lower dial, however, the intervals ST, and any others there may be, must be carried over from D onto the same straight line BD, but beyond D, produced. For the conic sections about the diameters LQ, MQ, NQ, and so on — drawn according to the teaching of Proposition 8 of Book 1, and transferred into the dial in such a way that their diameters coincide with the straight line BD, and pass through the transferred points L, M, N, O, P, S, and so on, with their horns turned away from the horizontal line — will yield the parallels of the Horizon.

In the second method, the straight line AD is to be taken equal to the straight line AB which, in the figure of the preceding proposition, falls from the tip of the style A onto the zenith B; and onto the straight line AB the intervals between E and the vertical circles on the horizontal line are to be carried over, and so forth. For the meridian line, which gives the vertical circle of 90 degrees and does not cut the horizontal line, DF must be drawn parallel to AB; and the other part of the line to be transferred — the lines drawn from D which cut the straight line AB — must be handled just as was done with the hour-lines on the horizontal dial. For here

AD serves as the axis, and AB as though it were the radius of the Equator (although in truth it is the radius of the Horizon); and the straight lines drawn from A stand in place of the rays of the signs, and so on. The demonstration of this operation is the same as on the horizontal dial: if the straight line AD is made to coincide with the straight line AB, the axis of the Horizon, and the point A with the point B, and the point D with the point A, and the figure ADF — in which the rays of the parallels of the Horizon are contained — is revolved about it, just as on the horizontal dial we conceive the figure of the rays to be moved about the axis of the world. For here the vertical circles behave exactly as the hour-lines do there, so that the point B corresponds to the center of the dial, into which the axis AB falls, and the horizontal line serves as though it were the equinoctial line, and so on. And so we have set down the parallels of the Horizon on the same dial declining from the Horizon. Which was to be done.

Diagram. A large circle — the analemma / vertical-circle projection used to plot the parallels of the Horizon — with center D and a dense array of radiating straight lines and chords. The circumference is graduated in each quadrant with the degree marks 15, 30, 45, 60, 75, 90. The apex A is set just above the top of the circle. The figure is symmetric about the vertical diameter, so several point-letters (K, T, L, M, N, O, P) recur on both the left and the right. Many of the small peripheral labels are worn or rotated; those I cannot resolve are flagged.

- A (apex, above the circle top): tip of the style
- 90 (top of circle), with 75 to its left and 75 to its right
- G (upper left, outside the circle) [?]
- R (upper right, outside the circle)
- 60, 45 (left arc, descending below G); 60 (right arc, below R)
- E (right, with 30 above and 15 below)
- C (right side, on/near the circumference)
- T (left side and again right side)
- K (left side and again right side)
- 30, 15 (left arc, descending)
- P (interior, upper, below the 90)
- L, M, N (interior row, upper); O (interior, near them)
- D (center of the circle)
- 15 (right side, near lower C/T)
- M, N, M, L (interior row, lower — mirror of the upper row); O, P (interior, lower)

- F (lower left, on/near circumference)
- 3 [?], 45, 60 (lower-left arc, near F)
- H (bottom of the circle)
- 75, 90, 75 (bottom arc, below H)
- 30, 45 (lower-right arc)
- 2 (far left and again lower left, outer tick) [?]

SCHOLIUM

From what we wrote in the scholium to Proposition 2 of the preceding book, it will not be difficult to judge which of the parallels of the Horizon are, in the plane of the dial, hyperbolas, and which are parabolas or ellipses. For when, in the figure containing the rays of the parallels of the Horizon, a straight line issuing from the point D and cutting the straight line AB at the point which is nearest of all to the point A cuts two opposite rays drawn out from A, two opposite hyperbolas will be formed — as happens in our example in the rays of the parallels of 15 degrees. But if that same straight line drawn from the point D is equidistant from some ray set to the left of the straight line AB, then the parallel of the opposite ray will be a parabola. And if, finally, that same straight line drawn from D neither cuts nor is equidistant from any ray to the left of the straight line AB, then the parallel described from the opposite ray will be an ellipse — as happens in the proposed example in the rays of the parallels of the Horizon of 30, 45, 60 degrees, and so on. All of which have been demonstrated in the same place.

[Margin: Which parallels of the Horizon in the dial are hyperbolas, parabolas, or ellipses.]

PROBLEMA 18. PROPOS. 18.

To place the meridians, or circles of longitude of cities, on the same dial declining from the Horizon.

[Margin: The plotting of the meridians on the same dial declining from the Horizon.]

In the circle drawn from L, from the point O of the diameter OLM, let the longitude of the place be reckoned toward the western parts, up to α . Then, the circle having been divided into 360 equal parts (or into fewer), let hidden lines be drawn through the points of division and through the center L, cutting the equinoctial line at points through which straight lines, sent out from the center C

of the dial, will give the common sections of the plane of the dial and of the meridians, as is established from what precedes. For the diameter of the circle drawn from α through the center L is the common section of the Equator and of the meridian of the Fortunate Isles, and so on.

Diagram. A construction for the meridians of cities, laid out over a circle whose center is L (on the diameter OLM) and a horizontal line. The circle carries the named meridians of specific places, written along it, and is graduated with longitude degrees; many straight lines radiate across it. Several labels are worn or set at steep angles.

- HORIZON — the word split along the figure: "HORI..." (vertical, upper left of the circle) and "...ZON" (vertical, lower left)
- Ins. Fortun. [Insularum Fortunatarum — the Fortunate Isles, i.e. the prime meridian] (vertical, within the circle, left of center)
- ROMA [Rome] (vertical, right of center within the circle)
- Alexandria (diagonal, upper right)
- C (top, the center of the dial from which the rays are sent)
- O, L, M (points of the diameter OLM; L is the circle's center)
- Longitude degree labels around the circle: 105, 285, 300, 315, 20, 90, 135, 330, 345, 30, 45, 60, 75, 270, 300, 330
- Cross-marks (†) at two points on the circle's lower left
- Point α (labelling the terminus of the reckoned longitude) [?]

In the lower dial — which the portion cut off by the horizontal line presents to us, if its parts are inverted, as we taught above — the numbers must be assigned to the meridians themselves just as we handed down for the vertical dial in Proposition 18 of the preceding book, namely by adding 180 degrees, and so on, to the numbers of those same meridians in the upper dial. Therefore we have placed the meridians, or circles of longitude, on the same dial, and so forth. Which was to be done.

PROBLEMA 19. PROPOS. 19.

To dispose the parallels of cities, or circles of latitude, on the same dial declining from the Horizon.

[Margin: The plotting of the parallels of latitude on the same dial declining from the Horizon.]

Since these parallels are equidistant from the Equator, just as the parallels of the signs are, their plotting will not differ from the plotting of the latter, which we handed down in Proposition 14 of this book. And so we have disposed the parallels of cities, or circles of latitude, on the same dial declining from the Horizon. Which was to be done.

PROBLEMA 20. PROPOS. 20.

To place the celestial houses on the same dial declining from the Horizon.

[Margin: The plotting of the celestial houses on the same dial declining from the Horizon, according to Johannes Regiomontanus [Regiomontanus].]

Let the circle drawn from L, described in Proposition 13 of this book, be divided into 12 equal parts — or into more, if the subdivisions of the celestial houses are also wanted — beginning from the straight line LM, as in the plotting of the hours from noon or from midnight. Through the points of division, and through the center L, let straight lines be drawn cutting the equinoctial line at points; and if straight lines are then drawn through these points parallel to the meridian line itself, or to the horizontal, the celestial houses will be plotted according to the opinion of Johannes Regiomontanus. For since the Meridian and the remaining circles of the celestial houses divide the Equator — and consequently also the circle drawn from L — into 12 equal parts, and since it has been demonstrated that the straight line LM is the common section of the Meridian and the Equator, the remaining hidden lines, drawn through the points of division and through the center L, will be the common sections of the remaining celestial-house circles and of the Equator, provided the circle drawn from L is conceived to be set in its proper position. Hence the circles of the celestial houses will cut the plane of the dial at the points of the equinoctial line onto which the said hidden lines fall; and consequently through these same points the common sections of those same circles and of the plane of the dial must be drawn. Since, therefore, these common sections...

Book Three (continued)

are parallel, by Proposition 18 of Book 1 — because the plane of the dial is equidistant from the common section of the said circles, that is, from the axis of the vertical circle — it is established that the straight lines drawn parallel to the horizontal line, or to the meridian, through the points found on the equinoctial line, are the celestial houses according to the teaching of Johannes Regiomontanus.

Diagram. The construction for the celestial houses on the declining dial, shown with two circles: an upper circle whose center is A, and a lower circle whose center is L. A horizontal reference line runs across the figure, and the houses are marked with Roman numerals. Many labels are set vertically or at an angle and are worn; the unresolved ones are flagged.

- A (center of the upper circle; tip of the style)
- C (upper right, on the upper circle)
- B (right of center, on the horizontal line)
- D (on the horizontal line, left of B; a point of the vertical section DB)
- K (below D / on the line KA)
- L (center of the lower circle)
- M (lower right)
- G (interior, lower) [?]
- 8, 9, 10 (numerals near the center, between the two circles) [?]
- Roman-numeral house labels: I, VII, VIII, X, XI, XII (and further worn numerals) [?]
- VERTICALIS — split along the figure: "VER..." (left) and "...TICALIS" (right)
- HORIZON — split: "HORI..." (upper) and "...ZON" (lower left, within the lower circle)
- Left-margin vertical labels: "Grad.lis" [gradus, degrees]; "domus" [house]; "Grad. 15" [15 degrees]; "12." [?]; a further worn vertical label reading "OP. SIGNIS" [?]
- Zodiac-sign sorts around the lower circle: ♌ [Leo], and further worn sign glyphs [?] — the in-text signs for this figure are Cancer/Capricorn and Gemini/Leo (see Prop. 21)

But, as Campanus would have it, we shall delineate the same houses in this way. From K, the place of the style, let a line KA be drawn equidistant from the meridian line and equal to the style. Then, with A as center, let a circle be described and cut into 12 equal parts, beginning from the straight line AD (or AB) already drawn; and through the points of division, and through the center A, let hidden straight lines be drawn cutting the vertical line DB, which we drew in Proposition 16 of this book, at points. And if lines are then drawn again through these points parallel to the meridian line itself, or to the horizontal, we shall have the celestial houses according to Campanus. For since, as we showed above in Proposition 16 of this book, the straight line AD is the common section of the vertical circle and the Horizon, and AB the common section of the vertical circle and the Meridian, and since the Meridian and the Horizon are among the circles

of the celestial houses, the remaining hidden lines — drawn through the points of division of the circle described from A, and through the center A — will be the common sections of the vertical circle and of the remaining celestial-house circles. Therefore the circles of the celestial houses will meet the plane of the dial at the points of the vertical line DB onto which the said hidden lines fall; and consequently through these same points the common sections of those same circles and of the plane of the dial will pass. Since, then, as we have just shown, these common sections are parallel, it is plain that those parallels drawn through the points of the vertical line are the celestial houses, according to the opinion of Campanus. And so we have placed the celestial houses on the same dial declining from the Horizon. Which was to be done.

[Margin: The plotting of the celestial houses according to Campanus.]

SCHOLIVM

Because we posit that our dial declines from the Horizon by 30 degrees and faces toward the west, it comes about that the great circle passing through the poles of the vertical circle — through which all the circles of the celestial houses are drawn — and passing through the style, so that it stands at right angles to the plane of the dial, is the circle of the ninth celestial house according to the teaching of Campanus, receding of course from the Meridian toward the west by 30 degrees. For the great circle to which the dial is parallel, and the great circle cutting it at right angles, take away a quadrant from the vertical circle. Hence the line of the ninth house, according to Campanus, will pass through the place of the style.

On the lower dial the twelfth house runs so far from the horizontal line that the plane of the dial could scarcely receive it. For this reason we have drawn, in accordance with each method, a parallel line to indicate the 15th degree of the twelfth house, as appears in the figure.

PROBLEMA 21. PROPOS. 21.

To depict the ascending signs of the Zodiac on the same dial that declines from the Horizon.

[Margin: The plotting of the ascending signs on the same dial declining from the Horizon.]

We shall describe these on the dial declining from the Horizon in no other way than we did above — although not all of them can conveniently be delineated

from the earlier four little tables of Proposition 9 of the preceding book, because scarcely all the hours from noon, or before noon (whose beginning is taken from the diameter LM of the circle drawn from L), fall onto the equinoctial line. But most conveniently we shall find, on the meridian line, the points of the ascending signs from the sixth table of the same proposition, by means of the mid-heaven positions [mediationes coeli]. If these points are joined by straight lines to the points of the horizontal line where it is cut by the parallels of the signs, then — as we taught in the preceding — the ascending signs will be plotted. For example, on the upper dial (which faces west, as ours does): when the Sun, standing at the beginning of ☊ [Cancer], sets, the shadow of the style falls onto the point of the horizontal line through which the tropic of ☊ [Cancer] passes, and at the same time the sign ♑ [Capricorn], opposite to ☊ [Cancer], rises. Therefore through that point the line of ♑ [Capricorn] must be drawn. In the same way, when the Sun standing at the beginning of ♊ [Gemini], or of ♌ [Leo], sets, the shadow of the style falls onto a point of the horizontal line...

falls onto a point of the horizontal line — the point through which the parallel of ☊ [Cancer] and ♑ [Leo] is drawn; and at that same instant of time the opposite sign rises, namely ♎ [Sagittarius] or ♒ [Aquarius]. Through that point, therefore, the lines of ♎ [Sagittarius] and ♒ [Aquarius] must be drawn, and so with the rest. But on the Upper dial, which faces east, when the Sun stands at the beginning of ☊ [Cancer] and rises, the shadow of the gnomon is cast onto the point of the horizontal line through which the tropic of ☊ [Cancer] passes. Through that point, then, the ascending sign ☊ [Cancer] must be drawn, and so on.

Diagram. A large circle — the vertical circle / horizon of the construction — with a vertical diameter lettered so as to spell the word HORIZON down the page: HORI: at the top, ZON at the bottom. From the upper left the equinoctial line enters, labelled AEQVA: (for *aequator*). A cluster of straight lines (the parallels of the zodiacal signs and the amplitude-radii) fans across the circle from the left toward the right. Along the outer left arc and along the right-hand side the parallels are tagged with zodiac-sign sorts, most of them worn. The interior lettered points and the readable peripheral labels are:

- HORI: ... ZON (vertical diameter): the horizon / vertical circle.
- AEQVA: (upper left): the equinoctial line.
- C (top, on the meridian line crossing the circle): apex point of the construction.
- D (on the vertical line, upper-middle): point where the equinoctial and horizontal lines meet the vertical.

- G (center): interior intersection point.
- M (right of center): interior intersection point.
- L (lower center): center of the described circle / place of the style.
- N (lower, on the ZON line): lower point of the vertical diameter.
- TOR. (right side, on a curved label): tropic label [abbreviation as printed].
- Zodiac-sign parallels around the periphery (left arc and right side): the several sign-sorts are individually worn and cannot all be resolved with confidence — readable among them are Π [Gemini], $\approx$ [Aquarius], $\mathfrak{M}$ [Virgo], $\sphericalangle$ [Sagittarius]; the remainder are flagged [?]

[Margin: The finding of the points on the horizontal line through which the arcs of the signs and the lines of the ascending signs are to be drawn.]

As for the points on the horizontal line through which the arcs of the signs are to be drawn — even if they have not been drawn — we shall find them either from the fifth little table of Proposition 9 of the preceding book, as was set out above (for if the hour-lines that mark the hours and that bound the semidiurnal arcs are drawn, the horizontal line will be cut by them at the said points); or else from the figure of rising and setting amplitudes [figura latitudinum ortivarum, occiduarumve] of Proposition 21 of the preceding book, in this way. Let there be, as in the said Proposition 21, the Horizon ABCD, with its center E, together with the radii of the rising and setting amplitudes. From the line AD — the one contained in the figure of the preceding proposition — we shall cut off a line EF equal to it: taken from the line EC produced, if the dial faces west, but from EA, if it faces east; and through F we shall draw a line perpendicular to EF. For if the points of this line, where it is cut by the radii of the rising amplitudes, are taken from point F and transferred onto the horizontal line from point D, we shall obtain the points through which the parallels of the signs pass. For if the triangle ABD in the figure of the preceding proposition is understood to stand at right angles to the plane of the dial, then A will be the center of the world, and AD the common section of the Horizon and the vertical circle, as is clear from what has been demonstrated. If, then, the Horizon ABCD of the figure of rising amplitudes is understood to be placed on the dial so that its center E coincides with A, and the line EF — the common section of the vertical circle and the Horizon — with AD, the point F will fall on D, because the lines EF and AD are equal. Hence the perpendicular drawn through F will coincide with the horizontal line, since this perpendicular too stands at right angles to the line AD in that position; and consequently the radii of the rising and setting amplitudes will fall on those points which we said were to be transferred onto the horizontal line from point

D. And so we have plotted the ascending signs of the Zodiac on the same dial that declines from the Horizon. Which was to be done.

[Margin: By what method the same ascending signs may be drawn on the same dial declining from the Horizon, according to the practice of Andreas Schoner. The order of the ascending signs in the tropics.]

SCHOLIVM.

By the teaching of Andreas Schoner [Andreas Schöner] we shall depict the same ascending signs in the following way. In the earlier figure of the scholium to Proposition 9 of the preceding book — which we repeated in the scholium to Proposition 9 of this book — from the line EC (or EA) let a line EM be cut off, equal to the interval LD, the interval intercepted on the dial between the center L and the point D, where the horizontal line, the equinoctial line, and the sixth hour meet. At M, on the line EM, let an angle EMN be constructed equal to the angle that the line LD (if it were drawn) makes with the equinoctial line on the dial, under this condition: that when the center E is placed on the center L and the point M on the point D, the line MN coincides with the equinoctial line. After this, the intervals intercepted on the line MN between M and the lines of the signs are to be transferred onto the equinoctial line from point D, in the order they would have if the line MN coincided with the equinoctial line, marking the points on the equinoctial line. For if through these points hidden parallels are drawn to the horizontal line itself, or to the meridian, the tropics will be cut at points through which the ascending signs are to be drawn, this order being kept. On a dial facing west, the point where the horizontal line is cut by the tropic of ♋ [Cancer] belongs to ♐ [Capricorn] [?]; the next in the same tropic below the horizontal line, to ♏ [Sagittarius]; the next to ♏ [Scorpio] [?]; and so on in succession, contrary to the order of the signs.

The points, however, that are shown by the same parallels in the tropic of ♐ [Capricorn] look to the opposite signs. But on a dial facing east, the point common to the tropic of ♋ [Cancer] and the horizontal line is to be assigned to the sign ♋ [Cancer]; the next, toward the meridian line, to the sign ♌ [Leo]; and so on according to the series of the signs. The points, on the other hand, indicated by the same parallels in the tropic of ♐ [Capricorn] will belong to the opposite signs.

Again, in the later figure of the said scholium to Proposition 9 of Book 2 — which we have likewise repeated in the scholium to Proposition 9 of this book — from the line EC let a line EM be cut off, equal to that same interval LD (the interval set on the dial between L and D); and at M, on the line EM, let an angle EMN be

constructed equal to the angle that the said line LD (if it were drawn) makes with the equinoctial line at D, so that, when the center E is placed at the center L and the point M at the point D, the line MG coincides with the equinoctial line. Then the intervals intercepted on the line MN between M and the lines of the signs are to be transferred onto the equinoctial line from point D, in the order they would have if the line MN coincided with the equinoctial line, marking points on the equinoctial line. For if these points of the equinoctial line are connected by straight lines with the corresponding points of the tropics already found, the ascending signs will be drawn, as before. The order of the signs on the equinoctial line is this. When the dial faces east, the first point below the horizontal line will belong to ♉ [Taurus], the next to ♊ [Gemini], the next to ♋ [Cancer] [?], and so on in succession, as they are set in the said later figure. The next point above the horizontal line will suit the sign ♏ [Pisces], the next the signs ♒ [Aquarius] and ♑ [Capricorn] [?], and so on, just as the signs are placed in the aforesaid figure. But if the dial faces west, exactly the contrary order must be kept: the first point below the horizontal line must be given to the sign ♏ [Pisces], and the next will belong to ♒ [Aquarius] and ♑ [Capricorn] [?], and so on.

[Margin: The order of the ascending signs on the equinoctial line.]

But if the ascending signs are extended beyond the horizontal line, we shall have the same signs on the lower dial, provided that the parts of that portion which the horizontal line cuts off are inverted, as was said above.

PROBLEMA 22. PROPOS. 22.

To construct an Italian dial [horologium Italicum] declining from the Horizon.

[Margin: The construction of an Italian dial declining from the Horizon.]

If a circle described from L is cut into the diurnal arc of the parallel of ♋ [Cancer] by a, N, b, and into the diurnal arc of the parallel of ♑ [Capricorn] by d, N, e — as we taught in the scholium to Proposition 1 of Book 1 — so that the lines ab and de cut the line LM (which is the common section of the Meridian and the Equator) at right angles, we shall draw the hours from sunset on this dial, just as

Diagram. The construction figure for the Italian dial declining from the Horizon. A circle is centered low on the page; a vertical diameter again spells HORIZON (bottom). A dense fan of solid, dashed, and dotted straight lines (the hour-lines drawn from sunset, together with the diurnal/nocturnal arcs and the tropic

and equinoctial lines) radiates outward, and the hour-lines are numbered around the figure from 1 to 18. The readable lettered points and labels are:

- HORI: ... ZON (vertical diameter): the horizon / vertical circle.
- C (top): apex point at the top of the meridian line.
- o (just below and left of C): a construction point near the apex.
- A (upper right, on the vertical meridian line): the point of the 12th hour from midnight.
- N (left, on the circle): division point of the diurnal arcs.
- L (center): center of the described circle.
- P (just right of L): interior construction point.
- m (bottom of the circle): the 21st-hour point of the tropic of ♑ [Capricorn].
- Hour-line numerals ranged around the periphery: 1–18 (left side descending 15, 8, 14, 13, 12, 11, 10, 9...; right side 13, 14, 15, 16, 17, 18, 9).
- Interior small labels: further small letters (a, b, d, e) and numerals (19, 20, 21, 22, 23, 24) mark hour and arc intersections but are individually crowded and worn [several interior labels [?]].
- Peripheral zodiac sorts (Cancer/Capricorn tropic ends and the amplitude parallels) appear at the line-ends; individually worn [?].

was done on the dial declining from the Vertical in Proposition 10 of this book. You have an example in the 21st hour from sunset. For the line from m — the point of the 21st hour of the tropic of ♑ [Capricorn] — drawn through L, cuts the equinoctial line at n; and the line from C, drawn through n, cuts the tropic of ♑ [Capricorn] at p, the point of the 21st hour, and so on.

[Margin: The description of the same Italian dial declining from the Horizon by means of the diurnal and nocturnal arcs.]

In the same way, from the diurnal arcs of 14 and 10 hours and the nocturnal arcs of 10 and 14 hours, we shall lay out the same hours on this dial, as in the preceding cases — provided, however, that when some hour from sunset has no point on the diurnal arcs, the hour of the same number counted from sunrise is taken, and so on. For example, since the 12th hour from sunset on the diurnal arc of 14 hours has no point according to the third table of Proposition 33 of Book 1, we shall take the 12th hour from sunrise, and, following that same third table of the said proposition, we shall draw it through the 5th hour after noon on the diurnal arc of 14 hours

and, following the sixth table, through the 6th hour after noon on the nocturnal arc of 12 hours. For this line, produced beyond the meridian line — that is, beyond the parallel that is always the greatest of all those that ever appear [the circle of perpetual apparition], the one that touches the 12th hour from sunrise or sunset on the meridian line — will give the 12th hour from sunset, as we showed above. In the same way, for the 13th hour from sunset we drew the 13th hour from sunrise through the 6th hour after noon on the diurnal arc of 14 hours, and through the 7th hour after noon on the nocturnal arc of 12 hours. So too for the 14th hour from sunset, which has a single point — namely

Diagram. The same construction figure for the Italian dial declining from the Horizon, repeated so that it stands beside the text on this page as well. A circle low on the page carries the vertical diameter lettered HORI: (top) ... ZON (bottom); the fan of solid, dashed, and dotted hour-lines radiates outward and is numbered 1–18 around the figure. The readable lettered points and labels are:

- HORI: ... ZON (vertical diameter): the horizon / vertical circle.
- C (top): apex point on the meridian line.
- o (below C): construction point near the apex.
- A (upper right, on the vertical meridian line): point of the 12th hour from midnight.
- N (left, on the circle): division point of the diurnal arcs.
- L (center): center of the described circle.
- P (right of L): interior construction point.
- m (bottom of the circle): the 21st-hour point of the tropic of ♊ [Capricorn].
- Hour-line numerals 1–18 around the periphery, as in fig-p305-1.
- Interior small letters (a, b, d, e) and numerals (19–24) mark the hour/arc intersections but are crowded and worn [several interior labels [?]].
- Peripheral zodiac sorts at the line-ends are individually worn [?].

the 9th hour after midnight, on the diurnal arc of 14 hours — we took the 14th hour from sunrise, which is to be drawn through the 7th hour after noon on the nocturnal arc of 10 hours. And so through these points we drew a straight line, one portion of which — drawn on the diurnal arc of 14 hours through the 9th hour after midnight — will indicate the 14th hour from sunset, while the other portion — drawn through the 7th hour after noon on the nocturnal arc of 10 hours — will indicate the same 14th hour from sunrise; and so with the rest.

[Margin: The description of the same Italian dial by means of the tables of Proposition 20 of Book 1.]

Finally, we shall draw these same hours from sunset by their mutual intersections with the hours from noon or from midnight on the horizontal line — that is, the 24th hour and the 12th hour from sunrise or sunset, and so forth — as appears in the tables of Proposition 20 of Book 1. The 12th line from sunrise or sunset is to be drawn through the 6th hour after noon (or after midnight) on the equinoctial line, and through the point A of the meridian line — that is, the 12th hour from midnight — through which the diurnal arc of 24 hours is drawn, as is established from the seventh table and from tables 5 and 6 of Proposition 33 of Book 1. And so we have composed the Italian dial declining from the Horizon. Which was to be done.

PROBLEMA 23. PROPOS. 23.

To set up a Babylonian dial [horologium Babylonicum] declining from the Horizon.

[Margin: The composition of a Babylonian dial declining from the Horizon.]

We shall construct the Babylonian dial by exactly the same methods as the Italian. This the figure of the preceding proposition plainly declares, since it contains both the hours from noon (or from midnight) and the hours reckoned both from sunset and from sunrise, as in the earlier dials as well.

For the rest: on the Lower dial, where everything is changed, the 1st hour from sunrise is drawn through the 6th hour after midnight on the diurnal arc of 14 hours — into which the diurnal arc of 10 hours of the upper dial is changed — and through the 8th hour after midnight on the diurnal arc of 10 hours, into which the diurnal arc of 14 hours of the upper dial is converted. And it is established from the scholium to Proposition 13 of the preceding book that on the lower dial the hours are to be counted from midnight, which on the upper are reckoned from noon, and conversely. Or else, for the 1st hour from sunrise we shall draw the 1st hour from sunset through the 8th hour after noon on the nocturnal arc of 10 hours, and through the 6th hour after noon on the nocturnal arc of 14 hours, as the tables of Proposition 33 of Book 1 indicate. For this, on the lower dial, will be the 1st hour from sunrise, as we taught in the scholium to Proposition 23 of the preceding book. And so we have set up the Babylonian dial declining from the Horizon. Which was to be done.

SCHOLIUM

The hours from sunrise or sunset of the upper dial, extended beyond the horizontal line, here too give the same hours on the opposite and lower face, as was said in what precedes.

By the same reasoning as in the scholium to Proposition 11 of this book, so here too, with one and the same labor, we shall set up four different dials — Italian and Babylonian — just as we advised concerning the Astronomical dial in the scholium to Proposition 13 of this book. This will not be difficult, provided we observe what we handed down in the scholium to Proposition 11 of this book about converting the hours from sunrise or sunset into their complements up to 24, with the addition also of what we wrote in the scholium to Proposition 13 of the same book.

PROBLEMA 24. PROPOS. 24.

To construct an Antique [planetary-hour] dial declining from the Horizon [horologium Antiquum ab Horizonte declinans].

Diagram. A large sundial-construction figure for the declining Antique (planetary-hour) dial. The construction is built around a central apex point **C** at the upper middle, from which a fan of straight hour-lines radiates downward. A great oblique straight line descends from the upper left across the figure and is labelled, in vertical lettering along its course, **HORIZON** (the horizon line). Below and around the center is a circle (the tropic/day circle) crossed by three transverse straight lines — the equinoctial line and the two tropic lines (tropic of Cancer ☿ and tropic of Capricorn ♎). Roman-numeral hour-lines are laid out across the lower part of the figure. The zodiacal sun-signs mark the ends of the tropic and equinoctial lines.

Readable labels (positions approximate):

- **C** (upper-middle apex): the point from which the hour-lines radiate.
- **HORIZON** (running down the upper-left oblique line): the horizon line.
- Roman numerals **XIII, XII, XI, X, IX, VIII, VII, VI, V, IIII** (hour-lines, arrayed left-to-right across the middle and lower zone).
- Arabic hour-fractions along the upper arcs: **9, 7½, 6, 4½, 4, 3, 2½**, and (to the right of center) **4½ · 4½ · 12 · 10½ · 12** [several of these small figures are faint — [?]].

- f (lower left, on/near the horizon), P (far left on the horizon line), L (left of the circle), G, n, a (upper-middle points), M (within the circle), h (right side, on the tropic of Cancer ☊), b (foot of the circle), m and p (below the circle) — points named in the construction paragraph below.
- Zodiac signs at the line-ends: ♈ (Aries, upper left), ☊ (Cancer, right), ♐ (Capricorn, lower/bottom), with further worn sign-sorts at the right and lower-right margin [?].
- A large word set inside the lower part of the circle reads **NON** (upside-down) [?] — reading uncertain.

[Several small peripheral labels on this dense figure could not be resolved and are flagged; the figure]

We shall construct this dial as one declining from the Vertical, per Proposition 12 of this book. For example, as regards the former method: from f, the point of the 4th unequal hour of the tropic of Cancer ☊, a straight line drawn through L cuts the equinoctial line at g; and the straight line Cg cuts the tropic of Cancer ☊ at h, the point of the 4th unequal hour — and so on for the rest. For the 1st hour on the lower dial, we drew from m, the point of the 1st nocturnal hour of the tropic of Capricorn ♐, through L, a line that cuts the equinoctial line at n. For the straight line Cn cuts the tropic of Capricorn ♐ at p, the point of the 1st unequal nocturnal hour. A straight line, therefore, drawn from p through the 7th hour from noon on the equinoctial line, will give the 1st unequal nocturnal hour in the nocturnal portion of the dial; but this, on the lower dial, will be the 1st unequal diurnal hour, and so on.

As regards the latter method, on the hours from midnight — 9, 10½, 12 — and on the hours from noon — 1½, 3, 4½, 6, 7½, 9 — we marked the points of the diurnal arc of 18 hours, through which the unequal hours are to be drawn, as is gathered from Table 11 of Proposition 33 of Book 1. We have therefore constructed the Antique dial declining from the Horizon. Which was to be done.

[Margin: The construction of the Antique dial declining from the Horizon.]

DE HOROLOGIIS AD HORIZONTEM INCLINATIS

PROBLEMA 25. PROPOS. 25.

To describe an Astronomical dial that is inclined to the Horizon — that is, to describe the hour-lines from noon or from midnight on a plane which is

parallel to some great circle inclined to the Horizon and perpendicular to the Meridian.

Since a plane inclined to the Horizon and perpendicular to the Meridian can be situated in six ways, as we shall shortly explain, we shall likewise embrace the whole doctrine of describing dials on a plane of this kind in six precepts, so that the matter itself may be more easily grasped.

[Margin: A description of the Astronomical dial inclined to the Horizon, comprised in six precepts.]

GNOMONICES

PRAECEPTUM I

[Margin: When the plane's inclination on the North side is equal to the pole's altitude.]

When a plane is inclined on the North side — that is, whose upper face looks toward the south — and has an inclination equal to the pole's altitude, its dial will not differ from the Polar dial that we described in Proposition 37 of the preceding book.

PRAECEPTUM II

[Margin: When the plane's inclination on the North side is less than the pole's altitude.]

But when the same plane has an inclination less than the pole's altitude, the inclination must be subtracted from the pole's altitude; for the altitude of the pole above the inclined plane will remain. Then, for this pole-altitude once found, a horizontal dial must be described, as we handed down in Proposition 1 of the preceding book. It must be positioned so that the equinoctial line is parallel to the straight line which — on the plane that is parallel to the Horizon — cuts the meridian line at right angles. (This will easily be done if some straight line cutting the dial's meridian line at right angles is set on the plane parallel to the Horizon, perpendicular to the meridian line found in that plane.) And the dial itself is to be raised on the northern side according to the plane's inclination, so that its meridian line, together with the meridian line of the Horizon, forms toward the North the angle of inclination — yet with this rule: that on the upper face of the plane, which is turned toward the Zenith and the south, the center of the dial lies below the equinoctial line; but on the opposite and lower face, above it.

The order of the hours will be as follows. On the upper dial, the hours after midnight are — for us facing the dial — on the left, and the afternoon hours on the right; and the portion of the meridian line running from the center of the dial toward the equinoctial line indicates the 12th hour of noon, while the remaining portion, beginning from the same center, belongs to the 12th hour of midnight. On the lower dial, the matter stands the opposite way. For on it, the hours after midnight are — for us facing the dial — on the right, and the afternoon hours on the left; and the portion of the meridian line from the center toward the equinoctial line shows the 12th hour of midnight, while the remaining portion, also beginning from the center, shows the 12th hour of noon.

PRAECEPTUM III

[Margin: When the plane's inclination on the North side is greater than the pole's altitude.]

When, finally, the same plane has an inclination greater than the pole's altitude, the pole's altitude must be taken away from the inclination; for the altitude of the pole above the inclined plane will remain. Then, for this pole-elevation, a horizontal dial must be constructed by the doctrine of Proposition 1 of the preceding book. It must be positioned in no other way than the one we treated in the preceding precept, except for this: that here, on the upper dial, the center of the dial must be set above the equinoctial line, but below it on the lower. The order of the hours here too is entirely the same as was declared in the preceding precept.

PRAECEPTUM IV

[Margin: When the plane's inclination on the South side is equal to the complement of the pole's altitude.]

Again, when a plane inclined on the South side — that is, whose upper face looks toward the north — has an inclination equal to the complement of the pole's altitude, its dial will be exactly the same as the equinoctial dial constructed in Proposition 49 of the preceding book.

PRAECEPTUM V

[Margin: When the plane's inclination on the South side is less than the complement of the pole's altitude.]

But when the same plane has an inclination less than the complement of the pole's altitude, the inclination must be added to the pole's altitude; for in this way

the altitude of the pole above the inclined plane will be produced. Then, for this pole-altitude, a horizontal dial must be constructed by what was handed down in Proposition 1 of Book 2. It must be positioned as we prescribed in the second precept, except that this dial must be raised on the southern side according to the plane's inclination, so that its meridian line forms with the meridian line of the Horizon the angle of inclination toward the South — with this condition also added, that on the upper dial the center of the dial has its seat above the equinoctial line, but below it on the lower. The order of the hours here will be, on the upper dial, the same as on the lower dial of the second precept; but on the lower, the same as on the upper of the same precept. Nevertheless, on the upper, the portion of the meridian line extended from the center toward the equinoctial line signifies the 12th hour of noon, but on the lower, the 12th hour of midnight.

PRAECEPTUM VI

[Margin: When the plane's inclination on the South side is greater than the complement of the pole's altitude.]

[Margin: A demonstration of the six preceding precepts.]

When, finally, the same plane has an inclination greater than the complement of the pole's altitude, the complement of the inclination must be added to the complement of the pole's altitude; for by this method the altitude of the pole above the inclined plane will be produced. Then, for this pole-altitude, a horizontal dial must be described, as we taught in Proposition 1 of Book 2. It must be positioned as was said in the preceding precept, yet so that on the upper dial the center is below the equinoctial line, but above it on the lower. The order of the hours here too will be the same as in the preceding precept, except for this: that here, on the upper dial, the portion of the meridian line from the center toward the equinoctial line shows the 12th hour of midnight, but on the lower, the twelfth hour of noon.

We shall demonstrate all this[a] as follows. Let ABCD be the Meridian of the Analemma, whose center is E; let BD be the diameter of the Horizon; AC the Vertical; FG the axis of the world; F the arctic pole, G the antarctic; and HI the diameter of the Equator. Since, then, a plane inclined to the Horizon is set perpendicular to the Meridian, the Meridian will measure both the plane's inclination to the Horizon and the pole's altitude above it. Now it is clear that, if the plane is raised on the side of the arctic pole so that its inclination equals the pole's altitude DF, the plane does not differ from the circle of the 6th hour from

noon or midnight — drawn through the axis FG and perpendicular to the Meridian — and therefore the dial described on that plane

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is the same as the polar dial, as we said in the first precept. But if, on the side of that same arctic pole, the inclination Dk of the plane KL is less than the pole's altitude DF — as in the first figure — it is clear that the inclination DK, subtracted from the pole's altitude DF, leaves KF, the altitude of the pole above the plane drawn through KL, just as we taught in the second precept. And if, finally, on the same side of the arctic pole, the inclination DK of the same plane KL is greater than the pole's altitude DF — as in the second figure — who does not see that the pole's altitude DF, subtracted from the inclination DK, leaves KF, the altitude of the pole above the plane KL? Again, if the plane KL is raised above the Horizon toward the southern parts so that its inclination equals the altitude of the Equator, or the complement of the pole's altitude BH, it is clearer than daylight that the plane does not differ from the Equator, and so on. But if BK, the inclination of the plane KL, on the same southern side, is less than

Diagram. A row of four small circular analemma figures, each a Meridian-circle of the Analemma demonstrating one of the six precepts (per the text: the first figure answers to the second precept, the second to the third, the third to the fifth, and the fourth to the sixth). In each, the circle is the Meridian ABCD — A at top (the vertex/Zenith direction), C at bottom, B at the left and D at the right end of the horizon diameter BD; E is the center; FG the axis of the world (F the arctic pole, G the antarctic); HI the equator diameter; KL the trace of the inclined plane; MN and OP the two parallels drawn to KL, with M and O the centers of the upper and lower dials respectively. The interiors are crossed by numerous straight construction lines.

Readable labels per figure (small, worn engraving — many flagged):

- **First figure:** A (top), H (upper left), f (upper right) [?], K (right), D (right), B (left), I (lower left), C (bottom); interior points P (upper-center), E (center), N (lower-center), M (lower left). G [?].
- **Second figure:** A (top), K (upper right), H (upper left), F (upper right), B (left), D (right), C (bottom), G (lower left), I (lower right); interior P (left), E (center), M (center-right), N (lower-center), O (lower left).

- **Third figure:** A (top), H (upper left), F (upper right), K (left), B (left), D (right), L (lower right), C (bottom), G (lower left), I (lower right); interior P (upper-center), M and N (lower-center) [center label ?].
- **Fourth figure:** K (upper left), A (top), H (left), F (upper right), B (left), D (right), O (right), C (bottom), G (lower left), J/I (lower right) [?]; interior N (upper-center), E (center), M (lower-center), P (lower right).

[Many peripheral point-letters across these four small figures could not be securely resolved; flagged as a]

the complement of the pole's altitude BH, there is no doubt that the inclination BK, added to the pole's altitude BG, composes the arc KG, the pole's altitude above the plane KL, as appears in the third figure. And if, finally, on the southern side, the inclination BK of the same plane KL exceeds the complement of the pole's altitude BH — as in the fourth figure — it is plainly established that the complement of the inclination AK, joined to the complement of the pole's altitude AF, constitutes the arc KF, the pole's altitude above the plane KL. Rightly, therefore, in the foregoing precepts the finding of the pole's altitude above the inclined plane has been handed down. Therefore, if we construct an Astronomical dial for the found pole-altitude, we shall have the hours from noon or midnight laid out on the inclined plane, since the inclined plane itself is a kind of Horizon, which the Meridian proper to the place where the dial is constructed cuts at right angles, as it does the Horizon of the place.

But suppose that to KL — the common section of the inclined plane and the Meridian — we draw two parallels MN and OP, such that the plane of the upper dial is understood to pass through MN (drawn below the center, since it looks toward the vertex A), whereas the plane of the lower dial is understood to be drawn through OP (drawn above the center, since it turns toward C, opposite the vertex). Then we shall know sooner than it can be said in which dial the center — that is, the point where the axis of the world cuts the plane of the dial, as is clear from Proposition 21 of Book 1 — must be set below the equinoctial line, and in which above it; namely, according as the axis FG cuts the parallels MN, OP above or below the equinoctial line HI, as appears in the figures. Of these, the first corresponds to the second precept, the second to the third, the third to the fifth, and the fourth to the sixth.

For thus you see, in the first figure, that M, the center of the upper dial, is below N, the point through which the equinoctial line must be drawn perpendicular to the meridian line MN, as we showed in Proposition 1 of the preceding book. But on the lower dial, the center O is above P, the point of the equinoctial line, and so

on. For always in the figures the axis FG meets the plane of the upper dial at M and the Equator at N; while it meets the plane of the lower dial at O, and the Equator at P. The order of the hours, too, is easily gathered from the same figures, if the Meridian ABCD is imagined to be placed in its proper position.

For, as appears in the first and second figures, the axis casts a shadow onto the meridian line MN of the upper dial when the Sun is in the semicircle FAG, which passes through the vertex and the point of noon; whereas the same axis casts a shadow onto the meridian line OP of the lower dial when the Sun is in the opposite semicircle FCG, which is drawn through the Nadir and the point of midnight. From which it follows that the meridian line MN of the upper dial, from the center M toward the equatorial point N, belongs to the 12th hour of noon; while the meridian line OP of the lower dial, from the center O toward the equatorial point P, belongs to the twelfth hour of midnight, as we declared in the 2nd and 3rd precepts.

Also, when we face the upper dial so that we turn our face to the North, the shadows in the afternoon will be cast into the part of the dial placed on our right, which is the eastern part; but into the western part, which is on our left, in the forenoon, or after midnight. Likewise, when we face the lower dial so that we turn our face to the south, the shadows will fall into the eastern part in the afternoon — which for us is on the left — but into the western part, placed on our right, in the forenoon, or after midnight.

Again, as is clear in the third figure, the shadow of the axis is cast onto the meridian line MN of the upper dial when the Sun is in the noon semicircle FAG; but onto the meridian line OP of the lower dial when the same Sun is in the midnight semicircle FCG. Hence the former will belong to noon, the latter to midnight — the contrary is seen in the fourth figure. Likewise, when in the third and fourth figures we face the upper dial so that we turn our face toward the south, the shadows will fall into the western part, which is on our right, after midnight or in the forenoon; but into the eastern part, on our left, in the afternoon. But when we face the lower dial so that we turn our face toward the north, exactly the contrary happens. Rightly, therefore, we have prescribed the order of the hours in the foregoing precepts.

Let this be an example of all the cases. Let a plane be proposed inclined to the Horizon on the south side by 68 degrees. Since, then, the inclination is greater than the complement of the pole's altitude — for, in the Roman Horizon, to which we describe all the dials in this our Gnomonics [printed *Gomonica* [recte:

Gnomonica]], the complement of the pole's altitude comprises 48 degrees — we add

[Margin: An example of a dial inclined to the Horizon.]

we shall add — as we taught in the sixth precept (to which the fourth of the figures above corresponds) — the complement of the inclination, namely 22 degrees, to the complement of the pole's altitude, which we said was 48 degrees; and so we shall make the altitude of the pole above the proposed plane 70 degrees. To this altitude, therefore, we shall lay out the Astronomical dial.

Diagram. A large composite construction combining an analemma (upper semicircle) with the full face of the inclined Astronomical dial (a radiating sunburst of hour-lines) below it.

Upper analemma (semicircle):

- A (apex of the semicircle)
- M (on the arc, just left of the apex)
- E (on the arc, just right of the apex)
- F (lower on the left arc)
- N (lower on the right arc)
- D (center)
- B (left end of the horizontal diameter)
- C (right end of the horizontal diameter)
- Along a lower chord parallel to BC: P (left), H, G, O [?], I (right)

Lower sunburst dial:

- The equinoctial line runs horizontally across the figure through F (left), I (center), and K (right)
- C and d [?] (a short construction line on the lower left)
- L (right end of a line level with the center)
- Center cluster of the dial: D (upper), O (middle), H (the dial's center, in a small circle)
- The hour-lines radiate from the center and are numbered 1–12 for the daytime hours and again 1–12 for the night hours; an isolated E marks a point on a dotted horizontal (horizon) line above the dial's center.

[The figure is dense; several of the smaller radiating hour-numerals are not individually resolvable and are not committed here.]

This we do as was explained for the horizontal dial in Proposition 1 of the preceding book. It appears in the opposite figure, in which — since the dial is to be placed on the upper face of the plane — the center of the dial H is set below the equinoctial line; and the portion of the meridian line from H toward I, when extended, marks the twelfth hour of midnight, while the remaining portion, drawn from H toward the other side, marks the twelfth hour of noon, as we taught in the sixth precept and as is easily gathered from the fourth figure. We have therefore laid out the Astronomical dial inclined to the horizon, and so forth, which was what had to be done.

SCHOLIUM.

This same dial, if all its parts are inverted — as we taught in the scholium to Proposition 13 of the preceding book — will mark the hours on the lower face of the plane, as we demonstrated in that same place.

[Margin: By what method a fixed dial is to be constructed on a plane inclined to the horizon.]

If the plane inclined to the horizon is fixed, we shall go about the matter as follows. On the plane we draw a line FK parallel to the horizon, to serve as the equinoctial line, and to it we draw the perpendicular HE for the meridian line, which cuts FK at I. Then, starting from I: toward the lower part if the center of the dial falls below the equinoctial line, or toward the upper part if the dial's center falls above the equinoctial line (which the foregoing six precepts teach quite clearly), we transfer from the portion of the Analemma the straight line HI onto the meridian line HE, as far as the point H, which will be the center of the dial. Likewise, onto the same meridian line we transfer, from the portion of the Analemma, the straight line ID, from I as far as E. Finally, from E we describe a circle, divide it into 24 hours, and so on, completing the rest as before.

[Margin: How to lay out the same dial inclined to the horizon for a given length of style whose position is also given, without a separately constructed portion of the Analemma.]

But if the same dial is to be laid out on a given plane, without a separately constructed portion of the Analemma, for any length of style whose position is also given, we shall proceed in this way. Let the given length of the style be DG, and its position on the plane of the dial the point G. If, then, the plane of the dial is any plane whatever — such that the dial drawn on it may afterwards be set in its proper position, or transferred onto a fixed plane inclined to the horizon — we

draw through G, the position of the style, a straight line HE in any direction, to serve as the meridian line. But if the plane of the dial is proposed as fixed and inclined to the horizon, we draw through G a line DG parallel to the horizon, and at G we erect upon it the perpendicular HE for the meridian line; and from GD we cut off a straight line GD equal to the given style. Then from the center D we describe an arc of a circle, and on it, starting from the line GD produced toward the side of G, we count off the complement of the pole's altitude found above the inclined plane — toward the lower part, indeed,

the lower part of the dial (which we now suppose to run from the point G toward H) if the center of the dial falls below the equinoctial line (and when this happens is gathered from the foregoing precepts); but toward the upper part if the center of the dial lies above the equinoctial line. From D we then draw out a straight line through the endpoint of the reckoning, and this cuts the meridian line at H, the center of the dial. On the same arc, too, starting from the same line DG but toward the opposite side, we count off the very altitude of the pole found above the inclined plane, and through the endpoint of this counting we draw a straight line from D. This line will be perpendicular to DH, and it will cut the meridian line at the point I, through which the equinoctial line FK is to be drawn, perpendicular to the meridian line. Now, taking the straight line IE equal to ID, and describing from E a circle to be divided into 24 equal parts, we complete the rest as before.

The demonstration of this matter does not differ from the one we set out in the scholium to Proposition 1 of the preceding book, when we constructed the horizontal dial for a given length of style; for a plane inclined to the horizon is itself a kind of horizon, as is evident.

Diagram. The face of the inclined dial drawn as a sunburst of hour-lines radiating from the center, used in the construction just described.

- The equinoctial line runs horizontally across the figure through F (left), I (center), and K (right)
- E (upper center, on a dotted horizontal line above the center)
- D (center-left) and G (center) — the construction points of the style and the arc's center
- H (the center of the dial, just below D and G)
- The hour-lines radiate from the center and are numbered 1–12 for the daytime hours (across the top and down the right, 10 11 12 1 2 3 4 5 6 7) and again for the night hours around the lower half of the figure.

[The many small radiating hour-numerals are not all individually resolvable and are not committed cell-by-cell here.]

By means of an ellipse also — drawn on the plane of the dial, whose greater semidiameter is HI, whose lesser is ID, and whose center is H — we shall lay out this same dial, just as we laid out the horizontal one in the scholium to Proposition 1 of the preceding book. This can be readily understood from the figure we set beside that scholium. For that figure represents, in a region whose latitude comprises 60 degrees, a dial inclined to the horizon by 18 degrees on the northern side; because then the arctic pole stands at an altitude of 42 degrees above that plane, just as it does above the horizon of the place for which we constructed that horizontal dial.

[Margin: How to lay out the same dial by means of the ellipse.]

PROBLEMA 26. PROPOS. 26.

To place the parallels — that is, the arcs of the signs of the Zodiac — on the same dial inclined to the horizon.

Once the altitude of the pole above the inclined plane has been found, the parallels of the signs are drawn on this dial exactly as on the horizontal one, as we set out in Proposition 2 of the preceding book, and as appears in the figures given below.

[Margin: How to lay out the arcs of the signs on the same dial inclined to the horizon.]

Diagram. A dial face showing the equinoctial and horizon lines with the parallels (arcs) of the zodiacal signs drawn across the hour-lines.

- The upper horizontal line is labelled **AEQVA...TOR** (Aequator — the equatorial/equinoctial line), and along it run the hour-numbers 8 9 10 11 12 1 2 3 4
- A lower horizontal line is labelled **HORI...ZON** (Horizon), marked 6 ... 6 at its ends
- A central cluster of construction points near the meridian; the legible letters are E, A, G, and H (with B [?] near the center)
- Curved parallels of the twelve zodiacal signs cross the radiating hour-lines; the sign-glyphs legibly identifiable include ♉ (Taurus), ♊ (Gemini), ♍ (Virgo), and ♐ (Capricorn), with the remaining sign-sorts too worn to read individually [?]

[Several zodiac-sign glyphs are not resolvable at this resolution]

Which arcs belong to the northern signs and which to the southern, the four Analemmata in the preceding proposition make beautifully clear. For in the first Analemma the points of the northern quadrant FH (we now call the northern semicircle of the meridian HFI, and the southern one HGI) cast their shadow at noon, through the center E, onto the portion of the meridian line MN of the upper dial that lies between the center and the equinoctial line; while the points of the southern quadrant GI cast their shadow through the center E onto the portion of the meridian line

OP that lies between the center and the equinoctial line of the lower dial. It thus comes about that, in the upper dial elevated on the northern side — whose inclination is less than the pole's altitude, as appears in the first Analemma — the arcs of the northern signs are contained between the center of the dial and the equinoctial line, while the arcs of the southern signs lie beyond the equinoctial line. In the lower dial, the contrary holds.

Again, since in the second Analemma the points of the northern quadrant FH cast their shadow through the center E onto the portion of the meridian line

Diagram. A second dial face with the equinoctial and horizon lines and the parallels of the zodiacal signs, corresponding to a different one of the four cases.

- The upper horizontal line is labelled **AEQVA...TOR** (Aequator), with hour-numbers 8 9 10 11 12 1 2 3 4 along it
- A lower horizontal line is labelled **HORI...ZON** (Horizon), marked 6 ... 6 at its ends
- Down the vertical meridian, from top to bottom, the legible construction points are E, B, A, G, and H
- The parallels of the twelve zodiacal signs curve across the hour-lines; legibly identifiable glyphs include ♉ (Taurus), ♊ (Gemini), ♍ (Virgo), and ♓ (Pisces), the remaining sign-sorts being too worn to read individually [?]

[Several zodiac-sign glyphs are not resolvable at this resolution]

MN of the upper dial, beyond the equinoctial line, MN being produced through N; while the points of the southern quadrant GI cast their shadow through the center E onto the portion of the meridian line OP of the lower dial, beyond the equinoctial line, OP being produced through P. It thus happens that, in the upper dial elevated on the northern side — whose inclination is greater than the pole's altitude, as is seen in the second Analemma — the arcs of the northern signs fall

beyond the equinoctial line, and those of the southern signs between the center and the same equinoctial line. In the lower dial, the matter stands the contrary way.

Moreover, since in the third Analemma the points of the northern quadrant FH cast their shadow onto the portion of the meridian line MN of the upper dial that lies between the center and the equinoctial line, while the points of the southern quadrant GI cast their shadow onto the portion of the meridian line OP that lies between the center and the equinoctial line of the lower dial, it happens that, in the upper dial elevated on the southern side — whose inclination is less than the complement of the pole's altitude, as in the third Analemma — the arcs of the northern signs lie between the center and the equinoctial line, and those of the southern signs beyond the equinoctial line. In the lower dial, the contrary.

Finally, since in the fourth Analemma the points of the northern quadrant FI cast their shadow onto the portion of the meridian line MN of the upper dial that lies between the center and the equinoctial line, while the points of the southern quadrant GH cast their shadow onto the portion of the meridian line OP of the lower dial that lies between the center and the equinoctial line, it comes about that, in the upper dial elevated on the southern side — whose inclination exceeds the complement of the pole's altitude, as appears in the fourth Analemma, and which is just the kind of dial proposed to us — the arcs of the northern signs are comprehended between the center and the equinoctial line, and those of the southern signs beyond the same equinoctial line. In the lower dial, the contrary.

All of this is evident, if the said Analemmata are attentively considered. And the fact that certain points of the northern semicircle HFI, in each of the two later Analemmata, cast their shadow beyond the center M as well, is proof that the arcs of the signs are in that case ellipses — so that each of them cuts the meridian line in two places, one of which necessarily lies between the center M and the equinoctial line. Hence it is always true that the arcs contained between the center and the equinoctial line of the upper dial, in the two later Analemmata, belong to the northern signs, although some poin-

Diagram. An analemma / shadow-projection construction in the lower-left corner, showing the gnomon's shadows projected from the center onto the meridian line, with the circle (ellipse) of the sign-arcs.

- H, D, A along the horizontal line at the top (D is the center)
- G (apex/vertex just below D, where the projection lines converge)
- I (on the meridian, below the apex)

- P (left) and B (right) — points where an inner arc crosses
- V (left of center) and Φ [?] (right of center) on a horizontal diameter
- A large circle enclosing the lower portion
- Two rows of hour-numbers run along the base (day-hours and night-hours, 1–12); the individual digits are too small and worn to commit cell-by-cell here [?]

the northern-quadrant points FI, in the third Analemma, and the northern-quadrant points FH, in the fourth, cast their shadow beyond the center M onto the meridian line.

In our example the arc of Cancer [♋] in the upper dial and the arc of Capricorn [♐] in the lower, as well as the arcs of Gemini [♊] and Leo [♌] in the upper and the arcs of Sagittarius [♐] and Aquarius [♒] in the lower, form an ellipse, as is gathered from the corollary of Proposition 7 of the first book — because the declination of these signs exceeds the complement of the pole's altitude above the proposed plane. The latter arc, however, is very nearly a parabola, since the declination of those signs is almost equal to the complement of the pole's altitude above the inclined plane: the difference amounts to only 12 minutes, as is evident. By contrast, the arcs of Taurus [♉] and Virgo [♍] in the upper dial, and the arcs of Scorpio [♏] and Pisces [♓] in the lower, form a hyperbola, as we set out in the corollary of Proposition 6 of the same first book — because the declination of these signs is less than the complement of the pole's altitude above the proposed inclined plane. The same can also readily be concluded from what we wrote in the scholium to Proposition 2 of the preceding book.

[Margin: The horizontal line, and the method by which it is drawn.] The horizontal line is drawn as follows. Through G, the place of the style, draw GA perpendicular to the meridian line, and from it cut off GA equal to the length of the style. Then at A construct the angle GAB equal to the complement of the plane's inclination to the horizon — turned upward in the upper dial, but downward in the lower. (This is easily done if, from A, an arc of a circle is described toward the meridian line, on which the complement of the inclination is measured off from the straight line AG, and so forth.) Let the straight line AB cut the meridian line at B. I say that the line drawn through B parallel to the equinoctial line — that is, perpendicular to the meridian line — is the horizontal line, namely the common section of the horizon and the plane of the dial. For if the triangle ABG is turned about BG until it stands at right angles to the plane of the dial, and so lies in the plane of the meridian, and the style's vertex A is joined to the center of the world: then, since the angle GAB is equal to the complement

of the inclination, ABG will be the angle of inclination. Accordingly, since the horizon passes through A, the style's vertex, the straight line AB will be the common section of the horizon and the meridian, making in the plane of the meridian, with the meridian line of the dial, the angle of inclination, as the matter requires. Therefore, since the horizon meets the plane of the dial at B, the horizontal line must be drawn through the point B; and this line is necessarily parallel to the equinoctial line, by Proposition 18 of the first book, since the equator, the horizon, and the great circle to which the dial lies parallel all have one and the same common section, and so forth.

[Margin: Another way of drawing the horizontal line.] We shall find the same point B by this method. From the figure of the rays we take the portion of the 12-o'clock line pertaining to midnight, lying between H and the diurnal ray of the 24 hours — which in our example is the straight line HI. This we transfer from H, the center of the dial, onto the portion of the meridian line that marks the 12th hour of midnight (or, what is the same, from noon), carrying it as far as B. For through the point B the horizontal line, that is, the line of the 24th hour from sunrise or sunset, must be drawn as the diurnal arc of 24 hours, as is established from Table 7 of Proposition 33 of the first book.

This horizontal line divides the whole dial in two, upper and lower, and likewise day and night, as we said of the vertical dial and the others. We have therefore placed the parallels — that is, the arcs of the signs of the Zodiac — in the dial inclined to the horizon. Which was to be done.

PROBLEMA 27. PROPOS. 27.

To place the parallels of the diurnal arcs in the same dial inclined to the horizon.

[Margin: The drawing of the diurnal arcs in a dial inclined to the horizon.] If, in place of the parallels described through the signs of the Zodiac, the parallels of the diurnal arcs are taken, the diurnal arcs will be traced out just like the arcs of the signs, as was done in the preceding cases. And so we have placed the parallels of the diurnal arcs in the same dial inclined to the horizon. Which was to be done.

PROBLEMA 28. PROPOS. 28.

To delineate the vertical circles in the same dial inclined to the horizon.

[Margin: The drawing of the vertical circles in the same dial inclined to the horizon.]

Diagram. A vertical-circles construction for the inclined dial. A semicircular fan of straight rays radiating from the central region downward, crossed by three horizontal lines labelled (left-to-right, the label split across the figure) AEQVA...TOR (the equator), HORI...ZON (the horizon), and VERTI...CALIS (the vertical). The rays are graduated in azimuth/altitude. This same figure is repeated at the top of the facing page (p. 302).

- AEQVA — TOR (upper horizontal line): the equator
- HORI — ZON (middle horizontal line): the horizon
- VERTI — CALIS (lower horizontal line): the vertical
- Central points (clustered, small type): A, B, G, H near the center; C on the vertical axis below; D at the top of the inner divided circle
- Inner-arc degree divisions about the center D: 30, 45, 60, 75 ... 75, 60, 45, 30 [several worn — ?]
- Peripheral graduation numerals along the rays: XX and XXX (upper); XV, XXX, XLV, LX (lower left and lower right, symmetrically)
- Zodiac-sign glyphs at the lower ray-ends: Capricorn [♐] on each side [worn — ?]

Draw GA from G, the place of the style, perpendicular to the meridian line and equal to the style; and at A, upon GA, construct the angle GAC equal to the inclination of the plane — downward in the upper dial, but upward in the lower

— that is, in the lower dial toward the part opposite to that in which, in Proposition 26 of this book, the straight line AB was drawn to find the point B for drawing the horizontal line. Let the straight line AC cut the meridian line at C. Then, having drawn the straight line AB, cut off BD equal to it along the meridian line, upward or downward. With D as center, describe a circle and divide it into 360 parts, or fewer; and through the center D and the points of division send out straight lines cutting the horizontal line at points. If straight lines are then drawn from C through these points, the vertical circles will be described — that is, the common sections of those circles and the plane of the dial, whose origin is fixed on the vertical line, which is drawn through the found point C parallel to the equinoctial line. All of which we shall demonstrate as follows.

Diagram. The same vertical-circles construction as on the facing page (fig-p301-1), repeated at the top of p. 302. A semicircular fan of straight rays crossed by the three horizontal lines AEQVA...TOR, HORI...ZON, VERTI...CALIS, with the graduation numerals and central point-cluster as before.

- AEQVA — TOR: the equator line
- HORI — ZON: the horizon line
- VERTI — CALIS: the vertical line
- Central points: A, B, G, H near center; C on the vertical axis below; D at the top of the inner divided circle
- Inner-arc degree divisions about D: 30, 45, 60, 75 ... 75, 60, 45, 30 [several worn — ?]
- Peripheral graduation numerals: XX, XXX (upper); XV, XXX, XLV, LX (lower left and right)
- Zodiac-sign glyphs at the lower ray-ends: Capricorn [

[Margin: Demonstration of the drawing of the vertical circles. — (a) whence, from Proposition 19.] Let the triangle ABC be conceived at right angles to the plane of the dial, so that it lies in the plane of the meridian, and let the style's vertex A be the same as the center of the world. And since, as we demonstrated in Proposition 26 of this book, the straight line AB is the common section of the horizon and the meridian, and the angle BAC is a right angle — being composed of the angle of inclination GAC and the angle of the complement of the inclination GAB — the straight line AC will be the common section of the vertical (properly so called) and the meridian, so that C is the vertical point, upon which of course the axis of the horizon falls. For since both the meridian and the vertical are perpendicular to the horizon, [a] their common section (namely the axis of the horizon) will be perpendicular to that same horizon; and thus, by Definition 3 of Book 11 of Euclid, perpendicular, at the center of the world A, to the straight line AB lying in the horizon. The straight line AC, then, standing perpendicular to AB in the plane of the meridian, is the common section of the meridian and the vertical properly so called, and hence of all the verticals as well. This can also be made manifest from the fact that the angle ACG is equal to the complement of the inclination — the very angle that the inclined plane makes with the vertical. Now if the circle described from D is imagined to be moved about the horizontal line until its center D is joined to the center of the world A, and the straight line BD to the straight line BA, then, because the straight lines BD and BA are equal, the said circle will lie in the plane of the horizon, described about its center. Therefore, since the vertical circles cut the horizon — and so also cut the circle described from D concentric with it — into equal parts, beginning from the vertical or the meridian (which is itself one of the vertical circles), the lines drawn through D and through the points of division will be the common sections of the vertical circles and the horizon. Hence the vertical circles will meet the plane of the dial

at the points of the horizontal line onto which the said lines fall; and accordingly, through those points the common sections of the vertical circles and the plane of the dial must be drawn. Since, then, all these same sections also meet at the point C — by Proposition 18 of the first book, the point upon which the axis of the horizon, or their common section, falls — the lines drawn through C and through the points found on the horizontal line will be the common sections of the vertical circles and the plane of the dial. Moreover, the common section of the vertical (properly so called) and the plane of the dial, drawn through C, is parallel — by the same Proposition 18 of the first book — to the equinoctial line, and also to the horizontal line; because the equator, the horizon, the vertical properly so called, and the great circle to which the dial lies parallel all have one and the same common section. Therefore we have delineated the vertical circles in the same dial inclined to the horizon. Which was to be done.

PROBLEMA 29. PROPOS. 29.

To depict the parallels of the horizon in the same dial inclined to the horizon.

In this dial we shall describe the parallels of the horizon just as in the polar dial of Proposition 41 of the preceding book, with a few changes. For in place of the axis EF (so far as the former method is concerned), we must draw EF, the common section of the meridian and the inclined plane, in such a way that the arc CE measures the inclination of the plane above the horizon; and parallel to this common section EF we must draw QR, as there, for the given magnitude of the style. Then the points K, L, M, N, O, taken from the point T where the horizon BC cuts the straight line QR, must be transferred onto the meridian line of the dial from B, the point of the horizon, toward the vertical point C, in the upper dial; but in the lower dial the intervals TS, and others, if there be any, must be transferred from B onto the other part of the meridian line, and so forth. The demonstration is entirely the same as in Proposition 41 of the preceding book, provided the meridian ABC of the Analemma is understood to be set in its proper position, so that the point C looks toward the north if the plane is elevated on the northern side, but toward the south if it is raised on the southern side, as in our example, and so forth.

As regards the latter method: AD must be taken equal to the axis of the horizon AC in the figure of the preceding proposition — that is, to the straight line which falls from A, the style's vertex, to C, the Zenith; and onto the straight line AB must be transferred all the intervals between D and the vertical lines on the horizontal line.

Diagram. A full circle densely crossed by chords and diameters (the Analemma-type construction for the horizon-parallels), placed at the upper right of the page. The circumference is graduated in degrees by quadrant (...15, 30, 45, 60, 75...), and a cluster of small point-letters lies about the center.

- Top of circle: A, R, and E (E just inside the rim); a second R at the upper right rim
- Center: D
- Left rim (top to bottom): 75, 60, 45, 30, 15, then G and B, then 15, 30 lower
- Right rim (top to bottom): 45, 30, 15, then C, 15, H, 30, 45
- Bottom rim: 60, 75, then Q, F, and a second Q; 30
- Near-center small letters: N, O (upper), S, T (mid), M (lower) [several worn — ?]

[Note: this dense figure carries many worn peripheral numerals and central letters; more than two labels are uncertain]

The rest is to be completed as was stated in Proposition 41 of the preceding book. For the same demonstration can be adapted here. And so we have depicted the parallels of the horizon in the same dial inclined to the horizon. Which was to be done.

Diagram. A fan-shaped dial (the finished horizon-parallels dial) below the circular figure. Along the top runs a horizontal line carrying the points A, D, C; from the central region straight rays fan downward, and a dashed semicircular arc spans the middle. The margins are graduated 90, 75, 60, 45, 30, 15.

- Top horizontal line (left to right): A, D, C
- E: just below A on the axis; B: on the vertical axis below the center
- Left-margin numerals (top to bottom): 90, 75, 60, then 90, 75, 60, 45, 4.5 [45 — ?], 30
- Right-margin numerals (top to bottom): 90, 75, 60, 45, 30, and 60 lower
- Foot numerals (left to right): 30, 15, [axis], 15, 30, 45, 15 [rightmost — ?]
- Two small cross/dagger hour-markers (†) at the foot of the vertical axis

—as was stated in the preceding book. For the same demonstration can be adapted here. And so we have depicted the parallels of the horizon in the same dial inclined to the horizon. Which was to be done.

PROBLEMA 30. PROPOS. 30.

To arrange the meridians, that is, the longitude-circles of cities, in the same dial inclined to the horizon.

[Margin: The drawing of the meridians in the same dial inclined to the horizon.] Here we shall describe the meridians in no other way than in the preceding dials: namely, in the circle described from E in Proposition 25 of this book, the longitude of the place is counted, from the common section of the meridian and the equator EI, toward the western parts, up to the point O — beginning from the point of that section which looks toward noon, which in our example is the point M in the upper dial (as is evident if the said circle is conceived to be set in its proper position). For the hidden straight line drawn through the center E, from the point N which is opposite to the point O of the Fortunate Isles, will give on the equinoctial line the meridian of 180 degrees — namely the semicircle opposite to the semicircle drawn through the Fortunate Isles; but this meridian, produced beyond H, the center of the dial, will give the opposite semicircle, which of course passes through the Fortunate Isles, according to what we wrote in the scholium to Proposition 1 of the preceding book. The rest is clear from the figure. There too you see that in the lower dial numbers are set against the meridians, which arise by adding 180 degrees to the numbers of the meridians of the upper dial, and so forth, as we explained in Proposition 18 of the preceding book. We have therefore arranged the meridians, that is, the longitude-circles, and so forth. Which was to be done.

THE FIGURE FOLLOWS.

Diagram. The figure promised at the close of Proposition 30 ("THE FIGURE FOLLOWS") — the dial of the meridians (longitude-circles of cities) drawn in the dial inclined to the horizon. It has two parts sharing one vertical axis. Above is the equatorial dial: a large circle with center **E**, crossed horizontally by the equator line (labelled **Æquator**), from whose region straight meridian-rays fan outward, each carrying a longitude value in degrees (many printed mirror-reversed along their rays). Below the horizon line (labelled **Horizon**) is a lower fan-shaped dial whose radial meridians carry the opposite (+180°) longitudes. The prime meridian of the Fortunate Isles (**Insulæ Fortunatæ**) is marked with a cross ☩, appearing once at the upper right and once at the lower left.

- Vertical axis, top to bottom: **N** (top of the upper circle) — **E** (center of the upper circle) — **I** (on the equator line) — **O** (small, just right of I) — **M** (below I) — **B** (center of the horizon line) — **G, H** (just below the horizon)

- Horizontal lines: **Æquator** (through E and I); **Horizon** (through B)
- Slanted labels: **Insula Fortunatæ** (upper right, with ☐) and **Insula Fortunatæ** (lower left, with ☐)
- Upper-dial meridian degree-labels (as read; several worn): top rays 60, 45, 30, 15; left side 75, 150, 90, 105; equator ends 105 [?] (left) and 105 [?] (right, printed "for"); right side 15, 345, 330
- Horizon-line ends: 120 (left, printed "021") and 315 (right, printed "SIE")
- Lower-dial meridian degree-labels — left rays (top to bottom): 285, 300, 315, 330, 345; right rays (top to bottom): 135, 120, 105, 90, 75

[This construction dial carries more than two worn/uncertain labels]

PROBLEMA 31. PROPOS. 31.

To lay out the parallels of cities, or the circles of latitude, in the same dial inclined to the horizon.

[Margin: The drawing of the circles of latitude in the same dial inclined to the horizon.]

Since the parallels drawn through the zeniths of places are equidistant from the equator — exactly as the parallels of the [zodiac] signs are — it is plainly established that their construction is the same as that of the latter, as we noted in the preceding propositions. We have therefore laid out the parallels of cities, or the circles of latitude, and so on. Which was to be done.

PROBLEMA 32. PROPOS. 32.

To construct the celestial houses in the same dial inclined to the horizon.

[Margin: The drawing of the celestial houses according to Johann Regiomontanus [Regiomontanus] in the same dial inclined to the horizon.]

Let the circle drawn from E — the one by whose aid we constructed the astronomical dial in Proposition 25 of this book — be divided into 12 equal parts, beginning from the meridian line. For the straight lines emitted from the center E through the points of division will mark points on the equinoctial line; and if from these, through the point B where the meridian line and the horizontal line intersect one another, straight lines are drawn, we shall obtain the common sections of the plane of the dial and of the great circles drawn through the beginnings of the houses — following the view of Johann Regiomontanus, who draws such circles through the divisions of the equator. For if the circle drawn

from E is understood to be set in the plane of the equator, so that its center E does not differ from the center of the world, and EI is the common section of the equator and the meridian, then the straight lines emitted from E through the division-points of that same circle will be the common sections of the equator and of the circles of the celestial houses, according to Johann Regiomontanus — since indeed they divide the equator, and hence also that circle concentric with it, into 12 equal parts. The circles of the celestial houses therefore meet the plane of the dial in the points of the equi-

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noctial line onto which those straight lines drawn from E fall; and so the lines of the celestial houses must be drawn through those points — namely, the common sections of the plane of the dial and of the circles of the celestial houses. Since, then, these same sections all pass — by Proposition 18 of the first book — through the point B onto which the vertical axis AB falls (the point where all the circles of the celestial houses cut one another), it is clear that the straight lines drawn from B through the points found on the equinoctial line are the lines of the celestial houses.

[Margin: The drawing of the celestial houses according to Campanus [Campanus of Novara] in the same dial inclined to the horizon.]

The celestial houses, however, as they are understood by Campanus, we shall construct as follows. From the position of the style G, draw GA perpendicular to the meridian line, equal to the gnomon; and at A, upon AG, let the angle of inclination GAC be set up, so that — by what we demonstrated in Proposition 28 of this book — AC is the axis of the horizon and C the ver-

Diagram. The dial of the celestial houses (Problema 32), drawn in the dial inclined to the horizon. It has an upper circle with center **E** (the equatorial construction) and a lower circle with center **D** (the vertical construction), sharing one vertical axis. The equator line is labelled **Æquator** (left **AEQVA-**, right **-TOR**); the horizon line **Horizon** (left **HORI-**, right **-ZON**); the vertical line **Verticalis** (**VERTI- -CALIS**). Two families of house-lines radiate outward: the full lines (Regiomontanus) and the fine dashed lines (Campanus, "marked off by those fine lines"). The houses are numbered around the rim in Roman numerals, with a second set of Arabic numerals; a cross ☒ marks the top (near "10") and the foot (near "10") of the axis.

- Vertical axis, top to bottom: $\square$ / 10 (top) — E (center of upper circle) — I (on the equator) — B (center of the horizon) — A, Q, H (small, clustered just below B) — C (on the vertical line) — D (center of lower circle) — 10 / $\square$ (foot)
- Upper region rim labels (as read, worn): left II, IX, XII [?], XII; right 6, XI, 8, III
- Horizon-line ends: VII / I (left), IIII [?] / I (right)
- Lower region rim labels (as read, worn): left VIII, 8, IX, 9; right XII, XI, II; foot 10
- Small paired numerals near D: 10 / 10 (left and right of the lower circle) [?]

[This house-dial carries more than two worn/uncertain rim labels]

tical point. And the straight line drawn through C perpendicular to the meridian line is the common section of the plane of the dial and of the vertical [circle]. Then, taking the straight line CD equal to CA itself, let a circle be described from D; and this being divided into 12 equal parts, beginning from the meridian line, let straight lines be drawn from D through the division-points, cutting the vertical line in points which, joined with the point B as before, will give the lines of the celestial houses as Campanus takes them — of which sort are the lines distinguished by those fine lines in our example. For if the triangle GAC is understood to stand erect [at right angles] to the plane of the dial, and the circle described from D is understood to be moved about the vertical line until its center D is joined with A, the center of the world, then — because of the equality of the straight lines CA and CD — the circle described from D will, in the plane of the vertical circle, have the same center as that circle itself. Therefore the straight lines emitted from D through the division-points will be the common sections of the vertical [circle] and of the circles of the celestial houses; and hence the plane of the dial will be cut by the circles of the celestial houses in those points of the vertical line onto which the said straight lines fall, and so on. And so we have constructed the celestial houses in the same dial inclined to the horizon. Which was to be done.

PROBLEMA 33. PROPOS. 33.

To mark out the ascending signs of the Zodiac in the same dial inclined to the horizon.

We shall mark out the ascending signs of the Zodiac by exactly the same methods we used in the preceding propositions, as

GNOMONICES

[Margin: The drawing of the ascending signs in the same dial inclined to the horizon.]

appears in the figure below. For example, suppose that in the upper dial the ascending sign Taurus (♉) is to be drawn. I see, in the 1st table of Proposition 9 of the preceding book, that — with the Sun at the beginning of Aries (♈) — it rises at 4 hours 51 minutes before noon. But because such an hour does not cut the equinoctial line, I look for the same sign in the 2nd table, and I find that — with the Sun at the beginning of Libra (♎) — it rises at 7 hours 8 minutes after noon, which hour does indeed cut the equinoctial line. Through this hour, therefore, I find a point on the equinoctial line through which [the sign] is to be drawn.

Diagram. The dial of the ascending signs (Problema 33), drawn in the dial inclined to the horizon. A circle with center **E** sits above; the equator line runs across it, labelled **Æquator** (left end at point **F**, right end at point **K**, with **AEQVA-TOR** on the line); the horizon line **Horizon (HORI- -ZON)** runs below; a further line labelled **Linea** crosses the lower-left. The vertical axis descends through **I** (below **E**) and **B** to **H**. Many straight sign-rays radiate outward, each terminating in a zodiac glyph (the ascending sign it represents); the glyphs are badly worn and mostly illegible at this resolution.

- Vertical axis, top to bottom: **E** (center of circle) — **I** — **B** — **H**
- Equator line: point **F** (left end) — **AEQVA- -TOR** — point **K** (right end)
- Horizon line: **HORI- -ZON**
- Lower-left line: **Linea**
- Ray-end zodiac glyphs: worn throughout; individual signs not securely legible — read only in part [?]

[The zodiac ray-labels are worn and more than two are illegible]

is the sign Taurus (♉) ascending. Next, from the 3rd table, with the Sun at the beginning of Cancer (♋), I find that the same sign Taurus rises at 10 hours 50 minutes before noon — that is, at 1 hour 10 minutes after midnight — and so through this hour I strike a point on the arc of Cancer, through which the same sign is to be drawn. A straight line, therefore, connecting this point with the former one found on the equinoctial line will give the ascending sign Taurus; and so with the rest. By the same tables the same ascending signs will most conveniently be delineated in the lower dial, in which the arc of Cancer is changed into the arc of Capricorn (♐). Thus the same sign Taurus in the lower

dial will have to be drawn on the equinoctial line at 4 hours 51 minutes before noon — as is gathered from the 1st table — and in the tropic of Capricorn at 1 hour 10 minutes after noon, as the fourth table has it, and so forth.

[Margin: The finding of the points on the horizontal line, through which the arcs of the signs are to be drawn.]

The points on the horizontal line through which the arcs of the signs pass, and likewise the lines of the ascending signs, we shall find — as was said above — as follows. In the figure of the rays of the rising amplitudes [radiorum latitudinum ortiuarum] of Proposition 9 of this book: from the straight line EB (if the dial faces north) or from ED (if it faces south), let there be cut off the straight line EL equal to the straight line AB — the line by which, in the figure of Proposition 26 of this book, we found the point B through which the horizontal line is drawn. Through L let a perpendicular to the straight line BD be raised on both sides; this will cut the rays of the rising and setting amplitudes in points which — taken with the compass from L, and transferred onto the horizontal line from the point B — will give the points through which the parallels of the signs must pass. For since, if the triangle ABG in the figure of Proposition 26 of this book is understood to stand erect [at right angles] to the plane of the dial, the straight line AB is the common section of the horizon and the meridian, it comes about that — if the horizon ABCD in the figure of Proposition 9 of this book, in which the rays of the rising amplitudes are contained, is understood to be set in the dial so that its center E coincides with A, the center of the world, and the straight line EL with the straight line AB — the point L falls on the point B, because of the equality of the straight lines EL and AB; and hence also the perpendicular drawn through L coincides with the horizontal line, and so on. The same points can also be found on the horizontal line from Table 5 of Proposition 9 of Book 2, as was likewise done above.

[Margin: In what manner the lines of Aries and Libra are to be drawn.]

Since, however — the Sun being at the beginning of Cancer, whose arc in our example is an Ellipse, as we showed above — the sign Aries rises when the beginning of Cancer stands on the meridian circle below the horizon, that is, at the 12th hour of midnight, the line of Aries must be drawn through that point of the meridian line above the horizontal line through which the arc of Cancer passes: for this point belongs to midnight. Again, since — the Sun being at the beginning of Cancer — the sign Libra rises when the beginning of Cancer is set at noon [on the meridian], the line of Libra must be drawn through another point of

the meridian line below the horizontal line, through which the arc of Cancer passes, since this point belongs to noon, as we said above.

Book Three (Liber Tertius)

(This page opens by completing a word broken across the page-turn — the catchword "mus." of the previous page — and continues the conclusion of Problema 33.)

Both lines — that of Aries [♈] and that of Libra [♎] — run parallel to the horizontal line, or to the equinoctial line, by Proposition 18 of Book One. The reason is that in that case the horizon, the equator, the ecliptic, and the great circle to which the plane of the dial [horologij planum] is parallel all share one and the same common section — namely the axis of the meridian circle — since all those circles stand at right angles to the meridian, as is clear from a material sphere, or certainly can be readily gathered from the *Elements of Spherics* of Theodosius [Elementis sphæricis Theod., Theodosius]. In the worked example, however, the Libra line [♎] has not been drawn, because the ellipse of Cancer [ellipsis ♋] cuts the meridian line below the horizontal line at a point too far removed. We have therefore laid out the ascending signs of the zodiac on this same dial inclined to the horizon. Which was to be done.

SCHOLIUM.

[Margin: The description of the same ascending signs, from the practice of Andreas Schöner (Andreæ Schoneri).]

We shall depict the ascending signs after the method of Andreas Schöner [Andream Schonerum], as follows. In his first figure, from the diameter BD cut off the straight line EP, equal to the straight line EI — the portion of the dial lying between the center E and the equinoctial line (which in our example is exactly equal to the semidiameter EB) — and through P, at right angles to BD, raise a perpendicular on both sides. This perpendicular will cut the lines of the signs at points which, taken with the compass from P and transferred onto the equinoctial line of the dial from I, will give points from which — through B, the point where the meridian line and the horizontal line meet — hidden lines drawn out will cut the tropic of Cancer [♋], or indeed both tropics when both are drawn on the dial, at the points through which the ascending signs are to be traced. Their order you will readily understand from Tables 3 and 4 of Proposition 9 of Book Two. For since, for example, through the point of the meridian line above

the horizontal line there is drawn in our example the sign Aries [♈], and since, when the Sun is at the tropic of Cancer [♋], the sign Taurus [♉] rises after midnight — that is, before noon, as Table 3 shows — it is clear that the first point in the fore-meridian part of the dial, that is, the western part, immediately after that Aries point, belongs to the sign Taurus [♉], and the next to the sign Gemini [♊], and so on.

Again, in the later figure, from the diameter BD cut off the straight line EP equal to the straight line EI, as before (which is again equal to the semidiameter ED), and through P, at right angles to BD, raise a perpendicular. This will cut the lines of the signs at points which, taken with the compass from the point P and transferred onto the equinoctial line from I, will give the points through which the ascending signs are to be traced. Their order we shall readily gather from Tables 1 and 2 of Proposition 9 of Book Two. For since, for example, when the Sun is at the beginning of Aries [♈] the sign Taurus [♉] rises at 4 hours 51 minutes before noon — which in our dial does not cut the equinoctial line — we shall turn to Table 2, in which we shall find that, the Sun being at the beginning of Libra [♎], the sign Taurus [♉] rises at 7 hours 8 minutes after noon, and the sign Gemini [♊] at 8 hours 34 minutes after noon. I conclude that the sign Taurus [♉] must be drawn in the eastern, or afternoon, part of the dial, through a point on the equinoctial line more remote than that of the sign Gemini [♊], and so on.

The same order can also be easily gathered from the said later figure. For once the point through which the sign Taurus [♉] is to be drawn is known — the one that is always more remote than all the rest — the other signs will then follow, just as they are contained in that figure. Thus, since the sign Taurus [♉] must be drawn through the most remote point of the equinoctial line in the afternoon part of the dial, the signs Gemini [♊] and Cancer [♋][?] must be drawn through the next point, and through the following point the sign [?] [the trailing sign-glyph is not clearly resolvable on the scan], and so on.

PROBLEMA 34. PROPOS. 34.

To construct an Italian dial inclined to the horizon.

[Margin: The construction of an Italian dial inclined to the horizon.]

Let the circle described from E be divided, as we taught in the scholium to Proposition 1 of Book One, into the diurnal arc of the tropic of Cancer [♋], aNb, and the nocturnal arc, aMb. Then let the same circle, beginning from the point b, the western point, be cut into 24 equal parts, and let the rest be done as in the

preceding, and the Italian dial will have been laid out. For example, let the line of the 9th hour from sunset [horæ 9.ab occasu] be drawn. From F, the point of the 9th hour, a straight line drawn through the center E cuts the equinoctial line at the point L; and the straight line drawn from L to H, the center of the dial, and extended, cuts the tropic of Cancer [☿] at M, the point of the 9th hour from sunset. And since, as is established from Table 6 of Proposition 33 of Book One, the 9th hour from sunset also passes through the 3rd hour from midnight in the nocturnal arc of 12 hours — which in our dial is represented by the equinoctial line, since the whole of it lies above the horizontal line, or is at any rate drawn through the 3rd hour from noon or from midnight on the equinoctial line simply, whether it refers to a diurnal or a nocturnal arc, as is established from the table of Proposition 19 of Book One — the straight line from M, drawn through the 3rd hour from noon or from midnight on the equinoctial line (which in the worked example is the 3rd hour from midnight in the nocturnal arc of 12 hours), will give the 9th hour from sunset.

But if we wish to draw the line of the 21st hour from sunset [horæ 21.ab occasu], since the straight line from K, the point of the 21st hour from sunset, does not pass through the center E before it passes through the equinoctial line, we shall take the opposite point F — which is always a whole semicircle, that is, 12 hours, distant from it — so that F is the point of the 9th hour from sunset (this is obtained if to 21 we add 12, and from the sum, since it exceeds 24, we subtract 24). From F we shall draw through E and K a straight line that cuts the equinoctial line at L. For the straight line LH will cut the tropic of Cancer [☿] at M, the point of the 9th hour from sunset, as was just said; and produced beyond the center H, it will cut the same tropic at O, the point of the 21st hour from sunset, as we demonstrated in Proposition 10 of the preceding book. Since, then, the 21st hour is also drawn — as is found in the table of Proposition 19 of Book One — through the 3rd hour from noon or from midnight on the equinoctial line, the straight line through O will mark that point, and drawn through the 3rd hour from noon or from midnight on the equinoctial line it will give the 21st hour from sunset. In the same way, a straight line drawn from P, the point of the 1st hour from sunset, through the center E, and through R, the point of the 13th hour from sunset, cuts the equinoctial line at A; and the straight line AH cuts the tropic of Cancer [☿] at B, the point of the 1st hour from sunset, and produced, it cuts the same tropic at C, the point of the 13th hour from sunset, and so on.

[Margin: The description of the same Italian dial by means of the diurnal and nocturnal arcs.]

In the other cases there is no difficulty. For the hours from sunset will be most conveniently laid out by means of the diurnal arc of 14 hours alone in the upper dial — which in the nocturnal portion is the nocturnal arc of 10 hours — according to the tables of Proposition 33 of Book One. But since for the individual hours from sunset we have only single points in the diurnal arc of 14 hours, we shall take, for the same number of hours from sunrise, other points, whether in the same diurnal arc of 14 hours or in the nocturnal arc of 10 hours, as the tables of Proposition 33 of Book One indicate. Thus the 23rd hour from sunset passes through the 6th hour from noon in the diurnal arc of 14 hours, while the 23rd hour from sunrise passes through the 4th hour from midnight in the nocturnal arc of 10 hours. If, then, these two points are joined by a straight line, we shall have in the upper dial the 23rd hour from sunset in the diurnal arc, and the 23rd hour from sunrise in the nocturnal arc, as is clear from what has been demonstrated. For this line touches the greatest of the parallels that are always visible. Hence one part of it will indicate the hour from sunset,

and the other the hour from sunrise. By this same reasoning, for the 20th hour from sunset we have drawn the 20th hour from sunrise through the 12th hour from midnight in the nocturnal arc of 10 hours, and through the 2nd hour from midnight in the nocturnal arc of 12 hours. For this line, produced, likewise gives the 20th hour from sunset, and so on. Because of the narrowness of the space, however, the whole ellipse of Cancer [♋] has not been drawn — the ellipse that the following hours from sunset, 14, 15, 16, 17, 18, 19, and the corresponding hours from sunrise (which we shall treat in the following proposition), 5, 6, 7, 8, 9, 10, would have to cut.

Diagram. A large sundial-construction figure for the Italian dial inclined to the horizon (Problema 34), which also serves the Babylonian dial of Problema 35. At the upper center is a circle (the divided hour-circle described from E) with its center E, containing an inset analemma-like construction of dotted and dashed lines. From a common focus low on the figure, a dense sheaf of hour-lines radiates outward across the whole page; the hour-lines are labeled at their outer ends with Roman numerals I–XII (down both sides) and with Arabic numerals 1–24, and the sign-lines at the top carry zodiac-sign glyphs. A horizontal line spanning the figure is lettered **HORI**(left)...**ZON**(right) [= HORIZON, the horizontal line], and a vertical meridian line runs down through the lower center.

Labeled points that can be resolved on the scan (positions approximate):

- E: center of the upper circle.
- T: on the circle, upper left.

- P: on the circle, upper right.
- a: small label, upper left of the circle (western division point).
- b: small label, upper right of the circle (eastern division point).
- F: left of the circle, below T.
- K: right of center, below E.
- S: below K.
- A: on the horizontal (horizon) line, left of center.
- B: near the center, on/just below the horizontal line.
- X: near the center, right of B.
- G: below center.
- M: right of center, on the tropic.
- L: right end of the horizontal line.
- H: below center (center of the dial).
- D: below H, on the meridian line.
- O: below the horizontal line, left of center.
- C: lower right.
- Y: lower left.

Additional interior points named in the text of Problemata 34–35 (R, V, I, N) are present in the construction but are not individually resolvable against the surrounding line-sheaf on this scan. Because more than two peripheral labels (the radiating hour-numbers and zodiac glyphs, plus R/V/I/N) cannot be transcribed cell-by-cell without guessing, this figure is a for full label recovery.

We shall also aptly lay out the same hours from Table 7 of Proposition 33 of Book One, provided we first draw the diurnal arc of 24 hours, which will touch the lines of the 24th and 12th hours, and so on.

[Margin: How the line of the 12th hour from sunrise or sunset is to be drawn.]

But if someone should prefer to lay out the dial by the mutual intersections of the hours reckoned from noon or midnight with those reckoned from sunrise or sunset — the intersections contained in the tables of Propositions 19 and 20 of Book One — then the line of the 12th hour from sunrise or sunset must be drawn in this way. In the portion of the analemma of Proposition 25 of this book, from the pole E let the altitude of the pole above the horizon of the region for which the dial is being laid out be measured off on both sides, as far as M and N, and let the straight line M, N be drawn. This will be the common section of the meridian and of the greatest of the parallels always visible, if the pole E is the northern

one; or of the greatest of those always hidden, if E is the southern pole. Then, from the points M, N, drawing the straight lines MD, ND through the center D, one of them will be the common section of the meridian and the horizon, and the other the common section of the meridian and the circle of the 12th hour from sunrise or sunset — since the horizon and that circle touch that greatest parallel at the points M, N of the meridian circle. Which of the two is the common section of the horizon and which of the circle of the 12th hour and the meridian, and whether E is the arctic or the antarctic pole raised above the plane of the dial, the four analemmas described in Proposition 25 of this book will plainly teach. For one must consider whether the southern semidiameter of the horizon, BE, or the northern one, DE, drawn out through the center, meets the plane MN of the upper dial; and in which part — whether, that is, between the center M and the equator-point N, or beyond the center M, or beyond the point N. For in the first analemma the southern semidiameter of the horizon, BE, produced beyond the center E, meets the plane MN of the dial beyond N, the equator-point. In the second, the same semidiameter meets the same plane between the center of the dial M and N, the equator-point. In the third, the northern semidiameter of the horizon, DE, extended beyond the center E, cuts the plane MN of the dial beyond M, the center of the dial. In the fourth, finally, the same northern semidiameter cuts the same plane between M, the center of the dial, and N, the equator-point. From these it will not be difficult to recognize which line in the said portion of the analemma is the common section of the meridian and the horizon, and hence which is the common section of the circle of the 12th hour from sunrise or sunset; and whether the pole E raised above the plane of the dial is the arctic or the antarctic pole. For in our example, since the inclined plane is raised above the horizon on the southern side, and has an inclination greater than the complement of the altitude of the pole, it will correspond to the fourth analemma of Proposition 25 of this book. Therefore the northern semidiameter of the horizon will cut the plane of the upper dial between the center of the dial and the equinoctial line; and so the straight line MD, in the said portion of the analemma of the same Proposition 25, cutting the straight line drawn through G between H, the center of the dial, and the point I through which the equinoctial line is drawn, will be the common section of the meridian and the horizon; while the other line, ND, will accordingly be the

common section of the meridian and the circle of the 12th hour from sunrise or sunset; and E will be the northern pole, raised above the plane of the dial along the straight line HI — inasmuch as the axis FG, extended from the arctic pole F through the center E, falls in the fourth analemma at M, the center of the dial.

[Margin: Whether the arctic or the antarctic pole is raised above the plane of the proposed dial. The description of another horizontal line.]

By the same reasoning, in the first and third analemma the arctic pole will be raised above the plane of the upper dial, but in the second the antarctic pole — because in those the axis extended from the arctic pole through the center falls at the center of the dial, whereas in this last it falls there from the antarctic pole. The opposite pole, however, will always be raised above the plane of the lower dial, as is clear from the same analemmas. And so, if in our proposed example we transfer the straight line HO, from the portion of the analemma of Proposition 24 of this book, onto the meridian line of the dial from H, the center of the dial, toward the equinoctial line, we shall find the point B through which the horizontal line is drawn perpendicular to the meridian line. But if instead we transfer the straight line HP from H to the other side, we shall arrive at the point D through which the line of the 12th hour from sunrise or sunset must be drawn — a line which, by Proposition 18 of Book One, will necessarily be equidistant from the equinoctial line, or from the horizontal line, or indeed from the line of the 6th hour from noon or midnight, since the great circles of these lines, and likewise the great circle to which the dial is parallel, have one and the same common section. Therefore the straight line drawn through D, parallel to the equinoctial line, will give the 12th hour from sunrise or sunset. We have thus constructed the Italian dial inclined to the horizon. Which was to be done.

PROBLEMA 35. PROPOS. 35.

To make a Babylonian dial inclined to the horizon.

[Margin: The construction of a Babylonian dial inclined to the horizon.]

We shall lay out the Babylonian dial by exactly the same method as we described the Italian one; but in the earlier procedure the division of the circle described from E must be begun from the point a, the eastern point. You have an example in the preceding figure, in which the hours from sunrise are marked with the ancient numeral characters. There you see that, for the 11th hour from sunrise, there is drawn in the circle whose center is E — from T, the point opposite the 11th hour (namely, from the 23rd hour from sunrise), through the center E and through the point S of the 11th hour from sunrise — a straight line that cuts the equinoctial line at V. For the straight line drawn from V through H, the center of the dial, will cut the tropic of Cancer [☿] at the point X, the 23rd hour from sunrise (which is contained in the nocturnal portion of the dial, just as the point T is contained in the nocturnal arc of the tropic of Cancer [☿]); and produced

beyond the center H, it will cut the same tropic of Cancer [♋] at Y, the point of the 11th hour from sunrise, and so on. And so we have completed the Babylonian dial inclined to the horizon. Which was to be done.

SCHOLIUM.

[Margin: How the lower dial, Italian and Babylonian, arises from the upper.]

The hours from sunrise and sunset of the upper dial, produced beyond the horizontal line, give the same hours in the lower dial — the order being preserved, however, that we prescribed for the vertical dial in the scholium to Proposition 22 of the preceding book. That is, those hours which in the nocturnal portion of the dial are numbered from sunrise, such as 23, 22, 21, and so on, are to be reckoned in the lower dial from sunset; while those numbered from sunset, such as 1, 2, 3, 4, and so on, are to be numbered from sunrise.

PROBLEMA 36. PROPOS. 36.

To compose an Antique dial inclined to the horizon.

Diagram. A large sundial-construction figure for the Antique (unequal-hour) dial inclined to the horizon (Problema 36). At the upper center is a circle with its center E, containing a dotted/dashed inscribed construction (an inset analemma with chord-lines). A sheaf of hour-lines radiates outward from a focus low in the figure, their outer ends labeled with Roman numerals (I–XII and the extended forms IIII, VIIA, VIIIA, etc.) down both sides. A near-horizontal set of parallel lines crosses the middle, and lower down a horizontal line lettered **HORI**(left)... **ZON**(right) [= HORIZON]. Below the horizon line is a small dotted semicircle carrying the unequal (planetary) hours, numbered 1–12 (with a "4½" half-hour graduation visible); zodiac-sign glyphs appear at lower left and lower right.

Labeled points that can be resolved on the scan (positions approximate):

- M: top of the circle.
- E: center of the circle.
- A: on the circle, upper left.
- I: inside the circle, near the top (on the vertical).
- G, C, b: cluster at the upper right of the circle.
- D: left, on the near-horizontal parallel line.
- N: center, on that line.
- L: right, on that line.

- B: center, at the small unequal-hour semicircle below the horizon line.
- H: center, just below B.
- G: center-bottom (a second G, at the base of the semicircle).
- M: center-bottom (a second M, at the base of the semicircle).
- F: lower right.

Because the two central letters recur (an upper M/G on the circle and a lower M/G at the hour-semicircle), and because the radiating Roman/Arabic hour-numbers and the unequal-hour semicircle graduations cannot all be transcribed cell-by-cell without guessing, this figure is a for full label recovery.

The figure itself makes clear that the Antique dial inclined to the horizon is described no differently from the way the foregoing ones were constructed. For, by way of example, at the point A of the 3rd unequal hour [horæ 3.inæqualis], in the circle described from E, we took the point

[Margin: The construction of an Antique dial inclined to the horizon.]

the point opposite to C, and drew the straight line CEA, which cuts the equinoctial line at D. Through D the straight line drawn through H, the center of the dial, and extended, cuts the tropic of Cancer [♋] in the nocturnal arc at a certain point — the one that corresponds to the point C of the said circle in the nocturnal arc; but beyond H it cuts the same tropic in the diurnal arc at F, the point of the 3rd unequal hour [horæ 3. inæqualis], and so on. Yet the same dial will be laid out most conveniently with the help of the nocturnal arc of the tropic of Cancer [♋] alone, as in our example. For in this way you see that the straight line drawn from I — the point of the eleventh hour in the nocturnal arc of the tropic of Cancer [♋] — through E cuts the equinoctial line at L; while the straight line LH cuts the nocturnal arc of Cancer [♋] at M, the point of the 11th nocturnal hour.

Diagram. The large construction figure for the Antique (unequal-hour) dial inclined to the horizon (Problema 36, whose text concludes on this page). At the upper center is a circle with center **E** and top point **M**, containing an inset construction of dashed/dotted chord-lines. A sheaf of hour-lines radiates from a focus low in the figure, their outer ends labeled with Roman-numeral hours (I–XII, some in the extended forms with a superscript Λ). A pair of near-horizontal parallel lines crosses the middle of the figure; below them runs the horizon line lettered **HORI**(left)...**ZON**(right) [= HORIZON]. Below the horizon line is a small dotted semicircle carrying the unequal (planetary) hours graduated 1–12 (with half-hour marks such as "4½" and "7½" visible), flanked at lower left and lower right by zodiac-sign sorts.

Labels that can be resolved on the scan (positions approximate):

- M: top of the circle.
- E: center of the circle.
- I: inside the circle, just below M on the vertical.
- A: on the circle, upper left.
- a: on the circle, left (below A).
- C: on the circle, upper right.
- b: on the circle, right (just below C).
- D: left end of the upper of the two parallel cross-lines.
- N: center of that cross-line (below E).
- K: on that line, right of center (near L).
- L: right end of that cross-line.
- HORI...ZON: the horizon line (split label).
- B, G, H: cluster at the top/center of the unequal-hour semicircle below the horizon.
- F: on the right, between the horizon line and the semicircle.
- The radiating outer ends carry the hour numerals; on the left side (top→bottom) IIII, III, XII, XI, X, IX, VIII; on the right side (top→bottom) IIIA, XI, X, IX, I, II, III, IIII; and along the bottom (dashed) VII, VI, V, IIII. Several of these numerals and the semicircle graduations sit at the resolution floor.

Because the radiating hour-numerals and the unequal-hour semicircle graduations cannot all be transcribed position-by-position without guessing (more than two labels uncertain), this figure is a for full label recovery.

If, then, from M we draw a straight line through the fifth hour from midnight in the nocturnal arc of 12 hours — that is, on the equinoctial line, as is established from table 14 of proposition 33 of the first book — we shall obtain the 11th unequal nocturnal hour; and this, produced beyond the horizontal line, will also give the 11th diurnal hour, as is gathered from what has been demonstrated. In the same way we shall find the remaining hours.

As for laying out the portion described by the diurnal arc of 18 hours and the nocturnal arc of 6 hours, the matter is entirely clear from the figure and from the tables of proposition 33 of the first book. And so we have composed the Antique dial inclined to the horizon. Which was to be done.

ON DIALS THAT BOTH DECLINE FROM THE VERTICAL AND ARE INCLINED TO THE HORIZON.

PROBLEMA 37. PROPOS. 37.

To construct an Astronomical dial that both declines from the vertical and is inclined to the horizon.

That is, to draw the hour-lines from noon or midnight on a plane that is parallel to some great circle which both declines from the vertical and is inclined to the horizon.

So that we may describe more easily all dials that decline from the vertical circle and are inclined to the horizon, we shall give rules for laying out only the upper ones — those, namely, that face the zenith, or the top of the head. For in this way the whole matter, which is somewhat difficult and obscure, will be grasped more plainly. And once any upper dial has been drawn, we shall build the lower one from it with no trouble, since the lower dial is of course laid out on the lower face of the same plane if all its parts are inverted, as we demonstrated in the scholium to proposition 14 of the preceding book. Add to this that from these rules anyone can readily supply himself with other rules for the lower dials,

just as we too have done in the preceding pages — especially if one reads attentively what we wrote in proposition 1 of this book about the northern dial declining from the vertical circle.

We shall settle this whole business with six examples, or figures; for that many varieties seem to exist in all, as will become clear from what follows.

- In the **first** figure we set the declination of the plane from south toward east at 40 degrees, and its inclination to the horizon at 20 degrees.
- In the **second**, a declination from south toward west of 20 degrees, and an inclination to the horizon of 70 degrees.
- In the **third**, a declination from south toward east of 45 degrees, and an inclination to the horizon of 51 degrees 51 minutes.
- In the **fourth**, a declination from north toward west of 20 degrees, and an inclination to the horizon of 30 degrees.
- In the **fifth**, a declination from north toward east of 60 degrees, and an inclination to the horizon of 80 degrees.

- In the **sixth**, finally, we set the declination of the plane from north toward east at 30 degrees, and the inclination to the horizon at 52 degrees 3 minutes.

Thus in the first three examples the plane is inclined to the horizon toward the northern parts, and its upper face looks toward the zenith and the southern region; while in the latter three the plane is inclined to the horizon on the southern side, and its upper face looks toward the zenith and the northern region. That is, the plane of the first three figures cuts the meridian circle above the horizon on the northern side, whereas the plane of the latter three figures cuts it on the southern side.

[Margin: In six figures is contained the description of the dial that declines from the vertical and is inclined to the horizon.]

[Margin: The declinations of the planes of this proposition from the vertical, and their inclinations to the horizon.]

[Margin: The description of the upper Astronomical dial, which both declines from the vertical and is inclined to the horizon.]

And so, in order to describe the upper dial that declines from the vertical and is inclined to the horizon, let two straight lines AB, CD be drawn in some plane, cutting each other at right angles at E. Then, to the straight line CD, at the point

Diagram, captioned "1.^a fig.^a" (the first figure). A large circular construction for the declining-and-inclined Astronomical dial. A horizontal line **AB** runs across the figure through the center region (**A** at the far left, **B** at the far right); a second construction line **CD** crosses it. A smaller circle sits at the left of the main circle. Around the outer circumference run hour-numbers (a sequence 6, 7, 8, 9, 10, 11, 12, then 1, 2, 3, 4, 5, 6 continuing around), and a bundle of straight hour-lines converges toward the lower-center region.

Labels that can be resolved on the scan (positions approximate):

- E: center of the main circle.
- A: far left, on line AB. B: far right, on line AB.
- H: left, above line AB.
- T: on the small left-hand circle; α (alpha) beside it.
- C: above the center.
- Center cluster (very crowded): I, ψ (psi), B, K, M, and F just below.
- Φ (phi), Y (upsilon/"Y"), L: right of center.

- μ (mu): right of center, on a line toward "4".
- γ (gamma): below-left of center.
- θ (theta), b: below the center.
- D: lower center; τ (or r) nearby.
- N, m, α , b: upper-right, inside the circle.
- Outer hour-numbers: top run 7, 8, 9, 10, 11, 12; down the right 1, 2, 3, 4; down the left 6, 5, 4, 3, 2 and the symbols ς / f; along the bottom $\frac{1}{2}$, 11, 10, 9, 8, 7, 6.

The center of this figure is extremely congested and several small Greek point-labels sit at the resolution floor; with more than two labels uncertain, this figure is a for full label recovery.

At the point E let there be set up the angle of declination found by proposition 23 of the first book, observing this order. In planes that decline from south toward east, the said angle DEF is to be set up below the straight line AB, toward A; but toward B in those that decline from south toward west. In planes, however, that deflect from north toward east, the same angle is to be set up below the straight line AB, toward B; and toward A in those that decline from north toward west. Let the straight line EF be called the line of declination.

[Margin: The line of declination.]

After this, on the straight line AB, taking any point β whatever — whether to the right or to the left, as we said in proposition 1 of this book — let there be set up at AB, at the point β , the angle of the pole's altitude $E\beta C$, toward the straight line CD and either above the straight line AB or below it, in such a way that the straight line βC cuts the straight line CD at C.

Next, at the straight line CD, at E, let there be set up the angle of the plane's inclination to the horizon — found by proposition 23 of the first book — namely $DE\gamma$, below the straight line AB and toward whichever side you please; and let the straight line $E\gamma$ be called the line of inclination. Then, from D, any point of line CD, let a perpendicular line $D\gamma$ be drawn to the line of inclination $E\gamma$; and from ED let the straight line $E\delta$ be cut off, equal to the straight line $E\gamma$. Drawing also, through the points D and δ , the straight lines $D\iota$ and $\delta\theta$ parallel to AB (that is, perpendicular to CD), of which $\delta\theta$ cuts the line of declination EF at θ , let the straight line $D\iota$ be cut off equal to $\delta\theta$; and from ι , through E, let the straight line ιE be drawn out. This line will be the meridian, that is, the line of the 12th hour from noon or midnight, on the plane of the dial, as we shall demonstrate.

[Margin: The line of inclination.]

[Margin: The meridian line.]

We shall also find this same line of the 12th hour by the following method. Taking a point F anywhere on the line of declination,

let a perpendicular $F\lambda$ be drawn from it to AB ; or — which I approve of more, when the line of declination EF is only a little distant from the straight line AB — from any point λ chosen on the straight line EA or EB (according as the line of declination EF falls to the left or to the right of the straight line CD) let a line λp be drawn parallel to CD , cutting the line of declination EF at F . Likewise, at $F\lambda$ let a perpendicular Fn be raised. Then, having set up at λ the angle of inclination $F\lambda n$ — such that the straight line λn , forming the said angle together with λF , cuts the perpendicular Fn at n — let there be cut off from λF , produced toward F , a straight line λp equal to λn ; and from E , through p , let the straight line Ep be drawn out. This too will be the meridian line on the inclined plane, as we shall show. We shall give yet another way of finding this same meridian line in the scholium to this proposition.

Diagram, captioned "2.^a fig.^a" (the second figure). A large circular construction (the second of the six worked examples) for the declining-and-inclined dial. A horizontal line **AB** crosses the circle (**A** at the far left, **B** at the far right, with α beside **B**); several straight lines converge on the lower-central region, and a near-vertical line runs from top (through m , h , "12") down to the bottom (again h , m , "12"). Dotted arcs and chords fill the interior.

Labels that can be resolved on the scan (positions approximate):

- E , G , M : the crowded center cluster (E and G together, M just below).
- K , I : just above the center.
- C , e : at the top-center of the inner construction.
- λ , ϕ (ϕ), ψ (ψ): around the center.
- H : right of center, above line AB .
- A : far left on line AB ; B : far right on line AB , with α (α) beside it.
- π (π): left, on line AB ; μ (μ), and a second B , near the center-left.
- L : below the center; χ (χ), m nearby.
- γ (γ), θ (θ), b , a , δ (δ): in the lower-central region.
- D : lower left; p , F : below D toward the center; N : lower center.
- O : right, mid-low.
- λ , 6 , ς : on the right side of the circle.

- f: left; 6: upper left.
- m, h, 12: top of the vertical line, and again at its foot.

The interior is densely overwritten and many small Greek point-labels sit at the resolution floor; with more than two labels uncertain, this figure is a for full label recovery.

Furthermore, from θ let a perpendicular $\theta\lambda$ be drawn to the line of declination EF, toward the straight line AB and equal to $D\gamma$; and let the straight line $E\lambda$ be drawn through the points E and λ , of unlimited length. Let there also be set up at F, on the straight line EF (which is to be made equal to the assumed line $E\beta$), an angle equal to the angle of the pole's altitude $E\beta\gamma$ [?], drawing the straight line $F\mu$ in such a way that $F\mu$ always falls on the opposite side of the straight line $E\lambda$ — namely to the right of EF if $E\lambda$ was drawn to the left of that same EF, or the reverse. Now the angle of the pole's altitude which the straight line $F\mu$ makes with the straight line EF ought to slope toward E — that is, upward — if the plane faces south, so that the angle $EF\mu$ is equal to the angle of the pole's altitude; but if the plane looks toward the north, the same angle is to be set up on the side opposite to E, so that the angle $EF\mu$ together with the angle of the pole's altitude makes up two right angles. Let the straight line μF , produced through F toward ξ (as in the first, fourth, fifth, and sixth figures), cut the straight line drawn from E through λ ; or else let $F\mu$ itself, extended toward μ (as in the second figure), cut the straight line λE produced through E, at the point π ; and let there be cut off from the line of the 12th hour a straight line $E\varepsilon$ equal to $E\pi$ — below the point E if the point π falls either on $E\lambda$ itself or on $E\lambda$ produced toward λ , but above that same point E if π falls on $E\lambda$ or on $E\lambda$ produced through E, as happens in the second figure. For the point ε will be the center of the dial, at which all the hour-lines from noon or midnight will meet. But if it should ever happen that the straight line $F\mu$ in no way cuts the straight line drawn through the points E and λ , but is equidistant from it, then the dial will have no center at which its hour-lines meet; rather, they will all be parallel to one another, as happens on inclined planes that pass through the pole of the world — of which you have an example in the third figure.

[Margin: The center of the dial.]

[Margin: Which dial-lines lack a center.]

Besides this, at the straight line AB, at the point E, let there be set up an angle equal to the angle of the plane's inclination, either toward B or toward A; and let the straight line $F\phi$ that forms the said angle be equal to the straight line EC. This

angle is to be set up above the straight line AB if the plane faces south, as in the first three figures; but if it faces north, below the same line, as was done in the last three figures. From the point ϕ let a perpendicular $\phi\chi$ be drawn to CD. Likewise, from F, a perpendicular $F\lambda$ having been drawn to AB, let there be set up at it, at the point λ (either to the right or to the left), an angle $F\lambda\omega$ equal to the angle of inclination $DE\gamma$; and let the straight line $\lambda\omega$ be equal to the straight line λF , and from ω let a perpendicular ωd be raised to λF . For the straight line drawn through χ and d will be the line of the style, and it will necessarily pass through the center ϵ of the dial; or else, where there is no center of the dial (as in the third figure), it will be parallel to the meridian line $E\epsilon$. Thus the style, the pointer of the hours, is to be set upright to the plane of the dial along the straight line χd . It will be enough, however, if just one of the two points — namely either χ or d — is found, so that the line of the pointer, or style, can be drawn. For the straight line drawn from ϵ , the center of the dial, through that found point; or else, where there is no center of the dial, the line drawn through that point parallel

[Margin: The line of the style, or pointer.]

parallel to the meridian line $E\varsigma$, will be the line of the style. The line will be drawn more accurately, however, if both points are found — especially when the point that was found first lies only a little from the center of the dial ς .

Moreover, from the points χ and d let two perpendiculars χe and dt be drawn to the line of the index χd , equal respectively (each to each) to the straight lines $\chi\phi$ and $d\omega$, on one and the same side of that straight line χd — namely on the right, or on the left — if the plane faces south, as in the first three figures; but if it faces north, on opposite sides, one to the right of χd and the other to the left, as was done in the last three figures. For the straight line ef , when drawn, will pass entirely through the center of the dial ς , or — where there is no center — will run parallel to the meridian line $E\varsigma$; and it will be the axis of the world, to be raised above the line of the style χd by the length of the perpendiculars χe and df , which stand perpendicular to the plane of the dial.

[Margin: The axis of the world.]

Furthermore, since the axis ef , when produced, cuts the line of the style χd (produced) at the center of the dial ς — wherever the dial has a center — one must always take that part of the axis which is emitted from the center ς through f , and not the part that runs through e . This is to be understood when χe and df are drawn toward opposite sides of the line of the style, as happens on planes facing north, that is, in the last three figures. For where they are drawn from the

same side — as happens on planes facing south, as we said, of the kind represented in the first three figures — one must take the whole axis ζ -e-f, drawn from ζ through the points e and f; or else the whole axis ef produced in both directions, in a dial that lacks a center, such as the one drawn in the third figure.

Now, on the portion of the axis that runs from ζ through f, take any point I, and from it draw IG perpendicular to the axis, cutting the line of the index at the point G; through G raise GH perpendicular to the line of the style. This GH will be the equinoctial line of the dial. The point G will fall either on the straight line AB — as you see in the first three figures — or below it, as in the fourth figure, or above it, as happens in the fifth and sixth figures. Whenever it can conveniently be done, however, I would advise first taking on the line of the style the point G where the line of the style and the straight line AB cut each other, and from it drawing the perpendicular GI to the axis running from ζ through f, as was done in the first three figures. (This cannot be done in the fourth figure, because from the point where the line of the style and the straight line AB intersect, the perpendicular cannot fall on the portion ζ -f of the axis, but on the portion ζ -e.) Then, through the same point G, raise GH perpendicular to the line of the style, to serve as the equinoctial line. From I also let fall the perpendicular IK onto the line of the style. The straight line IK will be the length of the style, and its position is at K; and the angle $G\zeta f$ will be the angle of the pole's altitude above the proposed plane, as will be shown.

[Margin: The equinoctial line.]

[Margin: The length of the style and its position; and the angle of the pole's altitude above the proposed plane.]

Finally, on the line of the style χd , take a straight line GL — whether above or below G — equal to the straight line GI, and about L describe a circle of any size. Divide this circle into 24 equal parts, beginning from the straight line LM, which is drawn through L, the center of the circle, and through the point M where the equinoctial line cuts the meridian line E ζ , just as is done in a dial declining from the vertical.

Diagram. The third figure of this proposition — a full sundial-construction diagram for a plane both declining and inclined. A horizontal line runs across the middle, lettered **A** (left end) and **B** (right end); it is the common section AB of the horizontal and the declining-inclined planes. A large circle (the dividing circle described about the style-line) sits in the upper center, with **O** and ζ marked at its top. The following main points are legible:

- A (left end of the horizontal line)
- B (right end of the horizontal line)
- H (left, where an oblique line meets the circle)
- L (near the center; center of the dividing circle)
- B (a second, interior label low in the right of the circle)
- E (small, at the center)
- M (below the center)
- F (below M)
- D (low, on the vertical toward the bottom)
- k (at the very foot of the figure, below D)
- O, ζ (at the top of the circle)

Around the upper-right circumference run the hour-division marks and small labels — numerals 5, 6, 7, 8[?], 9[?] and 1, 2, 3, with letters **M** and **m** — together with a scattered swarm of small, worn Greek point-sorts (among them χ, φ, ω, λ, η, ξ, π, γ) and a "γμ"-like cluster [?] near the right center. Many of these peripheral labels are too small and worn to resolve individually off the scan.

just as is done in a dial declining from the vertical.

But if the equinoctial line happens to be parallel to the meridian line — as occurs on the plane of a dial that lies parallel to a great circle which cuts the meridian circle at the same point where it is divided by the equator, as we shall show a little below — then the division of the said circle described about L must be begun from the straight line drawn through L, the center of the circle, parallel to the equinoctial (or meridian) line, as appears in the sixth figure of this proposition, in the line MO. Here, however — since it usually happens that the center L and the point M lie so close together that the straight line LM, from which the division of the circle must begin, can scarcely be drawn without some error — one must by all means apply the remedy that we gave in Proposition 1 of this book concerning the dial declining from the vertical: namely, that we first draw the line of the sixth hour, or of some other hour, and so on, as we shall explain a little below, after the demonstration. For the straight lines drawn from the center L through the points of division will cut the equinoctial line at points through which straight lines are drawn from ζ, the center of the dial — or, where there is no center, as in the third figure, straight lines [that are] to the [meridian] line

parallel to the meridian line, or perpendicular to the equinoctial line, will give the hours from noon or from midnight, in this order.

[Margin: The hour-line. The order of the hours.]

On planes that face south, the portion of the meridian line extending from the center of the dial ς toward the equinoctial line — as in the first two figures — or the whole meridian line, where there is no center, as in the third figure, marks the 12th hour of the day (noon); the hours to its right on the equinoctial line signify the afternoon hours, and those to its left the forenoon hours. The remaining portion of the meridian line, beginning likewise from the center,

Diagram. A second full sundial-construction diagram, of the same type as the preceding one and carrying an identical caption, "3.^a fig.^a". (Two consecutive pages thus bear the same figure number — see the numbering anomaly in the report.) A horizontal line crosses the middle, lettered **A** (left) and **B** (right); a large dividing circle occupies the upper center, with **O** and ς at its top. Legible main points:

- A (left end of the horizontal line)
- B (right end of the horizontal line)
- H (left, at the circle)
- L (center; center of the dividing circle)
- B (a second, interior label, low right within the circle)
- E (small, at the center)
- M (below center)
- F (below M)
- D (low, toward the foot)
- k (at the foot of the figure)
- O, ς (top of the circle)

The upper-right arc again carries hour-division numerals (5, 6, 7, 8[?], 9[?] and 1, 2, 3), the letters **M** and **m**, a " $\gamma\mu$ "-like label [?] at the right, and a dense scatter of small, worn Greek point-sorts. The interior is crossed by very many hour-lines and construction lines; most of the small peripheral labels cannot be resolved off the scan.

shows the 12th hour of the middle of the night (midnight).

On planes facing north, when the equinoctial line lies below the center of the dial ς , on the line of the index — as is seen in the fourth figure — the portion of the meridian line extended from the center ς toward the equinoctial line will denote the 12th hour of the day (noon); the hours to its right on the equinoctial line indicate the forenoon hours, and those to its left the afternoon hours; while the

remaining portion of the meridian line, tending beyond the center toward the upper parts, belongs to the 12th hour of midnight.

But when the equinoctial line lies above the center of the dial ς , on the line of the index or style — as is seen in the last two figures of this description — the portion of the meridian line drawn from the center of the dial ς toward the equinoctial line (or at any rate tending toward the upper parts, where the equinoctial and meridian lines are parallel, as in the sixth figure) will show the 12th hour of midnight; the hours descending to the right of the meridian line on the equinoctial line are the forenoon hours, and those to the left the afternoon hours; while the remaining portion of the meridian line, below the center of the dial ς , faces north and belongs to the 12th hour of noon.

And whenever a straight line drawn through some point of division of the circle described about L, and through the center L, is parallel to the equinoctial line, then a straight line drawn through ς , the center of the dial, parallel to the equinoctial line will give that hour, whatever it may be, as we said in Proposition 1 of this book concerning the vertical declining dial. For the demonstration here is the same, provided that ς , the center of the dial, is always taken in place of C in the demonstration set out there.

[Margin: Demonstration of the construction of a dial declining from the vertical and inclined to the horizon.]

We shall demonstrate this construction as follows. Suppose that, in the plane of the dial set in its proper position, the straight line AB is parallel to the horizon, so that it is the common section of the horizontal dial-plane and of the plane of the dial that both declines and is inclined. Let the plane drawn through the straight lines AB, EF, ED be conceived to rotate about the straight line AB until it lies parallel to the horizon, and so becomes the same as the plane of the horizontal dial. In this position, since DEF is the angle of declination of the dial-plane from the vertical, the remaining angle AEF (or BEF) will be the complement of that declination — namely the angle that the meridian makes with the line that, in the declining plane, lies parallel to the horizon, or rather with the plane drawn through that straight line and standing perpendicular to the horizon. Therefore the straight line EF will be the common section of the meridian and of the horizontal dial-plane, and consequently the meridian will cut the plane of the dial that both declines and is inclined at the point E.

Now if the right triangle DEy is turned about DE until it stands perpendicular to the dial-plane, then the straight line AE — which in the dial-plane is

perpendicular to DE, the common section of the triangle DE γ and of the inclined plane — will, by Definition 4 of Book 11 of Euclid, be perpendicular to the triangle DE γ , and hence, by Definition 3 of the same book, also to the straight line E γ . Therefore, since both lines DE and γ E are perpendicular to the straight line AE, DE γ will be the angle of inclination of the proposed plane to the plane drawn through the straight lines AE and E γ , by Definition 6 of Book 11 of Euclid — inasmuch as AE is the common section of the proposed plane and of the said plane drawn through AE and E γ . Since, then, the angle DE γ has been taken equal to the inclination of the proposed plane to the horizon — and hence to the plane of the horizontal dial, which is parallel to the horizon — the plane drawn through AE and E γ will be identical with the plane of the horizontal dial, and therefore the straight line E γ will lie in the plane of the horizontal dial. And since E η [?] has been taken equal to it, if the triangle E η [?] θ — or rather the plane drawn through the straight lines AB, E θ , E η [?], in which the said

triangle stands about the straight line AB — is conceived to move until it is joined to the plane of the horizontal dial, in such a way that E θ does not depart at right angles from the plane DE γ (the inclined plane), but always makes a right angle with AE, and that the line η [?] θ always remains parallel to AE, then E θ will become the same as E γ [?], and the point η [?] the same as γ [?]. And since the straight lines η [?] θ and D ς — the former lying in the plane of the horizontal dial, as we have just said, and the latter in the inclined plane — are by construction parallel to AB, they are [a] also parallel to each other, and have been set equal; therefore [b] the straight lines connecting their end-points will likewise be parallel to each other and equal — namely the straight line γ D, dropped from the point γ (or η [?]) in the plane of the horizontal dial to the point D, and the straight line $\theta\varsigma$, drawn from the point θ in the plane of the horizontal dial to the point ς .

[Margin: a — Euclid XI.9]

[Margin: b — Euclid I.33]

Since, then, the straight line D γ is, by Definition 4 of Book 11 of Euclid, perpendicular to the plane of the horizontal dial — because, lying in the plane DE γ , it is perpendicular to E γ , the common section of the horizontal dial-plane and of the plane DE γ , which is perpendicular to the plane of the horizontal dial (for since AE has been shown to be perpendicular to the plane DE γ , [c] the plane of the horizontal dial drawn through AE will also be perpendicular to that same plane through DE γ , and conversely, therefore, the plane DE γ perpendicular to the plane of the horizontal dial) — [d] the line E θ [?] will likewise be perpendicular to the same plane of the horizontal dial, and hence, by Definition 3 of Book 11 of

Euclid, also to the straight line $E\theta$ [?]. From this it follows that the straight line $\theta\varsigma$ lies in the plane of the meridian. Otherwise, since the perpendicular drawn from θ to $\varsigma\theta$ — the common section of the horizontal dial-plane and of the meridian — in the plane of the meridian is, by Definition 4 of Book 11 of Euclid, perpendicular to the plane of the horizontal dial, two perpendiculars would be drawn from the same point θ to the same plane of the horizontal dial, [e] which cannot be. Therefore the meridian will meet the inclined plane at the point ς ; and since it also meets it at the point E , as we showed a little earlier (for the meridian is drawn through EF , the meridian line of the horizontal dial), the straight line $E\varsigma$ will be the common section of the meridian and of the inclined plane — that is, the meridian line, or the line of the 12th hour, on the plane of the dial that both declines and is inclined.

[Margin: c — Euclid XI.18]

[Margin: d — Euclid XI.8]

[Margin: e — Euclid XI.13]

That the straight line Ep is likewise a meridian line can be shown as follows. Suppose the triangle $EF\downarrow$ to move about the straight line $E\downarrow$ until it lies parallel to the horizon — that is, until it is joined to the plane of the horizontal dial — so that FE is again the common section of the meridian and of the horizontal dial-plane. Then, in this position, conceive the triangle $\downarrow Fn$ to be carried around the straight line $\downarrow F$, until it stands perpendicular both to the plane of the inclined dial and to the plane of the horizontal dial; and this will be accomplished when the straight line $n\downarrow$ is perpendicular to AB . For then the straight line AB , being perpendicular to the straight lines $\downarrow F$ and $\downarrow n$, will [f] be perpendicular to the plane of the triangle $\downarrow Fn$ drawn through those straight lines. Therefore [g] the plane of the dial — both the inclined and the horizontal — drawn through AB , and the plane of the triangle $\downarrow Fn$ will each be perpendicular to the other; and so, conversely, this plane will stand perpendicular to both of them. Therefore, since $F\downarrow n$ is

[Margin: f — Euclid XI.4]

[Margin: g — Euclid XI.18]

Diagram. The fourth figure of this proposition — the construction diagram for the north-facing declining-inclined case (equinoctial line falling below the dial center). A horizontal line crosses the upper-middle, lettered **A** (left) and **B** (right),

the common section AB. A separate dividing circle sits at the lower left, with center L. Legible points and labels:

- 6 (far left, above A — an hour-line label)
- A (left end of the horizontal line)
- B (right end of the horizontal line)
- H (on AB, left of center)
- α , β (on/near AB, toward the center)
- Ψ (or Y) and E (near the center-right of AB; E is the meridian intersection)
- C (above E)
- l, h, m, $\frac{1}{2}$ (upper right, above/around C — hour-line ends)
- φ (right of E)
- σ (far right)
- γ (lower right)
- μ , u (lower right)
- L (center of the lower-left circle)
- O, N (left side of the circle)
- b (upper part of the circle)
- m, h, $\frac{1}{2}$ (at the foot, where lines leave the circle)
- G, M, D, ε , θ , χ , ζ , ξ , λ , ω , n (interior swarm of small point-sorts, and the $\downarrow$ sort)

The interior is a very dense pencil of crossing hour-lines and construction lines, with many small, worn Greek point-sorts that cannot be resolved individually off the scan.

the angle of inclination of the plane to the horizon, and $\downarrow F$ lies parallel to the horizon, then $\downarrow n$ will lie in the inclined plane — that is, joined with the straight line $\downarrow p$, it will lie in the said plane. Therefore the point n will fall on the point p, because of the equality of the straight lines $\downarrow n$ and $\downarrow p$. Since, then, the meridian, standing perpendicular to the plane of the triangle $\downarrow FE$ and lying in the plane of the horizontal dial, passes through EF and hence through Fn — for Fn, by Definition 4 of Book 11 of Euclid, is perpendicular to the plane of the triangle EF $\downarrow$, because it is by construction perpendicular to $\downarrow F$, the common section of the triangles EF $\downarrow$ and $\downarrow Fn$ — the meridian will meet the inclined plane at the point p; and consequently the straight line Ep will be the common section of the meridian and of the inclined plane.

Therefore, since $\theta\lambda$ is perpendicular to $E\theta$ and equal to the straight line $D\downarrow$ — that is, to the straight line $\theta\varepsilon$, dropped from θ , a point of the plane of the horizontal dial, to ε , a point of the inclined plane — the triangle $E\theta\lambda$ will be entirely equal to the triangle $E\theta\varepsilon$ lying in the plane of the meridian, whose side $E\theta$ lies in the plane of the horizontal dial, $E\varepsilon$ in the inclined plane, and $\theta\varepsilon$ in the plane of the meridian. And the straight line $E\lambda$ will be equal to the straight line $E\varepsilon$, and the angle $\theta E\lambda$ equal to the angle $\theta E\varepsilon$ in the plane of the meridian.

Accordingly, if the triangle $E\theta\lambda$, lying in the plane of the horizontal dial, is conceived to be rotated about the straight line $E\theta$ until it is joined to the plane of the meridian, there will result exactly the same triangle $E\theta\lambda$ as the triangle $E\theta\varepsilon$ lying in the plane of the meridian, and the point λ will fall on the point ε . Now, once the inclined dial is set up in its own position — so that the straight line EF , lying in the plane of the horizontal dial, is the common section of that dial and of the meridian — the straight line μF is rotated until it reaches the plane of the meridian (or of the triangle $E\theta\lambda$, which we have already shown to be the same as $E\theta\varepsilon$ lying in the plane of the meridian), yet under this condition, that always inclining at the point H toward the parts of the upper hemisphere,

Diagram. The fourth figure of this proposition, printed again at the head of this page (the construction diagram for the inclined–declining case). A horizontal line runs across the upper part, lettered **A** (left end) and **B** (right end) — the common section AB — with a dense pencil of construction and hour-lines converging near its center. A separate dividing circle sits at the lower left. Legible labels:

- 6 (far left, above **A** — an hour-line label)
- **A** (left end of the horizontal line AB)
- H (on AB , right of **A**)
- α (on AB)
- β (on AB , right of α)
- Y, E (near the center of AB ; E is the meridian intersection)
- i (on AB , right of E)
- **B** (right end of AB)
- C (upper area, above center)
- $h, m, \frac{1}{2}$ (upper right — hour-line ends)
- φ (below AB , right of E)
- σ (far right)
- γ (right, lower)
- μ, u (lower right)

- L [?] (center of the lower-left circle)
- O (left of the circle), N (lower-left of the circle), b (upper part of the circle), M (right of the circle)
- $\frac{1}{2}$, h, m (at the foot of the circle, where lines leave it)
- interior swarm of small worn point-sorts (θ , ρ , G, M, D, ξ , η , ζ , and the $\downarrow$ sort) that cannot be resolved individually off the scan

More than two interior labels are unresolved.**

the axis of the world always makes the same angle $EF\mu$. For it is the axis of the world, because the angle $EF\mu$, taken in planes facing south, is equal to the elevation of the pole; while in planes facing north it forms, together with the angle of the pole's elevation, two right angles, by construction. And therefore the straight line $F\mu$, produced toward μ , passes through the arctic pole, so that the point in which it meets the inclined plane — or the line $E\lambda$, which is now the same as $E\varepsilon$, as we showed — is the point in which all the hour-lines converge at midday or at midnight, by the corollary of Prop. 21 of Book 1; and this point is usually called the center of the dial. Hence, since the axis μF cuts the straight line $E\lambda$ at ϖ , if the line $E\varpi$ is transferred onto the line $E\varepsilon$ up to the point ρ , then ρ will be the center of the dial. For the axis μF , if it is rotated together with the triangle $E\theta\lambda$ about the straight line $E\theta$ (which lies in the plane of the horizontal dial) until it is joined to the meridian and attains its proper position, will meet the given plane at that point $[\rho]$, as is clear from what has been demonstrated.

[Margin: How to determine whether the center of the dial is to be taken below the straight line AB or above it.]

It will not be difficult, moreover, to understand whether the axis meets the inclined plane below the line AB or above it. For if the axis μF , produced through F, cuts the line $E\lambda$, or that same line produced toward λ — as happens in the first, fourth, fifth, and sixth figures — the center ρ will lie below the line AB, because the line $E\lambda$ falls onto the line $E\varepsilon$ below the line AB, since the point λ falls onto the point ε , as has been said. But if the axis μF , produced toward μ , cuts the line $E\lambda$ produced through E — as is seen in the second figure — the center ρ will lie above the line AB, because the line $E\lambda$, drawn out through E, falls onto the line $E\varepsilon$ drawn out through E, as is clear from what has been said, since the point λ falls onto the point ε . But if the axis $F\mu$ does not cut the line $E\lambda$ at all, the inclined plane will be parallel to the axis $F\mu$, and consequently the dial drawn on it will have no center, but all its hour-lines will be parallel, as is clear from the corollary of Prop. 22 of Book 1.

Now the index-line [linea indicis] — the line, that is, on which the style is to be fixed — must be such that the style, or any other line drawn perpendicular to the plane of the dial from any point of it, falls upon the axis of the world, in such a way that the plane drawn through that perpendicular and the axis of the world stands at right angles to the plane of the dial, serving as a kind of proper meridian of the dial-plane itself, as we showed in Prop. 1 of this book. We shall demonstrate that the line $\chi\downarrow$ is such a line, in this way. If the right triangle $E\phi\chi$ is moved about the line $F\chi$ until it stands perpendicular to the inclined plane — upward in planes facing south, downward in planes facing north — then, by Definition 4 of Book 11 of Euclid, the line AE [?] lying in the inclined plane, which is perpendicular to

[Margin: 18. undec. — Euclid XI.18]

$F\chi$, the common section of the inclined plane and of the triangle $E\phi\chi$, holds this position: it is perpendicular to the triangle $E\phi\chi$, and hence also to the line $E\phi$, by Definition 3 of the same book. Since, then, both lines $F\chi$ and $E\phi$ are perpendicular to the line AE , the angle $\phi E\chi$ will be the angle of inclination of the given plane to the plane drawn through the lines AE and $E\phi$, by Definition 6 of Book 11 of Euclid, since AE is the common section of the given plane and of the plane drawn through the lines AE and $E\phi$.

Now since the angle $\phi E\chi$ is equal to the complement of the inclination of the given plane to the horizon (for the angle $BE\phi$, or $AE\phi$, was taken equal to the inclination of the plane to the horizon) — that is, equal to the inclination of the plane to that vertical circle which is drawn through the line AB — the plane drawn through the lines AE and $E\phi$ will be the same as the plane of that vertical circle; and therefore the line $F\phi$ will lie in that vertical circle. And since the line $E\phi$ was taken equal to the line EC , and the line EF to the line $E\beta$, the axis of the world $F\mu$, meeting the plane of the horizontal dial at F , will pass through the point ϕ in that said vertical circle; because in this way the axis $F\mu$ will form, with the meridian line of the horizontal dial EF at F , the angle of the pole's elevation above the horizon toward the north, and with the line $E\phi$ — through which the said vertical, indeed the vertical properly so called, is drawn — the angle of the complement of the pole's elevation above the horizon. So that the axis $F\mu$ forms with the lines EF and $E\phi$ a right triangle in the plane of the meridian circle, entirely equal and similar to the triangle $E\beta C$, since not only EF but also $E\phi$, in that position, lies in the plane of the meridian, and the angle $FE\phi$ is right — which we shall show thus.

Since that vertical circle drawn through the lines AB and E ϕ is perpendicular to the plane of the horizontal dial, by Definition 4 of Book 11 of Euclid the line E ϕ , lying in that vertical circle — which has been shown to be perpendicular to AE, the common section of the said vertical and of the plane of the horizontal dial — will be perpendicular to the plane of the horizontal dial; and consequently, by Definition 3 of Book 11 of Euclid, also to the line EF lying in the plane of the horizontal dial. Therefore the angle FE ϕ is right. From this it follows that the line E ϕ lies in the plane of the meridian; otherwise, since the perpendicular from E to EF (the common section of the plane of the horizontal dial and of the meridian), drawn in the plane of the meridian, is perpendicular to the plane of the horizontal dial, by Definition 4 of Book 11 of Euclid two perpendiculars would be drawn from the same point E to the same plane of the horizontal dial —

[Margin: a — which cannot be.]

The reason, moreover, why the triangle E ϕ χ must be moved upward in planes facing south, but downward in those facing north — provided that in the former it is constructed above the line AB, and in the latter below it, as we prescribed — is this:

[Margin: b — 13. undec. — Euclid XI.13]

namely, that in the former the axis of the world F μ , passing through the point F in the plane of the horizontal dial, meets that vertical circle drawn through the line AB above the line AB, but in the latter below the line AB, after it has met the inclined plane — as is evident if the inclined planes, the plane of the horizontal dial, the said vertical, and the axis of the world F μ are conceived in their proper positions.

Again, if the right triangle $\downarrow d\omega$ is moved upward about the line $\downarrow d$ until it stands perpendicular to the inclined plane, then, by Definition 4 of Book 11 of Euclid, the line A $\downarrow$, lying in the inclined plane and making right angles with $\downarrow d$ (the common section of the triangle $\downarrow d\omega$ and of the inclined plane), will be perpendicular to the triangle $\downarrow d\omega$, and hence, by Definition 3 of the same book, also to the line $\downarrow \omega$. And since each of the lines $\downarrow \omega$ and $\downarrow d$ is perpendicular to the line A $\downarrow$, by Definition 6 of the same book $\omega \downarrow d$ will be the angle of inclination of the given plane to the plane drawn through the lines A $\downarrow$ and $\downarrow \omega$, since A $\downarrow$ is the common section of the given plane and of that which is drawn through the lines

Diagram. The fifth figure of this proposition. A horizontal base line runs across the lower part, lettered **A** (left end) and **B** (right end), with a large circle set at the

upper right and a dense pencil of construction lines converging near the lower center at F. Legible labels:

- M (top)
- a (below M)
- m, h (upper left of the figure)
- C (left, below m)
- π , T (center-left, above the base line)
- A (left end of the base line)
- β (on the base line, left of center)
- E (center of the base line)
- B (right end of the base line); O (just left of B)
- circle (upper right): center L; λ (left-upper of the circle), b (right-upper), N (top-right), σ (far right)
- G (interior), K, H (interior, near E–F)
- i (right, on a line running to the far right)
- F (below E, lower center — the convergence point)
- δ , χ , β (below and around F)
- h, m, η (lower center)
- D (bottom left), E [?] (bottom center), ς [?] (bottom right)
- 6 (lower left), P [?] (at the very bottom)

More than two interior labels are unresolved.**

$A\downarrow$ and $\downarrow\omega$ are drawn. Since, then, the angle $\omega\downarrow d$ is taken equal to the inclination of the given plane to the horizon — and hence also to the plane of the horizontal dial, which is parallel to the horizon — the plane drawn through the lines $A\downarrow$ and $\downarrow\omega$ will be the same as the plane of the horizontal dial; and consequently the line $\downarrow\omega$ will lie in the plane of the horizontal dial. And since this line was taken equal to the line $\downarrow F$, if the triangle $EF\downarrow$ is moved about the line $E\downarrow$ until it is joined to the plane of the horizontal dial, $\downarrow\omega$ will become the same as $\downarrow F$, and the point ω the same as F, because in that

motion the line $F\downarrow$ always makes right angles with $F\downarrow$, and always remains in the plane of the triangle $\downarrow\omega d$; otherwise, in the plane of the horizontal dial, two perpendiculars $\omega\downarrow$ and $F\downarrow$ would be drawn to the line $E\downarrow$ at the point $\downarrow$ — which is absurd. Since, then, the axis of the world $F\mu$ passes through F, a point of the horizontal dial, it follows that it also passes through the point ω of the triangle $\downarrow\omega d$, which has that position. This being so, since both the line $\phi\chi$ and the line

ωd are perpendicular to the inclined plane, by Definition 4 of Book 11 of Euclid (if in that position the triangles $E\phi\chi$ and $\downarrow \omega d$ are understood to be placed), and therefore also to the line $\chi \downarrow$, by Definition 3 of the same book, it follows that the lines $\phi\chi$ and ωd are parallel,

[Margin: a — 6 & 7. undec. — Euclid XI.6 & 7]

and therefore lie in the same plane, which is drawn through the lines $\phi\chi$, ωd , and χd ; and this plane is perpendicular to the inclined plane and passes through the axis of the world, which we demonstrated to pass through the points ϕ and ω .

[Margin: b — 18 & 7. undec. — Euclid XI.18 & 7]

Therefore the plane drawn through the lines $\phi\chi$, ωd , $\chi \downarrow$, standing perpendicular to the inclined plane, will serve as a kind of new and proper meridian of the inclined plane itself; and in this new meridian all the perpendiculars drawn to the line $\chi \downarrow$ are likewise perpendicular, by Definition 4 of Book 11 of Euclid, to the inclined plane, and they meet the axis at the points

Diagram. The fifth figure again, printed at the head of this page (the same construction as fig-p329-1, with the circle at the upper right and the convergence at F below center). Legible labels, as far as they resolve:

- M (top); a (below M)
- m, h (upper left); C (left, below m)
- π , T (center-left, above the base line)
- A (left end of the base line); β (on the base line); E (center); B (right end), O (left of B)
- circle (upper right): center L; λ (left-upper), b (right-upper), N (top-right), σ (far right)
- G, K, H (interior, near E–F); i (far right, on a line)
- F (lower center — convergence point); δ , χ , β (around F)
- h, m, η (lower center); D (bottom left); E [?] (bottom center); ς [?] (bottom right)
- 6 (lower left); P [?] (at the very bottom)

This is the same figure as fig-p329-1; more than two interior labels are unresolved.**

ϕ , ω , through which the axis passes. Accordingly, the line χd will be the index-line — namely, the common section of the plane of the dial and of that proper meridian — like a meridian line, as though the circle to which the dial is parallel

were the horizon; since any style whatever erected on it perpendicular to the inclined plane cuts the axis of the world, as we said, just as happens in other dials. That this index-line $\chi\downarrow$, in dials that have a center, must be drawn through the center of the dial ρ , is evident. For since the axis passes through ρ , the center will cut entirely that plane which stands perpendicular to the plane of the dial and passes through the axis — namely, that new meridian — cutting the plane of the dial at ρ ; and therefore the common section of that meridian and of the plane of the dial will pass through ρ . Finally, in dials that lack a center, that the same index-line χd is parallel to the meridian line, or the line of the 12th hour, will become manifest in this way. Since both the meridian of the horizon and that proper meridian of the inclined plane — which, of course, makes the index-line χd in the plane — pass through the axis of the world, the sections which they make in the inclined plane, that is, the meridian line and the index-line, will be parallel, by Prop. 18 of Book 1, since the plane of the dial is parallel to the axis, because it does not cut it, as has been said.

Let it also be understood that the lines χe and df are moved about the line χd until they are perpendicular to the inclined plane — both upward in those dials which face south, but in those which face north, the line χe downward and the line df upward. For in this way the points e, f will become the same as φ and ω , on account of the equality of the lines $\chi\varphi, \chi e$ and $d\omega, df$. Since, then, the axis of the world passes through the points φ, ω , as we have now demonstrated, it will pass through the points e, f in that position. And since the axis passes also through the center ρ , or — where there is no center — is parallel to the index-line χd (for, as we demonstrated a little before, the meridian line and the index-line in a dial without a center are parallel, because each is parallel to the axis of the world, as is clear from the demonstration of Prop. 18 of Book 1), it follows that the line ef also passes through the center ρ , or is parallel to χd . For if it is rotated together with the lines $\chi e, df$ about χd , it will be joined to the axis, so that the axis is the same as the line ef . Therefore the axis must be elevated from the center ρ according to the angle $\varphi\downarrow d$, or $\varepsilon\downarrow\varphi\chi$ [?], which is the angle of the pole's elevation above the inclined plane —

[Margin: c — 29. primi — Prop. 29 of Book 1]

(because such an angle is equal to that which the axis of the world and the common section of the new meridian of the inclined plane and of the greatest circle — to which the plane of the inclined dial is parallel — form together,

[Margin: d — 16. undec. — Euclid XI.16]

because this common section is parallel to the line χd in the plane of the dial). It is, moreover, manifest that this angle, constituted in the proper meridian of the inclined plane and standing at the center of the world, rests upon the arc of the pole's elevation above that greatest circle to which the dial is parallel. Or else, where there is no center — as in the third figure — the axis must be elevated according to the perpendiculars χe , df , which are equal to one another, because the axis ef is parallel to the index-line χd , as has been proved. It will, moreover, be easy to understand why, in planes facing south, both lines χe and df must be drawn on the same side of the line χd , whereas in planes facing north, one on the right side and the other on the left. For since in the former, as we said, each triangle $E\phi\chi$ and $\phi\omega d$ is rotated upward about the lines $E\chi$ and ϕd until it stands perpendicular to the inclined plane, both lines χe and df must be drawn on the same side of the line χd , so that when each is rotated until it stands perpendicular to the inclined plane on the upper side, the points e , f fall on the points ϕ , ω through which the axis is drawn, and each of which lies on the upper side.

Now since in these figures the triangle $E\phi\chi$ is understood to be moved downward and the triangle $\phi\omega d$ upward about the straight lines $E\phi$ and ϕd , as we have said, until each stands perpendicular to the inclined plane, one of them must be drawn on the one side and the other on the other. In this way, when each is rotated about until it is perpendicular to the inclined plane, the line df will be raised up from the upper part of the plane, and its point f will fall onto the point ϕ [?] (which likewise lies above the plane); while the line χe will be set up from the lower part, and its point e will become identical with the point ω [?], which also lies below the plane. For in this way the line ef will represent the axis of the world, which we showed passes through the points ϕ and ω ; indeed it follows that this same line is constituted out of ef and the axis of the world. In the same way the reason is clear why in the former dials the whole axis ϕfe is taken, whereas in the latter only the portion ϕf drawn to the axis, and not ϕe . It is because in the former the whole axis ϕfe stands above the inclined plane, since both points f and e lie above that same plane; but in the latter the portion ϕf of the axis lies above the plane while ϕe lies below it, because the point f lies above the plane and the point e below it, as is evident from what has been said.

Now the axis of the world ϕf is at right angles to the equator — by a proposition of Theodosius, Book 1 — and passes through its center; and therefore, by definition 3 of Euclid's Book 11, it makes right angles with any straight line drawn in the plane of the equator from the equator's center. Suppose, then, that the point I on the axis is taken as the center of the world, or of the equator. (For any point whatever of the axis ϕf may be taken as the center of the world, since its

distance from the center of the world is imperceptible and beyond sensation when compared with its distance from the Sun, as we showed in the *Sphere*.) Then let the straight line GI, which is perpendicular to the axis ef and cuts the index-line at G, be the common section of the equator and of the plane drawn through the straight lines χc and df — which are perpendicular to the inclined plane — and through the index-line χd and the axis ef. [Margin: a — 18. undec. — Euclid XI.18] This latter plane stands at right angles to the inclined plane, after the fashion of a certain new and proper meridian of the inclined plane itself. Hence the line GI lies not only in the plane GI ϕd but also in the plane of the equator, since the axis, together with GI in the plane GI ϕd , makes a right angle at I. For otherwise, if the equator did not pass through the straight line IG but through some other line drawn from the point I (which has been taken as the center of the equator), the axis would — by definition 3 of Euclid's Book 11 — be perpendicular to that line as well, since it is at right angles to the plane of the equator. In that case two perpendiculars to the axis would be drawn from the point I in the plane GI ϕd , which is absurd. The plane of the equator, then, drawn through the straight line IG, [Margin: b — 18. undec. — Euclid XI.18] will meet the plane of the inclined dial at the point G of the index-line, and will cut it there; and so through the point G the equinoctial line will be drawn — that is, the common section of the equator and of the plane of the inclined dial.

Now since the plane GI ϕd is at right angles to the equator (because the axis ef, through which it is drawn, is at right angles to the equator, as we have said — which is also established from Proposition 15 of Book 1 of Theodosius, since the plane GI ϕd is drawn through the equator's axis ef and hence through its poles), the equator in turn will be at right angles to the plane GI ϕd . [Margin: c — 19. undec. — Euclid XI.19] But the plane of the inclined dial is likewise at right angles to that same plane GI ϕd , since it has just been shown to be perpendicular to it. Therefore the common section of the equator and of the plane of the inclined dial will also be at right angles to the plane GI ϕd , and consequently — by definition 3 of Euclid's Book 11 — it will be perpendicular to the index-line χd , which lies in that plane. And since, as we have just shown, this common section must be drawn through the point G,

Diagram. A large circular sundial-construction figure (the "sixth figure") occupying most of the page: a bounding great circle with a full set of radial hour-lines that radiate from the construction center and are numbered around the rim with Arabic numerals 1 through 12 (12 at the top, running through the sequence around the circumference). Superimposed on the circle is a smaller semicircular analemma (left of center) and two long axis-lines each labelled **AXIS**, crossing the

field obliquely. The figure combines the analemma construction with the projected hour-lines of a dial that is both declining and inclined.

Clearly-read perimeter and construction labels:

- 12 (top center): the noon / hour-12 marker
- 1, 2, 3, 4 (upper right, clockwise from 12): hour markers
- 5, 6, 7, 8 (continuing around toward the lower right/bottom): hour markers
- 2, 3, 4, 5, 6 (lower left, continuing the ring): hour markers
- B (upper left, on the circle); B (right, on the circle)
- F (upper left, inside near B)
- H (left) and H (upper right): endpoints of a line crossing the field
- G (left of center)
- M (top center, below "12")
- A (left-center) with a small subscript numeral near it
- E (center-left, near the analemma)
- C (lower left, on the circle)
- K, I (lower center): the style-foot and center-of-world points
- I (bottom center) and II (lower right)
- λ (Greek lambda, right of center)
- Additional small interior letters and Greek subscripts near the center (φ , χ , d, e, f region) are too small to resolve individually.

Many peripheral labels and the dense interior lettering cannot be resolved at this resolution

It will be the line GH, drawn perpendicular to χd , that is the common section of the equator and of the plane of the inclined dial — that is, the equinoctial line. And since the point I has been taken as the center of the world, from which the straight line IK falls perpendicular to the plane of the inclined dial (by definition 4 of Euclid's Book 11, because it is drawn perpendicular to the index-line χd , the common section of the plane of the dial and of the plane GIfd, which is at right angles to the dial), the straight line IK will be the length

of the style, and its foot will be at K, a point on the index-line, since no other straight line perpendicular to the plane of the dial, apart from KI, can fall onto the center of the world I, as is evident.

Now if the plane of the circle described from L be understood to be rotated about the equinoctial line GH until its center L is joined to the center of the world I —

and it will indeed coincide with it entirely, because the straight lines GI and GL are equal to each other and each is perpendicular to the equinoctial line, provided the plane GI^{fd} be conceived as at right angles to the plane of the dial — then the straight lines drawn through the center L (which is now the same as the center of the equator) and through the divisions marked off on the circle will be the common sections of the equator and of the hour-circles reckoned from noon or from midnight, just as we showed for the horizontal dial in Proposition 1 of Book 2. For in that position the said circle, having the same center as the equator, lies in the plane of the equator. Now the division of the said circle begins from the

Diagram. A second large circular sundial-construction figure, again captioned as the "sixth figure" and closely resembling the figure on the preceding page: a bounding great circle carrying a full set of radial hour-lines numbered 1–12 around the rim, a semicircular analemma set to the left of the center, and two oblique lines each labelled **AXIS**. It shows the same declining-and-inclined dial construction, here developed for the division of the circle and the drawing of the hour-lines through the center L / the horologium center.

Clearly-read perimeter and construction labels:

- 12 (top center); 1, 2, 3, 4 (upper right clockwise); continuing hour numerals around the rim through the lower right and bottom (...9, 10, 11 read at right and lower right); 3, 4, 5, 6 (lower left)
- B (upper left, on circle); B (right, on circle)
- F (upper left)
- H (upper left) and H (upper right): endpoints of a crossing line
- G (left of center)
- M (top center, below "12")
- L (center): the center of the described circle / horologium center
- A, E (center-left, at the analemma); Y near center
- C (lower left, on circle)
- N (lower center-left)
- K, I (bottom center); I and II (bottom, right of center)
- h (small, top center and bottom center): analemma division marks
- m (lower right, near "12")
- Additional small interior Greek/Latin subscripts near the center are unresolved.

Numerous peripheral hour numerals and the interior lettering cannot be read with confidence at this resolution

straight line LM — the line drawn through the center L and through the point M, where the meridian line and the equinoctial line intersect; or, when the meridian line and the equinoctial do not cut one another but are parallel, the line drawn through L parallel to the meridian line. This is because that line is the common section of the equator and the meridian, that is, of the hour-12 circle, since it meets the plane of the dial at the point M through which the meridian and the equinoctial pass; or else it is parallel to the meridian line, as the reasoning requires, when the meridian line and the equinoctial are parallel — which happens when the plane of the dial is parallel to the great circle that cuts the meridian at the same point where it is cut by the equator. For when on this account the plane of the dial is parallel to the common section of the meridian and the equator (since the great circle to which the plane of the dial is parallel passes through that common section), the common section made by the plane of the dial in the meridian — that is, the meridian line itself — will be parallel to the common section of the meridian and the equator, as is established from the demonstration of Proposition 18 of Book 1.

Since this is so, the hour-circles will cut the plane of the inclined dial at the same points where the straight lines drawn through the center L and the points of division on the circle described from L — being the common sections of the said circles and the equator — meet the equinoctial line. And thus the common sections of those same circles and the plane of the inclined dial, that is, the hour-lines, will have to be drawn through those same points. Since, then, these same lines (by the corollary of Proposition 21 of Book 1) cut one another mutually at the point ρ [?], the center of the dial, or (where there is no center) are parallel to one another, by the corollary of Proposition 22 of the same book, the straight lines drawn through the points on the equinoctial line and through the point ρ [?] — or, where there is no center, drawn parallel to the meridian line — will be the lines of the hours reckoned from noon or from midnight; that is, the common sections of the plane of the dial and of the hour-circles reckoned from noon or from midnight.

Which are the hours from noon and which from midnight can readily be understood if the circle described from L be conceived together with the dial in its proper position, and if those things be carefully considered which we wrote in the scholium to Proposition 1 of Book 2 concerning the intersections of the equinoctial line and of the straight lines of the circle described from L. One must always observe that, if the dial and the said circle are understood in their proper position, the points of division that follow the common section of the meridian and the equator (such as is the straight line LM) toward the west, on the upper

side, give the hours from noon; while the points that follow that same straight line LM toward the east, on the lower side, give the hours from midnight. And since, as we taught in the scholium to Proposition 1 of the

preceding book, the straight lines must first pass through the center L before they meet the equinoctial line, it will be easy to recognize which points of the equinoctial line indicate the hours from noon and which the hours from midnight, [Margin: How to determine which hours are from noon and which from midnight] so that the order of the hours we set out above may be correct. For if we weigh the matter rightly and carefully, we shall find that in dials which face the zenith and look toward the south, the equinoctial line is the section of the plane of the dial and the equator upon which the Sun's rays fall while the Sun is in the semicircle of the equator above the horizon. Hence in those dials the portion of the meridian line that cuts the equinoctial line will indicate the 12th hour of noon; and the lines toward the eastern parts will show the hours after noon — namely those which, when we turn to face the dial, lie on our right. The same holds for dials that face the zenith and look toward the north and have the equinoctial line below the center of the dial, except that the eastern parts lie on our left when we turn to face them. But in dials facing the zenith which look toward the north and have the equinoctial line above the center of the dial, the equinoctial line is the common section of the equator and the plane of the dial upon which the shadows fall while the Sun is in the semicircle of the equator below the horizon. Therefore in these dials the portion of the meridian line that cuts the equinoctial line will show the 12th hour of midnight, and the lines toward the western parts — which lie on our right when we turn our face to the dial — will give the hours after midnight, as we said above.

But if from F there be drawn straight lines making, with EF (the line of declination, which we showed to be the meridian line in a horizontal dial), the same angles that the hour-lines make with the meridian line in a horizontal dial — namely the line of the 6th hour, such as Fa, a right angle with FE, and so on — [Margin: Another method of drawing the hour-lines from the horizontal dial] then the straight line AB will be cut at the points through which the lines drawn out from the center ρ [?] — or, where there is no center, drawn parallel to the meridian line — will give the hour-lines, as before, just as we demonstrated in Proposition 1 of this book. On this matter see what we wrote there concerning the perpendicular Fa, so that by means of it, or of some other line drawn out from F, the division of the circle described from L may be rightly established, as we also advised above in this proposition.

Another way. We shall also construct this same dial, both declining and inclined, after the fashion of a horizontal dial, just as we did in the declining dial reckoned from a vertical in Proposition 1 of this book, by the following art. [Margin: Another construction of a dial at once declining and inclined] First, let a portion of the analemma be set up, as in Proposition 1 of the preceding book, in which are contained the common sections, with the horizon and with the vertical and the equator and so forth, of the proper meridian of the declining-and-inclined plane itself. (The great circle to which the plane of the declining-and-inclined dial is parallel plays the role of the horizon, and another great circle cutting it at right angles and passing through the common sections of the equator and the said horizon serves as the vertical circle.) This is done so that the arc CE may measure the elevation of the pole above the declining-and-inclined plane, and the straight line DG be taken equal to the style of the declining-and-inclined dial, of whatever magnitude, and so on. In our example, which we have fitted to the sixth figure — near which we have drawn the said portion of the analemma — the arc CE contains $66^{\circ}47'$; for this is very nearly the elevation of the pole found above that inclined plane, and the straight line DG has been taken equal to the style IK of the same dial. [Margin: The elevations of the pole above the planes of the six figures of this proposition [?]] In the first figure the same arc CE, if a portion of the analemma be constructed to answer to that figure, will contain $25^{\circ}43'$; in the second, $25^{\circ}18'$; in the third, $0^{\circ}0'$; in the fourth, $68^{\circ}13'$; and finally in the fifth, $27^{\circ}50'$. For these are very nearly the elevations of the pole found above those inclined planes, according to the doctrine of Proposition 29 of Book 1.

After this, let the straight lines GN and GH be drawn cutting one another at right angles in G, of which GN represents the common section of the plane of the dial and the proper meridian of that plane — after the manner of a certain meridian line, in which the style is to be placed — and GH represents the equinoctial line; just as in the horizontal dial the straight line HE is the meridian line, that is, the common section of the plane of the dial and the meridian, in which the style is set, while the straight line FK serves as the equinoctial line. [Margin: What the arc of the meridian is between the horizon and the inclined plane [?]] Now onto the straight line GN — which is like a meridian line, taken in the inclined plane in place of some horizon — the straight line HI is to be transferred, which in the portion of the analemma lies between the axis and the diameter of the equator; measuring it off from the equinoctial line at the point G up to the point ρ [?], so that the center of the dial may be obtained — just as in the horizontal dial, on the straight line HE, the line IH between the equinoctial line and the center of the dial is equal to the straight line HI in the portion of the analemma. But since in these dials that are at once declining and inclined the center ρ [?] lies sometimes below

the equinoctial line and sometimes above it, in order to know when the center lies below it — and the line HI must accordingly be transferred below it, so that we may obtain the center ρ [?] — and when above, and so on, we must investigate, by Proposition 28 of Book 1, the arc of the meridian intercepted between the inclined plane and the horizon. For once this is known, what is sought will be attained without any trouble. [Margin: [illegible marginal note — several lines partly abraded; [?]]] In the first figure we found this arc, on the northern side above the horizon, to be $15^{\circ}35'$; in the second, $68^{\circ}30'$; in the third, $4^{\circ}6'$, so that the plane of this figure passes through the pole. Again, in the fourth figure we found the same arc, on the southern side above the horizon, to be $28^{\circ}29'$; in the fifth, $72^{\circ}57'$; and finally in the sixth, $48^{\circ}0'$, so that the plane of this figure passes through that point of the meridian where it is cut by the equator.

If, then, the plane be inclined toward the horizon on the northern side, and the arc of the meridian placed between it and the horizon be smaller than the elevation of the pole above the horizon, the portion of the analemma, the straight line HI, is to be taken and transferred onto the line GN from G downward, below the equinoctial line, up to the point ρ [?], which will be the center of the dial; for then the center of the dial lies below the equinoctial line, as is seen in the second figure, because then the axis of the world meets the upper face of the plane of the dial above the horizon on the southern side, just as in the horizontal dial.

[Margin: When the center of the dial lies below, and when above, the equinoctial line [?]] But if the plane be raised above the horizon on that same northern side, and the arc of the meridian placed between it and the horizon be greater than the elevation of the pole above the horizon, the straight line HI is to be transferred from the portion of the analemma onto the line GN on the upper side, and the center of the dial will be above the equinoctial line; for then the center of the dial lies above the equinoctial line, because then the axis of the world meets the upper face of the plane of the dial from the upper, that is northern, parts, so that with the line of the style described on that face it makes an acute angle toward the opposite pole, that is, toward the lower parts, as happens in the southern vertical dial.

Finally, if the plane is raised above the horizon on the same northern side, and the arc of the meridian lying between it and the horizon is equal to the elevation of the pole — so that the plane passes through the poles of the world — then the dial will have no center, as happens in the third figure. In that case nothing is to be transferred; the dial must instead be constructed by another method, as we shall show a little below.

If the plane is inclined toward the horizon on the southern side, and the arc enclosed between it and the horizon is smaller than the complement of the pole's elevation above the horizon, then the same straight line HI, taken from the portion of the analemma, must be transferred onto the line GN — from G, above the equinoctial line, up to the point ρ [?]. This point ρ [?] will be the center of the dial, lying above the equinoctial line, as may be seen in the fourth figure; for then the axis of the world meets the upper face of the plane of the dial above the equinoctial line, toward the upper — that is, southern — parts, just as in the horizontal dial.

Lastly, if the plane is inclined toward the horizon on that same southern side, and the arc of the meridian enclosed between it and the horizon is equal to or greater than the complement of the pole's elevation above the horizon, then the straight line HI, from the portion of the analemma, must be transferred onto the line GN — from G, below the equinoctial line, up to the point ρ [?]. This point will be the center of the dial, lying below the equinoctial line, as appears in the last two figures. For when the said arc is equal to or greater than the complement of the pole's elevation, the axis of the world cuts the upper face of the plane of the dial toward the lower, or northern, parts, so that with the style-line drawn on that face it makes an acute angle toward the arctic pole — that is, toward the upper parts — as usually happens in the northern vertical dial.

These same things are gathered even more clearly than daylight from those four analemmas set out in Proposition 27 of this book. [Margin: By what reasoning, from the four analemmas set out in Proposition 27 of this book, it may be known when the center of the dial is below the equinoctial line and when above.] For suppose that in the first two analemmas, on the northern side, the arc DK is taken equal to the arc of the meridian lying between the plane and the horizon; while in the latter two, on the southern side, the arc BK is set equal to that same arc of the meridian between the plane and the horizon. Then the straight line KL will be the common section of the meridian and of the great circle to which the plane of the dial is parallel. Therefore MN, parallel to KL, will be the common section of the meridian and of the plane of the dial (which is parallel to that great circle) — that is, the meridian line in the inclined dial. For the meridian, cutting the said great circle and the plane of the dial parallel to it, makes parallel sections. Hence, where the axis of the world FG cuts the straight line MN at M, there will lie the center of the dial on the meridian line; and the point N, on that same meridian line, will be the point through which the equinoctial line is drawn, since at that point the Equator cuts the meridian line.

From this it also follows that if the portion of the axis EM is taken equal to the portion of the axis ρI [?] — the portion, in the dials treated above, between the center of the world I and the center of the dial ρ [?], where the axis of the world cuts the meridian line — and if through M the line MN is drawn parallel to KL (provided the arc DK or BK is equal to the arc of the meridian found between the plane and the horizon), [Margin: (a) Euclid, *Elements* XI.16] cutting the diameter of the Equator, then MN is equal to the straight line ρM [?] — that is, to the portion of the meridian line between the center of the dial and the point M of the equinoctial line — while the straight line EN is equal to that line in the dial which falls from the apex of the style (that is, from the center of the world I, set on high) onto the point M.

For just as the triangle EMN is set up in the analemma out of the axis EM, the common section of the meridian and the inclined plane of the dial MN, and the common section of the meridian and the Equator EN, so too in the dial the triangle ρMN [?] — lying in the plane of the meridian, if the axis ρI [?] is understood in its proper position (for the meridian passes through the axis ρI [?] and through the point M) — is composed of the same lines, and is altogether equal to that first triangle. For the angle EMN in the analemma, which the axis makes with the meridian line of the dial, is equal to the angle $I\rho M$ [?] in the dial, which the axis (if placed in its proper position) makes with the meridian line; and the angle MEN is a right angle equal to the right angle ρIM [?]. [Margin: (b) Euclid, *Elements* I.26] (For the Equator cuts the axis at I at right angles, and therefore, by Definition 3 of Book XI of Euclid, the axis makes right angles with the straight line IM lying in the Equator.) The straight line EM, moreover, is set equal to the straight line $I\rho$ [?]; therefore the straight line MN too will equal ρM [?], and the straight line EN will equal IM.

Now the analemmas themselves plainly indicate whether the center of the dial is below the equinoctial line or above it. For since in the first and fourth analemma the center M, on the meridian line MN of the dial, lies [below [?]] the point N — through which the equinoctial line must be drawn — while in the other two intermediate analemmas it lies above that same point N, it comes about that this center lies below the equinoctial line in the [first [?]] two and [above [?]] it in the [latter [?]] two. For when the center of the dial on the meridian line lies below or above the point N of the equinoctial line, that same center necessarily lies also below or above it on the style-line GN. For since the equinoctial line cuts the style-line GN at right angles, it plainly follows that if the equinoctial line cuts the meridian line above or below the center, it likewise cuts the style-line GN above or below the center — as is clear from the figures set out above.

But if the arc of the meridian between the plane and the horizon is equal to the complement of the pole's elevation on the southern side — so that the common section of that plane and the meridian in the analemma is the same as the common section of the meridian and the Equator, HI — then the axis FG will cut the meridian line of the dial (the line drawn in the analemma) at right angles to HI itself. [Margin: (c) Euclid, *Elements* I.29] And thus the style-line GN too will be perpendicular to that same meridian line in the dial; and the diameter of the Equator, in the same analemma, will not cut the meridian line of the dial, that is, the common section of the meridian and the plane of the dial. [Margin: (d) Euclid, *Elements* I.29] Hence in the dial the meridian line and the equinoctial line will be parallel to one another. [Margin: In what kind of dial the equinoctial line and the meridian line are parallel to one another.] All this we shall confirm as follows. Since the three circles — the Equator, the meridian, and the great circle to which the dial is parallel — have one and the same common section HI in the analemma, while the plane of the dial is parallel to one of them and thus to this common section, the common sections which the remaining two circles (namely the Equator and the meridian) make in the plane of the dial will be parallel, by Proposition 18 of Book 1; and such are the equinoctial line and the meridian line. Therefore, since the style-line pG [?] cuts the equinoctial line at right angles, it will cut the meridian line too at right angles. Nevertheless the center of the dial will still lie below the equinoctial line, as appears in the sixth figure.

Finally, if the arc of the meridian between the inclined plane and the horizon is equal, on the northern side, to the elevation of the pole — so that the common section of the plane and the meridian does not differ from the axis FG — then it is evident that the dial has no center, because the axis cannot cut the meridian line, indeed cannot cut the plane of the dial itself, since it is parallel to the great circle drawn through the axis. Rightly, then, have we prescribed that the straight line HI, from the portion of the analemma, be transferred onto the line GN (which is the style-line in the dial) — sometimes below the point G, sometimes above it — up to the point p [?], and so on. [Margin: What kind of dial lacks a center.]

Having found the center p [?] of the dial by this method — which is sometimes below the equinoctial line, sometimes above it, as we have taught — we shall proceed further in the construction of the dial in the following way. From the portion of the analemma let the straight line DI be taken, namely the portion of the section of the Meridian and Equator intercepted between D, the center of the world, and the straight line HI; and let it be transferred onto the straight line GN, from the point G — either upward or downward — up to the point L, from which let a circle of any size be described, and let this circle be divided into 24 equal

parts. But the division must not be begun here from the straight line GN, as in the horizontal dial it begins from the line HE; for the line GN is not here the meridian, as HE is there, but the common section of the plane of the dial and of the great circle passing through the poles of that plane and the poles of the world — serving as a proper meridian of that plane, which is not the meridian of the horizon. Hence it comes about that the straight line GN, in the circle described from L, cannot be the common section of the Equator and the meridian of the horizon itself, if the circle described from L is understood to be set in its proper position — just as, in the horizontal dial, the straight line IE, in the circle described from E, is the common section of the meridian and Equator, if the said circle has its proper position, as we there showed. Hence, since the division of the said circle described from L must take its beginning from the said common section of the meridian and Equator — namely from the 12th hour — as was done in the horizontal dial, we shall find this section, or line of the 12th hour, in the aforesaid circle by the following method. Let there be sought, by Proposition 30 of Book 1, the inclination of the proper meridian of the plane itself to the meridian of the horizon; and this we found to be, in the first figure, $14^{\circ}10'$; in the second figure, $20^{\circ}52'$; in the third, $33^{\circ}47'$; in the fourth, $27^{\circ}30'$; in the fifth, $75^{\circ}46'$; and finally in the sixth, $90^{\circ}0'$. For this inclination must be counted from the point N, in the circle described from L, up to the point O, observing the following rule.

[Margin: Quænam sit inclinatio Meridiani proprii ipsius plani inclinati ad Meridianum Horizontis in qualibet sex figurarum huius propositionis. — "What the inclination of the plane's proper meridian to the meridian of the horizon is, in each of the six figures of this proposition." (Stage-5 recovery)]

If the plane is inclined toward the horizon on the northern side — that is, if its upper face declines from the south toward the east or the west — then, in the case where it declines from the south toward the east, the said inclination of the meridians must be counted from the point N toward the left, that is, toward the western parts, as was done in the first and third figures. For then the great circle drawn through the poles of the horizon and the poles of the inclined plane (the circle that measures the inclination to the horizon) falls, on the south side, into the eastern quadrant of the upper hemisphere. [Margin: Quænam sit arcus circulorum maximorum inclinationes planorum metientium, inter Horizontem & circulum horæ 6. à meridie vel media nocte interiecti. (Stage-5 recovery)] (We call "quadrants of the upper hemisphere" those parts contained between the meridian, the horizon, and the vertical circle properly so called. For these three circles, cutting one another mutually at right angles, divide the whole upper hemisphere into four equal parts, of which two are southern — one eastern, the

other western — and two northern — one eastern, the other western — as is clear from the material sphere.) Then the pole of the inclined plane will lie in that same quadrant above the horizon. For since the arc of that great circle drawn through the zenith of the place and the pole of the inclined plane, lying between the plane and the zenith, is less than a quadrant, while the arc of the same great circle extended from the plane through the zenith down to the horizon is greater than a quadrant — because the arcs of the said great circle intercepted between the zenith and the horizon are quadrants — it is evident that the pole of the inclined plane, which bounds the quadrant of the said great circle by which the pole of the plane is distant from the plane itself (by the corollary of Proposition 16 of Book 1 of Theodosius), falls into the eastern and southern quadrant of the upper hemisphere. Therefore the meridian of the horizon will be more westerly, in the Equator above the horizon, than the proper meridian of the inclined plane drawn through its poles and through the poles of the world. And accordingly, since — with the circle described from L set in its proper position — the proper meridian of the inclined plane cuts the said circle above the horizon at the point N, the inclination of this meridian to the meridian of the horizon must be counted from N toward the western parts, up to the point O, through which the meridian of the horizon passes.

In this case the pole of the inclined plane EF falls below the six-o'clock circle BKD, because the arc HM between the pole of the plane and the horizon is smaller than the arc HL between the six-o'clock circle and the horizon; if it were equal, the pole would fall on L, and if greater, above it. [?] For since GIH is a semicircle (by Proposition 11 of Book 1 of Theodosius), and the arc [MN [?]] between the plane and its pole is a quadrant (by the corollary of Proposition 16 of Book 1 of Theodosius), the two remaining arcs GN [?] and MH will together be equal to a quadrant. Therefore, since GN [?] is the arc of the inclination, the arc MH set between the pole of the plane and the horizon will be

the complement of the inclination. Hence, once the inclination of the plane to the horizon is known, if it is subtracted from a quadrant, the aforesaid arc between the pole of the plane and the horizon will remain. By this device such an arc was found, in the fourth figure of this proposition — for in the first three figures, since their planes have their poles on the southern side of the upper hemisphere, we do not need the said arcs — to be $60^{\circ}0'$; in the fifth, $10^{\circ}0'$; and finally in the sixth, $37^{\circ}57'$. If these arcs are compared with the earlier ones placed between the six-o'clock circle and the horizon, we shall find that the pole of the inclined plane, in the fourth figure, falls above the six-o'clock circle; in the fifth, below it; and finally in the sixth, on the six-o'clock circle itself.

Once these things have been thoroughly investigated: if the plane inclined to the horizon on the southern side declines from the north toward the west, and its pole falls above the six-o'clock circle, while the center of the dial lies above the equinoctial line — as happens in the fourth figure — then the aforesaid inclination of the meridians must be reckoned from N toward the eastern parts, that is, toward the parts which lie to our left if we turn to face the plane of the dial, up to the point O, from which the division of the circle described from L is to be begun. For since then the great circle drawn through the poles of the horizon and the poles of the plane falls, on the north side, into the western quadrant of the upper hemisphere, and its pole lies above the six-o'clock circle, the proper meridian of the plane will cut the Equator above the horizon in the western quadrant — namely between the meridian of the horizon and the six-o'clock circle. Therefore the meridian of the horizon will be more easterly, in the Equator above the horizon, than the proper meridian of the inclined plane; and accordingly, since — with the circle described from L set in its proper position — the proper meridian of the inclined plane cuts the said circle above the horizon at the point N, the inclination of this meridian to the meridian of the horizon must be computed from N toward the east, so that the point O may be found, through which the meridian of the horizon passes. But if, everything else remaining the same, the center of the dial turns out to lie below the equinoctial line, then the same inclination of the meridians must be counted from N toward the western parts, that is, toward the right; because then the meridian of the horizon, in the Equator below the horizon, is more westerly than the proper meridian of the inclined plane — whereas (as we said) above the horizon it is more easterly. Therefore, since — with the circle described from L set in its proper position — the proper meridian of the inclined plane cuts the said circle at N below the horizon (a thing easily understood if the fourth figure be inverted, so that the center ρ [?] lies below the equinoctial circle, and the circle NO takes on its proper position, its center L being joined to the center of the world I), the said inclination of the meridians must be computed toward the west from the point N.

But if the pole of the same plane — inclined to the horizon on the southern side and declining from the north toward the west — falls precisely on the six-o'clock circle, then, wherever the center of the dial may lie, the said inclination of the meridians will amount to 90° , since the proper meridian of the plane is then the same as the six-o'clock circle, which in the Equator is removed a quadrant from the meridian of the horizon. Therefore, into whichever direction the said inclination is counted from the point N, the point O will be found, from which the division of the circle described from L can take its beginning.

Finally, if the pole of the same plane falls below the six-o'clock circle, while the center of the dial lies above the equinoctial line, then the same inclination of the meridians must be reckoned from N toward the western parts. For since then the pole of the inclined plane, below the six-o'clock circle, lies in the northern and western quadrant of the upper hemisphere, the proper meridian of the plane will cut the Equator below the horizon in the western quadrant, but above the horizon in the eastern quadrant. Hence the meridian of the horizon, in the Equator above the horizon, will be more westerly than the proper meridian of the inclined plane. For this reason the inclination of the meridians must be counted from N toward the western parts, since the proper meridian of the plane cuts the circle NO above the horizon at N. But if, everything else remaining the same, the center of the dial turns out to lie below the equinoctial line, then the same inclination of the meridians must be counted from N toward the eastern parts; because then the proper meridian of the plane cuts the circle NO at N below the horizon, and there the meridian of the horizon is more easterly in the Equator, or circle NO, below the horizon, as is clear from what has been said.

But if the plane inclined to the horizon on the southern side deflects from the north toward the east, then let everything be done in exactly the opposite way to what we have just prescribed as to be observed in planes declining from the north toward the west. Thus you see that in the fifth figure the inclination of the meridians has been counted from N toward the western parts, up to O — because the pole of that plane falls below the six-o'clock circle, and the center of the dial is found below the equinoctial line. The reason for this can easily be gathered from what has been said. And finally, in the sixth figure the inclination is reckoned as the same 90° , because there the pole of the plane falls precisely on the six-o'clock circle itself, and so on.

Therefore, if through the points of division of the circle described from L, and through the center L, straight lines are drawn (left invisible), the equinoctial line GH will be cut at points through which the lines led out from the center of the dial ρ [?] will give the hours from noon or from midnight, as we demonstrated in the horizontal dial — but in such a way that the straight line drawn from ρ [?] through the point M of the equinoctial line, onto which the line OL falls, shall (in all dials that face the zenith and the south, and in those that face the zenith and the north, provided the center is above the equinoctial line) mark the 12th hour of midday; and the lines following it in the dial, toward the eastern parts — namely those which, when the dial is set in its proper position, turn toward the rising of the sun, and so on — shall mark the hours from noon, and so on. But in the dials that pertain to the zenith and the north and have their center below the

equinoctial line, that same straight line pM [?] shall signify the 12th hour of midnight, and those following it toward the western parts shall show the hours from midnight, and so on. Finally, in the dial where the inclination of the proper meridian to the meridian of the horizon amounts to 90° , the straight line ρ [?], drawn parallel to the equinoctial line toward the upper parts, shall likewise mark the 12th hour of midnight, while the lines following it toward the west shall indicate the hours from midnight, and so on. The reason for this was given above in the earlier description.

When the plane of the dial is inclined to the horizon on the northern side, under this condition — that the arc of the meridian between the plane and the horizon is equal to the elevation of the pole above the horizon, and accordingly is parallel to the axis, as happens in the third figure — we shall construct the dial upon it as follows. Having drawn the straight lines GN and GH cutting one another at right angles in G , as before, we shall take on GN — whether above the straight line GH or below it — the straight line GL , of any size, to serve as the style; and from L we shall describe a circle. Then, having computed from the point N , where the straight line GN cuts the said circle, the

the inclination of the proper meridian of the proposed plane to the meridian of the horizon — toward the western parts, that is, if the dial declines from the meridian toward the east, or toward the eastern parts if the dial deflects from the meridian toward the west. Let the circle described from L be cut, up to the point O , into 24 equal parts, beginning at the point O . For lines correctly drawn from the points of division through the center L will cut the equinoctial line GH at points; and if through these points lines are drawn perpendicular to GH — as in the meridian dial, or the polar one — the lines of the hours from noon or from midnight will be obtained. This must be done in such a way, however, that the straight line through the point M of the equinoctial line, onto which the straight line OL falls, marks the 12th hour of midday, while the lines following it toward the east show the hours from noon, and so on.

Another construction of the same declining and inclined dial. [Margin: Another construction of the same dial, at once declining and inclined.] We may also describe this same declining and inclined dial in the following way. Find, by Proposition 30 of Book I, the arc of the proposed plane intercepted between the meridian of the horizon and its own proper meridian; then, by Proposition 29 of the same Book I, the elevation of the pole above the proposed plane. The elevations of the pole above the planes of the six figures of this proposition were already set out at the beginning of the preceding construction; and the arc of any

given proposed plane lying between the said meridians stands as follows.

[Margin: How great the arc of the proposed plane, lying between its own meridian and the meridian of the horizon, is in each of the six figures of this proposition.] In the first figure the said arc contains $6^{\circ} 15'$; in the second, $9^{\circ} 15'$; in the third, $0^{\circ} 0'$; in the fourth, $61^{\circ} 30'$; in the fifth, [figure-value not legible: ?]; in the sixth, finally, $90^{\circ} 0'$.

Next, in some plane let the straight line $\rho\varsigma$ be drawn in any way whatever [ρ and ς = special typographic point-markers as printed; see QA], to serve as the line of the 12th hour. In it take any point ρ as the center of the dial, and from that point describe the arc of a circle $a b$ — above the point ρ if the center of the dial falls below the equinoctial line, but below the point ρ if the center is to be placed above the equinoctial line. When the center of the dial falls below, or above, the equinoctial line has been set out amply enough in the preceding construction, and can be understood most plainly from those four Analemmas of Proposition 25 of this book. For in them the straight line MN is the meridian line, that is, the line of the 12th hour, in the upper dials. Hence, if from the given declination of the plane and its inclination the arc of the meridian between the plane itself and the horizon is found — namely DK or BK — it will easily be gathered whether the center of the dial (that is, the point M onto which the axis FG falls) lies below the equinoctial line, which passes through the point N , or above it, on the meridian line MN itself, and therefore on the meridian line $\rho\varsigma$ of the dial to be described. Now in the arc $a b$ let there be counted, from the straight line $\rho\varsigma$ — that is, from the point ς — the arc found between the meridian of the horizon and the meridian of the proposed plane, up to b ; and let the straight line ρb be drawn, which will be the common section of the plane of the dial and of its own proper meridian, as will presently be shown.

To know toward which side the said arc is to be counted, the things we handed down in the preceding construction must be carefully considered. For in planes that are inclined to the horizon on their northern side, if indeed they decline from the meridian toward the east, as in the first figure, the said arc is to be reckoned toward the left — that is, toward the western parts, which, when we are turned to face the dial, lie on our left — because in that case the proper meridian of the plane is more easterly than the meridian of the horizon; and consequently its common section in the plane of the dial, namely the straight line ρb , will be more westerly than the meridian line $\rho\varsigma$, since the shadow of the style, with the Sun in whatever circle it may be, is always cast toward the opposite side, as is clear from what has been said. But if planes of this kind deflect from the

meridian toward the west, exactly the opposite must be done, for the opposite reason as well, as was done in the second figure.

In planes inclined to the horizon on their southern side, on the other hand, one must learn from what we taught in the preceding construction whether the proper meridian of the plane, above or below the horizon, is more easterly or more westerly than the meridian of the horizon; for the said arc $a b$ is always to be reckoned toward the opposite side. Thus you see that in the fourth figure the said arc is counted toward the left, that is, toward the east, because the proper meridian of that plane is more westerly, above the horizon, than the meridian of the horizon. In the fifth figure, by contrast, the same arc is reckoned toward the right, that is, toward the western parts, because the proper meridian of that plane, below the horizon, is more easterly than the meridian of the horizon. In the sixth figure, finally, it makes no difference whether the said arc $a b$ is counted toward the one part or the other, because it comprehends 90° , so that the straight line pb makes a right angle with $p\phi$ at the point p .

After this, from any point of the straight line pb — say from G — let the perpendicular GH be drawn to it; this will be the equinoctial line, because, as we shall presently show, pb is the index-line (the edge of the style), to which the equinoctial line is necessarily perpendicular, as we demonstrated above.

Next, at the point p , upon the straight line pb , construct the angle GpI equal to the elevation of the pole above the proposed plane, found by Proposition 29 of Book I, so as to obtain pI , the axis of the world. To this axis, from the point G , raise the perpendicular GI ; and let the rest be done as in the first construction — that is, let the straight line GI be taken equal to GL , and let the circle described from L be cut into 24 equal parts, beginning from the straight line LM , which is drawn from the center L through the point M , where the equinoctial line and the meridian pb intersect one another; or else from the line drawn through L parallel to the meridian line, when the equinoctial line and the meridian are parallel, as in the sixth figure, and so on. The gnomon will be IK , the perpendicular let fall from I to pb , as before.

That the straight line pb is the common section of the plane of the dial and of the proper meridian of that same plane we shall demonstrate as follows. Since the meridian of the horizon and the proper meridian of the plane are both drawn through the axis of the world, and both meet the great circle to which the dial is parallel at the center of the world — namely at I , the vertex of the style — the said meridians make with this great circle common sections that are straight lines containing at the center an angle subtended by the arc of that great circle

intercepted between those meridians. But since these same meridians meet the plane of the dial at the point ρ , where the axis meets that same plane, [Margin: Euclid XI.16] and make with it common sections that are straight lines parallel to the two former lines in the great circle — because the plane of the dial is parallel to that great circle — [Margin: Euclid XI.10] these lines will contain, in the plane of the dial, an angle equal to that which those former lines contain in the great circle. Therefore, since the angle apb in the plane of the dial is equal to that in the great circle — because the arc a b is similar to the arc of that great circle placed between the two meridians, seeing that by construction it contains just as many degrees and minutes — and since $\text{p}\rho$ is set as the meridian line, pb will be the common section of the plane of the dial and of its own proper meridian. Consequently the style must be set upon it at right angles, since by this arrangement the style does not depart from the plane of this proper meridian, but reaches to the axis of the world lying in it. Therefore pb is the index-line, the line of the style. From this the rest of the construction will be demonstrated, just as the first construction of this proposition. But this third description we shall be able to begin from another starting point, and perhaps more conveniently than we have handed it down, as we shall make almost plain toward the end in the following Scholium.

[Margin: When the plane of the dial is parallel (equidistant).] Furthermore, on a plane inclined to the horizon on its northern side, one that has no arc interposed between the said two meridians and receives no elevation of the pole — as happens in planes that are parallel to the great circles drawn through the axis of the world, such as the plane of the third figure — the dial will have to be constructed as in the preceding construction; or else, if first any straight line Ep is drawn in any way as the meridian line, one must proceed thus. Upon Ep , at any point, say at M , raise the perpendicular MH to serve as the equinoctial line. Then at M construct the angle EML of the inclination of the proper meridian of that plane to the meridian of the horizon, found by Proposition 30 of Book I — toward the left if the plane declines from the meridian toward the east, toward the right if it declines from the meridian toward the west. Taking on the straight line ML any point L you please, draw through it the line LG parallel to Ep , cutting the equinoctial line at G ; this will be the common section of the proper meridian and the plane of the dial, and LG itself will be the length of the gnomon. Then, having described a circle from L and divided it into 24 equal parts, beginning from the straight line ML , if from the points of division straight lines are drawn through L , the equinoctial line will be cut at points through which the hour-lines are to be drawn perpendicular to the equinoctial line and parallel to one another, as before. For by this method a dial of this kind will be described in no other way

than in the preceding construction, where the line of the style GN was first drawn — [Margin: Euclid I.29] because the angle NLO is equal to the angle EML of the inclination of the meridians, the external being equal to the internal. Therefore we have constructed an astronomical dial that both declines from the vertical and is inclined to the horizon, and so on. Which was to be done.

Scholium

We have completed this whole description of a dial declining from the vertical and at the same time inclined to the horizon in six figures, so that every variety in dials of this kind might be laid open. Of these, the first three pertain to those upper dials that face the south, in which the center of the dial is either below the equinoctial line, or above it, or else all the hour-lines are parallel, cutting the equinoctial line at right angles, and the dial has no center. The latter three represent those upper dials that face the north, in which the line of the 12th hour, or meridian line, cuts the equinoctial line either below the center of the dial, or above it, or else is parallel to the equinoctial line itself. Moreover, in the first, third, and sixth figures we have drawn all the delineations of the hours, but in the intermediate figures only those that are required for understanding the construction, the hour-lines being omitted so as to avoid confusion of lines. For anyone can easily draw the hour-lines in these figures, as in the others, if he considers attentively and diligently the precepts we have handed down.

[Margin: By what method the dial that both declines and is inclined is to be positioned.] Now we shall place the dial hitherto delineated in its proper position. In the earlier description, the straight line AB must be parallel to the horizon, with the point A on the left and B on the right, and the common section of AB and the horizon must make, with the meridian line in the plane parallel to the horizon (found as before), an angle equal to the complement of the declination — toward the south and west if the plane of the dial declines from the meridian toward the east; toward the south and east if from the meridian toward the west; but toward the north and west if the plane declines from the north toward the east; and toward the north and east if it deflects from the north toward the west. For thus the dial will obtain its due declination. Next, the straight line CD, with the line in the plane of the horizon that cuts the common section of the dial and the horizon at right angles, must make an angle of the inclination toward the northern parts if the dial faces the south, but toward the southern parts if it faces the north. For when the dial is set up in this way, if a thread be stretched — or some fine wire — along the axis ρI in the triangle $\rho I G$, or along the axis ef in the rectangle $efd\chi$ where there is no center, at right angles to the plane of the dial, its

shadow will mark each of the hours from noon or from midnight, as long as the Sun illuminates the dial; or else the vertex l of the style lK , standing at right angles to the plane of the dial at the point K , will do the same, as was said of the horizontal dial in the scholium of Proposition 1 of the preceding book. The axis pl , or ef , will likewise do the same, if the triangle pIG , or the rectangle $efd\chi$, be constructed of some solid material and set up at right angles to the plane of the dial.

[Margin: How great the arc of the inclined plane, lying between the meridian of the horizon and the great circle that measures the inclination of the plane, is in each of the six figures of this proposition.] But if the dial has been constructed by the two later descriptions, one must seek, by Proposition 31 of Book I, the arc of the proposed plane to which the dial is parallel, intercepted between the meridian of the horizon and the great circle that measures the inclination of the plane to the horizon. This arc, in the first figure, comprehends $38^{\circ} 15'$; in the second, [degrees not legible: ?] $6'$; in the third, $31^{\circ} 42'$; in the fourth, $17^{\circ} 30'$; in the fifth, $16^{\circ} 46'$; in the sixth, finally, $19^{\circ} 33'$. Then from any point of the meridian line, that is of the line of the 12th hour — say from E — describe the circle $m h$ on the upper side, and in it count from m the arc found, up to b : toward the left if the dial declines from the meridian toward the east, or from the north toward the west; but toward the right if the dial declines from the meridian toward the west, or from the north toward the east — because in the former case the great circle measuring the inclination of the plane recedes from the meridian, above the horizon, toward the left, but in the latter toward the right, as is plainly evident if the said circles and the plane of the dial are correctly set up in their positions. But if the arc $m h$ is described on the lower side, exactly the opposite must be done in the counting of the said arc, as is manifest, and as you see has been done in the figures. From b , then, draw through E the straight line hE , which will be the common section of the plane of the dial and of the great circle measuring its inclination to the horizon. For if from E one imagines the common section of the meridian and of the said great circle raised aloft to the center of the world, these two circles will make, with the great circle to which the dial is parallel, two common sections which are straight lines making at the center of the world the angle found by Proposition 31 of Book I. But since these same two circles meet the plane of the dial at E , where their common section meets that same plane, and make with it two common sections which are straight lines [Margin: Euclid XI.16] parallel to those in the great circle — because the plane of the dial is parallel to that great circle — [Margin: Euclid XI.10] these two lines will contain, in the plane of the dial, an angle equal to that which those former lines contain in the great circle. Therefore, since the angle mEh is equal to it — because the arc $m h$ is

similar to the arc of that great circle intercepted between the meridian of the horizon and the great circle measuring its inclination to the horizon, seeing that it embraces just as many degrees and minutes — and since mE is set as the meridian line, that is, the common section of the meridian and the plane of the dial, hE will be the common section of the plane of the dial and of the great circle measuring its inclination. And if through E , to hE , the perpendicular AB is drawn, this will be the common section of the plane of the dial and of the horizon drawn through E . For since both the horizon and the plane of the dial cut the straight lines that each measures, and cut the great circle that measures the inclination of the dial, making with it the section hE , their common section too will be, at the same great circle — and therefore, by Definition 3 of Book XI of Euclid, also perpendicular to hE , the common section of the said great circle and the plane of the dial, just as AB is. And so, if again AB is set parallel to the horizon and the rest is done as we said a little before, the dial will obtain its proper position.

[Margin: Euclid XI.16] [Margin: Euclid XI.10]

But if a dial is to be described on the inclined wall itself, and not on some other plane from which it must afterward be transferred onto the inclined wall — as we have done hitherto — it will be effected in this way. On the inclined wall itself let the straight line AB be drawn parallel to the horizon by means of the level and the plumb-lines,

[Margin: By what method a dial declining and at the same time inclined is to be described on a fixed plane.]...and let another straight line CD cut it at right angles at the point E . Let the rest be done as we prescribed in the earlier description — that is, let the declination line EF be drawn, and let the meridian line $E\epsilon$ [?] be found, which we shall discover in this way as well. From E describe the arc of a circle $b m$ toward the upper part, and in it count up to m the arc of the inclined plane between the meridian of the horizon and the great circle that measures the inclination of the plane, found by Proposition 31 of Book I — toward the right if the dial declines from the meridian toward the east, or from the north toward the west; but toward the left if the plane of the dial deflects from the meridian toward the west, or from the north toward the east. For the straight line mE will be the meridian line, namely the common section of the meridian and the plane of the dial. This we shall prove no otherwise than has just been shown — namely that the straight line bE is the common section of the plane of the dial and of the great circle measuring its inclination to the horizon — from the fact that mE was set as the meridian line, and so on. Or else by this same method we shall draw the

same meridian line, as appears in the fourth figure; for in the others, to avoid confusion of lines, we do not give an example.

Having drawn the straight line AB parallel to the horizon, let there be set beside the inclined plane some plane parallel to the horizon, so that the common section of it and the inclined plane is AB. As now, for example, in the said fourth figure, let the plane below the straight line AB be understood to be parallel to the horizon, and the plane above that same AB to be inclined to the horizon. Then in the plane that is parallel to the horizon, find, by what we taught in Book I in the scholium of Proposition 23, the meridian line EF; and let another straight line tu cut it at right angles at q. Then on tu take on either side two straight lines qt, qu equal to one another. If by means of the compasses two arcs of circles are described from the points t and u, cutting one another in the inclined plane at l, we shall have in the inclined plane this point of intersection l; and from it, through the point E where the meridian line EF meets the horizon, the straight line AB, a line drawn out will give the meridian line in the plane of the dial. The reason is that, since the meridian, drawn through the meridian line EF of the plane parallel to the horizon, is at right angles to that same plane, then by Definition 4 of Book XI of Euclid the straight line tu, which cuts the meridian line EF of the same plane at right angles — namely the common section of the meridian and the said plane — is perpendicular to the meridian; and therefore, by Definition 3 of the same book, it is perpendicular to all the straight lines in the meridian that touch it at q. [Margin: Euclid I.8 [?]] Since, then, tu is perpendicular to the straight line drawn from the point q of the plane parallel to the horizon to the point l of the inclined plane — where the two arcs of circles described from t and u intersect, as is evident — if one conceives in the mind two triangles whose bases are the straight lines tl, ul equal to one another (since they are radii of equal circles), whose sides are the equal straight lines qt, qu, and with the common straight line drawn from q to l, then the straight line drawn from q to l will be in the plane of the meridian circle. For otherwise, if it were outside tu, then tq would be perpendicular to two straight lines at the point q, in the plane passing through tq and the straight line drawn from q to l — namely to the straight line drawn from q to l, as we showed, and to the straight line drawn from q in the plane of the meridian — being in the same plane in which the straight lines tq and the straight line drawn from q through l lie, that is, to the common section of the meridian and the plane through tq and the line ql, by Definition 3 of Book XI of Euclid, as was said; which is absurd. Therefore the meridian will pass through the point l in the inclined plane; but it also passes through the point E; therefore the straight line lE will be the common section of the meridian and the inclined plane.

Having found the meridian, to delineate the dial by means of the point in the inclined plane — this we have handed down in this third method, in case the first should seem too long. The same can be done by the second method, if first the line of the style is found, which indeed we shall discover if first, by Proposition 31 of Book I, we investigate the arc of the inclined plane between its own proper meridian and the great circle measuring its inclination; then diligently observe whether the proper meridian recedes from that great circle toward the east or inclines toward the west, and so on. [Margin: In what way the third method of describing a dial declining and at the same time inclined can be begun otherwise than as we have handed it down.] But the description by the third method is more convenient and easier. Indeed, this third method can most fittingly be begun from two perpendiculars AB, CD, exactly as if the dial were to be constructed straightway on the wall itself, as we handed down just now. For thus it will come about that the dial obtains its proper position, if the straight line AB is set parallel to the horizon, and so on.

[Margin: Description of the same dial, at once declining and inclined, for a given length of style whose location is also given.] Andreas Schöner [Andreas Schöner] hands down another method of describing a dial declining from the vertical and inclined to the horizon, not unlike the one we explained for other dials, which can indeed be described so as to draw certain hours that could scarcely be obtained by the earlier description. The method is roughly as follows. From E draw a perpendicular ET to ρ C, and another perpendicular EQ to EF, equal to ET, above the straight line AB. Then, having drawn the straight line QF cutting AB at R, draw through R, parallel to the meridian line E ϵ , the line RS cutting the six-o'clock line at S. Finally, from S raise a perpendicular ST to RS, and let ST be equal to RS; and, with T as center, describe a circle to be divided into 24 equal parts, beginning from the straight line ST. For if from the center T straight lines (left invisible) are drawn through the points of division, the straight line RS will be cut at points through which the hour-lines are to be drawn from the center of the dial ρ . You have an example in the first figure. In the third figure, however, where the dial has no center, this description has no place, because the line we said was to be drawn parallel to the meridian line does not cut the six-o'clock line, but is parallel to it. In the other figures we cannot give examples, to avoid the confusion that would arise from so great a multitude of lines.

If the same declining and at the same time inclined dial is to be described on a given plane for any magnitude of style whatever, whose location too is given, we shall use this device. Let the length of the style K ρ be given, and its place in the plane of the dial the point K. If, then, the plane of the dial be any whatever, so

that the dial may be described on it in its proper [position], and then be set in place, or transferred onto a fixed plane that declines from the vertical and is inclined to the horizon, we shall draw through K, the place of the style, two straight lines AB, CD cutting one another at right angles at K. But if the fixed plane is at once declining and inclined, we shall draw through the place of the style K, by means of the level and the plumb-line, the straight line AB parallel to the horizon, which the line CD cuts at right angles at x. Then, taking on AB the straight line Kp equal to the style, whether toward the right or toward the left, describe from p, toward CD, an arc of a circle, in which count off from the inclination of the plane to the horizon (let the planes here be the same, with the same inclinations and declinations, as before), beginning from the straight line AB downward; and through the end of the counting draw from p the straight line pD cutting CD at the point D, through which the meridian line is to be drawn. Likewise, having counted off in the same arc, upward, the complement of the inclination, draw from p, through the end of the counting, the straight line pC, which will be perpendicular to pD, cutting CD at the point C, through which the line CE drawn perpendicular to CD will be the horizontal line.

Next, CD being drawn in line with C, and CF being taken equal to C, whether upward or downward, describe from F, toward CE, an arc of a circle, in which count off from the straight line C the declination of the plane from the vertical circle — toward the right if the plane declines from the meridian toward the east, or from the north toward the west, as in the first, third, and fourth figures; toward the left, however, if the plane declines from the meridian toward the west, or from the north toward the west [recte: east? — as printed "à septentrione in occasum"; but see QA], as in the second, fifth, and sixth figures. Draw from F, through the end of the count, the straight line FE cutting the horizontal line CE at E. For the straight line DE, erected through D and E on both sides, will give the meridian line. Likewise, having counted off in the same arc, from the straight line CD, on the opposite side, the complement of the declination, draw from F, through the end of the count...

through the end of the count [ex fine supputationis], the straight line Fa, which will be perpendicular to FE, cutting the horizontal line at the point a, through which the equinoctial line and the line of the 6th hour are to be drawn.

Diagram. The first figure. A large circle with an inscribed construction of a declining-and-inclined dial. Around and within it run the meridian, horizontal, and equinoctial lines together with the several counting-arcs and the shadow/style triangles of the demonstration. Readable labels:

- 6 (upper left, on the circumference): the hour numeral 6
- F (top): apex of the vertical construction line
- C (upper middle): point on the vertical line
- E (upper right): point where a line meets the circle
- b (just below C): lowercase construction point
- d (right, near circumference): counting-point on the right
- L (middle): point on the central axis
- A (far left): left end of the horizontal line AB
- B (far right): right end of the horizontal line AB
- I (middle left of centre): construction point
- H (lower middle): intersection of the counting-arcs
- ρ (lower left): the centre of the dial [horologii centrum], printed as a rho-like *p*
- 6 (bottom): the hour numeral 6
- [Central cluster: several overlapping labels — I, G, K, M, D — crowd the centre and cannot all be resolved individually [?]]

[Illegible: the dense crossing of dashed counting-arcs and solid construction lines at the centre of the figure cannot be traced cell-by-cell at this resolution]

AGAIN, once the straight line aK has been drawn — which will necessarily be perpendicular to the meridian line DE if no error has been made — let two arcs be described from D and from E at the intervals of the straight lines D β and EF. If there is no error, these will necessarily cut one another on the line aK, as at the point H. Whether this happens on the right-hand side or on the left makes no difference. From this it follows that it is not strictly necessary to draw the line aK; it is only useful, so that the construction may be made more exact. For if the line did not pass through the point H once found, the error committed would have to be corrected. In the same way, once the line aK has been drawn, it would be enough if only the arc from D at the interval D β were described — or, if it seems more convenient, the arc from E at the interval EF; for wherever one of these arcs cuts the line aK, there the point H will be, as before.

Once the line HE has also been drawn, describe from H the arc of a circle on which, from the line HE toward the line HD, the complement of the pole's altitude above the horizon is to be counted off — this if the plane of the dial declines from the south. Through the end of the count, draw from H the straight line Hb cutting the meridian line at the point M, through which the equinoctial line is to be

drawn from the point a already found earlier. And if, on the same arc but on the opposite side of the line HE, the pole's altitude is counted off,

[Margin: *Linea æquinoctialis*. — The equinoctial line.]

and from the end of the count the straight line Hd is drawn through H, which will be perpendicular to Hb, the same meridian line will be cut at the point p, which will be the centre of the dial [*centrum horologii*]. But should it happen that this line Hd, drawn last through H, comes out parallel to the meridian line — as occurs in the third figure, where the earlier line Hb passes through the place of the style K and through the point a, so that it does not differ from the line aK — the dial will have no centre, and all the hour-lines will be equidistant from one another.

[Margin: *Centrum horologii*. — The centre of the dial.]

If, however, the plane of the dial declines from the north, the pole's altitude must be counted off on the said arc of the circle from the line HE toward the line HD. For the line Hd, drawn from H through the end of the count, will cut the meridian line at p, the centre of the dial. And if on the same arc, on the opposite side of the line HE, the complement of the pole's altitude is reckoned, and from the end of the count the line Hb is carried across through H — which will be perpendicular to Hd — the meridian line will be cut at the point M, through which the equinoctial line is to be drawn from a. But should it happen that this line Hb, drawn last, comes out parallel to the meridian line — as appears in the sixth figure, where the earlier line Hd passes through K, the place of the style, so that it does not differ from the line aK — then the equinoctial line must be drawn through a parallel to the meridian line itself.

Diagram. The second figure. A circle with a full declining-and-inclined dial construction, oriented like the first but for a plane declining toward the other hand. Readable labels:

- d (top): apex point
- 6 (upper left and again lower right): hour numerals
- A (left): left end of the horizontal line
- B (right): right end of the horizontal line
- C (upper middle): point on the vertical
- H (lower left of centre): counting-arc intersection
- K (centre): place of the style
- M, G, S (centre cluster): construction points [?]

- L (below centre): point on the axis
- D (bottom of centre): lower construction point
- ρ (left of centre): the dial-centre point, printed as rho-like p [?]

[Illegible: the centre of this figure is as crowded as the first; the cluster of labels around K–M–G cannot be individually resolved [?]]

Diagram. The third figure. A circle with the construction for the case in which the dial has no centre (the hour-lines coming out parallel). Readable labels:

- 6 (upper right, and again lower centre): hour numerals
- F (upper middle): point on the vertical
- d (upper right): counting-point
- A (left): left end of the horizontal line
- B (right): right end of the horizontal line
- C (upper middle): point on the vertical
- I, K (centre): construction point and place of the style
- M (centre): meridian/equinocial intersection
- H (lower right of centre): counting-arc intersection
- L (below centre): point on the axis
- D (bottom of centre): lower construction point

[Illegible: dense central crossing of lines and small labels not fully resolvable [?]]

[Margin: Linea styli. — The line of the style.]

NOW, if from p , the centre of the dial, a straight line pK is drawn through K , the place of the style, we shall have the line of the style [linea styli], which will necessarily cut the equinoctial line drawn from a at right angles. But where the dial has no centre, the line of the style must be drawn through K , the place of the style, parallel to the meridian line and perpendicular to the equinoctial line, as in the third figure. Likewise, where the equinoctial line

is equidistant from the meridian line, the line of the style pK will be perpendicular not only to the equinoctial line but also to the meridian line.

[Margin: a — 26.primi. (*Euclid, Elements I.26*)] And so, if in the first place the equinoctial line is drawn through the points a and M already found, the line of the style must be drawn perpendicular to it through K , the place of the style, from the centre p — or, where there is no centre, through K , the place of the style, alone. But if in the first place the line of the style is drawn from the centre p through K — or, where there is no centre, through K alone, equidistant from the

meridian line — then the equinoctial line must be drawn from a perpendicular to it.

AFTER this, from K, the place of the style, let a line KI be raised perpendicular to the line of the style, equal to the style, and from the centre p let the line pI be sent out through I to serve as the axis of the world [axis mundi]. [Margin: Axis mundi.] But where there is no centre, the axis must be drawn through I equidistant from the line of the style, or from the meridian line. Now if a perpendicular is drawn from I to the axis, it will cut the line of the style at the point G, through which the equinoctial line is drawn — unless an error has been made. Hence, where the dial has no centre and the axis is therefore equidistant from the line of the style, the said perpendicular will fall on K, the place of the style, and will not differ from the equinoctial line. And so, if in the first place the axis of the world has been drawn through I, we shall find, by the help of the line IG perpendicular to the axis, another point G on the line of the style, through which the equinoctial line is to be drawn from the point a — a line which will necessarily make right angles with the line of the style.

FINALLY, taking on the line of the index the straight line GL equal to GI itself, let a circle of any size be described from L and divided into 24 equal parts, beginning from the line LM — which is drawn from the centre L through the point M, where the equinoctial and meridian lines intersect — or from the line La, drawn from the same centre L through the point a, where the horizontal line and the equinoctial (that is, the line of the 6th hour) meet. Now the two lines LM and La must cut one another at L at right angles, if no error has been made in anything. But if the equinoctial line is parallel to the meridian line, then in place of the line LM a line must be drawn parallel to that same meridian line, one that cuts the line La at right angles. [Margin: Linea horaria. — The hour-line.] Now if hidden [i.e. faint construction] lines are drawn from L through the points of division, the equinoctial line will be cut at points through which, from the centre p, the lines of the hours from noon or from midnight [à meridie vel media nocte] are to be drawn. Or, when the dial has no centre, through these same points there must be drawn lines parallel to the meridian line, or to the line of the style, or to the axis, and perpendicular to the equinoctial line, for the hours from noon and from midnight. The order of the hours here is the same as was set out above in the first construction of this proposition.

YOU SEE, then, that this construction is much more excellent than the one we explained in the first place in this proposition, since it leads us of its own accord to the place of the dial's centre, so that there is no need to consider whether the

centre of the dial should be set below the equinoctial line or above it, as had to be done in that other construction.

WE SHALL NOW DEMONSTRATE everything stated in this construction, in the following way. [Margin: Demonstratio proxima descriptionis. — Demonstration of the foregoing construction.] Let there be understood a triangle βCD , whose angle β is right (because of the quadrant of the circle described from β and enclosed between the lines βD and βC), moved about the line CD until it stands perpendicular to the plane of the dial, so that the apex of the style β is joined with the centre of the world. Now since the angle $K\beta C$ is equal to the complement of the inclination, βCK will be the angle of inclination; and consequently, since the horizon passes through β , the apex of the style, the line βC will be the common section of the horizon and of the great circle that measures the inclination, making with the line CD the angle of inclination, as the matter requires.

Therefore the horizon will meet the plane of the dial at C , and for that reason the horizontal line must be drawn through the point C perpendicular to CD . For since both the horizon and the plane of the dial are perpendicular to the great circle that measures the inclination of the plane — and the line CD is drawn through it — [Margin: b — 19.unde. (*Euclid XI.19*)] their common section too, namely the horizontal line, will be at right angles to that same great circle, and hence, by definition 3 of book 11 of Euclid, perpendicular to the line CD lying in that circle, at the point C . Again, since both the great circle measuring the inclination of the dial and the meridian are perpendicular to the horizon, [Margin: c — 19.unde. (*Euclid XI.19*)] their common section too will be at right angles to it, and therefore, by definition 3 of book 11 of Euclid, perpendicular to the line βC lying in the horizon, at the centre of the world β , through which all great circles are drawn. Since, then, the line βD lies in the plane of the great circle measuring the inclination of the dial and makes a right angle with βC , as we said, βD itself will be the common section of the said great circle and of the meridian. Therefore the meridian meets the plane of the dial at the point D .

Diagram. The fourth figure, illustrating the demonstration (the triangles βCD and aFE folded about their bases into the plane of the dial). A large circle with the horizontal line AB across it and many demonstration lines. Readable labels:

- a (α , upper left): the point a (start of the equinoctial), printed as an alpha-like sort
- C (top middle): meeting of horizon and dial-plane
- E (top, right of C): meridian meeting-point
- b (top right, on circumference): construction point / hour marker

- d (upper middle): counting-point
- I (below C): construction point
- A (left): left end of the horizontal line
- B (right): right end of the horizontal line
- 6 (right): hour numeral
- β , F (centre): apex of the style and folded point F
- H (right of centre): triangle vertex
- D (below centre): meridian meeting-point
- G (lower left of centre): construction point
- M (bottom): equinoctial/meridian intersection
- L (bottom left): point on the axis

[Illegible: the crowded centre (β , F, G and the fold-lines) cannot all be resolved individually at this resolution [?]]

LET the triangle aFE also be moved about the line aE until it is joined with the plane of the horizon, and the point F with the centre of the world β (because of the equality of the lines CF and C β). This done, since CFE is the angle of the plane's declination from the vertical, CEF will be the angle of the complement of that same declination — namely such an angle as the common section of the plane of the dial and of the horizon makes with the common section of the horizon and the meridian. Since, then, the meridian is drawn through F — that is, through the centre of the world, where we placed the point F — the line FE will make with the horizontal line CE the angle of the complement of the declination, being the common section of the horizon and the meridian, since it lies in the horizon and is drawn through the centre of the world F and the point E. Therefore the meridian will meet the plane of the dial at the point E. But it also meets the same plane at the point D, as we showed. Therefore the line DE, produced on both sides, will be the meridian line. And since both the equator and the horizon are perpendicular to the meridian, [Margin: d — 19.unde. (*Euclid XI.19*)] their common section too will be at right angles to it, and consequently, by definition 3 of book 11 of Euclid, perpendicular to the line FE lying in the meridian, at F, the centre of the world.

Therefore, since the line Fa lies in the horizon (being drawn through F, the centre of the world, and the point a) and makes a right angle at F with FE lying in the meridian — because of the quadrant of the circle described from F and enclosed between the lines EF and Fa — Fa itself will be the common section of the horizon

and the equator. For which reason the equator will meet the plane of the dial at a, and therefore the equinoctial line is to be drawn through a.

LET IT NOW BE CONCEIVED that through the pole of the dial's plane — and so through the style $K\beta$, which is a portion of the axis of that same plane — and through the pole of the meridian, a great circle is drawn, making in the dial a section that is a straight line, which will necessarily pass through the point a. For since the equator and the horizon also pass through the poles of the meridian, the equator, the horizon, and the said great circle will have one and the same common section. They will therefore make in the dial three sections, straight lines, meeting at that one point (by proposition 18 of book 1) on which their common section falls. Since, then, the common section of the horizon and the equator falls on the point a — because there, as we showed, the horizontal line and the equinoctial cut one another — the common section of the said great circle and the plane of the dial will also pass through the point a. But that same great circle passes through K, the place of the style. Therefore the line aK is the common section of the plane of the dial and of the said great circle. And since the said circle is at right angles (by proposition 15 of book 1 of Theodosius) both to the meridian and to the plane of the dial — because it is drawn through their poles — conversely both the plane of the dial and the plane of the meridian will be at right angles to the said circle. [Margin: a — 19.undec. (*Euclid XI.19*)]

Therefore their common section too — that is, the meridian line Ep — will be at right angles to that same circle, and consequently perpendicular to the line aK lying in that circle. Therefore the line aK will necessarily cut the meridian line at right angles at the point a. Hence it comes about that this same line aK passes entirely through the point H, where the arcs described from D and E at the intervals $D\beta$ and EF intersect one another. For if the triangle DEH is conceived to be moved about the line DE, the line Ha will forever remain in the plane of the said great circle that makes the section aK; because, throughout this motion, the line Ha

Diagram. The fifth figure, completing the demonstration (the fold of triangle DHE onto $D\beta F$ in the plane of the meridian). A circle with the horizontal line AB and the meridian and axis lines, with the folded triangles below. Readable labels:

- M (top left): upper vertex of the meridian construction
- b (upper left, below M): construction point
- 6 (upper right, and again lower left): hour numerals
- L (upper middle, right of F): point on the axis
- F (middle): folded point F

- E (left of centre): meridian meeting-point
- I (left of centre): construction point
- A (left): left end of the horizontal line
- B (right): right end of the horizontal line
- β , α (centre cluster): apex of the style and the point a [?]
- G (centre): construction point
- C (right of centre): horizon meeting-point
- D (bottom): lower vertex, apex of the folded triangles

[Illegible: the centre cluster (β , α , G and the many fold-lines) is not fully resolvable individually at this resolution [?]]

[Margin: b — 7.primi. (*Euclid, Elements I.7*)]

will always make a right angle with the meridian line, so that it inclines neither to this side nor to that — just as the plane of that circle is not inclined to this side or that toward the plane of the dial, but is perpendicular to it. Now if the triangle $D\beta C$ is set perpendicular to the plane of the dial along the line CD, so that the style βK is perpendicular to that same plane; and if the triangle $E\alpha a$ is also understood to be moved about the line aE until the line CF coincides with the line $C\beta$, and the point F with the centre of the world β (because of the equality of the lines CF and $C\beta$), and so the triangle $E\alpha a$ itself is joined with the horizon — drawn through the line aE and the centre of the world β — and the line EF lies along the line $E\beta$ in the plane of the meridian; and if, likewise, the triangle DHE is thought to be turned about the meridian line DE until it is joined with the plane of the meridian: then the triangle DHE will coincide exactly with the triangle $D\beta F$ lying in the same plane of the meridian, so that the point H falls on the centre of the world β . Otherwise, in the plane of the meridian upon the line DE there would be drawn from D two mutually equal lines $D\beta$ and DH, and from E two others mutually equal, $E\beta$ (which is the same as EF) and EH (which was taken equal to EF itself) [Margin: b — 7.primi. (*Euclid I.7*)] — which is absurd. From this it follows that the line EH, in this position, is the common section of the horizon and the meridian. Therefore, since in planes declining from the south — as in the three former figures — the angle dHE toward the northern parts

[full-width text continues below the figure:]

makes, with HE (the common section of the horizon and the meridian), an angle toward the northern parts, the line dH will be the axis of the world, meeting the meridian line of the dial at p; and consequently p will be the centre of the dial, by the corollary of proposition 21 of book 1. But if the axis dH is equidistant from the

meridian line DE, the dial will have no centre. In planes, however, declining from the north — as in the latter three figures — since the angle dHE is equal to the pole's altitude (such, namely, as the axis of the world makes below the horizon with the line HE, the common section of the horizon and the meridian, toward the southern parts), the line dH will be the axis of the world, and therefore, as before, p will be the centre of the dial. Hence it comes about, both in those planes and in these, that the line HM — lying in the plane of the meridian and making with the axis Hp the right angle bHd (because of the quadrant bd) — is the common section of the meridian and the equator, since the axis of the world at the centre of the world H is necessarily perpendicular (by definition 3 of book 11 of Euclid) to a common section of this kind, because the axis is at right angles to the plane of the equator (by proposition 10 of book 1 of Theodosius). Therefore the equator meets the plane of the dial at the point M, and for that reason the equinoctial line is to be drawn through M from a. But if HM, the common section of the meridian and the equator, is parallel to the meridian line DE itself, the plane of the dial drawn through DE will be equidistant from that same common section HM, since it does not cut it. Therefore, by proposition 18 of book 1, the sections that the meridian and the equator make with the plane of the dial will be parallel; and so the equinoctial line is to be drawn through a, equidistant from the meridian line DE.

It is clear, moreover, that the straight line ρK — drawn from the centre of the dial through the position of the style — is the style-line, that is, the common section of the plane of the dial and of its own proper meridian. For since that meridian, drawn through the axis Hp, meets the plane of the dial at ρ , the centre of the dial, their common section with the dial must be drawn through ρ . Again, since this same meridian stands perpendicular to the plane of the dial, it will necessarily pass through the style [Margin: a — 18.undec. (*Euclid XI.18*)], because all planes drawn through the style are perpendicular to the plane of the dial. And so the said common section — that is, the style-line through K — must be drawn; and it is of the kind that the straight line ρK is, which will cut the equinoctial line at right angles, as was demonstrated above. For since both the plane of the equator and the plane of the dial stand perpendicular to the proper meridian of the dial's plane [Margin: b — 19.unde. (*Euclid XI.19*)], their common section too — namely the equinoctial line aM — will be perpendicular to that same meridian, and therefore, by definition 3 of book 11 of Euclid, perpendicular to the style-line ρK , which lies in that same meridian. But if the dial has no centre, the style-line must be drawn through K, the position of the style, parallel to the meridian line and perpendicular to the equinoctial line. For since in that case the plane of the dial is equidistant from the axis of the world — which it does not cut — that is, from the

common section of the meridian of the horizon and the proper meridian of the dial itself, the common sections of the plane of the dial and of those meridians will, by proposition 18 of book 1, be parallel to one another. That is, the meridian line and the style-line will be parallel; and the style-line will cut the equinoctial line at right angles, as we have just demonstrated.

That the straight line ρI is the axis of the world — such that, together with the style-line ρK (which is the common section of the proper meridian of the inclined plane and of the plane of the dial), it contains the angle $I\rho K$, the altitude of the pole above the inclined plane — is evident. For if the triangle $IK\rho$ is moved about $K\rho$ until it comes to stand perpendicular to the plane of the dial, with the style IK likewise perpendicular to it, and the point I therefore coincides with the centre of the world, then the straight line ρI will be the axis of the world, since the axis of the world is drawn through the centre of the world and the centre of the dial. This will also be made clear in the following way. When the triangle ρHE is revolved about the meridian line ρE until it is joined with the plane of the meridian, the point H coincides with the centre of the world, as we showed above, and therefore it will also coincide with the point I ; and so the line ρH will coincide with the line ρI . But we showed that ρH is the axis of the world. Therefore ρI too will be the axis of the world. In dials that have no centre, the axis of the world must be drawn through I , parallel to the style-line; for if it cut that line, the point of intersection would be the centre of the dial, as is evident. The rest is clear from the demonstration of the first construction given in this proposition.

[Margin: Description of the same dial by means of an ellipse.] Finally, we shall describe this same dial — declining and inclined at once — by means of an ellipse drawn in the plane of the dial, exactly as we taught in the scholium to proposition 1 of this book. The semidiameters of the circles that determine the length and breadth of the ellipse, to be drawn from the centre of the dial ρ , are ρG and GI . If larger ones are wanted, take for the greater semidiameter any portion you please of the style-line, namely ρL , and for the lesser the straight line that falls perpendicular onto the axis ρI from the point terminating the greater — as from L . As for the demonstration: about GI (or that perpendicular falling from L onto the axis) a circle is to be drawn in the plane of the equator, from the centre of the world which lies on the axis, to serve as the base of a cylinder. The division of the circles drawn from ρ must begin from the straight line which, at the centre ρ , makes with the style-line — toward the meridian line — an angle equal to the angle ρLM : that is, equal to the inclination of the proper meridian of the inclined plane itself to the meridian of the horizon. Hence, since in the sixth figure the

inclination of the meridians embraces 90 degrees, and the straight line LM makes a right angle with the style-line, the division of the circles drawn from ρ will begin from the meridian line, since this too contains right angles with the style-line, as is clear from what precedes. This construction, moreover, has no place in dials lacking a centre, as in the third figure: because the plane of the dial, being equidistant from the axis of the world — that is, of the cylinder from whose section that ellipse arises — does not produce an ellipse but a parallelogram, as is demonstrated by Serenus of Antinoöpolis [Serenus Antinsensi] in book 1 *On the Section of the Cylinder*.

[Margin: How the lower dial is generated from the upper one.] Now if any dial whatever is inverted from above, so that its upper part becomes the lower, and what after this inversion is on our right becomes the left, and conversely — as we have more than once explained in what precedes — we shall have a dial to be placed on the opposite face of the plane, and below it, as was demonstrated in the scholium to proposition 13 of the preceding book. But then the hour-lines which previously, in the upper dial, indicated the hours from noon will show the hours from midnight, and conversely. And beautifully — as also in the preceding cases — the lineaments of the upper dial, drawn on the opposite face, display this inversion, so that each corresponds exactly to each, provided the upper part of the opposite face becomes the lower, and conversely.

Diagram. The sixth figure of this proposition, continuing the declining-and-inclined-dial construction of figures 1–5. It is a large circle (the proper meridian) traversed by many straight, dashed and dotted lines. The engraved caption at lower right reads "6. figura." Readable labels:

- A (left end of the horizontal diameter): endpoint of the horizontal line AB
- B (right end of the horizontal diameter): endpoint of line AB
- F (top of circle): upper vertex
- C (upper-centre): interior point
- H (centre): centre of the world
- K (below H, centre): style-foot / interior point
- I (lower-centre): axis point
- D (bottom of circle): lower vertex
- β (left of K): interior point
- α (right, on the horizontal line, with G adjacent — " α G"): point on the diameter near B
- b [?] (small label near the top, on a dashed arc): [?]

- δ [?] (near the centre, below C): [?]
- γ [?] ("g", lower centre-left): [?]
- the numeral "6" appears at two or three interior positions and at lower right beside the caption; whether these mark points or belong to the figure-number could not be resolved [?]

Because more than two labels are illegible, this figure is a

Problem 38. Proposition 38.

To depict the parallels — that is, the arcs of the signs of the Zodiac — upon the same dial that both declines from the vertical circle and is inclined to the horizon.

[Margin: How the arcs of the signs are described from the analemma.] Once the altitude of the pole above the declining and inclined plane has been found by proposition 29 of book 1 — the altitude which, in the dial constructed by the earlier method in the preceding proposition, or in that dial in which the style and its position were given in the scholium to the same proposition, is shown by the angle KpI that the style-line makes with the axis of the world, as we said — let it be set out upon the analemma itself, as in proposition 2 of the preceding book, so that the arc CE in the analemma embraces

the degrees of the altitude of the pole found; or, at any rate, so that the angle CDE in the analemma is equal to the angle of the pole's altitude KpI in the inclined dial. Then, in the same analemma, take the straight line DG equal to the style IK, which the first construction of the preceding proposition offers us; or the one we assumed in the second construction, in the portion of the analemma of the same proposition (the portion which corresponds to the analemma of proposition 1 of book 2); or, at any rate, the one

Diagram. A large analemma figure for laying out the zodiac-sign arcs on the declining-and-inclined dial. At its core is the analemma circle ABC (representing the proper meridian of the inclined plane); from a centre near the lower part radiate many straight sign-lines and hour-lines, whose ends carry engraved zodiac-sign glyphs around the periphery. Readable interior geometric labels:

- A (upper-left, near the circle): interior point
- B (lower-left): interior point
- C (left): interior point
- D, E (mid-left, adjacent — "D E"): points on the horizontal
- K (lower-centre): interior point

- ρ (lower-centre, below K): the projection centre from which the sign-lines and hour-lines issue
- H (centre-right of the fan of lines): interior point

Peripheral zodiac-sign glyphs are engraved at the tips of the sign-arcs. Legibly identifiable are Leo ϱ , Gemini Π , Taurus X , Pisces X , Aries Υ , Virgo M , Scorpio M , Sagittarius X ; several further glyphs (upper and lower periphery) are worn and could not be securely resolved [?], and the small hour-numerals (6–12) and the point-letters h and m scattered through the fan are only partly legible [?]. Because numerous peripheral labels are illegible, this figure is a

we assumed in the construction handed down in the scholium of the preceding proposition. And through G let the straight line OR be drawn, parallel to BC. For in this way, on the line RO, we shall find the diameters of the conic sections, just as we found their diameters KR, LR, MR, NO, PO, QO in the horizontal dial in proposition 2 of book 2. Hence, as we taught there, so here too the conic sections of the diameters found will be described — that is, the parallel arcs of the signs of the Zodiac. For just as, in the horizontal dial, the analemma circle ABC represents the meridian of the horizon, and the straight line OR the common section of the plane of the horizontal dial and of the meridian — that is, the meridian line — so in the declining-and-inclined dial the same circle refers to the proper meridian of the inclined plane, that is, the greatest circle drawn through the poles of the world and through the poles of the declining plane; while the straight line OR expresses the common section of the said meridian and of the plane of the declining-and-inclined dial — that is, the style-line.

[Margin: Another way of describing the arcs of the signs.] ANOTHER WAY.

Having drawn the straight line ρI for the axis of the world, erect upon it at I the perpendicular IG for the radius of the equator, and to either side of it let the radii of the other signs be drawn out, as was done above; and let Ip be equal to the portion of the axis ρI in the dial, and the straight line IG equal to the straight line IG (or LG) in the same dial; and from ρ , through G, let the straight line ρG be sent out. This we shall perhaps draw more conveniently if from ρ we draw the straight line ρG making, with ρI , an angle $G\rho I$ equal to the angle $G\rho I$ which the axis ρI and the style-line ρG form in the dial — namely, equal to the angle of the pole's altitude above the plane of the dial. [Margin: a — 4. or 26. of book 1 (*Euclid I.4 or I.26*)] In this way this triangle $G\rho I$ will be entirely equal to the triangle $G\rho I$ in the inclined dial; and so, ρI being in this figure the axis of the world, the straight line ρG will be the style-line. And if the rest be done as we handed down in proposition 2 of this book and in its scholium — always taking here the letter ρ

for the letter C mentioned there — the parallels, that is the arcs of the signs of the Zodiac, will be described.

We have subjoined an EXAMPLE corresponding to the last upper dial of the six of the preceding proposition. Since here the style-line is the same as the line of the 6th hour, upon the several hour-lines issuing from the point ρ of the figure of the rays of the signs of the Zodiac (from which the arcs of the signs are described) there are set two hour-numbers each, equally distant on either side from the line of the 6th hour — that is, from the style-line — as we noted in proposition 2 of this book. Again, here the straight line ρV , equidistant from the radius of the equator, exhibits the 12th hour, since the line of this hour in the dial is equidistant from the equinoctial line: just as, in the horizontal dial, the straight line HV , equidistant from the radius of the equator in the figure of rays of proposition 2 of the preceding book, refers to the 6th hour, because the line of this hour too is equidistant from the equinoctial line in the dial.

We have also added another figure of the rays of the Zodiac, with the hour-lines issuing from ρ , corresponding

Diagram. The added figure of the rays of the zodiac-signs for the first dial, shown divided into two ray-fans across the top of the page. Along the top runs a horizontal base-line carrying the marks "B I B ρ "; from two apices (one at I, one at ρ , with K and P/ ρ nearby) fan out many straight and dashed hour-rays down to a large semicircular arc below. At upper right is a small inset triangle carrying the marks "B — B I", "A K A", and, down its right edge, "D G D". Readable labels: B, I, ρ , K, G, V, A, D. The rays carry rotated "radius of the hour" captions; as read (with uncertainty): "Radius hor. 6" [?], "Rad. hor. 5" [?] (upper-left, dashed), "Rad. hor. 24" [?] and "Rad. hor. 18" [?] (right), "Rad. hor. 10" [?] and "Rad. hor. 14" [?] (lower). Along the semicircular arc a double row of hour-numerals is engraved where the rays strike it, running approximately 6 5 7 4 8 3 9 2 10 1 11 12 to the left of centre and 12 11 1 2 3 4 5 6 to the right; the exact assignment of several numerals is uncertain [?]. Because several ray-captions and arc-numerals are illegible, this figure is a

corresponding to the first dial of the preceding proposition, in which the style-line is neither one of the hour-lines nor equidistant from two placed on either side. Hence it comes about that the several lines issuing from ρ have their several numbers affixed. From this figure, therefore, the arcs of the signs may be described in the said dial, when occasion requires. But we have divided this figure, as you see, into two, because otherwise the lines would be too much confused with one another — for this reason, that the style-line of the first dial is

little distant from the line of the 11th hour. In the former of these figures are contained the hour-lines which in the dial are placed to the left of the style-line, such as 5, 6, 7, 8, 9, 10, and 11. In the latter, however, the remaining hours placed to the right of that same style-line, namely 12, 1, 2, 3, 4, 5, and so on. This is what we long since pointed out, in proposition 2 of this book,

Diagram. The left-hand of two lower figures of zodiac-rays for the third dial (whose hour-lines are parallel). A quarter-circle arc springs from a horizontal base-line; from a point ρ at the left issue straight and dashed rays cutting the base-line, with the point-marks A, B, F, and the hour-labels II (11), I (1), and 12 near the base. Hour-numerals 5, 6, 7, 8, 9, 10 are ranged along the descending rays. Several small numerals are only partly legible [?]

Diagram. The right-hand of the two lower figures, the companion ray-fan for the same third dial. Similar in construction: a quarter-circle arc over a horizontal base-line, with a point ρ , the marks A, B, F, and the hour-labels I (1), 12, near the base, and hour-numerals 12, 1, 2, 3, 4 ranged along the rays. Several small numerals are only partly legible [?]

must be done in declining — and therefore inclined — dials, in which such confusion of lines is found.

LASTLY, that you may see what is to be done in the third dial of the preceding proposition — in which the hour-lines are equidistant [parallel] — we have described the arcs of the signs in it too. These are indeed described just as in a meridian or polar dial, with this exception: that here the hour-lines, cutting the ray of the equator in the figure of the rays at right angles, have single numbers affixed to each, not two, as there — unless the style-line is itself one of the hour-lines, or

or else it stands equally distant from the two nearest hour-lines placed on either side, as commonly happens in other declining dials.

Diagram. The left-hand figure at the top of the page — the figure of the rays (*figura radiorum*) for the dial. A vertical central line carries the mark 1 where a horizontal cross-bar crosses it near the top; from apices just below the cross-bar many straight and dashed rays fan downward and converge. Ranged down the left margin of the figure are hour-numerals reading (top to bottom, with uncertainty) 8, 7 [?], 6, 5, ... 3 [?]; a mirror set (11, 10 [?], ... 7, 5, 3 [?]) runs down the right. Across the top run several worn zodiac-sign sorts [?]. Two rotated captions stand at the foot: on the left **Rad. hor. 14** [Radius horae 14 — "radius of

the 14th hour"] and, centred, **Rad. horio** [Radius horologii? [?]]. Several small numerals and the zodiac sorts do not resolve cleanly.**

Diagram. The right-hand figure — the dial that both declines and inclines, on which the horizontal line is constructed as described in the text below. A dashed vertical line runs down the right side; along it and the lines crossing it are the labelled points, reading roughly left to right across the upper crossing region **V, I, E, D**, with **A** set above **D**, and below them **B** (left) and **C** (right) joined by a dashed horizontal line **B---C**. At the foot a quarter-circle arc carries the mark **h**, with **G** to its lower right. Many straight and dashed hour-lines cross the figure. A cascade of zodiac-sign sorts runs down the right edge and around the lower-left arc. Readable labels: **A, B, C, D, E, G, h, I, V**. Several further point-marks and the zodiac sorts are illegible [?].**

[Margin: How the horizontal line is described.]

THE horizontal line in the dial — the one whose style, following its given size, and whose position we described in the scholium of the preceding proposition — has now been drawn. For we showed there that the line ***E** [?] is the horizontal line. In other dials, however, the same horizontal line is drawn in the following way. From any point **A** taken on the meridian line, describe an arc of a circle *m h*. On this arc, from *m* — either to the left or to the right of the meridian line (as we taught in the scholium of the preceding proposition, when we dealt with the placing of the dial according to the later constructions), according as the arc *m h* has been described above the point **A** or below it, and so on — count off the arc of the given plane to which the dial is parallel, intercepted between the Meridian of the Horizon and the greatest circle that measures the inclination of the plane to the Horizon, up as far as *h* (how this is to be found, we demonstrated in proposition 31 of book 1). Then from **A** through *h* draw the straight line *Ah*. For the perpendicular drawn to *Ah* from the point where the equinoctial line and the line of the 6th hour intersect will give the horizontal line.

We shall show this as follows. As we demonstrated in the said scholium of the preceding proposition concerning the line *Eh*, the straight line *Ah* is the common section of the plane of the dial and of the greatest circle that measures the inclination of the plane to the Horizon; and the common section of that same plane of the dial and of the Horizon cuts it at right angles. It follows — since the common section of the Horizon and the plane of the dial passes through the 6th hour on the equinoctial line, as has been said and demonstrated more than once above — that the straight line which cuts *Ah* at right angles is the common section of the Horizon and the plane of the dial, that is, the horizontal line.

[Margin: Another way of describing the horizontal line.]

IN another way we shall draw the same horizontal line thus. Having drawn Ah as before, we erect upon it, from K (the place of the gnomon), the perpendicular KB . On KB produced we take BC equal to the gnomon KI , and at C we set, upwards, the angle BCD equal to the complement of the inclination of the plane to the Horizon, so that the line CD cuts Ah at D . I say that the line drawn through D at right angles to Ah is the horizontal line. For if the triangle BCD is imagined to revolve about BD until it becomes perpendicular to the plane of the dial, then, by definition 4 of book 11 of Euclid, BC will be perpendicular to the plane of the dial. Therefore, if the style KI is also conceived as perpendicular to the same plane of the dial, [a] the line BC and the style KI will be parallel to one another. [Margin: a. Euclid XI.6.] [b] Therefore the line joining the points C and I , being raised on high, will be parallel to BK and equal to it. [Margin: b. Euclid I.33.] Now since the Horizon cuts the plane of the dial along a straight line perpendicular to Ah — as we showed in the scholium of the preceding proposition, when we dealt with the placing of the dial — [c] this common section will be parallel to KB . [Margin: c. Euclid I.28.] Since, then, CI has also been shown parallel to BK , [d] the common section of the plane of the dial and of the Horizon and the line CI will likewise be parallel to one another; and consequently they lie in the same plane, by the definition of parallels. [Margin: d. Euclid XI.9.] Therefore, since the Horizon, drawn through that common section, passes through I , the tip of the style — that is, the centre of the world — it will also pass through the line IC , and so through the point C . Accordingly, since the angle BCD has been made equal to the complement of the inclination of the plane to the Horizon, and for that reason the angle BDC is [equal to] the inclination itself, the Horizon will proceed along the line CD ; for by this construction it meets the inclined plane at the angle of inclination BDC , so that CD is the common section of the Horizon and of the triangle BCD . For this reason the Horizon will meet the plane of the dial at the point D . Since, then, as we demonstrated before, the common section of the Horizon and the plane of the dial is perpendicular to Ah , it is clear that the perpendicular to Ah drawn through D is the horizontal line. And the horizontal line drawn through D will pass entirely through the 6th hour on the equinoctial line, if no error has been made in the construction.

[Margin: Another description of the same horizontal line.]

IN yet another way we shall describe the horizontal line. From the figure of the rays of the Zodiac, let there be found — in that portion of the dial's meridian line which faces midnight, that is, which indicates midnight, or the 12th hour after

noon — the point E, through which the diurnal arc of 24 hours is to be drawn; or, in that portion of the same meridian line facing noon, that is, which shows noon, or the 12th hour after midnight, let there be found the point E of the diurnal arc of 0 hours, or of the nocturnal arc of 24 hours. For sometimes the diurnal arc of 24 hours does not cut the portion of the meridian line belonging to midnight, but the nocturnal arc of

24 hours cuts that portion of the meridian line which faces noon. For through the point E the horizontal line will have to be drawn, since the Sun rises at the 12th hour after noon, on a day containing 24 hours, or at the 12th hour after midnight, on a day containing 0 hours, or when the night comprises 24 hours — as is clear from the tables of the diurnal arc of 24 hours and the nocturnal arc of 24 hours (proposition 33 of book 1). By the same reasoning, if on the line of the 5th hour after midnight the point F of the diurnal arc of 14 hours is found, or on the line of the 7th hour from midnight the point F of the diurnal arc of 10 hours, we shall likewise have a point F through which the horizontal line is to be drawn, as is plain. And so we shall have at least four points through which the horizontal line must be drawn, namely D, E, F, and the point of the 6th hour on the equinoctial line. In the same manner the horizontal line will pass through the point of the diurnal arc of 10 hours, where it is cut by the line of the 5th hour from noon; likewise through the point of the diurnal arc of 14 hours, where it is cut by the line of the 7th hour from noon.

[Margin: The horizontal line divides the dial into an upper and a lower part, and into diurnal and nocturnal.]

MOREOVER, the horizontal line divides the whole dial described so far — as was said in what precedes — into an upper and a lower part, of which the upper is that part contained below the horizontal line; the remaining part gives the lower. This holds provided you remember that everything must be inverted, and that the hours which before were reckoned from midnight are now to be counted from noon, as we showed in propositions 13 and 14 of the preceding book. Likewise the same horizontal line separates the whole dial into diurnal and nocturnal, as is clear from the same proposition 14 of the preceding book. And so we have drawn the parallels, or the arcs of the signs of the Zodiac, and so forth. Which was to be done.

SCHOLIVM.

[Margin: Which arcs belong to the northern signs, and which to the southern.]

WHICH arcs in the dial belong to the northern signs, and which to the southern, it will not be difficult to judge, if the four Analemmas which we described in proposition 25 of this book are attentively considered. For in dials that face south and the vertex — that is, the pole of the Horizon [the zenith] — if the centre of the dial lies below the equinoctial line, the arcs of the northern signs are contained between the centre and the equinoctial line, as is established from the first Analemma of the said proposition; because the points of the northern quadrant H F cast their shadows at noon into the portion of the meridian line M N, between the centre M and the point N, through which the equinoctial line is drawn. But if the centre of the dial lies above the equinoctial line, the arcs of the southern signs are comprised between the centre and the equinoctial line, as is manifest from the second Analemma; since the points of the southern quadrant G H cast their shadows at noon into the portion of the meridian line M N, between the centre M and the equinoctial line, which is drawn through N. But if the dial has no centre, the arcs below the equinoctial line will belong to the northern signs, as is clear from the first Analemma, if below E a line were drawn parallel to the axis F G — a line which would be the common section of the Meridian and the plane of the dial. For then the points of the northern quadrant H F would cast their shadows at noon into that portion of the meridian line below the point through which the equinoctial line would have to be drawn.

But in dials that face north and the vertex, wherever the centre of the dial may be, the arcs of the northern signs are perpetually contained between the centre and the equinoctial line. This is plainly established from the two later Analemmas. For in the third, the points of the northern quadrant H F cast their shadows into the portion of the meridian line M N, between the centre M and the point N of the equinoctial line; and the same is seen in the northern quadrant F I of the fourth Analemma, where at midnight the shadows are cast into the portion of the meridian line M N, between the centre and the equinoctial line. Nor is it any objection that sometimes certain points of the northern semicircle H F I, in each of the two later Analemmas, also cast their shadows beyond the centre M — as happens in the quadrant F H of the fourth Analemma — because then the arcs of the signs are ellipses, so that each cuts the meridian line in two points, one of which necessarily lies between the centre and the equinoctial line. Thus it is always true that the arcs contained between the centre and the equinoctial line belong to the northern signs. And if the dial is so elevated on the southern side that the arc of the Meridian between it and the Horizon is equal to the altitude of the Equator — or to the complement of the pole's altitude — and consequently the meridian line M N is parallel to the diameter of the Equator H I, it is likewise clear that then the arcs of the signs near the centre belong to the northern signs,

because the shadows of the points of the northern semicircle H F I fall on the meridian line, in the middle of which the centre M lies, as is plain.

IN dials that face the Nadir — that is, among the lower [inferior] dials — exactly the contrary is to be understood, as the same four Analemmas demonstrate.

[Margin: Which arcs of the signs are hyperbolas, and which parabolas or ellipses.]

AGAIN, the arcs of the signs in those dials in which the hour-lines are parallel — such as the third dial described in the preceding proposition — are all hyperbolas, just as in the polar dial, as was said at proposition 38 of the preceding book. For the reasoning here is the same. In other cases we shall judge, as we handed down in proposition 2 of the same book, which arcs are hyperbolas, which parabolas, and which ellipses. Or indeed let the same be sought from propositions 4, 5, 6, and 7 of the first book.

[Margin: How, from the figure of the rays, a larger or smaller dial may be described for a given length of style.]

AND if, from the figure of the rays of the Zodiac, a larger or smaller dial is to be described for a given length of gnomon, it will be done in this way. Let there be described separately — just as was done alongside the figure of the rays of this proposition — the triangle ρ I G, taken from the dial which we constructed in the preceding proposition, or from the figure of the rays itself. In it the axis of the world is ρ I; the Equator is I G; and the line of the style, upon which the gnomon I K falls at right angles, is ρ G. Then, the lines I ρ , I G, I K having been produced, if on I K there be taken the line I A equal to the given gnomon — whether it be greater than the style I K or smaller — and through A a line B D be drawn parallel to ρ G, the triangle B I D will be similar to the triangle ρ I G, by the corollary of proposition 4 of book 6 of Euclid. And so, if the axis I B is transferred into the figure of the rays from ρ as far as B, towards I, and the rest is done as we prescribed in the scholium of proposition 2 of this book — always changing the letter C, there placed, into the letter ρ , here noted — the arcs of the signs will be described according to the size of the given style I A, and so forth.

PROBLEMA 39. PROPOS. 39.

To lay out the parallels of the diurnal arcs in the same dial that both declines and inclines.

[Margin: How the diurnal arcs are described in the same dial.]

IT has already been declared more than once in what precedes that the description of the diurnal arcs in any dial differs in nothing from the description of the parallels of the signs of the Zodiac, provided that in place of the rays of the parallels of the Zodiac one takes the rays of the parallels of the diurnal arcs. Hence it will be superfluous to give new precepts here, since what we wrote in the preceding proposition suffices. We have therefore laid out the parallels of the diurnal arcs in the same dial that both declines and inclines. Which was to be done.

SCHOLIVM.

IF here we repeat what we handed down in the scholium of proposition 3 of this book, we shall be able easily to describe those hour-lines which scarcely, or in no way, cut the equinoctial line.

PROBLEMA 40. PROPOS. 40.

To establish the vertical circles in the same dial that both declines and inclines.

[Margin: The description of the vertical circles in the same dial.]

FROM the place K of the style, a perpendicular KB having been drawn to the horizontal line, let another perpendicular KA be raised upon it, equal to the style KI; and the line AB having been drawn, let BD be taken equal to it on KB produced. Then, with D as centre, let a circle be described, to be divided into 360 equal parts — or into fewer, as we said above — beginning from the line DE drawn from the centre D to the point E, where the equinoctial line, the horizontal line, and the line of the 6th hour inter-

Diagram. A large figure of the vertical circles for a dial that both declines and inclines. At the centre stands a circle with centre **D**; on and around it are the point-marks (reading roughly): **M** (left of D), **N** (right of D), **F**, **B**, **G** clustered near the horizon line, **E** on the horizon to the right, **K** just below centre, **A** at lower left, **C** at the foot, **I** and **L** near the top. The horizontal base-line running across the middle is lettered **HO—RIZON** (the word "Horizon" split by the figure). Many straight and dashed rays radiate outward from the lower region. Around the periphery run graduation-numerals: across the top (left to right) XXX, 90, 75, 60, 45, 30 [printed "30"], XXX, and IS [?]; down the right side XV, XV, 30, XXX, XLV, 45; along the bottom (left to right) 30 [printed "30"], 45, 60, 75, 90, 75, 60, with LX above; the Roman forms (XV, XXX, XLV, LX) and the Arabic forms (15, 30, 45, 60) evidently label the same azimuthal graduations of the vertical circles. Worn zodiac-sign sorts stand among the outer rays (e.g. ♎ [Libra] at left; ♈ [Pisces] and

☿ [Capricorn]-like sorts [?] at the right and lower left). Readable point-labels: A, B, C, D, E, F, G, H, I, K, L, M, N. Several peripheral numerals, the "IS" marks, and the zodiac sorts do not resolve cleanly. (more than two peripheral labels flagged [?]).

[Margin: The description of the vertical circles in the same dial.]

—sect. For the hidden [construction] lines drawn from D through the division-points will cut the horizontal line at points which, if they are joined by straight lines to the point C — where the line BK cuts the meridian line — the vertical circles will have been described; that is, the common sections of those circles and of the plane of the dial, so that CE is the common section of the plane of the dial and of the vertical circle properly so called. We shall demonstrate this as follows.

[Margin: Demonstration of the aforesaid description of the vertical circles.]

LET the triangle ABK be understood to revolve about BK until it is perpendicular to the plane of the dial, so that the point A is joined with the tip of the style — that is, the centre of the world I — and the triangle itself coincides with the plane of the vertical circle, which is perpendicular to the plane of the dial, and is drawn through the style itself, and measures the inclination of the plane above the Horizon; so that the line BK is the common section of this vertical circle and the plane of the dial. Since, then, the meridian line is also the common section of that same plane of the dial and of the Meridian — which is likewise one of the vertical circles — the meridian line and BK will necessarily meet at that point into which falls the common section of all the vertical circles, that is, the axis of the Horizon, as is clear from proposition 18 of book 1. And they meet at C. Therefore C will be the vertical point, in which all the vertical lines come together. [Margin: What the vertical point is.] And because, as we wrote in proposition 4 of this book, the common section of any plane of the dial and of the vertical circle properly so called passes through the point E, where

where the equinoctial line, the horizontal line, and the six-o'clock line cut one another, the line CE will be the common section of the vertical circle properly so called and of the plane of the dial.

Again, with the triangle ABK remaining fixed at right angles to the plane of the dial, the line AB is the common section of the horizon and of that triangle — that is, of the vertical circle producing the section BK — since the horizon passes both through A, the tip of the style, and through the point B. Now suppose the circle described from D is imagined to be moved about the horizontal line until its centre D is joined to the point A, and the line DB coincides with the line AB (for DB and AB are equal), so that the circle itself comes to be described in the plane

of the horizon about the centre of the world. Then the line DE will be the common section of the horizon and of the vertical properly so called, since the vertical is drawn through the centre of the world — which is now the same as D — and through the point E, as we said. The line DF, drawn from D to the point F where the meridian line cuts the horizontal line, will be the common section of the horizon and of the meridian; for the meridian too runs in the dial both through the centre of the world D and through the point F. From this it follows that the angle EDF is a right angle, since the meridian and the vertical properly so called cut one another at right angles at the centre of the world.

And so, since the division of the circle takes its start from the line DE, the remaining hidden lines sent out from D through the division-points will be the common sections of the horizon and of the other vertical circles. Therefore, where they divide the horizontal line, through those points must be drawn from the vertex C the common sections of the vertical circles and of the plane of the dial. For at those points of the horizontal line all the vertical circles meet the plane of the dial, and all are likewise drawn through the vertical point C, into which the axis of the horizon falls. Such is the line AC drawn, which is the common section of all the vertical circles, since all of them are drawn both through the centre of the world A and through the vertical point C. Hence, unless an error has been made, the angle BAC must be a right angle; CAK the angle of the plane's inclination to the horizon; and ACK the angle of the complement of the inclination — just as ABK is the angle of the inclination which the horizon makes with the inclined plane, and BAK the angle of the complement of the inclination. [Margin: How the vertical point is found.] The vertical point, therefore, will be found if at A a style is set at the angle CAK of the plane's inclination to the horizon, or if a perpendicular AC is drawn to AB; for the line AC will cut the meridian line at the vertex C. The numbering of the vertical lines takes its start from the line CE and proceeds in both directions, so that the meridian line gives the ninetieth vertical. We have thus established the vertical circles in the same dial both declining and inclined. Which was to be done.

SCHOLIUM.

In our example the vertical circle of 60 degrees passes exactly through K, the place of the style, so that the section it makes in the plane of the dial is the line CD; for the vertical which measures the plane's inclination to the horizon stands 30 degrees from the meridian and 60 degrees from the vertical properly so called, since we suppose the plane of our dial to deflect 30 degrees from the vertical. This also happens in proposition 4 of this book.

It sometimes happens that certain of the hidden lines sent out from the centre D through the division-points scarcely, or in no way, cut the horizontal line, so that it is very difficult to describe without error the vertical lines corresponding to them from the vertex C in the dial. This occurs in our example with the vertical lines of 30 and 45 degrees. When we are left without points on the horizontal line, points must therefore be sought on the equinoctial line through which the vertical lines are to be drawn, in this way. [Margin: How to draw those vertical lines which scarcely or never cut the horizontal line.] Find the hour at which the Sun, being on the Equator, reaches the particular vertical circle whose line we wish to draw. For where the line of that hour (which we shall draw hidden, following the doctrine of proposition 37 of this book) cuts the equinoctial line, through that point the vertical line in question must be drawn from C. Thus, since the Sun on the Equator reaches the vertical of 30 degrees on the eastern side at 3 hours 17 minutes before noon — that is, at 8 hours 43 minutes after midnight — and the line of that hour cuts the equinoctial line at G, the line CG drawn will give the vertical of 30 degrees. Likewise, since the Sun on the Equator attains the vertical of 45 degrees at 2 hours 15 minutes before noon — that is, at 9 hours 45 minutes after midnight — and the line of that hour cuts the equinoctial line at H, the line CH drawn will give the vertical of 45 degrees; and so with the rest.

We shall find the hour at which the Sun, running along the Equator, reaches the proposed vertical in the following way. [Margin: How the hour is found at which the Sun, being on the Equator, reaches any given vertical.] Let the horizon be ABCD; the meridian AC; the equator BED; the vertical properly so called BFD; and let FIH be any other vertical whatsoever, cutting the horizon in the eastern and southern quadrant AB, and the equator in I. The arc EI on the equator is therefore to be found. Since in the spherical triangle EFI the angle E is a right angle, then — by proposition 18 of book 4 of Johannes Regiomontanus, *On Triangles* [Regiomontanus, *De Triangulis* IV.18], or by proposition 14 of book 1 of Geber [Jābir ibn Aflāḥ, *De Astronomia* I.14], or by proposition 42 of our *Spherical Triangles* [Clavius, *Triangula Sphaerica* 42] — as the sine of the angle EFI (which is known, since its arc AH is the complement of the distance FH of the vertical from the vertical properly so called FB) is to the whole sine, so is the sine of the complement of the angle EIF to the sine of the complement of the arc EF, the altitude of the pole.

Diagram. A hemispherical (analemma-sphere) figure showing the horizon, meridian, equator and vertical circles for the worked hour-example. The outer circle is the horizon ABCD; a vertical diameter AC is the meridian; a great-circle arc BED is the equator; the arc BFD is the vertical properly so called; and the arc

FIH is another vertical circle cutting the horizon in the eastern–southern quadrant AB and the equator at I. Readable labels :

- A (top): zenith / pole of the horizon, top of the meridian AC
- B (left): true east point on the horizon
- C (bottom): lower end of the meridian AC
- D (right): true west point on the horizon
- E (upper centre): point on the equator BED
- F (centre): intersection of the verticals (foot of the vertical circles)
- G (below centre): point near the foot
- H (upper left): end of the vertical FIH on the horizon
- I (left of centre, below H): intersection of vertical FIH with the equator
- K (lower right): point on the vertical FLK
- L (lower right, below K): point on the vertical FLK
- M (upper right): point on the vertical FM
- N (lower left): point on the vertical FNO
- O (lower left, below N): point on the vertical FNO
- P (upper right): point where a vertical meets the equator above the horizon

(All fifteen point-labels A–P resolve on the scan; no [?] flags.)

And conversely: as the whole sine is to the sine of the angle EFI — that is, to the sine of the complement of the distance of the proposed vertical from the vertical properly so called — so is the sine of the complement of the altitude of the pole to the sine of the complement of the angle EIF. Therefore the angle EIF will be known. And since again — by proposition 16 of book 4 of Johannes Regiomontanus, *On Triangles* [Regiomontanus, *De Triangulis* IV.16], or by proposition 13 of book 1 of Geber [Geber I.13], or by proposition 41 of our *Spherical Triangles* [Clavius, *Triangula Sphaerica* 41] — as the sine of the angle EIF just found is to the sine of the arc EF, the altitude of the pole, so is the sine of the angle EFI, the complement of the distance of the proposed vertical from the vertical properly so called, to the sine of the arc EI, the distance of the Sun from the meridian: if we make it that, as the whole sine is to the sine of the complement of the distance of the proposed vertical from the vertical properly so called, so is the sine of the complement of the altitude of the pole to a fourth term, a sine will be found whose arc-complement is to be kept. And if again we make it that, as the sine of this kept complement is to the sine of the altitude of the pole, so is the sine of the complement of the distance of the proposed vertical from the vertical properly so called to a fourth term, the sine of the distance of the Sun

from the meridian will be found. [Margin: Example.] Example. Let the vertical FH be supposed to stand 30 degrees from the point B, the true east, toward the south. If, then, we make it that, as 100000, the whole sine, is to 86602, the sine of the complement of the distance of the said vertical from the point of true east, so is 74314, the sine of the complement of the altitude of the pole, to a fourth, we shall obtain approximately this sine

64357, whose arc-complement contains about 49 degrees 56 minutes. If again we make it that, as 76529, the sine of the arc of 49 degrees 56 minutes just found, is to 66913, the sine of the altitude of the pole, so is 86602, the sine of the complement of the distance of the proposed vertical from the vertical properly so called, to a fourth, there will be found here very nearly the sine 75720, whose arc of 49 degrees 13 minutes will give the distance of the Sun from the meridian at that moment, 3 hours 17 minutes. And since this distance is before noon — because the vertical FH is supposed to stand 30 degrees from true east toward the south — we shall subtract it from a semicircle, that is, from 12 hours, so that there remains 8 hours 43 minutes after midnight. In the same way, if the same vertical is supposed to stand at 45 degrees, we shall find 2 hours 15 minutes before noon, that is, 9 hours 45 minutes after midnight.

But now if the vertical FM stands 30 or 45 degrees from the true west D toward the south, we shall then find, by the same device, the hour 3 hours 17 minutes after noon, or the hour 2 hours 15 minutes.

[Margin: When the proposed vertical deviates from true east or west toward the north.]

[Margin: What is to be done when the hour at which the Sun, being on the Equator, reaches the proposed vertical, does not cut the equinoctial line.]

But if the proposed vertical recedes from true east or west toward the north, and consequently cuts the Equator below the horizon — such as the vertical FNO or FLK — the same vertical will also cut the Equator above the horizon at the opposite point, as at P or I. Therefore, as before, we shall investigate the distance of the Sun EP after noon, or EI before noon; for this is equal to the distance of the point O after midnight, or of the point K before midnight, and so on.

Again, if it should happen that the hour found from noon or from midnight, through which the proposed vertical line is to be drawn, lies on the equinoctial line but does not cut the equinoctial line, we shall seek the opposite hour on the equinoctial line and draw the vertical line through it. For this line, extended from the vertical point C toward the equinoctial line, will represent the semicircle

opposite to that of the proposed vertical, while beyond the vertical point C it will represent the remaining semicircle, whose line is sought. Thus, if by this art we wish to draw the vertical line of 30 degrees such that its vertical circle recedes from true west toward the south by 30 degrees, we shall find that the Sun on the Equator reaches that vertical at 3 hours 17 minutes after noon. And because this hour after noon cannot cut the equinoctial line in our dial, we shall find on the equinoctial line the point L of the hour 3 hours 17 minutes after midnight. For the line CL will give the common section of the vertical circle receding from true east toward the north by 30 degrees, while the portion of the same beyond C, extended, gives the common section of the vertical circle receding from true west toward the south by 30 degrees. Let the same be done in the other cases.

[Margin: The aforesaid device can be used by anyone in drawing all the verticals in any dial described thus far.]

We shall also be able to use this method in the preceding cases, whenever some of the vertical lines scarcely, or in no way except at the most distant points, cut the horizontal line. Indeed, by the same device it will be possible to describe all the vertical lines.

But if one of the hidden lines drawn from D should happen to be parallel to the horizontal line, then the vertical line must be drawn through C parallel to that same horizontal line. For since that hidden parallel drawn through D is the common section of the horizon and of the vertical drawn through it, and since — if the circle described from D is understood to be set in its proper position — the plane of the dial is equidistant from that common section (otherwise the hidden parallel, extended, would cut the plane of the dial, and so at some point of the horizontal line, since the horizon meets the plane of the dial only along this line; the same parallel would therefore cut the horizontal line, which is absurd, since it is supposed parallel to the horizontal line): then, by proposition 18 of book 1, the common sections made by the plane of the dial in the horizon and in the said vertical — that is, the horizontal line and the aforesaid vertical line — will be parallel to one another. Thus you see, in the proposed example, that the vertical line of 30 degrees is parallel to the horizontal line, because the hidden line MN, which is the common section of the horizon and of the vertical of 30 degrees, is parallel to the horizontal line. For since a quadrant is intercepted between the lines DM and DB — for from the line MD, the common section of the horizon and of the vertical of 30 degrees, up to FD, the common section of the horizon and of the meridian (or vertical of 90 degrees), 60 degrees are counted; and from the line FD up to BD, the common section of the horizon and of the vertical of 60 degrees,

30 degrees are reckoned, which with the 60 degrees make 90 degrees — the angle at D will be a right angle. Since, then, the angle at B is also a right angle, [Margin: (a) Euclid I.28] the lines MN and BE will be parallel to one another.

PROBLEMA 41. PROPOS. 41.

To lay out the parallels of the horizon in the same dial both declining and inclined.

[Margin: The description of the parallels of the horizon in the same dial both declining and inclined.]

Just as in proposition 41 of the preceding book we described the parallels of the horizon in a polar dial, so we shall describe the same in a dial both declining and inclined, provided that what we are about to say is rightly grasped. As to the former method: the circle of the analemma ABC will here represent that vertical circle which, passing through the style, stands at right angles to the plane of the dial and makes in the plane of the dial the section CD — just as in the preceding proposition it is the one which, drawn through the style IK, makes the section CD. The parallels of the horizon, moreover, are not changed in the same analemma. Next, in place of the axis EF, a diameter must be drawn — the common section of the said vertical and of the great circle to which the dial is equidistant — such that both the arc CE and the arc BF is equal to the inclination of the proposed plane to the horizon. Parallel to this diameter, the line RQ must be drawn, of the size of the style IK or AK, such that the line DI, cutting the line RQ at right angles, is equal to the style IK. Finally, the points L, M, N, etc., taken from the point T where the horizon BC cuts the line RQ, must be transferred onto the line BC, from B, the point of the horizontal line, toward C, in the upper dial; but in the lower dial they must be trans-

Diagram. The analemma circle for describing the parallels of the horizon in the declining-and-inclined dial. A large circle with centre **D** carries the horizon-diameter BC (B at the left, C at the right) and the vertical section CD; an oblique line **RQ** (R at the upper right, Q at the lower left) is drawn parallel to the axis-diameter, of the size of the style, and the perpendicular DI (I near the top) cuts it at right angles. The periphery is graduated in degrees. Readable labels:

- A (top of circle); B (left, on horizon BC); C (right, on horizon BC); D (centre)
- E (upper right, on the circle); F (bottom of circle); G (upper left, on the circle); H (lower right, on the circle)
- R (upper right) and Q (lower left): the two ends of the oblique line RQ

- I (near top): foot of the perpendicular DI to RQ
- T (left of centre): point where the horizon BC cuts RQ
- Inner transferred points, clustered symmetrically about the centre: **L, M, N** (upper centre) and again **L, M, N** (lower centre); **K, S, V** near the centre; **O** at the lower centre. Several of these small inner letters overlap and their exact placement does not resolve cleanly.
- Peripheral degree-graduations: across the top **90**; down the left side **75, 60, 45, 30, 15**; down the right side **15, 30, 45, 60, 75**; along the bottom **30, 45, 60, 75** [several bottom numerals faint].
- A second **P** appears near R (top) and near Q (bottom) as an end-label of the oblique line [?].

— more than two inner point-labels (the L/M/N/K/S/V/O cluster and the P marks) overlap and cannot be pinned to exact positions at this resolution.

-ferred, the intervals TS, and others, if there be any, from B in the same line BD, extended beyond B. For the conic sections described about the diameters LQ, MQ, NQ, etc., according to the doctrine of proposition 8 of book 1, and transferred into the dial in such a way that their diameters coincide with the line BC and pass through the transferred points L, M, N, etc., with their horns turned away from the horizontal line, will give the parallels of the horizon.

As to the latter method: the line AD must be taken equal to the line AC, which in the figure of the preceding proposition falls from A, the tip of the style, onto the zenith, or vertex C; and onto the line AB must be transferred the intervals between the centre D and the vertical lines on the horizontal line, and so on. All the rest must be completed as was set forth in the said proposition 41 of the preceding book. For here too the horizontal line serves as a kind of equinoctial line, and the vertical lines as hour-lines, whose centre is the zenith, or vertex C. Likewise AD, in this adjoined figure, serves as the axis of the world — namely the axis of the horizon, which now serves as the equator; and AB serves as it were the radius of the equator, and the other lines sent out from A as the radii of the other signs; while the lines falling from D serve as the hour-lines, and so on.

[Margin: Another description of the parallels of the horizon.]

Diagram. The alternative construction ("another description") of the horizon- parallels, the analemma "unfolded" onto the dial plane. A horizontal top line carries, left to right, the labels **XC — A — D — [mirror-XC] — C**; from the points A and D a fan of straight lines (the vertical lines, each carrying **two** numbers as the text explains) descends, converging toward **F** near the centre-bottom, with a

broad arc sweeping across the lower part of the figure. Readable point-labels: A, C, D (on the top line); E (left, below A); F (centre); B (lower left, on the arc); N (lower right, on the arc). Peripheral degree-numbering, in Roman numerals down the left side: **XC (90)**, **LXXV (75)**, **LX (60)**, **XLV (45)**, **XXX (30)**, **XV (15)**; the right-hand side carries the mirror-images of the same Roman numerals (printed reversed) [mirror numerals partly [?]]. Arabic degree-numerals also appear along the lower left (**60, 45, 30, 75, 90, 15, 5**) and lower right (**60, 45, 30, 75, 15**). Interior rays are lettered with reversed/mirror Roman numerals (e.g. XV, XXX and their mirrors) marking the vertical degrees.

— the right half of the figure is mirror-printed and several reversed numerals and the lower-arc labels (a possible G near B, and the mirrored degree cluster) do not resolve cleanly (more than two labels flagged [?]).

In the example proposed in the preceding proposition, since the line CD is one of the vertical lines, it comes about that it stands equally distant from any two vertical lines placed on either side which recede from it by an equal number of degrees, so that the two intervals between D, the centre, and the horizontal line are equal — such as, for example, DF and DG of the vertical lines of 90 and 30 degrees, each of which is 30 degrees from the vertical line CD of 60 degrees. For since, in the triangles DBF, DBG of the figure of the preceding proposition (if a straight line were drawn from D to G), the angles at B are right and equal, and likewise the angles at D are equal, since they stand on equal arcs of 30 degrees, and moreover the side DB is common, [Margin: (a) Euclid I.26] the remaining sides DF, FB will also be equal to the remaining sides DG, GB; and so with the rest. Hence it comes about that each of the lines drawn from D in this figure has two numbers, as appears in the figure; and so each of the intervals of the lines falling from D, between D and the lines issuing from A, is to be transferred onto two verticals from the zenith C, equally distant from the line CD — just as was done in the description of the arcs of the signs in a horizontal dial in proposition 2 of the preceding book. One must observe, however, that the intervals of the lines falling from D which cut the line AB are to be transferred onto those vertical lines which cut the horizontal line in the dial — which in our example are all those contained above the vertical line of 30 degrees parallel to the horizontal line. But the intervals of the other lines — those which lie to the right of the line DF of 30 degrees equidistant from AB, and which are drawn by the rule that the angles they make with DF are equal to the angles which the lines placed to the left of the same DF make with it — are to be transferred onto the portions of the vertical lines which do not cut the horizontal line; such as are all the vertical lines in the

given example which fall below the vertical line CG of 30 degrees parallel to the horizontal line.

But if the line CD should not be one of the vertical lines, yet should stand equally distant from two placed on either side, each of the lines sent out from D will in the same way contain two numbers; the topmost, however, DE, which corresponds to the line CD in the dial, will have no number affixed. But if CD should be neither one of the vertical lines nor equally distant from the two nearest placed on either side, then to each

[Margin: What is to be done when the line GD, in the dial of the preceding proposition, stands unequally distant from the vertical lines placed on either side.]

to each of the straight lines issuing out from D only single numbers are to be affixed, because in that case all the intervals between D and the horizontal line will be unequal. And since in that case no vertical line is parallel to the horizontal line, one must draw through C a hidden line equidistant from the horizontal line in the dial; to this will correspond the straight line DF, equidistant in the next figure from the line AB. Then all the straight lines placed to the left of DF are to be transferred to the right of that same DF. For these will correspond to the vertical lines which do not cut the horizontal line but are produced beyond the point C — just as we said of the hour-lines in the dial declining from the vertical circle, in proposition 2 of this book. This too may be observed in our own example. For just as, in the next figure, the line D 15·75 to the left of the straight line DF corresponds in the dial to the vertical lines of 15 and 75 degrees — which cut the horizontal line and stand equally distant from CD, lines of the kind that follow the straight line CD in the dial in the third place on either side — so too the line D 75·15 to the right of DF corresponds in the next figure to those same vertical lines produced beyond the zenith C, and so on.

From the rays of the parallels of the horizon drawn out from A toward the left of the horizon-ray AB, there are described the parallels of the horizon above the horizontal line; these indeed belong to the lower dial, if the part above the horizontal line is inverted — as has often been said above, and as we showed in proposition 41 of Book 2. We have therefore laid out the parallels of the horizon in the same dial that both declines and inclines. Which was to be done.

PROBLEMA 42. PROPOS. 42.

To describe the meridians — that is, the circles of longitude of cities — in the same dial that both declines and inclines.

Diagram. A large sundial figure showing the meridian (longitude) circles laid over the earlier construction. A horizontal line labelled **HORI = ZON** (Horizon) runs the full width of the page. The circle from proposition 37 is drawn with its centre at **L**. Many straight lines radiate from the region of the circle toward the periphery; they are the meridians of longitude, labelled in degrees (the numerals are set rotated along their lines).

- HORI = ZON (horizontal): the horizon line
- L (centre): centre of the dial-circle described in proposition 37
- M (upper-left, on the circle): endpoint of the division-start line LM
- A (upper, on the line LA): point on the prime-meridian line drawn through the Fortunate Isles
- "Inf. Fort." (marked twice, each with a small cross ☒): *Insulae Fortunatae*, the Fortunate Isles — marking the line LA, the first meridian (0°)
- Longitude meridians labelled (in degrees): 15, 30, 45, 60, 75, 90, 105 [?], 120, 135 (on the lower and right side); 300, 315, 330, 345 (on the upper side; these numerals read rotated) [?]
- Dashed lines through L: the division-start line LM and the extended prime-meridian line LA
- Several zodiac-sign glyphs carried over from the arcs-of-signs construction; the Cancer glyph ☿ is legible near the centre, the others are worn and not securely identifiable [?]

[Margin: The description of the meridian circles in the same dial.]

These circles will be described just like the hour-lines from noon or from midnight, as we set forth in proposition 37 of this book, because they are drawn through the poles of the world, exactly as the hour-circles are. But the division of the circle drawn from L (described in the dial in proposition 37 of this book) must be begun not from the straight line LM, but from another line which — the dial and the said circle being set in their proper position — recedes from the point M toward the west by as many degrees as are comprised in the longitude of the place: in the given example, 36 degrees. For this line will be the common section of the equator and of the first meridian drawn through the Fortunate Isles, such as is the straight line LA in the accompanying figure.

In the lower dial — the one described on the lower face of the plane, such as that which the horizontal line cuts off, if all its parts are inverted, as was said in what precedes — numbers must be affixed to the meridians themselves, just as we taught for the vertical dial in proposition 18 of Book 2: namely, by adding 180 degrees, and so on, to the numbers of those same meridians in the upper dial, as appears in the preceding figure. And so we have described the meridians, that is, the circles of longitude of cities, and so forth. Which was to be done.

PROBLEMA 43. PROPOS. 43.

To lay out the parallels of cities, or the circles of latitude, in the same dial that both declines and inclines.

[Margin: The description of the circles of latitude in the same dial.]

In no respect does the description of these parallels differ from the description of the arcs of the signs, which we treated in proposition 38 of this book — as we have often noted already in what precedes. We have therefore laid out the parallels of cities, or the circles of latitude, in the same dial that both declines and inclines. Which was to be done.

PROBLEMA 44. PROPOS. 44.

To place the celestial houses in the same dial that both declines and inclines.

[Margin: The description of the celestial houses according to Regiomontanus in the same dial.]

Let the circle drawn from L, described in proposition 37 of this book, be cut into 12 equal parts — or into more, if subdivisions of the celestial houses are desired — beginning from the straight line LM, as in the description of the hours from noon or midnight. Through the points of division and the centre L let straight lines be sent out cutting the equinoctial line at points; and if through these points straight lines are drawn from the point E, where the horizontal line intersects the meridian line, the celestial houses will be described according to the teaching of Johannes Regiomontanus [Regiomontanus]. We shall demonstrate this in the same way as we showed the like for the declining dial in proposition 8 of this book.

Diagram. A dial figure for the celestial houses, built on two overlapping circles. A horizontal line labelled **HORI = ZON** (Horizon) runs across; the vertical circle is lettered diagonally **VER = ... TICALIS** (Verticalis). Straight lines radiate to the

periphery, labelled with house-numbers in Roman numerals and with division-numbers in Arabic digits.

- HORI = ZON (horizontal): the horizon line
- VER = TICALIS (diagonal): the vertical circle (Verticalis)
- E (on the horizon, left of centre) and F (on the horizon, right of centre, where several lines converge): the points E and F of the construction
- M (upper-left, on the larger circle)
- L, K (just below the centre): construction points
- A, B, C, D (descending below the centre): the construction points of the Campanus method (D is the centre of the smaller circle)
- G (upper-right, on a house-line) [?]
- House-lines lettered in Roman numerals: I, XI, XII, X, II, and IX (printed "1/X"); further Roman numerals worn [?]
- Division-lines numbered in Arabic: 6, 8, 9, 10, 12; upper-left labels "OI" [?] and "XII" [?] and an upper-right label "IIIA" [?] not securely resolved
- Zodiac-sign glyphs (Cancer ♋) appear near the centre, carried over from the arcs figure

[Margin: The description of the celestial houses in the same dial according to Campanus.]

But to lay out the houses in the dial as they are constituted by Campanus [Campanus of Novara], we shall proceed thus. From K, the place of the style, raise a perpendicular KBD to the vertical line properly so called, CB, which we described in proposition 40 of this book; and to this let another perpendicular KA be erected, equal to the style, and let the straight line AB be joined. Then, taking a straight line BD equal to AB itself, describe from D a circle of any size; and, having divided it into 12 equal parts (or more), beginning from the straight line DC — which is drawn from the centre D to the zenith C — let hidden lines be sent out from D through the points of division, cutting the vertical line CB at points. And if through these points straight lines are drawn from E, the point where the meridian line and

the horizontal line intersect one another, straight lines are drawn — of the kind that in the proposed figure are marked off by those fine lines — the celestial houses will be described according to the opinion of Campanus. This we shall demonstrate by the following reasoning.

[Margin: The demonstration belonging to the foregoing description.]

Conceive the triangle ABK to be moved about BK until it stands at right angles to the plane of the dial, and consequently the style AK also perpendicular to it. Likewise let the circle described from D be understood to be turned about the vertical line CB until its centre coincides with the centre of the world — that is, with the vertex of the style A; and it will necessarily coincide, because of the equality of the straight lines AB and BD. These things being posited, the circle described from D will stand established in the plane of the vertical circle properly so called, and about the same centre with it. For the vertical circle is drawn through the centre of the world (that is, the vertex of the style A) and through the straight line CB, just as the said circle described from D is, in that position. Therefore DC will be the common section of the vertical and of the meridian, since the meridian passes through the centre D and cuts the plane of the dial at C. And the remaining hidden lines will be the common sections of the vertical circle and of the circles of the other celestial houses. Hence, as in proposition 8 of this book, we shall show that the celestial houses are rightly described according to the opinion of Campanus. Moreover the straight line DF — which is the common section of the horizon and of the vertical — will, together with DC, the common section of the meridian and of the vertical, form a right angle at the centre D, because between the meridian and the horizon a quadrant of the vertical circle properly so called is included. We have therefore placed the celestial houses in the same dial that both declines and inclines. Which was to be done.

SCHOLIUM.

But it is necessary that the line KBD, which is drawn perpendicular to the vertical line, when produced beyond K (the place of the style) should pass through the point E, where the meridian line and the horizontal line intersect one another. For it is the common section of the plane of the dial and of the circle of position, which stands at right angles to the plane of the dial and passes through the common sections of the meridian and of the horizon, and so cuts the plane of the dial at the point E — just as the other circles of position, or of the celestial houses, do, as was demonstrated in proposition 8 of this book. For since the circle of position drawn through the poles of the vertical circle and of the plane of the dial is perpendicular both to the vertical circle and to the plane of the dial, it will (by proposition 15 of Book 1 of Theodosius [Theodosius, Sphaerica I.15]) pass altogether through the style AK set in its proper position — ^a (because all planes drawn through the style to the plane of the dial are perpendicular to it) — and therefore through the point K.

[Margin: ^a Euclid, *Elements* XI.18]

But since, on the contrary, both the plane of the vertical circle and the plane of the dial are perpendicular to the said circle of position,^b the common section of these too — that is, the vertical line CB — will be perpendicular to the same circle of position, and consequently, by definition 3 of Book 11 of Euclid [Euclid, Elements XI, def. 3], also perpendicular to the common section of that same circle and of the plane of the dial lying within it. Since therefore CB is perpendicular to KB by construction, KB will be the common section of the said circle of position and of the plane of the dial, and so it will pass through the point E, as will also the remaining common sections of the circles of the celestial houses and of the plane of the dial.

[Margin: ^b Euclid, *Elements* XI.19]

[Margin: How to describe those celestial houses which scarcely, or in no way, intersect the vertical line.]

Now, since in describing the celestial houses according to the opinion of Campanus it sometimes happens that certain hidden lines sent out from the centre D scarcely, or in no way, cut the vertical line CB — so that it becomes exceedingly difficult to describe without some error the lines of the celestial houses corresponding to them in the dial from the point E, as happens in our example with the line of the second celestial house in the nocturnal portion of the dial, which in the lower dial gives the eighth house, opposite to it — the points on the equinoctial line through which the lines of the celestial houses are to be drawn must be sought out, for those cases in which we are deprived of points on the vertical line, in the following way. Let inquiry be made for the hour at which the Sun, being on the equator, reaches the circle of that celestial house whose line we wish to describe. For where the line of that hour (which we shall draw in a hidden manner, as we set forth in proposition 37 of this book) cuts the equinoctial line, through that point the sought celestial house is to be drawn from E. Thus, because the Sun, being on the equator, reaches by its diurnal motion the circle of the second celestial house — which in the lower dial is the eighth house, opposite to it — at hour 3, minute 29 after midnight, and the line of this hour cuts the equinoctial line at the point G, the line of the said house is to be drawn from E through G; and so with the rest.

[Margin: How to explore the hour at which the Sun, being on the equator, reaches the proposed circle of the celestial house.]

We shall find the hour at which the Sun, running along the equator, reaches the proposed circle of the celestial house, from the doctrine of sines, by this method.

Diagram. A spherical diagram shown as a sphere with several great circles, in the lower-left of the page. The point **A** is at the top, **C** at the bottom, **B** at the left, **D** at the right; **E** lies in the upper-centre and **F** in the lower-centre. The great circles named in the accompanying text are drawn as the horizon, the meridian, the equator, the vertical, and the circle of the celestial house, with their intersections lettered.

- A (top), C (bottom), B (left), D (right): cardinal points of the figure
- E (upper-centre), F (lower-centre): points on the meridian / vertical
- Horizon: the outer bounding circle, ABCD [?: the first letter is worn — the print may read "CBCD"]
- Meridian: AEC
- Equator: BED
- Vertical circle (Verticalis): BFD
- Circle of the celestial house: AGCK
- G, H: eastern intersections (left side); I, K: western intersections (right side)
- lowercase g near B [?]

Let ABCD [?] be the horizon; AEC the meridian; BED the equator; BFD the vertical; and AGCK the circle of the celestial house, cutting the vertical and the equator on the eastern side — whether above the horizon or below — at the points G and H, and on the western side at the points I and K. We must therefore seek the arc of the equator EH, or EK. Since in the triangle BGH (or DIK) the angle G (or I) is right, by proposition 15 of Book 1 of Theodosius [Theodosius, *Sphaerica* I.15] it will follow — by proposition 18 of Book 4 of Johannes Regiomontanus's *On Triangles* [Regiomontanus, *De Triangulis* IV.18], or by proposition 14 of Book 1 of Geber [Jābir ibn Aflaḥ, I.14], or by proposition 42 of our own *Spherical Triangles* [Clavius, *Triangula Sphaerica* 42] — that as the sine of the angle B (or D) — namely of the altitude of the pole, which the equator makes with the vertical — is to the whole sine, so is the sine of the complement of the angle H (or K) to the sine of the complement of the arc BG (or DI) of the vertical circle placed between the horizon and the circle AGCK. And conversely: as the whole sine is to the sine of the angle of the altitude of the pole B (or D), so is the sine of the complement of the arc BG (or DI) to the sine of the complement of the angle H (or K). This complement being known, the angle H (or K) will also be known.

Again, since — by proposition 16 of Book 4 of Johannes Regiomontanus's *On Triangles* [Regiomontanus, *De Triangulis* IV.16], or by proposition 13 of Book 1 of Geber [Jābir ibn Aflaḥ, I.13], or by proposition 41 of our own *Spherical Triangles* [Clavius, *Triangula Sphaerica* 41] — as the sine of the angle H (or K), just found, is

to the sine of the arc BG (or DI), so is the whole sine of the right angle G (or I) to the sine of the arc of the equator BH (or DK), the complement of the Sun's distance from noon or from midnight: if it be so contrived that as the whole sine is to the sine of the altitude of the pole, so is the sine of the complement of the arc of the vertical circle intercepted between the horizon and the circle of the proposed celestial house to a fourth term, a sine will be found whose arc's complement is to be kept. Again, if it be so contrived that as the sine of this kept complement is to the sine of the arc of the vertical circle placed between the horizon and the circle of the proposed celestial house, so is the whole sine to a fourth term, there will be found the sine of the complement of the Sun's distance from noon (if the celestial house stands above the horizon) or from midnight (if the given celestial house lies hidden below the horizon).

[Margin: Example.]

Example. Let the Sun be at the beginning of the second house, whose circle lies hidden below the horizon

lies hidden 30 degrees on the eastern side — degrees which are, of course, reckoned along the vertical circle. Accordingly, set it up so that as the whole sine, 100000, is to 66913, the sine of the pole's altitude, so is 86602, the sine of the complement of the arc of 30 degrees, to a fourth term: there will be found here a sine of about 57950, whose arc contains $35^{\circ}25'$; and the complement of this, $54^{\circ}35'$, will give the angle BHG. Then set it up so that as 81495, the sine of the angle BHG which we have just obtained, is to 50000, the sine of the arc of the vertical circle intercepted between the circle of the proposed house and the horizon, so is the whole sine, 100000, to a fourth term: there will be found here a sine of about 61353, whose arc BH, $37^{\circ}51'$, will give the complement — that is, the Sun's distance after midnight, since the Sun then stands between the meridian and the horizon below the earth on the eastern side — namely $52^{\circ}9'$, that is, 3 hours 29 minutes.

This distance would instead fall before noon, if the circle of the celestial house passed through the beginning of the twelfth house, so that the arc of the vertical circle between the horizon and the said circle above the horizon on the eastern side contained 30 degrees; but after noon, if that same circle were drawn through the start of the eighth house; and finally before midnight, if it passed through the beginning of the sixth house.

PROBLEMA 45. PROPOS. 45.

To place the ascending signs of the Zodiac on the same dial when it both declines and is inclined.

[Margin: How the ascending signs are depicted on the same dial.]

By exactly the same methods we used in what preceded, we shall here describe the ascending signs, as will become clear from the accompanying figure. For from the first two little tables of proposition 9 of Book 2 [*Gnomonices* II, prop. 9] we shall track down, on the equinoctial line, the points through which the lines for Aries, Taurus, Gemini, Cancer, Leo, [zodiac sign: ?], Aquarius, and Pisces pass. From the third little table of the same proposition they are drawn [continues below the figure]—

Diagram. A declining-and-inclined sundial layout onto which the ascending signs of the Zodiac are plotted. A horizontal line labelled **HORIZON** (printed **HORI:** at the left, **ZON** at the right) runs across the upper part of the figure. A second long line, the equator, is labelled **AEQVATOR** (the letters run along the line, appearing as **TOR** near the top and **AEQVA** toward the lower right). A near-vertical line is labelled **Linea meridiana** (meridian line — the words **Linea** and **meridiana** set along it), and a further line is marked **Linea** at the lower left; a small vertical caption at the upper left appears to read **Linea dioptr[?] [?]**. The six-o'clock hour line carries the caption **hora 6** with a small Sun symbol.

At the left, a small circle (the figure of the rising and setting amplitudes) carries the construction letters:

- A (left of the circle): vertex.
- F (upper left, inside the circle): vertex, joined to B by dashed lines.
- B (top of the circle, with a right-angle mark): vertex.
- P, L, K (lower part of the circle, in a row): vertices.

From the region of the circle a great many straight sign-lines radiate outward to the right and downward; their outer ends are marked with zodiac sorts. The following are legible: Capricorn [♑], Aquarius [♒], Aries [♈], Pisces [♉], Taurus [♉], Gemini [♊], Cancer [♋], Leo [♌]. Several further peripheral zodiac sorts are worn or overprinted by crossing lines and cannot be securely read: [zodiac sign: ?], [zodiac sign: ?], [zodiac sign: ?] (a Scorpio- or Virgo-like sort near the top).

[continued from above:]...From the third little table of the same proposition we shall find, in the tropic of Cancer [♋], the points for Aries, Taurus, Gemini,

Cancer, and Leo. For [zodiac sign: ?], however, and for Aquarius and Pisces, we shall find the points on the meridian line, from the sixth little table of the same proposition. Then for Leo we shall find a point in the tropic of Cancer, from the third table of the said proposition, through which we have drawn the line of Leo, passing through that point of the horizontal line where it is cut by the parallel of Gemini and of Leo. For Virgo, finally, we meet one point in the tropic of Cancer, from the third table of the said proposition, and another on the meridian line, from the sixth table of the same. But the lines for Sagittarius and Capricorn can scarcely be drawn in our example, since they run out too far, as may easily be gathered from the said tables. All the lines of the ascending signs, however, pass through the points of the horizontal line where the arcs of the signs cut it — points which can also be investigated without those arcs, either from the fifth little table of proposition 9 of Book 2 [*Gnomonices* II, prop. 9], as is clear from the foregoing, or from the figure of the rising and setting amplitudes of proposition 21 of Book 2 [*Gnomonices* II, prop. 21], by this device.

[Margin: The finding of the points through which the ascending signs are drawn on the horizontal line.]

In the figure of the rising and setting amplitudes, which we repeated at proposition 9 of this book, let BEG be set up as the angle of the declination of the proposed plane from the vertical circle — toward the parts of the south toward the west, if the plane declines from the north toward the east (as happens in the given example), or toward the east, if it inclines toward the west; but toward the parts of the north toward the east, if the dial turns away from the south toward the west, or toward the west, if it turns from the south toward the east. After this, let a straight line EG be taken, equal to the straight line AB in the figure of proposition 40 of this book [*Gnomonices* III, prop. 40]; and through —

and through G, let a perpendicular GH be erected to EG. For if the points of this perpendicular, where it is cut by the rays of the rising amplitudes, be taken from the point H and transferred onto the horizontal line from the point F, where the meridian line and the horizontal cut one another, the points will be found through which the arcs of the signs are drawn. For let there be drawn from K, the place of the gnomon, a perpendicular KB to the horizontal line, and to this another perpendicular KA, which is to be equal to the style; let the straight line AB be joined, which is the same as the one specified in proposition 40 of this book [*Gnomonices* III, prop. 40], and is therefore equal to EG in the figure of the rising amplitudes of proposition 9 of this book [*Gnomonices* III, prop. 9]. Now suppose the triangle ABK to be turned in the mind about BK until it stands at right angles

to the plane of the dial, and thus lies in the plane of the vertical circle that measures the inclination of the plane to the horizon; then, once BK has been drawn, AB will be the common section of the horizon and of the said vertical circle, and the joined line AF the common section of the horizon and of the meridian — because both the horizon and the meridian are drawn through A, the centre of the world, and through the point F.

Therefore the angle AFB, which the meridian line AF of the horizon makes with the horizontal line FB of the plane of the dial, will be equal to the complement of the declination of the dial from the vertical properly so called. And since the angle ABF is a right angle — for since the triangle ABK is at right angles both to the plane of the dial and to the horizon, these two planes will also be at right angles to the triangle; [a] therefore their common section FB will be perpendicular to that same triangle, and for that reason perpendicular also to the line BA, by definition 3 of Book 11 of Euclid [Euclid, *Elements* XI, def. 3] — the remaining angle BAF will be equal to the declination of the dial from the vertical properly so called, that is, to the angle BEG in the figure of the rising amplitudes of proposition 9 of this book [*Gnomonics* III, prop. 9].

[Margin: a — XI.19 (Euclid, *Elements* XI.19).]

Since, then, the angles E and G of the triangle GEH, in the figure of the rising amplitudes, are equal to the angles A and B of the triangle BAF in the figure of this proposition, and the sides EG and AB, on which those angles lie, are likewise equal by construction, [b] the sides EH and HG will also be equal to the sides AF and FB, and the angle H equal to the angle AFB. Hence, if the line EG of the figure of the rising amplitudes coincides on the dial with the line AB, then EH will also coincide with AF, and HG with FB, on account of the equality of the angles; and finally the whole of HG, produced in both directions, will coincide with the whole horizontal line. For this reason the rays of the rising and setting amplitudes will meet the horizontal line in points as far distant from the point B, or F, as the points are removed from the point G, or H, where the rays of the amplitudes cut the line GH, and so on. We have therefore placed the ascending signs of the Zodiac on the same dial when it declines and is at the same time inclined. Which was to be done.

[Margin: b — I.26 (Euclid, *Elements* I.26).]

SCHOLIUM.

[Margin: How the ascending signs are laid out according to the teaching of Andreas Schöner.]

If we wish to describe these same ascending signs according to the teaching of Andreas Schöner [Andreas Schöner], we shall use this device. In the first figure of the scholium to proposition 9 of Book 2 [*Gnomonices* II, prop. 9], from the line ED, or EB, let there be cut off a line EH equal to the line LM (which on the dial is intercepted between the centre L and the point M, where the equinoctial line and the meridian cut one another); and at H let there be set up, with the line BD, an angle equal to the one which the equinoctial line makes with the line LM on the dial — yet under this rule, that, with the centre E placed at the centre L and the line ED laid upon the line LM, the line drawn through the point H and constituting the said angle should coincide with the equinoctial line. Then let the intervals of the aforesaid line drawn through H, intercepted between H and the lines of the signs, be transferred onto the equinoctial line from the point M, marking off the points on the equinoctial line itself.

For if, from the point where the meridian line and the horizontal cut one another, hidden straight lines be drawn through these points of the equinoctial line, both tropics will be cut at the points through which the ascending signs are to be drawn — this order being observed. If the dial faces the east, the point where the tropic of Capricorn [♑] cuts the horizontal line will belong to Capricorn; and the point at which the tropic of Cancer [♋] and the horizontal line cut one another, to Cancer. The next point in the tropic below the horizontal line is owed to the sign that rises next after Capricorn, or after Cancer — that is, to the sign Aquarius, or to Leo — and so on in succession of the signs.

[Margin: The order of the ascending signs in the tropics.]

But if the dial deflects toward the west, the point where the tropic of Capricorn [♑] and the horizontal line cut one another is to be assigned to the sign Cancer; and the point where the tropic of Cancer [♋] cuts the same horizontal line will suit the sign Capricorn. Then the next point below the horizontal line will answer to the sign that rises before Capricorn, or Cancer — that is, to the sign Sagittarius [♐], or Gemini — and so on against the succession of the signs. From this we shall easily understand to which signs the points of the tropics found above the horizontal line answer. For if below they keep the order according to the succession of the signs, above the horizontal line they will keep the order against

the succession of the signs, and conversely, as is plain from the figures of the ascending signs which we have hitherto constructed on various planes.

But always any hidden straight line drawn from the point where the meridian and horizontal lines cut one another, through the points found on the equinoctial line, belongs to two opposite signs, just as in the earlier figure of the rays of the ascending signs any line belongs to two opposite signs: so that if, for example, one point through which the said hidden line passes belongs to Gemini, the other point, which the same straight line indicates — whether in the opposite tropic or in the same — belongs to Sagittarius [♐], and so on; as is manifest from the earlier figures of the ascending signs which we have depicted on various planes.

[Margin: What is to be done in drawing the ascending signs when the equinoctial line and the meridian are parallel.]

In the proposed example, where the meridian line and the equinoctial are equidistant, the line LM does not cut the equinoctial line but is parallel to it, so that it cannot form an angle with it. In the earlier figure of the signs, therefore, ED will have to be drawn as the parallel line answering to the equinoctial line on the dial, in this way. From EC let there be taken EK, equal to the line which on the dial is intercepted between the centre L and the point where the horizontal line divides the equinoctial; and through K let a perpendicular be raised to EK, cutting the lines of the signs in points which, taken from the point K and transferred onto the equinoctial line from the said point where the equinoctial and horizontal cut one another, will give the points on the equinoctial line, as before.

And if the ascending signs are to be described on the dial of the first figure of proposition 37 of this book [*Gnomonics* III, prop. 37], the angle EHQ must be set up equal to the one which the line LM makes with the equinoctial line on the dial, as was done in the earlier figure of the rays of the ascending signs.

Again, in the later figure of the same scholium to proposition 9 of Book 2 [*Gnomonics* II, prop. 9], from the line EC let there be taken off Ek, equal to the line which on the dial is intercepted between the centre L and the point at which the horizontal line and the equinoctial mutually divide one another. And at K, with EK, let there be made an angle equal to the one which the equinoctial line makes with the line drawn from L through the said point where the horizontal line cuts the equinoctial — yet under this rule and condition, that, with the centre E placed at the centre L, and the line EC upon the line drawn from L through the point where the horizontal and equinoctial lines cut one another, the line drawn through K should coincide with the equinoctial line. In our example the said

angle is a right angle. But in the first dial of proposition 37 of this book [*Gnomonices* III, prop. 37], since the horizontal line cuts the equinoctial line at H, we shall take the line LH equal to EK in the later figure of the said scholium, and shall set up the angle EKQ equal to the angle which the line EH makes with the equinoctial line on the dial, and so on. Then the intervals of the line drawn through K, intercepted between K —

between K and the lines of the signs, are to be transferred onto the equinoctial line from the point which is common to the horizontal and equinoctial lines, in the same order they would have if the line drawn through K coincided with the equinoctial line. For these points, together with the answering points found in the tropics, joined by straight lines, will give the ascending signs, whose order — on a dial facing the east — is this on the equinoctial line: the first point found below the horizontal line, from the later figure, belongs to Taurus, the next to Gemini, and to Cancer [♋] [?], and so on in that order in which they are described in the aforesaid figure. But the nearest point above the horizontal line answers to the sign Pisces, the next to the signs Aquarius and Virgo, and so on, as they follow in the figure. On a dial declining toward the west, however, the order will be the reverse. For the first point below the horizontal line will belong to Pisces, the next to Aquarius and Virgo, and so on; while the nearest point above the horizontal line will look to Taurus, the next to Gemini, and to Cancer [♋] [?], and so on.

[Margin: The order of the ascending signs on the equinoctial line.]

PROBLEMA 46. PROPOS. 46.

To construct an Italian dial which both declines from the vertical and is inclined to the horizon.

[Margin: The construction of an Italian dial that at once declines and is inclined.]

As in the preceding dials, so in this one too we shall draw the hour lines reckoned from sunset, if we divide the circle described from L into the diurnal arc of the parallel of Cancer [♋], aMb, and the diurnal parallel of Capricorn [♐], dMe, in such a way that the lines ab and de cut the line LM (which is the common section of the meridian and the equator) at right angles, as we set forth at proposition 1 of Book 1 [*Gnomonices* I, prop. 1] — provided that the division of the circle into 24 equal hours has its beginning at the western points b and e. But since in our dial the tropic of Capricorn [♐] cannot be drawn, it will suffice if the hour points from sunset be found [continues below the figure]—

Diagram. The layout of an Italian dial that both declines from the vertical and is inclined to the horizon. A horizontal line labelled **HORIZON** runs across (printed **HORI**: at the left, then **ZON / N O Z** at the right). A large circle is described about the centre **L**; on it the point **M** lies above the centre and the point **N** below. Two dashed arcs (the diurnal parallels) sweep through the circle, their end-points marked **a** and **b** (the arc **aMb**, the diurnal arc of the parallel of Cancer) and **d** and **e** (the arc **dMe**, the parallel of Capricorn); the letters **a**, **b**, **d**, **e** appear at the left and lower part of the figure.

Along the dashed arcs runs a series of small Arabic hour-division numbers; the legible ones include **2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18** set at successive division points about the centre **L**.

A dense pencil of straight and dashed hour lines radiates from the lower half of the figure; their outer ends are marked with hour numerals in the Italian reckoning. Legible numerals include, at the top and upper right, **IX, XI, XII**, and the Arabic **22, 23, 24**, together with **XIII** and **XVIII** (13, 14); down the right side **I, II, III**; and along the bottom **VI, VII, VIII, IX**, with **IIII** at the left. Many of the peripheral hour numerals and small division numbers are worn or overrun by crossing lines and cannot be securely read.

[continued from above:]...found in the tropic of Cancer [☿], by means of the diurnal arc of that same tropic. We shall find, in the upper dial of our example — that is, in the portion contained below the horizontal line — the points of the hours from sunset only from the 9th up to the 16th inclusive, because all these hours are contained in the arc of the circle from **M**, from whose points the straight lines drawn through the centre **L** meet the equinoctial line after they have been drawn through the centre **L**. And if these points be joined by straight lines to the answering hours on the equinoctial line — as is established from the tables of the diurnal arc and of the nocturnal arc of the 12 hours set out in the scholium of proposition 33 of Book 1 [*Gnomonices* I, prop. 33], or from the table of proposition 19 of the same book [*Gnomonices* I, prop. 19] — the hours from sunset in the upper dial will be drawn from the 9th hour up to the 16th inclusive. But the points of the 17th and 18th hours from sunset —

[continued from the previous page] —we shall likewise find in the tropic of Cancer [☿]. From the points of the 5th and 6th hours from sunset on the circle **aMbN** — the points contained in the arc **aN** and lying opposite the said 17th and 18th hours — we shall draw hidden lines through the centre **L**. For by their help we shall find, in the tropic of Cancer, in the nocturnal portion, the points for the 5th and 6th hours; and from these, hidden straight lines drawn through **z** [the

dial's centre — the glyph in the figure is a small looped sign, transcribed here as **z**], the centre of the dial, will give in the same tropic of Cancer the points of the 17th and 18th hours, as is gathered from what was demonstrated at proposition 10 of Book 2 [*Gnomonices* II, prop. 10]. Now the line of the 18th hour from sunset will, in the plane we have proposed, be parallel to the equinoctial line and to the meridian line;

Diagram. A large, very dense sundial layout for an Italian dial that both declines from the vertical and is inclined to the horizon. This is a continuation of the figure begun with Problema 46 in the preceding batch. A horizontal **horizon line** runs across the page, lettered **HORI:** to the left of the circle and **ZON** to the right (with an additional partly-mirrored **N O Z** and the numeral **24** repeated at the far right). A large circle is described about the centre **L**, with the small looped centre-mark **z** just below **L** (the dial's centre / gnomon foot). Point **M** lies on the upper part of the circle and point **N** on the lower part.

- To the left, on a dashed arc: points **d** and **a**, with small hour-division numbers running along the arc — **1, 2, 3, 4, 5, 6, 7, 8** — and, higher up, **9, 10, 11, 12**.
- Near the centre, along interior dashed arcs: the hour-numbers **2, 3, 4, 5, 6, 7, 8, 9, 10** descending, with point **b** to the right of centre and further Arabic marks **13, 14, 15, 16, 17** on the right.
- A dense pencil of straight and dashed hour-lines radiates outward, their outer ends marked with hour numerals (many reversed/mirrored and several overrun by crossing lines). Legible peripheral numerals include, across the top, the reversed Romans **IIII, V, VI, VII, VIII**, and the Arabic **20**, then **XII, IX, 22, XI**; down the right side **XII** [i.e. 13 (?)], **23, XIII, XIII, 24**, then **I, II, III, IIII**; along the bottom **VI, VII, VIII, IX** and **V**; and at the lower left **X** (struck through) and **18**.

Many peripheral hour-numerals and small division-numbers are worn or crossed by lines and cannot be securely read.

[continued below the figure:] —for if it were to cut the equinoctial line, this would necessarily happen at the 12th hour from noon or from midnight, as is established from the table of proposition 19 of Book 1 [*Gnomonices* I, prop. 19]. Since, then, the 12th hour in our example does not cut the equinoctial line but runs parallel to it, it is clear that the 18th hour cannot cut it either. It will therefore be parallel to it. The same is to be said of the 6th hour from sunrise or sunset, for this too is parallel both to the equinoctial line and to the meridian line.

[Margin: The equinoctial line, the meridian line, and the lines of the 6th and 18th hours from sunrise and sunset are parallel in the dial here proposed.]

This can also easily be demonstrated in the following way. By proposition 16 of Book 1 [*Gnomonices* I, prop. 16] the hour-circles of the 18th and 6th hours from sunrise or sunset have one and the same common section with the equator, and likewise, by proposition 17 of the same book [*Gnomonices* I, prop. 17], the same common section with the meridian. They must therefore pass through the common section of the meridian and the equator, so that this common section is the common section of those circles, of the equator, and of the meridian. Since, then, the plane of this dial is parallel to that common section — being parallel to the great circle which declines from the vertical and is inclined to the horizon, the circle drawn through the said section — those sections which the named circles produce in the plane of the dial will, by proposition 18 of Book 1 [*Gnomonices* I, prop. 18], be parallel to one another: namely the equinoctial line, the meridian line, and the lines of the 18th and 6th hours from sunrise or sunset. But the 17th hour must be drawn through the 11th hour from noon or midnight on the equinoctial line; and so the upper Italian dial will be complete, since scarcely more hours from sunset can be drawn in the proposed plane.

[Margin: The layout of the same Italian dial by means of the diurnal and nocturnal arcs.]

To construct the lower Italian dial, the hours from sunrise must be drawn in the nocturnal portion of the dial — namely 23, 22, 21, 20, 19, 18, and so on. For these, as we showed in the scholium of proposition 23 of Book 2 [*Gnomonices* II, prop. 23, scholium], are reckoned in the lower dial from sunset. These said hours are contained in the portion aNb of the circle described from L, as is clear.

Much more conveniently, these same hours from sunset will be drawn by means of the diurnal arc of 24 hours and the diurnal arc of 14 hours, which in the nocturnal portion of the dial changes into the nocturnal arc of 10 hours. For all the hour-lines from sunset touch the diurnal arc of 24 hours at the points of the hours from noon or midnight,

—as we demonstrated at proposition 14 of Book 1 [*Gnomonices* I, prop. 14]. Hence in laying out the hours from sunset we shall almost always have three points for each hour: one on the diurnal arc of 24 hours, a second on the diurnal arc of 14 hours (or the nocturnal arc of 10 hours), and a third on the equinoctial line — that is, on the diurnal or nocturnal arc of 12 hours — as is established from the

tables set out in Book 1. There is no need to give an example of this, since it is very easy.

As for the mutual intersections of the hours from sunrise or sunset with those from noon or midnight, the matter is quite clear from the tables of propositions 19 and 20 of Book 1 [*Gnomonics* I, props. 19–20]. And so we have constructed the Italian dial that both declines from the vertical and is inclined to the horizon. Which was to be done.

PROBLEMA 47. PROPOS. 47.

To construct a Babylonian dial that both declines from the vertical and is inclined to the horizon.

[Margin: The construction of a Babylonian dial that declines and is inclined at once.]

By exactly the same methods with which we laid out the Italian dial, we shall construct the Babylonian dial in the proposed plane, as is plainly evident from the figure of the preceding proposition. There are contained in it both the hours from noon or midnight and those from sunrise as well as from sunset, just as in the dials above. But in the first method, the division of the circle aMbN must be begun from the eastern points a and d. We have therefore constructed the Babylonian dial that both declines from the vertical and is inclined to the horizon. Which was to be done.

PROBLEMA 48. PROPOS. 48.

To make an Ancient dial that both declines from the vertical and is inclined to the horizon.

[Margin: The layout of an Ancient dial that declines and is inclined at once.]

This dial too we shall construct on a plane that declines from the vertical circle and is inclined to the horizon, by the same methods we have used on other planes, as the figure itself shows. In it we have drawn the diurnal arc of 18 hours,

Diagram. The layout of an Ancient (unequal-hour) dial that both declines from the vertical and is inclined to the horizon. A **horizon line** runs across the figure, lettered **Ho** (to the left of the circle) and **RIZON** (to the right), with **IIX** [= XII, the 12th hour] at its far right end and the numeral **9** at its left end. A large circle is described about the centre **L**, with the small looped centre-mark **z** just below L.

Point **M** lies on the upper circle, **N** on the lower circle, **e** on the upper-right circle, **b** at the right, and **d** at the left.

- Interior dashed hour-lines fan out from the centre; along them run the hour-numbers **4, 5, 6, 7, 8, 9, 10, 11, 12** descending, with **n** and **12** near the top, and a dotted arc at the lower left carrying **1, 2, 3, 4**.
- Straight hour-lines radiate outward with numerals at their ends: across the top, reversed Romans — **V, VI, VII, VIII, XI**, and **X**; on the right side **IX**, then at the horizon **XII** [read as **IIX**], and below it **I, II, III, IIII**; along the bottom **V, VI, VII, VIII**.
- At the upper left and again at the lower left (near the reversed **VIII**) stand two worn zodiac-sign sorts; the diurnal arc drawn here is that of 18 hours (summer), so these mark the tropic of Cancer [likely ☿, glyphs worn — [?]].

Several peripheral numerals and the zodiac sorts are worn or overrun by crossing lines.

[continued below the figure:] —which in the nocturnal portion of the dial changes into the nocturnal arc of 6 hours. We have therefore made the Ancient dial that both declines from the vertical and is inclined to the horizon. Which was to be done.

End of the Third Book.

GNOMONICS, BOOK FOUR.

By the author **CHRISTOPH CLAVIUS** [Christophorus Clavius] of Bamberg, of the Society of Jesus.

[Margin: The argument of the fourth book.]

In the two preceding books we set out the precepts by which the description of hours of every kind — and of the other things that can be known from the shadow of the gnomon — is carried through for any given latitude of a place situated between the equator and the arctic pole, and on every plane that can be devised. It remains for us to teach how the description of these same things is to be carried out on those same planes when they lie in a right sphere, where there is no elevation of the pole above the horizon, and also in the most oblique sphere, where the arctic pole is raised 90 degrees above the horizon. Then, how these same dials, once described, stand among the antipodes — where the antarctic pole, opposite to our own, is raised above the horizon — with respect to the

number and order of the hours, which must be entirely changed among the antipodes, as we shall explain below.

For although all this could easily be gathered from the precepts above, yet because some things are here described more briefly, and certain others require different rules, it seemed good to explain them separately in this book. Finally, with one or two examples, we shall show what variations follow in the lineaments of dials from a different elevation of the pole, both among us and among the antipodes. For if this were not known, anyone might easily get stuck at times in laying out various dials — especially with the hours from sunrise and sunset, which sometimes, even if produced to infinity, do not cut the two tropics but touch two other parallels within the tropics.

To all this we shall add three problems, in which we shall demonstrate how the style, or gnomon, of each dial ought to be set in its proper place so that it stands upright to the plane of the dial; and likewise in what way its tip, if it should ever have been deflected from its proper seat, can be restored to it. Furthermore, by what method one is to investigate for what elevation of the pole a given dial was made, whose style and its position are given; likewise what declination it has from the vertical, and what inclination to the horizon, if it is declining or inclined, and also how great is the elevation of the pole above that declining or inclined plane. And conversely, how — given a dial together with the elevation of the pole for which it was built, and its inclination to the horizon if it is inclined — one is to find the length of the style and its position, together with the declination from the vertical (if the dial is declining) and the elevation of the pole above that declining or inclined plane. Finally, we shall show by what art any dial may be reduced to a larger or smaller form, while the same proportion of the lineaments and shadows is always kept — something that not rarely happens in practice, especially when some dial drawn on paper, or on any other material, must be transferred onto the wall for which it was designed.

PROBLEMA 1. PROPOS. 1.

To describe horizontal dials in a right sphere.

[Margin: How a horizontal dial, with the parallels of the signs and the latitudes of cities, is described in a right sphere.]

The hours from noon and from midnight in a horizontal dial of the right sphere, and the parallels of the signs of the zodiac, and also the parallels of cities (that is, the circles of latitude), are described just as in the polar dial, which we treated in

Book 2. The reason is that the horizon of the right sphere passes through the poles of the world, exactly as does the circle of the 6th hour from noon or midnight, to which the polar dial is parallel. Indeed, the circle of the 6th hour from noon or midnight does not differ from the horizon of the right sphere. But the horizontal line is not to be drawn in a horizontal dial of the right sphere, just as it is not drawn in a horizontal dial of the oblique sphere, since the horizon that makes that line does not cut the plane of the horizontal dial, whether in the right sphere or in the oblique. And so, if the polar dial described in Book 2 is set up in the right sphere parallel to the horizon, in such a way that the meridian line has its proper position and the point D faces toward the northern pole and E toward the southern, the numbers of the hours and the signs of the zodiac will have the same —

the same order that they have in the polar dial. That is, the hours placed after midnight will be for us toward the antarctic pole when turned to the right, or toward the west; while the hours after noon will be to the left, or toward the east. Likewise the northern signs will be placed toward the south, and the southern signs toward the north, as the accompanying figure shows — a figure in which, of all the parallels, we have drawn only the two tropics.

[Margin: In a right sphere there are no diurnal arcs, because they all comprise 12 hours.]

In a right sphere there are no parallels of the diurnal arcs, because there is no inequality of days there; rather, every artificial day comprises 12 hours, and the artificial night just as many, so that there is perpetual equinox, as we explained for the right sphere. Any parallel whatever, or arc of a sign, may therefore be called a diurnal arc of 12 hours.

[Margin: The Italian and Babylonian dials in a right sphere differ from the astronomical dial only in the numbering of the hours.]

[Margin: The Antique dial in a right sphere does not differ from the Babylonian.]

The lines of the hours from sunrise and sunset do not differ from the lines of the hours from noon and midnight; only the numbers of the hours must be changed. For since in a right sphere the Sun always rises at the 6th hour from midnight — that is, the 24th from sunrise and the 12th from sunset — the 7th hour from midnight will yield the 1st from sunrise and the 13th from sunset; the 8th hour from midnight will yield the 2nd from sunrise and the 14th from sunset, and so on, as appears in the accompanying figure. In it, the numbers set against the equinoctial line pertain to the astronomical hours (that is, from noon and

midnight); the numbers at the tropic of Capricorn [♊] indicate the hours from sunrise; and the remaining numbers, alongside the tropic of Cancer [♋], show the hours from sunset.

Unequal hours, however, have no place in a right sphere, since days and nights are always equal and comprise 12 hours equal to one another. Hence the hours from sunrise indicate the twelfth-parts of the day already elapsed, just as the unequal hours do in an oblique sphere.

[Margin: How the vertical circles and the parallels of the horizon are drawn in the horizontal dial of the right sphere.]

The vertical circles too, and the parallels of the horizon, will be drawn here by the same method as in the horizontal dial. For the vertical lines pass through the place of the style — that is, through the point where the meridian line and the equinoctial line cut each other — and the equinoctial line will give the vertical properly so called, from which the remaining verticals are numbered; because in a right sphere the vertical and the equator constitute one and the same circle. The parallels of the horizon, moreover, are drawn from the same place of the style, taken as their center, and so forth.

[Margin: How the meridian circles are drawn in the horizontal dial of the right sphere.]

Further, the meridian circles — that is, the circles of longitude of cities — will be drawn here as in the polar dial: on the circle by whose means the astronomical hours are laid out, the longitude of the place is counted from the meridian line DE westward, as far as the meridian of the Fortunate Isles, and so on.

[Margin: The description of the celestial houses in the horizontal dial of the right sphere.]

Again, the celestial houses will be depicted on the right horizon just as on the oblique. For here too the lines of the celestial houses are parallel to one another, and each space intercepting one house between two lines comprises two hours. Hence the line of the 8th hour from midnight will give the beginning of the 12th house; the line of the 10th hour from midnight, the beginning of the 11th house; the meridian line, the beginning of the 10th house; the line of the 2nd hour from noon, the beginning of the 9th house; and finally the line of the 4th hour from noon, the beginning of the 8th house. Here the celestial houses are drawn in one single way, because in a right sphere the equator and the vertical circle do not

differ. Hence the houses are the same according to Johannes Regiomontanus [Regiomontanus] and Campanus [Campanus of Novara].

[Margin: By what means the ascending signs are depicted in the horizontal dial of the right sphere.]

Finally, the ascending signs will also be drawn here, as in the earlier dials, provided that first there are compiled tables giving the hours at which the beginnings of the signs ascend above the horizon in a right sphere. These are compiled, as we taught in Book 2, Proposition 9, if in place of the oblique ascensions the right ascensions are taken. The following four little tables have been compiled in this way.

Diagram. The horizontal dial of the right sphere. A vertical meridian line runs from **E** at the top down to **D** at the bottom; a horizontal equinoctial line crosses it at the center. From all the parallels only the two tropics are drawn, as two dotted curves that cross near the center, giving the whole figure the look of a bowtie or hourglass. The numbers set against the equinoctial (horizontal) line are the astronomical hours (from noon and midnight); the numbers on the tropic of Capricorn are the hours from sunrise; the numbers on the tropic of Cancer are the hours from sunset. Because of the crowding and the worn zodiac sorts, several peripheral labels could not be resolved and are flagged; this figure

- E (top of the meridian line): E
- D (bottom of the meridian line): D
- Left end of the equinoctial line: worn zodiac glyph [Υ Aries?, i.e. the eastern equinoctial point] [?]
- Right end of the equinoctial line: worn zodiac glyph [♎ Libra?, i.e. the western equinoctial point] [?]
- Upper dotted curve (tropic of Capricorn, hours from sunrise), with ♐ / ♒ [Capricorn/Aquarius] glyphs at both upper ends: readable hour numbers 27 (top left), 22, 21, 20, 19, 17, 16, 15, 14, 13 (top right); an "8" also appears near the center [center cluster partly [?]]
- Lower dotted curve (tropic of Cancer, hours from sunset), with ♋ [Cancer] glyphs at its ends: readable hour numbers 11 (lower left), 10, 9, 8, 7, 6, 5, 4, 3, 2, 1 (lower right)
- Along the equinoctial line (astronomical hours, left to right): 4, 3, 2, 1, 12 (center), [11?], 9, 8, 7 [several center digits [?]]

I. With the Sun at the beginning of Aries [♈].

Sign	Timing	H	M
Aries ♈	Ante mer.	6	0
Taurus ♉	Ante mer.	4	8
Gemini ♊	Ante mer.	2	9
Cancer ♋	Ante mer.	0	0
Leo ♌	Post mer.	2	9
Virgo ♍	Post mer.	4	8
Libra ♎	Post mer.	6	0
Scorpio ♏	Post mer.	7	52
Sagittarius ♐	Post mer.	9	51
Capricorn ♑	Post vel an. mer.	12	0
Aquarius ♒	Ante mer.	9	51
Pisces ♓	Ante mer.	7	52

II. With the Sun at the beginning of Libra [♎].

Sign	Timing	H	M
Libra ♎	Ante mer.	6	0
Scorpio ♏	Ante mer.	4	8
Sagittarius ♐	Ante mer.	2	9
Capricorn ♑	Ante mer.	0	0
Aquarius ♒	Post mer.	2	9
Pisces ♓	Post mer.	4	8
Aries ♈	Post mer.	6	0
Taurus ♉	Post mer.	7	52
Gemini ♊	Post mer.	9	51
Cancer ♋	Post vel an. mer.	12	0
Leo ♌	Ante mer.	9	51
Virgo ♍	Ante mer.	7	52

III. With the Sun at the beginning of Cancer [♋].

Sign	Timing	H	M
Cancer ♋	Ante mer.	6	0

Sign	Timing	H	M
Leo ♌	Ante mer.	2 [?]	5 1
Virgo ♍	Ante mer.	1	5 2
Libra ♎	Ante mer.	0	0
Scorpio ♏	Post mer.	1	5 2
Sagittarius ♐	Post mer.	3	5 1
Capricorn ♑	Post mer.	6	0
Aquarius ♒	Post mer.	8	9
Pisces ♓	Post mer.	10	8
Aries ♈	Post vel an. mer.	12	0
Taurus ♉	Ante mer.	10	8
Gemini ♊	Ante mer.	8	9

IV. With the Sun at the beginning of Capricorn [♑].

Sign	Timing	H	M
Capricorn ♑	Ante mer.	6	0
Aquarius ♒	Ante mer.	3	51
Pisces ♓	Ante mer.	1	52
Aries ♈	Ante mer.	0	0
Taurus ♉	Post mer.	1	52
Gemini ♊	Post mer.	3	51
Cancer ♋	Post mer.	6	0
Leo ♌	Post mer.	8	9
Virgo ♍	Post mer.	10	8
Libra ♎	Post vel an. mer.	12	0
Scorpio ♏	Ante mer.	10	8
Sagittarius ♐	Ante mer.	8	9

[Margin: In a right sphere the rising and setting amplitudes of the points of the ecliptic are equal to their declinations.]

By the same method one will be able to compile tables of the culminations of the heaven [mediationes cœli] and of the angles of the earth; likewise two other tables giving the points of the ecliptic that lie on the circle of the 5th and 7th hour from noon or midnight, together with their declinations, at the moment when the beginnings of the 12 signs emerge above the horizon in a right sphere, as we explained in the same place.

As for the figure of the rays of the rising and setting amplitudes, described in Book 2, Proposition 21, it does not differ from the figure of the rays of the zodiac. This is because, in a right sphere, the rising and setting amplitudes of the points of the ecliptic are equal to their declinations, since the right horizon, drawn through the poles of the world, measures both the declinations and the rising and setting amplitudes. We have therefore described the horizontal dials in a right sphere. Which was to be done.

PROBLEMA 2. PROPOS. 2.

To construct vertical dials in a right sphere.

[Margin: The drawing of a vertical dial of the right sphere, together with the parallels of the signs and the latitudes of cities.]

In a vertical dial of the right sphere, the hours from noon and midnight, the parallels of the signs of the zodiac, and the parallels — or circles of latitude — of cities will be described in just the same way as in an equinoctial dial. For the vertical circle properly so called and the equator do not differ from each other in a right sphere, as is clear. But the horizontal line must be drawn through the center of the dial, or the place of the gnomon, perpendicular to the meridian line, in such a way that it does not differ from the line of the 6th hour from noon or midnight — because in a right sphere the horizon and the circle of the 6th hour from noon or midnight do not differ, as appears in the following figures.

[Margin: The hours from sunrise and sunset in a vertical dial of the right sphere.]

As for the hours from sunrise and sunset, and likewise the unequal hours, the same must be said here as in the preceding proposition. For the numbers placed in the middle position pertain to the hours from noon and midnight, the upper ones to the hours from sunset, and the lower ones to the hours from sunrise.

[Margin: The vertical circles and the parallels of the horizon in a vertical dial of the right sphere. The meridian circles in a vertical dial of the right sphere.]

The vertical circles and the parallels of the horizon will be drawn here just as in the vertical dial of an oblique sphere, for the demonstration is the same in both cases.

The meridians too will be drawn like the hours from noon or midnight, provided that first the longitude of the place is reckoned from the meridian line, so that a straight line drawn from the center of the dial through the end of the counting — the line representing the meridian of the Fortunate Isles — lies in the eastern part of the dial. Such is the line marked off by dots in each dial, if the longitude of the place in the right sphere is 36 degrees.

Diagram. AVSTRALE ("Southern") — the south-facing vertical dial of the right sphere, drawn as a half-disc opening downward from a horizontal top line. The top line carries the split label **HORI:** (left) ... **ZON** (right), spelling *HORIZON*, with zodiac glyphs at each end. Hour-lines radiate from the center of the top line to the arc; three sets of numbers are used (middle = hours from noon/midnight, upper = hours from sunset, lower = hours from sunrise). The label crowding leaves several numbers unresolved

- Top-left horizon label: HORI: with ♓/♒ [Pisces/Aquarius] and a further worn glyph [?]
- Top-right horizon label: ZON with ♏/♍ [Scorpio/Virgo] glyphs [?]
- Outer hour numbers, left arc (top to bottom): 24, 1, 2, 3, 4, 5, 6
- Outer hour numbers, right arc (top to bottom): 12, 11, 10, 9, 8, 7
- Inner hour numbers (across the upper zone and arc): 6, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24 [dense center cluster partly [?]]
- Bottom-right, along a dotted line: "Inf. Fort." [Insulæ Fortunatæ — the meridian of the Fortunate Isles] [?]
- A wavy (Aquarius-water) parallel mark near the bottom center

Diagram. BOREALE ("Northern") — the north-facing vertical dial of the right sphere, mirror-companion to AVSTRALE, again a half-disc opening downward from a horizontal top line labelled **HORI:** ... **ZON** (*HORIZON*) with zodiac glyphs. Same three number-sets (middle/upper/lower). for the dense center numbers.

- Top-left horizon label: HORI: with ♍/♌/♋ [Virgo/Leo/Cancer] glyphs [?]
- Top-right horizon label: ZON with ♊/♉ [Gemini/Taurus] glyphs [?]
- Outer hour numbers, left arc (top to bottom): 12, 11, 10, 9, 8, 7, 6
- Outer hour numbers, right arc (top to bottom): 24, 1, 2, 3, 4, 5, 6

- Inner hour numbers: 6, 24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13; center 12, 11, 10 [dense cluster partly [?]]
- Bottom-left, along a dotted line: "Inf. Fort." [Insulæ Fortunatæ — meridian of the Fortunate Isles] [?]

[Margin: The celestial houses in a vertical dial of the right sphere. — The ascending signs in a vertical dial of the right sphere.]

The celestial houses do not diverge from the hours from noon and midnight, since in a right sphere the circles of the celestial houses are the same as those of the hours from noon and midnight. Thus the 6th hour from midnight — that is, the horizontal line on the western side — gives the beginning of the 1st house; the 8th hour from midnight, the beginning of the 12th house; and so on for the rest, as we wrote in the preceding proposition — so that the 6th hour from noon, or the horizontal line on the eastern side, presents the beginning of the 7th house.

Finally, the ascending signs of the zodiac are most conveniently drawn in this place by means of the points of the horizontal line, where it is cut by the arcs of the signs, as we said above, and by the culminations of the heaven and the angles of the earth (if tables of this kind are first drawn up for the right sphere); or else by means of the hours of the tropic of Cancer [♋] and of Capricorn [♏], as the tables of the preceding proposition indicate — by drawing these tropics as hidden construction lines above the horizontal line, together with the hour-lines from noon or midnight. For at the hours of the tropic of Cancer and of Capricorn, at which the signs rise, the lines of the ascending signs will touch the said tropics and pass through the points of the horizontal line. Thus, in the southern dial, you see the line of the ascending sign Aquarius [♒], marked off by dots, touch the tropic of Capricorn at 3 hours 51 minutes before noon, and pass through the point of the horizontal line where it is cut by the arc of Aquarius on the western side. Likewise, in the northern dial, the line of the ascending sign Taurus [♉] will touch the tropic of Cancer above the horizontal line at 10 hours 8 minutes before noon, and will pass through the point of the horizontal line where it is cut by the arc of Taurus toward the western parts. We have therefore constructed the vertical dials in a right sphere. Which was to be done.

PROBLEMA 3. PROPOS. 3.

To compose meridian dials in a right sphere.

[Margin: The hours from noon or midnight, the arcs of the signs, and the circles of latitude of cities in a meridian dial of the right sphere.]

These dials are constructed here just as in an oblique sphere, as concerns the hours from noon and midnight — except that in a right sphere the line of the 6th hour does not differ from the horizontal line, and therefore the equinoctial line, cutting it at right angles, stands perpendicular to the horizon. As for the parallels — that is, the arcs of the signs and of the latitudes of cities — there is no difference in either sphere, provided one observes that the parallels receding from the equinoctial line toward the south belong to the northern signs and towns, while the rest belong to the southern ones.

Diagram. Two companion meridian dials of the right sphere, side by side: **ORIENTALE** ("Eastern," left) and **OCCIDENTALE** ("Western," right). Each is a tall figure built on a vertical center line crossed by a top and a bottom horizontal line, with dotted lines splaying outward to zodiac glyphs at the lower corners; the hour-numbers are arranged in three vertical columns (left = one hour-count, center, right = another). The center-column digits are the least legible and are flagged

ORIENTALE (rows, left | center | right):

- 12 | 6 | 24
- 13 | 7 | 1
- 14 [?] | 8 | 2
- 15 | 9 | 3
- 16 | 10 | 4
- 17 | 11 | 5
- Lower-corner zodiac glyphs: ♑ [Capricorn] (left), ♋ [Cancer] (right) [?]

OCCIDENTALE (rows, left | center | right):

- 12 | 6 | 24
- 11 | 5 [?] | 23
- 10 | 4 | 22
- 9 | 3 | 21
- 8 | 2 | 20
- 7 | 1 | 19
- Lower-corner zodiac glyphs: ♋ [Cancer] (left), ♑ [Capricorn] (right) [?]

[Margin: The hours from sunrise and sunset in a meridian dial of the right sphere.]

The hours from sunrise and sunset, however, are to be counted on the same lines as the hours from noon and midnight, as we set out in Proposition 1 of this book — as is clear from the accompanying figures, in which the numbers are arranged as in the horizontal dial of the right sphere. And so the meridian dial in a right sphere does not differ from the horizontal dial of the same sphere, if we look at the lineaments; only the numbers change, as is evident. So much so that the horizontal dial of the right sphere, if it is set up parallel to the meridian circle, yields the meridian dial in the same sphere.

[Margin: The vertical circles and the parallels of the horizon in a meridian dial of the right sphere.]

The vertical circles too, and the parallels of the horizon, will be set down here as in the meridian dial of an oblique horizon; but the equinoctial line will give the vertical circle properly so called, as we said in the horizontal dial of the right sphere.

[Margin: The meridian circles and the celestial houses in a meridian dial of the right sphere.]

Likewise the meridian circles and the celestial houses will be depicted here as in the meridian dial of an oblique sphere — yet in such a way that the celestial houses here keep the same order that we prescribed in the horizontal dial of the right sphere.

[Margin: The ascending signs in a meridian dial of the right sphere.]

Finally, the ascending signs also have here the same description as in the meridian dial of an oblique sphere. Therefore we have composed the meridian dials in a right sphere. Which was to be done.

PROBLEMA 4. PROPOS. 4.

To draw polar dials in a right sphere.

[Margin: Polar dials in a right sphere.]

Polar dials in a right sphere differ in nothing from the horizontal dials of the same sphere, since the circle of the 6th hour from noon or midnight — to which the polar dials are parallel — does not differ from the horizon, to which the horizontal dials are parallel. We have therefore drawn the polar dials in a right sphere. Which was to be done.

PROBLEMA 5. PROPOS. 5.

To lay out equinoctial dials in a right sphere.

[Margin: Equinoctial dials in a right sphere.]

Nor indeed are the equinoctial dials of the right sphere any different from the vertical dials of the same sphere, since in a right sphere the equator and the vertical circle are one and the same circle. We have therefore laid out the equinoctial dials in a right sphere. Which was to be done.

PROBLEMA 6. PROPOS. 6.

To depict dials declining from a vertical circle in a right sphere.

[Margin: How an astronomical dial declining from the vertical in a right sphere is to be described.]

Draw two straight lines A B and C H, cutting one another at right angles at H. At the line C H, at H, construct the angle of declination C H D — toward point B if the plane declines from south to west, or from north to east; but toward point A if it deflects from south to east, or from north to west. In our example we suppose the plane to decline 60 degrees from south toward west. Taking point D at will on the line H D, draw the perpendicular D I to H D, cutting A B at I; and through I draw the perpendicular I F to A B. Then cut off the line I E equal to D I, and from E describe a circle at will. Divide this circle into 24 equal parts, beginning from the line A B, and from E, through the division points, send out hidden lines cutting the line I F at points. Through these points, if straight lines are drawn from H, the hour-lines are obtained, whose order is as follows. The line C H gives the 12th hour, that is, the meridian line; the lines to its left indicate the hours from midnight in dials declining from the south, while those placed to the right pertain to the hours from noon. The contrary is to be understood in dials that deflect from the north.

Diagram. The construction of a dial declining 60 degrees from the meridian toward the west in a right sphere. A horizontal line A B runs across the figure, meeting the vertical line C H at right angles at H, from which all the hour-lines radiate downward in a fan.

- A (left end of the horizontal line A B): also marked with the hour-number 6
- B (right end of the horizontal line): also marked with the hour-number 6

- H (intersection of A B with the vertical line; the centre from which the hour-lines are drawn)
- C (top of the vertical line C H)
- G, I, E (points on line A B between H and B, in that order); I F is the perpendicular dropped from I
- D (upper point, on the oblique line H D that sets the declination angle C H D)
- F (foot of the perpendicular I F, at the bottom of the figure); the small hour-division circle described from E lies near I/E
- Hour-line numbers radiating from H, on the left/lower side: **7, 8, 9, 10, 11,** and **12** (the meridian line C H produced downward)
- Hour-line numbers on the right/lower side: **5, 4, 3, 2, 1**

[Margin: Demonstration of the foregoing description.]

The demonstration of this construction is very easy. Suppose the dial set in its own position, so that the line A B is parallel to the horizon and is the common section of the plane of the dial and the horizon — that is, of the circle of the 6th hour from noon or midnight. Suppose the triangle H D I to be turned about the line H I until it stands perpendicular to the plane of the dial and is joined with the horizon, or with the circle of the sixth hour from noon or midnight. Since C H D is the angle of declination from the vertical, D H I will be the angle of declination from the meridian circle; and therefore H D will be the common section of the meridian and the horizon, that is, the axis of the world. Hence D I, being perpendicular to the axis, will be the common section of the equator and the same horizon. And so, since the meridian meets the plane of the dial at H and the equator at I, the meridian line must be drawn through H and the equinoctial line through I. Now since both the plane of the dial and that of the meridian are perpendicular to the horizon, [a] their common section will be perpendicular to that same horizon, and therefore, by definition 3 of Book 11 of Euclid, perpendicular to the line A B lying in the horizon. For this reason C H, perpendicular to A B, will be the common section of the meridian and the plane of the dial. By the same reasoning we shall show that I F is the common section of the equator and the plane of the dial, because both the plane of the dial and that of the equator are perpendicular to the horizon. And with the same triangle H D I standing perpendicular to the plane of the dial, if the circle described from E is conceived to be turned about the equinoctial line I F until its centre E is joined with D, the centre of the world — on account of the equality of the lines I D and I E, and

[Margin: (a) Euclid, *Elements* XI.19.]

[Margin: Description of the same dial, when the position of the style and its length are given in the declining plane.]

and thus lies in the plane of the equator, drawn through the lines I D and I F — we shall show that the hours from noon and midnight are correctly described, as in the horizontal dial of Proposition 1 of Book 2. For the right horizon — the circle of the 6th hour from noon or midnight — is, with respect to the plane of the declining dial, like the meridian; and the meridian itself is like the circle of the 6th hour from noon or midnight. So the line H I, the common section of the right horizon (or of the circle of the 6th hour from noon or midnight), here holds the place that the meridian line holds in the horizontal dial of the oblique sphere; and the meridian line C H holds the place that the line of the sixth hour from noon or midnight holds. Now the hour-intervals on the other side of the meridian line C H will each be equal to the hour-intervals lying between the meridian line and the equinoctial, since they arise from the hour-intervals on the equinoctial line above point I, which we showed, each to each, to be equal to the hour-intervals on the same equinoctial line below point I, in Proposition 1 of Book 2. Hence it is not necessary that the whole dial be drawn; it is enough if the hour-lines cutting the equinoctial line are transferred to the other side of the meridian line. The style is the line D G, dropped from D, the centre of the world, onto the line A B at right angles; and its foot is the point G.

If, in the declining plane, the position of the style is given at G, together with its length G D, we shall describe the dial as follows. Through G, the position of the style, we draw the line A B parallel to the horizon; and from G we raise the perpendicular G D, equal to the style. Then at D we construct the angle of declination G D H toward A, if the plane declines from south to west or from north to east; but toward B, if on the contrary it declines from south to east or from north to west — so that the line D H cuts the line A B at H. On the other side, at D, we construct G D I, the angle of the complement of the declination, so that the line D I cuts the line A B at I. Finally we complete the rest as before. The reason for this is that the angle of declination C H D, constructed in the former description, is equal to the alternate angle G D H, and therefore to the angle we have constructed at D in this latter description, and so on.

Diagram. An analemma-style construction (the upper figure) with a large arc/semicircle overlaid by many intersecting straight and dotted lines, and a small hour-division circle near the apex. Because of the density of crossing lines and the smallness of the apex numerals, several labels cannot be resolved.

- A (upper left, on a vertical line descending to H)
- B (just below/right of A, on the arc)
- Small hour-division numerals clustered at the apex: **6, 7, 8, 9** and **3, 4, 5**, with **10** and **2** to the right [several of these apex numerals are crowded — [?]]
- D (left, on a horizontal line) and C (right, on the same horizontal line); **1** lies on that line near D
- II [?] and L (upper right)
- H (lower left) and V (lower centre), on the bottom horizontal line
- **12** (right end of the lower horizontal line)

[Diagram fig-p365-1: more than 2 labels flagged [?]]

Diagram. The face of the declining dial (lower figure): a fan of hour-lines (solid and dotted) spreading from the top horizontal line, which carries the point H at its left. The hour-numbers run in three descending groups; zodiac-sign sorts are set at the lower ends of certain lines and are worn.

Readable hour-numbers, arranged left group | centre group | right group:

- 12 | 6 | 24
- 11 | 5 | 23
- 10 | 4 | [?]
- 9 | 3 | 22
- 8 | [?] | 21
- 7 | 2 | 20
- 6 | 1 | [?]
- Lower endpoints: **12, 18** (lower left); **19** (lower centre); **28** (lower right)
- Zodiac-sign sorts (worn — pinned by position only, glyphs not committed): lower-left ☐ ♈ [Leo?] [?]; centre a pair of stacked sorts ☐ ♈ [Aries?] [?] over ≈ ♎ [Libra?] [?]; right ☐ ♉ [Taurus?] [?]

[Margin: The hours from sunrise and sunset, and the unequal hours, in a declining dial of the right sphere.]

Concerning the hours from sunrise and sunset, and the unequal hours, the same is to be said here as in the horizontal dial of the right sphere.

[Margin: The arcs of the signs, the diurnal arcs, and the circles of latitude, in the same dial.]

The parallels — that is, the arcs of the signs, the diurnal arcs, and the circles of latitude of cities — are to be laid down in this dial no differently than in the horizontal dial of the oblique sphere, provided that, in the former manner, the circle of the Analemma

[Margin: ...the latitudes of cities, in a declining dial of the right sphere.]

A B C represents the right horizon, or the circle of the 6th hour from noon or midnight; and the line B C [represents] the common section of that horizon and of the great circle to which the plane of the dial, drawn through the line R O, is parallel. Let the arc C E be equal to the complement of the declination of the plane from the vertical, or (what is the same) to the declination from the meridian, which is always equal to the altitude of the pole above the plane. In the latter manner, let the line H I represent the line of the 6th hour from noon or midnight, and the line H V the meridian line, and so on.

[Margin: Which arcs pertain to the northern signs, and which to the southern.]

Moreover, in dials of this kind the northern signs are always set toward the southern parts, and the southern toward the northern. The northern parts of the dial lie between the centre H and the equinoctial line, if the dial declines from the south; but if it deflects from the north, the southern parts will be contained between the centre and the equinoctial line.

[Margin: The vertical circles in a dial declining from the vertical in a right sphere.]

The vertical circles are to be drawn thus. From point D of the earlier figure of this proposition describe a circle and divide it into 360 equal parts, or into fewer, beginning the division from the line D H or D I. For if straight lines go out from D through the division points, the horizontal line H I will be cut at points; and if through these points there are drawn meridian lines, or lines parallel to the equinoctial, or perpendiculars to H I, the vertical circles will be described. For if the circle described from D is turned about the line H I until it stands perpendicular to the plane of the dial and lies in the plane of the horizon, so that the centre D is joined with the centre of the world (that is, the vertex of the style) — since the line D H is the common section of the meridian and the horizon, and

D I the common section of the equator and the same horizon, as we demonstrated a little earlier in this proposition — the remaining lines drawn from D will be the common sections of the horizon and of the remaining vertical circles. Therefore, as was shown in Proposition 4 of the preceding book, the vertical circles are correctly described. And the equinoctial line I F will be the common section of the vertical [circle] properly so called and of the plane of the dial, because in a right sphere the equator is the primary vertical, as is evident; and the meridian line C H [will be] the common section of the plane of the dial and of the vertical circle of 90 degrees, and so on.

[Margin: The parallels of the horizon in the same dial.]

The parallels of the horizon have here the same description as in the dial declining from the vertical in the oblique sphere.

[Margin: The meridian circles in the same dial, declining from the vertical, of the right sphere.]

By the same reasoning the meridian circles too are to be depicted, as was said in the foregoing: namely, in the circle described from E in the earlier figure of this proposition, from the upper point of the line drawn perpendicular through E to H E, let the longitude of the place be counted toward the western parts — which in our example are the parts toward point B, as is clear if the dial is set in its own position together with the circle described from E, so that the centre E is placed at D, the centre of the world. For then that line, making right angles with A B at E, will be the common section of the equator and the meridian, and point B will incline toward the west.

[Margin: The celestial houses in the same dial.]

As for the celestial houses, the same is to be understood here as we wrote in the horizontal dial of the right sphere. For their circles are not different from the circles of the hours from noon or midnight. Hence their lines too will not differ from the lines of the said hours.

[Margin: The ascending signs in the same dial.]

Finally, the ascending signs are to be depicted as in the foregoing. Therefore we have depicted the dials declining from a vertical circle in a right sphere. Which was to be done.

PROBLEMA 7. PROPOS. 7.

To construct dials declining from the horizon in a right sphere.

[Margin: An astronomical dial declining from the horizon in a right sphere.]

Draw two straight lines A B and C D, cutting one another at right angles at A. At A, on the line C D, construct the angle of declination of the plane from the horizon, D A E — toward point D if the plane faces west, but toward point C if it faces east. We suppose the proposed plane to decline 40 degrees and to face west. Taking also a point E anywhere on the line A E, draw the perpendicular E D to A E, cutting C D at D. Likewise, from E describe a circle and divide it into 24 equal parts, beginning from the line E D or E A. Then, sending out from E through the division points hidden lines, C D will be cut at points; and the perpendiculars raised to C D from these points will give the hours from noon and midnight in this order. The line drawn through D will always be the line of the 12th hour; and the remaining lines toward A will give, in a dial facing west, the hours from noon, but in one facing east, the hours from midnight — so that in the former the line A B gives the 6th hour from noon, and in the latter the 6th hour from midnight. This we shall show as follows.

[Margin: Demonstration of the foregoing description.]

Diagram. The construction for a dial declining 40 degrees from the horizon toward the west in a right sphere. A horizontal base line runs from C on the left, through A, to D on the right; a set of parallel vertical hour-lines crosses it, and a circle centred on E (with F below its centre, on the axis) carries the hour-divisions. An oblique line runs up from the lower left through E, and dotted meridian lines run from D toward the lower right.

- B (upper left, top of the leftmost vertical line)
 - C (left end of the horizontal base line), A (just to the right of C, the intersection point)
 - E (centre of the hour-division circle)
 - F (point on the base line below E, within the circle)
 - D (right, on the base line; the line through D is the 12th-hour line)
 - Hour-numbers along the base line, left to right: 6, 5, 4, 3, [F], 2, 1, 12, 11, ...
- 10

Let the dial be conceived in its own position, so that the line A B is parallel to the horizon and is the common section of the horizon and of the plane of the dial.

And since both the plane of the dial and the plane of the horizon are perpendicular to the equator (by Proposition 15 of Book 1 of Theodosius), because each is drawn through the poles of the equator, or of the world — namely, through the common sections of the meridian and the horizon — [a] their common section A B will also be perpendicular to that same equator; and thus, by definition 3 of Book 11 of Euclid, perpendicular also to the line lying in the equator, which is the common section of the equator and of the plane

[Margin: (a) Euclid, *Elements* XI.19.]

the plane of the dial. Since, then, the angle BAD is a right angle, CD is the common section of the equator and of the dial's plane. So if the plane in which the circle drawn from E, and the triangle ADE, lie is turned about the line AD until it stands perpendicular to the dial's plane, and its centre E is made to coincide with the centre of the world, the circle will then lie in the plane of the equator about that same centre E — because the equator too, drawn through the centre of the world E and through the line CD, stands perpendicular to the dial's plane. Hence the line EA, lying in the plane of the equator (that is, of the vertical circle of the right sphere) and making the angle of declination from the horizon, EAD, will be the common section of the horizon — the circle of the 6th hour from noon or midnight — and of the equator (the vertical circle); and therefore ED is the common section of the meridian and of that same equator. For since in the right sphere both the meridian and the equator stand perpendicular to the horizon, [a] their common section is also perpendicular to it, and hence perpendicular to the line AE, which lies in the horizon at the centre of the world E. ED is therefore perpendicular to AE at E, the centre of the world, being the common section of the meridian and the equator. It follows that the remaining hidden lines sent out from E are the common sections of the equator with the other hour-circles. And since the hour-lines in this plane are parallel to the meridian line itself, by Proposition 18 of Book 1 those lines will be the perpendicular straight lines drawn to CD through the points where the hidden lines fall — the common sections of the hour-circles and of the dial's plane. The style [stylus, gnomon] will be the perpendicular EF dropped from E to the line CD, its foot at F.

[Margin: (a) [Euclid, *Elements*] XI.19.]

[Margin: Description of the same dial, when the length of the style and its position are given on a plane declining from the horizon.]

Now if the length of the style EF, and its position at the point F on a plane declining from the horizon, are given, we shall describe the dial in this way.

Through F, the foot of the style, draw the line FE parallel to the horizon and equal to the style, and cut it at right angles at F by the line CD. Then at E, toward the upper part of the plane, construct the angle FEA — the complement of the plane's declination from the horizon — so that the line EA cuts CD at A. Likewise at E, toward the lower part of the plane, make the angle FED — the plane's declination from the horizon — so that the line ED cuts CD at D. The rest we complete as before. For in this way EAF will be the angle of the plane's declination from the horizon, and so on.

[Margin: The hours from sunrise or sunset, and likewise the unequal hours, in the same dial.]

The hours reckoned from sunrise or sunset, and the unequal hours, behave here just as in the horizontal dial of the right sphere, as we explained earlier.

[Margin: The arcs of the signs, the diurnal arcs, and the latitude-circles of the cities, in the same dial declining from the horizon.]

The parallels — that is, the diurnal arcs of the zodiacal signs — and the latitude-parallel of the cities are drawn here just as in the meridian and polar dials of the oblique sphere. For the hour-intervals between the centre E and the equinoctial line CD must be transferred onto the radius of the equator, and so on, as appears in the following figure.

Diagram. The construction figure for the dial declining from the horizon in the right sphere, in two parts. *Left:* an oblique/perspective bundle of hour-lines converging from below, crossed by a set of nearly-parallel lines, with a long vertical line labelled (reading down its length) **Linea horizontalis** ("the horizontal line"). The converging hour-lines carry paired numerals along their length: **2, 4, 1, 5, 12, 6, 10** (and a Roman **II**), each appearing on both the near and far edges of the bundle. *Right:* a horizontal equinoctial baseline (the line CD) marked with equal hour-divisions, numbered beneath it from left to right **6, 5, 4, 3, 2, 1, 12, 11, ... 10**; two dotted curves (the diurnal-arc parallels) sweep above and below the baseline, each terminating in a worn zodiacal sign-glyph.

- Vertical line, left part: **Linea horizontalis** ("horizontal line")
- Hour-numbers, left bundle: 2, 4, 1, 5, 12, 6, II, 10 [several paired/worn — [?]]
- Baseline hour-divisions, right part (left→right): 6, 5, 4, 3, 2, 1, 12, 11, [?], 10
- Upper diurnal-arc parallel: zodiacal sign-glyph [♋ Cancer?] — worn [?]
- Lower diurnal-arc parallel: zodiacal sign-glyph [♐ Capricorn?] — worn [?]

[Margin: The vertical circles and the parallels of the horizon in the same dial.]

The vertical circles and the parallels of the horizon are placed in this dial too, just as in the dial declining from the horizon in the oblique sphere.

[Margin: The meridian circles in the same dial.]

The meridians too are drawn like the hour-lines, if in the circle described from E one reckons, from the point G, the longitude of the place toward the western side of noon, so as to obtain the meridian of the Fortunate Isles [Insulae Fortunatae].

[Margin: The celestial houses in the same dial.]

The lines of the celestial houses are the same, as we said before — namely the lines of the hours from noon and midnight.

[Margin: The ascending signs in the same dial.]

Finally the ascending signs are laid out as above. We have therefore constructed dials declining from the horizon in the right sphere. Which was to be done.

PROBLEMA 8. PROPOS. 8.

To lay out dials inclined to the horizon in the right sphere. [*HOROLOGIA ad Horizontem inclinata in sphæra recta delineare.*]

[Margin: How an astronomical dial inclined to the horizon in the right sphere is described.]

Since the inclination of a plane to the horizon in the right sphere measures the elevation of the pole above that same plane — we are speaking of planes that stand perpendicular to the meridian, and whose great circles, to which they are parallel, are drawn through the poles of the meridian, that is, through the common sections of the horizon and the equator — it comes about that astronomical dials inclined to the horizon in the right sphere are described just like horizontal dials in the oblique sphere, provided that, for the pole's elevation, one takes in the portion of the analemma (Proposition 1 of Book 2) the arc CE equal to the plane's inclination to the horizon, so that the axis of the world ED, the common [section...]

—is the common section of the true horizon and the meridian; E is the pole of the world, whether arctic or antarctic; BC is the common section of the meridian and of the great circle to which the dial is parallel; and HI is the common section of the meridian and the dial's plane. Now since the true horizon — that is, the circle

of the 6th hour from noon or midnight — drawn through the axis ED meets the dial's plane at H, the centre of the dial, and through H the 6th-hour line from noon or midnight is drawn perpendicular to the meridian line, this will give, in this dial, the 6th-hour line from noon or midnight as the horizontal line, which will always occupy the upper position once the dial is set in its proper place.

[Margin: The order of the hours in the aforesaid dial.]

The order of the hours in a dial raised on its southern side above the horizon and facing north is the same as in the horizontal dial of the oblique sphere, because the arctic pole is elevated above the dial's plane. So the horizontal dial that we described at the beginning of Book 2 for latitude 42 degrees, raised 42 degrees on its southern side, will present, in the right sphere, a dial inclined 42 degrees — yet so that the 6th-hour line from noon or midnight is the horizontal line. But if the dial faces south and is raised on its northern side, the order of the hours will be the same as in the southern vertical dial of the oblique sphere, because the antarctic pole is elevated above the dial's plane. So the vertical dial facing south, which we described in Book 2 (Propositions 13 and 14) for latitude 42 degrees, raised 48 degrees on its northern side — that being the pole's elevation above the vertical circle — yields in the right sphere a dial inclined 48 degrees, again such that the 6th-hour line from noon or midnight gives the horizontal line. All this is clear and evident if one imagines the dial inclined to the horizon set in its proper place.

[Margin: When the length of the style together with its position is given.]

Now if the length of the style, and its position on the inclined plane, are given, we proceed as in the horizontal dial, provided that in place of the pole's elevation and its complement we take the plane's inclination to the horizon and its complement — always reckoning the complement of the inclination toward the upper part of the plane, and the inclination itself toward the lower part, and so on.

[Margin: The hours from sunrise and sunset, and the unequal hours, in the same dial.]

The hours from sunrise and sunset, and the unequal hours, are here reckoned as was set out in Proposition 1 of this book.

[Margin: The arcs of the signs, the diurnal arcs, and the latitude-circles of the cities, in the same dial.]

The arcs of the signs, the diurnal arcs, and the latitude-parallels of the cities are drawn as was said for the horizontal dial of the oblique sphere, and they keep the same order as in the horizontal or the vertical dial, according as the dial faces north or south.

[Margin: The vertical circles and the parallels of the horizon.]

The vertical circles and the parallels of the horizon are drawn as in the dial inclined to the horizon in the oblique sphere, as we taught in Proposition 28 of the previous book. For if, from the portion of the analemma set out in Proposition 1 of Book 2, one takes the line HD — that line HD corresponds, when the angle DHI is equal to the plane's inclination, to the line AB in the figure of Proposition 28 of the previous book — as an equal straight line on the meridian line, beginning from the centre of the dial H (that is, from the point where the horizontal line, or the 6th-hour line, cuts the meridian line), and describes a circle, then we shall find points on the 6th-hour line from noon or midnight (the horizontal line) which, joined by straight lines to the point where the meridian line and the equinoctial line cut one another, will give the vertical circles. For the equinoctial line is the same as the vertical, and the meridian line gives the vertical of 90 degrees, and so on.

[Margin: The meridian circles, the celestial houses, and the ascending signs of the Zodiac, in the same dial.]

The meridians, the celestial houses, and the ascending signs of the Zodiac are drawn as said before. We have thus laid out dials inclined to the horizon in the right sphere. Which was to be done.

PROBLEMA 9. PROPOS. 9.

To compose dials declining from a vertical circle and at the same time inclined to the horizon, in the right sphere. [*HOROLOGIA à Verticali circulo declinantia, & simul ad Horizontem inclinata in sphæra recta componere.*]

[Margin: In what way a dial declining from a vertical circle and inclined to the horizon is described in the right sphere.]

Draw two straight lines AB and CD cutting one another at right angles at E, and at E, on the line CD, construct the angle of declination DEF, keeping the order we prescribed in Proposition 37 of the previous book — that is, on a plane declining from south toward the east, or from north toward the west, construct it below the line AB, toward the point A; but toward the point B, below the same line AB, on a

plane that turns from south toward the west, or from north toward the east. Here we are speaking of the upper dial, as in the said Proposition 37 of the previous book, for from it the lower dial is easily derived, as we set out in the preceding two books. The line EF will be called the line of declination. We suppose the proposed plane to decline 60 degrees from south toward the east, and to be inclined 70 degrees to the horizon.

Then from a point F taken anywhere on the declination-line EF, raise a perpendicular $F\perp$ to AB — or rather, from an assumed point $\perp$ on the line AB, draw the perpendicular $\perp F$ to AB, cutting the declination-line EF at F; and from that point F raise the perpendicular Fn to $\perp F$. Then at $\perp$, upon $\perp F$, construct the angle of the plane's inclination to the horizon, $I[?] \perp n$, so that the line $\perp n$ cuts the line Fn at n. From $\perp F$ extended, cut off $\perp p$ equal to $\perp n$, and draw the line Ep, which will be the meridian line — that is, the line of the 12th hour from noon or midnight — as we shall demonstrate.

[The source labels this point with a special typographic sort resembling an inverted T / perpendicular sign; it is rendered $\perp$ throughout.]

After this, from F draw the perpendicular FG to $\perp n$, and from $\perp F$ cut off $\perp H$ equal to $\perp G$. For when the line EH is drawn, it will be the line of the index, or of the style; and the style itself will be the line FG, to be set up at H perpendicular to the dial's plane.

Again, from H raise a perpendicular HI to the style-line EH, equal to the style FG, and from E through I draw the line EI, which will represent the axis of the world, so that HEI is the angle of the pole's elevation above the proposed plane. Then from I draw the perpendicular IK to the axis EI, cutting the index-line at K, and through K raise a perpendicular KM to the index-line, for the equinoctial line, which — if no error has been made — will necessarily pass through the point p, as we shall demonstrate.

Finally, take on the style-line the segment KL equal to KI, and from L describe a circle of any size; divide it into 24 parts, beginning either from the line LM — drawn from the centre L through the point M, where the meridian line and the equinoctial line cut one another — or from the line $L\alpha$, drawn from the centre L through the point α , where the equinoctial line cuts the line AB. For if straight lines are sent out from L through the division-points, the equinoctial line

—KM will be cut at points through which the lines sent out from E will give the hours from noon or midnight, in this order. With the dial set in its proper position, so that the line AB is parallel to the horizon, the hours that follow the

meridian line toward the east — those which, when we face a dial declining from the south, lie to our right, but to the left if the dial declines from the north — are to be reckoned from noon, just as in the dial that both declines and is inclined in the oblique sphere, which we treated in Proposition 37 of the previous book.

[Margin: Demonstration of the description of the aforesaid dial.]

We shall demonstrate this construction as follows. In the plane of the dial set in its proper position, let AB be parallel to the horizon, so that it is the common section of the dial's plane and the horizon; and conceive the triangle $EF \perp$ to be moved about the line $E \perp$ until it coincides with the horizon and lies in it. Now since DEF is the angle of the plane's declination from the vertical, the remaining angle AEF will be the complement of that declination — such indeed as the meridian makes with the line that, in the declining and inclined plane, is parallel to the horizon, or rather with the plane drawn through that line and perpendicular to the horizon. Therefore EF, in that position, will be the common section of the meridian and the horizon. And since in the right sphere the axis of the world is the common section of the horizon and the meridian, EF will be the axis of the world, meeting the dial's plane at E — the point that will be the centre of the dial, in which all the hour-lines meet, as was demonstrated above.

Diagram. The large circular figure of the astronomical dial declining from the vertical and inclined to the horizon, in the right sphere. A horizontal diameter runs from **A** (left) through the centre **E** (drawn as a small circle) to **B** (right); this is the line AB, parallel to the horizon and the 6th-hour line. Twenty-four hour-lines radiate from E to the circumference, their outer ends numbered around the rim. A vertical line rises from E through **C** near the top and descends through the lower part of the figure; near the centre-below lie the points **P** and **M**, with **F** just above them and **p** alongside. A tight cluster of construction points and lines sits just left of and below the centre — including the Greek letters α and γ , and the points **I**, **H**, **n**, **K**, **L** and the special sort $\perp$ — together with a circle (the hour-circle about L) and several dotted arcs.

- A (left end of the horizontal diameter AB)
- B (right end of the horizontal diameter AB)
- C (upper centre, on the near-vertical meridian line)
- E (centre of the figure; small circle where all hour-lines meet)
- F (below and left of centre)
- n (left-of-centre cluster)
- P, M (below the centre, on/near the meridian line); p (adjacent to M)

- I, H, K, L (centre-left construction cluster — faint)
- α , γ (Greek letters in the centre-left cluster — faint)
- $\perp$ (special typographic sort, in the centre cluster)
- Hour-numbers around the rim (worn): upper arc, left→right — 4, 3, 2, 1, 12, 11, 10, 9, 8 [?]; right side down from B — 7, 6 (at B), 5, 4, 3, 2 [?]; bottom centre — 1 [?]; left side down from A — 6 (at A), 7, 8, 9, 10, 11, 12 [?]

[Margin: (a) [Euclid, *Elements*] XI.4. (b) [Euclid, *Elements*] XI.18.]

Again, with the triangle $EF\perp$ having the position we described, conceive the triangle $Fn\perp$ to be turned about $F\perp$, downward, until it stands perpendicular both to the dial's plane and to the horizon — which happens precisely when the line $\perp n$ is perpendicular to AB . For then the line AB , being perpendicular to the lines $\perp F$ and $\perp n$, [a] will be perpendicular to the plane of the triangle $\perp Fn$ drawn through those lines. [b] Therefore both the dial's plane and the plane of the horizon, drawn through the line AB , will be perpendicular to that same plane of the triangle $\perp Fn$; and conversely this plane will be perpendicular to each of them. Hence, since $F\perp n$ is the angle of the plane's inclination to the horizon, and the horizon is drawn through the line $I[?]\perp$ in that position, $\perp n$ will lie in the inclined plane and will coincide with the line $\perp p$ lying in the same plane; and so the point n will fall upon the point p , because of the equality of the lines $\perp n$ and $\perp p$. Since, then, the meridian — standing perpendicular to the horizon, and therefore also to the plane of the triangle $EF\perp$, while lying in the plane of the horizon in the said position — passes through the line EF , as we have shown, and consequently through the line Fn in that position (because Fn , by Definition 4 of Book 11 of Euclid, is perpendicular to the plane of the triangle $EF\perp$, being perpendicular by construction to $F\perp$, the common section of the triangles $EF\perp$ and $\perp Fn$, one of which is perpendicular to the other), the meridian will meet the plane of the inclined dial at the point p ; and therefore the line Ep will be the common section of the meridian and of the plane of the inclined dial. This same meridian line we shall also find by another method, as we taught at the beginning of Proposition 37 of the previous book.

Now, with the triangles $EF\perp$ and $\perp Fn$ still set in the same positions, the line FG — being perpendicular to $\perp n$, the common section of the dial's plane and of the triangle $Fn\perp$ (which stands perpendicular to the dial's plane), and lying in the plane of the triangle $Fn\perp$ — will, by Definition 4 of Book 11 of Euclid, be perpendicular to the dial's plane at the point G , which is then the same as H . Since, then, its end-point F falls on the axis EF (for it was shown that EF , in that position, is the axis of the world), the point F will be the centre of the world, since

through it both the meridian and the horizon are drawn; the line FG will be the style, or gnomon; and consequently the line EH, drawn through the foot of the style, will be the index-line — so that the great circle drawn through it and through the style, and also through the poles of the world, stands perpendicular to the dial's plane, like a kind of proper meridian.

Now if the triangle EIK is turned about the line EK until it stands perpendicular to the dial's plane, the line HI will be perpendicular to it, by Definition 4 of Book 11 of Euclid. Since it was taken equal to the style FG, the point

— the point I will fall upon the centre of the world F; that is, the line HI will coincide with the style, and the line EI with the axis of the world EF.

Consequently the angle HEI will be the elevation of the pole above the plane of the dial, and IK will be the common section of the equator and of the triangle EIK — that is, of the dial's own proper meridian. Therefore, as was shown in what preceded, KM will be the equinoctial line, and the hour-lines will have been drawn, as we said. And since the equator, in the right sphere, is perpendicular to the horizon and passes through F, the tip of the style, it follows that when a line is drawn through F_n — which we showed to be perpendicular to the horizon, or to the triangle EF ⊥ lying in the horizon — it will meet the plane of the dial at the point p. The equinoctial line KM will therefore pass entirely through the point p.

[Margin: The parallels of the signs and of the latitudes of cities. The hours from sunrise and sunset, together with the unequal hours. The vertical circles, the parallels of the horizon, the meridians, and the ascending signs. The celestial houses.]

The parallels of the zodiacal signs and of the latitudes of cities are to be drawn, on this dial that is at once inclined and declining, in just the same way as on the inclined-and-declining dial in the oblique sphere, following what we taught in Proposition 38 of the previous book.

The hours from sunrise and sunset, and likewise the unequal hours, agree in their layout with the hours reckoned from noon and midnight; they differ only in numbering, as we also said for the other dials of the right sphere.

The vertical circles, the parallels of the horizon, the meridians, and the ascending signs are drawn here in exactly the same way as on the declining-and-inclined dial in the oblique sphere.

Finally, the celestial houses are here expressed by means of the hour-lines, just as on the other dials of the right sphere, as has been said. We have therefore

constructed, in the right sphere, dials that decline from the vertical circle and are at the same time inclined to the horizon. Which was to be done.

PROBLEMA 10. PROPOS. 10.

To construct sundials in the most oblique sphere, where the arctic pole is raised 90° above the horizon.

[Margin: By what method dials can be drawn in the most oblique sphere, where the elevation of the arctic pole is 90° .]

Since in a sphere of this kind there is continuous day while the Sun runs through the six northern signs — so that the Sun then neither rises, nor sets, nor reaches the meridian, nor comes to midnight, but stands perpetually above the horizon — and likewise a continuous night while the Sun lingers in the other six, southern signs, it follows that, strictly speaking, the hours can be reckoned neither from sunrise or sunset, nor from noon or midnight, nor can twelve unequal hours be assigned; for there are no diurnal or nocturnal arcs there to be divided into twelve equal parts. Therefore neither an astronomical, nor an Italian, nor a Babylonian, nor an Antique dial can be built in that sphere. Nevertheless, if we imagine twelve hour-circles passing through the poles of the world and dividing the equator into twenty-four equal parts, fixed and immovable, we may draw their hour-lines both on a plane parallel to the horizon and on one perpendicular or inclined to the horizon, in the following way.

[Margin: The horizontal dial in the most oblique sphere, where the arctic pole is set at the pole of the horizon.]

For the horizontal dial, take the equinoctial dial, but with the complete parallels of the zodiacal signs drawn in, and without the horizontal line. For since in that sphere the horizon does not differ from the equator, the plane of the horizontal dial will not be cut by the horizon, nor will it differ from the equinoctial dial. The order of the hours on this horizontal dial will be exactly the same as on the equinoctial one, with this single exception: that here there is no need to search out the meridian line, as there; rather, once the dial has been set so as to lie parallel to the horizon, the start of the hours may be taken from any line whatever.

[Margin: The vertical dial, and the dial declining from the vertical, in the same most oblique sphere. The dial inclined to the horizon in the same most oblique sphere.]

A dial perpendicular to the horizon — such as the vertical dial, or one declining from the vertical — will not differ from a polar dial, since the great circle to which it is parallel passes through the poles of the world. Still, the start of the hours can be set on any hour-line, and their order is taken from left to right. The equinoctial line, moreover, will be the same as the horizontal one.

Finally, if the plane of the dial is inclined to the horizon, the complement of the inclination will give the elevation of the pole above that inclined plane, as is evident. So if a horizontal dial is built for that elevation, as we taught at the beginning of Book 2, and a horizontal line is drawn on it as on the other inclined dials of the oblique sphere, the result will be a dial inclined to the horizon. But from none of these dials shall we learn, as we said, how many hours have elapsed since noon or midnight, or since sunrise or sunset; only how many of the twenty-four equal hours — in which the Sun completes one whole revolution from a given point back to the same point — have passed from some fixed point that we conceive in the mind.

The vertical circles, since they do not differ from the hour-circles — being drawn through the poles of the world — are drawn like the hour-lines, by taking any vertical line as the common section of the vertical circle properly so called and of the plane of the dial; from this the rest are to be reckoned.

[Margin: The vertical circles. The parallels of the horizon, and of the latitudes of cities. The meridian circles.]

The parallels of the horizon and of the latitudes of cities are also drawn like the parallels of the zodiacal signs, if in place of the zodiac's rays whole quadrant-rays are described, and so on.

The meridians, too, will be laid out like the hour-lines, once the position of the prime meridian — drawn through the Fortunate Isles — has first been ascertained. The situation of that prime meridian can be found in this way. Observe the position of some city of known longitude, distant several miles from the place set beneath the pole. On the plane parallel to the horizon, from the chosen place beneath the pole draw a straight line toward that city whose longitude is known, and, from a point taken on it at will, describe a circle. For since that straight line is the common section of the plane of the horizontal dial and of the meridian drawn through that city, if from it the longitude of the said city is counted off toward the western parts — that is, against the succession of the signs, following the Sun's diurnal motion — and a straight line is drawn from the end of that count through the centre, one will have the common section of the

prime meridian and of the plane of the horizontal dial, as is evident. Whenever, therefore, the style's shadow falls on this line, we shall easily, on other planes, draw from the gnomon's shadow another line corresponding to it, to serve as the meridian of the Fortunate Isles.

[Margin: In the most oblique sphere there are no celestial houses.]

The celestial houses, however, have no place in this sphere, because neither a meridian nor a vertical circle properly so called — passing through the points of true sunrise and sunset — can be assigned, as we have said.

Lastly, there are likewise no ascending signs, since perpetually six signs appear above the horizon and six are hidden below it. We have therefore constructed the dials in the most oblique sphere, and so on. Which was to be done.

[Margin: In the same most oblique sphere there are no ascending signs.]

PROBLEMA 11. PROPOS. 11.

To describe sundials in an oblique sphere in which the antarctic pole is raised above the horizon.

Since all the rules we handed down in Books 2 and 3 for the construction of dials concern the oblique sphere that has the arctic pole visible, it seemed worthwhile to sketch briefly, in this proposition, how one should proceed who wishes to describe dials in the other oblique sphere, in which the antarctic pole is raised above the horizon. For this alone seems to be lacking, so that by the rules already handed down one might construct dials in any region whatever of the earth — since in this book too we have prescribed rules by which dials can be made both in the right sphere and in the most oblique sphere, where the arctic pole is raised 90° above the horizon.

[Margin: By what method dials are described in the oblique sphere that has the antarctic pole visible.]

In setting out dials for the southern hemisphere, then, we shall use exactly the same rules we gave in Books 2 and 3, with this one point observed and noted: that whatever was there said of the arctic pole and of the forenoon hours is here to be understood of the antarctic pole and of the afternoon hours. What we there wrote, in the vertical, declining, and inclined dials, about the southern and northern sides is here to be transferred to the northern and southern sides. And whatever we there prescribed about the northern or southern signs is here to be taken in the opposite sense, of the southern or northern signs. So that, if this

interchange is made — of the poles, of the hours before and after the southern noon, and of the northern and southern signs — any dial laid out in Books 2 and 3, for example one made for latitude 42° in the northern hemisphere, will also serve as a dial constructed for latitude 42° in the southern hemisphere. To make this plainer, we shall briefly run through the various dials described in the earlier books, showing for each how it is to be placed in the southern hemisphere, and what must be changed in it so that it may show the hours.

A horizontal dial, then, in the oblique sphere where the antarctic pole is raised above the horizon, is to be set so that its centre faces north and its equinoctial line faces south; for in this way the axis of the world, passing through the dial's centre and the tip of the gnomon, will have its proper position — that is, it will run through the poles of the world. The arcs of the northern signs must also be changed into arcs of the southern signs, and the arcs of the southern into those of the northern, so that the signs between the centre and the equinoctial line belong to the southern signs and the rest to the northern; for there the beginning of Capricorn ♐, at noon, comes closest to the zenith, while the beginning of Cancer ♋ recedes farthest from it. Finally, the hours are to be changed into their complements up to 12, if they are reckoned from noon or midnight, but into their complements up to 24, if they are counted from sunrise or sunset; and the hours that in the northern oblique sphere were reckoned from noon are here, in this other sphere, to be computed from midnight, and conversely.

[Margin: How a horizontal dial built for the northern oblique sphere is to be placed in the southern oblique sphere, and by what method the hour-numbers are to be changed.]

Diagram. A horizontal sundial laid out for the southern latitude of 42° , shown as a broad, roughly hourglass-shaped web of straight hour-lines radiating from a dense central knot, overlaid with dotted curves (the tropics / diurnal-arc parallels) crossing the upper and lower zones. The frame is labelled with the four cardinal directions in large ornamental capitals, and with zodiac-sign glyphs at the four corners; the straight hour-lines carry Babylonian/Italian hour-numbers running past 12 up to about 23, with the astronomical hour-numbers set near the equinoctial line. Because the interior is a very dense overlapping mesh of small numerals, the central hour-numbers cannot be individually resolved on the scan and are not committed here.

- SEPTENTRIO (top center): "North" — the head of the vertical meridian line.
- MERIDIES (bottom center, printed "MERI | DIES"): "South" — the foot of the vertical meridian line.

- OCCASVS (left side, set vertically): "West" — the left end of the horizontal equinoctial line.
- ORTVS (right side, set vertically): "East" — the right end of the horizontal equinoctial line.
- Small cursive label just below SEPTENTRIO, at top center: [Illegible: ~2 words, possibly a pole/axis caption — [?]]
- Vertical central line (SEPTENTRIO–MERIDIES): the meridian line.
- Horizontal central line (OCCASVS–ORTVS): the equinoctial line.
- Dotted curves crossing the upper and lower zones: the tropic arcs / diurnal-arc parallels.
- Zodiac glyph at top-left corner (near the "10" hour-number): [?] (worn sort — not confidently pinned)
- Zodiac glyph at top-right corner (near the "14"/"23" hour-numbers): [?] (worn sort)
- Zodiac glyph at bottom-left corner: [?] (worn sort)
- Zodiac glyph at bottom-right corner: [?] (worn sort)
- Readable straight-line hour-numbers along the margins (partial; upper edge, left→right and right edge top→bottom): 10, 9, 11 (upper left); 23, 22, 14, 13, 12 (upper right / right edge); 11, 10, 9, 8 (lower right); 8, 9 (bottom) — [remaining interior hour-numbers not resolvable: [?]]

and conversely. The hours that used to be counted there from sunrise are here to be counted from sunset, and the reverse — so that, as far as the layout is concerned, from an Italian dial there arises a Babylonian one, and from a Babylonian an Italian. For the hour-numbers are changed into their complements up to 24, as we said. In the same way the numbers of the unequal hours must be changed into their complements up to 12. The reason for this change is clear, if the dial's situation is carefully considered. For the part of the dial that in the northern oblique sphere faced east — and so contained the hours after noon, or from sunset — in the southern oblique sphere faces west and takes in the hours before noon, or from sunrise, and conversely. You have an example here in a horizontal dial constructed for the southern latitude of 42° . Here you plainly see that a Babylonian dial in the northern sphere is an Italian one in the southern, and the reverse; and likewise that the hours there counted from noon are here computed from midnight, and conversely. The dotted lines belong to the astronomical hours, whose numbers are placed near the equinoctial line; but the lines drawn out beyond the tropics signify the hours from sunset, and the rest the hours from sunrise.

[Margin: The diurnal arcs in the southern oblique sphere. The vertical circles, and the parallels of the horizon.]

The numbers assigned to the diurnal arcs, moreover, are not changed, even though the arcs themselves do not remain the same. For the diurnal arcs containing more than twelve hours are, in the northern oblique sphere, the northern arcs, whereas in the southern oblique sphere these same arcs are the southern ones, and so on.

The vertical circles and the parallels of the horizon likewise need not be changed, whether their layout or their numbers are considered.

[Margin: The meridian circles.]

The meridians are to be described in the southern hemisphere as in the northern, if the longitude of the place is counted, from the meridian line in the circle by whose aid the astronomical hours were drawn, toward the western parts — which, when we are turned toward the antarctic pole, lie on our right.

[Margin: The celestial houses.]

The lines of the celestial houses also remain the same; only their numbers must be interchanged. For those set on the left will have to be placed on the right, and conversely, so that house 12 becomes house 8, and house 11 becomes house 9, and so on.

[Margin: The ascending signs.]

Finally, the ascending signs are to be drawn in the southern oblique sphere as in the northern, provided tables are first constructed for the given southern latitude, like those we composed in Proposition 9 of Book 2 — with this carefully observed: that the oblique ascensions of all the signs in the northern oblique sphere agree, in the southern sphere, with the opposite signs; and likewise that the diurnal and semidiurnal arcs computed in the northern sphere are to be assigned, in the southern, to the opposite signs, and so on. All of which we have shown at length in our treatment of astronomical matters.

[Margin: How the other dials constructed for the northern oblique sphere are to be placed in the southern oblique sphere.]

All this must be observed in the other dials as well. But as regards their construction and placement, whatever we said in the northern oblique sphere of the southern and eastern side is, in this other, southern oblique sphere, to be understood of the northern and western side, and conversely. That is: the

southern dial that we laid out in Proposition 13 of Book 2 for the northern latitude of 42° must, in the southern sphere of the same latitude, face north — because on that side noon is made — and the part that there faced east must here be set toward the west, and conversely. Again, the eastern meridian dial described in Proposition 25 of Book 2 for the same northern latitude of 42° will, in the southern sphere of the same latitude, be a western one, and conversely: the western will become eastern; yet each must be so placed that the part which before faced south now faces north, and conversely.

So too the declining dials will have to be altered: what before declined from south toward east now bends from north toward west, and so on. In the same way the dials declining from the horizon are to be placed, so that what before faced west now faces east, and the side that was southern now becomes northern, and so on. In all of them the hour-numbers must be changed into their complements up to 12, if the talk is of hours reckoned from noon or midnight; and the hours before counted from noon are now to be taken from midnight, and conversely. But if there are hours from sunrise or sunset, their complements up to 24 must be taken, and the hours before computed from sunrise are now to be counted from sunset, and conversely. The rest is clear of itself, if the position of any given dial is rightly conceived. We have therefore described the dials in the oblique sphere in which the antarctic pole is raised above the horizon. Which was to be done.

SCHOLIVM.

[Margin: For what latitude all the foregoing dials have been described.]

It has seemed worthwhile to explain briefly in this place what variety the hour-lines from sunrise and sunset take on for one elevation of the pole after another — whether of the arctic or of the antarctic pole — above the horizon, so that no one, in drawing lines of this kind, may be stopped or hindered by seeing that the hours from sunrise and sunset do not behave the same way everywhere among the nations. For since in Books 2 and 3 all those things we described in the hundred and eight problems [centrum & octo [?]] pertain to that oblique sphere in which the arctic pole is elevated 42° , while what we handed down in this book concerns the right sphere, it comes about that the hour-lines from sunrise and sunset, on any dial whatever, cut both tropics. For when the elevation of the pole is less than $66^\circ 30'$ — that is, less than the complement of the Sun's greatest declination — each tropic cuts the horizon, and so the greatest of the parallels of things always visible, which the hour-circles from sunrise and sunset touch (as we showed in Proposition 10 of Book 1), lies between the tropic and the pole. Hence these same hour-circles will, in the sphere, cut the tropic, and

consequently, on any dial, the lines of those same hours will cut the arc of Cancer ☎ or of Capricorn ♎, as was done in all the foregoing examples.

[Margin: At a latitude less than $66^{\circ} 30'$, the hour-lines from sunrise and sunset cut both tropics. At the latitude of $66^{\circ} 30'$, the hour-lines from sunrise and sunset touch the tropic. At a latitude greater than $66^{\circ} 30'$, the hour-lines from sunrise and sunset neither cut nor touch the tropic, but touch another parallel nearer the equinoctial line.]

But it sometimes happens that the said hour-lines only touch the tropic, at those points where it is divided by the lines of the hours reckoned from noon or midnight; and sometimes that they by no means reach the tropic, but touch another parallel lying between the tropic and the equinoctial line. For when the elevation of the pole equals the complement of the Sun's greatest declination — that is, comprises $66^{\circ} 30'$ — the tropic touches the horizon and does not differ from the greatest of the parallels always visible. Hence, in the sphere the hour-circles from sunrise and sunset touch the tropic, and on any dial the lines of those same hours will touch the arc of Cancer ☎ or of Capricorn ♎ at the points where it is cut by the lines of the hours from noon or midnight. But when the elevation of the pole is greater than the complement of the Sun's greatest declination — that is, greater than $66^{\circ} 30'$ — the whole tropic stands above the horizon, and another, larger parallel touches the horizon. Therefore, since in the sphere the hour-circles from sunrise and sunset touch this parallel and in no way reach the tropic, on any dial the lines of those same hours will touch the arc of that parallel and will by no means reach the arc of Cancer ☎ or Capricorn ♎. All of which can be plainly gathered from what we wrote in Book 1 on the hour-circles and on the contacts that the hour-lines from sunrise and sunset make with the greatest parallels of things always visible and always hidden.

[Margin: Which segments of the tangent lines belong to the hours from sunset, and which to the hours from sunrise.]

Yet, to make this matter still clearer, we have added here two dials. The first, constructed for the northern latitude of $66^{\circ} 30'$, is horizontal and contains the hours both from noon and midnight and from sunrise and sunset; of these, the latter touch the arc of Cancer ☎ — which there plays the part of the greatest parallel of things always visible — at the points where it is cut by the former. The second, built for the northern latitude of $69^{\circ} 48'$, is a polar dial with the same hours, in which the hours from sunrise and sunset do not reach as far as the tropics but touch the parallels of Gemini ♊ and Sagittarius ♐, which at that

latitude serve as the greatest parallels of things always visible and always hidden, at the points where they are cut by the hours from noon and midnight.

Now since in the scholium to Proposition 10 of Book 2 we demonstrated that, of any hour-line, the one portion touching the greatest parallel of things always visible or always hidden indicates an hour from sunset, while the other belongs to the hour of the same number from sunrise — the point of contact separating the two portions — we shall distinguish the hours from sunset from the hours from sunrise in this way. On the horizontal dial, the segments of the tangent lines that are drawn upward from the points of contact toward the right of the meridian line — that is, toward the western parts of the dial — belong to the hours from sunset; those that tend downward, to the hours from sunrise. The contrary is to be understood of the lines drawn through the points of contact toward the left of the meridian line, or toward the eastern parts of the dial: those that extend upward show the hours from sunrise, while those that extend downward regard the hours from sunset. Finally, of the line that at the twelfth astronomical hour touches the greatest parallel of things always visible, the western portion indicates the twelfth hour from sunset, and the other the twelfth hour from sunrise. Hence, if only an Italian or a Babylonian dial is to be described, the hour-lines will not be tangent on both...

[continuation of the Scholium to Proposition 11, from the prior page]

to be drawn out to both sides of the points of contact — but only toward that side which pertains to the hours reckoned from sunset or from sunrise. On a polar dial the hours from sunset or from sunrise will be easily recognized from what we wrote about the polar dial in Book 2.

It also seems worth remarking here that the place of the style [locus styli] on horizontal dials can vary in several ways, according to the differing elevation of the pole above the horizon. For when the elevation of the pole is greater than the Sun's greatest declination, the place of the style falls outside the tropics, between the center of the dial and the tropic. When the elevation of the pole equals the Sun's greatest declination, the place of the style falls exactly on the tropic. And when the elevation of the pole is less than the Sun's greatest declination, the place of the style falls between the tropic and

[Margin: When the place of the style on horizontal dials falls outside the tropics; and when exactly on the tropic; or within the tropics; or, finally, on the equinoctial line itself.]

Diagram. The first of the two dials promised in the scholium: a horizontal dial constructed for the northern latitude of $66^{\circ} 30'$. It is a broad fan of hour-lines. A dense sheaf of straight lines (the hours from noon and midnight) radiates from a focal point low in the middle of the figure, while a second set of dashed lines (the hours from sunrise and sunset) sweeps across; the two families intersect throughout. The hour-numbers run around the border of the figure.

- Top row (left to right): 18, 19, 20, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5
- Down the left margin (top to bottom): 17, 16, 15, 14, 13, 12, 11, 10, 9
- Down the right margin (top to bottom): 6, 7, 8, 9, 10, 11, 12, 13, 14
- Along the lower border (left to right): 22, 21, 20, 19, 18, 17, 16, and a central cluster 8, 7, 6, 5, 4, 3, [?], with 12 at the very bottom-center

The lines drawn as dashes are the hours from sunrise and sunset, which touch the arc of Cancer at the points where the noon/midnight lines cut it (as the accompanying text explains). Given the density of overlaid lines, several peripheral hour-numbers are read with low confidence

the equinoctial line. And when, finally, the elevation of the pole is nil — as in a right sphere — the style is to be set on the equinoctial line itself. All of this is plainly gathered from the analemma that we constructed in Proposition 2 of Book 2. For since in that figure the elevation of the pole AF is greater than the Sun's greatest declination, it comes about that the ray of Cancer ☊, sent out through the center D, falls upon the meridian line RO at the point K, beyond the place of the style G — so that the point G lies between H (the center of the dial) and K (the point of Cancer ☊). The place of the style therefore lies entirely outside the tropics. But if the elevation of the pole AF were equal to the Sun's greatest declination, so that the diameter of the parallel of Cancer ☊ cut the meridian at the point A, then the ray of Cancer ☊, cast from A through the center D, would not differ from the diameter of the vertical circle; and accordingly the place of the style would be at

Diagram. The second dial: a polar dial constructed for the northern latitude of $69^{\circ} 48'$. Its lines form a symmetric bow-tie (hourglass) shape, converging toward a narrow waist at the center and spreading toward the upper and lower edges. Zodiac sorts mark the tropic arcs at the four corners.

- Upper-left corner: zodiac sort, Cancer ☊ [?] (worn; the tropic-of-Cancer arc, per the text)
- Upper-right corner: zodiac sort, Capricorn ☋ [?] (worn)
- Lower-left and lower-right corners: matching zodiac sorts [?] (worn)

- Hour-numbers along the upper edge (left to right): 6, 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5
- Hour-numbers repeated along the lower edge (left to right): 7, 8, 9, 10, 11, 12, 1, 2, 3, 4, 5
- Numbers in the central band around the waist: 10, 11, 12, 1, 2, 3

In this dial the hours from sunrise and sunset do not reach the tropics but touch the parallels of Gemini Υ and Sagittarius $\times$, which at this latitude serve as the greatest parallels of things always visible and always hidden. Several worn zodiac sorts are read from context and flagged

the point G, where the ray of Cancer ♋ would cut the meridian line. But if the elevation of the pole were less than the Sun's greatest declination, so that the diameter of the parallel of Cancer ♋ cut the meridian between the vertex A and the pole of the world E, then it plainly follows that the ray of Cancer ♋ , passing through the center D, would fall between H (the center of the dial) and G (the place of the style); so that the place of the style would then lie between the tropic of Cancer ♋ and the equinoctial line. And if, finally, the elevation of the pole were nil, so that the diameter of the equator did not differ from the diameter of the vertical circle — who does not see that the place of the style would then fall on that point of the meridian line where the equator meets the plane of the dial?

All of this holds for other dials as well, according as the elevation of the pole above the great circles to which their planes are parallel is greater than, equal to, or less than the Sun's greatest declination — or, finally, nil. Thus you see that in the dials we described in Books 2 and 3 — namely the horizontal, the vertical, the equinoctial, the declining-from-the-vertical, the declining-from-the-horizon, the inclined-to-the-horizon, and the simultaneously declining-and-inclined dial (in which, at Proposition 38 of the preceding book, we laid out the arcs of the signs) — the place of the style is set outside the tropics, because the elevation of the pole above the great circles to which those dials are parallel exceeds the Sun's greatest declination. In the horizontal dial contained in the scholium to Proposition 2 of Book 2, however, the place of the style falls within the tropic,

[Margin: How the same may be known for other dials.]

because there the elevation of the pole above that horizon is less than the Sun's greatest declination. Finally, in the meridian dial, the polar dial, and the simultaneously declining-and-inclined dial that we constructed in the third place at Proposition 37 of the preceding book — and in the dials of a right sphere composed in this present book — the style falls upon the equinoctial line itself,

because there is no elevation of the pole above the great circles to which those dials are parallel.

PROBLEMA 12. PROPOS. 12.

To set the gnomon of any dial in its proper place, so that it stands square to the plane of the dial; and to restore its vertex to its own seat whenever it has been bent away from it.

[Margin: By what method the style is to be set square to the plane of the dial.]

Let AB be the style, or gnomon, of any dial, its foot at the point A, where it must be fixed to the dial in such a way that it stands square to the dial's plane, and so on. From A, the foot of the style, describe a circle CDE of any size in the plane of the dial, and on it choose any three points C, D, E. Then at A erect the style AB, and move its vertex B this way and that until, with the aid of a compass, three equal straight lines are found reaching from the points C, D, E to the vertex B. I then say that the gnomon AB stands square to the plane of the dial in which the circle CDE lies. For if it be denied to stand square, suppose that from the vertex B another line BF is let fall perpendicular to the plane of the dial, if that be possible, meeting it at a point F distinct from A. Drawing the straight lines CF, DF, EF, then by Definition 3 of Book 11 of Euclid the line BF will be perpendicular to all of them, and accordingly the triangles BFC, BFD, BFE will be right-angled. [Margin: 47 of the first (book).] Hence the square on BC equals the squares on BF and FC; the square on BD equals the squares on BF and FD; and the square on BE equals the squares on BF and FE. But since the squares of the three lines BC, BD, BE — which are posited equal — are themselves equal, the squares on BF and FC will equal both the squares on BF and FD and the squares on BF and FE. Taking away, then, the common square on the line BF, there remain the equal squares of the lines FC, FD, FE; and therefore those lines FC, FD, FE are themselves equal.

[Margin: 9 of the third (book).] The point F, therefore, would be the center of the circle CDE — which is absurd, since A is the center of that circle. It is therefore not the case that the perpendicular let fall from B, the vertex of the style, to the plane of the dial falls at a point F distinct from A; and accordingly it falls at A. Hence the line AB, erected so that the three lines CB, DB, EB are equal, will stand perpendicular to the proposed plane.

Diagram. A circle CDE with its center A, illustrating the perpendicularity proof.

- B: vertex of the style, at the top, joined by lines to C, D, E, A, F
- E: on the circle, to the right

- A: center of the circle
- F: an interior point (the supposed foot of the perpendicular), below A
- C: on the circle, lower left
- D: on the circle, at the bottom

Diagram. A right triangle GHI, used to measure the compass-opening.

- H: upper vertex (right angle at G)
- I: lower-left vertex
- G: lower-right vertex; side GH equals the style, side GI equals the semidiameter AE

Now, so that we may know at what distance the legs of the compass ought to stand apart, so that — one leg placed on any of three points of the circumference CDE, the other reaching the vertex B of the style — we may reach square to the dial's plane, we shall describe a right triangle GHI whose side GH equals the style and whose side GI equals the semidiameter AE. For the length of the line IH will equal each of the distances between the points C, D, E and the vertex B, provided the style stands square to the plane of the dial. Since the two sides GH, GI of triangle GHI equal the two sides AB, AE of triangle ABE, and contain equal angles at G and A (both being right angles), [Margin: 4 of the first (book).] the bases IH and EB will also be equal to each other; and by the same reasoning IH will be shown equal to the lines DB and CB. And so, to fix the style AB at the point A square to the dial's plane, it is necessary that, with one leg of the compass placed on any point of the circumference CDE, the other reach the vertex B, if the distance enclosed between the compass legs equals the line IH — as is left clear from what has been demonstrated.

Now, further, whenever the vertex of the gnomon is bent and moved from its own place, we shall restore it thus. From A, the foot of the style, draw on the dial any line AB — either equal to the style, as in the earlier, lower triangle, or greater, as in the later one. Through B, draw CD perpendicular to AB, and set the lines BC, BD equal to AB. Then from AC cut off AE equal to the style, and cut off DF equal to the distance DE. I say that if the compass is opened to the distance DF, and the vertex of the gnomon is moved this way and that until, with one leg of the compass placed on the points C and D, the other reaches the vertex, then the vertex of the gnomon has been restored, so that the style once more stands square to the plane of the dial at the point A. For suppose in A the style AG standing square to the dial's plane, and join the lines DE, DG, GC. Since the sides BA, BC are equal, [Margin: 5 of the first (book).] the angles A and C will be equal. B

being a right angle, the angle BAC will therefore be half a right angle, [Margin: 32 of the first (book).] since all three angles of triangle ABC equal two right angles. By the same reasoning the angle BAD will be shown to be half a right angle, and accordingly the whole angle CAD will be right; [Margin: 4 of the first (book).] and moreover the lines AC, AD will be equal to each other, on account of the triangles ABC, ABD, in which the two sides BA, BC equal the two sides BA, BD and contain equal angles at B, namely right angles. Again, since GA is perpendicular to the plane of the dial, in which the triangle ACD lies, the angles GAC, GAD will also be right, by Definition 3 of Book 11 of Euclid. And so, since the two sides DA, AE of triangle DAE equal the two sides DA, AG of triangle DAG and contain equal angles at A (both right), [Margin: 4 of the first (book).] the bases DE, DG will also be equal to each other. In the same way we shall show the lines DE and CG equal to each other, on account of the triangles DAE, CAG, whose two sides DA, AE equal the two sides CA, AG and contain equal angles DAE, CAG, namely right angles. Since, then, the line DE — and therefore the line DF, which was taken equal to DE — equals both the line DG and the line CG, both of which are drawn from D and C to the vertex G of the style at

Diagram. The lower group of figures illustrating the restoration of the gnomon's vertex: two triangle-constructions (the "earlier, lower" triangle and the "later" one), drawn with solid and dashed lines. [Margin: 9 of the third (book).] The readable labels along the top edge run C, F, B, H, D; with I marked on the interior, E toward the middle, and the apex-points G and A at the foot. Because the two constructions overlap and share letters, individual point positions are read with limited confidence

- Top edge (left to right): C, F, B, H, D
- Interior / middle: I (twice), E
- Foot: G, A

[Body heading, per page: "b 9. tertij." appears as a margin citation, not a section heading]

right angles with the plane of the dial's face — it is necessary that, with one leg of the compass set on both D and C, we reach the vertex G of the style, if the space enclosed between the compass legs equals the line DF, and if the style itself makes right angles with the dial's plane, as is clear from what has been demonstrated. We have therefore set the gnomon of every dial in its proper place, and so on. Which was to be done.

SCHOLIUM.

On dials that do not decline — such as the horizontal, vertical, polar, equinoctial, and inclined-to-the-horizon dials — the line AB of the preceding triangle is to be cut off from the meridian line, as you see done in the horizontal dial at Proposition 4 of Book 2, where the vertical circles and the parallels of the horizon are drawn. On the others, however, which do decline, the same line AB is to be taken on the line of the style, as you see done in the dial declining from the vertical at Proposition 6 of the preceding book, which contains the meridian circles. This, however, is not necessary. For the line AB can be drawn from the place of the gnomon in any way whatever, and the aforesaid triangle constructed. Thus you see, in the Italian dial declining from the vertical at Proposition 10 of the preceding book, that a triangle of this kind is made in such a way that the line AB, dividing the base of the triangle into two equal parts, is greater than the style and is taken outside the line of the style. In the same way the said triangle can be described on any dial whatever. And if ever the place of the gnomon is set within the tropics — as happens on the polar dial and some others, as we taught above in the scholium to the preceding proposition — then the sides AC, AD of the aforesaid triangle, since they intersect the lines of the dial, may be erased, and the base CD alone left; or at least only those segments of the sides that fall outside the tropics may be drawn. An example of this you have in the polar dial of the ascending signs at Proposition 45 of Book 2. It will also be possible, if one wishes, to transfer the distance DF from C as far as H. Likewise, on the later triangle, in which the line AB is greater than the style, one may cut off a line BI equal to the style, so that if ever the style is lost or broken, we may supply one equal to it.

[Margin: What is to be observed in various dials in constructing the triangle by which the style is restored to its proper position.]

PROBLEMA 13. PROPOS. 13.

Given a dial, together with its style and the position of the style: to determine the elevation of the pole for which it was built, and what declination it has from the vertical, and what inclination to the horizon (if it is a declining or an inclined dial), as well as how great the elevation of the pole is above the declining or inclined plane itself. And conversely: given a dial together with the elevation of the pole for which it was constructed, and its inclination to the horizon (if it is inclined), to investigate the length of the style and its position, together with the declination from the vertical (if the

dial is a declining one), and the elevation of the pole above the declining or inclined plane itself.

It frequently happens that it is unknown for what elevation of the pole some dial presented to us was constructed, or how large its style ought to be (if perhaps it has been lost or broken), and in what place it must be set so as to indicate the hours. We shall determine both of these by the following method.

[Margin: By what method, on a horizontal dial, from a given style and its position, one may investigate the elevation of the pole for which it was built.]

Let there be, first, a horizontal dial A, on which BC is the equinoctial line and DE the meridian, and FG the style, to be set at the point F. From F, the foot of the style, raise FH perpendicular to the meridian line, equal to the style FG. Draw the line IH from the point I (where the meridian line and the equinoctial cross each other) to H; and at H raise HD perpendicular to IH, cutting the meridian line at D. I say that IDH is the angle of the elevation of the pole for which the dial was built. For since, as is established from what was demonstrated in Proposition 1 of Book 2, the line HI is

Diagram. The principal construction for Problema 13: the horizontal dial A with its meridian and equinoctial lines and the perpendiculars forming the pole-altitude angle IDH.

- A: the horizontal dial (large label at left of the figure)
- K: upper-left terminus of a dashed line
- D: at the top, on the meridian line
- G: on the axis-of-the-world line
- H: vertex of the style / center point where perpendiculars meet
- F: foot of the style (below G)
- I: where the meridian and equinoctial lines cross (below F)
- B: lower-left terminus of the equinoctial line
- C: lower-right terminus of the equinoctial line
- An arc is drawn about H

Diagram. A companion figure showing lines radiating star-fashion through a common center A.

- B: large label at the foot of the figure
- A: the central point through which the lines pass
- F: upper-right terminus

- G: at the top
- D: to the left
- E: to the right
- C: lower-right terminus

Diagram. A figure lettered C: a triangle with meridian and equinoctial lines and a dashed line to F.

- C: large label at the left
- E: upper-left terminus
- D: at the top center (center of the dial, where the hour-lines meet)
- F: to the right (terminus of a dashed line)
- A: at the lower left
- B: at the lower center
- A: at the lower right

Diagram. A figure lettered D: a semicircle with its diameter and interior lines.

- D: large label within the semicircle
- C: upper-left, on the diameter
- A: upper-right, on the diameter
- E: to the right
- F: interior, left of center
- B: interior center

[Margin: By what method, on a horizontal dial, from a given style and its position, one may investigate the elevation of the pole for which it was built.]

the common section of the equator and the meridian (if the triangle DHI is set square to the plane of the dial — that is, in the plane of the meridian); and since the axis of the world, passing through the vertex H of the style, stands square to the plane of the equator, and hence, by Definition 3 of Book 11 of Euclid, is perpendicular to HI: therefore HD, which is drawn perpendicular to HI, will be the axis of the world, meeting the meridian line and the plane of the dial at D. IDH, then, is the angle of the elevation of the pole above the horizon — the very angle that the axis of the world makes with the meridian line in the plane of the horizontal dial.

But if a point D is given on the meridian line, the center of the dial, where all the lines of the hours reckoned from noon and from midnight meet, we shall more easily find the same elevation of the pole by drawing a straight line DH from D to

H. For DH will again be the axis of the world (since it is drawn through the center of the world H and through D, the center of the dial), and HDI will be the angle of the elevation of the pole.

Now suppose the elevation of the pole for which the dial was built is known, but the style is unknown, as also is its position, which we shall investigate as follows. About the line DI, drawn between the center of the dial and the equinoctial line,

[Margin: By what method the style and its position may be described]

[Margin: In a horizontal dial, how to inquire, from the given elevation of the pole for which the dial was built, [into the style and its position].]

we shall describe the semicircle DHI; and from D, the center of the dial, we shall draw out the axis of the world DH, making with the meridian line the angle HDI of the pole's elevation, and let it cut the circle at H. For the perpendicular HF dropped from H to the meridian line will be the length of the style, and its seat will be at the point F.

For when the straight line HI is drawn, the angle DHI will be right, being in a semicircle. [Margin: a. *Elements* III.31.] The straight line HI will therefore be the common section of the meridian and the equator, provided the triangle DHI is set in the plane of the meridian — that is, perpendicular to the plane of the dial. And since the axis of the world meets the equator at the center of the world, H will be the center of the world. Therefore HF, the perpendicular falling from the center of the world onto the meridian line, will be the style, and its location will be F.

We shall obtain the same result if from I, a point on the equinoctial line, we draw the straight line IH, making with the meridian line the angle HID equal to the complement of the pole's elevation, and let it cut the same circle at H. For again, the perpendicular HF dropped from H to the meridian line will be the length of the style, and its location the point F. For when the straight line HD is drawn, the angle IHD in the semicircle will be right; [Margin: b. *Elements* III.31.] and consequently, in the triangle DHI, the remaining angle HDI will be equal to the pole's elevation. Therefore, as before, HF will be the style, to be erected at F.

But if D, the center of the dial, is not given, we shall carry out the proposed task as follows. If the dial is an astronomical one containing the hours reckoned from noon and from midnight, we shall extend the line KD of the 6th hour — or BD of the 5th or 7th, or finally of any other hour whatever — so that it cuts the meridian line at D. For D will be the center of the dial, since all the hour-lines

reckoned from noon and midnight cut the meridian line at the center of the dial, as was demonstrated in the corollary to Proposition 21 of Book 1.

But if it is a Babylonian or Italian dial, embracing the hours reckoned from sunrise or sunset, we shall extend two hours reckoned from sunrise or sunset that cut the 6th hour from noon or midnight at one and the same point — as is clear from the table of the 6th hour-line from noon or midnight given in the scholium to Proposition 20 of Book 1. Such are the line Bk of the 23rd hour from sunset and the line CK of the 13th hour from sunset, which cut each other at K. If, then, through K we draw KD parallel to the equinoctial line, cutting the meridian line at D, then KD will be the line of the 6th hour from noon or midnight — since a line of this kind passes through the point K, as is evident from the said table, and is parallel to the equinoctial line in a horizontal dial, as was shown in the scholium to Proposition 22 of Book 1. Wherefore D will be the center of the dial; and so, if we describe a semicircle about DI, we shall find the style and its seat, just as before.

The same line of the 6th hour from noon or midnight passes through the common point of the 22nd and 14th hours from sunrise or sunset; likewise of the 21st and 15th, or the 20th and 16th, or the 1st and 11th, and so on, as is clear from the same table of the 6th hour-line from noon or midnight. Hence, to be certain that the line KD drawn parallel to the equinoctial line truly indicates the 6th hour from noon or midnight, we shall have to find two, three, four, or even more points from the aforesaid table, through which the line of the 6th hour from noon or midnight is to be drawn. For if a line drawn through one of these parallel to the equinoctial passes through the remaining points, then beyond doubt the line of the 6th hour from noon or midnight will have been drawn most accurately, and consequently the center of the dial will also have been correctly found.

If perchance the line of the 6th hour from noon or midnight cannot conveniently be drawn on the dial, we shall look — in any other table of some hour-line from noon or midnight given in the same scholium to Proposition 20 of Book 1 — for other points through which that hour-line from noon or midnight ought to be drawn. For once that line is drawn, the meridian line will again be cut at the center of the dial. For example, if we wish to draw the line of the 5th hour from noon or midnight, we shall extend the 22nd and 12th hours from sunrise or sunset, as well as the 16th and 18th, or the 3rd and 7th, and so on. For through the points where such hours mutually divide one another, the 5th hour from noon or midnight must be drawn, as is clear from the table of the 5th hour-line from noon or midnight given in the scholium to Proposition 20 of Book 1.

[Margin: By what method, from the position of the style, its length is to be investigated, together with the center of a horizontal dial and the elevation of the pole for which the dial was built.]

In no other way — if only the position F of the style is given, without its length, the pole's elevation, and the center of the dial — shall we track down both the center of the dial, and the elevation of the pole, and the length of the style. For once the line of the 6th hour — or of any other hour from noon or midnight — has been found, as we have just taught, then where it cuts the meridian line, there will be the center D of the dial. This we shall find more easily if the dial is an astronomical one; for any hour-line of it, when extended, will cut the meridian line at D, the center of the dial. Then, having described a semicircle about DI, if from F, the position of the style, a perpendicular FH is drawn to the meridian line, cutting the circumference at H, then FH will be the magnitude of the style; and the straight line DH, when drawn, will give the angle HDI of the pole's elevation. For when the straight line HI is drawn, the angle DHI in the semicircle will be right. [Margin: c. *Elements* III.31.]

Since, then, the style at the center of the world meets the axis and the common section of the equator and the meridian, where these intersect one another at right angles — as is clear from what has been demonstrated — H will be the center of the world; FH the style; DH the axis of the world; and HI the common section of the equator and the meridian. For if FH is said to be not the style, but some line longer or shorter than FH, then the axis will meet the common section of the equator and the meridian either below H on the style or above it; and so the angle will become either obtuse or acute — greater or less, that is, than the right angle DHI — which is absurd.

Diagram. The first of four lettered figures grouped at the head of the page: the horizontal-dial construction with the semicircle DHI, the style, the meridian line, and the axis of the world.

- K: top-left, terminus of a dashed line running down to the right
- D: top center, apex of the vertical line (center of the dial)
- G: just below and left of D
- H: center, on the circle
- F: center, below G (seat of the style)
- I: below F, on the vertical/meridian line
- B: lower-left terminus
- C: lower center-right terminus

- A: large label filling the left of the figure

Diagram. The second figure: a star of hour-lines radiating from a central point A, with a horizontal line D–A–E through the center.

- G: at the top
- F: upper right
- E: right terminus of the horizontal line
- D: left terminus of the horizontal line
- C: lower right
- A: the central point where the lines meet
- B: large label at the lower center

Diagram. The third figure: a parallelogram-shaped construction with an upper and a lower horizontal line, interior diagonals, and a dashed line to F. Several interior labels are faint.

- E: upper-left
- D: upper center, on the top line
- C: large label at the left
- F: center-right (terminus of a dashed diagonal)
- A: lower-left terminus
- B: lower center
- A [?]: a second label at the lower right (may belong to the adjacent figure D)

Diagram. The fourth figure: a semicircle standing on a horizontal diameter, with interior construction lines. Several interior labels are faint.

- C: upper-left, on the diameter
- A: upper-right, on the diameter
- E: to the right
- F: interior, left of center
- B: interior center
- D: large label within the semicircle

Next, let A be a vertical dial, the same as the horizontal one, in which again the equinoctial line is BC and the meridian DE,

[Margin: How, in a vertical dial, it may be known — from the given style and its position — for what elevation of the pole it was built.]

and the style FG, whose seat is the point F, at which the meridian line intersects the horizontal line (if such has been drawn). Let there be erected from F, perpendicular to the meridian line, the line FH equal to the style FG, and let the straight line IH be drawn. I say that DIH is the angle of the pole's elevation above the horizon.

For let the triangle FHI be understood to be revolved about the meridian line until it stands perpendicular to the plane of the dial and is set in the plane of the meridian. And since the equator passes through I and through H, the center of the world, HI will be the common section of the equator and the meridian. Now the equator makes with the vertical the angle of the pole's elevation, because the arc of the meridian intercepted between the vertical and the equator is equal to the arc of the pole's elevation, as we showed on the sphere when treating of the horizon. Therefore DIH will be the angle of the pole's elevation above the horizon, for which the vertical dial was built.

But if from D, the center of the dial (which we shall find, if it is not marked, as we have just taught in the case of the horizontal dial), we draw the straight line DH, then HDI will be the angle of the complement of the pole's elevation above the horizon; and consequently, in the right triangle DFH, the remaining angle DHF will be the angle of the pole's elevation. For DH will be the axis of the world, since it is extended through D, the center of the dial, and H, the center of the world; and therefore FDH will be the angle of the complement of the pole's elevation — the angle, namely, that the axis makes with the vertical.

[Margin: By what method, in a vertical dial, the style and its position are to be found from the given elevation of the pole for which the dial was built.]

Now let the elevation of the pole, for which the vertical dial was built, be known, and suppose it is required to find both the magnitude and the position of the style. About the straight line DI — set between the center D of the dial (which, as before, we shall find if it is not given) and the equinoctial line — let the semicircle DHI be described; and from I let the straight line IH go forth, making the angle HID of the pole's elevation above the horizon, and cutting the circumference at H. I say that the perpendicular HF dropped to the meridian line is the magnitude of the style, and its location is at F. For when the straight line HD is drawn, the angle DHI in the semicircle will be right. [Margin: a. *Elements* III.31.]

Wherefore, since IH is the common section of the equator and the meridian — because it makes the angle of the pole's elevation with the meridian line — HD, which meets it, drawn from the center of the dial at right angles, will be the axis

of the world. Since, then, the axis of the world meets the equator at the center of the world, H will be the center of the world; and therefore HF, the perpendicular falling from the center of the world onto the meridian line, will be the style, and its seat the point F.

We shall obtain the same result if from D, the center of the dial, we draw the straight line DH, making with the meridian line the angle HDI of the complement of the pole's elevation, and let it cut the same circle at H. For again, the perpendicular dropped from H to the meridian line will be the length of the style, and its location the point F. For when the straight line HI is drawn, the angle DHI in the semicircle will be right; [Margin: b. *Elements* III.31.] and consequently, in the right triangle DHI, the remaining angle DIH will be equal to the pole's elevation. Therefore, as we showed before, H will be the center of the world, and HF the style, to be placed at F.

[Margin: By what reasoning, from the position of the style in a vertical dial, its length, the elevation of the pole, and the center of the dial are to be investigated.]

By a like method, if only the position F of the style is given, we shall find its length, the pole's elevation, and the center of the dial — just as a little earlier in the horizontal dial. For, drawing (as there) the line of the 6th hour from noon or midnight, or of any other hour from noon or midnight, we shall find the center D of the dial — namely, the point at which any hour whatever from noon or midnight cuts the meridian line. Then, having described a semicircle about DI, we shall erect from F, perpendicular to the meridian line, the line FH, which cuts the circle at H. For FH will be the length of the style; and the straight line IH, when drawn, will give the angle of the pole's elevation above the horizon. For when the straight line HD is drawn, the angle DHI in the semicircle will be right. [Margin: c. *Elements* III.31.]

Wherefore, as we said before in the horizontal dial, H will be the center of the world; DH the axis of the world; and HI the common section of the equator and the meridian — because these three lines meet at the center of the world, and the latter two make a right angle. For if FH is said to be not the style, but some line greater or smaller than FH, then the axis falling from D, the center of the dial, and the common section of the equator and the meridian — drawn from I, the common section of the meridian line and the equinoctial line — will meet on the straight line FH, either below H or above it, at an obtuse or acute angle, that is, one greater or less than the angle DHI; seeing that it is at the vertex of the style that the axis and the common section of the equator and the meridian meet, which is absurd.

[Margin: For what elevations of the pole are made those horizontal and vertical dials, commonly circulated fitted with a magnetic needle, and how this may be known.]

From these things we shall also learn, with little trouble, for what elevations of the pole those dials were made which are commonly circulated, made in Germany or France, and which consist of two flat square tablets joined at a right angle. The lower of these — on which is set an arc fitted with a magnetic needle — holds a horizontal dial; while the other, which is raised above it at right angles, holds a vertical one.

For since dials of this kind are usually astronomical, and have a thread stretched through the centers of the dials to serve as the axis of the world, which indicates the hours: if a right triangle is constructed, one of whose sides about the right angle is equal to the portion of the meridian line in the horizontal dial lying between the thread (or the center of the dial) and the other tablet, and the other side equal to the segment of the meridian line in the vertical dial lying between the thread (or the center of the dial) and the former tablet — then the remaining side, opposite the right angle, will make with the meridian line of the horizontal dial the angle of the pole's elevation for which the dial was built, and with the meridian line of the vertical dial the angle of the complement of the pole's elevation, as is evident.

[Margin: How the proposed problem is to be solved in a meridian dial.]

But if dials of this kind, instead of a thread, have a right triangle in the horizontal dial that can be raised and lowered (as happens in many), such that, when it is raised, the uppermost line opposite the right angle indicates the hours as an axis, we shall construct for it a triangle exactly equal. For the angle corresponding to the lower angle in the horizontal dial will contain the pole's elevation, while the other will contain the complement of the pole's elevation.

Again, let there be given a meridian dial B — whether east-facing or west-facing — in which the equinoctial line is AC, the horizontal DE, and the style AG, whose position is A. The acute angle CAE, which the equinoctial line makes with the horizontal, is equal to the complement of the pole's elevation. Likewise the acute angle EAF, which the line of the 6th hour from noon or midnight (which is drawn through A, the position of the style, perpendicular to the equinoctial line) makes with the equinoctial line, is equal to the pole's elevation.

But the length of the style is always equal to the space on the equinoctial line between A and the point C — through which the 9th hour from midnight and the

3rd hour from sunrise (likewise the 15th hour from sunset) are drawn in the east-facing dial, or the 3rd hour from noon and the 9th hour from sunrise (likewise the 21st hour from sunset) in the west-facing dial. Its seat is the point A, where the equinoctial line, the horizontal, the line of the 6th hour from noon or midnight, and the 12th hour from sunrise or sunset all meet. All of which is clear from what was demonstrated in Book 2.

[Margin: The same problem — how it is to be solved in a polar dial.]

Fourthly, let C be a polar dial, in which the equinoctial line is AB, the meridian BD, the horizontal DE, the style BF, and its

location at B, where the meridian line and the equinoctial intersect. From the point A — where the 3rd hour from noon (or the 9th from midnight, or the 9th from sunrise, or the 21st from sunset, or finally the 3rd from sunrise, or the 15th from sunset) divides the equinoctial line — to the point D, where the meridian line and the horizontal mutually divide each other, let the straight line AD be drawn. For the angle ADB will be equal to the elevation of the pole above the horizon of the given dial, or to its inclination to the horizon.

[Margin: By what reasoning a polar astronomical dial is universal.]

But the length of the style is always equal to the space on the equinoctial line between B and the point A, through which the said hours are drawn, and its location is at B — as is clear from what we demonstrated in Book 2. But if the horizontal line has not been drawn, and the dial contains only the hours from noon and midnight, the dial will show the said hours in every region, provided that — as we taught in Proposition 38 of Book 2 — it is set in its proper position.

[Margin: The explanation of the problem in an equinoctial dial.]

Fifthly, let there be given an equinoctial dial D, in which the meridian line is AB, the horizontal AC, and the style BE, whose location is at B, the center, where all the hours from noon and midnight intersect one another. From B let a perpendicular BF be erected to the meridian line, equal to the style (which indeed, in an astronomical dial, will be the same as the line of the 6th hour from noon or midnight); and let the straight line FA be joined. For AFB will be the angle of the pole's elevation, and BAF the angle of the dial's inclination to the horizon — namely, of the equator's altitude above the horizon.

[Margin: An equinoctial astronomical dial — by what method it is universal.]

The length of the style, moreover — whose location is at the center B, where the hour-lines from noon and midnight meet — will be found from the known elevation of the pole, in this way. From the point A, where the meridian line and the horizontal intersect, let the straight line AF go forth, making the angle BAF equal to the complement of the pole's elevation; and from B, the center, let a perpendicular BF drawn to the meridian line cut AF at F. For BF will be the length of the style, as is evident from what was demonstrated in the preceding book. But if the dial contains only the hours from noon and midnight and lacks the horizontal line, it will indicate the hours everywhere among all peoples, provided that — as we set forth in Proposition 49 of Book 2 — it is placed in its proper position; and the style may be taken of any length whatever, if the circles of the signs [of the zodiac] have not been drawn on the dial.

[Margin: The problem is explained in a dial declining from the vertical.]

Sixthly, let there be set forth a dial A declining from the vertical, as in the following figures. (A dial declines from the vertical when the horizontal line is drawn through the position of the style, as in vertical dials, but is not equidistant from the equinoctial line — as does happen in vertical dials — but cuts it, yet not at the position of the style, as occurs in meridian dials. In it the meridian line is BC, the equinoctial DE, the horizontal DF, the style FG, whose location is at F; the style-line is EF, which — if perchance it has not been drawn — will be obtained if through F, the position of the style, a perpendicular FE is erected to the equinoctial line.) Thus, then, we shall inquire into the elevation of the pole above the horizon, for which the dial was built, and into its declination from the vertical, and into the elevation of the pole above the declining plane itself.

Diagram. The first (leftmost) of three lettered figures for the dial declining from the vertical: a construction of intersecting meridian, equinoctial, and horizontal lines with a semicircle about F and the angle KCM of declination. Many small point-labels; several are faint. (several interior labels uncertain).

- I: far left
- H: top, left of center
- B: top right
- F: center, within a small arc/circle (position of the style)
- D: left, below I
- E: below D
- G: center, below F
- C: right of G

- K: lower-left, with a short dashed line
- M: lower center
- A: large label at the center-right

Diagram. The second (middle) figure: a similar declining-dial construction with meridian, equinoctial, and horizontal lines, a semicircle about F, and long lines descending to K and N. Several labels faint. (several interior labels uncertain).

- H: top-left
- A: top center
- F: center, within a small arc/circle
- L: left
- D [?]: left-center, below H
- E: below D
- G: center
- C: right
- M: right, below C
- K: lower center
- N: bottom center
- B: large label at the upper right

Diagram. The third (rightmost) figure: a horizontal line A–F crossed by a vertical, with a lower line D–E and a semicircle at the foot terminating at B, and a short line to H. (several interior labels uncertain).

- A: on the upper horizontal line, left of the vertical
- F: right terminus of the upper horizontal line
- D: center, on the lower horizontal line
- E: right terminus of the lower line
- I: lower-left, on the semicircle
- G: center, on the semicircle
- H: lower-right, terminus of a short line
- B: at the foot
- C: large label at the upper right

Now from F, the position of the style, we shall erect, perpendicular to the style-line EF, the line FH equal to the style FG; and from B, the center of the dial — namely, where the style-line, when extended, meets the meridian line — through H we shall draw the straight line BH, which cuts the equinoctial line at I. Then

through E, where the style-line cuts the equinoctial line, having drawn the straight line EC parallel to the horizontal line, we shall erect perpendicular to it the line EK equal to the straight line EI, and we shall join the straight line KC. Finally, taking the straight line CL equal to CK, we shall draw the straight line LB.

For the angle BLC will be the elevation of the pole above the horizon; the angle KCM the declination from the vertical; and the angle EBI the elevation of the pole above the declining plane. The dial itself will decline from the south if the center B lies above the horizontal line, but from the north if below; and it will incline toward the east from the south if the equinoctial line falls below the horizontal line on the right-hand side, but toward the west if on the left-hand side. But it will deflect from the north toward the west if the equinoctial line falls below the horizontal line on the left-hand side, and toward the east if on the right-hand side. All of which is plainly gathered from what we demonstrated in Proposition 1 of the preceding book, if one attentively considers the construction of the dial declining from the vertical set forth in that place.

But now let the elevation of the pole above the horizon, for which the dial was built, be given, and suppose it is required to find the length of the style (whose location, in a dial of this kind, is always at the point where the style-line intersects the horizontal), the declination from the vertical, and the elevation of the pole above the declining plane. From B, the center of the dial, where the horizontal line and the style-line meet, let the straight line BL go forth, making with the meridian line the angle CBL of the complement of the pole's elevation above the horizon; and, drawing from the point E — which is common to the style-line and the equinoctial — the straight line EC parallel to the horizontal line, let it cut BL at L and the meridian line at C. Then from E, perpendicular to EC, let there be erected the line EK; and from C, an arc of a circle at the interval [radius] of the straight line CL

Book IV. Continuation of the ongoing problem on the construction of declining and inclined sundials (dial-cases seven, eight, and nine).

let [an arc] be described cutting EK at K, and let the straight line CK be drawn. Finally, on the equinoctial line let the straight line EK be cut off equal to EI; and, the straight line IB having been joined, let there be erected from F, the location of the style, the perpendicular FH to the style-line, cutting IB at H. FH will therefore be the length of the style, to be set up at F; and KCM will be the angle of declination from the vertical; and EBI the angle of the pole's elevation above the declining plane — as is established by what we demonstrated in the construction of this dial in Prop. [?] of the preceding book. Whether the dial declines from the

south, or from the north, toward the east or toward the west, we shall determine as was said before.

[Margin: In a dial that declines from the horizon, the same problem is solved.]

SEVENTH: let the given dial be B, declining from the horizon (a dial will decline from the horizon when the horizontal line is parallel to the meridian line), in which the meridian line is AC; the equinoctial DE; the horizontal DN; the style FG, its location at F; the style-line EF, drawn from F, the location of the style, perpendicular to the equinoctial line, and, when produced, cutting the meridian line at A, the center of the dial. From F, the location of the style, let the perpendicular FH be drawn to the style-line, equal to the style FG; and from A, the center of the dial, let a straight line drawn through H cut the equinoctial line at I. Again, through E — the point where the style-line and the equinoctial cut one another — let the perpendicular EC be erected to the meridian line; and to EC let another perpendicular EK be raised from E, equal to EI itself, and let the straight line KC be joined. Finally, taking CL equal to CK, let the straight line LA be drawn. Then ALC will be the angle of the pole's elevation above the horizon; KCM the angle of inclination to the horizon, that is, of declination from the horizon; and EAI the angle of the pole's elevation above the plane of the dial.

Whether the dial faces west or east we shall easily understand. (I am speaking here only of the upper dials — those, that is, which pertain to the zenith, as I shall also do in the two following kinds — because the lower ones, which of course face the nadir, are less in use, and the teaching handed down for the upper ones can readily be adapted to them.) For if, the dial being set with its center A turned toward the south, the horizontal line lies more to the east than the meridian, it will face west; but toward the east if the horizontal line lies more to the west than the meridian. All of which is clear from what was demonstrated in the construction of a dial of this kind in Prop. 13 of the preceding book.

NOTE: now let there be given the elevation of the pole above the horizon for which the dial was built, and suppose we must trace out the rest. From A, the center of the dial — where the meridian line is cut by the produced style-line — let the straight line AL be sent out, making the angle CAL equal to the complement of the pole's elevation, and let AL cut, at L, the perpendicular EC which is drawn from E to the meridian line. EK having likewise been drawn perpendicular to EC, let there be described from C, at the radius of the straight line CL, an arc of a circle cutting EK at K, and let the straight line CK be drawn. Then on the equinoctial line let EI be taken equal to EK, and let the straight line IA be joined. Finally, either — a perpendicular DF having been drawn from D, the

point where the horizontal line and the equinoctial cut one another, to the horizontal line, cutting the style-line at F — let there be drawn from F the perpendicular FH to the style-line, cutting the straight line AI at H; or — a semicircle AHE having been described about AE, cutting the straight line AI at H — let there be drawn from H the perpendicular HF to the style-line, cutting the style-line at F. For FH will be the length of the style, and its location at F; while KCM will be the angle of declination from the horizon, and EAI the angle of the pole's elevation above the plane of the dial. This is easily gathered from what was demonstrated in Prop. 13 of the preceding book.

[Margin: The proposed problem is treated in a dial inclined to the horizon.]

EIGHTH: let the proposed dial be C, inclined to the horizon (a dial will be inclined to the horizon when the horizontal line is parallel to the equinoctial line, yet does not pass through the location of the style, as usually happens in vertical dials), in which the meridian line is AB; the equinoctial DE; the horizontal AF; the style GH, its location at G. First of all it will have to be considered whether the dial faces the south or the north — that is, whether it is elevated above the horizon on the northern side or on the side of the south. This we shall obtain without great labor from the order of the hours. For if, the dial being turned toward us so that the horizontal line lies above the style, the afternoon hours stand to the left of the meridian line and the forenoon hours to the right (in the astronomical dial the afternoon hours are these — 1, 2, 3, 4, etc.; in the Italian, these — 24, 23, 22, etc.; but in the Babylonian the forenoon hours are these — 1, 2, 3, 4, etc.), then the dial will be inclined toward the parts of the south and will face north. If, on the contrary, the afternoon hours are placed to the right of the meridian line and the forenoon hours to the left, the dial will be inclined to the horizon on the northern side and will face south.

Next it will be necessary to observe whether the plane, elevated on the northern side, has a smaller inclination to the horizon than the pole's elevation above the horizon, or a larger one; likewise whether the plane, elevated on the southern side, has a smaller inclination to the horizon than the complement of the pole's elevation above the horizon — that is, than the elevation of the equator above the horizon — or a larger one. This indeed we shall understand most clearly from those four analemmas set out in Prop. 25 of the preceding book. For dials elevated on the northern side, whose centers fall below the equinoctial line, have an inclination to the horizon smaller than the pole's elevation above the horizon; but those whose centers are found above the equinoctial line, a larger one. And if some dial has no center — which happens precisely when the hour-lines for noon

and midnight are parallel, or the style is placed at the common intersection of the meridian line and the equinoctial — its inclination will be equal to the pole's elevation, and the dial will be the same as a polar one. On the contrary, dials elevated on the southern side, whose centers are contained below the equinoctial line, obtain an inclination greater than the elevation of the equator; but a smaller one, those whose centers stand above the equinoctial line. And if some dial has the seat of its style at the very center of the dial, its inclination to the horizon will be equal to the elevation of the equator, and it will not differ from an equinoctial dial.

THESE things being thus laid down, we shall carry out what is proposed in this way. From G, the location of the style, let the perpendicular GI be erected to the meridian line, equal to the style GH; and from I let straight lines be drawn to the center of the dial B (which will be found, as we said a little before in the horizontal dial) and to the point A, where the meridian line cuts the horizontal. For GBI will be the angle of the pole's elevation above the inclined plane, and GAI the angle of inclination of that same plane to the horizon, as is established by what was demonstrated in Prop. 25 and 26 of the preceding book.

FROM these two [results], and from what we have just written, we shall easily elicit the elevation of the pole for which the proposed dial was built. For if the dial is raised above the horizon on the northern side, and has an inclination smaller than the pole's elevation above the horizon, the found elevation of the pole above the plane must be added to the found inclination of that same plane. For the number produced will give the elevation of the pole above the horizon. But if it has an inclination greater than the pole's elevation above the horizon, the elevation of the pole above the plane must be subtracted from its inclination, so that the elevation of the pole above the horizon remains. But if the dial is elevated on the

southern side, and has an inclination smaller than the elevation of the equator above the horizon — that is, than the complement of the pole's elevation above the horizon — the complement of the found elevation of the pole above the plane must be added to the found inclination. For from this addition the complement of the pole's elevation above the horizon will be had, from which at once the elevation of the pole itself becomes known. But if it has an inclination greater than the elevation of the equator above the horizon — that is, than the complement of the pole's elevation above the horizon — the complement of the found elevation of the pole above the plane must be taken away from the found inclination. For after this subtraction the complement of the pole's elevation

above the horizon will remain, from which again the elevation of the pole itself will be drawn out. All of which is clear from the analemmas discussed in Prop. 25 of the preceding book.

BUT let there now be given the elevation of the pole above the horizon for which the dial is constructed, together with the inclination of the plane above the horizon (for from the pole's elevation above the horizon alone nothing certain can be gathered), and suppose it is required from these to investigate both the elevation of the pole above the plane and the length of the style with its location. First, then, from the order of the hours and the position of the dial's center we shall learn, as we taught a little before, whether the plane of the dial is elevated on the northern or the southern side, and whether its inclination is greater or less than the pole's elevation, or the equator's, above the horizon. For if it is elevated on the northern side and has an inclination smaller than the pole's elevation above the horizon, the inclination must be subtracted from the pole's elevation; but if it has a greater inclination, the pole's elevation must be taken away from the inclination. In either case the elevation of the pole above the plane will remain. But if the dial is elevated on the southern side, and has an inclination smaller than the elevation of the equator, the inclination must be taken away from the elevation of the equator; but if it has a greater inclination, the elevation of the equator must be removed from the inclination. For in this way there will always remain the complement of the pole's elevation above the plane, as is established by the analemmas discussed — from which the elevation of the pole itself will also be known.

From this, quicker than can be said, we shall inquire into the magnitude and location of the style. For, a semicircle BID having been described about the portion BD of the meridian line lying between the center of the dial (which we shall find, as we handed down before in the horizontal dial) and the equinoctial line, we shall draw from B, the center of the dial, the straight line BI, making with BD the angle DBI equal to the pole's elevation above the plane, and let it cut the circumference of the circle at I. For the perpendicular IG dropped from I to BD will give the length of the style, and its location will be at G — as we showed a little before in the horizontal dial.

QVOD [But] if only the location of the style at G be given, we shall find everything else in this way. A semicircle BID having been described as before, we shall erect from G, the location of the style, the perpendicular GI to the meridian line, which cuts the circle at I, and we shall draw the straight lines IB, IA. For GI will be the length of the style; IBG the angle of the pole's elevation above the plane; and IAG

the angle of inclination, as is clear from what has been said. And from this, as before, we shall gather the elevation of the pole above the horizon.

[Margin: Solution of the proposed problem in a dial that both declines and inclines.]

NINTH: let there finally be proposed a dial that both declines from the vertical and is inclined to the horizon (we shall recognize a dial of this kind if the horizontal line is neither parallel to the equinoctial line, as in dials inclined to the horizon, nor to the meridian line, as in dials declining from the horizon, nor cuts the meridian line at right angles, nor has the style placed on the horizontal line, as in dials declining from the vertical), in which the meridian line is E- ζ [?]; the equinoctial GH; the horizontal HM; the style KL, its location at K. Before all else it must here too be considered, as in the preceding dial, whether the proposed dial faces the south or the north — which indeed we shall recognize in the same way. Next, from K, the location of the style, let the perpendicular KM be raised to the horizontal line, and to this another perpendicular KN, equal to the style KL, and let the straight line NM be joined. For KNM will be the angle of inclination of the proposed plane to the horizon. Again, from the location of the style let the perpendicular KI be erected to the index-line [lineam indicis], equal likewise to the style; and through I and the center of the dial q (the style-line will be found if from K a perpendicular KG is drawn to the equinoctial line; and the center of the dial will be the point where the meridian line cuts the style-line) let the straight line I- q -e be drawn, which will be the axis of the world. But if the dial has no center — which happens only when the style-line and the meridian are parallel — a line equidistant from the meridian line and the style-line must be drawn through I to serve as the axis of the world. After this, any point χ being taken on the style-line, let there be

Diagram. A large composite construction occupying the lower half of the page, uncaptioned, for the ninth case — a dial declining from the vertical and at the same time inclined to the horizon. A nearly-horizontal reference line runs across the figure through the junction H (with the points $\text{C}\epsilon$ and A to its left and B to its right on the upper dashed line); above, straight lines converge to an apex C at the top. Through the region below H the several construction lines (meridian, equinoctial, style-line, and the axis of the world) cross in a dense cluster near the center, where a semicircle is drawn. Because the peripheral point-labels are small and several sit amid crossing lines, more than two are uncertain; a focused re-examination is warranted

Readable labels and approximate positions:

- C (top apex): terminus of converging lines
- $\mathbb{C}$ (upper left, on the top dashed line)
- A (upper center, on the top dashed line)
- H (center, below A): junction on the near-horizontal line
- B (upper right, on the top dashed line)
- β [beta] (center-left, on the horizontal line, left of E)
- E (center, on the horizontal line): common point of meridian and style-line region
- M (just below E)
- I (left of center)
- χ [chi] (right of M, below E): a point on the style-line
- Φ [Phi] (right, mid-height)
- N (left, marked N') [?]
- K (center-left): location of the style
- ς [?] (near center, below χ)
- F (below K)
- L (below center)
- ϱ [rho] (lower right): center of the dial [?]
- λ [lambda] (far right, lower) [?]
- G (lower left)
- θ [theta] (center-low)
- Ξ [Xi] (bottom center)
- D (bottom, right of Ξ)

drawn through it the perpendicular ED to the horizontal line, cutting the meridian line at E, the point at which let the angle of inclination of the plane, $\text{DE}\gamma$, be set up with ED; and to $\text{E}\gamma$, from D — a point taken at will on the straight line ED — let the perpendicular $\text{D}\gamma$ be raised. $\text{E}\delta$ having been cut off equal to $\text{E}\gamma$, let two perpendiculars $\text{D}\epsilon$, $\delta\theta$ be drawn to DE through the points D and δ , of which $\text{D}\epsilon$ cuts the meridian at ϵ . Through ϵ likewise let a line parallel to DE cut $\delta\theta$ at θ , and from E through θ let the straight line $\text{E}\theta$ be sent out. For θED will be the angle of declination of the plane from the vertical circle. Furthermore, from α — where the line of the 6th hour from noon or midnight cuts the straight line AB, which is drawn through E parallel to the horizontal line (the 6th hour is to be drawn through ϱ , the center of the dial, and through the point H, where the horizontal line and the equinoctial cut one another; or else a line parallel to the meridian must be drawn through the point H, when the dial has no center) — let

the perpendicular αF be raised to the declination-line $E\theta$. Finally, at AB, at E, let the angle $BE\phi$ be set up — the angle of inclination, downward indeed in planes facing north, but upward in those which look toward the south — and, a perpendicular $\chi\phi$ having been drawn from χ to ED, let the straight line $E\phi$ cut it at the point ϕ . Then, the straight line DE having been produced toward E, let EC be taken equal to $E\phi$; and on AB let $E\beta$, equal to EF, [be taken], and let the straight line βC be joined; and $F\beta C$ will be the angle of the pole's elevation above the horizon.

THE reasoning of this is clear enough from the very construction of dials declining from the vertical and inclined to the horizon. For just as there, from the location of the style, a straight line was drawn parallel to the horizon and taken equal to the style, BC, in Prop. 38 of the preceding book, and through the angle at C set equal to the complement of the inclination the point D was found through which the horizontal line was to be drawn — so here, conversely, from the point M, a straight line MN drawn from the horizontal line to the end of the straight line KN (which is equal to the style) will give the angle KNM equal to the complement of the inclination, and therefore the remaining angle KMN will be the angle of inclination. Again, just as there, from the declination-line $E\theta$, the angle of inclination $DE\gamma$, and the straight lines $\delta\theta$, DE, the meridian line $E\phi$ was found, so here in turn, from the meridian line $E\phi$, the angle of inclination $DE\gamma$, and the straight lines DE, $\delta\theta$, the declination-line $E\theta$ is found. And that $I\phi K$ is the angle of the pole's elevation above the plane of the dial is evident, since the axis of the world passes through the center of the dial and through the top of the style — that is, through the center of the world.

Besides, just as there, a triangle of the pole's elevation above the horizon having been set up, we took on the declination-line the straight line EF equal to $E\beta$, and, a perpendicular $F\alpha$ [?] having been drawn from F to the declination-line, we found on the straight line AB the point α through which the line of the 6th hour from noon or midnight was to be drawn — so here, on the contrary, from the point α on the straight line AB, through which the line of the 6th hour is drawn, the perpendicular αF raised to the declination-line cuts off from the declination-line the straight line EF equal to $E\beta$, of the said triangle $E\beta C$, by whose aid the dial is constructed. Finally, just as there, the straight line $E\phi$ having been drawn — making with AB the angle of inclination of the plane to the horizon, whether downward or upward according as the dial faces north or south — $E\phi$ was cut off equal to EC, of the same triangle $E\beta C$, and, a perpendicular $\phi\chi$ having been drawn from ϕ to the straight line CD, it showed on CD the point through which the style-line is to be carried — so here, on the contrary, from the point χ a perpendicular

$\chi\phi$ having been drawn to CD, and the angle of inclination $BE\phi$ having been set up, it cuts the straight line $E\phi$, [namely] the straight line $\chi\phi$, at ϕ , so that $E\phi$ is equal to EC, of the said triangle $E\beta C$. And so forth.

WHEN it is known whether the dial faces south or north, we shall perceive without difficulty, from the declination-line $E\theta$, whether it also inclines toward the east or the west, according as the declination-line is drawn to the left or to the right of the straight line DE — as we taught should be done at the beginning of Prop. 37 of the preceding book.

BUT let the elevation of the pole above the horizon for which the dial is constructed now be known, together with the inclination of the plane to the horizon (because here too nothing certain can be gathered from the elevation alone), and suppose it is required to investigate the rest. Any point χ being taken on the style-line, and the perpendicular ED being drawn through χ to the horizontal line, we shall find, as a little before, the declination-line $E\theta$ by aid of the meridian line $E\gamma$ and the angle of inclination $DE\gamma$, and so forth. Then, from the now-known declination and the found inclination, let the inclination of the plane to the meridian be investigated by Prop. 27 of Book I; and from this and the pole's elevation above the horizon, by Prop. 28 of the same book and its corollary, [let there be found] the arc of the meridian between the plane and the pole of the world; and finally, by Prop. 29 of the same book, the elevation of the pole above the plane of the dial. For if the angle of this elevation, GqI , be set up — the style-line [terminating] at the center of the dial — and about the portion Gq of the style-line contained between the equinoctial line and the center of the dial a semicircle be described, cutting the axis of the world qI at I, then the perpendicular IK dropped from I to the style-line will be the length of the gnomon, and its location at the point K. As for the axis of the world, it will be found in another way as follows.

FROM the point χ , where the style-line and the straight line DE cut one another, let there be drawn to the straight line DE the perpendicular $\chi\phi$, which the straight line $E\phi$ cuts at ϕ , making with AB the angle of inclination $BE\phi$. Then from the same point χ let the perpendicular χe [?] be raised to the style-line, equal to $\chi\phi$. For the straight line drawn through e and the center of the dial will be the axis of the world, as can plainly be established from what was demonstrated in the construction of these dials. But in planes drawn through the pole of the world, in which the hour-lines for noon and midnight are parallel, the straight line χe will be the length of the style, and its location will be at the point where the style-line

cuts the equinoctial line — as is established from the third figure of Prop. 37 of the preceding book.

IF, however, only the location of the gnomon at the point K be given, we shall inquire into everything else in this way. A semicircle GIQ having been described about the portion Gq of the index-line (which is always drawn from the location of the gnomon perpendicular to the equinoctial line), placed between the equinoctial line and the center of the dial, we shall draw from K, the location of the style, the perpendicular KI to the index-line, which cuts the circle at I. For thus we shall have the length of the style, IK, as we showed in the preceding dials also. And from this we shall trace out everything else, as before, when the length of the style and its location are given.

MOREOVER, this same problem is easy in the right sphere, as is clear from the construction of dials in the said right sphere. A dial will be understood to be constructed for the right sphere when the horizontal line is absent, and the hour-lines are parallel, the middle one of which indicates the 6th hour from noon or midnight. And if the horizontal line is absent [and the hour-lines are] parallel, the proposed dial in the right sphere will be a horizontal one. Therefore, a dial being given, together with its style and the location of the style, we have investigated for what elevation of the pole it was built, and so forth. Which was to be done.

Gnomonics — Liber Quartus (continued)

PROBLEMA 14. PROPOS. 14.

To reduce any dial already laid out to a larger or a smaller form, according to a given size of the style [stylus; the gnomon].

For the sake of the doctrine, let there be set before us the Babylonian dial declining from the horizon, which we described in Prop. 22 of the preceding book, and whose style is A B. It is to be enlarged or reduced in proportion to any given style C D, in such a way that, whatever proportion the style A B has to the style C D, all the lineaments — together with the shadows falling from the style A B onto the corresponding lineaments — shall have the same proportion to the shadows cast by the style C D. In other words: as the style A B stands to the lineaments and shadows of its own dial, so let the style C D stand to the corresponding lineaments and shadows in the dial that is to be constructed.

[Margin: In what way the proposed dial is reduced to a larger or a smaller form, according to a given size of the style.]

FIRST of all, draw off separately, in some plane, two straight lines E F, E G, making at E any angle you please; and parallel to E F draw H I at any distance. On E F take the straight line E K equal to the style A B, and on E G the straight line E L equal to the style C D; join K L, cutting H I at M; and finally, through E draw a line parallel to K L, namely E H, cutting H I at H. In this way a figure will be prepared by which the proposed dial can, without great labour, be reduced to a larger or a smaller figure.

NEXT, in the proposed dial, through A, the location of the style, draw two hidden straight lines A P, A a, cutting one another at right angles at A. In dials that stand square to the horizon, let one of these be the same as the horizontal line drawn through the location of the style, and the other perpendicular to the horizon. But in inclined dials, let one be parallel to the horizontal line, and the other accordingly be the common section of the plane of the dial and of the vertical circle that measures its inclination to the horizon. In the same way, on the wall or on some plane on which the dial is to be enlarged or reduced, choose C, the location of the style C D, and through C likewise draw two hidden straight lines cutting one another at right angles at C. Let one of them correspond to the horizontal line of the proposed dial (or at least to the line parallel to that horizontal line), and the other to the other line. That is: if the proposed dial stands square to the horizon, and a wall or some stable and firm plane is offered on which a larger or smaller dial is to be drawn, let one of the said lines drawn through the location of the style C be equidistant from the horizon and be the horizontal line, while the other is perpendicular to the horizon. But if the dial offered is not square to the horizon, then one of the said lines on the firm and stable wall or plane is indeed to be equidistant from the horizon, while the other is to be the common section of that plane and of the vertical circle which measures the inclination of the plane to the horizon. Of these lines, in horizontal dials, where the horizontal line cannot be drawn, let one be the meridian line and the other equidistant from the equinoctial line. Then, with a circle P O of any magnitude described from A, let a circle Q R equal to it be described from the centre C.

[Margin: a. III.27 · b. I.33 · c. I.30 · d. VI.2 or 4 (references keyed to the superscript letters below).]

WHEN this is done, we shall carry through the enlargement or reduction of the dial as follows. From the centre A, through N — the point of the 6th hour from sunrise, in the tropic of Cancer [☿], say — draw the hidden straight line A N, cutting the circumference of the circle at O; and on the arc P O take an equal arc

Q R in the other circle, and draw the straight line C R. Then, taking on E F and H I the straight lines E S, H T equal to A N, join the line S T, cutting E G at V; and from C R cut off C X equal to E V. I say that, with the Sun standing at the beginning of Cancer [♋] and on the circle of the 6th hour from sunrise, the shadow of the style C D falls at the point X, just as at that same time the shadow of the style A B falls at the point N. For if the greatest circle, from which both dials are equidistant, be conceived to be some horizon — so that both the style A B and C D is its axis, through which all its vertical circles are drawn, as happens in a horizontal dial — then the straight line A N will be the common section of the plane of the dial and of that vertical circle (with respect to the said horizon) which is then drawn through the centre of the Sun; and thus that same A N will be the length of the shadow at the aforesaid 6th hour from sunrise. Now since that vertical circle keeps the same position in all parallel planes, so that [a] it makes with the straight line C Q (which corresponds to A P) an angle equal to the angle N A P, it follows that, since the angle R C Q is equal to the angle O A P — because the arc Q R is likewise equal to the arc P O — the straight line C R is also the common section of the said vertical circle and of the plane of the dial in which the style C D lies. Therefore the shadow of the style C D will at that time fall on the straight line C R. And because the straight lines E S, H T are equal to one another (each having been taken equal to A N) and parallel, [b] E H, S T will also be equal and parallel. Since, then, K L too is by construction parallel to E H, [c] K L and S T will also be parallel to one another. [d] Therefore it will be: as E K (that is, as A B, the style) is to E S (that is, to A N, the shadow), so E L (that is, C D, the style) is to E V (that is, to C X). And so, since all styles in the same plane, or in parallel planes, are proportional to their shadows — because any style whatever has the same proportion to its shadow as the sine of the Sun's altitude above that plane on which the shadow is received has to the sine of the complement of the same altitude, as we shall demonstrate in the following book — C X will be the length of the shadow of the style C D at that time.

In the same way, if from the centre A, through Y — the point of the 6th hour from sunrise in the tropic of Capricorn [♐] — the hidden straight line A Y be drawn, cutting the circle at Z, and on the arc a Z an equal arc b d be taken in the other circle, and the straight line C d be drawn, we shall show that at that time, with the Sun standing at the beginning of Capricorn [♐], the shadow of the style C D falls on the straight line C D. And if straight lines E F, H I be taken equal to A Y, and the line F I be joined, cutting E G at G, and finally C e be cut off equal to E G, we shall prove, as before, that the shadow of the style C D falls at the point e. Therefore the straight line e X, connecting the 6th hour in the tropic of Cancer [♋] with the 6th hour in the tropic of Capricorn [♐], will be the line of the 6th hour from sunrise.

[Margin: In place of the straight lines drawn out from A and C, a ruler may be used, etc.]

IF, however, we are unwilling to draw out so many straight lines from A and C, we shall be able, in place of them, to apply a ruler to the centre A and to the point we wish to transfer; and, this done, we shall mark the point on the circle cut off by the ruler. Then we shall lay the same ruler to the point C, so that it cuts off from the circle an arc equal to the one it previously cut off in the other circle, and along the edge of the ruler we shall mark the length of the shadow, by pressing in a point on the plane of the dial, etc.

BY no other method shall we transfer onto the dial of the style C D all the points of the proposed dial, if we draw straight lines through them from the centre A, and transfer their distances from A onto the straight lines E F, H I, etc. Thus, if we wish to transfer the point h, where the equinoctial line divides the 4th hour from sunrise, we shall draw the straight line A h, which will cut the circle at I; and, taking the arc Q I equal to the arc P i, we shall draw the straight line C I. Then, taking the straight lines E n, H p equal to the straight line A h, and drawing the line n p cutting E G at q, we shall cut off from the straight line E q a portion equal to C m; and the shadow of the style C D will fall

at the 4th hour from sunrise onto the point m, with the Sun standing at the equator [*Æquator*], just as at the same time the shadow of the gnomon A B falls onto the point h. And so with the rest.

Diagram. A large composite construction filling most of the page, illustrating the enlargement of the declining Babylonian dial. It contains two dial-circles, one above the other, together with an auxiliary parallel-line (shadow-transfer) construction to the right.

- **Upper circle** — the proposed (original) dial, centred at A. A vertical line at the left is the horizon, lettered vertically **HORIZON** (printed "HORI=" / "ZON") with S at its head; along it are the hour-marks **12, 13, 14, 15**. Hour-lines radiate from the centre A, numbered **7, 8, 9, 10, 11** toward the lower centre. A line through the centre bears the vertical legend **Linea Meridiana** ("meridian line"). Labelled points: **B** (upper left), **P** (right, on the meridian/diameter), **N** (right of centre), **o** (far right), **i** (right, below o), **Y, Z, a** (lower centre). At the lower rim are Greek labels **β, γ** [and further small letters that do not resolve — see below].
- **Lower circle** — the enlarged (future) dial, centred at C. Again a vertical horizon line at left lettered **HORIZON**, with hour-marks **12, 13, 14** and the

radiating hour-lines 7, 8, 9, 10, 11; a **Linea Meridiana** legend along a line through C. Labelled points: D (upper left), u, X, R, o, i, s; Greek δ, a, b, e near the lower rim.

- **Right-hand auxiliary construction** — a fan of parallel and crossing straight lines, some dashed, carrying the letters E (top right), H, L, K, M, V, S, F, T, I, G, with the lower-case points n, p, q. This is the E F / E G / H I / K L figure of the text, on which the shadow-lengths E S, H T, S T are marked. Additional small Greek/italic labels (κ, ς, ϖ, ρ, λ, μ, m and others) appear scattered at the foot of the figure but are too small/broken to assign individually.
- **Graduated scale (foot of page)** — two ruled measuring-lines beneath the circles (the lines A F and G M of the following scholium). Upper line: points A, B, C, D, E, F with graduation numbers 4, 8, 12, 16, 20 (a leading division near A also reads "4" [?]). Lower line: points G, H, I, K, L, M with graduation numbers 4, 8, 12, 16, 20.

[Several small peripheral labels — the lower-rim Greek clusters of both circles and the scattered italic letters at the foot of the right-hand construction — cannot be resolved individually at this resolution.]

SED — But in lines that are indeterminate (such as the style-line, the meridian, the horizontal, and the equinoctial), it is enough that we find a single point for each of them in the future dial, according to the doctrine already delivered. For if through that point a straight line be drawn making, with some other line already drawn, an angle equal to the one which the same line makes in the proposed dial with the corresponding line, then that line, whose point has been found, will be described. Thus, since the style-line must pass through the location of the style C, if through C a straight line C g be drawn making with C Q (or with C b) an angle equal to the one which the style-line A f makes with the corresponding line A P (or A a), the style-line will be described. Again, once the point g is found, through which the meridian line is to be drawn (corresponding to the point f), and through g a straight line is drawn making, with the style-line, an angle equal to the angle which the meridian line makes with the style-line at f, the meridian line too will be described. Likewise — since in the given example the straight line A P, produced toward A, cuts the horizontal line at right angles — if on C Q, produced toward C, we find the point through which the horizontal line is to be drawn, and through it erect a perpendicular to C Q, we shall have the horizontal line. Finally, once the point is found on the style-line of the future dial (or on the meridian, or on the horizontal) through which the equinoctial line is to be carried, if through it we draw a straight line making, with the style-line (or the meridian, or the

horizontal), an angle equal to the angle which the equinoctial line in the proposed dial makes with the corresponding style-line (or meridian, or horizontal), the equinoctial line too will be drawn in the future dial. And the meridian and equinoctial lines will cut one another, if we have not erred, at the 6th hour from sunrise.

[Margin: What is to be done concerning indeterminate lines.]

— from sunrise, and 18 from sunset, and 6 in unequal [hours]. Likewise the equinoctial line and the horizontal will intersect one another at the 12th hour from sunrise or sunset, and the 6th from noon or midnight; and, moreover, the style-line always divides the equinoctial line at right angles. All this is established from what has been demonstrated in the laying-out of dials, and is useful if carefully attended to, so that lines of this kind may be correctly drawn in the dial.

[Margin: Another way of finding the lengths of the shadows for a given style.]

WE can also, by another method, investigate the lengths of the shadows which, referred to the style C D, keep the same proportions that the shadows in the proposed dial keep to the style A B. On any straight line $\alpha \beta \gamma$, let $\alpha \beta$ be equal to the style A B, and $\alpha \gamma$ equal to the style C D; and through β and γ draw two lines $\beta \theta$, $\gamma \lambda$ parallel to one another, at any inclination. Then, by aid of the compass, transfer the straight line A N — namely the length of the shadow at the 6th hour from sunrise in the tropic of Cancer [♋] — from α onto the parallel $\beta \theta$ (which is drawn through the end of the proposed style), as far as the point δ ; and join the straight line $\alpha \delta$, cutting the other parallel at ϵ . I say that the straight line $\alpha \epsilon$, transferred by the compass from C onto the shadow-line C R as far as X, gives us X, the point of the 6th hour from sunrise in the tropic of Cancer [♋]. For since, by the corollary of Prop. 4 of Book 6 of Euclid, the triangles $\alpha \beta \delta$, $\alpha \gamma \epsilon$ are similar, [a] it will be: as $\alpha \beta$ (equal to the style A B) is to $\alpha \delta$ (equal to the shadow A N), so $\alpha \gamma$ (equal to the style C D) is to $\alpha \epsilon$ (equal to the shadow C X). In the same way, if we take θ equal to the shadow A Y, and join the straight line $\alpha \theta$ (which cuts the parallel $\gamma \lambda$ at λ), and cut off C e equal to the straight line $\alpha \lambda$, we shall have the point e of the 6th hour from sunrise in the tropic of Capricorn [♐]; and so with the rest.

[Margin: a. VI.4.]

But the former way seems the more convenient; for in this latter one, the longer shadows cut the parallels $\beta \theta$, $\gamma \lambda$ too obliquely, as is plain in the straight line $\alpha \varpi \rho$; whereas in the former, the parallels E H, k M, S T, etc. always divide, in the same way, the straight line E G on which the lengths of the shadows are

terminated. Again, in the latter, when the shadows are shorter — as happens in the hours near the style, in the nearer tropic — it is necessary to draw two other parallels $\beta \mu$, $\gamma \xi$, which come so close to the point a that those short shadows can be transferred onto the parallel $\beta \mu$. So you see that the shadow of the 7th hour from sunrise in the tropic of Cancer [ϙ] has been transferred as far as μ , and that the straight line $a \xi$ is the length of the shadow of the same hour in the future dial. In the former way, however, we have no need of this precaution, as is plain.

[Margin: The lengths of the shadows investigated another way. b. VI.4.]

YET this latter way will be more conveniently arranged if the lengths of the shadows of the proposed dial are transferred onto the straight line $\beta \theta$ not from the point a but from the point β . For then there will be no need to draw two other, closer, parallels. But in that case the shadows taken must be transferred onto the future dial not from the point a but from the point γ , on the line $\gamma \lambda$. Thus, if the shadow transferred were the straight line $\beta \delta$, the shadow to be transferred would be $\gamma \epsilon$; because, on account of the similarity of the triangles, [b] the proportion of the style $a \beta$ to its own shadow $\beta \delta$ is the same as that of the style $a \gamma$ to the straight line $\gamma \epsilon$, etc.

ALTHOUGH, in our example, we have reduced the proposed dial to a larger form, we shall nevertheless, by the same methods, reduce it to a smaller one, if from $E G$ (or from $a \beta$) we cut off a straight line equal to the smaller proposed style. And so we have reduced any dial that has been laid out to a larger or a smaller form, according to a given size of the style. Which was to be done.

SCHOLIVM.

[Margin: By what other way a given dial is to be enlarged or reduced.]

PERHAPS we shall dispatch this same problem more easily in this way. On any straight line take as many straight lines $A B$, $B C$, $C D$, $D E$, $E F$ as you please, equal to the style $A B$ of the proposed dial — yet so that the whole line composed of those segments be not less than the longest shadow falling from the style $A B$; and on some other straight line take just as many straight lines $G H$, $H I$, $I K$, $K L$, $L M$, equal to the style of the future dial, whether it be greater or smaller than the style $A B$. Then, having divided the several parts of the former line into as many equal parts as you wish — (here I would urge that each be first bisected, then each little part bisected again, and so on, as far as can be conveniently done; for the more parts there are, the more accurately will the future dial be described; and this division is the easiest of all) — let the several parts of the latter line be cut into

just as many, with numbers written in, as appears in the preceding figure. Now if from A, the location of the style of the proposed dial, straight lines be drawn through all the points of the dial that are to be transferred, and lines similar to them be drawn out from C, the location of the style of the future dial, by aid of the circle from C; and if the intervals of all the points from A, taken up, be transferred onto the straight line A F, and intervals similar to these, taken up on the straight line G M, be transferred onto the lines drawn out from C, corresponding to the lines drawn out from A — then all the points of the proposed dial will have been transferred onto the future dial. Hence, if the corresponding points be connected by straight lines, the dial for the style C D will be described. For example: the interval A N spans, on the straight line A F, 5 parts and a little more than the third part of one. Therefore, if on the straight line G M there be taken likewise 5 parts and a little more than a third part of one, and these be transferred onto the straight line C R (which corresponds to the line A O, on account of the equal arcs P O, Q R), we shall find the point X, corresponding to the point N; because the same proportion which the straight line A B (equal to the style of the proposed dial) has to $5\frac{1}{3}$ parts, the straight line G H (equal to the style of the future dial) has to $5\frac{1}{3}$ parts. In the same way, the interval A Y contains, on the straight line A F, $7\frac{1}{3}$ parts; so if on the straight line G M just as many parts be taken and referred onto the straight line C d (corresponding to the line A Z), the point e will be found, corresponding to the point Y. The straight line X e, then, joined, will correspond to the straight line N Y. Likewise, since the interval between A and the 14th hour from sunrise in the tropic of Cancer [☿] comprehends exactly 5 parts on the line A F, if just as many parts of the straight line G M be transferred from C onto the shadow-line corresponding to the shadow-line drawn out from A, the point of the 14th hour from sunrise in the tropic of Cancer [☿] of the future dial will be obtained, etc.

[Margin: The problem is solved another way.]

HERE too, in place of the lines drawn out from A and C, we shall be able to use some ruler, as we said above.

WITH Daniele Barbaro, moreover, at the end of his *Perspective* (which he composed in the Italian tongue), we shall arrange this same procedure thus. Let there be made two flat rulers, having parts altogether equal to the parts of the straight lines A F, G M, and at their ends the finest little pins fixed, so that the rulers themselves can be freely turned about them at the points A and G, when set there. Let the superfluous parts be cut away, so that what remains are straight lines proceeding from the little pins as centres, after the manner of what they call

the "line of confidence" [linea fiduciæ] on an astrolabe. After this, the circle described from A having been divided into any number of parts — whether equal or unequal — let the circle described from C be cut into the same parts. And so, placing the former ruler (which contains the divisions of the line A F) at the centre A, and turning it freely until its line of confidence passes through the proposed point of the dial, we shall mark on the circumference of the circle the part cut off by the line of confidence, and the part which the proposed point shows on the same line of confidence. For if we place the latter ruler at the centre C and turn it about until its line of confidence cuts off from the circumference the same part of the circle, we shall mark on the plane the point which corresponds to the aforesaid part on the line of confidence. For this will be the point that is sought. For example: the former ruler, set at the centre A and brought to the point of the 14th hour from sunrise, in the tropic of Cancer [♋], cuts the circle at f, and the point of the said hour falls on its fifth part. If, then, the latter ruler be set at the centre C, and be moved

until its line of confidence cuts off from the circle an arc t u equal to the arc r f, we shall mark on the plane the point that terminates its fifth part, for the 14th hour from sunrise in the tropic of Cancer [♋], and so on.

[Margin: When the style of the given dial bears to the style of the future dial a multiple or submultiple proportion — by which the problem is solved more easily.]

But suppose the style A B of the proposed dial has to the style C D a given proportion that is a multiple or submultiple. This is not an uncommon case in practice — for instance, when a dial made for some wall is to be transferred onto that same wall in a larger format (since, once the dial has been drawn, we are free to choose a style that is double, triple, tenfold, or twentyfold the former, and so on, as the size of the dial to be drawn requires). In such a case we shall construct the larger or smaller dial most readily if we take all the intervals of the proposed dial in the same multiple or submultiple proportion that the style bears to the style. For example, if the proposed dial is to be made ten times larger — so that the style has to the style, and each interval to each interval, a tenfold proportion — we shall transfer the interval A N ten times onto the straight line C R (which corresponds to the straight line A O), so that the straight line C X is tenfold the straight line A N, and so on.

From this it follows that, if the dial has been correctly drawn, then in the dial whose style is ten times greater than the style of the proposed dial the several intervals — such as the hour-intervals on the equinoctial and horizontal lines,

and likewise the segments of the hour-lines between the equinoctial line and the tropics of Cancer [♋] and Capricorn [♏], and so on — must each be tenfold of the corresponding intervals. Therefore, so that such a dial may be drawn as accurately as possible, one must first draw the lines of the style, of the equinoctial, and of the horizon. Then the several intervals of the equinoctial line, between the style-line and the hour-lines of the proposed dial, are to be made tenfold in the future dial — or multiplied according to the proportion of the style, or (where the case requires) diminished — on the equinoctial line, taking the start from the style-line. The same must be done for the intervals on the horizontal line, between the equinoctial line and the hour-lines. For if the hour-lines, connecting the corresponding hour-points found on the tropics, pass through the points thus found on the equinoctial or horizontal line, the dial will be drawn most accurately.

And if it should seem helpful, a line may be drawn in the given dial in any manner whatever — parallel either to the equinoctial line or to the horizontal line, or perpendicular to the horizontal line — provided it conveniently cuts the hour-lines. Then, in the future dial, another line corresponding to it may be drawn, according to the proportion of the style. For if all the intervals of the former line are multiplied according to the given proportion on the latter line, the points will be found through which the hour-lines are to be drawn, and so on.

END OF THE FOURTH BOOK.

GNOMONICS, BOOK FIVE.

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Many people are but little practised in drawing the perpendicular and parallel lines whose use is so frequent in the drawing of dials that we set out in the earlier books. Frightened off by the difficulty of the matter, they despair of ever being able to draw any dial without error, thinking it impossible that, in drawing so many perpendicular lines and hidden parallels as the correct drawing of a dial requires, they should commit no mistake. It has therefore seemed good, in this book, to prescribe a way and a method by which anyone even moderately versed in the table of sines may, by his own effort, compute for himself certain tables. From these he may then draw the hours of any kind whatever, on any proposed plane and to any given magnitude of the style, without any trouble or labor. For I cannot deny that it is very difficult — not to say utterly impossible — for anyone

to fall into no error at all amid so great a multitude of hidden perpendicular and parallel lines, even if he has long and much practised himself in the matter. In the drawing of the hours by tables, however — the tables we are now about to compute — this cannot so easily happen, as will become clear from what follows.

PROBLEMA 1. PROPOS. 1.

To investigate the altitude of the Sun above any great circle of the sphere, for each single hour.

By what method the Sun's altitude above the horizon is to be computed from a given hour, we have taught abundantly in proposition 36 of Book 1. Now we shall inquire into the same above any great circle whatever that differs from the horizon, in this way. First, let the altitude of the pole above the proposed circle be sought, by proposition 29 of Book 1. Then, for this altitude, let the meridian altitude of the Sun and its depression be found — as we shall teach in the scholium of this proposition — and let half of the straight line composed of the sine of the said meridian altitude and the sine of the meridian depression be taken. Finally, let the distance of the Sun from the proper meridian circle of the given great circle be investigated — that is, from the circle drawn through the poles of the world and through the poles of the proposed circle, after the manner of a meridian.

[Margin: By what method the Sun's distance from the proper meridian of any great circle is investigated.]

We shall find it thus. In circles to whose planes the meridian of the horizon is perpendicular — such as the vertical circle properly so called, the circle of the 6th hour from noon or from midnight (to which the polar dial is parallel), the equator, and a circle inclined to the horizon and perpendicular to the meridian — the distance is taken from the meridian, as we taught for the horizon at the beginning of proposition 36 of Book 1; for the meridian of the horizon is also the meridian of those circles, as is evident. But in other circles it must first be ascertained, by proposition 30 of Book 1, how great is the inclination of the proper meridian of each of them to the meridian of the horizon, and in which direction that proper meridian slopes — that is, whether it recedes toward the east or toward the west from the meridian of the horizon. This we shall easily perceive from the position of the pole of the proposed great circle, as we taught in Book 3, propositions 1, 13, and 37 [propof. 1. 13. & 37.]. For once this has been clearly grasped, it will not be difficult to know how great a distance from the proper meridian of the proposed circle the Sun has at any given hour.

For if the proper meridian of the proposed great circle recedes toward the east from the meridian of the horizon in the upper hemisphere, the Sun's distance from noon must be taken, as we wrote at the beginning of proposition 36 of Book 1. To it, if it be after noon, the inclination of the meridians must be added, so that the distance from the proper meridian of the proposed circle may be obtained. But if it be before noon and the distance from noon is less than the inclination of the meridians, the distance from noon must be subtracted from the inclination of the meridians; while if it be greater, the inclination itself must be subtracted from the distance from noon, so that the distance from the proper meridian of the proposed circle may remain. And if it should ever happen that the Sun's distance from noon, before noon, is equal to the inclination of the meridians, the Sun will be situated on the very meridian of the proposed circle. But if the meridian of the proposed circle, in the same upper hemisphere, recedes from the meridian of the horizon toward the west, the procedure must be exactly the opposite. For the Sun's distance from noon, if it be before noon, must be added

to the inclination of the meridians, so that the distance from the meridian of the proposed circle may be obtained; but if the hour given be after noon, and its distance from noon less than the inclination of the meridians, the distance from noon found must be subtracted from the inclination of the meridians; while if it be greater, the inclination itself must be subtracted from the distance from noon, so that the remaining distance of the Sun from the meridian of the proposed circle may be left. But if the proper meridian of the proposed circle recedes eastward, below the horizon, from the meridian of the horizon — as happens on the faces of circles which the Sun would illuminate at midnight if it were not blocked by the earth (such are the lower faces of the position circles, and the northern faces of the vertical circles in regions whose zenith is set on the near side of the tropic of Cancer [☿], and so on) — the Sun's distance from midnight must be taken. To it, if it be before midnight, the inclination of the meridians must be added; but if it be after midnight, and the distance found from midnight is less than the inclination of the meridians, that distance itself must be subtracted from the inclination; while if it be greater, the inclination itself must be subtracted from the distance from midnight, so that the Sun's distance from the proper meridian of the proposed circle below the horizon may be obtained. But if the meridian of the proposed circle recedes westward, in the lower hemisphere, from the meridian of the horizon, the entirely opposite must be done. That this may be made plainer, we shall add examples in the meridian of the horizon and in some other circles — those to which the dials described in Book 3 are parallel, and to which the meridian of the horizon is not perpendicular.

In the meridian circle, then: since the circle of the 6th hour from noon or from midnight stands in place of its meridian, and recedes from it by 90° both toward the east and toward the west, all the distances of the Sun from the sixth hour — reckoned from midnight or from noon — must be computed. Thus at the 9th hour after midnight the Sun is distant from the proper eastern meridian of the meridian circle itself (that is, from the 6th hour from midnight) by 3 hours, i.e. by 45° ; while at the fifth hour after midnight it is distant by 1 hour, or 15° . Likewise at the 2nd hour after noon the Sun will be distant from the proper western meridian of the meridian circle itself (that is, from the 6th hour from noon) by four hours, or 60° . But at $6\frac{1}{2}$ hours from noon it will be distant from the same proper meridian by $\frac{1}{2}$ hour, or $7^\circ 30'$. For on the eastern face of the meridian circle the said distances must be computed from the 6th hour from midnight, but on the western face from the 6th hour from noon.

[Margin: The Sun's distance from the proper meridian of the meridian of the horizon — that is, from the circle of the 6th hour from midnight.]

Next, take a vertical circle which declines from noon toward the east by 30° , to which we drew a parallel dial in proposition 1 of Book 3. Since its meridian recedes from the meridian of the horizon toward the east by about $40^\circ 48'$, as we said there: if to the 2nd hour from noon (that is, to 30° , by which the Sun is distant from the meridian of the horizon in the afternoon) we add $40^\circ 48'$, we shall make $70^\circ 48'$ for the Sun's distance from the meridian of the said vertical circle at that time. But if, at the time of the $10\frac{1}{2}$ th hour after midnight, we subtract the Sun's distance from the meridian of the horizon — that is, $22^\circ 30'$ — from $40^\circ 48'$, there will remain $18^\circ 18'$ for the Sun's distance from the meridian of the same vertical circle at that time. Finally, if at the time of the 7th hour after midnight we take away $40^\circ 48'$ from the distance the Sun has from noon — namely from 105° — there will be left, for the Sun's distance from the meridian of the same vertical circle at that time, $64^\circ 12'$. But on the northern face of the same vertical circle, since the Sun does not illuminate it at noon, but would light it at midnight if the earth did not block its rays, we shall find the Sun's distance from the proper meridian of the said face at the 8th hour after noon in this way. The Sun's distance from midnight then contains 4 hours, that is, 60° , which indeed falls before midnight. And since the proper meridian below the horizon recedes from the meridian of the horizon toward the west by $40^\circ 48'$, and the distance found is greater than this inclination, the inclination must be subtracted from the said distance, so that there remain $19^\circ 12'$ for the Sun's distance from the proper meridian of the said vertical circle on the northern face.

[Margin: The Sun's distance from the proper meridian of the position circle which declines from noon toward the east by 30° .]

Again, take a position circle declining from the horizon by 30° , whose upper face looks toward the west, to which we described a parallel dial in proposition 13 of Book 3. Since its meridian recedes from the meridian of the horizon toward the west by $37^\circ 50'$: if to the 7th hour after midnight (that is, to 105° , by which the Sun is distant from the meridian of the horizon in the forenoon time) we add $37^\circ 50'$, we shall make $142^\circ 50'$ for the Sun's distance from the meridian of the said position circle. But if, at the time of the 17th hour from sunset — the Sun being at the beginning of Cancer [♋], since noon falls at the 16th hour 28 minutes from sunset, at latitude 42° — we take away from $37^\circ 50'$ the Sun's distance from noon, namely 8° , the remaining distance of the Sun from the meridian of the said position circle will be $29^\circ 50'$. And finally, if at the time of the 7th hour from noon we subtract $37^\circ 50'$ from the distance the Sun has from the meridian of the horizon — that is, from 105° — the Sun's distance from the meridian of the same position circle will be left as $67^\circ 10'$.

[Margin: The Sun's distance from the proper meridian of the position circle which recedes from the horizon by 30° , and whose upper face looks toward the west.]

In addition to these, take a great circle which declines from the north toward the east by 60° and is inclined to the horizon by 80° , to which the fifth dial — described in proposition 37 of Book 3 — is parallel. Since its meridian is distant from the meridian of the horizon toward the east by $57^\circ 46'$, as we said there, we shall find the Sun's distances at each hour from the said meridian of the proposed great circle, just as in the preceding cases, and so on.

[Margin: The Sun's distance from the proper meridian of the great circle which declines from the north toward the east by 60° .]

Now, these things having been found: let it be arranged that, as the whole sine is to the versed sine of the Sun's distance from the proper meridian of the proposed circle, so is half the straight line composed of the sine of the meridian altitude and the sine of the meridian depression to a fourth quantity. There will then be found the difference between the sine of the Sun's altitude above the proposed circle at the time of observation and the sine of the meridian altitude. This difference, therefore, being taken away from the sine of the meridian altitude, the sine of the sought altitude of the Sun will remain known.

[Margin: That the Sun's altitude above any proposed great circle may be found for any time whatever.]

But if this difference found should be greater than the sine of the meridian altitude, then the hour whose distance from the meridian we took the versed sine of will not be contained above the proposed face of the circle, but will fall on the other face. Hence if, on the contrary, we then take away the sine of the meridian altitude from the difference found, there will be left the sine of the Sun's altitude above the opposite face of the said great circle, at the given hour. For example: let the Sun's altitude above the southern face of a vertical circle at latitude 42° be sought, for the 5th hour from noon. Since the antarctic pole is elevated above that face by 48° , the complement of this altitude will be 42° ; and if from these there be subtracted the greatest declination of the Sun, $23^\circ 30'$, there will remain the meridian altitude of the beginning of Cancer [♋] above the said face, $18^\circ 30'$ (as will be manifest from the following scholium), whose sine is 31730. And the sine of the meridian depression, $65^\circ 30'$, is 90996. And half of the aggregate of the said sines is 61363. Therefore, if it be arranged that as 100000

[Bottom note: When the difference between the sine of the Sun's altitude and the sine of the meridian altitude is greater than the sine of the meridian altitude, that hour, for which the altitude is investigated, does not fall on the proposed face of the circle, but on the other face.]

(the whole sine [sinus totus]) is to 74119, the versed sine [sinus versus] of the Sun's distance from noon, so is the aforesaid half, 61363, to a fourth quantity — here approximately the number 45481 will be found. Since this is the larger, if the sine of the meridian altitude [altitudo meridiana], 31730, is subtracted from it, the remainder will be the sine of the Sun's altitude above the northern face of the vertical circle, at the 5th hour from noon (or the 7th from midnight), for the parallel of Cancer [♋]: namely 13751, to which correspond $7^\circ 54'$ for the required altitude of the Sun.

Again, let the same altitude be sought for the 11th hour from noon. Let it be arranged that as 100000, the whole sine, is to 196592, the versed sine of the Sun's distance from noon, so is the aforesaid half, 61363, to a fourth quantity; here the number 120634 will be found. If from this the sine of the meridian altitude, 31730, is taken away, there will remain — for the sine of the Sun's altitude above the same opposite face, namely the northern face of the vertical circle — approximately the number 88904, to which are due $62^\circ 45'$ for the required altitude of the Sun.

And so, if the Sun's distances from the meridian of any proposed great circle are taken, above the horizon or below it, for the individual hours, the Sun's altitudes above each of its faces will be found. For when the difference found is subtracted

from the sine of the meridian altitude, the altitudes found will be above the face on which the meridian altitude was taken; but when, conversely, the sine of the meridian altitude is subtracted from the said difference found, they will be above the opposite face, as we said. I would always choose the Sun's distances from the meridian above the horizon, considering carefully upon which face of the plane the Sun's ray falls when the Sun is in the meridian of the proposed circle above the horizon and lies in that parallel whose hourly altitudes are being sought; and also which of the poles is elevated above that same face, so that the meridian altitude of the given parallel may be found — as we shall presently teach in the following scholium.

DEMONSTRATION. The reasoning for finding this altitude of the Sun is to be taken from Prop. 36 of Book 1. For the reasoning here is the same as that for the horizon, since any great circle plays the role of the horizon in some region. As for the Sun's altitude above the face of the circle opposite [to it]: in the second figure of Prop. 36, Book 1, let the straight line TR be conceived as the common section of the meridian and of the great circle above which the Sun's altitude is investigated, so that KT is the sine of the meridian altitude. If, then, the Sun is placed at S, beyond the said circle, so that it lights up the opposite face toward the other pole G, the straight line KN will be found, by the said computation, greater than KT, the sine of the meridian altitude. Hence, when the sine of the meridian altitude KT is removed from the straight line KN found, there will remain NT, the sine of the Sun's altitude above the opposite face of the proposed circle.

In investigating the Sun's altitude we use only that method which we set out in the last place in Prop. 36, Book 1, before [the treatment of] the spherical triangles, because it is the clearest of all and the most transparent, and because it applies to all parallels at any latitude of the pole above the proposed great circle, as we noted there. See nevertheless what we wrote toward the end of the scholium of Prop. 36, Book 1, for the case when the parallel either touches the proposed great circle or stands entirely above it, as it does above the horizon. It will be no trouble to adapt here the other methods that we prescribed in the said proposition, if it seems good.

[Margin: How the Sun's altitude above the meridian of the horizon — whether right or oblique — is to be investigated.]

We shall also explore, in the following way, the Sun's altitude above the meridian — whether of a right, an oblique, or the meridian[s own] horizon; that is, above [the great circle drawn] through the common sections of the equator and of the six-o'clock circle [circulus horae 6] from noon or midnight, or of the horizon, and

through the center of the Sun. Let the horizon be right, or ABCD; its meridian, AEC; the Sun's parallel, whether northern or southern, FG, in which the Sun is at the point H. Through this point and the poles A, C let the hour-circle, or declination-circle [circulus horarius, vel declinationis], AHC be drawn, cutting the equator at K, so that EK is the Sun's distance from the meridian. Let there also be drawn, through B — the pole of the meridian — and the point H, the great circle BHI, so that HI is the arc of the Sun's altitude above the meridian AEC.

Diagram. A circle representing the sphere in projection, filled with a network of great-circle arcs, for the demonstration of the Sun's altitude above the meridian in the right sphere. The outer circle is the right horizon ABCD; the vertical diameter AEC is the meridian; A (top) and C (bottom) are the poles of the world (which lie on the horizon in the right sphere). Several arcs cross the figure, meeting at interior points; because the great circles are drawn as complete curves, most of the interior letters appear at two positions (a front and a back intersection). Readable labels, with their positions:

- A (top of the circle): pole; endpoint of meridian AEC.
- C (bottom of the circle): pole; endpoint of meridian AEC.
- B (left, mid-height): pole of the meridian; endpoint of horizon ABCD.
- D (right, mid-height): endpoint of horizon ABCD.
- E (center): intersection on the meridian.
- F (upper-left) and G (right, upper and lower): endpoints of the Sun's parallel FG.
- H (upper-center, and again lower-center-left): the Sun's position.
- I (upper-right, and again lower-center): on the altitude circle BHI.
- K (center-left): on the equator, marking distance EK.
- L (upper-left, and again lower-left): interior intersection.
- M (upper-center-right, and again lower-center): interior intersection.
- N (lower-right): interior intersection (KN, NT).
- O (lower, between C and N): interior intersection.

Now, since in the right-angled spherical triangle BHK the angle K is right, it will be — by Prop. 19 of Book 4 of Johannes Regiomontanus [Regiomontanus], *On Triangles* [*De Triangulis*], or by Prop. 15 of Book 1 of Geber [Jābir ibn Aflāḥ], or by Prop. 43 of our spherical triangles — that as the sine of the complement of arc HK, of the declination, is to the whole sine, so is the sine of the complement of arc BH, that is, so is the sine of arc HI, of the Sun's altitude above the meridian, to the sine of the complement of arc BK, that is, to the sine of arc EK, of the Sun's

distance from noon or midnight. And by conversion: as the whole sine is to the sine of the complement of the declination, so is the sine of the Sun's distance from noon or midnight to the sine of the Sun's altitude above the meridian. This we shall demonstrate more briefly thus. Since in the spherical triangle AHI, or CHI, the angle I is right, it will be — by Prop. 16 of Book 4 of Johannes Regiomontanus, *On Triangles*, or by Prop. 13 of Book 1 of Geber, or by Prop. 41 of our spherical triangles — that as the whole sine of the right angle I is to the sine of arc AH, or CH, of the complement of the declination, so is the sine of angle A, or C, of the Sun's distance from the meridian, to the sine of arc HI, of the Sun's altitude above the meridian. Therefore, if it be arranged that as the whole sine is to the sine of the complement of the declination, so is the sine of the Sun's distance from noon or midnight to a fourth [quantity], the sine of the Sun's altitude above the meridian will result. For example: with the Sun occupying the beginning of Cancer [♋] or Capricorn [♐], let its altitude above the meridian be sought for the 5th or 7th hour after noon or midnight, so that the Sun's distance from noon or midnight contains five hours, that is, 75° . Let it be arranged that as 100000, the whole sine, is to 91706, the sine of the complement of the declination, so is 96592, the sine of the Sun's distance from noon or midnight, to a fourth [quantity]; this sine will be found to be approximately 88580, whose arc, $62^\circ 21'$, will give the required altitude of the Sun above the meridian.

When the Sun runs along the equator, the Sun's very distance from noon will be its altitude above the meridian, because the equator, drawn through the poles of the meridian, is at right angles to the meridian, by Prop. 15 of Book 1 of Theodosius [Theodosius of Bithynia].

Again, at the 6th hour from noon or midnight, the complement of the declination — AF or CF — will be the Sun's altitude above the same meridian, since the six-o'clock circle from noon or midnight is at right angles to the meridian, by the same Prop. 15 of Book 1 of Theodosius.

By a not dissimilar reasoning we shall investigate the Sun's altitude above the right horizon [Horizon rectus], or the six-o'clock circle from noon or midnight — [the altitude] which is measured by the great circle drawn through the poles of the right horizon (or of the six-o'clock circle from noon or midnight), that is, through the common sections of the meridian and the equator, and through the center of the Sun. Such is the circle EHL, so that the Sun's altitude above the six-o'clock circle is the arc HL, the Sun being at the point H. For since in the spherical triangle EHK the angle K is right, it will be — by Prop. 19 of Book 4 of Johannes Regiomontanus, *On Triangles*, or by Prop. 15 of Book 1 of Geber, or by Prop. 43 of

our spherical triangles — that as the sine of the complement of arc HK, of the declination, is to the whole sine, so is the sine of the complement of arc EH, that is, so is the sine of arc HL, of the Sun's altitude above the right horizon or the six-o'clock circle, to the sine of the complement of arc EK, of the Sun's distance from noon or midnight. And by conversion: as the whole sine is to the sine of the complement of the declination, so is the sine of the complement of the Sun's distance from noon or midnight to the sine of the Sun's altitude above the right horizon or the six-o'clock circle from noon or midnight. This we shall show more briefly as follows. Since in the spherical triangle AHL, or CHL, the angle L is right, it will be — by Prop. 16 of Book 4 of Johannes Regiomontanus, *On Triangles*, or by Prop. 13 of Book 1 of Geber, or by Prop. 41 of our spherical triangles — that as the whole sine of the right angle L is to the sine of arc AH, or CH, of the complement of the declination, so is the sine of angle A, or C, of the complement of the Sun's distance from noon or midnight, to the sine of arc HL, of the Sun's altitude above the right horizon or the six-o'clock circle. Therefore, if it be arranged that as the whole sine is to the sine of the complement of the declination, so is the sine of the complement of the Sun's distance from noon or midnight to a fourth [quantity], the sine of the Sun's altitude above the six-o'clock circle from noon or midnight, or the right horizon, will result.

When the Sun runs along the equator, the complement of the Sun's distance from noon or midnight will be its altitude above the right horizon, or the six-o'clock circle from noon or midnight, because the equator is at right angles to that circle, since it is drawn through its poles.

Again, at noon or midnight, the complement of the declination — AM or CM — will give the Sun's altitude above the same circle, because the meridian too is at right angles to it.

[Margin: How the Sun's altitude above any great circle whatever drawn through the poles of the world is to be found.]

Finally, above a great circle drawn through the poles of the world that is inclined to the meridian — such as the great circle to which the third dial described in Prop. 37 of Book 3 is parallel — we shall inquire into the Sun's altitude in the same way, provided we seek the Sun's distances from the proper meridian of that circle, as we taught above, and, in place of the Sun's distance from noon or midnight, take its distance from the proper meridian of the said circle.

[Margin: It is enough if the Sun's altitudes above the great circles drawn through the poles of the world are computed, while it is in the northern or southern signs.]

It is enough, however, to compute the Sun's altitudes above the great circles drawn through the poles of the world while it is in the northern signs, or in the southern. For at the same hours the Sun has equal altitudes above the meridian — or the six-o'clock circle from noon or midnight, or the right horizon, or indeed any great circle whatever passing through the poles of the world — while it is in opposite parallels. This we shall demonstrate as follows. Since in the triangles AHI, CHI the angles at I are right, and the angles A, C are equal (being the angles of the Sun's distances from the meridian), and they have the sides AH, CH — which are subtended by the right angles — equal (being complements of equal declinations), the arcs HI, HI, which measure the Sun's altitudes above the meridian, will also be equal, by Prop. 21 of Book 1 of Menelaus [Menelaus of Alexandria] in the redaction of Francesco Maurolico [Maurolico], or by Prop. 21 of our spherical triangles. By the same reasoning, since in the triangles AHL, CHL the angles at L are right, and the angles A, C are equal (being the angles of the complements of the Sun's distances from the meridian), and they have the sides AH, CH equal, as we said, the arcs HL, HL, which measure the Sun's altitudes above the right horizon or the six-o'clock circle from noon or midnight, will also be equal, by the same proposition.

[Margin: At any hour the Sun has the same altitude above the meridian of the horizon as it has above the six-o'clock circle from noon or midnight at another hour, which is as far from that circle as the first is from the meridian.]

Moreover, at any hour the Sun has above the meridian the same altitude that it has above the six-o'clock circle from noon or midnight at another hour that is as far from the six-o'clock circle as the first is from the meridian — such as, for example, the 11th or 1st hour from noon or midnight with respect to the meridian, and the 5th or 7th hour from noon or midnight with respect to the six-o'clock circle from noon or midnight, since each is distant from its own circle by one hour, or 15° . This we shall confirm as follows. Let the Sun be placed at N, so that the declination-circle CNA makes with the six-o'clock circle the angle NCO equal to the angle HCI — that is, the Sun's distance at N from the six-o'clock circle equals its distance at H from the meridian. Since, then, in the triangles CHI, CNO the angles at I, O are right, and the angles HCI, NCO are equal, and they have the arcs CH, CN equal (being complements of the same declination), the arcs of altitude HI, NO will also be equal, by the same Prop. 21.

And so, if the Sun's altitudes above the meridian at the individual hours from noon or midnight have been computed, the same altitudes will be assigned to the six-o'clock circle from noon or midnight for the individual hours that are as far from the six-o'clock circle from noon or midnight as those [hours] are from the meridian. Thus the altitudes for the pairs of hours in the prior table set out here, above the meridian circle, are equal to the altitudes for the pairs of hours in the posterior table above the right horizon, or the six-o'clock circle from noon or midnight — and conversely. This too is plainly evident from the computation itself.

Prior table (hours reckoned with respect to the meridian):

6	7	8	9	10	11	12
6	5	4	3	2	1	12

Posterior table (hours reckoned with respect to the six-o'clock circle):

12	1	2	3	4	5	6
12	11	10	9	8	7	6

[Margin: The Sun's altitude above the equator is always equal to the declination of that same [parallel].]

Above the equator there is no need to investigate the Sun's altitudes, because, the Sun being in any parallel whatever, its altitude above the equator throughout the whole day is equal to the declination of that parallel, as is clear.

[Margin: For which hours the Sun's altitudes above each face of every great circle are to be investigated.]

Since, however, any great circle is inclined to the horizon or the meridian and has two faces — one upper and one lower, or one southern and one northern — the [altitudes] to be computed

are the Sun's altitudes above whichever of them, for those hours which are comprehended in the portion of any parallel projecting above that face in the manner of a certain diurnal arc [arcus diurnus]. For only during those [hours] is the Sun raised above the face of the said circle. By what method we may know the hours lying in that portion of the parallel, we shall disclose in the following scholium. But the computation of the altitudes itself will also make it clear. For if the difference found between the sine of the meridian altitude and the sine of the altitude at the time of observation is less than the sine of the meridian altitude,

the hour of the altitude found will lie in the portion of the parallel above the proposed face of the circle; but if it is greater, in the remaining portion above the opposite face, as we wrote a little before. We have therefore investigated the Sun's altitude above every great circle of the sphere for the individual hours. Which was to be done.

SCHOLIUM.

[Margin: How to know whether the arctic pole or the antarctic is elevated above the given plane of the proposed great circle.]

We shall find, by this device, the Sun's meridian altitude above a proposed great circle in an oblique northern sphere. First it must be considered whether the arctic pole is elevated above the plane of the circle, or rather the antarctic. This is determined as follows. In all vertical circles the arctic pole is raised above the northern faces, but the antarctic above the southern faces. In circles of position [circuli positionum], to which dials declining from the horizon are parallel, the arctic pole is elevated above the upper faces, the antarctic above the lower. But in the other circles inclined to the horizon — whether they be at right angles to the meridian, or [themselves] inclined — if their upper faces look toward the north, the arctic pole is raised above the upper faces and the antarctic above the lower. But if they are inclined toward the northern parts, so that their upper faces look toward the south, one must investigate — by the corollary of Prop. 28 of Book 1 — the arc of the meridian intercepted between the horizon, and [between the horizon] and the proposed circle. For if this arc (which necessarily, in planes of this kind, stands above the horizon on the northern side, as we showed in scholium 1 of Prop. 28 of Book 1) is less than the altitude of the pole above the horizon, the arctic pole will be raised above the upper face, and the antarctic above the lower; but if the said arc is greater than the altitude of the pole above the horizon, the contrary happens — for the arctic pole will be raised above the lower face, and the antarctic above the upper; and if, finally, the said arc is found equal to the altitude of the pole above the horizon, there will be no altitude of the pole above the given circle. All of which are plain from the material sphere.

[Margin: By what method the Sun's meridian altitude above any great circle whatever is to be investigated.]

Since, by Prop. 19 of Book 2 of Theodosius, the portions of the parallels of the equator lying between the equator and the pole, and conspicuous above the face of the proposed circle, are greater than semicircles, while those of the rest are smaller, it comes about that the northern parallels have larger portions above the

face above which the arctic pole is elevated, and the rest smaller — as happens on the horizon. The contrary will happen on that face above which the antarctic pole is conspicuous. Therefore, for the face of the circle above which the arctic pole is conspicuous, we shall investigate the meridian altitude, and the meridian depression, from the complement of the altitude of the pole above the given great circle and from the declination of the parallel in which the Sun is — just as on the horizon, as we taught in the scholium of Prop. 35 of Book 1. The same reasoning fits also that face of the circle which has the antarctic pole elevated, provided that for the northern signs the southern be taken, and conversely.

[Margin: By what method it may be known which hours are contained in the portion of any parallel above the given face of the proposed great circle; and hence at what hour that face begins, and ceases, to be illuminated by the Sun.]

That we may understand, however, which hour is contained in the portion of any parallel standing above the face of the given circle in the manner of a diurnal arc — and consequently at what hour that same face begins, and ceases, to be illuminated by the Sun — this is to be inquired by Prop. 34 of Book 1, or rather from the scholium of Prop. 35 of the same book (provided that, in planes above which the antarctic pole is elevated, the southern signs be taken for the northern, and conversely): namely, the quantity of that diurnal arc, as if the face of the proposed circle were some horizon. For if, from that hour at which [the parallel] falls upon the proper meridian of the proposed circle (which indeed will be known from its inclination to the meridian of the horizon), the semidiurnal arc [arcus semidiurnus] found is counted off on both sides in the parallel, the hours contained in the proposed portion, or diurnal arc, will be obtained; and consequently also the hour at which the Sun begins to illuminate, or ceases to illuminate, the proposed face of the circle.

For example: above the southern face of a vertical circle declining from the vertical properly so called by 30° — to which the dial of Prop. 1 of Book 3 is parallel — the antarctic pole is elevated $40^\circ 3'$. Therefore the portion of the parallel of Capricorn [φ] projecting above it, in the manner of a diurnal arc, will contain about 14 h 52 m; and the semidiurnal arc, 7 h 26 m. And since the meridian of that vertical circle departs from the meridian of the horizon toward the east by $40^\circ 48'$ — that is, about 2 h 43 m — it falls at 9 h 17 m from midnight upon the said meridian. Accordingly, if we count off the semidiurnal arc of 7 h 26 m on both sides from that hour, we shall arrive at 1 h 51 m from midnight, and at 4 h 43 m from noon. Therefore, at all hours from 1 h 51 m from midnight up to 4 h 43 m from noon, that face of the vertical circle would be illuminated by the Sun

placed at the beginning of Capricorn, were the earth not in the way. But since the Sun at the beginning of Capricorn, at latitude 42° , rises at 7 h 32 m from midnight and sets at 4 h 28 m from noon, it is clear that that face is then illuminated by the Sun the whole day, except for the 15 m during which it ceases to be illuminated before sunset.

The portion of the parallel of Cancer [♋], existing above the same face as a diurnal arc, will embrace 9 h 8 m; the semidiurnal arc, 4 h 34 m. Therefore, since the meridian of the said vertical recedes from the meridian of the horizon toward the east by 2 h 43 m, as we said, and falls upon the same meridian at 9 h 17 m from midnight — if we count the semidiurnal arc on both sides from this hour, we shall arrive at 4 h 43 m from midnight, and at 1 h 51 m from noon.

Accordingly, the same face of the said vertical will be lit by the Sun placed at the beginning of Cancer, at all hours from 4 h 43 m from midnight up to 1 h 51 m from noon. Hence, since the Sun at the beginning of Cancer, at latitude 42° , rises at 4 h 28 m from midnight and sets at 7 h 32 m from noon, it is established that the Sun does not then illuminate that face throughout the whole day, but only from 15 m after sunrise up to 1 h 51 m from noon.

Although, however, because of the obstruction of the earth the Sun illuminates neither face of the proposed circle unless it is above the horizon, nevertheless the Sun's altitudes above each face are to be computed also for those hours in which the Sun is below the horizon, because through them the dials are described more easily and more expeditiously, as will be evident from what follows.

Again, above the northern face of the same vertical circle the arctic pole is elevated by just as many degrees as the antarctic above the former [southern] face — namely $40^\circ 3'$. Therefore the portion of the parallel of Cancer projecting above it, in the manner of a diurnal arc, will comprehend 14 h 52 m; the portion of the parallel of Capricorn, 9 h 8 m. And the meridian of the said [northern] face recedes from the meridian of the horizon, below the horizon, by 2 h 43 m toward the west, so that at 9 h 17 m from noon it falls upon the said meridian.

Accordingly, as before, we shall find the hours at which the Sun, placed at the beginning of Cancer or Capricorn, would illuminate the said northern face, were it not hindered by the earth.

Following Federico Commandino, we shall also investigate by means of the analemma how many hours, and at what hour, are contained above a given face of any great circle, and hence at what hour the Sun begins — or ceases — to illuminate that face. The procedure is as follows.

[Margin: "At what hour [the hours] are contained above the proposed face of a given great circle, and hence at what hour it begins or ceases to be illuminated — how this is ascertained from the analemma."]

Let the analemma ABCD be described, with center E, together with the parallels of the signs. (We have drawn only three parallels — namely F G, for Cancer [♋]; K L, for Taurus [♉]; and H I, for Gemini [♊][?] — so that a multitude of them should not breed confusion; and these three will serve in place of all. For F G will also perform the office of the parallel of Capricorn [♏], since Cancer and Capricorn have equal declinations from the equator. K L will likewise stand in for the parallel of Virgo [♍], of Pisces [♐], and of Scorpio [♏], because these signs and Taurus decline equally from the equator. And finally H I will also be equivalent to the parallel of Leo [♌], of Sagittarius [♐][?], and of Aquarius [♒][?], since the declination of Gemini and of these signs is the same.) About the diameters of the parallels, semicircles are to be described.

Let the axis of the world be B D; the horizon, M N; the vertical circle, [the one through points] 12 and 13. Then from the points O, P, Q — where the horizon cuts the diameters of the parallels — let perpendiculars Q R, P S, O T be drawn to those diameters. Now since both the horizon and the parallel of Cancer [♋] stand at right angles to the meridian, their common section will likewise be at right angles to it; [Margin: (note a) "19. See below."] and therefore, by definition 3 of Euclid, Book XI, it is perpendicular to both F G and M N. Hence Q R, which in the plane of the parallel is perpendicular to F G, will be the common section of the horizon and the parallel of Cancer; and thus F R will be the semidiurnal arc of Cancer [♋], while G R is the semidiurnal arc of Capricorn [♏].

In the same way, P S will be the common section of the horizon and the parallel of Taurus [♉], with K S the semidiurnal arc of Taurus and of Virgo [♍], and L S the semidiurnal arc of Scorpio [♏] and of Pisces [♐]. Likewise O T will be the common section of the horizon and the parallel of Gemini [♊], with H T the semidiurnal arc of Gemini and of Leo [♌][?], and I T the semidiurnal arc of Sagittarius [♐][?] and of Aquarius [♒][?]. Lastly E D will be the common section of the horizon and the equator — that is, of the parallel of Aries [♈] and Libra [♎] — with A D the semidiurnal arc of Aries [♈], and C D the semidiurnal arc of Libra [♎].

In the same manner, if from the points V, X, Y [Y] — at which the vertical circle [12–13] cuts the parallels — perpendiculars V Z, X α, Y β are drawn to the parallels, these will be the common sections of the vertical circle and the parallels, so that the arcs R Z, S α, T β are intercepted between the horizon and

the vertical circle; and the plane of the vertical circle, while the Sun runs across them, is illuminated by the Sun on the side of the south.

Now let $\gamma\delta$ be the common section of the meridian and of a given circle inclined to the meridian — which is easily obtained once, by proposition 28 of Book I, the arc of the meridian $N\delta$ between that circle and the horizon has been found. Then, the inclination of that circle to the meridian having been found by proposition 27 of Book I, let there be found, by proposition 25 of Book I, the minor diameter of the ellipse which the perpendiculars dropped from the circumference of the inclined circle onto the plane of the meridian produce; let this be $\varepsilon\theta$, cutting the major diameter $\gamma\delta$ at right angles at the center E. About the given diameters $\gamma\delta$ and $\varepsilon\theta$ let the ellipse be described according to what we set out in the scholium of proposition 26 of Book I, cutting the diameters of the parallels at the points $\lambda, \mu, \xi, \pi, \rho, \sigma, \varphi$, and $[\omega]$ — from which points perpendiculars are drawn to the diameters: $\lambda \rightarrow 3, \mu \rightarrow 4, \xi \rightarrow 7, \pi \rightarrow 8, \rho \rightarrow 9, \sigma \rightarrow 10, \varphi \rightarrow 11$, and $[\omega] \rightarrow \omega$.

I say that the inclined circle cuts the parallels at the points 3, 4, 7, 8, 9, 10, 11, ω . For, the semicircle of Cancer $[\mathfrak{C}]$, F R G, being set in its proper position — namely at right angles to the meridian — the perpendicular dropped from 3 onto the plane of the meridian falls upon F G, the common section of that plane and the meridian, [Margin: (note b) "38. See below."] and it is therefore perpendicular to the straight line F G by Euclid, Book XI, definition 3. Consequently that perpendicular must fall upon the point λ , lest from the point 3 two perpendiculars be said to be drawn to the straight line F G — which is impossible, as we demonstrated (following Proclus) at Euclid, Book I, proposition 16. The straight line 3λ is therefore at right angles to the plane of the meridian. And hence, since by proposition 24 of Book I the perpendiculars dropped from the circumference of the inclined circle onto the plane of the meridian fall also upon the ellipse, the circumference of the inclined circle will cut the parallel of Cancer, F R G, at the point 3 — the point from which, namely, the perpendicular drawn onto the plane of the meridian falls upon λ . And so for the rest.

The inclined circle therefore cuts off: from the parallel of Cancer $[\mathfrak{C}]$, the arc F 3; from the parallel of Capricorn $[\mathfrak{K}]$, the arc G 4; from Taurus $[\mathfrak{T}]$ and Virgo $[\mathfrak{M}]$, the arc K 7; from Scorpio $[\mathfrak{M}]$ and Pisces $[\mathfrak{P}]$, the arc L 8; from Aries $[\mathfrak{A}]$ and Libra $[\mathfrak{L}]$, the arc A 9, or C 10; from Gemini $[\mathfrak{G}]$ and Leo $[\mathfrak{L}]$, the arc I ω ; from Sagittarius $[\mathfrak{S}]$ and Aquarius $[\mathfrak{A}]$, the arc H 11 — these being the arcs, namely, that are intercepted between the meridian and the inclined circle.

Hence, once the hours are distributed — whether the beginning is made from the meridian, namely from the point F or G and so on, or from the point R or S and so on, after the manner of the Babylonians or the Italians — it will be plainly established at what hour, or fraction of an hour, [the inclined circle] falls upon point 3, or 7, or 9, and so forth.

For the rest, so that we may know whether point 3, or 7, or 9, and so on — and therefore the hour at which the parallel is cut by the inclined circle — lies on the eastern or the western side, we must carefully examine the situation and position of the inclined circle. For once this is known we shall easily understand the matter, as we shall teach a little further on, in the solution of this same problem by the doctrine of sines.

[Margin: "When the inclined circle cuts the proposed parallel of the Sun in two places."]

But if the ellipse should ever cut the diameter of a parallel in two places short of the point at which that diameter is divided by the diameter of the horizon, then two arcs will be cut off from the parallel — one toward the parts of the east, the other toward the parts of the west. And if the upper face of the circle looks toward the west, the point nearer the meridian will be the eastern one, and the more remote the western; whereas, on the contrary, if it looks toward the east [the reverse holds], as will become clearer from what follows. This, moreover, happens for the most part in planes passing through the zenith and having a small declination from the vertical circle, and in certain others — as can be understood from a material sphere.

Diagram. A large analemma projection. An outer circle A B C D with center E, containing the semicircular projections of three declination-parallels drawn about their diameters, the axis of the world B D, the horizon M N, the vertical circle through the numbered points 12 (upper) and 13 (lower), the equator and the inclined-circle ellipse, and a dense net of intersecting straight lines. The rim carries the zodiacal signs.

- A (top): endpoint of meridian/outer circle
- B, D: axis of the world (B upper-left, D lower-right)
- C: lower rim
- E (center): center of the projection
- F, G: diameter-endpoints of the Cancer [♋]/Capricorn [♐] parallel
- K, L: diameter-endpoints of the Taurus [♉] parallel
- H, I: diameter-endpoints of the Gemini [♊] parallel

- M, N: horizon endpoints
- O, P, Q: points where the horizon cuts the parallel-diameters
- R, S, T: feet of the perpendiculars Q R, P S, O T
- 12, 13: the vertical circle's poles/points
- Interior points labelled with Arabic numerals 3, 4, 7, 8, 9, 10, 11 and Greek letters λ , μ , ξ , π , ρ , σ , ϕ , ω , α , β (small — several not individually resolvable)
- Rim: zodiacal glyphs (ϖ , φ , χ , $\mathfrak{M}$, $\mathfrak{H}$, Υ , $\mathfrak{X}$, $\mathfrak{I}$, $\mathfrak{S}$, $\mathfrak{Q}$, $\mathfrak{M}$, $\mathfrak{L}$ — small, partly unresolved)

[Many small interior/rim labels are at the resolution floor]

[Margin: "When the circle stands at right angles to the meridian: by what method one inquires at what hour [the hours] are contained above either face."]

In circles standing at right angles to the meridian — such as the vertical circle properly so called, and all circles to which sundials inclined to the horizon are parallel — the proposed problem is of no difficulty from the analemma; for the diameter of the meridian cuts the diameter of any parallel at the point where the parallel is cut by the said circle, both before noon and after noon. You have an example in the adjoining figure, in the vertical circle whose diameter is 12–13, where the perpendiculars V Z, X α , E D, Y β mark the points at which the parallels are cut by the vertical circle; because, as has been demonstrated, those perpendiculars are the common sections of the vertical circle and the parallels.

The reasoning is the same if the diameter $\gamma \delta$ be taken as the common section of the meridian and of some great circle to which a sundial inclined to the horizon is parallel, so that the elevation of the pole above it is the arc A D. For the perpendiculars drawn to the same diameters from the points where $\gamma \delta$ cuts the diameters of the parallels will be the common sections of the parallels and of the said great circle — as will be demonstrated, just as was stated concerning the vertical circle and the horizon.

From these things it can readily be understood at what hour the Sun begins to illuminate the upper or the lower face of an inclined circle, and for which hours the Sun's altitudes are to be computed. For when the upper face of the circle looks toward the west, the Sun will begin to illuminate the upper face when it stands at the point which was found by the demonstration above — as at point 3, or 7, or 9, and so on.

Hence the Sun's altitudes above the upper face must be investigated for each hour from that point up to sunset — except when the parallel is cut by the

inclined circle in two places above the horizon: for then the hours are to be taken only from that point up to the other, more westerly point, because there the Sun ceases to illuminate the upper face. The Sun's altitudes above the lower face, on the other hand, must be sought for the hours from sunrise up to the said point; and, if the parallel is cut in two places above the horizon by the inclined circle, also for the hours from the more westerly point up to sunset — because at the former point the Sun ceases to illuminate the lower face, and begins again to illuminate the same at the latter point.

But when the upper face of the circle looks toward the east, the Sun will cease to illuminate the upper face when it stands at the point found by the preceding demonstration. Therefore the Sun's altitudes above the upper face must be sought for each hour from sunrise up to that point. And if the inclined circle cuts the parallel in two places, only those hours are to be taken which are comprised between the more easterly and the more westerly point, because at the former point the Sun begins to illuminate the upper face. Above the lower face, on the other hand, the Sun's altitudes are to be found for the hours from the point found up to sunset; and, if the parallel is cut in two places by the inclined circle, also for the hours from sunrise up to the more easterly point — because at this point the Sun ceases to illuminate the lower face, while at the more westerly point it begins to illuminate the same again.

But this same result we shall attain more expeditiously by the doctrine of sines [per sinuum doctrinam], in the following manner. Let the horizon be A B C D; the meridian A E C; the equator B E D; the parallel — whether northern or southern — F H G; and the great circle inclined to the meridian (whether it be at right angles to the horizon or not) R I, cutting the equator at the points K, L, the northern parallel at the points M, N, and the southern at the points R, S, and the meridian at the point I.

We must therefore investigate both arcs E K, E L in the equator, and both H M, H N in the northern parallel, and both H R, H S in the southern parallel — that is, the distance of the Sun before or after noon, when it stands in the inclined plane and in the said parallels. Now since the great circle inclined to the meridian [cuts] any parallel [in two points], of which the one nearer to the semicircle of the meridian above the horizon [is nearer], and the other more remote, we shall first inquire into that arc of the parallel which is intercepted between the nearer point and the meridian; for from this we shall without difficulty elicit the other, which is intercepted between the more remote point and the meridian.

[Margin: "Whether the arc of the proposed parallel between the meridian (above the horizon) and the inclined circle on the eastern side is smaller than the other on the western side, or the reverse. Likewise, when the parallel is cut by the inclined plane in two places on the same side — whether eastern or western — how this is recognized."]

In order that we may know whether the nearer point is eastern or western, the situation of the inclined circle must be carefully examined. For from it we shall easily understand whether the smaller arc of the parallel lying between the inclined circle and the semicircle of the meridian above the horizon is eastern or western — by this device. When the great circle is at right angles to the meridian — whether it passes through the zenith or not — it will necessarily pass through the common sections of the equator and the horizon, namely through the poles of the meridian, by proposition 14 of Book I of Theodosius; and it will cut any parallel in two points equally distant on either side from the meridian, because the meridian, standing at right angles to the parallel and to that great circle, bisects their segments by proposition 9 of Book II of Theodosius.

But when the same great circle is inclined to the meridian and passes through the zenith — that is, is at right angles to the horizon — then, if its southern face turns toward the west and the zenith lies between the arctic pole and the proposed northern parallel, both that northern parallel and the opposite southern one will be cut in two points, of which the eastern is nearer to the semicircle of the meridian above the horizon than the western, on account of the acute angle which the declining circle makes with the meridian toward the eastern and southern parts. But if the zenith lies between the equator and the northern parallel, that northern parallel will be cut in two western points, and the opposite southern one in two eastern points, if the southern face of the declining circle turns toward the east. For when the zenith lies between the arctic pole and the northern parallel, both that northern parallel and the opposite southern one will be cut in two points,

Diagram. The same analemma projection as on p. 379 (the "adjoining figure" of the text). Outer circle A B C D with center E; the three parallel-semicircles (F G, K L, H I); horizon M N; vertical circle through 12 (top) and 13 (bottom); axis B D; the inclined-circle ellipse and the perpendicular net; zodiac glyphs on the rim.

- A (top rim); B (left, axis); C (bottom rim); D (right, axis)
- E (center)
- F, G: Cancer [♋]/Capricorn [♏] parallel diameter
- K, L: Taurus [♉] parallel diameter

- H, I: Gemini [II] parallel diameter
- M, N: horizon endpoints
- O, P, Q, R, S, T: horizon-cut points and perpendicular feet
- 12, 13: vertical-circle points
- Interior Greek/Arabic point labels as on p. 379 (small — several unresolved)

[Repeat of the p. 379 analemma; small interior/rim labels at the resolution floor]

points of which the western is nearer to the semicircle of the meridian above the horizon than the eastern, because the declining circle makes an acute angle with the meridian toward the western [and southern] parts. But with the zenith between the equator and the northern parallel, that northern parallel will be cut in two eastern points, and the opposite southern one in two western points.

Again, when the great circle inclined to the meridian does not pass through the zenith: if it is inclined on the northern side and its upper face looks toward the west, and the arc of the meridian between the inclined circle and the horizon is smaller than the arc of the meridian on the northern side between [the northern parallel and the horizon] — so that the inclined plane cuts the meridian between the horizon and the northern parallel — then both that northern parallel and the opposite southern one will be cut in two points, of which the eastern is less distant from the semicircle of the meridian above the horizon than the western, because the inclined circle constitutes an acute angle with the meridian [toward the east]. But if the arc of the meridian between the inclined circle [and the horizon], on the northern side, [lies] between the proposed northern parallel and the horizon in such a way [that it falls] between the equator and the northern parallel, that northern parallel will be cut in two [western points], and the opposite southern one in two eastern points. The contrary will happen if the [upper] face [turns toward the east].

Exactly the same must be said of a circle inclined on the southern side. For if its upper face looks toward the west, and the arc of the meridian on the southern side between it and the horizon is smaller than the arc of the meridian [between the southern parallel and the horizon], both that southern parallel and the opposite northern one will be divided in two points, of which [the eastern] is less distant from the semicircle of the meridian above the horizon than the western, because the inclined circle makes [an acute angle] with the meridian toward the eastern and northern parts. But if [the arc of the meridian] between the inclined circle and the horizon is larger than the arc of the meridian on the southern side

between the [southern] parallel [and the horizon], so that the inclined plane cuts the meridian between the horizon and the southern parallel, then that southern parallel will be cut in two western points, and the [northern] parallel [in two eastern points]. The contrary of these will happen if the upper face of the inclined circle turns toward the east.

[Sense preserved throughout; bracketed spans mark words recovered by context where the print runs into the gutter or the cropped edge of the leaf.]

That which was said above — when the plane was inclined on the northern side — must also be said [here]: it cut the meridian between the arctic pole and the northern parallel, or between [the horizon and] the northern parallel, or between the equator and the northern parallel. All of which can easily be gathered and understood from a material sphere.

Diagram. A set of five closely-related spherical diagrams (two upper rows plus a lower pair), each showing a sphere with the horizon, meridian, equator, a declination-parallel and a great circle inclined to the meridian, illustrating the successive cases of the case-analysis in the text (which face turns east/west; where the zenith and the intersection points fall). Common labels across the figures: A at the top (pole/zenith end of the meridian), C at the bottom, B and D on the horizon, E at the center, F/G/H flanking a parallel, and R, S, K, L, M, N, P, T, I, O marking the intersection points of the inclined great circle with equator and parallels.

- Upper-left sphere: A (top), F, H, G (upper arc), B, R, T, K (left), E (center), D (right), M, O, I (lower), N, C (bottom)
- Upper-middle sphere: A (top), F, R, H, G, S; K, P, T, E, D, L, B; N, H, I, O, C
- Upper-right sphere: A (top), R, H, G; T, K, E, P; I, H, M, N; O, C, D, L
- Lower-left sphere: A (top), F, S, G; Q, L, B, V, E, T, K, D, P; N, E, O, M; C (bottom)
- Lower-middle sphere: A (top), R, H, G, F; T, I, K, P, E, D; H, M, O; C (bottom)

[Five overlapping figures with many small, partly-overlapping point-labels; numerous labels not individually resolvable]

[Margin: "At which hour before and after noon the inclined circle intersects the Sun's parallels, and hence how many hours are found above each face of that circle, and at what hour the Sun begins or ceases to illuminate each — by what route this is inquired through sines."]

And so, by this reasoning, we shall investigate the smaller arcs of the parallels between the inclined circle [and the semicircle of the meridian] above the horizon, which we suppose to be E K, H M, H R. From the arctic pole of the world O, through the points M, R, let there be drawn the great circles of declination O M, O R, cutting the equator at P and T. And since, in the spherical triangle E I K — whose angle E is right, and whose angle I (the inclination of the inclined circle to the meridian) is known by proposition 27 of Book I, together with the arc of the meridian E I between the plane and the equator —

[Margin: "How large the arc of the meridian is between the inclined plane and the equator above the horizon — how it is understood."]

For since, by the corollary of proposition 28 of Book I, the arc of the meridian between the inclined plane and the arctic pole is known, and since between the arctic pole and the equator a quadrant of the meridian is intercepted, the arc of the meridian between the inclined plane and the equator will also be known. For if the plane is inclined on the northern side, yet below the pole, the arc of the meridian between the plane and the equator above the horizon will be composed of a quadrant and the arc of the meridian between the arctic pole and the inclined plane. But if the plane is inclined on the northern side, above the pole — that is, in such a way that the arc of the meridian between the horizon and the inclined plane is greater than the elevation of the pole above the horizon — [and such that the arc of the meridian between the horizon and the inclined plane is greater than the altitude of the equator] — then the arc of the meridian between the plane and the equator is left, if the arc of the meridian between the plane and the arctic pole is taken away from the quadrant. And finally, if the plane is inclined on the southern side, but below the equator — that is, so that the arc of the meridian between the plane and the horizon is smaller than the altitude of the equator — the arc of the meridian between the plane and the equator will be had, if a quadrant is subtracted from the arc of the meridian between the plane and the arctic pole. [This may be done] by proposition 18 of Book IV of Johannes Regiomontanus, *De triangulis*; or by proposition 14 of Book I of Geber [Jābir ibn Aflah]; or by proposition 42 of our own *Spherical Triangles* —

[The scholium continues beyond this page.]

[The scholium of Problem 1 of Book V continues from the previous page. The sentence begun there completes here with a spherical-triangle proportion.]

[Margin: The angle which the inclined plane makes with the equator.]

[of our Spherical Triangles], it holds that: as the sine of angle EIK — the inclination of the plane to the meridian — is to the whole sine [radius], so the sine of the complement of angle EKI, which the plane makes with the equator, is to the sine of the complement of the arc of the meridian EI intercepted between the plane and the equator. By inversion, then, it will be: as the whole sine is to the sine of the angle of inclination of the plane to the meridian, EIK, so the sine of the complement of the arc of the meridian EI, between the plane and the equator, is to the sine of the complement of angle EKI, which the plane makes with the equator. Therefore, if we arrange it so that as the whole sine is to the sine of the angle of inclination of the plane to the meridian, so the sine of the complement of the arc of the meridian EI, between the plane and the equator, is to a fourth term, we shall find the sine of the complement of angle EKI; and hence the angle EKI itself, which the plane makes with the equator, will be known.

Again, since in the same right-angled spherical triangle EIK — by proposition 16 of Book IV of Johannes Regiomontanus [Regiomontanus], *On Triangles*, or by proposition 13 of Book I of Geber [Jābir ibn Aflaḥ], or by proposition 41 of our own *Spherical Triangles* — it holds that: as the sine of angle EKI, which the plane makes with the equator, is to the sine of the arc of the meridian EI between the plane and the equator, so the sine of angle EIK, the inclination of the plane to the meridian, is to the sine of the arc of the equator EK, the Sun's distance from the meridian. If we arrange it so that as the sine of the found angle [EKI], which the plane makes with the equator, is to the sine of the arc of the meridian [EI] between the plane and the equator, so the sine of the inclination of the plane to the meridian is to a fourth term, we shall find the sine of the smaller distance of the Sun from the meridian — the distance at which the Sun, being on the equator, begins to illuminate one face of the inclined plane and ceases to illuminate the other.

[Margin: The smaller distance of the Sun from the meridian on the equator, when the Sun lies on the inclined circle.]

Diagram. The first of five worn woodcut sphere-diagrams (this one at the top left of the page) illustrating the cases of the inclined plane cutting the meridian. The sphere shows the meridian as a vertical great circle (A at the top, C at the bottom), a roughly horizontal great circle across the middle, and a set of oblique circles — the equator, the horizon, the inclined plane, the declination-circles, and the day-parallel — intersecting near the centre E. Readable letter-labels: A (top of meridian), C (bottom of meridian), E (centre), B (left), D (right), and around the

sphere F, G, H, I, K, M, O, P, R, T. Several labels are abraded and their exact placement cannot be verified at this resolution. [— many labels [?].]

Diagram. The second sphere-diagram (top centre), of the same construction. Readable letter-labels: A (top), C (bottom), E (centre), B, D, L, and around the rim F, G, H, I, K, M, N, O, P, R, T, V. Exact roles of the peripheral labels are not verifiable. [— many labels [?].]

Diagram. The third sphere-diagram (top right), of the same construction. Readable letter-labels: A (top), C (bottom), E (centre), and around the sphere D, F, G, H, I, K, L, M, N, O, P, Q, R, S, T. [— many labels [?].]

Diagram. The fourth sphere-diagram (lower left, drawn larger), of the same construction, with the meridian A (top) ... C (bottom) and centre E. Readable letter-labels: A, C, E, B, D, and around the sphere F, G, H, I, K, L, M, N, O, P, Q, S, T, V. [— many labels [?].]

Diagram. The fifth sphere-diagram (lower right), of the same construction. Readable letter-labels: A (top), C (bottom), E (centre), B, D, and around the sphere F, G, H, I, K, L, M, O, P, R, S, T, V. [— many labels [?].]

Next, since in the right-angled spherical triangle KMP — by proposition 16 of Book IV of Regiomontanus, *On Triangles*, or by proposition 13 of Book I of Geber, or by proposition 41 of our *Spherical Triangles* — it holds that: as the sine of angle PKM, which the inclined plane makes with the equator (and which we have already determined), is to the sine of the arc MP, the declination of the parallel, so the whole sine is to the sine of the arc KM of the inclined plane intercepted between the equator and the parallel. If we arrange it so that as the sine of the found angle, which the inclined plane makes with the equator, is to the sine of the declination of the parallel, so the whole sine is to a fourth term, we shall find the sine of that arc of the inclined plane which lies between the equator and the parallel.

[Margin: The arc of the inclined plane between the equator and the parallel.]

Again, since in the same triangle — by proposition 19 of Book IV of Regiomontanus, *On Triangles*, or by proposition 13 of Book I of Geber, or by proposition 43 of our *Spherical Triangles* — it holds that: as the sine of the complement of the arc KM, just found, is to the sine of the complement of the arc MP, the declination of the parallel, so the sine of the complement of the arc KP is to the whole sine. By inversion it will be: as the sine of the complement of the arc MP, the declination of the parallel, is to the sine of the complement of the arc KM,

just found, so the whole sine is to the sine of the complement of the arc kP. If, therefore, we arrange it so that as the sine of the complement of the declination of the parallel is to the sine of the complement of the arc of the inclined plane intercepted between the equator and the parallel (just found), so the whole sine is to a fourth term, we shall find the sine of the complement of that arc of the equator KP which lies between the plane and the declination-circle; and hence the arc KP itself will be known — an arc equal to the three others KT, LQ, LV.

[Margin: The arc of the equator between the inclined plane and the declination-circle of the parallel drawn through the Sun, when the Sun lies on the inclined plane.]

For since the two angles PKM, TKR at the vertex K, and likewise the two angles QLN, VLS at the vertex L, are equal by proposition 6 of our *Spherical Triangles*; and since the angle EKI is equal to the angle ELI by proposition 13 of the same *Spherical Triangles* — the two angles at the vertex K will be equal to the two angles at the vertex L. Since, then, the angles at P, T, Q, V are right, the two angles MPK, PKM of triangle KPM will be equal to the two angles RTK, TKR of triangle KRT, and likewise the two angles NQL, QLN of triangle LNQ [equal] to the two angles SVL, VLS of triangle LSV. Moreover, the arcs MP, RT, NQ, SV — opposite the equal angles at K and L — are equal, because they measure the declinations of opposite parallels, which are equal. Therefore, by proposition 22 of our *Spherical Triangles*, the remaining arcs too will be equal, namely the arcs PK, TK, QL, VL.

From this arc of the equator between the inclined plane and the declination-circle of the parallel drawn through the Sun (when the Sun is on the inclined plane) — such as PK, or TK, or QL, or VL, which we have just found — we shall investigate the smaller distance of the Sun from the meridian, that is, the arc HM and the arc HR.

[Margin: By what method the smaller distance of the Sun from the meridian is investigated when the Sun lies on the inclined plane, which cuts the meridian between the arctic pole and the horizon.]

Whenever the plane, raised on the northern side above the horizon, cuts the meridian between the arctic pole and the horizon, as in the first figure, the arc of the equator PK just found — which lies between the inclined plane and the declination-circle — must, when the Sun is northern, be added to the arc of the equator EK situated between the plane and the meridian; but when the Sun is southern, the arc Tk, which we showed equal to the arc PK, must be subtracted

from the same arc of the equator EK. For thus, from that addition, there will arise the arc EP, which by proposition...

[proposition] 10 of Book II of Theodosius [Theodosius of Bithynia], [applied] to the arc HM (which measures, in the northern parallel, the Sun's distance from the meridian), is similar. And by subtraction there will be left the arc ET, which by the same proposition 10 of Book II of Theodosius is similar to the arc HR (the Sun's distance from the meridian in the opposite southern parallel).

But when the inclined plane cuts the meridian between the arctic pole and the northern parallel, as in the second figure, the arc PK, found from the arc [previously] found, must be subtracted, so that the arc EP is left; and thus [EP], similar to HM, becomes known, measuring in the northern parallel the Sun's distance from the meridian. But the arc TK, which is equal to the arc PK, must be added to the same arc EK, so that the arc ET arises — known, and similar to the arc HR — measuring the Sun's distance from the meridian in the opposite southern parallel.

Finally, when the plane cuts the meridian between the equator and the northern parallel, as in the third figure, if the arc of the equator EK, intercepted between the plane and the meridian, is subtracted from the arc PK of the same equator set between the plane and the declination-circle, there will remain the arc EP, which is similar to the arc HM (the Sun's distance from the meridian in the northern parallel). But if to the same arc of the equator EK the arc TK is added, which was shown equal to the arc PK, the arc ET will be produced, which is similar to the arc HR (the Sun's distance from the meridian in the opposite southern parallel).

[Margin: When the plane inclined on the northern side cuts the meridian between the arctic pole and the northern parallel.]

But if the plane is raised on the southern side above the horizon, and cuts the meridian between the horizon and the southern parallel, as in the fourth figure, we shall investigate the Sun's distance from the meridian in the same way as in the first figure, where the plane, inclined on the northern side, cuts the meridian between the horizon and the arctic pole. But if the plane cuts the meridian between the equator and the southern parallel, as in the fifth figure, the arc of the equator EK, set between the plane and the meridian, must be subtracted from the arc TK, which lies between the plane and the declination-circle, so that the arc ET is left, similar to the arc HR (the Sun's distance from the meridian in the southern parallel). And to the same arc EK the arc PK must be added, so that the arc EP is produced, similar to the arc HM (the Sun's distance from the meridian in the

opposite northern parallel). Finally, if the inclined plane, on the southern side, cuts the meridian between the equator and the northern parallel, or between the zenith of the place and the northern parallel, the Sun's distance from the meridian must be sought as in the third figure — where the plane inclined on the northern side cuts the meridian between the equator and the northern parallel — or as in the second figure, where the plane inclined on the northern side cuts the meridian between the arctic pole and the northern parallel.

[Margin: When the plane inclined on the northern side cuts the meridian between the equator and the northern parallel.]

[Margin: When the plane inclined on the southern side is set to the horizon, wherever it may cut the meridian — by what method the smaller distance of the Sun from the meridian is found when the Sun is on that plane.]

Thus far we have investigated the smaller arc of the Sun's distance from the meridian in any parallel from the smaller arc of the equator intercepted between the inclined plane and the meridian, such as EK. But if we subtract the said arc of the equator EK from the semicircle KL, there will remain the greater arc of the equator EL, which is intercepted on the other side between the plane and the meridian. Through this we shall explore, in exactly the same way, the greater distance of the Sun in any parallel — that is, the arc HN or HS — namely by adding the arc QL or VL to the arc EL, or by subtracting it, as has been said.

When the plane cuts the meridian between the equator and the northern parallel, as in the third figure — so that the parallel itself is cut by that plane at two points, one western and one eastern — the arc QL must be subtracted from the arc EL, so that the arc of the equator EQ is left, similar to the arc HN, that is, to the greater distance of the Sun from the meridian in the northern parallel. From this arc HN, if the smaller distance from the meridian HM is subtracted, there will be left, known, the arc MN of the northern parallel; and since, by proposition 19 of Book II of Theodosius, this is equal to the alternate segment of the opposite southern parallel, then if to the arc HR — the Sun's smaller distance from the meridian in the southern parallel — an arc equal to MN is added, the greater distance of the Sun from the meridian in the opposite southern parallel will be obtained.

Finally, when the plane cuts the meridian between the equator and the southern parallel, as in the fifth figure — so that the parallel itself is cut by that plane at two points, one eastern and one western — the arc VL must likewise be subtracted from the arc EL, so that the arc of the equator EV is left, similar to the arc HS, that is, to the greater distance of the Sun from the meridian in the

southern parallel. From this arc HS, if the smaller distance of the Sun from the meridian HR is subtracted, there will be left, known, the arc RS of the southern parallel; and since, by proposition 19 of Book II of Theodosius, this is equal to the alternate segment of the opposite northern parallel, then if to the arc HM — the Sun's smaller distance from the meridian in the northern parallel — an arc equal to RS is added, the greater distance of the Sun from the meridian in the opposite northern parallel will be obtained.

[Margin: When the parallel is cut at two points by the inclined plane, whether on the eastern or the western side — by what method the greater distance of the Sun from the meridian is inquired.]

We shall illustrate the whole matter with a single example. Let a plane be set erect to the horizon but declining from the true vertical by 30 degrees, from the meridian toward the east — the kind of dial-plane we described in proposition 1 of Book III — so that it is inclined to the meridian by 60 degrees. Let it be: as 100000, the whole sine, is to 86602, the sine of the plane's inclination to the meridian, so 74314, the sine of the complement of the arc of the meridian between the plane and the equator, is to a fourth term; and there will be found here approximately the sine 64357, to which corresponds an arc of 40 degrees 4 minutes, whose complement, 49 degrees 56 minutes, gives the angle EKI which the plane makes with the equator.

Again let it be: as 76529, the sine of the found angle EKI, is to 66913, the sine of the arc of the meridian between the equator and the plane, so 86602, the sine of the plane's inclination to the meridian, is to a fourth term; and there will be found here approximately the sine 75720, to which corresponds an arc of 49 degrees 13 minutes. Such is the arc of the equator EK, comprising 3 hours 17 minutes, by which the plane is illuminated by the Sun after noon on the southern side; while on the northern side it begins to be illuminated by the Sun at 3 hours 17 minutes from the meridian.

After this, let it be: as 76529, the sine of the found angle EKI which the plane makes with the equator, is to 39874, the sine of the declination of the parallel of Cancer [♋] or Capricorn [♐], so 100000, the whole sine, is to a fourth term; and there will be found here approximately the sine 52103, to which corresponds an arc of 31 degrees 24 minutes. Such is the arc KM of the inclined plane, intercepted between the equator and the parallel of Cancer.

Again let it be: as 91706, the sine of the complement of the declination of the parallel of Cancer or Capricorn, is to 85355, the sine of the complement of the arc

KM just found (which we said comprises 31 degrees 24 minutes), so 100000, the whole sine, is to a fourth term; and there will be found here approximately the sine 93075, to which corresponds an arc of 68 degrees 33 minutes, whose complement, 21 degrees 27 minutes, gives the arc of the equator KP, which is drawn between the inclined plane and the declination-circle set through the Sun on the inclined plane and the parallel of Cancer — an arc to which the arcs KT, LQ, LV have been shown equal.

And so, since the proposed example answers to the second figure — the southern face of the plane being turned toward the east — if the arc KP, 21 degrees 27 minutes, is subtracted from the arc of the equator EK, 49 degrees 13 minutes, there will remain the arc EP, and hence the arc HM similar to it, of 27 degrees 46 minutes, which comprises 1 hour 51 minutes. Therefore at 1 hour 51 minutes after noon the Sun, being on the parallel of Cancer, ceases to illuminate the southern face of the proposed plane.

But if we subtract the arc of the equator EK from the semicircle kL, there will remain the arc EL, 130 degrees 47 minutes; and if from this the arc LQ, 21 degrees 27 minutes, is again subtracted, there will remain the arc EQ, or its similar HN, 109 degrees 20 minutes, comprising 7 hours 17 minutes. Therefore at 4 hours 43 minutes after midnight the Sun, being on the parallel of Cancer, begins to illuminate the southern face of the proposed plane.

Now if we add the same arc KT, or LV, 21 degrees 27 minutes, to the arc EK, 49 degrees 13 minutes, and to the arc EL, 130 degrees 47 minutes, we shall find the arc HR, 70 degrees 40 minutes — that is, 4 hours 43 minutes — and the arc HS, 152 degrees 14 minutes — that is, 10 hours 9 minutes. Therefore the Sun, being on the parallel of Capricorn, ceases to illuminate the southern face of the plane at...

4 hours 43 minutes after noon, and begins to illuminate it at the first hour [numeral:?] after midnight. These distances from the meridian we have also found above.

But to inquire into the Sun's altitudes it is not altogether necessary to investigate how many hours are contained above the face of the proposed circle; for the computation itself will teach us whether the given hour falls above the offered face or above the opposite one — according as the difference between the sine of the meridian altitude and the sine of the Sun's altitude at the time of observation is smaller, or greater than the sine of the meridian altitude, as we said a little earlier in this proposition.

Diagram. The same five-figure set as is repeated here (top row of three spheres, lower row of two). This first sphere (top left) shows the meridian A (top) ... C (bottom) with centre E, and the oblique great circles (equator, horizon, inclined plane, declination-circles, parallels). Readable letter-labels: A, C, E, B, D, F, G, H, I, K, M, O, P, R, T. [— many labels [?].]

Diagram. Second sphere (top centre), same construction. Readable letter-labels: A, C, E, B, D, L, F, G, H, I, K, M, N, O, P, R, T, V. [— many labels [?].]

Diagram. Third sphere (top right), same construction. Readable letter-labels: A, C, E, D, F, G, H, I, K, L, M, N, O, P, Q, R, S, T. [— many labels [?].]

Diagram. Fourth sphere (lower left, larger), same construction, meridian A ... C with centre E. Readable letter-labels: A, C, E, B, D, F, G, H, I, K, L, M, N, O, P, Q, S, T, V. [— many labels [?].]

Diagram. Fifth sphere (lower right), same construction. Readable letter-labels: A, C, E, B, D, F, G, H, I, K, L, M, O, P, R, S, T, V. [— many labels [?].]

But so that the altitudes of all the hours may be computed more conveniently and easily, the distances from the meridian of the proposed circle above the horizon should be taken for all 24 hours of one parallel, and the altitudes of those same hours computed — of which some will fall above one face of the given circle and some above the opposite face — provided that those which before were reckoned from the meridian are now reckoned from midnight, and vice versa, and those which before were reckoned from sunset are now reckoned from sunrise, and vice versa. For these altitudes will be equal to the altitudes of the same hours of the opposite parallel. Likewise, those which before fell above one face of the given circle will now fall above the opposite face — as we shall presently demonstrate, if first, that the matter may be better understood, we illustrate it with a few examples.

[Margin: It is enough if the Sun's altitudes are computed for the hours of one parallel; for these are equal to the Sun's altitudes for the same hours of the opposite parallel.]

First, then, above the face of the vertical circle properly so called: on the southern side the antarctic pole is elevated 48 degrees, at latitude 42 degrees, and the complement of this altitude is 42 degrees. Therefore the meridian altitude of Cancer will comprise 18 degrees 30 minutes, to which corresponds the sine 31730; while the meridian depression is 65 degrees 30 minutes, to which corresponds the sine 90996. The half of the sum of the sine of the meridian

altitude of Cancer and the sine of the depression will be 61363. The distances which the hours reckoned from sunset have from the meridian above the horizon, in the parallel of Cancer (for the Sun, being at the beginning of Cancer, illuminates the southern face of the vertical circle both before and after noon), are the following — because the hour-circle of 16 hours 28 minutes and the parallel of Cancer cut the meridian above the horizon at one and the same point, so that the 16th hour is distant from the meridian toward the east by 0 hours 28 minutes, that is, 7 degrees 0 minutes; while the 17th hour is distant from it toward the west by 0 hours 32 minutes, that is, 8 degrees 0 minutes.

[Margin: Examples of the Sun's distances from the proper meridian of various great circles.]

The tables follow.

(Continuing the Scholium of Problema 1 of Book V. The Sun stands at the beginning of Cancer, at latitude 42°; the following tables — introduced by the "SEQVVNTVR TABELLÆ" of the previous opening — give, for each hour, the distance of the hour-circle from the meridian, and then the Sun's altitudes. Throughout, the zodiac glyphs are: ☊ = Cancer, ♈ = Aries, ♎ = Libra, ♐ = Capricorn. The paired headers "G." / "M." stand for Gradus (degrees) and Minuta (minutes); each cell is transcribed as printed, in the form "degrees. minutes.")

Table — Hours in Cancer (☊), reckoned from sunset; distances from the meridian (first part).

	1											
Horæ ☊, ab occ.	6	15	14	13	12	11	10	9	8	7	6	5
Distantiæ à	7.	22	37	52	67	82	97	112	127	142	157	172
Meridiano (G. M.)	0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0

Table — Hours in Cancer (☊), reckoned from sunset; distances from the meridian (second part).

	1											
Horæ ☊, ab occ.	7	18	19	20	21	22	23	24	1	2	3	4
Distantiæ à	8.	23	38	53	68	83	98	113	128	143	158	173
Meridiano (G. M.)	0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0

By the same reasoning, the distances from the meridian that the hours reckoned from sunrise have, in the same parallel of Cancer, are as follows — because the meridian cuts the parallel of Cancer at the hour of 7 hours 32 minutes after

sunrise. Therefore the 7th hour from sunrise is distant from the meridian toward the east by 0 hours 32 minutes, that is, 8 degrees 0 minutes; while the 8th hour from sunrise is distant from it toward the west by 0 hours 28 minutes, that is, 7 degrees 0 minutes.

Table — Hours in Cancer (♋), reckoned from sunrise; distances from the meridian (first part).

Horæ ☿, ab or.	7	6	5	4	3	2	1	24	23	22	21	20
Distantiæ à	8.	23	38	53	68	83	98	113	128	143	158	173
Meridiano (G. M.)	0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0

Table — Hours in Cancer (♋), reckoned from sunrise; distances from the meridian (second part).

Horæ ☿, ab or.	8	9	10	11	12	13	14	15	16	17	18	19
Distantiæ à	7.	22	37	52	67	82	97	112	127	142	157	172
Meridiano (G. M.)	0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0

The distances from the meridian that the hours reckoned from noon and from midnight have — the same for all parallels, since the meridian cuts every parallel at the 12th hour of noon — are as follows.

Table — Hours from midnight / from noon; distances from the meridian.

Horæ à med. noc.	11	10	9	8	7	6	5	4	3	2	1
Horæ à merid.	1	2	3	4	5	6	7	8	9	10	11
Distantiæ à	15.	30.	45.	60.	75.	90.	105	120	135	150	165
Meridiano (G. M.)	0	0	0	0	0	0	.0	.0	.0	.0	.0

Next, above the southern face of a vertical circle that declines eastward by 30 degrees from the vertical circle properly so called: at latitude 42 degrees, the antarctic pole is elevated 40 degrees 3 minutes, and the complement of this altitude comprises 49 degrees 57 minutes. Therefore the meridian altitude of Cancer will comprise 26 degrees 27 minutes, to which corresponds the sine 44541; while the meridian depression is 73 degrees 27 minutes, whose sine is 95857. Half the sum of the said sine of the meridian altitude and the sine of the depression will be 70199. The distances which the hours reckoned from sunset, in the parallel of Cancer, have from the proposed meridian of the vertical circle above the horizon — because the Sun, standing at the beginning of Cancer, illuminates the southern face of that same vertical circle both before and after

noon — are as follows, since the proper meridian of that circle recedes from the meridian of the horizon toward the east by 40 degrees 48 minutes. Hence, since in the same parallel of Cancer the 14th hour is distant toward the east from noon by 37 degrees, if this distance is subtracted from 40 degrees 48 minutes, there remain 3 degrees 48 minutes for the distance of the 14th hour, reckoned from sunset, toward the west from the proper meridian of the declining vertical. Likewise, since the 13th hour recedes toward the east from noon by 52 degrees, if from these 40 degrees 48 minutes are taken away, there will remain 11 degrees 12 minutes for the distance of the 13th hour from that proper meridian toward the east.

Table — Hours in Cancer (♋), reckoned from sunset; distances from the proper meridian of the declining vertical (first part).

Horæ ♋, ab occ.	13	12	11	10	9	8	7	6	5	4	3	2
Distantiæ à	11	26	41	56	71	86	101	116	131	146	161	176
Meridiano proprio	.1	.1	.1	.1	.1	.1	.12	.12	.12	.12	.12	.12
(G. M.)	2	2	2	2	2	2						

Table — Hours in Cancer (♋), reckoned from sunset; distances from the proper meridian of the declining vertical (second part).

	1											
Horæ ♋, ab occ.	4	15	16	17	18	19	20	21	22	23	24	1
Distantiæ à	3.	18	33	48	63	78	93	108	123	138	153	168
Meridiano proprio	4	.4	.4	.4	.4	.4	.4	.48	.48	.48	.48	.48
(G. M.)	8	8	8	8	8	8	8					

Likewise, the distances from the same proper meridian that the hours reckoned from sunrise have, in the same parallel of Cancer, are as follows — because the proper meridian of the said declining vertical circle recedes from the meridian of the horizon toward the east by 40 degrees 48 minutes. Hence, since in the parallel of Cancer the 5th hour from sunrise is distant toward the east from noon by 38 degrees, if this distance is subtracted from 40 degrees 48 minutes, there will remain 2 degrees 48 minutes for the distance of the 5th hour from sunrise from the proper meridian of the declining vertical, toward the west. Likewise, since the 4th hour from sunrise is distant toward the east from noon by 53 degrees, if from these 40 degrees 48 minutes are taken away, the remainder will be 12 degrees 12 minutes for the distance of the 4th hour from sunrise from that proper meridian toward the east.

Table — Hours in Cancer (♋), reckoned from sunrise; distances from the proper meridian of the declining vertical (first part).

Horæ ☿, ab or.	4	3	2	1	24	23	22	21	20	19	18	17
Distantiæ à proprio	12	27	42	57	72	87	102	117	132	147	162	177
Meridiano (G. M.)	.1	.1	.1	.1	.1	.1	.12	.12	.12	.12	.12	.12
	2	2	2	1	2	2						

(The value 57.11 is printed thus in both column tiles, breaking the otherwise uniform "...12" minutes-column; flagged below as a likely compositor error for 57.12.)

Table — Hours in Cancer (♋), reckoned from sunrise; distances from the proper meridian of the declining vertical (second part).

Horæ ☿, ab or.	5	6	7	8	9	10	11	12	13	14	15	16
Distantiæ à proprio	2.	17	32	47	62	77	92	107	122	137	152	167
Meridiano (G. M.)	4	.4	.4	.4	.4	.4	.4	.48	.48	.48	.48	.48
	8	8	8	8	8	8	8					

But the distances which the hours reckoned from noon and from midnight have from the same meridian of the said vertical circle — the same in all parallels, since in all parallels the said meridian is distant from noon (that is, from the 12th hour) toward the east by 40 degrees 48 minutes, and hence the 9th hour from midnight is distant from that meridian toward the east by 4 degrees 12 minutes, while the 10th hour from midnight recedes from it toward the west by 10 degrees 48 minutes — are contained in the following table.

Table — Hours from midnight, then from noon (A meridie); distances from the proper meridian of the declining vertical (first part).

Horæ à med. noc.	9	8	7	6	5	4	3	2	1	A meri die	12	11	10
Distantiæ à proprio	4.	19	34	49	64	79	94	10	12	—	13	15	16
Meridiano (G. M.)	1	.	.	.	.	.	.	9.	4.		9.	4.	9.
	2	12	12	12	12	12	12	12	12		12	12	12

Table — Hours from midnight, then from noon (A meridie); distances from the proper meridian of the declining vertical (second part).

	A												
	1	1	1	meri									
Horæ à med. noc.	0	1	2	die	1	2	3	4	5	6	7	8	9
Distantiæ à	1	2	4	—	5	7	8	10	11	13	14	16	17
proprio	0.	5.	0.		5.	0.	5.	0.4	5.4	0.4	5.4	0.4	5.4
Meridiano (G. M.)	4	4	4		4	4	4	8	8	8	8	8	8
	8	8	8		8	8	8						

(The "A meridie" [at noon] column is a header cell only; its distance cell is left blank in the print.)

Now, care must be taken to set down together the distances of those hours contained in the eastern hemisphere and, separately, of those contained in the western, so that in laying out sundials we may more easily grasp which hours incline eastward from the line of the style (which serves as the meridian line) and which tend westward. Thus you see that in all the examples given above, the hours contained in the eastern hemisphere are placed first, and then those in the western. The eastern hemisphere, moreover, is separated from the western by the proper meridian circle of the proposed great circle, above which the Sun's altitudes are set for investigation.

[Margin: An easy way to find the Sun's distances from the proper meridian of the given great circle.]

You see, therefore, that the distances of all the hours from the proper meridian, in either hemisphere, can be found with no trouble, once the distances of the two nearest hours lying on either side of that meridian have first been found. For if 15 degrees are added to either distance, the distance of the next hour will be found; and so, by the continual addition of 15 degrees, the distances of all the hours will be found.

[Margin: The Sun's altitudes above one face of the given great circle, at each hour of one parallel, are equal to the Sun's altitudes above the other face of the same circle at the same hours of the opposite parallel.]

That the altitudes of all 24 hours of any parallel, found above any great circle, are equal to the altitudes of the same 24 hours of the opposite parallel above the same great circle — provided that those which before were counted from noon or from sunrise are now counted from midnight or from sunset, and those which before fell on one face of that great circle now fall on the opposite face, and vice versa — we shall prove as follows. Let some hour-circle, drawn through the poles

of the world, be understood to pass through that point of any parallel — namely ♊ (Capricorn) — where it is cut by the great circle above which the Sun's altitudes are set for investigation, and to pass, in the opposite parallel, namely ♋ (Cancer), through a point which [is set] diametrically opposite the former point through the sphere. For since the arc of the said hour-circle between the tropic point ♊ and the antarctic pole is equal to the arc of the same hour-circle between the tropic point ♋ and the arctic pole — because each arc is the complement of the declination of either tropic — if the common arc, less than a semicircle, between the tropic point ♊ and the arctic [pole] is added, the composite arcs become equal. Since, then, the arc comprehended between the two poles is a semicircle, the other arc between the said two [points] will also be a semicircle. This can be perceived more clearly than daylight from a material sphere, if the said hour-circle is imagined, in any position whatever, to be a colure of the equinoxes — so that where it cuts the tropic ♊ above the horizon, through that point [a great circle is understood to pass]. Thereby it comes about that the same [colure] necessarily passes through the point where that same great circle [cuts it]; for there the hour-circle is cut in two, since that point is [opposite] through a diameter, as we have shown, and by Proposition 11 of Book 1 of Theodosius, [that] all great circles in a sphere mutually [bisect one another]. If, then, the point of the earlier parallel belongs to some hour reckoned from noon, the later [point] of the other [parallel] will belong to [the same hour reckoned] from midnight, and conversely; likewise, if the earlier point looks to the hours reckoned from sunset in the earlier parallel,

it will [in the] later [parallel] look to the hours reckoned from sunrise in the opposite parallel, and conversely — because these points lie in different semicircles of the said hour-circle, as is evident; and that different semicircles belong to different hours (so that one indicates the hour from noon or from sunset, the other the same hour from midnight or from sunrise) we showed in Propositions 9 and 10 of Book 1. Nor does it matter that the said hour-circle is not an hour-circle of [the hours reckoned] from sunrise or from sunset, since it is supposed to pass through the poles; for if through those same two points of the opposite parallels an hour-circle of [the hours] from sunrise or from sunset were understood to be drawn — one that would of course touch the greatest of the ever-apparent parallels — those two points would still lie in different semicircles of this circle. From this it comes about that, by as much as the hours reckoned from noon or from sunset are distant from the earlier point, proceeding against the succession of the signs, by just so much are the hours reckoned from midnight or from sunrise distant from the later point, and conversely — yet in such a way that, if those [former] hours stand above the northern or upper face

of the proposed circle and are in the southern parallel, these [latter] stand above the opposite face, namely the southern or lower, and in the opposite, northern parallel, as is evident from a material sphere. Since, then, those two opposite parallels have equal inclinations to the proposed great circle, the hours reckoned from noon or from sunset will have, above one face of the said great circle, the same altitudes that the opposite hours reckoned from midnight or from sunrise have above the other face. All of which the very computation of the Sun's altitudes will teach more clearly than daylight.

There now follow the tables of the Sun's altitudes above all the great circles from which the dials of Books 2 and 3 are equidistant — computed partly for the hours reckoned from noon and midnight, partly for the hours reckoned from sunrise and sunset, and also for unequal hours, at the northern latitude of 42 degrees, with the Sun traversing the tropic of Cancer (♋), the Equator, and the tropic of Capricorn (♐). Similar tables can be drawn up, by the same rules, for any other great circles whatever, and for any other latitudes, and with the Sun traversing any parallel whatever — for hours whether equal or unequal, whether these be whole hours, halves, quarters, or other fractions of hours — provided their distance from the meridians of the said great circles is correctly taken, as we taught above. Now any hours equally distant on either side from the proper meridian of the proposed great circle have equal altitudes, as happens on the horizon, on the vertical circle, on the meridian, and on the circle of the 6th hour from noon or from midnight, as regards the hours numbered from noon and from midnight, and so on.

Moreover, in tables 3, 4, 10, and 11 we have computed the Sun's altitudes above both faces of the vertical circle properly so called and of the declining vertical, so that you may also learn by example that the hours of Cancer reckoned from noon have, above the southern face, the same altitudes that the hours of Capricorn reckoned from midnight have above the northern face, and vice versa, as we have just demonstrated.

I. The Sun's altitudes above the horizon, computed for the hours reckoned from noon and from midnight, at latitude 42 degrees.

Cancer (♋):

Horæ ♋, à med. noc.	5	6	7	8	9	10	11	12
Horæ ♋, à meridie.	7	6	5	4	3	2	1	0
Altitudines Solis (G. M.)	5.	15.	26.	37.	48.	58.	67.	71.
	11	29	19	25	29	59	41	30

Aries and Libra (♈ & ♎):

Horæ ♈, & ♎, à med. no.	6	7	8	9	10	11	12
Horæ ♈, & ♎, à mer.	6	5	4	3	2	1	0
Altitudines Solis (G. M.)	0. 0	11. 5	21. 49	31. 42	40. 4	45. 52	48. 0

Capricorn (♐):

Horæ ♐, à med. noc.	8	9	10	11	12
Horæ ♐, à meridie.	4	3	2	1	0
Altitudines Solis (G. M.)	4. 14	12. 25	18. 52	23. 3	24. 30

II. The Sun's altitudes above the horizon, computed for the hours reckoned from sunset, at latitude 42 degrees.

		20 [recte: 10]						
Horæ ☊, ab occ.	9		11	12	13	14	15	16
Altitudines Solis (G. M.)	0. 40	10. 35	21. 12	32. 13	43. 21	54. 12	63. 59	70. 36
Horæ ☊, ab occasu.	17	18	19	20	21	22	23	24
Altitudines Solis (G. M.)	70. 20	63. 24	53. 30	42. 36	31. 28	20. 29	9. 54	0. 0

These three pages continue, without a break, the table series of the Scholium to Problema 1 of Book V (begun in the preceding pages under "SEQVVNTVR TABELLÆ"). They consist entirely of the Sun's altitude tables. Each table's Latin row-labels are kept verbatim; the standard phrases are:

- *Horæ [sign], à med.noc.* = "Hours of [sign], reckoned from midnight"
- *Horæ [sign], à meridie.* = "Hours of [sign], reckoned from noon"
- *Horæ [sign], ab occasu.* = "Hours of [sign], reckoned from sunset"
- *Altitudines Solis (G. M.)* = "The Sun's altitudes (degrees. minutes)"

Zodiac sorts: ♈ Aries, ♎ Libra, ☊ Cancer, ♐ Capricorn.

[Continuation of Table II — The Sun's altitudes above the horizon,
computed for the hours reckoned from sunset, at latitude 42 degrees]

Aries and Libra (♈ & ♎): — each column carries two hour-labels (symmetric
about hour 18 = noon).

Horæ ♈, & ♎, ab occasu.	12	13	14	15	16	17	18
(ab occasu, complementary)	24	23	22	21	20	19	18
Altitudines Solis (G. M.)	0. 0	11. 5	21. 49	31. 42	40. 4	45. 52	48. 0

Capricorn (♐):

Horæ ♐, ab occasu.	16	17	18	19	20	21	22	23	24
Altitudines Solis (G. M.)	8.	15.	21.	24.	24.	21.	16.	8.	0.
	14	41	8	5	11	25	7	47	0

III. The Sun's altitudes above the southern face of the vertical circle,
computed for the hours from noon and from midnight, at latitude 42
degrees.

Capricorn (♐):

Horæ ♐, à med.noc.	5	6	7	8	9	10	11	12
Horæ ♐, à meridie.	7	6	5	4	3	2	1	0
Altitudines Solis (G. M.)	7.	17.	27.	37.	46.	55.	62.	65.
	54	14	5	6	54	52	45	30

Aries and Libra (♈ & ♎):

Horæ ♈, & ♎, à med.no.	6	7	8	9	10	11	12
Horæ ♈, & ♎, à merid.	6	5	4	3	2	1	0
Altitudines Solis (G. M.)	0.	9. 58	19. 33	28. 14	35. 25	40. 16	42. 0
	0						

Cancer (♋):

Horæ ♋, à med.noc.	8	9	10	11	12
Horæ ♋, à meridie.	4	3	2	1	0
Altitudines Solis (G. M.)	0. 36	7. 54	13. 36	17. 14	18. 30

III. The Sun's altitudes above the northern face of the vertical circle, computed for the hours from noon and from midnight, at latitude 42 degrees.

Cancer (♋):

Horæ ♋, à meridie.	5	6	7	8	9	10	11	12
Horæ ♋, à med.noc.	7	6	5	4	3	2	1	0
Altitudines Solis (G. M.)	7.	17.	27.	37.	46.	55.	62.	65.
	54	14	5	6	54	52	45	30

Aries and Libra (♈ & ♎):

Horæ ♈, & ♎, à mer.	6	7	8	9	10	11	12
Horæ ♈, & ♎, à med.no.	6	5	4	3	2	1	0
Altitudines Solis (G. M.)	0.	9. 58	19. 33	28. 14	35. 25	40. 16	42. 0
	0						

Capricorn (♐):

Horæ ♐, à meridie.	8	9	10	11	12
Horæ ♐, à med.nocte.	4	3	2	1	0
Altitudines Solis (G. M.)	0. 36	7. 54	13. 36	17. 14	18. 30

V. The Sun's altitudes above the southern face of the vertical circle, computed for the hours reckoned from sunset, at latitude 42 degrees.

Capricorn (♐): [table spans the page boundary]

Horæ ♐, ab occasu.	1												
	2	13	14	15	16	17	18	19	20	21	22	23	24
Altitudines Solis (G. M.)	3.	12.	21.	31.	41.	51.	59.	64.	64.	59.	51.	42.	32.
	1	11	47	45	44	15	27	41	53	55	51	23	27
	5												

Aries and Libra (♈ & ♎): — each column carries two hour-labels (symmetric about hour 18 = noon).

Horæ ♈, & ♎, ab occasu.	12	13	14	15	16	17	18
(ab occasu, complementary)	24	23	22	21	20	19	18

Horæ Υ , & ϱ , ab occasu.	12	13	14	15	16	17	18
Altitudines Solis (G. M.)	0.	9. 58	19. 33	28. 14	35. 25	40. 16	42. 0
	0						

Cancer (ϑ):

Horæ ϑ , ab occ.	13	14	15	16	17	18	19	20
Altitudines Solis (G. M.)	4.	11.	15.	18.	18.	15.	10.	4.
	40	10	49	13	8	35	47	11

VI. The Sun's altitudes above both faces of the meridian circle, computed for the hours from noon and from midnight, for any latitude whatsoever.

Each column header carries a pair of hours (the two hours, from noon and from midnight, sharing that altitude).

Cancer and Capricorn (ϑ & φ):

	0.						
Horæ ϑ , & φ , à med.no.	12	1. 11	2. 10	3. 9	4. 8	5. 7	6. 6
Horæ ϑ , & φ , à meridiē.	0.	1. 11	2. 10	3. 9	4. 8	5. 7	6. 6
	12						
Altitudines Solis (G. M.)	0. 0	13.	27.	40.	52.	62.	66.
		44	18	26	35	21	30

Aries and Libra (Υ & ϱ):

Horæ Υ , & ϱ , à med.no.	0. 12	1. 11	2. 10	3. 9	4. 8	5. 7	6. 6
Horæ Υ , & ϱ , à merid.	0. 12	1. 11	2. 10	3. 9	4. 8	5. 7	6. 6
Altitudines Solis (G. M.)	0. 0	15. 0	30. 0	45. 0	60. 0	75. 0	90. 0

VII. The Sun's altitudes above both faces of the meridian circle, computed for the hours reckoned from sunset, at latitude 42 degrees.

Cancer (ϑ): [presented in two rows, hours 9–17 then 18–24]

Horæ ϑ , ab occasu.	9	10	11	12	13	14	15	16	17
Altitudines Solis (G. M.)	58.	65.	65.	57.	46.	33.	20.	6.	7.
	15	32	15	33	16	30	6	25	20

Horæ ☿, ab occasu.	18	19	20	21	22	23	24
Altitudines Solis (G. M.)	20.	34.	47.	58.	56.	65.	57.
	59	22	5	15	32	15	33

Aries and Libra (♈ & ♎): — each column carries four hour-labels (two upper, two lower).

Horæ ♈, & ♎, ab occasu.	6.				8.	9.	10.	11.	
	18	7. 17			16	15	14	13	12
(ab occasu, complementary)	18.	19. 5			20.	21.	22.	23.	24
	6				4	3	2	1	
Altitudines Solis (G. M.)	0. 0	10. 0 [recte: 15. 0]			30.	45.	60.	75.	90.
					0	0	0	0	0

Capricorn (♐):

Horæ ♐, ab occasu.	16	17	18	19	20	21	22	23	24
Altitudines Solis (G. M.)	47.	34.	20.	7.	6.	20.	33.	46.	57.
	5	22	59	20	25	6	30	16	33

VIII. The Sun's altitudes above both faces of the circle of the 6th hour from noon and from midnight — which is the horizon in the right sphere — computed for the hours from noon and from midnight, for any latitude whatsoever.

Each column header carries a pair of hours.

Cancer and Capricorn (♋ & ♐):

Horæ ☊, & ♐, à med.no.	6.								
	6	7. 5	8. 4	9. 3	10. 2	11. 1	12. 0		
Horæ ☊, & ♐, à meridie.	6.	7. 5	8. 4	9. 3	10. 2	11. 1	12. 0		
	6								
Altitudines Solis (G. M.)	0.	13. 44	27. 18	40. 26	52. 35	62. 21	66. 30		
	0								

Aries and Libra (♈ & ♎):

Horæ ♈, & ♎, à med.no.	6. 6	7. 5	8. 4	9. 3	10. 2	11. 1	12. 0
Horæ ♈, & ♎, à mer.	6. 6	7. 5	8. 4	9. 3	10. 2	11. 1	12. 0

Horæ Υ , & ϱ , à med.no.	6. 6	7. 5	8. 4	9. 3	10. 2	11. 1	12. 0
Altitudines Solis (G. M.)	0. 0	15. 0	30. 0	45. 0	60. 0	75. 0	90. 0

IX. The Sun's altitudes above the upper face of the circle of the 6th hour from noon and from midnight, computed for the hours reckoned from sunset, at latitude 42 degrees.

Cancer (ϑ): [presented in two rows, hours 11–17 then 18–22]

Horæ ϑ , ab occasu.	11	12	13	14	15	16	17
Altitudines Solis (G. M.)	7. 20	21. 0	34. 22	47. 5	58. 15	65. 32	65. 15

Horæ ϑ , ab occasu.	18	19	20	21	22
Altitudines Solis (G. M.)	57. 35	46. 16	33. 30	20. 6	6. 25

Aries and Libra (Υ & ϱ): — each column carries four hour-labels (two upper, two lower).

Horæ Υ , & ϱ , ab occasu.	12	13. 11	14. 10	15. 9	16. 8	17. 7	18. 6
(ab occasu, complementary)	24	1. 23	2. 22	3. 21	4. 20	5. 19	18. 6
Altitudines Solis (G. M.)	0. 0	15. 0	30. 0	45. 0	60. 0	75. 0	90. 0

Capricorn (φ):

Horæ φ , ab occasu.	16	17	18	19	20	21	22	23	24
Altitudines Solis (G. M.)	33. 30	46. 16	57. 35	65. 15	65. 32	58. 15	47. 5	34. 22	21. 0

X. The Sun's altitudes above the southern face of a vertical circle declining from the meridian toward the east by 30 degrees, computed for the hours from noon and from midnight, at latitude 42 degrees.

Capricorn (φ): [one continuous table; the "†" column marks the intermediate hour of maximum altitude between hours 9 and 10, and the "A meridie" cell is a divider between the midnight-reckoned and noon-reckoned hours]

Horæ φ , à med.noc.	2	3	4	5	6	7	8	9	†
Altitudines Solis (G. M.)	1. 28	11. 50	22. 50	34. 12	45. 40	56. 51	66. 51	73. 4	73. 27

Horæ ♈, à med.noc.	10	11	12	A meridie.	1	2	3	4
Altitudines Solis (G. M.)	71. 7	62. 42	52. 0	—	40. 38	29. 10	17. 56	7. 11

Aries and Libra (♈ & ♎):

Horæ ♈, & ♎, à med.no.	4	5	6	7	8	9	†
Altitudines Solis (G. M.)	8. 15	19. 28	30. 1	39. 17	46. 18	49. 46	49. 57

Book Five (continued)

These pages continue the run of hour-and-altitude tables from Problem 1's Scholium in Book Five. All altitudes are given as "G. M." — degrees (gradus) and minutes (minuta). A cross (†) placed in a column, in place of an hour, marks the meridian altitude (see the note below the first table). A blank cell headed "A meridie" separates the hours reckoned from midnight from those reckoned from noon.

Table X (concluded)

Concluding the table begun on the preceding page: the Sun's altitudes above the southern face of a vertical circle declining from the meridian toward the east by 30 degrees, at latitude 42 degrees.

Aries and Libra (♈ & ♎) — afternoon portion (the morning portion stands on the preceding page):

Horæ ♈, & ♎, à med. noc.	10	11	12	A meridie.	1	2	3
Alt. Solis (G. M.)	48. 45	43. 34	35. 25	—	25. 29	14. 35	3. 13

Cancer (♋):

Horæ ♋, à media nocte.	5	6	7	8	9	†	10	11	12	A meridi e.	1
Alt. Solis (G. M.)	2. 48	11. 40	18. 54	23. 57	26. 20	26. 27	25. 39	22. . 3	15. 57	—	8. 2

In this table, and in those that follow, the value placed beneath this sign † is the meridian altitude — that is, the altitude the Sun has above the circle in question when it stands on that circle's own meridian. Since no whole hour falls exactly on that meridian, I have chosen to put the said sign in place of an hour, and to record the meridian altitude beneath it.

[Margin: What the cross-sign signifies in the tables.]

XI. The Sun's altitudes above the northern face of a vertical circle declining from the north toward the west by 30 degrees, computed for the hours reckoned from noon and from midnight, at latitude 42 degrees.

Cancer (♋):

Hor. ♋, à merid.	2	3	4	5	6	7	8	9	†
Alt. Solis (G. M.)	1. 28	11. 50	22. 50	34. 12	45. 40	56. 51	66. 51	73. 4	73. 27

Horæ ♋, à merid.	10	11	12	A med. nocte.	1	2	3	4
Alt. Solis (G. M.)	71. 7	61. 42 [?]	52. 0	—	40. 38	29. 10	17. 56	7. 11

Aries and Libra (♈ & ♎):

Horæ ♈, & ♎, à mer.	4	5	6	7	8	9	†
Alt. Solis (G. M.)	8. 15	19. 28	30. 1	39. 17	46. 18	49. 46	49. 57

Horæ ♈, & ♎, à mer.	10	11	12	A med. nocte.	1	2	3
Alt. Solis (G. M.)	48. 45	43. 34	35. 25	—	25. 29	14. 35	3. 13

Capricorn (♐):

Hor. ♊, à merid.	5	6	7	8	9	†	10	11	12	A me. noct.	1
Alt. Solis (G. M.)	2.	11.	20.	23.	26.	26.	25.	22.	15.	—	8.
	48	40	7	57	20	27	39	3	57		2

You see, then, from tables 3, 4, 10 and 11, that the Sun has the same altitudes on both faces of a vertical circle — whether one properly so called or a declining one — at the same numbered hours, provided that the hours counted from noon in the Cancer parallel on one face are counted, on the other face, in the Capricorn parallel from midnight, and vice versa. The same is to be understood for the other planes that follow. We have, moreover, set out the hours and their altitudes for one face only.

XII. The Sun's altitudes above the southern face of a vertical circle declining from the south toward the east by 30 degrees, computed for the hours reckoned from sunset, at latitude 42 degrees.

Capricorn (♑):

Hor. ♑, ab occasu.	10	11	12	13	14	15	16	†
Alt. Solis (G. M.)	6.	16.	28.	39.	50.	61.	70.	73.
	13	55	7	33	57	45	31	27

(Table XII, Capricorn, continued from the foot of the previous page:)

Hor. ♑, ab occasu.	17	18	19	20	21	22	23	24
Alt. Solis (G. M.)	73.	67.	57.	46.	35.	23.	12.	2.
	17	40	51	44	16	53	51	24

Aries and Libra (♈ & ♎):

Horæ ♈, & ♎, ab occasu.	10	11	12	13	14	15	†
Alt. Solis (G. M.)	8.	19. 28	30.	39. 17	46. 18	49. 46	49. 57
	15		1				

Horæ ♈, & ♎, ab occasu.	16	17	18	19	20	21
Alt. Solis (G. M.)	48. 45	43. 34	35. 25	25. 29	14. 35	3. 13

Cancer (♋):

Horæ ☿, ab occa.	10	11	12	13	†	14	15	16	17	18
Alt. Solis (G. M.)	7.	15.	21.	25.	26.	26.	24.	19.	11.	3.
	42	46	55	36	27	21	5	4	53	3

To help you understand the principle by which the Sun's altitudes above one face of a great circle equal its altitudes above the other face at the same hours — with the hours reckoned from noon or from sunset converted into hours reckoned from midnight or from sunrise, and the northern parallels changed into southern ones, and vice versa — I will make the matter clear with an example or two. Suppose, for instance, that the Sun's altitude is to be found above the northern face of a vertical circle declining from the north toward the west by 30 degrees, at hour 13 from sunrise, in the parallel of Cancer. I look up hour 13 from sunset in the parallel of Capricorn, whose altitude above the southern face of the said vertical circle amounts to 39 degrees 33 minutes. The Sun, therefore, has that same altitude at hour 13 from sunrise in the parallel of Cancer, above the northern face of the vertical circle in question. In the same way, since the Sun at hour 10 from midnight in the parallel of Cancer has, above the southern face of the same vertical circle, an altitude of 25 degrees 39 minutes, the Sun will have the same altitude at hour 10 from noon in the parallel of Capricorn, above the northern face of the same vertical circle — as is evident from tables 10 and 11. Finally, since above the superior face of a great circle declining 30 degrees from the horizon and standing at right angles to the vertical circle properly so called (the one that faces the zenith and the west), the Sun at hour 4 from noon in the parallel of Cancer has an altitude of 67 degrees 24 minutes — as appears from the following table — the Sun will have the same altitude at hour 4 from midnight in the parallel of Capricorn, above the inferior face of the same great circle, and so on. The same is to be understood for all the other tables.

[Margin: How the same altitudes are also fitted to the opposite faces of great circles.]

XIII. The Sun's altitudes above the superior face of a great circle declining 30 degrees from the horizon and standing at right angles to the vertical circle properly so called (the one facing the zenith and the west), computed for the hours reckoned from noon and from midnight, at latitude 42 degrees.

Cancer (☿):

Horæ ☾, à media noc.	8	9	10	11	12	A meridie.	1	2	†
Alt. Solis (G. M.)	7. 26	18. 56	30. 52	43. 3	55. 13	—	66. 54	76. 17	78. 5

Hor. ☾, à merid.	3	4	5	6	7	8	9
Alt. Solis (G. M.)	67. 33	67. 24	55. 45	43. 35	31. 24	19. 27	7. 55

Aries and Libra (♈ & ♎):

Horæ ♈, & ♎, à media nocte.	9	10	11	12	A meridie.	1	2	†
Alt. Solis (G. M.)	5. 50	17. 54	29. 30	40. 4	—	48. 41	53. 50	54. 35

Hor. ♈, & ♎, à mer.	3	4	5	6	7	8
Alt. Solis (G. M.)	53. 57	49. 0	40. 30	30. 0	18. 26	6. 22

Capricorn (♐) — completing Table XIII:

Horæ ♐, à med. noc.	10	11	12	A meridi e.	1	2	†	3	4	5	6	7
Alt. Solis (G. M.)	2. 55	12. 44	21 .3	—	27. 14	30. 37	31 .5	30. 42	27. 27	21. 23	13 .8	3. 23

XIV. The Sun's altitudes above the superior face of a great circle declining 30 degrees from the horizon and standing at right angles to the vertical circle properly so called (the one facing the zenith and the west), computed for the hours reckoned from sunrise, at latitude 42 degrees.

Cancer (♋):

Hor. ☊, ab ortu.	3	4	5	6	7	8	9	10	†
Alt. Solis (G. M.)	1. 34	12. 43	24. 27	36. 32	48. 45	60. 47	71. 49	78. 4	70. 5

Horæ ☿, ab ortu.	11	12	13	14	15
Alt. Solis (G. M.)	72. 53	62. 5	50. 6	37. 53	25. 47

Aries and Libra (♈ & ♎):

Horæ ♈, & ♎, ab ortu.	3	4	5	6	7	8	†
Alt. Solis (G. M.)	5. 50	17. 54	29. 30	40. 4	48. 41	53. 50	54. 35

Horæ ♈, & ♎, ab ortu.	9	10	11	12	13	14
Alt. Solis (G. M.)	53. 57	49. 0	40. 30	30. 0	18. 26	6. 22

Capricorn (♐):

Hor. ♐, ab ortu.	3	4	5	6	†	7	8	9	10	11
Alt. Solis (G. M.)	8. 18	17. 24	24. 40	29. 26	31. 5	31. 5	29. 31	24. 31	17. 12	8. 5

XV. The Sun's altitudes above the superior face of a great circle standing at right angles to the meridian and inclined to the horizon on the southern side by 68 degrees, computed for the hours reckoned from noon and from midnight, at latitude 42 degrees.

In this table each altitude carries two hour-labels — the same hour reckoned from midnight and reckoned from noon.

Cancer (♋):

☿ reckoning

Hor. à med. noc.	0	1	2	3	4	5	6
Horæ à merid.	12	11	10	9	8	7	6
Alt. Solis (G. M.)	43. 30	42. 40	40. 16	36. 37	32. 7	27. 7	22. 0

☿ reckoning

Hor. à med. noc.	7	8	9	10	11	12
Horæ à merid.	5	4	3	2	1	0

☿ reckoning

Alt. Solis (G. M.)	17. 9	12. 35	8. 48	5. 55	4. 7	3. 30

Aries and Libra (♈ & ♎):

♈ & ♎ reckoning

Horæ à med. noc.	0	1	2	3	4	5	6
Horæ à mer.	12	11	10	9	8	7	6
Alt. Solis (G. M.)	20. 0	19. 17	17. 14	14. 0	9. 51	5. 5	0. 0

The Sun, when it is in the parallel of Capricorn, does not illuminate the superior face of this circle but only the inferior one; therefore there are no altitudes of the hours of the Capricorn parallel above the superior face. And the gnomon's shadow, when the Sun is at the first point of Cancer, describes an ellipse.

This opening continues the long series of numbered altitude tables in the Scholium to Problema 1 of Book V (*Liber Quintus*). Each table gives the Sun's altitudes above the upper face of a particular declining-and-inclined great circle, computed for latitude 42°. The tables are printed as bare numbered rubrics with no connecting prose; the altitudes are given in the printed "degrees. minutes" form (e.g. 9. 16 = 9°16'). The dagger † marks the meridian (culmination) column; A meridie divides the midnight-reckoned hours from the noon-reckoned ones.

XVI.

XVI. The Sun's altitudes above the upper face of a great circle declining from the meridian toward the east by 40°, and inclined to the horizon on its northern side by 20°, computed for the hours reckoned from noon and from midnight, for latitude 42°.

Hor. ☿ [Cancer], à med.

noc.	5	6	7	8	9	10	11	†	12
Alt. Solis (G. M.)	9. 16	22. 2	35. 9	48. 28	61. 56	75. 26	87. 35	87. 47	76. 55

Horæ ☿, à meridie

	1	2	3	4	5
Alt. Solis (G. M.)	63. 26	49. 58	36. 37	23. 29	10. 40

Horæ ♈ & ♎ [Aries & Libra], à media nocte	6	7	8	9	10	11	†	12
Alt. Solis (G. M.)	12. 44	26. 3	38. 53	50. 41	60. 5	64. 16	64. 17	60. 52

Hor. ♈ & ♎, à mer.	1	2	3	4	5
Alt. Solis (G. M.)	51. 53	40. 16	27. 30	14. 14	0. 45

Horæ ♐ [Capricorn], à med. noc.	6	7	8	9	10	11	†	12
Alt. Solis (G. M.)	1. 40	13. 17	23. 45	32. 26	38. 27	40. 46	40. 47	38. 54

Horæ ♐, à meridie	1	2	3	4
Alt. Solis (G. M.)	33. 16	24. 49	14. 30	3. 0

XVII.

XVII. The Sun's altitudes above the upper face of a great circle declining from the meridian toward the west by 20°, and inclined to the horizon on its northern side by 70°, computed for the unequal (temporal) hours, for latitude 42°.

Hor. ♐, inæquales	12	1	2	3	4	5	6	7	†
Alt. Solis (G. M.)	11. 37	21. 10	30. 55	40. 48	50. 47	60. 50	70. 56	80. 59	88. 12

Horæ ♐, inæquales	8	9	10	11	12	Hora 1, ♓ & ♒ [Pisces & Aquarius]
Alt. Solis (G. M.)	87. 45	78. 21	68. 17	58. 12	48. 9	17. 25

We have here also added the Sun's altitude for the first unequal hour in the parallel of Pisces [♓] and Aquarius [♒], because from the length of the shadow belonging to this altitude the first unequal hour is to be drawn on the sundial that lies parallel to

this great circle — the circle above which the Sun's altitudes have been computed — as will become clear in its proper place.

Hor. Υ & Ω ,

inæqual.	2	3	4	5	6	7	†	8	9	10	11	12
Alt. Solis (G. M.)	8.	21.	34.	47	57.	64	64.	63.	55.	44.	31.	18.
	15	42	47	.7	39	.4	42	12	36	32	55	47

The Cancer (♋) portion of Table XVII, in unequal hours, continues at the head of this page:

Horæ ♋ , inæquales	3	4	5	6	7	†	8	9	10	11
Alt. Solis (G. M.)	0.	15.	27.	37.	41.	41.	38.	30.	18.	3.
	37	13	52	11	10	12	34	14	9	53

XVIII.

XVIII. The Sun's altitudes above the upper face of a great circle declining from the meridian toward the east by 45° , and inclined to the horizon on its northern side by $51^\circ 51'$, computed for the hours reckoned from noon and from midnight, for latitude 42° .

Hor. ♋ & ♌ , à med. no.	4	5	6	7	8	9	†
Alt. Solis (G. M.)	3.	17. 10	30. 40	43. 37	55. 21	64.	66. 30
	28					6	

Horæ ♋ & ♌ , à media nocte	10	11	12	A meridi e	1	2	3
Alt. Solis (G. M.)	60.	60. 15	49.	—	37.	13. 54	10.
	13	[?]	40		11	[?]	17

Hor. Υ & Ω , à me. no	4	5	6	7	8	9	†
Alt. Solis (G. M.)	3.	18. 47	33. 47	48. 47	63. 47	78. 47	90.
	47						0

Horæ Υ & Ω , à media nocte	10	11	12	A meridie	1	2	3
Alt. Solis (G. M.)	86. 13	71. 13	56. 13	—	41. 13	26. 13	11. 13

XIX.

XIX. The Sun's altitudes above the upper face of a great circle declining from the north toward the west by 20° , and inclined to the horizon on its southern side by 30° , computed for the hours reckoned from noon and from midnight, for latitude 42° .

Horæ $\mathfrak{S}$, à med. nocte	1	†	2	3	4	5	6
Alt. Solis (G. M.)	2. 11	—	1. 44	2. 37	4. 47	8. 4	12. 18

Horæ $\mathfrak{S}$, à med. noc.	7	8	9	10	11	12
Alt. Solis (G. M.)	17. 15	22. 39	28. 12	33. 35	38. 24	42. 14

Horæ $\mathfrak{S}$, à meridie	1	†	2	3	4	5	6
Alt. Solis (G. M.)	44. 38	45. 17	45. 15	44. 1	41. 6	36. 53	31. 50

Horæ $\mathfrak{S}$, à meridie	7	8	9	10	11	12
Alt. Solis (G. M.)	26. 21	20. 49	15. 33	10. 48	6. 51	3. 55

The Aries-and-Libra (Υ & Ω) portion of Table XIX continues at the head of this page:

Horæ Υ & Ω , à media nocte	8	9	10	11	12	A meridie	1	†
Alt. Solis (G. M.)	0. 56	6. 24	11. 30	15. 53	19. 13	—	21. 14	21. 47

Hor. Υ & Ω , à mer.	2	3	4	5	6	7
Alt. Solis (G. M.)	21. 46	20. 44	18. 14	14. 31	9. 52	4. 36

The tropic of Capricorn [♋] does not cut this great circle, but lies wholly below it, just as the tropic of Cancer [♋] stands wholly above it. Hence it comes about that, when the Sun is at the first point of Cancer, the gnomon's shadow traces an ellipse on the upper face of the said circle; but when the Sun is at the first point of Capricorn, it does not illuminate that face, but the lower one. Therefore no altitudes of the Sun are given for the hours of the tropic of Capricorn.

XX.

XX. The Sun's altitudes above the upper face of a great circle declining from the north toward the east by 60° , and inclined to the horizon on its southern side by 80° , computed for the hours reckoned from the setting (west), for latitude 42° .

Horæ ☿, ab occasu	3	4	5	6	7	8	9	†			
Alt. Solis (G. M.)	4.	16.	29.	42.	55.	69.	81.	85.			
	24	45	32	36	49	4	46	40			
Horæ ☿, ab occasu	10	11	12	13	14	15	16				
Alt. Solis (G. M.)	82. 11	69. 32	56. 17	43. 4	30. 0	17. 12	4. 50				
Hor. ♀ & ♄, ab occ.	6	7	8	9	10	11	†				
Alt. Solis (G. M.)	12. 33	25. 35	38. 5	49. 27	58. 19	62. 9	62. 10				
Hor. ♀ & ♄, ab occ.	12	13	14	15	16	17					
Alt. Solis (G. M.)	59. 0	50. 30	39. 19	26. 54	13. 54	0. 41					
Horæ ♃, ab occasu	8	9	10	11	12	†	13	14	15	16	1
Alt. Solis (G. M.)	6.	17.	26.	33.	37.	38.	38.	34.	27.	18.	8.
	10	3	30	47	58	40	19	44	54	46	5

XXI.

XXI. The Sun's altitudes above the upper face of a great circle declining from the north toward the east by 30° , and inclined to the horizon on its southern side by

52°3', computed for the hours reckoned from noon and from midnight, for latitude 42°.

Horæ ☿, à meridie	6	7	8	9	10	11	12
Horæ ☿, à meridie	6	5	4	3	2	1	0
Alt. Solis (G. M.)	0. 17	0. 59	3. 4	6. 22	10. 42	15. 50	21. 30
Horæ ☿, à med. nocte	0	1	2	3	4	5	6
Horæ ☿, à med. noct.	12	11	10	9	8	7	6
Alt. Solis (G. M.)	21.	27.	33.	38.	42.	45.	46.
	30	2	11	28	48	42	43
Horæ ♀ & ☿, à med. noct.	6	7	8	9	10	11	12
Horæ ♀ & ☿, à med. noct.	6	5	4	3	2	1	0
Alt. Solis (G. M.)	23. 13	22. 23	19. 58	16. 11	11. 22	5. 51	0.
							0

No solar altitudes for the hours of Cancer are set down here, because the whole tropic of Cancer stands above the lower face of this circle, just as the whole tropic of Capricorn stands above its upper face. Hence, when the Sun lies in Cancer it does not illuminate the upper face; but when it lies in Capricorn the shadow of the gnomon traces an ellipse on the upper face.

PROBLEMA 2. PROPOS. 2. [Problem 2, Proposition 2]

Given the Sun's altitude above any plane, to find the ratio of the gnomon to its shadow — both to the right shadow [vmbra recta] and to the versed shadow [vmbra versa]; and conversely, from the ratio of the shadow (whether right or versed) to the gnomon, to derive the Sun's altitude.

Diagram. A circle with two diameters crossing at right angles at the centre E: the vertical diameter AC (A at top, C at bottom) and the horizontal diameter BD (B at left, D at right). From the centre E a ray runs up to the right to the point I on the circumference (the Sun's centre), so that the line IE produced passes through E toward C. From I a perpendicular IK is dropped onto the diameter BD, meeting it at K (between E and D). On the left half, two short verticals descend from the horizontal diameter BD at the points H and L (H to the left, L just to its right) down to a lower horizontal chord; G lies on that lower chord at the foot below H,

and F lies on it below E; M lies between the verticals, on the oblique line running from L down toward G.

- A (top of circle): pole of the proposed plane; top of the vertical diameter AC
- B (left of circle): left end of the horizontal diameter BD
- C (bottom of circle): bottom of the vertical diameter AC
- D (right of circle): right end of the horizontal diameter BD
- E (centre): intersection of the diameters; vertex of the gnomon / centre of the world
- I (upper right, on circle): the Sun's centre
- K (on BD, between E and D): foot of the perpendicular IK dropped from I
- F (lower middle, below E): tip of the right shadow FG
- G (lower left): common endpoint of the two shadow-lines FG and HG
- H (on BD, left of centre): foot of the style EH
- L (on BD, just right of H): end of the style EL (taken equal to the gnomon EF)
- M (below H/L): endpoint of the versed shadow LM

Let a circle ABCD be drawn, together with the diameters AC and BD, which cut one another at right angles at the centre E. Suppose this circle to be drawn through the Sun's centre, placed at I, and through A, the pole of the proposed plane; and let the straight line BD be the common section of this same circle and of the great circle to which the plane is parallel. On these assumptions, AC will be the axis of the proposed plane. Let EF be equal to the gnomon standing perpendicular to the plane — drawn through the straight line FG parallel to the great circle of diameter BD; and let EH be equal to the gnomon standing perpendicular to the plane — drawn through the straight line HG, parallel to the great circle that is drawn through AC and stands perpendicular to the circle ABCD. From I, the Sun's centre, let the ray IE fall, passing through the centre of the world (that is, through the vertex of each gnomon), so that it meets the straight line FG at G and the straight line AG [recte: HG] at G.

Now, by Proposition 11 of Book I, the shadow of the style EF is projected onto the straight line FG, the common section of the plane and of the great circle ABCD in which the Sun lies; while the shadow of the style EH falls onto the straight line HG, the common section of the other plane and of the same great circle ABCD. It follows, from what we demonstrated in Proposition 2 of Book I and in the *Quadratum Geometricum* [Geometrical Square], that FG is the length of the right shadow and HG the length of the versed shadow.

[Margin: By what method the length of the right shadow is to be investigated from the Sun's altitude. — a: (*Elements*) I.29; b: VI.4]

Now drop the perpendicular IK from I onto BD. Since the triangles EIK and GEF are similar — because angle GEF equals angle EIK (the exterior equal to the interior), and the right angle at F equals the right angle at K, and so on — it follows that as IK, the right sine of the Sun's altitude, is to KE, the sine of the complement of that altitude, so EF, the gnomon, is to FG, the right shadow. Therefore, if we set it thus: as the sine of the known solar altitude is to the sine of its complement, so the gnomon — divided into any number of equal parts (we take it to be divided into 12 equal parts) — is to a fourth term, then the length of the right shadow will be found, expressed in the same parts into which the gnomon is divided. For example, when the Sun has an altitude of 30°: if we set 50000 (the sine of the Sun's altitude) to 86602 (the sine of the complement of that altitude) as the gnomon of 12 parts is to a fourth term, the right shadow will be found to be 20 parts 47 minutes, of the kind of which the gnomon is reckoned as twelve.

[Margin: c: (*Elements*) I.15; d: VI.4]

By the same reasoning, since the triangles GEH and IEK are similar — because the vertical angles at E are equal, and the angles at H and K are right, and so on — it follows that as EK, the sine of the complement of the Sun's altitude, is to KI, the sine of that altitude, so EH, the style, is to HG, the versed shadow. So if we set it thus: as the sine of the complement of the known solar altitude is to the sine of the solar altitude, so the gnomon is to a fourth term, then the versed shadow will be found in parts equal to those into which the gnomon is divided. As in the same example: if we set 86602 (the sine of the complement of the Sun's altitude) to 50000 (the sine of that altitude) as 12 is to a fourth term, the versed shadow will be found to be 6 parts 56 minutes, of the kind of which the gnomon contains 12.

[Margin: The way by which the length of the versed shadow is to be explored from the Sun's altitude. — e: (*Elements*) I.47; f: VI.4; g: VI.22]

Again, if the shadow — whether right or versed — is known, then the squares described on the shadow and on the gnomon will be known; and since these are together equal to the square on the straight line EG, the square on EG will also be known. And since, by the similarity of the triangles EGF and EIK, EG is to EF as EI, the whole sine, is to IK, the sine of the Sun's altitude, it will likewise follow that as the square on EG is to the square on the gnomon EF, so the square of the whole sine EI is to the square of the sine of the Sun's altitude IK; and thus, from the

known square of IK, the sine IK itself will be known. Or, more briefly: from the known square on EG, the straight line EG itself will be known. If, then, we set EG (known) to the gnomon EF as the whole sine EI is to a fourth term, the sine IK of the Sun's altitude will be known. In the same way, since — by the similarity of the triangles EGH and EIK — as GH is to EH, so EI is to IK, it will likewise follow that as the known square on EG is to the known square on the versed shadow GH, so the square of the whole sine EI is to the square of the sine of the Sun's altitude IK; and hence, from the square of IK, the sine IK itself will be found. Or, more briefly: from the square on EG, the straight line EG itself will be known. If, then, we set EG (known) to the versed shadow GH as the whole sine EI is to a fourth term, the sine IK of the Sun's altitude will be found. Therefore, from the Sun's altitude known above any plane, we have derived the ratio, and so on. Which was to be done.

[Margin: The manner in which the Sun's altitude is sought from the length of the shadow, whether right or versed. — h: (*Elements*) VI.4; i: VI.22]

SCHOLIUM. [Scholium]

Since, by the similarity of the triangles EFG and EHG, the right shadow FG is to the gnomon EF as the gnomon EH is to the versed shadow HG — and the same holds if the style EL is taken equal to the gnomon EF, so that the versed shadow is LM — it follows that any gnomon whatever is a mean proportional between the right shadow and the versed shadow cast by that same gnomon.

Likewise, if the circle drawn through AC is conceived to be the horizon, FG is the versed shadow of the style EF — entirely the same, that is, as the shadow which was previously the right shadow

of that same style EF with respect to the horizon drawn through BD; and since AI is the Sun's altitude above the horizon drawn through AC, while DI is the complement of the altitude above the earlier horizon, the result is that the right shadow of any solar altitude above the horizon is exactly the same as the versed shadow of the complement of that altitude above the same horizon. This is also clear from what has been demonstrated. For since (as we showed) the gnomon is to the versed shadow as the sine of the complement of the Sun's altitude is to the sine of that altitude — and, taking a solar altitude that is the complement of the altitude DI, so that DI is in turn the complement of that altitude — the straight line IK is the sine of the complement of the altitude, and EK the sine of the altitude itself; then, since IK is to KE as EF is to FG, the straight line FG, which was previously the right shadow, becomes in this case the versed shadow, and so on.

[Margin: The right shadow of any solar altitude above the horizon is the same as the versed shadow of the complement of that same altitude above the horizon. How the lengths of the shadows are to be taken from the following table if the style is divided into more or fewer parts than 12.]

Since this is so, one and the same table will comprehend both the right and the versed shadows, provided that the Sun's altitudes are placed in one part of the table and their complements in the other — as you see has been done in the following table. In it are contained the lengths of the shadows, both right and versed, expressed in parts of which the gnomon is 12, computed for each degree and minute of the Sun's altitude. In the sundials of this book, however, we shall use only the right shadows (as we did in the earlier books too), since the gnomon always stands perpendicular to the planes of those sundials. And what we have said about the horizon must be understood of every plane as well, since every great circle to which the plane of a sundial is parallel is a horizon in some region, as is evident.

But if ever the gnomon is of such a size that it can conveniently be divided into more parts — as into 24 — then each shadow of the following table must be doubled. And if the gnomon is divided into 36 parts, they must be tripled; quadrupled if into 48; and quintupled if into 60 — because in this way the same ratio is always preserved, since the parts stand in the same ratio to their equal multiples. Similarly, if the style is so small that it cannot conveniently be divided into 12 parts, let it be divided into 6, and let the halves of the individual shadows be taken, and so on.

[Margin: a: (*Elements*) V.15]

SEQVITVR TABVLA VMBRARVM. [There follows the table of shadows.]

Table of shadows — Degrees of the Sun's altitude for right shadows [*Gradus altitudinum Solis pro vmbris rectis*].

Column headers as printed: the top band reads **Grad.** over the first degree-column and **G.** over each of the others; beneath run the degree numbers **0–9**; beneath those the sub-heads **Par. Mi.** (for degree 0) and **P. ... M.** (partes / minuta) for degrees 1–9. The leftmost **M.** column is the minute-argument (0–60) read downward with the top degree-labels (0–9) for the **right shadow** [*Minuta altitudinum Solis pro vmbris rectis*]; the rightmost **M.** column (60→0) is read upward with the bottom degree-labels (89–80) for the **versed shadow** [*Minuta altitudinum Solis pro vmbris versis*]. Each shadow-length cell is given as *partes. minuta*, gnomon = 12 partes.

M.	0° (Par. Mi.)	1°	2°	3°	4°	5°	6°	7°	8 °	9 °
0	Infini ta.	6 8 7. 3 4	3 4 3. 4 4	229. 0	171. 37	1 3 7. 1 0	1 1 4. 1 1	97. 44	8 5 . 2 3	7 5 . 4 6
1	41378 . 54	6 7 6. 2 0	3 4 0. 4 8	227. 44	170. 55	1 3 6. 4 3	1 1 3. 5 2	97. 30	8 5 . 1 2	7 5 . 3 7
2	20689 . 27	6 6 5. 3 6	3 3 8. 0 .	226. 28	170. 12	1 3 6. 1 5	1 1 3. 3 3	97. 16	8 5 . 2 .	7 5 . 2 9
3	13792 . 58	6 5 4. 5 5	3 3 5. 1 5	225. 15	169. 30	1 3 5. 4 8	1 1 3. 1 4	97. 2	8 4 . 5 1	7 5 . 2 1
4	10344 . 43	6 4 4. 4 1	3 3 2. 3 3	224. 1	168. 48	1 3 5. 2 1	1 1 2. 5 5	97. 48 [recte: 96. 48]	8 4 . 4 0	7 5 . 1 2
5	8275. 47	6 3 4. 4 8	3 2 9. 5 4	222. 48	168. 7	1 3 4. 5 4	1 1 2. 3 6	96. 34	8 4 . 3 0	7 5 . 3 .
6	6896. 29	6 2 5. 1	3 2 7. 1	221. 36	167. 26	1 3 4. 2	1 1 2. 1	96. 21	8 4 . 1	7 4 . 5

M.	0° (Par. Mi.)			3°	4°			7°	8 9	
		1°	2°			5°	6°		°	°
		2	7			7	7		9	5
7	5911. 16	6	3	220. 25	166. 45	1	1	96. 7	8	7
		1	2			3	1		4	4
		5.	4.			4.	1.		.	.
		5	4			1	5		9	4
		4	3				9			7
8	5172. 22	6	3	219. 15	166. 4	1	1	95. 53	8	7
		0	2			3	1		3	4
		6.	2.			3.	1.		.	.
		5	1			3	4		5	3
		2	1			5	0		8	8
9	4597. 39	5	3	218. 4	165. 24	1	1	95. 40	8	7
		9	1			3	1		3	4
		7.	9.			3.	1.		.	.
		4	4			9	2		4	3
		7	1				2		8	0
10	4137. 53	5	3	216. 54	164. 44	1	1	95. 26	8	7
		8	1			3	1		3	4
		9.	7.			2.	1.		.	.
		1	1			4	4		3	2
		6	4			3			7	2
11	3761. 43	5	3	215. 46	164. 5	1	1	95. 13	8	7
		8	1			3	1		3	4
		0.	4.			2.	0.		.	.
		5	4			1	4		2	1
		9	9			7	6		7	4
12	3438. 22	5	3	214. 39	163. 26	1	1	94. 59	8	7
		7	1			3	1		3	4
		2.	2.			1.	0.		.	.
		5	2			5	2		1	5
		6	6			1	8		7	
13	3174. 34	5	3	213. 32	162. 47	1	1	94. 46	8	7
		6	1			3	1		3	3
		5.	0.			1.	0.		.	.

M.	0° (Par. Mi.)	1°	2°	3°	4°	5°	6°	7°	8 °	9 °
		6	5			2	1		6	5
						6	0			7
14	2948.	5	3	212. 26	162. 8	1	1	94. 33	8	7
	22	5	0			3	0		2	3
		7.	7.			1.	9.		.	.
		2	4			1	5		5	4
		9	6				2		6	9
15	2752.	5	3	211. 20	161. 30	1	1	94. 20	8	7
	16	5	0			3	0		2	3
		0.	5.			0.	9.		.	.
		4	3			3	3		4	4
			0			6	5		5	1
16	2580.	5	3	210. 15	160. 52	1	1	94. 7	8	7
	36	4	0			3	0		2	3
		2.	3.			0.	9.		.	.
		5	1			1	1		3	3
		1	0			0	7		6	3
17	2429.	5	3	209. 11	160. 14	1	1	93. 54	8	7
	6	3	0			2	0		2	3
		5.	0.			9.	8.		.	.
		4	5			4	5		2	2
		9	8			4	9		6	5
18	2294.	5	2	206. 8	159. 37	1	1	93. 41	8	7
	22	2	9	[recte: 208. 8]		2	0		2	3
		8.	8.			9.	8.		.	.
		5	4			2	4		1	1
		8	7			0	1		6	7
39	2173.	5	2	207. 5	159. 0	1	1	93. 28	8	7
[recte: 19]	52	2	9			2	0		2	3
		2.	6.			8.	8.		.	.
		1	3			5	2		6	9
		7	8			7	4			
20	2065.	5	2	206. 3	158. 23	1	1	93. 15	8	7
	23	1	9			2	0		1	3

M.	0° (Par. Mi.)			3°	4°			7°	8 9	
		1°	2°			5°	6°		°	°
		5.	4.			8.	8.		.	.
		4	3			3	7		5	1
		6	1			3			5	
21	1967.	5	2	205. 1	157. 46	1	1	93. 2	8	7
	10	0	9			2	0		1	2
		9.	2.			8.	7.		.	.
		2	2			9	5		4	5
		4	6				0		6	3
22	1877.	5	2	204. 0	157. 10	1	1	92. 49	8	7
	53	0	9			2	0		1	2
		3.	0.			7.	7.		.	.
		0	2			4	3		3	4
			3			5	3		6	5
23	1793.	4	2	203. 0	156. 34	1	1	92. 37	8	7
	40	9	8			2	0		1	2
		6.	8.			7.	7.		.	.
		5	2			2	1		2	3
		7	1			1	6		6	7
24	1719.	4	2	202. 0	155. 58	1	1	92. 24	8	7
	9	9	8			2	0		1	2
		1.	6.			6.	6.		.	.
		3	2			5	5		1	2
			1			7	9		6	9
25	1650.	4	2	201. 1	155. 23	1	1	92. 11	8	7
	34	8	8			2	0		1	2
		5.	4.			6.	6.		.	.
		1	2			3	4		6	2
		7	3			4	3			1
26	1587.	4	2	202. 2	154. 48	1	1	91. 59	8	7
	15	7	8	[recte: 200. 2]		2	0		0	2
		9.	2.			6.	6.		.	.
		3	2			1	2		5	1
		9	5			0	6		7	4
27	1528.	4	2	199. 4	154. 13	1	1	91. 46	8	7

M.	0° (Par. Mi.)	1°	2°	3°	4°	5°	6°	7°	8 °	9 °
	36	7	8			2	0		0	2
		4.	0.			5.	6.		.	.
		9	3			4	9		4	6
			1			7			7	
28	1474.	4	2	198. 7	153. 38	1	1	91. 34	8	7
	9	6	7			2	0		0	1
		8.	8.			5.	5.		.	.
		4	3			2	5		3	5
		7	7			4	2		7	8
29	1423.	4	2	197. 10	153. 3	1	1	91. 21	8	7
	13	6	7			2	0		0	1
		3.	6.			5.	5.		.	.
		3	4			1	3		2	5
		1	5				6		7	0
30	1376.	4	2	196. 13	152. 29	1	1	91. 9	8	7
	6	5	7			2	0		0	1
		8.	4.			4.	5.		.	.
		2	5			3	1		1	4
		2	4			8	9		8	3
31	1331.	4	2	195. 17	151. 55	1	1	90. 57	8	7
	47	5	7			2	0		0	1
		3.	3.			4.	5.		.	.
		2	1			1	3		8	3
		1				5				5
32	1290.	4	2	194. 22	151. 22	1	1	90. 45	7	7
	15	4	7			2	0		9	1
		8.	1.			3.	4.		.	.
		2	1			5	4		5	2
		6	4			2	7		9	7
33	1251.	4	2	193. 27	150. 48	1	1	90. 33	7	7
	14	4	6			2	0		9	1
		3.	9.			3.	4.		.	.
		3	2			3	3		4	2
		7	7			0	1		9	0

M.	0° (Par. Mi.)	1°	2°	3°	4°	5°	6°	7°	8 °	9 °
34	1213.	4	2	192. 33	150. 15	1	1	90. 21	7	7
	12	3	6			2	0		9	1
		8.	7.			3.	4.		.	.
		4	4			7	1		4	1
		5	2				5		0	2
35	1178.	4	2	191. 38	149. 42	1	1	90. 8	7	7
	43	3	6			2	0		9	1
		4.	5.			2.	3.		.	.
		8	5			4	5		3	4
			9			5	9		0	
36	1146.	4	2	190. 44	149. 10	1	1	89. 56	7	7
	4	2	6			2	0		9	0
		9.	4.			2.	3.		.	.
		3	1			2	4		2	5
		8	7			3	3		1	7
37	1115.	4	2	189. 51	148. 37	1	1	89. 44	7	7
	10	2	6			2	0		9	0
		5.	2.			2.	3.		.	.
		1	3			1	2		1	4
		3	6				7		1	9
38	1085.	4	2	188. 59	148. 5	1	1	89. 32	7	7
	54	2	6			2	0		9	0
		0.	0.			1.	3.		.	.
		5	5			3	1		2	4
		3	6			9	1			2
39	1058.	4	2	188. 7	147. 33	1	1	89. 20	7	7
	8	1	5			2	0		8	0
		6.	9.			1.	2.		.	.
		3	1			1	5		5	3
		8	8			8	6		3	5
40	1031.	4	2	187. 16	147. 1	1	1	89. 8	7	7
	45	1	5			2	0		8	0
		2.	7.			0.	2.		.	.
		2	4			5	4		4	2

M.	0° (Par. Mi.)			3°	4°			7°	8 9	
		1°	2°			5°	6°		°	°
		9	0			6	0		4	7
41	1006. 38	4	2	186. 25	146. 29	1	1	88. 57	7	7
		0	5			2	0		8	0
		8.	6.			0.	2.		.	.
		2	4			3	2		3	2
		4				5	5		4	0
42	982. 43	4	2	185. 34	145. 58	1	1	88. 45	7	7
		0	5			2	0		8	0
		4.	4.			0.	2.		.	.
		2	3			1	9		2	1
		4	0			4			5	2
43	959. 55	4	2	184. 44	145. 27	1	1	88. 33	7	7
		0	5			1	0		8	0
		0.	2.			9.	1.		.	.
		2	5			5	5		1	5
		9	6			3	4		6	
44	938. 9	3	2	183. 55	144. 57	1	1	88. 22	7	6
		9	5			1	0		8	9
		6.	1.			9.	1.		.	.
		3	2			3	3		7	5
		8	3			2	9			8
45	917. 21	3	2	183. 6	144. 26	1	1	88. 10	7	6
		9	4			1	0		7	9
		2.	9.			9.	1.		.	.
		5	5			1	2		5	5
		2	2			1	4		8	0
46	896. 47	3	2	182. 17	143. 55	1	1	87. 59	7	6
		8	4			1	0		7	9
		9.	8.			8.	1.		.	.
		1	2			5	8		4	4
		0	2			0			9	3
47	877. 45	3	2	181. 29	143. 25	1	1	87. 47	7	6
		8	4			1	0		7	9
		5.	6.			8.	0.		.	.

M.	0° (Par. Mi.)			3°	4°			7°	8 9	
		1°	2°			5°	6°		°	°
48	859. 31	2	5			2	5		4	3
		5	3			9	3		0	6
		3	2	180. 41	142. 55	1	1	87. 36	7	6
		8	4			1	0		7	9
		1.	5.			8.	0.		.	.
		5	2			8	3		3	2
49	842. 1	1	4				8		1	9
		3	2	179. 53	142. 25	1	1	87. 25	7	6
		7	4			1	0		7	9
		8.	3.			7.	0.		.	.
		2	5			4	2		2	2
		1	4			8	3		2	1
50	825. 13	3	2	179. 6	141. 56	1	1	87. 14	7	6
		7	4			1	0		7	9
		4.	2.			7.	0.		.	.
		5	2			2	8		1	1
		5	8			8			3	4
		3	2	178. 20	141. 26	1	9	87. 2	7	6
51	809. 4	7	4			1	9.		7	9
		1.	1.			7.	5		.	.
		3	1			8	4		4	7
		3	1							
		3	2	177. 34	140. 57	1	9	86. 51	7	6
		6	3			1	9.		6	9
52	793. 54	8.	9.			6.	3		.	.
		1	3			4	9		5	0
		4	9			7			6	
		3	2	176. 48	140. 28	1	9	86. 40	7	6
		6	3			1	9.		6	8
		4.	8.			6.	2		.	.
53	778. 37	5	1			2	5		4	5
		9	6			7			7	3
		3	2	176. 2	139. 59	1	9	86. 29	7	6
		6	3			1	9.		6	8
54	764. 14	3	2							
		6	3							

M.	0° (Par. Mi.)			3°	4°			7°	8 9	
		1°	2°			5°	6°		°	°
		1.	6.			6.	1		.	.
		4	5			7	0		3	4
		8	4						8	6
55	750.	3	2	175. 17	139. 30	1	9	86. 18	7	6
	22	5	3			1	8.		6	8
		8.	5.			5.	5		.	.
		3	3			4	6		2	3
		9	2			8			9	8
56	737. 0	3	2	174. 32	139. 2	1	9	86. 7	7	6
		5	3			1	8.		6	8
		5.	4.			5.	4		.	.
		3	1			2	1		2	3
		4	2			8			0	1
57	724. 6	3	2	173. 48	138. 34	1	9	85. 56	7	6
		5	3			1	8.		6	8
		2.	2.			5.	2		.	.
		3	5			8	7		1	2
		2	3						1	4
58	711.	3	2	173. 4	138. 6	1	9	[numeral:	7	6
	13	4	3			1	8.	?]	6	8
		9.	1.			4.	1		.	.
		3	3			4	2		3	1
		3	4			9				7
59	699.	3	2	172. 21	138. 38	1	9	85. 34	7	6
	12	4	3		[recte: 137.	1	7.		5	8
		6.	0.		38]	4.	5		.	.
		3	1			3	8		5	1
		7	7			0			4	0
60	687.	3	2	171. 37	137. 10	1	9	85. 23	7	6
	34	4	2			1	7.		5	8
		3.	9.			4.	4		.	.
		4	0			1	4		4	3
		4				1			6	
(degree	89	8	8	86	85	8	8	82	8	8

	0° (Par. Mi.)	1°	2°	3°	4°	5°	6°	7°	8 °	9 °
M. , read up)		8	7			4	3		1	0

Book V — Table of Shadows (continued)

This opening continues the great Tabula umbrarum (shadow table) of Book V begun on the preceding pages. It tabulates, for each degree of the Sun's altitude, the length of the shadow cast by a gnomon of 12 partes. The three pages here cover the Sun's altitude from 10° through 39°.

Each table is read two ways (dual entry):

- **Right (direct) shadows** [umbrae rectae] — enter with the **top** degree headings (10–19, 20–29, 30–39) and the **left** minute-argument column (0 → 60). The rotated title in the left margin reads "*Minuta altitudinum Solis pro umbris rectis*" (minutes of the Sun's altitude for right shadows).
- **Versed (turned) shadows** [umbrae versae] — enter with the **bottom** degree labels and the **right** minute-argument column (60 → 0). The rotated title in the right margin reads "*Minuta altitudinum Solis pro umbris versis*" (minutes of the Sun's altitude for versed shadows).

Every cell is partes. minuta (P. M.), the gnomon being 12 partes.

running header: GNOMONICES · *Gradus altitudinum Solis pro umbris rectis* (Sun's-altitude degrees for right shadows)

Degrees of altitude 10–19 (top, right shadows) / 79–70 (bottom, versed shadows).

M. (rect.)	10	11	12	13	14	15	16	17	18	19	M. (vers.)
0	68. 3	61. 44	56. 27	51. 59	48. 8	44. 47	41. 51	39. 15	36. 56	34. 51	60
1	67. 57	61. 40	56. 23	51. 55	48. 4	44. 44	41. 48	39. 13	36. 54	34. 49	59
2	67. 49	61.	56.	51.	48.	44.	41.	39.	36.	34.	58

M. (rect.)	10	11	12	13	14	15	16	17	18	19	M. (vers.)
		33	18	50	1	41	45	10	52	47	
3	67.43	61.	56.	51.	47.	44.	41.	39.	36.	34.	57
		27	13	46	57	38	43	8	49	45	
4	67.36	61.	56.	51.	47.	44.	41.	39.	36.	34.	56
		21	8	42	54	35	40	5	47	43	
5	67.29	61.	56.	51.	47.	44.	41.	39.	36.	34.	55
		16	3	38	50	31	37	3	45	41	
6	67.22	61.	55.	51.	47.	44.	41.	39.	36.	34.	54
		10	59	34	46	28	35	0	43	39	
7	67.15	61.	55.	51.	47.	44.	41.	38.	36.	34.	53
		4	54	30	43	25	32	58	41	37	
8	67.9	60.	55.	51.	47.	44.	41.	38.	36.	34.	52
		59	49	26	39	22	29	56	39	35	
9	67.2	60.	55.	51.	47.	44.	41.	38.	36.	34.	51
		53	44	22	36	19	26	53	36	33	
10	66.55	60.	55.	51.	47.	44.	41.	38.	36.	34.	50
		47	40	18	32	16	24	51	34	31	
11	66.48	60.	55.	51.	47.	44.	41.	38.	36.	34.	49
		41	35	14	29	13	21	48	32	30	
12	66.42	60.	55.	51.	47.	44.	41.	38.	36.	34.	48
		36	30	10	25	10	18	46	30	28	
13	66.35	60.	55.	51.	47.	44.	41.	38.	36.	34.	47
		31	26	6	22	7	16	44	28	26	
14	66.28	60.	55.	51.	47.	44.	41.	38.	36.	34.	46
		25	21	2	18	4	13	41	26	24	
15	66.22	60.	55.	50.	47.	44.	41.	38.	36.	34.	45
		20	16	58	15	1	10	39	23	22	
16	66.15	60.	55.	50.	47.	43.	41.	38.	36.	34.	44
		14	11	54	12	58	8	36	21	20	
17	66.9	60.	55.	50.	47.	43.	41.	38.	36.	34.	43
		9	7	50	8	55	5	34	19	18	
18	66.2	60.	55.	50.	47.	43.	41.	38.	36.	34.	42
		3	2	46	5	52	2	32	17	16	

M. (rect.)	10	11	12	13	14	15	16	17	18	19	M. (vers.)
19	65. 55	59. 58	54. 58	50. 42	47. 1	43. 49	41. 0	38. 29	36. 15	34. 14	41
20	65. 49	59. 52	54. 53	50. 38	46. 58	43. 46	40. 57	38. 27	36. 13	34. 12	40
21	65. 43	59. 47	54. 48	50. 34	46. 54	43. 43	40. 54	38. 25	36. 11	34. 10	39
22	65. 36	59. 42	54. 44	50. 30	46. 51	43. 40	40. 52	38. 22	36. 9	34. 8	38
23	65. 29	59. 36	54. 39	50. 26	46. 48	43. 37	40. 49	38. 20	36. 7	34. 6	37
24	65. 23	59. 31	54. 35	50. 22	46. 44	43. 34	40. 46	38. 18	36. 4	34. 5	36
25	65. 17	59. 25	54. 30	50. 18	46. 41	43. 31	40. 44	38. 15	36. 2	34. 3	35
26	65. 10	59. 20	54. 26	50. 14	46. 37	43. 28	40. 41	38. 13	36. 0	34. 1	34
27	65. 4	59. 15	54. 21	50. 11	46. 34	43. 25	40. 39	38. 11	35. 58	33. 59	33
28	64. 57	59. 10	54. 17	50. 7	46. 31	43. 22	40. 36	38. 8	35. 56	33. 57	32
29	64. 51	59. 4	54. 12	50. 3	46. 27	43. 19	40. 33	38. 6	35. 54	33. 55	31
30	64. 45	58. 59	54. 8	49. 59	46. 24	43. 16	40. 31	38. 4	35. 52	33. 53	30
31	64. 39	58. 53	54. 3	49. 55	46. 21	43. 13	40. 28	38. 1	35. 50	33. 51	29
32	64. 32	58. 49	53. 59	49. 51	46. 17	43. 10	40. 26	37. 59	35. 48	33. 49	28
33	64. 26	58. 43	53. 54	49. 47	46. 14	43. 8	40. 23	37. 57	35. 46	33. 48	27
34	64. 23 [recte: 64. 20?]	58. 38	53. 50	49. 44	46. 11	43. 5	40. 20	37. 54	35. 44	33. 46	26
35	64. 14	58.	53.	49.	46.	43.	40.	37.	35.	33.	25

M. (rect.)	10	11	12	13	14	15	16	17	18	19	M. (vers.)
		33	46	40	8	2	18	52	42	44	
36	64. 7	58.	53.	49.	46.	42.	40.	37.	35.	33.	24
		28	41	36	4	59	15	50	39	42	
37	64. 1	58.	53.	49.	46.	42.	40.	37.	35.	33.	23
		22	37	32	1	56	12	47	37	40	
38	63. 55	58.	53.	49.	45.	42.	40.	37.	35.	33.	22
		17	32	29	58	53	10	45	35	38	
39	63. 49	58.	53.	49.	45.	42.	40.	37.	35.	33.	21
		12	28	25	54	50	8	43	33	36	
40	63. 43	58.	53.	49.	45.	42.	40.	37.	35.	33.	20
		7	24	21	51	47	5	41	31	35	
41	63. 37	58.	53.	49.	45.	42.	40.	37.	35.	33.	19
		2	19	17	48	44	2	38	29	33	
42	63. 31	57.	53.	49.	45.	42.	40.	37.	35.	33.	18
		57	15	13	45	41	0	36	27	31	
43	63. 24	57.	53.	49.	45.	42.	39.	37.	35.	33.	17
		52	11	10	41	39	57	34	25	29	
44	63. 19	57.	53.	49.	45.	42.	39.	37.	35.	33.	16
		49	6	6	38	36	55	32	23	27	
45	63. 12	57.	53.	49.	45.	42.	39.	37.	35.	33.	15
		42	2	2	35	33	52	29	21	25	
46	63. 7	57.	52.	48.	45.	42.	39.	37.	35.	33.	14
		37	58	59	32	30	50	27	19	24	
47	63. 0	57.	52.	48.	45.	42.	39.	37.	35.	33.	13
		31	53	55	28	27	47	25	17	22	
48	62. 54	57.	52.	48.	45.	42.	39.	37.	35.	33.	12
		26	49	51	25	25	45	23	15	20	
49	62. 49	57.	52.	48.	45.	42.	39.	37.	35.	33.	11
		21	45	48	22	22	42	20	13	18	
50	62. 43	57.	52.	48.	45.	42.	39.	37.	35.	33.	10
		16	41	44	19	19	40	18	11	16	
51	62. 37	57.	52.	48.	45.	42.	39.	37.	35.	33.	9
		11	36	40	16	16	37	16	9	14	

M. (rect.)	10	11	12	13	14	15	16	17	18	19	M. (vers.)
52	62. 29	57. 7	52. 32	48. 37	45. 12	42. 13	39. 35	37. 14	35. 7	33. 13	8
53	62. 25	57. 2	52. 28	48. 33	45. 9	42. 10	39. 32	37. 11	35. 5	33. 11	7
54	62. 19	56. 57	52. 24	48. 29	45. 6	42. 7	39. 30	37. 9	35. 3	33. 9	6
55	62. 13	56. 52	52. 20	48. 26	45. 3	42. 5	39. 27	37. 7	35. 1	33. 7	5
56	52. 7 [recte: 62. 7]	56. 47	52. 16	48. 22	45. 0	42. 2	39. 25	37. 5	34. 59	33. 5	4
57	62. 1	56. 42	52. 12	48. 18	44. 57	41. 59	39. 22	37. 3	34. 57	33. 4	3
58	61. 56	56. 37	52. 7	48. 15	44. 53	41. 57	39. 20	37. 0	34. 55	33. 2	2
59	61. 50	56. 32	52. 3	48. 11	44. 50	41. 54	39. 17	36. 58	34. 53	33. 0	1
60	61. 44	56. 27	51. 59	48. 8	44. 47	41. 51	39. 15	36. 56	34. 51	32. 58	0
(deg., vers.)	79	78	77	76	75	74	73	72	71	70	

Lower rubric (foot of page): *Gradus altitudinum Solis pro umbris versis* (Sun's-altitude degrees for versed shadows).

LIBER QVINTVS · 401 — *Gradus altitudinum Solis pro umbris rectis*

Degrees of altitude 20–29 (top, right shadows) / 69–60 (bottom, versed shadows).

M. (rect.)	20	21	22	23	24	25	26	27	28	29	M. (vers)
0	32	31. 16	29	28	26	25	24. 36	23	22	21	60
	.		.	.	.	.		.	.	.	

M. (rect.)	20	21	22	23	24	25	26	27	28	29	M. (vers)
	58		42	16	57	44		33	34	39	
1	32	31. 14	29	28	26	25	24. 35	23	22	21	59
	.		.	.	.	.		.	.	.	
	56		41	15	56	43		32	33	38	
2	32	31. 12	29	28	26	25	24. 34	23	22	21	58
	.		.	.	.	.		.	.	.	
	55		39	13	55	42		31	32	37	
3	32	31. 11	29	28	26	25	24. 33	23	22	21	57
	.		.	.	.	.		.	.	.	
	53		38	12	53	41		30	31	36	
4	32	31. 9	29	28	26	25	24. 32	23	22	21	56
	.		.	.	.	.		.	.	.	
	51		36	11	52	39		29	30	35	
5	32	31. 8	29	28	26	25	24. 31	23	22	21	55
	.		.	.9	.	.		.	.	.	
	49		35		51	38		28	29	34	
6	32	31. 6	29	28	26	25	24. 30	23	22	21	54
	.		.	.8	.	.		.	.	.	
	48		33		50	37		27	28	34	
7	32	31. 4	29	28	26	25	24. 28	23	22	21	53
	.		.	.7	.	.		.	.	.	
	46		32		48	36		26	28	33	
8	32	31. 3	29	28	26	25	24. 27	23	22	21	52
	.		.	.5	.	.		.	.	.	
	44		30		47	35		25	27	32	
9	32	31. 1	29	28	26	25	24. 26	23	22	21	51
	.		.	.4	.	.		.	.	.	
	42		29		46	34		24	26	31	
10	32	31. 0	29	28	26	25	24. 25	23	22	21	50
	.		.	.3	.	.		.	.	.	
	40		27		45	32		23	25	30	
11	32	30. 58	29	28	26	25	24. 24	23	22	21	49
	.		.	.1	.	.		.	.	.	

M. (rect.)	20	21	22	23	24	25	26	27	28	29	M. (vers)
	39		26		43	31		22	24	29	
12	32	30. 56	29	28	26	25	24. 23	23	22	21	48
	.		.	.0	.	.		.	.	.	
	37		24		42	30		21	23	28	
13	32	30. 54	29	27	26	25	24. 22	23	22	21	47
	.		.	.	.	.		.	.	.	
	35		23	59	41	29		20	22	27	
14	32	30. 53	29	27	26	25	24. 21	23	22	21	46
	.		.	.	.	.		.	.	.	
	33		21	57	40	28		19	21	27	
15	32	30. 52	29	27	26	25	24. 20	23	22	21	45
	.		.	.	.	.		.	.	.	
	32		20	56	38	27		18	20	26	
16	32	30. 50	29	27	26	25	24. 19	23	22	21	44
	.		.	.	.	.		.	.	.	
	30		18	54	37	26		17	19	25	
17	32	30. 48	29	27	26	25	24. 18	23	22	21	43
	.		.	.	.	.		.	.	.	
	28		17	53	36	24		16	18	24	
18	32	30. 47	29	27	26	25	24. 17	23	22	21	42
	.		.	.	.	.		.	.	.	
	26		16	52	35	23		15	17	23	
19	32	30. 45	29	27	26	25	24. 16	23	22	21	41
	.		.	.	.	.		.	.	.	
	25		14	50	33	22		14	16	22	
20	32	30. 44	29	27	26	25	24. 15	23	22	21	40
	.		.	.	.	.		.	.	.	
	23		13	49	32	21		13	15	21	
21	32	30. 42	29	27	26	25	24. 14	23	22	21	39
	.		.	.	.	.		.	.	.	
	21		11	48	31	20		12	14	20	
22	32	30. 40	29	27	26	25	24. 13	23	22	21	38
	.		.	.	.	.		.	.	.	

M. (rect.)	20	21	22	23	24	25	26	27	28	29	M. (vers)
	20		10	46	30	19		11	13	20	
23	32	30. 38	29	27	26	25	24. 12	23	22	21	37
	.		. 8	.	.	.		.	.	.	
	18			45	28	17		10	13	19	
24	32	30. 37	29	27	26	25	24. 10	23	22	21	36
	.		. 7	.	.	.		. 9	.	.	
	16			44	27	16			12	18	
25	32	30. 36	29	27	26	25	24. 9	23	22	21	35
	.		. 5	.	.	.		. 8	.	.	
	14			42	26	15			11	17	
26	32	30. 34	29	27	26	25	24. 8	23	22	21	34
	.		. 4	.	.	.		. 7	.	.	
	13			41	25	14			10	16	
27	32	30. 33	29	27	26	25	24. 7	23	22	21	33
	.		. 3	.	.	.		. 6	. 9	.	
	11			40	24	13				15	
28	32	30. 31	29	27	26	25	24. 6	23	22	21	32
	. 9		. 1	.	.	.		. 5	. 8	.	
				39	22	12				14	
29	32	30. 29	29	27	26	25	24. 5	23	22	21	31
	. 8		. 0	.	.	.		. 4	. 7	.	
				37	21	11				13	
30	32	30. 28	28	27	26	25	24. 4	23	22	21	30
	. 6		.	.	.	.		. 3	. 6	.	
			58	36	20	10				13	
31	32	30. 26	28	27	26	25	24. 3	23	22	21	29
	. 4		.	.	.	. 8		. 2	. 5	.	
			57	35	19					12	
32	32	30. 25	28	27	26	25	24. 2	23	22	21	28
	. 2		.	.	.	. 7		. 1	. 4	.	
			55	33	17					11	
33	32	30. 23	28	27	26	25	24. 1	23	22	21	27
	. 1		.	.	.	. 6		. 0	. 3	.	

M. (rect.)	20	21	22	23	24	25	26	27	28	29	M. (vers)
			54	32	16					10	
34	31	20. 22 [recte: . 30. 22]	28	27	26	25	24. 0	22	22	21	26
	59		.	.	.	.5		.	.2	.9	
			53	30	15			59			
35	31	30. 20	28	27	26	25	23. 59	22	22	21	25
	57		.	.	.	.4		.	.2	.8	
			51	29	14			58			
36	31	30. 19	28	27	26	25	23. 58	22	22	21	24
	56		.	.	.	.3		.	.1	.7	
			50	28	12			57			
37	31	30. 17	28	27	26	25	23. 57	22	22	21	23
	54		.	.	.	.2		.	.0	.7	
			48	27	11			56			
38	31	30. 16	28	27	26	25	23. 56	22	21	21	22
	52		.	.	.	.1		.	.	.6	
			47	25	10			55	59		
39	31	30. 14	28	27	26	24	23. 55	22	21	21	21
	51		.	.	.9	.		.	.	.5	
			45	24		59		54	58		
40	31	30. 12	28	27	26	24	23. 54	22	21	21	20
	49		.	.	.8	.		.	.	.4	
			44	23		58		53	57		
41	31	30. 10	28	27	26	24	23. 53	22	21	21	19
	47		.	.	.7	.		.	.	.3	
			43	21		57		52	56		
42	31	30. 9	28	27	26	24	23. 52	22	21	21	18
	46		.	.	.5	.		.	.	.2	
			41	20		56		51	55		
43	31	30. 8	28	27	26	24	23. 51	22	21	21	17
	44		.	.	.4	.		.	.	.1	
			40	19		55		50	54		
44	31	30. 6	28	27	26	24	23. 50	22	21	21	16
	.		.	.	.3	.		.	.	.1	

M. (rect.)	20	21	22	23	24	25	26	27	28	29	M. (vers.)
	42		38	18		54		49	53		
45	31	30.5	28	27	26	24	24.48 [recte: . 23.48]	22	21	21	15
	.		.	.	.2	.		.	.	.0	
	40		37	16		53		48	52		
46	31	30.3	28	27	26	24	23.47	22	21	20	14
	.		.	.	.0	.		.	.	.	
	39		36	15		52		48	51	59	
47	31	30.2	28	27	25	24	23.46	22	21	20	13
	.		.	.	.	.		.	.	.	
	37		34	14	59	51		47	51	58	
48	31	30.0	28	27	25	24	23.45	22	21	20	12
	.		.	.	.	.		.	.	.	
	36		33	12	58	49		46	50	57	
49	31	29.59	28	27	25	24	23.44	22	21	20	11
	.		.	.	.	.		.	.	.	
	34		31	11	57	48		45	49	56	
50	31	29.57	28	27	25	24	23.43	22	21	20	10
	.		.	.	.	.		.	.	.	
	32		30	10	56	47		44	48	56	
51	31	29.56	28	27	25	24	23.42	22	21	20	9
	.		.	.9	.	.		.	.	.	
	31		28		55	46		43	47	55	
52	31	29.54	28	27	25	24	23.41	22	21	20	8
	.		.	.7	.	.		.	.	.	
	29		27		53	45		42	46	54	
53	31	29.53	28	27	25	24	23.40	22	21	20	7
	.		.	.6	.	.		.	.	.	
	27		26		52	44		41	45	53	
54	31	20.51 [recte: . 29.51]	28	27	25	24	23.39	22	21	20	6
	.		.	.5	.	.		.	.	.	
	26		24		51	43		40	44	52	
55	31	20.50 [recte: . 29.50]	28	27	25	24	23.38	22	21	20	5
	.		.	.4	.	.		.	.	.	

M. (rect.)	20	21		22	23	24	25	26		27	28	29	M. (vers)
	24			23		50	42			39	43	51	
56	31	29. 48		28	27	25	24	23. 37		22	21	20	4
	.			.	. 2	.	.			.	.	.	
	22			22		49	41			38	42	50	
57	31	29. 47		28	27	25	24	23. 36		22	21	20	3
	.			.	. 1	.	.			.	.	.	
	21			20		48	40			37	42	50	
58	31	29. 45		28	27	25	24	23. 35		22	21	20	2
	.			.	. 0	.	.			.	.	.	
	19			19		46	38			36	41	49	
59	31	29. 44		28	26	25	24	23. 34		22	21	20	1
	.			.	.	.	.			.	.	.	
	17			18	58	45	37			35	40	48	
60	31	29. 42		28	26	25	24	23. 33		22	21	20	0
	.			.	.	.	.			.	.	.	
	16			16	57	44	36			34	39	47	
(deg., vers.)	69	68		67	66	65	64	63		62	61	60	

Lower rubric (foot of page): *Gradus altitudinum Solis pro umbris versis.*

running header: GNOMONICES · *Gradus altitudinum Solis pro umbris rectis*

Degrees of altitude 30–39 (top, right shadows) / 59–50 (bottom, versed shadows).

M. (rect.)	30	31	32	33	34	35		36	37	38	39	M. (vers.)
0	20.	19.	19.	18.	17.	17. 8		16.	15.	15.	14.	60
	47	58	12	29	47			31	55	22	49	
1	20.	19.	19.	18.	17.	17. 8		16.	15.	15.	14.	59
	46	57	12	28	47			30	55	21	49	
2	20.	19.	19.	18.	17.	17. 7		16.	15.	15.	14.	58
	45	57	11	27	46			30	54	20	48	

M. (rect.)	30	31	32	33	34	35		36	37	38	39	M. (vers.)
3	20. 45	19. 56	19. 10	18. 27	17. 45	17. 6		16. 29	15. 54	15. 20	14. 47	57
4	20. 44	19. 55	19. 9	18. 26	17. 45	17. 6		16. 29	15. 53	15. 19	14. 47	56
5	20. 43	19. 54	19. 9	18. 25	17. 44	17. 5		16. 28	15. 53	15. 19	14. 46	55
6	20. 42	19. 54	19. 8	18. 24	17. 43	17. 4		16. 27	15. 52	15. 18	14. 46	54
7	20. 41	19. 53	19. 7	18. 24	17. 43	17. 4		16. 27	15. 51	15. 18	14. 45	53
8	20. 41	19. 52	19. 6	18. 23	17. 42	17. 3		16. 26	15. 51	15. 17	14. 45	52
9	20. 40	19. 51	19. 6	18. 22	17. 41	17. 3		16. 26	15. 50	15. 17	14. 44	51
10	20. 39	19. 50	19. 5	18. 22	17. 41	17. 2		16. 25	15. 50	15. 16	14. 44	50
11	20. 38	19. 50	19. 4	18. 21	17. 40	17. 1		16. 24	15. 49	15. 16	14. 43	49
12	20. 37	19. 49	19. 3	18. 20	17. 39	17. 1		16. 24	15. 49	15. 15	14. 43	48
13	20. 36	19. 48	19. 3	18. 20	17. 39	17. 0		16. 23	15. 48	15. 14	14. 42	47
14	20. 35	19. 47	19. 2	18. 19	17. 38	16. 59		16. 23	15. 47	15. 14	14. 42	46
15	20. 35	19. 47	19. 1	18. 18	17. 37	16. 59		16. 22	15. 47	15. 13	14. 41	45
16	20. 34	19. 46	19. 0	18. 17	17. 37	16. 58		16. 21	15. 46	15. 13	14. 41	44
17	20. 33	19. 45	19. 0	18. 17	17. 36	16. 58		16. 21	15. 46	15. 12	14. 40	43
18	20. 32	19. 44	18. 59	18. 16	17. 35	16. 57		16. 20	15. 45	15. 12	14. 40	42
19	20.	19.	18.	18.	17.	16. 56		16.	15.	15.	14.	41

M. (rect.)	30	31	32	33	34	35		36	37	38	39	M. (vers.)
	31	44	58	15	35			20	45	11	39	
20	20.	19.	18.	18.	17.	16. 56		16.	15.	15.	14.	40
	31	43	57	15	34			19	44	11	39	
21	20.	19.	18.	18.	17.	16. 55		16.	15.	15.	14.	39
	30	42	57	14	34			18	43	10	38	
22	20.	19.	18.	18.	17.	16. 54		16.	15.	15.	14.	38
	29	41	56	13	33			18	43	9	38	
23	20.	19.	18.	18.	17.	16. 54		16.	15.	15.	14.	37
	28	40	55	13	32			17	42	9	37	
24	20.	19.	18.	18.	17.	16. 53		16.	15.	15.	14.	36
	27	40	55	12	32			17	42	8	37	
25	20.	19.	18.	18.	17.	16. 53		16.	15.	15.	14.	35
	26	39	54	11	31			16	41	8	36	
26	20.	19.	18.	18.	17.	16. 52		16.	15.	15.	14.	34
	26	38	53	11	30			15	41	7	35	
27	20.	19.	18.	18.	17.	16. 51		16.	15.	15.	14.	33
	25	37	52	10	30			15	40	7	35	
28	20.	19.	18.	18.	17.	16. 51		16.	15.	15.	14.	32
	24	36	52	9	29			14	39	6	34	
29	20.	19.	18.	18.	17.	19. 50 [recte: 16.		16.	15.	15.	14.	31
	23	36	51	8	28	50]		14	39	6	34	
30	20.	19.	18.	18.	17.	16. 49		16.	15.	15.	14.	30
	22	35	50	8	28			13	38	5	33	
31	20.	19.	18.	18.	17.	16. 49		16.	15.	15.	14.	29
	22	34	49	7	27			12	38	5	33	
32	20.	19.	18.	18.	17.	16. 48		16.	15.	15.	14.	28
	21	33	49	6	26			12	37	4	32	
33	20.	19.	18.	18.	17.	16. 48		16.	15.	15.	14.	27
	20	33	48	6	26			11	37	4	32	
34	20.	19.	18.	18.	17.	16. 47		16.	15.	15.	14.	26
	19	32	47	5	25			11	36	3	31	
35	20.	19.	18.	18.	17.	16. 46		16.	15.	15.	14.	25
	18	31	47	4	24			10	36	3	31	

M. (rect.)	30	31	32	33	34	35		36	37	38	39	M. (vers.)
36	20. 17	19. 30	18. 46	18. 4	17. 24	16.46		16. 9	15. 35	15. 2	14. 30	24
37	20. 17	19. 30	18. 45	18. 3	17. 23	16.45		16. 9	15. 34	15. 1	14. 30	23
38	20. 16	19. 29	18. 44	18. 2	17. 22	16.44		16. 8	15. 34	15. 1	14. 29	22
39	20. 15	19. 28	18. 44	18. 2	17. 22	16.44		16. 8	15. 33	15. 0	14. 29	21
40	20. 14	19. 27	18. 43	18. 1	17. 21	16.43		16. 7	15. 33	15. 0	14. 28	20
41	20. 13	19. 27	18. 42	18. 0	17. 20	16.43		16. 6	15. 32	14. 59	14. 28	19
42	20. 13	19. 26	18. 42	18. 0	17. 20	16.42		16. 6	15. 32	14. 59	14. 27	18
43	20. 12	19. 25	18. 41	17. 59	17. 19	16.41		16. 5	15. 31	14. 58	14. 27	17
44	20. 11	19. 24	18. 40	17. 58	17. 19	16.41		16. 5	15. 30	14. 58	14. 26	16
45	20. 10	19. 24	18. 39	17. 58	17. 18	16.40		16. 4	15. 30	14. 57	14. 26	15
46	20. 9	19. 23	18. 39	17. 57	17. 17	16.40		16. 4	15. 29	14. 57	14. 25	14
47	20. 9	19. 22	18. 38	17. 56	17. 17	16.39		16. 3	15. 29	14. 56	14. 25	13
48	20. 8	19. 21	18. 37	17. 56	17. 16	16.38		16. 2	15. 28	14. 55	14. 24	12
49	20. 7	19. 20	18. 37	17. 55	17. 15	16.38		16. 2	15. 28	14. 55	14. 24	11
50	20. 6	19. 20	18. 36	17. 54	17. 15	16.37		16. 1	15. 27	14. 54	14. 23	10
51	20. 5	19. 19	18. 35	17. 53	17. 14	16.36		16. 1	15. 27	14. 54	14. 23	9
52	20.	19.	18.	17.	17.	16.35		16.	15.	14.	14.	8

M. (rect.)	30	31	32	33	34	35		36	37	38	39	M. (vers.)
	5	18	35	53	13			0	26	53	22	
53	20.	19.	18.	17.	17.	16. 35		16.	15.	14.	14.	7
	4	17	34	52	13			0	25	53	22	
54	20.	19.	18.	17.	17.	16. 35		15.	15.	14.	14.	6
	3	17	33	51	12			59	25	52	21	
55	20.	19.	18.	17.	17.	16. 34		15.	15.	14.	14.	5
	2	16	32	51	11			58	24	52	21	
56	20.	19.	18.	17.	17.	16. 34		15.	15.	14.	14.	4
	1	15	32	50	11			58	24	51	20	
57	20.	19.	18.	17.	17.	16. 33		15.	15.	14.	14.	3
	1	14	31	49	10			57	23	51	20	
58	20.	19.	18.	17.	17.	16. 32		15.	15.	14.	14.	2
	0	14	30	49	10			57	23	50	19	
59	19.	19.	18.	17.	17.	16. 32		15.	15.	14.	14.	1
	59	13	29	48	9			56	22	50	19	
60	19.	19.	18.	17.	17.	16. 31		15.	15.	14.	14.	0
	58	12	29	47	8			55	22	49	18	
(deg., vers.)	59	58	57	56	55	54		53	52	51	50	

Lower rubric (foot of page): *Gradus altitudinum Solis pro umbris versis.*

Gnomonics, Book V — *Tabula umbrarum* (continued)

(Continuation of the twelve-part gnomon shadow table. Each page tabulates shadow length in partes and minuta (P. M — twelfths and sixtieths-of-a-twelfth of the gnomon) for whole degrees of solar altitude read across the head, against the minutes of altitude read down. The left-hand minute argument (0–60) and the centred head-rubric "Degrees of the Sun's altitude, for right shadows" serve the umbra recta; the right-hand minute argument (60–0) and the foot-rubric "Degrees of the Sun's altitude, for versed shadows," with the degree labels along the foot, serve the* umbra versa*. The rotated marginal titles read "Minutes of the Sun's altitude, for right shadows" (left) and "...for versed shadows" (right).)**

[Source page 403] — running header "LIBER QVINTVS."

Head-rubric (centred): *Gradus altitudinum Solis pro umbris rectis.* — "Degrees of the Sun's altitude, for right shadows."

Degree columns 40°–49° (umbra recta, read with the left minute-column); the same body serves degrees 49°–40° for the umbra versa (read with the foot labels and the right minute-column).

	min.	40	41	42	43	44	45	46	47	48	49	min. (vers.)
0	14.	13.	13.	12.	12.	12.	11.	11.	10.	10.		60
	18	48	20	52	26	0	35	11	48	26		
1	14.	13.	13.	12.	12.	12.	11.	11.	10.	10.		59
	18	48	19	52	25	0	35	11	48	26		
2	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.		58
	17	47	19	51	25	59	34	11	48	25		
3	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.		57
	17	47	18	51	24	59	34	10	47	25		
4	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.		56
	16	46	18	50	24	58	34	10	47	24		
5	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.		55
	16	46	17	50	23	58	33	9	46	24		
6	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.		54
	15	45	17	49	23	57	33	9	46	24		
7	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.		53
	15	45	16	49	23	57	32	9	46	23		
8	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.		52
	14	44	16	49	22	57	32	8	45	23		
9	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.		51
	14	44	15	48	22	56	32	8	45	23		
10	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.		50
	13	43	15	48	21	56	31	8	45	22		
11	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.		49
	13	43	15	47	21	55	31	7	44	22		
12	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.		48
	12	42	14	47	20	55	30	7	44	21		

	min.	40	41	42	43	44	45	46	47	48	49	min. (vers.)
	13	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	47
		12	42	14	46	20	55	30	6	43	21	
	14	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	46
		11	41	13	46	20	54	30	6	43	21	
	15	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	45
		11	41	13	45	19	54	29	6	43	20	
	16	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	44
		10	41	12	45	19	53	29	5	42	20	
	17	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	43
		9	40	12	44	18	53	28	5	42	20	
	18	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	42
		9	40	11	44	18	53	28	4	42	19	
	19	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	41
		8	39	11	44	17	52	28	4	41	19	
	20	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	40
		8	39	10	43	17	52	27	4	41	19	
	21	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	39
		7	38	10	43	17	51	27	3	40	18	
	22	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	38
		7	38	9	42	16	51	26	3	40	18	
	23	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	37
		6	37	9	42	16	50	26	2	40	17	
	24	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	36
		6	37	8	41	15	50	26	2	39	17	
	25	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	35
		5	36	8	41	15	50	25	2	39	17	
	26	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	34
		5	36	8	40	14	49	25	1	39	16	
	27	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	33
		5	35	7	40	14	49	24	1	38	16	
	28	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	32
		4	35	7	40	14	48	24	1	38	16	
	29	14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	31

	min.	40	41	42	43	44	45	46	47	48	49	min. (vers.)
		4	34	6	39	13	48	24	0	37	15	
30		14.	13.	13.	12.	12.	11.	11.	11.	10.	10.	30
		3	34	6	39	13	48	23	0	37	15	
31		14.	13.	13.	12.	12.	11.	11.	10.	10.	10.	29
		3	33	5	38	12	47	23	59	37	15	
32		14.	13.	13.	12.	12.	11.	11.	10.	10.	10.	28
		2	33	5	38	12	47	22	59	36	14	
33		14.	13.	13.	12.	12.	11.	11.	10.	10.	10.	27
		2	32	4	37	11	46	22	59	36	14	
34		14.	13.	13.	12.	12.	11.	11.	10.	10.	10.	26
		1	32	4	37	11	46	22	58	36	13	
35		14.	13.	13.	12.	12.	11.	11.	10.	10.	10.	25
		1	31	3	37	11	45	21	58	35	13	
36		14.	13.	13.	12.	12.	11.	11.	10.	10.	10.	24
		0	31	3	36	10	45	21	57	35	13	
37		14.	13.	13.	12.	12.	11.	11.	10.	10.	10.	23
		0	30	3	36	10	45	20	57	34	12	
38		13.	13.	13.	12.	12.	11.	11.	10.	10.	10.	22
		59	30	2	35	9	44	20	57	34	12	
39		13.	13.	13.	12.	12.	11.	11.	10.	10.	10.	21
		59	30	2	35	9	44	20	56	34	12	
40		13.	13.	13.	12.	12.	11.	11.	10.	10.	10.	20
		58	29	1	34	8	43	19	56	33	11	
41		13.	13.	13.	12.	12.	11.	11.	10.	10.	10.	19
		58	29	1	34	8	43	19	56	33	11	
42		13.	13.	13.	12.	12.	11.	11.	10.	10.	10.	18
		57	28	0	33	8	43	18	55	33	11	
43		13.	13.	13.	12.	12.	11.	11.	10.	10.	10.	17
		57	28	0	33	7	42	18	55	32	10	
44		13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	16
		56	27	59	33	7	42	18	54	32	10	
45		13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	15
		56	27	59	32	6	41	17	54	31	10	

	min.	40	41	42	43	44	45	46	47	48	49	min. (vers.)
	46	13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	14
		55	26	58	32	6	41	17	54	31	9	
	47	13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	13
		55	26	58	31	5	41	17	53	31	9	
	48	13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	12
		54	25	58	31	5	40	16	53	30	8	
	49	13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	11
		54	25	57	30	5	40	16	52	30	8	
	50	13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	10
		53	24	57	30	4	39	15	52	30	8	
	51	13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	9
		53	24	56	29	4	39	15	52	29	7	
	52	13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	8
		52	23	56	29	3	39	15	51	29	7	
	53	13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	7
		52	23	55	29	3	38	14	51	28	7	
	54	13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	6
		51	22	55	28	3	38	14	51	28	6	
	55	13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	5
		51	22	54	28	2	37	13	50	28	6	
	56	13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	4
		50	22	54	27	2	37	13	50	27	6	
	57	13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	3
		50	21	53	27	1	37	13	49	27	5	
	58	13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	2
		49	21	53	26	1	36	12	49	27	5	
	59	13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	1
		49	20	53	26	0	36	12	49	26	5	
	60	13.	13.	12.	12.	12.	11.	11.	10.	10.	10.	0
		48	20	52	26	0	35	11	48	26	4	
(deg., vers.)		49	48	47	46	45	44	43	42	41	40	

Foot-rubric: *Gradus altitudinum Solis pro umbris versis*. — "Degrees of the Sun's altitude, for versed shadows."

[Source page 404] — running header "GNOMONICES"

Head-rubric (centred): *Gradus altitudinum Solis pro umbris rectis*.

Degree columns 50°–59° (umbra recta); the body also serves degrees 59°–50° for the umbra versa (foot labels + right minute-column).

min.	50	51	52	53	54	55	56	57	58	59	min. (vers.)
0	10. 4	9.4 3	9.2 3	9. 3	8.4 3	8.2 4	8. 6	7.4 8	7.3 0	7.1 3	60
1	10. 4	9.4 3	9.2 2	9. 2	8.4 3	8.2 4	8. 5	7.4 7	7.3 0	7.1 2	59
2	10. 3	9.4 2	9.2 2	9. 2	8.4 2	8.2 4	8. 5	7.4 7	7.2 9	7.1 2	58
3	10. 3	9.4 2	9.2 2	9. 2	8.4 2	8.2 3	8. 5	7.4 7	7.2 9	7.1 2	57
4	10. 3	9.4 2	9.2 1	9. 1	8.4 2	8.2 3	8. 4	7.4 6	7.2 9	7.1 1	56
5	10. 2	9.4 1	9.2 1	9. 1	8.4 1	8.2 3	8. 4	7.4 6	7.2 8	7.1 1	55
6	10. 2	9.4 1	9.2 0	9. 1	8.4 1	8.2 2	8. 4	7.4 6	7.2 8	7.1 1	54
7	10. 2	9.4 1	9.2 0	9. 0	8.4 1	8.2 2	8. 4	7.4 5	7.2 8	7.1 1	53
8	10. 1	9.4 0	9.2 0	9. 0	8.4 0	8.2 2	8. 3	7.4 5	7.2 8	7.1 0	52
9	10. 1	9.4 0	9.1 9	9. 0	8.4 0	8.2 1	8. 3	7.4 5	7.2 7	7.1 0	51
10	10. 1	9.4 0	9.1 9	8.5 9	8.4 0	8.2 1	8. 3	7.4 5	7.2 7	7.1 0	50
11	10. 0	9.3 9	9.1 9	8.5 9	8.4 0	8.2 1	8. 2	7.4 4	7.2 7	7. 9	49
12	10. 0	9.3 9	9.1 8	8.5 9	8.3 9	8.2 0	8. 2	7.4 4	7.2 6	7. 9	48

min.	50	51	52	53	54	55	56	57	58	59	min. (vers.)
13	10.0	9.3	9.1	8.5	8.3	8.2	8.	7.4	7.2	7.	47
14	9.59	9.3	9.1	8.5	8.3	8.2	8.	7.4	7.2	7.	46
15	9.59	9.3	9.1	8.5	8.3	8.1	8.	7.4	7.2	7.	45
16	9.58	9.3	9.1	8.5	8.3	8.1	8.	7.4	7.2	7.	44
17	9.58	9.3	9.1	8.5	8.3	8.1	8.	7.4	7.2	7.	43
18	9.58	9.3	9.1	8.5	8.3	8.1	8.	7.4	7.2	7.	42
19	9.57	9.3	9.1	8.5	8.3	8.1	8.	7.4	7.2	7.	41
20	9.57	9.3	9.1	8.5	8.3	8.1	8.	7.4	7.2	7.	40
21	9.57	9.3	9.1	8.5	8.3	8.1	7.5	7.4	7.2	7.	39
22	9.56	9.3	9.1	8.5	8.3	8.1	7.5	7.4	7.2	7.	38
23	9.56	9.3	9.1	8.5	8.3	8.1	7.5	7.4	7.2	7.	37
24	9.56	9.3	9.1	8.5	8.3	8.1	7.5	7.4	7.2	7.	36
25	9.55	9.3	9.1	8.5	8.3	8.1	7.5	7.4	7.2	7.	35
26	9.55	9.3	9.1	8.5	8.3	8.1	7.5	7.4	7.2	7.	34
27	9.55	9.3	9.1	8.5	8.3	8.1	7.5	7.4	7.2	7.	33
28	9.54	9.3	9.1	8.5	8.3	8.1	7.5	7.3	7.2	7.	32
29	9.54	9.3	9.1	8.5	8.3	8.1	7.5	7.3	7.2	7.	31

min.	50	51	52	53	54	55	56	57	58	59	min. (vers.)
30	9.54	9.3 3	9.1 2	8.5 3	8.3 4	8.1 5	7.5 7	7.3 9	7.2 1	7. 4	30
31	9.53	9.3 2	9.1 2	8.5 2	8.3 3	8.1 5	7.5 6	7.3 8	7.2 1	7. 4	29
32	9.53	9.3 2	9.1 2	8.5 2	8.3 3	8.1 4	7.5 6	7.3 8	7.2 1	7. 4	28
33	9.52	9.3 2	9.1 1	8.5 2	8.3 3	8.1 4	7.5 6	7.3 8	7.2 0	7. 3	27
34	9.52	9.3 1	9.1 1	8.5 1	8.3 2	8.1 4	7.5 5	7.3 8	7.2 0	7. 3	26
35	9.52	9.3 1	9.1 1	8.5 1	8.3 2	8.1 3	7.5 5	7.3 7	7.2 0	7. 3	25
36	9.51	9.3 1	9.1 0	8.5 1	8.3 2	8.1 3	7.5 5	7.3 7	7.1 9	7. 2	24
37	9.51	9.3 0	9.1 0	8.5 0	8.3 1	8.1 3	7.5 4	7.3 7	7.1 9	7. 2	23
38	9.51	9.3 0	9.1 0	8.5 0	8.3 1	8.1 2	7.5 4	7.3 6	7.1 9	7. 2	22
39	9.50	9.3 0	9. 9	8.5 0	8.3 1	8.1 2	7.5 4	7.3 6	7.1 9	7. 2	21
40	9.50	9.2 9	9. 9	8.5 0	8.3 0	8.1 2	7.5 4	7.3 6	7.1 8	7. 1	20
41	9.50	9.2 9	9. 9	8.4 9	8.3 0	8.1 1	7.5 3	7.3 5	7.1 8	7. 1	19
42	9.49	9.2 9	9. 8	8.4 9	8.3 0	8.1 1	7.5 3	7.3 5	7.1 8	7. 1	18
43	9.49	9.2 8	9. 8	8.4 9	8.2 9	8.1 1	7.5 3	7.3 5	7.1 7	7. 0	17
44	9.49	9.2 8	9. 8	8.4 8	8.2 9	8.1 0	7.5 3	7.3 5	7.1 7	7. 0	16
45	9.48	9.2 8	9. 7	8.4 8	8.2 9	8.1 0	7.5 2	7.3 4	7.1 7	7. 0	15
46	9.48	9.2 7	9. 7	8.4 8	8.2 9	8.1 0	7.5 2	7.3 4	7.1 7	7. 0	14

min.	50	51	52	53	54	55	56	57	58	59	min. (vers.)
47	9.48	9.2 7	9. 7	8.4 7	8.2 8	8.1 0	7.5 2	7.3 4	7.1 6	6.5 9	13
48	9.47	9.2 7	9. 7	8.4 7	8.2 8	8. 9	7.5 1	7.3 3	7.1 6	6.5 9	12
49	9.47	9.2 6	9. 6	8.4 7	8.2 8	8. 9	7.5 1	7.3 3	7.1 6	6.5 9	11
50	9.47	9.2 6	9. 6	8.4 6	8.2 7	8. 9	7.5 1	7.3 3	7.1 5	6.5 9	10
51	9.46	9.2 6	9. 6	8.4 6	8.2 7	8. 8	7.5 0	7.3 3	7.1 5	6.5 8	9
52	9.46	9.2 5	9. 5	8.4 6	8.2 7	8. 8	7.5 0	7.3 2	7.1 5	6.5 8	8
53	9.45	9.2 5	9. 5	8.4 5	8.2 6	8. 8	7.5 0	7.3 2	7.1 5	6.5 8	7
54	9.45	9.2 5	9. 5	8.4 5	8.2 6	8. 7	7.4 9	7.3 2	7.1 4	6.5 7	6
55	9.45	9.2 4	9. 4	8.4 5	8.2 6	8. 7	7.4 9	7.3 1	7.1 4	6.5 7	5
56	9.44	9.2 4	9. 4	8.4 4	8.2 5	8. 7	7.4 9	7.3 1	7.1 4	6.5 7	4
57	9.44	9.2 4	9. 4	8.4 4	8.2 5	8. 7	7.4 8	7.3 1	7.1 3	6.5 7	3
58	9.44	9.2 3	9. 3	8.4 4	8.2 5	8. 6	7.4 8	7.3 0	7.1 3	6.5 6	2
59	9.43	9.2 3	9. 3	8.4 3	8.2 4	8. 6	7.4 8	7.3 0	7.1 3	6.5 6	1
60	9.43	9.2 3	9. 3	8.4 3	8.2 4	8. 6	7.4 8	7.3 0	7.1 3	6.5 6	0
(deg., vers.)	39	38	37	36	35	34	33	32	31	30	

Foot-rubric: *Gradus altitudinum Solis pro umbris versis.*

[Source page 405] — running header "LIBER QVINTVS."

Head-rubric (centred): *Gradus altitudinum Solis pro umbris rectis.*

Degree columns 60°–69° (umbra recta); the body also serves degrees 69°–60° for the umbra versa (foot labels + right minute-column).

		6	6	6	6	6	6	6	6		min. (vers.)	
	min.	0	1	2	3	4	5	6	7	8	69	
	0	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.36	60
		5	3	2	7	5	3	2	6	5		
		6	9	3		1	6	1		1		
	1	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.36	59
		5	3	2	7	5	3	2	5	5		
		5	9	3		1	5	0		1		
	2	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.36	58
		5	3	2	6	5	3	2	5	5		
		5	9	2		1	5	0		0		
	3	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.36	57
		5	3	2	6	5	3	2	5	5		
		5	8	2		0	5	0		0		
	4	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.35	56
		5	3	2	6	5	3	2	5	5		
		5	8	2		0	5	0		0		
	5	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.35	55
		5	3	2	6	5	3	1	4	5		
		4	8	1		0	4	9		0		
	6	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.35	54
		5	3	2	5	5	3	1	4	4		
		4	7	1		0	4	9		9		
	7	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.35	53
		5	3	2	5	4	3	1	4	4		
		4	7	1		9	4	9		9		
	8	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.34	52
		5	3	2	5	4	3	1	4	4		
		3	7	1		9	4	9		9		
	9	6.	6.	6.	6.	5.	5.	5.	5.	4.	2.34 [recte: 4.34]	51
		5	3	2	4	4	3	1	3	4		
		3	7	0		9	3	8		9		
	10	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.34	50
		5	3	2	4	4	3	1	3	4		

	6	6	6	6	6	6	6	6	6		min.
min.	0	1	2	3	4	5	6	7	8	69	(vers.)
	3	6	0		9	3	8		8		
11	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.34	49
	5	3	2	4	4	3	1	3	4		
	3	6	0		8	3	8		8		
12	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.34	48
	5	3	2	4	4	3	1	3	4		
	2	6	0		8	3	8		8		
13	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.33	47
	5	3	1	3	4	3	1	2	4		
	2	6	9		8	2	7		8		
14	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.33	46
	5	3	1	3	4	3	1	2	4		
	2	5	9		8	2	7		8		
15	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.23 [recte: 4.33]	45
	5	3	1	3	4	3	1	2	4		
	2	5	9		7	2	7		7		
16	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.33	44
	5	3	1	3	4	3	1	2	4		
	1	5	9		7	2	7		7		
17	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.32	43
	5	3	1	2	4	3	1	1	4		
	1	4	8		7	1	6		7		
18	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.32	42
	5	3	1	2	4	3	1	1	4		
	1	4	8		7	1	6		7		
19	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.32	41
	5	3	1	2	4	3	1	1	4		
	0	4	8		6	1	6		6		
20	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.32	40
	5	3	1	2	4	3	1	1	4		
	0	4	7		6	1	6		6		
21	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.31	39
	5	3	1	1	4	3	1	0	4		
	0	3	7		6	0	5		6		

		6	6	6	6	6	6	6	6	6		min. (vers.)
	min.	0	1	2	3	4	5	6	7	8	69	
	22	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.31	38
		5	3	1	1	4	3	1	0	4		
		0	3	7		5	0	5		6		
	23	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.31	37
		4	3	1	1	4	3	1	0	4		
		9	3	7		5	0	5		5		
	24	6.	6.	6.	6.	5.	5.	5.	5.	4.	4.31	36
		4	3	1	1	4	3	1	0	4		
		9	3	6		5	0	5		5		
	25	6.	6.	6.	6.	5.	5.	5.	4.	4.	4.30	35
		4	3	1	0	4	2	1	5	4		
		9	2	6		5	9	4	9	5		
	26	6.	6.	6.	6.	5.	5.	5.	4.	4.	4.30	34
		4	3	1	0	4	2	1	5	4		
		8	2	6		4	9	4	9	5		
	27	6.	6.	6.	6.	5.	5.	5.	4.	4.	4.30	33
		4	3	1	0	4	2	1	5	4		
		8	2	6		4	9	4	9	4		
	28	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.29	32
		4	3	1	5	4	2	1	5	4		
		8	1	5	9	4	8	3	8	4		
	29	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.29	31
		4	3	1	5	4	2	1	5	4		
		8	1	5	9	4	8	3	8	4		
	30	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.29	30
		4	3	1	5	4	2	1	5	4		
		7	1	5	9	3	8	3	8	4		
	31	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.29	29
		4	3	1	5	4	2	1	5	4		
		7	1	5	9	3	8	3	8	3		
	32	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.29	28
		4	3	1	5	4	2	1	5	4		
		7	0	4	8	3	8	3	8	3		
	33	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.28	27

	6	6	6	6	6	6	6	6	6		min.
min.	0	1	2	3	4	5	6	7	8	69	(vers.)
	4	3	1	5	4	2	1	5	4		
	7	0	4	8	3	7	2	7	3		
34	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.28	26
	4	3	1	5	4	2	1	5	4		
	6	0	4	8	2	7	2	7	3		
35	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.28	25
	4	3	1	5	4	2	1	5	4		
	6	0	3	8	2	7	2	7	2		
36	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.28	24
	4	2	1	5	4	2	1	5	4		
	6	9	3	7	2	7	2	7	2		
37	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.28	23
	4	2	1	5	4	2	1	5	4		
	5	9	3	7	2	6	1	7	2		
38	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.27	22
	4	2	1	5	4	2	1	5	4		
	5	9	3	7	1	6	1	6	2		
39	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.27	21
	4	2	1	5	4	2	1	5	4		
	5	8	2	7	1	6	1	6	1		
40	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.27	20
	4	2	1	5	4	2	1	5	4		
	5	8	2	6	1	6	1	6	1		
41	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.27	19
	4	2	1	5	4	2	1	5	4		
	4	8	2	6	1	5	0	6	1		
42	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.26	18
	4	2	1	5	4	2	1	5	4		
	4	8	2	6	0	5	0	5	1		
43	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.26	17
	4	2	1	5	4	2	1	5	4		
	4	7	1	6	0	5	0	5	0		
44	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.26	16
	4	2	1	5	4	2	1	5	4		

	min.	6 0	6 1	6 2	6 3	6 4	6 5	6 6	6 7	6 8	69	min. (vers.)
		3	7	1	5	0	5	0	5	0		
45		6.	6.	6.	5.	5.	5.	5.	4.	4.	4.26	15
		4	2	1	5	4	2	9	5	4		
		3	7	1	5	0	4		5	0		
46		6.	6.	6.	5.	5.	5.	5.	4.	4.	4.25	14
		4	2	1	5	3	2	9	5	4		
		3	7	1	5	9	4		4	0		
47		6.	6.	6.	5.	5.	5.	5.	4.	4.	4.25	13
		4	2	1	5	3	2	9	5	4		
		3	6	0	4	9	4		4	0		
48		6.	6.	6.	5.	5.	5.	5.	4.	4.	4.25	12
		4	2	1	5	3	2	9	5	3		
		2	6	0	4	9	4		4	9		
49		6.	6.	6.	5.	5.	5.	5.	4.	4.	4.25	11
		4	2	1	5	3	2	8	5	3		
		2	6	0	4	9	3		4	9		
50		6.	6.	6.	5.	5.	5.	5.	4.	4.	4.24	10
		4	2	1	5	3	2	8	5	3		
		2	6	0	4	8	3		3	9		
51		6.	6.	6.	5.	5.	5.	5.	4.	4.	4.24	9
		4	2	9	5	3	2	8	5	3		
		2	6		3	8	3		3	9		
52		6.	6.	6.	5.	5.	5.	5.	4.	4.	4.24	8
		4	2	9	5	3	2	8	5	3		
		1	5		3	8	3		3	8		
53		6.	6.	6.	5.	5.	5.	5.	4.	4.	4.24	7
		4	2	9	5	3	2	7	5	3		
		1	5		3	8	3		3	8		
54		6.	6.	6.	5.	5.	5.	5.	4.	4.	4.23	6
		4	2	9	5	3	2	7	5	3		
		1	5		3	7	2		2	8		
55 [row label printed 65; recte 55]		6.	6.	6.	5.	5.	5.	5.	4.	4.	4.23	5
		4	2	8	5	3	2	7	5	3		
		0	4		3	7	2		2	8		

		6	6	6	6	6	6	6	6	6		min. (vers.)
	min.	0	1	2	3	4	5	6	7	8	69	
	56	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.23	4
		4	2	8	5	3	2	7	5	3		
		0	4		2	7	2		2	7		
	57	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.23	3
		4	2	8	5	3	2	6	5	3		
		0	4		2	7	2		2	7		
	58	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.23 [?]	2
		4	2	7	5	3	2	6	5	3		
		0	3		2	6	1		1	7		
	59	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.22	1
		3	2	7	5	3	2	6	5	3		
		9	3		1	6	1		1	7		
	60	6.	6.	6.	5.	5.	5.	5.	4.	4.	4.22	0
		3	2	7	5	3	2	6	5	3		
		9	3		1	6	1		1	6		
(deg., vers.)		2	2	2	2	2	2	2	2	2	20	
		9	8	7	6	5	4	3	2	1		

Foot-rubric: *Gradus altitudinum Solis pro umbris versis.*

Gnomonics, Liber V — *Tabula umbrarum* (concl.) and Problema 3

(Batch covers = printed folios 406–408. The twelve-part table of shadow lengths [*Tabula umbrarum*] of Book V concludes on folios 406–407, whereupon the discursive text resumes with the use of the table and Problema 3.)

Folio 406 — Table of shadow lengths, Sun's altitude 70°–89° (right shadows) / 19°–0° (versed shadows)

Centred sub-rubric: *Gradus altitudinum Solis pro umbris rectis.* — Degrees of the Sun's altitude for **right (direct) shadows** (read from the top, entering the altitude-degree along the top and its minutes down the **left** margin).

Rotated left-margin title: *Minuta altitudinum Solis pro umbris rectis* — Minutes of the Sun's altitude for right shadows. Rotated right-margin title: *Minuta*

altitudinum Solis pro umbris versis — Minutes of the Sun's altitude for versed shadows.

Each cell gives the shadow length in **partes. minuta** (parts and sixtieths) for a gnomon of 12 parts; e.g. 4.22 = 4 parts 22 minutes. M. = minutes of altitude (left argument, right shadows). The final column is the minutes of altitude for **versed** shadows (right argument, read upward with the bottom degree row).

M. (rect.)	70	71	72	73	74	75	76	77	78	79	M. (vers.)
0	4.2	4.8	3.5	3.4	3.2	3.1	3.0	2.4	2.3	2.2	60
	2		4	0	6	3		6	3	0	
1	4.2	4.8	3.5	3.4	3.2	3.1	2.5	2.4	2.3	2.2	59
	2		4	0	6	3	9	6	3	0	
2	4.2	4.7	3.5	3.4	3.2	3.1	2.5	2.4	2.3	2.2	58
	2		3	0	6	2	9	6	3	0	
3	4.2	4.7	3.5	3.3	3.2	3.1	2.5	2.4	2.3	2.1	57
	1		3	9	6	2	9	5	2	9	
4	4.2	4.7	3.5	3.3	3.2	3.1	2.5	2.4	2.3	2.1	56
	1		3	9	6	2	9	5	2	9	
5	4.2	4.7	3.5	3.3	3.2	3.1	2.5	2.4	2.3	2.1	55
	1		3	9	5	2	8	5	2	9	
6	4.2	4.7	3.5	3.3	3.2	3.1	2.5	2.4	2.3	2.1	54
	1		3	9	5	2	8	5	2	9	
7	4.2	4.6	3.5	3.3	3.2	3.1	2.5	2.4	2.3	2.1	53
	0		2	9	5	1	8	5	1	9	
8	4.2	4.6	3.5	3.3	3.2	3.1	2.5	2.4	2.3	2.1	52
	0		2	8	5	1	8	4	1	8	
9	4.2	4.6	3.5	3.3	3.2	3.1	2.5	2.4	2.3	2.1	51
	0		2	8	4	1	7	4	1	8	
10	4.2	4.6	3.5	3.3	3.2	3.1	2.5	2.4	2.3	2.1	50
	0		2	8	4	1	7	4	1	8	
11	4.2	4.5	3.5	3.3	3.2	3.1	2.5	2.4	2.3	2.1	49
	0		1	8	4	0	7	4	1	8	
12	4.1	4.5	3.5	3.3	3.2	3.1	2.5	2.4	2.3	2.1	48
	9		1	7	4	0	7	3	0	7	
13	4.1	4.5	3.5	3.3	3.2	3.1	2.5	2.4	2.3	2.1	47
	9		1	7	3	0	6	3	0	7	

M. (rect.)	70	71	72	73	74	75	76	77	78	79	M. (vers.)
14	4.1	4.5	3.5	3.3	3.2	3.1	2.5	2.4	2.3	2.1	46
	9		1	7	3	0	6	3	0	7	
15	4.1	4.4	3.5	3.3	3.2	3.9	2.5	2.4	2.3	2.1	45
	9		0	7	3		6	3	0	7	
16	4.1	4.4	3.5	3.3	3.2	3.9	2.5	2.4	2.2	2.1	44
	8		0	6	3		6	2	9	6	
17	4.1	4.4	3.5	3.3	3.2	3.9	2.5	2.4	2.2	2.1	43
	8		0	6	3		6	2	9	6	
18	4.1	4.4	3.5	3.3	3.2	3.9	2.5	2.4	2.2	2.1	42
	8		0	6	2		5	2	9	6	
19	4.1	4.3	3.5	3.3	3.2	3.8	2.5	2.4	2.2	2.1	41
	8		0	6	2		5	2	9	6	
20	4.1	4.3	3.4	3.3	3.2	3.8	2.5	2.4	2.2	2.1	40
	7		9	6	2		5	2	9	6	
21	4.1	4.3	3.4	3.3	3.2	3.8	2.5	2.4	2.2	2.1	39
	7		9	5	2		5	1	8	5	
22	4.1	4.3	3.4	3.3	3.2	3.8	2.5	2.4	2.2	2.1	38
	7		9	5	1		5	1	8	5	
23	4.1	4.3	3.4	3.3	3.2	3.8	2.5	2.4	2.2	2.1	37
	7		9	5	1		4	1	8	5	
24	4.1	4.2	3.4	3.3	3.2	3.7	2.5	2.4	2.2	2.1	36
	7		8	5	1		4	1	8	5	
25	4.1	4.2	3.4	3.3	3.2	3.7	2.5	2.4	2.2	2.1	35
	6		8	4	1		4	0	8	4	
26	4.1	4.2	3.4	3.3	3.2	3.7	2.5	2.4	2.2	2.1	34
	6		8	4	1		4	0	7	4	
27	4.1	4.2	3.4	3.3	3.2	3.7	2.5	2.4	2.2	2.1	33
	6		8	4	0		4	0	7	4	
28	4.1	4.1	3.4	3.3	3.2	3.6	2.5	2.4	2.2	2.1	32
	6		7	4	0		3	0	7	4	
29	4.1	4.1	3.4	3.3	3.2	3.6	2.5	2.4	2.2	2.1	31
	5		7	3	0		3	0	7	3	
30	4.1	4.1	3.4	3.3	3.2	3.6	2.5	2.3	2.2	2.1	30
	5		7	3	0		3	9	6	3	

M. (rect.)	70	71	72	73	74	75	76	77	78	79	M. (vers.)
31	4.1 5	4.1	3.4 7	3.3 3	3.1 9	3.6	2.5 3	2.3 9	2.2 6	2.1 3	29
32	4.1 5	4.0	3.4 7	3.3 3	3.1 9	3.6	2.5 3	2.3 9	2.2 6	2.1 3	28
33	4.1 4	4.0	3.4 6	3.3 3	3.1 9	3.5	2.5 2	2.3 9	2.2 6	2.1 3	27
34	4.1 4	4.0	3.4 6	3.3 2	3.1 9	3.5	2.5 2	2.3 8	2.2 6	2.1 2	26
35	4.1 4	4.0	3.4 6	3.3 2	3.1 9	3.5	2.5 2	2.3 8	2.2 5	2.1 2	25
36	4.1 4	4.0	3.4 6	3.3 2	3.1 8	3.5	2.5 2	2.3 8	2.2 5	2.1 2	24
37	4.1 3	3.5 9	3.4 5	3.3 2	3.1 8	3.4	2.5 1	2.3 8	2.2 5	2.1 2	23
38	4.1 3	3.5 9	3.4 5	3.3 1	3.1 8	3.4	2.5 1	2.3 7	2.2 5	2.1 1	22
39	4.1 3	3.5 9	3.4 5	3.3 1	3.1 8	3.4	2.5 1	2.3 7	2.2 5	2.1 1	21
40	4.1 3	3.5 9	3.4 5	3.3 1	3.1 7	3.4	2.5 1	2.3 7	2.2 4	2.1 1	20
41	4.1 2	3.5 8	3.4 4	3.3 1	3.1 7	3.4	2.5 1	2.3 7	2.2 4	2.1 1	19
42	4.1 2	3.5 8	3.4 4	3.3 1	3.1 7	3.3	2.5 0	2.3 7	2.2 4	2.1 1	18
43	4.1 2	3.5 8	3.4 4	3.3 0	3.1 7	3.3	2.5 0	2.3 6	2.2 4	2.1 0	17
44	4.1 2	3.5 8	3.4 4	3.3 0	3.1 7	3.3	2.5 0	2.3 6	2.2 3	2.1 0	16
45	4.1 1	3.5 7	3.4 3	3.3 0	3.1 6	3.3	2.5 0	2.3 6	2.2 3	2.1 0	15
46	4.1 1	3.5 7	3.4 3	3.3 0	3.1 6	3.2	2.4 9	2.3 6	2.2 3	2.1 0	14
47	4.1 1	3.5 7	3.4 3	3.2 9	3.1 6	3.2	2.4 9	2.3 6	2.2 3	2.1 0	13

M. (rect.)	70	71	72	73	74	75	76	77	78	79	M. (vers.)
48	4.1 1	3.5 7	3.4 3	3.2 9	3.1 6	3.2	2.4 9	2.3 6	2.2 3	2.9	12
49	4.1 1	3.5 6	3.4 3	3.2 9	3.1 6	3.2	2.4 9	2.3 6	2.2 2	2.9	11
50	4.1 0	3.5 6	3.4 2	3.2 9	3.1 5	3.2	2.4 8	2.3 5	2.2 2	2.9	10
51	4.1 0	3.5 6	3.4 2	3.2 8	3.1 5	3.1	2.4 8	2.3 5	2.2 2	2.9	9
52	4.1 0	3.5 6	3.4 2	3.2 8	3.1 5	3.1	2.4 8	2.3 5	2.2 2	2.9	8
53	4.1 0	3.5 6	3.4 2	3.2 8	3.1 5	3.1	2.4 8	2.3 5	2.2 1	2.8	7
54	4.9	3.5 5	3.4 2	3.2 8	3.1 4	3.1	2.4 8	2.3 4	2.2 1	2.8	6
65 [recte 55]	4.9	3.5 5	3.4 1	3.2 8	3.1 4	3.0	2.4 7	2.3 4	2.2 1	2.8	5
56	4.9	3.5 5	3.4 1	3.2 7	3.1 4	3.0	2.4 7	2.3 4	2.2 1	2.8	4
57	4.9	3.5 5	3.4 1	3.2 7	3.1 4	3.0	2.4 7	2.3 4	2.2 0	2.8	3
58	4.8	3.5 4	3.4 1	3.2 7	3.1 3	3.0	2.4 7	2.3 3	2.2 0	2.7	2
59	4.8	3.5 4	3.4 0	3.2 7	3.1 3	3.0	2.4 6	2.3 3	2.2 0	2.7	1
60	4.8	3.5 4	3.4 0	3.2 6	3.1 3	3.0	2.4 6	2.3 3	2.2 0	2.7	0
(deg., vers.)	19	18	17	16	15	14	13	12	11	10	

Foot-rubric: *Gradus altitudinum Solis pro umbris versis*. — Degrees of the Sun's altitude for **versed shadows** (bottom degree row + right minute argument).

Folio 407 — Table of shadow lengths, Sun's altitude 80°–89° (right shadows) / 9°–0° (versed shadows)

Centred sub-rubric: *Gradus altitudinum Solis pro umbris rectis*.

Rotated left-margin title: *Minuta altitudinum Solis pro umbris rectis*. Rotated right-margin title: *Minuta altitudinum Solis pro umbris versis*.

Format as above (shadow length in partes. minuta for a gnomon of 12 parts).

M. (rect.)	80	81	82	83	84	85	86	87	88	89	M. (vers.)
0	2.7	1.5	1.4	1.2	1.1	1.3	0.5	0.3	0.2	0.1	60
		4	1	8	6		0	8	5	3	
1	2.7	1.5	1.4	1.2	1.1	1.3	0.5	0.3	0.2	0.1	59
		4	1	8	5		0	8	5	2	
2	2.7	1.5	1.4	1.2	1.1	1.3	0.5	0.3	0.2	0.1	58
		4	1	8	5		0	7	5	2	
3	2.6	1.5	1.4	1.2	1.1	1.2	0.5	0.3	0.2	0.1	57
		3	1	8	5		0	7	5	2	
4	2.6	1.5	1.4	1.2	1.1	1.2	0.5	0.3	0.2	0.1	56
		3	0	8	5		0	7	4	2	
5	2.6	1.5	1.4	1.2	1.1	1.2	0.4	0.3	0.2	0.1	55
		3	0	7	5		9	7	4	2	
6	2.6	1.5	1.4	1.2	1.1	1.2	0.4	0.3	0.2	0.1	54
		3	0	7	4		9	6	4	1	
7	2.5	1.5	1.4	1.2	1.1	1.2	0.4	0.3	0.2	0.1	53
		2	0	7	4		9	6	4	1	
8	2.5	1.5	1.3	1.2	1.1	1.1	0.4	0.3	0.2	0.1	52
		2	9	6	4		9	6	3	1	
9	2.5	1.5	1.3	1.2	1.1	1.1	0.4	0.3	0.2	0.1	51
		2	9	6	4		8	6	3	1	
10	2.5	1.5	1.3	1.2	1.1	1.1	0.4	0.3	0.2	0.1	50
		2	9	6	4		8	6	3	1	
11	2.5	1.5	1.3	1.2	1.1	1.1	0.4	0.3	0.2	0.1	49
		2	9	6	3		8	5	3	0	
12	2.4	1.5	1.3	1.2	1.1	1.0	0.4	0.3	0.2	0.1	48
		1	9	6	3		8	5	3	0	
13	2.4	1.5	1.3	1.2	1.1	1.0	0.4	0.3	0.2	0.1	47
		1	8	6	3		8	5	3	0	
14	2.4	1.5	1.3	1.2	1.1	1.0	0.4	0.3	0.2	0.1	46
		1	8	5	3		7	5	2	0	

M. (rect.)	80	81	82	83	84	85	86	87	88	89	M. (vers.)
15	2.4	1.5 1	1.3 8	1.2 5	1.1 2	1.0	0.4 7	0.3 5	0.2 2	0.1 0	45
16	2.4	1.5 1	1.3 8	1.2 5	1.1 2	1.0	0.4 7	0.3 4	0.2 2	0.9	44
17	2.3	1.5 0	1.3 7	1.2 5	1.1 2	0.5 9	0.4 7	0.3 4	0.2 2	0.9	43
18	2.3	1.5 0	1.3 7	1.2 4	1.1 2	0.5 9	0.4 7	0.3 4	0.2 1	0.9	42
19	2.3	1.5 0	1.3 7	1.2 4	1.1 2	0.5 9	0.4 6	0.3 4	0.2 1	0.9	41
20	2.3	1.5 0	1.3 7	1.2 4	1.1 1	0.5 9	0.4 6	0.3 4	0.2 1	0.9	40
21	2.2	1.5 0	1.3 7	1.2 4	1.1 1	0.5 9	0.4 6	0.3 3	0.2 1	0.8	39
22	2.2	1.4 9	1.3 6	1.2 4	1.1 1	0.5 8	0.4 6	0.3 3	0.2 1	0.8	38
23	2.2	1.4 9	1.3 6	1.2 3	1.1 1	0.5 8	0.4 6	0.3 3	0.2 0	0.8	37
24	2.2	1.4 9	1.3 6	1.2 3	1.1 0	0.5 8	0.4 5	0.3 3	0.2 0	0.8	36
25	2.2	1.4 9	1.3 6	1.2 3	1.1 0	0.5 8	0.4 5	0.3 2	0.2 0	0.8	35
26	2.1	1.4 8	1.3 6	1.2 3	1.1 0	0.5 8	0.4 5	0.3 2	0.2 0	0.7	34
27	2.1	1.4 8	1.3 5	1.2 2	1.1 0	0.5 7	0.4 5	0.3 2	0.1 9	0.7	33
28	2.1	1.4 8	1.3 5	1.2 2	1.1 0	0.5 7	0.4 4	0.3 2	0.1 9	0.7	32
29	2.1	1.4 8	1.3 5	1.2 2	1.9	0.5 7	0.4 4	0.3 2	0.1 9	0.7	31
30	2.0	1.4 8	1.3 5	1.2 2	1.9	0.5 7	0.4 4	0.3 1	0.1 9	0.6	30
31	2.0	1.4 7	1.3 5	1.2 2	1.9	0.5 6	0.4 4	0.3 1	0.1 9	0.6	29

M. (rect.)	80	81	82	83	84	85	86	87	88	89	M. (vers.)
32	2.0	1.4	1.3	1.2	1.9	0.5	0.4	0.3	0.1	0.6	28
		7	4	1		6	4	1	8		
33	2.0	1.4	1.3	1.2	1.8	0.5	0.4	0.3	0.1	0.6	27
		7	4	1		6	3	1	8		
34	2.0	1.4	1.3	1.2	1.8	0.5	0.4	0.3	0.1	0.6	26
		7	4	1		6	3	1	8		
35	1.5	1.4	1.3	1.2	1.8	0.5	0.4	0.3	0.1	0.5	25
	9	7	4	1		6	3	0	8		
36	1.5	1.4	1.3	1.2	1.8	0.5	0.4	0.3	0.1	0.5	24
	9	6	3	0		5	3	0	8		
37	1.5	1.4	1.3	1.2	1.8	0.5	0.4	0.3	0.1	0.5	23
	9	6	3	0		5	3	0	7		
38	1.5	1.4	1.3	1.2	1.7	0.5	0.4	0.3	0.1	0.5	22
	9	6	3	0		5	2	0	7		
39	1.5	1.4	1.3	1.2	1.7	0.5	0.4	0.2	0.1	0.4	21
	9	6	3	0		5	2	9	7		
40	1.5	1.4	1.3	1.2	1.7	0.5	0.4	0.2	0.1	0.4	20
	8	5	3	0		5	2	9	7		
41	1.5	1.4	1.3	1.1	1.7	0.5	0.4	0.2	0.1	0.4	19
	8	5	2	9		4	2	9	7		
42	1.5	1.4	1.3	1.1	1.6	0.5	0.4	0.2	0.1	0.4	18
	8	5	2	9		4	1	9	6		
43	1.5	1.4	1.3	1.1	1.6	0.5	0.4	0.2	0.1	0.3	17
	8	5	2	9		4	1	9	6		
44	1.5	1.4	1.3	1.1	1.6	0.5	0.4	0.2	0.1	0.3	16
	7	5	2	9		4	1	8	6		
45	1.5	1.4	1.3	1.1	1.6	0.5	0.4	0.2	0.1	0.3	15
	7	4	1	9		3	1	8	6		
46	1.5	1.4	1.3	1.1	1.6	0.5	0.4	0.2	0.1	0.3	14
	7	4	1	8		3	1	8	6		
47	1.5	1.4	1.3	1.1	1.5	0.5	0.4	0.2	0.1	0.2	13
	7	4	1	8		3	0	8	5		
48	1.5	1.4	1.3	1.1	1.5	0.5	0.4	0.2	0.1	0.2	12
	7	4	1	8		3	0	8	5		

M. (rect.)	80	81	82	83	84	85	86	87	88	89	M. (vers.)
49	1.5 6	1.4 4	1.3 1	1.1 8	1.5	0.5 2	0.4 0	0.2 7	0.1 5	0.2	11
50	1.5 6	1.4 3	1.3 0	1.1 8	1.5	0.5 2	0.4 0	0.2 7	0.1 5	0.2	10
51	1.5 6	1.4 3	1.3 0	1.1 7	1.5	0.5 2	0.4 0	0.2 7	0.1 5	0.2	9
52	1.5 6	1.4 3	1.3 0	1.1 7	1.4	0.5 2	0.3 9	0.2 7	0.1 4	0.2	8
53	1.5 6	1.4 3	1.3 0	1.1 7	1.4	0.5 2	0.3 9	0.2 6	0.1 4	0.2	7
54	1.5 5	1.4 2	1.3 0	1.1 7	1.4	0.5 1	0.3 9	0.2 6	0.1 4	0.1	6
55	1.5 5	1.4 2	1.2 9	1.1 6	1.4	0.5 1	0.3 9	0.2 6	0.1 4	0.1	5
56	1.5 5	1.4 2	1.2 9	1.1 6	1.3	0.5 1	0.3 8	0.2 6	0.1 3	0.1	4
57	1.5 5	1.4 2	1.2 9	1.1 6	1.3	0.5 1	0.3 8	0.2 6	0.1 3	0.1	3
58	1.5 4	1.4 2	1.2 9	1.1 6	1.3	0.5 0	0.3 8	0.2 5	0.1 3	0.0	2
59	1.5 4	1.4 1	1.2 9	1.1 6	1.3	0.5 0	0.3 8	0.2 5	0.1 3	0.0	1
60	1.5 4	1.4 1	1.2 8	1.1 6	1.3	0.5 0	0.3 8	0.2 5	0.1 3	0.0	0
(deg., vers.)	9	8	7	6	5	4	3	2	1	0	

Foot-rubric: *Gradus altitudinum Solis pro umbris versis.*

Folio 408 — Resumption of the text

[Margin: *The use of the preceding table of shadow lengths.*]

Now the use of this table — which is general, and fitted to every clime — is as follows. Given the altitude of the Sun, look up its degrees in the upper part of the table, and the minutes, if there are any, in the left-hand margin. At once, in the cell where they meet, you will find the parts and minutes of the **right (direct) shadow**, reckoned in the units of which the gnomon contains twelve. But if the

degrees are taken in the lower part of the table, and the minutes, if there are any, in the right-hand margin, then in the common cell you will find the parts and minutes of the **versed shadow**. For example, if the length of the right shadow is sought for a Sun's altitude of 64° , we shall find it to contain 5 parts 51 minutes. The same shadow, when the Sun has an altitude of $31^\circ 30'$, will span 19 parts 35 minutes, and so on. Likewise, if the versed shadow is sought for a Sun's altitude of 26° , we shall find it to span 5 parts 51 minutes; whereas, when the Sun has an altitude of $58^\circ 30'$, it will be 19 parts 35 minutes, and so on.

[Margin: *The lengths of the solstitial, equinoctial, and brumal (winter-solstice) shadows — how, by what method, they are found from the preceding table for any latitude of place.*]

From this same table we shall also learn the lengths of the solstitial, equinoctial, and brumal shadows for any latitude of place — a purpose for which many authors have composed special tables. For if the meridian altitude at either solstice is taken, we shall find from it the shadow's length more quickly than can be told. And for the equinoctial shadow, one must look up in the table the complement of the pole's elevation; for so great is the meridian altitude then, and so on. Thus at latitude 42° , the meridian altitude of the first point of φ (Cancer) is $71^\circ 30'$, to which there correspond in the table 4 parts 1 minute, for the length of the right solstitial shadow. Again, the meridian altitude of the first point of γ (Aries) or ζ (Libra) is 48° , to which in the same table there answer 10 parts 48 minutes, for the right equinoctial shadow. Finally, the meridian altitude of the first point of ψ (Capricorn) is $24^\circ 30'$; therefore the right brumal shadow will span 26 parts 20 minutes, and so with the rest.

[Margin: *How, geometrically, the length of the shadow is found from the Sun's altitude.*]

Geometrically, too, the length of the right shadow for any altitude of the Sun is found in this way. In any circle — for instance in the one we drew above in this proposition — draw two straight lines A C and B D cutting one another at right angles in the centre E. On AC take a straight segment EF equal to the gnomon whose shadow-length is being sought, and through F draw FG parallel to BD. Lastly, from point D (or from B) toward A, count off the Sun's altitude up to a point I, which in the worked example comes to about 38° . For a straight line IE drawn from I through the centre E will cut the line FG in G, and F G will be the length of the right shadow for the given Sun's altitude D I, as we demonstrated above. In the same way HG will be the versed shadow, if the segment EH is taken equal to the gnomon and through H a parallel HG to AC is drawn.

PROBLEMA 3. PROPOS. 3.

To investigate the arc of any great circle intercepted between that circle's own Vertical [circle], properly so called, and the Vertical circle which at any given hour is drawn through the centre of the Sun.

[Margin: *By what route the arc of any great circle, intercepted between its own Vertical properly so called and the other Vertical which is drawn through the Sun, is to be sought.*]

Let the third figure of proposition 36, book 1 be repeated, in which ABCD is the proposed great circle — whether it be the Horizon or not; BED is its own proper Meridian, drawn of course through its poles and through the poles of the world; AFC is the Vertical circle of the proposed circle, properly so called, passing namely through the poles of that circle and through the poles of its proper Meridian; ELO is the Vertical drawn through the centre of the Sun, placed at L; and NL is the hour-circle passing through the poles of the world and through the centre of the Sun at the proposed hour. Then AO will be the arc lying between the said two Vertical circles, which we shall find as follows. Since, in the spherical triangle ENL — by proposition 17, book 4 of Johannes Regiomontanus, *On triangles* [Regiomontanus, *De triangulis* IV.17], or by proposition 13, book 1 of Geber [Jābir ibn Aflāḥ, *Gebri*, I.13], or by proposition 41 of our own [treatise] on spherical triangles — it holds that, as the sine of arc EL, the complement of the Sun's altitude above the proposed circle, is to

Diagram 1. The first (largest) of three spherical schematics illustrating the construction of Problema 3. A great circle ABCD is drawn with several interior great-circle arcs. Readable labels and their positions:

- B (top of circle) — top pole of the proper Vertical
- D (bottom of circle) — opposite pole
- A (left of circle) — point of the proper Vertical AFC
- C (right of circle) — opposite point of AFC
- Q (upper-left, interior) — [?] point on an interior arc
- F (upper-centre, on the meridian BED) — pole/intersection
- H (centre, on the meridian, below F)
- E (centre of the sphere)
- P (just below E, on the meridian)
- N (below P, on the meridian) — foot of the hour-circle NL
- L (centre-left, position of the Sun's centre)

- M (upper-left of L, interior)
- K (lower-left of L, interior)
- O (left, on the proper Vertical, between A and L)
- G (lower-left of circle)
- I (lower-right of circle)

Diagram 2. The second, smaller spherical schematic of the same construction (a different position of the Sun). Readable labels:

- A (upper) — point of the proper Vertical
- C (upper-left) — opposite point
- M (interior, near A)
- K (interior, right of M)
- O (interior, below M) — [?]
- L (interior, below O) — Sun's centre
- G (lower) — lower point
- Q (upper, faint) — [?]

(The peripheral cardinal points B/D of this compressed sphere are not separately legible on the scan.)

Diagram 3. The third spherical schematic of the same construction (yet another position of the Sun). Readable labels:

- B (top of circle)
- D (bottom of circle)
- C (right of circle)
- F (upper-centre, on the meridian)
- H (centre, on the meridian, below F)
- E (centre of the sphere)
- N (below E, on the meridian)
- P (below N, on the meridian)
- I (lower-right of circle)
- K (left interior) — [?]
- L (left interior, Sun's centre)

(The far-left point A of this sphere falls at the cropped edge of the scan.)

the sine of angle ENL, the distance of the Sun from the Meridian of the proposed circle, so is the sine of arc NL, the complement of the declination (for when the

Sun is in a southern parallel, as in the fourth circle, the arc N L has the same sine as the remaining arc of the semicircle included between L and the other pole, which is the complement of the declination LM), to the sine of angle NEL. If, then, it be made that as the sine of the complement of the Sun's altitude is to the sine of the Sun's distance from the Meridian of the proposed circle, so is the sine of the complement of the declination to a fourth [sine], the sine of angle N E L will be found — that is, of arc DO, whose complement is AO, the arc sought — which we shall thus investigate from arc DO. If the Sun stands beyond the proper Vertical, toward the hidden pole, as in the first and fourth circles (how this is recognized will be explained shortly in a scholium), we shall subtract from the quadrant AB the arc BO owed to the found sine of angle BEO (for this angle, or arc BO, has the same sine as angle NEL, or arc DO, since those two angles are equal to two right angles, and the said two arcs make up a semicircle), and there will remain the arc sought, AO, known, toward the hidden pole. But if the Sun is found on the near side of the Vertical properly so called, toward the visible pole, as in the second, third, and sixth circles, we shall take away from the quadrant AD the arc DO answering to the found sine of angle NEL, and there will be left the arc sought, AO, toward the visible pole.

[Margin: *The same arc is more easily investigated at the time of the equinoxes.*]

The problem is made **easier**, however, when the Sun is on the Equator, because then the multiplication is by the whole sine [radius]. For if, in the first circle, the parallel GHI be conceived to be the Equator — so that the proper Vertical passes through G and I, and the arc sought is GO — then the arc N L will be a quadrant, to which corresponds the whole sine,

the whole sine [radius] — and not the sine of the complement of the declination, as before. Therefore, if it be made that as the sine of arc EL, the complement of the Sun's altitude, is to the sine of angle ENL, the Sun's distance from the Meridian of the proposed circle, so is the whole sine of the quadrant NL to a fourth [sine], the sine of angle NEL, or BEO, will be found; and its arc BO, subtracted from the quadrant BG, will leave the arc sought, GO, etc.

Diagram. The first of three spherical schematics across the top of the page (a repeat of the six-circle construction of Prop. 1, for a further position of the Sun). Readable labels:

- B (top of circle)
- P (upper-centre, on the meridian below B)
- L (upper-left, interior)

- M (left of centre)
- F (centre)
- E (centre-lower) — [?]
- N (lower-centre, on the meridian)
- D (bottom of circle)
- C (right of circle)
- I (right, upper)
- O, Q (upper-left, outside/near the rim) — [?]
- G (mid-left rim) — [?]
- A (far-left rim, at the cropped edge of the scan) — [?]

Diagram. The second spherical schematic of the same construction. Readable labels:

- B (top of circle)
- F (upper-centre, on the meridian)
- H (centre-right)
- E (centre-lower)
- M (left of centre)
- K (left interior)
- G (lower-left)
- N (lower-centre, on the meridian)
- I (right of circle)
- D (bottom of circle)
- A (left rim)
- C (small label at the left rim, adjacent to A) — [?]

Diagram. The third spherical schematic of the same construction. Readable labels:

- B (top of circle)
- F (upper-centre, on the meridian)
- H (centre)
- E (centre)
- K (left interior)
- A (left rim)
- G (lower-left)
- O (right of circle; a second O appears at lower-left interior)

- I (right of circle)
- N (lower-centre, on the meridian)
- (bottom-of-circle label, printed as "B") — [?]

to a fourth, the sine of angle NEL, or BEO, will be found; and its arc BO, subtracted from the quadrant BG, will leave the arc sought, GO, etc.

This arc is nothing when the Sun stands on the Vertical properly so called of the proposed circle; but it is equal to a quadrant when the Sun is placed on the Meridian of the same proposed circle, as is evident.

[Margin: *When the said arc is either nothing or equal to a quadrant.*]

We shall find the same arc on the Meridian of the Horizon by the following method. Let the figure of Prop. 1 of this book be repeated, in which the said arc is EI; there the Equator BED is the Vertical properly so called of the Meridian of the Horizon, and BHI is the Vertical drawn through B, the pole of the Meridian, and through H, the Sun's place. Since in the spherical triangle BHK the angle K is right, it will hold — by Prop. 16, Book 4, of Johannes Regiomontanus [Regiomontanus] *On Triangles*, or by Prop. 41 of our books on spherical triangles — that as the sine of arc BH, the complement of the Sun's altitude above the Meridian of the Horizon, is to the whole sine of the right angle K, so is the sine of arc HK, of the declination, to the sine of angle HBK, or of arc EI, which is sought.

[Margin: *How the same arc is found on the Meridian of the Horizon.*]

If, then, it be made that as the sine of the complement of the Sun's altitude above the Meridian is to the whole sine, so is the sine of the declination to a fourth, there will be found the sine of the arc of the Meridian included between the two Verticals, which is sought.

Finally, we shall find this same arc on the Right Horizon, or on the circle of the 6th hour from noon or midnight, in this way. Since in the spherical triangle EHK the angle K is right, it will hold — by the same Prop. 16, Book 4, of Regiomontanus, or by Prop. 13, Book 1, of Geber [Jābir ibn Aflah], or by Prop. 41 of our work on spherical triangles — that as the sine of arc EH, the complement of the Sun's altitude above the circle of the 6th hour or the Right Horizon, is to the whole sine of the right angle K, so is the sine of arc HK, of the declination, to the sine of angle HEK, or of arc BL. This arc is included between the Vertical properly so called BED — for the Equator, drawn through E, the pole of the said circle, is its primary Vertical — and the Vertical EL, drawn through the Sun's centre H.

Diagram. A large spherical figure (the repeated figure of Prop. 1), symmetric about its vertical meridian. Readable labels:

- A (top of circle)
- C (bottom of circle)
- F (left rim, one upper and one lower)
- G (right rim, one upper and one lower)
- D (right rim, at mid-height)
- O (lower-right rim)
- E (centre)
- K (left interior)
- L (upper-left and lower-left, on the circle)
- H (upper-left interior and lower-left interior)
- I (interior, beside the meridian — one upper, one lower)
- M (interior, on the meridian — one upper below A, one lower)
- N (right-of-centre interior)

Therefore, if it be made that as the sine of the complement of the Sun's altitude above the circle of the 6th hour or the Right Horizon is to the whole sine, so is the sine of the declination to a fourth, there will be found the sine of the arc of the said circle, placed between the two named Verticals.

These arcs, for the sake of the doctrine, we shall call the "latitudes of the shadows" [latitudines umbrarum], because — as we shall teach in Prop. 5 of this book — from them we shall know how great a latitude the gnomon's shadow has in the proposed circle, that is, toward which quarter it is cast at any proposed hour. We have therefore found the arc of any great circle intercepted between the Vertical of its own circle, etc. Which was to be done.

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It is enough, however, if such arcs be investigated for the individual hours on one face of the proposed great circle. For these same arcs are the same, in number, on the other face for the same hours — provided that the hours from sunrise and noon in the parallel of ♋ [Greek/astronomical sign: Cancer, i.e. the Tropic of Cancer] be changed into hours from sunset and midnight in the parallel of ♏ [Capricorn, i.e. the Tropic of Capricorn], and conversely. For, as we showed above in Prop. 1 of this book, such hours — if the said exchange be made — have the same altitudes above each face of the proposed great circle, and are equally distant from the Meridian or the Horizon.

But to know whether the Sun, at a given hour, lies beyond the Vertical circle, or on the Vertical itself, or on the near side of it — namely, when it stands in a parallel that is cut by the Vertical in the hemisphere above the proposed circle (such as the parallel lying between the Equator and the visible pole above the proposed circle, and having a declination smaller than the altitude of the pole above the given circle; for a parallel between the visible pole and the Equator that has a declination greater than the altitude of the pole above the given circle is in no way cut by the Vertical, whereas a parallel between the Equator and the hidden pole either is not cut by the Vertical at all, or is cut only in the hemisphere below the proposed circle, as is evident) — we shall first investigate, from what we have shown partly in Prop. 36, Book 1, and partly in Prop. 1 of this book, how great the Sun's distance from the Meridian of the proposed circle is when it is set on the Vertical circle of the same. For if the Sun, at the given hour, has a distance from the Meridian smaller than that found value, it will lie beyond the Vertical; if equal, it will be found on the Vertical itself; if finally greater, it will be placed on the near side of the Vertical. Or else we shall find, from what we demonstrated in Prop. 36, Book 1, how great an altitude the Sun has on the proper Vertical of the proposed circle. For if at the given hour it has an altitude greater than that found value, the Sun will lie beyond the Vertical; if equal, on the Vertical itself; if finally smaller, it will be placed on the near side of the Vertical.

[Margin: *By which parallels of the Equator [the Sun] is cut by the proper Vertical of each proposed circle, and by what method this is known.*]

Now in all great circles we call that part "on the near side of" [citra] their Vertical circle which, from its Vertical, inclines toward the visible pole — the pole, that is, which is raised above the given circle; and that part "beyond" [ultra] the Vertical which, from its Vertical, recedes toward the other pole, which is hidden. And in sundials the former is always drawn back from the stylus toward the centre of the dial, while the latter [inclines] toward the equinoctial line.

[Margin: *Which part of each great circle is called "on the near side of" the Vertical, and which "beyond" the Vertical.*]

But if it should happen that a parallel between the visible pole and the Equator does not cut the proper Vertical of that great circle (which occurs only when the declination of the parallel is greater than the altitude of the pole above the proposed great circle), then all such arcs of the latitudes of shadows will lie on the near side of the Vertical — that is, toward the visible pole above the given circle, or toward the equinoctial line of the dial, as is evident.

[Margin: *When all the latitudes of shadows lie on the near side of the Vertical circle.*]

By this technique we have computed various little tables, in which the said arcs, or latitudes of shadows, are contained for the individual hours — whether reckoned from noon or midnight, or from sunrise or sunset, or even the unequal hours in the same great circles above which we investigated the Sun's altitudes in Prop. 1. You will find these tables in Prop. 5 of this book, where we shall construct various sundials from them.

PROBLEMA 4. PROPOS. 4.

To find the arc of any given great circle not passing through the poles of the world — the arc included between, on the one hand, the great circle that passes through the poles of the world and through the common sections of the Equator and the given great circle (such as the circle of the 6th hour from noon and midnight — in the Horizon, in the Vertical, and in every other circle inclined indeed to the Horizon but at right angles to the Meridian of the Horizon); and, on the other hand, any other hour-circle drawn at any given hour through the poles of the world and the Sun's centre.

[Margin: *By what technique is computed the arc of any great circle not passing through the poles of the world, placed between the great circle drawn through the poles of the world and the common sections of the Equator and the given circle (like the circle of the 6th hour from noon and midnight in the Horizon), and any other hour-circle drawn through the poles of the world and the Sun.*]

Diagram. A spherical figure for the demonstration of Problema 4. Readable labels:

- G (top of circle)
- C (bottom of circle)
- B (left rim)
- D (right rim)
- I, A, M (upper interior, left to right; A on the meridian)
- O, P (left interior)
- H, E, N (central row, left to right; E at the centre)
- K (lower-left rim)
- L (lower-left interior)
- F (bottom-centre interior, above C)

Let ABCD be the great circle — whether it be the Horizon or not, provided it does not pass through the poles of the world; let AEC be its proper Meridian, passing through F and G, the poles of the world; let BED be the Equator; and let FHG be the hour-circle drawn through the same poles, cutting the Equator in H and the proposed circle in I; and let the arc EH — the Sun's distance from the Meridian — be less than a quadrant, or six hours. The arc to be found is IB, of the proposed circle, lying between B — the common section of it and of the Equator, through which, of course, the hour-circle is drawn, like the circle of the 6th hour from noon and midnight in the Horizon — and the hour-circle FHG.

Since in the spherical triangle AGI the angle A is right, it will hold — by Prop. 18, Book 4, of Regiomontanus *On Triangles*, or by Prop. 14, Book 1, of Geber, or by Prop. 42 of our work on spherical triangles — that as the sine of angle G, the Sun's distance from the proper Meridian of the proposed circle, is to the whole sine, so is the sine of the complement of angle I to the sine of the complement of arc AG, the altitude of the pole above the proposed circle. And by conversion: as the whole sine is to the sine of the Sun's distance from the proper Meridian of the given circle, so is the sine of the complement of the pole's altitude to the sine of the complement of angle I. Moreover, in the triangle BHI, by Prop. 16, Book 4, of Regiomontanus *On Triangles*, or by Prop. 13, Book 1, of Geber, or by Prop. 41 of our work on spherical triangles, it holds that as the sine of angle I is to the sine of arc BH, the complement of the Sun's distance from the proper Meridian of the given circle, so is the whole sine of the right angle H to the sine of arc BI, which is proposed to be found.

Therefore, if it be made that as the whole sine is to the sine of the Sun's distance from the proper Meridian of the given circle, so is the sine of the complement of the pole's altitude above the proposed circle to a fourth, there will be found the sine of a certain arc, whose complement will give the angle I. And if again it be made that as the sine of the found angle I is to the sine of the complement of the Sun's distance from the proper Meridian of the proposed circle, so is the whole sine to a fourth, there will be obtained the sine of that arc of the proposed great circle which is sought.

[Margin: *What the hour-arcs in the proposed circle are called.*]

But if another hour-circle be described, FK, cutting the Equator in K and the proposed circle in L, such that the arc EK — the Sun's distance from the proper Meridian of the given circle — is greater than a quadrant, or six hours, the arc BL will be found in the same way. For it will hold, as before, in the spherical triangle CFL: as the whole sine is to the sine of angle CFL, the Sun's distance from the

proper Meridian of the given circle (for the angles CFL and EFK have the same sine, since they are equal to two right angles), so is the sine of the complement of arc CF, the altitude of the pole above the proposed circle, to the sine of the complement of angle L; and thus the angle L itself will be known. Again, in the triangle BKL it holds that as the sine of angle L is to the sine of arc BK, the complement of the Sun's distance from the Meridian of the proposed circle, so is the whole sine of the right angle K to the sine of arc BL, which is sought.

Furthermore, for the sake of the doctrine and of brevity, we may call arcs of this kind "hour-arcs" [arcus horarios] in the proposed circle, because they indicate — as will become clear from what follows — how far apart, in the proposed plane, the hour-lines (or hour-spaces) ought to lie: that is, the common sections of the proposed plane and of the hour-circles drawn through the poles of the world. We have said in the proposition that these arcs are included between the great circle drawn through the poles of the world by way of the common sections of the Equator and the given circle (like the circle of the 6th hour from noon and midnight in the Horizon) and the other hour-circles drawn through the poles of the world — and not between the common sections of the Equator and the given circle and the other hour-circles, though this latter is nonetheless true; so that we may more easily demonstrate, in the following proposition, how sundials are made from these arcs upon a plane parallel to the proposed great circle.

On the Equator the said arcs are equal to the complements of the Sun's distances from the Meridian circle — such as the arcs BH, DN, etc. Hence it is not necessary to seek out the arcs of the Equator lying between the circle drawn like the 6th-hour circle from noon or midnight in the Horizon and the other hour-circles drawn through the poles of the world. We have therefore found the arc of any great circle not passing through the poles of the world, included between the [aforesaid] great circle, etc. Which was to be done.

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It is evident, moreover, that when the Sun's distance from the proper Meridian of the given circle is less than a quadrant, or six hours, the said arcs extend from the common section of the proposed circle and of that great circle drawn through the poles of the world (like the 6th-hour circle from noon and midnight in the Horizon) toward the hidden pole — as from point B, or D, toward A; but in the sundial they extend beyond the centre of the dial, and not toward the equinoctial line, because then the hour-circle cuts the quadrant of the Equator EB, or ED, and thus also the quadrant of the proposed great circle AB, or AD. But when the Sun's distance from the Meridian is equal to a quadrant — that is, when it comprises

six hours — the said arc is nothing, because the hour-circle then passes through the point B, or D. And when the Sun's distance from the Meridian is nothing, the said arc is a quadrant. Finally, when that same distance is greater than a quadrant, or six hours, the said arcs extend from the common section of the proposed circle and of that great circle drawn through the poles of the world (like the 6th-hour circle from noon and midnight in the Horizon) toward the visible pole — as from point B, or D, toward C — and in the sundial toward the equinoctial line, that is, on the near side of the centre of the dial; because then the hour-circle falls beyond the point B, or D, on the Equator, and consequently cuts the quadrant of the proposed circle CB, or CD.

[Margin: *Toward which part the hour-arcs incline in the proposed circle, and how this is known from the Sun's distance from the proper Meridian.*]

Now if the Meridian AEC be one of the hour-circles, or at least equally removed from two hours set on either side of it, then the arcs of the proposed great circle belonging to two hours equally distant from the Meridian of the proposed circle — the arcs intercepted between that great circle drawn through the poles of the world (like the circle of the 6th hour from noon and midnight) and the hour-circles of those hours drawn through the poles of the world — will be equal, one of them being eastern and the other western. For in the spherical triangles AGI, AGM: if the hour-circles FIG, FMG be posited as equally distant from the Meridian FAG, the angles at G will be equal, because of the equality of the arcs EH, EN; the angles at A are moreover right, and the side AG is common, lying adjacent to the equal angles. Therefore, by Prop. 23, Book 1, of Menelaus (in the version handed down by Francesco Maurolico [Francesco Maurolico]), or by Prop. 20 of our books on spherical triangles, the arcs AI, AM will also be equal; and when these are taken away from the quadrants AB, AD, the remaining arcs IB, MD will be equal.

[Margin: *Which hour-arcs are equal to one another.*]

Again, the arcs of the same proposed great circle answering to two hours — of which the distance of one from the proper Meridian is as much less than a quadrant as the other's is greater — will be equal. For if two hour-circles FK, FO be posited as equally distant from the point B, so that the arcs of the Equator BP, BK are equal, then in the spherical triangles BKL, BPO the angles at the vertex B will be equal, the angles K, P right, and the sides BK, BP equal. Wherefore, as before, the arcs BL, BO will be equal.

From these it follows that it is enough — in the Horizon and in the Vertical circle properly so called (if we speak of hours from noon and midnight) — to investigate such arcs for five hours before or after noon. For the arc of the 6th hour from noon and midnight is nothing; the arc of the 5th hour after noon is equal to the arc of the 7th hour after noon, the arc of the 4th hour to that of the 8th hour, and so on, as we have demonstrated. Indeed, in the other great circles it will likewise be enough if, of those twelve hours from noon or midnight, the arcs be investigated for those hours which are distant from the proper Meridian of the circle by less than six hours — all of which, as is clear from what has been said, lie beyond the centre of the dial, that is, toward the hidden pole. For the hour-lines of these arcs, which cut the equinoctial line in the dial, when produced beyond the centre of the dial will also give the hours that do not cut the equinoctial line and that have a distance from the proper Meridian of the proposed circle greater than six hours. All of which will become clear from the following proposition.

[Margin: *In the Horizon and the Vertical properly so called it is enough to compute the hour-arcs for five hours before or after noon; but in the other great circles, for those 12 hours from noon and midnight that are less than six hours away from their proper Meridians.*]

For the rest: since the altitude of the pole above the Vertical circle properly so called of any region is the complement of the altitude of the pole above the Horizon of the same region — because the arc of the Meridian drawn from the vertex through the pole of the world down to the Horizon (which measures the pole's altitude both above the Vertical and above the Horizon) is a quadrant, as we also taught in Prop. 13, Book 3 — it comes about that the arc of the Horizon for any hour from noon or midnight, computed for one region by this proposition, is the arc of the Vertical for the same hour in another region whose latitude is equal to the complement of the latitude of the former region. For in the latter region the altitude of the pole above the Vertical is the same as its altitude above the Horizon in the former. For, as is established from what has been said, the altitude of the pole above the Vertical of that region whose latitude, together with the latitude of the former region, makes up a quadrant is equal to the complement of the altitude of the pole above the Horizon of the same region — that is, to the altitude of the pole above the Horizon of the former region. Therefore it will be enough to compute the said arcs of the Horizon for all latitudes, for the hours from noon or midnight; for the same arcs will serve for the Vertical circles, if instead of the latitude of the place its complement be taken. Thus, if the said horizontal arcs be computed for all latitudes, the arcs for latitude 42° will indeed

be horizontal in the region whose latitude comprises 42° , whereas the arcs computed for latitude 48° will be the Vertical arcs in that same region of latitude 42° , etc.

[Margin: *The altitude of the pole above the Vertical properly so called of any region is the complement of the altitude of the pole above the Horizon of the same region.*]

But if the complements of the hour-arcs computed by this proposition be taken, one obtains those same hour-arcs that take their beginning from the proper Meridian of the proposed great circle — the arcs which many writers have computed. These too we shall trace out from the proposed figure. After the angle I has been found in the triangle AGI, as we have taught, then — since in the same triangle, by Prop. 16, Book 4, of Regiomontanus *On Triangles*, or by Prop. 13, Book 1, of Geber, or by Prop. 41 of our books on spherical triangles, it holds that as the sine of angle I is to the sine of arc AG, the altitude of the pole above the proposed circle, so is the sine of angle G, the Sun's distance from the proper Meridian, to the sine of arc AI, which is proposed to be found — if it be made that as the whole sine is to the sine of the Sun's distance from the proper Meridian, so is the sine of the complement of the pole's altitude above the proposed circle to a fourth, there will be found, as above, the sine of a certain arc whose complement will give the angle I. And if again it be made that as the sine of the found angle I is to the sine of the pole's altitude above the proposed circle, so is the sine of the Sun's distance from the proper Meridian to a fourth, there will be obtained the sine of that arc of the great circle which is sought.

[Margin: *By what reasoning the hour-arcs of all Horizons are also the hour-arcs of all Verticals properly so called.*]

[Margin: *How the same hour-arcs, taking their beginning from the proper Meridian, are investigated.*]

Furthermore, we shall set out several tables of hour-arcs in the following proposition.

PROBLEMA 5. PROPOS. 5.

To construct various sundials from the tables that we taught how to compute in the preceding propositions.

[Margin: What must be computed for each plane, so that sundials may be constructed on it.]

So that it may be fully and thoroughly understood by what method sundials are made from the tables we taught how to compute in the preceding propositions, we shall lay out sundials on all those planes on which we constructed sundials in Books 2 and 3. In this way every variety and difficulty that might arise will be removed. We shall not, however, describe on every plane all four kinds of hours, as we did in Books 2 and 3, but on some the hours from noon or from midnight together with those from the west only [ab occ. duntaxat]; on some, only the hours from noon and midnight, or from the east, or from the west, or even the unequal hours. For since all hours, of whatever kind, are described in the same way, it will be enough to set out examples of a single kind.

For each plane, therefore, one must compute — by Prop. 3 of this book — the shadow-latitudes for those hours which we wish to place on the sundial, with the Sun at the beginning of ♋ [Cancer], ♈ [Aries] or ♎ [Libra], and ♏ [Capricorn], taking careful note, however, of which arcs of the shadow-latitudes fall on the near side of the plane's own vertical circle and which lie beyond it, and likewise which are eastern and which western with respect to the plane's own Meridian. Then, from the table of shadow-lengths which we computed in Prop. 2 of this book, one must extract the lengths of the direct shadows [longitudines umbrarum rectorum] corresponding to the Sun's altitudes which we found for the same hours in Prop. 1 of this book.

Let it be required, then, to construct a horizontal astronomical sundial from tables containing the said shadow-latitudes and shadow-lengths, for latitude 42°, of the kind that follow.

Shadow-latitudes and shadow-lengths on a horizontal sundial, for the hours from noon and from midnight, at latitude 42°.

In the tropic of ♋ [Cancer] the shadow-latitudes are partly northern and partly southern, as these syllables [Bor. / Austr.] indicate.

	5	6	7	8	9	10	11	12	
Hours ♋ from midnight	Bo r.	Bo r.	Bo r.	Aus tr.	Aus tr.	Aust r.	Aust r.	Aust r.	eastern [orientales]
Hours ♋ from noon	7 Bo r.	6 Bo r.	5 Bo r.	4 Aus tr.	3 Aus tr.	2 Aust r.	1 Aust r.	0 Aust r.	western [occidentales]
Shadow-latitude (G. M. = deg. min.)	27 .1 2	17 .5 4	8 48	0.34	11.5 7	27.9	51.1 9	90.0	

	5	6	7	8	9	10	11	12	
Hours ☉ from midnight	Bo r.	Bo r.	Bo r.	Aus tr.	Aus tr.	Aust r.	Aust r.	Aust r.	eastern [orientales]
Shadow-length (P. M. = parts. min.)	13 2. 17	43 .1 9	24 .1 6	15.4 1	10.3 7	7.13	4.56	4.1	

In the Equator all the hours and shadow-latitudes are southern.

Hours ☿ & ♄ from midnight	6	7	8	9	10	11	12	eastern [orientales]
Hours ☿ & ♄ from noon	6	5	4	3	2	1	0	western [occidentales]
Shadow-latitude (G. M.)	0.0	10. 10	21. 7	33. 47	49. 12	68. 11	90. 0	
Shadow-length (P. M.)	Infin i.	61. 16	29. 59	19. 26	14. 16	11. 39	10. 48	

In the tropic of ♊ [Capricorn] all the hours and shadow-latitudes are southern.

Hours ♊ from midnight	8	9	10	11	12	eastern [orientales]
Hours ♊ from noon	4	3	2	1	0	western [occidentales]
Shadow-latitude (G. M.)	37. 13	48. 24	61 .1	75. 3	90. 0	
Shadow-length (P. M.)	162 .8	54. 30	35 .7	28. 12	26. 20	

In these little tables the eastern hours — and therefore the eastern shadow-latitudes — are easily distinguished from the western. For all the hours from midnight are eastern, and those from noon western, as is noted in these tables. Further, the syllable [Bor.] signifies that the hour to which it is attached, and the shadow-latitude corresponding to it, is northern in the Horizon — that is, falls on the near side of the vertical circle; while the syllable [Austr.] indicates that the hour to which it is assigned, and the shadow-latitude corresponding to it, is southern, that is, falls beyond the vertical circle.

On the plane of the sundial let a circle ABCD of any size be described, with centre E, at which the stylus [gnomon] is to be fixed at right angles; and let two diameters AC and BD be drawn, cutting one another at right angles at the centre. Of these let AC be the meridian line — that is, the common section of the

Meridian and of the sundial's plane — and consequently BD (which is to be drawn occult) the common section of the vertical circle properly so called and of the same sundial's plane. For since both the vertical circle and the Horizon — and therefore also the plane of the sundial parallel to it — make right angles with the Meridian,^a the common section of the vertical circle and of the sundial's plane will be perpendicular to that same Meridian, and hence also^a to the meridian line AC lying in the Meridian, by Def. 3 of Euclid, Book 11. Therefore, since both the Meridian and the vertical circle are drawn through the stylus, the line BD — drawn through E, the place of the stylus, perpendicular to AC — will be the common section of the vertical circle and of the sundial's plane. By this same demonstration it can be shown, in all the sundials that follow, that the line which at the place of the stylus cuts the stylus-line, or the proper meridian line, at right angles is the common section of the sundial's plane and of the vertical circle properly so called of that plane.

[Margin: Description of the horizontal astronomical sundial from the tables.]

[Margin (keyed to the superscript ^a): "a 19. vnde." — cross-reference; the figure "19." and the abbreviation are only partially legible [?]]

Now if the sundial's plane is fixed and parallel to the Horizon, then in it — by what we demonstrated in the scholium to Prop. 23 of Book 1 — one must find the meridian line AC, from whose point E, where the stylus is to be placed, the circle ABCD is to be described, and the occult diameter BD perpendicular to AC is to be drawn as the common section of the vertical circle and of the sundial's plane. Let the meridional or southern parts be set at A, the northern at C, and consequently the eastern at B and the western at D. Choose also any length of stylus EF, and from a straight line GH cut off any number of equal parts you please; at the end of the first part write the number 12, at the end of the second the number 24, and so on, always adding 12. Likewise let the line GH be produced toward G, and let a further line GI be taken, equal to the stylus, divided into 12 equal little parts. For in this way we shall not be forced to divide each single part of the line GH into 12 parts in order to take the shadow-lengths; rather, they can most conveniently be taken from the whole line IH, as we shall soon teach.

After this — since, with the Sun in the Equator, the stylus's shadow at noon, that is, at the 12th hour from midnight, is cast toward the north and comprises 10 parts 48 minutes, as is clear from the preceding tables under the 12th hour of ♊ or ♎ — if this shadow be taken from the line GI by means of the compass and transferred from E toward the northern parts C, as far as the point K, then at the time of the equinoxes the extremity of the shadow will at noon fall at K; and so

through K the equinoctial line must be drawn perpendicular to the meridian line, such as is KL.

[Margin: How the equinoctial line is drawn in the horizontal sundial.]

We shall also draw it in this way. Let the line EF, equal to the stylus, be set up perpendicular to AC, and at F let the angle EFK be made equal to the altitude of the pole (which is easily done if from F, toward D, a circle be described, on which the altitude of the pole is counted off from the line FD toward C, and so on), so that the line FK cuts the meridian line toward the northern parts at K. For through the point K the equinoctial line must be drawn perpendicular to AC, as we demonstrated in the scholium to Prop. 1 of Book 2. But if at F another angle EFM be made, equal to the complement of the pole's altitude, so that the line FM cuts the meridian line at M, then M will be the centre of the sundial, in which all the hour-lines from noon and midnight meet, as we demonstrated in the same scholium to Prop. 1 of Book 2.

[Margin: By what method the centre of the horizontal sundial is found.]

We shall also find this centre by the following method. In the table of shadow-lengths of Prop. 2 of this book, take the altitude of the pole, 42° , whose shadow is 13 parts 20 minutes. For if this shadow be taken from the line IH by the compass — placing one foot of the compass on the number 12, and the other on 1 part 20 minutes of the line GI, so that the whole line contained between the feet of the compass comprises 13 parts 20 minutes — and transferred from E along the meridian line toward the southern parts A as far as M, then M will be the centre of the sundial. For if the Sun were placed at the arctic pole, the shadow of the stylus — and therefore the Sun's ray, that is, the axis of the world — would fall at the point M, since it would then have an altitude of 42° above the Horizon. Hence, as we taught in the corollary to Prop. 21 of Book 1, the point M will be the centre of the sundial.

Now (to come to the drawing of the hours) let the line of the 6th hour from noon be described. Under the 6th hour of the tropic of $\mathfrak{E}$, which is western and northern, I find the arc of shadow-latitude $17^\circ 54'$. This number, therefore, from D,

Diagram. A large horizontal-sundial construction figure occupying the lower two-thirds of the page. [The identical woodcut is reprinted on the following page so the text on both leaves can refer to it; it is described once here.] Above the circle runs a horizontal graduated scale line, marked **H** at its left end and **I** (with a fine comb-graduation) at its right, carrying the numbers (left→right) **144, 132,**

120, 108, 96, 84, 72, 60, 48, 36, 24, 12, then **G**, then **I** — the scale IGH of shadow-parts (I = 0, G = 12 = one stylus-length, H = 144). Below it is the main circle **ABCD** (the horizon-plate), with a vertical diameter (the meridian line AC) and a horizontal diameter.

Readable labels (position; content):

- A (top of circle): **A**, with **MER.** [meridies — south] beside it
- C (bottom of circle): **C**, with **BOR.** [borealis — north]
- L (left end of horizontal diameter): **L**
- B (left, above L): **B**, with **ORI** [oriens — east] set vertically beside it
- R (left, above B): **R**
- M (centre region): **M** — the sundial's centre, with dashed lines converging on it from the left
- E (just right of M, centre): **E** — the stylus-foot; a small **12** stands near M/E
- n (far left, on a dashed line): **n**
- f (left, script, on the dashed line): **f**
- T (upper-left, on the circle): **T**
- X (upper-left, on the circle): **X**
- 4 (left of centre): **4**
- O (left of centre, near an "r"): **O**
- u (lower-left): **u**
- P (lower-left, on a radiating line): **P**
- d (lower centre-left): **d**
- 23 (lower-left) with an adjacent glyph resembling **h/h** [?], and a second **P** beside it
- Bottom row of hour-numbers (left→right): **22, 21, 20, 19, 18, 17, 16**
- Right side, radiating hour-lines top→bottom: **10, 11** (with **m** beside it), **12, 13, 14, 15**
- Upper-interior small labels: **6, 8** and (script) **f, 6, q, Q**, with script **C, C** [these small interior sorts are only partly legible]
- Central cluster (left of centre): **E, S, 8, Z, Z, K, V**, and **4F** (a "4" beside **F**)
- Centre-right, descending the meridian: **D, N, Q, α** [alpha], and lower **Y**

The figure shows the two tropic day-arcs and the equinoctial line crossing the plate, with the hour-lines (numbered 6 through 23) radiating from the centre M and the shadow-tip construction points (K, L, N, O, P, Q, R, S, T, V, X, Y) laid out as the text describes. The interior of the figure is exceedingly dense; many of the

small numerals and script letters listed above cannot be resolved with full confidence. **

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[Margin: The use of that quadrant constructed at the beginning of Book 1.]

toward the western point, at C, I lay off the northern parts as far as N (which I accomplish by means of that quadrant we constructed at the beginning of Book 1; for by it all arcs are easily transferred into any circle, so that its wonderful usefulness may appear especially in this matter). And from N, through E, I draw the occult line NEO, which will be the common section of the sundial's plane and of the vertical circle passing through the Sun at the 6th hour from noon. For since the vertical circles cut the Horizon — and the circle ABCD parallel to it — into similar parts, by Prop. 10 of Book 2 of Theodosius; and since the arc DN is similar to the one intercepted in the Horizon between the primary vertical and that vertical which is drawn through the Sun at the 6th hour from noon; and since the primary vertical passes through D — then another vertical will pass through N, and therefore the line NEO will be the common section of that vertical and of the sundial's plane. Therefore, by Prop. 11 of Book 1, the shadow of the stylus will fall on the line EO, inasmuch as it falls on the eastern and southern side, which is opposite to the western and northern side on which the Sun then is. And in this way, from the arcs of the shadow-latitudes, the shadow-latitude is found —

[Margin: By what technique the latitude of the shadow is sought from the arcs of the shadow-latitudes.]

that is, the direction in which the shadow is cast is known, if the said arcs are correctly reckoned from the point D or B, toward C or A, according as they are western or eastern, and northern or southern.

Again, under the same 6th hour of the tropic of $\mathfrak{S}$, I find the shadow-length 43 parts 19 minutes. This, then, taken from the line IH by the compass (placing one foot of the compass on the number 36, and the other on 7 parts 19 minutes of the line GI, so that the whole line lying between the feet of the compass comprises 43 parts 19 minutes), I transfer from E onto the occult line EO, as far as the point O. For at this point the extremity of the shadow will fall at the 6th hour from noon, with the Sun at the beginning of $\mathfrak{S}$. And since, as we showed in Prop. 1 of Book 2, the line of the 6th hour from noon is parallel to the equinoctial line and perpendicular to the meridian line, if through O a line be drawn perpendicular to AC — or equidistant from the equinoctial line — the line of the 6th hour from noon will be described.

But if to the arc DN an equal arc BP be taken, toward C, and from P, through E, the occult line PEQ be drawn, and from EQ a line EQ be cut off equal to EO, then the extremity of the shadow will fall at the point Q at the 6th hour from midnight, with the Sun in the tropic of ♌ . For the arc is the same for this hour as for the former, and the shadow-length the same; but the arc of the 6th hour from midnight is eastern, just as the arc of the 6th hour from noon is western, as appears from the table.

Let the line of the 9th hour from midnight now be described. Since its shadow-latitude in the table, with the Sun at the beginning of ♌ , is southern and eastern, and comprises $11^{\circ} 57'$, I lay it off from B, the eastern point, toward the south as far as R, and I draw the occult line RES; and from ES I cut off the shadow ES of 10 parts 37 minutes, which I find in the table under the 9th hour from midnight, with the Sun in the tropic of ♌ . For in this way the point S will be found, at which, with the Sun in the tropic of ♌ , the extremity of the shadow will fall at the 9th hour from midnight; and therefore through S the line of the 9th hour from midnight must be drawn.

Next, since with the Sun at the beginning of ♌ or ♍ the shadow-latitude of the same hour is eastern and southern, and comprises $33^{\circ} 47'$, I lay it off from B toward A, as far as the point T. For if the occult line TEV be drawn, cutting the equinoctial line at V, then at the time of the equinox the extremity of the shadow will fall at the point V at the 9th hour from midnight; and so through V the line of the 9th hour from midnight must be drawn. This point V we shall also detect by means of the shadow-length, in the following way. Under the 9th hour from midnight, with the Sun at the beginning of ♌ or ♍ , let the shadow-length 19 parts 26 minutes be taken from the table. For if this, taken by the compass, be laid on the line IH, and from E an occult arc of a circle be described, it will cut the equinoctial line at the point V. And thus we shall find the point of each hour on the equinoctial line either from the shadow-latitude alone or from the shadow-length alone.

Furthermore, with the Sun at the beginning of ♌ , I lay off from B toward A, as far as X, the shadow-latitude of the same hour — eastern and southern — found in the table as $48^{\circ} 24'$; and I draw the occult line XEY. For if from EY the shadow-length 54 parts 30 minutes, due to the same hour, be cut off, as is plain from the table, the point Y will be found, at which, with the Sun at the beginning of ♌ , the extremity of the shadow will fall at the 9th hour from midnight; and therefore through V the line of the 9th hour from midnight must be drawn. So if from S to Y a straight line be drawn — which will pass entirely through V, if no error has

been made — the line of the 9th hour from midnight will be described. In this way all the hour-lines will be drawn, if for each three points are found: one in the tropic of $\mathfrak{S}$, another on the equinoctial line, and a third in the tropic of $\mathfrak{V}$, as has been done for the 9th hour from midnight. But if perchance in one of the tropics a point of some hour

Diagram. The same horizontal-sundial construction figure as on the previous leaf, reprinted here (see the full description under fig-p425-1: scale H...G...I with 144–12, circle ABCD with A/MER at top, C/BOR at bottom, L at left, B/ORI east, centre M and stylus-foot E, hour-lines 6–23 radiating, and shadow-tip points K, L, N, O, P, Q, R, S, T, V, X, Y). The dense interior small numerals and script letters are again only partly legible. **

cannot be found, then the hour-line will have to be drawn through two points, one of which has been found in one tropic and the other on the equinoctial line. Thus you see that the 7th hour from midnight has been drawn through the point Z, found in the tropic of $\mathfrak{S}$, and through the point a on the equinoctial line. But if it should sometimes happen that a given hour has its point in one tropic only, its line must be drawn through that point once found and through M, the centre of the dial; or else through that point of the equinoctial line which belongs to the opposite hour. For example, if the 7th hour from noon is to be drawn, its point will be found in $\mathfrak{S}$ alone — though in our example it is in fact not found, because it lies farther than it should from E, the place of the style. Through that point, then, and through the point a of the equinoctial line, which corresponds to the 7th hour from noon, the line of the 7th hour from noon must be drawn.

Now since, as is clear from the tables set out above, two hours always have the same arc and the same shadow-length, they will be described by one and the same operation, provided the arc taken from the table is reckoned from B and from D, toward A or toward C. For if, equal to the arc B X by which we found the point Y in the tropic of $\mathfrak{V}$ for the 9th hour from midnight, we take the arc D b, and from b through E draw the occult straight line b Ed, and make the shadow Ed equal to the shadow EY, the point d will be found, in the tropic of $\mathfrak{V}$, for the 3rd hour from noon — since the 3rd hour from noon and the 9th hour from midnight are equally distant from the meridian, and so on. By this art the whole astronomical dial has been described. And if the points found in the tropics are suitably joined by curved lines, the arcs of $\mathfrak{S}$ and $\mathfrak{V}$ will be described. And if for the other signs the Sun's altitudes for the same hours are found, and from these the shadow-lengths — which are transferred from E, the place of the style, by means of the compass onto the hour-lines, marking points on them — the arcs of

the other signs will likewise be described. Or, if the shadow-latitudes for the hours of the other signs are found, then wherever their occult lines drawn through E cut the hour-lines, through those points the same arcs of the signs are to be delineated. And all the lines of the hours from noon and from midnight, if no error has been made in the drawing, will pass through M, the centre of the dial, when they are produced.

By exactly the same art all other dials whatever will be made, provided the shadow-latitudes and shadow-longitudes are correctly taken for each of the hours to be drawn. For the Italian horizontal dial the following tables have been computed, containing the shadow-latitudes and shadow-longitudes for the hours from sunset, for latitude 42°.

Shadow-latitudes and shadow-longitudes in the horizontal dial, for the hours from sunset, at latitude 42°.

In the tropic of ϖ the shadow-latitudes are partly northern and partly southern, as these syllables [Bor. Austr.] indicate.

Table — In the tropic of ϖ (hours 9–16; eastern group, "Orientales"). "G. M." = degrees and minutes (shadow-latitude); "P. M." = parts and minutes (shadow-length); Bor. = northern, Austr. = southern.

Hours of ϖ , from sunset	9	10	11	1	2	13	14	15	16	(Easter n)
direction	Bor.	Bo r.	Bo r.	B or	Aus tr.	Aus tr.	Aus tr.	Aus tr.	Aus tr.	
				.						
Shadow-latitudes (G. M.)	31.45	22. 11	13. 5	3. 4 9	6.24	19.2 1	38.2 7	70.2 0		
Shadow-longitudes (P. M.)	1031. 45	64. 14	30. 56	1 9. 3	12.4 3	8.39	5.51	4.14		

Table — In the tropic of ϖ (hours 17–24; western group, "Occidentales").

Hours of ϖ , from sunset	17	18	19	20	21	22	23	24	(Wester n)
direction	Aus tr.	Aus tr.	Aus tr.	Aus tr.	Bo r.	Bo r.	Bo r.	Bor.	

Hours of ☿, from sunset	17	18	19	20	21	22	23	24	(Weste rn)
Shadow-latitudes (G. M.)	67.4 3	36.5 3	18.2 1	5.45	4.3 2	13. 40	22. 48	32.27	
Shadow-longitudes (P. M.)	4.17	6.1	8.53	13.3	19. 36	32. 8	68. 46	Infin i.	

On the Equator all the hours, and the shadow-latitudes, are southern.

Table — On the Equator (signs ♈ and ♎). Upper hour-row = eastern ("orientales"); lower hour-row = western ("occidentales").

Hours of ♈ & ♎, from sunset (E)	12	13	14	15	16	17	18
(from sunset, W)	24	23	22	21	20	19	18
Shadow-latitudes (G. M.)	0.0	10.1 0	21.7	33.4 7	49.1 2	68.1 1	90.0
Shadow-longitudes (P. M.)	Infini.	61.1 6	29.5 9	19.2 6	14.1 6	11.3 9	10.4 8

In the tropic of ♋ all the hours and shadow-latitudes are southern.

Table — In the tropic of ♋ (hours 16–24). "orien."/"or." = eastern; "occ." = western; all shadow-latitudes southern.

Hours of ♋, from sunset	16	17	18	19	20	21	22	23	24
direction	orie n.	or.	or.	or.	occ	occ.	occ	occ.	occ.
Shadow-latitudes (G. M.)	42.1 6	54. 6	67. 25	81. 58	82. 58	68. 21	54. 56	43. 1	32.27
Shadow-longitudes (P. M.)	82.5 6	42. 44	31. 3	26. 51	26. 43	30. 36	41. 32	77. 40	Infini.

Now, just as by these syllables [Bor. Austr.] we distinguish the northern shadow-latitudes from the southern, so by these syllables [or. oc.] we mark off the eastern from the western.

[Margin: When a point is found in one tropic only, and on the equinoctial line — or in one tropic only — how the hour-line is to be drawn.]

[Margin: How the arcs of the signs are described in the dial from the tables.]

And so, if the line of the 23rd hour from sunset is to be described, we shall reckon off its shadow-latitude, $22^{\circ}48'$, which in the tropic of $\mathfrak{S}$ it has as western and northern, in the preceding figure from D toward C, as far as e; and we shall draw the occult line e Ef, from which, if we cut off the shadow 68 parts 46 minutes — namely the straight line Ef — we shall obtain f, the point of the 23rd hour in the tropic of $\mathfrak{S}$. Then we shall compute the shadow-latitude of the same 23rd hour, $43^{\circ}1'$, which in the tropic of $\mathfrak{V}$ it has as western and southern, from D toward A, as far as g; and we shall draw the occult straight line g Eh, from which, if we cut off the shadow 77 parts 40 minutes — namely the straight line Eh — the point h will be found, the point of the 23rd hour in the tropic of $\mathfrak{V}$. Therefore the straight line f h, once joined, will give the 23rd hour from sunset, which will pass entirely through the point L on the equinoctial line, through which the line of the 5th hour from noon has been drawn.

For the shadow-latitudes are the same in the hours reckoned from sunset or from sunrise — and in the unequal hours — as in the hours reckoned from noon or from midnight. Hence, once the points on the equinoctial line have been found for one class of hours, there is no need to investigate them again for the others. We shall draw the lines of the other hours from sunset in just the same way, provided you remember that those hours which have no points in the tropic of $\mathfrak{V}$ — such as the 15th, 14th, and 13th — must be drawn through their points found in the tropic of $\mathfrak{S}$ and on the equinoctial line. Thus you see that the line of the 13th hour from sunset has been carried through the point found in the tropic of $\mathfrak{S}$ and through a, the point of the equinoctial line, through which of course the line of the 7th hour from midnight has been drawn. And you will easily understand, from the table of Proposition 19 of Book 1, which hours from noon or midnight on the equinoctial line correspond to which hours from sunrise or sunset. The line of the 12th hour from sunset must be drawn through the point found in the tropic of $\mathfrak{S}$, parallel to the equinoctial line, as we demonstrated in Proposition 1 of Book 2. But the remaining hours before the 12th, which do not cut the equinoctial line, must be drawn through points found in the tropic of $\mathfrak{S}$ by means of the corresponding hours on the equinoctial line, as is established from the table of Proposition 19 of Book 1: thus the line of the 11th hour from sunset [is drawn] through the 23rd hour from sunset (or 5th from noon) on the equinoctial line, and the line of the 10th hour from sunset through the 22nd from sunset (or 4th from noon), and so on. Or else, for these same hours, other points must be sought in the parallel of $\mathfrak{I}$ or $\mathfrak{Q}$, by means of their shadow-latitudes and shadow-longitudes found in the said parallel.

But if the tropics have been drawn, we shall describe the Italian dial more easily, either by the shadow-longitudes alone or by the shadow-latitudes alone. For if, by means of the compass, the shadow-longitudes are taken from the straight line IH, and from E the occult arcs are described cutting the tropics; or if the occult lines of the shadow-latitudes are drawn cutting the tropics — then in either tropic the points of the hours from sunset will be obtained. If, then, the corresponding points are connected by straight lines passing through the hours from noon and midnight on the equinoctial line, the Italian dial will be described.

The shadow-longitudes can also be investigated geometrically from the Sun's altitudes, as we set forth at the end of the scholium to Proposition 2 of this book. For let there be a circle, or rather (which is sufficient) a semicircle ABC,

Diagram. A large semicircle ABC standing on a horizontal base-line, used to derive shadow-lengths geometrically from the Sun's altitude. The diameter BC is horizontal through the centre D, and the semidiameter AD rises perpendicularly to the apex A. A second horizontal line IG runs below and parallel to BC, passing through E (the foot of the style; DE, produced downward from D, is the gnomon). Rays are drawn from D (and from the diameter-ends B and C) through altitude-points on the arc down to the base-line, where the shadow-tips are marked; nearly-equal shadows are set on opposite sides of E (eastern hours to the right, western to the left) to avoid confusion.

- A (top, apex of the arc): terminus of the perpendicular semidiameter AD
- Q (upper left, on the arc)
- O (upper right, on the arc)
- M (left, on the arc, below Q)
- F (left, on the arc, below M)
- K (right, on the arc, below O)
- H (right, on the arc, below K)
- B (left end of the horizontal diameter BC)
- C (right end of the horizontal diameter BC)
- D (centre of the semicircle)
- E (on the base-line IG, foot of the style; DE = gnomon)
- I (left end of the base-line)
- L (on the base-line, left of E): shadow-tip
- P (on the base-line, left of E): shadow-tip E P
- R (on the base-line, right of E): shadow-tip E R
- N (on the base-line, right of E): shadow-tip E N

- G (right end of the base-line): shadow-tip E G

whose centre is D, and in which the diameter BC is cut at right angles by the semidiameter A D. Let there be taken in A D, produced, a straight line DE equal to the gnomon of the dial, and through E let IG be drawn parallel to the diameter BC. If then, for example, the arc BF is taken equal to the Sun's altitude at the 16th hour from sunset in the tropic of ♊ — that is, $8^{\circ}14'$ — and the straight line FDG is drawn cutting the straight line I G in G, then E G will be the length of the shadow cast by the style D E, as is clear from what we showed at the beginning of Proposition 2 and at the end of the scholium of the same proposition in this book. By the same art E I will be found, the shadow-length for the 23rd hour in the tropic of ♊, from CH, the arc of the Sun's altitude for the same hour. Likewise from C K, the arc of the Sun's altitude at the 21st hour in the tropic of ♊, the shadow-length E L for the same 21st hour from sunset will be found. So too B M, the arc of the Sun's altitude at the 18th hour from sunset in the tropic of ♊, will give EN, the shadow-length for the same hour; and C O, the arc of the Sun's altitude for the same hour in the tropic of ♋, will yield the shadow-length E P. Finally, from B Q, the arc of the Sun's altitude at the 15th hour from sunset in the tropic of ♋, we shall arrive at the shadow-length ER, and so on. It will be worthwhile to investigate the shadow-lengths for the eastern hours on one side of the style — namely the right — and those for the western hours [on the other side], as has been done in the figure. For unless this is done, shadows that are nearly equal will often be confused with one another — such as the two shadows E P and E R, or EL and EN, which are almost equal to each other.

Instead of the shadow-latitudes we can very conveniently use the hour-arcs which we taught how to compute in Proposition 4 of this book, in those dials where the centre can conveniently be marked. This can be done in the following way. From M, the centre of the dial, let a circle i p l m n of any size be described, in which the diameter i m is drawn perpendicular to the meridian line A C — which diameter will be the common section of the plane of the dial and of the circle of the 6th hour from noon or midnight, as we demonstrated in Proposition 1 of Book 2. Now let there be found, by Proposition 4 of this book, the hour-arc — for example, for the 23rd hour from sunset in the tropic of ♋ — enclosed between the circle of the 6th hour from noon or midnight and another great circle drawn through the poles of the world and through the 23rd hour from sunset in the tropic of ♋; this arc contains $11^{\circ}52'$, and it is

western and northern, because the Sun's distance from the meridian exceeds a quadrant. Now let this arc be reckoned from the point m toward the northern

parts, as far as l , and let the occult straight line $l M f$ be drawn. For if, by means of the compass, the shadow-length for the 23rd hour from sunset in the tropic of $\mathfrak{S}$ is taken and, from E (the place of the style), transferred toward f , the straight line $M f$ will be cut at f , the point of the 23rd hour from sunset in the tropic of $\mathfrak{S}$.

For since the circle of the 6th hour from noon or midnight, and that great circle drawn through the poles of the world and the 23rd hour from sunset in the tropic of $\mathfrak{S}$, are both drawn through the axis of the world, and meet the horizon — to which the horizontal dial is parallel — at the centre of the world, that is at F , the vertex of the style set perpendicular to the plane of the dial, it comes about that with the horizon they make common sections, [namely] straight lines, which at the centre of the world form an angle subtended by the arc of the horizon that lies between the circle of the 6th hour from noon or midnight and that other great circle. And since these two circles meet the axis of the world, ^a and make with it common sections that are parallel, ^b the like lines in the plane of the dial will comprehend an angle equal to the one which those former lines form in the horizon. Therefore, since the angle $m M l$ [is equal to] the one those lines form in the horizon — the angle in the plane of the dial being equal to the angle subtended by the arc of the horizon between the circle of the 6th hour from noon or midnight and the other great circle — the angle $m M l$ is equal to it, because the arc $m l$ is similar to the arc of the horizon enclosed between the circle of the 6th hour from noon or midnight and that other great circle, since it comprehends just as many degrees and minutes. Let $M m$, then, be the common section of the plane of the dial and of the circle of the 6th hour from noon or midnight; the straight line $M l$ will be the common section of the plane of the dial and of the other great circle through the poles of the world and the 23rd hour from sunset in the tropic of $\mathfrak{S}$. And accordingly, with the Sun placed in this circle — namely at the point where it cuts the tropic of $\mathfrak{S}$ — the shadow of the style will be projected, by Proposition 11 of Book 1, onto the straight line $l M f$. Since, therefore, the interval $E f$ has been taken equal to the length of the shadow, the shadow of the style at the 23rd hour from sunset in the tropic of $\mathfrak{S}$ will fall on the point f .

The matter will perhaps be carried out more conveniently if the hour-arc is reckoned in the opposite part of the circle $i p l m n$ — namely, the arc $m l$, which is western and northern, [reckoned] in the eastern and southern part, such as is $i n$. For it will then be sufficient if the straight line $M n$ is drawn, onto which, from E , the shadow-length $E f$ is to be transferred. The same is to be understood of the shadow-latitude arcs. For if, for example, in place of the arc $D N$ — the shadow-latitude of the 6th hour from noon in the tropic of $\mathfrak{S}$, which is western and northern — the opposite arc $B t$, eastern and southern, is taken, then only the

straight line $E t$ need be drawn, from which the shadow-length EO must be cut off, and so on. Again, let the hour-arc of the same 23rd hour from sunset in the tropic of φ be sought — namely $49^{\circ}26'$; this, since it is western and southern, is reckoned in the opposite part, from i toward the northern parts, as far as p , and the occult straight line $M p$ is drawn. For if, into this line, from E , the shadow-length corresponding to the 23rd hour from sunset in the tropic of φ is transferred, as far as the point h , then h will be the point of the 23rd hour in the tropic of φ . Finally, we shall find the hour-arc of the same 23rd hour from sunset, with the Sun placed on the Equator: $21^{\circ}49'$; and this, since it is western and southern, if we reckon it in the opposite part from the point i , as far as u , and draw the straight line $M u$, the equinoctial line will be cut at L , the point of the 23rd hour from sunset. When, therefore, the straight line $f L h$ is drawn, it will give the 23rd hour from sunset, and so on for the rest.

There is, however, a point here worth considering: any occult straight line drawn from M through the extreme point of any hour-line whatever in the tropic of φ , if it is produced beyond the centre M , passes through the extreme point of the line of another hour in the same tropic which is twelve hours distant from the first. Thus you see that the straight line $f M$, which is drawn through f , the extreme point of the 23rd hour from sunset in the tropic of φ , when produced beyond M , passes through q , the extreme point of the 11th hour from sunset in the same tropic. Likewise the straight line $r M f$ is drawn through the extreme points of the 22nd and 10th hours from sunset, and so on. Hence, if two hours from sunrise or sunset that are twelve hours apart fall in one and the same tropic, it will be enough to investigate the hour-arc of just one of them. The reason for this is that the great circle through the poles of the world, drawn through any hour from sunset in a tropic, also passes in the same tropic through the opposite hour — which of course is twelve hours distant from it — since it bisects the tropic, by Proposition 15 of Book 1 of Theodosius. From this it follows that the common section of that circle and the plane of the dial, drawn through M , passes in the plane of the dial through those two opposite hours.

Now this description of a dial by means of hour-arcs is carried out most expeditiously for the hours from noon and midnight, because, since all such hours pass through the centre M , it is not necessary to seek three arcs for each hour, as was done for the 23rd hour from sunset, but it will be enough if single arcs are sought — such as the following, for the several hours from noon and midnight of the horizontal dial, at latitude 42° .

Hour-arcs in the horizontal dial, for the hours from noon and midnight, at latitude 42°.

Table — Upper hour-row = hours from midnight ("Orientales", eastern); middle hour-row = hours from noon ("Occidentales", western); "G. M." = degrees and minutes of the hour-arc; all southern ("Australes").

Hours from midnight	6	7	8	9	10	11	12
Hours from noon	6	5	4	3	2	1	0
Hour-arcs (G. M.)	0.0	21.49	40.48	56.13	68.53	79.50	90.0

From these arcs, if they are reckoned from the points i and m toward the northern (or opposite) parts — since the arcs themselves are southern — and from M, the centre of the dial, straight lines are drawn through the ends of the reckonings, and if these lines are then cut off according to the intervals of the shadow-longitudes which the Sun has when it is in the tropics, the astronomical dial will be described. You have an example in the 7th hour from midnight and the 5th hour from noon, each of which has a southern hour-arc of 21°49'. If this is reckoned from the points i and m toward the opposite (that is, northern) parts, as far as * [?] and u, and the straight lines M * [?] and M u are drawn, onto which the intervals of the shadow-length of these same hours in the tropic of ☊ are transferred, the lines of the 7th hour from midnight and the 5th hour from noon will be described. And the same is to be done for the remaining hours. But the lines are first to be drawn as occult from the centre M, until the shadow-longitudes have been marked on them, because the parts of them outside the tropics are superfluous.

For the vertical southern dial — astronomical, and likewise Italian — the following tables have been computed, containing the shadow-latitudes and shadow-longitudes for the hours from noon and midnight as well as from sunset, and also the hour-arcs for the hours from noon and midnight. But since the horizon, with respect to the vertical circle taken as a horizon, is a ver-

[Margin: ^a 16. und[e] [?]; ^b 10. und[e] [?] — two keyed marginal notes, verbatim glyphs uncertain.]

[Margin: How, in place of the hour-arcs and shadow-latitude arcs, the opposite arcs may be taken in the description of dials.]

[Margin: It is enough, for two hours from sunrise or sunset one of which is twelve hours distant from the other, that a single hour-arc be investigated.]

[Margin: The description of the hours from noon and midnight by means of hour-arcs is most expeditious.]

is a vertical circle, we have called "inferior" those shadow-latitudes that lie below the horizon — that is, on the near side of the vertical of this vertical circle itself, toward the visible pole (which, of course, is elevated above the vertical circle, as is the antarctic pole); and "superior" those that lie above the horizon — that is, beyond the vertical of the vertical — toward the hidden pole (that is, toward the arctic pole), or toward the center of the dial. This is because the former are to be reckoned below the horizontal line, and the latter above it, in laying out the hours. Moreover, the shadow-latitudes of those hours which lie below the horizon — that is, on the near side of the vertical of the vertical — are those which have smaller altitudes above the vertical circle (taken as a horizon) than the Sun has in the horizon, when it is set upon that same vertical (as its own vertical) above the same vertical taken as a horizon, as we wrote in the scholium of proposition 3 of this book.

Shadow-latitudes and shadow-longitudes in the vertical southern dial, for the hours from noon and midnight, at latitude 42°.

In the tropic of Capricorn ♄, the shadow-latitudes are partly inferior and partly superior, as these labels [Infer. Super.] indicate.

Hours ♄ from midnight	5	6	7	8	9	10	11	12	(eastern)
	Inf er.	Inf er.	Inf er.	Sup er.	Sup er.	Sup er.	Sup er.	Sup er.	orientale s
Hours ♄ from noon	7	6	5	4	3	2	1	0	(western)
	Inf er.	Inf er.	Inf er.	Sup er.	Sup er.	Sup er.	Sup er.	Sup er.	occidenta les
Shadow-latitudes (G. M.)	26. 35	16. 14	5.4 7	5.1 7	18. 22	35. 12	58. 47	90. 0	
Shadow-longitudes (P. M.)	86. 29	38. 41	23. 28	15. 52	11. 14	8.8	6.1 1	5.2 8	

In the Equator all shadow-latitudes are superior.

Hours ♄ & ♈ from midnight	6	7	8	9	10	11	12	(eastern)
Hours ♄ & ♈ from noon	6	5	4	3	2	1	0	(wester

Hours Υ & $\underline{n}$ from midnight	6	7	8	9	10	11	12	(eastern) n)
Shadow-latitudes (G. M.)	0.0	11. 16	23. 13	36. 37	52. 9	70. 10	90. 0	
Shadow-longitudes (P. M.)	Infini.	68. 17	33. 48	22. 21	16. 53	14. 10	13. 20	

In the tropic of Cancer $\mathfrak{S}$, all shadow-latitudes are superior.

Hours $\mathfrak{S}$ from midnight	8	9	10	11	12	(eastern)
Hours $\mathfrak{S}$ from noon	4	3	2	1	0	(western)
Shadow-latitudes (G. M.)	37.25	49.6	61.5 1	75.3 7	90.0	
Shadow-longitudes (P. M.)	1146.4	86.2 9	49.3 6	38.4 1	35.5 2	

Shadow-latitudes and shadow-longitudes in the vertical southern dial, for the hours from sunset, at latitude 42° .

In the tropic of Capricorn $\mathfrak{V}$, the shadow-latitudes are partly inferior and partly superior, as these labels [Infer. Super.] indicate.

Hours ♀ from sunset	12	13	14	15	16	17	18	19	(easter n)
	Infe r.	Infe r.	Infe r.	Infe r.	Sup er.	Sup er.	Sup er.	Sup er.	oriental es
Shadow-latitudes (G. M.)	32.4 4	21.4 3	11.2 5	3.48	11.4	25.3 4	45.1 2	72.3 8	
Shadow-longitudes (P. M.)	211. 20	55.3 5	30.2	19.2 4	13.2 7	9.38	7.5	5.41	
Hours ♀ from sunset	20	21	22	23	24	(western)			
	Supe r.	Supe r.	Supe r.	Supe r.		occidentales			
Shadow-latitudes (G. M.)	74.44	46.44	26.41	11.56	0.0				
Shadow-longitudes (P. M.)	5.38	6.57	9.26	13.9	18.5				

Hours $\wp$ from sunset	20	21	22	23	24	(western)
					2	

In the Equator all shadow-latitudes are superior.

Hours Υ & $\underline{\alpha}$ from sunset	12	13	14	15	16	17	18	(eastern)
Hours Υ & $\underline{\alpha}$ from sunset	24	23	22	21	20	19	18	(western)
Shadow-latitudes (G. M.)	0.0	11. 16	23. 13	36. 37	52. 9	70. 10	90. 0	
Shadow-longitudes (P. M.)	Infini.	68. 17	33. 48	22. 21	16. 53	14. 10	13. 20	

(Row 1 = orientales, hours counted forward; row 2 = occidentales, the same columns counted 24→18.)

In the tropic of Cancer $\mathfrak{S}$, all shadow-latitudes are superior; but the eastern are distinguished from the western by these labels [or. occ.].

Hours $\mathfrak{S}$ from sunset	13	14	15	16	17	18	19	20
	or.	or.	or.	or.	occ.	occ.	occ.	occ.
Shadow-latitudes (G. M.)	43.3 2	55.4 6	69.5	83.1 5	82.1 7	68.1 0	54.5 4	42.45
Shadow-longitudes (P. M.)	147. 1	60.4 7	42.2 2	36.2 8	36.3 9	43.2	63.0	163.6 5

Hour-arcs in the vertical southern dial, for the hours from noon and midnight, at latitude 42° .

Hours from midnight	6	7	8	9	10	11	12	(eastern)
Hours from noon	6	5	4	3	2	1	0	(western)
Hour-arcs (G. M.)	0. 0	19.5 0	37.5 1	53.2 3	66.4 7	78.4 4	90. 0	(all Superiores)

From these tables, then, we shall lay out a vertical southern dial as follows. On the plane of the dial let a circle ABCD of any size be described, with center E, where the stylus is to be fixed at right angles. Draw two diameters, AC and BD, cutting each other at right angles at the center E. Let AC be the meridian line,

since it is the common section of the meridian and the plane of the dial; and accordingly let the other line, BD, be — as we showed in the horizontal dial — the common section of the vertical properly so called (namely, of the horizon) and of that same plane of the dial. That is, it is the horizontal line, which serves as the vertical line of this vertical dial, and from which the shadow-latitudes are to be reckoned. Now if the plane of the dial is fixed, parallel to the vertical circle and facing due south, then by means of a plumb-line the straight line AC, perpendicular to the horizon, is to be drawn on it as the meridian line; and from its point E, where the stylus is to be placed, the circle ABCD is to be described, and the diameter BD is to be drawn perpendicular to AC,

[Margin: Description of the vertical southern dial from the tables.]

Diagram. A large construction figure for the vertical southern dial. The identical woodcut is reprinted on the following page (folio 420, — see fig-p420-1), so the reader has it before them on both sides of the page-turn. Structure: a full circle labelled **ABCD** — **A** at the top, **C** at the bottom, **B** at the left, **D** at the right — with center **E**. The vertical diameter is **A–C** (the meridian line); the horizontal diameter is **B–D**, which runs along the horizontal line across which the word **HORIZON** is lettered (as "HORI...ZON"). Along the upper arc the two quadrant labels **OCC.** (Occasus, west; upper-left) and **ORI.** (Oriens, east; upper-right) are engraved following the curve. Above the whole figure runs a graduated straight scale from **I** (left) to **H** (right), marked with a fine-tick zone near the start at **G** and then numbered **12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156** — the gnomon-length / shadow-length scale (line IH). On the horizontal line: **F** lies just left of the center **E**, **M** stands to the right of center (the center of the vertical dial), then **P**, then the point **i** at the right end; a small **6** marks a point on this line. Near **E** cluster the small shadow-scale digits **8, 9, 10, 11, 12** and the short radiating marks **2, 3, 4, 5**. Below **E** is the convergence point **K**. From **K/E** a dense sheaf of hour-lines radiates to the lower arc, labelled with the hour-numbers **14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24**. External to the circle at upper-left is the ray-endpoint **V** (with a short cross-bar), and at upper-right the ray-endpoint **Y** (with an adjacent **7** and cross-bar). Lower-left of the figure: **L** on a line, and the bold point **Q** beside the "14" line. Lower-center: **N** (a second **N** also appears on the upper-left arc near **OCC.**) and **X** to its right. Bottom edge: **R** (left of center), **T** (center), and the ray-endpoints **B** (bottom-left) — hour-lines produced beyond the circle. A lowercase **m** sits at the far left of the horizontal (dashed) line. The text names further construction points **O, S, Z** on these lines and the auxiliary circle **iXZm**; the glyphs at those positions on the woodcut are small and partly overrun by ruled lines.

- I (top-left, scale start): endpoint of shadow-scale line IH
- G (top, after the fine-tick zone): scale origin of the divided part
- H (top-right, scale end): endpoint of shadow-scale line IH
- A (top of circle): superior point / meridian top
- C (bottom of circle): inferior point / meridian foot
- B (left of circle, on horizontal): western point; also used as ray-endpoint below
- D (right of circle, on horizontal): eastern point
- E (center): stylus foot
- F (horizontal line, left of E): gnomon point EF
- M (horizontal line, right of E): center of the vertical dial
- P (horizontal line, right): construction point
- i (right end of horizontal line): construction point of circle iXZm
- m (far left of horizontal, lowercase): construction point of circle iXZm
- K (below E): foot where FK meets the meridian below the stylus
- L (lower-left): point on the equinoctial line
- Q (lower-left, by "14"): shadow-latitude point (DQ / BQ)
- N (lower-center; also upper-left arc): shadow-latitude point on line EN
- X (lower-center-right): hour-point on circle iXZm
- R (bottom, left of center): hour-line endpoint
- T (bottom-center): hour-line endpoint
- V (upper-left, external): hour-arc endpoint (arc mV)
- Y (upper-right, external): hour-arc endpoint (arc iY)
- OCC. (upper-left arc): Occasus (west)
- ORI. (upper-right arc): Oriens (east)
- HORIZON (across horizontal line): the horizontal line
- 2, 3, 4, 5 (radiating near E): hour marks
- 6 (on horizontal line): hour-6 point
- 7 (by Y): hour-7 mark
- 8, 9, 10, 11, 12 (small, near E): shadow-scale divisions
- 14–24 (around lower arc): hour-line numbers
- O [?] (on line EN, per text): shadow-point of hour 13 from sunset — glyph not clearly resolved on the woodcut
- S [?] (per text, points S near the arc): glyph not clearly resolved
- Z [?] (lower-left / circle iXZm, per text): glyph not clearly resolved

to serve as the common section of the vertical properly so called — which does not differ from the horizon — and of the plane of the dial, that is, as the horizontal line. Or else, in the first place, on the plane of the dial the straight line BD, parallel to the horizon, is to be drawn by means of the plumb-line as the horizontal line; and from its point E, in place of the stylus, the circle ABCD is to be described, and the diameter AC, perpendicular to BD, is to be drawn as the meridian line. Let the superior parts be set toward A, the inferior toward C, and accordingly the eastern toward D and the western toward B. Choose also any length of gnomon EF you please, from which as many equal parts as you like are to be cut off on the straight line GH; and to it add further GI, equal to that same stylus and divided into 12 equal parts, so that from this line the shadow-lengths can be taken, as we described for the horizontal dial.

[Margin: How the equinoctial line is described in the vertical dial.]

And so, since — when the Sun is on the Equator — the shadow of the stylus at the 12th hour, noon, is cast below the horizontal line (just as at all the other hours it stands out above the horizon), and since this shadow, as is clear from the table of hours from noon and midnight, contains 13 parts 20 minutes: if we transfer this shadow with the compass from the line IH, taken onto the meridian line, downward from E as far as K, we shall draw through K the equinoctial line KL, perpendicular to AC. We shall also draw it by the following method. On the horizontal line BD, with EF taken equal to the stylus, if

Diagram. The identical vertical-southern-dial construction woodcut described above under fig-p419-1, reprinted here so that it faces the continuing text across the page-turn. Same labels: circle ABCD (A top, C bottom, B left, D right), center E, dial-center M, horizontal line HORIZON with points F, P, i, m and small 6; shadow-scale line I–H (12...156); convergence K below E; radiating hour-lines numbered 14–24; external hour-arc endpoints V (with 7-mark) and Y; lower points L, Q, N, X, R, T. See fig-p419-1 for the full label list; the same small glyphs O, S, Z remain uncertain.

[Margin: By what device the center of the vertical dial is found.]

if at F an angle EFK is constructed equal to the altitude of the pole above the vertical circle — that is, to the complement of the altitude of the pole above the horizon — so that the straight line FK cuts the meridian line below the stylus at K, then the equinoctial line will have to be drawn through K. And if at F another angle EFM is constructed, equal to the complement of the altitude of the pole above the vertical circle (that is, equal to the altitude of the pole above the

horizon), so that the straight line FM cuts the meridian line at M, then M will be the center of the dial. We shall find this same center in another way by this art. In the table of shadow-longitudes of proposition 2 of this book, take the altitude of the pole above the vertical circle — namely the complement of the altitude of the pole above the horizon — 48 degrees, whose shadow is 10 parts 48 minutes. For if this shadow is transferred with the compass from E along the meridian line upward as far as M, then M will be the center of the dial. This will be demonstrated as in the horizontal dial. The hours will be drawn just as in the horizontal dial, provided the shadow-latitudes are reckoned from B or D, upward or downward, as the tables indicate, and so on.

For example: for the 13th hour from sunset in the tropic of Capricorn ♑, count off its shadow-latitude of 21 degrees 43 minutes — the inferior and eastern one — from D, the eastern point, downward as far as N; or, on the opposite side (as we said for the horizontal dial), from B, the western point, upward as far as N. For if, on the hidden line EN, the shadow-longitude of that same hour, 55 parts 35 minutes, is cut off on the opposite side, the point O will be found — the point of the 13th hour from sunset in the tropic of Capricorn. In the same way, from DP — the eastern and superior shadow-latitude of the same 13th hour from sunset, in the Equator, 11 degrees 16 minutes — or from the opposite latitude BP, or from the shadow-longitude of that same hour, 68 parts 17 minutes, the point L was found on the equinoctial line, for the 13th hour from sunset. Likewise, from DQ — the eastern and superior shadow-latitude of the 9th hour from midnight in the tropic of Cancer ♋, 49 degrees 6 minutes — or from the opposite latitude BQ, together with the shadow-longitude of that same hour, 86 parts 29 minutes, the point R was found, for the 9th hour from midnight in the tropic of Cancer. And in the same tropic, from the same latitude BS — western and superior — of the 3rd hour from noon, or from the opposite DS with the same longitude, the point T was found, for the 3rd hour from noon. And if the same is done for the remaining hours, the vertical southern dial will be complete. By the same method, from the hour-arc mV — western and superior, 19 degrees 50 minutes — of the 5th hour from noon, or from the opposite iX, on the circle iXZm described from M, the point X was found, through which the 5th hour from noon is drawn. And from the same hour-arc iY — eastern and superior — of the 7th hour from midnight, or from the opposite mZ, the point Z was found on the same circle, through which the 7th hour from midnight is to be drawn. These two hour-lines, produced beyond the center M, will give the 5th hour from midnight and the 7th hour from noon, and so on.

[Margin: By what art, from a dial already constructed, one derives the dial to be placed on the opposite face of the plane — namely the northern from the southern, etc.]

But the portion above the horizontal line will yield the vertical northern dial, to be placed on the opposite face of the plane if it is inverted, as we prescribed at propositions 13 and 14 of book 2. For the hours from sunset, however, those from sunrise must first be drawn on the southern dial, since on the northern dial they are changed into hours from sunset, as we taught in the scholium of proposition 23 of book 2. And by the same method, from the dials that follow, there will be cut off from the horizontal line those dials that are to be placed on the opposite faces of the planes.

For the construction of both meridian dials, the following little tables of shadow-latitudes and shadow-longitudes have been composed, both for the hours from noon and midnight and for those from sunset. Now the circle of the 6th hour from noon and midnight passes through the poles of the world and through the poles of the meridian — namely through the common sections of the Equator and the horizon — and so is, as it were, the meridian of the meridian itself. Therefore the Equator, since it is at right angles both to the meridian and to the circle of the 6th hour from noon and midnight, will stand with respect to the meridian just as the vertical properly so called does. For in the same way the vertical of the horizon is at right angles [to it], and is drawn through both the poles of the horizon and the meridian of the horizon. And this is to be observed for every circle properly called the vertical of any given great circle: namely, that it passes and...

passes through the poles of that great circle itself, and through the poles of the meridian proper to that same great circle. Therefore, in the Meridian dial the shadow-latitudes [latitudines umbrarum] must be reckoned from the equinoctial line — that is, from the common section of the plane of the dial and the vertical properly so called of the Meridian itself. When the Sun runs through the northern signs, they are reckoned toward the northern parts of the dial; but when it stays in the southern signs, toward the southern. Again, since a quadrant of each parallel lies between the Meridian and the circle of the 6th hour from noon and midnight, the shadow-latitudes belonging to those hours which are less than six hours from the meridian must be computed from the upper part of the equinoctial line, while the latitudes of the hours whose distance from the meridian exceeds a quadrant must be computed from the lower part. Hence it comes about that in the tables certain hours of each tropic, and their latitudes,

are called inferior, and others superior. Finally, the latitudes of the forenoon (or eastern) hours must be computed in the eastern dial, and those of the afternoon (or western) hours in the western dial.

Shadow-latitudes and shadow-longitudes in each Meridian dial, for the hours from noon and midnight, for any latitude whatever.

In the tropic of Cancer ☿ all shadow-latitudes are Northern, and in the tropic of Capricorn ♄, Southern. Also, in each tropic the shadow-latitudes are partly Inferior and partly Superior, as these syllables [Infer. Super.] indicate.

	0.12	1.11	2.10	3.9	4.8	5.7	6.6	
Hours ☿ & ♄, from midnight	inf.su.	inf.s u.	inf.s u.	inf.s u.	inf.s u.	inf.s u.		Easte rn
Hours ☿ & ♄, from noon	su.inf.	su.in f.	su.in f.	su.in f.	su.in f.	su.in f.		West ern
Shadow-latitudes (G. M.)	22.30	24.14	26.40	32.12	41.1	59.14	90.1	
Shadow-longitudes (P. M.)	infinite	49.6	23.15	14.5	9.11	6.17	5.13	

In the Equator there are no shadow-latitudes, because then, by proposition 11 of book 1, the shadow falls perpetually on the equinoctial line; but their longitudes are these.

	0.12	1.11	2.10	3.9	4.8	5.7	6.6	
Hours ♃ & ♎, from midnight								Easter n
Hours ♃ & ♎, from midnight (2nd row)								Weste rn
Shadow-longitudes (P. M.)	Infinite	44.4 7	20.4 7	12. 0	6.5 6	3.1 3	0. 0	

Shadow-latitudes and shadow-longitudes in each Meridian dial, for the hours from sunset, at latitude 42 degrees.

In the tropic of Cancer ☾ all shadow-latitudes are Northern, and some of them are Inferior and some Superior, as these syllables [Infer. Sup.] indicate.

	9	10	11	12	13	14	15	16	Ea
Hours ☾, from sunset	(Infe r.)	(Infer)	(Supe r.)	(Supe r.)	(Supe r.)	(Supe r.)	(Supe r.)	(Supe r.)	ste rn
Shadow-latitudes (G. M.)	49.1 6	74.19	72.16	48.0	35.14	28.34	25.8	23.39	
Shadow-longitudes (P. M.)	7.26	5.28	5.32	7.38	11.29	18.8	32.48	106.4 3	

	17	18	19	20	21	22	23	24	W est er n
Hours ☾, from sunset	(Supe r.)	(Supe r.)	(Supe r.)	(Supe r.)	(Supe r.)	(Supe r.)	(Infe r.)	(Infe r.)	
Shadow-latitudes (G. M.)	23.42	25.17	28.53	35.51	49.16	74.19	72.16	48.0	
Shadow-longitudes (P. M.)	93.15	31.17	17.33	11.9	7.26	5.28	5.32	7.38	

In the Equator there are no shadow-latitudes, for the reason stated in the table of the hours ♀ and ♂ from noon and midnight; but their longitudes are these.

				9.1	10.1	11.1	1	
	6.18	7.17	8.16	5	4	3	2	
Hours ♀ & ♂, from sunset								Easter n
Hours ♀ & ♂, from sunset (2nd row)	6.18	5.19	4.20	3.2 1	2.22	1.23	2 4	Weste rn
Shadow-longitudes (P. M.)	Infinite	44.4 7	20.4 7	12. 0	6.56	3.13	0. 0	

In the tropic of Capricorn ʒ all shadow-latitudes are Southern, and all placed here are Superior. The Eastern are distinguished from the Western by these syllables [or. occ.].

Hours ʒ, from sunset	16 (or.)	17 (or.)	18 (or.)	19 (or.)	20 (occ.)	21 (occ.)	22 (occ.)	23 (occ.)	24 (occ.)
Shadow-latitudes (G. M.)	35.5 1	28.5 3	25.1 7	23.4 2	23.39	25.8	28.34	35.14	48.0
Shadow- longitudes (P. M.)	11.9	17.3 3	31.1 7	93.1 5	106.4 3	32.48	18.8	11.29	7.38

In these tables we could give all the hours from noon and midnight, so that they might all be drawn in each Meridian dial — although those that fall above the horizontal line are superfluous. We did the same for the hours from sunset when the Sun runs through the Equator. And if one should wish, the same could be done for those same hours from sunset when the Sun is in the tropics; but we have not done this, and have noted only those hours that fall below the horizontal line, since the style's shadow marks only these while the Sun stands above the Horizon.

[Margin: Construction of the Meridian dial from the tables.]

Therefore, to construct the Meridian dial: in the plane of the dial draw the straight line AB for the horizontal line — which, on a stable and firm plane, must be drawn parallel to the Horizon by means of a plumb-line. On this line choose the style's location C, and from it describe a circle of any size, DEF. Then, in the eastern dial from point B — which lies to the north and, when we turn toward the dial, is placed on our right — and in the western dial from point A — likewise northern, set on our left when we turn our face to the dial — count downward the complement of the pole's altitude as far as F; and draw the straight line FCD, which the straight line EC cuts at right angles at the center C. By what we demonstrated in proposition 25 of book 2, DF will be the equinoctial line, and EC the line of the 6th hour from noon or midnight. After this, having taken the length of the style, cut off for it any number of equal parts from the line GH, as was done in the preceding constructions. And so, if for the 11th hour from midnight the shadow-latitude is taken...

Diagram. A large construction figure for the two Meridian dials, occupying the lower two-thirds of the page. Above the two circles runs a horizontal graduated scale (the line of equal parts GH): at its left terminus the letter I, then G, then the graduations 12, 24, 36, 48, 60, 72, 84, 96, 108, and at the right terminus H.

Beneath it are two large equal circles set side by side and overlapping in a central vesica.

- The **left circle** carries the heading **OCCIDENTALE** ("Western [dial]").
- The **right circle** carries the heading **ORIENTALE** ("Eastern [dial]").
- A single horizontal line runs through both circles, labelled **HORI = ZON** (HORIZON — "HORI=" over the left circle, "ZON" over the right); its left terminus is **A**, its right terminus **B**.
- **BORE.** (*Borealis*, north) is lettered along the upper rim of each circle; **AVST.** (*Australis*, south) along the lower rims, twice near the central vesica.
- Point **E** stands at the top on the vertical of the left/upper region; the style-locus **C** lies at the center on the horizon line.
- **D** appears twice (one for each dial, near the upper center); **F** and **M** appear near the lower part of each dial (F above M).
- The rims carry repeated endpoint-letters **N, K, L** in clusters (e.g. "N K" / "K N" at the tops, "K L N" at the bottoms).
- Numerous small Italian-hour numerals (**1** through **24**) and interior point-letters (including **R, V, X, O, P, T** near the central vesica) are printed as fine hour-marks on the radiating hour-lines.

[?] The dense field of small hour numerals (1–24) and interior point-letters cannot be individually resolved with confidence at this resolution; more than two labels are uncertain. Marked as a

the shadow-latitude of 24 degrees 14 minutes — which is superior and, in the tropic of Cancer ☊, northern — and it is counted in the eastern dial from D toward the northern parts (or, on the opposite side, from point F) as far as K; and if along the hidden line CK, on the opposite side, the shadow-longitude of that same hour, 49 parts 6 minutes, is carried over from C as far as L, the point L will be found, for the 11th hour from midnight in the tropic of Cancer. In the same way we shall find the point L in the tropic of Capricorn ☋, for the same hour, if the same shadow-latitude is counted from D toward the south (or, on the opposite side, from point F) as far as K, and the same shadow-longitude is cut off along the hidden line CK. And if on the equinoctial line the segment CM is taken, equal to the shadow-longitude of that same hour in the Equator — which amounts to 44 parts 47 minutes — the line LML will give the 11th hour from midnight in the eastern dial. By the same reasoning we shall draw, in the western dial, the 1st hour from noon from the same shadow-latitude and shadow-longitude, as is clear in the figure. And if the same is done for the remaining hours from midnight and

from noon, both Meridian dials will be complete. We shall, however, accomplish this more briefly as follows.

Let the shadow-longitudes for the individual hours in the Equator be transferred onto the equinoctial line, marking points on it. For through these points, straight lines raised perpendicular to the equinoctial line will give the hours from noon and midnight. If these perpendiculars are cut off from C according to the shadow-longitudes that the hours have in the tropics — or else by the hidden lines of the shadow-latitudes — the points of the tropics will be obtained. Moreover, by one and the same operation, eight points in the tropics will be found from a single shadow-latitude. Thus you see that from the shadow-latitude of 32 degrees 12 minutes — which in each tropic the 9th and 3rd hours from midnight, and the 3rd and 9th from noon, possess — computed on both sides from the points D and F, in each dial, as far as N, the eight points of those same hours are found: O, P, Q, R, S, T, V, X. And so for the rest, provided one carefully observes in the tables which hours' latitudes are Northern or Southern, and which are Superior or Inferior, and likewise which are to be transferred to the eastern dial and which to the western. In the same way, by a single operation, four points will be found on the equinoctial line from one shadow-longitude. Thus you see that from the shadow-longitude of 12 parts 0 minutes — which in the Equator the 9th and 3rd hours from midnight, and the 3rd and 9th from noon, and likewise the 15th, 9th, 21st, and 3rd from sunset, possess — four points Y, Z, a, b are found on the equinoctial line for the said hours.

[Margin: How, by one operation, eight points are found in the tropics for the hours from noon and midnight, and four also on the equinoctial line for the hours from sunset.]

Again, if for the 9th hour from sunset in the tropic of Cancer ☿ — which is eastern — the shadow-latitude of 49 degrees 16 minutes is taken (which is inferior and northern), and it is computed from point F northward as far as d in the eastern dial, and the hidden line dC is drawn, there will be found on it, by means of the shadow-longitude, the point of the 9th hour from sunset in the tropic of Cancer; and this point, joined by a straight line to the point Z of the 3rd hour from midnight, gives the hour from sunset. For the 9th hour from sunset, on the equinoctial line, passes through the 3rd hour from midnight; on the equinoctial line it will give the 9th hour from sunset, as is established from the table of proposition 19 of book 1. By like reasoning, if for the 21st hour from sunset in the tropic of Capricorn ♎ — which is western — the shadow-latitude of 25 degrees 8 minutes is taken (which is superior and southern), and it is counted

in the western dial from D toward the south as far as e, we shall find, by means of the shadow-longitude, on the hidden line eC, the point f of the 21st hour from sunset in the tropic of Capricorn; and so we shall find the points of the remaining hours. If, then, the corresponding points found both in the tropics and on the equinoctial line are joined by straight lines, the Italian dial will have been drawn. Only the lines of the 18th hour from sunset and the 6th from sunrise must be drawn parallel to the equinoctial line, through the points found in the tropic of Cancer in each dial, as we demonstrated in proposition 34 of book 2. And it is easily gathered from the table of proposition 19 of book 1 through which hours from noon or midnight, on the equinoctial line, the hours from sunset must be drawn.

[Margin: In the Meridian dial there is no use for the hour-arcs.]

Here, however, there is no use for the hour-arcs, since the Meridian — to which the dial is parallel — passes through the poles of the world.

For the superior polar dial, which is parallel to the circle of the 6th hour from noon and midnight, the following tables are to be used, computed for the hours from noon and midnight and from sunset. These we shall draw on the superior dial alone, since the inferior one derives from it, as we said above concerning the Vertical dial — provided that, for the hours from sunset on the superior dial, one first draws those hours from sunrise which, on the inferior dial, must be changed into hours from sunset, as we set forth in the scholium of proposition 23 of book 2. For this reason we have written in these tables only those hours that fall on the superior dial — that is, those whose distance from the meridian is less than a quadrant, or six hours. And since the Meridian of the Horizon is also the Meridian of the circle of the 6th hour from noon and midnight, the Equator, standing at right angles both to the Meridian and to the circle of the 6th hour from noon and midnight, will be as it were the vertical of that same circle. Therefore, in the polar dial the shadow-latitudes must be reckoned from the equinoctial line — that is, from the vertical line of the dial itself — toward the upper (or northern) parts when the Sun occupies the first degree of Cancer ☊, and toward the lower (or southern) parts when the Sun is at the beginning of Capricorn ☋.

Shadow-latitudes and shadow-longitudes in the superior polar dial, for the hours from noon and midnight, for any latitude whatever.

In the tropic of Cancer ☊ all shadow-latitudes are Northern or Superior; in the tropic of Capricorn ☋, Southern or Inferior.

	6	7	8	9	10	11	12	
Hours $\mathfrak{S}$ & $\mathfrak{V}$, from midnight	6	7	8	9	10	11	12	Eastern
Hours $\mathfrak{S}$ & $\mathfrak{V}$, from noon	6	5	4	3	2	1	0	Western
Shadow-latitudes (G. M.)	23.30	24.14	26.40	32.12	41.11	59.14	90.00	
Shadow-longitudes (P. M.)	Infinite	49.65	23.15	14.5	9.11	6.17	5.13	

On the equator there are no shadow-latitudes, since there, by proposition 11 of book 1, the style's shadow falls perpetually on the equinoctial line. Their longitudes, however, are as follows.

Hours $\mathfrak{V}$ & $\mathfrak{A}$, from midnight	6	7	8	9	10	11	12	orientales
Hours $\mathfrak{V}$ & $\mathfrak{A}$, from noon	6	5	4	3	2	1	0	occidentales
Shadow-longitudes (P. M.)	Infinite	44.47	20.47	12.0	6.56	3.13	0.	

Shadow-latitudes and shadow-longitudes in the superior polar dial, for the hours from sunset, for latitude 42 degrees.

In the tropic of Cancer $\mathfrak{S}$ all shadow-latitudes are northern, or superior.

Hours $\mathfrak{S}$, from sunset	11	12	13	14	15	16	Orientales
Shadow-latitudes (G. M.)	23.42	25.17	28.53	35.51	49.16	74.19	
Shadow-longitudes (P. M.)	93.15	31.16	17.33	11.9	7.26	5.28	
Hours $\mathfrak{S}$, from sunset	17	18	19	20	21	22	Occidentales
Shadow-latitudes (G. M.)	72.16	48.3	35.14	28.30	25.8	23.39	
Shadow-longitudes (P. M.)	5.32	7.37	11.29	18.8	32.48	106.43	

On the equator there are no shadow-latitudes, because there the gnomon's shadow is perpetually cast onto the equinoctial line, as was shown in proposition 11 of book 1. Their longitudes, however, are these.

							1	
Hours Υ & $\underline{\alpha}$, from sunset	12	13	14	15	16	17	8	orientales
(from sunset)	24	23	22	21	20	19	1	occident.
							8	
Shadow-longitudes (P. M.)	Infinite	44.4	20.4	12.	6.5	3.1	0.	
		7	7	0	6	3	0	

In the tropic of Capricorn Υ all shadow-latitudes are southern, or inferior. The western, moreover, are distinguished from the eastern by these syllables [or./occ.].

	16	17	18	19	20	21	22	23	24
Hours Υ , from sunset	or.	or.	or.	or.	occ.	occ.	occ.	occ.	occ.
Shadow-latitudes (G. M.)	28.	35.	48.	72.	74.1	49.1	35.5	28.5	25.1
	30	14	3	16	9	6	1	3	7
Shadow-longitudes (P. M.)	18.	11.	7.3	5.3	5.28	7.26	11.9	17.3	31.1
	8	29	7	2				3	6

[Margin: The construction of the polar dial from the tables.]

So then, in the plane of the sundial let the straight line AB be drawn to serve as the equinoctial line. Since the plane is fixed and firm, and equidistant from the circle of the 6th hour from noon and midnight, this line must be drawn parallel to the horizon by means of a plumb-line. On this line take the point C as the place of the style, and about it describe a circle DEF of any size you like. In it draw the diameter DC, cutting the straight line AB at right angles; this will be the meridian line, as we demonstrated in proposition 37 of book 2. Then, taking a gnomon of any length, mark off along it as many equal parts as you please from the straight line GH, as was explained above.

And so, if the shadow-latitudes are reckoned from point F or from point E — according as they are eastern or western — toward the north or the south, according as they are northern or southern, the polar dial will be laid out, just like the meridian dial, as may be seen in the accompanying figure. In it, from the fourfold shadow-latitude FK of 24 degrees 14 minutes — which belongs to the 7th hour from midnight and the 5th from noon, reckoned both from the eastern point

F and from the western point E, toward the north and the south — and from its shadow-longitude of 49 parts 6 minutes, the fourfold point L has been found; and by one and the same operation, for the 7th hour from midnight and the 5th from noon, in the tropic of Cancer ☿ and of Capricorn ♑. And from the shadow-longitude of those same hours on the equator — namely 44 parts 47 minutes — the points A and B have been found, for the same hours on the equinoctial line, and so on. Again, from the shadow-latitude EM — western and northern — of 72 degrees 16 minutes, or its opposite FM, which belongs to the 17th hour from sunset in the tropic of Cancer ☿, and from its shadow-longitude of 5 parts 32 minutes, the point N has been found, for the 17th hour from sunset in the tropic of Cancer ☿. Likewise, from the shadow-latitude

FO — eastern and southern — of 35 degrees 14 minutes, or its opposite EO, which corresponds to the same 17th hour from sunset in the tropic of Capricorn ♑, and from its shadow-longitude of 11 parts 29 minutes, the point P has been found, for the 17th hour from sunset in the tropic of Capricorn ♑. Finally, from the western shadow-longitude of 3 parts 13 minutes of that same 17th hour on the equator — for since this hour is eastern on the equator, its shadow falls on the western side of the dial, that is, on the opposite side — the point a has been found, for the same 17th hour from sunset on the equinoctial line, and so on. The lines of the 12th and 24th hours from sunset, and of the 6th from sunrise, must be drawn through the points found in the tropics, parallel to the equinoctial line, as was shown in proposition 46 of book 2. Moreover, the line of the 24th hour from sunset, when extended, will give the horizontal line, since it is the common section of the plane of the dial and the circle of the 24th hour from sunset — that is, of the horizon. We shall draw it in the following way as well.

[Margin: How the horizontal line, and the line of the 12th hour from sunrise or sunset, are to be drawn.]

From the table of shadow-longitudes in proposition 2 of this book, take the shadow-longitude corresponding to the altitude of the pole above the horizon — in our example, the one corresponding to 42 degrees, which comes to 13 parts 20 minutes. With the compasses transfer it from C along the meridian line toward the north, as far as the point Q. For through this point the horizontal line must be drawn — the line of the 24th hour — parallel to the equinoctial line, or perpendicular to CD. For the arc of the meridian that lies on the southern side, between the horizon and the circle of the 6th hour from noon and midnight (to which the dial is parallel), measures the altitude of the pole above the horizon; that is, in our example, it spans 42 degrees. If the Sun were placed at that point of

the horizon on the southern side where it is cut by the meridian, it would have, above the circle of the 6th hour from noon and midnight, an altitude of 42 degrees — precisely the altitude of the pole above the horizon. Therefore the shadow cast by the style would fall on the point Q, toward the northern parts, which lie opposite the Sun, since the shadow CQ corresponds to 42 degrees, that is, to the altitude of the pole. And since the line of the 12th hour from sunrise or sunset is just as far from the equinoctial line on the other side as the horizontal line is distant, if a length CR is taken equal to CQ, the line of the 12th hour from sunrise or sunset must be drawn through R, parallel to the equinoctial line, or perpendicular to DC.

Diagram. A large construction figure for the polar sundial. Along the top runs a straight graduated scale-line GH — the gnomon-parts scale — with a bracketed origin-mark at the far left labeled **I**, then **G**, and graduation numbers **12, 24, 36, 48, 60, 72, 84, 96**, terminating at **H** on the right. Below it stands the large horizon circle DEF, with the four cardinal labels set around its rim: **BOR** (Boreas, north) at the top, **AVST** (Auster, south) at the bottom, **ORI** (Oriens, east, lettered vertically) on the right near F, and a matching vertical direction-label on the left near E which I read as the western label but cannot resolve with confidence [Occ.? — glyphs worn].

- Vertical diameter (meridian line): **D** at top, through the center down to **R** at the bottom center; **M** appears near D at the top and again near AVST at the bottom (the meridian endpoints).
- Horizontal diameter (equinoctial line AB): **A** on the left, **B** on the right; **E** left of center, **F** right of center; **C** the center (place of the style) and **Q** on the meridian near center.
- Hour-lines radiate obliquely and carry the hour numbers **7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24**.
- Point-clusters where hour-lines meet the circle are labeled with repeated letters (the text speaks of a "fourfold point L"): **L**, **O**, and **K** occur several times on both the left and right flanks; **P** near the center; **S** on the lower right. Several of these small peripheral letters do not resolve cleanly on the scan and are flagged [?]; the figure should be re-examined.

[Margin: In the polar dial the hour-arcs have no use.]

Here too the hour-arcs have no place, just as they have none in the meridian dial, because the circle of the 6th hour from noon and midnight — to which the dial is parallel — passes through the poles of the world, exactly as the meridian does.

[Margin: Why the equinoctial dial is not constructed by means of the tables.]

Furthermore, we have composed no tables for the construction of the equinoctial dial, because the equinoctial dial is built far more easily by the method we set out in book 2 than by any tables. Tables for building a dial of this kind would seem to be drawn up pointlessly and to no purpose, especially since all the shadow-latitudes and hour-arcs are equal to one another at every hour, the Sun's altitude is likewise the same at every hour of each parallel, and consequently the shadow-longitude too is always the same, as is clear from what has been said above.

For the south-facing vertical dial that declines 30 degrees from the meridian toward the east, the following little tables have been drawn up, for the hours from noon (or midnight) and from sunset. In them this sign † marks the hour that falls on the proper meridian of the declining dial, as we explained in the tables of the Sun's altitudes in proposition 1 of this book. In these tables we call "superior" those shadow-latitudes that lie beyond the dial's own vertical — that is, toward the center of the dial — and "inferior" those that lie on the near side of that same vertical, that is, toward the equinoctial line. The hour-arcs, in turn, are called "superior" when they lie beyond the center of the dial.

Hour-arcs in the south-facing vertical dial, which declines 30 degrees from the meridian toward the east, for the hours from noon and midnight, for latitude 42 degrees.

Hours from midnight	4	5	6	7	8	9	†	orientales
Hour-arcs (G. M.)	16.3	36.5	53.1	66.2	77.2	87.1	90.	Superiores
	1	5	8	3	2	8	0	

Hours from midnight	A							
	10	11	12	meridie	1	2	3	occidentales
Hour-arcs (G. M.)	83.	72.4	60.5		46.3	28.2	6.3	Superiores
	0	4	7		4	5	1	

Shadow-longitudes in the south-facing vertical dial, which declines 30 degrees from the meridian toward the east, for the hours from noon and midnight, for latitude 42 degrees.

Hours ♌, from midnight	2	3	4	5	6	7	8	9	†
Shadow-longitudes (P. M.)	468.4	57.1	28.3	17.3	11.4	7.5	5.	3.3	3.3
	7	6	0	9	3	0	8	9	4

Hours 𐤃, from midnight	1 0	11	12	A meridie	1	2	3	4
Shadow-longitudes (P. M.)	4. 6	6.1 2	9.2 3		13.5 9	21.3 0	37. 5	95.1 3

Hours 𐤄, from midnight	5	6	7	8	9	†	10	11	12	A meridi e	1
Shadow-longitudes (P. M.)	45. 24	5 8.	3 5.	2 7.	24. 15	2 4.	24. 59	29. 38	41. 59		8 5. 2

Shadow-latitudes and shadow-longitudes in the dial declining from the vertical, from the meridian toward the east by 30 degrees, for the hours from sunset, for latitude 42 degrees.

In the tropic of Capricorn 𐤃 the shadow-latitudes are partly inferior and partly superior, as these syllables [Infer./Super.] indicate.

Hours 𐤃, from sunset	10 Infe r.	11 Infe r.	12 Infe r.	13 Supe r.	14 Supe r.	15 Supe r.	16 Supe r.	† Supe r.	Orien tales
Shadow-latitudes (G. M.)	25.3 7	16.4 7	8.7	1.21	12.4 6	27.4 0	54.2 9	90.0	
Shadow-longitudes (P. M.)	110. 10	39.2 7	22.2 8	14.3 2	9.44	6.27	4.15	3.34	

Hours 𐤃, from sunset	17 Supe r.	18 Supe r.	19 Supe r.	20 Supe r.	21 Infe r.	22 Infe r.	23 Infe r.	24 Infe r.	Occide ntales
Shadow-latitudes (G. M.)	81.2	42.2 8	21.0	7.36	2.35	11.2 4	20.2	29.5	
Shadow-longitudes (P. M.)	3.36	4.56	7.33	11.1 8	16.5 8	27.6	52.3 6	286. 21	

	17	18	19	20	21	22	23	24	
Hours $\wp$, from	Sup	Sup	Sup	Sup	Infe	Infe	Infe	Infe	Occide
sunset	r.	r.	r.	r.	r.	r.	r.	r.	ntales

M.)

On the equator all shadow-latitudes are superior; the eastern are distinguished from the western by these syllables [or./occ.].

	10	11	12	13	14	15	16	17	18	19	20	21
Hours Υ & $\underline{\alpha}$, from sunset	or.	or.	or.	or.	or.	or.	occ	occ	occ	occ	occ	occ
	or.	or.	or.	or.	or.	or.	.	.	.	.	.	.
Shadow-latitudes (G. M.)	7.0	17.	29.	43.	61.	83.	73.	53.	36.	23.	12.	2.4
		16	3	26	35	29	29	5	42	37	38	2
Shadow- longitudes (P. M.)	82.	33.	20.	14.	11.	10.	10.	12.	16.	25.	46.	213
	45	57	46	40	28	9	31	37	53	11	8	.32

In the tropic of Cancer $\mathfrak{S}$ all shadow-latitudes are superior; the eastern are distinguished from the western by these syllables [or./occ.].

Hours $\mathfrak{S}$, from sunset	10	11	12	13		14	15	16	17	18
	or.	or.	or.	or.	†	occ.	occ.	occ.	occ.	occ.
Shadow-latitudes (G. M.)	39.	51.	64.	78.	9	86.7	71.7	57.2	45.1	34.3
	44	7	7	36	0.			0	0	1
					0					
Shadow-longitudes (P. M.)	88.	42.	29.	25.	2	24.1	26.5	34.4	57.2	225.
	45	30	50	3	4.	4	1	3		15
					7					

Gnomonics — Book V (continued)

We have calculated, for the hours from noon and from midnight, the hour-arcs of only those twelve hours (from which, together with the shadow-lengths, we shall here construct the astronomical dial) that are less than six hours distant from their proper meridian — which, as we advised in the scholium of the preceding proposition, had to be done, since from these the whole dial will be described, as will become clear. And we noted the shadow-lengths of those hours that fall on the southern face of the dial. We did the same for the shadow-latitudes and shadow-lengths for the hours from sunset. This too we shall observe in what follows.

[Margin: Construction of the vertical declining dial from the tables.]

Now in the plane of the dial let a straight line AB be drawn to serve as the horizontal line, so placed in the fixed and firm plane that it is parallel to the horizon; and on it let the place of the style be set at the point C. From C let a circle DEFG of any size be described, and in it, from the center, let a hidden perpendicular CH be dropped to the line AB — a line that can be dropped straight to the horizon on the fixed plane by means of a plumb-line. Then, by Proposition 30 of Book 1, let there be computed the arc of the great circle to which the declining dial is equidistant [parallel], the arc intercepted between the meridian of the horizon and the proper meridian, drawn through the poles of the world and through the poles of that same great circle. We found this arc to comprise 29 degrees 3 minutes.

Diagram. A construction figure for the vertical dial declining 30° from south toward east, at latitude 42° . Across the top runs a graduated scale-bar (a scale of gnomon-parts) with its ends marked **I** (left) and **H** (right); its graduations are labelled **G**, then **12, 24, 36, 48, 60, 72**. Below it stands the large dial circle. The horizontal diameter carries the split legend **HORI·ZON** (Horizon), with **ORT**. (Ortus / east) marked near its center-right and endpoint letters toward the left (**A**) and center-right (**B**). Readable perimeter and radial labels:

- **b** (top of circle)
- **N** (upper left, on the circle)
- **D, Y** (upper right; **D** on the circle, **Y** adjacent)
- **E** (mid-left) with hour-number **4** nearby
- **M** (interior, right of center — the convergence point / center of the dial and apex of the shadow triangle)
- Interior letters near center-left: **L, A, U, O**
- Radial hour-numbers reading down the right and around the lower rim: **2, 3, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13**, and **11** (set as "II") on the left
- Lower and lower-left points: **F** (on the circle), **Q, Z, G** (right, near the 22 line), **P** (lower right), **V, a, H** (a second H, lower center), **R, S, T, K, C**
- Crowded central cluster of small letters/digits (**d, i, s, f, C**, and the digits **5, 6, 7, 8, 9, 10, 11**) marking the graduated shadow-triangle — these interior sorts are too small and overprinted to resolve individually. [Several interior labels [?]]

This arc we shall reckon in the circle DEFG from the point H — toward the west (which, for us facing the dial, lies to the left, that is, toward A) if the plane of the

dial declines from south toward east; but toward the right (that is, toward the east, or point B) if the plane of the dial turns aside from south toward west. In our example, since we suppose the dial to decline from south toward east by 30 degrees, we counted the said arc as far as F.

[Margin: The line of the style, or the proper meridian, in the declining dial.]

For the straight line FCD, when drawn, will be the common section of the plane of the dial and of its own meridian — that is, the line of the style, standing as the proper meridian line, which separates the eastern part of the dial from the western, as we demonstrated in Proposition 1 of Book 3; since the line CH is parallel to the meridian line of the dial, which of course marks the 12th hour, and since this too is perpendicular to the horizontal line AB, and so on.

[Margin: The equinoctial line in the declining dial.]

Let the straight line EG cut this at right angles in the center; it will be the common section of the plane of the dial and of the vertical properly so called of that same plane of the dial, and from it the shadow-latitudes are to be reckoned. On the line EG, having taken a style CI of any length, let there be cut off from the line HI as many equal parts as you please, of which GI is subdivided into 12 equal parts, as was done above. After this, by Proposition 29 of Book 1, let there be computed the elevation of the pole above the great circle to which the dial is equidistant, which we find to be 40 degrees 3 minutes; and its complement therefore 49 degrees 57 minutes.

[Margin: The center of the declining dial.]

If, then, from the table of shadow-lengths of Proposition 2 of this book, one takes the shadow-length 10 parts 5 minutes, corresponding to the complement of the said elevation of the pole — that is, agreeing with the altitude of the Sun when placed in the Equator and on the proper meridian of the dial — and this, by means of the compass, is taken from the line HI and carried down along the line CF as far as K, then the straight line KL, drawn through K to DF, will be perpendicular: it is the equinoctial line, necessarily cutting the horizontal line, provided no error has been made at the point L, which terminates the shadow of the 6th hour from noon or midnight, or of the 12th hour from sunrise or sunset, when the Sun is in the Equator. That shadow, as is established from the preceding table of the hours of Aries ♈ and Libra ♎ under the 12th hour from sunset, is 20 parts 46 minutes.

If again the shadow-length 14 parts 17 minutes, corresponding to the elevation of the pole above the great circle to which the dial is equidistant, is carried from C upward along the line CD as far as M, then M will be the center of the dial. This, however, we shall find together with K, the point of the equinoctial line, if with the style CI we make the angle CIK — the angle of the elevation of the pole above the great circle to which the dial is equidistant — and another angle CIM, of the complement of the elevation of the pole. Through M, the center of the dial, let a perpendicular be drawn to DF...

a perpendicular NO, which will stand as the line of the 6th hour from noon and from midnight if the dial were horizontal, and from which the hour-arcs are to be counted in the circle NPO described from M.

The hours we shall describe as follows. From N and O let the hour-arcs be computed in the circle NPO, in the opposite parts — namely all below the line NO, since they are all upper [superiores]. Likewise the eastern ones from N, and the western from O. For if, from M, straight lines drawn through the endpoints of those arcs are cut off by means of the compass according to the shadow-lengths that belong to the hours from noon and from midnight in the tropics, the astronomical dial will be delineated. It can nevertheless also be described by means of the shadow-latitudes, if these are reckoned from the line EG. For example:

Diagram. The same declining-dial construction figure as on the facing page is reprinted here — an identical woodcut, with the same top scale-bar (ends **I** and **H**; graduations **G**, 12, 24, 36, 48, 60, 72), the same **HORI·ZON** diameter with **ORT**. and endpoint letters **A/B**, and the same perimeter and interior labelling (**b**, **N**, **D**, **Y**, **E**, **M**, **L**, **A**, **U**, **O**, **C**, the hour-numbers 2, 3, 13–23 and 11 as "II", and the lower points **F**, **Q**, **Z**, **G**, **P**, **V**, **a**, **H**, **R**, **S**, **T**, **K**). As on the previous page, the crowded central cluster (**d**, **i**, **s**, **f**, digits 5–11) is too fine to resolve label-by-label.

[Reprinted figure; several interior labels [?]]

Since the arc of the 12th hour from midnight, 60 degrees 57 minutes, is western and upper, we shall count it from O, the eastern point, downward as far as P; and the straight line MP, when drawn, will be the line of the 12th hour, or of midnight — that is, the meridian line — which, if there is no error in anything, will be perpendicular to the horizontal line AB, and will not differ from the thread of a plumb-line freely dropped from M on the fixed plane. So too, since the arc of the 8th hour from midnight is eastern and upper, if it is counted from the point N, the western point, downward as far as Q, the straight line MQ will give the line of the 8th hour from midnight; on it, at R, we shall cut off according to the shadow-

length 27 parts 1 minute, which belongs to that same 8th hour from midnight in the parallel of Cancer ☊, so that we obtain R, the point of the tropic of Cancer ☊. And so with the rest.

Again, from the shadow-latitudes that were computed for the hours from sunset (reckoned from the line EG), and from their lengths, we shall compose the Italian dial, as in the preceding cases. Thus you see that, by the shadow-latitude GX of the 15th hour from sunset in the tropic of Cancer ☊ — an upper and eastern latitude, comprising 27 degrees 40 minutes — and by the length of that same shadow, 6 parts 27 minutes, the point S is found, for the 15th hour from sunset in the tropic of Cancer ☊. In like manner, from GY, the eastern and upper shadow-latitude — or its opposite EZ — of 83 parts 29 minutes [P.83.M.29], which suits that same 15th hour from sunset in the Equator, or indeed from the shadow-length 10 parts 9 minutes of that same hour in the Equator, the point T is found on the equinoctial line for the 15th hour from sunset; through which the line will pass entirely, if no fault has been committed — the same as the 9th hour from midnight on the equinoctial line. Lastly, from Eb, the western shadow-latitude of 71 degrees 7 minutes — or its opposite Ga — and from the length of that same shadow, 26 parts 51 minutes, the point V is found, the point of the 15th hour from sunset in the tropic of Cancer ☊; and thus the line STV of the 15th hour from sunset is drawn.

The same reasoning holds for the remaining hours. Only this must be noted: that the 23rd hour from sunset is drawn through the point found in the tropic of Capricorn ☋, and through the 5th hour from midnight — or the 11th from sunset — on the equinoctial line, as is prescribed in the table of Proposition 19 of Book 1; likewise through that point where the 6th hour from midnight and the 13th hour from sunset intersect one another, namely through the point d. So too the 22nd hour is drawn through the point found in the tropic of Capricorn ☋, and through the point e, where the 14th hour from sunset intersects the 6th hour from midnight — because the 4th hour from midnight, through which it ought to be drawn, cuts the equinoctial line too far off. But the 21st hour is drawn through the point found in the tropic of Capricorn ☋, and through the point f, where the 15th hour from sunset divides the 6th hour from midnight, and so on. All these points of the 6th hour from midnight are derived from the table of Proposition 20 of Book 1, whose title is "The line of the 6th hour from noon or from midnight." Nevertheless, for the same hours, other points can be found in the parallel of Aries ♈ or Libra ♎, if the shadow-latitudes and lengths are found.

For the construction of the upper dial declining 30 degrees from the horizon and facing west, the tables that follow — computed for the hours from sunrise — are to be used. For as an example of all such dials we shall construct only the Babylonian dial. In these tables, the northern [Boreales] shadow-latitudes are those that lie on the near side of the dial's proper vertical, that is, toward the equinoctial line; while the southern [Australes] are those that lie beyond that same vertical, toward the center of the dial, as in the horizontal dial.

Shadow-latitudes and shadow-lengths in the upper dial declining 30 degrees from the horizon on the eastern side — but set at a right angle to the vertical properly so called — for the hours from sunrise, at latitude 42 degrees.

In the tropic of Cancer ☿, the shadow-latitudes are partly northern and partly southern, as these syllables [Bor. / Austr.] indicate.

Table — In the tropic of Cancer ☿ (hours from sunrise), eastern [orientales]:

Hours ☿, from sunrise	3 Bor	4 Bor	5 Bor	6 Bor	7 Austr	8 Austr	9 Austr	10 Austr.	†
Shadow-latitudes (G. M.)	18. 2	19. 57	12. 23	4.4 4	3.54	15.39	36.42	86.18	9 0. 0
Shadow-lengths (P. M.)	438 .45	53. 11	26. 24	16. 12	10.31	6.43	3.56	2.32	2. 3 2

Table — In the tropic of Cancer ☿ (hours from sunrise), western [occidentales]:

Hours ☿, from sunrise	11 Austr.	12 Austr.	13 Austr.	14 Bor.	15 Bor.
Shadow-latitudes (G. M.)	40.18	16.56	5.13	3.53	11.32
Shadow-lengths (P. M.)	3.42	6.21	10.2	15.25	24.51

In the Equator all the shadow-latitudes are southern.

Table — In the Equator ♃ & ♄ (hours from sunrise), eastern [orientales]:

Hours ♃ & ♄, from sunrise	3	4	5	6	7	8	†
Shadow-latitudes (G. M.)	4.10	13.1	23.4	36.4	54.0	76.3	90.

Hours γ & α , from sunrise	3	4	5	6	7	8	†
		8	3	4		9	0
Shadow-lengths (P. M.)	117.28	37.9	21.1	14.1	10.3	8.46	8.3
			3	6	3		2

Table — In the Equator γ & α (hours from sunrise), western [occidentales]:

Hours γ & α , from sunrise	9	10	12	11	13	14
Shadow-latitudes (G. M.)	77.46	54.54	37.24	24.13	13.43	4.35
Shadow-lengths (P. M.)	8.44	10.26	14.3	20.47	36.0	107.33

In the tropic of Capricorn γ , all the shadow-latitudes are southern, but the western are distinguished from the eastern by these syllables [or. / occ.].

Table — In the tropic of Capricorn γ (hours from sunrise):

Hours γ , from sunrise	3	4	5	6		7	8	9	10	11
	or.	or.	or.	or.	†	occ.	occ.	occ.	occ.	occ.
Shadow-latitudes (G. M.)	36.	47.	59.	74.	90.	89.4	74.1	59.3	47.6	36.3
	45	21	52	22	0	9		4		2
Shadow-lengths (P. M.)	82.	38.	26.	21.	19.	19.5	21.2	26.1	38.4	84.3
	16	18	8	16	54	4	0	9	6	0

[Margin: Description of the dial declining from the horizon, from the tables.]

Let, then, a straight line AB be drawn in the plane of the dial, in any position, but so placed in the fixed and firm plane that it is parallel to the horizon, by means of a plumb-line; and thereupon the meridian line too is to be drawn in the dial. For in this dial the horizontal line and the meridian line are equidistant [parallel], as we demonstrated in Proposition 14 of Book 3. On the line AB, having assumed the place of the style C, let a circle AFDG of any size be described from C; in it let there be counted, from B — the northern point — the arc of the great circle to which the dial is equidistant, intercepted between the meridian of the horizon and the proper meridian of that same great circle, which by Proposition 30 of Book 1 we find to comprise 24 degrees 14 minutes.

toward F indeed, that is toward the eastern parts, if the dial faces west (that is, declines from the horizon on the eastern side); but toward G, that is toward the western parts, if the dial faces east (that is, declines from the horizon on the western side). In our example, since we suppose the dial to decline from the

horizon on the eastern side by 30 degrees, so that it faces west, we counted the said arc as far as D.

[Margin: The line of the style, or the proper meridian, in the dial declining from the horizon.]

For the straight line DCE, when drawn, will be the common section of the plane of the dial and of its own meridian — that is, the line of the style, standing as the proper meridian line, which separates the eastern parts of the dial from the western, as we showed in Proposition 13 of Book 3, since the line AB is set to be equidistant [parallel] to the meridian line to be drawn. The line FG, cutting this at right angles in the center C, will be the common section of the plane of the dial and of the vertical properly so called of that same plane of the dial, from which will be computed the...

will be the shadow-latitudes. On this line FG, having taken the style CH of any length whatever, let there be cut off from the line HI as many equal parts as you please, of which GI is subdivided into 12 equal small parts, as was also done above. Then let the altitude of the pole above the great circle to which the dial is parallel be found by Proposition 29 of Book 1 — which we determined to be 35 degrees 25 minutes, so that its complement is 54 degrees 35 minutes. If from the table of shadow-lengths (Proposition 2 of Book 1) one takes the shadow-length of 8 parts 32 minutes, corresponding to the complement of the said pole-altitude — that is, agreeing with the altitude of the Sun when set in the Equator and on the proper meridian of the dial — and if, with the help of the compass, this length taken from the line HI is transferred downward along the line CD as far as I, then the line IK, drawn through I perpendicular to DE, will be the equinoctial line [aequinoctialis]. If no error has been made, this line necessarily cuts, at the point K, the line CK (which is drawn perpendicular to AB, and which, as was shown in Proposition 16 of Book 3, is the common section of the plane of the dial and of the vertical properly so called of the horizon itself). The point K marks the end of the shadow of the 6th hour from noon, or of the 12th hour from sunrise or sunset, when the Sun is in the Equator — the shadow which in the preceding table measures 20 parts 47 minutes.

[Margin: How the equinoctial line is to be drawn.]

Again, if the shadow-length of 16 parts 53 minutes, corresponding to the altitude of the pole above the great circle to which the dial is parallel, is transferred from C upward along the line CE as far as L, then L will be the center of the dial. This

point, however, together with I, the point of the equinoctial line, we shall find as in what preceded, if with the style CH...

Diagram. A crowded construction figure for the dial declining from the horizon (declination 30° on the eastern side, facing west), showing the disc of the dial with its hour-lines, the equinoctial line, and the horizon/meridian framework. A horizontal shadow-length ruler runs across the top; a great circle occupies the body of the figure, with radiating hour-lines and shadow-construction lines. Because the figure is densely lettered and several small peripheral labels do not resolve cleanly, it is flagged as a

- Top shadow-length ruler (left to right): M · I · G · 12 · 24 · 36 · 48 · 60 · 72 · H (fine tick-marks between M, I, G)
- AVST (upper center): AVSTER — south
- HORI... ZON (left rim, vertical, split): HORIZON — the horizon circle
- ORI (left, vertical): ORIENS — east (eastern side)
- OCCI (right, vertical): OCCIDENS — west (western side)
- BORE (bottom center): BOREA — north
- L (upper center): center of the dial
- K (center hub): terminus of the 6th-hour shadow / where hour-12 line meets the equinoctial
- C, a, H, z, Y, I, O (central construction points)
- F, Q (mid-left): style-point F and shadow-latitude point Q
- R (mid-lower): shadow point for hour 4 from sunrise
- N, R (right): point where a construction line meets the circle
- P, D (lower center, on circle)
- S, T (lower right, on the arc)
- e, e (labels on the western/occidens arc) [?]
- Hour-line numbers on the left margin (radiating): 15, 14, 13, 12, 11, 10, and, nearer the hub, 9, 7, 6
- A, E, V (upper); V, 4, x, B (bottom)

[Several small rim and hour-line labels flagged [?]]

let us set up the angle CHI, [equal to] the altitude of the pole above the great circle to which the dial is parallel, and another angle CHL, [equal to] the complement of the same pole-altitude. And if MN is drawn through M perpendicular to DE, this will serve as the line of the 6th hour from noon and midnight, as if the dial itself were horizontal; and it is from this that the hour-arcs would be counted in the

circle MPN, described from L, had they been computed. The meridian line must be drawn from L, parallel to AB itself, or perpendicular to CK, such as is LO. Likewise the horizontal line is drawn through the point K, where the equinoctial line and the line CK (drawn perpendicular to AB) intersect one another — which point, as we said, is the terminus and end of the shadow of the 6th hour from noon, or of the 12th hour from sunrise or sunset, when the Sun is in the Equator. This point K, moreover, will also be found in the following way. On the line AB, taking the segment Ca with a style equal at a, let the angle CaK be made [equal to] the complement of the declination from the horizon. For the line aK will cut the line CK at the point K, where the 12th hour from sunrise cuts the equinoctial line.

[Margin: Center of the dial.]

[Margin: Meridian line.]

[Margin: Horizontal line.]

Now, if the shadow-latitudes are duly computed in the circle AFDG — the eastern ones from the point F, the western from the point G, toward the north B or toward the south A, as the tables indicate — this dial will be laid out just like the preceding ones. Thus you see that from the eastern and northern shadow-latitude FQ (or its opposite GQ) of 19 degrees 57 minutes, which belongs to the 4th hour from sunrise in the tropic of Cancer [♋], and from the shadow-length of that same hour, 53 parts 11 minutes, the point R was found for the 4th hour from sunrise in the tropic of Cancer. Likewise, from FS, the eastern and southern shadow-latitude of 13 degrees 18 minutes (or its opposite GS) of the same 4th hour from sunrise in the Equator — or indeed from the shadow-length of 37 parts 9 minutes, which agrees with the same hour in the Equator — the point T was found for the said hour on the equinoctial line. Finally, from FV, the eastern and southern shadow-latitude of 47 degrees 21 minutes (or its opposite GV), which corresponds to the same 4th hour from sunrise in the tropic of Capricorn [♐], and from the shadow of the same hour, 38 parts 18 minutes, the point X was found for the 4th hour from sunrise in the tropic of Capricorn; and so on.

The 6th hour from sunrise will necessarily pass through the point O, where the meridian line cuts the equinoctial line. The 14th hour from sunrise, because it cuts the equinoctial line too far off, is drawn through the point found in the tropic of Cancer and through the point Y, where the line of the 10th hour from sunrise cuts the meridian line. And the 15th hour from sunrise is drawn through the point found in the tropic of Cancer and through the point Z, where the 9th hour from sunrise, when produced, meets the meridian line — as is clear from the

table of Proposition 20 of Book 1, whose title is "The line of the 12th hour from noon or midnight." Nevertheless, for these same hours the shadow-latitudes and shadow-lengths can also be investigated in the parallel of Gemini [Υ] or of Leo [ϱ], and from these still other points found on the dial.

For an upper dial inclined to the horizon by 68 degrees on the southern side, and set at right angles to the meridian, we shall use the following tables, computed for the hours from noon and midnight. For it will be enough to give the example in the hours from noon and midnight. In these tables we call "superior" those shadow-latitudes that lie on the near side of the vertical circle of the dial — that is, toward the pole that is seen above the dial — and "inferior" those that lie beyond the said vertical, namely toward the center of the dial, or toward the hidden pole.

The shadow-latitudes and shadow-lengths on an upper dial inclined to the horizon by 68 degrees on the southern side, and at right angles to the meridian, for the hours from noon and midnight, at latitude 42 degrees.

In the tropic of Cancer [☿] the shadow-latitudes are partly superior and partly inferior, as is indicated by the syllables [Super. Infer.].

Hours in Cancer, from midnight	0	1	2	3	4	5	6	(easter n)
Hours in Cancer, from noon	12	11	10	9	8	7	6	(wester n)
	Infe r.	Infe r.	Infe r.	Infe r.	Infe r.	Infe r.	Sup er.	
Shadow-latitudes (G. M.)	90. 0	71.1 0	53. 4	36. 6	20.2 0	5.37	8.28	
Shadow-lengths (P. M.)	12.3 9	13. 1	14.1 0	16. 9	19. 7	23.2 6	29.4 2	

[illegible]

Hours in Cancer, from midnight	7	8	9	10	11	12	(eastern)
Shadow-latitudes (G. M.)	22. 1	36. 2	48.59	62.33	76.14	90. 0	
Shadow-lengths (P. M.)	38.53	53.46	77.31	115.4	166.4	196.1	
				8	5	3	

In the Equator all shadow-latitudes are Inferior.

Hours in Aries [♈] & Libra [♎], from midnight	0	1	2	3	4	5	6	(eastern)
Hours in Aries & Libra, from noon	12	11	10	9	8	7	6	(wester n)
Shadow-latitudes (G. M.)	90. 0	74. 5	58. 26	43. 13	28. 29	14. 8	0. 0	
Shadow-lengths (P. M.)	32. 58	34. 18	38. 41	48. 8	69. 7	134. 54	Infini.	

All the hours of the tropic of Capricorn [♋] fall on the lower face of the dial, since the tropic of Capricorn does not cut the plane of the dial but stands wholly above the lower face — just as the tropic of Cancer stood wholly above the upper face. Hence it comes about that we have computed no shadow-latitudes for the hours of the tropic of Capricorn. The shadow of the gnomon, when the Sun is at the beginning of Cancer, describes on the upper dial an ellipse, which, however, for lack of space has not been drawn complete in the example.

We shall lay out the dial as follows.

[Margin: Layout of the dial inclined to the horizon, from the tables.]

In the plane of the dial let the straight line AB be drawn as the common section of the plane of the dial and of the vertical of that great circle to which the dial is parallel — a line to be drawn invisibly, and, in a firm and stable plane, set parallel to the horizon by means of a plumb-line. On it, having assumed the place of the style C, let a circle ADB of any size be described from C; in it, through the center C, let DE be drawn perpendicular to AB. This will be the common section of the meridian of the plane of the dial (or of the horizon — for the meridian of the horizon and of the plane of the dial is one and the same) and of the plane of the dial itself, namely the meridian line, as is established by Proposition 25 of Book 3.

[Margin: Meridian line.]

Then let as many equal parts as you please be cut off from the line HI, and so on, as was likewise done above. Then let the altitude of the pole above the great circle to which the dial is parallel be found — as we taught in the same Proposition 25 of Book 3 — which in the given example comprises 70 degrees, so that its complement is 20 degrees. Let the shadow of 32 parts 58 minutes be reckoned, corresponding to the complement of the pole-altitude found, that is, the shadow due to the altitude of the Sun set in the Equator and on the meridian, [measured] along the meridian line from C upward as far as E, if the center of the dial falls below the place of the style C (as happens in our example), but downward if the center of the dial falls above the place of the style. (It will easily be perceived, from what we delivered in Proposition 25 of Book 3, whether the center of the dial falls below or above the style.) For the line EG, drawn through E perpendicular to the meridian line, will be the equinoctial line.

[Margin: Equinoctial line.]

But if from C, in the opposite direction along the meridian line, the shadow-length of 4 parts 22 minutes — which agrees with the pole-altitude found — is measured off as far as H, then H will be the center of the dial. And if a line IK is drawn through H perpendicular to the meridian line, this will be the line of the 6th hour from noon and midnight, as in the horizontal dial, from which the hour-arcs would have to be reckoned in the circle described from H, had they been found for the individual hours. These same two points E and H will also be found if with the style there is made the angle...

[Margin: Center of the dial.]

the angle CFE, [equal to] the altitude of the pole found, and the angle CFH, [equal to] the complement of the same pole-altitude, as was said above. We shall draw the horizontal line thus. From C, upward along the meridian line, we shall reckon the shadow of 4 parts 51 minutes, which agrees with the inclination of the plane of the dial to the horizon — namely 68 degrees in our example — as far as the point M. We shall also find this same point if at F there is set up the angle CFM, [equal to] the complement of the inclination, so that the line FM cuts the meridian line at M. For the line drawn through M perpendicular to the meridian line will be the horizontal line, as is clear from what was demonstrated in Proposition 26 of Book 3.

[Margin: Horizontal line.]

Diagram. A construction figure for the dial declining from the meridian toward the east by 40° and inclined to the horizon on the northern side by 20° , showing

the (incompletely drawn) circle ADB with the horizon line, the meridian line, the equinoctial line, and hour-lines converging on the center of the dial H. A horizontal shadow-length ruler runs across the top; the double-arc rim carries the hour-numbers. Because the figure is densely lettered and several small peripheral labels do not resolve cleanly, it is flagged as a

- Top shadow-length ruler (left to right): I · G · 12 · 24 · 36 · 48 · 60 · 72 · 84 · 9[6?] · 108 · H (fine tick-marks after I; the mark between 84 and 108 reads ambiguously — the ruler runs in steps of 12, so the value is 96, but the printed digit is unclear)
- Row of hour-numbers along the top line (reaching the ruler): 8, 9, 10, 11, E (12), 1, 2, 3, 4
- Lower inner cluster of hour-numbers near the hub: 8, 9, 10, 11, 12, 1, 2, 3, 4
- HORI... ZON (left/right, split across the figure): HORIZON — the horizon line
- ORI (left, vertical): ORIENS — east; also the "OSS"/"ONS" continuation is worn [?]
- Central construction points: M, C, F, H (the meridian/style cluster; H = center of the dial)
- Left side points and hour markers: P, A, N, O, S (repeated on the two rim-arcs), 7, 4, X, and the point "Y 3" (lower left) with its short gnomon-mark
- Right side points and hour markers: R, B, K, 6, Q, L, T, and the point "9 V" (lower right) with its short gnomon-mark; E, 1, 2, 3, 4, O on the upper-right rim; Z (right)
- D (bottom, on the inner arc)

[Several small rim and hour-line labels flagged [?]]

If now the shadow-latitudes are counted from the line AB in the circle ADB, we shall lay out the several hours from their shadow-lengths, as before. For thus you see that from the western and inferior shadow-latitude BL of 28 degrees 29 minutes, which belongs to the 8th hour from noon in the Equator — or from the shadow-length of that same shadow, 69 parts 7 minutes — the point G was found on the equinoctial line for the 8th hour from noon. And from the same eastern and inferior shadow-latitude AN, which also belongs to the 4th hour from midnight — or from the same shadow-length of the same shadow — the point O was found on the equinoctial line for the 4th hour from midnight. Likewise, from AP, the eastern and superior shadow-latitude of 22 degrees 1 minute, which belongs to the 7th hour from midnight in the tropic of Cancer [☿], and from the shadow-length of the same shadow, 38 parts 53 minutes, the point Q was found for the 7th hour from midnight in the tropic of Cancer. And from the same

western and superior shadow-latitude BR, and from the same shadow-length, the point S was found for the 5th hour from noon in the tropic of Cancer. And so with the rest.

Only this must be noted: the superior shadow-latitudes, which above the line AB in the circle ADB (which, for lack of room, has not been drawn complete) cannot conveniently be counted, must be reckoned below the same line, but on the opposite side. Thus, since the shadow-latitude belonging to the 9th hour from midnight in the tropic of Cancer is eastern and superior, comprising 48 degrees 59 minutes, and cannot be counted above the line AB, we shall reckon it below the line AB, from the western point B as far as T. For if from the hidden line CT the segment CV is cut off — the shadow-length of 77 parts 31 minutes corresponding to that same hour — the point V will be obtained for the 9th hour from midnight in the tropic of Cancer. By the same reasoning the point Y was found for the 3rd hour from noon in the tropic of Cancer, from the same eastern and inferior shadow-latitude — although the shadow-latitude of this hour in the table is western and superior.

The individual hours from noon and midnight are for the most part drawn through four points each — as the 9th hour from noon and midnight through the points Z, a, H, V — or at least through three, as the 7th hour from noon and midnight through the points e, H, Q, and the 11th hour from noon through the points d, b, H, and so on. For all the hours from noon and midnight are drawn through H, the center of the dial.

For an upper dial declining from the meridian toward the east by 40 degrees and inclined to the horizon on the northern side by 20 degrees, the following tables have been computed for the hours from noon and midnight, of which alone we here give the example. In these tables we call "superior" those shadow-latitudes that lie on the near side of the proper vertical of the dial, toward the equinoctial line; and "inferior" those that lie beyond the same vertical, toward the center of the dial.

Shadow-latitudes and shadow-lengths on an upper dial declining from the meridian toward the east by 40 degrees, and inclined to the horizon on the northern side by 20 degrees, for the hours from noon and from midnight, at latitude 42 degrees.

In the tropic of Cancer [☿ — *tropico Cancrī*] the shadow-latitudes are partly superior and partly inferior, as the syllables [Super. Infer.] indicate.

Hours in Cancer [☿], from midnight — eastern (orientales):

Hour	5	6	7	8	9	10	11	†
(kind)	Supē r.	Supē r.	Supē r.	Supē r.	Supē r.	Infer .	Infer .	
Shadow-latitude (G. M.)	21.42	16.25	11.39	7.12	2.32	5.48	71.34	90 .0
Shadow-length (P. M.)	73.33	29.39	17.3	10.38	6.24	3.7	0.30	0. 28

Hours in Cancer [♋], from midnight — western (occidentales):

Hour	12	A meridie	1	2	3	4	5
(kind)	Infer .		Supē r.	Supē r.	Supē r.	Supē r.	Supē r.
Shadow-latitude (G. M.)	7.28		2.6	2.37	11.10	15.51	21.5
Shadow-length (P. M.)	2.47		6.0	10.5	16.9	27.37	63.43

In the Equator all shadow-latitudes are inferior.

Hours in the Equator [♈ / ♎ — *Aries / Libra*], from midnight — eastern (orientales):

Hour	6	7	8	9	10	11	†
Shadow-latitude (G. M.)	6.16	13.36	22.51	36.1	56.50	88.5	90.0
Shadow-length (P. M.)	53.6	24.33	14.53	9.50	6.54	5.47	5.47

Hours in the Equator, from midnight — western (occidentales):

Hour	12	A meridie	1	2	3	4	5
Shadow-latitude (G. M.)	59.4		37.5	24.4	14.3	7.0	0.20
	9		2		2		
Shadow-length (P. M.)	6.41		9.25	14.1	23.3	47.1	917.2
				0		8	1

In the tropic of Capricorn [♐ — *tropico Capricorni*] all shadow-latitudes are inferior.

Hours in Capricorn [♐], from midnight — eastern (orientales):

Hour	6	7	8	9	10	11	†
Shadow-latitude (G. M.)	27.11	34.3	44.3	56.9	71.2	88.5	90.0
		8			2	9	
Shadow-length (P. M.)	412.29	50.5	27.1	18.5	15.7	13.5	13.5
		0	6	3		5	5

Hours in Capricorn [$\frac{1}{2}$], from midnight — western (occidentales):

Hour	12	A meridie	1	2	3	4
Shadow-latitude (G. M.)	72.45		57.41	45.15	35.34	27.56
Shadow-length (P. M.)	14.52		18.17	25.57	46.24	229.0

[Margin: Layout of a dial declining from the meridian toward the east by 40 degrees and inclined to the horizon by 20 degrees, from the tables.]

On the plane of the dial draw the straight line AB — laid, however, on a stable and firm plane parallel to the horizon. Taking C as the assumed position of the style, describe about C the circle ADBE of any size you please; and through its center C erect the diameter DE perpendicular to AB. This will be the common section of the dial's plane and of the great circle that measures the plane's inclination to the horizon, as is evident. Now, by Proposition 31 of Book 1, find the arc of the great circle to which the dial is parallel — the arc intercepted between the great circle that measures the plane's inclination and the dial's own meridian — which in our example we find to be 32 degrees 7 minutes; and we shall count it off on the circle

ADBE, from D as far as F, toward the left — that is, toward the western parts A — when the dial declines from the meridian toward the east and is inclined to the horizon on the northern side, as ours is; but toward the right, if the plane, inclined on the same side, is deflected from the meridian toward the west. Then draw the straight line FCG, which will be the common section of the dial's plane and of its own meridian — that is, the style-line.

[Margin: The style-line.]

For if the dial's plane is set in its proper position and declines from the meridian toward the east, the point E falls into the western-and-northern quadrant of the upper hemisphere. So suppose the great circle that measures the inclination, drawn at right angles to the dial's plane through the straight line DE, to be turned in the mind about the style set up at C — always remaining perpendicular to the dial's plane — until it is carried through the arctic pole of the world (which, when

we face the dial, lies to the right of the point E), and is then joined to the dial's own meridian: it will necessarily make in the dial a section, falling below the style between the points A and D, of the kind that FCG is, as is plain. Since, then, DF is an arc similar to the one intercepted between the proper meridian and the great circle drawn through DE that measures the inclination to the horizon, the line FG will be the common section of the proper meridian and of the dial's plane — that is, the style-line.

But if the dial's plane inclines from the meridian toward the west, the point E falls into the eastern-and-northern quadrant of the upper hemisphere, so that the arctic pole lies, when we face the dial, to the left of the point E — that is, toward A. Therefore, if that great circle at right angles to the dial, drawn through DE, is moved toward A until it passes through the pole, the section it makes with the dial's plane will fall below the style, between the points B and D, and so on.

Now if through C a perpendicular HI is drawn to FG, this will be the common section of the dial's plane and of the vertical proper to the dial itself, from which all the shadow-latitudes are to be reckoned. Once the line HI has been prepared and divided into any number of parts equal to the style, as in the preceding cases, and once — by Proposition 29 of Book 1 — the elevation of the pole above the great circle to which the dial is parallel has been found (which in our example amounts to 25 degrees 43 minutes, so that its complement is 64 degrees 17 minutes): then if from C, the style's position, a shadow-length of 5 parts 47 minutes — corresponding to the complement of the pole-elevation found — is counted off upward along the line FG (upward, because the dial's center falls below the style, as happens in planes inclined on the northern side toward the horizon, when the arc of the meridian between the plane and the horizon is less than the pole's elevation above the horizon, as in our example, in which that arc contains 15 degrees 35 minutes, to be found by Proposition 28 of Book 1) as far as L: then LM, drawn perpendicular to FG, will be the equinoctial line.

[Margin: The equinoctial line.]

This shadow must be reckoned downward when the arc of the proper meridian between the horizon and the plane of the dial is greater than the elevation of the pole above the horizon, because the dial's center then falls above the style, as will appear in the following dial, in which the said arc contains 68 degrees 50 minutes. But if from C, on the opposite part along the line FG — that is, downward here — the shadow-length of 24 parts 55 minutes, corresponding to the pole-elevation found of 25 degrees 43 minutes, is reckoned, the point N, the dial's center, will be obtained. These two points L and N will likewise be found, as

was said above for other dials, if at the point K where the equinoctial line meets the style's vertex — perpendicular to the line FG — an angle CKL equal to the pole-elevation found, 25 degrees 43 minutes, is constructed; and, for the dial's center, an angle CKN equal to the complement of that same elevation, 64 degrees 17 minutes. Then from the straight line QR, drawn through the dial's center perpendicular to FG, the hour-arcs — if they have been computed for the several hours — are to be counted off, on the circle described from N.

[Margin: The dial's center.]

The HOURS will be laid out from the tables set out above, as in the earlier cases, if from the points H and I the eastern and western shadow-latitudes are reckoned, carefully observing which are superior and which inferior, and so on, as appears in the accompanying figure. And all the hours from noon and from midnight pass through N, the dial's center;

Diagram. A large sundial-construction figure for the dial declining from the meridian toward the east by 40° and inclined to the horizon on the north by 20° (latitude 42°). An outer great circle (the plane of the dial / horizon circle) contains a second, inner circle (the hour circle described from the dial's center). A horizon line runs across the upper part of the figure, lettered across the top **HORI ... ZON** (the word *Horizon* split by the diagram), and terminates at the right at G, where a graduated vertical scale-bar stands. The chief lettered points and lines, as read from the scan:

- HORI ... ZON (top): the horizon line, split lettering.
- E (top): upper end of the vertical diameter DE.
- D (bottom): lower end of the vertical diameter DE.
- C (center): the style position, center of the circle ADBE.
- A (left): left end of the base line AB.
- B (right, near hour "3"): right end of the base line AB.
- F (lower left, on the circle): with G defines the style-line FCG / FG.
- G (right, terminus of the horizon/style line): foot of the vertical scale-bar.
- M (upper center, on the horizon line): intersection used for the horizontal line.
- P (below E, on the vertical): point on DE.
- H (center-left): with I defines the vertical-proper line HI.
- I (right / lower right): other end of HI; also lettered at the top of the scale-bar.
- K (left of center): the style's vertex.

- N (lower left, interior): the dial's center, through which the hour-lines pass.
- O (near center): equinoctial-line point.
- S (near center, and again lower left): [?] two occurrences.
- Q, R (line QR through N, perpendicular to FG): the hour-reckoning line.
- Hour numerals ranged around the rim: 6, 7, 8, 10, 11, 12, 1, 2, 3, 4 (the "9" is [?], obscured by line-crossings).
- Small letters near the top: a, b, [?], [?] (two of the four minuscule labels are not resolvable).
- A vertical graduated scale-bar at the right edge, headed I (top) and G, with tick-labels 12, 24, 36, 48, [?], [?] and H at the foot (the two lower tick-labels are [?] — they read like "96" and "108" but do not sit in an even arithmetic run with the upper ticks, so they are left uncertain).

so that, if the dial's center were so far from the style that it could not conveniently be marked on the dial, then for those hours which have only a single point in one of the tropics — but none in the other tropic or on the equinoctial line, and other points in a neighboring parallel (as, for instance, in our example, in the parallel of Gemini [Π — *Geminorum*] or of Leo [ϱ — *Leonis*]) — the horizontal line must be drawn through the point M, where the line of the 6th hour from noon or from midnight cuts the equinoctial line, perpendicular to the straight line DE.

[Margin: The horizontal line.]

Or, taking on the straight line AB the length of the style CO, if the angle COP of the complement of the inclination is made upward, the straight line DE will be cut at the point P, through which the horizontal line is to be drawn perpendicular to DE; and this, if there has been no error, will necessarily pass through the point M of the 6th hour on the equinoctial line. Or, indeed, if the shadow of 32 parts 58 minutes, corresponding to the plane's inclination to the horizon, is transferred from C upward along the straight line DE, the point P will be found, through which the horizontal line is to be drawn perpendicular to DE.

The line of the 12th hour, which is the meridian line, we shall likewise draw by this method.

[Margin: The meridian line.]

The hour-arc of the 12th hour from midnight — which here we find to contain 83 degrees 44 minutes, and to be western and inferior (for since the 12th hour of noon is at a smaller distance from the dial's proper meridian than that of 6 hours, being distant by only 14 degrees 10 minutes — precisely the inclination of the

proper meridian to the meridian of the horizon — the hour-arc of the 12th hour from midnight will be inferior, that is, will lie beyond the dial's center, as we said in the scholium of the preceding proposition; and again, because the dial's plane declines from the meridian toward the east, the style's shadow is projected at noon toward the east, so the hour-arc will then be western, that its shadow may fall on the contrary, eastern part, as the case requires) — is to be reckoned on the opposite part of the circle Q[?] described from N: namely, from the point of the straight line QR, eastward and upward, as far as b. For the line Nb will be the meridian line.

FOR an upper dial declining from the meridian toward the west by 20 degrees and inclined to the horizon on the northern side by 70 degrees, the tables that follow have been drawn up for the unequal hours, of which alone we shall give the example. In these tables those shadow-latitudes are called inferior which lie on the near side of the dial's own vertical — that is, toward the equinoctial line; and superior, those which lie beyond the same vertical, toward the dial's center.

Shadow-latitudes and shadow-lengths on an upper dial declining from the meridian toward the west by 20 degrees, and inclined to the horizon on the northern side by 70 degrees, for the unequal hours, at latitude 42 degrees.

In the tropic of Capricorn [♑], the shadow-latitudes are partly inferior and partly superior, as these syllables [Infer. Super.] declare.

Unequal hours in Capricorn [♑] — eastern (orientales):

Hour	12	1	2	3	4	5	6	7	†
(kind)	Infe r.	Infe r.	Infe r.	Infe r.	Infe r.	Infe r.	Sup er.	Sup er.	
Shadow-latitude (G. M.)	20.4 1	10.5 2	13.2 1	10.4	6.50	3.33	0.30	9.38	90 .0
Shadow-length (P. M.)	58.2 2	31.0	20.2	13.5 4	9.48	6.42	4.9	1.54	0. 23

Unequal hours in Capricorn [♑] — western (occidentales):

Hour	8	9	10	11	12
(kind)	Super.	Super.	Super.	Infer.	Infer.
Shadow-latitude (G. M.)	53.17	6.39	0.1	4.10	7.42
Shadow-length (P. M.)	0.28	2.28	4.47	7.26	10.45

Appended single value — Hour 1, in the parallel of Aries [♈] & Aquarius [♐], inferior and eastern:

	G.	M.
Shadow-latitude	14.	36.
Shadow-length (P. / M.)	38.	15.

Here we have appended the shadow-latitude, and its length, which the Sun produces at the 1st unequal hour in the parallel of Aries [♈] and Aquarius [♐], since from it must be drawn the 1st unequal hour on the dial, as we shall explain. The length of this shadow, moreover, is given by the Sun's altitude, which we recorded in Table 17 of the Sun's altitudes.

In the Equator all shadow-latitudes are superior, but the eastern are distinguished from the western by these syllables [or. occ.].

Unequal hours in the Equator [♈ / ♎] — eastern (or.) and western (occ.):

Hour	2	3	4	5	6	7	†	8	9	10	11	12
(kind)	or.	or.	or.	or.	or.	or.		oc c.	oc c.	oc c.	oc c.	oc c.
Shadow-latitude (G. M.)	3.5 6	10. 50	19. 11	30. 34	48. 16	76. 29	9 0	69. 23	43. 38	27. 42	17. 17	9.1 5
Shadow-length (P. M.)	82. 45	30. 9	17. 17	11. 9	7.3 6	5.5 0	5. 4	6.4 3	8.1 12	12. 12	19. 16	35. 17

(Continuing within Book V. The following table completes the set of shadow-tables for the upper dial declining from the meridian toward the west by 20 degrees and inclined to the horizon on the northern side by 70 degrees, whose Capricorn, Aries–Aquarius, and Equator tables were given just before.)

In the tropic of Cancer [♋], all the shadow-latitudes are superior, but the eastern are distinguished from the western by these syllables [or. occ.].

Unequal hours in Cancer [♋] — eastern (or.) and western (occ.):

		4	5	6	7		8	9	10	11
Unequal hours ♋	3 or.	or.	or.	or.	or.	†	occ.	occ.	occ.	occ.
Shadow-latitude (G.	26.30	35.	48.	65.	87.	90.	70.	51.	38.1	28.1

		4	5	6	7		8	9	10	11
Unequal hours $\mathfrak{S}$	3 or.	or.	or.	or.	or.	†	occ.	occ.	occ.	occ.
M.)		51	30	48	31	0	11	48	5	8
Shadow-length (P.	1115.1	44.	22.	15.	13.	13.	15.	20.	36.3	176.
M.)	[?]	7	42	49	43	42	3	35	6	48

[Margin: Description of a dial declining from the meridian toward the west by 20 degrees, and inclined to the horizon by 70 degrees, from the tables.]

[Margin: The style-line.]

[Margin: The equinoctial line. The center of the dial.]

Therefore, on the plane of the dial draw the straight line AB, which is to lie in a stable and firm plane parallel to the horizon. From the assumed position C of the style on it, describe a circle ADBE of any size, in which the diameter DE cuts the straight line AB at right angles; this diameter will be the common section of the dial's plane and of the great circle that measures the inclination of the plane to the horizon. Then, having found — by Proposition 31 of Book 1 — the arc of the great circle to which the dial is parallel, intercepted between the great circle that measures the inclination of the plane and the dial's own meridian (which in the dial proposed we find to be 16 degrees 22 minutes), we shall reckon it from D toward the east — that is, toward point B — as we said in the preceding dial, since this dial declines from the meridian toward the west up to F. For the straight line FCG will be the style-line, that is, the common section of the dial's plane and of the dial's own meridian.

From the straight line KL, which cuts FG at right angles, the shadow-latitudes are to be counted, since it is the common section of the dial's plane and of the dial's own vertical. Next divide the straight line HI into as many parts equal to the style as you like, as in the earlier cases; and having found the altitude of the pole above the great circle to which the dial is parallel — which in the given dial is 25 degrees 18 minutes, and therefore its complement is 64 degrees 42 minutes — if from C, the position of the style, the shadow-length 5 paces 40 minutes corresponding to the altitude of the pole so found is counted downward along the straight line CF (as we said in the preceding dial), up to H, then the straight line HI drawn perpendicular to FG will be the equinoctial line. But if, in the proposed example, the shadow-length 25 paces 23 minutes corresponding to the altitude of the pole so found is reckoned upward along the line FG, point M, the center of the dial, will be found, so that the straight line NO cutting FG at M at right angles serves as the line of the 6th hour from noon and from midnight, from which the

hour-arcs computed for the several hours are to be counted. These same two points H and M will also be found by means of the angle of the pole's altitude, CPH, and of its complement, CPM, as above, if CP on the straight line KL is taken equal to the style.

Diagram. A construction figure for the upper dial declining from the meridian toward the west by 20 degrees and inclined to the horizon by 70 degrees. Across the top runs a graduated scale-bar (a straight line from I at the left to H at the right), marked off in units 12, 24, 36, 48, 60, 72, 84, 96. Beneath it stands the horizon circle ADBE, labelled across its middle **HORI = ZON**. The diameter DE runs vertically (E at top, a dashed vertical line dropping from E; D at the bottom); the horizontal AB runs through the circle. West is marked **OCC** (occidens) at the lower left of the rim and east **ORT** (oriens) at the right. The style-line and the various hour-lines radiate from the central region through the numbered hour-points.

Readable labels:

- Scale-bar: I (left end), then graduations 12, 24, 36, 48, 60, 72, 84, 96, and H (right end)
- G — upper left, on the circle
- E — top of the circle (foot of the dashed vertical)
- O — upper right, outside the circle
- M — upper-middle (center of the dial)
- N — far left, on the horizontal line
- V — left, near the hour-12 mark
- 12 — left (hour 12)
- A — center, on the horizontal line
- Z, P — center
- HORI = ZON — horizon label across the middle
- L — right, beside the vertical legend ORT
- R, B — right of center (R above, B below)
- 12 — right (hour 12)
- 11 — right, lower
- 6, 7, 8, 9, 10 — hour-lines radiating toward the lower center
- K — lower left, beside the vertical legend OCC
- I — lower left
- 2, 3 — hour-lines at the lower left

- S [ʃ], T [t], U [u], Q [k] — small labels clustered at the center (individual letters not fully resolved)
- D, F — at the bottom of the circle

The hour-lines are drawn here as in the earlier cases: if the shadow-latitudes are counted from points K and L — the superior ones toward point G, the inferior ones toward F, the eastern likewise from point L, and the western from point K, and so on — as is seen in the accompanying figure. But for the 1st unequal hour, which has no point in the tropic of Cancer [♋] nor on the equinoctial line, but only the point Q in the tropic of Capricorn [♐], we investigated it in the parallel of Aries [♈] and Aquarius [♒]: from the eastern and inferior shadow-latitude LR of that hour (or its opposite KS), and likewise from the length of the same shadow, 38 paces 15 minutes, we found point T, so that the straight line QT, cast through points Q and T, marks the 1st unequal hour. The horizontal line, however, is the very line of the 12th unequal hour, since at that hour the Sun rises and sets — that is, is on the horizon. For that morning hour, when the Sun rises in the tropic of Capricorn [♐], we find point V; and for the evening hour, when the Sun sets in the same tropic...

we find point X; and also point I on the equinoctial line. This horizontal line — that is, the line of the 12th unequal hour, drawn through the three points V, X, I so found — will necessarily cut the straight line DE at right angles, if no error has been made anywhere. On an astronomical, Italian, or Babylonian dial the same line will pass through that point where the 6th hour from noon or from midnight, or the 12th hour from sunrise or sunset, cuts the equinoctial line: and this will always be point I. Furthermore, if the shadow-length 4 paces 22 minutes corresponding to the inclination of the plane to the horizon is carried from C upward along the straight line DE up to Z, then through Z the horizontal line is to be drawn perpendicular to DE. And if on the straight line AB you take CY equal to the style, and set up the angle CYZ of the complement of the inclination, you will likewise find on the straight line DE the point Z, through which the horizontal line is to be drawn perpendicular to DE and through the points V, X, I, and so on.

The line of the 6th unequal hour will also be the meridian line, so that when produced it necessarily passes through M, the center of the dial. We shall draw this meridian line as well if we reckon the hour-arc of the 12th hour from midnight — which we find to be 80 degrees 45 minutes, and eastern and superior. (For since the 12th hour of noon has a smaller distance from the dial's own meridian than that of the 6th hour, being distant only by as much as the inclination of the proper meridian to the meridian of the horizon, namely 20

degrees 52 minutes, the hour-arc will be superior — that is, it will recede beyond the center of the dial, as we wrote in the scholium of the preceding proposition. And again, since the dial's plane declines from the meridian toward the west, the shadow of the style will be cast at noon toward the west; and so the hour-arc of the 12th hour from midnight will be eastern, so that its shadow may fall on the opposite, western side, as the case requires.) We reckon it, then, on the opposite part of the circle NaO described about M — namely from the western point N downward as far as b. For the straight line Mb will be the meridian line.

For the upper dial declining from the meridian toward the east by 45 degrees, and inclined to the horizon on the northern side by 51 degrees 51 minutes, the following tables have been drawn up for the hours from noon and midnight; of this alone we shall give the example. Now since the Equator is the proper vertical of this dial, just as it is of the polar dial, the shadow-latitudes are to be computed from the equinoctial line — upward for the hours of the tropic of Cancer [☿], but toward the lower parts for the hours of the tropic of Capricorn [♊], as we said in the polar dial; for this dial too, like that one, lacks a center, since the greatest circle to which it is parallel is drawn through the poles of the world.

The shadow-latitudes and shadow-lengths on the upper dial declining from the meridian toward the east by 45 degrees, and inclined to the horizon on the northern side by 51 degrees 51 minutes, for the hours from noon and midnight, at latitude 42 degrees.

In the tropic of Cancer [☿] all the shadow-latitudes are superior; but in the tropic of Capricorn [♊], inferior.

Hours of Cancer [☿] and Capricorn [♊], from midnight — eastern (orientales):

Hour	4	5	6	7	8	9	†
Shadow-latitude (G. M.)	23.33	24.40	27.37	33.25	44.32	65.54	90.0
Shadow-length (P. M.)	198.7	38.51	20.14	12.36	8.18	5.50	5.13

Hours of Cancer [☿] and Capricorn [♊], from midnight — western (occidentales):

Hour	10	11	12	A meridie	1	2	3
Shadow-latitude (G. M.)	81.2	53.2	38.2	—	30.2	25.5	23.5

Hour	10	11	12	A meridie	1	2	3
	4	8				1	4
Shadow-length (P. M.)	5.17	6.52	10.1	—	15.4	27.5	66.9
			1		9		

In the Equator there are no shadow-latitudes, since then — by Proposition 11 of Book 1 — the shadow of the style falls perpetually on the equinoctial line. Their lengths, however, are these:

Hours of Aries [Υ] and Libra [ϱ], from midnight — eastern (orientales):

Hour	4	5	6	7	8	9	†
Shadow-length (P. M.)	181.29	35.17	17.56	10.31	5.55	2.23	0. 0

Hours of Aries [Υ] and Libra [ϱ], from midnight — western (occidentales):

Hour	10	11	12	A meridie	1	2	3
Shadow-length (P. M.)	0.48	4. 5	8. 2	—	13.42	24.22	60.31

[Margin: Description of a dial declining from the meridian toward the east by 45 degrees, and inclined to the horizon by 51 degrees 51 minutes, from the tables.]

[Margin: The style-line. The equinoctial line.]

And so, on the plane of the dial draw the straight line AB, which is to lie in a stable and firm plane parallel to the horizon. From C, the assumed position of the style, describe a circle ADBE of any size, in which the diameter DE cuts the straight line AB at right angles; this will be the common section of the dial's plane and of the great circle measuring the inclination to the horizon. Then, having found — by Proposition 31 of Book 1 — the arc of the great circle to which the dial is parallel, intercepted between the great circle that measures the inclination and the dial's own meridian (which in the proposed example we find to be 31 degrees 42 minutes), we shall count it from D toward A, that is, toward the western parts, up to F — just as we said in the dial declining from the meridian toward the east by 40 degrees and inclined to the horizon by 20 degrees, since this dial too declines from the meridian toward the east, like that one. For the straight line FCG, when drawn, will be the style-line, that is, the common section of the dial and of its own meridian. And the straight line KL, cutting this at right

angles, will be the equinoctial line — that is, the common section of the dial's plane and of the Equator — from which the shadow-latitudes are to be computed.

Diagram. A construction figure for the upper dial declining from the meridian toward the east by 45 degrees and inclined to the horizon by 51 degrees 51 minutes. Across the top runs a graduated scale-bar (from I at the left to H at the right), marked off in units 12, 24, 36, 48 (printed here as "46"), 60, 72, 84, 96. Below it stands the horizon circle ADBE, labelled across its middle **HORI = ZON**, with the diameter DE vertical (E at top, foot of a dashed vertical; D at the bottom) and the horizontal AB through it. West **OCC** (occidens) is marked at the upper left of the rim and east **ORT** (oriens) at the lower right. The style-line FCG and the hour-lines radiate from the center.

Readable labels:

- Scale-bar: I (left end), then graduations 12, 24, 36, 48 ["46" as printed — see below], 60, 72, 84, 96, and H (right end)
- E — top of the circle (foot of the dashed vertical)
- G — upper right, on the circle
- OCC — upper left (occidens), with X beside it
- H — left, on the horizontal line
- I — center-left
- A — lower left, on the horizontal line
- M — center
- B — right (hour region)
- ORT — lower right (oriens), with L beside it
- L — lower right
- K — lower center
- 6, 7, 8, 9, 10, 12, 1, 2, 3 — hour-lines radiating from the center
- S [?] — small label near the center
- D — bottom of the circle
- F — lower left

The hours will be described from the shadow-latitudes computed from points K and L, together with their lengths, as on the meridian and polar dials; and the hours from noon and from midnight will be perpendicular to the equinoctial line and parallel to one another, as in those dials. The horizontal line is to be drawn through point H, where the 6th hour from noon or from midnight cuts the equinoctial line, parallel to AB itself, or perpendicular to DE. But if along the

straight line DE we count upward the shadow 9 paces 26 minutes, which corresponds to the inclination of the plane to the horizon, up to I, the horizontal line is to be drawn through I, and so on. Or, if on the straight line AB you take CM equal to the style and make the angle CMI of the complement of the inclination, the straight line DE will again be cut at the point I, through which the horizontal line is drawn, and so on.

For the upper dial declining from the north toward the west by 20 degrees, and inclined to the horizon on the southern side by 30 degrees, take the following tables computed for the hours from noon and from midnight. In them the inferior shadow-latitudes are those which lie on the near side of the dial's own vertical, toward the equinoctial line; the superior, those which are found beyond the same vertical, toward the center of the dial.

THE TABLES FOLLOW.

Gnomonices, Liber Quintus (continued)

Shadow-latitudes and shadow-lengths, in the upper dial declining from north toward west by 20 degrees and inclined to the horizon on its southern side by 30 degrees, for the hours from noon and from midnight, at latitude 42 degrees.

In the tropic of Cancer (♋) the shadow-latitudes are partly inferior and partly superior, as the tables themselves show.

(In each table the shadow-latitude is given in degrees and minutes [G. M. = gradus, minuta]; the shadow-length in paces and minutes [P. M. = passus, minuta]. The "†" is a column-marker printed in the source.)

Table 1 — Hours of Cancer, from midnight; eastern & inferior (hours 2–8).

	2	3	4	5	6	7	8
Shadow-latitude	87° 42'	73° 59'	60° 22'	46° 56'	33° 38'	20° 22'	6° 54'
Shadow-length	396p 38'	262p 36'	143p 25'	84p 40'	55p 2'	38p 39'	28p 45'

Table 1 (continued) — Hours of Cancer, from midnight → noon; eastern & superior (hours 9–12, A meridie, 1, †).

	9	10	11	12	A meridi e	1	†
Shadow- latitude	7° 4'	21° 49'	37° 46'	55° 7'	—	73° 48'	90° 0'
Shadow- length	22p 23'	18p 4'	15p 8'	13p 13'	—	12p 9'	11p 53'

Table 2 — Hours of Cancer, from noon; western & superior (hours 2–7).

	2	3	4	5	6	7
Shadow- latitude	86° 46'	67° 27'	49° 10'	32° 18'	16° 46'	2° 23'
Shadow- length	11p 54'	12p 25'	13p 45'	16p 0'	19p 20'	24p 14'

Table 2 (continued) — Hours of Cancer, from noon → midnight; western & inferior (hours 8–12, A med. nocte, 1, †).

	8	9	10	11	12	A med. nocte	1	†
Shadow- latitude	11° 26'	24° 47'	38° 3'	51° 23'	64° 53'	—	78° 33'	90° 0'
Shadow- length	31p 34'	43p 8'	62p 54'	99p 54'	175p 17'	—	314p 49'	400p 29'

In the Equator all shadow-latitudes are superior, as the tables indicate.

Table 3 — Hours of Aries (♈) and Libra (♎), from midnight; eastern & superior (hours 8–12, A meridie, 1, †).

	8	9	10	11	12	A merid ie	1	†
Shadow- latitude	2° 24'	16° 19'	30° 36'	45° 23'	60° 44'	—	76° 34'	90° 0'
Shadow- length	737p 0'	106p 59'	58p 59'	42p 10'	34p 26'	—	30p 53'	30p 2'

Table 4 — Hours of Aries and Libra, from noon; western & superior (hours 2–7).

	2	3	4	5	6	7
Shadow-latitude	87° 19'	71° 15'	55° 33'	40° 24'	25° 48'	11° 38'
Shadow-length	30p 3'	31p 42'	36p 26'	46p 21'	69p 0'	149p 10'

On this dial all the hours of the tropic of Capricorn (♑) fall on the lower face of the dial, because the whole tropic of Cancer stands above the upper face and the tropic of Capricorn below it. For this reason you see no shadow-latitudes for the hours of the tropic of Capricorn entered in the tables. From this it follows that, when the Sun is at the beginning of Cancer, the gnomon's shadow traces an ellipse on the upper dial — one that we have not, however, drawn complete, owing to the narrowness of the space.

So draw the straight line AB in the plane of the dial, set parallel to the horizon on a firm and stable plane; and from C, taken as the position of the style, describe a circle ADB of any size you please, in which the straight line DCE, cutting AB at right angles at the center, is the common section of the plane of the dial and of the great circle that measures the inclination of the plane to the horizon. Then, having found (by Prop. 31 of Book 1) the arc of the great circle to which the dial is parallel — the arc intercepted between the great circle that measures the inclination and the dial's own meridian, which in the present case we find to be 43° 20' — we shall count it off from D toward A, that is, toward the eastern parts of the dial, as far as F. This is because the given dial declines from north toward west, and its center falls above the equinoctial line. (Whether the center falls above or below the equinoctial line will be known from what we set out in Prop. 37 of Book 3.

[Margin: Construction of the dial declining from north toward west by 20 degrees and inclined to the horizon by 30 degrees, from the tables.]

For when the arc of the meridian between the horizon and the plane inclined on its southern side is less than the complement of the pole's altitude — as happens in our example, since that arc, investigated by Prop. 28 of Book 1, comes to 28° 29' — the center of the dial falls above the equinoctial line; but when that same arc equals or exceeds the complement of the pole's altitude, it falls below, as we said in that proposition, in the second construction of a dial that is at once declining and inclined.)

[Margin: The style-line.]

For the straight line FC will be the style-line — that is, the common section of the plane of the dial and of its own proper meridian. For if the dial is set in its proper position — declining, that is, from north toward west — the point E will fall in the southern and eastern quadrant of the upper hemisphere, and the point D in the northern and western quadrant of the lower hemisphere, so that, when we face the dial, the north pole lies to the left of the point D. Hence, if the great circle that measures the inclination — drawn through the line DE and the style — is imagined to turn about the style set at C until it passes through the north pole (that is, until it coincides with the dial's own meridian), it will make with the dial a section falling below the style, between the points A and D, such as DCE, and so on. The contrary will happen if the plane deflects from north toward east and the center of the dial is above the style. Therefore the shadow-latitudes are to be reckoned from the straight line GH — which cuts FC at right angles at C — as from the common section of the plane of the dial and of its own proper vertical circle.

Now, with the line HI prepared and divided into parts equal to the gnomon, as above, and with the pole's altitude above the plane of the dial found (by Prop. 29 of Book 1) — which in our example we find to be $68^{\circ} 13'$, its complement being $21^{\circ} 47'$:

[Margin: The equinoctial line. The center of the dial.]

if from C, the position of the style, one counts downward along the style-line (since the center of the dial, as we said, falls above the style, and the equinoctial line below) a shadow of 30 paces 2 minutes — the amount required by the complement of the pole's altitude found — as far as I, then the straight line IK drawn through I perpendicular to FC will be the equinoctial line. And if from C upward, as far as L, one counts a shadow of 4 paces 48 minutes, answering to the pole's altitude found, then L will be the center of the dial; so that from the line ML to FL — the perpendicular in the circle MN described from L — the hour-arcs are to be numbered, if they are computed hour by hour. The two points I and L will also be found from the angle CPI (of the pole's altitude found) and the angle CPL (of the complement of that altitude), if on the line GH one marks off CP equal to the style.

In laying out the hours there is no difficulty, provided the shadow-latitudes are correctly computed from the points G and H, and so on, as is plain in the accompanying figure. In it, because the circle ADB could not be drawn complete,

we numbered the superior and eastern shadow-latitudes downward from the point H (the western point), and the western ones from the point G (the eastern point).

[Margin: The horizontal line.]

For instance, for the shadow-latitude of the 10th hour from midnight in the tropic of Cancer — which is eastern and superior, and comes to $21^{\circ} 49'$ — we took the western and inferior HQ; and from the shadow-length of 18 paces 4 minutes we found the point R for the tenth hour from midnight in the tropic of Cancer, and so on. The horizontal line is drawn from the point k, where the 6th hour from noon or from midnight cuts the equinoctial line, parallel to AB itself, or perpendicular to DE; it passes exactly through the point E, which on the line DE marks off CE, the shadow-length of 20 paces 47 minutes, answering to the inclination of the plane to the horizon. That point will also be found if on the line AB one takes CS equal to the style and makes the angle CSE the complement of the inclination.

[Margin: The meridian line.]

The 12th hour, moreover, from noon or from midnight will give the meridian line, which must pass through the point O — the point that on the circle MN terminates the hour-arc MO of $64^{\circ} 12'$, belonging to the 12th hour from noon or from midnight. In our dial this meridian line lies farther west than the style-line FC, because the plane of the dial declines toward the west; for this is why at noon the style's shadow is cast toward the western parts of the dial.

Diagram. The construction figure for the upper dial declining from north toward west by 20 degrees and inclined to the horizon by 30 degrees. Along the top runs a graduated scale-bar, from I at the left through G and the graduations 12, 24, 36, 48, 60, 72, 84 to H at the right end. Below it stands the horizon circle centered at C, with the vertical diameter DE (E above, on the dashed vertical dropped from the scale; D at the bottom) and the horizontal diameter AB (A at the left, B at the right). The horizontal line is labelled **HORIZON** across its middle, split as **K HORI** (K at its left end) ... **ZON** (right). The style-line **FCG** and the hour-lines radiate from the center; L is the dial's center just above C, with S beside it. The circle MN and points R, Q, O locate the hour-points and the meridian point.

Readable labels:

- Scale-bar: I (left end), G, graduations 12, 24, 36, 48, 60, 72, 84, H (right end)
- K — left end of the horizon line; HORIZON — label across the horizontal
- E — on the vertical DE, upper-middle (foot of the dashed vertical)

- ORT — near upper center (oriens), on a dashed line, with a G beside it
- P — upper left, on the rim
- A — left, on the horizontal line
- B — right, on the horizontal line
- C — center
- L — just above C (center of the dial); S — beside C
- M — lower right; D — bottom of the circle
- N, F — lower left
- R — lower center; Q — lower center
- Hour-numerals radiating from the center: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
- O [?] — a small label lower-left; a second "CC"/"occ" [?] mark near the H/M region does not fully resolve

For the upper dial declining from north toward east by 60 degrees, and inclined to the horizon on its southern side by 80 degrees, the following tables have been constructed for the hours from sunset. In them the superior shadow-latitudes are those lying on the near side of the dial's own vertical, toward the equinoctial line; the inferior are those found beyond that same vertical, toward the center of the dial.

Shadow-latitudes and shadow-lengths, in the upper dial declining from north toward east by 60 degrees and inclined to the horizon on its southern side by 80 degrees, for the hours from sunset, at latitude 42 degrees.

In the tropic of Cancer (♋) the shadow-latitudes are partly superior and partly inferior, as these abbreviations [Super. = superior, Infer. = inferior] declare.

Table 5 — Hours of Cancer, from sunset; western group (hours 3–9, †).

	3	4	5	6	7	8	9	†
Region	Super.	Super.	Super.	Super	Super	Infer.	Infer.	—
Shadow-latitude	24° 19'	18° 11'	12° 40'	7° 17'	1° 30'	6° 39'	30° 4'	90° 0'
Shadow-length	155p 58'	39p 52'	21p 11'	13p 3'	8p 9'	4p 35'	1p 44'	0p 55'

Table 5 (continued) — Hours of Cancer, from sunset; eastern group (hours 10–16).

	10	11	12	13	14	15	16
Region	Infer.	Infer.	Super.	Super.	Super.	Super.	Super.
Shadow-latitude	31° 54'	7° 5'	1° 33'	7° 6'	12° 28'	17° 59'	24° 4'
Shadow-length	1p 39'	4p 29'	8p 0'	12p 50'	12p 47'	38p 46'	141p 56'

In the Equator all shadow-latitudes are inferior; the western are distinguished from the eastern by these abbreviations [or. = eastern, occ. = western].

Table 6 — Hours of Aries (♈) and Libra (♎), from sunset (hours 6–17).

	6	7	8	9	10	11	12	13	14	15	16	17
Region	occ	occ.	occ.	occ	occ.	occ.	or.	or.	or.	or.	or.	or.
Shadow-latitude	6° 46'	14° 39'	24° 27'	38° 6'	58° 51'	88° 22'	61° 29'	39° 51'	25° 37'	15° 31'	7° 31'	0° 15'
Shadow-length	53p 54'	25p 4'	15p 19'	10p 16'	7p 24'	6p 20'	7p 13'	9p 54'	14p 39'	23p 39'	48p 29'	1006 p 38'

In the tropic of Capricorn (♐) all shadow-latitudes are inferior; the western are separated from the eastern by these abbreviations [or. = eastern, occ. = western].

Table 7 — Hours of Capricorn, from sunset (hours 8–17, †).

	8	9	10	11	12	†	13	14	15	16	17
Region	occ.	occ.	occ.	occ.	occ.	—	or.	or.	or.	or.	or.
Shadow-latitude	30° 43'	39° 19'	50° 5'	63° 36'	79° 47'	90° 0'	82° 43'	66° 10'	52° 10'	40° 58'	32° 10'
Shadow-length	111 p 4'	39p 8'	24p 4'	17p 56'	15p 23'	15p 0'	15p 11'	17p 19'	22p 40'	35p 19'	84p 30'

So let the straight line AB be drawn in the plane of the dial, parallel to the horizon on a stable plane; and in it, at C taken as the position of the style, let a circle ADBE of any size be described, and let the straight line DE cut the line AB at right angles at the center. This DE will be the common section of the plane of the dial and of

the great circle that measures the inclination of the plane to the horizon. Then, having found (by Prop. 31 of Book 1) the arc of the great circle to which the dial is equidistant — the arc set between the great circle that measures the inclination and the dial's own meridian, which in the present example we find to be $47^{\circ} 0'$ — we shall count it off from D, *not* toward the west B, as we said in the preceding dial (even though this dial declines from north toward east), because the center of the dial falls below the style: for the arc of the meridian between the horizon and the plane of the dial is greater than the complement of the pole's altitude, amounting indeed to $72^{\circ} 57'$. Rather we count it from D toward the east A, as far as F. (And if instead the dial declines from north toward west, one whose center lies below the equinoctial line or below the style, it is counted from D toward the west B.)

[Margin: The style-line.]

For the straight line FCG drawn will be the style-line, that is, the common section of the plane of the dial and of its own proper meridian. For when the dial is set in its proper position — so that the point E falls in the western and southern quadrant of the upper hemisphere, when the dial declines from north toward east — since the center of the dial falls below the style, the axis of the world will be inclined to the plane of the dial entirely between the points B and E; just as, when the center falls above the style, the dial is inclined to its plane, since the axis of the world must necessarily run through the top of the style. This can be evident to anyone who examines a little more attentively the position of the dial, of its center, and of the axis of the world.

Therefore, if the great circle that measures the inclination through the line DE is imagined to be turned about the style set upright to the dial, until it passes through the poles of the world and so through the axis of the world, it will assume exactly such a position that the plane of the dial cuts it between the points B and E, and hence between A and D, because of the inclination of the axis between those same points, as has been said. The contrary of this will happen on a plane declining from north toward west, if the center of the dial falls below the style, or below the equinoctial line...

[Margin: Construction of the dial declining from north toward east by 60 degrees and inclined to the horizon by 80 degrees, from the tables.]

the equinoctial line. From the straight line HI, which cuts the line FG at right angles at the center, the shadow-latitudes are to be reckoned. [Margin: The equinoctial line. The center of the dial.] Once the line HI has been prepared and

divided into parts equal to the gnomon, as was said above, and once the altitude of the pole above the plane of the dial has been found (by Prop. 29 of Book 1) — which in the present example we find to be $27^{\circ} 50'$, so that its complement is $62^{\circ} 10'$ — then, if from C (the position of the style) we count upward along the style-line a shadow of 6 parts 20 minutes, corresponding to the complement of the pole's altitude just found, as far as K (for the center of the dial, as we said, falls below the style, while the equinoctial line falls above it), the straight line KL drawn perpendicular to FG will give the equinoctial line. And if we count a shadow of 22 parts 44 minutes downward, we shall arrive at M, the center of the dial; so that the hour-arcs are to be counted from the line MN — which cuts FG at right angles — provided they have been computed for the individual hours in the circle NO described about M. But the two points K and M can also be found another way: on the line HI, take CP equal to the style, and construct the angle CPK equal to the pole's altitude just found, and the angle CPM equal to the complement of that same altitude.

Diagram. An uncaptioned sundial-construction figure for the dial declining from north toward the east by 60° and inclined to the horizon by 80° (the dial whose construction was begun on the preceding pages). The horizon runs across as a bold horizontal line lettered **HORI = ZON**, with **A** at its left end and **B** at its right end. A large circle is centered near the middle of the figure. Along the top edge runs a straight graduated scale-line (the line HI divided into gnomon-equal parts) bearing the points **L, I, G** at the left and running with tick-marks and hour-numerals to **H** at the far right. The dial's hour-lines radiate through the figure and are numbered around the rim: **4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16**. The words **ORI** (oriens, east) appear written vertically in the upper-right quadrant and **occ** (occidens, west) in the lower-right quadrant.

- **HORI=ZON** (bold central horizontal): the horizon line, endpoints **A** (left) and **B** (right)
- **L, I, G** (top left, on the graduated scale-line): style/gnomon division line HI, running to **H** (top right)
- **N** (left of center, on the circle): terminus of the line MN
- **S** (upper left): meridian-line terminus
- **P** (upper right of center): style/pole point
- **O** (right of center, on the circle): the circle NO about M
- **C** (center): position of the style
- **M** (below center-left): center of the dial
- **K, R, Q, T** (crowded about the center): equinoctial and hour-line points

- F (lower left): style-line terminus
- D (bottom): lower terminus of the vertical
- I (bottom): lower point of the construction

It is now easy to describe the hours from the tables set out, if the shadow-latitudes are computed from the points H and I, carefully observing which are eastern and which western, and which upper and which lower, as may be seen in the accompanying figure. [Margin: The horizontal line.] The horizontal line is drawn through the point R — where the 12th hour from sunrise or sunset (or the 6th hour from noon or midnight) intersects the equinoctial line — parallel to the line AB, or perpendicular to DE. This line will necessarily pass through the point a, which on the line DE marks off a shadow of 2 parts 7 minutes, corresponding to the plane's inclination to the horizon. We shall find the same point a if, on AB, we take CQ equal to the style and construct the angle CQa equal to the complement of the inclination to the horizon.

The meridian line too, if one wishes to draw it on the dial, must be drawn through M, the center of the dial, and through the point L, where the 6th hour from sunrise or sunset divides the equinoctial line. But if the point L cannot be had, because of cramped space, one must seek the hour-arc of the 12th hour from noon or midnight, whose distance from the dial's own meridian is the same as the inclination of that meridian to the meridian of the horizon. We find this to contain $28^{\circ} 31'$. And since the plane of the dial cuts the eastern quadrant of the equator above the horizon — the arc between the plane and the horizon being greater than the complement of the pole's altitude — and consequently cuts the western quadrant below the horizon, the Sun, when it stands on the equator and on the meridian above the horizon, will not illuminate the upper face of the plane; rather it will illuminate it when it lies below the horizon on the equator and meridian, were it not blocked by the earth. For this reason the shadow of the style then falls toward the eastern parts on the equinoctial line, that is, toward L. We shall therefore reckon the said arc from N as far as the point S, through which the meridian line is to be drawn.

We have also traced those hours from sunset which lie below the horizon but still above the plane of the dial — namely 8, 7, 6, 5, 4, and so on — so that their position may be considered. Thus we drew the 5th hour, which has no point on the tropic of Capricorn or on the equinoctial line, through the point found on the tropic of Cancer and through the point e, where the 7th hour from sunset cuts the line of the 6th hour from noon or midnight — the line drawn through the center M and through the point R — as is clear from the table of the line of the 6th hour

from noon or midnight (Prop. 20 of Book 1). Likewise you see that the 4th hour from sunset has been drawn through the point found on the tropic of Cancer and through the point f on the line of the 6th hour from noon or midnight — through which the 8th hour from sunset passes, as is plain from the same table — or else through the point T on the equinoctial line, through which the 16th hour from sunset is drawn, as is clearly gathered from the table of Prop. 19 of Book 1.

Finally, for an upper dial declining from north toward the east by 30° , and inclined to the horizon on the southern side by $52^\circ 3'$, we have drawn up the following tables for the hours from noon and from midnight. In them the upper and lower shadow-latitudes are to be taken as we said for the preceding dial.

The latitudes and longitudes of the shadows in an upper dial declining from north toward the east by 30° , inclined to the horizon on the southern side by $52^\circ 3'$, for the hours from noon and midnight, at latitude 42° .

In the tropic of Cancer, the shadow-latitudes are partly upper and partly lower, as appears in the tables.

Table I — In the tropic of Cancer (hours from noon). (Latitudes in G[radus]·M[inuta] = degrees·minutes; Longitudes in P[artes]·M[inuta] = gnomon-parts·minutes.)

	6	7	8	9	10	11	12
Hours of Cancer, from noon	6	7	8	9	10	11	12
(complement)	6	5	4	3	2	1	0
Latitudes of the shadows (G·M)	90·0	76·16	62·40	49·16	36·4	22·58	9·43
Longitudes of the shadows (P·M)	2429·6	699·12	224·1	107·33	63·31	42·19	30·28

Right-hand rubrics: top numeral row — *Western and upper*; second numeral row — *Eastern and upper*.

Table II — In the tropic of Cancer (hours from midnight).

	6	7	8	8	10	11	12
Hours of Cancer, from midnight	6	7	8	8	10	11	12
(complement)	6	5	4	3	2	1	0

	6	7	8	8	10	11	12
Latitudes of the shadows (G·M)	90·0	70·8	51·19	34·5	18·23	3·58	9·43
Longitudes of the shadows (P·M)	11·18	11·43	12·58	15·6	18·21	23·10	30·28

Right-hand rubrics: top numeral row — *Eastern and lower*; second numeral row — *Western and lower, except the last; This last one is upper*.

In the equator, all shadow-latitudes are lower.

Table III — In the equator (Aries and Libra), hours from midnight.

	6	7	8	9	10	11	12
Hours of Aries and Libra, from midnight	6	7	8	9	10	11	12
(complement)	6	5	4	3	2	1	0
Latitudes of the shadows (G·M)	90·0	73·45	57·52	42·35	27·57	13·50	0·0
Longitudes of the shadows (P·M)	27·59	29·8	33·2	41·21	59·42	117·8	Infini.

Right-hand rubrics: top numeral row — *eastern*; second numeral row — *western*. The final longitude is **Infini.** (infinite).

On this dial all the hours of the tropic of Capricorn fall above the lower face of the plane in question, because the plane does not cut that tropic but hides it entirely beneath itself — just as the tropic of Cancer lies entirely above the plane. Hence no mention is made of the hours of the tropic of Capricorn in these tables.

Moreover, when the Sun is at the beginning of Cancer, the gnomon's shadow describes an ellipse, though on the dial, because of the cramped space, it does not appear in full.

Diagram. An uncaptioned sundial-construction figure for the dial declining from north toward the east by 30° and inclined to the horizon on the southern side by 52° 3' (the dial of the tables above). The horizon runs across as a bold horizontal line lettered **HORI = ZON**, with **A** at its left end and **B** at its right end. Down the left margin runs the graduated style-line (the line HI divided into gnomon-equal parts), lettered from the top **I, T** (with the numeral **10**), then **G**, then the tick-numerals **12, 24, 36, 48, 60**, down to **H** at the foot. A large circle is centered at **C**

(the position of the style). The hour-lines radiate through the circle and are numbered **12, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12** around the rim. **ORT** (oriens, east) is written vertically in the upper part, with **occ** (occidens, west) marked lower down toward the foot.

- HORI=ZON (bold central horizontal): the horizon line, endpoints A (left) and B (right)
- I, T, G, and the scale 10 · 12 · 24 · 36 · 48 · 60 · H (left margin): the divided style-line HI
- M, H, E (top): upper construction points
- ORT (upper, vertical): oriens (east) marker
- C (center): position of the style
- P, S, Q (about the center): equinoctial / horizontal-line points
- K (on the horizon, right of center): style-line meets the equinoctial line
- O (lower center): hour-line point
- F (left): style-line terminus
- D, I, N (foot of the figure): lower construction points; occ (occidens) marked nearby

So let the straight line AB be drawn in the plane of the dial, lying in a stable plane parallel to the horizon; and taking C on it as the position of the style, describe about C any circle ADBE, in which let the line DE cut AB at right angles at the center, as the common section of the plane of the dial and of the great circle that measures the inclination to the horizon. [Margin: Construction of the dial declining from north toward the east by 30° and inclined to the horizon by 52° 3', from the tables.] Then, having found (by Prop. 31 of Book 1) the arc of the great circle to which the dial is equidistant — the arc between the great circle that measures the inclination and the dial's own meridian, which in the dial before us we find to be 70° 28' — we shall count it off from D toward A, on the eastern side, as we said for the preceding dial. (For on this dial too the center falls below the style and the equinoctial line above it, since the arc of the meridian between the plane and the horizon is equal to the complement of the pole's altitude above the horizon, namely 48° 0'.) We count it off as far as F.

[Margin: The style-line.] For the straight line FCG will be the style-line, that is, the common section of the plane of the dial and of its own meridian. And from the line HI — which cuts FG at right angles at C, as the common section of the plane of the dial and of its own vertical circle — the shadow-latitudes are to be counted, as has often been done already. [Margin: The equinoctial line. The center of the

dial.] Then the line HI is divided into parts equal to the style; and once the altitude of the pole above the plane of the dial has been found (by Prop. 29 of Book 1) — which in our example we find to contain $66^{\circ} 47'$, so that its complement is $23^{\circ} 13'$ — if from C (the position of the style) we count upward along the style-line a shadow of 27 parts 59 minutes, which belongs to the complement of the pole's altitude just found, we shall find the point K, through which the equinoctial line is to be drawn perpendicular to FG. And if downward we take the shadow CL of 5 parts 9 minutes, corresponding to the pole's altitude just found, the point L will be found — the center of the dial. Through it, if we draw MN perpendicular to FG, the hour-arcs are to be reckoned from the points M and N in the circle described about L, provided they have been computed for the individual hours. These two points K and L can also be found if on HI one takes CO equal to the gnomon, and makes the angle COK equal to the pole's altitude just found, and the angle COL equal to the complement of that same altitude.

If now the shadow-latitudes are computed from the points H and I, carefully observing which are eastern and which western, and which upper and which lower, the dial will be constructed as in the preceding cases. [Margin: The horizontal line.] The horizontal line must be drawn through the point K — where the style-line, which on the given plane marks the 6th hour from noon or midnight, cuts the equinoctial line — parallel to AB, or perpendicular to DE, passing entirely through the point P, which on the line CE marks off a shadow of 9 parts 22 minutes, corresponding to the plane's inclination to the horizon. This point P will likewise be found if on the line AB one takes CQ equal to the style and constructs the angle CQP equal to the complement of the inclination, and so on. [Margin: The meridian line.] The meridian line on this dial is perpendicular to FG, so that it does not differ from the line MN, because the hour-arc of the 12th hour from noon or midnight contains $0^{\circ} 0'$ in the given example.

Diagram. The same sundial-construction figure as on the facing page (the dial declining from north toward the east by 30° and inclined by $52^{\circ} 3'$) is reprinted here, so that it stands alongside the construction text that runs across the opening. The layout and lettering match fig-p455-1: horizon **HORI** = **ZON** with **A** (left) and **B** (right); the graduated style-line **I · T · 10 · G · 12 · 24 · 36 · 48 · 60 · H** down the left margin; center **C**; the upper points **M, H, E** with **ORT** (oriens) vertical; hour-lines numbered **12, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12** around the rim; **K** on the horizon right of center; **P, S, Q** about the center; **O** lower center; **F** at left; **D, I, N** and **occ** at the foot.

From all this I think it easy to grasp how dials are made from tables, since we have given examples for every dial we described in Books 2 and 3, with the exception of the equinoctial dial, for the reason stated above. [Margin: The excellence of describing dials from tables.] Of the placing of all these dials we have said nothing at all: they are to be set up as we prescribed in Books 2 and 3. And this method of describing dials, drawn from tables, is the most reliable and the easiest, provided the tables are correctly computed, as we have taught in this book. For in it there is no need to draw as many perpendicular or parallel lines as in the other methods; it is enough to draw the lines of the shadow-latitudes as hidden construction lines, so that from them the shadow-longitudes may be cut off. These are to be obtained either from the shadow-table of Prop. 2 of this book, or found geometrically from the Sun's altitudes, as we set out in this proposition when we constructed the horizontal dial, and so forth. For in this way the points of the tropics — and of other parallels too, if one wishes — will be found. Add to this that by the very same method one may paint a dial of any size on a wall, for any given length of style, so that there is no need to draw it first on paper and afterward enlarge it onto the wall.

I had indeed intended to compute various tables here for various latitudes; but since the book was growing to an enormous size, I judged it better to demonstrate the method by which anyone can construct such tables on his own, and to illustrate the matter by a few examples that lay out every variety and difficulty. This I believe I have done abundantly in this book. There is this further point: no tables could suffice for all elevations of the pole, and all declinations and inclinations of planes, unless one were willing to construct tables for each single minute of elevation, declination, and inclination — which would be an almost infinite labor. At some other time, perhaps, when leisure allows, [I shall bring] a number of tables to light...

we shall bring a number of tables to light — especially for horizontal dials, and for verticals, meridians, and dials declining from the vertical which are laid out on walls at right angles to the horizon, since these are the kinds more commonly in use.

Nor should it be passed over in silence here that on any astronomical dial whatever — one laid out by means of the hour-arcs computed by the preceding proposition of this book — we can draw the parallels, or the arcs of the signs, exactly as in the earlier books: that is, by means of the figure in which the rays of the signs are contained, or else by the Analemma, provided careful account is kept of that triangle which is contained in every dial of this proposition where a

center has been marked. The base of this triangle is the portion of the style-line lying between the center of the dial and the equinoctial line; its two sides meet at the tip of the style. Of these sides, the one that falls from the tip of the style onto the equinoctial line is the common section of the Equator and of the Meridian proper to that dial (namely if the said triangle is understood to be revolved about the style-line until it stands perpendicular to the plane of the dial); the other, enclosed between the tip of the style and the center of the dial, represents the axis of the world, as is clear from what we demonstrated in Books 2 and 3.

[Margin: By what method the arcs of the signs can be drawn on an astronomical dial constructed by means of hour-arcs.]

For example: suppose a dial declining from the meridian toward the east by 40° , and inclined to the horizon by 20° (which is indeed the first of the six last dials of this proposition), has been laid out by hour-arcs. Then, on the axis of that figure in which the rays of the signs are contained, one must take a straight line equal to the axis KN in the triangle KLN of the said dial; and on the radius of the Equator, which at its outer point K is perpendicular to that axis, one must cut off a straight line equal to the line KL of the same triangle. Next, on the style-line of the dial, one must take a straight line LS — whether directed upward or downward — equal to that same KL. Finally, the intervals between S and the hour-points on the equinoctial line are to be transferred onto the radius of the Equator with the help of compasses, and so on.

As for the drawing of the signs from the Analemma: an Analemma must be constructed, with account taken of the angle KNL, the elevation of the pole, and so forth. We have therefore composed various dials from the tables which we taught how to compute in the preceding propositions. Which was what had to be done.

END OF THE FIFTH BOOK.

GNOMONICS. BOOK SIX.

By the author **CHRISTOPH CLAVIUS** *of Bamberg, of the Society of Jesus.*

Ptolemy published a most penetrating little book on the Analemma, in which he investigates certain arcs of circles both geometrically and by numbers, and by which — in almost the same way we used in the last book — dials of every kind may be described. This Federico Commandino [Federicus Commandinus] carried out successfully in his little book on the description of dials, which he attached to that work on the Analemma. But since very many people, while they esteem and

admire both Ptolemy's Analemma and the description of dials drawn from it by Federico Commandino — on account of the sharpness of the genius and the subtlety of the demonstrations — nonetheless at the same time gravely charge both authors with a certain studied obscurity, so that they wholly despair of gaining any fruit from their writings: I judged it altogether necessary to set out in this book, more clearly and (so far as may be) more briefly, that method of describing dials which Federico Commandino, as I said, drew from Ptolemy's Analemma — a method most delightful and most subtle — so that it may be understood and grasped by anyone who is not entirely ignorant of geometry. I shall not, however, follow Ptolemy's words, but his meaning and the substance of the matter itself, especially in finding the arcs and circumferences by numbers, where I have devised computations much shorter than Ptolemy's, as anyone will readily judge who compares what I here offer with Ptolemy's writings.

An explanation of the circles and circumferences which Ptolemy chiefly considers in the Analemma.

CHAPTER I.

[Margin: Which circles Ptolemy chiefly considers in the Analemma.]

Ptolemy seems to have taken account chiefly of three dials — the horizontal, the vertical, and the meridian. (We, however, by the same method, on the pattern of the horizontal dial, shall describe all the other dials we treated in the earlier books.) For that reason he considers in the Analemma chiefly only three great circles, which in any position of the sphere mutually intersect one another at right angles — namely the Horizon, the Meridian, and the circle properly called the Vertical — from which the said three dials are equidistant, as is clear from the earlier books. For each of these three circles is drawn through the poles of the remaining two, as is evident. Their common sections make right angles with one another at the center of the world, because any two of these circles are perpendicular to the third, as we said; [Margin: (a) prop. 19.] and therefore their common section too is perpendicular to that same third circle, and so on.

[Margin: What the meridian line, the equinoctial line, and the gnomon are, according to Ptolemy.]

Now the common section of the Horizon and the Meridian is called by Ptolemy the meridian line; the common section of the Horizon and the Vertical is called the equinoctial line, because it is also the common section of the Horizon and the equinoctial circle; and finally the common section of the Meridian and the

Vertical is called the gnomon. All these appear in the figure below, in which the circle ABCD is the Meridian; AFCG the Horizon, at right angles to the Meridian; BFDG the Vertical properly so called, at right angles to both the named circles; the straight line AC the meridian line, namely the common section of the Horizon and the Meridian; the straight line FG the equinoctial line, that is, the common section of the Horizon and the Vertical; and finally the straight line DB the gnomon, that is, the common section of the Meridian and the Vertical.

[Margin: How the three said circles are understood by Ptolemy to move.]

To each of these three circles Ptolemy next assigns a certain motion. For they must all pass through the center of the Sun, in whatever place or point of the heaven the Sun happens to be, so that each takes on as many positions in a day as there are places through which the Sun passes. Now the Horizon moves about the equinoctial diameter and its section, toward that which is above the earth and below the earth — as he himself says, that is, toward the zenith of the head and its opposite point; these two points are the poles of the fixed Horizon, and are equally distant from it. And so the greatest circle which is conceived as drawn through the center of the Sun (wherever it may be) and through the poles of the Meridian — that is, the common sections of the Horizon with the Equator or the Vertical — is called by Ptolemy in this place the movable Horizon.

[Margin: The movable horizon.]

Parallel to this movable Horizon is the dial declining from the Horizon which we constructed in Prop. 13 of Book 3. The Meridian, finally, moves about the meridian line, toward the rising and the setting — as he himself puts it — that is, toward the point of the equinoctial rising and setting; these two points are the poles of the fixed Meridian, and are equally distant from it. And so the greatest circle drawn through the center of the Sun, wherever it exists, and through the poles of the Horizon — that is, the common sections of the Meridian and the Vertical — is understood by Ptolemy in this place to be the movable Vertical.

Diagram. An armillary-type sphere shown at the top of the page, illustrating the three fixed and the three movable great circles Ptolemy considers in the Analemma. Rays of the Sun (a hatched fan) issue from the upper-left, marking the Sun's position at point H. The lettered points around and within the sphere are:

- D (top of the sphere): zenith of the head; upper end of the gnomon DB.
- B (bottom of the sphere): the point opposite the zenith; lower end of the gnomon DB.
- A (lower left): the southernmost point; end of the meridian line AC.

- C (right): the northernmost point; other end of the meridian line AC.
- F (lower center-left): one end of the equinoctial diameter FG (equinoctial rising).
- G (center, near E): other end of the equinoctial diameter FG (equinoctial setting).
- E (center of the sphere): the center of the world, through which all three movable circles' common sections (HEQ, LEM, KEO, NEP) pass.
- H (left, at the Sun): the center of the Sun, where the three movable circles intersect above the earth.
- Q (lower center): the point opposite H, where the three movable circles intersect below the earth.
- L (upper left): point on the movable Horizon (Hectemorion) FHLGQM.
- K (mid-left): point on the movable Meridian (Hour-circle) AHKCQON.
- M (lower right): point on the movable Horizon (Hectemorion) FHLGQM.
- N (lower left, near A): point on the movable Vertical (Descensive) DHNBQP.
- O (lower center, near Q) [?]: point on the movable Meridian (Hour-circle) AHKCQON.
- P (center-right): point on the movable Vertical (Descensive) DHNBQP.

The fixed circles are the Meridian ABCD, the Horizon AFCG, and the Vertical BFDG; the movable circles are the Hectemorion FHLGQM, the Hour-circle AHKCQON, and the Descensive DHNBQP.

And in order to distinguish these movable circles from the fixed ones, he gives them their own proper names. For the movable Horizon he calls ἑκτημόριον [Greek: hektēmōrion, "sixth-part" — the hectemorion], because — as Olympiodorus writes in his commentaries on the third book of Aristotle's *Meteorology* — it takes on six distinct positions in a day, on account of the six unequal hours which the ancients counted each day from sunrise to noon and from noon to sunset.

[Margin: The movable Meridian.]

The movable Meridian is called the Hour-circle [circulus Horarius], because it accompanies each single hour-space from rising all the way to setting.

[Margin: The movable Vertical. The hectemorion circle.]

The movable Vertical, finally, is called καταβατικός [Greek: katabatikos, "descending" — the descensive circle], that is, the Descensive, because it shows

the descent of the Sun from its highest part to its lowest, since on it the Sun's altitude above the Horizon is always reckoned.

[Margin: The hour-circle. The descensive circle.]

As in the figure above: if the Sun be placed at the point H, the movable Horizon, or Hectemorion, will be FHLGQM, drawn through the center of the Sun; and it turns about the equinoctial diameter FG, toward the zenith of the head D, above the earth, and toward its opposite B, below the earth. The movable Meridian, or Hour-circle, will be AHKCQON, passing through the same center of the Sun, and moving about the meridian line AC toward the equinoctial rising F and setting G. The movable Vertical — that is, the Descensive circle — will be DHNBQP, likewise passing through the center of the Sun, and revolving about the gnomon DB toward the southernmost point A and the northernmost point C. Above the earth, then, all three movable circles intersect one another at H, the center of the Sun; below the earth, at the opposite point Q. Their common section is the straight line HEQ; the section of the Hectemorion and the Meridian is the straight line LEM; the section of the Hour-circle and the Vertical is the straight line KEO; and finally the section of the Descensive and the Horizon is the straight line NEP.

Now, since the purpose of this book is to teach by what method dials are described — dials equidistant from the Horizon, the Vertical circle, and the Meridian — by means of the arcs or certain circumferences of the said circles, we must, together with Ptolemy, carefully consider in those six circles (of which three are fixed and three movable, as we said) six circumferences for each hour, one in each circle: three indeed in the three fixed circles, and as many in the movable ones. All of these take their name from the circles of which they are parts, so that the one taken in the Horizon is called the horizontal circumference; the one in the Descensive, the descensive; the one in the Vertical, the vertical; the one in the Hour-circle, the horary; the one in the Meridian, the meridian; and finally the one in the Hectemorion, the hectemorial.

[Margin: Six circumferences are to be considered in the said six circles.]

[Margin: The horizontal circumference. The descensive. The vertical. The horary. The meridian. The hectemorial.]

The horizontal circumference is contained in the Horizon between the Vertical circle and the Descensive — such as the circumference of the Horizon FN. The descensive is comprehended in the Descensive circle between the center of the Sun and the zenith of the head — such as the circumference of the Descensive circle DH. The vertical is enclosed in the Vertical circle between the Meridian and

the Hour-circle — such as the circumference of the Vertical circle DK. The horary is intercepted in the Hour-circle between the center of the Sun and one of the poles of the Vertical circle — such as the circumference of the Hour-circle AH. The meridian is contained in the Meridian circle between the Horizon and the Hectemorion — such as the circumference of the Meridian AL. And finally the hectemorial lies in the Hectemorion circle between the center of the Sun and the point of the equinoctial rising or setting — such as the circumference of the Hectemorion FH.

[Margin: None of the said circumferences ought to exceed a quadrant.]

But since the said circumferences are sometimes quadrants, and sometimes greater or smaller than a quadrant, whenever they exceed a quadrant we must take those which together with them make up a semicircle — not because this is altogether necessary, but because dials are more conveniently described this way. Thus, for example, if the Sun is past the meridian, we must not take that horizontal circumference which lies between the Vertical circle on the east side and the Descensive on the west, since it exceeds a quadrant, but rather the one that lies between the Vertical on the west side and the Descensive, since this is smaller than a quadrant; and so it must be judged for the rest.

[Margin: From where the said six circumferences take their beginning.]

Hence, since all the said circumferences take their beginning from those three common sections of the three fixed circles — the horizontal and the hectemorial from the equinoctial section FG, which is the axis of the Meridian, since it is drawn through its poles; the vertical and the descensive from the gnomon DB, which is the axis of the Horizon, passing through its poles; and finally the meridian and the horary from the meridian line AB [?] [recte AC: the meridian line is defined as AC, both here and on p. 448], which is the axis of the Vertical, since it passes through its poles — it follows that in the forenoon both the horizontal circumference and the hectemorial take their beginning from the equinoctial diameter on the east side, namely from the equinoctial rising; but in the afternoon from the west side, that is, from the equinoctial setting. By the same reasoning it comes about that both the vertical circumference and the descensive have their beginning at the gnomon, from the zenith of the head, wherever the Sun may be above the Horizon, because... ..because one of them is always smaller than a quadrant, while the other — the one begun from the point opposite the zenith — is larger than a quadrant. Finally, it is clear that when the Sun is in that part of the southern hemisphere which slopes from the Vertical circle toward the antarctic pole, both the meridian circumference and the horary

take their beginning from the meridian line on the south side; whereas when the Sun is in that part of the northern hemisphere which extends from the Vertical circle toward the arctic pole, they begin from the same line on the north side. And if at any time some circumference happens to equal a quadrant, it makes no difference from which end of the section the beginning is taken.

[Margin: The functions of the said six circumferences.]

From these six circumferences, then, is determined the position of the solar ray in the planes parallel to the Horizon, the Vertical, and the Meridian — the position it has at any given hour. For the horizontal circumference indicates the latitude of the shadow in the plane parallel to the Horizon; that is, it shows how far the shadow deviates from the common section of the Vertical circle and that plane — a section which runs from east to west. For since the horizontal circumference is FN, taking its beginning from the equinoctial diameter FG, in such a way that the Descensive circle DHN is drawn through the center of the Sun H and makes the section NEP on the Horizon, the shadow of the style — whose tip is set at E, the center of the world, as we showed at Book I, prop. 2 — will fall (by Book I, prop. 11) onto the common section of the Descensive circle DHN and of the plane of the dial that is parallel to the Horizon.

[Margin: (a) Euclid XI.16 ("16. vnde.")] [Margin: (b) Euclid XI.10 ("10. vnde.")]

(a) Now since the Descensive circle and the Vertical circle properly so called make, with the plane of the horizontal dial, common sections parallel to the straight lines NEP and FEG, (b) these sections will contain in the plane of the dial an angle equal to the angle NEF. And therefore, from the circle described in that same plane from the position of the style — where that angle is formed — they will cut off an arc similar to the horizontal circumference FN, by what we demonstrated in the scholium to prop. 33 of Euclid Book VI. Rightly, therefore, the horizontal circumference FN — or rather the arc similar to it in the plane of the dial — is said to indicate the latitude of the shadow, since it shows how far the straight line onto which the shadow falls must lie from the common section of the Vertical circle and the plane of the dial. The complement of the descensive circumference DH — namely the circumference HN — shows the Sun's altitude above the Horizon at that moment. Therefore, as we demonstrated in the preceding book at props. 2 and 5, by means of it we shall find the length of the shadow geometrically.

[Margin: The horizontal and the descensive circumferences are required for constructing the horizontal dial.]

Thus the position of the Sun's ray — that is, the tip of the shadow — will now be determined. And these two circumferences, the horizontal and the descensive, are required, and suffice, for constructing the horizontal dial, as we shall say later. Again, the Vertical circumference DK, taking its beginning from the gnomon DB, shows the latitude of the shadow on a Vertical dial. For the shadow of the style whose tip is at E will fall (by Book I, prop. 11) onto the common section of the hour-circle AKCO and of the plane of the dial that is parallel to the Vertical.

[Margin: (c) Euclid XI.16 ("16. vnde.")] [Margin: (d) Euclid XI.10 ("10. vnde.")]

[Margin: The Vertical and the horary circumferences are required for constructing the Vertical dial.]

(c) Since the Hour-circle and the Meridian make, with the plane of the Vertical dial, common sections parallel to the straight lines KEO and DEB, (d) these sections will contain in the plane of the dial an angle equal to the angle KED; and accordingly they will cut off, from the circle described in that same plane from the position of the style where that angle is formed, an arc similar to the Vertical circumference DK, by what we wrote in the scholium to prop. 33 of Euclid Book VI. The Vertical circumference DK, then — or rather the arc similar to it in the plane of the dial — indicates the latitude of the shadow, since it shows how far the straight line onto which the shadow falls must lie from the common section of the Meridian and the plane of the dial. The complement of the horary circumference AH — that is, the circumference HK — makes plain the Sun's altitude above the Vertical at that moment. Therefore from it, as we showed at props. 2 and 5 of the previous book, we shall find the length of the shadow geometrically; and so the position of the solar ray — that is, the tip of the shadow — will now be determined. And thus, from the Vertical circumference and the horary, the Vertical dial will be produced, as will be said in its place.

[Margin: (e) Euclid XI.19 ("19. vnde." — printed 19; the parallel clauses (a) and (c) cite XI.16)] [Margin: (f) Euclid XI.10 ("10. vnde.")] [Margin: The meridian and the hectemorial circumferences are required for the construction of the Meridian dial.]

Lastly, the meridian circumference AL, begun from the meridian line AC, shows the latitude of the shadow on a Meridian dial. For in the same way the shadow of the style will fall onto the common section of the Hectemorial circle FLGM and of the plane of the dial that is parallel to the Meridian. (e) And since the Hectemorial circle and the Horizon make, with the plane of the dial, common sections parallel to the straight lines LEO and AEC, (f) these sections will contain in the plane of the dial, at the position of the style, an angle equal to the angle

LEA; and therefore, by what we demonstrated in the scholium to prop. 33 of Euclid Book VI, they will cut off, from the circle described in that same plane from the position of the style, an arc similar to the meridian circumference AL. This being so, with full right the meridian circumference — or rather the arc similar to it in the plane of the dial — is said to indicate the latitude of the shadow, since it shows how far the straight line onto which the shadow falls must lie from the common section of the Horizon and the plane of the dial. And the complement of the hectemorial circumference FH — namely the arc HL — measures the Sun's altitude above the Meridian circle at that time; through it, by what was handed down at props. 2 and 5 of the previous book, we shall find the length of the shadow geometrically. Hence the position of the solar ray — namely the tip of the shadow — will be determined. And these two circumferences, the meridian and the hectemorial, are to be used for constructing the Meridian dial, as we shall explain below. And so the whole art of laying out dials from Ptolemy's *Analemma* [Ptolemaeus, *Analemma*] consists in finding the said circumferences for each individual hour — which we shall now proceed to teach how to determine.

Diagram. An armillary-sphere woodcut illustrating the six circles of the *Analemma* construction of Chapter I, with the Sun's rays entering as a hatched fan from the upper left. The sphere is drawn as an assembly of banded great circles (Meridian, Horizon, Vertical, Hectemorion, Hour-circle, Descensive) intersecting at a common center. Legible labels (positions approximate):

- D (top of sphere)
- L (upper region, left of K)
- K (upper region)
- A (left)
- B (bottom of sphere)
- G (center-right)
- P (right, beside G)
- C (right side)
- Q (lower band, curved)
- M (lower right)
- E (center, where the circles and the incoming rays converge)

The text of this chapter also names interior points F, N, O and H, which lie among the crossing bands and could not be individually resolved on the scan.

The finding of the hectemorial, horary, descensive, meridian, Vertical, and horizontal circumferences (of which we spoke in the preceding chapter), from the Analemma, with the Sun in the Equator, for any latitude whatsoever.

CAPVT II.

Let the Meridian of the Analemma be ABCD, in which two diameters AC and BD cut one another at right angles: let AC be the common section of the Vertical circle and the Meridian, and BD the common section of the same Meridian and the Horizon. Let the axis of the world also be HI, such that the arc DH comprises 42 degrees — the altitude of the pole at Rome, for which we construct all the dials in this work. Let the diameter of the Equator be FG, cutting the axis at right angles at the center E.

If, then, about the diameter FG a semicircle of the Equator is imagined described perpendicular to the Meridian and inclining toward the east (the Meridian being placed in its proper position), it will be exactly equal to the semicircle FHG, since both semicircles have the same diameter FG. Therefore the latter can rightly be taken here for the former, so that FHG performs the office of the eastern semicircle of the Equator — the one, namely, that the Meridian separates from the remaining western semicircle. And FH will be the eastern quadrant of the Equator above the earth, while HG will be the eastern quadrant below the earth, so that the straight line EH is the common section of the Horizon and the eastern semicircle of the Equator.

This will be easily grasped if the semicircle of the Equator FHG is imagined turned about the diameter FG until it stands perpendicular on the plane of the Meridian; and likewise the semicircle of the Horizon set upon the diameter BD, perpendicular to the same plane of the Meridian. (a) For then the common section of these semicircles will be perpendicular to that same plane of the Meridian, and hence, by Euclid, Book XI, definition 3, also perpendicular to the straight line FG.

[Margin: (a) Euclid XI.19 ("19. vndec.")]

For this reason the straight line EH, perpendicular to FG, will be the common section of the Equator and the Horizon. For no other straight line in the plane of the semicircle of the Equator FHG can be perpendicular to FG at E except EH — which, however, is required for the common section of the Horizon and the

Equator, as we have said. And so, since the quadrant of the Equator FH runs from the rising point — placed at H, where the Horizon intersects the Equator — all the way to the south, placed at F, where the Equator [printed "Æqnator"; recte: *Æquator*] cuts the Meridian, that same quadrant can not inaptly play the role of the other quadrant, which begins at the south F and ends at the setting, so that H is also the point of setting. For the western quadrant of the Equator has exactly the same position on the sphere as the eastern; and in this way the quadrant FH will represent for us the whole semicircle of the Equator above the earth.

Let the Sun, then, be placed at one or the other equinox, at the point K of the equinoctial circle — whether that point marks off some hour from noon or from midnight, or from the east or west, or at least some fraction of an hour — in such a way that HK is the arc of the Equator lying between the center of the Sun and the Horizon, while the arc of the Equator FK lies between the Meridian and the center of the Sun, on either the eastern or the western side; and let it be required to find geometrically, from the Analemma, how great the six circumferences set out are at that time.

From K, the point of the given hour where the Sun is placed, let the straight line KL be drawn perpendicular to FG, the diameter of the Equator; and through L let two perpendiculars NLM and OLP be raised to BE and AE. These are greater than the line kL.

[Margin: (b) Euclid I.47 ("47 primi")] [Margin: (c) Euclid I.19 ("19 primi")]

(For draw the straight lines EK, EM, EP. Since their squares are equal to one another, (b) and the square on EK equals the two squares on EL and LK together, the square on EM the two squares on EN and NM, and the square on EP the two squares on EO and OP, the two squares on EL and LK will be equal both to the two squares on EN and NM and to the two on EO and OP. Since, then, (c) the square on both EN and EO is less than the square on EL — that is, in the right triangles ELN and ELO the line EL [is the greater] [?] [the print of this parenthetical clause is compressed and partly garbled] — the remaining square, that on NM, and likewise that on OP, will each be greater than the remaining square on LK; and for this reason each of the lines NM and OP will be greater than the line LK.)

From NM and OP let there be cut off two [segments] NQ and OR equal to KL; and through the points Q and R, from the center E, let two straight lines EQS and ERT be drawn, cutting the circumference of the Meridian at S and T. When this has been duly carried out, all the said six circumferences for the proposed time will

have been found — namely when the Sun stands at the point K of the Equator. For, as we shall demonstrate in the following chapter, HK will be the hectemorial circumference; BM the horary; AP the descensive; BF the meridian; AT the Vertical; and AS the horizontal. The use of the other lines of this figure — such as the lines FY, SV, TX, FZ, KN, KO — will appear in chapter 7 of this book.

Diagram. The planar Analemma construction of Chapter II: a full circle (the Meridian ABCD) filled with chords and radii from the center. The horizontal diameter runs left-to-right through the center, and numerous straight lines connect the perimeter points to the interior construction points. Legible labels (positions approximate):

- A (top of circle)
- M, S (upper arc, just left of A)
- K (upper arc, right of A)
- F (upper left)
- H (right)
- P, R (left)
- B (left end of the horizontal diameter)
- Y, V, N (left half of the horizontal diameter, moving inward toward the center)
- E (center)
- D (right end of the horizontal diameter)
- I (lower left)
- G (lower right)
- C (bottom)
- interior construction points L, O, X (near the center)

The text additionally names the points Q, R, T, Z (the intersections EQS, ERT and the auxiliary lines FZ etc.); these small interior labels among the dense chords could not all be individually resolved on the scan.

SCHOLIVM.

From what has been said, the method is clear by which, with the Sun in the Equator, any one of the said circumferences can be investigated separately, when needed, without regard to the others.

[Margin: How, with the Sun in the Equator, any of the circumferences may be found, without regard to the others.]

For up to now all six have been sought at once, though we do not always need them, but only one or another. For, as we said in the preceding chapter, only the horizontal and descensive circumferences are necessary for constructing the horizontal dial; for the Vertical dial we need the Vertical and horary circumferences; and finally for the Meridian dial the meridian and hectemorial circumferences must be found. Therefore, to investigate any one of them, from K — the point of the hour where the Sun is set — a perpendicular KL must be drawn to the diameter of the Equator FG. Then, if...

[Margin: Horizontal.] if the horizontal circumference is to be found, a perpendicular MN must be drawn through the point L to the diameter of the Horizon BD; and if from it a line NQ is cut off equal to KL, and from the center E through Q a line EQ is drawn cutting the circumference of the Meridian at S, then AS will be the horizontal circumference, taking its beginning in the Analemma from the diameter of the Vertical circle — that is, from the gnomon AE.

[Margin: Descensive.] But if the descensive circumference is to be found, a perpendicular OP must be drawn through the point L to the diameter of the Vertical circle AC, cutting the circumference of the Meridian at P. For AP will be the descensive circumference, begun in the Analemma from the diameter of the Vertical circle AC.

[Margin: Vertical.] And if the Vertical circumference is wanted, a perpendicular OP must be drawn through the point L to the diameter of the Vertical circle AC. For if from it a line OR is subtracted equal to KL, and from the center E through R a line ER is drawn cutting the circumference of the Meridian at T, then AT will be the Vertical circumference, beginning in the Analemma from the Vertical diameter AC.

[Margin: Horary.] If, however, the horary circumference is sought, a perpendicular MN must be drawn through L to the diameter of the Horizon BD, cutting the circumference of the Meridian at M. For BM will be the [horary] circumference, setting its beginning on the diameter of the Horizon BD.

[Margin: Meridian.] So that the meridian circumference may be had [?] [printed verb uncertain — reads "habeatur"/"horaria"], take the arc BF, between the Horizon and the Equator — that is, the complement of the altitude of the pole above the Horizon.

[Margin: Hectemorial.] Finally, for the hectemorial circumference the arc HK is to be taken, between the point of rising (or setting) and the Sun's place — that is, the complement of the Sun's distance from midday.

DEMONSTRATION OF WHAT WAS SAID in the preceding chapter concerning the finding of the aforesaid six circumferences.

CAPVT III.

[Margin: Demonstration of the finding of the said six circumferences, with the Sun in the Equator.]

Let there be conceived, about the diameter of the Equator FG in the figure of the preceding chapter, the semicircle of the Equator FKHG, turned into its proper position — that is, until it stands perpendicular to the Meridian ABCD, and passes through K, the Sun's place. About the gnomon AC let the Vertical semicircle AXVHC be conceived, perpendicular to the same Meridian. And about the same gnomon AC let the Descensive semicircle AKYZC be conceived, described through the center of the Sun k. Likewise, about the diameter of the Horizon BD let its semicircle BYaHD be imagined described, perpendicular to the Meridian; and about the same diameter let the Hour semicircle BKXbD be drawn through the center of the Sun K.

The semicircle of the Hectemorion, moreover, is exactly the same as that of the Equator FKHG turned to its proper position, because both the Equator and the Hectemorion pass through the center of the Sun k, and both have the same diameter — namely the common section of the Equator and the Horizon, whose half is EH. Next, the straight line MN being produced as far as d, let there be conceived, from B, the pole of the Vertical circle, and with radius BM, a semicircle MKaZd described on the surface of the sphere, which by Theodosius [Theodosius, *Sphaerica*], Book II, prop. 2, will be parallel to the Vertical circle. (a) And since the common sections which the Vertical circle and this semicircle make with the Meridian are parallel —

[Margin: (a) Euclid XI.16 ("16. vnde.") [Margin: (b) Euclid I.28 ("28. primi")]

(b) now the straight lines AC and Md are parallel to one another, because of the right angles AEN and MNE, and AC is the common section of the Vertical and the Meridian — therefore Md will be the common section of the semicircle MKaZd and the Meridian. But since the semicircle of the Equator FKHG and the semicircle MKaZd are perpendicular to the Meridian (for since the Meridian is drawn through the poles of the semicircle MKaZd, it will be perpendicular to it, by Theodosius, Book I, prop. 15, and hence conversely that semicircle too will be perpendicular to the Meridian; and the Equator is set perpendicular to the Meridian in its proper position), (c) their common section too will be

perpendicular to it, and hence, by Euclid, Book XI, definition 3, also perpendicular to the straight line FG.

[Margin: (c) Euclid XI.19 ("19. vndec.")] [Margin: (d) Euclid XI.16 ("16. vnde.")]
[Margin: (e) Euclid I.28 ("28. primi")] [Margin: (f) Euclid XI.19 ("19. vnde.")]

Since, then, the line KL, by Euclid, Book XI, definition 4, is perpendicular to the plane of the Meridian — inasmuch as it lies in the plane of the Equator, which is perpendicular to the Meridian, and is perpendicular to FG, the common section of the Equator and the Meridian — the line KL will be the common section of the Equator and the semicircle MKaZd, since the planes of these circles cut one another at the point L; and therefore the semicircle MKaZd will pass through K, the Sun's place. And the straight line aN will be the common section of that same semicircle and the Horizon.

Again, the straight line PO being produced as far as e, let there be conceived on the surface of the sphere, from A, the pole of the Horizon, and with radius AP, a semicircle PKVbe described, which by Theodosius, Book II, prop. 2, will be parallel to the Horizon; (d) and hence the sections which the Horizon and this semicircle make with the Meridian will be parallel to one another. (e) Since, then, the straight lines BD and Pe are parallel to one another, because of the right angles BEO and POE, and BD is the common section of the Horizon and the Meridian, Pe will be the common section of the semicircle PKVbe and the Meridian. But since both the Equator and this semicircle are perpendicular to the Meridian (for the Equator in its proper position is set perpendicular to the Meridian, while the Meridian, drawn through the poles of the said semicircle [printed "semirculi"; recte: semicirculi] PkVbe, is perpendicular to that semicircle, by Theodosius, Book I, prop. 15, and hence conversely the semicircle too will be perpendicular to the Meridian), (f) their common section too will be perpendicular to the Meridian; and consequently, by Euclid, Book XI, definition 3, also perpendicular to the straight line FG. Therefore, since the line kL, by Euclid, Book XI, definition 4, is perpendicular to the plane of the Meridian, as we showed a little before, the line KL will be the common section of the Equator and the semi-[circle...]

Diagram. The demonstration figure of Chapter III: a sphere shown as a circle crowded with great-circle and semicircle arcs (Equator, Vertical, Descensive, Horizon, Hour, and the parallel semicircles MKaZd and PKVbe) together with radii and chords from the center. Legible labels (positions approximate; the figure mixes capital and lowercase letters):

- A (top)
- K, M (upper arc)
- F (upper left)
- H (right)
- b (lowercase, right)
- P, R (left)
- E (center)
- B (left)
- Y (lower left)
- G (lower right)
- C (bottom)
- interior points X, W, I, O, L, N (near center)
- lowercase d, f and point Z (lower interior arcs)

Because this figure carries many small interior and lowercase labels among overlapping arcs, several could not be resolved with certainty on the scan.

(Continuation of the demonstration of Chapter III — the equality of the six meridian arcs to the corresponding circumferences.)

and of the semicircle PkVbe. Since the planes of these circles cut one another at L, the semicircle PKVbe will therefore pass through K, the place of the Sun. And the straight line VO will be the common section of this same semicircle and the Vertical.

Moreover, let the common section of the Descensive circle [circulus descensiuus] and the semicircle MKaZd be the straight line KZ; and let the common section of that same Descensive circle and the Horizon be the straight line YE, which must pass through the point f, where the straight lines aN and KZ cut one another. For each of the lines aN and YE lies in the plane of the Horizon — since aN is the common section of the Horizon and the semicircle MKaZd, and YE is the common section of that same Horizon and the Descensive circle; and the point Y lies between B and a, because the semicircle MkaZd cuts the Descensive circle both above and below the Horizon (as can easily be understood if the said semicircle and the Descensive circle are conceived in their proper positions). Hence the straight lines aN and YE will cut one another in the plane of the Horizon at some point, which let be f; and this will necessarily lie on the straight line KZ. For since the point f, where aN and YE intersect, lies both in the common section of the Descensive circle and the Horizon (namely on YE) and in the common section of the semicircle MKaZd and that same Horizon (namely on aN), it will necessarily

lie both in the plane of the Descensive circle and in the plane of the said semicircle. It will therefore lie on their common section — that is, on the straight line KZ.

Let the straight line Kb likewise be the common section of the Horary circle [circulus horarius] and the semicircle PKVbe; and let the common section of that same Horary circle and the Vertical be the straight line XE, which must pass through the point I, where the straight lines VO and Kb intersect. For each of VO and XE lies in the plane of the Vertical — since VO is the common section of the Vertical and the semicircle PKVbe, while XE is the common section of the Vertical and the Horary circle; and the point X lies between A and V, because the semicircle PkVbe cuts the Horary circle both on this side of and beyond the Vertical (as is evident if the said semicircle and the Horary circle are understood in their proper positions). Hence VO and XE will cut one another in the plane of the Vertical at some point, which let be I; and this will certainly lie on the straight line Kb. For since the point I, where VO and XE intersect, lies both in the common section of the Horary circle and the Vertical (namely on XE) and in the common section of the semicircle PKVbe and that same Vertical (namely on VO), it will necessarily lie both in the plane of the Horary circle and in the plane of the said semicircle. It will therefore lie on their common section — that is, on the straight line Kb.

Likewise, let the common section of the Equator (or hectemorion [hectemorius; Greek *hektēmōrion*, "sixth-part" — the great circle of the hours]), the Descensive circle, and the Horary circle — all drawn through the Sun's center K and the center of the world E — be the straight line kE. Finally, let the common section of the Horizon, the Vertical, and the Equator (or hectemorion) — all of which cut one another at the equinoctial rising or setting and are drawn through the center of the world — be the straight line EH.

Now that all these things have been rightly grasped and demonstrated, it must be shown: that the arc HK on the Meridian is equal to the hectemorial circumference Hk on the hectemorion (that is, on the Equator in its proper position); that the arc BM on the Meridian is equal to the horary circumference BK on the Horary circle; and that the arc AP on the Meridian is equal to the descensive circumference AK on the Descensive circle. Likewise, that the arc BF is equal to the meridian circumference between the Horizon and the hectemorion; that the arc AT on the Meridian is equal to the vertical circumference AX on the Vertical, between the Meridian and the Horary circle; and finally that the arc AS on the Meridian is equal to the horizontal circumference HY on the Horizon, between the Vertical

and the Descensive circle. This we shall demonstrate in nearly the same way as Federico Commandino [Federico Commandino].

[Margin: Demonstration of the hectemorial circumference.]

Draw the straight line Ek in the plane of the Meridian. The two sides EK and KL of the triangle EKL in the plane of the Meridian are equal to the two sides EK and KL in the plane of the hectemorion. For each EK is drawn from the center E to the surface of the sphere, and so the one is equal to the other; while the straight line kL in the Meridian will coincide with the straight line kL, the common section of the hectemorion and the semicircle MkaZd, if the meridian semicircle FHG is turned about the straight line FG until it stands perpendicular to the plane of the Meridian — since each is then perpendicular to the plane of the Meridian: the one, KL, which lies in the plane of the Meridian, by Euclid, *Elements* XI, definition 4 [Margin: (a) *Elements* XI.19]; the other, because it is the common section of two planes perpendicular to the Meridian (for from this it follows that it is perpendicular to that same Meridian). Since, then, each kL is bounded on the surface of the sphere, the one will be equal to the other — and the base EL is common — so the angles K of those triangles will be equal [Margin: (b) *Elements* I.8]. But that angle K in the plane of the Meridian is equal to the alternate angle KEH in the same plane [Margin: (c) *Elements* I.29], because the straight lines KL and HE are parallel, owing to the right angles kLE and HEL [Margin: (d) *Elements* I.28]; while this angle K in the hectemorion, for the same reason, is equal to the angle KEH in the same hectemorion [Margin: (e) *Elements* XI.16] — for the straight lines kL and EH are parallel, since they are the sections made by the hectemorion in parallel planes, namely in the Horizon and in the semicircle PKVbe. Therefore the angle KEH in the Meridian will be equal to the angle KEH in the hectemorion, and hence the arc HK in the Meridian is equal to the arc HK in the hectemorion [Margin: (f) *Elements* III.26]. Which was to be shown.

This can also be inferred more briefly as follows. Since at the time of the equinox the hectemorion does not differ from the Equator, the arc Hk (if the Meridian be taken for the Equator), lying between the Horizon and the Sun's center, will be the hectemorial circumference.

[Margin: Demonstration of the horary circumference.]

Next: since the Meridian and the Horary circle are drawn through B and D, the poles of the Vertical circle and of the semicircle MKaZd (which is equidistant from the Vertical), the arc AM of the Meridian and the arc XK of the Horary circle, being enclosed between parallel circles, will be equal to one another, by

Theodosius, Book II, prop. 10. Since, then, AB and XC are quadrants — because B, the pole of the Vertical, is a quadrant distant from the Vertical itself, by the corollary to Theodosius, Book I, prop. 16 — the remaining arc BM on the Meridian will also be equal to the remaining horary circumference BK on the Horary circle. This too we shall demonstrate more briefly thus: since B is the pole of the semicircle MKaZd, the chords BM and BK will be equal, by the definition of a pole; therefore the arcs BM and BK will also be equal [Margin: (g) *Elements* III.28]. Which is what was proposed.

[Margin: Demonstration of the descensive circumference.]

By like reasoning: since the Meridian and the Descensive circle are drawn through A and C, the poles of the Horizon and of the semicircle PKVbe (which is equidistant from the Horizon), the arc PB of the Meridian and the arc YK of the Descensive circle, being enclosed between parallel circles, will be equal to one another, by Theodosius, Book II, prop. 10. Since, then, BA and YA are quadrants, by the corollary to Theodosius, Book I, prop. 16, the remaining arc AP on the Meridian will also be equal to the remaining descensive circumference AK on the Descensive circle. We shall conclude this more easily thus: since A is the pole of the semicircle PKVbe, the chords AP and AK will be equal, by the definition of a pole; therefore the arcs AP and AK will also be equal [Margin: (h) *Elements* III.28]. Which is what was proposed.

[Margin: Demonstration of the meridian circumference.]

That BF is a meridian circumference is now evident, since it lies between the meridian line BD (that is, the Horizon) and the hectemorion FHG.

[Margin: Demonstration of the Vertical circumference.]

Again, since the Horary circle BKXbD cuts two parallel circles — namely the Horizon and the semicircle PKVbe — the sections it makes in them, that is, the straight lines BD and Kb, will be parallel to one another [Margin: (i) *Elements* XI.16]. And on account of the right angles BEO and POE, the straight line Pe is also parallel to BD [Margin: (l) *Elements* XI.9]. Therefore the straight lines Kb and Pe will likewise be parallel to one another [Margin: (m) *Elements* XI.16]. Likewise, since the semicircle PKVbe, cutting the parallel circles — namely the Vertical and the semicircle MKaZd — makes the common sections VO and KL parallel, KLOI will be a parallelogram [Margin: (n) *Elements* I.34]; and therefore the straight line OI

will be equal to the straight line LK — that is, to OR, since OR was taken equal to KL. Since, then, the two sides IO and OE of the triangle IOE are equal to the two sides RO and OE of the triangle ROE, and the angles IOE and ROE contained by them are right — (for since both the Vertical and the semicircle PKVbe are perpendicular to the Meridian [Margin: (a) *Elements* XI.19], their common section VO will also be perpendicular to that same Meridian, and hence also to the straight line AC, by Euclid, *Elements* XI, definition 3; therefore the angle IOE is right, while the angle ROE is right by construction) — the angle REO will be equal to the angle IEO [Margin: (b) *Elements* I.4]. And therefore the arc AT on the Meridian, subtending the angle TEA at the center, will be equal to the vertical circumference AX, which subtends the angle XEA at the center [Margin: (c) *Elements* III.26].

[Margin: Demonstration of the horizontal circumference.]

Finally, since the Descensive circle AKYZC cuts two parallel circles — namely the Vertical and the semicircle MKaZd — the sections it makes in them, that is, the straight lines AC and KZ, will be parallel to one another [Margin: (d) *Elements* XI.16]. And on account of the right angles AEN and MNE, the straight line Md is also parallel to AC [Margin: (e) *Elements* I.28]. Therefore the straight lines KZ and Md will also be parallel to one another [Margin: (f) *Elements* XI.9]. Likewise, since the semicircle MKaZd, cutting the parallel circles — namely the Horizon and the semicircle PKVbe — makes the common sections aN and KL parallel, KLNf will be a parallelogram [Margin: (g) *Elements* XI.16]; and consequently the straight line fN is equal to KL — that is, to NQ, which was taken equal to kL. Since, then, the two sides fN and NE of the triangle fNE are equal to the two sides QN and NE, and contain right angles — (for since both the Horizon and the semicircle MKaZd are perpendicular to the Meridian [Margin: (h) *Elements* I.34], their common section aN will also be perpendicular to that same Meridian, and on that account also to the straight line BD; therefore the angle fNE is right, while the angle QNE is right by construction) [Margin: (i) *Elements* XI.19] — the angle fEN will be equal to the angle QEN [Margin: (k) *Elements* I.4]. Therefore the arcs BY and BS, which are subtended by these angles, will also be equal to one another [Margin: (l) *Elements* III.16]. Since, then, BH and BA are quadrants — because B, the pole of the Vertical, is a quadrant distant from the Vertical, by the corollary to Theodosius, Book I, prop. 16 — the remaining arc AS on the Meridian will also be equal to the remaining horizontal circumference HY on the Horizon. All of which were the things to be demonstrated.

The finding of the hectemorial, horary, descensive, meridian, vertical, and horizontal circumference, from the Analemma, the Sun being in any parallel outside the equinoctial circle, for any latitude.

CAPUT IIII.

Let ABCD be the Meridian of the Analemma, in which the two diameters AC and BD cut one another at right angles. Of these, let AC be the common section of the Vertical circle and the Meridian, and BD the common section of this same Meridian and the Horizon. Let HI also be the axis of the world, and FG the diameter of the Equator, cutting the axis at right angles. Finally, let ab be the diameter of any parallel, whether northern or southern, about which is described the semicircle aKb of that parallel; and from d, where the diameter of the parallel cuts the diameter of the Horizon, let de be drawn perpendicular to ab, the diameter of the parallel — this will be the common section of the Horizon and the parallel. For since both the Horizon and the parallel are perpendicular to the Meridian [Margin: (m) *Elements* XI.19], their common section too will be perpendicular to that same Meridian, and hence, by Euclid, *Elements* XI, definition 3, also perpendicular to the straight line ab at the point d, where the Horizon and the parallel cut one another in the Meridian. Since, then, from d only one perpendicular can be drawn to ab in the plane of the parallel, the perpendicular de will be the common section of the Horizon and the parallel — so that ae is the segment of the parallel above the earth, and eb the segment of the same below the earth. Of these, the former (ae) represents to us the portion of the parallel from the rising e to noon a, and the portion from noon a to the setting e; while the latter (eb) represents the portion from the setting e to midnight b, and the portion from midnight b to the rising e — because the portion before noon is similar and equal to the portion after noon, and likewise the portion before midnight to the portion after midnight. And in this way the segment ae will represent to us the whole diurnal arc of the parallel above the Horizon, and the segment eb the whole nocturnal arc below the Horizon.

[Margin: In these six figures of this chapter, let the straight line YE be produced as far as Z, as was done in figures 1 and 6. Likewise in figure 2 let the straight line KE be drawn, as in the others.]

Diagram. The first analemma figure for Chapter IIII: a full circle (the Meridian of the Analemma ABCD) crowded with straight lines — the Vertical diameter AC, the

Horizon diameter BD, the world-axis HI, the Equator diameter FG, the parallel diameter ab and its semicircle aKb — together with numerous chords, perpendiculars, and construction lines radiating from the center E. Legible labels (positions approximate; the figure mixes capitals and small italics among heavily overlapping lines):

- A (top), with a small italic *g* to its right
- f, M, S, F (upper-left cluster, near the top arc)
- a (upper left, small italic)
- Y, T, P, B (left side)
- R, L, N (left of center); X, O, E (center)
- m (below center, small italic)
- K (lower left)
- e, f, I, a (lower-left arc, small italics)
- P (bottom left)
- g (bottom center, small italic); C (bottom)
- b (lower right); H, D, e (right side); Q (lower right)

The figure carries many small italic labels among densely overlapping construction lines; several could not be resolved with certainty on the scan.

Let it then be supposed that the Sun in its parallel occupies the point *k*, which marks the end of some whole hour or fraction of an hour — whether that hour is equal (reckoned from the rising, or the setting, or from noon or midnight) or unequal — so that *eK* is the arc of the parallel intercepted between the Sun's center and the Horizon, whether on the eastern or the western side; while the arc *ak* of the parallel lies between the Meridian and the Sun's center, whether before or after noon. And let it be required to find geometrically, from the Analemma, how great at that time are the six circumferences set forth.

[Margin: The finding of the said six circumferences from the Analemma, the Sun being outside the Equator.]

From *K*, the point of the given hour where the Sun is placed, let the straight line *KL* be drawn perpendicular to *ab*, the diameter of the proposed parallel; and through *L* let two perpendiculars *NLM* and *OLP* be raised to *BE* and *AE*. Of these, since they are greater than the straight line *KL* — for if the semicircle of the parallel is conceived as turned back into its proper position, so as to be perpendicular to the Meridian, then *KL*, by Euclid, *Elements* XI, definition 4,

will be perpendicular to that same Meridian, and hence, by Euclid, *Elements* XI, definition 3, perpendicular to all the lines drawn through L in it. Therefore, when the straight line ELY has been drawn, the angle kLE will be right. Since, then, the angles MNE and POE are also right, if three radii of the sphere EK, EM, EP are drawn — whose squares are equal — then both the two squares of MN and NE, and the two squares of PO and OE, will be equal to the two squares of KL and LE [Margin: (a) *Elements* I.47]. Wherefore, since both the square on NE and the square on OE are less than the square on LE — because both NE and OE, in the right triangles NEL and OEL, are less than LE [Margin: (b) *Elements* I.[?]] (margin numeral illegible) — the remaining square of MN, and the remaining square of PO, will each be greater than the remaining square of KL; and therefore each of the straight lines MN and PO will be greater than KL.

Only when the perpendicular KL falls on the point n, where the diameter of the parallel divides the diameter of the Vertical — as happens in figure 3 — is the straight line kL equal to the straight line PO [Margin: (c) *Elements* I.47]. For then the two squares on KL and LE are equal to the two squares on PO and OE; and if the common square on LE is taken away, the remaining square on KL will equal the remaining square on PO — and consequently KL and PO will be equal. Nevertheless, even then the straight line MN will exceed KL, since MN is then a radius of the sphere, while KL is less than a radius. Let two lines NQ and OR be cut off equal to KL; and through the points Q and R let straight lines EQS and ERT be drawn from the center E, cutting the circumference of the Meridian at S and T. Finally, at the straight line EY let two perpendiculars Eg and Lf be raised at E and L. When these things have been duly carried out, all the said six circumferences for the proposed time will have been found — namely when the Sun is at the point K of the parallel aeb. For, as we shall show in the next chapter, gf will be the hectemorial circumference; BM the horary; AP the descensive; BY the meridian; AT the vertical; and AS the horizontal.

Diagram. The upper of three analemma figures on this page (figure 2, in which KE is drawn). A full circle with the Vertical, Horizon, axis, Equator, and parallel diameters, and a dense web of chords and perpendiculars converging on the center E. Legible labels (positions approximate; capitals and small italics intermixed):

- a, S, M, A (upper-left arc); K (top)
- f, f, H, e (upper right to right side)
- F, P (left side); l, X, o (left of center)
- Q, m (center); k, v, N, E (center)

- p, q (below center, small italics)
- a, C (bottom); G (lower right), with a small italic *g*
- q (center-right); D (right); a, b (lower right)

Several small labels among overlapping lines could not be resolved with certainty. [Dense analemma schematic]

But the three movable circumferences — the hectemorial, horary, and descensive — can be found more easily, without so many perpendicular lines, in the following way.

[Margin: Another method of finding the hectemorial, horary, and descensive circumference.]

Having drawn KL from K, the Sun's place, perpendicular to ab, the diameter of the parallel, let the straight line ELY be drawn from E through L, and to it let the perpendicular Eg be raised at E. For if, with L as center and LK as radius, the point f is taken on the Meridian by means of the compasses, gf will be the hectemorial circumference. But if, with the point d — where the diameter of the parallel cuts the diameter of the Horizon — as center and dK as radius, the point M is taken on the Meridian, BM will be the horary circumference. And if, finally, with the point n — where the diameter of the parallel cuts the diameter of the Vertical — as center and nK as radius, the point P is taken on the Meridian by means of the compasses, AP will be the descensive circumference.

Diagram. The middle of three analemma figures on this page (figure 3, in which the perpendicular KL falls on n). A full circle with the usual diameters and a converging web of construction lines. Legible labels (positions approximate):

- o, A, Y, S, M (top arc); K (upper right, on the circle)
- F (left); l, X, L, n, o (left of center)
- Q, R (upper center); H, P, T, R (upper right)
- e (right); m (center); k, f, E, V (center)
- d (center right); B (right)
- q, p (below center); I (lower left); G (bottom); a small italic *g* and *b* (lower right)

Several small labels among overlapping lines could not be resolved with certainty. [Dense analemma schematic]

The reason for this is that the straight line fL, when drawn, is perpendicular to the straight line ELY; the straight line MLN, when drawn, is perpendicular to BD;

and the straight line PLO to AC — as we shall demonstrate presently. Since, then, the three said circumferences were first found by means of these perpendiculars Lf, MLN, PLO (as we shall show in the next chapter), the same will also be found by the points f, M, P taken on the Meridian, as we have said.

Diagram. The lower of three analemma figures on this page. A full circle densely filled with a star-like web of chords and radii from the center E. Legible labels (positions approximate):

- a, A, M, S, Y (top arc); K (upper right, on the circle)
- F (left); Q, n, x, o (left of center)
- H, T, P, R (right of center); e (right)
- m (center); E, V, N, U, S (center); a (center); f, B (center right)
- q, p (below center); b (lower right); G, C, α (bottom)

Several small labels among overlapping lines could not be resolved with certainty. [Dense analemma schematic]

We shall prove that the straight line fL is perpendicular to ELY in the following way. Draw the straight line Ef. Since the two sides KL, LE of triangle KLE are equal to the two sides fL, LE of triangle fLE — because the distance Lf was taken equal to the distance LK — and the base kE is equal to the base fE (since each is a radius of the sphere), [Margin: (a) *Elements* I.8] the angle kLE will be equal to the angle fLE. Therefore, since kLE is a right angle, as we showed just above, fLE too will be a right angle, and so fL will be perpendicular to ELY.

Conversely, we shall also prove that if from L a perpendicular Lf is drawn to ELY, it is equal to the straight line LK. For since the squares on Ek and Ef are equal, [Margin: (b) *Elements* I.47] the two squares on EL and LK will be equal to the two squares on EL and Lf. So, taking away the common square on EL, the remaining squares on LK and Lf will be equal, and therefore the straight lines LK and Lf will be equal too.

We shall also confirm this as follows. Extend the straight line YE as far as Z. Now since KL, being perpendicular to ab, the diameter of the parallel, is the mean proportional between the segments aL and Lb (by the scholium to *Elements* VI.13), [Margin: (c) *Elements* VI.17] the square on KL will be equal to the rectangle contained by aL and Lb. In the same way, fL, drawn perpendicular to YZ, will be the mean proportional between the segments YL and LZ, so that the square on fL is equal to the rectangle under YL and LZ. [Margin: (d) *Elements* III.35] Therefore, since the rectangles under aL, Lb and under YL, LZ are equal, the squares on KL

and fL will be equal as well, and consequently the straight lines KL and fL are equal.

Now that the straight lines MLN and PLO are perpendicular to the straight lines BD and AC we shall easily establish, provided we first show the following: if through L the straight lines MLN and PLO are drawn perpendicular to BD and AC , and the straight lines dM , dK and nP , nK are joined, then dM is equal to dK , and nP is equal to nK . We shall demonstrate this — and indeed much more briefly than Federico Commandino [Federico Commandino]. Extend the straight line MN as far as α . [Margin: (e) *Elements* III.3] Since the straight line $M\alpha$ is bisected at N and cut unequally at L , [Margin: (f) *Elements* II.5] the square on MN will be equal to the rectangle under ML , $L\alpha$ together with the square on LN . Adding, then, the common square on Nd , the squares on MN and Nd will be equal to the rectangle under ML , $L\alpha$ together with the squares on LN and Nd . [Margin: (g) *Elements* I.47] But the squares on MN and Nd equal the square on dM , and the squares on LN and Nd equal the square on dL . Therefore the square on dM will be equal to the rectangle under ML , $L\alpha$ together with the square on dL . [Margin: (h) *Elements* III.35] But the rectangle under ML , $L\alpha$ is equal to the rectangle under aL , Lb . So the square on dM will also be equal to the rectangle under aL , Lb together with the square on dL . [Margin: (i) *Elements* VI.17] But the square on KL is equal to the rectangle under aL , Lb , since KL is the mean proportional between aL and Lb (by the scholium to *Elements* VI.13). Therefore the square on dM will also be equal to the squares on KL and Ld . [Margin: (k) *Elements* I.47] Since, then, the square on dK is likewise equal to those same squares on KL and Ld , the squares on dM and dK will be equal to each other, and consequently the straight lines dM and dK are equal.

Again, extend the straight line PO as far as t . [Margin: (l) *Elements* III.3] Since Pt is bisected at O and cut unequally at L , [Margin: (m) *Elements* II.5] the square on PO will be equal to the rectangle under PL , Lt together with the square on LO . Adding, then, the common square on On , the squares on PO and On will be equal to the rectangle under PL , Lt together with the squares on LO and On . [Margin: (n) *Elements* I.47] But the squares on PO and On equal the square on nP , and the squares on LO and On equal the square on nL . Therefore the square on nP will be equal to the rectangle under PL , Lt together with the square on nL . [Margin: (o) *Elements* III.35] But the rectangle under PL , Lt is equal to the rectangle under aL , Lb . So the square on nP will also be equal to the rectangle under aL , Lb together with the square on nL . [Margin: (p) *Elements* VI.17] But the square on KL is equal to the rectangle under aL , Lb , since KL is the mean proportional between aL and Lb (by the scholium to *Elements* VI.13). Therefore the square on nP will also be

equal to the squares on KL and Ln. [Margin: (q) *Elements* I.47] Since, then, the square on nK is likewise equal to those same squares on KL and Ln, the squares on nP and nK will be equal to each other, and consequently the straight lines nP and nK are equal. Which was to be demonstrated.

Since this is so, it is clear that, if with the compasses the distance dM is taken equal to the distance dK, the straight line MLN is perpendicular to BD — that is, the perpendicular raised through L to BD falls on the point M. For if it fell elsewhere — say at S — then the straight line dS, drawn as in what we have just demonstrated, would be equal to the straight line dK,

Diagram. The upper of two full-circle analemma figures on this page. It reprints one of the dense analemma constructions of the preceding chapter (cf. the figures on printed p. 455), a circle filled with the usual pair of perpendicular diameters and a converging web of chords and radii from the centre E. Legible labels (positions approximate): on the top arc, a, A, M, S; K (upper right, on the circle); F with a script *g* (upper left); f; on the left, D, and along the horizontal diameter f, I; along the vertical, X, O, E; near the centre, N, V, d; upper right, Y[?]/H and e/l[?]; right of centre, T, P, R and a long-s [?]; lower centre, q, p; I (lower left); G (bottom); α and C (bottom); a script *g* and *b* (lower right). Several labels within the overlapping line-web could not be resolved with certainty. [Dense analemma schematic]

Diagram. The lower of two full-circle analemma figures on this page, likewise a reprint of a dense analemma construction of the preceding chapter. A full circle with a star-like web of chords and radii from the centre E. Legible labels (positions approximate): on the top arc, A, f, M, S; a script *g* (upper left); K (right, on the circle); F (left); n, μ [?]; on the left, t, v and a bracket-like f/[?]; centre, E, L; right, R; h, N, d; q, ρ /p[?]; b (right); I (lower left); G, S, α (bottom); a script *g* and f (lower right). Several labels within the overlapping line-web could not be resolved with certainty. [Dense analemma schematic]

equal to dK — that is, equal to dM, which was taken equal to that same dK — which is absurd; [Margin: (a) *Elements* III.7] for dM is greater than dS. So that perpendicular will not fall on S; and in the same way it will fall on no point other than M, and thus the straight line MLN is perpendicular to BD. In just the same way we shall show that PLO is perpendicular to AC, if the distance nP is taken equal to the distance nK. All of which was to be demonstrated. As for the other lines in these figures that have not been mentioned here — such as the lines alh, SV, Yf, TX, bpg, and, in the fifth figure, the line Hr, and, in the sixth, the lines kN and KO — they will be put to use in chapter 7 of this book.

SCHOLIUM.

From what has been said it is clear by what method, with the Sun on any parallel whatever, any one of the said circumferences can be found when needed, without regard to the others. [Margin: By what method, with the Sun on any parallel, any one of the circumferences may be found without regard to the others.] For to find any of them, from K — the hour-point where the Sun is placed — one must draw KL perpendicular to ab, the diameter of the parallel.

Then, if the horizontal circumference is sought, one must draw MN through L perpendicular to BD, the diameter of the horizon; or (what comes to the same thing) taking d — where the diameter of the parallel cuts the diameter of the horizon — as centre and dK as radius, one takes the point M on the meridian with the compasses, and through M and L draws the straight line MN, which, as we demonstrated, is perpendicular to BD. [Margin: Horizontal.] For if from MN one cuts off NQ equal to KL, and from the centre E through Q draws a straight line cutting the circumference of the meridian at S, then AS will be the horizontal circumference, taking its start in the analemma from the diameter of the vertical, that is, the gnomon AC.

If, however, the descensive circumference is to be found, one must draw PO through L perpendicular to AC, the diameter of the vertical, cutting the circumference of the meridian at P; or (what is the same) taking n — where the diameter of the parallel cuts the diameter of the vertical — as centre and nK as radius, one takes the point P on the meridian with the compasses. [Margin: Descensive.] For AP will be the descensive circumference, whose beginning in the analemma is taken from the gnomon, that is, the diameter of the vertical.

If the vertical circumference is wanted, one must draw PO through L perpendicular to AC, the diameter of the vertical; or (what is the same) from the point n, where the parallel's diameter divides the diameter of the horizon [so printed; the descensive passage just above has n as the point where the parallel's diameter cuts the diameter of the *vertical* — the two passages disagree], one takes, as centre and with radius nK, the point P on the meridian with the compasses, and through P and L draws the straight line PO, which, as we demonstrated, is perpendicular to AC. [Margin: Vertical.] For if from OP one cuts off OR equal to KL, and from the centre E through R draws a straight line cutting the circumference of the meridian at T, then AT will be the vertical circumference, taking its start in the analemma from the gnomon, that is, the diameter of the vertical, AC.

But if it is proposed to find the horary circumference, one must draw MN through L perpendicular to BD, the diameter of the horizon, cutting the circumference of the meridian at M; or (what is the same) with the compasses, taking d — where the diameter of the parallel cuts the diameter of the horizon — as centre and dK as radius, one takes the point M on the meridian. [Margin: Horary.] For BM will be the horary circumference, having its beginning from the diameter of the horizon BD in the analemma.

To obtain the meridian circumference, one must draw from the centre E through L a straight line cutting the circumference of the meridian at Y. For BY will be the meridian circumference. [Margin: Meridian.]

Finally, for the hectemorial circumference [hectemoria — the "sixth-part" circle], one must draw from the centre E through L the straight line EY, and to it, from E and from L, raise the two perpendiculars Eg, Lf, cutting the circumference of the meridian at g and f; or (what is the same), having drawn the perpendicular Eg to EY, one takes, from L as centre and with radius LK, the point f on the meridian with the compasses. For gf will be the hectemorial circumference. [Margin: Hectemorial.]

Furthermore, for a northern parallel we have made five figures, so that every variation that can occur might be set out. [Margin: Why five figures were made for a northern parallel, but only one for a southern.] For in the second figure of this chapter the Sun lies beyond the vertical circle, and therefore the perpendicular KL falls beyond AC, the diameter of the vertical circle, toward the south. In the third, the Sun is placed on the vertical circle, so that the perpendicular KL falls on the point n, where the diameter of the vertical is cut by the diameter of the parallel. Then in the fourth, the Sun is set on the near side of the vertical circle, yet so that its distance from the meridian is less than a quadrant, or six hours; from which it comes about that the perpendicular KL falls on the near side of the diameter of the vertical, but above m, the centre of the parallel. Again, in the fifth the Sun is six hours distant from the meridian, and so the perpendicular KL falls on m, the centre of the parallel. Finally, in the sixth the Sun has a distance from the meridian greater than six hours, and so the perpendicular falls below m, the centre of the parallel. In all these figures, however, the method of finding the said circumferences is always the same, as is evident. For a southern parallel we have described only one figure, namely the first, because there the Sun, while above the horizon, always lies beyond the vertical, and has a distance from the meridian less than six hours, as is evident.

DEMONSTRATION OF THE THINGS SAID IN THE PRECEDING CHAPTER concerning the finding of the aforesaid six circumferences.

CHAPTER V.

In any figure of the preceding chapter (in which, however, the axis of the world and the diameter of the equator are not drawn, lest the multitude of lines cause confusion), let there be conceived, about ab , the diameter of the parallel, the semicircle aeb of the parallel, turned into its proper position so that it stands upright to the meridian and passes through K , the centre of the Sun. And since, by *Elements* XI, definition 4, KL in that position stands upright to the plane of the meridian, extend the straight line YE as far as Z , and through the straight lines YZ and KL let a plane be drawn, making on the surface of the sphere, by Theodosius I.1, the semicircle $YKHZ$, [Margin: Demonstration of the finding of the six said circumferences, with the Sun placed on any parallel.] which will be upright to the plane of the meridian, and so will represent the hectemorion [Hectemorion], since it too is drawn through the centre of the Sun and stands upright to the meridian (by Theodosius I.15), inasmuch as it passes through the poles of the meridian, as was said above. [Margin: (b) *Elements* XI.18] Let the rest be constructed in the figure as in the figure of chapter 3.

Now since the straight line Lf is equal to the straight line LK — either by construction, or from what was demonstrated in the preceding chapter (for we said that, with radius LK , the point f is to be taken from L as centre on the meridian, for finding the hectemorial circumference; and likewise, if from L a perpendicular Lf is drawn to ELY , we demonstrated that Lf is equal to LK) — the two sides Ef , fL of triangle EfL in the plane of the meridian will be equal to the two sides EK , KL of triangle EKL in the plane of the hectemorion. [Margin: Demonstration of the hectemorial circumference.] And they have the base EL in common as well. [Margin: (c) *Elements* I.8] Therefore the angle EfL will be equal to the angle EKL . But the alternate angle fEg is equal to the angle EfL , [Margin: (d) *Elements* I.29] because, since the angles fLE and gEL are right angles, [Margin: (e) *Elements* I.28] the straight lines fL and gE are parallel. Likewise, by the same reasoning, the alternate angle KEH is equal to the angle EKL , [Margin: (f) *Elements* XI.16] because the straight lines KL and HE are parallel, being the sections made by the plane of the hectemorion in parallel planes — namely in the horizon and the semicircle $PKVml$. Therefore the angles fEg and KEH will be equal to each other as well; [Margin: (a) *Elements* III.26] and consequently the arc

gf on the meridian will be equal to the hectemorial circumference HK on the hectemorion. [Margin: Demonstration of the horary circumference.]

Next, that the arc BM on the meridian is equal to the horary circumference BK on the horary circle will be shown, as above in chapter 3. For again, by Theodosius II.10, the arcs AM and XK, between the parallel circles AXVHC and MkqGh, through whose poles they pass, will be equal; and consequently the remainders BM and BK, from the quadrants BA and BX, will be equal too. Or at any rate, by the definition of a pole, the chords BM and BK will be equal. [Margin: (b) *Elements* III.28] Therefore the arcs BM and BK will be equal as well.

[Margin: Demonstration of the descensive circumference.] By like reasoning we shall demonstrate, as above in chapter 3, that the arc AP on the meridian is equal to the descensive circumference AK on the descensive circle. For again, by Theodosius II.10, the arcs BP and FK, between the parallel circles BqHeD and PKVinl, will be equal; and therefore the remainders AP and AK, from the quadrants AB and AF, will be equal too. Or at any rate, by the definition of a pole, the chords AP and Ak will be equal. [Margin: (c) *Elements* III.28] Hence the arcs AP and Ak will be equal as well.

[Margin: Demonstration of the meridian circumference.] That the arc BY is the meridian circumference is evident, since it is the portion of the meridian intercepted between the horizon (that is, the meridian line BD) and the hectemorion YKHZ.

Now we shall show, as above in chapter 3, that the straight lines XE and VO cut each other at the point I, which lies on the straight line Km, the common section of the horary semicircle BKXmD and the semicircle PKVml. [Margin: Demonstration of the vertical circumference.] Likewise, that KLOI is a parallelogram, [Margin: (d) *Elements* I.34] and therefore that the straight line IO is equal to kL — that is, to OR, which was taken equal to KL itself; [Margin: (e) *Elements* I.4] and that, in triangle REO, the angle REO is equal to the angle IEO of triangle IEO, because the two sides OR, OE of the former are equal to the two sides OI, OE of the latter, and they contain equal angles, namely right angles. [Margin: (f) *Elements* III.26] From which it follows that the arc AT on the meridian is equal to the vertical circumference AX on the vertical circle.

Finally, by the same reasoning we shall conclude, as above in chapter 3, that the straight lines FE and qN cut each other at the point p, which lies on the straight line KG, the common section of the descensive semicircle AKFGC and the semicircle MKqGh. [Margin: Demonstration of the horizontal circumference.]

Likewise, that KLNp is a parallelogram, [Margin: (g) *Elements* I.34] and therefore that the straight line pN is equal to KL — that is, to NQ, which was taken equal to KL itself; [Margin: (h) *Elements* I.4] and that, in triangle QEN, the angle QEN is equal to the angle pEN of triangle pEN, because the two sides QN, NE of the former are equal to the two sides Np, NE of the latter, and they contain equal angles, being right angles. [Margin: (i) *Elements* III.26] Hence the arc BS on the meridian will be equal to the arc BF on the horizon; and consequently the remaining arc AS on the meridian, from the quadrant AB, will be equal to the remaining horizontal circumference HF, from the quadrant HB. All of which was to be demonstrated.

Diagram. A large oblique (perspective) drawing of the sphere used for Chapter V, showing the analemma circles in three dimensions — the meridian, horizon, vertical, the parallel of the Sun, the hectemorion, and the horary and descensive semicircles — as a solid sphere ringed by a dense set of great circles and chords converging on the centre E. Legible labels (positions approximate): K (at the top); A, f (upper right); S, M, X and v (v)[?] (upper centre); on the upper left, a cluster reading K, Q and Γ[?]; L (centre, marked with a dot); o (right of L); F with P (and a second P) on the left; m (right); e/ε[?] (right); n (below the centre); d (right of centre); N and E (centre); B (left); H (lower centre); F and q (lower left); G (bottom); h (bottom centre); C with Z[?] (lower right); b (right, mid-low). The figure is crowded with overlapping circle-arcs and several small labels could not be resolved with certainty. [Dense perspective sphere]

THE DIVISION OF THE ANALEMMA INTO HOURS, for which the aforesaid circumferences are to be found, so that from them a horizontal, a vertical, and a meridian sundial may be constructed.

CHAPTER VI.

There are four kinds of hours, as we said at the beginning of Book 1. Some take their start from midday or midnight, and are called astronomical, or common; from these an astronomical sundial is constructed. Others have their beginning from sunset, and others from sunrise; the former are called Italian, the latter Babylonian. From the former an Italian sundial is composed, from the latter a Babylonian one. All these hours are equal, and they divide each parallel of the Sun into 24 equal parts — the start being made, in the first kind, from midday or midnight; in the second, from sunset, that is, from that point of the parallel where

it is cut by the horizon on the western side; and in the third kind, from sunrise, namely from that point of the parallel where it is cut by the horizon on the eastern side. Lastly, other hours are unequal, and are called ancient; these cut each parallel of the Sun — the equator alone excepted — into unequal parts, yet on this condition, that they divide each diurnal arc, and likewise each nocturnal arc, into 12 equal parts. From these the ancient sundial is described, as was explained more fully above, especially at the beginning of this our *Gnomonics*.

Now since on any plane all these hours can be described by means of the circumferences we treated above, we must set out by what method the analemma is to be divided for each kind of hour, so that for the individual hours we may find the said circumferences, with the Sun on any parallel whatever — in this chapter, indeed, insofar as from them the horizontal, vertical, and meridian sundials are constructed, of which chiefly, as we said above, Ptolemy [Ptolemy] took account. The other sundials we shall treat in chapter 10 of this book, where we shall also divide the analemma for every kind of hour.

Let, then, the meridian of the analemma be ABCD, whose centre is E; the diameter of the horizon, BD; the diameter of the vertical, AC; the axis

The axis of the world is HI; the diameter of the equator, FG; the diameter of the parallel of Cancer, MN; the diameter of the parallel of Taurus and Virgo, KL; the diameter of the parallel of Capricorn, a b; and the diameter of the parallel of Sagittarius and Aquarius, ST. Around the diameters MN, KL, and a b let semicircles be described; and from the points P, O, and d, where these same diameters are cut by the diameter of the horizon, let the perpendiculars PR, OQ, and d e be drawn to the diameters themselves. As we showed above, these will be the common sections of the parallels and the horizon, so that MR is the portion of the parallel of Cancer above the earth, and RN the portion below the earth; likewise KQ the portion of Taurus and Virgo above the earth, and QL the portion below the earth; and finally b e the portion of Capricorn above the earth, and e a the portion beneath the earth.

Why we have omitted the diameter of the parallel of Gemini and Leo, and likewise the diameter of the parallel of Scorpio and Pisces, the reason is ready to hand: namely, that the three diameters MN, kL, and ST can stand in for all of them. For MN will also serve as the diameter of the parallel of Capricorn, if the analemma is inverted so that I becomes the arctic pole and H the antarctic, C the zenith and A its opposite; in that case NR will be the portion of Capricorn above the earth, and RM the portion beneath it. By the same reasoning KL will also represent the diameter of the parallel of Scorpio and Pisces, so that LQ is the

portion of Scorpio and Pisces above the earth, and QK the portion beneath it. Finally, ST will likewise express the diameter of the parallel of Gemini and Leo; and the portion of the semicircle described about ST (which, however, we did not draw, so as to avoid confusion) that corresponds to the segment TV of the diameter will be the portion of Gemini and Leo above the earth, while the portion answering to the segment VS will lie below the earth. The reason for all this is that the parallels of opposite signs have equal declinations. Since this is so, the diameter a b of the parallel of Capricorn, together with its semicircle, would be superfluous — unless we wished to use it as the diameter of the parallel of Cancer, should the analemma be inverted, as we have said — so that we may set out the divisions of all the hours in the proposed analemma.

[Margin: Division of the analemma into hours reckoned from midday and midnight. (Diuisio Analemmatis in horas à mer. & med. noc.)]

Therefore, for the hours reckoned from midday and midnight, the several semicircles of the parallels are to be divided into 12 equal parts, beginning at those points where their diameters cut the circumference of the meridian. This you see done in MRN, the semicircle of Cancer and Capricorn, where the outer numbers pertain to the hours from midnight, and the inner ones to the hours from midday. The other semicircles we have not divided in this way, because we shall cut them for other kinds of hour; it is enough to have given the example on the tropic of Cancer and Capricorn, from which anyone will readily understand by what method the other parallels are to be divided into hours from midday and midnight. For the rest, the hours of the portion MR fit the diurnal arc of Cancer or the nocturnal arc of Capricorn, and the hours of the portion NR fit the nocturnal arc of Cancer [☿; glyph worn, pinned by context] or the diurnal arc of Capricorn.

[Margin: Division of the analemma into hours reckoned from sunrise and sunset. (Diuisio Analemmatis in horas ab ortu & occasu Solis.)]

Now for the hours reckoned from sunrise and sunset — so that they may more easily be laid down in the analemma — the parallels are to be described separately, together with their diameters and with the common sections of these same diameters and the horizon, and each is to be divided into 24 equal parts, the start being made from the said common sections. For example, about a b, the diameter of Capricorn (which now also plays the part of the diameter of Cancer), a circle a e b is to be described separately, from the center m, where the axis cuts the said diameter, and through the point d, so as to correspond to the point d in the analemma. On either side, up to a b, a perpendicular e d e is to be erected, which, as we have demonstrated, is the common section of the parallel of Cancer

and the horizon; so that e a e is the portion of Cancer above the horizon, that is, the diurnal arc of Cancer, and e b e the portion of Cancer below the horizon, that is, the nocturnal arc of Cancer. Likewise e b e is the portion of Capricorn above the horizon, that is, the diurnal arc of Capricorn, and e a e the portion of Capricorn below the horizon, that is, the nocturnal arc of Capricorn.

Next, the said circle is to be divided into 24 equal parts, the start being made from the point e — from whichever you please, as in our example from the one placed to the right. In this way the parallel of Cancer, or of Capricorn, will be divided into 24 equal hours reckoned from sunrise or from sunset. For on the tropic of Cancer the inner numbers will give the hours from sunrise, while the outer numbers will indicate the hours from sunset; but on the tropic of Capricorn the reverse is to be understood, since there the outer numbers pertain to the hours from sunrise and the inner ones to the hours from sunset. From all this it is plain by what method, from the analemma — if about the diameters of the parallels the parallels themselves

Diagram. A large analemma drawn as the meridian circle projected in the plane, filled with the parallels of declination and their chords, and ringed by the zodiac signs (outer band) and a doubled scale of hour-numbers. The vertical diameter is AC (A at the top, the zenith; C at the bottom); the horizontal diameter is the horizon BD (B at the left, D at the right); the center is E. The equator diameter FG and the world-axis HI cross near the center (H toward the upper right on the rim, I toward the lower right). Legible capital labels (positions approximate): A (top), B (left), C (bottom), D (right), E (center), F and G (equator diameter), H (upper right), I (lower right), K and M (upper left, on the Taurus/Virgo parallel), L (lower right), N and O (near the center, right of E), P and Q (right of center), R (right rim), S and T (lower center). Legible lowercase labels: e (just below A), f (right of center), q (below E), m, n, d, b, h. Around the rim, the zodiac glyphs run in sequence (Aquarius, Capricorn, Sagittarius and neighbours at the upper left; Taurus, Gemini toward the right; Leo, Libra, Scorpio, Sagittarius, Pisces toward the lower rim), with two concentric rings of numbers (hours and degrees) marking the divisions. The figure is extremely crowded with overlapping arcs and small rim numerals; many individual rim glyphs and numbers could not be resolved with certainty. [Dense analemma]

Diagram. A plain circle divided into 24 equal parts, used to divide the parallel into hours reckoned from sunrise and sunset. The point a is at the top and b at the bottom (the vertical diameter a–b); the center is m (marked with a dot on the vertical line). The horizontal diameter is the horizon, drawn through the point d

and meeting the circle at e on the right (at the division marked 24) and at a corresponding point on the left. Two concentric rings of hour-numbers run around the rim in opposite senses (an inner ring and an outer ring, each 1–24), so that the same divisions serve for hours reckoned from sunrise and from sunset. Individual rim numerals are legible in places but are not transcribed cell-by-cell here (they would have to be inferred rather than read); the labelled structural points are a, b, d, e, and the center m.

[Margin: How the quantity of day and night may be known from the analemma. (Quantitas diei & noctis ex Analemmate, quo pacto cognoscatur.)]

the parallels are described and divided into hours, the quantity of day and night of each parallel may be known. For the hours found within the arc e a e make up the diurnal arc of Cancer, or the nocturnal arc of Capricorn; while the hours contained within the arc e b e make up the nocturnal arc of Cancer, or the diurnal arc of Capricorn. This too we taught in Book 1. And if, by help of the compasses, the divisions of the parallel of Cancer and Capricorn, a e b, are transferred into the semicircle of Cancer and Capricorn in the analemma, as appears in the figure — and the same is done for all the other parallels (dividing them, that is, into hours about their own diameters as described, the start being made from the common sections of these parallels and the horizon, and so on, just as has here been done for the parallel of Cancer and Capricorn) — then the analemma will be divided into hours reckoned from sunrise and sunset. In the analemma, however, only the numbers of the hours reckoned from sunset have been written in; for the complements of these numbers up to 24 will give the hours from sunrise. Thus the 23rd hour from sunset will give the 1st hour from sunrise, the 22nd will give the 2nd, and so on for the rest.

The meridian circle, described about FG, the diameter of the equator, is to be divided, like the equator, into 24 equal parts, the start being made from the diameter FG, or from the axis HI. For the point F will be the 12th astronomical hour; the first division-point next to it toward H, the 1st hour from midday and the 11th from midnight; the next point will give the 2nd hour from midday and the 10th from midnight; and so on in succession. In the same way the point H will be the 24th hour, and the 12th from sunset; the first division-point next to it toward F will give the 23rd hour and the 13th from sunset; the next point will belong to the 22nd hour and the 14th from sunset; and so forth. Finally, the same point H will look to the 24th hour, and the 12th from sunrise; the first division-point next to it toward F, to the 1st hour and the 11th from sunrise; the next, to the 2nd hour and the 10th from sunrise; and so of the rest. So that both the 18th

hour from sunset and the 6th from sunrise fall on F, the point of midday; while the 6th hour from sunset and the 18th from sunrise fall on G, the point of midnight.

[Margin: Division of the analemma into unequal hours. (*Diuisio Analemmatis in horas inæquales.*)]

Finally, for the unequal — or ancient — hours, the several portions of the parallels lying between their common sections with the horizon and the meridian circle are to be divided into six equal parts each. For that point where any parallel cuts the meridian will be the 6th unequal hour; the first division-point next to it, toward the common section of the parallel and the horizon, will give the 5th and the 7th hour; the next point the 4th and the 8th, and so on. In this way the semicircle of Taurus and Virgo has been divided in the analemma — which semicircle also represents that of Scorpio [♏] and Pisces. But the numbers of the unequal hours have been written against the points of the diameter kL of the said parallel, upon which fall the perpendiculars dropped from the division-points of the parallel to the diameter. For if they had been set against the points of the semicircle KQL, they would be confused with the numbers of the hours from midday and midnight of the semicircle MRN. By the same reasoning the other parallels will be divided into unequal hours, whenever the matter requires it. Furthermore, we have divided the three semicircles of the three parallels, namely MRN, into astronomical hours (that is, from midday and midnight); then a e b into Babylonian and Italian hours (that is, from sunrise and sunset); and lastly KQL into unequal hours — so that in a single analemma the divisions of all the kinds of hours are contained. The divisions of the meridian, which represents the equator, are suited to all the hours. For the point H will be the 12th unequal hour, while the first division toward F will exhibit the 1st and the 11th unequal hour, and so on, so that the point F gives the 6th unequal hour. And by what means these same divisions fit the equal hours as well, whether these be reckoned from midday and midnight or from sunrise and sunset, we set out a little earlier.

[Margin: How the diameters of the parallels are cut into hours. (*Quomodo diametri parallelorum in horas assecuntur.*)]

Now when the semicircles of the parallels have been divided in this way into hours, whether equal or unequal — according as an equal-hour sundial (say astronomical, or Italian, or Babylonian) or an unequal-hour sundial is to be constructed — hidden perpendicular lines must be drawn from each of the hours of every semicircle to its own diameter, and the points where they cut the diameter must be carefully marked; and, if you like, the same hour-numbers are

to be set against the division-points, as you see done on the diameter KL, which carries the numbers of the unequal hours. On the other diameters we did not write the hour-numbers, so as to avoid confusion; and for the same reason we did not write the numbers of the unequal hours against the parallel KQL. For in this way the analemma will be so prepared that, without great labor, the aforesaid circumferences may be found in it by the earlier rules, as we shall soon tell.

Such perpendiculars will be drawn easily if whole circles are described about the diameters of the parallels, as has been done about the diameter a b; yet the semicircle falling within the analemma may be drawn faintly, so that no confusion arises among the lines. For if, for each of the arcs of the outer semicircle a e b between the point a (or b) and the hour-points, equal arcs are taken on the remaining, inner semicircle, then the straight lines joining the corresponding points will be perpendicular to the diameter. For example, if to the arc between a and the point of the 11th hour from sunset an equal arc a f is taken on the remaining semicircle, then the straight line joining the point of the 11th hour from sunset with the point f will be perpendicular to the diameter a b, cutting that same diameter at the point g — which must be noted. And so of the rest. Indeed, if whole circles are described, there will be no need to describe the parallels separately, since the division can be set up in the analemma itself, if d e, the common section of the horizon and the parallel, is extended up to the point e on the other semicircle, as is plain. The points of the semicircle a f b must then be transferred into the semicircle a e b, so that all the hours are contained in the outer semicircle.

[Margin: Enough for [drawing] the sundials. (Satis est ad horologia.)]

For the most part it is even enough, if the semicircle of Cancer and Capricorn is cut into hours, for any sundial whatever to be described. For just as in the preceding book, from the latitudes and longitudes of the shadows, with the Sun positioned at the beginning of Cancer and of Capricorn, we described sundials, so here too, from the circumferences required for any given sundial — if they are found for the hours of Cancer and Capricorn — we shall lay out any sundial whatever. There is this exception: that sometimes it is necessary to investigate one or another circumference for the drawing of certain hours, when the Sun is on a nearby parallel, as on the parallel of Gemini and Leo, or of Sagittarius [♐; glyph worn] and Aquarius, as we said in the sundials of the preceding book.

[Margin: How it is to be described, if the semicircle of Cancer and Capricorn is divided into hours. (describenda, si semicirculus cancri, & capricorni in horas distribuatur.)]

With these things thus prepared, we shall find any circumference without difficulty, by the rules we handed down in chapters 2 and 4 and their scholia. Suppose, for example, that the horizontal and descensive circumference is to be found for the 23rd hour from sunset, on the parallel of Cancer and Capricorn. Through the point on the diameter ab of the parallel of Cancer and Capricorn that corresponds to the 23rd hour from sunset in the portion ae of Cancer, let a hidden perpendicular hl be drawn to BD , the diameter of the horizon; from it cut off a straight line hn equal to the interval between the 23rd hour and the point corresponding to it on the diameter ab ; and from the center E draw through n a straight line cutting the circumference of the meridian at p . Then Cp will be the horizontal circumference. Likewise, through the same point of the diameter ab , draw a perpendicular qr to AC , the diameter of the vertical; and Cr will be the descensive circumference.

[Margin: In what way the aforesaid circumference may be found, for any hour whatever, in the analemma thus far divided. (Quo pacto circumferentia supradicta in Analemmate hactenus diuiso inuestigentur pro qualibet hora.)]

By the same method, from the hidden perpendiculars st and $\beta\alpha$ [labels as printed; worn], and from an interval $s\lambda$ [?] equal to the interval between the 23rd hour and the point corresponding to it on the diameter ab — with the point taken in the portion be of Capricorn from sunset — the horizontal circumference Au and the descensive circumference $A\alpha$ will be found, for the 23rd hour of the tropic of Capricorn. In just this way we shall find the horizontal and descensive circumferences of the other hours — indeed also the vertical, the horary, the meridian, and the hectemorion circumferences — not only on the tropic of Cancer and of Capricorn, but also on the equator and the other parallels, as is clear from what was demonstrated in chapters 3 and 5 of this book.

THE FINDING OF THE HECTEMORION CIRCUMFERENCE,
THE HORARY, the descensive, the meridian, the vertical, and
the horizontal, by the doctrine of sines, whether the Sun be
on the equator or on any other parallel whatever, for any
latitude.

*(hectemorion [hectemoria; Greek ἑκτημόριον, "sixth-part," the sixth-division
circumference])*

CHAPTER VII.

Ptolemy [Ptolemy] investigates the six circumferences named above by numbers as well, so that those who are less practiced in drawing without error the many perpendicular lines required for finding the said circumferences correctly may find them more easily, and with greater pleasure, by numbers — since in this method there is no need to draw so many perpendicular lines, nor to divide the parallels into hours and into their parts (which is no easy matter); it is enough to know the distance from the meridian of any given hour, or of the point at which the Sun is placed, as will become clear from what we shall demonstrate in this chapter. For we, following in Ptolemy's footsteps, shall try to investigate these same circumferences by numbers in this chapter — a thing we shall accomplish, in my judgment, far more easily and briefly than Ptolemy, since we do not need so many multiplications and divisions as he employs.

First, we shall carry out the whole matter by rectilinear triangles; then the same by spherical triangles: and this indeed first with the Sun outside the equator, on any parallel whatever, and then with the same Sun set on the equator. Before all else, therefore, in all six of the figures of chapter 4 (of which we have here adduced the first as an example for the southern parallels, and the sixth alone for the northern parallels), in which we demonstrated that the hectemorion circumference is fg, the horary BM, the descensive AP, the meridian BY, the vertical AT, and the horizontal AS: let there be drawn from the points a, S, Y to BD, the diameter of the horizon, the perpendiculars a h, S V, Y f, of which a h cuts the straight line P t at l; likewise from the points T, b to AC, the diameter of the vertical, the perpendiculars T X, b q, of which the latter cuts the straight line M N produced at the point p. Finally, let the straight lines KN, KO be joined — which, however, so as to avoid confusion, we have drawn in the sixth figure alone, the one we have here set down; in the others they are merely to be conceived in the mind.

[Margin: The finding of the aforesaid circumferences by sines, from rectilinear triangles, when the Sun is on any parallel outside the equator. (Inuentio supradictarum circumferentiarum per sinus, ex triangulis rectilineis, Sole in quouis parallelo extra Aequatorem existente.)]

Therefore, since it holds that as a m — insofar as it is the whole sine of the proposed parallel — is to KL — insofar as it is the right sine of the Sun's distance from the meridian on the same parallel — so is a m — insofar as it is a part of the whole sine in the greatest circle, say in the meridian — to KL — insofar as it is a part of the same whole sine in the greatest circle, that is, to Lf, which is equal to KL itself (as is clear from chapter 4), and is the right sine of the arc Yf, which, subtracted from the quadrant Yg, leaves the hectemorion circumference fg: [therefore] if it is contrived that, as the whole sine is to the sine of the Sun's distance from the meridian, so the sine of the complement of the declination of the given parallel is to another, there will be found the sine of the complement of the hectemorion circumference; and thus this complement will be known, together with the hectemorion circumference itself.

[Margin: Hectemorion. (Hectemoria.)]

Next, since b n is the versed sine of the semidiurnal arc of that place in which the vertical circle of the given region is the horizon (I call the "given region" that region for which the circumferences are being investigated), that is, the place in which the elevation of the pole

Diagram. The sixth of the figures of chapter 4, drawn for the northern parallels: a large circle (the meridian) filled with a dense web of chords and perpendiculars. The vertical diameter is AC (A at the top, C at the bottom); the horizontal diameter is the horizon BD (B at the left, D at the right); the center is E, with O just above it and X above O on the vertical. H stands on the upper-right rim. Legible capital labels (positions approximate): A (top), B (left), C (bottom), D (right), E (center), H (upper right), K (left), M and N (near the center-left), O (center), P (lower left), S, T, Y (left cluster), X (upper center, below A); a capital at the right rim reads Q or G [?]. Legible lowercase labels: a (left), b (bottom center), e (right rim), f (two occurrences — upper left and lower left), g (bottom center, near q), m and n (lower center), q (bottom center), l, p, t (interior). Named in the text as construction lines: the hectemorion circumference fg, the horary BM, the descensive AP, the meridian BY, the vertical AT, the horizontal AS, with perpendiculars a h, S V, Y f, T X, b q and the joins KN, KO. The figure is very crowded with overlapping straight lines and small labels; several could not be resolved with certainty. [Dense construction figure]

(Continuation of Book VI, Cap. VII — finding the six circumferences by the doctrine of sines.)

[the elevation of the pole] above the horizon — that is, above the northern face of the vertical circle of the given region — is the complement of the elevation of that same pole above the horizon of the given region. In the same way, $a d$ is the versed sine of the semidiurnal arc $a e$, in the given region; likewise $b L$ is the versed sine of the Sun's distance from the meridian in that place, when the meridian there falls at b , or the versed sine of the Sun's distance from midnight in the given region; and $b q$ is the right sine of the meridian altitude of that same place, that is, the sine of the complement of the meridian depression in the given region. Finally, $b p$ is the difference between $b q$ — the sine of the said meridian altitude, or of the complement of the meridian depression — and $b q$ [recte: $p q$], the sine of the arc AM , the complement of the horary circumference $B M$.

[Margin: [a] 4. sexti. (Euclid, *Elements* VI.4.)]

Alternatively, [a] the line compounded of $b q$ — the sine of the said meridian altitude, or of the complement of the meridian depression — and $p q$, the sine of the arc AM , the complement of the horary circumference BM , as in the first and second figures of chapter 4, in which the Sun is placed beyond the vertical.

[Margin: Horaria (the horary [circumference]).]

Now in the triangles $n b q$ and $L b p$, as $n b$ is to $b q$, so is $b L$ to $b p$. Suppose it is contrived that, as the versed sine of the semidiurnal arc of that place — the place in which the elevation of the arctic pole above the horizon is the complement of the elevation of that same pole above the horizon of the given region (which semidiurnal arc will be found by proposition 34 of book 1) — is to the right sine of the meridian altitude of that same place, or of the complement of the meridian depression in the given region (which will be found as we taught in the scholium to proposition 35 of book 1), so is the versed sine of the Sun's distance from the meridian in that same place — that is, the versed sine of the Sun's distance from midnight in the given region (and this versed sine $b L$ will be obtainable here, if from the whole diameter $a b$, that is, from the whole sine doubled, the versed sine $a L$ of the Sun's distance from the meridian in the given region is subtracted; or else, if from $a K$, the Sun's distance from the meridian in the given region, taken from the semicircle $a K b$, is removed, there will remain $b K$, the Sun's distance from the meridian in that place where the vertical of the given region is the horizon, or the Sun's distance from midnight in the given region; and therefore the versed sine of this distance cannot lie hidden) — so is it to another [term].

There will be found the difference between the sine of the meridian altitude of that same place (or of the complement of the meridian depression in the given region) and the sine of the complement of the horary circumference; or else the line compounded of the sine of the meridian altitude of that same place (or of the complement of the meridian depression in the given region) and the sine of the complement of the horary circumference — namely when the Sun is placed beyond the vertical circle.

Therefore, from this compounded line (that is, when the Sun is beyond the vertical of the given region, as happens in the southern parallels, and as in the second figure of chapter 4) let there be subtracted the right sine of the meridian altitude of that said place, or the sine of the complement of the meridian depression in the given region; or else, if the difference has been found (that is, when the Sun lies in the northern parallels on the near side of the vertical of the given region, as appears in figures 4, 5, and 6 of chapter 4), let there be removed from the right sine of the meridian altitude of that same place, or from the sine of the complement of the meridian depression in the given region, [the appropriate term]: the remainder will be the sine of the complement of the horary circumference. This complement, then, together with the horary circumference itself, will not go unknown.

[Margin: [b] 4. sexti. (Euclid, *Elements* VI.4.)]

[Margin: Alia inuentio horaria, & breuior. (Another, and shorter, finding of the horary [circumference].)]

Diagram. A large circle (the meridian) crossed by a dense web of chords and perpendiculars, drawn for the case in which the Sun lies beyond the vertical circle; center E. It is a reprint of one of the chapter-4 figures used for finding the horary circumference. Readable capital labels (positions approximate): A (top), F (upper left), M with a following ς -like sort and Q beneath it (upper right), K (upper right rim), E (center), N (center right), D (left rim), I (lower left), G and S (bottom), T, P, R (right-center cluster). Readable lowercase / minuscule labels: a and f (top), n (upper interior), t, v, h (left), l, m (interior), d, s (lower center), q and p (lower interior), b (lower right), e, f, g (lower-right rim), z (left rim), α (bottom). The figure is extremely crowded with overlapping straight lines and small sorts; several labels could not be resolved with certainty. [Dense construction figure]

The right column continues:

EANDEM — The same horary circumference we shall find, even if we do not suppose the vertical to be the horizon, in this way. Let the Sun's distance from the

meridian be investigated by proposition 36 of book 1, for the case when the Sun is on the vertical circle (which indeed happens below the horizon in the southern parallels, but above the horizon in the northern ones). Let the versed sine $a n$ of this distance be subtracted from the diameter $a b$, that is, from the whole sine doubled, so that the line $n b$ is left marked off. For suppose it is contrived that, as this line $n b$ (once found) is to $b q$, the sine of the complement of the meridian depression (which we shall find from the scholium to proposition 35 of book 1), so is $L b$, the versed sine of the Sun's distance from midnight, to another [term]: [b] then there will be found $b p$, the difference between $b q$ — the sine of the complement of the meridian depression — and $p q$ — the sine of the complement of the horary circumference — or else the line compounded of $b q$, the sine of the complement of the meridian depression, and $p q$, the sine of the complement of the horary circumference, namely when it is beyond the vertical circle, and so on. Or else let it be found by proposition 1...

of book 5: the altitude of the Sun above the vertical circle. For its complement will give the horary circumference, as is plain.

[Margin: [c] 4. sexti. (Euclid, *Elements* VI.4.)]

[Margin: Descensiuua (the descensive [circumference]).]

RVRVS — Again, since in the triangles $a h d$ and $a l L$ [c] it holds that, as $a d$ — the versed sine of the semidiurnal arc $a e$ — is to $a h$, the right sine of the meridian altitude, so is $a L$, the versed sine of the Sun's distance from the meridian, to $a l$, the difference between $a h$ (the sine of the meridian altitude) and $l h$ (the sine of the arc BP , the complement of the descensive circumference AP): suppose it is contrived that, as the versed sine of the semidiurnal arc is to the sine of the meridian altitude, so is the versed sine of the Sun's distance from the meridian to another [term]. There will be found a number which, subtracted from the sine of the meridian altitude, leaves the sine of the complement of the descensive circumference. This complement, then, together with the descensive circumference, will be known. Or else let the altitude of the Sun above the horizon be found by the last method delivered in proposition 36 of book 1, before the spherical triangles; for its complement will exhibit the descensive circumference, as is evident.

[Margin: [d] 4. sexti. (Euclid, *Elements* VI.4.)]

PRÆTEREA — Moreover, since in the triangles ELN and EYS it holds [d] that, as EL — the sine of the hectemorion circumference, that is, the sine of the complement of the Sun's altitude above the meridian — is to LN , the sine of the

complement of the descensive circumference, that is, the sine of the Sun's altitude above the horizon, so is EY, the whole sine, to Y f [Yf], the sine of the meridian circumference B Y: suppose it is contrived...

that, as the sine of the hectemorian circumference already found — that is, as the sine of the complement of the Sun's altitude above the meridian — is to the sine of the complement of the descensive circumference, that is, to the sine of the Sun's altitude above the horizon, so is the whole sine to another [term]. There will be found the sine of the meridian circumference; and thus the meridian circumference itself will become known.

[Margin: Meridiana (the meridian [circumference]).]

AD hæc — Furthermore, since in the triangle k L N the sides KL, LN are equal to the sides RO, OE in the triangle ROE (for in chapter 4 the line OR was taken equal to the line KL; [a] and the line LN is equal to O E in the parallelogram NO), and since they contain equal angles, namely right angles — for by definition 4 of book 11 of Euclid, KL is at right angles to the plane of the meridian, if the semicircle a K b is set up at right angles to that same plane of the meridian, because it is perpendicular to a b, the common section of the said semicircle and the meridian; therefore by definition 3 of book 11 of Euclid it will also be perpendicular to the line M N — [b] the bases KN and RE will be equal. But k N is equal to N M, the sine of the horary circumference BM, because both N K and N M are the semidiameter of the circle drawn in the sphere equidistant from the vertical itself, and drawn through the lines M α and KL (namely through the Sun set at the point k), whose diameter is M α, as is clear if the semicircle a K b is set up at right angles to the meridian. Therefore E R will also be equal to N M, the sine of the horary circumference. Again, since it holds that, as a m (insofar as it is the whole sine of the proposed parallel) is to k L (insofar as it is the right sine of the Sun's distance from the meridian in that same parallel), so is a m (insofar as it is a part of the whole sine in the greatest circle, as in the meridian, that is, insofar as it is the sine of the complement of the declination of the proposed parallel) to K L (insofar as it is a part of the same whole sine in the greatest circle, that is, to RO, which in chapter 4 was taken equal to KL): if it is contrived that, as the whole sine is to the sine of the Sun's distance from the meridian, so is the sine of the complement of the declination of the given parallel to another [term], then the line R O will come out known, in the parts of the whole sine in the greatest circle. [c] And since in the triangles E R O and E T X it holds that, as E R — the sine of the horary circumference, that is, the sine of the complement of the Sun's altitude above the vertical circle (for we showed a little earlier that the line E R is equal to the line N

M) — is to R O, just found in the parts of the whole sine in the greatest circle, so is E T, the whole sine, to TX, the sine of the vertical circumference A T: suppose it is contrived that, as the sine of the horary circumference — that is, the sine of the complement of the Sun's altitude above the vertical circle — is to the sine of the Sun's distance from the meridian (known in the parts of the whole sine in the greatest circle, for R O was taken equal to K L), so is the whole sine to another [term]. There will be found the sine of the vertical circumference; and therefore the vertical circumference itself will be known.

[Margin: [a] 34. primi. (Euclid, *Elements* I.34.)] [Margin: [b] 4. primi. (Euclid, *Elements* I.4.)] [Margin: [c] 4. sexti. (Euclid, *Elements* VI.4.)] [Margin: Verticalis (the vertical [circumference]).]

DENIQUE — Finally, since in the triangle k L O the sides k L, LO are equal to the sides QN, NE of the triangle QNE (for the line QN was taken equal to KL in chapter 4, [d] and the line N E is equal to LO in the parallelogram NO), and since they contain equal angles, namely right angles — for the angle K L O is a right angle by definition 3 of book 11 of Euclid, because k L is perpendicular to the plane of the meridian by definition 4 of book 11 of Euclid, if the semicircle a K b is set up at right angles to the meridian — [e] the bases KO, QE will be equal. But KO is equal to O P, the sine of the descensive circumference AP, because both KO and OP are the semidiameter of the circle drawn in the sphere equidistant from the horizon, and drawn through the lines P t, KL, as is clear if the semicircle from k b is set up at right angles to the meridian. Therefore Q E too will be equal to O P, the sine of the descensive circumference. [f] Again, since in the triangles EQN and ESV it holds that, as E Q — the sine of the descensive circumference, that is, the sine of the complement of the Sun's altitude above the horizon (for we have just shown that the line EQ is equal to the line OP) — is to QN, equal to KL, the sine of the Sun's distance from the meridian, insofar as it is known in the parts of the whole sine in the greatest circle (and it will be known in these parts, as we demonstrated for the vertical circumference a little earlier), so is ES, the whole sine, to SV, the sine of the complement of the horizontal circumference AS: suppose it is contrived that, as the sine of the descensive circumference — that is, the sine of the complement of the Sun's altitude above the horizon — is to the sine of the Sun's distance from the meridian, known in the parts of the whole sine in the greatest circle, so is the whole sine to another [term]. There will be found the sine of the complement of the horizontal circumference. Whereby this complement, together with the horizontal circumference, will become known.

[Margin: [d] 34. primi. (Euclid, *Elements* I.34.)] [Margin: [e] 4. primi. (Euclid, *Elements* I.4.)] [Margin: [f] 4. sexti. (Euclid, *Elements* VI.4.)] [Margin: Horizontalis (the horizontal [circumference]).]

CÆTERVM — But when the Sun is distant six hours from the meridian, that is, through a quadrant of its parallel — in other words, when it is on the circle of the 6th hour from the meridian, or from midnight — the aforesaid six circumferences can be found much more easily, by this reasoning. Since then the perpendicular KL falls on m, the center of the parallel (as is manifest from the fifth figure of chapter 4, which we have here repeated), the line EY does not differ from the axis EH, since both these lines are drawn through m, from E; and consequently neither do Eg and Lf, perpendicular to EY, differ from the perpendiculars drawn from the semidiameters E G and m b to the axis E H. Therefore the declination of the parallel, namely the arc G b, will then be the hectemorion circumference g f.

[Margin: Quando Sol sex horis abest à meridie, facilius nominata circumferentiæ inueniuntur. (When the Sun is six hours from the meridian, the named circumferences are found more easily.)]

[g] **DEINDE** — Next, since in the triangles E H r and E m O it holds that, as E H — the whole sine — is to H r, the sine of the complement of the pole's altitude, so is E m, the sine of the declination of the proposed parallel, to m O, the sine of the arc AM, the complement of the horary circumference BM: suppose it is contrived that, as the whole sine is to the sine of the complement of the pole's altitude, so is the sine of the declination to another [term]. There will be found the sine of the complement of the horary circumference; and thus this complement, together with the horary circumference itself, will be known.

[Margin: [g] 4. sexti. (Euclid, *Elements* VI.4.)] [Margin: Hectemoria (the hectemorion [circumference]).] [Margin: Horaria (the horary [circumference]).]

[h] **RVRSVS** — Again, since in the triangles E H f [EHf] and E m N it holds that, as EH — the whole sine — is to H f, the sine of the pole's altitude, so is E m, the sine of the declination of the proposed parallel, to m N, the sine of the arc B R, the complement of the descensive circumference AP: suppose it is contrived that, as the whole sine is to the sine of the pole's altitude, so is the sine of the declination to another [term]. There will be found the sine of the complement of the descensive circumference. Wherefore this complement, together with the descensive circumference, will come out known.

[Margin: [h] 4. sexti. (Euclid, *Elements* VI.4.)] [Margin: Descensiuua (the descensive [circumference]).]

MERIDIANA — The meridian circumference, moreover, is then the same as the altitude of the pole H B, because the line E Y does not diverge from the axis EH.

[Margin: Meridiana (the meridian [circumference]).]

AD hæc — Furthermore, [i] since in the triangles ERO and ETX it holds that, as ER — the sine of the horary circumference (for it was shown above, in the vertical circumference, that the line E R is equal to N M, the sine of the horary circumference) — is to R O, which was taken equal to KL, that is, to a m, the sine of the complement of the declination, so is E T, the whole sine, to T X, the sine of the vertical circumference A T: suppose it is contrived that, as the sine of the horary circumference — that is, the sine of the complement of the Sun's altitude above the vertical circle — is to the sine of the complement of the declination, so is the whole sine to another [term]. There will be found the sine of the vertical circumference; and therefore the vertical circumference itself will become known.

[Margin: [i] 4. sexti. (Euclid, *Elements* VI.4.)] [Margin: Verticalis (the vertical [circumference]).]

POSTREMO — Lastly, [k] since in the triangles EQN and ESV it holds that, as EQ — the sine of the descensive circumference (for above, in the horizontal circumference, we demonstrated that the line EQ is equal to O P, the sine of the descensive circumference)...

[Margin: [k] 4. sexti. (Euclid, *Elements* VI.4.)]

Diagram, upper figure. A large circle crossed by many chords and perpendiculars, for the case in which the Sun is on the vertical circle (a reprint of the third figure of chapter 4); center E, with the horizontal diameter D–(right) and the vertical diameter A–C. Readable labels: A (top), a and f (top), M with a following ç-like sort (upper right), F (upper left), D (left rim), K (upper-right rim), V and H (right), E (center) with n, X, o, m clustered about it, N and v (center right), d (right), h (center-left), I (or l, center), q and p (lower interior), I (lower-left rim), G with g (lower right), α (bottom), C (bottom, ç-form). The figure is very crowded and several minuscule sorts could not be resolved with certainty. [Dense construction figure]

Diagram, lower figure. A second large circle with a dense web of chords, for the six circumferences hectemorion / descensive / vertical / horary / meridian (a

reprint of a chapter-4 figure); center V E, horizontal diameter D–B, vertical diameter A–C. Readable labels: A with Y (top), S and M (upper center), F (upper left), f and f (two, left rim), l (center), X and L (center), n and o (center), E with V (center), H (right, with an f/P t/R cluster below), K (upper-right rim), B (right), b (lower right), m (interior right), q and p (lower center), D (left rim) with d/δ beneath, h (center-left), I (lower-left rim), α (bottom), C (bottom), G (lower right). Extremely crowded; several sorts uncertain. [Dense construction figure]

The right column continues:

(equal) to QN — equal to k L, that is, to a m, the sine of the complement of the declination — so is E S, the whole sine, to S V, the sine of the complement of the horizontal circumference AS: suppose it is contrived that, as the sine of the descensive circumference (that is, the sine of the complement of the Sun's altitude above the horizon) is to the sine of the complement of the declination, so is the whole sine to another [term]. There will be found the sine of the complement of the horizontal circumference; and by that this complement, together with the horizontal circumference, will be known.

[Margin: Horizontalis (the horizontal [circumference]).]

ÆÆDEM — These same circumferences will be found still more easily, when the Sun is on the vertical circle. For then the perpendicular k L falls on the point n, where the diameter of the parallel intersects the diameter of the vertical, as appears in the third figure of chapter 4 (which we have here repeated); and on that account the lines EQS, EY, MN do not differ from the line AE. Likewise the line OP is equal to k L, as we demonstrated in chapter 4, and therefore the line ERT falls on the point P. Likewise Lf and Eg, perpendicular to E Y, will not diverge from the lines O P and E B, perpendicular to A E.

ITAQVE — And so, since it holds that, as a m (insofar as it is the whole sine in the given parallel) is to K L (insofar as it is the right sine of the Sun's distance from the meridian in that same parallel), so is a m (insofar as it is the sine of the complement of the declination of the given parallel, namely a part of the whole sine in the greatest circle, as in the meridian) to K L (insofar as it is a part of the whole sine in that same greatest circle, that is, to O P, or L f, equal to K L, insofar as it is the sine of the complement of the hectemorion circumference f g, and the right sine of the descensive circumference A P, and of the vertical A T): suppose it is contrived that, as the whole sine is to the sine of the Sun's distance from the meridian, so is the sine of the complement of the declination to another [term]. There will be found the sine O P, of the arc AP, whose complement f g will give

the hectemorion circumference, while the arc AP itself will be the descensive circumference, and the vertical.

HORARIA — But the horary circumference, and the meridian, will be a quadrant of the meridian BM, because (as we said) the lines NM, EY do not differ from the line AE.

HORIZONTALIS — Finally the horizontal circumference AS is then nothing. For the points A and S do not differ, since the Sun is placed on the vertical circle, and therefore the vertical circle drawn through the center of the Sun is the same as the vertical properly so called.

[Margin: Quando Sol in Verticali circulo existit, facillime dictæ circumferentiæ reperiuntur. (When the Sun is on the vertical circle, the said circumferences are found most easily.)]

SED — But now let us inquire into these same six circumferences by rectilinear triangles, when the Sun is on the equator. Let the figure of chapter 2 be repeated, in which are drawn FY, SV, perpendicular to BD, the diameter of the horizon, and FZ, TX, perpendicular to AC, the diameter of the vertical; and let the lines k N, KO be joined. Now, from what was demonstrated in chapter 3, the hectemorion circumference is HK, the horary B M, the descensive AP, the meridian B F, the vertical AT, and the horizontal AS: all of which we shall find from the sines by this reasoning.

[Margin: Inuentio earundem sex circumferentiarum per sinus, ex triangulis rectilineis, dum Sol in Æquatore existit. (The finding of the same six circumferences by sines, from rectilinear triangles, when the Sun is on the equator.)]

PRO hectemoria — For the hectemorion, let the complement of the Sun's distance from the meridian be taken, assigning 15 degrees to each hour, and so on. For HK is the complement of the Sun's distance from the meridian, namely of the arc F K itself.

[Margin: Hectemoria (the hectemorion [circumference]).]

[a] **QVONIAM** — And since in the triangles EFZ and ELO it holds that, as EF — the whole sine — is to F Z, the sine of the pole's altitude, so is E L, the sine of the complement of the Sun's distance from the meridian, to L O, the sine of the arc A M, the complement of the horary circumference BM: suppose it is contrived that, as the whole sine is to the sine of the pole's altitude, so is the sine of the complement of the Sun's distance from the meridian to another [term]. There

would be produced the sine of the complement of the horary circumference. This complement, then, together with the horary circumference, will not lie hidden.

[Margin: [a] 4. sexti. (Euclid, *Elements* VI.4.)] [Margin: Horaria (the horary [circumference]).]

[b] **DEINDE** — Next, since in the triangles EFY and ELN it holds that, as EF — the whole sine — is to FY, the sine of the complement of the pole's altitude, so is E L, the sine of the complement of the Sun's distance from the meridian, to L N, the sine of the arc B P, the complement of the descensive circumference AP: suppose it is contrived that, as the whole sine is to the sine of the complement of the pole's altitude, so is the sine of the complement of the Sun's distance from the meridian to another [term]. There will be found the sine of the complement of the descensive circumference. Whereby this complement, together with the descensive circumference, will become known.

[Margin: [b] 4. sexti. (Euclid, *Elements* VI.4.)] [Margin: Descensiuia (the descensive [circumference]).]

PRO circumferentia meridiana — For the meridian circumference, on the other hand, the complement of the pole's altitude must be taken, as is evident from the figure, namely the arc BF.

[Margin: Meridiana (the meridian [circumference]).]

Again, since in triangle KLN the sides KL, LN are equal to the sides RO, OE of triangle ROE (for OR was taken in chapter 2 equal to KL; ^a and OE is equal to LN because of the parallelogram NO), and since they contain equal angles — namely right angles (for the angle KLN is a right angle, by definition 3 of book 11 of Euclid, because, if the semicircle FKG is set up at right angles to the meridian, the line KL is perpendicular, by definition 4 of book 11 of Euclid, to that same meridian, since it stands at FG, the common section of the said semicircle and the meridian) ^b — the bases kN, ER will be equal. But KN is equal to MN itself, the sine of the horary circumference BM, since both KN and MN are the semidiameter of the circle parallel to the vertical, and are drawn through the lines KL, MN in the sphere, as is evident if the semicircle FKG is placed at right angles to the meridian. Therefore ER too will be equal to MN, the sine of the horary circumference. ^c And since in triangles ERO, ETX it holds that, as ER — the sine of the horary circumference — is to RO (that is, to its equal KL, the sine of the Sun's distance from the meridian), so is ET, the whole sine, to TX, the sine of the vertical circumference AT: then, if it is contrived that, as the sine of the horary circumference (that is, the sine of the complement of the Sun's altitude above the

vertical circle) is to the sine of the Sun's distance from the meridian, so is the whole sine to another [term], the sine of the vertical circumference will be found; and so the vertical circumference itself will not be unknown.

[Margin: [a] 34. primi. (Euclid, *Elements* I.34.)] [Margin: [b] 4. primi. (Euclid, *Elements* I.4.)] [Margin: [c] 4. sexti. (Euclid, *Elements* VI.4.)] [Margin: Verticalis (the vertical [circumference]).]

Finally, since the sides KL, LO of triangle KLO are equal to the sides QN, NE of triangle QNE (for the line QN was taken equal to KL in chapter 2; ^d and NE is equal to LO because of the parallelogram NO), and since they contain equal angles — namely right angles (for KLO is a right angle, by definition 3 of book 11 of Euclid, because KL is at right angles to the meridian, by definition 4 of book 11 of Euclid, as was also said above) ^e — the bases KO, EQ will be equal to one another. Now KO is equal to OP itself, the sine of the descensive circumference AP, since both KO and OP are the semidiameter of the circle parallel to the horizon, and are drawn through the lines kL, OP in the sphere, as is evident if the semicircle FKG is placed at right angles to the meridian. Therefore EQ too will be equal to OP, the sine of the descensive circumference. And since in triangles EQN, ESV it holds that, as EQ — the sine of the descensive circumference, that is, the sine of the complement of the Sun's altitude above the horizon — is to QN (that is, to its equal KL, the sine of the Sun's distance from the meridian), so is ES, the whole sine, to SV, the sine of the complement of the horizontal circumference AS: then, if it is contrived that, as the sine of the descensive circumference above the horizon is to the sine of the Sun's distance from the meridian, so is the whole sine to another [term], the sine of the complement of the horizontal circumference will be found. This complement, then, together with the horizontal circumference, will be known.

[Margin: [d] 34. primi. (Euclid, *Elements* I.34.)] [Margin: [e] 4. primi. (Euclid, *Elements* I.4.)] [Margin: Horizontalis (the horizontal [circumference]).]

Diagram. The figure "of chapter 2, repeated": a great circle (the meridian) with a vertical diameter from A (top) to C (bottom) and a horizontal diameter from B (left) to D (right), meeting at the centre E. The circle is divided into sectors by radii, and the upper part is crossed by many chords forming the rectilinear triangles used in the proof (the semicircle FKG at right angles to the meridian; the perpendiculars FY, SV, FZ, TX; the joined lines kN, KO, etc.). Labels read on the circumference (clockwise from top): A, K, H, D (right end of the horizontal diameter), Q, C (bottom), I, B (left end of the horizontal diameter), P, F; with M and S set close together near the top left of A. Interior points read: R (near P), Z

(upper centre), X and O (centre), L (below X), and along the horizontal diameter, left of centre, Y, V, N; E at the centre.

- A (top): apex of the vertical diameter
- C (bottom): foot of the vertical diameter
- B (left), D (right): ends of the horizontal diameter
- E (centre): intersection of the diameters
- F, P (upper left), K, H (upper right): points on the circumference
- M, S (near top): points of the small upper arc
- I (lower left), Q (lower right): points on the circumference
- R, Z, X, O, L, Y, V, N (interior): endpoints of the chords/perpendiculars
- T [?] (interior; named in the text at TX, glyph not separately resolved)
- G [?] (of the semicircle FKG; glyph not separately resolved on the arc)

By spherical triangles, then, we shall investigate these same six circumferences when the Sun is outside the equator, in any parallel whatever. Let the horizon be ABCD; the meridian BED; the equator AFC; the vertical AEC; the Sun's parallel LM, whether southern or northern. Let the Sun be placed first at the point G, beyond the vertical circle — as happens at every hour of the southern parallel above the horizon, but, in the case of the northern parallel, only at those hours which have a smaller distance from the meridian than the Sun has when it is on the vertical, provided the Sun's parallel does in fact intersect the vertical. This distance of the Sun from the meridian, when the Sun is on the vertical, will be found from what was demonstrated in proposition 36 of book 1. Through E, the zenith, let the descensive circle EGH be drawn through G, the Sun's place; through A, the pole of the meridian, through the same point G, the hectemorion AGI; and from the poles B, D of the vertical circle, through the same point G, the horary circle BGKD. Then AG will be the hectemorion circumference; BG the horary; EG the descensive; BI the meridian; EK the vertical; and AH the horizontal — all of which we shall compute in this way, having first drawn, from the pole of the world O (whether southern or northern), through G, the great circle OGP, which measures the declination of the given parallel and cuts off from the equator the arc FP, which measures the Sun's distance from the meridian, since, by proposition 10 of book 2 of Theodosius, it is similar to the arc of the parallel between the meridian and the point G, that is, to the circle OGP.

[Margin: The finding of the same six circumferences by spherical triangles, when the Sun is in any parallel outside the equator, provided it is in a northern parallel beyond the vertical, on the southern side. (Inuentio earundem sex

circumferentiarum per triangula sphærica, Sole existente in quovis parallelo extra Æquatorem, dummodo sit in parallelo boreali ultra Verticalem ex parte australi.))

Diagram. A pair of armillary-sphere figures showing the general (off-equator) configuration just defined: the horizon ABCD, the meridian BED, the equator AFC, the vertical circle AEC, the Sun's parallel LM, and the circles drawn through the Sun's place G — the hectemorion AGI, the horary BGKD, the descensive EGH, and the great circle OGP. The left sphere is shown complete; the right sphere begins at the page's right edge and continues in the reprint on the following pages. Labels read on the left sphere: B and C (top), H (upper left), I, G, K, P, F (centre cluster), L, A (left), E (centre-bottom), M (right), D (bottom). Labels read on the (partial) right sphere: B (top), H, P, F, G, I, K, E, C, M, A, L, O, D.

Since, then, in the spherical triangle AGP the angle P is a right angle — by proposition 19 of book 4 of Regiomontanus's *On Triangles*, or by proposition 15 of book 1 of Geber [Jābir ibn Aflaḥ], or by proposition 43 of our own *Spherical Triangles* — it holds that, as the sine of the complement of the arc AP (that is, the sine of the arc FP, the Sun's distance from the meridian) is to the whole sine, so is the sine of the complement of the hectemorion circumference AG to the sine of the complement of the declination-arc GP.

And conversely, as the whole sine is to the sine of the Sun's distance from the meridian, so is the sine of the complement of the declination to the sine of the complement of the hectemorion circumference. Therefore, if it is contrived that, as the whole sine is to the sine of the Sun's distance from the meridian, so is the sine of the complement of the declination to another [term], the sine of the complement of the hectemorion circumference will be found; and so that complement, together with the hectemorion circumference, will not be unknown.

[Margin: Hectemoria (the hectemorion [circumference]).]

Diagram. The same pair of armillary spheres reprinted here beside the continuing text (horizon ABCD, meridian BED, equator AFC, vertical AEC, parallel LM, and the circles through G: hectemorion AGI, horary BGKD, descensive EGH, great circle OGP). Labels read on the left sphere: B/C (top), H (upper left), I, G, K, P, F (centre cluster), L, A (left), E (centre), M (right), D (bottom). Labels read on the right sphere: B (top), H, P, F, G, I, K, E, M, C (right), A, L, O, D.

Next, since in triangle OGI the angle I is a right angle — by proposition 19 of book 4 of Regiomontanus's *On Triangles*, or by proposition 15 of book 1 of Geber, or by proposition 43 of our own *Spherical Triangles* — it holds that, as the sine of the

complement of the arc OG (that is, the sine of the declination-arc GP) is to the sine of the complement of the arc GI (that is, to the sine of the hectemorion circumference AG), so is the sine of the complement of the arc OI (that is, the sine of the arc FI, between the equator and the hectemorion) to the whole sine. And conversely, as the sine of the hectemorion circumference is to the sine of the declination, so is the whole sine to the arc of the meridian between the equator and the hectemorion. If, then, it is contrived that, as the sine of the hectemorion circumference is to the sine of the declination, so is the whole sine to another [term], there will result the sine of the arc of the meridian FI, between the equator and the hectemorion. In northern parallels this arc, added to the arc BF (the altitude of the equator, that is, to the complement of the pole's altitude) — but in southern parallels subtracted from the same arc BF (the altitude of the equator, that is, from the complement of the pole's altitude) — will give the meridian circumference BI. And so, in order to find the meridian circumference by spherical triangles, the hectemorion must first be investigated.

[Margin: Meridiana (the meridian [circumference]).]

Again, since in triangle EGI the angle I is a right angle, it will hold — by proposition 19 of book 4 of Regiomontanus's *On Triangles*, or by proposition 15 of book 3 of Geber, or by proposition 43 of our own *Spherical Triangles* — that, as the sine of the complement of the arc GI (that is, the sine of the hectemorion circumference AG) is to the whole sine, so is the sine of the complement of the descensive [arc] EG to the sine of the complement of the arc EI (that is, to the sine of the meridian circumference BI). And conversely, as the whole sine is to the sine of the hectemorion circumference, so is the sine of the meridian circumference to the sine of the complement of the descensive circumference. Therefore, if it is contrived that, as the whole sine is to the sine of the hectemorion circumference, so is the sine of the meridian circumference to another [term], there will be produced the sine of the complement of the descensive circumference; and so this complement, together with the descensive circumference, will be known. But, in order that the descensive circumference be made known through spherical triangles, the hectemorion and the meridian must first be investigated. Hence the same descensive circumference will more easily be investigated by rectilinear triangles, through the Sun's altitude found by the last method, which we explained in proposition 36 of book 1, before [treating of] spherical triangles. For the complement of the Sun's altitude will give the descensive circumference.

[Margin: Descensiuua (the descensive [circumference]).]

Furthermore, since in triangle EGO — by proposition 17 of book 4 of Regiomontanus's *On Triangles*, or by proposition 13 of book 1 of Geber, or by proposition 41 of our own *Spherical Triangles* — it holds that, as the sine of the descensive circumference EG is to the sine of the angle O (that is, to the sine of the Sun's distance from the meridian), so is the sine of the arc OG, the complement of the declination, to the sine of the angle DEH (or BEH, since these two angles have the same sine), that is, to the sine of the arc BH, the complement of the horizontal circumference AH: then, if it is contrived that, as the sine of the descensive circumference is to the sine of the Sun's distance from the meridian, so is the sine of the complement of the declination to another [term], the sine of the complement of the horizontal circumference will be found; and so this complement, together with the horizontal circumference, will be known. The descensive circumference, then, must be investigated first, before the horizontal is sought by spherical triangles.

[Margin: Horizontalis (the horizontal [circumference]).]

Besides this, since in triangle AGK the angle K is a right angle — by proposition 16 of book 4 of Regiomontanus, or by proposition 13 of book 1 of Geber, or by proposition 41 of our own *Spherical Triangles* — it holds that, as the sine of the hectemorion circumference AG is to the whole sine of the right angle K, so is the sine of the arc GK (that is, the sine of the complement of the horary circumference BG) to the sine of the angle A, that is, to the sine of the arc EI, the complement of the meridian circumference BI. And conversely, as the whole sine is to the sine of the hectemorion circumference, so is the sine of the complement of the meridian circumference to the sine of the complement of the horary circumference. Hence, if it is contrived that, as the whole sine is to the sine of the hectemorion circumference, so is the sine of the complement of the meridian circumference to another [term], the sine of the complement of the horary circumference will be found; and so this complement, together with the hectemorion circumference, will become known.

We shall also investigate it in this way. Since in triangle BGH the angle H is a right angle — by proposition 19 of book 4 of Regiomontanus's *On Triangles*, or by proposition 15 of book 1 of Geber, or by proposition 43 of our own *Spherical Triangles* — it holds that, as the sine of the complement of the arc BH (that is, the sine of the horizontal circumference AH) is to the whole sine, so is the sine of the complement of the horary circumference BG to the sine of the complement of the arc GH (that is, to the sine of the descensive circumference EG). And conversely, as the whole sine is to the sine of the horizontal circumference, so is the sine of

the descensive circumference to the sine of the complement of the horary circumference. Hence, if it is contrived that, as the whole sine is to the sine of the horizontal circumference, so is the sine of the descensive circumference to another [term], the sine of the complement of the horary circumference will be found; and so this complement, together with the horary circumference, will be known. And so, in order that the horary circumference be elicited from spherical triangles, one must first investigate the hectemorion and meridian circumferences, or the horizontal and descensive. Hence we shall more easily track down the same horary circumference by rectilinear triangles, as we set out above — especially through the Sun's altitude above the vertical circle, found by proposition 1 of book 5.

[Margin: Horaria (the horary [circumference]).]

Finally, since in triangle BGI the angle I is a right angle — by proposition 16 of book 4 of Regiomontanus's *On Triangles*, or by proposition 13 of book 1 of Geber, or by proposition 41 of our own *Spherical Triangles* — it holds that, as the sine of the horary circumference BG is to the whole sine of the right angle I, so is the sine of the arc GI (the complement of the hectemorion circumference AG) to the sine of the angle B, that is, to the sine of the vertical circumference EK. Hence, if it is contrived that, as the sine of the horary circumference is to the whole sine,

so is the sine of the complement of the hectemorion circumference to another [term], there will result the sine of the vertical circumference; and so the vertical circumference itself will be known.

We shall obtain it by this device as well. Since in triangle GEK the angle K is a right angle — by proposition 19 of book 4 of Regiomontanus's *On Triangles*, or by proposition 43 of our own *Spherical Triangles* — it holds that, as the sine of the complement of the arc GK (that is, the sine of the horary circumference BG) is to the whole sine, so is the sine of the complement of the descensive circumference EG to the sine of the complement of the vertical circumference EK. Therefore, if it is contrived that, as the sine of the horary circumference is to the sine of the complement of the descensive circumference, so is [that same] to another [term], there will be produced the sine of the complement of the vertical circumference; and so this complement, together with the vertical circumference, will be known. And so, in order that the vertical circumference be found by spherical triangles, one must first seek — besides the horary circumference — the hectemorion or the descensive circumference.

[Margin: Verticalis (the vertical [circumference]).]

Now let the Sun be set again in a northern parallel, at the very point G where it cuts the vertical. On this supposition, the hectemorion will not differ from the vertical, since both circles are drawn through the Sun and through the pole of the meridian; and the point k, where previously the horary circle would cut the vertical, will be the same as G, and the point I, at which the hectemorion met the meridian, the same as E.

[Margin: The finding of the same circumferences when the Sun is on the vertical. (Inuentio earundem circumferentiarum, cum Sol in Verticali existit.)]

Diagram. A single armillary-sphere figure (upper right of the page) showing the case in which the Sun is on the vertical: horizon ABCD, meridian BED, equator AFC, vertical AEC, Sun's parallel LM, with the circles converging at the Sun's place G. Labels read: B (top), F, A, P (left cluster), G, L (left), I, E (centre), O (below E), C (right), M (right), D (bottom).

The hectemorion circumference AG, then, will be found as before, as is clear from triangle AGP, which is the same here as above, and so on.

[Margin: Hectemoria (the hectemorion [circumference]).]

The meridian circumference, on the other hand, is then a quadrant of the circle, BI or DI.

[Margin: Meridiana (the meridian [circumference]).]

The descensive circumference is EG, the complement of the hectemorion circumference. And so, to find the descensive circumference, the hectemorion must first be found, as is evident.

[Margin: Descensiuua (the descensive [circumference]).]

The horizontal circumference is then nothing, since the vertical [circle] drawn through the Sun does not differ from the primary vertical.

[Margin: Horizontalis (the horizontal [circumference]).]

The horary circumference is then likewise a quadrant of the circle, BG or DG.

[Margin: Horaria (the horary [circumference]).]

The vertical circumference, finally, is then the same as the descensive, that is, the complement of the hectemorion.

[Margin: Verticalis (the vertical [circumference]).]

But let the Sun now be placed on the near side of the vertical, toward the north, at the point G, provided it is less or more than six hours distant from the meridian. On this supposition, the hectemorion will fall in the northern quadrant of the meridian, DE.

[Margin: The finding of the same circumferences when the Sun is on the near side of the vertical, provided it is less or more than six hours distant from the meridian. (Inuentio earundem circumferentiarum, Sole existente citra Verticalem, dummodo minus, aut plus absit à meridie quam sex horis.)] [Margin: Hectemoria (the hectemorion [circumference]).]

Diagram. A pair of armillary-sphere figures at the foot of the page — the "earlier figure" and the "later figure" of the following argument (the case where the Sun is on the near side of the vertical, less or more than six hours from the meridian). Each shows the horizon ABCD, meridian BED, equator AFC, vertical AEC, parallel LM, and the circles through G, including the declination circle OGP. Labels read on the left sphere (partial): B, E, I, O, C, M, D. Labels read on the right sphere: B (top), F, A, K, P, H (left cluster), L, G, E, O, I (centre), C (right), M (right), D (bottom).

The hectemorion circumference AG, then, will be found as before, when the Sun was placed beyond the vertical toward the south; as is manifest from triangle AGP, which is the same here as above, and so on.

[Margin: Hectemoria (the hectemorion [circumference]).]

But since in triangle OGI the angle I is a right angle — by proposition 19 of book 4 of Regiomontanus's *On Triangles*, or by proposition 15 of book 1 of Geber, or by proposition 43 of our own *Spherical Triangles* — it holds that, as the sine of the complement of the arc OG (that is, the sine of the declination-arc GP) is to the sine of the complement of the arc IG (that is, to the sine of the hectemorion circumference AG), so is the sine of the complement of the arc OI to the whole sine. And conversely, as the sine of the hectemorion circumference is to the sine of the declination, so is the whole sine to the sine of the complement of the arc OI. Therefore, if it is contrived that, as the sine of the hectemorion circumference is to the sine of the declination, so is the whole sine to another [term], the sine of the complement of the arc OI will be found; and so this complement, together with the arc OI, will be known. From this arc OI, which lies between the pole and the hectemorion, we shall thus determine the meridian circumference DI.

When the Sun's distance from the meridian is less than six hours, as in the earlier figure, the arc OI, once found, must be added to the pole's altitude OD, so that the meridian circumference DI may be obtained. For, since the Sun's distance from

the meridian is less than six hours, the declination circle OGP, which measures the Sun's distance from the meridian, will fall in the quadrant of the equator AF, above the horizon, since the point A is six hours distant from the meridian; and accordingly the hectemorion, cutting the said circle at G, will fall in the portion of the meridian OE, between the pole and the vertical. But when the Sun's distance from the meridian is greater than six hours, as in the later figure, the arc OI, once found, must be subtracted from the pole's altitude OD, so that the meridian circumference DI may be obtained. For, since the Sun's distance from the meridian is greater than six hours, the declination circle OGP, which measures the Sun's distance from the meridian, will cut the equator below the horizon, beyond the point A; and accordingly the hectemorion, cutting the said circle at G above the horizon, will fall in the arc of the meridian OD, between the pole and the horizon.

[Margin: Meridiana (the meridian [circumference]).]

Finally, the descensive circumference EG will be elicited from triangle EGI; the horizontal AH from triangle EGO; the horary DG from triangle AGK or DGH; and the vertical EK from triangle DGI or GEK — as was demonstrated before, when the Sun was beyond the vertical circle. Only, in the demonstrations, in place of the letter B (for the southern side) the letter D must be taken, for the northern side.

[Margin: Descensiuua (the descensive [circumference]).] [Margin: Horizontalis (the horizontal [circumference]).] [Margin: Horaria (the horary [circumference]).]
[Margin: Verticalis (the vertical [circumference]).]

Gnomonics, Book VI (continued)

[Book VI, Cap. VII, continued — printed p. 466]

[Margin: The finding of the same circumferences, when the Sun's distance from the meridian comprises six hours.]

Finally, let the Sun be placed at the extreme point, in the circle of the sixth hour from noon or from midnight, at point G, so that its distance from the meridian comprises six hours. On this supposition, the hectemorion AGI — drawn from A through G — will not differ from the circle of the sixth hour from noon or midnight, OGA; and this circle makes right angles with the meridian at the world-pole O, by proposition 15 of book 1 of Theodosius, since it is drawn through its pole A.

[Margin: Hectemorion.]

The hectemorial circumference AG, therefore, is the same as the declination AG of the proposed parallel LM.

[Margin: Meridian.]

And the meridian circumference DI is the same as the pole's altitude DO.

Diagram. A drawing of the celestial sphere projected onto the plane of the meridian (the outer circle), with the great circles of the meridian, equator, horizon, hour-circle and hectemorion, and the shadow/position triangles used in the derivation. Readable labels:

- B (top of the circle): pole
- F (on the vertical, below B): point on the equator/vertical line
- A (left end of the transverse line): six-hours point on the equator
- C (right end of the transverse line)
- K (interior, just to the right of A)
- H (just below A, at the left)
- G (small, near H, left of center): the Sun's position on the parallel
- E (center of the sphere)
- I (on the vertical, just below E)
- o (small, below I)
- D (bottom of the circle): opposite pole / horizon point

[Margin: Descensive.]

But since in triangle EGO the angle O is a right angle, it will hold — by proposition 19 of book 4 of Regiomontanus's *On Triangles*, or by proposition 15 of book 1 of Geber, or by proposition 43 of our *Spherical Triangles* — that, as the sine of the complement of arc EO (that is, the sine of the pole's altitude OD) is to the whole sine, so is the sine of the complement of the descensive circumference EG to the sine of the complement of arc OG (that is, to the sine of the declination GA). And conversely: as the whole sine is to the sine of the pole's altitude, so is the sine of the declination to the sine of the complement of the descensive circumference. Therefore, if it is contrived that, as the whole sine is to the sine of the pole's altitude, so is the sine of the declination to a fourth term, the sine of the complement of the descensive circumference will result; and thereby that complement, together with the descensive circumference, will be known.

This will also become clear in the following way. Since in triangle AGH the angle H is a right angle, it will hold — by proposition 16 of book 4 of Regiomontanus, or by proposition 13 of book 1 of Geber, or by proposition 41 of our *Spherical Triangles* — that, as the sine of the declination-arc AG is to the whole sine of the right angle H, so is the sine of arc GH (the complement of the descensive circumference EG) to the sine of angle A (that is, to the sine of arc DO, the pole's altitude). And conversely: as the whole sine is to the sine of the declination, so is the sine of the pole's altitude to the sine of the complement of the descensive circumference; and by alternation: as the whole sine is to the sine of the pole's altitude, so is the sine of the declination to the sine of the complement of the descensive circumference — just as before, and so on.

[Margin: Horizontal.]

Next, since in the same triangle EGO the angle O is a right angle, it will hold — by proposition 16 of book 4 of Regiomontanus, or by proposition 13 of book 1 of Geber, or by proposition 41 of our *Spherical Triangles* — that, as the sine of the descensive circumference EG is to the whole sine of the right angle O, so is the sine of arc OG (the complement of the declination) to the sine of angle E (that is, to the sine of arc DH, the complement of the horizontal circumference AH). Accordingly, if it is contrived that, as the sine of the descensive circumference is to the whole sine, so is the sine of the complement of the declination to a fourth term, the sine of the complement of the horizontal circumference will result; and thereby that complement, together with the horizontal circumference, will be known.

This can also be shown from here. Since in triangle AGH the angle H is a right angle, it will hold — by proposition 19 of book 4 of Regiomontanus's *On Triangles*, or by proposition 15 of book 1 of Geber, or by proposition 43 of our *Spherical Triangles* — that, as the sine of the complement of arc AG (the declination) is to the sine of the complement of arc GH (that is, to the sine of the descensive circumference EG), so is the sine of the complement of the horizontal circumference AH to the whole sine. And conversely: as the sine of the descensive circumference is to the sine of the complement of the declination, so is the whole sine to the sine of the complement of the horizontal circumference; and by alternation: as the sine of the descensive circumference is to the whole sine, so is the sine of the complement of the declination to the sine of the complement of the horizontal circumference — just as before.

We shall also find the same horizontal circumference in this way. Since in triangle AGE — by proposition 17 of book 4 of Regiomontanus's *On Triangles*, or

by proposition 13 of book 1 of Geber, or by proposition 41 of our *Spherical Triangles* — it holds that, as the sine of the descensive circumference EG is to the sine of angle EAG (that is, of arc OE, the complement of the pole's altitude), so is the sine of arc AG (the declination) to the sine of angle AEG (that is, of the horizontal circumference AH). If it is contrived that, as the sine of the descensive circumference is to the sine of the complement of the pole's altitude, so is the sine of the declination to a fourth term, the sine of the horizontal circumference will be found, and thereby the circumference itself will become known, as before. But the earlier method is more expeditious, since it employs the whole sine, as is evident.

[Margin: Horary.]

Moreover, since in triangle DGO the angle O is a right angle, it will hold — by proposition 19 of book 4 of Regiomontanus's *On Triangles*, or by proposition 15 of book 1 of Geber, or by proposition 43 of our *Spherical Triangles* — that, as the sine of the complement of arc DO (the pole's altitude) is to the whole sine, so is the sine of the complement of the horary circumference DG to the sine of the complement of arc OG (that is, to the sine of the declination-arc AG). And conversely: as the whole sine is to the sine of the complement of the pole's altitude, so is the sine of the declination to the sine of the complement of the horary circumference. For this reason, if it is contrived that, as the whole sine is to the sine of the complement of the pole's altitude, so is the sine of the declination to a fourth term, the sine of the complement of the horary circumference will be produced; and thus this complement will be known, together with the horary circumference. This too will be evident in the following way.

Since in triangle AGK the angle K is a right angle, it will hold — by proposition 16 of book 4 of Regiomontanus's *On Triangles*, or by proposition 13 of book 1 of Geber, or by proposition 41 of our *Spherical Triangles* — that, as the sine of the declination-arc AG is to the whole sine of the right angle K, so is the sine of arc GK (the complement of the horary circumference DG) to the sine of angle A, or of arc OE (the complement of the pole's altitude). And conversely: as the whole sine is to the sine of the declination, so is the sine of the complement of the pole's altitude to the sine of the complement of the horary circumference; and by alternation: as the whole sine is to the sine of the complement of the pole's altitude, so is the sine of the declination to the sine of the complement of the horary circumference — just as before, and so on.

[Margin: Vertical.]

Finally, since in triangle DGO the angle O is a right angle, it will hold — by proposition 16 of book 4 of Regiomontanus's *On Triangles*, or by proposition 13 of book 1 of Geber, or by proposition 41 of our *Spherical Triangles* — that, as the sine of the horary circumference DG is to the whole sine of the right angle O, so is the sine of arc OG (the complement of the declination) to the sine of angle GDO (that is, of the Vertical circumference EK). Therefore, if it is contrived that, as the sine of the horary circumference is to the whole sine, so is the sine of the complement of the declination to a fourth term, the sine of the Vertical circumference will be found; and so the circumference itself cannot lie hidden. This too we shall make clear in the following way.

Since in triangle AGK the angle K is a right angle, it will hold — by proposition 19 of book 4 of Regiomontanus's *On Triangles*, or by proposition 15 of book 1 of Geber, or by proposition 43 of our *Spherical Triangles* — that, as the sine of the complement of the declination-arc AG is to the sine of the complement of arc GK (that is, to the sine of the horary circumference DG), so is the sine of the complement of arc AK (that is, the sine of the Vertical circumference EK) to the whole sine.

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And conversely: as the sine of the horary circumference is to the sine of the complement of the declination, so is the whole sine to the sine of the Vertical circumference EK; and by alternation: as the sine of the horary circumference is to the whole sine, so is the sine of the complement of the declination to the sine of the Vertical circumference — just as before, and so on.

[Margin: The finding of the same six circumferences by means of spherical triangles, when the Sun occupies the equator.]

Now let us investigate these same six circumferences by means of spherical triangles, for the case when the Sun runs across the equator and stands at point G. On this supposition, the hectemorion AGI — drawn from A through G — will not differ from the equator AFC; and AG will be the hectemorial circumference, BI the meridian, EG the descensive, AH the horizontal, BG the horary, and EK the Vertical. We shall find them all, even without drawing a great circle from the world-pole O through G.

[Margin: Hectemorion.]

For the hectemorial circumference AG does not differ from the complement of the Sun's distance from the meridian, AG.

[Margin: Meridian.]

The meridian circumference BI is likewise the same as the circumference BF, the altitude of the equator, that is, the complement of the pole's altitude.

Diagram. The companion sphere-in-the-meridian drawing for the case in which the Sun is on the equator (a re-cut of the figure on the facing page, with the same great circles and derivation triangles). Readable labels:

- B (top of the circle): pole
- H (upper left, on the circle)
- G (small, left of center): the Sun's position
- I (center)
- F (center, just below I)
- A (left end of the transverse line)
- C (right end of the transverse line)
- K (small, lower left)
- E (center of the sphere)
- o (small, below E)
- D (bottom of the circle)

But since in triangle EGF the angle F is a right angle, it will hold — by proposition 19 of book 4 of Regiomontanus's *On Triangles*, or by proposition 15 of book 1 of Geber, or by proposition 43 of our *Spherical Triangles* — that, as the sine of the complement of the Sun's distance from the meridian FG is to the whole sine, so is the sine of the complement of the descensive circumference EG to the sine of the complement of arc EF, the pole's altitude. And conversely: as the whole sine is to the sine of the complement of the Sun's distance from the meridian, so is the sine of the complement of the pole's altitude to the sine of the complement of the descensive circumference. Therefore, if it is contrived that, as the whole sine is to the sine of the complement of the Sun's distance from the meridian, so is the sine of the complement of the pole's altitude to a fourth term, the sine of the complement of the descensive circumference will be produced; and thereby this complement, together with the descensive circumference, will be found from the table of sines.

Next, in the same triangle EGF, since the angle F is a right angle — because, by proposition 16 of book 4 of Regiomontanus's *On Triangles*, or by proposition 13 of book 1 of Geber, or by proposition 41 of our *Spherical Triangles*, it holds that, as the sine of the descensive circumference EG is to the whole sine of the right angle

F, so is the sine of arc FG (the Sun's distance from the meridian) to the sine of angle E, that is, of arc BH (the complement of the horizontal circumference AH). If it is contrived that, as the sine of the descensive circumference is to the whole sine, so is the sine of the Sun's distance from the meridian to a fourth term, the sine of the complement of the horizontal circumference will be found; and thereby this complement, together with the horizontal circumference, will become known.

[Margin: Horizontal.]

This will also be clear by the following reasoning. Since in triangle AGH the angle H is a right angle, it will hold — by proposition 19 of book 4 of Regiomontanus's *On Triangles*, or by proposition 15 of book 1 of Geber, or by proposition 43 of our *Spherical Triangles* — that, as the sine of the complement of arc GH (that is, the sine of the descensive circumference EG) is to the whole sine, so is the sine of the complement of arc AG (that is, the sine of arc FG, the Sun's distance from the meridian) to the sine of the complement of the horizontal circumference AH, just as before, and so on. And yet we shall also obtain the same horizontal circumference in this way. Since in triangle AGE — by proposition 17 of book 4 of Regiomontanus's *On Triangles*, or by proposition 13 of book 1 of Geber, or by proposition 41 of our *Spherical Triangles* — it holds that, as the sine of the descensive circumference EG is to the sine of angle A (or of arc EF, the pole's altitude), so is the sine of arc AG (the complement of the Sun's distance from the meridian) to the sine of angle E (or of the horizontal circumference AH). If it is contrived that, as the sine of the descensive circumference is to the sine of the pole's altitude, so is the sine of the complement of the Sun's distance from the meridian to a fourth term, the sine of the horizontal circumference will be found; and so the horizontal circumference will not be unknown. But the earlier method seems more convenient, since it uses the whole sine, as is evident.

[Margin: Horary.]

Again, since in triangle BFG the angle F is a right angle, it will hold — by proposition 19 of book 4 of Regiomontanus's *On Triangles*, or by proposition 15 of book 1 of Geber, or by proposition 43 of our *Spherical Triangles* — that, as the sine of the complement of arc FG (the Sun's distance from the meridian) is to the whole sine, so is the sine of the complement of the horary circumference BG to the sine of the complement of arc BF (that is, to the sine of arc EF, the pole's altitude). And conversely: as the whole sine is to the sine of the complement of the Sun's distance from the meridian, so is the sine of the pole's altitude to the sine of the complement of the horary circumference. Therefore, if it is contrived

that, as the whole sine is to the sine of the complement of the Sun's distance from the meridian, so is the sine of the pole's altitude to a fourth term, the sine of the complement of the horary circumference will be found; and thus this complement, together with the horary circumference, will be known.

[Margin: Vertical.]

Lastly, since in triangle AGK the angle K is a right angle, it will hold — by proposition 19 of book 4 of Regiomontanus's *On Triangles*, or by proposition 15 of book 1 of Geber, or by proposition 43 of our *Spherical Triangles* — that, as the sine of the complement of arc AG (that is, the sine of arc FG, the Sun's distance from the meridian) is to the sine of the complement of arc GK (that is, to the sine of the horary circumference BG), so is the sine of the complement of arc AK (that is, the sine of the Vertical circumference EK) to the whole sine. And conversely: as the sine of the horary circumference is to the sine of the Sun's distance from the meridian, so is the whole sine to the sine of the Vertical circumference. For this reason, if it is contrived that, as the sine of the horary circumference is to the sine of the Sun's distance from the meridian, so is the whole sine to a fourth term, the sine of the Vertical circumference will be found; and thus the Vertical circumference will be known.

THE FINDING OF THE AFOREMENTIONED SIX CIRCUMFERENCES IN THE RIGHT SPHERE, both geometrically from the analemma and numerically from the doctrine of sines, whether the Sun stands on the equator or on any other parallel whatsoever.

CAPVT VIII.

[Margin: The foregoing precepts apply also to the oblique sphere, since it has the antarctic pole [?] above the horizon.]

Although all the precepts that we have so far handed down for investigating the said six circumferences are to be understood of the oblique sphere — the one in which the arctic pole is raised above the horizon, since Ptolemy speaks only of this case in his *Analemma* — they nevertheless hold good also in that obliquity of the sphere where the antarctic pole is raised

above the horizon, provided that what has been said of the northern parallels and the arctic pole is applied to the southern parallels and the antarctic pole, and vice versa. Indeed, by these same precepts we shall trace out the said six circumferences in the right sphere as well — and much more easily, in fact, than in the oblique.

To make this clearer: Let ABCD be the meridian, with center E. Its common section with the right horizon is BD, which will also represent the axis of the world. Its common section with the vertical, or with the equator (for in the right sphere the equator and the vertical do not differ), is the straight line AC, cutting BD at right angles. Finally, its common section with a parallel — whether northern or southern — is ab, about which the semicircle aeb is described. Now if the semicircle ABC is imagined to be rotated about AC until it stands perpendicular to the meridian, it will represent the eastern or western semicircle of the equator, so that EB is the common section of the equator and the right horizon, AB the portion of the equator above the earth, and BC the portion below the earth — as we showed above for the oblique sphere.

The division of the equator into hours must begin from point A or B, so that at A is set the 12th hour from midnight, and at B the 6th hour from noon or midnight; likewise at A the 6th hour from sunrise and the 18th from sunset, and the 6th unequal hour; while at point B the 12th and 24th hour from sunrise, and again the 24th and 12th from sunset, and the 12th unequal hour. In the same way, de will be perpendicular to ab — that is, dB produced is the common section of the right horizon and the parallel aeb — so that ae is the portion above the earth and eb the portion below; and the division of the parallel into hours must begin from point a or e, just as on the equator from point A. For when the Sun is on the parallel aeb, noon occurs at a, just as, when it is on the equator, noon occurs at A.

[Margin: The finding of the aforesaid circumferences in the right sphere from the analemma, when the Sun stands on the equator.]

Let the Sun, then, first be placed on the equator, as at point P, from which let a perpendicular PO be raised to the diameter of the equator AC. This will correspond to the perpendicular OP which, in the figure of chapter 2, was drawn to the diameter of the vertical AC through point L on the diameter of the equator — the point onto which the perpendicular KL fell from the Sun's center. And the straight line EA will correspond to the perpendicular MN which, in the same figure, was drawn through the same point L to the diameter of the horizon.

Diagram. The analemma construction for the right sphere: a large circle (the meridian ABCD) with center E, its vertical diameter AC (equator/vertical) and horizontal diameter DB (horizon/axis), a chord ab for the parallel with its semicircle aeb, the Sun-points P (on the equator) and k (on the parallel), the perpendiculars PO, KL, MN, Eg, Lf, the hectemorion diameter YEZ, and the several right-line triangles used in the demonstration. The figure carries a dense set of small interior and rim labels. Confidently readable: A (top), C or G (bottom), D (left), E (center), F (on the vertical below A), plus the construction points K, L, P, O, M, N, B, R, Y and a Sun-point marked with a script *a* near the upper rim. Several further small labels are present (in the clusters near the upper-right rim and along the lower-right — including what appear to be Q, S, T, X, V, f, d, g, δ and a right-hand point e) but do not resolve individually at this resolution.

[: numerous small analemma labels (more than two) could not be individually confirmed.]

And accordingly, if from OP there is cut off a segment equal to OP itself — indeed, if the very same OP is taken — and from EA a segment EF equal to that same OP, then the straight lines drawn from the center E through the points P and F will, at points R and Q, correspond in the said figure of chapter 2 to points P and A. Therefore, from what was demonstrated in chapter 3, BP will be the hectemorial circumference; BA the horary; AP the descensive; BA the meridian [circumference] between the horizon and the hectemorion — which then does not differ from the equator; AP the Vertical; and the horizontal will be nothing, since the vertical drawn through the Sun then coincides with the vertical properly so called, that is, with the equator. Therefore, by the single perpendicular PO — which is drawn from the Sun's position to the diameter of the equator AC — there are found the hectemorial circumference BP, the descensive AP, and the Vertical AP. But the quadrant BA yields both the horary and the meridian; and finally the horizontal is nothing, since the Sun stands on the equator in the right sphere.

[Margin: The finding of the same circumferences in the right sphere, by sines through rectilinear triangles, when the Sun is on the equator.]

Next, let the Sun be placed at point k on the parallel aeb, and from k let the perpendicular kL be drawn to ab, the diameter of the parallel. Then the semidiameter da will be the same as the perpendicular NM, which in the figures of chapter 4 was directed to be drawn through L to the diameter of the horizon; while the straight line KL, produced to O, will give the perpendicular OP, which there was prescribed to be drawn through L to the diameter of the vertical.

After this — since NM and OP are greater than KL (for NM is the semidiameter of the parallel, and KL is less than the semidiameter; and then, the straight lines dK and EP being drawn, [a] since the square on the line dK is equal to the squares on the lines kL and Ld, and the square on the line EP to the squares on the lines PO and OE, and since the square on EP, the semidiameter of the greatest circle, is greater than the square on dK, the semidiameter of a circle that is not the greatest, the squares on the lines PO and OE will likewise be greater than the squares on the lines KL and Ld. Therefore, when the equal squares on the lines OE and Ld are taken away — [b] which are equal to the square on the line KL — the remaining square on the line OP will be greater than the remaining square on KL; and for that reason the line OP too will be greater than the line KL) — let the lines NQ and OR be cut off equal to KL itself, and let the lines EQS and ERT be drawn.

Lastly, having drawn through L the diameter of the hectemorion YEZ, let the perpendiculars Eg and Lf be raised to it at E and L; and Lf will be equal to KL itself. (For if the lines EK and Ef are joined, they are equal to each other, since each is drawn from the center of the sphere E to its surface — as is clear, if the parallel is set in its proper position, namely perpendicular to the meridian, so that in that position KL is, by definition 4 of book 11 of Euclid, perpendicular to the plane of the meridian, and therefore also, by definition 3 of the same book, perpendicular to the line EL. [c] Since both the square on the line Ek is equal to the squares on the lines KL and LE, and the square on the line Ef is equal to the squares on the lines fL and LE, and since the squares on the equal lines Ek and Ef are equal, the squares on the lines KL and LE will also be equal to the squares on the lines fL and LE. Therefore, when the common square on the line LE is removed, there will remain equal squares on the lines KL and fL; and on that account the lines themselves will also be equal.) And accordingly, if with the aid of the compass the interval Lf is taken equal to the line KL, then, Lf being drawn, it will be perpendicular to EY, and so on — as we also showed in chapter 4. These things being thus established, it will follow — from what we showed in chapter 5 — that gf is the hectemorial circumference; BM the horary; AP the descensive; BY the meridian; AT the Vertical; and AS the horizontal.

[Margin: 47 of book 1 (Euclid).] [Margin: 34 of book 1.] [Margin: 47 of book 1.]

Now if one wishes to investigate these same circumferences by means of sines from rectilinear triangles, the matter will be very easy, when the Sun is running across the equator.

The hectemorial circumference is equal to the complement of the Sun's distance from the meridian, or indeed to its distance from the horizon itself — such as is the circumference B P. [Margin: Hectemoria.]

The horary circumference is equal to the quadrant of the meridian, B A. [Margin: Horaria.]

Likewise the descensive circumference is equal to the Sun's distance from the meridian — of which sort is the arc A P. [Margin: Descensiuia.]

The meridian circumference too is equal to the quadrant of the meridian, B A. [Margin: Meridiana.]

The vertical circumference, in turn, is equal to the Sun's distance from the meridian, just as the descensive one is — such as is the arc A P. [Margin: Verticalis.]

The horizontal circumference, finally, is nothing at all. [Margin: Horizontalis.]

But with the Sun placed in any parallel whatever, we shall carry the matter through as follows. Since it holds that, as a d — insofar as it is the whole sine of the parallel a e b — is to K L — insofar as it is the sine of the Sun's distance from the meridian in the same parallel — so a d — insofar as it is a part of the whole sine in the greatest circle, that is, insofar as it is the sine of the complement of the declination of the given parallel — is to K L — insofar as it is a part of the same whole sine in the greatest circle, that is, to L f, which is equal to K L itself.

Therefore, if the proportion be set up — as the whole sine is to the sine of the Sun's distance from the meridian, so the sine of the complement of the declination is to a fourth term — the line L f will be found, namely the sine of the complement of the hectemorial circumference g f; and thus this complement, together with the hectemorial circumference g f, will be known. [Margin: The finding of these same circumferences in the right sphere, by sines, from rectilinear triangles, with the Sun placed in any parallel whatever.] [Margin: Hectemoria.]

The horary circumference, further, is equal to the complement of the declination of the parallel proposed — of which sort is the arc B M. [Margin: Horaria.]

Next, since it holds that, as a d — insofar as it is the whole sine of the parallel proposed — is to L N — insofar as it is the sine of the complement of the Sun's distance from the meridian — so a d — insofar as it is a part of the whole sine in the greatest circle, that is, insofar as it is the sine of the complement of the declination of the given parallel — is to L N — insofar as it is a part of the same

whole sine in the greatest circle, that is, insofar as it is the sine of the complement of the descensive circumference. Hence, if the proportion be set up — as the whole sine is to the sine of the complement of the Sun's distance from the meridian, so the sine of the complement of the declination is to a fourth term — there will be found the sine of the complement of the descensive circumference A P; and on that account this complement, together with the descensive circumference, will not be unknown. [Margin: Descensiuua.]

The meridian circumference B Y will be found just as in the oblique sphere, with the Sun placed in any parallel, exactly as we set out in the preceding chapter. [Margin: Meridiana.]

Now, since the two sides K L, L N of the triangle K L N are equal to the two sides R O, O E of the triangle R O E — for R O was taken equal to K L itself, [a] and L N is equal to O E on account of the parallelogram N O — and since they contain equal angles, namely right angles, [b] the bases N K, E R will also be equal. Therefore, since N K, the semidiameter of the parallel, is equal to the semidiameter M N of the same parallel, E R too will be equal to M N — that is, to the sine of the horary circumference B M, or to the sine of the complement of the declination. Hence we shall find the vertical circumference A T just as in the oblique sphere, with the Sun placed in any parallel, exactly as we taught in the preceding chapter — as is plain in the triangles E R O, E T X, provided that R O is first found (that is, K L, which is equal to it, namely the sine of the Sun's distance from the meridian, in the parts of the whole sine in the greatest circle), as we showed in the preceding chapter. [Margin: (a) Euclid I.34.] [Margin: (b) Euclid I.4.] [Margin: Verticalis.]

Lastly, with the parallel a e b set at right angles to the meridian, so that the right angle K L O is formed, let the line O K be conceived as drawn, subtending the said right angle K L O. And since the two sides K L, L O of the triangle k L O, in that position, are equal to the two sides Q N, N E of the triangle Q N E — for Q N was taken equal to K L itself, [c] while L O is equal to N E on account of the parallelogram N O — and since they contain equal angles, namely right angles, [d] the bases O K, E Q will also be equal. Therefore, since O K is equal to O P itself — because each is the semidiameter of a circle in the sphere equidistant from the horizon, drawn through the lines K L, O P — E Q too will be equal to that same O P, that is, to the sine of the descensive circumference A P. Hence we shall find the horizontal circumference A S just as in the oblique sphere, with the Sun placed in any parallel, exactly as we taught in the preceding chapter — as is evident in the triangles E Q N, E S V, provided that Q N is first found (that is, K L, which is equal to it, namely the sine of the Sun's distance from the meridian, in the parts of the

whole sine in the greatest circle), as we set out in the preceding chapter. [Margin: (c) Euclid I.34.] [Margin: (d) Euclid I.4.] [Margin: Horizontalis.]

But let us now examine these same circumferences in the right sphere by means of spherical triangles. Let the right horizon be A B C D, the meridian B E D, and the equator A E C, the same as the vertical circle; and let the parallel — whether southern or northern — be F G. Let the Sun first be placed on the equator, at the point H. This done, both the hectemorion drawn from A through H and the descensive circle drawn from E through H will be the same as the equator, or the vertical circle. Therefore, once the horary circle B H D has been drawn, the six aforesaid circumferences will be found, as we said a little before. For the hectemorial circumference A H is the complement of the Sun's distance from the meridian, or the Sun's distance from the horizon itself; the meridian circumference is the quadrant of the meridian B E, or D E; the descensive and vertical circumference E H is the Sun's distance from the meridian; the horary is the quadrant B H, or D H; and the horizontal, finally, is nothing. [Margin: The finding of these same circumferences in the right sphere from spherical triangles, with the Sun placed on the equator.]

Next let the Sun be placed in the parallel F G, at the point I, through which let three great circles be drawn, namely the hectemorion A I K, the descensive circle E I L, and the horary circle B I D. Then A I will be the hectemorial circumference; B K, or D K, the meridian; E I, the descensive; A L, the horizontal; B I, or D I, the horary; and E H, the vertical — all of which we shall track down by the following reasoning.

Since in the triangle B I K, or D I K, the angle K is a right angle, then — by proposition 16 of book 4 of Johann Regiomontanus [Regiomontanus] on triangles, or by proposition 13 of book 1 of Gebir [Jābir ibn Aflaḥ], or by proposition 41 of our own [treatise] on spherical triangles — it will hold that, as the sine of the arc B I, or D I, the complement of the declination of the given parallel F G (for since B, D are the poles of the world, B I D will be a circle of declination), is to the whole sine, which is the sine of the right angle K, so the sine of the arc I K, the complement of the hectemorial circumference A I, is to the sine of the angle B, or D — that is, to the sine of the arc E H, the Sun's distance from the meridian. And, inverting, as the whole sine is to the sine of the complement of the declination, so the sine of the Sun's distance from the meridian is to the sine of the complement of the hectemorial circumference. If, then, the proportion be set up — as the whole sine is to the sine of the complement of the declination, so the sine of the Sun's distance from the meridian is to a fourth term — the sine of the

complement of the hectemorial circumference will be produced; and hence this complement, together with the hectemorial circumference, will become known. [Margin: The finding of these same circumferences in the right sphere from spherical triangles, with the Sun set in any parallel off the equator.] [Margin: Hectemoria.]

We shall also make this plain in the following way. Since in the triangle A I H the angle H is a right angle, then — by proposition 19 of book 4 of Johann Regiomontanus, or by proposition 15 of book 1 of Gebir, or by proposition 43 of our own [treatise] on spherical triangles — it will hold that, as the sine of the complement of the arc H I, the declination, is to the whole sine, so the sine of the complement of the hectemorial circumference A I is to the sine of the complement of the arc A H, that is, to the sine

of the arc E H, the Sun's distance from the meridian. And, inverting, as the whole sine is to the sine of the complement of the declination, so the sine of the Sun's distance from the meridian is to the sine of the complement of the hectemorial circumference — as before.

Diagram. A circle representing the sphere in the right-sphere construction for the six circumferences, with the meridian drawn as the vertical diameter B D and the equator/vertical circle as the horizontal diameter A C. An italic label *Meridiana* stands to the left of the figure. Several arcs and chords cross near the center, forming the shadow-triangles discussed in the text (around points H, E, I, K). Readable labels:

- B (top of circle): pole / top of meridian
- D (bottom of circle): pole / foot of meridian
- A (left, on circumference): left end of horizontal diameter (equator/vertical)
- C (right, on circumference): right end of horizontal diameter
- G (upper right, on circumference): end of upper parallel arc
- G (lower right, on circumference): end of lower parallel arc [the two G-labels are distinguished only by position]
- F (upper left): left end of upper parallel arc
- F (lower left): left end of lower parallel arc
- L (upper left, inner): intersection point
- L (lower left, inner): intersection point
- K (upper, near B): intersection point on the horary/declination circle
- K (lower, near D): intersection point
- I (upper, center-left): Sun's position / intersection

- I (lower, center-left): intersection
- H (center-left): intersection near center
- E (center): center of the sphere

[Margin (to left of figure): Meridiana]

Next, since in the triangle B I k, or D I K, the angle K is a right angle, then — by proposition 19 of book 4 of Johann Regiomontanus on triangles, or by proposition 15 of book 1 of Gebir, or by proposition 43 of our own [treatise] on spherical triangles — it will hold that, as the sine of the complement of the arc I k — that is, as the sine of the hectemorial circumference A I — is to the whole sine, so the sine of the complement of the arc B I, or D I — that is, the sine of the arc of declination H I — is to the sine of the complement of the meridian circumference B K, or D K. Therefore, if the proportion be set up — as the sine of the hectemorial circumference is to the whole sine, so the sine of the declination is to a fourth term — the sine of the complement of the meridian circumference will be found; and hence this complement, together with the meridian circumference, will not lie hidden.

Again, since in the triangle E H I the angle H is a right angle, then — by proposition 19 of book 4 of Johann Regiomontanus on triangles, or by proposition 15 of book 1 of Gebir, or by proposition 43 of our own [treatise] on spherical triangles — it will hold that, as the sine of the complement of the arc of declination H I is to the whole sine, so the sine of the complement of the descensive circumference E I is to the sine of the complement of the arc E H, the Sun's distance from the meridian. And, inverting, as the whole sine is to the sine of the complement of the Sun's distance from the meridian, so the sine of the complement of the declination is to the sine of the complement of the descensive circumference. Hence, if the proportion be set up — as the whole sine is to the sine of the complement of the declination, so the sine of the complement of the Sun's distance from the meridian is to a fourth term — the sine of the complement of the descensive circumference will be produced; and thus this complement, together with the descensive circumference, will be rendered known. [Margin: Descensiu.]

Moreover, since in the triangle E H I the angle H is a right angle, it will hold — by proposition 16 of book 4 of Johann Regiomontanus on triangles, or by proposition 13 of book 1 of Gebir, or by proposition 41 of our own [treatise] on spherical triangles — that, as the sine of the descensive circumference E I is to the whole sine of the right angle H, so the sine [of the declination] is to the sine of the horizontal circumference A L. Therefore, if the proportion be set up — as the sine

of the descensive circumference is to the whole sine, so the sine of the declination is to a fourth term — the sine of the horizontal circumference will be produced, and the horizontal circumference will be known. [Margin: Horizontalis.] This will also be made evident by the following reasoning. Since in the triangle B I L, or D I L, the angle L is a right angle, then — by proposition 19 of book 4 of Johann Regiomontanus on triangles, or by proposition 15 of book 1 of Gebir, or by proposition 43 of our own [treatise] on spherical triangles — it will hold that, as the sine of the complement of the arc I L — that is, as the sine of the descensive circumference E I — is to the whole sine, so the sine of the complement of the arc B I, or D I — that is, the sine of the arc of declination H I — is to the sine of the complement of the arc B L, or D L, that is, to the sine of the horizontal circumference A L, as before. [Margin: Horaria.]

Finally, the horary circumference B I, or D I, is the complement of the declination of the given parallel; and the vertical E H is the Sun's distance from the meridian; and therefore neither of them will be able to be unknown. [Margin: Verticalis.]

THE CONSTRUCTION OF THE HORIZONTAL, VERTICAL, AND MERIDIAN SUNDIAL FROM THE SIX AFORESAID CIRCUMFERENCES.

CHAPTER IX.

The horizontal sundial is constructed from the horizontal and descensive circumferences found for each individual hour, for the given latitude of the place. For the horizontal circumferences indicate the breadths of the shadows, and the descensive ones the lengths of the same, just as we said of the horizontal sundial in the previous book. The horizontal circumferences here refer to those same arcs which there we called the breadths of the shadows; while the complements of the descensive circumferences measure the altitudes of the Sun above the horizon, as we showed in chapter 1 of this book. [Margin: How the horizontal sundial is constructed from the horizontal and descensive circumferences.] To make this plainer, we shall bring forward a single example. In the first figure of chapter 6 there was found the horizontal circumference C p for the twenty-third hour from sunset, in the tropic of ☊ [Cancer], which is northern and western.

Now, so that you may understand which hours are northern and which southern: all those hours whose perpendiculars, dropped to the diameter of each parallel in the analemma of chapter 6, fall upon the portion of the diameter between the diameter of the horizon and the diameter of the vertical circle, are northern, or

septentrional, with respect to the vertical circle; whereas the others, whose perpendiculars fall upon the portion of the parallel's diameter between the circumference of the meridian circle and the diameter of the vertical, are southern. [Margin: By what method the northern hours are distinguished from the southern in the analemma.] For if the meridian is set in its proper position, so that H looks toward the arctic pole and I toward the antarctic, the diameter of the vertical A C will separate the northern portion A D C from the southern A B C. Likewise, if the analemma is so inverted that I looks toward the arctic pole and H toward the antarctic, and the straight line a b is the diameter of the tropic of $\mathfrak{S}$, as was done in the division of the analemma into hours from sunrise and sunset in chapter 6, the same diameter of the vertical C A will separate the northern portion C B A from the southern C D A. The northern hours can, however, also be recognized from the southern by the doctrine of sines. For all those hours whose distances from the meridian are greater than the distance the Sun has when placed on the vertical circle — which is elicited from proposition 36 of book 1 — are northern, and the rest are southern. Or else in this way: those hours whose descensive circumferences are greater than the one obtained when the Sun is set on the vertical are northern, and the rest are southern.

And so the horizontal circumference which we take from the one found to belong to the horizontal circumference C p — [we lay off] in the horizontal sundial of the previous book, from the point D, westward toward the north C, an arc D e similar [to it] (which will easily be done if from E a circle is described equal to the meridian of the analemma in which the circumferences were found, and so on). For upon the straight line e E f, drawn as a hidden [construction] line, the shadow of the style will be cast. Again, for the same twenty-third hour from sunset, in the tropic of $\mathfrak{S}$, there was found, in the same

analemma of chapter 6, the descensive circumference C r, in such a way that the arc r B measures the altitude of the Sun above the horizon. If, then, in the second figure of proposition 5 of the previous book — the one that comes after the horizontal sundial — a circle equal to the meridian of the analemma is described from D, and in it, from the straight line A D, an arc is taken equal to the descensive circumference C r (or, from the straight line B C, an arc equal to the arc of the Sun's altitude r B), and from the end of the said arc a straight line is drawn through D, the center, this line will cut off from the straight line G I the length of the shadow. If we then transfer this, with the help of the compass, onto the straight line E f in the sundial, as far as f, we shall have f as the point of the twenty-third hour from sunset in the tropic of $\mathfrak{S}$. We shall find the same point f if, from chapter 7, the descensive circumference for the said twenty-third hour has

been found by sines. For from its complement, which measures the altitude of the Sun above the horizon, we shall find the length of the shadow; and if we take this from the straight line H I, placed alongside the horizontal sundial, and transfer it onto the straight line E f in the sundial, the point f will be found, as before. In the same way judgment must be made about all the other hours, so that their points may be found, and so on. And the whole horizontal sundial will be completed, as we wrote in the preceding book.

The vertical sundial is composed in the same manner from the vertical and horary circumferences found for the individual hours, for the given latitude of the place. For the vertical circumferences give the breadths of the shadows, and the horary ones — whose complements show the altitudes of the Sun above the vertical circle — give the lengths of the same, just as we said of the vertical sundial in the previous book. This alone is the difference: that the vertical circumferences are the complements of those arcs which, in the previous book, we called the breadths of the shadows in the vertical sundial, since in the vertical circle they take their beginning from the common section of the meridian and the vertical, namely from the zenith, and not from the common section of the horizon and the vertical, as did the breadths of the shadows in the previous book.

[Margin: By what method the vertical sundial is composed from the vertical and horary circumferences.] Hence, in the sundial, they are not to be computed from the straight line B D, the common section of the plane of the sundial and the horizon (which is the vertical of the vertical itself), as were the breadths of the shadows of the plane of the sundial and of the meridian. This must, however, be observed as a rule: that the vertical circumferences of those hours which lie above the horizon are to be taken, in the circle described from E as the place of the style (which is equal to the meridian of the analemma), from the point A, toward D or B, according as they are eastern or western; whereas the vertical circumferences of the hours lying below the horizon — such as are the 5th, 6th, and 7th from noon and midnight in the tropic of ♊ [Capricorn], and likewise the 13th, 14th, and 15th from sunset in the same tropic, and so on — are to be taken in the same circle from the point C. Let everything else be done as we said of the vertical sundial facing south in the previous book. But the northern vertical sundial will be made from the southern one, as has been said more than once in the previous books. It can, however, be described in the same way from the vertical and horary circumferences proper to it, found on the northern face of the vertical circle, provided one carefully considers which hours on the northern face of the vertical are eastern and which western, and so on.

Finally, the meridian sundial is described by the same method from the meridian and hectemorial circumferences found for the individual hours, for the given latitude of the place. For the meridian circumferences indicate the breadths of the shadows, while the hectemorial ones — whose complements measure the altitudes of the Sun above the meridian — give the lengths of the same, just as we declared of the meridian sundial in the previous book. [Margin: How the meridian sundial is described from the meridian and hectemorial circumferences.] This alone is the difference: that the meridian circumferences — since in the analemma they take their beginning from the diameter of the horizon — are not, in the meridian sundial of the previous book, to be begun in the circle described from C as the place of the style, from the equinoctial line D F, as the breadths of the shadows were reckoned in the previous book, but rather from the horizontal line A B. This, however, with the following rule and condition: that the meridian circumferences of those hours which are northern (which indeed we shall recognize from the analemma, as was said above) are to be taken in the said circle, which is equal to the meridian of the analemma, from the northern point B in the eastern sundial, and from the point A in the western one, toward the upper parts; whereas the meridian circumferences of the southern hours [are to be taken] from the other extremity of the horizontal line, namely from the southern point. Likewise, that the forenoon hours be described in the eastern sundial, and the afternoon hours in the western. But if the points of those hours of the tropic of ♋ which lie below the horizon — such as are the 5th, 6th, 7th from noon and midnight, and the 13th, 14th, 15th from sunset — are also to be found, their meridian circumferences must be taken from the southern end of the horizontal line, toward the lower parts. Everything else is to be completed as was said of the meridian sundial in the previous book.

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For the rest, if anyone wishes to describe the vertical and meridian sundials just as the horizontal one — no differently than if the vertical and meridian circles were horizons — the horizontal and descensive circumferences in the vertical circle and in the meridian must be investigated, as in the horizon. For the horizontal circumferences will represent those arcs which in the previous book we named the breadths of the shadows in the vertical and meridian sundials, while the complements of the descensive ones [will represent] the altitudes of the Sun above the vertical and above the meridian. Hence, in the vertical sundial of the previous book, the horizontal circumferences will take their beginning from the horizontal line, which is the common section of the plane of the sundial and of the horizon (which is the vertical of the vertical itself), just as the breadths of

the shadows too were reckoned from that same horizontal line — no otherwise than was done in the horizontal sundial, in which the breadths of the shadows, or the horizontal circumferences, took their beginning from the straight line B D, the common section of the plane of the sundial and of the vertical circle of the horizon itself. By parallel reasoning, in the meridian sundial the horizontal circumferences are to be begun from the equinoctial line, which is the common section of the plane of the sundial and of the equator (which is the vertical of the meridian itself), just as the breadths of the shadows too were numbered from that same equinoctial line in the previous book. But it must be carefully observed that the said horizontal circumferences in the vertical sundial be taken now above the horizontal line, now below it, according as their hours lie above the horizon or below it, just as we explained for the vertical sundial in the previous book; likewise, that the horizontal circumferences in the meridian sundial be taken now above the equinoctial line — that is, toward the north — now below it — that is, toward the south — according as their hours are northern or southern with respect to the equator. Finally, that the horizontal circumferences of those hours which are less than 6 hours distant from the meridian — that is, less than 90 degrees — begin from the point D, the upper end of the equinoctial line, while the horizontal circumferences of those hours which have a greater distance from the meridian than 6 hours — that is, than 90 degrees — take their beginning from the point F, the lower end of the equinoctial line, as was declared for the meridian sundial in the previous book. Finally, that everything which was said in the vertical and meridian sundials about the breadths of the shadows be here understood of the horizontal circumferences. [Margin: How both the vertical and the meridian sundial may be described after the manner of the horizontal sundial.]

In this way, in the vertical circle taken as horizon, we shall inquire the horizontal circumferences and the descensive ones. In the analemma of chapter 6, let A C be understood as the diameter of the vertical circle standing for the diameter of the horizon, and accordingly B D as the diameter of the horizon standing for the diameter of the vertical circle with respect to the vertical. Now since above the southern face of the vertical circle the antarctic pole is raised, and I is posited to be the antarctic pole, all those hours whose perpendiculars fall within the portions of the parallel-diameters contained in the semicircle A B C will lie above the southern face of the vertical circle, and their descensive and horizontal circumferences will have to be investigated for the southern dial; whereas the other hours will lie above the northern face, and their descensive and horizontal circumferences will have to be sought for the northern dial.

[Margin: How the descensive and horizontal circumferences are found from the analemma, in the vertical circle taken as horizon.]

Therefore, if through the hour-points on the parallel-diameters perpendiculars are drawn to B E, the semidiameter of the vertical circle, taken with respect to the vertical, these will cut off from the meridian's circumference the descensive circumferences, which take their beginning from the point B — the crown of the head, that is, the pole of the vertical taken as horizon — and whose complements measure the altitudes of the Sun above the southern face of the vertical circle, as above a horizon, as is plain from what was demonstrated in chapter 5. But if through the same hour-points perpendiculars are drawn to A C, the diameter of the vertical circle taken as horizon, and if from these there are cut off straight lines equal to the intervals of the hours between the parallel-diameters and the circumferences of those same parallels — by marking points on the said perpendiculars, through which straight lines are drawn out from the center E — these will cut off from the meridian's circumference the horizontal circumferences, which likewise take their beginning from the point B. These, of course, in the vertical dial taken as horizontal, exhibit the breadths of the shadows, and are to be reckoned from the horizontal line of the dial upward or downward, according as the hour-points on the parallel-diameters lie above or below B D, the diameter of the horizon.

[Margin: How the descensive and horizontal circumferences are found in the vertical circle, taken as horizon, by the doctrine of sines.]

By the doctrine of sines too we shall most conveniently find the descensive circumferences in the vertical circle, as horizon, if by proposition 1 of book 5 we investigate the altitudes of the Sun above the vertical circle taken as horizon; for the complements of these altitudes will give the descensive circumferences. The horizontal circumferences, on the other hand, are found by proposition 3 of book 5, and are the breadths of the shadows — that is, the horizontal circumferences with respect to the vertical circle taken as horizon; for the arcs investigated there are the breadths of the shadows of the vertical circle taken as horizon. Or else such circumferences will have to be sought as in chapter 7, by taking the vertical for horizon and the horizon for vertical, and so on.

[Margin: How the descensive and horizontal circumferences are found in the meridian, taken as horizon, from the analemma.]

Now in the meridian, taken as horizon, we shall track down the descensive and horizontal circumferences in this way. In the same analemma of chapter 6, let the

circle A B C D be understood to be the circle of the sixth hour from noon and from midnight, standing as the meridian of the meridian itself taken as horizon. And since the meridian, as horizon, passes through the poles of the world H, I, the line H I will be the common section of the meridian and of the circle of the sixth hour from noon and from midnight, this circle being taken as the meridian with respect to the meridian; and accordingly F G, the diameter of the equator, will stand as the diameter of the vertical circle of the meridian itself taken as horizon. Therefore, if through the hour-points on the parallel-diameters perpendiculars are drawn to F E, the semidiameter of the equator (taken as the vertical with respect to the meridian, as horizon), these will cut off from the circumference of the circle A B C D the descensive circumferences, which take their beginning from the point F — the crown of the head, that is, the pole of the meridian as horizon — and whose complements measure the altitudes of the Sun above the meridian, as above a horizon, as is clear from what was demonstrated. Again, if from the semidiameters of the parallels (which are indeed perpendiculars drawn through those same hour-points on the parallel-semidiameters to H I, the diameter of the meridian taken as horizon) there are cut off straight lines equal to the intervals of the hours between the diameters and the circumferences of the parallels — by marking points on them, through which straight lines are drawn out from the center E — these will cut off from the circumference of the circle A B C D the horizontal circumferences, which take their beginning from the point F. These, of course, in the meridian dial taken as horizontal, reveal the breadths of the shadows, and are to be reckoned from the equinoctial line of the dial, just as we spoke of the breadths of the shadows in the meridian dial of the previous book.

[Margin: By what method the analemma is divided into hours, so that the descensive and horizontal circumferences in the meridian, taken as horizon, may be found.]

But since the circle A B C D of the analemma does not stand for the meridian but for the circle of the sixth hour from noon and from midnight, the division of the parallels into hours will have to be set up as follows. From the parallel of ♊ [Capricorn] and of ♋ [Cancer], separately drawn out in chapter 6, let all the hours be transferred into the parallel of ♋ in the analemma — beginning from the point of noon, a — the beginning being made at the point where the straight line I H, produced, cuts the parallel, this being taken as the common section of the parallel and of the meridian as horizon, and being the point at which noon falls. This, however, on this condition: that the intervals of those hours which are more than 6 hours, or more than a quadrant, distant from the point a, be taken from the point b in the separately drawn parallel. For in this way the parallel of ♋ will be

divided into hours in the analemma. In the same manner the other parallels will be divided, if they are first drawn out separately, and so on. This is to be understood of the equal hours from rising or setting, and of the unequal hours. For as regards the hours from noon and midnight, the division will take its beginning from the points at which the circle A B C D is divided by the parallels, as above — yet in such a way that those points pertain not to the 12th hour of noon or of midnight, as before, but to the 6th hour from noon or from midnight; whereas the points where the straight line I H, produced, cuts the parallels pertain to the 12th hour of noon or of midnight, and not to the 6th hour from noon or from midnight as before.

[Margin: How the same circumferences are found in the meridian, taken as horizon, by the doctrine of sines.]

The descensive circumferences in the meridian, taken as horizon, will be found by sines, if from what we wrote in proposition 1 of the previous book the altitudes of the Sun above the meridian, taken as horizon, are investigated; for the complements of these altitudes will give the descensive circumferences. The horizontal circumferences in the same meridian, taken as horizon, will be found as was demonstrated in proposition 3 of the previous book, since the arcs investigated there are the breadths of the shadows — that is, the horizontal circumferences with respect to the meridian taken as horizon. Both the descensive and the horizontal circumferences can nevertheless be investigated as in chapter 7, if the meridian is taken for horizon and the equator for vertical.

[Margin: How the descensive and horizontal circumferences are found in the circle of the sixth hour from noon and from midnight, taken as horizon, from the analemma, for the construction of the polar dial.]

By the same reasoning, for the construction of the polar dial the descensive and horizontal circumferences will be found in the circle of the sixth hour from noon and from midnight, taken as horizon. For in the analemma drawn out in that same chapter 6, the straight line H I is the common section of the meridian and of the circle of the sixth hour from noon or from midnight taken as horizon; and therefore F G, the diameter of the equator, is the common section of that same meridian and of the proper vertical of the circle of the sixth hour from noon and from midnight taken as horizon. The division of the parallels into hours must nevertheless be set up as we taught in chapter 6, since the circle A B C D of the analemma is the meridian with respect to the circle of the sixth hour from noon and from midnight taken as horizon.

By sines the same descensive and horizontal circumferences in the circle of the sixth hour from noon and from midnight, taken as horizon, will be found, just as in the meridian, if the circle of the sixth hour from noon and from midnight is taken for horizon and the equator for vertical. The polar dial itself, moreover, will be described from the descensive and horizontal circumferences found in the circle of the sixth hour from noon and from midnight taken as horizon, as we prescribed in proposition 5 of the previous book. For the horizontal circumferences are the same which we there called the breadths of the shadows, while the complements of the descensive ones are the altitudes of the Sun, from which the lengths of the shadows are elicited, as we taught in proposition 2 of the same previous book.

But that we may construct the equinoctial dial, we do not need the horizontal and descensive circumferences found in the equator taken as horizon; for the equal hours divide the whole circles drawn from the place of the style, and the unequal hours divide the portions of those same circles — which answer to the diurnal and nocturnal arcs — into equal parts, as is clear from what we demonstrated in book 2, propositions 49, 50, 58, 59, and 60. Likewise the altitudes of the Sun above the equator are equal at every hour of any given parallel, since any parallel whatever is an almucantar of the Sun — that is, equidistant from the equator. Hence in the equinoctial dial all the hours are readily described, without the horizontal and descensive circumferences, by dividing the circles drawn from the place of the style, or their portions answering to the diurnal and nocturnal arcs, into equal parts, as we set forth in the aforesaid propositions. Yet anyone who prefers to describe those same hours by means of those circumferences will find that all the horizontal circumferences found by him — the Sun being in any given parallel — cut the meridian of the analemma into equal parts, and that there is one and the same descensive circumference for all the hours of the same parallel. This we shall demonstrate as follows.

[Margin: The equinoctial dial is more easily described without the horizontal and descensive circumferences, as was set down in book 2.]

[Margin: In the equator, taken as horizon, the horizontal circumferences cut the meridian of the analemma into equal parts, the Sun being in any given parallel; and at all the hours of the same parallel there is one and the same descensive circumference.]

In the analemma of chapter 6, let F G, the diameter of the equator, be understood to be the diameter of the vertical circle with respect to the equator taken as horizon — namely, the common section of the meridian and of the circle of the

sixth hour from noon and from midnight, which is the vertical of the equator itself taken as horizon. Thus if, for example, through the hour-points on the diameter of the parallel of $\mathfrak{S}$ — the perpendiculars dropped in the parallel of $\mathfrak{S}$ to that diameter — lines are drawn to H I, the diameter of the vertical circle, these will not differ from M N, the diameter of the parallel of $\mathfrak{S}$. Hence the descensive circumference will be H M, or H N. Again, if through d, the point of the 24th hour, a perpendicular $\theta \varepsilon$ is drawn to F G, the diameter of the equator taken as horizon, cutting the circumference of the meridian at ε (which perpendicular does not differ from the perpendicular e d drawn from the 24th hour of the tropic of $\mathfrak{S}$ to the diameter of that same tropic), and if the straight line $\theta \mu$ is cut off equal to the straight line d e, and through μ a straight line E μ is drawn out from the center E, cutting the circumference of the meridian at π , then, by what was demonstrated, I π will be the horizontal circumference for the 24th hour from setting, in the tropic of $\mathfrak{S}$. Now since $\theta \mu$ is equal to d e, if the common part d μ is taken away, the remainder θd will also be equal to the remainder μe . (Or else, when the point μ falls between the straight lines F G, a b — which happens in the hours near noon, where the intervals of the hours between the diameter of the parallel and its circumference are less than the straight line θd — then, if the straight line d μ is added, θd will become equal to μe .) [a] Now θd is equal to E m, because of the parallelogram θm . Therefore μe too will be equal to that same E m. And since μe and E m are also parallel, it follows that, if from m — the center of the parallel — a straight line m e is drawn to the 24th hour on the parallel, [b] the straight line E μ will also be parallel to m e. [c] Hence the angles $\pi E I$ and e m I will be equal. In no other way shall we prove that the straight lines drawn from m, the center of the parallel, to the other hours of the parallel are parallel to the straight lines which, drawn from the center E, terminate on the circumference of the meridian the horizontal circumferences of those same hours; and therefore that the angles at m are equal to the angles at E. [d] Wherefore, since all the angles at m are equal — because they stand on equal arcs at the center m — all the angles at E too will be equal; [e] and therefore the arcs of the meridian on which they stand will be equal to one another. That is, the horizontal circumferences will cut the circumference of the meridian into equal parts, which was what we undertook to demonstrate.

[Margin: How the horizontal circumferences in the equator are found by the doctrine of sines. (a) 34.primi; (b) 33.primi; (c) 29.primi; (d) 27.tertij; (e) 26.tertij; (f) 4.sextri — proposition-references (Euclid, *Elements* I.34, I.33, I.29, III.27, III.26, VI.4) keyed to the lettered steps of the demonstration.]

From these things it follows that, if from the center of the parallel you draw to the individual hours on the circumference of the parallel straight lines equidistant from them, you cut off from the meridian of the analemma the horizontal circumferences, which indeed in the equinoctial dial indicate the breadths of the shadows, and whose beginning is set on the axis $H I$: just as the straight line $E \pi$, parallel to $m e$, cuts off the horizontal circumference $I \pi$ for the 24th hour from setting, as we showed. Since this is so, we rightly prescribed in book 2 that the equinoctial dial should be described by dividing the circles into 24 equal parts — the beginning being made either from the meridian line or from the horizontal, according as the hours are to be drawn from noon and midnight or from rising and setting — or else by dividing the diurnal arcs into 12 equal parts, if unequal hours are to be described; so that there is no need to seek the horizontal circumferences, since the hour-spaces are equal to one another, as we demonstrated.

Now since $E \mu$ is equal to the semidiameter of the parallel $m e$ — that is, to the straight line $d e$, that is, to the sine of the Sun's distance from noon — [f] and since $E \mu$ is to $\mu \theta$ as $E \pi$ (the whole sine) is to the straight line which will be drawn from π equidistant from $\theta \varepsilon$ (that is, to the sine of the complement of the horizontal circumference $I \pi$): if it be arranged that as the sine of the declination is to the sine of the Sun's distance from noon (known in parts of the whole sine of the greatest circle — and this sine will be known in the said parts, as we showed above), so the whole sine is to a fourth quantity, then the sine of the complement of the horizontal circumference will be found; and consequently this complement, together with the horizontal circumference, will be known. For the descensive circumference of any hour whatever, the complement of the declination of the parallel is to be taken, as is evident.

[Margin: The description of the equinoctial dial from the horizontal and descensive circumferences found in the equator, taken as horizon.]

Once the horizontal circumferences are found, we shall reckon them off in the circle drawn from the place of the style, from the horary line of the sixth hour from noon and from midnight — the line which cuts the meridian line at right angles at the place of the style, and which stands as the vertical line of the equinoctial dial itself — that we may construct the equinoctial dial: upward indeed, in the upper equinoctial dial, for the hours which have a greater distance from noon than 6 hours, but downward if a lesser distance; and the contrary in the lower dial. For through the points found in this way the horary lines are to be drawn from the center, that is, from the place of the style.

For the rest, everything we have said in this chapter and its scholium about the description and the descensive [circumferences] of the dials will be better understood if there be recalled to mind those things which we handed down in the previous book concerning the description of those same dials. For the horizontal circumferences are here the same which there were the breadths of the shadows, and the descensive ones the same which were the complements of the altitudes of the Sun, and so on. Wherefore, if anything is here understood less rightly, recourse will have to be made to the previous book; for we judged it superfluous to repeat in this place what we wrote there.

ON DECLINING DIALS — WHETHER FROM THE VERTICAL OR FROM THE HORIZON — AND ON DIALS INCLINED TO THE HORIZON, AND ON THOSE WHICH BOTH DECLINE FROM THE VERTICAL AND ARE AT THE SAME TIME INCLINED TO THE HORIZON.

CAPVT X.

That at last, by means of the analemma, a dial declining from the vertical or from the horizon — and likewise one inclined to the horizon, or one both declining from the vertical and inclined to the horizon at the same time — may be described, there must first be investigated, by proposition 29 of book 1, the altitude of the pole above the greatest circle from which the dial to be described is equidistant; and the analemma must be built for this altitude, observing carefully whether the arctic pole or the antarctic

is raised above that face of the greatest circle from which the dial is equidistant — which indeed we shall recognize from what we wrote at the beginning of the scholium of proposition 1 of the previous book. As an example, let there be set before us this analemma answering to that greatest circle from which a dial declining from the vertical circle by 30 degrees is equidistant, which we also described in proposition 1 of book 3 and in proposition 5 of book 5.

Diagram. A large analemma circle for a dial declining 30° from the vertical. The circle A B C D is drawn about the center E, with the vertical diameter running from A at the top down through E, and the great-circle diameters and parallels crossing at E. Legible letter-labels around the rim and interior, read off the scan under magnification: **A** (top, on the vertical diameter), **F** (upper left, on the circle), **M** (left), **B** (left, with lowercase **u** just below it), **P** (lower left), **T** (lower-left

interior, near the parallel-chord), **N** (bottom), **C** (bottom, beside **N**), **I** (bottom, on the vertical diameter), **G** (lower right, on the circle), **L** (lower right), **D** (right), **R** (right, near the "24/1" rim marks), **H** (upper right), **K** (top, beside **A**). Interior points **O** (left of center) and **Q** (right of center) sit on the oblique diameters. The named diameters/chords of the construction are **K L** (diameter of the parallel of φ), **M N** (diameter of the parallel of $\ominus$), **F G** (diameter of the equator), **H I** (the axis of the world, through the poles), together with **B D** and **A C**. Two concentric rings of hour-numbers 1–24 run around the rim (an outer series for the hours from setting and an inner series for the hours from rising), and further number-series run along the parallel-chords marking the hour-divisions; the individual rim and chord numerals are only partially legible at this resolution and are **not** transcribed cell-by-cell here to avoid pattern-fill. The points **S**, **V**, **X** referenced in the text (the separately drawn parallel of φ , **K R L R**, with **R Q R** and **X V X**) are not confidently locatable as distinct labels on the figure at this resolution.

- **A** (top, on vertical diameter): vertex point
- **K** (top, beside **A**): endpoint of parallel-diameter **K L**
- **F** (upper left, on circle): endpoint of equator-diameter **F G**
- **M** (left, on circle): endpoint of parallel-diameter **M N**
- **B** (left, on circle): endpoint of diameter **B D**
- **u** (left, just below **B**): [?] (lower-case label)
- **P** (lower left): endpoint of the **N P** / seminocturnal segment
- **T** (lower-left interior): nocturnal-arc point
- **I** (bottom, on vertical diameter): pole (endpoint of axis **H I**)
- **C** (bottom, beside **N**): endpoint of diameter **A C**
- **N** (bottom): endpoint of parallel-diameter **M N**
- **G** (lower right, on circle): endpoint of equator-diameter **F G**
- **L** (lower right): endpoint of parallel-diameter **K L**
- **D** (right): endpoint of diameter **B D**
- **R** (right, near rim marks 24/1): parallel/hour point
- **Q** (interior right of center): parallel-hour point
- **O** (interior left of center): parallel-hour point
- **H** (upper right, on circle): pole (endpoint of axis **H I**)
- **E** (center): center of the analemma

For the altitude of the pole **D H** amounts to 40 degrees 3 [?] minutes — namely, as much as we found it above the said circle, as is clear from what precedes. And since the antarctic pole is raised above the southern face of that circle, if we posit

H to be the antarctic pole, then kL will be the diameter of the parallel of φ and MN the diameter of the parallel of ϑ ; so that KR is the semidiurnal arc of φ , and LR the seminocturnal arc, while MP is the semidiurnal arc of ϑ , and NP the seminocturnal arc — if the aforesaid greatest circle were the horizon on which the antarctic pole were visible. But conversely, since the arctic pole is raised above the northern face of the same circle, if H is posited to be the arctic pole, then kL will be the diameter of the parallel of ϑ and MN the diameter of the parallel of φ ; so that kR is the semidiurnal arc of ϑ , and LR the seminocturnal arc, while MP is the semidiurnal arc of φ , and NP the seminocturnal arc.

The circle $ABCD$, moreover, does not stand for the meridian of the place in which the dial is constructed, but for the meridian of that place in which the said greatest circle would be the horizon — passing, that is, through the poles of the world and through the poles of that greatest circle. Of this greatest circle, taken as horizon, and of the said meridian the common section is BD ; and of that same meridian and of the proper vertical of that greatest circle the common section is AC .

Next, the analemma so constructed must be divided into hours, so that the horizontal and descensive circumferences at the individual hours may be found. This division (since it differs from the one we set forth in chapter 6, because the circle $ABCD$ is not the meridian of the place in which the hours are to be described) is to be set up as follows.

[Margin: The division of the analemma into hours for the construction of declining and inclined dials.]

About the diameter KL , for example, of the parallel of φ , let the parallel of φ , $KRLR$, be described separately, together with RQR , the common section of the proposed greatest circle (taken as horizon) and of the parallel of φ , as in the analemma; so that RKR is the portion of φ above the southern face of the said greatest circle, as above the horizon, and RLR the portion of φ above the opposite northern face — that is, below the southern face. Then, once the inclination of the proper meridian of the proposed circle to the meridian of the horizon of that place in which the dial is to be built has been found by proposition 30 of book 1 — an inclination which we found to contain 40 degrees 48 minutes, as we said in proposition 1 of book 3 — we shall reckon it off from the point K , to the right or to the left, as far as S , and from S , through the center of the parallel, we shall draw the straight line ST for the common section of the parallel and of the meridian of the horizon, together with XVX , the common section of the parallel and of the meridian of the horizon; so that SV is equal to

the segment N P of the diameter of $\odot$ in the analemma of chapter 6, and X S X is the diurnal arc of $\odot$, and X T X the nocturnal. Now since the meridian of the horizon is more to the west in the equator, and hence in all the parallels, than the proper meridian of the proposed greatest circle from which the dial is equidistant, as we wrote in proposition 1 of book 3 — and since K is the point at which the proper meridian of the given circle cuts the parallel of $\odot$, while S is the point where the meridian of the horizon divides the same parallel — the point X to the right of S will be western, and the other point X to the left of the same S eastern. Therefore, for the unequal diurnal hours, the diurnal arc of $\odot$, X K S X, is to be cut into 12 equal parts, the beginning being made from the eastern point X, which is placed to the left of S. Likewise the nocturnal arc of $\odot$, X R L T R X, is to be cut into 12 equal parts for the unequal nocturnal hours, the beginning being made from the point X placed to the right of S, which is western. For the hours from noon and from midnight, the whole circle is to be divided into 24 equal parts, the beginning being made from the point S of noon, so that the afternoon hours tend from S toward the right, through X, R, L, as far as T; the forenoon hours, or those after midnight, from the point T toward the left, through R, X, K, as far as S. Then, for the hours from rising, the same circle is to be distributed into 24 equal parts, the beginning being made from the eastern point X, which lies to the left of S, so that the point X indicates the 24th hour from rising, while the first point of division toward S pertains to the 1st hour from rising, the next to the 2nd, and so on. Finally, for the hours from setting, the same circle is to be divided into 24 equal parts, the beginning being taken from the western point X, which is placed to the right of S, so that the point X denotes the 24th hour from setting, while the first point of division toward S pertains to the 23rd hour from setting, the next to the 22nd, and so on. We have set our example in the hours from setting and from rising, as is plain from the figure, in which the outer numbers pertain to the hours of $\odot$ from setting, the inner ones to the hours of $\odot$ from rising.

[Margin: Which hours fall on the southern face of the proposed greatest circle, and which on the northern, and how this may be recognized from the adjoining figure.]

Now all the hours which are comprehended in the portion R k R fall on the southern face of the proposed greatest circle, whereas those contained in the portion R L R do not fall on the said face, but on the opposite northern one — the Sun being at the beginning of $\odot$. Therefore all the hours of the diurnal arc of $\odot$ can be described in the proposed dial, which faces south; but of the hours of the nocturnal arc of $\odot$, those can be described which are comprehended between the

two straight lines X V X, R Q R, namely the 10th, 11th, 12th, 13th, 14th, and 15th — all of which are contained in the tropic of ♊ (dial of proposition 10 of book 3, and declining dial of proposition 5 of book 5), the 10th hour excepted,

which, on account of the narrowness of the space, could not be drawn in, since it runs out too far. From this figure, so constructed, we shall easily recognize how great is the diurnal arc of Capricorn above the southern face of the proposed circle, and how great the nocturnal; and likewise at what hour the Sun, standing at the beginning of Capricorn, begins to illuminate that same face, or ceases to, if it be not obstructed by the earth. For in this way you see the Sun rise above the southern face a little before the 9½ hour from setting, and set — that is, rise above the opposite northern face — somewhat after the 24th hour from setting. The same may be tried out at the other hours.

Now if the circle KRLR be taken for the parallel of Cancer, then RLR will be the portion of Cancer above the southern face of the proposed greatest circle, and RkR the portion above the opposite, northern face, as is plain from the adjacent analemma; while XTX will be the diurnal arc of Cancer, and XSX the nocturnal. Now the meridian of the horizon lies farther west in the parallel of Cancer than does the proper meridian of the proposed greatest circle, just as in the other cases, as we said above. And L is the point at which the proper meridian of the proposed greatest circle, acting as the horizon, cuts the parallel of Cancer above the horizon (for this meridian cuts off the greater portion of the parallel of Cancer below the horizon, as is clear from a material sphere), while T is the noon point above the horizon of the place in which the dial is to be laid out. So the point X to the left of T will be the western one, and the other point X to the right of T the eastern. Therefore, for the unequal diurnal hours, the diurnal arc of Cancer, XLTX, is to be cut into 12 equal parts, beginning from the point X, the eastern one, which is placed to the right of T. Likewise the nocturnal arc of Cancer, XkSX, is to be divided into 12 equal parts for the unequal nocturnal hours, beginning from the western point X, which is placed to the left of T.

For the equal hours from noon and from midnight, the whole circle is to be cut into 24 equal parts, beginning from the point T of noon, so that the afternoon hours run from T toward the left through R, X, K, as far as S; while the forenoon hours, or those after midnight, run from S toward the right through X, R, L, as far as T. Next, for the hours from rising, the same circle is to be distributed into 24 equal parts, beginning from the point X, the eastern one, which of course lies to the right of T, so that the point X marks the 24th hour from rising, while the first point of division toward T belongs to the 1st hour from rising, the next to the 2nd,

and so on. Finally, for the hours from setting, the same circle is to be cut into 24 equal parts, beginning from the point X, the western one, which of course lies to the left of T, so that the point X marks the 24th hour from setting, while the first point of division toward T belongs to the 23rd hour from setting, the next to the 22nd, and so on. We have set our example, for the hours from setting and from rising, in the adjacent figure; for the inner numbers belong to the hours of Cancer from setting, the outer to the hours of Cancer from rising.

Diagram. An uncaptioned circle (the hour-circle for the parallels of Cancer and Capricorn, as discussed in the text). The rim carries two concentric sets of hour-numbers, each running 1–24, with the letter-points set at the ends and crossings of the interior chords.

- K (top of circle): endpoint of the vertical diameter.
- L (bottom of circle, printed as "S L"): endpoint of the vertical diameter; the point S stands immediately beside it at the bottom.
- R (left end of the horizontal diameter): endpoint of the horizontal diameter through the centre Q.
- R (right end of the horizontal diameter): opposite endpoint of the horizontal diameter.
- Q (centre): centre of the circle, where the vertical diameter KL and horizontal diameter RR cross.
- X (upper left, near the rim numbers 15/16/24): endpoint of an oblique chord.
- X (right, near the rim numbers 24/9): opposite endpoint of that oblique chord.
- V (interior, upper right of centre): crossing-point of the oblique chords.
- S (upper right, near the rim number 20): point where a chord meets the rim.
- T (lower left, near the rim numbers 7/8/16): noon point on the rim.

The rim is graduated into 24 divisions carrying two rings of numerals 1–24: per the text, the inner ring = hours of Cancer reckoned from setting, the outer ring = hours of Cancer reckoned from rising. (Individual rim numerals not transcribed cell-by-cell; the two 1–24 rings and the lettered points are the informational content.)

All the hours contained in the portion RLR of Cancer fall upon the southern face of the proposed greatest circle, whereas those comprehended in the portion RKR do not fall upon that face but upon the opposite, northern one, the Sun being at

the beginning of Cancer. And so not all the hours of the diurnal arc of Cancer can be drawn in on the proposed dial, which faces south, but only those contained in the portion RLR — namely the 10th, 11th, 12th, 13th, 14th, 15th, 16th, 17th, and 18th, all of which have been drawn, in the tropic of Cancer, on the declining dial of proposition 5 of the preceding book, the 10th and 18th hours excepted, whose points could not be marked because they run out too far. From this too we shall recognize how great is the diurnal arc of Cancer above the southern face of the proposed circle, and how great the nocturnal; and likewise at what hour the Sun, standing at the beginning of Cancer, begins to illuminate that same face, or ceases to. For in this way you see the Sun rise above the southern face a little after the 9th hour from setting, and set — that is, rise above the opposite northern face — a little after the 18th hour from setting. The same may be tried out at the other hours.

Lastly, the divisions of this circle are to be transferred, by aid of the compass, by starting from the points K, L, or R — onto KRL, the parallel of Capricorn of the analemma, if the hours of Capricorn are in question, beginning likewise from the points k, L, or R that correspond to those; or, if it is the hours of Cancer that are treated, onto MPN, the parallel of Cancer, beginning from the points M, N, or P, which correspond to the points L, K, R of the aforesaid circle. For if perpendiculars are drawn from the divisions of the parallels KRL, MPN to their proper diameters, and the same is done in all the other parallels as well, and in the equator too (which the circumference of the meridian of the analemma represents), the analemma will be divided into hours for the construction of a dial declining from south to east by 30 degrees. Now for these hours the horizontal and descensional circumferences are to be found, as we taught above, exactly as if BD were the diameter of some horizon above which the antarctic pole is elevated, and AC the diameter of the vertical with respect to that horizon. Once these circumferences are found, the dial will be drawn, as we taught in proposition 5 of the preceding book. For the horizontal circumferences found here are the arcs which there we called the latitudes of the shadows; and from the complements of the descensional circumferences we shall know the altitudes of the Sun above the proposed plane, and hence the lengths of the shadows for each hour. Moreover, the hours which lie short of, or beyond, the proper vertical of the proposed greatest circle (taken as horizon) will be recognized here from the analemma, as above. For those whose perpendiculars fall in the segments of the diameters between the diameter of the vertical AC and the semicircle of the meridian ADC — in which the visible pole above the given plane is contained — lie short of the vertical, and therefore their horizontal circumferences are to be reckoned in the dial from the vertical line (which is drawn through the place of

the style perpendicular to the style-line) toward the equinoctial line; whereas those whose perpendiculars fall in the segments of the diameters between the diameter of the vertical AC and the semicircle of the meridian ABC — in which the hidden pole is contained — lie beyond the vertical,

[Margin: Description of a declining dial from horizontal and descensional circumferences.]

and their horizontal circumferences are to be reckoned in the dial from the same vertical line toward the centre of the dial. And from a dial declining from south toward east we shall also make another, which declines from north toward west, just as we explained above.

[Margin: How the dial on the opposite face of a given plane is to be described from its own horizontal and descensional circumferences.]

But if someone should wish to describe the dial on the opposite, northern face from its own horizontal and descensional circumferences, the same circle KRLR — drawn separately about the diameter of the parallel — is to be divided into hours in almost the same way as we set out a little before. For RkR will be the portion of Cancer above the northern face of the proposed greatest circle, and RLR the portion above the opposite southern face, as is plain from the analemma, if H be taken for the arctic pole, which is elevated above the northern face of the proposed circle. Now the point L, where the parallel of Cancer is cut by the proper meridian of the proposed greatest circle, is above the horizon — just as the point T, where the meridian of the horizon cuts the diurnal arc XTX of that same parallel, stands above the horizon, since noon occurs at it, the Sun being at the beginning of Cancer; and so the point K, where the same parallel is cut by the proper meridian of the proposed greatest circle, will be below the horizon. This will be readily perceived if the parallel of Cancer, separately drawn nearby, be set in such a position that the straight line XVX is equidistant from the horizon, and the point S tends downward and T upward, and likewise the straight line ST represents the common section of the meridian of the horizon and the parallel of Cancer, and the semicircle SKT tends to the west and the semicircle SLT to the east. For in this way the meridian of the horizon will cut the diurnal arc of Cancer, XTX, above the horizon at T, and will cut the smaller portion of Cancer, RLR — which the proposed greatest circle cuts off from the tropic of Cancer toward the south — at the point L above the horizon, tending toward the east, as the matter requires.

And so, for the unequal hours, the portion of Cancer XTX above the horizon is to be divided into 12 equal parts, beginning from the eastern point X, which, to us turned toward the circle in the position just described, lies to the left. For the equal hours from noon and from midnight, the whole circle is to be cut into 24 equal parts, beginning from the points T and S, so that the afternoon hours are comprehended in the western semicircle TKS, and the forenoon hours, or those after midnight, in the eastern semicircle SLT. Next, for the hours from rising, the same circle is to be distributed into 24 equal parts, beginning from the eastern point X and proceeding through L toward T. Finally, for the hours from setting, the division of the same circle into 24 equal parts must be begun from the western point X. In the figure the inner numbers regard the hours from setting, the outer the hours from rising. And when the Sun is at the beginning of Cancer, the diurnal hours in the parallel of Cancer, comprehended between the straight lines XVX and RQR, fall upon the northern face of the proposed plane — namely the hours 9, 19, 20, 21, 22, 23, 24 from setting, and the hours 10, 11, 12, 13, 14, 15 from rising, together with all the nocturnal hours — as is evident from the adjacent figure. But if the circle KRLR be taken for the tropic of Capricorn, then, as has been said, RKR will be the portion of Capricorn above the southern face of the proposed circle, and RLR the portion above the opposite northern face; likewise XSX will be the diurnal arc of Capricorn, and XTX its nocturnal arc. And no hour of the diurnal arc of Capricorn, XSX, falls in the portion of Capricorn RLR that stands above the northern face of the proposed circle, but only the hours of the nocturnal arc of Capricorn, XTX, which are contained in the portion RLR, as is plain.

And so, if the hours lying above the northern face of the given plane are transferred from the circle kRLR into the analemma, as before, and perpendiculars are drawn from the hours to the diameters of the parallels, and the same is done in the other parallels and in the equator too, the analemma will be divided into hours for the construction of a dial declining from north to west by 30 degrees. Now for these hours the horizontal and descensional circumferences are to be investigated, as before, exactly as if BD were the diameter of some horizon above which the arctic pole I is elevated.

[Margin: A circumference found for the hours from noon, or from rising, or from setting, in any parallel, upon one face of the proposed plane, is likewise the circumference for the hours from midnight, or from setting, or from rising, in the opposite parallel, upon the other, opposite face of the plane.]

Now the outer numbers in the separately-drawn parallel, when it is taken for the tropic of Capricorn, regard on the southern face the hours from setting, but on the northern face — when the same parallel is taken for the tropic of Capricorn — the hours from rising; likewise the inner numbers, on the southern face, when the same parallel is taken for the tropic of Cancer, regard the hours from rising, but on the northern face, when the parallel is taken for the tropic of Cancer, they regard the hours from setting — as is manifest from what has been said. Hence it comes about that the horizontal and descensional circumferences found for the hours of Capricorn from setting on the southern face of the proposed plane are likewise the circumferences for the hours of Cancer from rising on the opposite, northern face of the same plane; and those which were there found for the hours from rising are here the circumferences for the hours from setting, and conversely — since in the same manner in which the parallel of Capricorn is divided into hours from setting above the southern face of the proposed greatest circle, the parallel of Cancer is divided into hours from rising above the northern face, and so on. The same is to be said of the hours from noon and from midnight. For the circumferences found for the hours from noon in the tropic of Capricorn, above one face of the plane, will be the circumferences for the hours from midnight in the opposite tropic of Cancer, above the other, opposite face of the plane. Wherefore, since the complements of the descensional circumferences measure the altitudes of the Sun above the given plane, the altitudes of the Sun for the same hours will be the same above both faces of the proposed plane — provided that the hours which are taken on one face of the plane, in one parallel, from noon or from rising or from setting, be received on the other, opposite face of the plane, in the opposite parallel, from midnight or from setting or from rising. This too we showed in proposition 1 of the preceding book. And the same is to be said of the other planes, which not only decline but are at the same time inclined, or which are only inclined to the horizon.

Now if, in the dial, the style-line be drawn in the contrary way, as we prescribed in the preceding book, and the centre of the dial too be sought on the opposite side, then from the circumferences found there will be delineated a declining northern dial — that is, one on the opposite face of the plane — just as the southern one was, provided that the horizontal circumferences of the eastern hours be reckoned from the vertical line (which cuts the style-line at right angles at the place of the style) on the eastern side, and those of the western hours on the western side, beyond or short of the vertical line — that is, toward the centre of the dial or toward the equinoctial line — according as the hours are beyond or short of the vertical circle of the proposed plane; and these hours will readily be recognized from the analemma of this chapter. For since the other pole, I —

namely the arctic, in the given example — is elevated above the opposite face of the plane, all those hours whose perpendiculars fall in the segments of the diameters between the diameter of the vertical AC and the semicircle ABC (in which the visible pole above the given face of the proposed plane is contained) will be placed short of the vertical, and the others beyond it, and so on — as was said in the earlier cases too, as is plain if the analemma of this chapter is inverted so that C becomes the vertex of the head and A the point opposite the vertex.

[Margin: The finding of horizontal and descensional circumferences by sines, in declining planes.]

And when the analemma has been divided into hours, as has been said, and the distance of any hour from the proper meridian of the declining plane is known (which indeed will be known from the inclination of this meridian to the meridian of the horizon, as we handed down in proposition 1 of the preceding book), the horizontal and descensional circumferences can also be investigated from the doctrine of sines, both by rectilinear triangles and by spherical ones, taking the proposed greatest circle for some horizon. Or, if by proposition 1 of the preceding book the altitudes of the Sun above the proposed greatest circle are found for each hour, their complements will be the descensional circumferences; and the latitudes of the shadows, found by proposition 3 of that same preceding book, will be the horizontal circumferences.

Now just as we divided the analemma into hours for a greatest circle that declines from the vertical and stands square to the horizon, so too we shall divide it in the same way for a greatest circle that is inclined to the horizon — whether it declines from the vertical or not — if it be carefully weighed whether the proper meridian of the given greatest circle cuts, in the sphere, the parallel which we drew separately, above the horizon or not, and how great an inclination that same meridian has to the meridian of the horizon. And by the same procedure we shall depict the upper dial, as was handed down in the preceding book, from which the lower will be deduced, as we have often said. Or indeed we shall describe the lower dial from its own horizontal and descensional circumferences, as we did the upper, provided only that we draw the style-line in the contrary way and describe the horizontal line below the style, and not above it, as in the upper dial, and so on.

[Margin: Examination of the division made in the separately-drawn parallel in this chapter.]

The division of the separately-drawn parallel can also be examined, if from the inclination of the proper meridian of the given greatest circle to the meridian of the horizon one finds the hour, or that particle of an hour, which in that parallel falls upon the proper meridian of the proposed plane. For if that hour, or particle of an hour, agrees with the point K or L in the parallel according to the division that has been made, the division was rightly established; if not, by no means. For example: the proper meridian of the greatest circle declining from south to east by 30 degrees recedes from the meridian of the horizon, above the horizon toward the east, by 40 degrees 48 minutes — that is, by about 2 hours 43 minutes. Likewise the meridian of the horizon cuts the diurnal arc of Capricorn at hour 19 minute 32 from setting, and at hour 4 minute 28 from rising. If, then, from these hours we subtract 2 hours 43 minutes, there will remain hour 16 minute 49 from setting, and hour 1 minute 45 from rising. To these hours, therefore, the point K in the parallel must correspond. But if the meridian receded from the meridian of the horizon toward the west, the inclination of the meridians would have to be added to hour 19 minute 32, and so on. Again, since the meridian of the horizon cuts the diurnal arc of Cancer at hour 16 minute 28 from setting, and at hour 7 minute 32 from rising, if from these hours are taken away the 2 hours 43 minutes that are due to the inclination of the meridians, the remainder will be hour 13 minute 45 from setting, and hour 4 minute 49 from rising, to which the point L in the same parallel must correspond, and so with the rest.

Besides this, if in the analemma of the scholium to proposition 1 of book 5 the ellipse [figure point-labels worn — cannot be read with certainty: [?]] has been described, account being taken of the inclination of the proposed greatest circle to the meridian of the horizon, and if, for example, the semicircle of Cancer and of Capricorn, FRG, be divided into hours as we prescribed in chapter 6, it will there at once appear by how great an interval any given hour in the portion of Capricorn RG stands from the point 4, where the proposed plane cuts the parallel of Capricorn; and likewise how great is the distance of any given hour in the portion of Cancer RF from the point 3, where the parallel of Cancer is cut by the same plane. If, then, distances of this kind agree with the distances of the hours in the parallel drawn separately in this chapter — the distances which they have from the straight line RQR, the common section of the proposed plane and the parallel — the division will have been rightly established, and so on.

Scholium

Before we set an end to this sixth book, it has seemed good to show in a few words by what method, from a horizontal dial alone, there can be described —

with wonderful ease and pleasure — all the dials that stand square to the horizon: such as the vertical dial of either kind, both meridian dials, and all that decline from the vertical properly so called, of which sort are those that are usually painted on the walls of buildings, and which have a more frequent use than the others that are inclined to the horizon.

[Margin: By what method dials standing square to the horizon can be described from a horizontal dial — namely the vertical dial of either kind, both meridian dials, and all that decline from the vertical properly so called.]

Let there then be an Italian horizontal dial in which the style is AB, and its foot at A, and so on. Let it now be proposed to describe from it a dial declining from south to east by 30 degrees. At A let the angle BAC be constructed equal to the complement of the declination — such indeed as the greatest circle to which the dial is equidistant makes with the meridian — so that the straight line CAD is the common section of the plane of the horizontal dial and of that declining vertical to which the dial is equidistant, whose southern face looks from south toward east, and whose northern face from north toward west. From A draw AE perpendicular to CD; and from it cut off, toward the north, the straight line AE equal to the style of the future dial, of whatever magnitude it be proposed to be. Take also EF equal to the style AB of the horizontal dial, and through E and F draw EG, FH parallel to CD.

If the dial is to be described on the meridian line, then, beginning from the point A, a straight line equal to the style of the future dial is to be drawn either toward the north or toward the south, according as the dial to be described is southern or northern; and through the extremity of the style a straight line is to be drawn perpendicular to the meridian line, and parallel to it another line, distant from it by as much as is the gnomon of the horizontal dial — just as in the given example the straight line FH is equidistant from EG by the interval of the style EF. But if a meridian dial is to be described, one must take, from the straight line perpendicular to the meridian line at A, the length of the style, and through the end of this assumed style a line is to be drawn parallel to the meridian line — on the side of the west, if an eastern dial is wanted, but on the side of the east, if a western one is to be described; and parallel to this parallel, another parallel is to be drawn by the interval of the style of the horizontal dial. But let us return to the proposed example.

Let the plane through the straight lines EF, EG be turned about the straight line EG until it stands square to the plane of the horizontal dial. This done, by definition 4 of book 11 of Euclid, FE will be at right angles to the same plane. And

the style BA too stands square to the same plane. [a] Therefore the straight lines AB and EF will be parallel to each other; and since they are set equal, [b] the straight line joining the points B and F will also be parallel and equal to AE. And since AE, [by definition 4 of book 11 of Euclid,] is perpendicular to the plane drawn through the straight lines EF, EG, FH, [c] the straight line BF will likewise be perpendicular to the same plane. Wherefore the plane drawn through the straight lines EF, EG, FH will be the plane of the dial declining from south to east by 30 degrees, since — the line drawn square to AE being perpendicular to that same plane, and BF (equal to the assumed style AE) thus being the gnomon of the future declining dial, whose foot is at F, its vertex B being set at the centre of the world, namely at the vertex of the style in the horizontal dial — the straight line EG is the common section of the plane of the horizontal dial and of the plane of the declining dial, and of the horizon (since the plane drawn through the straight lines AE, EG, which are parallel, is equidistant from the plane of the horizontal dial). And FH will be the horizontal line — that is, the common section of the plane of the declining dial and of the horizon — since the plane drawn through the straight lines BF, FH (which are parallel to the straight lines AE, EG) is equidistant from the plane of the horizontal dial, being drawn through the centre of the world B and through the straight lines BF, FH, and so does not differ from the horizon; [d] and this because the horizon, drawn through the centre of the world B, is equidistant from the horizontal dial.

[Margin: a. Euclid XI.6 (6. vndec.)] [Margin: b. Euclid I.33 (33. primi)] [Margin: c. Euclid XI.8 (8. vndec.)] [Margin: The horizontal line in the dial, which...] [Margin: d. Euclid XI.15 (15. vndec.)]

[Margin: How the meridian line is to be drawn.]

And so we shall draw the meridian line in a declining dial as follows. From the point I, where the meridian line of the horizontal dial cuts the straight line EG, let the perpendicular IK be raised to EG; this line, I say, is the meridian line. For since the plane of the declining dial stands at right angles to the plane of the horizontal dial, the straight line KI will — by definition 4 of book 11 of Euclid — be perpendicular to that same plane; [a] and therefore the plane drawn through the straight lines KI and IA, being drawn to that same plane, will be at right angles to it. Since, then, the Meridian likewise stands at right angles to that same plane — being drawn through the meridian line AI — the Meridian will not differ from the plane drawn through the straight lines AI and IK; and for that reason IK will be the common section of the Meridian and of the plane of the declining dial.

That is to say, IK is the meridian line in the declining dial. In this meridian line we shall find the centre of the dial in the following way.

[Margin: How the centre of the dial is to be found.]

[Margin: a. Euclid XI.18 (18. vndec.)]

From I, let the perpendicular IL be raised to the meridian line AI of the horizontal dial. Now, with the circle BM described from A, the place of the style, so that all lines drawn from A to the circumference of this circle (which will often have to be drawn) are equal to the style AB, let the perpendicular AM be drawn from A to AI, up to the circumference of the said circle, and toward the same side to which the straight line IL was drawn. From N, the centre of the horizontal dial, let a straight line drawn through M cut the straight line IL at L; and equal to that same IL let a segment IK be cut off on the meridian line of the declining dial, toward the horizontal line FH. I say that K is the centre of the dial declining from the south. For if the triangle NLI is rotated about NI until it stands at right angles to the plane of the horizontal dial — so that the points M and L are set above the dial and not below — the straight lines LI and MA will, by definition 4 of book 11 of Euclid, be perpendicular to that same plane. Since, then, IK too is perpendicular to that same plane, IL will be the same as IK, and the point L the same as K.

But since AM is the style of the horizontal dial, NML will be the axis of the world. For the axis of the world, drawn through M, the vertex of the style, meets the plane of the dial at the centre of the dial. The axis of the world, therefore, will meet the plane of the declining dial at K; and consequently K is the centre of that dial, as is established from the corollary of proposition 21 of book 1. From K, the centre of the dial, the straight line KF, drawn through F, the place of the style, will be the line of the style — that is, the common section of the declining dial and of its own Meridian. Although in the given example this line happens to be parallel to the meridian line AI in the horizontal dial, and to cut the equinoctial line of that same dial at right angles, you must not on that account suppose that this necessarily happens so in every dial; for it happened by chance in our example, whereas in other declining dials you will find the contrary.

[Margin: Which is the line of the style.]

Diagram. A large declining-sundial construction figure filling the lower two-thirds of the page. The four cardinal directions are engraved around the frame: at the top **F · MERID·** (*meridies*, south), down the right-hand side **OCCASVS** (west), across the bottom **SEPTEN·** (*septentrio*, north), and up the left-hand side **ORTVS**

(east). A small circle (the style-circle BM) sits just left of centre, around the central point A; through and around it radiate the meridian, equinoctial, hour- and vertical-lines of the construction, with many auxiliary (dotted "occult") lines.

Readable labels:

- Cardinal frame: F, MERID·, OCCASVS, SEPTEN·, ORTVS
- Around the central style-circle: A (centre), B, N, P, M, and e, T, P, R below it
- Upper right: C, X, and the point V, O, H
- Descending series of hour-points down the right side: 10, 11, 12, 13, 14, 15, 16
- Left side: H, G, D, L, and 23
- Lower-central cluster (dense, dotted construction): 22, 21, 20, 19, 18, 17, with points I, E, F and lowercase a, b, d, f, h, i, l, m, n
- Bottom: K, and a second G at lower left; near F 17 a small Greek-letter cluster (γ [?], and further minuscule letters that do not resolve) [?]

[The same figure is reprinted — see fig-p492-1.]

[Margin: How the equinoctial line is to be drawn.]

Now since the equinoctial line cuts the line of the style at right angles — as we demonstrated in proposition 1 of book 3 — the equinoctial line in the declining dial must be drawn through the point O, where the Equator, carried through the equinoctial line of the horizontal dial, meets the plane of the declining dial: it will be the straight line OP, drawn from O perpendicular to the line of the style. This equinoctial line, in our example, does not differ from the equinoctial line of the horizontal dial, because, as we said, the line of the style cuts the equinoctial line of the horizontal dial at right angles — which, however, does not happen in other declining dials. We shall find, on KI (the meridian line of the declining dial) produced, the point P through which the equinoctial line is to be drawn, in this way. From A, along the meridian line AI of the horizontal dial, let the perpendicular AQ be raised up to the circumference of the circle BMQ — not toward the same side to which IL was drawn, but toward the contrary side — and from Q let a straight line be drawn through the point R, where the meridian line and the equinoctial line intersect one another in the horizontal dial, until it cuts, at the point S, the straight line IL (which was drawn from the point I, where the meridian line of the horizontal dial meets the plane of the declining dial, perpendicular to that same meridian line). For if the straight line IS, produced from KI toward the centre A, is cut off equal to IP, the equinoctial line in the declining dial must be drawn from O through P.

For let the plane in which the triangles AQR and RSI lie be conceived as rotated about the meridian line AI until it stands at right angles to the horizontal dial, so that the point Q comes to lie above the dial and S below. This done, both AQ and IS will be at right angles to the plane of the horizontal dial, by definition 4 of book 11 of Euclid. And, by the same definition, the meridian line IP in the declining dial is at right angles to that same plane. Therefore the straight line IS will coincide with the straight line IP, and the point S with the point P. For since the ray of the Sun at noon, passing through Q, the vertex of the style — when the Sun is in the Equator — falls at the point R, and, produced below the plane of the horizontal dial, at the point S or P, in the declining dial the

Equator will meet the plane of the declining dial at P, seeing that the ray of the Sun, passing through the centre of the world, does not then depart from the plane of the Equator. The equinoctial line, therefore, must be drawn through P. We shall find yet another point, H, in the horizontal line FH, through which the same equinoctial line is to be drawn, in this way. From A let AV be drawn parallel to the equinoctial line, cutting the straight line EG at V. (This is easily drawn if the straight line TA is taken equal to OV; [a] for so it will come about that the straight lines AV and TO, joining the extremities, are equal and parallel to one another.) And from V, to EV, let the perpendicular VH be raised, cutting the horizontal line at the point H, through which the equinoctial line is to be drawn. For since the plane of the declining dial stands at right angles to the plane of the horizontal dial, the straight line HV will — by definition 4 of book 11 of Euclid — be perpendicular to that same horizontal dial. Since, then, the style BA too is at right angles to that same plane, BA and HV will be parallel to one another; [b] and they are also equal to one another [c] (for EF, to which HV in the parallelogram FV is equal, was taken equal to the style AB). [d] Therefore the straight line joining the extreme points B and H will be equal and parallel to AV. Since, then, TO is also parallel to that same AV by construction, [e] the straight line BH will also be parallel to that same TO. Consequently BH and TO will lie in one and the same plane — namely in the plane of the Equator, which is drawn through the straight line TO (that is, through the equinoctial line of the horizontal dial) and through B, the vertex of the style. Therefore the Equator meets the plane of the declining dial in the equinoctial line. And so, if the three points H, O, P lie in a straight line, the equinoctial line is drawn through the points H, O, P.

[Margin: a. Euclid I.33 (33. primi)] [Margin: b. Euclid XI.6 (6. vndec.)] [Margin: c. Euclid I.34 (34. primi)] [Margin: d. Euclid I.33 (33. primi)] [Margin: e. Euclid XI.9 (9. vndec.)]

The point H, however, in the horizontal line, is more easily found in this way. From N, the centre of the horizontal dial, let the perpendicular NX be raised to the meridian line NI — serving as the line of the 6th hour from noon and midnight — cutting the straight line EG at X. For a straight line drawn from K, the centre of the declining dial, to X, will be the line of the 6th hour from noon or midnight in the declining dial, since the circle of the 6th hour from noon or midnight meets this dial at the point X and at the centre of the dial. This straight line KX, then, will cut the horizontal line at the point H, through which the equinoctial line is to be drawn. For the equinoctial line, the line of the 6th hour from noon or midnight, and the horizontal line always intersect one another at one and the same point, as is clear from what was demonstrated in book 3.

By the same device by which we described the equinoctial line on the declining dial, we shall depict all the hour-lines, together with the points of the tropics and of the other parallels. To make this clearer, we shall illustrate the matter with one or two examples. Let, say, the line of the 23rd hour from the west be drawn. Since the line of the 23rd hour in the horizontal dial does not cut the straight line EG, we shall produce it until it cuts EG itself at the point G, through which the 23rd hour from the west must necessarily be drawn on the declining dial, since at that point the circle of the 23rd hour, carried along by the line of the same hour, meets the plane of the declining dial. And in this way all the hours must be drawn on the declining dial through those points at which the hour-line in the horizontal dial cuts the straight line EG.

[Margin: How the hour-line is to be marked out.]

Next, on the straight line EG we shall take Ga equal to the straight line DA — the [distance] between the line of the 23rd hour and A, the place of the style, in the horizontal dial — and from a we shall draw the perpendicular aY to EG, which cuts the horizontal line FH at the point Y, through which the 23rd hour from the west must also be drawn, so that the straight line YG produced marks the 23rd hour from the west in the declining dial. For since the plane of the declining dial stands at right angles to the plane of the horizontal dial, Ya will — by definition 4 of book 11 of Euclid — be perpendicular to that same plane; and the style AB is likewise at right angles to that same plane. [f] Therefore aY and AB will be parallel to one another. Since, then, they are also equal to one another — because EF, [g] to which aY in the parallelogram aF is equal, was taken equal to the style AB — the straight line connecting the points B and Y will also be equal and parallel to the straight line Aa. [h] But the straight line Aa, joining the points A and a, is also parallel to DG, since the straight lines DA and Ga are equal and

parallel by construction. [i] Therefore the straight line connecting the points B and Y will also be parallel to DG; and consequently BY and DG will lie in one and the same plane — namely in the plane of the circle of the 23rd hour from the west, which is drawn through the straight line DG and through B, the vertex of the style. Wherefore the circle of the 23rd hour will meet the plane of the declining dial at the point Y.

[Margin: f. Euclid XI.6 (6. vndec.)] [Margin: g. Euclid I.34 (34. primi)] [Margin: h. Euclid I.33 (33. primi)] [Margin: i. Euclid XI.9 (9. vndec.)]

By the same reasoning the line of the 22nd hour from the west must be drawn through the points b, d, if the straight line eA — between the 22nd hour and the place of the style in the horizontal dial — is taken equal to bf (that is, to the line of the 22nd hour from the west). Through A let the hidden parallel Af be drawn, and let fd be drawn perpendicular to EG, cutting the horizontal line at d. Likewise the line of the 15th hour from the west must be drawn through the points i, g, seeing that the line of the 15th hour in the horizontal dial cuts the straight line EG at i, and Ah, parallel to the line of the 15th hour, cuts that same EG at h, and hg is drawn perpendicular to EG, cutting the horizontal line at g. In no other way is it done for whatever line in the horizontal dial has been described — whether it be one of the number of the hour-lines, or another of the verticals, or the meridian, or one of the celestial houses, or of the ascending signs — provided that it, when produced, cuts the straight line EG. In the declining dial we shall draw it through that point at which the line of the horizontal dial to be described in the declining one intersects the straight line EG, and through that point of the horizontal line onto which falls the perpendicular to EG drawn from the point onto which falls the line drawn through A parallel to the proposed line in the horizontal dial, as is manifest from what has been demonstrated.

Now the lines of the hours from noon and midnight, and the common sections of the Meridians — that is, of the circles of longitude — with the plane of the dial, all pass through K, the centre of the dial. Hence it is enough if the centre of the dial, K, is found. For straight lines drawn through K, the centre of the dial, and through the points at which the straight line EG is cut by the lines of the hours from noon and midnight, or by the common sections of the Meridians and of the plane of the horizontal dial, or by the common sections of the Meridians and of the plane of the declining dial, will give the hours from noon and midnight. They will, however, be drawn more accurately if a third point in the horizontal line is found for each line. Again, since the vertical lines in the declining dial are all parallel and perpendicular to the horizontal line, as we showed in proposition 4 of book 3,

the vertical lines must be drawn in the declining dial through those points of the straight line EG at which it is cut by the vertical lines of the horizontal dial, [and drawn] perpendicular to the horizontal line. Finally, since the lines of the celestial houses are drawn in the declining dial

[Margin: By what method any line whatever of the horizontal dial is depicted on the declining dial.]

through the point at which the meridian line cuts the horizontal line, such lines must be drawn in the preceding figure through the point Z, and through those points of the straight line EG at which the same lines meet the straight line EG in the horizontal dial.

Finally, in the lines transferred from the horizontal dial onto the declining one, we shall find the points through which the arcs of the signs are to be drawn, in this way. Let, say, on the line db of the 22nd hour from the west, the point through which the tropic of Capricorn [♋] is to be drawn have to be found. From A, the place of the style in the horizontal dial, through the point of the 22nd hour in the tropic of Capricorn [♋], let the hidden straight line A22 be drawn, cutting the produced straight line EG at the point m; through this point let the perpendiculars mn and ml be drawn to the straight lines Am and EG. Likewise, from A, toward the same Am, on the side opposite to that on which mn was drawn, let the perpendicular Ap be drawn, up to the circumference of the circle BM. Then let a hidden straight line be drawn from p through the same point of the 22nd hour in the tropic of Capricorn [♋], which cuts the perpendicular mn at n. If the straight line mn is cut off equal to ml, then through the point l, in the declining dial, must be drawn the parallel of Capricorn [♋] — that is [the point where], with the Sun at the beginning of Capricorn [♋], the shadow of the gnomon at the 22nd hour from the west will fall at the point l — so that necessarily the line db of the 22nd hour, drawn earlier, passes through the point l, if there has been no error. Hence it would be enough if, from the point m onto which falls the hidden straight line drawn from A through the 22nd hour in the tropic of Capricorn [♋], the perpendicular ml were drawn to EG: for where this cuts the line db of the 22nd hour, there will be the point of the tropic of Capricorn [♋]. But the hour-line will be described more exactly if the point l is found, as we have taught. For if the straight line ml is taken equal to the straight line mn,

[Margin: By what method the arcs of the signs are transferred from the horizontal dial onto the declining dial.]

and the three points l, b, d lie in a straight line, the line of the 22nd hour from the west will be drawn most exactly through these three points. And if the point of the tropic of Cancer [Ϛ] is also found, there will be four points through which the proposed hour-line is to be drawn. In the 22nd hour from the west this fourth point cannot be found in our example, because the straight line drawn from A through the point of Cancer [Ϛ], at the said hour, does not cut the straight line EG. Hence the Sun, being at the beginning of Cancer [Ϛ] at the 22nd hour from the west, does not illuminate the plane of the declining dial, [which declines] from south toward east. Now, that the extremity of the gnomon's shadow at the 22nd hour from the west, when the Sun is at the beginning of Cancer [Ϛ], falls at the point l, we shall demonstrate by the same method by which a little earlier we showed that the extremity of the shadow at noon, when the Sun is at the equinoxes, falls at the point P on the meridian line KI. For if the plane in which the triangles Ap22 and 22nm lie is conceived as rotated about the straight line Am until it stands at right angles to the horizontal dial — so that the point p comes to lie above the horizontal dial and n below — then, by definition 4 of book 11 of Euclid, both Ap and mn will be at right angles to that same horizontal dial; and, by the same definition, ml too will be perpendicular, in the plane of the declining dial, to the horizontal dial. The straight line mn, therefore, and the straight line ml, and the point n and the point l, will coincide, on account of the equality of the straight lines mn and ml. Now since the ray of the Sun, passing through p, the vertex of the style, at the 22nd hour from the west in the tropic of Capricorn [Ϟ], falls at the point 22, the same [ray], produced below the plane of the horizontal dial, will fall at the point n, or l, in the declining dial; and consequently the circle of the [22nd] hour will meet

Diagram. The same declining-dial construction figure as reprinted (from the same woodblock). The cardinal directions are engraved around the frame — **F · MERID ·** (south) at the top, **OCCASVS** (west) down the right side, **SEPTEN ·** (north) along the bottom, **ORTVS** (east) up the left side — with the central style-circle at **A** and the same dense web of meridian, equinoctial, hour- and vertical-lines and dotted auxiliary lines. The readable labels match those listed for fig-p490-1 (cardinal frame; A, B, N, P, M, e, T, P, R about the centre; C, X, V, O, H upper right; hour-points 10–16 down the right; H, G, D, L, 23 at left; the dense lower cluster 22, 21, 20, 19, 18, 17 with points I, E, F, K and lowercase a, b, d, f, h, i, l, m, n; a small unresolved Greek-minuscule cluster near F 17 [?]).

[Margin: By what method all the points of the horizontal dial are transferred onto the declining dial.]

the plane of the declining dial at the point l, at the 22nd hour from the west, seeing that the ray of the Sun, passing through the centre of the world, does not then depart from the plane of the said circle. By the same reasoning all the other points of the horizontal dial will be transferred onto the declining dial, if hidden straight lines are drawn from A (such as was, at the 22nd hour [♊], the straight line Am), and through the points at which the straight line EG is cut by the said hidden straight lines (such as the point m), perpendiculars are drawn both to those hidden straight lines and to the straight line EG (of which sort are the perpendiculars mn, ml), in which [perpendiculars] are found the points onto which the shadow is projected — [perpendiculars] first drawn from A to those same hidden lines up to the circle BM, and so on. This one thing only must be observed: that when a point to be transferred from the horizontal dial lies between A and the straight line EG — as was the point of the 22nd hour [♊] in our example — the perpendicular to EG (such as ml) must be drawn toward A, namely below the straight line EG, in the declining dial; because the ray of the Sun then cuts the plane of the horizontal dial at the given point before it meets the plane of the declining dial at right angles to the plane of the horizontal dial, and so will meet the declining dial below the straight line EG. But when the straight line EG lies between A and the point to be transferred — of which sort in our example are the points of the hours 18, 17, 16, and so on, from the west in the tropic of Capricorn [♋] — the perpendicular to EG must be drawn toward the horizontal line, namely above the straight line EG, in the declining dial; because the ray of the Sun then meets the declining dial above the straight line EG, before it cuts the plane of the horizontal dial at the given point, as is evident. But in which direction the perpendiculars to EG must be drawn is beautifully indicated by the hour-lines themselves, already drawn. For the said perpendiculars must be raised toward that side where they can cut the hour-lines. It makes no difference, however, in which direction the perpendicular is drawn to the hidden straight line drawn from A through the point that is to be transferred from the horizontal dial onto the declining one, provided that another perpendicular from A to the same hidden straight line is drawn in the contrary direction, as was done in the example.

By this reasoning, in each of the hour-lines that cut the equinoctial line in the declining dial (such as are those through whose points, in the equinoctial line of the horizontal dial, straight lines drawn from A intersect the straight line EG), the points can be found through which the equinoctial line is to be drawn. In this same example of ours we drew, from A, through the point of the 15th hour from the west in the tropic of Cancer [♋], the straight line Aq, which cuts the straight line EG at q; and through q, to Aq, we raised the perpendicular qr, which the

straight line tr — drawn from t , the extremity of the style (which is perpendicular to Aq) — cuts at r , through the point of the 15th hour from the west in the tropic of Cancer [$\mathfrak{S}$]; and off the straight line qr , from q , we cut a segment qf equal to the perpendicular to EG , at the point f , where this perpendicular intersects the line gh of the 15th hour, drawn earlier. For through f the tropic of Cancer [$\mathfrak{S}$] will pass, on the line of the 15th hour from the west. But for the tropic of Capricorn [$\mathfrak{V}$] we find no point in the 15th hour; instead we ended it in the horizontal line, because this hour, in the horizontal dial, has no point in the tropic of Capricorn [$\mathfrak{V}$].

[Margin: A dial declining from the north [toward the west].]

If a dial declining from the north toward the west is to be described, one must take, on the straight line AE , toward the south, a straight line AE equal to the style equal to the style of the future declining dial, and the straight line EF equal to the style of the horizontal dial. Then, through E and F , straight lines EG , FH are to be drawn parallel to AD ; of these, EG will be the common section of the horizontal dial and of the declining dial, while the straight line FH is the horizontal line. The rest will have to be completed as we said for the dial declining from south toward east.

Now if, in the horizontal dial, the greatest of the ever-apparent parallels [i.e. the circle of perpetual apparition] had been drawn, we would at once distinguish, in the dial declining from the north, the hours reckoned from the east from those reckoned from the west. For those hours which, in the Italian horizontal dial, touch the said parallel before they meet the straight line EG belong, in the declining dial, to the hours from the east — as is clear from what we wrote in the scholium of proposition 10, Book 2.

[Margin: How a declining dial is described from a horizontal one.]

For the rest, this discovery of ours does not differ much from the device which Guidobaldo del Monte [Guidus Ubaldus e Marchionibus Montis], a man most outstanding in mathematical matters, thought out and most kindly sent to me. By that device, from the horizontal and descensional circumferences of the Sun found on the horizon, we shall construct the same dials on a plane erect to the horizon. Now although he himself first describes, in the plane of the horizon, the ellipses which the perpendiculars falling onto the plane of the horizon from the hour-points of all the parallels make in that plane (as we demonstrated in proposition 24, Book 1), we, however, will explain the matter itself more easily from the horizontal and descensional circumferences of the Sun, in this way.

[Margin: By what method the same dials, set erect to the horizon, are constructed from the horizontal and descensional circumferences found on the horizon.]

Let ABCD be the horizon, whose centre is E. Let the common section of it and the prime vertical (the vertical circle properly so called) — or equally of the equator — be BD; and the common section of it and the meridian be AC. Suppose a dial is to be described declining both from south toward east and from north toward west. Draw the diameter FG, which is to be the common section of the horizon and of the great circle from which the dial to be described is equidistant, such that DG, BF are arcs of declination, and the southern face of the said great circle declines from south toward east, while the northern face declines from north toward west. To FG, through the centre E, draw the perpendicular HI, and on either side of it cut off the straight lines EH, EI equal to the style of the future dial; and through H and I draw HK, IK parallel to FG. Then HK will be the common section of the horizon and of the plane of the dial declining from south toward east, and IK the common section of the horizon and of the plane of the dial declining from north toward west.

Now let us reckon off, from the western point D, the horizontal circumference of the 24th hour from the west, in Cancer [☿], up to L, toward the north — because this is an afternoon hour and northern in the tropic of Cancer [☿], as is clear from the analemma. This circumference is the western amplitude [latitudo occidua] of the Sun when it is at the beginning of Cancer [☿]. From L, through the centre E, let a hidden straight line be drawn cutting IK at the point M — onto which the Sun's ray, drawn through the centre E (that is, through the tip of the style EI), then falls when the Sun is at the point L on the horizon, at the 24th hour from the west, in the northern dial.

Again, take DN: let the horizontal circumference of the 23rd hour from the west, in Cancer [☿] — which is likewise northern, and western, or afternoon — [be reckoned to N]; and let a hidden straight line be drawn from N through E, cutting EK at the point O, from which are raised both OP, perpendicular to NO, and OR, perpendicular to IK. Let NQ be the complement of the descensional circumference of the same 23rd hour, and from Q, through E, let a hidden straight line be sent out cutting OP at P; and on OR let a segment be taken equal to OP.

Diagram. A large circular construction for two declining dials laid out from the horizontal, framed by the four cardinal directions. The circle ABCD is the horizon, with centre E, from which a fan of solid and dotted straight lines radiates. Around the frame, in large capitals: **MERID.** at the top (south — a small ornamental

symbol and what appears to be the numeral "16" stand between "ME" and "RID" [?]), **SEPT.** at the bottom (north), **ORT.** at the left (east, set vertically), and **OCCAS.** at the right (west, set vertically). Two long straight lines (the traces HK and IK of the two declining planes) run obliquely across the upper part of the figure through the outer points, with dotted perpendiculars dropped to them.

Readable letter-labels (position → note):

- E (centre): common centre of the radiating lines and of the horizon.
- A (top rim, below MERID): meridian endpoint (south).
- Left/lower-left construction points: T, K (upper left), O, M, P, R, u, V — endpoints of the hidden construction lines.
- Left rim: B, I, F (with a small "h" near the top).
- Right and lower-right rim: f, e, Y (with small letters Z / a adjacent [?]), G, D, B, N, L, X.
- Lower rim / lower construction: s, H, m, b, d, and a second K at lower left.
- A row of small letters (approximately p, f, q, g, r [?]) sits between A and the upper rim.

For if the circle ABCD is turned about NO until it stands erect to the horizon — and so becomes the vertical circle passing through the Sun at that moment, such that the point Q lies above the horizon — then the arc NQ will be the Sun's altitude, and the straight line OP will, by Euclid, Book 11, definition 4, be perpendicular to the horizon. But by the same definition OR too — in the plane of the declining dial (for we now understand that the plane of the declining dial is drawn through the straight lines IK, OR, and is erect to the horizon) — stands at right angles to the horizon. Therefore the straight line OP will coincide with OR, and the point P with the point R. Hence the Sun's ray QEP, drawn through the centre E, will fall onto the point P, that is, R.

By similar reasoning, from the horizontal circumference DS of the 22nd hour from the west, in Cancer [☿], and from SX, the complement of the descensional circumference, the point V of the 22nd hour from the west in the tropic of Cancer [☿] will be found — if the straight line SEK is drawn, and KV perpendicular to IK, and the point T (at which the straight line XE cuts the straight line KT) is transferred onto the straight line KV, up to V; and so on for the rest.

By the same art, from the horizontal circumference DY of the 24th hour from the west, in Cancer [☿] — which is southern and western — the point of the 24th hour from the west in the tropic of Cancer [☿] will be found on the horizontal line HK of the dial declining from south toward east, namely where the straight

line TE cuts the horizontal line HK; this point, however, is not contained within our example, on account of the narrowness of the space. Likewise, from the horizontal circumference DZ of the 23rd hour from the west, in Cancer [☊], and the complement Za of the descensional circumference, the point b was found, for the 23rd hour from the west, in the tropic of Cancer [☊]. And from the horizontal circumference De, and the descensional complement ef, of the 22nd hour from the west, in Cancer [☊], the point d was found, for the 22nd hour from the west, in the tropic of Cancer [☊]. And thus, for any hour whatever, we shall find a point on any parallel whatever, if from D, or E, we reckon the horizontal cir-

[Margin: In what way, for any hour, a point may be found on any parallel whatever in the declining dial.]

-cumference toward A or toward C, according as the circumference is western or eastern, and southern or northern; and if, from the endpoint of this circumference, we draw a hidden straight line through the centre E, which cuts the horizontal line at a point, from which two perpendiculars are to be raised — one to this hidden line just drawn, and the other to the horizontal line; and if, finally, from the endpoint of the horizontal circumference, in whichever direction, we count off the complement of the descensional circumference, and from its endpoint draw, through the centre, a straight line that cuts, at a point, the hidden perpendicular raised to the hidden line drawn from the endpoint of the horizontal circumference — [that point] being transferred onto the perpendicular drawn to the horizontal line. For this will be the point of the proposed hour for that parallel for which the horizontal circumference, and the complement of the descensional, were found.

[Margin: How, for any hour, a point may be found on any parallel in the declining dial.]

Now the perpendicular to the horizontal line must always be drawn toward the side of the centre, in both dials; but the other perpendicular, to that hidden line drawn from the endpoint of the horizontal circumference, may be drawn out in whichever direction, provided that the complement of the descensional circumference is counted from the endpoint of the horizontal circumference toward the contrary side, so that the hidden straight line drawn through the centre from the endpoint of this complement can cut this perpendicular — as you see done in the examples. Furthermore, only those hours can be drawn in the southern dial whose horizontal circumferences have such a position on the horizon that the straight lines drawn from their endpoints, produced beyond the centre E, cut the horizontal line HK; the others cannot. And the same must be said

of the northern dial. Hence it has come about that the hours 24, 23, 22 in Cancer [♋], from the west, were drawn in the northern dial but not in the southern; and likewise the hours 24, 23, 22 in Capricorn [♏], from the west, in the southern dial but not in the northern, and so on.

[Margin: How it may be known, from the horizontal circumferences, at what hour [the lines] can be drawn in the proposed dial.]

But so that you may have an example of some hour-line that has a point both in the tropic of Cancer [♋] and in the tropic of Capricorn [♏]: we shall find the point g, for the 16th hour from the west in the tropic of Cancer [♋], from the horizontal circumference of that same 16th hour — Bh — which is southern and eastern, and from hl, the complement of the descensional circumference. (For although the straight line lE, produced, does not, in the example, cut the perpendicular drawn to hm, nevertheless we transferred the point where it would cut it onto the perpendicular mg, up to g.) Likewise, from Bp, the horizontal circumference of the same 16th hour in Capricorn [♏] from the west — which is likewise southern and eastern — and from pq, the complement of the descensional circumference, we find the point r, for the 16th hour from the west in the tropic of Capricorn [♏], so that rg is the line of the 16th hour from the west.

[Margin: In what way the equinoctial line is drawn.]

Finally, so that the equinoctial line may be drawn, two, or three, or more points must be investigated for two, or three, or more hours, with the Sun on the equator; for through these the equinoctial line will have to be drawn. Thus you see that the point f was found on the line rg, for the 16th hour from the west, with the Sun on the equator, from the horizontal circumference Bt of the same hour — which is eastern and southern — and from tu, the complement of the descensional circumference. For to the straight line tEα the perpendicular αβ was drawn, which the straight line uEβ cuts at β, and the point β was transferred onto αf, the perpendicular to HK; and this point f will necessarily fall on the straight line rg, if there has been no error. By this device the whole dial will be completed, as far as concerns the hours and the arcs of the signs.

[Margin: A comparison between the two methods by which, in this scholium, declining dials have been constructed.]

For the other things which we described in the dials of Books 2 and 3 cannot be laid out by this method unless one had first found the horizontal and descensional circumferences, for those other descriptions too, in the horizontal dial — which was done neither by Ptolemy nor by anyone else, because it would

first have to be investigated in what way the parallels of the Sun are divided by the vertical circles, by the parallels of the horizon, by the circles of the celestial houses, and so on; which is exceedingly arduous and difficult. For in order that such circumferences may be investigated for the individual hours, one must first know where the hours intersect the parallels of the Sun, and how far they are, from the south or from sunrise or sunset, on their own parallels. Indeed it was for this reason that we taught, in the preceding [books], how to cut the parallels into hours.

Since this is so, it is clear that our discovery, by which a declining dial is described from a horizontal dial, is much more universal than this device by which a declining dial is depicted from the horizontal and descensional circumferences — for by our discovery all the points of the horizontal dial, even if they do not belong to the hours, are transferred into the declining dial, which cannot be done by this other device, since by what method the horizontal and descensional circumferences are to be sought for all the points not belonging to the hours has been handed down by no one. Indeed, on this account too I judge that the earlier method, explained by us, is to be preferred to the later — because it transfers all the hour-lines, or any others whatever, from the horizontal dial into the declining dial, even if the points through which the parallels of the signs are to be drawn have not been found for them. Thus by the first method the line of the 22nd hour from the west can there be drawn through the point l found in the tropic of Capricorn [♋]; but by the second method it cannot be drawn here, through the point d of the same hour found in the same tropic of Capricorn [♋], unless first another point of the same hour is found with the Sun in another sign, as at the beginning of Libra [♎] or of Aquarius [♒].

In this one respect, however, the later method seems to surpass the earlier: that from the horizontal and descensional circumferences there can be drawn, in the declining dial, even those hours which are little distant from sunrise or sunset — say the hours $23\frac{1}{4}$, $23\frac{1}{2}$, $23\frac{3}{4}$, 9, $9\frac{1}{4}$, $9\frac{1}{2}$, and $9\frac{3}{4}$ from the west, and so on — which can scarcely be done by our earlier way, because such hours, since in them the shadow of the gnomon in the horizontal dial is too long, can only with great difficulty be described in the horizontal dial. So much so that, for a complete and perfect description of the hours in declining dials, both ways and methods seem to have to be employed.

END OF THE SIXTH BOOK.

GNOMONICS, BOOK SEVEN.

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Since, in the preceding books, the demonstrations have for the most part been intermingled with the descriptions of the dials — so that to some the subject may seem treated at greater length than is fitting — I think it will be worthwhile if, in this seventh book, for the sake of those who either take less pleasure in geometrical demonstrations, or have not trained themselves in mathematical matters enough to be able to follow them, or finally wish to have the rules for describing dials gathered into one place without demonstrations, I set these out separately, and very briefly present the chief headings by which all the kinds of hours — of which we have treated most fully in the preceding books — can be described on any plane surface whatever, for a given size of style. For thus it will come about that, since the precepts of the descriptions are separated from the demonstrations, and written out separately and distinctly, no impediment or delay at all can hold back the dial-maker in describing the hours. For in the preceding books the course of the description of the dials must again and again be interrupted on account of the multitude of demonstrations, which are everywhere entangled with the precepts; so that scarcely any dial can be described without trouble. And in this matter I shall gratify very many who have earnestly begged this of me.

The method we shall follow in delivering this brevity of precepts will be as follows. Those precepts which are altogether necessary for describing the hours we shall repeat here, so that there is no need to transfer anything from what was written in the preceding books hither for the description of any given dial — except for certain things which make for fuller instruction, or do not pertain to the hours, in which cases the reader will have to be referred back to the earlier books. Nor indeed does it seem necessary here to repeat the division of the hours and of the dials, nor the nature of the hour-circles, nor their position in the heavens, since all these have been explained copiously by us in Book 1 — the division of the hours and of the dials at its beginning, and the position of the hour-circles in propositions 9 and 10. Therefore we shall proceed to the description of the dial.

ON THE ASTRONOMICAL HORIZONTAL DIAL.

CHAPTER I.

[Margin: Description of the astronomical horizontal dial.]

We shall describe the astronomical horizontal dial in this way. In the plane of the dial let a straight line HN be drawn, in any position, for the meridian line; or, if the plane is fixed and equidistant from [parallel to] the horizon, let the meridian line HN be found on it by the methods we set out in our commentaries on the Sphere, where we explained the offices of the meridian circle, or at least as we taught in the scholium of proposition 23, Book 1, of this Gnomonics. Then, having chosen the place of the style at the point G, and having drawn the straight line GD perpendicular to HN, let GD be taken equal to the proposed style of whatever size. With D as centre, let the arc of the circle ABC be described; and drawn toward the southern parts — that is, toward A — let the complement of the pole's altitude, BA, be counted off, and toward C the pole's altitude itself, BC; and let the straight lines DA, DC be drawn (of which the former represents the axis of the world, the latter the common section of the equator and the meridian), cutting the meridian line at the points H and I. Through the point I let the perpendicular FK be raised to the meridian line.

[Margin: The centre of the dial.] [Margin: The equinoctial line.]

H will be the centre of the dial, in which of course all the hour-lines will intersect one another; and the straight line Fk will be the equinoctial line — namely the common section of the equator and of the plane of the dial — on which, when the Sun is at the equinoxes, the shadow of the style falls at each hour and each moment of the hours. Also, taking the straight line IE equal to ID, describe from E a circle at any interval; and dividing it into 24 equal parts, beginning from the meridian line HN, draw through the division-points and the centre E hidden straight lines cutting the equinoctial line Fk at points, through which — and through the point H — straight lines drawn out will give the lines of the hours from noon or midnight, in the plane which is equidistant from [parallel to] the horizon.

[Margin: The hour-line.]

But for the sixth hour, the line CL is to be drawn through H, perpendicular to HN, per-

perpendicular to HN — that is, parallel to FK. Now these hour-lines can be extended through the point H infinitely in both directions, although for the sake of neatness we have terminated them at the circumference of a circle described from centre H at whatever interval you please. You will find the demonstration of this construction in the scholium to proposition 1, Book 2.

[Margin: Which hours are before noon, and which after noon.]

The portion of the meridian line running from the centre H toward the equinoctial line marks the 12th hour of midday, and the remaining portion, beginning from the same centre H, marks the 12th hour of midnight. The hours placed to the right of the meridian line — that is, toward the point L — are the forenoon hours, while the afternoon hours lie to the left of the same meridian line, that is, toward the point C. Thus the hour next to the 12th hour of midday on the side of C is the 1st hour after noon, the next the 2nd, and so on for the rest, as the figure shows.

Diagram. The astronomical horizontal dial completing Chapter I. A large circular (here drawn as a full round) dial with its centre at **H**, from which the hour-lines radiate. The vertical meridian line runs through **M** (top) and **N** (bottom); the horizontal line through the centre carries **F** on the left and **K** on the right, with **L** just above K on the right and **C** above the centre-line on the left (C lies on the 6-o'clock line). Near the centre are the construction points **G**, **D**, **I**, with **E** on the meridian below the centre, and **A** on the upper-left rim; **B** marks a dashed line crossing the left half. The circumference is numbered with the hours (the numerals 1 through 12 appear around the rim, the forenoon hours toward L and the afternoon hours toward C).

- H (centre): meeting-point of all the hour-lines
- M (top of meridian) / N (foot of meridian, marked with the 12th hour)
- F (left) — K (right): the horizontal (equinoctial) line
- L (right, above K), C (left, on the 6-o'clock line)
- A (upper-left rim), B (dashed line, left half)
- D, G, I (near centre); E (on the meridian below centre)
- Rim: hour-numbers 1–12 around the circumference
- Two worn zodiac sign-glyphs mark dashed arcs on the left [zodiac glyph: ⚈ Taurus (?), worn] [?]

Now, if no error has been made, the intervals of the hours in the portion IF of the equinoctial line will be equal, one by one, to the intervals of the hours in the other portion Ik. Likewise the arcs of the circle described from H that lie between

the meridian line HN and the hours on the left side will be equal, one by one, to the arcs of the same circle intercepted between the meridian line and the hours on the right side. Therefore one must take care that intervals of this kind are equal to one another, if an accurate marking of the hours is desired.

[Margin: How half-hours and quarter-hours are to be marked on the dial.]

But if one wishes to mark on the dial the half-hours as well, and the quarter-hours — indeed the eighth-parts, and so on — then each of the parts of the circle described from E must be divided into two equal parts, or four, or eight, and so on; the rest is then completed as before. This is to be observed in the other dials that follow as well.

[Margin: How the style is to be placed.]

The style, or gnomon GD, must be erected upon the dial at right angles from the point G, as we showed in proposition 12, Book 4. For in this way the tip of the shadow, falling upon the hour-lines, will indicate the hours, and the apex of the style itself will touch the axis of the world, which is extended from H through the arctic pole to the point D, so that HD represents the axis of the world. And if the triangle HDI is erected at right angles upon the plane of the dial along the meridian line HN, the shadow of the axis HD will likewise indicate the same hours — provided the dial is set on a plane parallel to the horizon in such a way that the line HN represents the meridian line, with the point H turned toward the south and I toward the north.

[Margin: How the horizontal dial is to be positioned so that it indicates the hours.]

By what device the elevation of the pole is to be found — for in order that the dial be correctly laid out in the given region, one must know the elevation of the pole above the horizon of that place, together with its complement, as is clear from the construction already given — we have taught in the *Use of the Astrolabe*, in the *Cosmography*, and in our *Commentaries on the Sphere* (where we treated of the offices of the meridian circle), and also in the 2nd scholium to proposition 28, Book 1, of this Gnomonics.

ON DESCRIBING THE ARCS OF THE SIGNS ON THE HORIZONTAL DIAL.

CHAPTER II.

The ray of the Sun, when the Sun is on the equinoctial circle, describes by its daily motion a circle about the centre of the world — namely the Equator itself. But when the Sun dwells on any other parallel, on this side of the equinoctial circle or beyond it, the same ray, drawn out through the centre of the world, describes two conical surfaces joined together at the centre of the world as at their apex, whose bases are two opposite and equal parallels. For instance, when the Sun is at the beginning of Cancer (♋) or Capricorn (♏), the bases of the said conical surfaces are the tropic of Cancer and the tropic of Capricorn — all of which we demonstrated in Book 1, proposition 3. And since the plane of any dial, cutting these conical surfaces, makes either a circle, a parabola, a hyperbola, or an ellipse (because it does not pass through their apex, that is, through the centre of the world), as is established from the *Conics* of Apollonius [Apollonius of Perga] and as we showed in the same Book 1, propositions 4, 5, 6, and 7; and since the shadow of the gnomon does not depart from these sections when the Sun is on that parallel which is the base of the conical surface, as we demonstrated in the same Book 1, proposition 12 — it follows that the arcs of the signs which the gnomon's shadow describes on the dial, when the Sun is at the beginnings of the signs, are either circles, or parabolas, or hyperbolas, or ellipses; except when the Sun is on the equinoctial circle, for then the gnomon's shadow describes a straight line,

as is clear from corollary 2 of proposition 11 of the same Book 1. In which particular dial these arcs of the signs are circles, parabolas, hyperbolas, or ellipses we have shown fully in Book 1, propositions 4, 5, 6, and 7, and in their scholia. For now, let it be taken as certain that arcs of this kind, on the equinoctial dial, are circles everywhere among all nations; whereas on the horizontal dial of a latitude less than $66^{\circ} 30'$, and both on the meridian dial and on the polar dial of any region whatever, and on the vertical dial of that place where the elevation of the pole above the horizon is greater than $23^{\circ} 30'$, they are hyperbolas.

[Margin: Construction of the figure of the rays of the Zodiac, from which the arcs of the signs on the horizontal dial are drawn.]

Therefore, in order to mark out these arcs — whatever their shape may be — we must first prepare the figure of the rays of the Zodiac together with the hour-

lines, in the following way. Let a line AH be drawn crosswise, in any position, to represent the axis of the world, and let DC be raised perpendicular to it, for the equinoctial ray. Then from D let the arc of a circle NO be described; on it, reckoning off the maximum declination of the Sun, $23^{\circ} 30'$, toward both sides of the Equator's ray DC as far as the points N and O, let the straight line NO be drawn, cutting DC at the point R. From R as centre, upon the diameter NO, let a circle be described and divided into 12 equal parts, beginning from the line NO (or from the Equator's ray DC). After this, let any two points equally distant from the point N or O be joined by straight lines cutting the arc NO at points; and through these points, straight lines drawn out from D will give the rays of all the signs of the Zodiac, as appears in the figure.

[Margin: The drawing of the rays of the signs in the figure of the rays of the Zodiac.]

After this, from the triangle HDI of the figure of the preceding chapter, let there be transferred onto the axis AH of the figure just constructed — from D leftward — the line DH, and onto the Equator's ray DC — downward — the line DI; and from H, through I, let the line HI be drawn out, of whatever length, for the 12th hour, which perhaps we shall more conveniently draw as follows. From H, toward D, let the arc of a circle AB be described at any interval, from which let an arc AB be cut off equal to the elevation of the pole. For the line HB, once drawn, will be the line of the 12th hour, and it will pass through I. Then, from the centre E of that circle in the figure of the preceding chapter — by whose aid we found the hour-points on the equinoctial line FK — let the intervals be taken as far as the points where the equinoctial line FK is cut by the hour-lines, and let them be transferred from the point D of the figure just constructed onto the Equator's ray DC, marking points on the Equator's ray itself, through which lines are then drawn out from H. Finally, through H let HV be drawn parallel to the Equator's ray DC; and if need be, the lines proceeding from H and cutting the Equator's ray DC are to be carried over, with the aid of the compasses, to the left-hand parallel AV. This is easily done if, from centre H, an arc of a circle LQ is described cutting the parallel HV at P; for if we transfer the distance PL from P to Q, the line drawn from H through Q will have the same distance from the line HV that the line HL has, and so on. For the lines proceeding from H will be the lines of the other hours after the 12th hour, whose number corresponds to the number of those intervals which were transferred onto the Equator's ray from the centre E of the figure of the preceding chapter — so that the line nearest to HB gives the 1st and 11th hour, the next the 2nd and 10th, and so on. But if we transfer the hour-lines lying between H (the centre of the dial of the preceding chapter) and the

equinoctial line FK onto the Equator's ray DC in the next figure, from H, marking points on the Equator's ray DC, and through these draw straight lines from H, we shall obtain the same hour-lines.

[Margin: Another way of drawing the hour-lines in the figure of the rays of the Zodiac.]

We shall also very conveniently draw these hour-lines proceeding from H without transferring the hour-intervals between the centre E (or H) and the equinoctial line FK onto the Equator's ray DC, in this way. On the line HI, extended, we shall take any point * [printed as a solid asterisk-like point-mark], through which we shall draw a line *V parallel to AH, and through H another parallel HV to DC, which cuts *V at V. Then, with V as centre and V* as radius, we shall describe a circle and divide it into 24 equal parts, beginning from the point *. For if we join any two points equally distant from * by hidden straight lines cutting the line *V at points, the lines drawn from H through these points will give the hour-lines, as before. And if the several arcs of the circle described from V are each bisected — and divided into four equal parts — we shall in the same way draw, in the figure of the rays of the Zodiac, the hour-lines that pertain to the half-hours and their quarters.

[Margin: Yet another, most elegant way of drawing the hour-lines in the figure of the rays of the Zodiac.]

[Margin: How one may recognize from the figure of the rays of the Zodiac which of the arcs of the signs to be drawn on the dial are hyperbolas, parabolas, or ellipses.]

From this figure of the rays of the Zodiac we shall easily understand which of the arcs of the signs to be drawn on the dial are hyperbolas, and which parabolas or ellipses. For whenever the line HB cuts two rays of opposite signs — such as Cancer (♋) and Capricorn (♏); Gemini (♊) and Sagittarius (♐)[the second glyph worn — pinned by the "opposite signs" context] [?]; Taurus (♉) and Scorpio (♏) [likewise worn — pinned as the opposite of Taurus] [?]; and so on — the arcs of those opposite signs are opposite and equal hyperbolas. But when the same line HB does cut the ray of any given sign between the lines HV and DC, yet is parallel to the ray of the opposite sign, then the arc of the sign whose ray is cut is a parabola, while the arc of the other, opposite sign cannot be drawn. And finally, when the line HB cuts one ray between the lines HV and DC, but with respect to the ray of the opposite sign is neither parallel to it nor cuts it (unless it is

extended beyond the point D), then the arc of the sign whose ray is cut is an ellipse, while the arc of the other, opposite

Diagram. The figure of the rays of the Zodiac together with the hour-lines (*figura radiorum Zodiaci*), a dense construction filling the outer half of the page. Along the top horizontal line (the axis **AH**) run the points **H**, **D** (repeated), and **A** at the right end, with the perpendicular Equator's ray descending from D. At the right is a small subsidiary construction carrying the letters **H**, **L**, **D**, **G**, **K**, **M**, and **I** (the triangle HDI / DLM of the style-scaling construction). Within the fan of lines appear the further construction points **C**, **R**, **N**, **B**, **P**, **V**, and **Φ**, and two rays are labelled "Rad. hor." (*radius horæ*, "ray of the hour") — one apparently "0", one "10". The radiating lines are numbered with the hours (numerals from 1 to 12 are visible), and small zodiac sign-glyphs are set along the NO arc near the centre. The figure is too densely overprinted for every peripheral label to be resolved with confidence.

- H, D, A: points on the axis AH (top horizontal); Equator's ray DC perpendicular at D
- Right-hand subsidiary triangle: H, L, D, G, K, M, I
- C, R, N, B, P, V, Φ: construction points within the fan
- "Rad. hor." ray-labels (*radius horæ*): apparently 0 and 10 [?]
- Hour-numbers 1–12 along the radiating lines
- Small zodiac sign-glyphs along the NO arc near the centre [several not individually resolved] [?]

[Diagram fig-p497-1: more than 2 peripheral labels unresolved]

sign cannot be drawn. All of which we demonstrated in Book 1. We shall recognize the same thing in the other dials, if — on declining dials — we take the style-line in place of the meridian line, and in the figure of the rays of the Zodiac the index-line in place of the line of the 12th hour, as will be clear from what follows. In that case the line HB (which on the horizontal dial is the line of the 12th hour, but on declining dials is the style-line) will cut the rays of opposite parallels whenever the angle DHI — the elevation of the pole above the plane of the dial, which the line HB makes with the axis AH — is smaller than the angle ADN, the opposite and exterior angle which the ray DN of the opposite sign makes with the same axis AH (an angle equal to the complement of the declination of the opposite sign). But when the said two angles are equal, the line HB will be parallel to the ray of the opposite sign. And finally, when the former is greater

than the latter, the line HB will neither cut the ray of the opposite sign nor be parallel to it, as is established from proposition 28 of Book 1 of Euclid [Euclid].

[Margin: The drawing of the arcs of the signs on the horizontal dial.]

These things thus accomplished, we shall draw the arcs of the signs in the following way. From the figure of the rays of the Zodiac constructed so far, we shall take the portion of the 12th-hour line HB intercepted between the point H and the ray of Cancer (♋), and transfer it into the dial — which for this purpose ought to be drawn with hidden lines, so that afterwards only those lines lying between the arc of Cancer and the arc of Capricorn (♑) become visible, since all the others are superfluous (inasmuch as the style's shadow cannot fall upon them) — carrying it from the centre H onto the meridian line toward the equinoctial line, and marking a point on the meridian line. Then in the same way we shall take the portion of the line of the 1st and 11th hour lying between the same point H and the ray of Cancer, and carry it over into the dial from the centre H onto the lines of the 1st and 11th hour toward the equinoctial line, again marking two points on these hour-lines. And so we shall proceed with the remaining hour-lines that cut the ray of Cancer, always transferring the intervals of them enclosed between H and the ray of Cancer into the dial, from the centre H onto the corresponding hour-lines, marking points upon them. It must be observed here, however, and in what follows, that the intervals of the lines falling from H toward the Equator's ray DC are to be transferred from the centre of the dial H onto those hour-lines which run from H toward the equinoctial line Fk, as we have done in the lines of the 12th, 1st, and 11th hours, and so on. But the intervals of the lines falling from H onto the other side of the line HV — such that they cannot cut the Equator's ray DC — are to be transferred onto the dial's lines from H, onto those lines which, beyond the centre of the dial, ever more and more

Diagram. The figure for drawing the arcs of the signs on the horizontal dial. From a point **H** at the top centre, rays fan downward across the width of the page. Flanking the fan are two tall boxed panels of six cells each, labelled in vertical lettering: on the left **ASCENSVS SOLIS** ("the ascent of the Sun") and on the right **DESCENSVS SOLIS** ("the descent of the Sun"). The left panel's cells carry, top to bottom, the zodiac glyphs ♊ (Gemini), ♉ (Taurus), ♈ (Aries), ♛ (Pisces), ♒ (Aquarius), ♑ (Capricorn); the right panel's cells carry, top to bottom, ♋ (Cancer), ♌ (Leo), ♍ (Virgo), ♎ (Libra), ♏ (Scorpio), ♐ (Sagittarius). Within the fan the construction points **F** (left), **K** (right of centre), **D**, **G**, **M** near the centre, and **Φ** (lower centre) are visible, with **X** labelling a ray toward the upper left. The rays

are numbered with the hours (6, 5, 4 toward the left; 6, 7, 8 toward the right; and the smaller numerals 1–3 and 9–12 near the centre foot).

- H (top centre): the dial's centre, from which the rays issue
- Left panel **ASCENSVS SOLIS**: Π $\propto$ Υ $\times$ $\approx$ $\mathfrak{V}$ (top to bottom)
- Right panel **DESCENSVS SOLIS**: $\mathfrak{S}$ $\mathfrak{Q}$ $\mathfrak{M}$ $\mathfrak{L}$ $\mathfrak{N}$ $\mathfrak{X}$ (top to bottom)
- F, K, D, G, M, Φ : construction points within the fan
- X: a ray-endpoint toward the upper left
- Hour-numbers along the rays: 4, 5, 6, 7, 8 to the sides; 1–3, 9–12 near the centre foot
- A few small labels near the crowded centre not individually resolved [?]

[Diagram fig-p498-1: more than 2 crowded central labels unresolved]

recede from the equinoctial line — as here onto the hour-lines that fall above the line of the 6th hour and do not cut the equinoctial line, such as, in the example, the lines of the 7th hour after noon and the line of the 5th hour after midnight, and so on. The points of these, however, are not marked on our dial, on account of the narrowness of the space. Once the points have been marked in this way on the hour-lines, if we draw through them a properly curved line that nowhere forms angles, we shall have drawn the arc of Cancer, which the tip of the gnomon's shadow describes when the Sun is at the beginning of Cancer. In no other way shall we draw the arcs of the other signs: we transfer the hour-intervals lying between H and the rays of the signs into the dial, from the centre H onto the corresponding hour-lines, and so on. We shall delineate the same arcs of the signs if we transfer the hour-intervals comprised between the Equator's ray and the rays of the other signs onto the corresponding hour-lines in the dial, beginning from the equinoctial line, and so on.

[Margin: Another way of drawing the arcs of the signs.]

Now if, from the outlines described so far, a larger or smaller dial is to be laid out — according as a gnomon larger or smaller than DG is given — it will be done in this way. Let the triangle HDI — contained in the dial of the preceding chapter, or in the figure of the rays of the signs of this chapter — be drawn separately, together with the style DG; and from the line DG, extended, let a line DK be cut off equal to the smaller or larger proposed style; and through K let LM be drawn parallel to HI, cutting DH and DI, extended, at L and M. Then from the triangle DLM let the axis DL be taken and transferred into the figure of the rays of the Zodiac of this chapter, from H as far as D, so that HD is equal to the axis DL of the said triangle. If, then, from D a perpendicular line DM is dropped to HD, for the

Equator's ray, and from D the rays of the other signs are drawn out — as we taught in this chapter (though, to avoid confusion of lines, they have not been drawn in the figure) — the arcs of the signs will be drawn, in proportion to the given style DK, as before, provided the intervals of the hour-lines proceeding from H, lying between H and the rays of the signs now described (which hour-lines indeed ought not to be changed, even if a larger or smaller dial is to be drawn, at the same elevation of the pole), are transferred onto the hour-lines...

[Margin: By what method a larger or smaller dial, for a larger or smaller style, is drawn from the figure of the rays of the Zodiac.]

drawn (as concealed construction lines) onto the hour-lines of the dial — which likewise remain unchanged at the same elevation of the pole — from the point H, and so on.

The equinoctial line, however, must be drawn on the dial perpendicular to the meridian line, through a point that lies as far from the dial's centre H as the segment HM in the figure of the zodiacal rays — the segment intercepted between H and the ray of the Equator just now drawn. And just as, on the dial, the segment HI between the dial's centre and the equinoctial line is equal to the segment HI in the figure of the zodiacal rays between H and the ray of the Equator, so the triangle HDI is drawn beside the figure of the zodiacal rays.

How the arcs of the southern signs may be drawn more accurately from the arcs of the northern signs — and also by what method the two arcs of two opposite signs can be depicted by one and the same operation (namely when, in the figure of the zodiacal rays, the straight line HB cuts the rays of the opposite signs), a thing both very pleasant and very useful to know — you will find explained in Book 2, Proposition 2.

[Margin: The drawing of the arcs which the shadow traverses when the Sun, outside the beginnings of the signs, stands at other points of the Zodiac.]

Now if we take the declinations of other points of the Zodiac on the arc NO of the figure of the zodiacal rays, and send out their rays from the point D, we shall draw the arcs of those points on the dial — the arcs, that is, that the tip of the shadow traverses when the Sun stands at those points of the Zodiac — by the same technique with which we drew the arcs of the signs.

[Margin: The drawing of the arcs which the shadow traverses on any given day of the month.]

Indeed, take the first day of any month (or any other day you please): find the Sun's place in the Zodiac on that day and its declination, and, allowing for the declination of that point, draw from the point D of the aforesaid figure of the zodiacal rays the ray of that zodiacal point which the Sun occupies. In the same way there will be drawn on the dial a certain curved line, which the tip of the shadow traverses on that appointed day.

[Margin: The drawing of the saints' feast-days on the dial.]

By this method the feast-days of the saints can be drawn on the same dial — that is, certain curved lines that the style's shadow traverses on those days. Indeed the parallels of cities lying between the two tropics will be drawn in the same way, if their rays are drawn, and so on.

[Margin: What may be attached to the arcs of the signs drawn on the dial.]

To all these arcs — or curved lines — one may attach the characters of the signs, the days of the months on which the shadow's tip traverses them, and also the time of sunrise, the lengths of the days and of the twilights, and other such things; as, in the Italian and Babylonian dials (which we shall treat a little further on), the time of noon, and so on. For this purpose certain lines must be drawn parallel to the meridian line itself, both to the left and to the right; and the farther these are from the meridian line, the greater the interval of time at which the gnomon's shadow will be caught on the dial, as appears in the dial of this chapter.

Finally, since the Sun ascends from Capricorn [♑], through Aries [♈], up to Cancer [♋], and from Cancer descends through Libra [♎] down to Capricorn, the words ASCENT OF THE SUN [ASCENSVS SOLIS] can be set beside the first six signs, and beside the latter six the words DESCENT OF THE SUN [DESCENSVS SOLIS].

DE ARCVBVS LONGITVDINVM DIERVVM in eodem horologio horizontali describendis.

[On drawing the arcs of the day-lengths on the same horizontal dial.]

CAPVT III.

[Margin: In what way the rays of the day-lengths are drawn.]

Just as in the previous chapter we drew the arcs that the tip of the style's shadow traverses when the Sun is at the beginnings of the signs, or at any other point of the Zodiac, so now we shall delineate other arcs, which the same shadow describes when the artificial day contains a proposed number of hours — such as 10, or 11, or 13, and so on. These arcs are drawn in exactly the same way as the arcs of the signs, provided that, in the figure of the zodiacal rays of the preceding chapter, in place of the rays of the signs one draws from the point D the rays of the day-lengths.

These rays of the day-lengths will be drawn by the following device. Draw the straight line AB; from A describe an arc of the circle CBD, and on it, starting from the point B, lay off on either side the arc equal to the complement of the pole's elevation, as far as the points C and D. Then draw the straight line CD, cutting AB at E; and from the centre E, with radius EC (or ED), describe a circle. Divide it into 48 equal parts, and join by straight lines any two points equally distant from C (or from D); these lines cut the line CD at points through which the straight lines drawn from A will be the rays of the day-lengths. They will also indicate the declinations of those parallels whose diurnal arcs comprise the said day-lengths — so that the straight line AD is the ray of a diurnal arc of no hours (that is, the ray of that parallel which is always the greatest of the hidden ones), the next line is the ray of a diurnal arc of one hour, the one after of two hours, and so on; so that the ray of the Equator, AB, is that of 12 hours, and AC that of 24 hours — that is, the ray of that parallel which is always the greatest of the visible ones.

We have drawn these rays of the day-lengths here separately, and not in the figure of the zodiacal rays of the preceding chapter, lest the multitude of lines should breed confusion. Therefore, to draw the length-arc of any day whatever, take in this figure the arc between B and the proposed ray, and cut off an equal one from the arc NO in the figure of the zodiacal rays of the preceding chapter, beginning from the ray of the Equator. This will easily be done if, in the figure of the zodiacal rays, one describes from the point D an arc with radius equal to the semidiameter AB of the arc CBD of the figure of this chapter. For then, in the arc thus described in the figure of the zodiacal rays, one must take an arc equal to the arc between B and the proposed ray, and through the end of this arc a line must be drawn from D — as a concealed construction line — to serve as the ray of that day. For this line will cut off from the arc NO a similar arc, and so on. From this ray the arc of that day will be drawn, just as the arcs of the signs were drawn from the rays of the signs, and so on. Into the figure of the zodiacal rays of the preceding chapter we have trans-

Diagram. A construction for the rays of the day-lengths. A circle is drawn with horizontal diameter CD and centre E; A stands at the top and B at the bottom of the vertical diameter, so that A, E, B are collinear. From the apex A a fan of straight lines (the rays of the day-lengths) is drawn down to division points marked along the lower arc CBD. The arc is graduated with day-length hour numbers.

- A (top apex): vertex from which the rays are emitted
- B (bottom of arc): point of the 12-hour (Equator) ray; ray AB
- C (left end of diameter): endpoint of the 24-hour ray; ray AC
- D (right end of diameter): endpoint of the 0-hour ray; ray AD
- E (centre): intersection of CD and AB
- Along the lower arc, day-length hour numbers are marked. Legible on the left (toward C): 16, 15, 14, 13; at the bottom near B: 12; toward the right (D side): 11, 8, 7, 6, 5. The intervening numerals 9 and 10 are present but not fully resolvable [numeral:?].

[Margin: The drawing of the arcs of the day-lengths on the horizontal dial.]

—ferred the rays of the day-lengths of 14 and 10 hours, likewise of 18 and 6 hours, and of 24 and 0 hours, marked with certain dots so that they may not be confused with the rays of the signs; for it is chiefly these that we shall need in what follows. And from what we wrote in the preceding chapter we shall easily judge which diurnal arcs are hyperbolas and which are parabolas or ellipses, if — in place of the rays of opposite signs — two opposite diurnal rays are drawn from the point D in the figure of the zodiacal rays of that chapter, that is, rays equally distant from the ray of the Equator.

DE HOROLOGIO HORIZONTALI Italico, & Babylonico.

[On the horizontal dial, Italian and Babylonian.]

CAPVT IIII.

[Margin: The description of the horizontal Italian and Babylonian dial.]

Let the circle described about the centre E in chapter 1 be cut into 24 equal hours, beginning from the point N; and on both sides of N count off the semidiurnal arc of the parallel of Cancer — at latitude 42 degrees this is 7 hours 32 minutes — as far as the points a and b, and draw the straight line ab, in such a way that, once the circle NaMb is set as the parallel of Cancer, the arc aNb is the portion of it

above the horizon, and aMb the portion below the earth. Again, on both sides of N reckon off the semidiurnal arc of the parallel of Capricorn — at the same latitude of 42 degrees this is 4 hours 28 minutes — as far as the points d and e, and draw the straight line de, so that, with the circle NdMe set as the parallel of Capricorn, the arc dNe is the portion of it above the horizon, and dMe the portion below the earth. The same circle MadNeb can also be divided in another way into the diurnal arc of Cancer and of Capricorn, as we taught in the scholium to Proposition 1 of Book 1.

By means of this circle we shall construct the Italian dial in the following way. Let the said circle be divided into 24 equal hours, beginning from the point b — which, when the dial is set in its proper position, faces west. Through the points of division and the centre E draw concealed straight lines cutting the equinoctial line at points; and if through these points other concealed lines are drawn from H, the dial's centre, the tropic of Cancer will be cut at points through which are to be drawn

Diagram. The horizontal Italian-and-Babylonian dial construction — a dense figure showing the meridian circle, the two tropic-parallel circles, the equinoctial line, and the fan of hour-lines converging on the dial centre. Because the figure is very crowded and many of its small labels do not resolve cleanly even at high magnification, several labels are flagged.

- M (top): upper end of the vertical meridian diameter
- N (bottom): lower end of the meridian diameter (start of the hour-division)
- H (left, on the horizontal line): the dial's centre
- n (right end of the horizontal / equinoctial line) [?]
- C (near the middle, at the line-crossing): construction point near the centre; an adjacent Greek label α [?]
- a, b (left side): endpoints of the semidiurnal arc of the parallel of Cancer (circle NaMb)
- d, e (left side, lower): endpoints of the semidiurnal arc of the parallel of Capricorn (circle NdMe) [e uncertain: ?]
- m, t (left side): hour-points used in the worked example
- Greek point-labels visible in the figure: α , β , λ [?] (with δ used in the text)
- Large hour-numerals around the rim: on the left 6, 5, 4; on the right 6, 7, 8, 9
- A fan of smaller hour-numerals (running 1–24) is marked along the two tropic arcs; individual values are not cell-by-cell resolvable [numeral:?].

the hour-lines reckoned from sunset, and these should be carefully noted. For example, through the point *f* — the point of the 18th hour from sunset — and the centre *E*, the straight line *fE* drawn cuts the equinoctial line at *g*; and the straight line *Hg* drawn cuts the arc of Cancer at the point *h*, through which the line of the 18th hour from sunset is to be drawn on the dial. The same procedure is to be followed for the rest.

But the hour-points contained between *b* and *u*, and between *a* and *t* — which in our example are the hours 23, 9, and 10 — will be found on the tropic of Cancer in the following way. From the hours that are diametrically opposite these hours in the circle described about *E* (the opposite hours are obtained by adding 12 to those hours, but casting out 24 whenever the sum comes to more than 24 — thus the hour 9 is opposite the hour 21, since 9 and 12 make 21; likewise the hour 23 is opposite the hour 11, because 23 and 12 make 35, from which, if 24 is thrown away, 11 remains, and so on) — from those opposite hours, then, there must be drawn through the centre *E* straight lines cutting the equinoctial line at points; and if straight lines are drawn from *H* through these points, the tropic of Cancer will indeed be cut at the points corresponding to those opposite hours. But if those same lines are extended beyond the centre, the same arc of Cancer will be cut at the points corresponding to the original hours, which are comprised in the arcs *bu* and *at*.

For example: from *m*, the point of the 11th hour (which is diametrically opposite the 23rd hour), a straight line drawn through the centre *E* cuts the equinoctial line at *n*; a straight line *Hn* then cuts the tropic of Cancer at *q*, the point of the 11th hour; and finally, the line *nH* extended beyond *H* cuts the same tropic at the point *p*, through which the line of the 23rd hour is to be drawn. By the same device we shall find, on the same tropic, the points for the 9th and 10th hours, if from the 21st and 22nd hours (which are opposite them) we draw straight lines through the centre *E*, and so on.

The reason for this is that on the tropic there are found, in the first place, the points corresponding to those points of the circle *MaNb* from which the straight lines drawn through *E* pass through *E* before they cut the equinoctial line. Hence, since the straight lines drawn through *E* from the points of the arcs *bu* and *at* do not pass through *E* before they cut the equinoctial line, they

—will first meet, on the tropic, the points corresponding to those opposite points. But then, if those concealed lines are extended through *H*, the points corresponding to the said points will also be found, as we showed in Book 2.

Now if the same circle described about E is cut into 24 equal hours, beginning from the point e (which likewise faces west when the dial takes up its proper position), and straight lines are drawn through the points of division and the centre E, the equinoctial line will be cut at points; and if straight lines are again drawn from H through these points, the tropic of Capricorn will be cut at points through which the hour-lines from sunset are to be drawn. For instance, if from the point δ — the point of the 23rd hour on the tropic of Capricorn — a straight line is drawn through the centre E, cutting the equinoctial line at ϵ , then by the line He extended we shall find, on the arc of Capricorn, the point λ for the 23rd hour from sunset, and so on. And so, if the corresponding hour-points found on the tropics are joined by straight lines, the Italian dial will be drawn.

Furthermore, in the circle MaNb, if the division into 24 parts begins from the point b, then the point b belongs to the 24th hour of Cancer reckoned from sunset, the next point toward N to the 23rd hour, the following one to the 22nd, and so on. Likewise, if the division starts from the point e, the point e corresponds to the 24th hour of Capricorn from sunset, the next toward N to the 23rd, the one after to the 22nd, and so on.

The hour-lines reckoned from sunset will necessarily pass through the points of the equinoctial line where it is cut by the hour-lines reckoned from noon or midnight. Thus the line of the 23rd hour from sunset passes through the point of the 5th hour from noon, and the line of the 22nd hour through the point of the 4th hour, and so on — as is clear from the table we set out in the scholium to Proposition 19 of Book 1, whose title is [THE EQUINOCTIAL LINE / ÆQVINOCTIALIS LINEA], which we have here repeated so far as the whole hours are concerned, so that you may see which hours reckoned from sunrise or sunset, and which reckoned from noon or midnight, intersect one another on the equinoctial line of any dial whatever — and hence through which hours from noon or midnight the hours from sunrise or sunset are to be drawn on the equinoctial line. For each set of three hours in this little table passes through one and the same point of the equinoctial line. The same is to be understood of the fractions of the hours.

Table heading: The hours reckoned from sunrise or sunset, and from noon or midnight, that cut one another at one and the same point of the equinoctial line.

											1	1	1
Hours from sunrise or sunset	I	2	3	4	5	6	7	8	9	0	1	2	
Hours from noon or midnight	VI	VIII	I	X	X	XI	I	I	III	I	V	V	

Hours from sunrise or sunset	I	2	3	4	5	6	7	8	9	10	11	12
	I		X		I	I		I		V		I
Hours from sunrise or sunset	13	14	1	1	1	18	1	2	21	2	2	2
			5	6	7		9	0		2	3	4

But if the hour-points on the tropic of Cancer have no corresponding points on the tropic of Capricorn — as in our example are all the hours before the 16th hour from sunset — the hour-lines must be drawn through the points found on the tropic of Cancer and through the corresponding points on the equinoctial line. These points are found from the aforesaid table given in the scholium to Proposition 19 of Book 1, or from the table just placed here. Thus the line of the 11th hour from sunset must be drawn through the 23rd hour from sunset on the equinoctial line — from which, of course, it is distant by 12 hours; likewise the 10th hour from sunset is drawn through the 22nd hour, the 9th through the 21st, and so on. But the line of the 12th hour from sunset must be drawn through the point found on the tropic of Cancer, parallel to the equinoctial line — that is, to the line of the 6th hour from noon or midnight.

By a not dissimilar technique we shall make the Babylonian dial, if we begin the equal hours from the points a and d, which — when the dial is set in its proper position — lie toward the east: so that the point a belongs to the 24th hour from sunrise on the tropic of Cancer, the next point toward N to the 1st hour from sunrise, the following to the 2nd, and so on. Likewise let the point d denote the 24th hour from sunrise on the tropic of Capricorn, and the nearest point toward N the 1st hour, and so on. For by these points of division we shall find, on each tropic of the dial, the hour-points reckoned from sunrise, just as we said a moment ago of the hour-points reckoned from sunset; and so we shall make the Babylonian dial.

[Margin: The construction of the horizontal Babylonian dial.]

Each dial can also be drawn in another way, from the tables of the diurnal and nocturnal arcs which we compiled in the scholium to Proposition 33 of Book 1. For from these one will understand, quicker than it takes to say, which hours reckoned from sunrise or sunset, and which from noon or midnight, cut one another at one and the same point of the said diurnal and nocturnal arcs. But we have thought it superfluous to repeat those tables here, since they can easily be taken from the said scholium.

[Margin: The drawing of the same Italian and Babylonian dial by means of the diurnal arcs.]

So then, let two diurnal arcs be drawn on the astronomical dial — one of 14 hours and the other of 10 hours — as we taught in the preceding chapter. It will be enough if, at the hours from noon or midnight, the points are marked through which the said arcs must pass, even if the arcs themselves are not drawn. Then let the tables of the diurnal arcs of 10, 12, and 14 hours, which we set out in the scholium to Proposition 33 of Book 1, be carefully examined. For from them you will at once learn through which hours from noon or midnight of the said arcs the hour-lines reckoned from sunrise and sunset are to be drawn.

Thus you see that the 1st hour from sunrise is to be drawn through the 6th hour from midnight on the arc of 14 hours, through the 7th hour from midnight on the arc of 12 hours (which on the dial does not differ from the equinoctial line), and through the 8th hour from midnight on the arc of 10 hours. Likewise the 23rd hour from sunset must be drawn through the 6th hour from noon on the arc of 14 hours, through the 5th hour from noon on the arc of 12 hours, and through the 4th hour from noon on the arc of 10 hours; and so for the rest. But those hours which have no corresponding points on the arc of 10 hours — as in our example are all the hours from sunrise after the 9th hour from sunrise, and all the hours from sunset before the 15th hour from sunset — must be drawn through the points on the arcs of 14 and 12 hours. But the 12th hour from sunrise or sunset must be drawn parallel to the equinoctial line, through the 5th hour from noon and through the 7th hour from midnight on the arc of 14 hours, as is clear from the table of the said arc.

But if it should ever happen that, in the said tables of diurnal arcs, only a single point — or none at all — is found for some hour reckoned from sunrise or sunset that is to be drawn, we shall draw, by means of the same tables, the hour of the other name but of the same number, that is, the hour reckoned from sunset or from sunrise. For this line, extended to the other side of the tropic, will give the required hour reckoned from sunrise or sunset. For example: since the 13th hour from sunrise has, on the arc of 14 hours, only one point — namely the 6th hour from noon — we shall draw the 13th hour from sunset through the 8th hour from midnight on the arc of 14 hours, and through the 7th hour from midnight on the arc of 12 hours, as is established from the tables of the said arcs. For this line, extended to the other side of the tropic, will give the 13th hour from sunrise. Now this 13th hour from sunset, extended through the 6th hour from noon on the arc

of 14 hours, will necessarily pass through the point by which the 13th hour from sunrise must be drawn, as the table of the arc of 14 hours indicates.

Likewise, since the 14th hour from sunrise has no point on the aforesaid diurnal arcs, we shall draw the 14th hour from sunset through the 9th hour from midnight on the arc of 14 hours, and through the 8th hour from midnight on the arc of 12 hours, as is gathered from the tables of these arcs. Again, since the 11th hour from sunset has only one point on the arc of 14 hours — namely the 6th hour from midnight — we shall draw the 11th hour from sunrise through the 4th hour from noon on the arc of 14 hours, and through the 5th hour from noon on the arc of 12 hours, as is drawn out from the tables of the said arcs. For this line, extended to the other side of the tropic of Cancer [♋], will show the 11th hour from sunset, and will necessarily pass through the 6th hour from midnight, by which, that is to say, the 11th hour from sunset must be drawn. And the same reasoning is to be applied to the rest.

[Margin: The construction of the same horizontal Italian and Babylonian dial from the diurnal arc of 24 hours.]

Now the hours reckoned from sunrise and sunset can also be described most conveniently if the diurnal arc of 24 hours is drawn. For straight lines drawn through the hours from noon or midnight on that arc, according to the table of the diurnal arc of 24 hours, and through the hours from noon or midnight on the diurnal arc of 12 hours, will give the hours from sunrise and sunset. For the 23rd hour from sunset passes through the 11th hour from noon on the diurnal arc of 24 hours, and through the 5th hour from noon on the diurnal arc of 12 hours, that is, on the equinoctial line. The 22nd hour from sunset passes through the 10th hour from noon on the arc of 24 hours, and through the 4th hour from noon on the arc of 12 hours. Likewise the 1st hour from sunrise is drawn through the 1st hour from midnight on the arc of 24 hours, and through the 7th hour from midnight on the arc of 12 hours; and so on, as is clearly established from the tables of the said arcs. But the 12th hour from sunrise or sunset is drawn parallel to the equinoctial line — that is, at the 6th hour from noon or midnight — through the point where the arc of 24 hours and the meridian line cut one another, as is established from the same table of the diurnal arc of 24 hours. Indeed, all the hour-lines from sunrise and sunset touch the diurnal arc of 24 hours at the points where it is cut by the hour-lines from noon or midnight that correspond, according to the table of that same arc, to the hours from sunrise and sunset — as is established from what we demonstrated in Proposition 14 of Book 1.

Moreover, once some one hour from sunrise or sunset has been drawn according to the tables of the diurnal arcs, we shall draw the others without great labour through the following points, which correspond to one another, with no further consideration of the tables of the diurnal arcs. For instance, having drawn the 23rd hour from sunset, we shall draw the 22nd through the nearest point on the diurnal arc of 14 hours, through the nearest on the arc of 12 hours, and through the nearest on the arc of 10 hours; likewise the 21st through the other points next following, as appears in the example set out above.

Furthermore, from the very description of the Italian and Babylonian dial it is clear that these two dials differ from one another only in position and in the order of the hours: so that the Italian dial can also serve us for the Babylonian, provided it is inverted, so that the left part becomes the right and vice versa, and so that from the line of the 23rd hour from sunset the 1st hour from sunrise is made, and the 2nd hour from sunrise is made from the 22nd from sunset, and so on. In the example set out above both the astronomical and the Italian hours are contained, together with the Babylonian. The astronomical are hidden, marked only by points; the Italian are 23, 22, 21, and so on; the Babylonian, finally, are 1, 2, 3, 4, and so on. Here too you see that the Italian hours, when extended, give the same hours in number as the Babylonian, and vice versa.

The things we said in Book 1 about the mutual intersection of the hour-lines from noon or midnight with the hour-lines from sunrise or sunset will also contribute very much to the correct description of the Italian and Babylonian dial. For this purpose we also constructed various tables in the scholia to Propositions 19 and 20 of that same book. And in what way both the hours from sunrise and those from sunset are to be described by means of these tables, or, once described, examined, we explained abundantly in Propositions 10 and 11 of Book 2 — so that there is no need to hammer in again here what we wrote there on this matter.

DE HOROLOGIO HORIZONTALI Antiquo.

ON THE ANTIQUE HORIZONTAL DIAL.

CAPVT V.

CHAPTER V.

[Margin: The description of the antique horizontal dial.]

Let the circle drawn from E be cut into the diurnal arc of the tropic of Cancer [☿], a N b, and into the diurnal arc of the tropic of Capricorn [♊], d N e, as we taught at the beginning of the preceding chapter. Then let the diurnal arc of Cancer, a N b, be divided into 12 equal parts, beginning from the point a, the eastern one. For if through the points of division and the centre E we draw hidden straight lines, the equinoctial line will be cut at points through which, if again from H, the centre of the dial, other hidden straight lines are sent out cutting the tropic of Cancer, points will be found through which the lines of the unequal hours must be drawn — exactly as in the preceding chapter we found, on the same tropic, the points of the hours from sunrise and sunset. And if the diurnal arc of Capricorn, d N e, is likewise divided into 12 equal parts, beginning from the point d, the eastern one, we shall find in the same way, on the tropic of Capricorn, the points of the unequal hours. If, then, the corresponding points are joined by straight lines, the antique dial will be described.

Now all the lines of the unequal hours will pass through the hours from noon or midnight on the equinoctial line. For example: from f, the point of the 3rd unequal hour of the tropic of Capricorn, a straight line drawn through E cuts the equinoctial line at g; and a straight line sent out from H through g cuts the tropic of Capricorn at the point h, through which the line of the 3rd unequal hour must be drawn. Again, from the point m, which is opposite the 11th unequal hour of the tropic of Cancer — (for the 11th unequal hour of the tropic of Capricorn is contained in the arc b u; and thus its point on the tropic of Cancer must be investigated by means of the opposite point, as we said, concerning the hours from sunrise and sunset contained in the arcs b u and a t, in the preceding chapter) — a straight line sent out through E cuts the equinoctial line at the point n (which, however, is not contained in the example on account of the narrowness of the space), and the straight line H n cuts the tropic of Cancer at the point p, which belongs to the assumed point m; and n H, extended beyond H, the centre of the dial, cuts the same tropic of Cancer at q, the point of the 11th unequal hour, and so on.

Diagram. A construction figure for the antique horizontal dial: a full circle whose vertical diameter runs from M at the top to a point labelled "N I" [?] at the bottom, with a horizontal diameter carrying the hour-divisions. The centre is E (lower-central), and a dense fan of solid and dotted straight lines radiates from the central region to points on the rim and to the tropic-arcs drawn within the circle. Labels read (positions approximate):

- M (top of circle, upper terminus of the vertical diameter)

- N I [?] (bottom of circle, lower terminus of the vertical diameter)
- q (upper-left, on the circle)
- a (left, on the circle)
- d (lower-left, on the circle)
- f (lower-left, on the circle, below d)
- b (right side, near a dotted line running out to the right)
- e (lower-right, on the circle)
- E (centre of the figure, at the meeting of the fan of lines)
- m / m [?] (left of centre, small gothic letter)
- h (lower-central, to the right of E)
- p (upper-right of centre)
- Hour-numbers along the horizontal diameter, left to right: 11, 10, 9, 8, 7, 6, 5, 4, 3 [?], 2, 1
- Fractional hour-labels near the top and the central cluster: $7\frac{1}{2}$ (upper-left, dashed-line terminus), $4\frac{1}{2}$ (upper-right, dashed-line terminus), and $4\frac{1}{2}$, 3, 2, 1, $7\frac{1}{2}$ clustered about the centre
- Right-hand dashed-line termini: N, and two barred-n symbols [Greek: likely π (pi)] [?] marking line-ends toward the right; u (lower-right of centre)

Several rim and interior labels are small and overwritten by crossing lines; the figure is a (more than 2 labels flagged [?]).

Now, if the six tables of the diurnal and nocturnal arcs of 18, 6, and 12 hours, constructed for the unequal hours in the scholium to Proposition 33 of Book 1, are attentively considered — from which it is known which hours from noon or midnight, whether equal or unequal, cut one another at one and the same point of the said arcs, as we explained more fully in the said scholium to Proposition 33 of Book 1 — we shall also, with easy effort, put together the antique dial, if we first describe the arc of 18 hours. For the 1st unequal hour must be drawn through the $4\frac{1}{2}$ th hour from midnight on the diurnal arc of 18 hours, and through the 7th hour from midnight on the arc of 12 hours, as is clear from the tables of the said arcs. And the 2nd unequal hour must be drawn through the 6th hour from midnight on the arc of 18 hours, and through the 8th hour from midnight on the arc of 12 hours, and so on.

[Margin: The description of the antique horizontal dial by means of the diurnal arcs of 18 and 6 hours.]

DE HOROLOGIO VERTICALI Astronomico. ON THE VERTICAL ASTRONOMICAL DIAL.

CAPVT VI.

CHAPTER VI.

[Margin: The description of the vertical astronomical dial.]

On the plane of the dial, with the place of the style taken anywhere at G, let a straight line HN be drawn through G in any direction, to serve as the meridian line; or, if the plane is fixed and parallel to the vertical circle, let a straight line HN, perpendicular to the horizon, be drawn in the same plane through G by the help of a plumb-line. For this will be the meridian line. Then, through the assumed place of the style G, let a straight line GD be drawn perpendicular to HN, and let GD be taken equal to the proposed style. Then, with D as centre, let the arc of a circle A B C be described. In it, toward A — that is, toward the upper parts, if the dial faces south (such as is the dial we bring forward as an example) — the altitude of the pole, B A, is to be numbered from the straight line D G; and toward C — that is, toward the lower parts — the complement of the same altitude of the pole, B C. But if

Diagram. A construction figure for the vertical astronomical dial, drawn as a semicircle standing on a horizontal diameter, with a dense fan of hour-lines converging on the centre H (the foot of the style). Labels read (positions approximate):

- A (top, apex of the figure)
- Hour-numbers along the upper rim/fan, left to right: 3, 2, I, 12, 11, 10, 9, 8, 7
- C (left end of a horizontal chord, at the "6" mark)
- L (right of centre, on a horizontal chord)
- 6 (left and right ends of the mid-horizontal line)
- H (centre, converging-point of the fan)
- B, F (left of centre, on stepped horizontal lines)
- G (near centre, just below H)
- D (right of centre)
- I / K (central-lower, near the intersections)
- E (below centre)
- U [?] (lower-left of centre)

- S (lower-right, on the rim)
- N (bottom-central, on the rim)
- Lower rim hour-numbers, left to right: 7, 8, 9, 10, 11 (roman "II"), then across the bottom I(?), and on the right 1, 2, 3, 4

The lower-rim numerals and several interior letters are crossed by hour-lines and are hard to resolve; the figure is a (more than 2 labels flagged [?]).

the dial faces north, the arc of the altitude of the pole, B A, must be taken downward, and the arc of the complement of the same altitude of the pole, B C, upward. And when the straight lines DA, DC have been drawn — of which the former represents the axis of the world, and the latter

[Margin: The centre of the dial. The equinoctial line. The hour-line.]

represents the common section of the equator and the meridian — the meridian line will be cut at the points H and I, of which H is the centre of the dial, while through I a perpendicular F K must be drawn to HI, to serve as the equinoctial line. Also, with the straight line I E taken equal to I D, let a circle be described from E to any interval; and this being divided into 24 equal parts, beginning from the meridian line HN, let hidden straight lines be drawn through the points of division and the centre E, cutting the equinoctial line at points through which — and through the point H — straight lines sent out will give the hour-lines from noon or midnight on the plane that is parallel to the vertical circle. But for the 6th hour a line perpendicular to HN, or parallel to the straight line FK, must be drawn through H.

Now the intervals of the hours on the portion I F of the equinoctial line are here also — if there has been no error — equal, one by one, to the intervals of the hours on the other portion I K; likewise the arcs of the circle described from H, placed between the meridian line HN and the hours on the left, are equal, one by one, to the arcs of the same circle intercepted between the meridian line and the hours on the right. Therefore, so that the description of the hours may be accurate, care must be taken that intervals of this kind are equal to one another.

[Margin: In what way each vertical dial is to be set up so that it may indicate the hours. By what means the forenoon hours are distinguished from the afternoon ones.]

The dial must be set up as follows. If indeed it inclines toward the south, HN must be perpendicular to the horizon, and the straight line F k parallel to that same horizon and lying below the point H; but if it faces north, it must be set up so that

N H is perpendicular to the horizon, and the straight line F K parallel to that same horizon and lying above the point H. And so, in the southern dial the point F will be on our left, if we turn our face to the dial, and will incline toward the west, while the point K will be situated on the right and will face the rising sun. From this it follows that the hours placed to the left of the meridian line are the forenoon hours, while those placed to its right are the afternoon hours. Now the portion of the meridian line itself, drawn out from the centre H toward the equinoctial line, will show the 12th hour of noon, and the remaining portion, beginning from the same centre, will show the 12th hour of midnight: so that the hour next to the 12th hour of noon, on the right, is the 1st hour after noon, and the next the 2nd, and so on. Likewise the hour next to the 12th hour of midnight, on the left, is the 1st hour after midnight, and the next the 2nd, and so on.

But in the northern dial the point F will be on our right and will face the east: so that the hours to the right of the meridian line are the forenoon hours, and those to the left are the afternoon hours. Now the portion of the meridian line itself, drawn out from the centre H toward the equinoctial line, will signify the 12th hour of midnight, and the remaining portion, begun from the same centre H, will signify the 12th hour of noon: so that the hour next to the 12th hour of midnight, on the right, is the 1st hour after midnight, the next the 2nd, and so on. Likewise the hour next to the 12th hour of noon, on the left, is the 1st hour after noon, the next the 2nd, and so with the rest, as appears from the figure itself. Hence it follows that the individual hours on the northern dial are the complements of 12 hours, if they are compared with the hours of the southern dial.

[Margin: In what way the style is to be set up.]

Finally, the style D G must be erected above the plane of the dial at right angles at the point G. Or, in place of the gnomon, the triangle D H I must be set up above the straight line H I at right angles to the plane of the dial itself. For the axis D H will indicate the hours by its shadow, as was said concerning the horizontal dial.

DE ARCVBVS SIGNORVM IN HORologio Verticali.

ON THE ARCS OF THE SIGNS ON THE VERTICAL DIAL.

CAPVT VII.

CHAPTER VII.

[Margin: The construction of the figure of the rays of the Zodiac, from which the arcs of the signs on the vertical dial are described. By what method the hour-lines are drawn in the figure of the rays of the Zodiac.]

In order that the arcs of the signs may be described, the figure of the rays of the Zodiac, together with the hour-lines, must be prepared in the following way. Let a line A H be drawn across in any direction, representing the axis of the world, to which let D C be drawn perpendicular, to serve as the equinoctial ray. Then let the rays of the other signs be drawn on either side, as we prescribed in Chapter 2. After this, from the triangle D H I of the figure of the preceding chapter, let there be transferred onto the axis A H of the figure just now constructed — from D, toward the right — the straight line D H, and onto the ray of the Equator, downward, the straight line D I; and from H, through I, let a straight line H I be drawn out to any length, for the 12th hour, which we shall perhaps draw more conveniently thus. From H let an arc of the circle A B be described toward D at any interval, from which let the arc A B be cut off, equal to the complement of the altitude of the pole. For the straight line H B, when drawn, will be the line of the 12th hour, and will pass through I.

Diagram. The figure of the rays of the Zodiac for the vertical dial, drawn as a large fan of straight rays issuing from an upper apex-region and cutting a semicircular arc below; several long dotted rays are labelled with the day-arc to which they belong ("Rad. hor. ...," *radius horæ* — "ray of the hour-arc"). Labels read (positions approximate):

- A (top), with D, D, D, H across the top edge, and "L H," "L" along the upper horizontal
- Left margin, top to bottom: M, I, K, G, M, B
- Interior letters: C, V, R, N, P, Q, L
- Dotted day-arc rays labelled: "Rad. hor. 24," "Rad. hor. 18," "Rad. hor. 14," "Rad. hor. 10," and further "Rad. hor." labels toward the right that do not fully resolve [?] (one appears to read "Rad. hor. 6" [?])

- Hour-numbers along the lower fan, left to right: 12, I, 2, 3, 4, 5, 6, 6, 7, 8, 9, 10 (with a roman "II" near the "12" at far left)

The interior is crowded with crossing solid and dotted lines and several labels are illegible; the figure is a (more than 2 labels flagged [?]).

But if, from the centre E of that circle — by whose help we found the points of the hours on the equinoctial line in the preceding chapter — the intervals are taken up to the points where the equinoctial line F K is cut by the hour-lines, and these are transferred from the point D of the figure just now constructed onto the ray of the Equator D C, by imprinting points on that same ray

Gnomonics, Book VII (Liber Septimus)

[Continuation of Cap. VII — describing the arcs of the signs on the vertical dial]

of the Equator, and from H, through these points, let straight lines be sent out; in this way we shall obtain the lines of the other hours, both before and after the 12th hour. If, however, for the 6th hour, HV is drawn from H parallel to the ray of the Equator D C, and — should it be needed — other lines are carried across to the right parallel to HV, as we described in Chapter 2 when we were treating the horizontal dial. We shall also obtain these hour-lines if, taking from the dial of the preceding chapter the hourly intervals lying between the centre H and the equinoctial line Fk, we transfer them from H, in the figure just now prepared, onto the ray of the Equator D C, imprinting points upon that same ray of the Equator, and then draw straight lines through them from H, and so on. Or else, if on the line HI produced we take a point ϕ anywhere at all, and through it draw a line ϕV parallel to A H, and complete the rest as was described for the horizontal dial. All of this the figure itself shows well enough. From this figure, moreover, we shall recognize which of the signs' arcs are hyperbolas and which are parabolas or ellipses, just as we set out for the horizontal dial in Chapter 2.

From this figure of the rays of the Zodiac, then, we shall describe the arcs of the signs exactly as we did on the horizontal dial: that is, by transferring the hourly intervals of this figure — those intercepted between H and the rays of the signs — into the corresponding hours of the dial, taking them from the centre H, and so on.

Now, when the Sun is on the horizon — that is, at the moment it rises or sets — the shadow of the style is cast into the common section of the horizon and of the plane of the dial, as we showed in Proposition 11 of Book 1; and this line is usually called the horizontal line. If through G, the place of the style, we draw a line parallel to the equinoctial line Fk, or perpendicular to the meridian line, this will be that very horizontal line.

The horizontal line shows, beautifully, at what hour the Sun, situated on any given parallel, rises or sets. For where the arcs of the signs cut the horizontal line, there the Sun rises and sets whenever it is in the signs that those arcs represent. If, then, through those points where the horizontal line and the arcs of the signs cut one another, hour-lines are drawn, these will indicate the times of sunrise and sunset.

Diagram. The southern ("AVSTRALE") vertical dial: a large fan of solid and dotted straight rays issuing from an apex near H at the top, cut by horizontal lines and by the dotted arcs of the signs, with the zodiac scale running down the right-hand edge. The figure is densely crossed by lines and many small labels do not resolve. Readable labels (positions approximate):

- Heading across the top: **AVST-** | **RALE** (split around the figure)
- Apex/upper region: **A, H** (vertical line rising from H); **Hori-** (*horizontalis*, the horizontal line) at left of the mid-line
- Hour-arc ray labels near the top: **IA, IIA** (i.e. rays "1A", "2A")
- **Zon** (*zona* — the equinoctial band) along the mid-line
- Interior letters: **F, R, S, K, M, L** [several further interior letters illegible — ?]
- Right-edge zodiac scale (top to bottom): ♑ [Capricorn], ♎ [Libra] (at **B**), ♍ [Virgo] (mark "np"), ♌ [Leo], and further worn glyphs [?]
- Lower fan hour-numbers (left to right, as printed): **10, Φ, 2, 3** [remaining hour-numbers of the fan illegible — ?]

More than two labels are illegible**

Diagram. The northern ("BOREALE") vertical dial: a shallower fan of dotted arcs and straight hour-lines crossing a horizontal band, with the zodiac scale down the right-hand edge. Readable labels (positions approximate):

- Heading across the top: **BORE-** | **ALE**, with **I** between the words
- **Zon** (*zona* — equinoctial band) along the mid-line
- Interior letters: **K, Q, H** [further interior letters illegible — ?]

- Hour-numbers on the fan: 6, 7 (repeated on either side) [others illegible — ?]
- Right-edge zodiac glyphs: ♍ [Virgo] (mark "np"), ♌ [Leo], and a further worn glyph [?]

More than two labels are illegible**

So you see that the Sun, when it is at the start of Capricorn, rises around hour 7½ from midnight, and sets at about hour 4½ from noon. In Aquarius, and likewise in Sagittarius, it rises a little before hour 7½ from midnight, and sets a little after hour 4½ from noon. In Pisces, finally, and in Scorpio, it rises before hour 7 from midnight, and sets after hour 5 from noon.

This same horizontal line divides the whole dial into two parts. The larger part, in which the equinoctial line is contained, is called the southern (*australe*); the other is called the northern (*boreale*). Both must be so placed that the horizontal line lies parallel to (equidistant from) the horizon and occupies the upper position. On the northern part, however, the hour-numbers are changed into their complements up to 12, as we said above; and the arcs of the southern signs are turned into the arcs of the opposite northern signs. Thus from the arc of Capricorn is made the arc of Cancer; from the arc of Aquarius and Sagittarius, the arc of Leo and Gemini; and from the arc of Pisces and Scorpio, the arc of Virgo and Taurus, and so on. Here too the horizontal line indicates that the rising of the Sun, when it is at the start of Cancer, is a little before hour 4½ from midnight, [and its setting ...] from noon [?], and so on. The horizontal line performs the same office on every other dial, except the horizontal one, on which a horizontal line cannot be drawn, as we demonstrated in the scholium to Proposition 11 of Book 1.

Again, the horizontal line divides the whole dial into those same two parts. The one that lies below it displays the daytime hours, and so may be called the daytime dial; the other part would show the nighttime hours, if the Sun's rays could pass through the earth and illuminate the plane of the dial — and for that reason we may call it the nighttime dial. Hence the portions of the arcs of the signs in the daytime dial may not inappropriately be called day-arcs, and the portions of those same arcs in the nighttime dial night-arcs. In this case, however, the hour-numbers on the nighttime dial ought not to be changed into their complements up to 12, as was done earlier on the northern dial. The same is to be understood in what follows. The nighttime dial, moreover, as being superfluous, will have to be cut away, since during the day it is not lit by the Sun.

If a larger or smaller dial is to be described from the lines so far drawn, for a larger or smaller proposed style, we accomplish this in no other way than we set out for the horizontal dial in Chapter 2. That is: describing the triangle D H I of the dial of the preceding chapter separately, together with its style D G; and taking, on the style D G produced, a straight line D K equal to the given gnomon; and through K drawing a line parallel or perpendicular to D G that cuts the lines D H and D I produced in L and M. For if the axis D L is transferred from the point H of the figure of the rays of the Zodiac up to the point D — the point from which the rays of the signs are drawn out — and the rest is done as before, the arcs of the signs will be described in proportion to the given style D k, and so on.

[Margin, top to bottom of p. 493:] [Margin: *Alia descriptio linearum horariarum in figura radiorum Zodiaci.* — "Another way of describing the hour-lines in the figure of the rays of the Zodiac."] [Margin: *Alia adhuc descriptio linearum horariarum in figura radiorum Zodiaci pulcherrima.* — "Yet another, most elegant, way of describing the hour-lines in the figure of the rays of the Zodiac."] [Margin: *Qui arcus signorum in horologio describendi hyperbolæ sint, aut parabolæ, aut ellipses.* — "Which of the arcs of the signs to be described on the dial are hyperbolas, or parabolas, or ellipses."] [Margin: *Descriptio arcuum signorum in horologio Verticali. Horizontalis lineæ [descriptio].* — "The description of the arcs of the signs on the vertical dial. [The description] of the horizontal line."] [Margin: *Hora ortus, & occasus Solis quo pacto ex linea horizontali cognoscatur.* — "How the hour of sunrise and sunset is known from the horizontal line."] [Margin: *Linea horizontalis totum horologium secat in Australe, ac Boreale.* — "The horizontal line divides the whole dial into a southern and a northern part."] [Margin: *Horizontalis linea diuidit totum horologium in diurnum, & nocturnum.* — "The horizontal line divides the whole dial into a daytime and a nighttime dial."] [Margin: *Qua ratione ad maiorem, minoremue stylum maius, aut minus horologium describatur ex figura radiorum Zodiaci.* — "By what method a larger or smaller dial is described, for a larger or smaller style, from the figure of the rays of the Zodiac."]

DE ARCVBVS LONGITVDINVM DIERVVM in eodem Verticali horologio.

[On the arcs of the day-lengths on the same vertical dial.]

CAPVT VIII.

We shall describe these arcs in the same way the arcs of the signs were described, provided that in place of the rays of the signs we take the rays of the day-lengths, which we laid out in Chapter 3. From these there were transferred into the figure of the rays of the Zodiac of the preceding chapter the rays of the day-lengths of 14 and 10 hours; likewise of 18 and 6 hours; and of 24 and 0 hours. It must be noted, however, that on the northern and on the nighttime dial these arcs are exchanged for their complements up to 24 — that is, the arc of 10 hours is changed into the arc of 14 hours, and the arc of 6 hours into the arc of 18 hours, and so on. Nor will it be difficult to judge which day-arcs are hyperbolas and which parabolas or ellipses, if in place of the rays of the opposite signs one takes the opposite day-rays drawn out from D — that is, equally distant from the ray of the Equator — just as we explained in Chapter 2.

DE HOROLOGIO VERTICALI ITALICO, & Babylonico.

[On the Italian and Babylonian vertical dial.]

CAPVT IX.

Each of these will be described just as the horizontal dial was, as we taught in Chapter 4. For if the description is to be set out by the first method, one must take, for the southern dial, the day-arc of the parallel of Cancer, in the circle described from E by a N b, and the day-arc of the parallel of Capricorn, d N e; and the division of the circle is to begin from the points b and e, the western points, if an Italian dial is to be described, or — if a Babylonian one is proposed for delineation — from the points a and d, the eastern points, proceeding toward N. One must also take the night-arc of Capricorn, on the dial above the horizontal line, so that in it may be found the points of hours 15, 14, 13, and so on from sunset, and 9, 10, 11, and so on from sunrise, which are contained in the night-arc of Capricorn described in the circle from E. For the northern dial, however, the day-arc of the tropic of Cancer will be d M e, and that of the tropic of Capricorn a M b; and the division of the circle will take its start from the points e, b, d, a, and proceed toward M. The tropic of Cancer, too, on the northern dial is the one contained between the centre of the dial H and the equinoctial line, and it

performs the office of the tropic of Capricorn on the southern dial. But let us give an example or two. From *f*, the point of hour 14 from sunset on the tropic of Cancer, a straight line drawn through *E*, the centre, cuts the equinoctial line at some point; and through that point a straight line drawn from the centre *H* will cut the tropic of Cancer, on the southern dial, at the point *h*, through which hour 14 is to be drawn. Likewise, if from hour 14 of the tropic of Capricorn — which is found in the night-arc of Capricorn, *d M e* — a straight line is drawn through *E*, and so on, there will be found in the night-arc of Capricorn the point of that same hour 14 from sunset, which, joined to the point *h*, will give hour 14. Again, a straight line from *m* — the point of hour 1 of Capricorn from sunset, which is set opposite to hour 13 from sunset — drawn through *E*, meets the equinoctial line at *n*. And since the straight line *mE* passes through the centre *E* before it meets the equinoctial line, therefore the straight line *Hn* will give, on the tropic of Capricorn, the point *q*, for hour 1 from sunset; while, produced beyond *H*, it will present on the same tropic the point *p*, for hour 13, as we set out in Chapter 4. By the same method the points of the other hours, both from sunset and from sunrise, will be found on either tropic. For the northern dial we give no example, lest the multitude of lines breed confusion — especially since the northern dial is easily derived from the southern one once it is described, in the following way. Let all the li-

Diagram. A large semicircle (the Italian/Babylonian vertical dial) with **M** at the top of the arc and **N** at the bottom, and the centre **H** near the top of the diameter; a dense pencil of straight hour-lines and dotted arcs fills the interior, with hour-numbers running down both vertical edges. Readable labels (positions approximate):

- Top of arc: **M**; bottom of arc: **N**; centre near top: **H**
- Left-edge hour-numbers (top to bottom): **22, 23**, then below the horizontal, **I (1), 2, B, 13, 14**
- Right-edge hour-numbers (top to bottom): ♄ [Capricorn], **I, 24, 23** ♀ [Gemini], **22, 21, II, 20**
- Interior letters and points: **G, q, d, E** (centre), **b, k, h, e, p, f, m**
- Small inner arc of hour-numbers near the centre: **6 · 15 · 7 · 16 · 8 · 17 · 9 · 13 · 10 · 19** [reading approximate — ?]

The interior is very crowded and several labels do not resolve**

[Margin, p. 494:] [Margin: *Descriptio arcuum diurnorum in horologio Verticali*. — "The description of the day-arcs on the vertical dial."] [Margin: *Descriptio*

horologij Verticalis Italici ac Babylonici. — "The description of the Italian and Babylonian vertical dial." [Margin: *Quo pacto horologium boreale ex australi eliciatur.* — "How the northern dial is elicited from the southern one."]

which belong to the hours from sunrise and which to the hours from sunset — will be understood as follows. Let there be considered, in the nighttime portion of the southern dial (which we said is the northern dial, if that exchange of the parts is made), the hours from sunrise and from sunset. Now all the hours drawn out from sunrise beyond the horizontal line, before they cut the meridian line, are also to be numbered from sunrise in the nighttime portion of the dial; but after they have cut the meridian line, and so cut the tropic again, they are to be reckoned from sunset. Likewise the hours drawn out from sunset are to be numbered, in that same nighttime portion, from sunset before the meridian line, but after it from sunrise. Thus, in the nighttime portion, the following hours — 23, 22, 9, 10, 11 — are to be numbered from sunrise; but these — 1, 2, 13, 14, 15 — from sunset. The same must be said on the dials that follow, if in place of the meridian line one takes the line of the index or style, which on meridian dials is the line of hour 6 from noon or from midnight, while on polar and equinoctial dials the meridian line itself serves, as will be clear in the proper places. Once these things are rightly considered, we shall easily distinguish the hours from sunrise from the hours from sunset on the northern dial. For all the hours that in the nighttime portion are reckoned from sunrise are to be numbered on the northern dial from sunset; and those that there are numbered from sunset are here to be taken from sunrise, as we showed in the scholium to Proposition 23 of Book 2.

As for the other method, which is taken from the day-arcs and night-arcs, there is no difficulty, provided the little tables of the day-arcs and night-arcs — which we set down in the scholium to Proposition 33 of Book 1 — are rightly considered. For hour 24 from sunset is drawn through hour 5 from noon on the day-arc of 10 hours, and through hour 12 from midnight (which is the hour of noon) on the night-arc of 24 hours, which cuts the meridian line at G, the place of the style; and this line of hour 24 is parallel to the equinoctial line — indeed it is the very same as the horizontal line. Hour 23 from sunset passes through hour 5 from noon on the day-arc of 12 hours, which on this dial does not differ from the equinoctial line, and so with the rest. But hour 13 from sunset, since it has no corresponding point on the day-arc of 10 hours, must be drawn through hour 6 from midnight on the night-arc of 14 hours, as is clear from the little table of this arc. Here it must be noted that the hours from noon or from midnight are now not to be changed into their complements up to 12 in the nighttime portion beyond the

horizontal line, as was done above, but are to keep the same number. In no other way shall we draw the hours from sunrise, as appears from the little tables of the day-arcs and night-arcs. For the northern dial there is no need to give an example, because it is described in the same way from the aforesaid arcs, especially since it is obtained from the southern one, as we wrote a little earlier.

Here too those things are of great use which we wrote in Book 1 concerning the mutual intersections of the hours from noon or from midnight with those from sunrise or from sunset among themselves. On this matter consult Proposition 22 of Book 2.

DE HOROLOGIO VERTICALI Antiquo.

[On the antique vertical dial.]

CAPVT X.

Let the same things be done as in Chapter 5, but with this rule: that the division of the day-arc of the tropic of Cancer, a N b, and of the tropic of Capricorn, d N e, into 12 equal parts should begin from the points a and d, the eastern points. For from the points of the divisions we shall find the points of the unequal hours on either tropic, and shall complete the antique dial, as we set out in Chapter 5. For example, on the southern dial a straight line drawn from f — the point of the 4th unequal hour of the tropic of Cancer — through E cuts the equinoctial line at g. A straight line drawn from H through g cuts the tropic of Cancer at h, the point of the 4th unequal hour. Likewise a straight line drawn from m — the point of the 4th unequal hour of the tropic of Capricorn — through E cuts the equinoctial line at n, and the straight line H n cuts the tropic of Capricorn at p, the point of the 4th unequal hour. The straight line joining p and h, therefore, will give the 4th unequal hour, and so on. For the northern dial we give no example, both because it will easily be described from what has been said, and also because the unequal hours on the southern dial, produced beyond the horizontal line, make the northern dial, if that exchange of the parts is made of which we spoke in the preceding chapter.

Diagram. The antique ("horologium verticale antiquum") vertical dial: an upper quadrant of hour-lines fanning from a centre near **H/P**, above a semicircular arc running down to **N** at the bottom, with the unequal (antique) hours marked inside and the ordinary hour-numbers running down both vertical edges.

Readable labels (positions approximate):

- **HORI-** (*horizontalis*) label at the upper left of the horizontal line

- Curved label crossing the upper region: **Arcus horarum** ("arc of the hours")
- Left-edge hour-numbers (top to bottom): **X, IX, IIX** (= VIII), then below the horizontal **I, II, III**
- Right-edge hour-numbers (top to bottom): **II, I, XII, XI, [Y?] [?], IX**
- **ZON** (*zona* — the equinoctial band) across the middle right
- Interior points and letters: **M** (top of arc), **H, P, F, G, a, b, d, e, f, h, m, p, q, n, I, E**, and **N** at the foot of the arc
- Unequal-hour numbers on the inner fan: **IIII, V, VI, VII, VIII** [reading approximate — ?]

Several labels do not resolve**

[Margin, p. 495:] [Margin: *Alia descriptio horologij Verticalis Italici, ac Babylonici ex arcubus diurnis atque nocturnis*. — "Another way of describing the Italian and Babylonian vertical dial, from the day-arcs and night-arcs."] [Margin: *Descriptio horologij Verticalis Antiqui*. — "The description of the antique vertical dial."]

[Margin: *Descriptio alia Verticalis horologij antiqui ex tabulis diurnis*. — "Another way of describing the antique vertical dial, from the day-arc tables."]

But if the day-arc of 6 hours is drawn — which on the northern dial will be the arc of 18 hours — we shall construct the same dial by means of the tables of day-arcs and night-arcs of 6, 12, and 18 hours, which we compiled in the scholium to Proposition 33 of Book I, exactly as was said concerning the horizontal dial.

ON THE ASTRONOMICAL MERIDIAN DIAL

CAPVT XI.

[Margin: *Descriptio horologij Meridiani Astronomici*. — "The description of the astronomical meridian dial."]

Draw a straight line AB anywhere. (If the plane of the dial is fixed and parallel to the meridian, this line must be drawn parallel to the horizon, using a plumb-line.) On it, choose the place of the gnomon at point A, and from A as center, with any radius, describe an arc of a circle BC — toward the left for the dial that faces east, but toward the right for the one that faces west. On this arc mark off the altitude of the equator — that is, the complement of the pole's altitude — as the arc BC, and through A and C draw the straight line AC as the equinoctial line, which the straight line DE cuts at right angles at A. Then take the straight line AE equal to the length of the gnomon (of whatever size you please), and from E as center,

with any radius, describe the circle FGHI. Divide this circle into 24 equal hours, beginning from the line FH, or from the line GI, which cuts FH at right angles.

After this, draw hidden construction-lines through the division-points and through the center E; these will cut the equinoctial line AC in points. Through these points, lines drawn parallel to DE — that is, perpendicular to the equinoctial line — will give the hour-lines reckoned from noon or from midnight. These parallels (or perpendiculars) are easily drawn: through any point of the line AD — say through D — draw a line parallel to AC. For if all the points of line AC are then transferred onto it from D, taking their intervals from point A, the straight lines joining each pair of corresponding points will be equally distant from the line DE, parallel to DE, and parallel to one another.

The order of the hours is this. The line DE gives the 6th hour on both dials. On the eastern dial the line next below it marks the 7th, the next the 8th, and so on; but on the western dial the line next below DE shows the 5th hour, the next the 4th, and so on — as is clear in the figures. Only the 12th hour cannot be drawn, since the meridian circle does not cut the plane of the dial but is parallel to it.

Diagram. The eastern-facing meridian dial (*horologium orientale*). A circle stands on the horizontal line AB, with the word **ORIEN** — **[C]** — **TALE** ("Eastern") set across its top, split by the figure. Readable labels (positions approximate):

- **ORIEN ... TALE** across the top (the point **C** stands between the two halves)
- **K, G** — upper arc
- **B** — on the horizontal line at the left, where the arc BC begins
- **A** — center, at the foot of the gnomon
- **E** — to the right of center
- **N, I, L, S** — lower right region
- **D** — lower left, with the hour-figures **2, 3, 4, 5, 6, 7, 8, 9** running along the adjacent line
- **H** — near the number-column
- A fan of straight hour-lines crossing the circle, with dotted construction-lines.

Several small letters do not fully resolve.**

Diagram. The western-facing meridian dial (*horologium occidentale*). Structure symmetrical to the first. Readable labels (positions approximate):

- **Occi...** at the left and **...dentale** [?] at the right — the word **OCCIDENTALE** ("Western") set across the top
- **r** — far left, on the horizontal line
- **K, G** — upper arc
- **C** — upper center
- **E** — center; **M, R** to its right
- **A** — just below E
- **Q** — below A
- **N, I, L** — lower left; **S** at the lower-left extremity
- **H** — lower right; **B** — right edge
- Hour-figures **2, 3, 4, 5, 6, 7, 8** running along the lower-right edge
- A fan of straight hour-lines with dotted construction-lines.

Several small letters do not fully resolve.**

It is also enough, for describing the meridian dial, to divide a single quadrant of the circle described about E into six equal parts — such as the quadrant FG. For if hidden lines are drawn through the division-points and through the center E, cutting the line AS in points through which the hour-lines (parallel to DE) are to be drawn, then, when these points are transferred from A onto the other side, onto the line AC, we shall have the further points through which the hour-lines are to be drawn — because each segment of the line AS is equal to the corresponding segment of the line AC.

Now if the eastern dial is set up on a plane that is parallel to the meridian and faces east, and the western dial on the same kind of plane facing west — so that the line AB is parallel to the horizon, with point A toward the north and point B toward the south (that is, the angle BAC, equal to the complement of the pole's altitude, is always set toward the south) — then the hours will be indicated by the gnomon AE, standing at right angles to the plane of the dial at point A. But if from any two points of the line DE — say from D and E — two perpendiculars EO and DP, equal to the gnomon AE, are raised to the plane of the dial, and the straight line OP is laid across their tops (or at least a thread is stretched from O to P), then that line or thread will serve as the axis, and its shadow cast onto the hour-lines will likewise indicate the hours from noon or from midnight. Likewise the gnomon AE, if no error has been made in laying out the dial, is always equal to that segment of the equinoctial line which lies between point A and the 3rd (or 9th) hour.

[Margin: *Quomodo vtrumque horologii Meridianu collocandu sit, vt horas indicet.* — "How each meridian dial is to be set up so as to indicate the hours."]

Diagram. A small bracket-shaped figure in the right margin representing the axis: two uprights **EO** and **DP** joined at top by the bar **OP**. Labels: **O** (top left), **P** (top right), **E** (foot left), **D** (foot right).

ON THE ARCS OF THE SIGNS ON THE MERIDIAN DIAL

CAPVT XII.

Draw the straight line DC anywhere as the radius of the equator, and on either side describe the radii of the other signs, as we taught in Chapter 2. Then draw the axis DG perpendicular to the equinoctial radius DC. Transfer onto the equatorial radius DC, from D, all the hour-intervals intercepted between E (the center of the circle FGHI in the preceding chapter) and the equinoctial line AC, marking points on the equatorial radius through which lines are drawn parallel to the axis DG; and to these assign the hour-numbers corresponding to the transferred hour-spaces.

[Margin: *Descriptio arcuu signorum in vtroq; horologio Meridiano.* — "The description of the arcs of the signs on each meridian dial."]

For if the segments of these parallels intercepted between the equatorial radius DC and the radii of the other signs are transferred onto the corresponding hour-lines of the dial, measured from the equinoctial line AC on both sides, marking points on the hour-lines, and if curved lines are then drawn through these points, the arcs of the signs will be described — the upper ones being the southern signs, the lower ones the northern. The line AB is the horizontal line; hence everything found above it must be cut away as superfluous. The parallels to the axis DG are easily drawn through the points marked on the equatorial radius DC, if all their distances, taken from point D, are transferred onto the line GF, drawn parallel to DC anywhere.

Diagram. The rectangular figure of the *arcs of the signs* on the meridian dial. A rectangular frame carries the axis and hour-lines; radii for the zodiacal signs fan downward from near the top-center point **D**. Readable labels (positions approximate):

- Top edge: **G** (left corner), **q**, **D** (center), **k**, **G** (right corner)
- Left edge, hour-figures descending: **12**, **11**, **10**, **9**, **8** and lower **7**; also **2**, **3**, **4** near the equinoctial line; **F** at the left of the equinoctial line

- Right edge, hour-figures: 5, 6, 7, 8, 9, 10; F at the right of the equinoctial line
- Interior points: **B, B, p, p, T, m, l, C** (center-bottom)
- Radial (day-radius) labels, running along the diagonal rays: **Rad. hor. 0, Rad. hor. 6, Rad. hor. 18, Rad. hor. 14, Rad. hor. 10, and Rad. hor. 2[4?]** [?]
- At the lower termini of the rays, a row of zodiac sign-glyphs (Capricorn ♑, Aquarius ♒, Pisces ♓, Aries ♈, Taurus ♉, Gemini ♊, Cancer ♋, Leo ♌, Virgo ♍, Libra ♎, Scorpio ♏, Sagittarius ♐); several individual glyphs do not resolve cleanly [?].

Dense figure; more than two labels uncertain.**

Diagram. The paired dial-faces beneath the horizon line, labelled **HORI — [B] — ZON** ("HORIZON") across the horizontal line AB. From a central point **B** two fans of straight lines diverge, one down to the left (ending near **C**) and one down to the right (ending near **C**). Readable labels (positions approximate):

- **HORI ... ZON** across the horizontal (horizon) line
- **B** — center of the horizon line
- **E, N, H** — points along the fans
- **C** — at the lower end of each fan
- Zodiac sign-glyphs at the ray-termini on both sides (Cancer ♋, Leo ♌, Virgo ♍, Libra ♎, Scorpio ♏, Sagittarius ♐, Capricorn ♑, and others); several do not resolve [?].

Dense figure; more than two labels uncertain.**

transferred onto the line GF, drawn parallel to DC anywhere. For the straight lines joining each pair of points on the lines DC and GF, being equally distant from the points D and G, will be parallel. As for the numbers set beside these parallel lines: those on the right belong to the hours of each meridian dial, while those on the left belong to the hours of each polar dial, of which more shortly. Here all the arcs of the signs are hyperbolas, as we said in Chapter 2.

Moreover, the upper portion of each dial — the part cut off by the horizontal line — gives the dial to be placed on the opposite face of the dial-plane, provided it is inverted: the part now uppermost becomes lowest and vice versa; what is on the right in that position is changed to the left; the hours formerly reckoned from noon are now counted from midnight and vice versa; and finally the northern

signs become southern and the southern northern — as we demonstrated in Proposition 26 of Book II.

Again, the upper portion of either dial would show the hours during the night, if the Sun's rays could fall upon it, as was said above about the vertical dial. For the upper part of the eastern dial contains the hours from midnight to sunrise, while the upper part of the western dial contains the hours from sunset to midnight.

ON THE ARCS OF THE LENGTHS OF THE DAYS ON THE MERIDIAN DIAL

CAPVT XIII.

[Margin: *Descriptio arcuum diurnorum in utroq; horologio Meridiano.* — "The description of the day-arcs on each meridian dial."]

These arcs, which are likewise all hyperbolas, are described just like the arcs of the signs, if in place of the radii of the signs the radii of the day-lengths are taken, as was said in the preceding chapters. In the figure of the zodiac-radii of the preceding chapter we drew the day-radii of hours 0, 24, 6, 18, 10, 14 — which are the straight lines drawn out from D and marked with dots.

ON THE ITALIAN AND BABYLONIAN MERIDIAN DIAL

CAPVT XIV.

[Margin: *Constructio horologij Meridiani Italici, & Babylonici.* — "The construction of the Italian and Babylonian meridian dial."]

In the circle FGHI — by means of which, in Chapter 11, we found the points on the equinoctial line in order to describe the astronomical meridian dial — take the day-arc of Cancer ☊, namely BGC, and the day-arc of Capricorn ☋, namely DGE. This is easily done: on that circle, from point G, count off on either side the semidiurnal arc of Cancer ☊ and of Capricorn ☋, up to the points B, C and D, E. Then divide the same circle into 24 equal parts. For the Italian dial, begin — on the eastern side — from point C, and on the western side from point B, so that both C and B mark the 24th hour from sunset, the next point toward G the 23rd hour, and so on.

Diagram. The large construction for the Italian and Babylonian meridian dial: two overlapping circles (left and right), each drawn about the meridian, densely

crossed by straight and dotted radial lines, with the two 24-hour divisions numbered around and across them. Readable labels (positions approximate):

- Left circle: **M, D, F, B** (upper left), **K** (left), **I**; inner hour-figures **24, 23, 22, 21, 20** and outer **6, 10, 18**; a lower **10, 7, 8, 18**
- Right circle: **L, 12** (top), **G, D, M, F, B** (upper right), **K** (right), **E, E, H, H, C, C, I**; hour-figures **16, 17, 18, 5, 4** and a **12** at the foot
- Central region: a tight web of crossing lines with small hour-numbers (11–24) around the mutual intersection.

Extremely dense; many labels do not resolve individually.**

The contrary is done for the Babylonian dial: that is, on the eastern side the division begins from point B, and on the western side from point G [?], so that both point B and point C give the 24th hour from sunrise, the next point toward G the 1st hour, and so on. For if, for the Italian dial, hidden lines are drawn through the center of the circle from the hours placed between points B and G on each dial, the equinoctial line will be cut in points; and if perpendiculars are raised to it through these points, the tropic of Cancer ☊ will be cut in the points of the hours from sunset. But if, for the Babylonian dial, hidden lines issue through the center of the circle from the hours set between points C and G on each dial, the equinoctial line will be divided in points; and if perpendiculars are drawn to it through these, cutting the tropic of Cancer ☊, we shall have the points of the hours from sunrise on the said...

on the said tropic. In just the same way, on the tropic of Capricorn ☋, we shall investigate the points of the same hours, provided that the division of circle FGHI begins, for the eastern Italian dial, at point E, and for the western at point D; while for the Babylonian dial it begins, for the eastern, at point D, and for the western at point E — always proceeding toward point G. And so, if the mutually corresponding hours on the two tropics are joined by straight lines, both the Italian and the Babylonian dial will have been drawn. Now the hours from sunrise or from sunset necessarily pass through the points of the hours from noon and from midnight on the equinoctial line, as was said above, and as is plainly gathered from the table you will find in Chapter 4.

But if some of the hours from sunrise or sunset have no corresponding points on either of the tropics, those hours are to be drawn through the corresponding hours on the equinoctial line. Thus, on the eastern dial, hours 16 and 17 from sunset — and likewise 4 and 5 from sunrise — are drawn through hours 10 and 11 from midnight on the equinoctial line; on the western dial, hours 19 and 20

from sunset, and likewise 7 and 8 from sunrise, through hours 1 and 2 from noon. The lines of hours 6 and 18, moreover, are to be drawn parallel to the equinoctial line — as we shall show in Book 2, Proposition 34 — through the points found on the tropics of each dial.

[Margin: *Alia descriptio horologij Meridiani & Babylonici per arcus diurnos, ac nocturnos.* — "Another way of describing the meridian [Italian] and Babylonian dial by means of the day-arcs and night-arcs."]

By means of the day-arcs and night-arcs of hours 14 and 10 we shall likewise describe each dial, just as we set out for the horizontal and vertical dials. The same can be done by means of the day-arc and night-arc of the 24 hours. But it must be observed that, of the day-arcs and night-arcs set out in the scholium to Proposition 33, Book 1, one should take, for the eastern dial, only those hours from sunrise and sunset to which, in those same tables, the hours from midnight — not those from noon — correspond, since the latter are not contained in that dial; and for the western dial, those to which the hours from noon, not from midnight, correspond. For example, hour 14 from sunset we shall, on the eastern dial, draw through hour 2 from midnight on the night-arc of the 24 hours, and not through hour 2 from noon on the day-arc of the 24 hours. Likewise, hour 14 from sunrise we shall, on the western dial, draw through hour 2 from noon on the day-arc of the 24 hours, and not through hour 2 from midnight on the night-arc of the 24 hours; and so on.

[Margin: *Qua ratione ex horologio Italico fiat Babylonicum, et contra, in facie opposita plani horologij Meridiani delineandum.* — "By what method a Babylonian dial is made from an Italian one, and conversely, to be laid out on the opposite face of the plane of the meridian dial."]

As for the mutual intersections of the hours from noon or midnight with those from sunrise or sunset on the horizontal line, one must take into account what we wrote in Propositions 34 and 35 of Book 2.

Furthermore, it is evident from the figure of this chapter that the lines of the hours from sunset on the eastern dial have the same position that the lines of the hours from sunrise keep on the western dial. The same holds true of the hours from sunrise on the eastern dial and the hours from sunset on the western dial. Hence an eastern Italian dial drawn on the opposite face of the dial-plane, so that line answers to line, will be a western Babylonian dial; and a western Italian dial will be an eastern Babylonian dial, and conversely — provided only that the

individual hour-numbers are changed into their complements up to 24, as 9 into 15, and 3 into 21, and so on.

ON THE OLD [ANTIQUE] MERIDIAN DIAL

CAPVT XV.

Let both the day-arc of Cancer ☿, BGC, and that of Capricorn ♊, DGE, in circle FGHI, be divided — for each dial — into 12 equal parts. For by their means we shall find, on the same tropic, the points through which the unequal hours are to be drawn, exactly as in the preceding chapter we located the points of the hours from sunrise and sunset on these same tropics. The unequal hours, moreover, will pass through the hours from noon or midnight on the equinoctial line, as we taught in the case of the horizontal and vertical dial.

Diagram. The construction for the Old (Antique) meridian dial, drawn as two large overlapping circles (a left face and a right face about the meridian), each crossed by a dense web of straight and dotted radial lines, with the unequal hours numbered around and across them. Readable labels (positions approximate):

- Circumference letters (shared across the twin circles): **D** (top), **F** (upper left and upper right), **G** (top, on each circle), **B** (left and right flanks), **E E** (the doubled label at the shared central region), **H** (lower flanks), **C** (bottom of each circle), **I** (lower left / lower right), **S** (lower right).
- Interior/hour figures read on the left circle: **8, 9, 10, 4, 3, 2, 7** and **II** (= 11).
- Interior/hour figures read on the right circle: **2, 3, 4, 10** and **II** (= 11).

Extremely dense; many interior hour-figures and line-endings do not resolve individually.**

If two day-arcs are drawn, one of 18 hours and the other of 6 hours, we shall construct the same Antique dial in the way it was done above for the horizontal and vertical dial, from the tables described in the scholium to Proposition 33, Book 1.

It is clear, moreover, from the figure and the description of each dial that the lines of the unequal hours have the same position and situation on both dials. Therefore, once one dial has been drawn, if on the opposite face the same lines are drawn, answering exactly to the earlier lines, the other dial too will be obtained — the hour-numbers, however, must be changed into their complements up to 12, as 1 into 11, and 2 into 10, and so on.

[Margin: *Constructio eiusdem horologii Meridiani Antiqui ex arcubus diurnis. Quo pacto ex horologio Antiquo orientali fiat occidentale, & contra.* — "The construction of the same Antique meridian dial from the day-arcs. By what method a western dial is made from an eastern Antique dial, and conversely."]

ON THE POLAR DIAL, Astronomical.

CAPVT XVI.

[Margin: *Qua ratione ex horologio Meridiano Polare efficiatur. Quo pacto collocandum sit horologium Polare, vt horas monstret.* — "By what method a polar dial is made from a meridian one. In what way the polar dial must be set up so that it may show the hours."]

The polar dial does not differ from the meridian dial, which we described in Chapter 11, except in the number of the hours and in their order. For in this dial, on both sides of the straight line DE, one takes all the hours that can fall upon the plane of the dial, and the line DE itself is no longer the 6th astronomical hour but the 12th, as appears in the accompanying dial. For if this dial is so set up that the line CS is parallel to the horizon, the line DE lies in the plane of the meridian circle, point E faces south and D faces north, and finally point D is raised from the northern side according to the elevation of the pole — that is, if the line DE, together with the meridian line, is established in the plane of the horizon toward the [regions]...

Diagram. The polar astronomical dial: a broad figure with the meridian line DE running vertically (D at the top, E at the foot within a small graduated circle marked **12**), the horizontal line CS crossing it (C at the left, S at the right), the central intersection **A**, and a fan of straight and dotted hour-lines. Along the right and left edges the sign-arcs of the zodiac are labelled with sign-glyphs (day-parallels for the signs). Readable labels (positions approximate):

- Meridian/horizon frame: **D** (top), **E** (foot, in the circle, beside **12**), **C** (left), **S** (right), **A** (center), **B** (upper right).
- Interior points/figures: **N**, **K**, and hour-figures **4**, **7**, **8**, **9**, **10**.
- Right and left margins: a descending series of zodiac sign-glyphs marking the day-arcs; the individual glyphs are largely unresolved [?] (wavy Aquarius-like $\approx$ and Pisces-like $\bowtie$ forms on the left; Scorpio-/Libra-like forms on the right).

Dense, with many small edge-glyphs and interior figures unresolved.**

the pointer AE, standing at point A at right angles upon the plane of the dial, will indicate the angle of the pole's elevation of point D; or else the line OP, as we taught in Chapter 13 — the hours from noon and from midnight being kept in this order, namely that the hours between DE (the meridian line, which indicates the 12th hour of midday) and point C, toward the left, are counted from midnight, while those drawn toward point S are counted from noon. Only the 6th hour from noon or midnight cannot be drawn on this dial, since the circle of the 6th hour does not cut the plane of the dial but is parallel to it.

And so, if a polar astronomical dial is to be described, two straight lines CS and DE must be drawn on the proposed plane, cutting each other at right angles in A, of which DE represents the meridian line — to such effect that, if the plane is firm and stable and parallel to the circle of the 6th hour from noon or midnight, one must first draw, with the help of a level and a plumb-line, on the given plane the line CS parallel to the horizon, and then raise to it the perpendicular DE for the meridian line, cutting the former in A. Next, on the line DE, one takes the segment AE equal to the proposed pointer; and, having described a circle of any size from E, the rest is to be completed as we set out for the meridian dial in Chapter 11, and so on.

[Margin: *Descriptio horologij Polaris Astronomici.* — "The description of the polar astronomical dial."]

ON THE ARCS OF THE SIGNS AND OF THE LENGTHS of the days on the polar dial.

CAPVT XVII.

[Margin: *Descriptio arcuum signorum, et longitudinum dierum in Polari horologio. Horizontalis linea in horologio Polari, quomodo describatur. Horologium Polare Superius, & inferius. Item Diurnum, & nocturnum quod.* — "The description of the arcs of the signs and of the lengths of the days on the polar dial. How the horizontal line on the polar dial is to be drawn. The upper and the lower polar dial. Likewise which is the day-dial and which the night-dial."]

The arcs of the signs and of the lengths of the days, which are all hyperbolas, are described here just as on the meridian dial, from the figure of the zodiac-radii in Chapter 12 — provided you remember that the line which there was assigned to the 6th hour here represents the 12th hour, as we likewise noted there. The horizontal line is drawn as follows. From point B, where hour 9 or 3 intersects the equinoctial line, let an arc of a circle be described toward the meridian line,

upward. (I call the "upper" part that which lies above the equinoctial line when the dial is set in its proper position, so that the hours from midnight are to our left and the afternoon hours to our right.) Taking on that arc a portion equal to the complement of the pole's elevation, from its end let a straight line be drawn to hour 9, or 3, on the equinoctial line; this line will cut the line of the 12th hour at point D. Through D, a line drawn parallel to the equinoctial line will be the horizontal, dividing the whole dial into two parts, of which the larger — that in which the equinoctial line is contained — is called the upper or day part, and the other the lower or night part. In the lower part, however, the hour-numbers are changed into their complements up to 12, the meridian line signifying the 12th hour of midnight, and the arcs of the southern signs become the arcs of the northern signs, and so on. But if the same dial is taken as a night-dial, this change need not be made. The arcs of the southern signs are those which lie above the equinoctial line, proceeding toward the horizontal line; those of the northern signs are the ones drawn below the equinoctial line. An example is given in the preceding chapter.

ON THE POLAR ITALIAN AND Babylonian DIAL.

CAPVT XVIII.

[Margin: *Compositio horologii Polaris Italici, ac Babylonici.* — "The construction of the polar Italian and Babylonian dial."]

Let the day-arc of Cancer ☊, aFb, in circle FGHI, and the day-arc of Capricorn ☋, dFe (both of which are obtained if from point F the semidiurnal arc of Cancer ☊ and of Capricorn ☋ is counted off on either side, up to the points a, b and d, e), be divided into 24 equal parts — beginning, for the Italian dial, at the western points a, d, but for the Babylonian at the eastern points b, e, always proceeding toward F. For if hidden straight lines are drawn from the points of division through the center, the equinoctial line will be cut in points; and if perpendicular lines are raised to it through these, they will indicate on each tropic the points of the hours from sunset and sunrise, as was done on the meridian dial. But if some hours do not have...

Diagram. The polar Italian and Babylonian dial: a large circle FGHI (F at the top, G at the left, H at the foot, I at the right) crossed by a broad fan of straight and dotted hour-lines radiating outward on both sides, with the horizon line running across labelled **HORI** on the left and **ZON** on the right (i.e. HORIZON). The hours from sunset/sunrise are numbered along the rays; a tight interlaced web of lines fills the region about the center. Readable labels (positions approximate):

- Circumference letters: **F** (top), **G** (left), **H** (foot), **I** (right), with **C** on the upper right and **S** at the lower right.
- Interior points: **D, K, L, b** (lowercase), and **a/d** among the divisions.
- Left-side ray numbers: **23, 24, 12, 7, 8** and **II** (= 11).
- Right-side ray numbers: **24, 6, 23, 22, 13, II** (= 11), and **01** (= 10).
- Interior web figures read: **8, 9, 10, 12, 13, 14, 20, 21, 2, 3**.

Very dense; many interior figures and ray-endings do not resolve individually.**

points on either of the tropics — as is the case with those hours contained in the arcs aG, bI — they must be drawn through the points of the hours from noon or midnight on the equinoctial line. For all the hours from sunrise and sunset pass through the said hours, as is plain from the table in Chapter 4. The line of the 12th hour, however, both from sunrise and from sunset, must be drawn parallel to the equinoctial line through the point found on the tropic of Cancer ☿: the one belonging to the 12th hour from sunrise is found to the right of the meridian line, while the one indicating the 12th hour from sunset lies to the left.

These lines of the hours from sunrise and sunset, extended beyond the horizontal line, exhibit the same hours on the lower polar dial, if that portion of the dial cut off by the horizontal line is inverted in all its parts, as we said in Chapter 9 concerning the vertical dial.

We shall construct the same dial by means of the day-arcs and night-arcs of hours 10 and 14, and also of the 24 hours, with the help of the tables belonging to those arcs, as was done in the dials above. Here this alone is to be noted: that the lines of hours 6 and 18 from sunrise or sunset do not meet the day-arc and night-arc of the 24 hours, but are [Greek: ἀσυμπτότους (asymptotous, "asymptotic" — lines that never meet; the print sets omicron, not the vulgate ω)], as is clear from the corollary to Proposition 15, Book 1 — which we also stated in the case of the meridian dial.

By what method the same dial is drawn with the help of the line of the 12th hour from sunrise or sunset and of the horizontal line, that is, of the 24th hour from sunrise or sunset, may be sought from Propositions 46 and 47, Book 2.

[Margin: *Quomodo ex Polari horologio Superiori efficiatur Polare inferius. Constructio eiusdem horologij Polaris Italici ac Babylonici ex arcubus diurnis, nocturnisque.* — "How a lower polar dial is produced from an upper polar dial. The construction of the same polar Italian and Babylonian dial from the day-arcs and night-arcs."]

Gnomonics, Liber Septimus (continued)

On the Antique Polar Dial. Chapter XIX.

[Margin: *Cōstructio horologij Polaris Antiqui.* — "Construction of the antique polar dial."]

Just as in the preceding chapters, so here too we shall lay out the unequal hours. We take the day-arc — both that of Cancer ☊, aFb, and that of Capricorn ☋, dFe — on the circle FGHI and cut it into 12 equal parts, from which we may seek the points of the unequal hours on each tropic. This we shall likewise carry out with the help of the day-arcs and night-arcs of hours 18 and 6 [numeral:?].

Diagram. The antique polar dial for unequal (temporal) hours. A circle FGHI is drawn with F at the top, G at the left (on the horizon), H at the foot, and I at the right. A horizontal line runs across the circle carrying the label **HORI** on the left and **ZON** on the right (i.e. HORIZON). A broad fan of straight and dotted hour-lines radiates outward on both sides beyond the circle; the equal-hour Roman numerals **IX** (upper left), **I** (upper right), **XII** (right, beside ZON), **XI** (lower right), **I** (lower left) and **III** (foot) are set at the ends of the outer rays. Along the upper circumference, running from F toward the right, are the Arabic figures 6 (at F), 5, 4, 3, 2; running from F toward the left, 7, 8, 9, 10. Readable interior labels: G and lowercase a, b near the left of center, and a divided lower arc bearing the figures 8, 9, 10, 11 and 1, 2, 3, 4, 5 together with a row **III · V · VI · VII · VIII · IX**. A dotted ray at the lower right ends at X.

The interior is a very dense interlaced web of straight and dotted lines; many of the small ray-figures and interior divisions do not resolve individually at native resolution.**

The figure set beside makes this clear. There too, the lines of the unequal hours, when produced beyond the horizontal line, exhibit the lines of those same hours on the lower dial — provided that the portion of the dial cut off by the horizontal line is inverted in all its parts, as we taught in Chapter 9.

On the Astronomical Equinoctial Dial, by the Arcs of the Signs and the Lengths of the Days. Chapter XX.

[Margin: *Compositio horologij Aequinoctialis Astronomici.* — "Composition of the astronomical equinoctial dial."]

Draw a straight line AB to serve as the meridian line, that is, the line of the 12th hour from noon or from midnight, and let another straight line CD cut it at right angles in E, to serve for the 6th hour before and after noon. From E describe a circle of any size and divide it into 24 equal parts. For if straight lines are drawn out through the center E and through the points of division, an equinoctial dial will have been described. Indeed, if at E a style EF of any length is fixed perpendicular to the plane of the dial, and the dial itself is so placed that the line CD is parallel to the horizon and the line AB is set in the plane of the meridian circle (as the common section of the plane of the dial and of the meridian), and finally the point A on the noon side is raised — with point B leaning toward the north, C toward the east, and D toward the west — through the complement of the pole's altitude above the horizon (that is, so that AB together with the meridian line in the plane of the horizon makes, on the side of B, an angle equal to the complement of the pole's altitude): then the style EF will show the hours from noon or from midnight, this order being kept among them, that the hours to the right of the meridian line AB are numbered from midnight, while those to the left are numbered from noon.

[Margin: *Descriptio arcuum signorum, et longitudinum dierū in horoloio [horologio] aequinoctiali.* — "Description of the arcs of the signs and of the lengths of the days on the equinoctial dial."]

The arcs of the signs and of the lengths of the days, all of which are circles on this dial, will be described as follows. In the figure of the Zodiac rays of Chapter 12, take on the axis DG a straight line Dq to the left, or Dh to the right, equal to the style of the dial; and through q, or through h, draw a line qn (or hl) parallel to the equatorial radius DC, cutting the rays of the signs and of the lengths of the days at the points p, o, n, etc. Then if from the center of the dial E circles are described according to the intervals qp, qo, qn, etc., the arcs of the signs and of the lengths of the days will have been described — such that on the upper dial all the arcs are northern, the most northern being the one nearest to the center E, while on the lower dial all are southern, the one nearest the center E being the most southern. Therefore on the former dial the circle described according to the semidiameter qp will be the arc of Cancer ☊ [Greek/Latin zodiac gloss: Cancer], the next of Gemini ♊ and Leo ♌, and the third of Taurus ♉ and Virgo ♍; while on the latter the first will be Capricorn ♐, the next Sagittarius ♐ and Aquarius ♒, the third Scorpio ♏ and Pisces ♓. But by what arrangement the lower takes its origin from the upper, the horizontal line will disclose, which we shall now delineate.

[Margin: *Horizontalis lineae descriptio in horologio aequinoctiali.* — "Description of the horizontal line on the equinoctial dial."]

On the line of the 6th hour, CD, take a straight line El equal to the length of the style EF, and from l describe an arc of a circle toward the meridian line. If on this arc a portion equal to the pole's altitude is taken, and from the end of it a straight line is drawn to l, it will cut the meridian line AB at the point m; and the straight line drawn through m so as to cut the meridian line at right angles will be the horizontal line, dividing the whole dial into two parts. The greater part, in which the center E lies, is called the upper or diurnal,

Liber Septimus (continued)

diurnal, the other, the lower, is called nocturnal. On the lower dial, however, the numbers of the hours are changed into their complements up to 12, and the arcs of the northern signs are exchanged for arcs of the southern signs — but not so on the nocturnal dial. The equinoctial circle cannot be marked out on this dial, because it does not cut the plane of the dial but is parallel to it. Hence its common section with the plane of the dial, which ought to be a straight line, vanishes, as we demonstrated in Proposition 11 of Book 1.

Diagram. The astronomical equinoctial dial. A circle is drawn with **A** at the top and **B** at the foot (the vertical meridian diameter, AB), and **C** at the left and **D** at the right (the horizontal 6-o'clock diameter, CD); the left end is labelled **C 6** and the right end **6 D**. The center is **E**. An oblique horizon line crosses the figure carrying **HORI** on the left and **ZON** on the right; it meets the meridian at the interior point **m**, and the interior point **P** lies near it. Concentric inner circles (the arcs of the signs and of the lengths of the days) bear zodiac glyphs around their rims; among the clearly readable ones are Pisces ♓, Aquarius ♒, Virgo ♍, Cancer ♋, Gemini ♊, Taurus ♉ and Leo ♌. The outer rim is divided into 24 parts, labelled with Arabic hour-figures (2, 3, 4, 5 on the lower left; 1 and 12 at the foot near B; 7, 8, 9, 10 on the lower right; and further figures around the top) together with Roman hour-numerals **I** and **II** near the top.

The zodiac rim and the interleaved 24-part hour divisions are dense and partly overprinted; individual rim positions of the zodiac glyphs and several of the small hour-figures do not resolve cell-by-cell at native resolution.**

Now since, when the Sun is on the equator, the shadow of the style is infinite — and even a little before and after the equinoxes it is nearly infinite — the hour-lines on the upper dial must be produced beyond the circle of Taurus ♉ and Virgo ♍

℞, as far as the size of the plane on which the dial is described allows, so that on those days before and after the equinoxes the gnomon may be able to indicate the hours. Or else a circle of some particular degree near the equinoctial circle must be described — namely at 25° Virgo ℞ and 5° Taurus ☿ [?], or at 20° Virgo and 10° Taurus ☿ [?], and so on. The same is to be understood as needing to be done on the lower dial among the southern signs, always taking account of the size of the plane of the dial. This too must be understood of the Italian and Babylonian hours.

[Margin: *Quid agendum sit, vt vmbra styli horas indicet prope tempus aequinoctiorum.* — "What is to be done so that the shadow of the style may indicate the hours near the time of the equinoxes."]

For the rest, on the upper dial the portion EB of the meridian line indicates the 12th hour of noon, and the remaining portion EA the 12th hour of midnight. On the lower dial the matter stands in the contrary way: for the portion EB signifies the 12th hour of midnight, and the remaining portion EA the 12th hour of noon.

On the Equinoctial, Italian, and Babylonian Dial. Chapter XXI.

[Margin: *Cōstructio horologij Aequinoctialis Italici ac Babylonici.* — "Construction of the equinoctial Italian and Babylonian dial."]

Let the several arcs of the signs — or at least two of them, an interior and an exterior, which is enough — be divided into 24 equal parts, the beginning being made from the horizontal line, on the left side for the Italian dial and on the right side for the Babylonian. Then through the nearest points of these arcs, above or below the horizontal line, draw a straight line; likewise through the next points in sequence, and so on, until straight lines have been drawn through all the points. For these straight lines will indicate the hours from sunrise and from sunset on the upper dial, and thereby the upper equinoctial Italian and Babylonian dial will have been described, in which the number and order of the hours is this. The line nearest above the horizontal line, which indicates the 24th hour — that is, on the nocturnal dial (the part cut off by the horizontal line) — shows on the left the 1st hour from sunset, the next 2, and so on; so that the line nearest below the horizontal line on the same left side indicates the 23rd hour from sunset, and so forth. Again, the first line below the horizontal line on the right side shows the 1st hour from sunrise, the next 2, and so on, as is clear from the figure. These lines, produced beyond the horizontal line, form the lower Italian and Babylonian dial, if all the parts of the lower dial are inverted, as we said in the case of the northern vertical dial and the lower polar dial.

[Margin: *Alia descriptio horologij Aequinoctialis Itali[c]i & Babylonici per arcus diurnos nocturnosque.* — "Another description of the equinoctial Italian and Babylonian dial by means of the day-arcs and night-arcs."]

Each dial can also be described by means of the day-arcs and night-arcs of hours 10, 14, and 24, as was done in the preceding chapters. For example, the 23rd hour from sunset is drawn through the 6th hour from noon on the day-arc of 14 hours; and since the same line, carried beyond the day-arc of 24 hours (which touches the horizontal line at the point m), indicates when produced the 23rd hour from sunrise (as we demonstrated in Proposition 10 of Book 2), while the 23rd hour from sunrise passes through the 4th hour from mid-

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night on the night-arc of 10 hours — for this reason, so that the 23rd hour from sunset may be correctly drawn through the 6th hour from noon on the day-arc of 14 hours, the 4th hour from midnight must also be taken, corresponding to the 23rd hour from sunrise, on the night-arc of 10 hours. By the same reasoning the 22nd hour from sunset is drawn through the 5th hour from noon on the day-arc of 14 hours, and through the 3rd hour from midnight of the night-arc of 10 hours, which corresponds to the 22nd hour from sunrise. And so on continually, always taking one point on the day-arc of 14 hours for any given hour from sunset, and another on the night-arc of 10 hours — or likewise on the day-arc of 14 hours — for the same hour from sunrise, as the tables of the scholium to Proposition 33 of Book 1 will indicate. Thus you see that the 11th hour from sunset is drawn through the 6th hour from midnight on the day-arc of 14 hours, and through the 4th hour from noon on the same arc, which agrees with the 11th hour from sunrise. For by this reasoning the Italian dial is easily composed. In the same way the Babylonian will be made, if for any given hour from sunrise one point is taken on the day-arc of 14 hours and another on the night-arc of 10 hours — or likewise on the day-arc of 14 hours — for the same hour from sunset. Thus you discern that the 1st hour from sunrise passes through the 6th hour from midnight on the day-arc of 14 hours; and the 1st hour from sunset through the 8th hour from noon on the night-arc of 10 hours; likewise the 13th hour from sunrise through the 6th hour from noon on the day-arc of 14 hours, and the 13th hour from sunset through the 8th hour from midnight on the same arc; and so with the rest.

But each dial will be delineated much more easily with the help of the day-arc of 14 hours and the day-arc of 24 hours. For example, the 23rd hour from sunset passes through the 6th hour from noon on the day-arc of 14 hours, and through

the 11th hour from noon on the day-arc of 24 hours. Likewise the 8th hour from sunrise is drawn through the 1st hour from noon on the former arc, and on the latter through the 8th hour from midnight. Finally, on the lower dial the 17th hour from sunset is drawn through the 10th hour from mid-

Diagram. The equinoctial Italian and Babylonian dial. The circle has **C** at the upper left of the rim and **B** at the left where the horizon meets the circle; the center is **E**, with the interior point **D** just below the center. An oblique horizon line runs across, labelled **HORI** on the left and (on the right) **NoZ 24 / ZON**, marking the horizontal line as the 24th hour; the interior point **A** lies on it near the center-right. On the right, the rays are marked with the Babylonian hours from sunrise **II, I** below the 24 line. On the left, near **B**, run the hours **24 · XIII[?]**, **23 · XII**, **22 · XI**. The double outer rim carries paired hour-figures — Arabic Italian/Babylonian hours (**13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24** ...) beside Roman numerals (**III, IIII, V, VI, VII, VIII** ...) — and zodiac glyphs are interspersed around the rim.

The rim is a very dense double band of paired Arabic-and-Roman hour-figures with interleaved zodiac glyphs, and the interior is a tight web of straight and dotted hour-lines; the individual rim numbers and their positions do not resolve reliably cell-by-cell at native resolution.**

night on the day-arc of 10 hours, and through the 5th hour from midnight on the night-arc of 24 hours, and so on — as the tables of these arcs, described in the scholium to Proposition 33 of Book 1, teach. It must be noted, however, that the hours from noon or from midnight are the same, provided that those which on the upper dial are counted from noon are, on the lower, reckoned from midnight, and conversely. This indeed happens because of the inversion of all the parts of the lower dial, an inversion that is altogether necessary on the Italian and Babylonian dial. Hence, when above in Chapter 20 we taught that the hours from noon or from midnight must, on the lower dial, be changed into their complements up to 12, this is to be understood as holding if the aforesaid inversion of the parts is not made. But it will be better to derive the lower Italian and Babylonian dial from the upper, just as we set forth a little before.

[Margin: *Linea horarum ab or. & occ. tangent circulum horarum 24. in horis à meri. & med. noc.* — "The lines of the hours from sunrise and sunset touch the circle of 24 hours at the hours from noon and midnight."]

It must not be passed over here that the lines of the hours from sunrise or sunset touch the circle of 24 hours at the points where they are cut by the lines of the

astronomical hours, and thus that they are there cut by them at right angles, as we demonstrated in Propositions 58 and 59 of Book 2.

But here too one may observe the common intersections of the hours from noon or from midnight with those from sunrise or sunset, which they make on the horizontal line (that is, the line of the 24th hour) and on the line of the 12th hour from sunrise or sunset; and likewise the mutual intersections which the hours from sunrise or sunset make on the line of the 6th hour from noon or from midnight — as is plain from the tables set out in the scholium to Proposition 20 of Book 1.

On the Antique Equinoctial Dial. Chapter XXII.

[Margin: *Descriptio horologij Aequinoctialis Antiqui.* — "Description of the antique equinoctial dial."]

Let the several portions of the arcs of the signs which the horizontal line cuts off be divided, on each dial (both the upper and the lower), into 12 equal parts. It is enough, however, to do this on the inner

on the inner and outer dial. Then draw a straight line through the points nearest below the horizontal line, and do the same in turn for the succeeding points that correspond to one another. Once this is done, the antique dial will have been drawn. For the line nearest below the horizontal line — on the right-hand side, that is, on the upper dial, but on the left on the lower — will give the first unequal hour, the next the second, and so on. Here too the hours on the upper dial, drawn out beyond the horizontal line, give the same hours on the lower, provided all its parts are inverted, as was said above. Thus each line of the unequal hours passes through six points of the divisions of the arcs of the signs, three of which lie on the upper dial and three on the lower.

Diagram. A full circle representing the antique (unequal-hours) equinoctial dial. Its horizontal diameter is the horizon line: at the left end it is labelled **HORI** (with **12** just below it), and at the right end **ZON** (with **12**; the right-hand legend is set reversed/inverted, appearing as "1Z ZON", marking the inverted lower dial). Around the rim run the hour-figures of the dial; along the interior run dotted hour-lines carrying fractional hour-values.

- Horizon line: **HORI** (left), **ZON** (right), **12** at each end.
- Left rim, descending from the horizon: **1(I)**, then Roman **II**, then **10, 9, 8, 7, 6** (at bottom).

- Right rim, descending from the horizon: reversed **10(OI)** and **11[?]** above the horizon, then below it **1(I)**, **2**, **3**, **4**, **5**.
- Interior dotted hour-lines (left group): fractional values $7\frac{1}{2}$, **6**, $4\frac{1}{2}[?]$, **3**, $1\frac{1}{2}$, and near the centre **12[?]**, **10 $\frac{1}{2}$ [?]**.
- Interior dotted hour-lines (right group): $4\frac{1}{2}$, **6**, $7\frac{1}{2}$, **9[?]**, **10 $\frac{1}{2}$ [?]**.
- Top rim: a band of zodiac glyphs, worn and not reliably resolvable at native resolution — [Zodiac glyphs: ?].

The rim hour-figures, the interior fractional dotted-hour values, and the top-rim zodiac glyphs are a dense, partly worn cluster that does not resolve cell-by-cell; several labels are flagged.**

The antique dial can also be described easily by means of the diurnal arc of 18 hours and the nocturnal arc of 6 hours, just as was done in the cases above. This is gathered from the little tables drawn up for this use in the scholium to Proposition 33 of Book 1, and it appears clearly in the accompanying figure.

[Margin: *Alia descriptio horologij Aequinoctialis Antiqui per arcum diurnum horarum 18. & nocturnum horarum 6.* — "Another description of the antique equinoctial dial by means of the diurnal arc of 18 hours and the nocturnal arc of 6 hours."]

ON THE ASTRONOMICAL DIAL THAT DECLINES FROM A VERTICAL CIRCLE. CHAPTER XXIII.

[Margin: *Descriptio horologij Astronomici à Verticali declinantis.* — "Description of the astronomical dial declining from the vertical."]

Before all else, the declination of the plane on which the dial is to be described must be investigated, as we taught in Proposition 23 of Book 1 and in its scholium. Since we there discussed the declination and inclination of planes at length, there seems no need to say anything at all about it here. Once the declination has been found, we shall proceed to describe the dial on a plane that declines from the vertical, in this way.

On the plane of the dial, taking the location of the style anywhere at **K**, let two straight lines **AB** and **DF** be drawn through **K**, cutting each other at right angles at **k**; or, if the plane is fixed and vertical to the horizon, let the line **AB** be drawn through **K** parallel to the horizon by means of a level and plumb-line, and let **DF**, perpendicular to the horizon, cut it at right angles. Or, at any rate, in the first

place let the perpendicular be drawn, which the line **AB**, parallel to the horizon, cuts at right angles at **K**.

On **FD**, taking any length of the style **KF**, whether directed upward or downward, describe from **F**, toward **AB**, the arc **DH** — toward **B** if the wall or plane of the dial declines from south toward the east, or from north toward the west; but toward **A** if it deflects from south toward the west, or from north toward the east. Then, once the declination of the wall **DH** has been counted off (which in our example we set at 30 degrees from south toward the east), and **FH** has been drawn cutting **AB** at **E**, draw through **E** the line **CE** parallel to **FD**, that is, perpendicular to **AB**. For **CE** will be the meridian line, or the line of the 12th hour.

[Margin: *Linea meridiana*. — "The meridian line."]

Next, on the arc **DH** produced on the other side of the line **FD**, with the complement of the declination **DN** reckoned off, draw the line **FN**; or indeed draw the line **Fα** perpendicular to **FE**, cutting **AB** at **α**. For through **α** both the equinoctial line and the line of the 6th hour must be drawn. Take also on **AB** a segment **Eβ**, either to the right or to the left of the meridian line **CE**, equal to **EF**. From **β** describe an arc of a circle toward **CE** — upward, on a dial declining and deflecting from the north; on this arc, the altitude of the pole having been counted off starting from the line **AB**, draw from **β** to the end of the reckoning a line cutting **CE** at the point **C**, which will be the centre of the dial, in which all the hour-lines will meet.

[Margin: *Centrum horologij*. — "The centre of the dial."]

Therefore, drawing from **C** through **K**, the place of the style, the line **Ck** — which may be called the line of the style — and letting the line drawn from **α** cut it at right angles at **G**, the line **αG** will be the equinoctial line.

[Margin: *Linea styli*. — "The line of the style."]

[Margin: *Linea aequinoctialis*. — "The equinoctial line."]

After this, erect from **K** the line **KI** perpendicular to **CK** and equal to the style **KF**.

[Margin, later hand (× keyed into the text): *Errorem hic fuisse commissum colligis ex principio capitis primi libri tertij*. — "You may gather that an error was committed here from the beginning of the first chapter of the third book." (manuscript annotation)]

[Margin: *Axis mundi*. — "The axis of the world."]

For the line **CI** drawn will give the axis of the world, which the line **GI** joined to it will cut at right angles, if no error has been made. Now on the line of the style **CK**, taking a segment **GL** equal to **GI**, describe from **L** a circle of any size, and divide it into 24 equal parts, beginning from the line **LM**, which is drawn from **L** through the point **M**, where the equinoctial line cuts the meridian; and this line **LM** will necessarily cut the drawn line **Lα** at right angles, if no error has been made.

[Margin: *Linea horarum à mer. & med. noc.* — "The line of the hours from noon and from midnight."]

For if hidden straight lines go out through the centre **L** and the points of division — the lines emitted from **C** through which will give the hours from noon and from midnight — they do so in this order. On the dial that faces south, the portion of the meridian line from the centre **C** toward the equinoctial line marks the 12th hour of noon, and the remaining portion beginning from the same centre **C** marks the 12th hour of midnight. The lines preceding the meridian line on the left — that is, toward the parts of **A** — indicate the hours after midnight, or before noon; and those that follow the same line on the right — that is, toward the parts of **B** — show the hours after noon. Their order and number advance along the line **AB**, or the equinoctial line, from left to right. On the dial...

Diagram. The construction of the declining astronomical dial. A small circle at the upper centre is the origin of the world's axis, its centre labelled **C** (with lowercase **a** just above it); from **C** and from the style-vertex **F** a fan of straight hour-lines spreads downward across a horizontal line **AB**. The horizontal line runs **A** (left) to **B** (right). An arc of a circle (drawn partly dotted) sweeps through the lower-centre region and carries several construction points.

- **A** — left end of the horizontal line **AB**.
- **B** — right end of the horizontal line **AB**.
- **C** — centre of the small circle (centre of the world/dial), upper centre; **a** (lowercase) marked just above.
- **F** — style-vertex, at the convergence of the ray-fan, upper centre-left.
- **E** — on the vertical below **F**.
- **α** — centre-left, on **AB**.
- **β** — below centre, near **AB**.
- **G, K, M** — construction points at centre-left/left.
- **N** — lower left, on the dotted arc.
- **L, D, H** — points along the dotted arc (**L** left, **D** lower-left, **H** lower-centre).
- **I** — bottom centre.

- **z** — ray-label, centre-right; **S[?]** — ray-label, upper right.
- Hour-figures on the rays: right side **1, 2, 3, 4**; left side **6, 7, 8, 9**; bottom **10[?], 11, 12**.

(The exact hour attached to two of the outer rays and the reversed lower-rim figures do not resolve cleanly; those are flagged. Letter-labels A–N, α , β read confidently.)

that faces north, the portion of the meridian line from the centre **C** toward the equinoctial line indicates the 12th hour of midnight, and the remaining portion the 12th hour of noon. The lines preceding the meridian line on **AB**, to the left, show the hours before noon, and those on the right the hours from midnight; their order and number likewise advance from left to right along the equinoctial line, or the line **AB**.

Moreover, these same hour-lines are most conveniently drawn by the following method. From the point **F** let straight lines go out making with **FE** the same angles that the hour-lines make with the meridian line on the horizontal dial — namely, the line of the 6th hour, which is **F α** , a right angle, and so on. This is easily done if, from the centre of the horizontal dial, an arc of a circle is described cutting all the hour-lines, together with an equal arc from **F** toward the line **AB**. For if all the hour-intervals from that first arc, beginning from the meridian line, are transferred to this one, beginning from the line **FE**, the lines emitted from **F** through the points of this arc will be the hour-lines answering to the hour-lines on the horizontal dial.

[Margin: *Qua ratione lineae horariae ex horologio horizontali describantur.* — "By what method the hour-lines are drawn from the horizontal dial."]

Where, then, these lines cut the line **AB**, the lines drawn from the centre **C** [recte: *centro*; the print reads "cento"] through those points will give the hour-lines, as before. And in this way the earlier description can easily be examined and corrected, if some error has by chance been committed in it. Indeed, this latter method of drawing the hour-lines is very useful for drawing those lines which cut the equinoctial line either in no way at all, or scarcely at all except at the most remote points — such as, in the proposed example, the lines of the 4th and 3rd hours.

But should anyone doubt whether the hour-lines are drawn straight by this method, we shall demonstrate it briefly in this way. Let the triangle **α FE** be conceived as moved about the line **α E** until it stands perpendicular to the plane of the dial, so that **F**, the vertex of the style, becomes the same as the centre of the

world, and the triangle itself lies in the plane of the horizon. This being posited, all the hour-circles from noon and from midnight will be drawn through **F**, the centre of the world. Since, then, the meridian is drawn through the line **FE**, the circle of the 6th hour will be drawn through the line **Fα**. And since the common sections of the hour-circles and of the horizon at the centre of the horizon make angles equal to those which the common sections of the same circles and of the plane of the horizontal dial make at the centre of the dial — into which the axis of the world falls — as is clear from what we demonstrated toward the end of Proposition 1 of Book 3, and from what we demonstrated in Proposition 5 of Book 5 when we described the horizontal dial by means of the hour-arcs: it is evident that the hour-circles meet the declining dial at the points of the line **AB** into which fall the straight lines that make at **F** the same angles that the hour-lines make on the horizontal dial at the centre of the dial, and so on.

[Margin: *Quando puncta L, M, praecedentis horologii ...* — "When the points L, M of the preceding dial..." (continued in the note on the next page)]

Again, since the points **L** and **M** are sometimes so little distant from one another that a line — from which the division of the circle described from **L** must begin — can scarcely be drawn through them without error (for it may easily deflect to one side or the other, whence a sometimes considerable error will arise in drawing the hour-lines) — we shall do better if first, from **α**, through the cen...

centre **L** we draw a straight line. For the line **LM** will necessarily cut this at right angles, if no error has been made. The same must also be observed in the following declining and inclined dials. For by this method it will come about that the hour-lines are drawn most accurately, provided the said two lines have cut each other at right angles. Therefore this operation must be carefully examined before we go any further in describing the dial.

[Margin: *...parum inter se distant, quid agendum, vt accurate horae depingantur.* — "...are little distant from one another: what is to be done so that the hours may be drawn accurately."]

By the same method, if the line **Fα** happens to cut the line **AB** too far from the point **E**, we shall be able to draw from **F** any other hour-line besides the line of the 6th hour — one that makes with **FE** the same angle which that hour makes with the meridian line on the horizontal dial, and that cuts the line **AB** conveniently. For if, through the point where it cuts **AB**, we draw a line to **C**, the centre of the dial, and another line through the point where this line cuts the equinoctial and through the centre **L**, we shall have another line from which we

may most conveniently begin the division of the circle described from **L**. If, then, one point of division falls precisely on the line drawn through **L** and **M**, the operation will have been rightly set up; but if not, some error will have been committed, which you must correct before you pass on to other things.

[Margin: *Qua ratione diuisio circuli ex P, descripti aliunde possit inchoari quam à recta LM.* — "By what method the division of the circle described from **P** [so printed; expected *L*] may be begun from somewhere other than the line **LM**."]]

Therefore, if this dial is placed on a wall that declines 30 degrees from south toward the east, so that the line **AB** is parallel to the horizon and the line **FD** is perpendicular to it, and a style is fixed at the point **K** perpendicular to the wall and equal to the line **KF**; or if the triangle **CIG** is erected at right angles upon the line of the style **CK** — then the tip of the shadow projected from the vertex of the style, or the shadow of the axis **CI**, will indicate the hours from noon or from midnight.

[Margin: *Quomodo horologium declinans à Verticali collocandum sit, vt horas indicet.* — "How the dial declining from the vertical is to be placed so that it may indicate the hours."]]

In what manner a dial is to be laid out on walls that are little distant from the meridian circle, let it be sought from the scholium to Proposition 1 of Book 3.

[Margin: *Quando murus parum à Meridiano abest.* — "When the wall is little distant from the meridian."]]

Always, in these dials and in all the others that follow, the angle contained between the axis and the line of the style — such as here the angle **ICK** — is the measure of the altitude of the pole above the plane of the dial, as we showed in Proposition 1 of Book 3.

[Margin: *Altitudo poli supra planum declinans.* — "The altitude of the pole above the declining plane."]]

Now this same dial, declining from south toward the east, will also show the hours on the opposite face of the wall — that is, on a wall that declines 30 degrees from north toward the west — if it is so placed that the line **AB** is parallel to the horizon and the centre **C** is set below it, and the things that in that position are to our right become left, and conversely. But then the line **CEM** will indicate the 12th hour of midnight, not of noon, as on the dial facing south. Thus, when the dial declining from south toward the east, which we have described up to now, is turned toward us so that its upper part becomes the lower and conversely — that

is, the point **A** is to our right and **B** to our left — if on the opposite face of the dial all the linework is drawn so that each single line answers exactly to each single line, and again the point **A** becomes to our left and **B** to our right, then a dial will have been drawn declining from north toward the west by as many degrees as the one earlier described declines from south toward the east, to be placed on the wall as it lies. For by this method the centre **C** will be below the line **AB**, and the things that were earlier to our left (the dial being of course turned toward us so that **A** verged to the right and **B** to the left) will have been made right, and conversely.

Diagram. The finished declining astronomical dial. The centre **C** is at the top, and from it a fan of straight hour-lines spreads downward, crossing the horizontal line **AB** (**A** at left, **B** at right) and continuing to the lower margin. A dotted line runs obliquely across the lower right (the equinoctial line).

- **C** — centre of the dial, at the top, where the hour-lines converge.
- **A** — left end of the horizontal line; **B** — right, on the same line.
- **E** — on the horizontal line, centre.
- **I, K** — upper-left construction points below **C**.
- α — on the horizontal line, centre (lowercase).
- **M** — below **E**.
- **F** — at the bottom, on the meridian.
- **D** — lower left (foot of a ray to the lower margin).
- Hour-figures on the rays: left side **6, 7, 8, 9**; centre-bottom **10, 11, 12**; right side **1, 2, 3, 4** (and a dotted equinoctial line trailing to the lower right, marked $s[?] / \Omega$).

(All letter-labels **A, B, C, D, E, F, I, K, M, α** read confidently; two lower-margin ray markers are flagged.)

Being about to construct [recte: CONSTRVCTVRI; the print reads "CONSTVCTVRI"] a dial declining from north toward the east or the west, we shall describe a dial declining from south toward the opposite quarter by as many degrees as the one to be described declines. (For the declination of a dial facing north is always the same as that of one facing south, although they do not both decline toward the same quarters — because, when the one declines toward the east, the other declines toward the west, and conversely, yet by an equal number of degrees.) Thus, if a dial declining from north toward the west is to be described, we shall describe one declining from south toward the east; but if one declining from north toward the east, we shall describe one declining from south toward the

west. For from this we shall gather what is proposed, if all its parts are inverted, as has been said.

[Margin: *Quid agendum sit, quando horologium declinans à Septentrione describendum proponitur.* — "What is to be done when a dial declining from the north is proposed for description."]

Moreover, when any dial declining from south or north toward the east or west has been constructed, if all its linework is drawn on some other plane so that the things situated to the right of its meridian line become left, and those to the left come out right, a dial will have been made declining likewise from south or north toward the contrary quarter — so that, if the former declines toward the east, this one verges toward the west, and con-...

[Margin: *Qua ratione ex horologio declinante à meri. vel Sept. in or. vel occ. fiat declinans à mer. vel Sept in occ. vel ortum, & contra.* — "By what method, from a dial declining from south or north toward the east or west, one may be made declining from south or north toward the west or east, and conversely."]

and conversely. Such is the linework of any dial when it is described on the opposite part of the plane, so that its several lines correspond one to one. By this technique you see that here a dial has been made declining from south toward the west by 30 degrees, out of a dial declining by the same number of degrees from south toward the east.

Diagram. A declining astronomical dial (the mirror-image of the one on the preceding page — this one declines from south toward the west by 30°). The center **C** stands at the top; from it a fan of straight hour-lines spreads downward, crossing the horizontal base line and running to the lower margin. A dotted line runs obliquely across the lower right (the equinoctial line).

- **C** — center of the dial, at the top, where the hour-lines converge.
- **A** — left end of the horizontal base line.
- **E** — on the horizontal line, left of center.
- **B** — on the horizontal line, right of center.
- **α** — right end of the horizontal line.
- **I, K** — upper construction points just below **C**.
- **M** — below **E**.
- **F** — lower center, on the meridian line.
- **D** — lower left, at the foot of a ray to the margin.

- one small lowercase cursive point on the horizontal line between **E** and **B** [?].
- Hour-figures on the rays (read partially; the crowded lower digits are difficult): upper and left rays **6, 7, 8, 9**; lower rays **10, 11, 12, 1, 2, 3, 4**; a marker resembling long-f / 5[?] at the lower right end of the dotted equinoctial line.

[Margin: *Quo pacto una opera fermè quatuor horologia declinantia à Verticali describantur.* — "How, at nearly a single working, four dials declining from the vertical are described."]

You see, then, that at nearly one working four dials are described: one declining from south toward the east, and a second from south toward the west, if the linework of the first is transposed so that right becomes left and conversely; a third from north toward the west, if the first is inverted so that upper becomes lower, and then whatever is on the right after this inversion is made left, and conversely; and a fourth from north toward the east, if the second is inverted in all its parts, just as we said a moment ago concerning the first. In all four the amount of the declination is always the same.

DE ARCVBVS SIGNORVM, ET LONGITVDINVM DIERVVM in horologio, quod à Verticali circulo declinat.

On the arcs of the signs and the lengths of days, in a dial that declines from a vertical circle.

CAPVT XXIV.

[Margin: *Constructio figuræ radiorum Zodiaci pro arcibus signorum, & longitudinum dierum describendis in horologio declinante à Verticali.* — "The construction of the figure of the rays of the Zodiac, for describing the arcs of the signs and the lengths of days in a dial declining from the vertical."]

Draw the straight line **CI** for the axis of the world, and at **I**, perpendicular to **CI**, erect **IG** for the radius of the Equator, on either side of which the rays of the other signs are to be drawn, as above. Take **IC** equal to the straight line **IC** in the axis of the dial of the preceding chapter, and **IG** equal to the straight line **IG** (or **GL**) in the same dial. From **C**, through **G**, draw the straight line **CG** for the style-line — which we shall perhaps draw more conveniently if from **C** we draw the straight line **CG** making with **IC** an angle **ICG** equal to the angle **ICK** that the axis **CI** makes with the style-line **CG** in the dial.

Next, from the center **L** of that circle by whose aid we found the hour-points on the equinoctial line in the dial, take all the intervals up to those points of the equinoctial line where it is cut by the hour-lines; and transfer them from **I** onto the equatorial radius **IG**, marking the points on the equatorial radius. For if from **C** straight lines are drawn through these points, they will be the lines of all the hours — on this side and beyond the style-line — that intersect the equinoctial line. To these, then, attach the numbers of the hours answering to the hours whose intervals have been transferred, noting carefully which lines answer to the hours placed to the left of the style-line and which to the hours set to its right. This the numbers attached to the lines readily indicate, since they correspond to the numbers of the hour-lines of the dial that cut the equinoctial line. For these lines alone are what the lines so far emitted from the point **C** through the points of the equatorial radius represent.

We shall obtain these same lines if, from the dial, we take the hourly intervals between the center **C** and the equinoctial line [**◀M**[?] — a marginal reference mark, followed by the letter **M**; reading uncertain], and transfer them, in the figure of the zodiac rays from **C**, onto the equatorial radius **IG**, marking on it the points through which the hour-lines drawn from **C** must pass, and so on.

[Margin: *Qua arte linea horaria, quæ in horologio lineam æquinoctialem non secat, ducendæ sint in figura radiorum Zodiaci.* — "By what device the hour-lines which, in the dial, do not cut the equinoctial line are to be drawn in the figure of the zodiac rays."]

But if at any time some hour-line in the dial does not cut the equinoctial line but is equidistant from it, then from **C** the line **CA** must be drawn parallel to the equatorial radius **IG**. For this will answer to that hour — just as in the horizontal and vertical dial: because the line of the 6th hour from noon or midnight is equidistant from the equinoctial line, the straight line **HV** was drawn in the figure of the zodiac rays from **H**, parallel to the equatorial radius, for the hour so cut, as is settled from chapters 2 and 7.

But to obtain the lines of the other hours — those which neither cut the equinoctial line nor are parallel to it, of which kind are the ones that, after they have cut the equinoctial line, are produced beyond the center **C** (as in our example the hours 4, 5, 6 and the rest after noon, and 1, 2, 3 before noon, for all these are produced beyond the center **C**) — we shall follow this course. Through **C**, the center of the dial, we shall draw a straight line parallel to the equinoctial line (in our example this need only be conceived in the mind, since it is not

actually drawn); and through **C**, in the figure of the zodiac rays, the straight line **CA** parallel to the equatorial radius **IG**, which cuts the former in **A**.

After this, since every hour-line makes equal angles at the vertex **C** with that straight line which in the dial is drawn through the center **C** equidistant from the equinoctial line, we shall transfer the lines that go out from **C** in the figure of the zodiac rays and cut the equatorial radius, to the other side of the straight line **CA**, by aid of an arc of a circle described from **C**, in the figure of the zodiac rays — just as we did in the horizontal and vertical dial in chapters 2 and 7 — so that the lines equally distant on either side from **CA** enclose, with the straight line **CA**, equal arcs of that circle described from **C**, and equal angles at **C**. For

[Margin: *15. primi.* — Euclid, *Elements* I.15 (vertical angles equal).]

For the lines thus transferred will answer to those hours that do not cut the equinoctial line but run out beyond the center **C**, so that two lines equally distant from the straight line **CA** belong to two hours of the same number. Of these, the one that cuts the equatorial radius — that is, is drawn from **C** toward the equatorial radius — represents the hour that intersects the equinoctial line, while the other, transferred to the opposite side, represents the same hour produced beyond the center **C**, which does not cut the equinoctial line: exactly as was done in the horizontal and vertical dial. For there too the lines transferred beyond the straight line **HV** represent the same hours by number, though they do not cut the equinoctial line; because, of course, any hour-line makes equal angles at the vertex — that is, at the center **H** — with the line of the 6th hour, just as here it does with that line which is drawn through **C**, the center of the dial, parallel to the equinoctial line, and to which the straight line **CA** answers in the figure of the zodiac rays, exactly as there the straight line **HV** answers to the 6th hour, being the line parallel to the equinoctial in the horizontal and vertical dial. Thus you see here that the line of the 3rd hour after midnight has been transferred beyond the straight line **CA**, because the 3rd hour after noon cuts the equinoctial line; and likewise the line of the 4th hour after noon, because the 4th hour after midnight cuts the equinoctial line. The same reckoning is to be kept in the rest.

[Margin: *Quando linea styli una est ex horarijs lineis, vel æqualiter à duabus hinc inde distat, pertinet quælibet linea horaria in figura radiorum Zodiaci ad duas horas.* — "When the style-line is one of the hour-lines, or is equally distant from two on either side, each hour-line in the figure of the zodiac rays belongs to two hours."]

Now if the style-line should itself also be an hour-line, or should stand equally distant midway between two hour-lines — which sometimes happens — then each hour-line drawn from **C** in the figure of the zodiac rays will serve for two hours equally removed from the style-line on either side, just as in the horizontal and vertical dial. Hence to each single line two numbers must be assigned, answering to those two hours equally spaced from the style-line, as was done in the horizontal and vertical dial.

Diagram. The large "figure of the rays of the Zodiac" (*figura radiorum Zodiaci*) for the declining dial — a dense construction with an upper horizontal line, a great arc/semicircle graduated with the hours, a nested shadow-scale ladder at the upper left, and a fan of hour-radii issuing from the apex **C**. Because of the very crowded interior, the letter-labels below are those read with confidence; several interior hour-digits and points could not be resolved and are flagged.

- **C** — apex at top center, from which the hour-radii issue.
- On the upper horizontal line: **B, I, B, C** (repeated **B/C** letters marking the ladder rungs meeting the top line).
- Upper-left shadow-scale ladder: outer points **I** (top left), then **D, G, D** descending the left edge; interior rungs lettered **A, B, C, B, A**.
- Ray-captions along the oblique radii: *Rad. hor. 24, Rad. hor. 18, Rad. hor. 16, Rad. hor. 14, Rad. hor. 10* [and further *Rad. hor.* captions, partly obscured].
- Interior points: **M, F, n, G, E, H, L, N, B, A, P, O**.
- Hours graduated around the arc and along the ray-ends: on the arc **1** through **12**; lower-left ray-ends **8, 9, 10, 11, 12**; lower-right ray-ends **1, 2, 3, 4, 5, 6** (many interior digits illegible [?]).

[Margin: *[figura radiorum Zodiaci ad duas horas — continuation of the note above]*]

It would not be beside the point — indeed it would be expedient — when the style-line stands unequally distant from the two nearest hours, to construct two figures of the zodiac rays. In one of them would be drawn the hour-lines from **C** answering to those hour-lines in the dial that are contained on one side, namely the left, of the style-line; in the other, the hour-lines answering to the dial's hour-lines on the other side of the style-line. For thus all confusion of lines would be removed — the confusion that necessarily follows when the style-line stands almost equally distant from the lines set on either side, since then the points found on the equatorial radius are scarcely distinguished from one another, as is plain. On this plan, in our example, in one of the figures would be drawn the lines

from **C** answering to these hours 1, 2, 3, 4, 5, 6, 7, 8, 9 from midnight, which are to the left of the style-line; and in the other would be contained the lines answering to the hours to the right of the same style-line, namely 10, 11, 12 from midnight and 1, 2, 3, 4, 5, 6, and so on, from noon. In each figure, however, **CG** would have to be drawn for the style-line. An example of this you will see set out in chapter 29, when we treat of dials that decline and are at the same time inclined.

[Margin: *Quando linea styli inæqualiter distat à duabus proximis horis, construendæ essent duæ figuræ radiorum, ut confusio linearum tolleretur.* — "When the style-line is unequally distant from the two nearest hours, two figures of rays should be constructed, so that the confusion of lines is removed."]

Most conveniently, too, we shall draw these same hour-lines going out from **C** in the following way. Having drawn the straight line **CG** as before, we shall take on it, produced, any point **B** whatever, through which we shall draw the line **BA** parallel to **IC** itself, and through **C** another line **CA** parallel to the equatorial radius, which cuts the former in **A**. Then, with **A** as center and **AB** as the interval, describing a circle, we shall cut it into 24 equal parts, the beginning being made from a point which we shall now teach how to find, in this way. Let the straight line **LM**, taken in the dial, be transferred onto the equatorial radius from **I** up to **E**, and through

[Margin: *Quo artificio aliter horariæ lineæ in figura radiorum Zodiaci ducantur.* — "By what other device the hour-lines are to be drawn in the figure of the zodiac rays."]

and through **E**, from **C**, draw the straight line **CE** for the line of the 12th hour, cutting the straight line **BA** in **D**; and from **D**, perpendicular to the straight line **BA**, erect **DF**, cutting the circumference of the circle in the point **F**, which will be the starting-point of the division of the circle. Or else let the interval between the center **L** and any hour on the equinoctial line — whichever seems more convenient — be transferred from **I** onto the equatorial radius, marking a point on it; and through this point, from **C**, let a straight line be drawn for that hour, cutting the straight line **BA** in a point; and if through this point a perpendicular is drawn to **BA** — upward, indeed, if the assumed hour is taken from that side of the style-line where the 12th hour is, but downward if from the other side — we shall again have a point on the circumference from which the division of the circle can take its start.

Indeed it will sometimes be very useful to investigate this second point. For if one point of the division begun from it falls on the point **F** which was found in the

first place, the matter will succeed as intended; but if not, an error committed somewhere must be corrected. It is also better to begin the division of the circle rather from this latter point once found than from the point **F**, when the line of the 12th hour is very near the style-line. For then the perpendicular **DF** cuts the circumference too obliquely, so that the point **F** can scarcely be taken without error.

I wish this same thing to be understood in all the dials that follow which decline and are inclined. If, therefore, from the points of the divisions of the circle, perpendiculars are dropped to the straight line **BA**, or lines parallel to **CA** itself, cutting the straight line **BA** in points, then the straight lines emitted from **C** through these points will give the hour-lines, as before. It will be easy to judge which lines answer to which hours in the dial, from the points of the divisions of the circle. For the points always, proceeding from **F** toward **B**, represent those hours which in the dial follow the 12th hour toward the style-line, as is clear from the figure.

Moreover, we shall easily draw in this way those lines perpendicular to the straight line **BA** from the points of the divisions of the circle. If perhaps two points nearest to the point **B** are equally distant from it (which will indeed happen when the style-line is also an hour-line, or has receded equally from two hour-lines), the straight lines joining two points equally distant from the point **B** will be perpendicular to **BA**. But if not, let the interval of any point, taken from **B** in the upper semicircle, be transferred into the lower semicircle from the same point **B**. For the straight line connecting these two points will be perpendicular to **BA**. The same will be done if the intervals of the points from the straight line **BA** in the lower semicircle are transferred into the upper semicircle from the same straight line **BA**, as you see done in our example.

Diagram. The large finished declining dial with the arcs of the signs. The word **HORIZON** runs across the figure as the horizontal line (split **HO ... RIZON** by the construction). The center **C** sits at upper center on the meridian; a fan of hour-lines and dotted sign-arcs spreads downward and to the right, with zodiac-sign glyphs set at the outer ends of the sign-rays around the periphery. Because of the density and the small worn sorts, the letter-points read with confidence are listed first, then the sign-glyphs; several are flagged.

- **C** — center of the dial, on the meridian near the top.
- **K, I** — construction points just left/below **C**.
- **HORIZON** — the horizontal (horizon) line across the figure.

- Hour-figures along the lines: **1** through **12** (read partially; the crowded interior digits near center are illegible [?]).
- Zodiac-sign glyphs at the ray-ends around the rim (pinned by their position; worn sorts flagged): upper left ♒ (Aquarius), ♓ (Pisces); left ♐ (Capricorn)[?], ♉ (Taurus), ♊ (Gemini), lower-left ♋ (Cancer); lower center-right ♌ (Leo), ♍ (Virgo); right ♏ (Scorpio)[?], ♐ (Sagittarius)[?]; and one further glyph at upper right [?].

[Margin: *Descriptio arcuum signorum in horologio declinante à Verticali.* — "The description of the arcs of the signs in a dial declining from the vertical."]

[Margin: *Quando linea styli inæqualiter à duabus horis hinc inde distat, quid faciendum, ut accuratius arcus signorum describantur.* — "When the style-line is unequally distant from two hours on either side, what is to be done so that the arcs of the signs may be more accurately described."]

From these things thus prepared, the arcs of the signs will be described in the same way as in the horizontal and vertical dial: namely, by transferring the intervals of the hours intercepted between **C** and the rays of the signs onto the corresponding hours of the dial from the center **C**; or else by transferring the hourly intervals lying between the equatorial radius and the rays of the signs onto the corresponding hour-lines of the dial, the beginning being made from the equinoctial line, and so on. If the style-line should be the same as some hour-line, or should have receded equally from two hours placed on either side, each of the aforesaid intervals must be transferred onto the two hour-lines equally distant from the style-line, just as was done in the horizontal and vertical dial. But if the style-line stands unequally removed from the two hour-lines placed on either side, then, so that the arcs of the signs may be described more accurately, all the hour-points of the equinoctial line may be transferred from one side (namely from that on which more of them stand — as in our example from the left side) to the other, the beginning always being made from the line of the index, or style, as we did in our dial. For if through these points, from **C**, we draw hidden lines, such as are those which

These, which are marked off by those small lines on the dial, will always give us two lines equally distant from the index-line, so that onto them the same hourly interval, taken from the figure of the zodiacal rays, may be transferred. Indeed, if in the dial a straight line is drawn through **C** parallel to the equinoctial line, and if onto it there are transferred, from **C** the center of the dial and to either side, the portions of the line **CA** intercepted in the figure of the zodiacal rays between **C** and the rays of the signs, then on that line we shall have the points through

which the arcs of the signs are to be drawn — just as in the horizontal and vertical dial upon the line of the sixth hour.

[Margin: *Qui arcus ad signa australia, & qui ad borealia spectent.* — "Which arcs look toward the southern signs, and which toward the northern."]

Here too the arcs above the equinoctial line belong to the southern signs, while the rest belong to the northern, just as in the vertical dial. Moreover, from what we wrote in chapter 2 we shall easily recognize which signs' arcs are hyperbolas and which are parabolas or ellipses — namely, according to whether the style-line CB cuts the rays of the opposite signs or not.

[Margin: *Plura scripta sunt prop.2.li.3. & in schol. eiusdē propo. de descriptione arcuum signorum.* — "More is written in proposition 2 of book 3, and in the scholium of the same proposition, on the description of the arcs of the signs."]

Further material bearing on the accurate description of the arcs of the signs you will find explained by us in book 3, proposition 2, and in the scholium of the same proposition.

[Margin: *Linea Horizontalis.* — "The horizontal line."]

The horizontal line is the straight line AB itself, which is drawn through the place of the style perpendicular to the meridian line. This horizontal line divides the whole dial into two parts: the one below it faces south, the other faces north.

[Margin: *Horizōtalis linea diuidit totū horologiū in australe, & boreale.* — "The horizontal line divides the whole dial into a southern and a northern part."]

Both parts must be positioned on the wall so that the horizontal line is parallel to the horizon and occupies the upper place. In the northern dial, however, the arcs of the southern signs are changed into the arcs of the opposite northern signs, as in the vertical dial, and the lineaments that lie on the left must be drawn on the right, and vice versa, as we taught in the preceding chapter. But when the arc [zodiac sort: ?] in the southern dial also cuts the horizontal line on the other side — as happens when the plane of the dial has a small declination from the vertical — then the portion of the northern dial which the horizontal line cuts off must also be drawn on the other side, just as in the vertical dial the northern one was drawn on both sides. Otherwise, in a northern dial that declines toward the east, the hours would not be shown when the Sun rises, nor in one that inclines toward the west when the Sun sets.

The former dial can also be called Diurnal, and the latter Nocturnal, as we taught in the case of the vertical dial.

[Margin: *Horologiū declinans diurnū, ac nocturnū*. — "The declining dial, diurnal and nocturnal."]

Now the arcs of the day-lengths will be described in exactly the same way, if in place of the rays of the signs one takes the rays of the day-lengths — provided one observes that these arcs, in a dial facing north, are converted into their complements up to 24, as we wrote for the vertical dial in chapter 8. In the figure of the zodiacal rays we drew the rays of the hours 0, 6, 8, 10, 14, 16, 18, and 24.

[Margin: *Arcus longitudinum dierū quo pacto describantur*. — "How the arcs of the day-lengths are to be described."]

But if, from the lineaments described so far, a larger or smaller dial is to be drawn for a given size of style, we shall accomplish it in this way. Let there be drawn separately (as was done next to the figure of the zodiacal rays) the triangle ICG, taken from the dial of the preceding chapter or from the figure of the zodiacal rays of the present chapter, in which IC is the axis of the world, IG the equator, and CG the style-line, while the gnomon IK makes right angles with the index-line CG. Then, taking on the style IK (when produced) a line IA equal to the given gnomon, and drawing through A a line BD parallel to CG: if the axis BI is transferred from C in the figure of the zodiacal rays as far as the point B, toward I, and from B the rays of the signs are drawn out, and everything else is done as before, then the arcs of the signs will be described for the size of the given style IA, just as before they were described for the given style IK. For the hour-lines proceeding from C in the figure of the zodiacal rays, and in the dial we described in the preceding chapter, remain entirely the same at the same elevation of the pole and the same declination of the plane. The equinoctial line, however, must be drawn perpendicular to the style-line, distant from the center of the dial by as great an interval as the line BD.

ON THE ITALIAN AND BABYLONIAN DIAL, which declines from the vertical circle.

CHAPTER XXV.

[Margin: *Compositio horologij Italici, ac Babylonici à Verticali declinātis*. — "The construction of the Italian and Babylonian dial declining from the vertical."]

Let the circle drawn from L — by whose aid we found the hour-points on the equinoctial line — be cut into the diurnal arc of the tropic of Cancer $\mathfrak{S}$, aNb, and the diurnal arc of the tropic of Capricorn $\mathfrak{V}$, dNe, in such a way that the lines ab and de cut the line MN at right angles; and let the semidiurnal arcs of Cancer and Capricorn be numbered from the point N on either side in the circle MaNb, so that the whole diurnal arcs may be obtained, as we set out in chapter 4. Then, for the Italian dial, let the circle MaNb be divided once, and again, into 24 equal parts — the first time with the beginning made at point a, the second time at point d — so that a is the 24th hour from sunset in the tropic of Cancer, and the next point toward N is the 23rd hour, and so on; while d is the 24th hour from sunset in the tropic of Capricorn, and the next point toward N is the 23rd hour, and so on. Again, let the same circle be divided once into 24 equal parts, the beginning taken at point b, and again into 24 equal parts, the beginning made at point e, so that b is the 24th hour from sunrise in the tropic of Cancer, and the next point toward N is the 1st hour, and so on; while point e is the 24th hour of Capricorn from sunrise, and the next point toward N is the 1st hour, and so on. For by means of these division-points we shall find, in each tropic, the hour-points from sunset and from sunrise, and we shall complete the Italian and Babylonian dial in no other way than we constructed the horizontal one in chapter 4.

For example: a straight line drawn from f, the point of the 16th hour of Cancer from sunset, through the center L, cuts the equinoctial line at point g; and if from C, the center of the dial, a straight line is sent through g, the tropic of Cancer will be cut at point h, the 16th hour from sunset. So too, a straight line drawn from m, the point of the 21st hour of Capricorn from sunset, through L, cuts the equinoctial line at point n; and a straight line drawn from C through n cuts the tropic of Capricorn at p, the point of the 21st hour from sunset. In like manner, a straight line from q, the 14th hour from sunset in the tropic of Capricorn — which indeed is contained in the nocturnal arc of Capricorn, dMe — drawn through L, cuts the equinoctial line at point r; and a straight line drawn from C through r cuts the tropic of Capricorn, in the nocturnal arc, at point t, the 14th hour from sunset.

In this way we shall find only the points of those hours which are contained in the semicircle ANB, whose diameter AB is equidistant from the equinoctial line — because the straight lines drawn from those points through L pass through the center L before they cut the equinoctial line. Hence we shall find the points of the other hours, just as we said concerning the horizontal dial

in chapter 4: by the hours diametrically opposite to them. For instance, if we wish to find the point of the 23rd hour from sunset in the tropic of Capricorn — to which there corresponds, in the circle, the point D lying outside the semicircle ANB — we shall draw, from E, the opposite point (namely the point taken from the 11th hour from sunset in the tropic of Capricorn, comprised in the nocturnal arc of Capricorn), a straight line through L and D, which will cut the equinoctial line at point R. (Because R is very remote, it is not included in the example.) If through R a straight line is drawn from C, the tropic of Capricorn will be cut, in the nocturnal arc, at point F, the 11th hour from sunset; while the line RC, produced beyond the center C, will cut the same tropic of Capricorn at P, the point of the 23rd hour. And so with the rest.

In the same way we shall determine the points of the hours from sunrise in each tropic. For the northern dial, however, we give no example, because the hour-lines from sunrise and sunset, produced beyond the horizontal line, make the northern dial, as we said of the vertical dial in chapter 9.

Diagram. A large, very dense construction of the declining Italian and Babylonian dial: a fan of straight and dotted/dashed hour-rays spreads outward from the center **C** (upper center, on the vertical meridian/gnomon line topped by **I**), crossing a horizontal line whose right-hand end is marked **24**. At the lower left sits the tropic/diurnal circle carrying the lettered division-points; radial lines cross its interior. Because of the crowding and the small, worn sorts, only the labels read with confidence are listed; the dense interior digits do not resolve cell-by-cell on the scan and are flagged rather than guessed. **More than two labels are uncertain.**

- **C** — center of the dial (upper center, on the meridian line).
- **I** — top of the vertical meridian/gnomon line above C.
- **K** — just below C.
- **L** — to the left of C (center of the construction circle).
- **M** — near the center-right of the lower circle.
- **24** — right-hand end of the horizontal line (hour-ray 24).
- **P** — on a right-hand ray; **2** set below it.
- Outer right-hand hour-rays labelled (arabic): **22, 3, 21, 20** [several worn — read with low confidence].
- Roman hour-numerals set along the rays around the dial: **I, II, III, IIII, VI, VII, VIII, IX, XII** (and one form resembling **XIII** [?]); several are worn — [?].

- Lower-left construction circle, lettered points: **N** and **m** (left), **f** (below **N**), **A** and **d** (lower left), **g**, **B**, **D**, **E**, **F** (lower right of the circle), **e**, **q** (bottom).
- Several **R** labels descend the left margin at the ray-ends [worn, count uncertain — [?]].
- Interior arabic numerals (roughly 6 through 19) crowd the circle and the ray-crossings; these are not individually legible even under magnification [numeral: ?].

[Margin: *Alia descriptio horologij Italici, ac Babylonici à Verticali declinantis, per arcus diurnos, & nocturnos.* — "Another description of the Italian and Babylonian dial declining from the vertical, by means of the diurnal and nocturnal arcs."]

As for the description of these same dials derived from the diurnal and nocturnal arcs, no difficulty whatever appears, if the tables described in the scholium of proposition 33 of book 1 are carefully considered. For example, the 11th hour from sunset is drawn through the 6th hour from midnight in the diurnal arc of 14 hours, and through the 5th hour from midnight in the nocturnal arc of 12 hours, and likewise through the 4th hour from midnight in the nocturnal arc of 14 hours. So too the 20th hour from sunset passes through the 1st hour from noon in the diurnal arc of 10 hours, and through the 2nd hour from noon in the diurnal arc of 12 hours. But the 21st hour from sunset, which in our example has only a single point — namely the 2nd hour from noon in the diurnal arc of 10 hours — will be described if the 21st hour from sunrise is drawn through the 4th hour from midnight in the nocturnal arc of 14 hours, and extended through the 2nd hour from noon in the diurnal arc of 10 hours. So too, for the 22nd hour from sunset, the 22nd hour from sunrise must be drawn through the 5th hour from midnight in the nocturnal arc of 14 hours and through the 4th hour from midnight in the nocturnal arc of 12 hours; for this line, produced, will give the 22nd hour from sunset. Finally, by the same reasoning, for the 23rd hour from sunset, the 23rd hour from sunrise must be drawn through the 6th hour from midnight in the nocturnal arc of 14 hours and through the fifth hour from midnight in the nocturnal arc of 12 hours, and so on.

ON THE ANCIENT DIAL, WHICH declines from the vertical circle.

CHAPTER XXVI.

[Margin: *Descriptio horologij Antiqui à Verticali declinātis.* — "The description of the ancient dial declining from the vertical."]

Let the diurnal arc — both that of Cancer, aNb, and that of Capricorn, dNe — be cut into 12 equal parts in the circle MaNb, in such a way that the points next following the points b and e toward N belong to the 1st unequal hour, the next to the 2nd, and so on. For from the division-points we shall find, in each tropic, the points of the unequal hours, as we taught for the vertical dial in chapter 10. For a straight line thrown out from f, the point of the 2nd unequal hour of the tropic of Cancer, through the center L, cuts the equinoctial line at point g; and a straight line sent out from C through g cuts the tropic of Cancer at h, the point of the 2nd unequal

hour. Likewise, a straight line drawn from m, the point of the 9th unequal hour of the tropic of Capricorn, through L, cuts the equinoctial line at point n; and a straight line drawn from C through n cuts the tropic of Capricorn at p, the point of the 9th unequal hour, and so with the rest. For the 11th unequal hour of the tropic of Capricorn — since it is not contained in the semicircle ANB, whose points alone are found in the tropics, as was said above — we shall take the point D, opposite to it, which falls in the 11th unequal hour of the nocturnal arc of the tropic of Cancer, as we showed in book 3, proposition 13. If from D we draw a straight line through L, the equinoctial line will be cut at a certain point (which, on account of the narrowness of space, is not contained in our example); and a straight line sent out from C through that point will give, in the nocturnal arc of Cancer, the point corresponding to point D, while the same line, thrown out beyond the center C, will present, in the diurnal arc of Capricorn, another point for the 11th unequal diurnal hour of the tropic of Capricorn. By the same reasoning, for the unequal hours 11, 10, and 9

Diagram. The companion construction for the *ancient* (unequal-hour) declining dial — closely similar to the figure on printed page 512, but here the rays carry Roman unequal-hour numerals. A fan of straight and dotted rays spreads from the center C (upper right of the figure, on the meridian line) across a horizontal equinoctial line; at the lower left is the tropic/diurnal circle with its lettered division-points A ... F, centered on L. Because of the crowding and worn sorts, only the confidently-read labels are listed; the dense interior digits are flagged, not guessed. **More than two labels are uncertain.**

- C — center of the dial (upper right, on the meridian line).
- L — center of the lower construction circle.
- K — near the center, above the circle.
- M — right of center within the circle.
- N — at the left of the circle.

- **P** — on a left/mid ray (and a second **P** lower).
- **R** — upper left, and again lower left (ray-ends) [worn — [?]].
- **A, d, g, n, h, m** — lettered points along the left of the equinoctial line / circle.
- **B, D** — at the right of the circle; **e, f, E** at the bottom of the circle.
- **S** — upper right, beside the ray marked **XI**.
- Roman hour-numerals on the right-hand rays: **XI, X, IX, VIII**; and within the fan **III, IIII, V, VI, VII**.
- Interior arabic numerals (about 2, 3, 6, 7, 8, 9, 10, 11, 12) crowd the center and the circle; these are not individually legible even under magnification [numeral: ?].

of the tropic of Cancer, which likewise fall outside the semicircle ANB, we shall take the points E, F, G opposite to them, which fall in the unequal hours 11, 10, and 9 of the nocturnal arc of Capricorn; and from these we shall draw straight lines through L, so as to find, in the nocturnal arc of Capricorn, the points K, P, Q corresponding to the points E, F, G of the nocturnal arc of the tropic of Capricorn, in the circle MaNb. And if at any time some hour should fall precisely on the point A or B, then a straight line drawn through C parallel to the equinoctial line will mark in the tropic of Capricorn two points; of these, the one that lies to the right of CL will pertain to the hour that falls on point A, while the other, which is set to the left of CL, toward the north, will pertain to the hour that falls on point B. If, therefore, the corresponding points in the tropics are joined by straight lines, the ancient dial will be described, and all the unequal hours will pass through the hours reckoned from noon or from midnight on the equinoctial line, as we wrote above. But since the points of hours 11, 10, and 9 of the arc of Capricorn have no corresponding points in the tropic of Cancer, we shall join them with the points of the same hours of the nocturnal arc of Capricorn — points found by means of the points E, F, G, which are set opposite to those hours in the diurnal arc of Cancer and fall in the unequal hours 11, 10, and 9 of the nocturnal arc of Capricorn.

[Margin: *Constructio eiusdem horologii Antiqui ex arcubus diurnis atque nocturnis. Qua ratione Boreale horologium Antiquum ex Australi ortum habeat.* — "The construction of the same ancient dial from the diurnal and nocturnal arcs. In what way the northern ancient dial takes its origin from the southern."]

Now, if the diurnal arc of 6 hours — which above the horizontal line is the nocturnal arc of 18 hours — is described, the same ancient dial will be constructed from the tables of unequal hours set out in the scholium of proposition 33 of book 1.

Moreover, the lines of the unequal hours, produced beyond the horizontal line, display the same hours in the northern dial, if all the parts are interchanged, as was said above.

ON DIALS DECLINING FROM THE HORIZON.

CHAPTER XXVII.

At the beginning of book 1 we called a dial "declining from the horizon" that one whose flat surface, on which it is drawn, is equidistant from (that is, parallel to) a great circle standing at right angles to the vertical circle properly so called, and passing through the common sections of the horizon and the meridian, so that the common section of that circle and the horizon is the meridian line. But this

[Margin: The dial declining from the horizon is of four kinds.]

Now this dial is of four kinds. For it either faces the zenith and the rising Sun, or it looks toward the nadir and the setting Sun, so that the angle of inclination which it forms with the horizon slopes toward the west. Or else it faces the zenith and the setting Sun, or it belongs to the nadir and the rising Sun, so that the angle of inclination is set toward the east. The first may be called the upper eastern dial, because it shows the hours as soon as the Sun rises, and is drawn on the upper part of the plane. The second is the lower western dial, because it is drawn on the lower part of the plane and shows the hours right up to sunset. The third is the upper western, and the fourth the lower eastern: for the third is constructed on the upper part of the plane and marks the hours up to sunset, whereas the fourth is drawn on the lower part of the plane and displays the hours immediately from sunrise.

[Margin: Description of the dial declining from the horizon.]

The upper dial declining from the horizon, therefore, is drawn just as the southern dial declining from the vertical, and the lower one just as the northern, with the following exceptions. In place of the declination from the vertical circle, one should reckon the inclination to the horizon, which we shall find by proposition 23 of book 1: from D toward the right, toward B, if the plane of the dial faces west and is the upper dial; but toward the left, toward A, if the plane of the dial is the upper one and faces east. If, however, the dial is the lower one and faces west, the inclination must be counted from D toward the left, toward A; and finally, if the dial is the lower one and belongs to the east, the inclination must be reckoned from D toward the right, toward B. Then, on the arc of the circle drawn from β , one must count off from the straight line AB the complement of the pole's

altitude — not the pole's altitude itself, as was done earlier in dials declining from the vertical — above the line AB if the dial is the upper one, but below it if it is the lower. Let everything else be done as in dials declining from the vertical, as the figure itself shows, in which the upper dial facing west is set out, with a declination from the horizon of 30 degrees.

[Margin: The order of the hours in the dial declining from the horizon.]

The order of the hours is this. The straight line CE is always the line of the twelfth hour, of midday; those preceding it toward the left, toward A, show the afternoon hours, while those following it toward the right, toward B, belong to the forenoon hours — this in the upper dial, whether it faces east or west. But in the lower dial, whether facing west or east, the straight line CE is the line of the twelfth hour of midnight, not of midday; those preceding it toward A show the forenoon hours, that is, the hours after midnight, while those following it toward B signify the afternoon hours.

[Margin: Description of the same dial declining from the horizon, derived from the vertical dial.]

As for the latter method of drawing the hour-lines — the one by which, as in chapter 24, we find on the straight line AB the points through which pass the hours emitted from the center C — it must be observed that the hour-lines going out from F and cutting the straight line AB are not to be taken from the horizontal dial, but from the vertical one, as we showed in book 2, proposition 13.

[Margin: The altitude of the pole above the plane declining from the horizon.]

The angle GCI here too is the magnitude of the pole's altitude above the plane of the dial, just as in dials declining from the vertical, as we said in chapter 24.

[Margin: Description of the arcs of the signs and of the lengths of the days in the dial declining from the horizon.]

The arcs of the signs and of the lengths of the days will be drawn here just as in the dial declining from the vertical. This is plain from the two figures that follow.

[Margin: Description of the horizontal line.]

The horizontal line is drawn here parallel to the straight line CE, through the point where the line of the sixth hour and the equinoctial line cut each other, as may be seen in the figure. This horizontal line divides the whole dial into two parts. The larger of these — in which, of course, the line of the twelfth hour is contained — faces the zenith and is the upper part; the other is the lower part.

This holds provided that, when we turn to face the dial toward the east or toward the west (according as the dial faces west or east), all the upper features become lower, and what are then on our right become on the left, and vice versa. Of this kind would be the features which, drawn on the opposite side of the upper dial (after the upper part has been made lower), would correspond exactly to the features lying beyond the horizontal line.

[Margin: The upper and the lower dial. How the dial is to be positioned so that it may indicate the hours.]

Moreover, each dial, both the upper and the lower, is to be positioned so that the horizontal line is parallel both to the horizon and to the meridian

Diagram. A large hour-line construction for the upper dial facing west, declining 30 degrees from the horizon. A long horizontal line runs across the figure with **A** at its left end and **B** at its right end. Above the line, near the center, the point **C** sits within a small radiating circle (the source of the hour-rays), with **E** just below it and β (beta) a little lower on the line CE. From C a pencil of straight hour-rays fans out; the outer rays are labelled with the hours **7** and **8** (upper right), **9** and **10** (right and lower right), **11** and **12** (bottom center), and **3**, **4**, **5** (lower left). A large arc (semicircle) sweeps through the lower half, with further labelled points **K**, **G**, **L**, **M**, **N**, **D**, **H**, and the hours **1** and **2** near D. Dotted arcs cross the solid construction.

- A (horizontal line, left end): endpoint of line AB
- B (horizontal line, right end): endpoint of line AB
- C (upper center, in radiating circle): center from which the hour-rays are emitted
- E (below C): point on the meridian ray CE
- β (on CE, below E): point from which the graduated arc is described
- K, G, L, M, N, D, H: construction points on the lower arc / hour-network
- 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12: hour-ray labels

found in the plane parallel to the horizon; and so that the straight line AB lies in the plane of the vertical circle properly so called — yet with this rule, that the horizontal line in any upper dial whatsoever be above the meridian line, but

Diagram. A second hour-line construction for the same declining dial, now carrying the arcs of the signs and the "radius of the hour" lines. A horizontal line runs across near the top, marked with points **I**, **B**, **C** and, at the far left, **I** again with **B**, **C**, **B** stacked; the top-right is closed by a great arc bearing the hour-

numbers 6, 7, 8, 9, 10. Below the horizontal line a pencil of hour-rays descends from the upper points, together with several dashed straight lines labelled "**Ra. hor. 0.**", "**Ra. hor. 6.**", "**Ra. hor. 14.**", "**Ra. hor. 18.**", "**Ra. hor. 24.**" (radius of hour 0, 6, 14, 18, 24) and short lines labelled "**Ro.**". Further interior points **D, G, K, A, E** are marked. Along the lower right run a series of zodiac-sign glyphs marking the sign-arcs; ♈ (Aries), ♉ (Taurus), ♊ (Cancer), ♋ (Leo), ♌ (Virgo) and ♍ (Capricorn) are identifiable, the remaining glyphs worn [?].

- I, B, C: points on the upper horizontal line
- A, D, E, G, K: interior construction points
- 6, 7, 8, 9, 10: hour-numbers on the upper arc
- Ra. hor. 0 / 6 / 14 / 18 / 24: "radius of the hour" lines (dashed)
- Ro.: radius lines
- ♈ ♉ ♊ ♋ ♌ ♍ [and further worn zodiac glyphs [?]]: sign-arc markers

inferior one below. Finally — so that, if the superfluous parts are cut away, the horizontal line always occupies the upper position, and the center C in the upper dial slopes toward the south, but toward the north in the lower dial; and so that the straight line AB, together with the line that cuts the meridian line at right angles, forms the angle of the plane's inclination to the horizon; and so that the common section of the plane of the dial and of the plane parallel to the horizon is the meridian line itself, in the plane parallel to the horizon, or equidistant from it.

Diagram. An Italian-and-Babylonian declining dial constructed on two circles. An upper small circle carries the points **A, B, C** (with dotted construction triangle); a large lower circle carries a diameter and the point **Φ** (phi), with the word for the horizon set vertically across the figure (legible fragments "HORI..." and "...ZON"). Between and around the circles runs a dense fan of solid and dashed hour-lines and sign-arcs, with hour-number labels (including **8**) and zodiac-sign glyphs down the right and lower right (Cancer ♊, Leo ♋, Virgo ♌, Scorpio ♏, Sagittarius ♐, Capricorn ♑, Aquarius ♒ identifiable; remaining glyphs and the vertical lettering "NON"/"ZON" worn [?]).

- A, B, C: points on the upper circle
- Φ: labelled point on the lower circle
- HORIZON (set vertically, split by the circles): the horizon line
- 8 [and further hour-numbers [?]]: hour-line labels
- ♊ ♋ ♌ ♏ ♐ ♑ ♒ [and further worn zodiac glyphs [?]]: sign-arc markers

[Margin: The diurnal and nocturnal dial.]

Again, the same horizontal line separates the whole dial into a diurnal and a nocturnal part, as was said in the case of the vertical dial.

[Margin: Which arcs in the dial declining from the horizon belong to the northern signs, and which to the southern.]

In every dial declining from the horizon, the arcs of the northern signs lie toward the south, and those of the southern signs toward the north — that is, in the upper dial the arcs of the northern signs are contained between the equinoctial line and the center, whereas in the lower dial the arcs of the southern signs are drawn toward the center, and so on. In the same way we shall here point out which arcs of the signs are hyperbolas, which parabolas, and which ellipses, as we said in chapter 24 for the dial declining from the vertical.

[Margin: By what method a dial facing east is made from one facing west, and vice versa.]

But if all the features of the upper dial facing west are drawn on another plane in such a way that those situated to the right of the meridian line become the left-hand ones, and vice versa, then there will be constructed an upper dial facing east, with the hour-numbers changed into their complements up to 12. Such are the features of the dial that faces west, drawn on the opposite side, so that each line corresponds exactly to each line. In the same way the dial that faces east can be changed into another that faces west, if that same interchange of lines and numbers is made. Indeed, the lower eastern dial too will by the same method be changed into the lower western, and vice versa.

[Margin: Construction of the Italian and Babylonian dial declining from the horizon.]

Both the Italian and the Babylonian dial will be drawn as we said above — namely, if the circle drawn from L is cut into the diurnal arc of Cancer ☊, aNb, and the diurnal arc of Capricorn ☋, dNe, as we taught before, and the division of that same circle takes its beginning, for the Italian dial, from the western points b and e, but for the Babylonian dial from the eastern points a and d. You have an example in the 21st hour from sunset. For from m, the point of the 21st hour from sunset on the tropic of Capricorn ☋, a straight line drawn through L cuts the equinoctial line at n; and a straight line drawn from C through n cuts the tropic of Capricorn ☋ at p, the point of the 21st hour from sunset, and so on.

[Margin: Another description of the Italian and Babylonian dial declining from the horizon, from the diurnal and nocturnal arcs.]

As for the construction derived from the diurnal and nocturnal arcs, there is no difficulty — provided, however, that whenever some hour reckoned from sunrise or sunset has no point in the diurnal or nocturnal arcs, one takes the hour of the same number reckoned from sunset or sunrise, and so on, as was said above. For example, since the 12th hour from sunset has no point in the diurnal arc of 14 hours — as is clear from table 3 of proposition 33 of book 1 — we shall take the 12th hour from sunrise; and, as that table 3 requires, we shall carry it through the 5th hour from noon in the diurnal arc of 14 hours, and through the 6th hour from noon (as table 6 of proposition 33 of book 1 requires) in the nocturnal arc of 12 hours. For this line, when produced, will also give the 12th hour from sunset. In the same way, for the 13th hour from sunset, the 13th hour from sunrise has been drawn through the 6th hour from noon in the diurnal arc of 14 hours, and through the 7th hour from noon in the nocturnal arc of 12 hours. Likewise, for the 14th hour from sunset — which has a single point, namely the 9th hour from midnight in the diurnal arc of 14 hours — we have taken the 14th hour from sunrise, which must be drawn through the 7th hour from midday in the nocturnal arc of 10 hours. For a straight line drawn through these two points — namely through the 9th hour from midnight and through the 7th hour from midday — will give, in the diurnal arc of 14 hours, the 14th hour from sunset, and in the nocturnal arc of 10 hours, the 10th hour from sunrise. Finally, for the 1st hour from sunrise, in the lower dial the 1st hour from sunset has been drawn, in the

Diagram. The companion Italian-and-Babylonian declining dial, built from the diurnal and nocturnal arcs. At the top a nearly vertical line rises to a gnomon-point **C**, with **A** below it and a dotted construction triangle; a long dotted horizontal line runs to the right carrying the sign-glyph ☊ (Cancer) and the hour-number **13**. Down the left side the hour-numbers **8, 14, 13, 12, 7, 6**, and the point **I** are marked, with the hours **14, 13, 12, 15, 16, 17, 18, 19, 20** running out along the right and lower rays. A large circle at the lower center carries the horizon lettering ("HORI..."/"...ZON") set vertically together with the vertical lettering "NON", the points **N, m, q** (rho), **L, P**, and the point **Φ** (phi) on a diameter. Zodiac-sign glyphs run down the right and lower right (Cancer ☊, Leo ♌, Virgo ♍, Libra ♎, Scorpio ♏, Sagittarius ♐, Capricorn ♑ identifiable; the rest worn [?]).

- **C** (top): gnomon-point / center of the hour-rays
- **A**: construction point below **C**
- **I, L, P, N, m, q**: construction points
- **Φ**: labelled point on the lower circle's diameter
- **HORIZON** (set vertically): the horizon line; "NON" lettering [?]

- 6, 7, 8, 12, 13, 14, 15, 16, 17, 18, 19, 20: hour-line labels
- ☿ ♈ ♉ ♊ ♋ ♌ ♍ ♎ ♏ [and further worn zodiac glyphs [?]]: sign-arc markers

in the nocturnal dial, through the 7th hour from noon in the nocturnal arc of 12 hours, and through the 8th hour from noon in the nocturnal arc of 14 hours. For in the lower dial this gives the 1st hour from sunrise, as we showed in the scholium to proposition 23 of book 2.

Moreover, the hours produced beyond the horizontal line likewise give, in the lower dial, the hours reckoned from sunrise and from sunset, just as we explained in the case of the vertical dial.

The ancient dial [i.e. the unequal-hour dial] is constructed exactly like one declining from the vertical. For example, from f — the point of the 4th unequal hour on the tropic of Cancer [☿] — a straight line drawn through L cuts the equinoctial line [linea aequinoctialis] at g; and the straight line C g cuts the tropic of Cancer at h, the point of the 4th unequal hour; and so on for the rest. For the 1st hour, in the lower dial, we have drawn from m — the point of the 1st nocturnal hour on the tropic of Capricorn [♏] — a straight line through L that cuts the equinoctial line at n. For the straight line C n cuts the tropic of Capricorn at p, the point of the 1st unequal nocturnal hour. Therefore the straight line drawn from p, through the 7th hour from noon in the nocturnal arc of 12 hours, will give the 1st unequal nocturnal hour in the nocturnal portion of the dial; and this same line, in the lower dial, will be the 1st unequal diurnal hour, and so on.

[Margin: Construction of the ancient dial declining from the horizon.]

Diagram. The construction of the ancient (unequal-hour) dial declining from the horizon, developed here from the diurnal arc of 18 hours. From a point C at the upper right, the center of the hour-rays (the gnomon point), a fan of straight and dotted rays runs down and to the left toward a large circle at the lower center. Along the upper rays out of C run the hour-numbers 9, 6, 4, 3 (with further fractional numbers — 2½[?], 12, 10½[?] — partly worn); the roman-numeral hour-lines XII, XI, X, IX, VIII, VII, VI, V, IIII are set inside and around the circle. The upper near-vertical line carries the vertical lettering "HORI[S?]" (worn [?]); at the lower left of the circle the lettering "NON" is set vertically beside a separate point N. Two oblique tropic-arcs cross the figure, marked with the sign-glyphs ☿ (Cancer, near the top and again near h) and ♏ (Capricorn, at the lower right); further sign-glyphs along the lower right are worn [?]. A large S stands at the lower right where a line runs out.

- C (upper right): center of the hour-rays / gnomon point

- n, a: points near the top of the circle
- G (center of circle), M (right), f (lower left), d (left), I (far left): construction points
- N (left of circle), m (bottom of circle), c (bottom left), b (lower right), h (right, on the horizontal line): construction points
- IIII, V, VI, VII, VIII, IX, X, XI, XII: unequal-hour lines
- 9, 6, 4, 3 (and worn fractional hour-numbers [?]): hour-line labels along the upper rays
- ♋ (Cancer), ♑ (Capricorn): tropic-arc markers; further worn sign-glyphs [?]
- "HORI[S?]" (vertical, worn [?]); "NON" (vertical): horizon lettering
- S (lower right): terminal label [?]

By the second, later method we have marked out — at the hours from midnight 9, 10½, 12, and at the hours from noon 1½, 3, 4½, 6, 7½, 9 — the points of the diurnal arc of 18 hours, through which the unequal hours are to be drawn, as is clear from the tables of the scholium to proposition 33 of book 1.

[Margin: Description of the same ancient dial declining from the horizon, from the diurnal arc of 18 hours. The dial inclined to the horizon is fourfold: upper southern; lower northern; upper northern; and lower southern, as it may be called. Six precepts for the construction of dials inclined to the horizon.]

ON DIALS INCLINED TO THE HORIZON.

CHAPTER XXVIII.

At the beginning of this *Gnomonics* we said that a dial is called inclined to the horizon when it lies equidistant from the great circle drawn at right angles to the meridian and through the common sections of the horizon, the equator, and the vertical circle, in such a way that the common section of that circle and of the horizon stands perpendicular to the meridian line. Now this is of four kinds. For either it faces the zenith and the south, or the nadir and the north, so that the lines constituting the angle of inclination which it makes with the horizon verge toward the north; or else it faces the zenith and the north, or the nadir and the south, so that the lines constituting the angle of inclination extend toward the south. The first may be called upper southern; the second, lower northern; the third, upper northern; and the last, lower southern. So that all these dials may be described more easily, the six precepts that follow are to be carefully committed to memory.

PRECEPT I.

When the plane of an upper southern or lower northern dial — that is, one whose angle of inclination verges toward the north — has an inclination equal to the pole's altitude, its dial does not differ from the polar dial, upper and lower, which we treated in chapter 16.

PRECEPT II.

But when the same plane has an inclination smaller than the pole's altitude, if the inclination is subtracted from the pole's altitude, there remains the altitude of the pole above the proposed plane; and for that altitude a horizontal dial is to be described, as we set out in chapter 1. This dial is then to be placed so that its equinoctial line lies equidistant from the straight line which, in the plane equidistant from the horizon, cuts the meridian line at right angles. (This is easily done if some straight line, cutting the dial's meridian line at right angles, is set in the plane parallel to the horizon, perpendicular to the meridian line found in it.) The dial itself is then to be raised on the northern side according to the inclination of the plane, so that its meridian line, together with the meridian line found in the plane equidistant from the horizon, contains toward the north the angle of inclination — yet on this condition: that on the upper face of the plane, which is turned toward the zenith and the south, the center of the dial lies below the equinoctial line, but on the lower and northern face, above the same line.

The order of the hours will be this. In the upper dial, when we have turned toward the dial facing north, the hours after midnight lie to the left of the meridian line and the afternoon hours to its right, so that the portion of the meridian line running from the center toward the equinoctial line marks the 12th hour of noon, and the remaining portion from the center, also beginning there, the 12th hour of midnight — just as in the horizontal dial. In the lower dial the matter stands the opposite way: because, when we have turned toward the dial facing south, the hours after midnight lie to the right of its meridian line and the afternoon hours to the left, so that the portion of the meridian line from the center toward the equinoctial line indicates the 12th hour of midnight, and the remaining portion from the center, also beginning there, the 12th hour of noon — as in the vertical dial facing north.

Furthermore, the arcs of the northern signs fall between the center and the equinoctial line in the upper dial, and those of the southern signs beyond the equinoctial line; but in the lower dial they stand the opposite way.

PRECEPT III.

Finally, when the plane's inclination exceeds the pole's altitude, if the pole's altitude is subtracted from the inclination, there remains the altitude of the pole above the proposed plane; and for that altitude a horizontal dial is to be built, following the teaching of chapter 1. It is to be placed just as in the preceding precept, with this exception: that here, on the upper and southern face of the plane, the center of the dial is to be set above the equinoctial line, but on the lower and northern face, below the same — as happens in vertical dials. The order of the hours is here entirely the same as that set forth in the 2nd precept.

The arcs of the southern signs are enclosed between the center and the equinoctial line in the upper dial, and those of the northern signs beyond the equinoctial line; but in the lower dial they stand in the opposite manner.

PRECEPT IV.

Again, when the plane of an upper northern or lower southern dial — that is, one whose angle of inclination verges toward the south — has an inclination equal to the complement of the pole's altitude, its dial is the same as the equinoctial dial, upper and lower, described in chapter 20.

PRECEPT V.

But when its inclination is less than the complement of the pole's altitude, if the inclination is added to the pole's altitude, there is produced the altitude of the pole above the plane itself; and for that altitude, following the teaching of chapter 1, a horizontal dial is to be constructed. It is to be placed as we said in the precept of chapter 2, except that this dial is to be raised on the southern side according to the inclination of the plane, so that its meridian line, together with the meridian line found in the plane equidistant from the horizon, constitutes the angle of inclination toward the south — with this further condition added: that on the upper and northern face of the plane the center of the dial has its seat above the equinoctial line, but on the lower and southern face, below the same.

The order of the hours will be this. In the upper dial, when we have turned toward the dial, the hours lying to the right of the meridian line are those after midnight, and those to its left are the afternoon hours — yet on this rule: that the portion of the meridian line from the center toward the equinoctial line marks the 12th hour of noon, and the remaining portion from the center, also beginning there, the 12th hour of midnight. But in the lower dial, when we have turned toward the dial, the hours after midnight are found to the left of the meridian

line, and the afternoon hours to the right, yet in such a way that the portion of the meridian line from the center toward the equinoctial line belongs to the 12th hour of midnight, and the remaining portion from the center, taking its beginning there too, indicates the 12th hour of noon.

The arcs of the northern signs, moreover, are those contained in the upper dial between the center and the equinoctial line, while those which lie beyond the said line belong to the southern signs — as in the horizontal dial. In the lower dial the matter stands the opposite way.

PRECEPT VI.

If, finally, the plane's inclination is greater than the complement of the pole's altitude, the complement of the inclination is to be added to the complement of the pole's altitude. For by this reckoning is produced the altitude of the pole above the proposed plane; and for that altitude, as we taught in chapter 1, a horizontal dial is to be composed, which must be placed as we taught in the preceding precept — yet on this condition: that on the upper and northern face of the plane the center of the dial is set below the equinoctial line, just as in the vertical dial verging toward the north; while in the lower

southern dial [it is set] above the same, as in the vertical southern dial. The order of the hours here is the same as in the preceding precept, with this difference: that here, in the upper dial, the portion of the meridian line from the center toward the equinoctial line indicates the 12th hour of midnight, and the remaining portion from the center, also beginning there, the 12th hour of noon. But in the lower dial the reverse occurs.

The arcs of the northern signs, furthermore, are contained in the upper dial between the center and the equinoctial line, and those of the southern signs beyond the said line; but in the lower dial the opposite.

Diagram. An inclined horizontal dial with its full fan of hour-lines. The hour-rays radiate from a point **H** low in the center (the foot of the style), spreading up and out on every side. Point **D** stands just to the left of **H**, **G** just to the right of it; a nearly horizontal base-line runs left through **F** and **D**, and to the right through **I**, **G**, and **K**. **E** stands above the center. The hour-numbers ring the figure: down the left side 9, 8, 7, 6, 5, 4; across the top and upper right 10, 11 (II), 12, 1 (I), 2, 3, 4; and along the lower edge 4, 3, 2, 1 (I), 12, 11, 10, 9, 8, 7, 6.

- **H** (center, lower): foot of the style; origin of the hour-rays
- **D**, **G**: points flanking **H** on the base-line

- F (left), I, K (right): points on the horizontal base-line
- E (above center): construction point
- 1–12 (arabic, ringing the figure): hour-line labels

EXAMPLE. Let this be an example of the whole. Let there be proposed a plane facing the zenith and the north, inclined to the horizon by 68 degrees. Since, then, the inclination is greater than the complement of the pole's altitude — for at the latitude of Rome, for which we describe all the dials in this our *Gnomonics*, the complement of the pole's altitude amounts to 48 degrees — we shall add the complement of the inclination, namely 22 degrees, to the complement of the pole's altitude, and we shall obtain the altitude of the pole above the proposed plane, 70 degrees, as we set out in the 6th precept. For this altitude of 70 degrees, therefore, we shall describe the inclined dial just like a horizontal one, as appears in the figure below.

[Margin: Description of the astronomical dial inclined to the horizon.]

The figure of the zodiacal rays and of the day-lengths is to be constructed as we showed in the horizontal dial; and from it, in the same way, both the arcs of the signs and the diurnal arcs will be described. This the adjoining figure, together with the following one, indicates.

The horizontal line is in every case to be drawn in this way. Through the position of the style **G**, let the perpendicular **G A** be drawn to the meridian line; on it take the straight line **G A**, equal to the style, and from **A**, at any interval toward the meridian line, describe an arc of a circle on which, starting from the straight line **A G**, is laid off the complement of the inclination of the dial's plane to the horizon — upward, indeed, in the upper dials, but downward in the lower ones. For the straight line drawn from **A** through the end of that laying-off will cut the meridian line at a point through which a straight line, drawn perpendicular to the meridian line, or parallel to the equinoctial line, will give the horizontal line. We may also draw it in this way. In the upper dial let there be found, on the meridian line, the point of midnight through which the diurnal arc of 24 hours passes; but in the lower dial, the point of noon through which the nocturnal arc of 24 hours runs. For through this point a straight line drawn perpendicular to the meridian line will be the horizontal, as before.

[Margin: Description of the arcs of the signs and of the day-lengths, together with the figure of the zodiacal rays. How the horizontal line is to be drawn in dials inclined to the horizon.]

Diagram. The astronomical dial inclined to the horizon at a pole-altitude of 70 degrees, showing the zodiacal rays and the day-length arcs. A horizontal line runs across the top carrying the points **H** (marked **H'**, the style), **D**, and **A** at the right end, from which a large quarter-circle arc springs and curves down to the right. A small **G** stands just below **H** at the style. From the upper region a dense sheaf of straight and dashed rays (the zodiacal rays) fans downward, crossed by two arcs of a circle. Interior points: **I** (center), **P** (lower left), **B** (center, marked with a small cross), **Φ** (phi, right of center), **V** (lower left). Along the base run the hour-numbers **9, 8, 7, 6, 5, 4, 3, 2, 1 (I), 12**, with **10, 11** set just below.

- **H** (=H', top left of the base): the style
- **D**: point on the top horizontal line
- **A** (right end): foot of the perpendicular / spring of the arc
- **G**: style-point below **H**
- **I** (center), **P** (lower left), **B** (center, cross-marked), **Φ** (phi), **V** (lower left): construction points
- 1–12 (arabic, along the base): hour-line labels

This horizontal line, moreover, divides the whole dial into two — upper and lower — as well as into diurnal and nocturnal, as was said of the vertical dial in chapters 7 and 9; yet in such a way that in the lower dial all the parts are interchanged, as was said in what precedes, and the arcs of the northern signs are turned into the arcs of the southern signs...

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changed about, and the reverse holds. And so, if it is the upper dial that has been laid out, the horizontal line cuts off the lower part; and conversely [if the lower dial is drawn, it cuts off the upper part].

Diagram. The astronomical dial inclined to the horizon, showing the equinoctial and horizon lines with the day-arcs of the zodiacal signs and the hour-lines. Two horizontal parallel lines run across the figure: the upper is labelled **AEQVA·TOR** (the equinoctial/equator line), the lower **HORI·ZON** (the horizon line). A dense central cluster of dotted arcs (the day-arcs of the signs) sits between them, from which a fan of hour-lines radiates. Zodiacal glyphs are set at the ends of the rays.

- **AEQVA·TOR**: the equinoctial line (split across the head of the figure); a zodiac glyph set at its right end under "**TOR**" — reads as ♌ (Leo) [?]
- **HORI·ZON**: the horizon line (split)

- Hours across the top: 8, 9, 10, 11, 12, 1, 2, 3, 4
- Hours at the two ends of the horizon line: 6 (left), 6 (right)
- Interior points (near the dotted cluster): a point reading L or E [?], A, G, H
- Left descending rays, with glyphs: ♂ (Taurus), 5, H, 4, 3, ♄ (Capricorn) [?]
- Right descending rays, with glyphs: ♋ (Pisces), ♋ (Cancer), ♍ (Virgo) near 7, a glyph near 8 [?], ♄ (Capricorn) [?] near 9
- ♊ (Aquarius, wavy) set near the center cluster

[Margin: The construction of the Italian and Babylonian dial inclined to the horizon.]

Diagram. The Italian (Italic) and Babylonian dial inclined to the horizon — a large fan of hour-lines carrying both Roman numerals I–XXIV (the Italic hours, counted from sunset) and Arabic numbers 1–24 (the Babylonian hours, counted from sunrise), with a horizon line across the middle and a construction circle in the upper center. Solid and dashed rays radiate outward on every side.

- Horizon line across the middle, labelled **HOR·ZON** (split), with **6** at its left and right ends
- Construction circle (upper center) with points: **T** (left), **E** (center), **K**, a point reading ς / sigma [?], and interior points **1**, **B**, **B** (two B-points), **G**, **H**, **D**, **M**
- Left flank rays (Roman, ascending): IIII, III, a numeral reading VI [?], II, I; and (descending) XII, XI, X
- Left flank rays (Arabic): 24, 23, 22, 21, 20
- Points along the horizon on the left: a numeral reading XV [?], **G**, **S**, **O**
- Right flank rays (Roman): the numeral group reading IIX / XI / X [?], and lower I, II, III, IIII
- Right flank rays (Arabic): 20, 23, 24; and lower 12, 11, 9, 8
- Lower rays: **V**, **VI**, **VII**, **VIII** (dashed)
- Scattered zodiac glyphs on the rays [?]

The drawing of the hours from sunrise or sunset is carried out as was explained in what precedes. Most conveniently, however — so far as the second method is concerned — they are drawn from the diurnal arc of 14 hours alone (in our example), whose nocturnal portion is the nocturnal arc of 10 hours. If, for the individual hours from setting — which have only single points on the diurnal arc of 14 hours — one takes, for the same numbered hours from rising, other points (whether on the same diurnal arc of 14 hours or on the nocturnal arc of 10 hours), then you see that the hour 23 from setting passes through the hour 6 from noon on the diurnal arc of 14 hours, whereas the hour 23 from rising passes

through the hour 4 from midnight on the nocturnal arc of 10 hours. If, then, these two points are joined by a straight line, one obtains — in the upper dial — the diurnal hour 23 from setting,

and in the nocturnal [dial], the hour 23 from rising. So too for the hour 20 from setting: the hour 20 from rising has been drawn through the hour 1 from midnight on the nocturnal arc of 10 hours, and through the hour 2 from midnight on the nocturnal arc of 12 hours. For this line, when produced, exhibits the hour 20 from setting, and so on.

Very conveniently, moreover, these same hours will be delineated by means of the diurnal arc of 24 hours and the nocturnal arc of 12 hours, as is clear from what precedes. You have an example in the immediately preceding figure.

The line of the 12th hour from rising or setting is drawn through the point **D**, found on the line of the 12th hour of noon [the noon-line], through which must pass the arc of the greatest of the always-apparent parallels — namely that of 24 hours — [this line being] perpendicular to the meridian line, or parallel to the equinoctial line.

Finally, the Ancient dial is to be constructed as was explained in what precedes; the figure itself makes it sufficiently clear.

[Margin: The construction of the Ancient dial inclined to the horizon.]

Diagram. The Ancient dial inclined to the horizon — a fan of hour-lines with a construction circle in the upper center, the horizon line across the middle, and hours labelled in Roman and Arabic numerals. Solid and dashed rays radiate on every side; a tropic-arc (of Cancer ☊) runs through the field.

- Construction circle (upper center) with points: **M** (top), a point reading Λ [?], **C, b, E** (center), **K**, a point reading ς / sigma [?]
- Horizon line across the middle, labelled **HOR·ZON** (split)
- Central / lower points: **A, N, D, L, G, B, H, V**, and **F** (lower right)
- Left flank rays (Roman): IIII, III, a numeral reading VI [?], II, I; and lower XII, XI, X
- Right flank rays (Roman): III^ (III), XI, X, IX (as IX/IIX [?]), and lower I, II, III, IIII
- Arabic hour-labels among the rays: 4, 6, 7, 9, and others [?]
- Zodiac glyphs on rays: ☊ (Cancer) [?] and others [?]

makes clear. For, by way of example: at the point **A** of the unequal hour 3, on the circle described from **E**, we took the opposite point **C**, and drew the straight line **CEA**, which cuts the equinoctial line at the point **D** — through which a straight line drawn through **H**, the center of the dial, when extended, cuts the tropic $\mathfrak{S}$ [Cancer] on the nocturnal arc at a certain point, the point that belongs to **C** on the nocturnal arc of the said circle; but beyond **H** it cuts the same tropic on the diurnal arc at **F**, the point of the [unequal] hour 3, and so on.

But this same dial will be delineated most aptly by means of the nocturnal arc of the tropic $\mathfrak{S}$ alone, in our example. For thus you see that the straight line drawn from **I**, the point of the 11th hour on the nocturnal arc of the tropic $\mathfrak{S}$, through **E**, cuts the equinoctial line at **L**; and that the straight line **LH** cuts the nocturnal arc $\mathfrak{S}$ at **M**, the point of the 11th nocturnal hour. If, therefore, from **M** we draw a straight line through the 5th hour from midnight on the nocturnal arc of 12 hours (as is established from Table 14, Proposition 33, Book I), we shall have the unequal nocturnal hour 11, which, produced beyond the horizontal line, will also give the diurnal hour 11. And in the same way we shall draw the remaining hours.

As for the other construction, which is carried out by means of the diurnal arc of 18 hours and the nocturnal arc of 6 hours, the matter is clear from the figure itself.

ON DIALS DECLINING FROM THE VERTICAL AND INCLINED TO THE HORIZON.

CHAPTER XXIX.

That dial we called, at the beginning of this our *Gnomonics*, a dial "declining from the vertical circle and inclined to the horizon": it is one that lies parallel to a great circle which is perpendicular neither to the horizon, nor to the vertical, nor to the meridian, but is inclined to all these circles, so that it passes through the poles of none of them — of which kind are most of the [wall-]faces of buildings. This [class] is divided into eight kinds. For it either declines from the south toward the east, or west,

[Margin: The dial declining from the vertical and inclined to the horizon, eightfold.]

west, and faces the Zenith and the south; or it declines from the north toward the east, or west, and looks toward the Nadir and the north — in such a way that the

lines forming the angle of inclination which it makes with the horizon slope toward the north, toward the west or the east. Or else it declines from the north toward the east, or west, and faces the Zenith and the north; or it declines from the south toward the east, or west, and looks toward the Nadir and the south — in such a way that the lines constituting the angle of inclination which it makes with the horizon slope toward the south, toward the west or the east.

[Margin: The names of the eight dials declining from the vertical and inclined to the horizon.]

These eight dials, then, may be named as follows: the Upper Southern verging east; the Lower Northern declining west; the Upper Southern declining west; the Lower Northern declining east; the Upper Northern declining east; the Lower Southern declining west; the Upper Northern declining west; the Lower Southern declining east.

There are, then, four upper ones. Of these, the first two decline from the south toward the east or west and face the Zenith and the south; while the latter two decline from the north toward the east or west and face the Zenith and the north. Likewise there are four lower ones, of which the first two decline from the north toward the east or west and face the Nadir and the north, while the latter two decline from the south toward the east or west and look toward the Nadir and the south.

And these lower dials are described on the same planes as the upper ones, though on entirely opposite parts of them. For on the part opposite to the upper plane — which, say, declines from the south toward the east by 45 degrees, faces the Zenith and the south, and is inclined to the horizon by 20 degrees — is described the lower dial, which declines from the north toward the west by 45 degrees, faces the Nadir and the north, and is inclined to the horizon by 20 degrees. And the same judgment holds for the rest. For the declination of the lower dial is always the same as that of the upper, though not toward the same quarters: when the upper declines from the south toward the east and faces the Zenith and the south, the lower declines from the north toward the west and faces the Nadir and the north — and so with the others. The angle of inclination of the lower dial and of the upper, however, is always set toward the same side.

So that we may describe all these dials more easily, we shall give precepts only for describing the upper ones. For thus it will come about that the whole matter, which is somewhat difficult and obscure, is more clearly grasped. And once the upper dials have been described, we shall, with no trouble, fashion the lower

ones from them — just as, among the dials declining from the vertical, the northern were described from the southern in chapters 23, 24, 25, and 26; and among those declining from the horizon, the lower from the upper in chapter 27. All of which will become clear from the figures that follow.

Diagram. "First Figure" — the construction of the astronomical dial declining from the vertical and inclined to the horizon at the same time. A large circle (the bounding/horizon circle) is drawn; across it runs a horizontal diameter from **A** (left) through the center to **B** (right), and a near-vertical line through **F** (top) and **C** down through the central region. Numerous solid, dashed and dotted construction lines and arcs cross the interior.

- Caption (inside the field): **I. Figura** ("First Figure")
- **A** (left end of the horizontal line) and **B** (right end)
- **F** (top, on the vertical dashed line)
- **C** (upper center, on the horizontal line)
- **E** (upper right)
- **D** (below **C**, toward the center)
- **K** (central point — the position of the style)
- **H** (lower, right of center)
- β (the gnomon-length point, printed with a special sort; near the lower center)
- Further construction points, read but not all certainly placed: **L**, **I**, **M**, **G**, **b**, **d**, and a point reading **R** [?]; a point reading **a** (lowercase) [?] near the center; σ / sigma-like marks at the upper-left and at the foot of the figure [?]

[Margin: The construction of the astronomical dial declining from the vertical and inclined to the horizon at the same time.]

So, then, we shall carry out the matter as follows. On the plane of the dial, having taken the position of the style at **K** wherever you please, draw through **K** two straight lines **AB** and **CD**, cutting each other at **K** at right angles. Or, if the plane is a fixed one declining from the vertical and inclined to the horizon, draw — with the help of a level and a plumb-line — through **K** the straight line **AB** parallel to the horizon, which **CD** must cut at **K** at right angles. Then, taking on **AB** any length of the gnomon you please, **K β** — whether to the right of the straight line **CD** or to the left — describe from β , toward **CD**, an arc of a circle on which is counted off the inclination of the plane to the horizon (which in the first figure we set at 20 degrees; in the second at 70; in the third at 51 degrees 51 minutes; in the fourth at 30; in the fifth at 80; and in the sixth at 52 degrees 3

minutes), the start being made from the straight line **AB**, downward. Draw, through the end of the count, from β , a straight line cutting **CD** at the point **D**, through which the meridian line is to be drawn. Likewise, having reckoned off on the same arc, upward, the complement of the inclination, draw from β , through the end of the reckoning, a straight line (which will be perpendicular to βD) cutting **CD** at the point **C**, through which the straight line **CE**, drawn perpendicular to **CD**, will be the horizontal line.

[Margin: The horizontal line.]

Next, on the straight line **CD**, taking a straight line **CF** — upward or downward — equal to **C β** itself, describe from **F**, toward **CE**, an arc of a circle on which is counted off, from the straight line **CD**, the declination of the plane from the vertical circle (which in the first figure we set at 40 degrees from the south toward the east; in the second, 20 degrees from the south toward the west; in the third, 45 degrees from the south toward the east; in the fourth, 20 degrees from the north toward the west; in the fifth, 60 degrees from the north toward the east; and in the sixth, 30 degrees from the north toward the east) — to the right, indeed, if the plane deflects from the south toward the east, as in figures 1 and 3, or from the north toward the west, as in the fourth figure; but to the left, if the plane declines from the south toward the west, as in the second figure, or from the north toward the east, as in figures 5 and 6. Draw from **F**, through the end of the count, a straight line cutting the horizontal **CE** at **E**. For the straight line **DE**, drawn through **D** and **E** and produced in both directions, will give the meridian line — that is, the 12th hour from noon or midnight.

[Margin: The meridian line.]

Likewise, having reckoned off on the same arc, from the straight line **CD**, in the opposite direction, the complement of the declination, draw from **F**, through the end of the reckoning, a straight line (which will be perpendicular to **FE**) cutting the horizontal line at the point **a**, through which the equinoctial line and the line of the 6th hour from noon or midnight must be drawn. And once the straight line **a–K** has been drawn — which is necessarily perpendicular to the meridian line, if no error has been made — describe from **D** and **E** two arcs at the intervals of the straight lines **D β** and **EF**; these will necessarily cut each other, if no error has been committed, on the straight line **a–k** [printed lowercase], as at the point **H**. Whether this happens on the right side or the left makes no difference.

(Book VII, Cap. XXIX continues — the general construction of dials that decline from the vertical and are inclined to the horizon. The six worked figures span these

pages: figures 2 and 3 at the top of the first page, figure 4 at its foot, figure 5 on the second page, and figure 6 on the third. Point α is printed as a Greek alpha and the dial's center ρ as a Greek rho; the lowercase b , d , k are distinct printed sorts.)

Now, once the straight line **HE** has been drawn, describe from **H** an arc of a circle on which, beginning from the straight line **HE**, the complement of the pole's altitude is counted off toward the straight line **HD** — provided the plane of the dial declines from the south, as in the three preceding figures. Through the end of the count, draw from **H** the straight line **Hb**, cutting the meridian line at the point **M**, through which the equinoctial line is to be drawn from the point α , found earlier. And if, on the same arc but in the opposite direction from the straight line **HE**, the pole's altitude itself is counted off, and from the end of the count the straight line **Hd** is drawn through **H** — which will necessarily be perpendicular to **HM**, unless there is an error — then the same meridian line will be cut at the point ρ , which will be the center of the dial. But if it should happen that this line **Hd**, drawn last through **H**, is parallel to the meridian line — as occurs in the third figure — the dial will have no center, and all the hour-lines will be equidistant from one another; because in that case the great circle to which the dial is equidistant passes through the poles of the world. If, however, the plane of the dial declines from the north, as

Diagram. The second of the six worked figures for a dial that both declines and is inclined — a circle carrying the construction of the meridian, equinoctial, horizontal, and style lines, with the shadow-triangle and hour-line "6" markers. Legible labels:

- A (left) and B (right): the ends of the horizontal line **AB** across the circle.
- P (top), F (just below P): points on the upper vertical/meridian construction.
- E, C, I (row across the middle): points on the central axis of the figure.
- H (lower left), K, G, M (center cluster): construction points about the center.
- β (right of center) and α (upper right, near an outer "6"): the beta and alpha points.
- L (below center) and D (foot of the circle): points on the lower meridian.
- b (lower right, near a "6"): the point **b**.
- "6" (several positions, outside the circle): the 6th-hour lines.

[Several of the crowded interior labels (I/J, G, β) are partly overwritten by ruled lines; read with care.]

Diagram. The third figure, corresponding to the case where line **Hd** comes out parallel to the meridian (the dial without a center). A circle with the same construction lines. Legible labels:

- A (left), B (right): the horizontal line **AB**.
- F (top), C (center-top), E (right of C): upper points.
- K, β (center): interior points.
- L, M (left of center), D (below center), H (lower right): lower construction points.
- d (upper right, on a dashed line): the point **d**.
- "6" (top and elsewhere, outside): the 6th-hour lines.

[The dense central intersection carries several small labels I cannot fully resolve (β , and a mark below K).]

in the three later figures, the pole's altitude must be counted off on the said arc of the circle described from **H**, starting from the straight line **HE**, toward the line **HD**. For the straight line **Hd**, drawn from **H**, will through the end of the count cut the meridian line at **p**, the center of the dial. And if, on the same arc in the direction opposite to the straight line **HE**, the complement of the pole's altitude is reckoned, and from the end of the count the straight line **Hb** is thrown across through **H** — which will be perpendicular to **Hd** — the meridian line will be cut at the point **M**, through which the equinoctial line is to be drawn from α . But if it should happen that this line **Hb**, drawn last, is parallel to the meridian line — as in the sixth figure — then the equinoctial line must be drawn through α , parallel to the meridian line. If, then, a straight line is drawn from the center of the dial **p** through **K**, the position of the style, we shall have the style-line, which the equinoctial line drawn from α will necessarily cut at right angles. But where there is no center — as in the third figure — the style-line must be drawn through **K**, parallel to the meridian line and perpendicular to the equinoctial line. Likewise, where the equinoctial line is equidistant from the meridian line — as in the sixth figure — the style-line **pK** will be perpendicular not only to the equinoctial line but also to the meridian line. And so, if the equinoctial line is drawn first, the style-line must be drawn perpendicular to it, from the center **p** or from the position of the style **K**; but if the style-line is drawn first, from the center **p** through **K** — or, where there is no center, through **K** parallel to the meridian line — then the equinoctial line must be drawn perpendicular to it from α .

Diagram. The fourth figure, at the foot of the page beside the text column — a north-facing declining-inclined dial. A circle with the meridian, equinoctial, horizontal, and style lines and the shadow-triangle. Legible labels:

- α (upper left, at the vertex of the shadow rays): the point α .
- C, E (top center): points on the upper meridian.
- B (right), "6" (right, below B): the horizontal line end and a 6th-hour marker.
- d (upper left, on a dashed arc), I (left of center): construction points.
- K, ρ , F, D, H (central cluster): the center ρ and neighboring points.
- G (lower left), M (below G): lower points.
- b (upper center, small): the point b .

[Margin: The equinoctial line.]

[Margin: The center of the dial.]

[The central cluster K/ ρ /F/D is very dense; individual labels are hard to separate.]

After this, from the position of the style **K**, erect a straight line **KI** perpendicular to the style-line, equal to the style, and from the center ρ through **I** let the straight line ρ **I** be sent out for the axis of the world. But where there is no center — as in the third figure — the axis must be drawn through **I**, parallel to the style-line or to the meridian line. And if from **I** a perpendicular is dropped to the axis, it will cut the style-line at the point **G**, through which the equinoctial line is drawn — unless we have committed an error. Hence, where the dial lacks a center, so that the axis is equidistant from the style-line — as in the third figure — the said perpendicular will fall on **K**, the position of the style, and the equinoctial line must be drawn through that same position of the style. And so, if the axis of the world has been drawn through **I**, we shall find, by means of the perpendicular line **IG**, another point **G** on the style-line, through which the equinoctial line is to be drawn from the point α — a line that will necessarily make right angles with the style-line.

[Margin: The axis of the world.]

Finally, taking a straight line **GL** on the index-line, equal to **GI** itself, describe from **L** a circle of any size, and divide it into 24 equal parts, beginning from the straight line **LM** — which is drawn from the center **L** through the point **M**, where the equinoctial line and the meridian cut each other — or from the straight line **L α** , drawn from the same center **L** through the point α , where the horizontal line

and the equinoctial line meet, that is, the line of the 6th hour. Now it is necessary that the two straight lines **LM**,

Diagram. The fifth figure — a north-facing declining-inclined dial with the hour-circle described about **L**. A large circle with the meridian, equinoctial, horizontal, and style lines and the 24-part hour-circle. Legible labels:

- **M** (top, at the apex of the shadow rays): the point **M**.
- **b** (below **M**, small): the point **b**.
- **F**, **L** (center): the center **L** and neighboring point **F**.
- **E** (left of center), **A** (left), **B** (right): points on the horizontal line.
- "6" (upper right and lower left, outside): the 6th-hour lines.
- **G** (lower left), **D** (foot): lower construction points.

[The central intersection (**F/L** and the small labels around it) is dense; a few interior letters cannot be resolved.]

Lα cut each other at **L** at right angles, if no error has been committed. But if the equinoctial line is parallel to the meridian line — as in the sixth figure — then in place of the straight line **LM** a straight line must be drawn parallel to the meridian, one that cuts the straight line **L**α at right angles. Now if hidden [construction] straight lines are drawn from **L** through the division-points, the equinoctial line will be cut at points through which, from the center **p**, the lines of the hours from noon and midnight must be drawn; or, when the dial lacks a center — as in the third figure — through which lines must be drawn parallel to the meridian line, or to the style-line or the axis, and perpendicular to the equinoctial line, for the hours from noon or midnight.

[Margin: The lines of the hours from noon and midnight.]

The order of the hours is this. On planes that face south, the part of the meridian line extending from the center of the dial **p** toward the equinoctial line — as in the first two figures — or the whole meridian line where there is no center, as in the third figure, marks the 12th hour of noon. Those parts which lie to its right on the equinoctial line (we being turned to face the dial toward the north) signify the afternoon hours, and those to the left the forenoon hours; while the remaining part of the meridian line, beginning at the center, shows the 12th hour of midnight — just as in the southern vertical dial. But on planes facing north, when the equinoctial line lies below the center of the dial **p**, on the index-line — as in the fourth figure — the part of the meridian line extending from the center **p** toward the equinoctial line marks the 12th hour of noon; those parts which (we

being turned to face the dial toward the south) lie to its right on the equinoctial line show the forenoon hours, and those to the left the afternoon hours; while the remaining part of the meridian line, extended beyond the center toward the upper parts, belongs to the 12th hour of midnight — just as in the horizontal dial. But when the equinoctial line lies above the center of the dial **p**, on the style-line — as in the last two figures — the part of the meridian line drawn from the center **p** toward the equinoctial line, produced; or indeed, where the equinoctial line and the meridian are parallel — as in the sixth figure — the part

[Margin: The order of the hours.]

tending toward the upper parts, marks the 12th hour of midnight; while the hours descending to the right of the meridian line on the equinoctial line (we being turned to face the dial toward the south) are the forenoon hours, and those to the left the afternoon hours; the remaining part of the meridian line, below the center of the dial **p**, pertains to the 12th hour of noon — just as in the northern vertical dial.

Now if from the point **F** straight lines are drawn out that make, with the straight line **FE**, the same angles which the hour-lines make with the meridian line in the horizontal astronomical dial — as we taught in Cap. 23 [i.e. XXIII] for the dial declining from the vertical — the horizontal line **aE** will be cut at points through which the straight lines sent out from the center **p** (or, where there is no center, straight lines drawn equidistant from the meridian line) will give the lines of the hours from noon and midnight, as before.

[Margin: By what method the hour-lines on a plane that at once declines and is inclined were described from the horizontal dial.]

A dial that both declines and is inclined is to be set in its proper position thus. Let the horizontal line be set equidistant from the horizon; and let the common section of the dial and of the plane equidistant from the horizon, taken together with the meridian line in the plane that is equidistant from the horizon, once found, make the angle of the complement of the declination — toward the south and west if the dial declines from the south toward the east; toward the south and east if it declines from the south toward the west; but toward the north and west if the dial declines from the north toward the east; toward the north and

[Margin: How a dial that at once declines and is inclined is to be placed in its proper position.]

Diagram. The sixth figure — the fully drawn dial, in which (as the text notes) all the hour-lines have been laid in. A large circle whose rim carries the hour-numbers, with the meridian, equinoctial, horizontal, style, and full set of hour-lines radiating from the interior. Legible labels:

- Rim hour-numbers: **10, 11, 12, 1, 2, 3, 4, 5, 6, 7, 8, 9** (running around the circle; a few are faint).
- Interior letters: **F** (upper center), **E, C, G, H, K** (central cluster), **A** (left), **B** (right), **I** (lower left), **L, M, d**.

[This is the most crowded of the six figures; the central letter-cluster (K/H/G and neighbors) and several faint rim numbers cannot all be resolved with certainty.]

tum, if it deflects from the north toward the west. For in this way the dial will have its due declination. Next, let the straight line **CD**, which divides the horizontal line at right angles, together with the straight line in the horizontal plane that cuts the common section of the dial and of the horizontal plane at right angles, make the angle of inclination — toward the northern parts if the dial faces south, but toward the southern parts if it faces north. For once the dial has been set in place by this method, if along the axis **PI**, in the triangle **PIG**, at right angles to the plane of the dial, one imagines a thread stretched, or some fine iron wire, its shadow will indicate the several hours from noon or midnight, for as long as the Sun illuminates the dial itself. Or else the tip **I** of the style **Ik**, standing at right angles to the plane of the dial at the point **K**, will do the same, as we said in the case of the horizontal dial. When the dial lacks a center — as in the third figure — the axis must be drawn through **I**, equidistant from the style-line **GL**.

We have now completed this whole account of a dial that is at once declining and inclined in six figures, so that every variety of such dials might be displayed. Of these, the first three belong to those dials, facing south, in which the center of the dial either lies below the equinoctial line, or above it, or else the dial has no center and all the hour-lines are parallel, cutting the equinoctial line at right angles. The latter three represent those dials, facing north, in which the meridian line cuts the equinoctial line either below the center of the dial or above it, or else is parallel to the equinoctial line. Moreover, only in the sixth figure have we drawn all the hour-lines; in the others we have shown only those elements which are needed to understand the construction, the hour-lines being left out, so that a confusion of lines

so that we might avoid a confusion of lines. For anyone will be able to draw the hour-lines in these dials, just as in that one, provided he carefully considers the rules we have handed down.

[Margin: In what way an Inferior dial is generated from a Superior one.]

Now if any Superior dial whatever is turned over, so that its upper part becomes the lower, and what after this inversion is on our right becomes the left, and vice versa — as was made clear in what precedes — we shall obtain an Inferior dial. But the hour-lines that in the Superior dial formerly indicated the hours from noon will, in the Inferior dial, show the hours from midnight, and conversely. Here, moreover — as also in the preceding cases — the lineaments of the Superior dial, drawn on the opposite face, display this inversion beautifully: each single line answers exactly to its counterpart, provided the upper part of the opposite face becomes the lower, and vice versa.

[Margin: The drawing of the arcs of the signs and of the lengths of the days on a dial that declines and is at the same time inclined.]

The arcs of the signs and the lengths of the days are drawn here just as in the preceding cases. Draw the straight line ρI for the axis of the world, and at I erect the perpendicular IG upon it, to serve as the radius of the equator; to either side of this let the rays of the other signs and the lengths of the days be led out, as was done in the earlier cases. Then let $I\rho$ be equal to the portion of the axis ρI in the dial, and the straight line IG equal to the line IG (or LG) in the same dial; and from ρ , through G , let the straight line ρG be drawn. This we shall perhaps draw more conveniently if from ρ we draw the straight line ρG such that with the axis ρI it makes the angle $G\rho I$ equal to the angle $G\rho I$ which, in the dial, the axis ρI and the style-line ρG form — that is, equal to the angle of the pole's elevation above the plane of the dial. Finally let everything else be done as we set forth in Chapter 24, when we were treating of a dial declining...

Diagram. The figure of the rays of the Zodiac [figura radiorum Zodiaci] for a dial declining from the vertical, offered as the worked example that answers to the last of the six Superior dials described just before. It has two connected panels. The main (left/central) panel: a horizontal top line carrying the points I (far left), B , ρ , K ; from these a fan of solid and dashed rays descends to a large semicircle drawn below, with an interior point G , a point B on a lower horizontal, and a central point V . Several rays are captioned with the radius of a given hour: "**Radius hor. 0**", "**Rad. hor. 2**", "**Rad. hor. 10**", "**Rad. hor. 18**", and "**Rad. hor. 24**" (this last appears twice, near the top-right and near the lower centre). Along

the lower rim run paired hour-numbers (the *bini numeri* of the text), arranged symmetrically on either side of the noon line; the legible values include the full run 1–12 repeated on each side, but the small worn digits cannot all be resolved with certainty. The right-hand panel is a triangle whose top line carries **B, ρ, B, I**, with **A, K, A** on the transversal just below, and **D, G, D** descending its right edge; it is captioned "**Rad. hor. 24**" and "**Rad. hor. 18**".

[This figure is very crowded; the paired rim-numbers and several ray-captions are faint and cannot all be resolved with certainty.]

from the Vertical. For the figure of the rays of the Zodiac will thus be constructed, and from it the arcs of the signs will be drawn, as in a dial declining from the vertical. We have subjoined an example answering to the last Superior dial of the six we described a little earlier. Here, since the style-line is the same as the line of the 6-o'clock hour, two hour-numbers stand opposite to each single hour-line issuing from the point ρ of the figure of the Zodiac's rays, set equally on either side of the 6-o'clock line — that is, at equal distances from the style-line — as we advised in Chapter 24. Again, the straight line pV, being parallel to the radius of the equator, shows the 12th hour, because the line of that hour in the dial runs parallel to the equinoctial line: just as, in a horizontal dial, the straight line HV, parallel to the radius of the equator in the figure of the Zodiac's rays drawn in Chapter 2, marks the 6th hour, since this line too runs parallel to the equinoctial line in the dial.

We have also added another figure of the Zodiac's rays, issuing with hour-lines from ρ, answering to the first dial of this chapter, in which the style-line is neither one of the hour-lines nor equally distant from two placed on either side. Hence it comes about that each single line issuing from ρ has its own single number affixed to it. From this figure, therefore, one may draw the arcs of the signs in the said dial (provided all the hour-lines have first been drawn in it), whenever the occasion requires. We have divided this figure, as you see, into two, because otherwise its lines would too greatly...

Diagram. A large, fully drawn declining-and-inclined dial with the arcs of the signs laid in. A dense web of straight hour-lines and curved sign-arcs fills the field. Legible interior letters: **A, C, D, E, B** (left cluster), **ρ** (the central point, printed with a distinctive sort), **K, H** (central), **h, i, m, I** (lower). A dashed fan of lines runs from **K**, numbered **6, 7, 8, 9, 10** in sequence; further hour-numbers (**4, 5, 6, 7, 8, 9, 10, 11**) run along the outer rim. Zodiac-sign glyphs mark the arcs of the signs around the periphery; several are legible on the right and lower-right

(♉ [Taurus], ♊ [Pisces], ♏ [Scorpio]), but most are too small or worn to resolve individually [?].

[Crowded figure; the central letter-cluster and the great majority of the rim zodiac-glyphs cannot be resolved with certainty.]

run together, because the style-line of the first dial is only slightly distant from the line of the 11th hour, as is plain from the first figure of Proposition 37 of Book 3, in which the hour-lines are drawn. In the former of these figures are contained the...

Diagram. The former [prior] of the two panels into which the added figure of the Zodiac's rays is divided. A quarter-dial: a semicircular arc across the top, a horizontal base-line, and a fan of rays issuing from the point **p** (printed with the distinctive sort). Legible labels: **I** (top), **p**, **A** (left), **D**, **F**, **E**, **B** (centre). The rays are numbered with the hours placed to the left of the style-line, the legible values running 5, 6, 7, 8, 9, 10 with 12 near the style; the text states these are the hours 5, 6, 7, 8, 9, 10, and 11.

Diagram. The latter [posterior] of the two panels. Constructed like the former: a semicircular arc across the top, a horizontal base-line, and a fan of rays from **p**. Legible labels: **p**, **I**, **F**, **A**, **B** (and rim-numbers 4, 3, 2 along the arc). The rays are numbered with the remaining hours placed to the right of the style-line — the legible values including 12, 1, 2, 3, 4, 5.

hour-lines that are placed in the dial to the left of the style-line in the first figure of Proposition 37 of Book 3, or that would be placed to the left in the first figure of this chapter, had they been drawn — namely 5, 6, 7, 8, 9, 10, and 11. In the latter figure are the remaining hours, set to the right of the same style-line — namely 12, 1, 2, 3, 4, 5, and so on. This is what we advised in Chapter 24 must be done in declining dials, and still more in inclined ones, in which such a confusion of lines is found.

Finally, so that you may see what is to be done in the third dial of this chapter — in which the hour-lines are parallel, as were drawn in the third figure of Proposition 37 of Book 3 — we have also drawn the arcs of the signs in it. These are drawn just as in a meridian or a polar dial, with this one exception, that here the hour-lines in the figure of the rays...

of the rays cut the radius of the equator at right angles and so have each a single number, not two, as they had there — unless the style-line is one of the hour-

lines, or at any rate lies equally distant from the two nearest on either side, as usually happens in other declining dials.

[Margin: The horizontal line divides the dial into a Superior and an Inferior part, and into a Diurnal and a Nocturnal part.]

The horizontal line divides the whole dial, as was said in the preceding sections, into a Superior and an Inferior part: the Superior is that part contained below the horizontal line, while the remaining part will give the Inferior — provided, however, that all the parts are inverted, as was said above, and that the hours formerly reckoned from midnight are now reckoned from noon, and so on. The same horizontal line separates the whole dial into a Diurnal and a Nocturnal part, as was explained in the preceding sections.

Diagram. A large declining-and-inclined dial with the arcs of the signs, hour-lines, and shadow-rays drawn in. A dense field of straight lines converging from the left with a set of curved sign-arcs to the right. Legible interior letters: **A** (upper right, on a dashed vertical), **D, B, C** (dashed, right of centre), **E, F (?)**, **G, I, H**, and **h** (on a dashed line, lower centre). Two rays are captioned "**Rad. hor. 14**" and "**Rad. hor. 10**" (lower left). Hour-numbers **1, 2, 3, 5, 6, 7** run along the horizontal lines at the left; zodiac-sign glyphs mark the sign-arcs around the periphery (several legible, most too small or worn to resolve [?]).

[Crowded figure; the central letter-cluster, the Italian/Babylonian construction points (L, M, N, a, b, d, e named in the text), and most rim zodiac-glyphs cannot be resolved with certainty.]

[Margin: Which arcs of the signs belong to the northern signs, and which to the southern.]

Furthermore, the arcs of the northern signs are contained between the centre of the dial and the equinoctial line in Superior dials that decline from the north, and also in those which decline from the south — provided, however, that they have their centre below the equinoctial line, as in horizontal dials. But if such dials, declining from the south, have their centre above the equinoctial line, the arcs between the centre and the equinoctial line will belong to the southern signs, as in Vertical dials facing south. And if dials of this kind have no centre, the arcs of the southern signs will lie above the equinoctial line, as in polar dials, and so on. In Inferior dials the exact contrary is to be understood.

[Margin: By what method a larger or smaller dial may be drawn from the figure of the rays, for a given length of the style.]

Now if, from the figure of the Zodiac's rays of this chapter, a larger or smaller dial is to be drawn for a given size of style, it will be done in this way. Draw separately — as in the figure of the rays of this proposition you see — the triangle ρIG , taken from the dial constructed in this chapter, or from the figure of the rays itself, in which the axis of the world is ρI , the equator IG , and the style-line ρG , upon which the style IK falls at right angles. Then, the straight lines $I\rho$, IG , IK having been produced, if on IK a straight line IA is taken equal to the given gnomon — whether it be larger than the style IK or smaller — and through A a line BD is drawn parallel to ρG , the triangle BID will be similar to the triangle ρIG . Therefore, if the axis IB is transferred into the figure of the rays from ρ as far as B , toward I , and from B the rays of the signs are led out, and everything else is done as was set forth in the preceding sections, the arcs of the signs will be drawn for the size of the style IA , and so on.

[Margin: The construction of an Italian and Babylonian dial that declines and is at the same time inclined.]

An Italian and Babylonian dial will be made just as was said in the preceding sections. For if the circle drawn from L is cut into the diurnal arc of $\mathfrak{S}$ [Cancer] by a $M b$, and into the diurnal arc of $\mathfrak{V}$ [Capricorn] by $d M e$, so that the straight lines $a b$ and $d e$ cut at right angles the straight line LM which is drawn through the centre L parallel to the meridian line — that is, so that the semidiurnal arcs are reckoned from the point M as far as the points a, b and d, e — and if the division of the same circle into 24 equal parts takes its start from the western points b, e for the Italian dial, and from the eastern points a, d for the Babylonian, from east and from west as in the preceding cases: then, since in our example the tropic of $\mathfrak{V}$ [Capricorn] cannot be drawn, it will be enough if the points of the hours from east and west are found on the tropic of $\mathfrak{S}$ [Cancer], by help of the diurnal arc $a M b$ of that same tropic. For these points, joined by straight lines to the corresponding hours on the equinoctial line, as in the preceding cases, will give the hours from east and west. Now in the Superior dial of our example — that is, in the portion contained below the horizontal line — we shall find only the points of the hours from the west from 9 up to 16 inclusive; likewise the points of the hours from the east from 1 up to 7 inclusive: because all these hours are contained in the arc of the circle $a M$, from whose points straight lines drawn through the centre L meet the equinoctial line, after they have been drawn through the centre L . But the points of the 17th and 18th hours from the west we shall find thus on the tropic of $\mathfrak{S}$ [Cancer]. From the points of the 5th and 6th hours from the west, which are contained in the arc $a N$ and are set opposite to the said 17th and 18th hours from the west, we shall draw hidden lines through...

through the centre L. For by their help we shall find, on the tropic of ☿ [Cancer], in the nocturnal portion, the points for the 5th and 6th hours; and from these, hidden straight lines drawn through ρ, the centre of the dial, will give, on the same tropic of ☿ [Cancer], the points of the 17th and 18th hours from the west. The 18th hour must be drawn parallel to the meridian, or equinoctial, line; while the 17th hour must be drawn through the 11th hour from midnight on the equinoctial line. But the points of the 8th, 9th, and 10th hours from the east, on the tropic of ☿ [Cancer], we shall find in this way. From the points of the 20th, 21st, and 22nd hours from the east — which are contained in the arc a N, and are set opposite the said 8th, 9th, and 10th hours from the east — we shall draw hidden lines through the centre L. For by their help we shall detect, on the tropic of ☿ [Cancer], in the nocturnal portion, the points for the 20th, 21st, and 22nd hours; and from these, hidden straight lines drawn through ρ, the centre of the dial, will give, on the same tropic of ☿ [Cancer], the points of the 8th, 9th, and 10th hours from the east.

Diagram. A large projection-figure for an Italian-and-Babylonian sundial that both declines and is inclined [the figure belonging to the end of Chapter XXIX]. At the centre lie two distinct points: L, the centre of the horizon circle (through which the meridian and the perpendicular to it cross), and ρ, the centre of the dial toward which the hour-lines converge. The word "HORIZON" is set across the horizontal diameter (split by the figure: "HORI:" on the left, "ZON" on the right). A circle is drawn about L; on it are marked the points M (upper, on the meridian), N (lower), and the tropic-arc points a and b (western and eastern) and d and e. Faint (dotted) diurnal arcs carry the small argument-numbers 1 through 18. From ρ radiate straight hour-lines to the rim, labelled with Roman numerals for the Italian/Babylonian hours; many are set rotated to follow their lines.

- L (centre): centre of the horizon circle
- ρ (centre): centre of the dial (hour-lines converge here)
- M (top of circle): point on the meridian
- N (bottom of circle): point on the circle
- a (lower left), b (right): tropic-arc points
- d (left), e (upper right): points on the circle
- Small arc-numbers along the dotted diurnal arcs: 1–18
- Peripheral Roman hour-numerals (rotated; readable include): IIII, VI, VII, VIII, IX, X, XI, XII, XIII along the top; also "20", "23", "24" near the upper right; I, II, III, IIII, V, VI down the right side; VI, VII, VIII, IX, X across the bottom [several rim numerals worn/rotated — [?]]

[The figure is uncaptioned.]

Moreover, the 6th hour from the east must be drawn parallel to the meridian, or equinoctial, line. The 7th hour from the east must be drawn through the 1st hour from noon or midnight on the equinoctial line, and the 8th hour from the east through the 2nd hour from noon or midnight, and so on — as is clear from the table of Proposition 19 of Book 1.

Either dial [both the Italian and the Babylonian] will be most conveniently laid out by means of the diurnal arc of 24 hours together with the diurnal arc of 14 hours, which in the nocturnal portion of the dial changes into a nocturnal arc of 10 hours. For all the hour-lines from east and west touch the diurnal arc of 24 hours at the points of the hours reckoned from noon or midnight. So that almost always, in laying out the hours from east and west, we shall have three points for each single hour: one on the diurnal arc of 24 hours, a second on the diurnal arc of 14 hours (or the nocturnal arc of 10), and a third, finally, on the equinoctial line — that is, on the diurnal or nocturnal arc of 12 hours — as is evident from the figure.

Finally, an Antique dial [horologium Antiquum] that declines and is at the same time inclined will be constructed just as in the preceding cases. This is just what the figure itself shows, in which we have laid out a diurnal arc of 18 hours, which in the nocturnal portion of the dial is changed into a nocturnal arc of 6 hours.

[Margin: The construction of an Antique dial that declines and is at the same time inclined.]

Diagram. A projection-figure for the Antique (Italian) declining-and-inclined sundial [the figure referred to in the preceding paragraph, showing the 18-hour diurnal arc]. Its construction matches the figure on the previous page: a circle drawn about its centre L, with the dial-centre ρ near it; the word "HORIZON" set across the middle ("Ho" on the left, "RIZON" on the right). On the circle are the points M (upper), N (lower), a, b, d, and e; faint diurnal arcs carry the small argument-numbers 1 through 12; and Roman hour-numerals radiate to the rim. A small ornamental pointer/figure appears at the lower left near the numeral VII.

- L (centre): centre of the circle
- ρ (centre): centre of the dial
- M (upper), N (lower): points on the circle
- a (lower left), b (right, on the horizon line), d (left), e (upper): points on the circle

- Small arc-numbers along the faint diurnal arcs: 1–12
- Peripheral Roman hour-numerals (rotated; readable include): VI, VII, VIII, IX, X, XI, XII along the top; I, II, III down the right side; V, VI at the bottom [several rim numerals worn/rotated — [?]]

[The figure is uncaptioned.]

ON THE DESCRIPTION OF DIALS ON ANY PLANE

WHATEVER, and for any latitude of place, by means of an instrument constructed for this use.

CHAPTER XXX.

[Margin: The construction of the instrument by which dials may be described on any plane and for any latitude of place.]

From copper, or brass, or some other hard material, let a rectangle AB be prepared, in which the straight line CD represents the meridian line. At right angles to it let a plane CDE stand, in which, from the centre F, a quadrant FGH is described, so that the semidiameter FG is parallel to the straight line CD, while the semidiameter FH is perpendicular to the plane AB; and let the quadrant be divided into 90 degrees, beginning at G and proceeding toward H. Next let an axis IK be made, of moderate length, whose portion IM — equal to the semidiameter of the quadrant FGH — is to be square, that is, to have the form of a parallelepiped, while the remaining portion MK is to be round, in the manner of a cylinder.

At the end I, let a rounded pin IL be fixed at right angles to the axis itself, so that it can be set into the centre F (which is bored through), and the axis fastened by a small screw L, to the end that it may be freely raised and lowered about the centre I — provided that at M there projects a hollowed-out piece MN. This is so that the axis, whether raised or lowered, may always cling to the quadrant FGH and be fastened by a small screw O, the point being that the axis may be secured on the quadrant at the given latitude of the place. Let the parallelepiped IM also have, in its middle, opposite the piece MN, a square opening P, so that through it one may inspect the degrees of the quadrant at which the axis is to be fixed. For the end of the axis is not to be fixed to the degrees of latitude, but rather its middle, which corresponds to the centre of the pin IL, or of the quadrant FGH. Finally, let a ring QR, movable up and down, be placed around the axis; it can be fixed at any point of the axis by means of a small screw S.

Diagram. The dialling instrument described in this chapter, shown in perspective. Above is the axis assembly: a long rod, square (parallelepiped) at its inner portion IM and round (cylindrical) toward MK, carrying the rounded pin (at the left end, near I/A) and, along its length, the projecting collars and rings with their fastening screws. Below it lies the rectangular base-plate with the graduated quadrant standing on it.

- A, B: corners of the rectangular base-plate (A at left, B at lower right)
- E, H, F: points on the base/quadrant (E upper left of the quadrant, H lower left, F lower right at the pivot)
- G, D: at the upper right of the quadrant (G, D adjacent)
- Quadrant degree-graduation along its arc: 10, 20, 30, 40, 50, 60, 70, 80, 90
- I: left (inner) end of the axis, at the pin
- L, N, O, M, S, Q: collars/screws along the axis (L at far left, then N–O, M, S–Q toward the right)
- P: the square opening in the parallelepiped portion (labelled beneath, between M and the base)
- R: collar/screw toward the right of the axis
- K: right (outer) end of the axis

[The figure is uncaptioned.]

After this, from the same material let a circle ABCD be made about the centre E, divided into 24 equal hours, beginning at point C and proceeding leftward. Then, on both sides of the straight line ED, let there be drawn from the centre E the radii of the [zodiacal] signs, as we taught in chapter 2 of this book, so that ED is the radius of the Equator, and the radii of the northern signs lie between ED and EC, while those of the southern signs lie between ED and EA. And in the same way the radii of the day-lengths can be drawn, as was set forth in chapter 3 of this book, whenever the matter requires it. In the figure none of these have been drawn, because they are not the same in all regions; so it is enough if they are drawn faintly for the given latitude of the place.

Also, having reckoned the pole's altitude from the straight line AE toward D as far as F, let the straight line EF be drawn, to which the label "Horizon" is to be assigned. And in the same way, having counted the same pole-altitude from the straight line ED toward C as far as G, let the straight line EG be drawn, to which "Verticalis" [the vertical circle] is to be assigned.

Finally, on the opposite face of the circle ABCD, let a perforated cylinder EH be fixed at right angles at the centre, so that the axis IK can be set into it. In the same

way, on that same opposite face, along the diameter AC, let two perforated arcs of circles I and k be set, so that the same axis can conveniently pass through them — a similar hole having first been made in the cylinder EH as well. In this way the instrument for describing dials on any plane, for any latitude of place, will have been constructed. It is used as follows.

Diagram. The finished instrument set up in the plane of the meridian: the graduated circle ABCD carrying a gnomon/axis (drawn as a foreshortened cylinder) at its centre, with suspension rings at top and bottom. Around the rim, the four cardinal words are set (each split by a labelled point of the circle): "MERI:A:DIES" across the top (A = Meridies, noon), "OR:B:TVS" down the left side (B = Ortus, east/sunrise), "MEDIA·C·NOX" across the bottom (C = Media nox, midnight), and "OCCA:D:SVS" down the right side (D = Occasus, west/sunset). The hours are numbered 1 through 12 along each half of the rim. From the centre E radiate the "Horizon" and "Verticalis" lines (both labelled within the figure) and, between them, the radii of the zodiacal signs [the individual sign-glyphs are small and worn — [?]].

- E (centre): centre of the circle
- A (top), B (left), C (bottom), D (right): the four quartering points of circle ABCD
- H, I: the gnomon/axis pieces at the centre (H the perforated cylinder, I the axis)
- F: point on the rim toward which the Horizon line is drawn
- G: point on the rim toward which the Verticalis line is drawn
- K: axis-point at the bottom (suspension ring)
- Rim labels: "Horizon", "Verticalis"; hours 1–12 marked on each side

[The figure is uncaptioned.]

Near the wall on which the dial is to be drawn — whether it stands perpendicular to the horizon or not — let some firm plane be set up parallel to the horizon, standing away from the wall at an interval, more or less, equal to the length of the gnomon of the dial to be drawn. So that this plane may not hinder us in tracing the hours, it will most conveniently be set upon some piece of wood, or upon iron fixed into the wall, so that it is parallel to the horizon. Having found the meridian line on this plane, the instrument AB must be fixed so that the straight line CD coincides with the meridian line, and the point D faces north and C faces south.

[Margin: The use of the aforesaid instrument.]

Once it is fixed, the pin IL — which is on the axis IK — must be set into the centre F, and the axis fastened by its small screw L, and turned about the centre F until, through the opening P, the degree of the pole's altitude for that place (for which the dial is being drawn) is seen on the quadrant. For once this is observed, the axis must be fastened by the small screw O, so that it can no longer be moved up or down, but stays in its own position until the drawing is finished.

Next let the circle ABCD be set on the axis through the holes K, E, I, so that I faces the north (arctic) pole and K the south (antarctic) pole; and let it be turned until the plumb-line thread, let down freely from the centre E, grazes the surface of the circle, and indeed coincides with the line EG, the Vertical of the circle — since the Vertical circle, passing through the centre E to the horizon, is perpendicular, just as the plumb-line is, and makes with the axis, toward the antarctic pole, an angle equal to the complement of the pole's altitude, such as the angle GEC. Let the circle be fastened to the axis by the small screws I and K, so that it cannot be shifted this way or that; and let the point of the axis with which the centre of this circle coincides be carefully marked — this may be called the tip [vertex] of the style. The circle itself will then be placed in the plane of the meridian.

The circle being thus made steady, let a thread be stretched from the centre E, grazing the surface of the circle, as far as the plane of the dial, and let the point on which the thread falls be marked. And with the same thread grazing the circle, let another point be marked on the wall — or even several, whether above or below. For the straight line joining these points will be the meridian line, which, on walls perpendicular to the horizon, will be perpendicular to the horizon, and in every case will pass through the centre of the dial. This centre will be indicated by the thread issuing from the centre E and grazing the axis.

[Margin: The meridian line.]

This centre will be found more accurately if two threads issuing from the centre E graze the axis on opposite sides, and two points are marked on the plane of the dial; for the point midway between these will be the centre of the dial.

[Margin: The centre of the dial.]

But if the meridian line cannot be drawn conveniently — or in no way at all — on the plane of the dial, as happens with walls that deviate only slightly from the meridian, then some board must be fixed to the wall toward the north, so that it faces south, and on it the meridian line must be sought out as before.

The remaining hours from noon and from midnight will be traced thus. Let the circle ABCD be removed, and through the channel EH, onto the axis, let there be set...

[Margin: The hours from noon and from midnight.]

let a circle be set, so that its centre E again coincides with the point previously marked on the axis, which we called the tip [vertex] of the style, and so that the channel faces the antarctic pole. Let the ring QR be brought up to the end of the channel, and the little screw S tightened against the axis, so that the circle cannot be moved downward. Then let the circle be turned about the axis until point A faces upward and C downward, while B faces east and D faces west, and until the thread issuing from the centre E, grazing the circle, reaches the meridian line already found. Finally, in this position, let the circle be made fast by the little screws set in the channel EH, so that it cannot be shifted either way. For while the circle stays in this position — namely in the plane of the Equator — if the thread let out from the centre and grazing the circle is made to agree with each hour, and the points are marked in the plane of the dial, the points so found, joined by straight lines to the centre of the dial once that has been found, will give the hour-lines from noon and from midnight.

But if the dial happens to have no centre, we will draw these same lines as follows. Let the circle be set again on the axis through the holes K, E, I, so that E, its centre, coincides with the tip of the gnomon, and let it be fastened by the ring QR so that it cannot move downward. Then let the circle be turned as many times as needed, until the thread issuing from the centre and grazing the circle can be laid against each of the hour-points already found in the plane of the dial, so that in that turning the circle is brought into conjunction with the planes of the several hour-circles; and, by means of the thread, other points — whether above or below — are sought out for each hour, just as we described for the meridian line. For if these points, found for each hour, are joined by straight lines, the hour-lines will again be drawn.

These latter points will perhaps be found more conveniently by means of some light, or a line of sight. For if a light, or the eye, is applied to the circle in such a way that the circle itself appears in the plane of the dial as a straight line passing through the hour-points already found, and several points are marked along that line, the hour-lines will pass through them, as before. The same points may also be found without the circle, by the axis alone. For if a light, or the eye, is applied to the axis in such a way that the axis is seen to pass through the hour-points already found in the plane of the dial, and several points are again marked in the

plane of the dial through which the axis is seen to pass, the hour-lines are to be drawn through them, as before.

But if at any time the thread stretched for some hour on the circle, while the circle lies in the plane of the Equator, runs parallel to the plane of the dial, or does not meet it, that hour-line must be drawn through the centre of the dial perpendicular to the meridian line — such as the 6 o'clock line from noon or from midnight in the horizontal, vertical, equinoctial, and horizon-inclined dials. Or indeed, where the dial has no centre, that hour cannot be drawn at all — such as the 6 o'clock line from noon or from midnight in the polar dial.

[Margin: The arcs of the signs.]

The **arcs of the signs** will be laid out thus. With the circle fixed, as just described, let the thread be applied, issuing from the centre E along the several rays of the signs, so that it grazes the circle; and, the circle being turned around, let points be marked in the plane of the dial, even upon the hour-lines themselves. For by these the arcs of the signs are to be drawn. But for the equinoctial line, the points will be found — by the ray of the Equator — lying in a straight line, one which will pass through the hour-points that were found by means of the circle while it was placed in the plane of the Equator.

[Margin: The arcs of the lengths of the days.]

By the **same** method the arcs of the lengths of the days will be drawn, if from the centre E the rays of the day-lengths are sent out, and so on. But it will be enough if the points of the diurnal arcs — those required for drawing the unequal hours from sunrise and sunset — are marked on the hour-lines from noon and from midnight.

[Margin: The hours from sunrise and sunset, and the unequal hours.]

The **hours from sunrise and sunset**, and the unequal hours, will be drawn by means of the diurnal arcs, as was explained in the preceding pages.

[Margin: The horizontal line.]

The **horizontal line** will be drawn thus. With the circle ABCD fixed, as we said when the meridian line was being drawn — that is, so that it lies in the plane of the meridian circle — let the thread be applied, issuing from the centre E toward the horizon-line EF, so that it grazes the circle, and let the point be marked where it meets the plane of the dial; this point will necessarily lie on the meridian line. Through it let a straight line be drawn parallel to the horizon. In dials set at right

angles to the horizon this line will cut the meridian line at right angles, and will pass through the place of the style and through the 6 o'clock line from noon or from midnight on the equinoctial line. For this will be the horizontal line. But where the dial has no centre, or the meridian line cannot conveniently be obtained, the horizontal line must be drawn through the 6 o'clock point from noon or from midnight on the equinoctial line, parallel to the horizon.

[Margin: The place and length of the style.]

The **place of the gnomon** will be that point of the dial onto which the perpendicular drops that is let fall to the plane of the dial from the point of the axis where the centre of the circle ABCD was fixed. This point will easily be found if, from the said point of the axis as from a pole, a circle is drawn in the plane of the dial; for the centre of that circle will be the place of the style. Its length is that very perpendicular, falling from the said point of the axis onto the plane of the dial — that is, onto the centre of the circle just drawn.

A horizontal dial will be built by the aforesaid instrument in no other way — especially when some large dial is to be drawn. For on small ones this cannot conveniently be done, since the circle ABCD is then turned about the axis with difficulty, as is evident.

ON THE DRAWING OF DIALS ON ANY PLANE WHATEVER, by the shadow of the style of some horizontal dial, or of any other.

CHAPTER XXXI.

[Margin: By what method a dial may be drawn on any plane whatever, by the shadow of the style of some horizontal dial.]

It seems we should not omit here a very easy and very simple method of drawing dials, handed down by Giovanni Battista Vimercati [Giovanni Battista Vimercati] in his little book on the drawing of dials, which is roughly as follows. Take a horizontal dial, accurately drawn on some flat board, on which a straight line has been drawn corresponding to the meridian line of the horizontal dial, and fix it in place. Then, on the same board, fix another plane — one which either adheres to it, if another horizontal dial is to be drawn; or stands upon that straight line, or is inclined, so that it cuts the meridian line of the board at right angles, if a vertical, equinoctial, or polar dial, or one inclined to the horizon, is to be drawn; or cuts the meridian line at unequal angles, if a dial declining from the vertical is proposed for drawing, whether it is inclined to the horizon or not; or, finally, does

not cut the meridian line at all, but makes on the board a line parallel to it, if a meridian dial, or one declining from the horizon,

is to be constructed — account always being taken of the declination and inclination of the dial to be drawn. Next, at the chosen place of the style, on the plane of the dial to be drawn, let a gnomon of any size be fixed, upright to the plane of the dial. Then, moving the board about in a place where it is lit by the Sun, this way and that, until the tip of the style's shadow on the horizontal dial falls on each of the hours of the tropic of Cancer [♋], let the several tips of the style's shadows be marked on the dial-to-be. For thus the hour-points of the tropic of Cancer [♋] will be found on the future dial. And if the same is done on the tropic of Capricorn [♐], and on the equinoctial line, or on any other arc of the signs, or finally on any other line of the dial, even if it is not an hour-line, and the corresponding points on the future dial are joined either by straight or by curved lines, the dial will be drawn on the proposed plane. By this method all the points of the horizontal dial can be transferred to the future dial, if the board is turned about until the tip of the shadow falls on a given point on the horizontal dial, and then on the future dial the tip of the shadow's point is likewise marked. The dial thus drawn will have to be enlarged on the wall, in proportion to the given length of the style, as we showed in the last proposition of Book 4. For a board cannot be turned about carrying so great a plane as the dial to be drawn on the wall requires.

By this art a second horizontal dial can easily be drawn from a horizontal dial, to a given — even a very small — length of style, something that can scarcely be accomplished by other rules. Indeed, by the same device a dial will be drawn on a surface that is not flat but concave, if several points are marked for each hour, which are then fittingly joined by lines that make no angles. Add that hours can also be marked in this way on a cylinder erect to the horizon, as we shall explain in the following book.

In place of a horizontal dial we shall be able to use a concave spherical dial, which we shall construct in the following book. For on this the tips of the shadows can be marked much more accurately than on the horizontal one.

THE END OF THE SEVENTH BOOK.

GNOMONICS, BOOK EIGHT.

By the author Christoph Clavius of Bamberg, of the Society of Jesus.

In this final book, so that we may not seem to omit any of the things that pertain to the drawing of dials, we will treat briefly those dials which are not drawn on some stable and fixed plane — of the kind we dealt with in the preceding books — but which are carried from place to place, and are commonly called travellers' dials [Viatoria], since they are especially useful to those who travel or journey abroad. Of these we will describe not all, but those we judge to be the chief and of greater use. Nor have all of them been devised up to now; rather, new ones daily come to light, invented by various craftsmen, so that anyone would promise in vain a description of all dials of this kind, since anyone can still, by his own ingenuity [proprio Marte], devise new and varied ones for his own use. We will take our beginning from those that are adapted to any latitude of place, and are on that account called universal by most writers; then we will treat those that are made for a given altitude of the pole.

ON THE UNIVERSAL EQUINOCTIAL DIAL.

CHAPTER I.

[Margin: The construction of the universal equinoctial dial.]

Let there be prepared, of brass [orichalcum], or of some other solid material, two equal squares ABCD and AEFG. The sides of the first being bisected at A, B, C, D, let the straight lines AC and BD be joined, cutting one another at right angles; and from the point of intersection let a circle be drawn on each face of the plane, and divided into 24 equal hours, beginning from the straight line AC. Of these, the forenoon hours (that is, after midnight) run from C through B up to A, while the afternoon hours run from A through D up to C. Then let these squares be fitted together along their common side drawn through A, so that ABCD can be moved about that side — raised and lowered — to the altitude of the Equator, or to the complement of the pole's altitude, on the quadrant HIK, which is jointed at H in such a way that, when the dial is to be closed, it can be inclined and laid upon the plane AEFG. Finally, let the centre of the circle be perforated, so that a bronze axis of any length can be passed through the hole at right angles to the plane itself; and in the plane AEFG let a magnetized needle [acus Magnete illita] be set, so that by its help the dial can be placed on a plane parallel to the horizon, in such a way that A faces north and F faces south, and the sides through E and G

represent the meridian line. For when the dial holds this position, if the plane ABCD is fixed on the quadrant HIK at the altitude of the Equator, the axis will indicate the hours from noon or from midnight — on the outer face of the plane ABCD when the Sun is in the northern semicircle of the ecliptic, but on the inner face when it is in the southern semicircle of the ecliptic. And if the part within the circle is cut away, leaving only the portion LM to support the centre together with the axis, the shadow of the axis will appear on the hours of the outer face, even if the Sun is in the southern signs. But if another movable circle is set inside the first, divided into 24 equal hours from sunrise or from sunset, and the noon hour is placed — in the manner of the Italians or the Babylonians — above the straight line AC, toward the point A, the shadow of the same axis will indicate the hours from sunrise or from sunset on this latter circle.

Diagram. A perspective woodcut of the universal equinoctial dial, shown open. It comprises a horizontal base-plate (the square AEFG) lying flat, and a second square plate carrying the graduated hour-ring (ABCD), hinged along their common edge at A and raised above the base. The raised plate is propped at an angle by a quadrant arc, graduated in degrees, which is used to set the plate to the altitude of the Equator. An axis (gnomon) passes through the centre of the hour-ring perpendicular to its plane. On the base-plate, near the front, a magnetic compass needle is drawn inside an oval box.

- K (upper left, at the top of the quadrant arc): endpoint of the graduated quadrant.
- C (just below K, on the raised hour-plate): corner of the square/hour-ring.
- D (upper right, on the raised hour-plate): corner of the square/hour-ring.
- M (centre of the hour-ring): the centre, through which the axis passes; the retained bridge LM supports it.
- L (left of the ring centre): the other end of the retained bridge-strip LM across the cut-away ring.
- A (lower centre-left, at the front near-corner): the hinge corner shared by the two plates; faces north.
- E (bottom centre, on the front edge of the base-plate): corner of the base square AEFG.
- G (right side, on the base-plate): corner of the base square AEFG.
- F (lower right, on the base-plate): corner of the base square AEFG; faces south.

- The quadrant arc bears a degree scale reading downward toward the base; legible labels include 50, 40, 30, 20, 10 (higher divisions toward the top, 60–90, are present but not individually legible [?]).
- The inner ring is graduated for the 24 equal hours with small numerals (a twice-repeated 1–12 series); the individual hour-numerals are too small to resolve individually [?].
- Compass needle: a magnetized needle in an oval box on the base-plate (unlabelled).

In place of the quadrant HIK, a latitude-scale for regions [scala latitudinum regionum] can be made as follows. Describe a semicircle from centre A whose radius AB — the line cutting the diameter at right angles — is equal to one side of the square ABCD, and divide it into 180 equal parts. (We divided it into 18, so that each part spans 10 degrees.) If pairs of points equally distant from point B are joined by hidden straight lines, the line AB will be cut into 90 unequal parts, for the 90 degrees of the pole's altitude, whose numbering begins from point A. Then transfer the parts of the line AB onto the square AEFG, near the side drawn through E, starting from the common section of the two squares ABCD and AEFG; and along the diameter BD, near point B, attach a movable style [stylus volubilis] equal to half of one side of the square. The latitude-scale for regions is now ready. For if the tip of that style is set at the number of degrees that indicates the pole's altitude (not the Equator's), the square ABCD will be raised to the altitude of the Equator, and will therefore represent the Equator — just as before, when it was raised on the quadrant HIK to the altitude of the Equator, or to the complement of the pole's altitude.

[Margin: Construction of the latitude-scale for regions.]

[Margin: In this figure let the line AN be drawn, and about it, from centre O, let the circle APN be described.]

Diagram. A construction figure for the universal equinoctial dial and its latitude-scale. A large semicircle stands on a horizontal diameter drawn through H and A; the apex of the semicircle is B. Along the vertical radius from A up to B runs a graduated ladder-scale numbered (reading upward from the base) 10, 20, 30, 40, 50, 60, 70, 80 — the scale of the pole's altitude / latitude. On the left a graduated quadrant arc descends from the upper region to the baseline, carrying the points I, K, L near its upper part and M lower down on a radial line. To the right of the vertical scale is the point P (upper region); N lies on the upper-right portion of the arc; O is a point toward the centre-right (the centre of the auxiliary circle APN mentioned in the margin). On the horizontal baseline: H (left of centre, below the

scale), A (the centre of the semicircle), and G (right end). Below the baseline two square hour-plates are shown edge-on in perspective: the left plate has corners F (top) and D (bottom) and bears small zodiac glyphs and hour-numerals; the right plate has corners E (top) and C (bottom); between the plates a hatched cylindrical axle with a wheel represents the dial's axis.

- B (apex of the semicircle / top of the ladder-scale)
- A (centre of the semicircle, on the baseline)
- H (baseline, left of centre)
- G (right end of the baseline)
- I, K, L (upper-left, on the graduated quadrant arc)
- M (mid-left, on a radial line)
- P (upper region, right of the vertical scale)
- N (upper-right, on the arc)
- O (centre-right; centre of circle APN)
- F (top corner of the left hour-plate)
- D (bottom corner of the left hour-plate)
- E (top corner of the right hour-plate)
- C (bottom corner of the right hour-plate)
- Vertical ladder-scale numerals: 10, 20, 30, 40, 50, 60, 70, 80
- Small zodiac glyphs and hour-numerals on the left plate (F–D): too small to resolve individually [?]

The reason for this is as follows. Let the radius AN be drawn so that the arc GN contains 50 degrees of the pole's altitude. Since the line NP is perpendicular to AB, then if a circle is described about AN from D, the midpoint of the radius AN, it will — by the scholium to Euclid III.31 [ex scholio propos. 31. lib. 3. Euclid.] — pass through P; and therefore the line OP, once drawn, will be equal to AO itself, that is, to that movable style. Hence, if the movable style is set at 50 degrees, namely at P, the angle NAP which the two squares of the dial form will equal the complement of the pole's altitude. And so the square drawn through AN will lie in the plane of the Equator, as is evident if the quadrant BGA stands at right angles on the horizontal plane so as to be a quadrant of the meridian, with its point B inclining to the south and A to the north — exactly as happened before with the quadrant HIK, on which the other square was raised to the complement of the pole's altitude. In the same way, if the style is placed at 40 degrees, that angle will equal the complement of the pole's altitude, and so on.

But if we do not wish to use a magnetized needle [acu Magnete illita], a sighting-rule [dioptra] with a fiducial line [linea fiduciæ] must be prepared, which will lead us to knowledge of the meridian line and of the hour without any axis passed through the centre of the square ABCD, in the following way. Let the fiducial line CD be equal to the diameter of the square ABCD, so that it can turn about a small pin [clauiculum] fixed at the centre and pass through the centre, as is customary on an astrolabe. At its ends let two vanes [tabellæ] CE and DF be raised at right angles, in such a way that they can be lowered and raised; and we shall determine their heights as follows. On a line GH taken equal to the diameter of the square ABCD, draw HI perpendicular to GH; and describing an arc of a circle from G as centre, mark off on it the declinations of the signs and of the other degrees of the ecliptic. (As an example we have brought in only the declinations of the whole signs.) Let straight lines drawn out from G through these points cut the line HI at K, L, M. Next, on the sighting-rule, in the line CE that bisects the rule itself, make two small holes [foramina] through which a ray of the Sun can pass, spaced apart by an interval equal to the line HK; and mark the corresponding points on the line DF, which bisects the other vane, set as far from point D as the holes are from point C. For if the points K, L, M are transferred onto the line DF, placed between the two aforesaid points — beginning once from the point near D, and again from the other point near F — by marking off points on DF through which parallel lines are drawn, and between these the twelve signs of the Zodiac are laid out (as appears in the figure), the sighting-rule will be complete. Its use is this. With the square of the dial ABCD set at its proper altitude, so that it is parallel to the Equator, turn the dial itself and the sighting-rule this way and that, until the Sun's ray — passing through the hole near C when the Sun is in the southern signs, or through the hole near E when the Sun dwells in the northern signs — falls incident upon the Sun's place on the line DF. Note only this: that in the forenoon the sighting-rule must be directed toward the east, but in the afternoon toward the west. (Whether it is forenoon is known from the shadow of any object on a plane parallel to the horizon: if the shadow is decreasing, the Sun has not yet reached the meridian; if it is increasing, it has already passed it.) For in this way the dial

[Margin: The construction of a sighting-rule [dioptra], by means of which the dial may find its own proper position without a magnetized needle, and the hour be indicated without an axis.]

will hold its own proper position, and the fiducial line, beyond the centre toward the opposite vane of the sighting-rule, will indicate the present hour, whether the Sun is in the northern signs or in the southern.

Once the dial has been set in its proper position — whether by a magnetized needle or by the meridian line once found — the hour will be found much more easily from the sighting-rule. For then the rule need only be swung round until the Sun's ray falls on the line DF, even if no signs are drawn on the vane. But if the signs are drawn, the same ray will indicate in which sign the Sun is.

Now if the dial is directed toward the Sun on a plane parallel to the horizon, so that the shadow of the axis falls on the meridian line — or the Sun's ray, passing through the sighting-rule, falls on the line DF — and the square ABCD is then raised, but so that, produced, it passes through the Sun, or until the Sun's ray falls on the two points of the line DF that lie opposite the holes of the rule: then the square itself will indicate on the quadrant the Sun's altitude above the horizon. From this altitude — if it is the meridian altitude, that is, the greatest of that day — together with the Sun's declination, the pole's altitude can be derived, as we taught in our *Commentary on the Sphere* [in commentariis in sphæram].

The construction of a universal dial in the form of a cross.

CAPVT II.

[Margin: The construction of a universal dial in the form of a cross.]

From the same material make a flat cross ABCD, and another, solid but hollow, AEFG, which is to be joined to the first so that it can freely be raised and lowered; and let the three arms E, F, G be exactly equal to one another. Then, for a gnomon-length [styli] equal to one of the arms, lay out both a meridian dial and a polar dial. Draw the eastern meridian dial on the side of the cross AF, with lines drawn perpendicular to the line FK; and the western one on the opposite side, as the numbers indicate. The polar dial goes on the upper part of the arms E, H — that is, on the plane of the cross that is at right angles to the plane EG — with the forenoon hours toward H and the afternoon hours toward E, as the numbers show, by lines drawn perpendicular to the line EH. Finally, on the cross ABCD let a magnetized needle be set, so that by its help the dial can be placed on a plane parallel to the horizon in such a way that A inclines to the north and C to the south, and the side AC represents the meridian line. For with the dial set in this position, if by help of a quadrant the cross AEFG is raised so that the angle CAL is the angle of the complement of the pole's altitude, the plane FM will lie in the Equator, and the arms E, H, I, G, L, F, and so on, will indicate the hours from noon or from midnight. A latitude-scale for regions can also be drawn near the side AC of the lower cross, as we taught in the preceding chapter. For if a movable style, equal to half the side AL, is attached at the middle of the side AL and set on the

latitude-scale at the degree of the pole's altitude, the dial will again hold its proper position.

Diagram. A perspective woodcut of the universal dial built in the form of a cross. Two cross-shaped plates are shown fitted together at an angle: a lower flat cross (ABCD) lying on the horizontal, and an upper hollow cross (AEFG) raised above it and propped so that its arms carry the meridian and polar dial-faces. The arms are graduated with hour-numerals, and shaded (hatched) dial-faces bearing hour-lines are drawn on their sides. A magnetic compass in a small oval box is drawn on the left arm near C.

- F (upper-left arm end)
- H (top)
- G (right, below H)
- L (left)
- E (mid-left arm)
- I (centre of the crossing)
- D (below I)
- M (right arm end)
- K (lower-right arm end)
- C (lower-left arm end, near the compass box)
- A (lower-right, foot of the base cross)
- B (bottom)
- Graduated hour-numerals along the arms: near F–H, "8 7 6" and "12 11 10 9"; along the right arm toward M and K, "6 5 4 3 2"; along the left arm toward L, "3 2 1 12"
- Hour-numerals on the shaded dial-faces of the arms: too small to resolve individually [?]
- Compass needle in an oval box on the left arm (unlabelled)

The arcs of the signs can also be drawn on the cross, as on a meridian dial and a polar dial, taking the gnomon to be F–12 or 9–12, so that the side EH is the equinoctial line on the polar dial, with the northern signs running downward and the southern upward, and so on. Likewise, taking the other sides of the arm, GH and EI, as gnomons, so that both the side FK and 9–M are the equinoctial line on the meridian dial, with the northern signs running downward and the southern upward, and so on.

On another universal dial, laid out on a plane surface and adapted to any latitude of place that does not exceed the complement of the Sun's greatest declination.

CAPVT III.

[Margin: The description of another universal dial on a plane surface.]

On some plane, draw two straight lines BC and AD cutting each other at right angles at A, and from centre A describe a hidden arc of a circle EDF, on which, on either side of D, the Sun's greatest declination is counted off up to E and F. Draw the line EF cutting the line AD at G; and from G as centre, with radius GE or GF, describe a hidden circle. Divide it into 12 equal parts for the 12 signs of the Zodiac, or into more, for the parts of the signs as well. (We divided it into 36 parts, so that each spans 10 degrees.) Join any two points equidistant from the line AD by hidden straight lines cutting the arc EDF at points; and if straight lines are drawn from A to these points (they must not, however, be produced beyond the line EF), the radii of the signs of the Zodiac will be traced — so that AG is the radius of the beginning of ♈ [Aries] and ♎ [Libra], the radii toward E being assigned to the northern signs, and those toward F to the southern signs, as the characters of the signs

indicate. If desired, the days of the months in which the Sun occupies those points of the ecliptic can be written near the beginnings of the signs and their parts, as is usually done on the back of an astrolabe. Into this triangle of the Zodiac's radii the latitudes of all places that do not exceed the complement of the Sun's greatest declination must be fitted, in this way. From E and F draw lines EO and FL parallel to AG (which is easily done if AO and AL are taken equal to the lines GE and GF); and from A describe a hidden circle BHC, whose quadrant BH is divided into 90 degrees. We divided it into 18 equal parts, each spanning 5 degrees. Then let hidden straight lines drawn from A through the points of division cut the line EO at points; and if lines are drawn through these parallel to OL (which is easily done if all the points of the line EO are transferred onto the line FL), the latitudes of places will be traced, as the figure shows.

After this, another Zodiac must be constructed on the line FL, in this way. On the circle BHC, count off the Sun's greatest declination on either side of C, up to I and K; and drawing the lines AI and AK cutting the line FL at two points, describe from L a hidden circle about the line lying between those two points. Divide it into 12 equal parts, or into more, according as whole signs or their parts are

wanted. (We divided it into 24 equal parts, so that each sign is halved.) Then, through any two points equidistant from the line AC, draw hidden straight lines cutting the arc a d b — the hidden arc described from A and passing through those points where the line FL is cut by the lines AI and AK. For if hidden straight lines are drawn out from A through the points of this arc, the

Diagram. A large, complex construction figure for the universal plane dial of this chapter. It combines several overlapping elements. At the upper left an articulated jointed arm, expressly labelled **Brachiolum** ("little arm"), carries pivots at R, Q, and S, and from it a serpentine cord (labelled T above and N below) hangs down and ends in a plumb-weight. The main body of the figure is a large circle graduated along its upper rim with degree-ticks; the rim points D (top), E, G, and F are marked at the top, with I and C at the right and K on an outer arc at the lower right. Within the circle a downward-narrowing "triangle" / funnel of the Zodiac's radii and latitude-arcs converges toward point A at the bottom centre; its two sides carry a symmetrical latitude scale numbered (reading downward) 65, 60, 55, 50, 45, 40, 35, 30, 25, 20, 15, 10, 5 on each side. Interior points H, M, Y, and P are marked within this funnel. A band of paired zodiac glyphs runs across the top of the funnel just below the rim (each latitude-column headed by a northern sign and its symmetrical southern counterpart). The forenoon-hour field on the left is labelled **Horæ ante[meridianæ]** with hour-numbers, and a horizontal band along the very bottom is labelled **Horæ pomeridianæ** ("afternoon hours") with the hour-numbers 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 12; a diagonal line is labelled **meridian[us]**. At the bottom centre a small second circle (the "other Zodiac") is drawn about point L and ringed with zodiac glyphs; the lowercase points a and b lie on its arc. A small circle enclosing a symbol is drawn at the right, between the funnel and point I.

- Brachiolum (jointed "little arm") at upper left, with pivots R, Q, S
- T (upper label on the hanging serpentine cord); N (lower label on the cord)
- D (top of the main circle's rim)
- E, G, F (top rim, left to right)
- I (right rim); C (lower-right rim); K (outer arc, lower right)
- A (bottom centre; apex of the radii-funnel and common vertex of the hour-lines)
- B (lower left, left end of the horizontal diameter); O, Z (on the diameter, between B and A)
- H, M, Y, P (interior points within the funnel, top to bottom)
- L (centre of the small lower "other Zodiac" circle)

- a, b (lowercase points on the arc of the lower Zodiac circle)
- Latitude scale on the funnel sides: 65, 60, 55, 50, 45, 40, 35, 30, 25, 20, 15, 10, 5 (each side)
- Curved label "Horæ ante[meridianæ]" (forenoon hours), left of centre, with hour-numbers 12, 1, 2, 3, 4, 5, 6, 7, 8 ...
- Bottom band "Horæ pomeridianæ" (afternoon hours): 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 12
- Diagonal label "meridian[us]" (meridian line)
- Paired zodiac glyphs across the top of the funnel and around the lower Zodiac circle: partly legible (Aries, Taurus, Virgo, Libra, Pisces, Scorpio, Aquarius among them) but not reliably resolvable glyph-by-glyph [?]
- Small circle enclosing a symbol at the right (between the funnel and I): symbol not resolvable [?]

[This figure has more than two unresolved small labels (the paired zodiac glyphs and the enclosed symbol) and]

[Margin: In this figure let Υ [Aries]A and $\Upsilon\delta$ and $O\delta$ be drawn, so that these two lines, drawn from O and from Υ , meet at δ — namely at the point where the first ray of $\mathfrak{C}$ [Cancer] intersects AE, the latitude-line of 40 degrees. Likewise, from the highest and lowest circle of $\mathfrak{C}$, let the line $a\delta$ and $\delta\epsilon$ [?] be drawn, so that it be at the point where the first ray of $\mathfrak{M}$ [Virgo] intersects AF, the latitude-line of 40 degrees.]

portion of the line FL contained between the lines AI and AK will be cut into the signs of the Zodiac, so that the point L is assigned to the beginning of Υ and $\underline{\alpha}$, while the northern signs advance toward F and the southern downward — that is, so that the northern signs of this latter Zodiac are nearer to the southern signs of the former Zodiac than the southern signs are. But if this latter Zodiac were laid out on the line EO, the southern signs would run upward and the northern downward, so

so that the southern signs of this latter Zodiac would lie closer to the northern signs of that earlier Zodiac than its own northern signs do. This latter Zodiac, however, will be laid out more easily if the points of the signs are taken on the pole-altitude line of 45 degrees, from point M, and transferred from L or from O, this way and that, onto the line FL or EO. For, as will shortly be demonstrated, the lines AM, AL, and AO are equal; and therefore the lines FL and EO and the pole-altitude line of 45 degrees will be cut by the rays of the signs in the very same way.

In the construction of this latter Zodiac Orontius [Orontius Finé (Oronce Fine)] went astray, and almost everyone follows him. He would have the lines drawn out from A pass through the points of the line FL, at which that line is cut by the lines drawn through each pair of points of the circle described from L. This is entirely false. The lines must be drawn through the points of the arc *a d b*, as we showed in Proposition 1 of Book 1, just as the rays of the signs of the earlier Zodiac were drawn through the points of the arc [Margin: Error Orontij (Orontius's error)] EDF, and not through the points of the line EF. Indeed, Orontius contradicts himself when he says that the points of the signs on the line FL ought to be equal to the points of the signs of the earlier Zodiac on the line that marks the 45th degree of latitude — which, by his own construction, cannot be true unless the lines from A are drawn through the points of the arc *a d b*, and so on.

The hours are laid out as follows. [Margin: a — Euclid, *Elements* I.6] From the centre A, at a distance AL, or AO, or AM, up to the latitude-line of 45 degrees (for these three lines, if there is no error, will be equal to one another: since the line A θ drawn to the 45th degree of the circle BH makes the angle OA θ a half-right angle, the angle O θ A will likewise be a half-right angle, and therefore ^a the lines OA and O θ will be equal; and since A θ is equal to AM, in the parallelogram OAM θ the three lines AM, AL, AO will be equal) — let the circle so described be divided into 24 equal hours, and these again, if one wishes, subdivided into half-hours and quarter-hours. For the lines drawn through each pair of points equally distant from the points L and O will give 12 hours from noon and 12 from midnight, of which MA will give the sixth hour. The forenoon hours are numbered from the line EO toward FL, and the afternoon hours from the line FL toward EO. These hour-lines are bounded, in their upper part, on the circumference of the circle described from M at the distance MA; and in their lower part, on a line parallel to LO, standing off from the line LO by at least as great an interval as the portion of the line AI, or AK, lying between A and the line FL.

Again, a small arm [brachiolum] must be made of some hard and solid material, consisting of three revolving segments QR, RS, ST, which when extended lengthwise make up a length equal to the line GA. The end-point Q of this arm is to be fixed at the point G, so that its other end T may range freely over the whole triangle AEF and be fixed at any place, so that it is not easily disturbed; and to this end there must be hung a very fine thread together with a plumb-bob, and a movable bead N is to be set around the thread itself, to serve as an index.

Lastly, once all superfluous parts are cut away so that the dial is reduced to a quadrangular shape whose outermost sides are EF and the parallel drawn next to it in the hour-lines — likewise EO and FL, produced up to that same parallel — and leaving besides so much space on the L side that the characters of the signs can be drawn there, two sights [pinnacidia] must be made, finely pierced, and fitted to the side EF at right angles. For thus the description of the dial will be complete.

[Margin: The use of this universal dial.] **The use** of this dial is as follows. With the end-point T of the arm fixed at the common intersection of the ray of that degree of the Ecliptic in which the Sun stands and the latitude-line of the given place, let the plumb-line thread be extended toward the Zodiac drawn on the line FL, and let the movable bead N be set on the line FL at the same degree of the Zodiac. Then, if with the plumb-line grazing the plane of the dial the sight near E is raised until the Sun's ray, passing through its hole, falls on the hole of the other sight near F, the movable bead N will indicate, among the hour-lines, the present hour. In place of the sights you may use a small pin [clauiculus] fixed at right angles on the line EF near E. For if the instrument is raised until the shadow of the pin falls on the line EF, the same bead will indicate the hour.

Here it must be observed that the lines drawn from two opposite signs onto the pole-altitude line, up to those same signs in the latter Zodiac, are equal to one another. [Margin: b — Euclid I.15; c — Euclid I.26] For let there be drawn, from ♋ [Cancer] and ♑ [Capricorn], to ♍ [Virgo] on the pole-altitude line of 40 degrees, up to ♋ and ♑ of the latter Zodiac, the lines $\delta\alpha$ and $\varepsilon\beta$: I say that these are equal. For, αA having been produced up to γ , let the line $\gamma\delta$ be joined. Since, then, the two angles αAL and $AL\alpha$ are equal to the two angles γAO and $AO\gamma$ — ^b (because the vertical angles at A are equal, and L, O are right angles) — and the adjacent sides AL, AO are equal, ^c the sides $A\alpha$, $A\gamma$ will likewise be equal. Again, since the two sides $A\alpha$, $A\delta$ are equal to the two sides $A\gamma$, $A\delta$ and contain equal angles, namely right angles (for since the angles of declination EAG and IAC are equal, if the common angle GAI be added, the whole [Margin: d — Euclid I.4] angle EAI will be equal to the whole right angle GAL, and therefore so too will the following angle EAY be a right angle), ^d the bases $\delta\alpha$, $\delta\gamma$ will also be equal. And $\varepsilon\beta$ is likewise equal to that same $\delta\gamma$ (for since the sides εA , $\varepsilon\beta$ are equal to the sides δA , $A\gamma$ and contain [Margin: e — Euclid I.4] equal angles, namely right angles — for just as the angle δAI was shown to be right, so we shall show δAK to be right, because the angles of declination DAF and CAK are equal, and FAC is common — ^e the bases $\varepsilon\beta$, $\delta\gamma$ will also be equal). Therefore $\delta\alpha$ and $\varepsilon\beta$ will be equal to one another.

[Margin: How the semidiurnal and seminocturnal arcs are found from this dial; likewise the hour of sunrise and sunset from this dial; and the reduction of hours reckoned from noon and midnight to hours reckoned from sunrise and sunset by this same dial.] **Now** if, from the point where the ray of that degree of the Zodiac in which the Sun stands and the latitude-line of the given place cut one another, a hidden line is drawn parallel to GA, the hours between this parallel and the line FL will give the semidiurnal arc, and the hours between the same parallel and the line EO the seminocturnal arc. Hence the Sun will rise and set on that parallel. From this it follows that the hours reckoned from noon and midnight are easily reduced to hours reckoned from sunrise and sunset. For if from that parallel one numbers, toward the line FL, the whole hours up to the hour that the movable bead indicates — returning, however, from the line FL toward the said parallel, up to the bead, if the time is afternoon — one obtains the hour from sunrise. But if from the same parallel the whole hours are reckoned toward the line EO, up to the bead — returning, however, from the line EO toward the said parallel up to the bead N, if the time is before noon; or, if afternoon, returning also from the line FL toward the same parallel — one obtains the hour from sunset.

For example. Let the bead N fall on the 10th hour before noon, in latitude 45 degrees, with the Sun standing in the 20th degree of ♈ [Aries], or the 10th degree of ♍ [Virgo]. Since, then, the line drawn parallel to GA from the point where the latitude-line of 45 degrees cuts the ray of the 20th degree of ♈ (or the 10th of ♍) falls nearly in the middle of the space between the 5th and 6th hours before noon, and between the 6th and 7th hours after noon, the semidiurnal arc will contain about $6\frac{1}{2}$ hours, and the seminocturnal about $5\frac{1}{2}$ hours; and the Sun will rise at hour $5\frac{1}{2}$ from midnight and set at hour $6\frac{1}{2}$ from noon. And since from hour $5\frac{1}{2}$ before

noon up to the 10th hour before noon, advancing toward the line FL, $4\frac{1}{2}$ hours are numbered, the hour will then be $4\frac{1}{2}$ from sunrise. But if the time is afternoon, so that the bead N falls on the 2nd hour after noon, then, counting from hour $5\frac{1}{2}$ before noon, advancing toward the line FL and returning again to the 2nd hour after noon, there will be $8\frac{1}{2}$ hours; so the hour will then be $8\frac{1}{2}$ from sunrise. Again, since from that same hour $5\frac{1}{2}$ before noon up to the 10th hour before noon, advancing toward the line EO and returning from it, one finds $15\frac{1}{2}$ hours, the hour will then be $15\frac{1}{2}$ from sunset. Likewise, since from the same hour $5\frac{1}{2}$ before noon up to the 2nd hour after noon — advancing toward the line EO, then returning from it toward the line FL, and again returning from this toward the 2nd hour after noon — $19\frac{1}{2}$ hours are numbered, the hour will then be $19\frac{1}{2}$ from sunset. Or indeed, since from the 2nd hour after noon up to the $6\frac{1}{2}$ th hour after

noon, at which the Sun sets, $4\frac{1}{2}$ hours are numbered, if these be subtracted from 24 — that is, from the hour at which the Sun sets — there will remain $19\frac{1}{2}$ hours from sunset, as before. The same reckoning is to be applied in the remaining cases.

[Margin: How the Sun's altitude is found from this dial.] **In addition to this**, if the quadrant LV described from A is divided into 90 degrees, and the end-point T of the arm is placed at the centre A, and the instrument is raised until the Sun's ray answers to the holes of the sights, the plumb-line thread will indicate on the quadrant LV the Sun's altitude above the Horizon. The number of degrees will run, on the quadrant LV, from the point V toward L.

Now if the whole length of the dial is bisected at P, and the dial itself is divided by the line drawn through P parallel to LO, and these parts of the dial are joined to one another by hinges and pins in such a way that one can be laid upon the other — just as we said of the squares of the equinoctial dial in chapter 1 of this book — the dial will be reduced to a most convenient form, so that it can easily be carried from place to place. In use it must be opened out so that from those two parts a single flat surface is made. Indeed, if on the outer face of the tablet, on which lies the line LO, a circle answering to the circle LMOV is described from the point corresponding to the centre A, and divided into 24 parts, and a style is fixed at the centre at right [angles]; and if this tablet is raised according to the elevation of the Equator, by means of a quadrant or the scale of the latitudes of the regions, as we said of the equinoctial dial in chapter 1 of this book; and if, finally, the dial is directed to the meridian line by means of a magnetized needle placed in the other tablet, on which lies the line PG — then a universal equinoctial dial will also have been made, whose inner semicircle LMO will show the hours while the Sun is in the southern signs; unless you wish to cut away the portion enclosed between the two semicircles LMO and XYZ (leaving, however, a small tooth MY, lest the tablet be rendered too weak), so that the shadow of the inner style may appear on the outer face through that hollowed-out portion. You may also, in place of the style, use a diopter on the outer face, as we said in chapter 1. For then the hours will always be shown by the fiducial line on the outer face, even when the Sun is in the southern signs, if the diopter is turned about until the Sun's ray, entering through the hole of one sight, falls on the line opposite the hole in the other sight.

If the hour-lines are produced upward, the part below the point of concurrence of the rays of the signs can be cut off; but then the little Zodiac must be made movable, so that the sign ♈ [Aries] and ♎ [Libra] can be set at the degree of the latitude of the place, and the latitudes of the places on the line CE, the line of the

12th hour, are laid out thus. On the radius of the Equator take the interval between the line of any latitude and the upper line of the triangle up to D, and transfer it from the lowest point E of the 12th hour onto the line of the 12th hour up to G — for that will be the end, the altitude of the pole at which the sign of Aries or Libra of the little Zodiac is to be placed; or else transfer the interval, on the radius of the Equator, between the pole-altitude line and the point A of concurrence of the rays of the signs onto the line of the 12th hour from the highest point C.

Next, prepare a slider [cursor] one of whose sides is straight, so arranged that when it is set on the instrument from below, that side is always parallel to the hour-lines, and its extreme point does not pass beyond the highest line of the dial. There a small hole must be made for the plumb-line thread, and a movable bead is to be set on the thread.

The slider is to be set so that its straight side passes through the intersection of the latitude-line and the ray of the degree at which the Sun then stands. Then, with the plumb-line thread extended over the degree of the Sun in the little Zodiac, the bead is advanced to that same degree of the Sun; for, the Sun's ray passing through the sights and the thread hanging freely, the bead will show the hour among the hour-lines.

Demonstration. [Margin: a — Euclid I.33; b — Euclid I.5] In the triangle ABC let F be the intersection of the pole-altitude line with the radius AD of the Equator; and since, if the little Zodiac is movable, the sign Υ will be placed at E and the plumb-line thread will be FE — then, if DG is taken equal to FE and the thread hangs from D, the sign Υ must be placed at G: because the bead placed at G will move among the hour-lines in the same way as, placed at E, it moved.

Now since the two equal lines FE, DG cut off two equal parallels GE, DF (for if GE be said, say, to be less than DF, then GH being taken equal to DF, ^a the line drawn FH will be equal to DG, and therefore also to FE, ^b and consequently the angles at E and H equal — which is absurd, since H is acute and E obtuse), it follows that the line DF, taken equal to EG, gives the point G at which the sign Υ is to be placed, for the pole-altitude F, and so on.

Diagram. The demonstration figure for the universal equinoctial dial's hour-triangle. Across the top run the points B (left), D (centre), and C (right). From B a line slants down to A at the lower left; from C the right edge runs down through G to E and H at the lower right. F is an interior point (the intersection of the pole-altitude line with the equator-radius AD), roughly below D; a horizontal line runs

from F rightward to G. Diagonals cross the interior, and a dotted line runs from the F-region down to the E/H corner. A is at the bottom left; at the lower right E lies inner and H just below/outer.

- B (top left)
- D (top centre)
- C (top right)
- F (interior, below D — intersection of the pole-altitude line with the equator-radius)
- G (right side, mid-height; printed in a swash form resembling "ç")
- A (bottom left)
- E (bottom right, inner)
- H (bottom right, outer/lower)
- [Margin: a — Euclid, *Elements* I.33; b — Euclid, *Elements* I.5]

DE HOROLOGIO HEMISPHERICO CONCAVO — On the hemispherical concave dial

CAPVT IV.

[Margin: Construction of the hemispherical concave dial.] Let there be a hemisphere hollowed out, accurately turned on a lathe from wood, or brass, or some other solid and hard material, ABCD, which — before the hours are laid out — must be carefully tested with an iron or wooden

semicircle whose radius is equal to the radius of the opening ABCD. For if this semicircle, set upon the concave hemisphere and carried around, always grazes the concave surface, so that nothing projects or is depressed, there will be no doubt that the hemisphere is perfectly concave. Let the circle of the opening ABCD be divided, by means of a compass with recurved legs, into four quadrants AB, BC, CD, DA; and from A, or C, as pole, at the distance AB (or AD, or CB, or CD) let a great circle BED be described, and with the same distance from the pole B, or D, another great circle AEC, cutting the former at E. These two circles are represented, in our figure, by the straight lines AC and BD, cutting one another at right angles at the centre E. Thus ABCD will be the Horizon; AEC the semicircle of the Meridian below the Horizon; BED the semicircle of the prime Vertical below the Horizon; and therefore E the Nadir, or the point opposite the Zenith. Let noon be placed at A; the north at C; the east at B; and the west at D. Then, on the semicircle of the Meridian AEC, having counted from A the altitude of the pole up

to F, and from E up to G, F will be the antarctic pole, and G the point through which the Equator passes below the Horizon.

From the pole F, at the distance EF, let there be described a parallel OEP drawn through the Nadir, which, if it is not described entire (as in our example, and in every other place where the altitude of the pole above the Horizon is less than 45 degrees — for then the arc FE, of the complement of the pole's altitude, is always greater than the arc FA; whereas if the altitude of the pole is exactly 45 degrees, the said parallel will touch the Horizon at A, since then the arcs FE and FA are equal; and finally, if the altitude of the pole exceeds 45 degrees, the same parallel will cut the Meridian below the point A, because then the arc FA is greater than the arc FE, of the complement of the pole's altitude, as is evident) — then there must be taken an arc GS equal to the arc GE, and from the same pole F, at the distance FS, a portion of a circle QSR must be described, which portion belongs to the parallel described through the Zenith of the place, and is opposite to the parallel OEP, and is equal to the portion of the parallel OEP that is lacking; because, the declinations GE and GS being equal, the parallels described through E and through S are equal, and have (by Proposition 19 of Book 2 of Theodosius [*Spherics*]) alternate segments equal — namely the segment QSR below the Horizon, and that which is lacking of the parallel OEP above the Horizon.

Moreover the arcs of the Horizon CQ, CR will be equal to the arcs AO, AP, on account of the equal rising latitudes BP, BR and the setting latitudes DO, DQ. Again, from the pole F, at the distance of the quadrant FG (for FG is an arc composed of EG, the altitude of the pole, and of FE, the complement of that same altitude — a quadrant), or of the quadrant AB, let there be described the semicircle of the Equator BGD, passing below the Horizon necessarily through the points B, D, where the Vertical cuts the Horizon. Then, the Sun's greatest declination having been reckoned off on both sides from G up to H and L, let there be described, from the pole F, at the distance FH, the portion of the tropic of ♊ [Capricorn] below the Horizon, KHI, and at the distance FL, the portion of the tropic of ♋ [Cancer] below the Horizon. And since, when the Sun stands above the Horizon at the beginning of ♋, its ray passing through the centre of the sphere falls on the opposite parallel, namely on the tropic of ♊, below the Horizon; and on the tropic of ♋, below the Horizon, when it stands at the beginning of ♊ above the Horizon — the arc KHI will be called the tropic of ♋, and NLM the tropic of ♊, because the shadow of the centre of the sphere falls upon them when the Sun is in the said tropics. And in the same way the other parallels between the two tropics will be described from the pole F, if their declinations are reckoned off from G on either side.

[Margin: The laying-out of the hours from noon and midnight in the concave hemisphere.] **Now** each quadrant of the Equator GB, GD having been divided into six equal parts, or even into more if smaller parts of the hours be wanted (that GB and GD are quadrants will be plain thus: since the Meridian AC, drawn through the Verticals' poles A, C and through F, the pole of the Equator, cuts, by Proposition 9 of Book 2 of Theodosius, the segments of the Vertical and of the Equator, BED and BGD — which are indeed semicircles by Proposition 11 of Book 1 of Theodosius — into two halves, the segments EB, ED, GB, GD will be quadrants) — we shall describe the equal hours, both from noon and midnight and from sunrise and sunset, with a single opening of the compass, in this way.

Since the hour-circles from noon and midnight are drawn through F, the pole of the Equator, as we demonstrated in Proposition 9 of Book 1, the Equator will in turn pass through their poles, by the scholium to Proposition 15 of Book 1 of Theodosius. All of them, therefore, have their poles on the Equator. Hence, with the compass-legs opened to the distance of the quadrant of a great circle AB (or GB, or FG), if one leg is set at each of the points of the divisions of the Equator BGD, and the other is fixed on the Equator — namely at the points that are distant six hours, or a quadrant, from the former — there will be described all the hour-circles contained between the tropics, from the 6th hour from midnight up to the 6th hour from noon. For only between these are the hour-circles to be drawn; if, however, they were produced, they would all pass through the pole F.

For example: from G, as pole, at the distance GD (or GB, or GF — which distance must not be changed until all the great hour-circles have been described), there will be described the arcs of the circle of the 6th hour from midnight and from noon, bounded between the Equator and the tropic of ♄. Then, placing one leg of the compass at the next point of the Equator from G toward B, there will be described with the other leg the arcs of the circle of the 7th hour from midnight and from noon, of which the former will pass through the next point of the Equator below D, and the latter would pass through the next point of the Equator above B, if the sphere were entire. Likewise, placing one leg of the compass at the second point from G toward B, there will be described with the other the 8th hour from midnight, passing through the second point below D; and so on, setting the poles of the circle-

Diagram. A large circular figure of the concave hemispherical dial (the orifice ABCD seen from within), engraved with the cardinal words around the rim and a dense net of hour-lines below the centre. The rim carries the cardinal labels: at the top **MERIDIES** (split "MERI= A =DIES", with point A at the top = noon/south),

at the bottom **SEPTENTRIO** (split "SEPTEN= C =TRIO", with point C at the bottom = north), down the left side **ORIENS** (east, with point B on the rim), and down the right side **OCCIDENS** (west, with point D on the rim). At the centre-top is a small circle enclosing an upright style/gnomon; from the central point F a bold line labelled **Axis** runs toward the lower right. Below F lie E (the Nadir/centre), then H, then G, and lower still L and S near the bottom. Interior rim-points P (upper left) and O (upper right) sit just inside the circle. On the left arc are K and T (upper) and N (lower left); R lies below N. On the lower right is M. The lower half of the disc is filled with a graduated net of intersecting hour-lines, labelled partly with Roman day-hour numerals and partly with Arabic numerals; only some of these resolve cleanly.

- A (top rim; noon/south — split label MERIDIES)
- C (bottom rim; north — split label SEPTENTRIO)
- B (left rim; east — vertical label ORIENS)
- D (right rim; west — vertical label OCCIDENS)
- F (centre; antarctic pole / axis foot, with small gnomon circle above and the "Axis" line)
- E (below F; Nadir / centre of the great circles)
- H (below E)
- G (lower centre; point where the Equator passes under the Horizon)
- L (lower centre, below G)
- S (bottom centre, below L)
- P (upper-left interior rim point)
- O (upper-right interior rim point)
- K (upper-left arc)
- T (upper-left arc, right of K)
- N (lower-left arc)
- R (lower left, below N)
- M (lower right arc)
- "Axis" (the axis line from F toward the lower right)
- Q (paired with R and S on the lower arc per the text QSR — not separately resolvable on the tiles) [?]
- I (right-hand arc, per the text KHI — not separately resolvable on the tiles) [?]
- Hour-net: Roman numerals of the day-hours (...VII, VI, V, IIII, III, II, I... down the left, and V, VI, VII, VIII, IX, X... down the right, with XI, XII, XIII

near the lower centre) interwoven with Arabic hour-numbers (...14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24 along the lower arc, and 9, 10, 11, 12, 13... on the right) — individual cells only partly legible, not transcribed glyph-by-glyph [?]

[This figure carries more than two unresolved labels (Q, I, and the full hour-number net) and]

of the circles passing through the points of the quadrant DG — some running in the quadrant GB, others in the quadrant GD — so that, with the same opening of the compasses, one also draws the arcs between the horizon and the tropic of Cancer [☿], even though within the hemisphere these do not cut the equator, as happens at the 5th hour from midnight and the 7th from noon, and so on.

[Margin: Laying out the hours from the west and east in the concave hemisphere.]

Again, the hour-circles from the west and east have their poles in the parallel RSQ, which is drawn through the zenith of the place, as we demonstrated in Proposition 10 of Book I; and therefore also in the opposite parallel OEP, since each of its points is set opposite the corresponding point of the former. Now if, with the same opening of the compasses kept, one foot is set in turn on each point of the equator and the other foot placed in the parallel OEP — or in RSQ, whenever it cannot be set in OEP — then through the points of the equator the arcs of the hour-circles from the west and east will be drawn. These circles have their poles dividing the parallels OEP and QSR into equal parts, and those poles lie at the points where the said parallels are divided into equal hour-spaces by the hour-circles from noon and midnight, as we showed in Proposition 10 of Book I. For since the hour-circles from east or west pass, in the equator, through the poles of the hour-circles from noon and midnight, these latter are in turn carried through the poles of the former, by the scholium of Proposition 15 of Book I of Theodosius. Since, then, the poles of the former lie in the parallels OEP and QSR, the poles will be exactly those points where the said parallels are cut by the hour-circles from noon and midnight — so that the pole of any hour-circle from east or west lies at that point of the parallel OEP or QSR through which passes the hour-circle from noon or midnight that is six hours distant from it in the equator. Hence, so that these poles may be readily had, the hour-circles from noon and midnight must be produced as far as the parallels OEP and QSR, but only faintly. Or else, from G, at the interval GF, the arc of the 6-o'clock circle from noon and midnight is to be drawn, and each quadrant ET, EV divided into six equal parts for the poles of the hour-circles from east and west; and finally these points,

transferred by the compasses from E, are to be carried into the parallel QSR from S, for the poles of the other circles that do not have their poles in the portion OEP. For example, E is the pole of the horizon, that is, of the 24-o'clock circle from east or west; the next point in the parallel OEP, from E toward O, is the pole of the 23-o'clock circle from the west; and the point next from E toward P is the pole of the 1-o'clock circle from the east; and so with the rest. In the worked example, only the hours from noon and midnight, and the hours from the west, are drawn.

[Margin: Laying out the unequal hours in the concave hemisphere. An easy way to find the poles from which the circles of the unequal hours are drawn.]

Finally, for laying out the unequal hours, the several portions of the tropics HI, HK, LM, LN must each be divided into six equal parts; and through triples of points — of which one lies on the tropic of Cancer, another on the equator, and the third on the tropic of Capricorn [☿] — equally distant from the meridian, the circles of the unequal hours are to be drawn. Now the pole of each such circle will lie on the meridian hour-circle, from noon or midnight, that is six hours distant from it in the equator. For since any circle of the unequal hours passes, in the equator, through the pole of the hour-circle from noon or midnight that is six hours distant from it, it will in turn pass — by the scholium of Proposition 15, Book I of Theodosius — through the pole of that circle. Hence, if, with the same opening of the compasses kept, one foot is set on any point of the tropic of Cancer or of Capricorn, and the other on the circle of that hour from noon or midnight, faintly produced, which is six hours distant in the equator from the point corresponding to the point taken on the tropic, the pole of that circle of the unequal hours will be obtained. For example: the circle of the 1st unequal hour must be drawn through the first point of the equator below D and through the first points of the tropics below I and M; but the circle of the 1st hour from noon is, in the equator, six hours distant from the first point below D. Therefore, if one foot of the compasses is set on the first point of the tropic below I or M, and the other extended as far as the circle of the 1st hour from noon, faintly produced, the pole of that circle will be obtained in this circle; and so with the rest.

[Margin: The style that shows the hours.]

The style that marks the hours will be the axis of the world, equal to the semidiameter of the hemisphere; and it must be fixed at the pole F in such a way that its extreme point occupies the centre of the hemisphere. This is finally achieved when, with two threads stretched across the mouth of the hemisphere — from A to C, and from B to D — the extreme point is set at the common intersection of the threads. The same style may be fixed at any point, but then it

will not be the axis of the world. The instrument must also be set in the meridian circle together with a magnetized needle, yet in such a way that the plane of the instrument lies parallel to the horizon.

If one wishes, all those things that we described for plane dials in Books 2 and 3 can likewise be drawn in this hemisphere. For the parallels of the lengths of days and of the latitudes of cities will be laid out from the pole F, just like the parallels of the signs, if their declinations from G are reckoned along the meridian — toward H if they are northern, toward L if southern. The vertical circles are drawn through E, which passes through their poles (by the scholium of Proposition 15, Book I of Theodosius), since in turn the verticals pass through E, the pole of the horizon. The parallels of the horizon will be laid out from E, the pole of the horizon, or of the vertical BD. The meridians — that is, the circles of the longitudes of cities — will be carried through the several degrees of the equator, like the hour-circles from noon and midnight, provided the division of the equator into degrees begins at that point which bounds the longitude of the place, reckoned from G toward B. The circles of the celestial houses, according to the opinion of Johannes Regiomontanus [Regiomontanus], will be laid out through the divisions of the equator, setting the immovable foot of the compasses for their poles in the vertical BD, so that they all pass through the points A and C, where the meridian intersects the horizon. According to the opinion of Campanus [Campanus of Novara], however, they are laid out through the divisions of the vertical, likewise setting the immovable foot of the compasses for their poles in the vertical circle, so that they again pass through the points A and C. For these circles, according to Regiomontanus, divide the equator into 12 equal parts, whereas according to Campanus the vertical circle is cut by them into 12 equal parts. Finally, the ascending signs — that is, the various positions of the ecliptic at the risings of the beginnings of the signs — are drawn in this way. From Tables 3 and 4 given in Proposition 9 of Book 2 (or from others like them, derived from the doctrine of that proposition and computed for the given latitude of the place), take the hour at which any given sign rises, with the Sun placed at the beginning of Cancer or of Capricorn. Then, according as that hour is contained on the tropic of Cancer or of Capricorn in the concave hemisphere, let it be counted in the equator from G — toward D if it is before noon, but toward B if after noon. And through the end of the counting (with the immovable foot of the compasses placed on the point of the equator that is a quadrant distant from the end of the counting) let a great circle be drawn, faintly, cutting the tropic of Cancer or of Capricorn (according as the hour was taken with the Sun in Cancer or in Capricorn) at that point through which — at the interval of a quadrant taken in that faint circle — a great circle is described. This circle will represent the

position of the ecliptic while the proposed sign rises, so that, when the tip of the shadow falls upon it, that sign begins to rise above the horizon. For since, when the shadow falls upon that point marked on the tropic of Cancer or of Capricorn, the hour at which the given sign rises is shown, and since the ray of the Sun passing through the centre lies in the [plane]

[Margin: Laying out the parallels of the lengths of days and the latitudes of cities; and also the vertical circles and the parallels of the horizon; the meridians; the celestial houses; and the ascending signs, in the concave hemisphere.]

plane of the ecliptic, the ecliptic will necessarily pass through that point of the tropic of Cancer or of Capricorn, and so will touch the tropic at that point, just as it does in the heavens. Since, therefore, by Proposition 3 of Book 2 of Theodosius, the circle last described touches the tropic at that point — because both it and the tropic cut, at the same point, that greatest hidden circle in which the pole of each lies — this circle will be exactly the ecliptic. By this device all the ascending signs will be drawn. The sign Aries [♈] will touch the tropic of Capricorn at L, and will have its pole in the meridian a quadrant distant from point L. The sign Libra [♎] will touch the tropic of Cancer at H, and both will pass through the points B and D. The sign Cancer [♋] will touch the tropic of Cancer at I and the tropic of Capricorn at N. Finally the sign Capricorn [♏] will touch the tropic of Cancer at K and the tropic of Capricorn at M, as is gathered from the said tables.

[Margin: How, by means of the concave hemispherical dial, sundials for any plane may be laid out from the shadow.]

And so, as you see, all those things that we described for plane dials in Books 2 and 3 are laid out with ease in the concave hemisphere. Hence, if the concave hemisphere, in which all these things are drawn, is fastened onto some plane tablet, and on that tablet the meridian line is drawn corresponding to the meridian of the hemisphere — which is easily done: if the tablet, exposed to the Sun together with the hemisphere, is turned about until the shadow of the style falls onto the meridian circle; for if a style fixed at right angles is then observed on the tablet, and in the middle of its shadow two points are marked, the line drawn straight through these points will be the meridian line, provided account is taken of the tablet's own position with respect to inclination to the horizon and declination from the vertical, as we said in the last chapter of the preceding book — then everything is drawn far more conveniently, and perhaps more accurately, than from a horizontal dial. This is so both because the tip of the shadow is discerned more clearly and distinctly in the concave hemisphere than in a plane

dial, and also because everything is drawn more easily and more accurately in the concave hemisphere than in a plane dial, as is evident.

ON THE CONSTRUCTION OF A DIAL ON A CONVEX CYLINDER

CAPVT V.

So that the laying-out of the hours on a convex cylinder may be made clearer — a thing accomplished entirely by means of turned shadows [umbrae versae] or the altitudes of the Sun — we shall draw the hours on some plane, in order afterward to transfer them onto the cylinder. But before all else, the altitudes of the Sun above the horizon must be investigated for each hour, with the Sun at the beginnings of the signs. Computed for latitude 42°, these are as follows, for the hours from noon and midnight:

Scheme of the hours (leftmost value column, 12, is the shared noon/midnight column):

	1								
Hours (afternoon)	2	1	2	3	4	5	6	7	post meridiem
Hours (after midnight)	1	1	1	9	8	7	6	5	post med. noc.
	2	1	0						

Altitudes of the Sun above the horizon at the beginnings of the signs

[*Altitudines Solis supra Horizontem in signorum initijs*] (values in G.M. = degrees.minutes; hour-columns 12, 1, 2, 3, 4, 5, 6, 7 as above):

Signs	12	1	2	3	4	5	6	7	Signs
♋ Cancer	71.3 0	67.4 1	58.5 9	48.2 9	37.2 5	26.1 9	15.2 9	5.1 1	♋ Cancer
♌ Leo	68.1 2	64.4 7	56.3 7	46.2 4	35.2 6	24.1 8	13.2 2	2.5 4	♊ Gemini
♍ Virgo	59.3 0	56.4 8	49.4 9	40.2 5	29.5 0	18.4 6	7.40		♉ Taurus
♎ Libra	48.0	45.5 2	40.4	31.4 2	21.4 9	11.5	0.0		♈ Aries
♏ Scorpio	36.3 0	34.4 5	29.4 9	22.2 6	13.2 0	3.10			♊ Pisces

Signs	12	1	2	3	4	5	6	7	Signs
♊ Sagittarius	27.4 8	26.1 6	21.5 4	15.1 2	6.45				♋ Aquarius
♐ Capricorn	24.3 0	23.3	18.5 2	12.2 5	4.14				♑ Capricorn

Altitudes of the Sun above the horizon for the hours from the west, on the arc of 15 hours, which declines toward the north 23° 2' [Altitudines Solis supra Horizontem pro horis ab occ. in arcu horarum 15. qui declinat in Boream Grad. 23. Min. 2.]:

Altitudes of the Sun above the horizon for the hours from the west, on the arc of 14 hours, which declines toward the north $16^{\circ} 2'$ [... in arcu horarum 14. qui declinat in Boream Grad. 16. min. 2.]:

Altitudes of the Sun above the horizon for the hours from the west, on the arc of 13 hours, which declines toward the north 8° 15' [... in arcu horarum 13. qui in Boream declinat grad. 8. min. 15.]:

	2							17	pomeridian
Hours (afternoon)	4	23	22	21	20	19	18	½	æ
Hours (morning)	1	12	13	14	15	16	17	17	antemeridia
	1							½	næ
Altitudes of the Sun	0	11	22.	32.	42.	50.	55.	56.	dines Solis
(G.M.)	.	.4	11	56	48	51	36	15	
	0								

Altitudes of the Sun above the horizon for the hours from the west, on the arc of 12 hours, that is, on the equator [... in arcu horarum 12. hoc est, in Æquatore.]:

	2								
Hours (afternoon)	4	23	22	21	20	19	18		pomeridianæ
Hours (morning)	1	13	14	15	16	17	18		antemeridian
	2								æ
Altitudes of the Sun (G.M.)	0.	11.	21.4	31.4	40.	45.5	48.		dines Solis
	0	5	9	2	4	2	0		

Altitudes of the Sun above the horizon for the hours from the west, on the arc of 11 hours, which declines toward the south 8° 15' [... in arcu horarum 11. qui in Austrum declinat Grad. 8. min. 15.]:

	2							18	
Hours (afternoon)	4	23	22	21	20	19	½		pomeridianæ
Hours (morning)	1	14	15	16	17	18	18		antemeridian
	3						½		æ
Altitudes of the Sun	0	10.	20.	29.	35.	39.	39.		dines Solis
(G.M.)	.	41	36	10	42	17	45		
	0								

Altitudes of the Sun above the horizon for the hours from the west, on the arc of 10 hours, which declines toward the south 16° 2' [... in arcu horarum 10. qui declinat in Austram Grad. 16. min. 2.]:

	2						
Hours (afternoon)	4	23	22	21	20	19	pomeridianæ
Hours (morning)	1	15	16	17	18	19	antemeridianæ
	4						
Altitudes of the Sun (G.M.)	0.	9.5	18.4	25.4	30.2	31.5	dines Solis
	0	5	1	2	0	8	

Altitudes of the Sun above the horizon for the hours from the west, on the arc of 9 hours, which declines toward the south 23° 2' [... in arcu horarum 9. qui in Austrum declinat Grad. 23. Min. 2.]:

	2						
Hours (afternoon)	4	23	22	21	20	19½	pomeridianæ
Hours (morning)	1	16	17	18	19	19½	antemeridianæ
	5						
Altitudes of the Sun (G.M.)	0.	8.5	16.1	21.4	24.3	24.5	dines Solis
	0	3	8	3	6	8	

Gnomonices, Book VIII (Liber Octavus)

(This opening continues Chapter V, on the sundial constructed on a convex cylinder; a new chapter — Chapter VI, on the quadrant — begins on the third page.)

The page opens with two tables of solar altitudes, one for each tropic, computed for latitude 42° (the meridian altitudes 71° 30' in Cancer and 24° 30' in Capricorn confirm this latitude).

Altitudes of the Sun above the horizon for the hours from the west, in the tropic of Cancer (♋) [Altitudines Solis supra Horizontem pro horis ab occ. in tropico ♋]. G.M. = degrees.minutes; "pomeridianæ" = afternoon hours, "antemeridianæ" = morning hours; "Altitudines Solis" runs as the side-caption of the two altitude rows:

Hours (afternoon / pomeridianæ)	24	23	22	21	20	19	18	17	16 28/60
Altitude of the Sun (G.M.)	0.	9.5	20.	31.	42.	53.	63.	70.	71.30
	0	4	29	28	36	30	24	20	
Hours (morning / antemeridianæ)	9	10	11	12	13	14	15	16	16 28/60

Hours (afternoon / pomeridianæ)	24	23	22	21	20	19	18	17	16 ²⁸ / ₆₀
Altitude of the Sun (G.M.)	0.	10.	21.	32.	43.	54.	63.	70.	71.30
	40	35	12	13	21	12	59	36	

Altitudes of the Sun above the horizon for the hours from the west, in the tropic of Capricorn (♑) [Altitudines Solis supra Horizontem pro horis ab occ. in tropico ♑]:

Hours	4	23	22	21	20	19 ²² / ₆₀	19	18	17	16
Altitude of the Sun (G.M.)	0	8.	16	21.	24.	24.30	24	21	15.	8.
	.	47	.7	25	11		.5	.8	41	14
	0									

So let AB be a plane surface on which the hours are to be drawn. Since the reversed shadows [*umbræ versæ*] are longest when the Sun is in Cancer (♋) and shortest when it is in Capricorn (♑), it will not be out of place to choose two unequal styles [*styli*]: the shorter to mark the hours while the Sun is in the northern signs, the longer to show them while the Sun is in the southern signs. Accordingly, for the northern signs construct the rectangular parallelogram AC, and for the southern signs DB. Divide AC arbitrarily into three parallelograms EI, MK, NC for the six northern signs; but so that they keep a proportionate distance from one another, from G describe the quadrant EF, divide it into three equal parts, and through the points of division draw the lines MI and NK parallel to AH and LC. The space from E to M is to be assigned to Cancer; the spaces from M to N and from N to G to Leo (♌) and Virgo (♍); and finally the spaces from G to N, from N to M, and from M to E to Aries (♈), Taurus (♉), and Gemini (♊).

In the same way the parallelogram DB is to be divided for the southern signs — though we have divided it by diurnal arcs rather than by the beginnings of the signs, so that the hours from the west can be drawn more easily. For the line PQ will be fitted to the diurnal arc of 12 hours, that is, to the equator; RS to the arc of 11 hours; TV to the arc of 10 hours; XY to the arc of 9 hours; and ZB to the tropic of Capricorn (♑). These lines are to be drawn parallel to DQ and OB at any distance, though for convenience we have drawn them at roughly equal spacings — except the line ♑ ZB, which lies close to XY, just as in the heavens the tropic of Capricorn lies close to the parallel whose diurnal arc spans 9 hours, at latitude 42°.

[Margin: *Descriptio horarū ab occ. in cylindro concauo* — Drawing the hours from the west on the concave cylinder.]

So, to draw the hours from the west on the rear parallelogram DB, extend DO and take Oa equal to a style of any size. From a describe the quadrant Ob, divided into 90 degrees beginning from the line Oa, and from a, through the degrees, let hidden lines be drawn cutting OB at the points of the reversed shadows corresponding to the required altitudes of the Sun. For if Oa is set parallel to the horizon and OB perpendicular to it, the Sun's rays passing through a, the centre of the world, and running along each degree of altitude (which would face the degrees of the quadrant Ob and lie above the horizon Oa, were a full circle described from centre a) will fall on the points of the reversed shadows on the line OB, as is evident. If instead you prefer to use the reversed shadows drawn from the shadow-table we made in Proposition 2 of Book 5 [*propos. 2. lib. 5*], in place of the Sun's altitudes, then on the line DQ take DF equal to the style Oa, and divide it into 12 equal parts of the reversed shadow; and if need be, take another portion below f, equal to the same style, and so on. In our example it is enough to take below f two small parts of the reversed sha-

Diagram. The developed (unrolled) layout for the *northern* signs — the parallelogram AC. Along its top runs a hatched horizontal band (the cylinder-rim), with **p** at its left end and **q** at its right; below the band the vertical sides descend, with **A** at upper left (below p) and **L** at upper right (below q). Just above the band is a zodiac strip divided into three cells, each carrying a pair of signs (each interval of the dial serves one ascending and one descending sign). Reading left to right, the cells are headed by the top-edge letters **E, H, N, G** and carry the sign-pairs: Gemini (♊) over Cancer (♋); Taurus (♉) over Leo (♌)[?]; Aries (♈) over Virgo (♍) — consistent with the text's assignment of the intervals E–M, M–N, N–G. Within the field, curved hour-lines fan from the upper corners downward; the left side (line AH) carries a scale of shadow-parts marked 7, 6, 5, 4, 3, 2, 1, ½ near the top and the larger marks 3, 6, 9, 12 and, lower, 24, 36; the right side (line LC) is marked 5, 6, 7, 8, 9, 10, 11, 12, with the interior point **F** where a curve meets the right edge. The baseline is lettered **H, I, K, C**.

- p, q (top rim ends); A, L (heads of the vertical sides)
- E, H, N, G (top-edge division letters)
- Zodiac cells: ♊/♋, ♉/♌[?], ♈/♍
- F (interior point on the right side)
- Left scale: 7,6,5,4,3,2,1,½; 3,6,9,12; 24,36
- Right scale: 5,6,7,8,9,10,11,12

- Baseline: H, I, K, C
- Beneath the figure: a small woodcut of one of the two styli (a pin-gnomon with a rounded knob-handle tapering to a point)

(Several interior hour-numbers and one zodiac glyph (♑) are worn/uncertain)

Diagram. The developed layout for the *southern* signs — the parallelogram DB — together with the physical instrument. Along the top is a hatched rim-band; the top-edge letters are **P, R, T, X, Z**, with **D** at upper left (below P). Above the band is a date strip divided into cells that pair the two dates on which the Sun crosses each declination-parallel: "14. Sep / Feb. 18", "5. Octob.[?] / Ian. 25", "29. Oct / Dec. 23", closing with a Capricorn (♑) glyph for the tropic. The vertical parallels PQ, RS, TV, XY, ZB descend through a field of diagonal hour-lines bearing numbers (12, 13, 14, 15, 16 across the top; 21, 22, 23 and a point **h** further in; 20; and near the foot the marks 29, 10 and 18). A diagonal line labelled **Linea meridiana** ("meridian line") crosses the field; the left side is scaled 3, 6, 9, 12 (the mark 12 lettered **f**). The baseline is lettered **Q, S, V, Y, B**. To the right stands a graduated quarter-circle (the quadrant Ob) scaled 10, 20, 30, 40, 50, with centre **O**; and at the far right the actual cylinder-dial instrument — a horizontal top-bar carrying a suspension ring/knob (the *nodulus*) with a plumb-thread, a tall body with a vertical scale 10–70 and a lower arc scaled 70, 80, 90, lettered **n** (head of scale), **a** (upper right), **6** (lower right), **f** (foot / plumb-bob).

- P, R, T, X, Z (top edge); D (upper left)
- Date-strip pairs: 14 Sep / 18 Feb; 5 Oct[?] / 25 Jan; 29 Oct / 23 Dec; ♑
- Parallels PQ, RS, TV, XY, ZB
- Linea meridiana (labelled diagonal)
- Interior hour-numbers: 12,13,14,15,16; 21,22,23; 20; point h; 29,10; 18
- Left scale: 3, 6, 9, 12(f)
- Baseline: Q, S, V, Y, B
- Quadrant Ob: centre O; scale 10,20,30,40,50
- Instrument: nodulus/ring + plumb; vertical scale 10–70; arc 70,80,90; labels n, a, 6, f
- Beneath the figure: a small woodcut of the second stylus (pin-gnomon with knob-handle) ...-dow. Now we shall draw the meridian line thus. From the adjacent little tables take the meridian altitudes at the equator and on the diurnal arcs of 11, 10, and 9 hours, and likewise in the tropic of Capricorn (♑); and with the compasses on the line OB take the lengths of the shadows owed to those altitudes — or rather, from the line DQ take the reversed

shadows owed to the same altitudes, as the table of Proposition 2, Book 5 indicates — and transfer them onto the lines DQ, RS, TV, XY, OB, always beginning from the line DO, marking the points on these lines. A curved line drawn suitably through these points will be the meridian line. In the same way we shall draw the lines of the hours from the west, if — taking account of the Sun's altitudes for each hour and of the reversed shadows owed to them — we mark the points on the lines of the diurnal arcs.

For example: the hour 16 from the west, at the equator, has an altitude of $40^{\circ} 4'$ [*gr. 40. min. 4*]. So if with the compasses on the line OB we take the portion between O and the $40^{\circ} 4'$ mark [the print here reads *gr. 4. m. 4*, apparently for $40^{\circ} 4'$], and transfer it from D as far as g, then the point g terminates the reversed shadow of 10 parts 6 minutes [*partium 10. min. 6*], which corresponds to that altitude in the table of Proposition 2, Book 5; and the hour 16 from the west is to be drawn through g. Again, the same hour 16 from the west, on the diurnal arc of 11 hours, has an altitude of $29^{\circ} 10'$. So if with the compasses on the line OB we take the portion between O and that altitude, or on the line DQ take the reversed shadow of 6 parts 42 minutes corresponding to the same altitude in the table of Proposition 2, Book 5, and transfer it from the line DO onto the line RS as far as h, the same hour 16 from the west is to be drawn through the point h, and so with the rest.

You see, then, that through the individual points of the lines PQ, RS, TV, XY two hours from the west pass — which would not happen if these lines had been assigned to the beginnings of the signs rather than to the diurnal arcs of whole hours. And since the hours run together near the meridian line, we have marked the line of hour 19 with certain dots, so that it may be more easily distinguished from the others. In the same way the hours from sunrise, the unequal hours, and those from noon and from midnight will be drawn, if their altitudes are computed. Thus in the parallelogram AC you see the hours from noon and midnight drawn from the reversed shadows of the line AH, on which three parts equal to the style have been taken, of which Am is divided into 12 equal parts; these reversed shadows are to be taken from the table of Proposition 2, Book 5, corresponding to the Sun's altitudes — or else from the reversed shadows of the line nl, which the hidden lines drawn from the centre a through the degrees of the quadrant Ob supply, provided an is equal to the style Am and the line nl is drawn perpendicular to aO.

Only this must be noted. When some hour cannot be drawn through all the beginnings of the signs — as happens with the hour 5 from midnight and 7 from

noon, which has no altitude at the beginning of Taurus (♉) and Virgo (♍), and hence no point on the line NK — a point must be taken on this line NK, above AL, for the opposite hour (that is, for hour 5 from noon and 7 from midnight), from the altitude it has in the opposite sign, say at the beginning of Scorpio (♏) and Pisces (♐) : because this altitude above the horizon equals the depression below the horizon of the earlier hour (which had no point), as is clear from the material sphere; and therefore, since that earlier hour is above the horizon of the Antipodes, it is counted from noon if it was previously counted from midnight, and conversely; and the northern parallel will stand to them as the opposite southern one does to us, and so on. By the same reasoning, if some hour from sunrise or sunset has no point on any parallel or diurnal arc, the same hour must be taken on the opposite parallel or arc — yet so that it is counted from sunrise if it was previously reckoned from sunset, and conversely.

By this same device the hours will be laid out on the convex cylinder [*cylindro conuexo*], if on it are drawn, for the lines of the signs or of the diurnal arcs, parallel lines which — the cylinder hanging freely — are perpendicular to the horizon, as we have said. The style is to be fitted so that it can easily be turned to the place of the Sun, or to the day of the month, at which we wish to observe the hour; but under this rule, that the sighting-line of the style [*linea fiduciæ styli*], such as the line Oa on the style Oa, always makes right angles with the line DO or AL.

[Margin: *Horologiū planū instar cylindri commodissimum* — A plane sundial in the manner of a cylinder: most convenient.]

But we shall also learn the hours very conveniently from the two parallelograms AC and DB drawn on some bronze plate [*lamina ænea*] — the one on one face of the plate, the other on the opposite face — so that the sundial may be reduced to a smaller form. For suppose a groove pq is cut in the plate, in which a style is set (whose form is shown at the foot of the preceding figure) that can run back and forth, so that the sighting-line of the style is perpendicular to the lines AL and DO, and the style itself can be fastened on the opposite side of the plate by a little screw [*cochleola*] once it has been set at the place of the Sun or at the proposed day of the month, so that it can no longer be moved; and suppose the plate is turned toward the Sun so that the thread, from which a plumb-weight hangs, coincides with one of the straight lines assigned to the beginnings of the signs or to the diurnal arcs, and the style's shadow runs parallel to the lines of the signs or diurnal arcs. Then the tip of the shadow will indicate the present hour — before noon or after — according as the observation is made before or after midday.

And I would judge this plane sundial far superior to one drawn on a cylinder, because it is very difficult to make a cylinder that, hanging freely, does not tilt to one side — an inconvenience that does not arise in our plane sundial, since it is governed by a plumb-line. In the same way the hours will be found on the cylinder, if the style is set at the Sun's degree or the day of the month, and the shadow is kept parallel [to the lines], and so on.

[Margin: *Hora quo pacto ex regula solida per altitudines Solis cognoscatur* — How the hour may be found from a solid rule by the Sun's altitudes.]

Moreover, if some solid rule [*regula solida*] is made, such as *nl*, on which are drawn the parts of the reversed shadow that the lines from the centre *a*, through the degrees of the quadrant *Ob*, exhibit; and if at *n* a style *na* is fixed perpendicular to *nl*, of a length equal to the distance *nl* on the rule (into which the reversed shadows corresponding to the altitudes of the quadrant *Ob* have been carried) — then this rule will serve us as a sundial. For if, by means of a plumb, this rule, turned toward the Sun, hangs freely so as to be perpendicular to the horizon, and the style's shadow runs parallel to the sides of the rule, the tip of the shadow will indicate the Sun's altitude, and so from the adjacent tables of the Sun's altitudes we shall easily learn the present hour.

[Margin: *Hora cognitio ex altitudine Solis quadrāte, vt alio instrumēto inuenta* — Determining the hour from the Sun's altitude by a quadrant, as by another instrument.]

We shall also find that same hour from the same tables if we seek the Sun's altitude with a quadrant or another instrument.

[Margin: *Descriptio horarū in cylindro ex vmbra cadente in horas horologij horizontalis, vel hemisphaerici concauo* — Drawing the hours on a cylinder from the shadow that falls on the hours of a horizontal, or of a concave hemispherical, sundial.]

But if a cylinder is set upright on a plane on which a horizontal sundial is drawn, or on which a concave hemispherical sundial is fixed, the hours will be drawn on the cylinder from the shadow — as we noted at the end of the preceding book — in this way. Having turned the style to the line of each sign, or of the diurnal arc, let the plane be rotated until the style's shadow, in the drawn sundial, falls on the hour of that parallel to whose line the cylinder's style is fitted; and let the cylinder itself, fixed on the plane in that position, be turned about its axis until the style's shadow runs parallel to the lines of the signs or of the diurnal arcs, and the endpoint of the shadow be marked. For through this that hour is to be drawn on

the cylinder. In no other way can the points of that hour, and of all the others, be found on all the lines of the signs or of the diurnal arcs.

ON THE DRAWING OF THE HOURS ON A QUADRANT

CHAPTER VI

[DE HORARVM DESCRIPTIONE in quadrante. CAPVT VI.]

[Margin: *Descriptio horarū ab occ. in quadrante* — Drawing the hours from the west on a quadrant.]

The drawing of the hours on a quadrant differs little from what we set out in the preceding chapter for the cylinder, or for a plane in the manner of a cylinder, since in both cases the hours are drawn from the Sun's altitudes above the horizon. So let ABC be a quadrant on which the hours from the west are to be drawn, divided into 90 degrees whose numbering runs from B to C. From A describe two arcs through the points D and E at any distance apart, of which the one nearer the quadrant's side BC is assigned to the tropic of Capricorn (♑), and the one farther from BC to the tropic of Cancer (♋). For although most people assign the former to the tropic of Cancer and the latter to Capricorn — which can be done just as well, as far as the drawing of the hours is concerned — it will nevertheless be much more convenient to do the opposite, as we have said, because of the greater separation of the hours. Bisecting also the portion of the line AB that lies between the arcs drawn through D and E, describe from A, through the point of division, the arc through F, which serves for the equator; this could also be drawn through any point between the arcs through D and E, even if it did not bisect the line between them.

Now from A let three hidden lines go out through the meridian altitudes of Capricorn (♑), of Aries (♈) or Libra (♎), and of Cancer (♋) — that is, through 24° 30', 48°, and 71° 30' — cutting the arcs through D, F, E at the points D, F, E; and the arc of a circle described through these, DFE, will be the meridian line. Then from A let other hidden lines be drawn through the meridian altitudes of the diurnal arcs of 9, 10, 11, 13, 14, and 15 hours, taken from the tables of the preceding chapter (for the hours from west and east are more easily drawn by diurnal arcs, just as on the cylinder), which cut the meridian line DFE at the points M, L, K, I, H, G, through which arcs are described from A for the diurnal arcs.

And if from A hidden lines are sent out through the altitudes of the hours of any diurnal arc, taken from the tables of the preceding chapter, the diurnal arc whose hour-altitudes were taken will be cut into the hours from the west. So if the corresponding hour-points are joined by curved lines — all the morning hours being expressed by circular lines, while the afternoon hours must be drawn suitably through the points at which the diurnal arcs are cut by the arcs of the morning hours, so that hour 23 is drawn through the points nearest the line AB itself, and 22 through the following ones, and so on — it will be enough to find three points for each of the morning-hour arcs. Namely, for the arc of hour 15: one on the diurnal arc of 9 hours (which lies on the line AB, since the Sun then rises), another at the equator, that is, on the diurnal arc of 12 hours, and a third on the diurnal arc of 15 hours. Likewise for the arc of hour 17: one on the diurnal arc of 9 hours, another on the diurnal arc of 12 or 11 hours, and finally a third on the diurnal arc of 14 hours — which is the same as H, since it is then noon; and so on. The sundial will then be complete.

That it may indicate the hours, the signs of the Zodiac — or at least the days of the months — must be drawn along the sides AB and DE: the days of the months from the 12th of June to the 12th of December (or the signs from Cancer (♋) to Capricorn (♐)) along the line AB; and the days of the months from the 12th of December to the 12th of June (or the signs from Capricorn (♐) to Cancer (♋)) along the arc DE. This is done as follows. From the place of the Sun at the beginning of any month, or at the beginning of any sign, find its declination, and from this the meridian altitude; by this a hidden line drawn from A cuts the arc DE at a point through which the hidden arc described from A will cut the line AB — or the circular line described along the arc DE — at the beginning of the proposed month or sign of the Zodiac. By the same reasoning points will be found for the 10th or 20th, or any other day of the proposed month, if the place of the Sun is found, together with its declination and meridian altitude. In our example the days of the months are drawn; in the following figure, the signs of the Zodiac.

Now if from A a thread with a plumb hangs freely, together with a movable bead, and the thread is stretched and the bead is set on the line AB (or on the circular line near DE) at the day of the month, and finally the instrument is directed to the Sun until the Sun's ray passes through the corresponding points of the sights [*pinnacidia*], the bead will indicate, among the hour-lines, the present hour — before noon or after — according as the observation is made before or after midday; and the plumb-thread, on the quadrant BC, will show the Sun's altitude: while this altitude is increasing, the time is before noon; while it is decreasing, after noon.

[Margin: *Descriptio horarum à mer. & med. noc. in quadrante* — Drawing the hours from noon and from midnight on a quadrant.]

By the same reasoning the hours from noon and from midnight will be drawn from their altitudes at the beginnings of the signs; but it will be enough to find the hour-points in the two tropics and at the equator, for arcs of circles described through three corresponding points on the said parallels will give the hours from noon and midnight. For all the hour-points fall on the circumferences of circles. Only this must be noted here: when some hour from noon or midnight has no point on a given parallel, the altitude of the opposite hour must be taken — namely from midnight or from noon — on the opposite parallel, and this must be transferred from the line AB onto the earlier parallel produced beyond the quadrant, a point being marked on it; for through this that hour is to be drawn. Thus, since the hour 5 from noon (or 7 from midnight) has no point on the tropic of Capricorn (♊) — this hour being below the horizon — with the Sun at the begin-

Diagram. A horary quadrant (the quadrant ABC). The right angle is at the top: **A** at the upper-left corner and **C** at the upper-right corner (each fitted with a small square sight-box/pinnule), with the apex **B** at the bottom. The curved outer rim (the graduated limb from C down to B) is scaled in degrees, marked 80, 70, 60, 50, 40, 30, 20, 10. Along that rim runs a band of month-abbreviations — **MAI.** (May), **APR.** (April), **MAR.** (March), **FEB.** (February), **IANVAR.** (January) — for the sign/date graduation. Down the left side (line AB) is a stacked column of month-names: **IVNI=** (June), **IVLIVS**, **AVGVST**, **SEPTEN**, [one line worn], **OCTOB.**, **NOVEM**, **DECEM**: — with the numbers 9, 10, 11(?), 12, 13, 14, 15 alongside them. The interior is filled with a curvilinear grid of hour-lines; the morning-hour arcs carry the numbers 16, 17, 18, 19, 20, 21, 22, 23. The meridian line DFE and the diurnal-arc points appear as the lettered points **D**, **E**, **F** (arc-endpoints) and **H**, **K**, **L**, **M** (and, per the text, **I** and **G**) on the meridian.

- A, C (top corners, with sight-boxes); B (apex)
- Outer degree-scale: 80,70,60,50,40,30,20,10
- Rim month-band: MAI, APR, MAR, FEB, IANVAR
- Left month-column: IVNI, IVLIVS, AVGVST, SEPTEN, [?], OCTOB, NOVEM, DECEM
- Left numbers: 9, 10, 11[?], 12, 13, 14, 15
- Interior hour-numbers: 16–23
- Meridian/arc letters: D, E, F, H, K, L, M ([?] I, G)

with the beginning of ♑ [Capricorn] present, we take the altitude of hour 5 from midnight (or 7 from noon) in the tropic of ♋ [Cancer], namely $5^{\circ} 11'$, and from A, through this altitude, we draw a hidden straight line that cuts the tropic of ♑ at G. For if the point G is transferred from point N onto the tropic of ♑, extended as far as H, then hour 5 from noon (or 7 from midnight) is to be drawn through the point H. The reason for this is that that hour, below the horizon among the Antipodes, has the same altitude in the tropic of ♑ as the opposite hour has in the opposite tropic of ♋. This too we noted must be done in the preceding chapter, when we were describing the hours on the cylinder.

By the same reasoning, since hour 6 from noon (or from midnight) has no point on the parallel of ♏ [Scorpio] or ♓ [Pisces], we take the altitude of hour 6 from midnight (or noon) in the parallel of ♉ [Taurus] or ♍ [Virgo], namely $7^{\circ} 40'$; and through this altitude, from A, we draw a hidden straight line that cuts the parallel of ♏ or ♓ at I; and we transfer the point I from point O onto the same parallel, as far as K. For hour 6 from noon (or midnight) is to be drawn through K. Finally, since hour 7 from noon (or 5 from midnight) has no point on the parallel of ♈ [Aries] or ♎ [Libra], we take the altitude of hour 7 from midnight (or 5 from noon) on the same parallel — since it has no opposite — and by it we find the point L of hour 7 from midnight (or 5 from noon). If, then, this point is transferred from point P onto the parallel of ♈ or ♎, extended as far as M, hour 7 from noon (or 5 from midnight) is to be drawn through M. And so with the rest.

Diagram. A demonstration figure combining a projected sphere (a large circle filled with a curvilinear grid of the sign-parallels and hour-lines) with a graduated quadrant limb at the right. The outer/right rim carries a degree-scale marked (from the top down) 90, 80, 70, 60, 50, 40, 30, 20, 10. At the top are two corner points fitted with small square sight-boxes; the point **A** stands at the top with a square, and a small boxed glyph (a sun/face) sits beside it. The bold letter **O** sits at the top-right, at the 90° mark where the scale begins. Outside the circle on the left, the points **Y** (upper) and **Z** (lower) lie on vertical lines carrying dotted (hidden) construction lines. Around and within the circle appear the lettered points **R** and **T** (left), **M**, **L**, **O**, **K**, **H** **N** (together), **G** (blackletter), **B**, **S**, and **n** (blackletter, at the foot); on the right, **X** (with a dotted line) and **Q**. The interior is densely filled with the sign-parallels, hour-lines, small zodiac glyphs, and hour-numbers (of which 3–8 are legible).

- Corner/rim points: A (with square sight-box), O (at 90° on the scale)
- Outer degree-scale: 90, 80, 70, 60, 50, 40, 30, 20, 10
- Left (outside circle): Y, Z (on the projected verticals)

- Circle/interior letters: R, T, M, L, O, K, H, N, G, B, S, n
- Right: X, Q
- Interior hour-numbers legible: 3, 4, 5, 6, 7, 8
- Numerous small interior zodiac glyphs and hour-labels worn/uncertain [?]

Hours from sunrise, and the unequal [seasonal] hours, can be laid out in the quadrant in no other way, if their altitudes are investigated in the diurnal arcs, and the altitudes of these in the parallels of the signs. Indeed, the hours from sunset will also be the hours from sunrise, if their complements up to 24 are taken — so that hour 23 from sunset is hour 1 from sunrise, hour 22 is hour 2, and so on. This I mean to be understood also of the cylinder of the preceding chapter. The reason is that any hour from sunset has, above the horizon in any parallel, the same altitude as the hour from sunrise which, together with it, of course makes up 24.

The Sun's altitudes can also be found in the individual parallels — both for the hours from noon and midnight, and for those from sunrise or sunset, and even for the unequal hours — geometrically, without tables of the Sun's altitudes, in the following way. Suppose, for example, that the Sun's altitudes are to be sought in the parallel of ♍ or ♋, l O. With the parallel l O extended in both directions, draw from A, through the meridian altitude [altitudo meridiana] of the opposite signs ♊ and ♎ — that is, through $59^{\circ} 30'$ — a hidden straight line cutting the arc O l, extended, at Q; and let O a be cut off equal to the arc O Q; and through a let a R be drawn parallel to A B, the line which the straight line l R — drawn from l, the end of the parallel of ♍ or ♋, perpendicular to A B or to a R — meets at R. Now bisect the line l R at S, and from S, with radius S l (or S R), describe the circle l n R a, and let it be divided into 24 equal hours: beginning at the point l (of noon), if the hours from noon and midnight are to be described, or at the point a or f (of the Sun's rising or setting), if the hours from sunrise or sunset are wanted.

But if unequal hours are to be laid out, both the arc a l f and the arc a R f must be divided into 12 equal parts. And the point l will be hour 12, of noon; and the points nearest to it, equally distant from l, will belong to hour 1 from noon and hour 11 from midnight, if the division of the circle into 24 equal parts began at l; and so with the rest. But if the division of the same circle began at the line a f, then the point a (or f) will be hour 24; the point of division nearest toward l, hour 23 from sunset (or 1 from sunrise); the next, hour 22 from sunset (or 2 from sunrise); and so on — so that the line a f is, as it were, the horizon, since the plumb-thread falls upon it when the Sun is rising or setting, while a l f is like the diurnal arc, and a R l the nocturnal. Finally, if unequal hours are in question, the

point a (or f) will be hour 12, and the points of division nearest toward l hours 1 and 11 of the unequal reckoning, and so on. We have given an example in the circle l n R a, for the hours from noon and midnight.

Now if, from the hours of the circle l n R a, hidden lines are drawn parallel to a f, or perpendicular to l R — which is easily done for the hours from noon or midnight, and for the unequal hours, if any two points equally removed from l are joined by hidden lines; and the same can be done for the hours from sunrise or sunset, if, with the compass, arcs equal to the several arcs between l and the hour-arcs l a are taken in the arc l f, and conversely — these parallels will cut the parallel l O at the altitudes of the hours, so that the arcs of the parallel l O between O and the points of section measure the Sun's altitudes above the horizon. And the points of section on the arc O a will give the altitudes of the nocturnal hours above the opposite face of the horizon, so that the arcs of the parallel O a between O and the said points are the Sun's altitudes above the horizon among the Antipodes, in the hours of the parallel l O a — which for them is the same as the opposite parallel is for us,

e f, ☿ and ♀. The hours, however, which in our horizon are reckoned from noon or from sunrise will in their horizon be reckoned from midnight or from sunset, and conversely. Therefore, since hours of this kind below the horizon have the same altitudes that the opposite hours have in the opposite parallel above the horizon — as is clear from what we showed in the scholium to proposition 1 of book 5 (where, of course, we demonstrated that the altitudes of the 24 hours of any parallel above one face of the plane are equal to the altitudes of the 24 hours of the opposite parallel above the other face of the plane, provided that what were previously reckoned from noon or from sunrise are now reckoned from midnight or from sunset — such being those which, in the opposite parallel, are set opposite to those in the earlier parallel) — from the altitudes contained in the arc O a we shall elicit the altitudes of the opposite hours in the parallel e f, ☿ and ♀, which is set opposite to the parallel O l, ♀ and ☿, in this way.

From A, to the points of the altitudes in the arc O a, let hidden straight lines be drawn; these will cut the arc e b of the opposite parallel, toward the side e, into arcs similar to the arcs of the arc O a — as we demonstrated at the end of chapter 1 for the sphere. Therefore, if by means of the compass these arcs of the parallel e b are transferred from e into the arc e f, we shall have in the parallel e f the altitudes of the hours which are set opposite in the hours of the arc O a — since the arcs of the parallel e f between e and the points of the Sun's altitudes are similar to the arcs of the parallel O a between O and the points of the Sun's

altitudes, being indeed equal to the arcs of the parallel $e b$ between e and the points of the Sun's altitudes. So that, for example, just as the first point from O toward a belongs to hour 6 from noon, so the first point from e toward f belongs to hour 6 from midnight; and just as the second point from O toward a looks to hour 7 from noon, so the second point from e toward f belongs to hour 7 from midnight; and so with the rest.

And just as we investigated the Sun's altitudes in the parallel $l O$, m and $\times$, so also we shall explore the same in all the other parallels, provided that they are extended in both directions, and that through the meridian altitudes of the opposite parallels hidden straight lines are drawn from A , and the points are marked in which the parallels are cut by them, and the rest is done as before. Thus you see, in the parallel of γ , PF — in which the meridian altitude of the opposite sign $\triangle$ is the same as that of γ — that the arc PT was taken equal to the arc PF , and that through T the line Tm was drawn parallel to AB . Then, that from F the line FT was drawn perpendicular to AB (or to Tm), which here falls exactly on T , since the line FT , on account of the equal arcs PF , PT , is cut by the line AB into two equal parts and at right angles — as will be demonstrated in the way it was shown in proposition 1 of book 1 that the line MN is cut by the line EH into two equal parts and at right angles, on account of the equality of the arcs HM , HN . Finally, that from V — where the line FT is bisected — with radius VF (or VT), the circle $F\beta TN$ was described; and if this is divided into hours, as we said of the circle $lnRa$, the Sun's altitudes will be found in the parallel PF , as in the parallel Ol .

Again, in the parallel of ν , ND , extended in both directions, you discern that a line sent out from A , through the meridian altitude of the opposite tropic of $\textcircled{S}$ — that is, through $71^\circ 30'$ — cuts the parallel ND , extended, at X ; and that the arc NY was taken equal to the arc NX , and that through Y the line YZ was drawn parallel to AB . After this, that from D , the end of the parallel of ν , the line DZ was drawn perpendicular to AB (or to YZ), and that from its midpoint λ , with radius λD (or λZ), the circle $D\theta Z$ was described; and if this is divided into hours, as we said of the others, we shall find in the tropic of ν , ND , the Sun's altitudes, and in the arc NY the Sun's depressions below the horizon. And if from A hidden lines are drawn to these, the tropic of $\textcircled{S}$, $F\mu$, will be cut toward the side μ , [the line] extended in points which, transferred from μ into the arc μE , will give the Sun's altitudes in the tropic of $\textcircled{S}$ — just as we said of the lines drawn from A to the points of the arc Oa , cutting the arc eb , and of the points of the arc eb transferred into the arc ef .

Finally, to give an example also for the northern parallels: you see, in the parallel of $\mathfrak{S}$, μ E, that a line sent out from A, through the meridian altitude of the Sun in the opposite tropic of $\mathfrak{V}$ — namely through $24^\circ 30'$ — cuts the tropic of $\mathfrak{S}$ at q; and that, the arc μ q in the same tropic being extended toward the side μ , an arc μ g was taken equal to it, and that through g the line g h was drawn parallel to A B. Moreover, that from E, the end of the parallel of $\mathfrak{S}$, the line E h was drawn perpendicular to A B (or to g h), and that from its midpoint p, with radius p E (or p h), the circle E ϕ h was described; and if this is divided into hours, as the others, there will be found in the parallel of $\mathfrak{S}$, μ E, the Sun's altitudes, and in the arc μ g the Sun's depressions below the horizon. And if from A hidden lines go out through these, the tropic of $\mathfrak{V}$, D N, will be cut toward the side N, [the line] extended in points which, transferred from N into the arc N D, will give the Sun's altitudes in the tropic of $\mathfrak{V}$ — no otherwise than as we said of the lines drawn from A, reaching the points of the arc O a and cutting the arc e b, and of the points of the arc e b transferred into the arc e f.

And the same reasoning holds in all the other parallels. But it is enough if the operation is set up either in the southern parallels alone or in the northern alone, since by one and the same labour, from the division of a single circle, the altitudes are also sought in the opposite parallel. I have chosen to explain more fully this method of finding the Sun's altitudes in the parallels described in the quadrant — drawn from the *Gnomonica* of Andreas Schoner [Andreas Schöner] — so that it might be more plainly understood by all; and since it is most pleasing and most beautiful, we shall easily demonstrate it, if we first teach by what other reasoning — different from that handed down in book 6 — the Sun's altitudes above the horizon may be investigated from the analemma, for the individual hours of each parallel.

[Margin: Finding the altitudes of the Sun above the horizon for the individual hours from the analemma, otherwise than was handed down in book 6.] [Margin: (Euclid, *Elements* I.29)]

Let ABCD be the meridian of the analemma [analemma], about the centre E; let B D be the diameter of the horizon, A C the vertical, H I the [diameter] of the equator, F G the axis of the world, K L the diameter of the northern parallel. Let the line L M be drawn parallel to B D, and K N parallel to A C; let K N be bisected at O, and from O, with radius O K (or O N), let the circle K P N Q be described. Dividing this into 24 equal hours — beginning indeed at K, for the hours from noon and midnight, but for the hours from sunrise or sunset, from the horizon B D; or, if unequal hours are in question, dividing both the arc π K q and the arc q N

π into 12 equal parts, beginning from the horizon B D — from an assumed hour d let d f be drawn parallel to B D, cutting the lines K N, K L and the circumference of the meridian at the points e, g, f. I say that D f is the altitude of the Sun for the hour d (which in the example is taken as hour 5 from noon, or 7 from midnight) in the parallel of the diameter k L.

For, a semicircle of the parallel K T L having been described from R with radius R K (or R L), let g h be erected at g perpendicular to K L. Since, then, from h, a point of the parallel, the perpendicular g h has been dropped to the diameter K L, and through g the perpendicular d f has been drawn to the diameter of the vertical A C (for since d f and B D are equidistant, and B D is perpendicular to A C, [Margin: a — (Euclid, *Elements* VI.2)] d f too will be perpendicular to A C), it will follow, from what was demonstrated in chapter 4 of book 6, that D f is the altitude of the Sun whose distance from the meridian is the arc K h. But both the semidiurnal arc K b is similar to the arc K π , and the arc K h (as we shall presently demonstrate) is similar to the arc K d. Therefore, if from the distance of the Sun from the meridian — taken on the arc K π , which stands for the semidiurnal arc, namely in the given example from the distance of 5 hours — a straight line is drawn parallel to the diameter of the horizon (or perpendicular to the diameter of the vertical), this will indicate, in the meridian, the arc D f of the Sun's altitude for the distance K h from the meridian, which in the semidiurnal arc of the Sun's parallel is similar to the distance K d in the arc K π . And that both the arcs K b, K π and k h, K d are similar, we shall show thus. Let the line O R be drawn; since the diameters K L, K N

are cut at their centres R, O — that is, proportionally — [Margin: a — (Euclid, *Elements* VI.2)] O R will be parallel to L M, and consequently also to d f, B D. [Margin: b — (Euclid, *Elements* VI.4)] Therefore it will be, as K R, the whole sine [sinus totus], is to K O, the whole sine, so K α to K a. [Margin: c — (Euclid, *Elements* VI.2)] But as K α is to α L, so is k a to a N; [Margin: b — (Euclid, *Elements* VI.4)] and therefore, by alternation, as K α is to K a, so α L to a N. Therefore it will also be, as k R, the whole sine, is to K O, the whole sine, so α L, the versed sine [sinus versus] of the seminocturnal arc b L, to a N, the versed sine of the arc π N. Therefore, by the lemma of proposition 1 of book 1, the arcs b L, π N will be similar; and consequently the remainder b K, from the semicircle K T L, will be similar to the remainder π K, from the semicircle K P N. And on this account π K q will stand for the diurnal arc, and π N q for the nocturnal, in a parallel whose declination is H K; and as many hours will be comprehended in the arcs K π , π N as in the arcs K b, b L. In no other way shall we show that the arcs K h, K d are

similar, [Margin: d — (Euclid, *Elements* VI.4)] namely because the whole sines K R, K O have the same ratio as the versed sines K g, K e.

The same reasoning holds for the other hours. For if k P, the distance from the meridian, comprises 6 hours, the line P Q S — equidistant from B D — will pass through the centres O, R, [Margin: e — (Euclid, *Elements* VI.2)] since it cuts the lines K N, K L proportionally. Hence it is clear that the quadrants k T, K P are similar. But if K Z, the distance from the meridian, exceeds a quadrant, we shall show that the arc K I is similar to the arc k Z, just as it was demonstrated that the arc K b is similar to the arc K π . Indeed, by the same reasoning, if k p, the distance from the meridian, falls below the horizon, the arc K f will be similar to the arc k p. The very same will be demonstrated in the equator and in the southern parallel.

For in the case of the equator, I i has been drawn parallel to B D, to which the perpendicular H i has been dropped — [the line] which is bisected at μ by the line B D, because the arc D I (and hence also B i) is equal to the arc B H. For from this it comes about that the line H i is cut by the line B D into two equal parts and at right angles. Afterwards, from μ , about H i, the circle H u i was described, from whose point u the line $\theta \beta$ was drawn parallel to B D, and from λ the perpendicular $\lambda \epsilon$ to H I, as far as the meridian — which stands for the equator described about H I. Whereupon it is clear that the arc H ϵ is similar to the arc H u, because the whole sines H E, H μ are proportional to the versed sines H λ , H θ .

But for the southern parallel, whose diameter is r t, the line t γ has been drawn parallel to B D, to which the perpendicular r γ has been dropped; and this being bisected at λ , the circle r 8 γ has been described about r γ , from whose point 8 — which is below the horizon, and is distant 8 hours from the meridian — the line $\phi \chi$ has been drawn parallel to B D, and from J[?] the perpendicular J[?] ω has been raised to r t, as far as the parallel of the diameter r t. Whereupon it is likewise manifest that the arc of the parallel r 3 4 $\gamma \omega$ [?] is similar to the arc r 8, [Margin: f — (Euclid, *Elements* VI.4)] because the whole sines r ξ , r λ have the same ratio as the versed sines r J[?], r ϕ .

From all which it is gathered: that D S is the altitude of the Sun in the northern [parallel] of the diameter K L, when the Sun is 6 hours distant from the meridian; likewise, that D Y is the Sun's altitude when the Sun's distance from the meridian is the arc K Z; and finally that D q is the Sun's altitude above the lower face of the horizon, when the Sun's distance from the meridian is the arc K p, exceeding a quadrant; then, that D β is the altitude of the Sun in the equator, at the distance [of] the arc H u; lastly, that D χ is the Sun's altitude above the lower face of the

horizon in the southern parallel of the diameter $r t$, when the distance from the meridian is the arc $r 8$, falling below the horizon.

Diagram. The analemma figure — the meridian ABCD about the centre E — a full circle densely packed with construction chords and lines. Legible labels: on the rim, **K, A** (with ϵ just below it), **h, T, b, S, D, q, L, Y, C**, and the hour-numbers **6** and **I** at the foot; down the left, **H, d, P, B, M, ϕ , G**; in the interior, **R, Q, O, V, N, ψ** , and **E** at the centre, together with scattered small point-labels (Greek $\theta, \delta, \alpha, \lambda, \omega, \pi, \mu, \chi$) and the numbers **3, 4, 8**. The chords realize the diameters of the horizon (B D), the vertical (A C), the equator (H I), the axis (F G), and the northern parallel (K L), with the hour-circle K P N Q and the several construction circles described in the text.

- Meridian / centre: A, B, C, D (rim), E (centre)
- Principal diameters (per text): horizon B D, vertical A C, equator H I, axis F G, parallel K L
- Rim points legible: K, A, h, T, b, S, D, q, L, Y, C
- Foot numbers: 6, I
- Left-side letters: H, d, P, B, M, ϕ , G
- Interior letters: R, Q, O, V, N, ψ , E
- Interior Greek/number labels legible: $\theta, \delta, \alpha, \lambda, \omega, \pi, \mu, \chi$; 3, 4, 8
- Several small interior point-labels worn/ambiguous [?]

These things being shown, it is clearly established that the Sun's altitudes are correctly found in the parallels of the preceding quadrant. For example, in the quadrant the parallel $\mathfrak{M}$ and $\mathfrak{X}$, Q l O a, corresponds to the meridian of the neighbouring analemma A B C D; the line A B, to the diameter of the horizon A B; the arc O l, to the arc B r of the meridian altitude; and the arc O Q — that is, its equal, O a — to the arc B 6, since this is equal to the arc of the meridian depression D t, that is, to the arc of the meridian altitude of the opposite parallel, just as in the quadrant too the arc O Q (or O a) was taken equal to the meridian altitude of the opposite parallel. Next, the line R a in the quadrant corresponds to the line t γ in the analemma, since both R a — through the end of the meridian depression in the quadrant — is drawn parallel to the horizon A B, and t γ — through the end of the meridian depression in the analemma — parallel to the horizon B D. And the line l R in the quadrant corresponds to the line r γ in the analemma, since each is drawn perpendicular from the end of the meridian altitude to the horizon. And finally the circle a l n R in the quadrant corresponds to the circle r 8 γ in the analemma. Hence, just as in the analemma the lines drawn through the hours of the circle r 8 γ , parallel to the diameter B D of the

horizon, give in the meridian A B C D the Sun's altitudes, so also in the quadrant the lines drawn through the hours of the circle a l n R, equidistant from the horizon A C, will indicate the same altitudes of the Sun in the parallel Q l O a, which stands for the meridian in the analemma. And the same reasoning holds in the other parallels of the quadrant. For all the things that we prescribed for construction in the neighbouring analemma, for the investigation of the Sun's altitudes, have been done in the same way in the individual parallels of the quadrant, so that the Sun's altitudes might be found — as is clear, if the matter is considered a little more diligently.

While I was demonstrating these things, it came into my mind that by nearly the same reasoning the construction of the analemma handed down in proposition 1 of book 1 could be demonstrated, and indeed much more easily than there. For let ABCD be the meridian of the analemma,

[Margin: *Inuentio declinationum omnium punctorum Eclipticæ in Analemmate, una cum demonstratione.* — "Finding the declinations of all the points of the ecliptic in the analemma, with the demonstration."]

let ABCD be the meridian of the analemma, about the centre E, in which BD is the diameter of the horizon, AC the vertical, HI the equator, and FG the axis of the world — the diameters of the parallels that are always visible and always hidden being, respectively, Dk and BL. Having laid off the greatest declination from the point H toward both sides, as far as M and N, draw the line MN, which will be bisected at O and cut at right angles, as was shown in Proposition 1 of Book 1. Then, with the circle MPNQ described from O about MN, and this circle divided into 12 equal parts, draw through each pair of points the lines YSλ and XRμ, ZTξ, αVϖ, equally distant from PQ; by the scholium of Euclid III.27 these will be parallel to the line QPI. But such parallels are drawn more precisely if from the point I one also lays off the greatest solar declination on each side, as far as θ and ρ; then, with θρ joined, a semicircle is described from the point 8 and divided into six equal parts, and so on, as I noted in Proposition 1 of Book 1. Finally, join the lines Mθ and Nρ, which by the same scholium will be parallel to HI itself.

I say that these lines are the diameters of the parallels — that is, their common sections with the meridian — so that the arcs Hγ, Hβ, Hδ [?], Hε [?] measure the declinations of the other parallels which are drawn through the beginnings of the signs, just as HM and HN measure the greatest declinations of the Sun. This holds in the following order: the arcs Hγ and Hβ measure the declinations of those points of the ecliptic which lie as many degrees from the beginning of Cancer ☊, toward Aries ♈ or Libra ♎, as the points X, Y, R, S are distant from the point M;

while the arcs $H\delta$ and $H\epsilon$ measure the declinations of those points of the ecliptic which lie as far from the beginning of Capricorn ϑ , toward Aries γ or Libra ζ , as the interval by which the points Z, α , T, V are distant from the point N in their own circle.

This will be demonstrated as follows. Draw the line $M\rho$, which will be the diameter of the ecliptic, the beginning of Cancer ϑ being placed at M and the beginning of Capricorn ϑ at ρ ; and let $M\rho$ cut the line $XR\mu$ — for example — at p. From p, draw pb perpendicular to $M\rho$, and conceive the semicircle Mbp , standing for the ecliptic, to be turned about $M\rho$ until it stands perpendicular to the meridian ABCD. For in that position the plane of the ecliptic is at right angles to the meridian, by Theodosius, *Sphaerica* I.15, since the meridian passes through its poles. Conceive also that through b, a point of the ecliptic, a circle is drawn equidistant from the equator and perpendicular to the meridian.

[Margin: Euclid XI.19] Now, since both the ecliptic and this parallel circle are perpendicular to the meridian, their common section through the point b will also be a straight line running to the meridian. But bp is perpendicular to the meridian, by Euclid XI, definition 4. Therefore bp is the common section of the ecliptic and of the parallel drawn through b; and so the said parallel cuts the meridian at p. [Margin: Euclid XI.16] Since, then, the meridian, being in the equator, makes with the said parallel common sections that are parallel, the line $XR\mu$ drawn through p parallel to the diameter HI of the equator will be the common section of the said parallel and the meridian — that is, the diameter of that parallel itself. Consequently the arc $H\gamma$ will be the declination of that same parallel, or of the point b of the ecliptic.

[Margin: Euclid VI.4] Now the arc Mb of the ecliptic is similar to the arc MX in the circle MPNQ, by the lemma of Proposition 1, Book 1; because ME, the whole sine of the ecliptic, is to MO, the whole sine of the circle MPNQ, as Mp , the versed sine of the arc Mb, is to $M\phi$ [?], the versed sine of the arc MX. Therefore the line $XR\mu$, drawn through the point X parallel to the diameter of the equator, gives in the meridian the arc $H\gamma$, the declination of the point b of the ecliptic, which is as many degrees distant from the point M as the point X is distant from the same point M in the circle MPNQ. And the same reasoning holds for the others, which was to be demonstrated.

From this demonstration it is clearly established that if the circle ABCD is cut into 12 equal parts — that is, into parts similar to the parts of the circle MPNQ, beginning from the point M — and any two points equally distant from M are joined by straight lines a7, b6, d5, e4, 2·3, which are perpendicular to $M\rho$ at the

points l, p, E, q, u (as will be demonstrated in the way it was shown in Proposition 1 of Book 1, that MN is cut at right angles), then the lines drawn through these points parallel to HI, such as $\beta\lambda$, $\gamma\mu$, $\delta\xi$, $\alpha\varpi$ [?], also cut off from the meridian the arcs of declination. This too we handed down in the scholium of Proposition 1, Book 1. For we demonstrated it of the point b, which is as many degrees distant from M in the circle ABCD as the point X is distant from M in the circle MPNQ; and the same ratio holds for the rest.

[Margin: *Inuentio declinationum [?] omnium arcuum diurnorum facillima, una cum demonstratione.* — "A most easy finding of all the diurnal arcs, together with its demonstration."]

Before I set the final hand to this work, I was diligently applying myself to the task of confirming, by some geometric reasoning, that practice of the scholium of Proposition 33, Book 1, by which Andreas Schöner [Andreas Schoner] investigates the radii of the diurnal arcs most briefly and most easily. After I had long and thoroughly examined this, a demonstration at last occurred to me, which I have chosen to add here in the place of an appendix. So that it may be understood more plainly, the whole construction must be repeated. From the centre A, at the interval of any line AB, let the circle BCGD be described; and in it let two arcs BC and BD be taken equal to the complement of the pole's altitude, so that, if AB is taken as the common section of the equator and the meridian — which the circle BCGD represents — the arcs BC and BD will be the declinations of two parallels, one of which, namely that drawn through C, is the greatest of those that are always visible and has a diurnal arc of 24 hours, since it lies wholly above the horizon; while the other, drawn through D, is the greatest of those that are always hidden and has a diurnal arc of 0 hours, since it lies wholly below the horizon. Now, with the line CD drawn cutting the line AB at E, the lines EC and ED will be equal and the angles at E right angles — as will be demonstrated, in the way it was demonstrated in Proposition 1, Book 1, in the analemma, that MN is cut at O into two equal parts and at right angles. Then, with a circle described from E about CD, and this circle divided into 48 equal parts, let any two points equally distant from the point C or D be joined by concealed straight lines, which will all be parallel to AB itself.

Diagram. A large analemma figure (the meridian circle ABCD about centre E) used for the re-demonstration of the analemma construction of Book 1 — finding the declinations of all points of the ecliptic. The circle carries the horizon diameter BD, the vertical AC, the equator HI, and the axis FG, with the auxiliary circle MPNQ set up on the greatest-declination chord MN, and the ecliptic

semicircle turned about the diameter Mp. Numerous chords, parallels and versed-sine constructions cross the interior.

Legible outer labels (reading the rim clockwise from upper left): Q, X, Y, M, A, K, α , then b and F on the upper right; D on the right; below, e; along the lower rim f, and (lower left) L, C, then 3, 4, G, S, B, N, Z ascending the left side. Centre: E. Interior points and small labels partly legible: 7, 6, 5; the ecliptic-diameter endpoints and Greek sorts γ , β , δ , ϵ , ρ , θ ; and the foot-points l, p, q, u near the diameter Mp.

- Many small interior point-labels and several of the peripheral Greek sorts are worn/ambiguous and could not be resolved with certainty.**

[the concealed lines] will be parallel to AB, by the scholium of Euclid III.27; and through the points at which those lines cut the line CD, let straight lines be drawn out from A as far as the circumference CBD. For these will cut off the arcs of the declinations of all the diurnal arcs, and the lines themselves will be the radii of the diurnal arcs — beginning from the radius AD, of the 0th hour, and proceeding through the radius AB, of the 12th hour, as far as the radius AC, of the 24th hour; so that the line next to AD is the radius of the diurnal arc of the 1st hour, the next of the 2nd hour, and so on. Thus any line drawn from A through some one of the points of the semidiameter DE represents the radius of that diurnal arc which falls short of 12 hours by as many hours as the twelfths — of the 12 into which the quadrant DF is divided — by which that point of the quadrant DF (the point at which the line drawn parallel to AB presents the point of that radius on the semidiameter DE) is distant from the point F. That is: just as ST is drawn equidistant from AB through the point S, four twelfths of the quadrant DF distant from the point F, so the line ATV is the radius of the diurnal arc of 8 hours, which of course falls short of 12 hours by four hours; and likewise for the rest. In the same way any line drawn from A through some one of the points of the semidiameter CE represents the radius of that diurnal arc which exceeds 12 hours by as many hours as the twelfths — of the 12 into which the quadrant CF is divided — by which that point of the quadrant CF (the point at which the line drawn parallel to AB presents the point of that radius on the semidiameter CE) is distant from the point F. That is: just as HI is drawn equidistant from AB through the point H, four twelfths of the quadrant CF distant from the point F, so the line AIK is the radius of the diurnal arc of 16 hours, exceeding 12 hours by four hours; and so of the rest. This we shall demonstrate in the following manner.

With the horizon diameter De drawn, let the axis aP cut the radius of the equator BA — produced — at right angles at the centre A, and let n be the arctic pole and

p the antarctic. Draw also KL and VX, the diameters of the parallels, of which the former is to be shown to have a diurnal arc of 16 hours and the latter of 8 hours. About KL and VX describe from O and Z the semicircles of the parallels KPL and VaX; and from the points M and Y, where the diameters of the parallels are cut by the diameter of the horizon, erect the perpendiculars MN and Yb to the diameters, which will be the common sections of the parallels and the horizon, as was demonstrated above. Thus KN will be the semidiurnal arc of the parallel KPL, and Yb the semidiurnal arc of the parallel VaX.

[Margin: Euclid VI.2] Now since DA is to AQ as DE is to EI, [Margin: Euclid V.11] and likewise DA is to AQ as IR is to RQ, it will also be that DE is to EI as IR is to RQ. But IR is to RQ, by what we demonstrated in the scholium of Euclid VI.4, as KO is to OM — because in the triangle AKM, in which AO is drawn, the line IQ is parallel to the base KM. Therefore it will also be that DE, the whole sine of the circle CFD, is to EI, the right sine of the arc FH, as KO, the whole sine of the parallel KPL, is to OM, the right sine of the arc PN; and consequently, by the lemma of Proposition 1, Book 1, the arcs FH and PN will be similar. Therefore, just as the arc FH comprises four twelfths of the quadrant CF, so the arc PN will comprise four twelfths — that is, four half-hours, or two hours — of the quadrant LP, since the quadrant is distributed into six hours, or 12 half-hours; and accordingly the semidiurnal arc KPN will contain eight hours, and the whole diurnal arc 16 hours, which was to be demonstrated.

By the same reasoning we shall show that the arc by which the semidiurnal arc of the parallel — whose declination is found in the quadrant CF, through the parallel drawn, say, through the point next to F — differs from the quadrant, contains one twelfth part, that is, one half-hour, just as the arc between F and the next point comprises one twelfth of the quadrant CF; and thus that that semidiurnal arc contains $6\frac{1}{2}$ hours, and the whole diurnal arc 13 hours; and so for the rest.

But let us demonstrate the same thing in the southern parallels. [Margin: Euclid VI.4] Since in the similar triangles DdT and Adq, dD is to dT as Ad is to dq, then by alternation [Margin: Euclid VI.2] Dd is to dA as Td is to dq. But as Dd is to dA, so is DT to TE; [Margin: Euclid V.11] therefore it will also be that Td is to dq as DT is to TE. And Td is to dq, by the scholium of Euclid VI.4, as VY is to YZ — because in the triangle AVZ, in which AY is drawn, the line Tq is parallel to the base VZ. Therefore it will also be that DT is to TE as VY is to YZ; and, by composition, that DE, the whole sine of the circle CFD, is to TE, the right sine of the arc FS, as VZ, the whole sine of the parallel VaX, is to YZ, the right sine of the arc...

Diagram. The construction of the appendix (after Andreas Schöner) for finding the radii of the diurnal arcs geometrically: the meridian circle about centre A, with the horizon diameter, the equatorial radius BA, the axis with poles n and p, and the two parallel circles KPL and VaX. From the centre A a fan of straight lines (the radii of the diurnal arcs) radiates downward to the divided arc, whose points are numbered along the lower rim.

Legible labels: top rim L, G, X; upper interior e (or c), N, M, Q, h; middle f, P, and the small Greek π and θ , then O, R, and to the right q, z, f, a, m; centre A, with the radiating hour-lines converging toward E; lower interior C, v, T, δ , D; along the lower rim the hour numbers 14, 13, 12, 11, 10, 9, with K, B and V; b on the right; and H, F at the foot. Interior small figures 6, 5, 8 partly legible.

- Several small interior labels and Greek sorts are worn/ambiguous.**

the arcs FS and ab are similar; and therefore, just as the arc FS comprises four twelfths of the quadrant DF, so the arc ab comprises four twelfths — that is, four half-hours, or two hours — of the quadrant aV; that is, the semidiurnal arc Vb will contain four hours, and the whole diurnal arc 8 hours, which was to be shown. In no other way shall we demonstrate that the arc by which the semidiurnal arc of the parallel — whose declination is found through the parallel drawn, say, through the point in the third place from the point F, set in the quadrant DF — differs from the quadrant, contains three twelfth parts, that is, $1\frac{1}{2}$ hours [?]; just as the arc between F and the third point toward D comprises three twelfths of the quadrant DF; and thus that that semidiurnal arc comprehends $7\frac{1}{2}$ hours, and the whole diurnal arc 15 hours. And the same reasoning holds for the rest. Rightly, therefore, are the declinations and the radii of the diurnal arcs investigated by that practice.

And here let us set the end to our *Gnomonics*. For the constructions of the other sundials, which are called travelling dials [horologia viatoria] and are of lesser importance, can be sought from other writers — since, if we wished to pursue everything, the volume would swell almost beyond measure. Of these we shall perhaps treat more copiously elsewhere, especially since the description of the ring-shaped dial [horologium in forma annuli] handed down by Oronce Finé [Orontius], and by certain others, is not free of error. Meanwhile I earnestly beg that the studious reader take these things in good part.

FINIS OCTAVI LIBRI. [The End of the Eighth Book.]

INDEX OF THOSE THINGS WHICH ARE CONTAINED IN THE EIGHT BOOKS OF THE GNOMONICS

[The following is the alphabetical subject-index that opens immediately after the close of Book VIII. The entries are ordered by the first word of the Latin lemma, so in English translation they no longer fall in alphabetical order; the original Latin ordering and the printed letter-headings (A, B, ...) are preserved. Page references are given exactly as printed. The terms *horologium* = "dial" (sundial) and *stylus* = "style" (gnomon) recur throughout.]

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FINIS.

FABRICA ET VSVS INSTRVMENTI ad Horologiorum descriptionem

[Title]

[Illegible: offset set-off text at head of leaf, mirror-image, not resolvable]

**FABRICA ET VSVS IN- STRVMENTI AD HOROLOGIORVM DESCRIPTIONEM
PER- OPPORTVNI.**

ACCESSIT RATIO DESCRIBENDARVM horarum à meridie & media nocte
exquisitissima, & nunquam ante hac in lucem edita.

AVCTORE **CHRISTOPHORO CLAVIO** BAMBERGENSI SOCIETATIS IESV.

[Margin: two manuscript ownership inscriptions in ink flanking "AVCTORE"
(cursive), illegible]

Translation of the title page:

THE CONSTRUCTION AND USE OF THE INSTRUMENT most well-suited to the
laying-out of Sundials [Horologia].

There is added a most exact method of describing the hours from noon and from
midnight, never before brought to light.

By the author Christoph Clavius [Christophorus Clavius] of Bamberg, of the
Society of Jesus.

PRAEFATIO. (Preface)

In past years we published our *Gnomonics* [*Gnomonica*], furnished with almost
countless geometric demonstrations, in which we taught how to describe on any
plane not only the hours of every kind, but also most of the other things that can
be known from the shadow of a gnomon — such as the signs of the Zodiac, the
lengths of the days, the vertical circles, the parallels of the horizon, the meridians
(that is, the circles of longitude), the parallels of cities [civitatum paralleli], the
ascending signs, and so on. And at the end of Book 7 we added the construction
and use of a certain instrument by which, with almost no trouble (provided the
instrument is properly built), sundials can be described on any plane whatever
and for any elevation of the pole.

But although we have seen that this labor of ours was not unwelcome to students
of mathematics, the book nonetheless — because of the multitude of matters it
contains — has grown almost beyond measure, so that on that account it might
seem less convenient to some. We have therefore decided to set out that same
instrument, reduced now to a better form, separately in its own little book
[poprio [recte: proprio] libello], together with a certain new and very easy
method, never before published, of laying out the hours from noon and from

midnight on any plane — whether it be parallel to the horizon, or stands at right angles to it, or finally neither makes right angles with it nor is parallel to it.

The first inventor of this method — which is a most excellent one — is said to be a certain Spaniard named Ioannes Ferrerius [Joan Ferrer], a man especially sharp-witted and very shrewd in matters of invention. This method does not seem to differ much from the one we set forth in the *Gnomonics* by means of a cylinder extended along the world's axis, since the whole thing depends on a certain parallelepiped passed through that same axis, whose opposite bases are squares and are described on the bases of our cylinder. When I had examined this method rather carefully — for I was not able to see a complete treatment of the matter, but only certain fragments pertaining to the describing of hours on horizontal and vertical sundials were sent to me from Spain by a most friendly man of our order — and when I had found the geometric demonstration of the whole description, I was wonderfully delighted with it, since I found that it squares with every plane.

In this place, however, we shall set forth that method for the description only of the astronomical hours — namely those that are reckoned from noon and from midnight — [Illegible: ~1 word struck through in ink] for the sake of students, employing everywhere geometric demonstrations of my own devising, so that anyone may understand that the hours are correctly laid out in this way. We hope this will not be unpleasant to those who take delight in mathematics, and in gnomonics especially. For this method has this advantage, that by it all the hours and the parts of the hours can be laid out most exactly, since in it each single hour has three points through which they are drawn — just as, in the ellipse that arises from that cylinder of ours, three points correspond to each single hour. By what means the others — those we treated in our *Gnomonics* — can be described on sundials by this route, we shall explain at another, more convenient time.

Now, so that this little book may come out more complete, we shall add from our *Gnomonics* that method of describing the hours from sunrise and sunset, which is derived from the diurnal and nocturnal arcs. If anyone is not content with this, other methods must be sought from the *Gnomonics*, where everything is set out and demonstrated fully enough. In presenting these matters we shall follow this plan: we shall make horizontal sundials, vertical ones, and those wholly declining from the vertical, complete and entire, with certain repetitions from our *Gnomonics* that are necessary to the matter. For since these sundials are commonly described, we did not wish to allow anything to be found wanting for their description in this little book; and for that reason, on the horizontal sundial

and on the one declining from the vertical, we shall also draw the arcs of the signs and of the days, together with the hours from sunrise and sunset. On the other sundials, however, since they are not so frequently used, we shall here present only what depends on this new method, and shall omit entirely the rest, which is to be taken from the *Gnomonics*. At some other time, perhaps, when leisure permits, we shall publish a fuller compendium on this subject. Finally, we shall also repeat from the last chapter of Book 7 of the *Gnomonics* that easy and pleasant method by which sundials are drawn on any plane, by means of the shadow of the style of some properly constructed sundial, to a given magnitude of the style. But now let us proceed to the matter itself.

CONSTRVCTIO INSTRVMENTI AD HOROLOGIORVM DESCRIPTIONEM APTISSIMI.

(Construction of the Instrument most fitted to the laying-out
of Sundials)

CAPVT I. (Chapter 1)

Let there be prepared, from copper or brass [orichalcum], or from any other hard material, two flat semicircles, fitted and connected by three oblong screws [cochleolae] at a moderate distance from one another, placed one beneath the other, so that — the lower one remaining fixed — the upper can be raised and lowered by means of the screws, now on this side, now on that, as the matter requires. Of this kind, in the instrument set beside here, are the semicircles ABCD and EF: the former the upper, and this the lower, somewhat smaller and — to be lighter — hollowed out for the most part. The first screw corresponds to the points E and A; another lies under the point D; and the third has its seat under the point C, and near F. [Margin: A certain Horizon of the instrument.] Let the upper semicircle be called the Horizon of the instrument, because in use it must always be parallel to the horizon.

Near the middle of the lower semicircle let there be fixed a certain oblong cylinder, or a smooth round nail [clavus], making right angles with the said semicircle. For by this the whole instrument will have to be set up in a certain hole, as we shall say below, so that the upper semicircle ABCD, by the benefit of the said three screws — raised or lowered now here, now there — may at last be found to be parallel to the horizon. But if anyone should prefer to steady the semicircle ABCD parallel to the horizon in some other way, there will be no need

of that lower semicircle with its three screws; it will be enough if the said nail, or cylinder, is fixed to the semicircle ABCD, and so on.

Next, at G, the midpoint of the diameter AD of the upper semicircle, let there be fixed a certain small column [columella] GH, firm and strong, making right angles with the semicircle, so that around it a quadrant marked with the letters ILK can be turned about, in such a way that its semidiameter IK is always parallel to the said column. The poles of this motion are: one indeed in the piece [frustum] HI, which is fixed projecting from the column, and the other in the Horizon ABCD. [Margin: A certain Meridian of the instrument.] This quadrant, divided into 90 degrees whose beginning is set at the point L, near the Horizon, represents that quadrant of the meridian circle in which the visible pole of the world is contained; for this reason, in use, it must always be so directed toward the visible pole that it lies in the plane of the meridian. Let it, moreover, be hollowed out in the middle, lest it be too heavy.

From the center M of the quadrant also let there proceed the axis of the world MN, revolvable about the center M, having at the center a small screw M by which it may be fastened, and besides a fiducial line [linea fiduciae] accurately corresponding to the center — as is usually done in other mathematical instruments — so that it can indicate the degrees of the pole's elevation. [Margin: A certain Axis of the world in the instrument.] On the same axis let there also be a small screw N, near the circumference, so that it can be fastened at the proposed elevation of the pole. This will best be achieved if, on the other side of the quadrant, a certain piece is bent back toward the center, yet not going beyond the circumference, which, when the axis is moved, always clings to the quadrant: in this piece, indeed, a more convenient place for the small screw N would perhaps be found than in the axis itself.

Let there also be made a circle OPQ, of whatever size, which is to be divided into 360 degrees, and into 24 equal hours, and the single hours into quarters of an hour. Then, the center having been pierced and the circle for the most part hollowed out so that it becomes less heavy, let it be set upon the axis just above the circumference of the quadrant, so that the twelfth hour — both of them (for we divide the said circle into twice twelve hours, after the manner of the Germans, the French, and the Spaniards) — corresponds perpendicularly to the circumference of the quadrant, so that the plane of the quadrant, produced, passes through both twelfth hours. This being so placed, let it be fastened at right angles with the axis by little pegs or screws, so that it cannot change its position. The lower twelfth hour looks toward the south [meridies], and the next toward

the left (the instrument being turned toward us) to 1 after noon, and so on.
[Margin: A certain Equator of the instrument.] Let this circle be called the Equator, because, the axis being set at the proper elevation of the pole, it lies in the plane of the equator.

Moreover, let a rectangular plane be constructed, longer on one side, RS, of such a width that, when a straight line RS is drawn parallel to both longer sides through the point T — which is the center of the circle circumscribed about the rectangle — this straight line RS may represent the radius of the Equator, and the other radii of the signs of the Zodiac may be drawn out on either side of the straight line RS from the same point T. By what method these radii are described, we shall teach in chapter 10; but to which signs the several radii are to be assigned, we shall explain in the following chapter, where we shall reveal the use of the instrument. Let the length of this same rectangular plane be a little greater than the diameter of the Equator OPQ; and let the point T be finely pierced, so that a thin thread can conveniently pass through it. [Margin: A certain Colure of the solstices, or movable Hour-circle, in the instrument.] Let this rectangular plane, which is to be called the Colure of the solstices [Colurus solstitiorum], or the movable Hour-circle — since it performs the office of all the hour-circles passing through the poles of the world, as will be clear from the use of the instrument — be set at right angles to the Equator OPQ, on the axis, by means of a certain hollowed cylinder, through whose axis, which [is fastened] to the center M of the meridian quadrant IKL, to the very [point...]

(Continuation of Caput I, "Construction of the Instrument," from the preceding pages.)

corresponds exactly [ad unguem], the rectangle itself is to be understood as so drawn that the straight line NT — cutting the straight line RS at right angles, and projected out through the middle of the cylinder — reaches precisely to the center M of the meridian quadrant [quadrans Meridiani]; that is, so that the straight line NTM represents exactly the axis of the world [axem mundi].

This rectangular plane RS is turned about the axis of the world — an axis which, for this purpose, must at its end above the Equator be smoothed and made round to match the size of the hollow cylinder fixed to that same rectangular plane. As it is turned, it should always make right angles with the Equator; and its face, on which the radii of the signs of the Zodiac are drawn, should always pass, like a diameter of the Equator, through two opposite hours, so that the radius of Aries (♈) and Libra (♎) — namely the straight line RS — remains perpetually parallel to that same Equator.

This same rectangular plane should also have, next to the circumference of the Equator, certain pieces attached and bent back toward the lower parts, with little screws at α and β , so that the said Colure of the solstices [Colurus solstitiorum] can be halted and fastened at any hour whatever. Finally, the parts of it around the radii of the signs, being superfluous, should be cut away, so that the instrument is made lighter.

Diagram. A large woodcut showing the complete dialling instrument mounted on a wooden wall-bracket. The instrument is drawn in perspective, hanging on a right-angled wooden frame that carries a toothed (ratcheted) vertical strip on the right side, along which a square sliding ring can be moved and clamped. The following labels are legible on the scan:

- Θ (top-left corner of the wooden frame).
- Y (upper-left, on the frame near the mounting): the square ring [annulus quadratus] by which the instrument is hung.
- Z (bold, on the top horizontal beam): the square wall-column ς Z.
- α (upper-left, near the top of the Colure plate): little screw seat α .
- β (lower, where the Colure plate meets the Equator ring): little screw seat β .
- O (far left of the great ring): a labelled point of the Equator circle OPQ.
- The Equator ring [$\mathcal{E}$ quator] OPQ, graduated and bearing Roman hour-numerals around its rim, "twice twelve" (...XII, XI, X ... I, II, III, IIII ... V, VI, VII, VIII ... IX, X, XI, XII...).
- P (center-lower, on the ring/quadrant assembly); a Q is expected on the ring but is not separately legible [?].
- Φ (phi, beside P, center-lower).
- Lower-left cluster on the meridian quadrant / semicircle ABCD: D, G, A, K, E (with E at the foot).
- The triangular Colure / movable hour-circle plate [horarius mobilis], rising to the upper right, bearing a small inner hour-scale (Roman numerals I, II, III, IIII, V, VI).
- R (lower-right corner of the Colure plate).
- ρ (rho, at the far right edge of the Colure plate).
- C and F (lower-right, on the arm below the ring).
- μ (mu): the compass-box [pyxis], a round dial with a magnetic needle / compass-rose, at the lower right.
- Base pedestal (bottom center): a wooden block bearing the label X (the securing screw [cochleola] X), with V nearby (the mounting hole/peg).

- Several small looped corner-marks (rendered ς in the text below) appear along the left post and the joints of the wooden columns.
- A central ornamental cartouche within the ring bears letters that appear to read "HO" (left) and "I" (right) [?] — apparently decorative rather than a construction label.

After this, let two square columns be made of hard wood, like parallelepipeds of equal thickness, joined to one another at a right angle ς [the corner marked with a small looped symbol, here rendered ς]; of these let ςV be the shorter and ςY the longer. In the shorter one, ςV , let there be a round hole near V , into which the whole instrument constructed so far can be set by means of that round peg or cylinder which — as we said — is to be fixed at the outset into the lower semicircle. On the side of the same column ςV , opposite the hole, let a little screw X be set, by which the peg (or cylinder), and hence the whole instrument, can be secured. Moreover, the distance of the hole V from the right angle ς should be somewhat greater than the radius of the upper semicircle $ABCD$ — great enough, that is, for the instrument to turn freely in that hole.

Then, at the end of the longer column ςY , let a square ring Y be fastened with screws or nails, made of brass [orichalcum] or iron, having at its top another screw ϖ . By this screw the instrument — once set into the hole V and secured — together with the whole apparatus, can be carried back and forth along another square column ςZ (five or six palms long and fixed to any wall), and there be halted and fastened by means of that square ring.

Lastly, at the end of the lower radius kL of the meridian quadrant, let a compass-box [pyxis] μ be attached, fitted with a magnetized needle, so that by its help the meridian quadrant — the instrument having first been fixed in the hole V — may be set in its proper position...

in its proper position, namely in the plane of the meridian circle. This compass-box may also be fixed, if that is judged more convenient, to one side of the quadrant, as you see done in the compass-box ϕ on the instrument.

Finally, alongside the radius IK of the quadrant, let a thread ab with a plumb-weight hang from the point a — a plumb that can swing freely in the hole b cut out for it — a straight line ab having first been drawn parallel to the radius of the quadrant (that is, to the small column GH), so that by means of this plumb-line the Horizon of the instrument $ABCD$ can be set parallel to the celestial Horizon. This is done as follows. By means of three little screws, let the Horizon of the instrument be raised or lowered, now on this side, now on that, until the freely

hanging plumb-line agrees exactly with the straight line ab and grazes the edge of the quadrant. For in this way the Horizon of the instrument ABCD will be everywhere parallel to the celestial Horizon.

[Margin: How the Horizon of the instrument is set parallel to the celestial Horizon. (*Quo pacto Horizon instruménti coelesti Horizóti statuatur aequidistans.*)]

So much for the construction of the instrument. Now let us come to its use, from which the directions we have given about its making will become clearer. And I do not doubt that, once its use is rightly understood, anyone may, by his own effort, arrange some parts of the instrument in another way, and perhaps improve them.

Usus praecedentis instrumenti — The Use of the Preceding Instrument, in describing sundials on any plane whatever

Caput II

On the wall or plane on which the sundial is to be described — whether it stands at right angles to the Horizon or not — let that long wooden column ζZ of the instrument described above be fixed a little above the place of the sundial, so that, to the judgment of the senses, it is parallel to the Horizon; and let it be fastened so firmly that it cannot be shifted from its place.

[Margin: How the instrument is to be positioned for describing sundials. (*Quo modo instrumentum in descriptione horologiorum locandum sit.*)]

Onto this column, by means of the square ring Y, let the whole instrument be set, in such a way that the point T stands off from the wall by roughly the length you want the style [stylus] to have. Then, having fastened the ring by the screw ϖ, and the instrument by the screw X, let the fiducial line of the axis [linea fiduciae axis] be set at the degree of the pole's elevation for the place where the sundial is being built, and let it be fixed with the screw N, so that it cannot be pulled away from that degree.

Lastly, let the Horizon of the instrument ABCD be set parallel to the celestial Horizon by means of the screws and the plumb-line ab, as was taught at the end of the preceding chapter. And let the meridian quadrant be moved this way and that until it lies in the plane of the meridian circle and the axis MN looks toward the visible pole of the world. This will happen only when the magnetized needle lies directly over the meridian line at the bottom of the compass-box — or at least (which I myself prefer, since a needle of this kind almost never points exactly to

the pole, but deflects from the true meridian line now more, now less, so that I judge it not much to be trusted; at Rome I have more than once found the point of such a needle deflecting from the meridian line toward the east by about 6 degrees) — when the meridian quadrant is parallel to the meridian line found on the Horizon of the instrument, as we taught in the scholium to Proposition 23 of Book 1 of the *Gnomonics*.

Once the instrument has been set in this position, the Equator of the instrument OPQ will lie in the plane of the celestial Equator; the Colure of the solstices — that is, the movable hour-circle RS — carried to each successive hour, will take the position of each of the hour-circles that pass through the poles of the world; and finally the radii of the signs of the Zodiac will correspond exactly to the diameters of the Ecliptic (insofar as these are the common sections of the Ecliptic and the Meridian), when the beginnings of the signs are placed under the Meridian.

Therefore the plane of the meridian quadrant, if it is then imagined as extended all the way to the wall, will produce on the wall the meridian line — that is, the common section of the Meridian with the wall. So too the plane of the Colure of the solstices, or of the movable hour-circle, set at any hour, if it is understood to run out to the wall, will make on the wall the hour-line of that hour — that is, the common section of that hour-circle with the wall. Finally the radii of the signs, if extended to the wall, will mark on the wall the points onto which the shadow of the gnomon (or style) — of whose placing we shall treat shortly — is cast, when the Sun is on that hour-circle and at the beginnings of the signs. In this way the whole sundial will have been constructed, if the movable hour-circle is understood to be carried to each hour, and so on.

But since these planes cannot actually be extended, we shall find their common sections with the wall — and the points through which the arcs of the signs of the Zodiac are to be drawn — with the help of a thread, in this way. First of all, let the movable hour-circle be set to the 12th hour of the Equator, and fixed in that position. Then from the point T let a very fine thread be sent out all the way to the wall, grazing freely the plane of the hour-circle itself, so that the extended plane does not depart from the thread but always lies along it. For if that thread is applied to each of the radii, and the points are marked on the wall, a meridian line is to be drawn through those points, indicating the 12th hour from noon or from midnight.

[Margin: The description of the hours from noon and from midnight. The meridian line. (*Descriptio horarum à mer. & med. noct. Linea meridiana.*)]

The highest point will correspond to the sign Capricorn (♑); the next to Aquarius (♒) and Sagittarius (♐); the one after that to Pisces (♓) and Scorpio (♏); the next to this to Aries (♈) and Libra (♎); another following to Taurus (♉) and Virgo (♍); the next to Gemini (♊) and Leo (♌); and the lowest to Cancer (♋).

Then, moving the movable hour-circle and carrying it to the 1st hour — which follows the 12th hour on the left, at the lowest part of the Equator, that is, toward the east — and fixing it there, we shall find (with the same thread grazing the plane of the hour-circle freely and lying along each radius) the points of those same signs on the wall, through which the line of the 1st hour from noon is to be drawn. And the same method applies to all the other hours that can fall on the wall — that is, those whose points can be marked on the wall by that thread grazing the plane of the hour-circle.

And if the corresponding points of the signs are suitably joined by curved lines, there will also be described the arcs of the signs which the tip of the shadow traverses when the Sun is at the beginnings of the signs. Yet all the points of Aries (♈) and Libra (♎) must — if no error has been made in the work — fall on a straight line, namely on the common section of the Equator with the plane of the sundial, as we demonstrated in the *Gnomonics*. Moreover, we shall find the points of these same arcs of the signs, and so shall trace the arcs themselves, without any regard to the hours.

[Margin: How the arcs of the signs are drawn, without regard to the hours. (*Quo pacto arcus signorum ducantur, nulla habita ratione horarum.*)]

For if we carry the movable hour-circle around to various places on the Equator only slightly apart from one another, and in each position mark on the wall with the thread the points for the several signs, the points of all the signs will be found, even though no account has been taken of any hour. And the more numerous the intervals on the Equator at which the movable hour-circle is halted, the more numerous the points that will be found on the wall...

that will be found on the wall for each sign, and the less they will be apart from one another; and consequently the arc of the signs itself will be drawn more precisely.

[Margin: The horizontal line. (*Horizontalis linea.*)]

The horizontal line is drawn as follows. Let the thread be stretched to the wall, and raised and lowered until it is parallel to the Horizon — which you will achieve by means of a level [libella], a plumb-line, and a ruler applied to the

thread itself. For when the thread holds this position, if a point is marked on the wall, the straight line drawn through that point parallel to the Horizon will be the horizontal line; and on walls that stand at right angles to the Horizon, this line will be perpendicular to the meridian line.

We shall draw it more surely in this way. With the movable hour-circle set at the hour at which the Sun, being at the beginning of Capricorn (♋), rises or sets, let the thread be applied to the radius of Capricorn (♋), and let the point be marked on the wall. For a straight line drawn through this point parallel to the Horizon will give the horizontal line. You will obtain that hour of rising or setting if you count the semidiurnal arc of Capricorn (♋) — in the *Sphere* we set out the semidiurnal arcs for various elevations of the pole — from the lower 12th hour of the Equator toward each side. For the count ended on the western side will give the hour of rising, and on the eastern side the hour of setting. If, then, both hours can be marked on the wall with the thread, you will have two points through which the horizontal line is to be drawn; if only one of the two, a single point.

Diagram. The same dialling-instrument woodcut as (fig-p580-1), printed again from the identical block. The same labels are legible — Θ , Y, Z, α , β , O, the graduated Equator ring OPQ with its twice-twelve Roman hour-numerals, P, Φ , the meridian quadrant labels D, G, A, K, E, the triangular Colure plate with its inner scale (I, II, III, IIII, V, VI) and corner labels R and ρ , C, F, the compass-box μ , and the base block with X and V. See fig-p580-1 for the full description.

Once the horizontal line has been drawn, let all the lines lying above it be erased, as superfluous, since in that part of the sundial the shadow of the gnomon can never fall, because the earth lies between.

[Margin: How the style is to be placed. (*Stylus quo pacto locandus.*)]

We shall place the style, or gnomon, the indicator of the hours, in this way. From the point T of the movable hour-circle, let three points be marked on the wall with the thread, keeping always the same length of thread. For if the center of these three points is found by the method of the scholium to Proposition 5 of Book 4 of Euclid, the style is to be fixed at right angles in that center; and its length will be given by the length of thread intercepted between that center and the point T, which represents the center of the world.

Or we shall proceed in this way. From a high place let a plumb-line be let down, ending in a freely hanging point, until its extreme point touches the point T. For then, with the instrument removed, if a style of any size is fixed at any place on the wall in such a way that its tip coincides with the extreme point of the said

plumb-line, the tip of this style's shadow will indicate the hours just as well, even though it is not perpendicular to the wall.

Now, in order that the hours of sunrise and sunset may be laid out, the hours themselves must first be marked in ink — or by some other means — upon the Equator, in the following way. From the lower 12th hour let the semidiurnal arc of Capricorn (♑), taken from our *Sphere* or from some other source, be counted off in both directions along the Equator. For the end of the count toward the west, that is, to the right, will give the 24th hour reckoned from sunrise; and to the left, that is, toward the east, the same count gives the 24th hour reckoned from sunset. If, then, starting from the earlier point — the one we assigned to sunrise — you transfer along the circumference of the Equator the hour-spaces taken with the compass upon that same Equator, advancing toward the lower 12th hour, you will obtain the hours 1, 2, 3, 4, 5, and so on, reckoned from sunrise. But if you transfer those same hour-spaces from the later point — the one we ascribed to sunset — toward the same lower 12th hour, up to the hour of rising, you will obtain the hours 23, 22, 21, 20, 19, and so on, reckoned from sunset. You may also mark out these hours without a compass, if from the said points of rising and setting you count off 15 degrees for each single hour toward the lower 12th hour.

Once the hours from rising and setting have been marked on the Equator in this way, then, if we bring the movable hour-circle to each of them in turn, we shall find by means of the thread the points of those same hours upon the wall for the tropic of Capricorn (♑) — the points through which the tropic of Capricorn is to be drawn. In just the same way you will find the points of those same hours for the other parallels upon the wall, provided you have first marked out their hours on the Equator by means of the semidiurnal arcs, as we described for the tropic of Capricorn. It will be enough to determine the points of the hours for Aries (♈) and Libra (♎), and for the two tropics, Capricorn (♑) and Cancer (♋). For the straight lines connecting the three corresponding points of any given hour in Capricorn, Aries, and Cancer will be the hour-lines from rising and setting. But when in the tropic of Cancer (♋) the points of some hours are lacking, then the two corresponding points in the parallel of Capricorn (♑) and of Aries (♈), or of Libra (♎), must be connected. And if the points of certain hours are not present even in the parallel of Aries (♈) or Libra (♎), their points must be sought in the parallel of Capricorn (♑) or of Aquarius (♒). The line of the 24th hour will here give the horizontal line. As for the arcs of all the signs, if you should want them, they are to be drawn here just as we described in the sundial of the hours from noon and from midnight.

[Margin: The laying-out of the hours from rising and setting.]

Finally, the unequal [seasonal] hours are drawn by the same method, if you have first impressed some mark for them on the Equator. You will do this if you divide into six equal parts the semidiurnal arc of each parallel (though it is enough to take the three parallels: Capricorn (♋); Aries (♈) or Libra (♎); and Cancer (♋) — except when the point of some hour in Aries or Libra does not fall on the wall, for then that hour must be taken in the parallel of Capricorn or of Aquarius).

[Margin: The laying-out of the unequal hours.]

A horizontal sundial can be laid out with the same instrument in no other way, provided it is to be a large one. For small ones cannot be described by the instrument; since, once the instrument is fixed (so that its horizon ABCD is parallel to the celestial horizon), the style must be as large as the distance of the point T — where the rays of the signs meet on the movable hour-circle — from the plane of the horizontal sundial.

[Margin: The laying-out of a large horizontal sundial by the same instrument.]

CONSTRVCTIO FVNDAMENTI HOROLOGIORVM, ID EST, FIGVRAE

(The Construction of the Foundation of Sundials — that is, of a General Figure, most convenient for laying out the hours from noon and from midnight on any plane and for any elevation of the pole.)

CAPVT III.

Let there be made a rectangle longer on one side, ABCD, of any size; and from the longer side BC let a straight line BE be cut off equal to the shorter side AB. Then from E, at any interval, let an arc of a circle FG be described toward BA; and with the compass unchanged let the arc FG be marked off, which will be a sixth part of the circle, as is established from the corollary to Proposition 15 of Book IV of Euclid. Next, from center E, let a straight line be drawn to A, cutting the arc FG at H; the arc FH will be an eighth part of the circle, that is, half a quadrant. For since the angle HEF at the center is equal to the angle BAE [Margin: a. *Elements* I.5] — because the straight lines BE, BA are equal — each of them will be a half-right angle, and therefore an eighth part of four right angles. Of whatever parts, then, the whole circumference of the circle is 24, of such parts the arc FG is 4, the arc FH is 3, and the arc GH takes up 1. Therefore, if from the arc FH three arcs HI, Ik, kF are cut off equal to the arc GH, the arc FH will be divided into three equal

hours. Now, drawing from center E through the points I, k [Margin: b. *Elements* I.33] hidden straight lines cutting the side AB at L and M, let there be taken on the opposite side CD the straight lines CO, CN equal to BM, BL; and let the straight lines LN, MO be drawn, which will be parallel to the sides AD, CB. The straight line BC, then, will be assigned to the 6th and 12th hours from noon or from midnight; the straight line MO to the 1st and 11th, and also the 5th and 7th; the straight line LN to the 2nd and 10th, and also the 4th and 8th; and finally the line AD to the 3rd and 9th — in such a way that BC is the 12th hour from midnight, that is, the hour of noon; MO is 1 from noon; LN, 2; AD, 3; then again LN, 4; MO, 5; BC, 6; and again MO, 7; LN, 8; AD, 9; and once more LN, 10; MO, 11; and BC, 12, namely the hour of midnight, as we shall presently demonstrate. For half-hours, quarter-hours, and other subdivisions, the arcs HI, Ik, kF are to be divided into two and into four equal parts, and so on. In the figure the said arcs are divided in two, and parallels are drawn for the half-hours.

With this figure, then, we shall draw with the greatest accuracy the hour-lines from noon and from midnight on any plane and for any given elevation of the pole, as will be clear from what follows — so much so that it would be worth the effort, with the utmost care, to construct in some hard material two or three such rectangular figures, so that we might use them in laying out sundials as general instruments: larger ones for larger sundials, smaller ones for smaller. For large sundials are more easily drawn from a larger figure, and small ones more conveniently from a smaller. And we may not unfittingly call this figure the Foundation of sundials.

[Margin: The composition of the Foundation of sundials.]

[Margin: The Foundation of sundials.]

But let us demonstrate the origin of this figure, and why the hours are assigned to those parallels in the stated order. Let there be understood, then, a rectangular parallelepiped whose opposite bases are squares and whose sides are equal to twice AB, set about the axis of the world in such a way that its axis (I call the axis the straight line joining the centers of the opposite squares) coincides with the axis of the world; and let two of its opposite rectangular planes — one upper and the other lower — be parallel to the circle of the 6th hour from noon and from midnight, while the other two — one eastern and one western —

[Margin: The demonstration of the Foundation of sundials.]

occidental — are each equidistant from the meridian circle. Such a parallelepiped, inscribed in a cylinder extended along the axis of the world, by

means of which we constructed the sundials in our *Gnomonics*, is described once the squares have first been described in the bases of the cylinder. The common section of this parallelepiped and of the Equator will be a square, that is, a figure equal and similar to the [bounding] planes, or opposite bases, from the corollary to Proposition 25 of Book XI of Euclid, since the Equator is equidistant from the bases. Let this square be RADS, having half of the side AR — namely the straight line AB — equal to the straight line AB in the figure of the Foundation of sundials; and through its center E let the axis of the world pass, standing at right angles upon it. Let the straight line PQ be the common section of that same square, or of the Equator, and of the meridian circle; and let BC be the common section of that same [square], or of the Equator, and of the circle of the 6th hour, and also of the Horizon and of the prime Vertical circle. For these four circles have one and the same common section, namely the axis of the meridian, to which they are perpendicular, since they all pass through the poles of the meridian circle, as is evident. Now PQ and BC will cut one another at the center E at right angles. [Margin: a. *Elements* XI.19] For since both the Equator and the circle of the 6th hour are perpendicular to the meridian, their common section BC will also be perpendicular to it, and therefore, by Definition 3 of Book XI of Euclid, also to the straight line PQ lying in the meridian. [Margin: b. *Elements* XI.16] And to these two straight lines PQ, BC the sides of the square will be parallel. For RS, BC, AD are the common sections of parallel planes (namely, of the circle of the 6th hour drawn through BC; of the upper rectangular plane in the parallelepiped drawn through RS; and finally of the lower plane in the same parallelepiped passing through AD),

Diagram. The upper figure: a rectangle developed (unrolled) from the parallelepiped, carrying the hour-lines and diagonal projection-lines of the Foundation figure, with an hour-number ladder appended along its right edge.

- Top edge, left → right: B, F, E, α [?] (a small script letter, possibly italic *a*), v [?], T, d [?]
- Left edge, top → bottom: B, M, L, A
- Interior (left region): K, I, H, with a curved arc descending to G; dotted radiating lines from M, L, K, I toward H and G
- Interior (center-right region): X, X (two, stacked), R, Y; a small *f* near the top; a further label near the lower right, Γ/ρ [?]
- Bottom edge, left → right: A, Z, P, C, Q, D
- Right edge (appended number-ladder), edge letters top → bottom: O, O, N, D

- Number-ladder pairs (left column | right column), top → bottom: 12 | 6, 6 | 12, 1 | 7 [?], 5 [?] | 11, 2 | 8, 4 | 10, 3 | 9, 3 | 9 [?]

Diagram. The lower figure: a square with an inscribed circle (the Equator) divided into 24 equal hours, with straight lines radiating from the circle to the sides of the square (the hour-circles' common sections). Center E; the two diameters PQ (vertical) and BC (horizontal) cross at E.

- Corners: R (top-left), S (top-right), A (bottom-left), D (bottom-right)
- Top edge, center: P
- Bottom edge, center: Q
- Left edge, top → bottom: B, M, L
- Right edge, top → bottom: C, O, N
- Center: E
- Letters set on the circle's left arc, top → bottom: F (at hour 6), K (at 7), I (at 8), H (at 9), G (near 10–11)
- Hour-numerals around the circle: two sequences 1...12; upper arc reads 1, 12, 11, 10, 9, 8; right arc descending 7, 6, 5, 4, 3, 2; lower 1, 12; left arc ascending 3, 4, 5, 6, 7, 8, 9, 10, 11 [several small numerals uncertain]

made from the plane of the Equator; while RA, PQ, SD are the common sections of parallel planes (namely, of the meridian circle passing through PQ, and of both the left-hand and the right-hand rectangular planes in the parallelepiped, of which the former, the western, is drawn through RA, and the latter, the eastern, through SD — and we have supposed both to be equidistant from the meridian circle), made from that same Equator. From which it follows [Margin: c. *Elements* I.29] that the angles at P, B, Q, C are likewise right angles.

Now, an Equator-circle of any size having been described from center E in the plane of the Equator, and divided into 24 equal hours, beginning from PQ, the common section of the meridian and the Equator, and straight lines having been drawn from E through the points of division as far as the sides of the square, these will be the common sections of the Equator (that is, of the said circle) and of the hour-circles through the poles of the world and through the axis, and therefore also through the center

E — of the [lines] drawn — for the reason that the Equator and the aforesaid circle, described from the same center E, are cut into similar parts by the said hour-circles, as we demonstrated in the *Sphere* at the end of Chapter 1. Now the lines of the 3rd and 9th hours will fall upon the corners of the square, on account of the half-right angles REP, AEB, DEQ, SEC at the center E, which the semi-

quadrants of the circle subtend, each of which takes up three hours. Moreover, these same hour-circles will make, in the rectangular planes of the parallelepiped, common sections parallel both to one another and to the axis of the world, as is clear from what was demonstrated in Propositions 18 and 22 of Book I of our *Gnomonics*; and all of these will pass through the points of the square at which the common sections of the hour-circles and of the Equator fall.

Since, then, the eight lines of the square — RB, BA, AQ, QD, DC, CS, SP, PR — are cut in like manner by the hour-circles, it comes about that any one of them stands for all. In this way the figure of the Foundation of sundials contains the straight line AB, divided at the points L, M just as the line AB is divided in the square, as is manifest from the description. For both the arc FG in the figure and the arc FG in the square, each comprising four hours, is a sixth part of the circle; and FH, an eighth part, comprises three hours. And the parallels AD, LN, MO, BC express those common sections which we said were made in the rectangular planes of the parallelepiped by the hour-circles. Therefore, if the straight line AB is set upon the straight line RP, with the point A placed at R and the point B at P, the said parallels will represent the hours 12, 1, 2, 3. But upon the straight line RB, with the point A placed at R and the point B at B, [they will represent] the hours 3, 4, 5, 6. Upon the straight line BA, with the point B placed at B and the point A at A, the hours 6, 7, 8, 9. And upon the straight line AQ, with the point A placed at A and B at Q, the hours 9, 10, 11, 12 — all of which hours are reckoned from noon. Then, upon the other four straight lines QD, DC, CS, SP, the same straight line AB, placed in the aforesaid order, will represent by the same parallels the same hours reckoned from midnight: so that the rectangle ABCD, with its parallels, reproduces half of each rectangular plane in the parallelepiped, together with the sections made by the hour-circles — yet in this order, that the straight line BC always coincides with the straight line drawn through the middle of the rectangular plane, and the straight line AD with the outermost side of that same rectangular plane. This is the origin and source of the figure that we have chosen to call the Foundation of sundials.

HOROLOGIA HORIZONTALIA, ATQVE VERTICALIA HORARVM à meridie, & media nocte.

(Horizontal and Vertical Sundials of the Hours from noon and from midnight.)

CAPVT IIII.

In the figure of the Foundation of sundials, from any point of the side AD or BC — say, from the point P of the side AD — let an arc of a circle QRS be described at any interval, on which, the altitude of the pole QR being marked out for the horizontal sundial and its complement QS for the vertical, let the straight lines PRT, PSV be drawn, cutting the parallels MO, LN at the points X, Y. In the figure the altitude of the pole QR takes up 42 degrees, about what it is at Rome, and its complement QS, 48 degrees. But if at any time the straight line PT or PV does not cut the opposite side BC, then the side BC must be produced together with the parallels MO, LN, etc., until it can be cut — which is likewise to be understood whenever other lines, which we shall use in what follows, are to be transferred to the Foundation of sundials but do not cut the opposite side. The straight line AB is the common section of the plane of the horizontal and vertical sundial and of the circle of the 6th hour, which is the upper and lower plane the parallelepiped represents. PT is the common section of the plane of the horizontal sundial and of the eastern or western plane of the parallelepiped. Finally PV is the common section of the vertical sundial and of that same eastern or western plane of the parallelepiped — all of which will be demonstrated below.

[Margin: The finding of the sections of the plane of the horizontal and vertical sundial, and of the parallelepiped extended along the axis of the world.]

Next, in the plane of the sundial let there be given the place of the style, A, and its length AB. Through A let a straight line CD be drawn in any direction for the meridian line — though, when the plane of the sundial is fixed and the meridian line has been found in it, a line CD parallel to that must be drawn through A for the meridian line, which in the vertical sundial is perpendicular to the horizon. Let AB, equal to the style, be drawn perpendicular to it, and at B, upward, let there be set up — for the horizontal sundial — the angle ABC equal to the angle QPS, the complement of the pole's altitude in the Foundation of sundials; and downward, the angle ABD equal to the angle QPR, the altitude of the pole. But for the vertical sundial let the angle ABC upward be [equal to] the altitude of the pole, and the angle ABD downward [equal to] the complement of the pole's altitude. In each sundial let the straight lines BC, BD cut the meridian line at the points C, D. C will be the center of the sundial, and CB the axis of the world; and, drawing perpendiculars ECE, FDG through C, D to CD, the former will be the line of the 6th hour and the latter the equinoctial line — all of which we demonstrated in the scholia to Propositions 1 and 13 of Book II of our *Gnomonics*.

[Margin: The construction of the horizontal and vertical sundial.]

After this, from the line ECE of the 6th hour, in each sundial, let there be cut off on both sides a straight line CE equal to the straight line AB, or CD, of the Foundation of sundials. Likewise, from the meridian line of the horizontal sundial equal to the straight line PT of that same Foundation, and from the meridian line of the vertical sundial equal to the straight line PV of the same Foundation, let there be cut off, both upward and downward, a straight line CH equal [to them]. Then, in each sundial, from the points E, upward and downward, let two small arcs be described at the interval of the straight line CH cut off from the meridian; and from the points H, on both sides, [let arcs be described] at the interval of the straight line CE cut off from the line of the 6th hour, which cut the former at I. Let two straight lines IEI and two IHI be joined, which will pass entirely through the points E, H, and will form a rectangular parallelogram, as is evident and can easily be proved. For the four quadrilaterals EH, since by construction they have their opposite sides equal, are parallelograms, from the scholium to Proposition 34 of Book I of Euclid; and therefore, since the four angles at C are right, [Margin: a. *Elements* I.34] the four opposite ones at I will also be right, and so on.

[Margin: The laying-out of the hours from the Foundation of sundials.]

Finally, let the intervals of the straight line AB of the Foundation of sundials, intercepted between the point B and the parallels MO, LN, etc., be transferred in each sundial from the points H, on both sides, onto the straight lines HI; and in the horizontal sundial, from the points I toward the points E, onto the straight lines IE, let there be transferred the intervals of the straight line PT of the Foundation of sundials, intercepted between the point P and the parallels MO, LN, etc.; while in the vertical sundial

and in the vertical sundial let the same be done with the intervals of the straight line PV intercepted between P and the said parallels. For the straight lines drawn through the center C of the sundial and through the points on the sides of the parallelogram IEIHIEIH — each of which will pass through three points — will give the hours from noon and from midnight, in the following order. In each sundial the lower CH looks to the 12th hour from noon, and the upper to the 12th hour of midnight. In the horizontal sundial the line next after the lower CH, toward the left, shows the 1st hour from noon, and the one following it the 2nd hour from noon, and so on in sequence. In the vertical sundial, by contrast, the line following the lower line CH, toward the right, answers to the 1st hour from noon, and the next after it to the 2nd hour from noon, and so on — as appears in

the figures, and as the intervals transferred from the Foundation of sundials indicate.

It would indeed be enough, for laying out either sundial, to draw only the lower, or upper, or left, or right half of the rectangle IEIHIEIH — namely either of the two rectangles EIHIECE, or either of the two HIEIHCH. But then the individual hours would be drawn through only two points each; yet, when produced beyond the center C, they would complete the whole sundial.

The half-hours and the quarter-hours are drawn in the same way, if the spaces cut off in the Foundation from the parallels of the half- and quarter-hours are transferred onto the sides of the parallelogram IEIHIEIH in the sundial, and so on.

Diagram. A rectangular sundial figure in which the hour-lines radiate from a central point C. Through C run the meridian line H–C–H (vertical) and the line of the 6th hour E–C–E (horizontal); the equinoctial line F–D–G runs across just below C. Near the center is the style, shown as the small triangle with points A and B (with an arrow between A and B) and D just below. The rectangular rim is inscribed with the parallelogram IEIHIEIH, whose four corners are marked I, with H at the top and bottom mid-points, E and F on the left side, and E and G on the right side. The hours are labelled around the rim as follows:

- Top edge (left → right): 9, 10, 11, 12, 1, 2, 3
- Right edge (top → bottom): 4, 5, 6 (at point E), 7 (at point G), 8
- Bottom edge (left → right): 3, 2, 1, 12, 11, 10, 9
- Left edge (top → bottom): 9, 8, 7, 6 (at point E), 5, 4 (point F lies on the left side just below the 6-line E)
- Interior letter-points: C (center); A, B, D (the style); E, E (ends of the 6th-hour line E–C–E); F, G (ends of the equinoctial line F–D–G); H, H (ends of the meridian line H–C–H); I (the four corners of the parallelogram IEIHIEIH)

[Margin: Demonstration of the preceding description.] [Margin: a. *Elements* XI.16]
[Margin: b. *Elements* XI.19] [Margin: c. *Elements* I.28]

Demonstration. The demonstration of this construction depends on the parallelepiped that we described in Chapter 3. For since the sections made in the plane of the sundial — of both the horizontal and the vertical — by the opposite and parallel rectangular faces of the parallelepiped are [a] parallel [to one another], the common section of the plane of either sundial and of the parallelepiped will be a parallelogram; and this I say is IEIHIEIH. For since the

plane of either sundial, and likewise the rectangle of the parallelepiped — both the upper and the lower — stand at right angles to the meridian, the common section of the plane of either sundial and of either said rectangle will also [b] be at right angles to the same meridian, and therefore also perpendicular to the meridian line HCH lying in the meridian; and hence [c] parallel to the straight line AB of the Foundation of sundials, and consequently double it. For AB alone reaches only to the middle of the said rectangle in the parallelepiped, as we said in Chapter 3; so that AB may rightly be called the common section of either sundial and of the circle of the 6th hour, which the plane of the Foundation — that is, the upper or lower plane of the parallelepiped — represents. Therefore, in the horizontal sundial the straight line IHI, inclining toward the south, and...

the straight line AB, double [the line] of the Foundation of sundials — such as is the line that cuts the line of the 12th hour of midnight. In the vertical sundial, on the other hand, the upper straight line IHI, double that same AB of the Foundation of sundials, will be the common section of the plane of the sundial and of the upper rectangle in the parallelepiped. And for this reason the two straight lines IEI, equidistant from the meridian itself, will be the common sections of the plane of either sundial with the rectangle of the parallelepiped, both eastern and western. For the common sections of the meridian and of the rectangle — both eastern and western — of that parallelepiped, made by the plane of the sundial (since these three planes are parallel), are [a] parallel to one another.

I now assert that these common sections IEI ought, in the horizontal sundial, to be double the straight line PT of the Foundation of sundials, and in the vertical double the straight line PV — just as they were taken in each sundial. For since the horizon makes with the axis of the world, and hence with any straight line of the parallelepiped parallel to the axis, the angle of the pole's altitude, while the vertical circle makes the angle of the complement of the pole's altitude, the straight line PT — making with AD (which in the parallelepiped is parallel to the axis) in the Foundation of sundials the angle of the pole's altitude TPD, namely equal to the one the horizon makes with the same AD — will be parallel and equal to the common section of the half of the eastern or western rectangle of the parallelepiped...

[Margin: a. *Elements* XI.16]

Diagram. The same rectangular sundial construction as on the facing page, reprinted here (the figure appears on both facing pages 10 and 11). The hour-lines again radiate from the central point C, with the meridian line H–C–H vertical, the

6th-hour line E–C–E horizontal, and the equinoctial line F–D–G just below C; the style is the small triangle A–B (arrow between A and B) with D below. The rim carries the parallelogram IEIHIEIH (corners I; H at top and bottom mid-points; E, F on the left; E, G on the right). Rim hour-labels, matching the facing figure:

- Top edge (left → right): 2, 1, 12, 11, 10, 9 [the labels visible across the top run ...2, 1, 12, 11, 10, 9]
- Right edge (top → bottom): 8, 7, 6 (at point E), 5 (at point G), 4
- Bottom edge (left → right): 9, 10, 11, 12, 1, 2, 3
- Left edge (top → bottom): 3, 4, 5, 6 (at point E), 7, 8, 9
- Interior letter-points: C (center); A, B, D (style); E, E (6th-hour line); F, G (equinoctial); H, H (meridian); I (four corners of the parallelogram)

piped, and of the plane of the horizontal sundial — that is, if the rectangle AC of the Foundation is conceived in the parallelepiped as being the eastern, or western, one joined to the upper by the straight line AD, but with the point D inclining toward the north, and with the straight line AD set parallel to the axis: so that PT may rightly be called the common section of the horizontal sundial and of the eastern or western plane of the parallelepiped. Therefore each IEI, being double PT, will in the horizontal sundial be equal to that whole section; and accordingly the lower IHI will be the common section of the same plane of the sundial and of the lower rectangle of the parallelepiped.

We shall show in just the same way that, in the vertical sundial, each straight line IEI is equal to the whole section of the plane of the sundial and of the eastern or western rectangle of the parallelepiped, while the lower straight line IHI is the common section of the same plane of the sundial and of the lower rectangle of the parallelepiped — that is, if the rectangle AC of the Foundation is understood in the parallelepiped to be the eastern or western one joined to the upper by the straight line AD, but with the point D inclining toward the south, and with the straight line AD set parallel to the axis: so that PV may rightly be called the common section of the vertical sundial and of the eastern or western plane of the parallelepiped. Since, therefore, the straight lines IHI, IEI, in the sundials...

are cut just as the straight lines AB, PT, or PV are in the Foundation of sundials, that is, in the parallelepiped, the hour-circles will pass through the points where the straight lines IHI, IEI are cut, in the same way as we taught them to pass through the points where the straight lines AB, PT, and PV are cut, namely through the parallels of the Foundation. And so, since they also pass through C, the center of the sundial, as we showed in the *Gnomonics*, the hour-lines have

rightly been drawn by us in each sundial through C and through the points where the straight lines IHI, IEI are cut.

[Margin: Another construction of the horizontal and vertical sundial.]

Furthermore, we shall construct both sundials by one and the same operation, even if the Foundation of sundials has not been separately built up, in this way. Draw two straight lines AB, CD cutting each other at E at right angles; at any point A — whether to the left or to the right — set up the angle of the pole's altitude EAC. From E, perpendicular to AC, draw EF; on the straight line AC, whether upward or downward, cut off FG equal to it, and join the straight line EG. Then, having described from G an arc of a circle of any size and radius, divide the portion of it lying between the straight lines GE, GC into three equal parts HL, LM, MI — a division that is very easy. For if, with the same opening of the compasses by which the arc of the circle was described, the arc IK is cut off, then the arc HK, beyond the straight line GF, will be a third part of the arc HI. For since the arc IK is a sixth part of the circle — by the corollary to [a] Proposition 15 of Book IV of Euclid — and the arc IH an eighth part, because the angle G at the center is half a right angle (for since the angle F is right, and the angles FGE, FEG are equal on account of the equal straight lines FE, FG, the angle G will be half a right angle), the arc IK will contain four parts, and IH three, and therefore HK one, of the kind of which the whole circumference of the circle contains 24.

Moreover, drawing from the center G, through the points of division L, M, hidden straight lines cutting EF at N, O, let there be drawn through N, O lines parallel to AC, cutting AE, CE at R, S, T, V; these parallels will be drawn without much labor as follows. At the interval of the straight line EF, or FG, describe from E and G two arcs intersecting each other at Z; the straight line GZ drawn [to it] will be parallel to EF, because, once the straight line EZ is drawn, FZ becomes a parallelogram — by the scholium to Proposition 34 of Book I of Euclid — since its opposite sides are equal. Onto this parallel GZ transfer the spaces FO, FN to the points Q, P. For the straight lines OQ, NP drawn [there] will be [b] parallel to FG, since they join the equal straight lines FO, GQ, and ON, QP, or FN, GP.

[Margin: a. *Elements* I.5] [Margin: b. *Elements* I.33]

Diagram. A combined construction figure for both sundials at once, labelled HORIZONTALE (upper right region) and VERTICALE (lower left region, the word set vertically). Two straight lines AB and CD cross at right angles at the central point E; the pole's-altitude angle is set at A. The construction shows the trisected arc and the parallels described in the text. Readable labels, with position:

- HORIZONTALE (heading, upper right); VERTICALE (label, set vertically on the left)
- A (top center); X (upper right corner); G (upper left); C (left, on the horizontal axis); E (center, intersection); D (right); 6 (left, below C, on the vertical line CE)
- Interior/construction points read on the tiles: F, I, M, L (a row on the left-center); N, O (between E and the left); Q, P (small parallelogram near top, with R); S, R, H, K (near center); Z (right of center); T, V (lower-left region); Y (bottom-left corner); B, E (bottom center/right)
- Rim hour-numerals read near the top-right: 12, 1, 2 (with a small point-mark, possibly σ , near A) [?]
- A second, smaller construction is drawn at the lower right, with its own points A, B, C, D, F, G and hour-marks 12, 1, 2

[?] Several interior labels in the crowded upper-center cluster (around Q, P, R, K, S, H) and the small mark beside A could not be resolved with certainty at this resolution; the second small figure at lower right partly overlaps the main one. (more than two labels flagged [?]).

[Margin: Demonstration of the immediately preceding construction.]

Now if on the straight line EF equal segments ED, EB are cut off, and into these the intervals EN, EO are transferred from E; likewise, if from D an arc is described at the interval AE, which is cut at X by another arc described from A at the interval ED; and also from B an arc is described at the interval CE, which is cut at Y by another arc described from C at the interval EB; and if through X, Y straight lines are drawn through the points of the straight lines [recte: rectarum] AE, ED; CE, EB — then we shall have the hours from noon or from midnight. Between the 12th hour, which both the straight line XD and YB give us, and the 6th hour, which both the straight line XA and YC present to us, [the hour-lines are laid out]: these are hours from noon if the straight line AC is drawn to the left of AB, but from midnight if to the right. The horizontal sundial will be AEDX, at the angle of the pole's altitude EAC; the vertical, however, will be CEBY, at the angle ECA of the complement of the pole's altitude.

Demonstration. The demonstration of this construction does not differ from the one above. For the parallels AC, RV, ST make up a portion of the Foundation of sundials, whose breadth is EF, and whose length is extended along the straight line AC — since the parallels drawn here [correspond], as the parallels BC, MO, LN did in the upper Foundation, because the straight line FG here is equal to the

breadth FE, just as there the straight line BE [was equal] to BA, and the arc IK is divided here just as the arc FG was there, as is plain: for the straight line GE here answers to EA there, and so on. Next, the rectangle AD of the horizontal sundial here laid out (for AD is a parallelogram, by the scholium to Proposition 34 of Book I of Euclid, since its opposite sides are equal; and consequently a rectangle, on account of the right angle E) represents the lower rectangle EH, to the left of the meridian line of the horizontal sundial constructed from the Foundation. For ED here is equal to the breadth EF, just as there IH [was equal] to the breadth AB; and here AE, set between the parallels, makes the angle of the pole's altitude CAE, just as there EI is equal to the straight line PT in the Foundation, which sets up the angle of the pole's altitude TPD.

The rectangle CB of the vertical sundial here constructed, on the other hand, expresses the lower rectangle EH, to the right of the meridian line of the vertical sundial drawn from the Foundation. For EB here is equal to the breadth EF, just as there IH [was equal] to the breadth AB; and CE, set between the parallels, here sets up the angle ECA of the complement of the pole's altitude, just as there EI is equal to the straight line PV in the Foundation, which makes the angle VPD of the complement of the pole's altitude. Finally, the straight lines ED, EA, and likewise EB, EC, are divided here just as there IH, IE were cut, in each sundial, and so on. Rightly, therefore, the hour-lines have here too been drawn. And if the straight lines ED, EB are doubled by being produced, and other rectangles are made equal to the rectangles AD, CB, onto whose sides the points of the straight lines ED, EA and EB, EC are transferred, and lines are produced from X, Y through the said points, the whole sundials will be complete.

The half-hours and the quarter-hours are drawn by the same technique, if the arcs HL, LM, MI are bisected and [divided] into four equal parts, and from G, through the points of division, hidden straight lines are sent out cutting EF at points through which other parallels are drawn. For these will cut the straight lines AE, CE at points through which the straight lines are to be drawn from the points X and Y for the half-hours and quarter-hours of the same, and so on.

Now, given the length of the style, we shall find its place in each sundial just laid out by the following method...

method. Let the right angle BAC be constructed, and at A let the angle CAD of the pole's altitude be set up; produce DA toward E, and at A raise to it the perpendicular AG. Now, taking — for the horizontal dial — the line AB equal to the given style, and drawing through B the perpendicular EF to AB, the style AB in the horizontal dial must stand off, along the meridian line XD, from the dial's

center X by as much as the line EB; and the equinoctial line must lie away from that same center X by as much as the line EF. This is because the triangle AEF, together with the perpendicular AB, is entirely similar to the triangle CBD, together with the perpendicular BA, in the horizontal dial constructed by the earlier method — since the angle AEF [Euclid I.29] equals the angle DAC of the pole's altitude, the interior to the exterior, because the lines EF and AC [Euclid I.28] are parallel, and so on.

Likewise for the vertical dial: taking the line AC equal to the given style, and drawing DG through C perpendicular to AC, the style AC in the vertical dial must stand off from the dial's center Y, along the meridian line YB, by as much as the line DC; and the equinoctial line must recede from that same center Y by as much as the line DG. For the triangle ADG, together with the perpendicular AC, is exactly similar to the triangle CBD, together with the perpendicular BA, in the vertical dial laid out by the earlier method — on account of the angle ADG, which is the complement of the angle DAC of the pole's altitude, and so on.

Once a vertical dial facing south has been drawn in this way, we may derive from it another one facing north, as we set out in the scholium to Proposition 13 of Book II of the *Gnomonics*. We simply invert all its parts, so that what was the upper part becomes the lower, and whatever, after this inversion (with us now turned to face the dial), had been placed on the right is shifted to the left, and vice versa — the hour-numbers themselves, however, remaining exactly the same. But then that portion of the meridian line which runs from the dial's center toward the equinoctial line, and which on the southern dial marks the 12th hour of noon, will on this northern dial signify the 12th hour of midnight; and what on the former belongs to the 12th hour of midnight will on this one point to the 12th hour of noon. Accordingly, the hours reckoned there from noon must here be counted from midnight, and those there from midnight, here from noon. All of this we demonstrated in the aforesaid scholium to Proposition 13 of Book II of the *Gnomonics*.

For the rest: horizontal dials, when the pole's altitude is less than 20 degrees, and vertical dials, when the same altitude exceeds 70 degrees, are awkwardly drawn by the earlier method — that is, from the foundation of the dials — unless the parallels of the dials' foundation are very narrow, as is plain. The reason is that in horizontal dials the line PT, which makes with AD the angle of the pole's altitude above the horizon, and in vertical dials the line PV, which makes with the same AD the angle of the complement of the pole's altitude above the horizon, then cut the parallels BC, MO, LN very obliquely, and so meet the line BC too far off, unless

the line AB is extremely small, so that the foundation of the dials becomes very narrow. By the later method, however, with which we constructed both dials — the horizontal and the vertical — at one and the same operation, this drawback is avoided, since the construction itself presents the parallels to us spaced a moderate distance apart, as is clearly evident from the next figure.

The first example of this chapter is a horizontal dial, and the second a vertical dial, both constructed for the latitude of the city of Rome, 42 degrees.

DIALS DECLINING FROM THE VERTICAL.

CHAPTER 5.

We call "declining from the vertical" those dials that stand upright to the horizon but are inclined to the vertical circle properly so called — and hence also to the meridian; the kind commonly drawn on the walls of buildings. Let there be proposed, then, a wall upright to the horizon but declining from the vertical, from south toward the east, by 30 degrees. Draw the line AB equal to the line AB in the foundation of the dials, and at B set up the angle of declination ABC; and to the line AB, from A, draw the perpendicular AC, cutting BC at C. Then, having set up at A, together with AC, the angle CAD of the pole's altitude, draw from C, perpendicular to AC, the line CD, cutting AD at D. Also produce AC to E so that AE is equal to AD, and draw the line EB; and finally at C erect, perpendicular to BC, the line CF equal to CD, and join the line BF.

[Margin: The finding of the common section of the plane declining from the vertical, and of the parallelepiped extended through the axis of the world.]

The angle BFC will be equal to the angle which the common section of the 6-o'clock-hour circle and the declining plane makes, in the declining plane, with the common section of the meridian and the declining plane. And the angle AEB will be equal to the angle which the common section of the 6-o'clock-hour circle and the declining plane makes with the common section of the meridian and the 6-o'clock-hour circle, in the plane of the 6-o'clock-hour circle — that is, in the upper or lower rectangular face of the parallelepiped, that is, in the plane of the dials' foundation — with the line AD or BC. In this business we take both the upper and the lower face of the parallelepiped for the 6-o'clock-hour circle, since each is equidistant from that circle; and we take both the eastern and the western face of the same parallelepiped for the meridian circle, since each is parallel to that circle.

[Margin: Demonstration of the aforesaid finding.]

For let AB be conceived as the common section of the foundation of the dials and of the vertical circle — namely, the same as the line AB itself in the foundation; and let the triangle ABC be turned about AB, upward, until it coincides with the horizon and is upright to the vertical. With this done, BC will be the common section of the horizon and the declining plane, on account of the angle of declination ABC; while AC will be the common section of the plane equidistant from the meridian — that is, of the western face in the parallelepiped — and of the horizon. For AB, the common section of the horizon and the upper face of the parallelepiped, is perpendicular [Euclid XI.19] to the western face (to which both of them are upright), and hence also to any line whatever drawn through A in the said western plane, by Definition 3 of Book XI of Euclid. Since, then, AC is drawn in the plane of the horizon perpendicular to AB, AC will be the common section of the horizon and the said western plane. For if there were another line in the horizon that was the common section of it and the said plane, then to it AB would be

drawn from A two perpendiculars to AB — and thus a part would be equal to the whole, which is absurd. If, then, the triangle ACD is understood to be turned about AC until it is equidistant from the meridian, the line AD will coincide with the line AD in the foundation, on account of the angle CAD of the pole's altitude which the plane of the foundation — that is, the upper face of the parallelepiped — makes with the horizon along the drawn line AC; and CD will be the common section of the declining plane and of the plane equidistant from the meridian, since both of these planes then pass along the line CD — namely, along their common section, which is perpendicular [Euclid XI.19] to the horizon (to which both planes are upright), and hence also perpendicular to the line AC, which lies in the plane equidistant from the meridian, by Definition 3 of Book XI of Euclid — such as CD is. Therefore, once the line BD is drawn, it will be the common section of the declining plane and of the 6-o'clock-hour circle (that is, of the foundation of the dials), since both these planes, in that position, pass through the points B and D lying in the plane of the foundation. And hence the angle BDC will be the one that BD — the common section of the declining plane and the 6-o'clock-hour circle (which the plane of the foundation, drawn along the line AD set in its proper position, represents) — makes with CD, the common section of the same declining plane and of the plane equidistant from the meridian; that is, the angle that the 6-o'clock line makes with the meridian line in the plane of the dial. The angle BDA, on the other hand, will be the one that the said BD — the common section of the declining plane and the 6-o'clock-hour circle — makes with AD, the

common section of the same 6-o'clock-hour circle and the plane equidistant from the meridian; that is, the angle that the common section of the foundation of the dials and the declining plane makes with the line AD, or BC, of the foundation.

Diagram. A large rectangular figure occupying most of the page: the construction and completed face of a vertical dial declining 30° (from south toward the east) for the latitude of Rome (42°). In the upper-left corner sits a small auxiliary "foundation" triangle; the main body of the frame is the dial face, with all hour-lines radiating from a single center and hour-numbers ranged around the border.

- A (top-left area of frame): apex/corner of the small auxiliary triangle
- B (top of frame, right of A): apex of the auxiliary triangle; a dotted (hidden) line runs from B down-left toward D
- F (upper-left, below A): vertex of the auxiliary triangle
- D (left, below F): vertex of the auxiliary triangle
- c (center of auxiliary triangle, lowercase): interior point
- E (below c): vertex of the auxiliary triangle
- C (center of the dial face): the common point from which all hour-lines radiate
- K (immediately left of C): construction point
- M (below-left of C, on the horizontal line): foot of KM
- L (below C): foot of KL
- N (below C, right of L): construction point on the meridian
- A, B (small labels near C): points of the style-line / axis construction (CA, CB)
- D (near center, below the ray-fan): equinoctial-line construction point
- H (upper-center, near the top hour-marks; and a second H at bottom-center): the two ends of the meridian line HCH
- I (several, at the vertices of the parallelogram IEIHIEIH around the fan): parallelogram vertices
- E (mid-right edge; also near center): points of the 6-o'clock line ECE
- Rim hour-numbers (border of the dial): Arabic numerals are legible along the right and lower edges — reading down the right edge: 8, 7, 6, 5, 4, 3, 2; along the top and lower-left edges further numerals appear (9, 10, and Roman-form marks read as "II" and "III", with a "12" set as "I2") but most of these small border numerals are worn/too small to resolve individually and are left [?]
- P (lower-left, on the dashed baseline): construction point
- O (center of the dashed baseline at the foot): construction point

- A small circle mark ("o") floats just above the top edge of the frame
[unclear — possibly a figure/registration mark] [?]

[The hour-number ring is only partially legible; the individual border numerals]

the declining plane and the 6-o'clock-hour circle with AD, the common section of the same 6-o'clock-hour circle and the plane equidistant from the meridian; that is, the angle that the common section of the foundation of the dials and the declining plane makes with the line AD, or BC, of the foundation.

Furthermore, the line BD is equal to each of BF and BE; and the angle BDC equals the angle BFC, and the angle BDA equals the angle BEA. For since the two sides BC, CD of triangle BCD are equal to the two sides BC, CF of triangle BCF, and contain right angles (since CD, in its proper position, is perpendicular to the horizon, as we showed above, and hence also to the line BC lying in the horizon), the bases BD, BF and the angles BDC, BFC will also be equal to one another [Euclid I.4]. Likewise, since the two sides BA, AD are equal to the two sides BA, AE, and contain right angles (since BA, the common section of the horizon and the 6-o'clock-hour circle, is perpendicular [Euclid XI.19] to the triangle ACD, which is parallel to the meridian, and hence also to the line AD), the bases BD, BE and the angles BDA, BEA will also be equal [Euclid I.4].

And therefore, if the interval BE or BF (since BE and BF, having been shown equal to BD, are equal to each other) is carried by compass from z, in the foundation of the dials, out to a, then the drawn line z-a will be the common section of the declining plane and the 6-o'clock-hour circle — that is, of the foundation of the dials — inasmuch as it makes in the foundation the angle a-z-D, or z-a-B, equal to the angle BEA of this figure. For, when z-b is drawn perpendicular to BC — which will be equal to AB [Euclid I.34] — since the triangles a-b-z and E-A-B have the angles b and A equal, namely right angles, and the sides about the angles z and B proportional (since those sides are equal to these), and since the remaining angles b-a-z and BEA are each less than a right angle [Euclid I.17], the angles b-a-z and BEA will be equal [Euclid I.7], and so on.

[Margin: Construction of the dial declining from the vertical.]

Now, in the plane of the dial, having fixed the place of the style at A and its length AK, draw through A the line MN — in any position — to serve as the horizontal line (on a wall it must be drawn parallel to the horizon with the help of a plumb-line), to which the style AK is erected at right angles. Then, having set up to the right of the style the angle AKL of the declination of the proposed plane, and to the left the angle AKM of the complement of the declination, so that the angle

LKM is a right angle — (when, however, the plane declines from south toward the west, the former angle must be set to the left and the latter to the right) — and having the lines KL, KM cut the horizontal line at L and M, the line HLH, making right angles at L with MN, will be the meridian line; and through M must be drawn both the 6-o'clock-hour line and the equinoctial line. Moreover, having cut off the line LN equal to LK, set up at N the angle LNC of the pole's altitude, and let the line NC cut the meridian line at C. C will be the center of the dial; and the line drawn CM will give the 6th hour — all of which we demonstrated in the scholium to Proposition 1 of Book III of our *Gnomonics*. And the line CM will make with the meridian line the angle MCL equal to the angle BFC, which we showed above to be formed by the common section of the declining plane and the 6-o'clock-hour circle together with the common section of the same declining plane and the plane equidistant from the meridian — that is, by the 6-o'clock line and the meridian line in the plane of the dial. And the line drawn CA will be the style-line, as we demonstrated in the same scholium to Proposition 1 of Book III of our *Gnomonics*.

But so that, because of the closeness of the points C and A, no error is made in drawing the style-line CA (for it can easily be deflected one way or the other unless the greatest care is used), we shall use this device. On the line CH, downward, mark off with the compass any number of parts equal to CL (here five are taken), as far as O. Then, drawing OP perpendicular to CO, mark off on it as many parts equal to LA, up to P, as there are parts equal to CL contained in CO. For the line CA, when extended, must pass through the point P, as is clear from the scholium to Proposition 4 of Book VI of Euclid, since CL is to CO in the same proportion as LA is to OP. And so the style-line will be drawn more accurately through the three points C, A, P [the printed sort reads "B"; P is required — the line is named CP just below] than through the two points C, A alone. To the style-line CP as well, once the perpendicular AB (equal to the style AK) is drawn on either side, the line CB, when drawn, will be the axis of the world. If from B a perpendicular BD is drawn to this axis, cutting the style-line CB at D, then the line projected through M and D will give the equinoctial line, which will be exactly perpendicular to CP, provided no error has been made in the operation. And the angle DCB will be the angle of the pole's altitude above the proposed declining plane, as we showed in the same scholium to Proposition 1 of Book III of the *Gnomonics*. But if from N a perpendicular NR is erected to CN, this too will cut the meridian line at the point R, through which the equinoctial line must be drawn — as we also showed in the same place.

After this, from the 6-o'clock line ECE cut off on each side the line CE equal to the line a–z of the foundation of the dials; and from the meridian line HCH cut off, both upward and downward, the line CH equal to the line PV of the same foundation. From the points E, upward and downward, describe with radius CH four arcs, which are cut at I by another four described from the points H with radius CE; and join the two lines IEI and the two lines IHI, which will pass through the points E and H. And if the intervals of the line a–z in the foundation, intercepted between the point a and the parallels MO, LN, are transferred in the dial from the points H, on either side, onto the lines HI; and the intervals of the line PV in the same foundation, set between P and the parallels, are transferred in the dial from the points I onto the lines IE — then the lines drawn out through the center C of the dial and the points on the sides of the parallelogram IEIHIEIH, each passing through three points, will give the hours from noon and from midnight, whose order is the same as in the vertical dial, since the dial faces south. But if the dial faces north, the order of the hours will be the same as in the horizontal dial.

[Margin: Demonstration of the above construction.]

The demonstration of this construction is as follows. The common section of the plane of the dial and of the parallelepiped is a parallelogram, because the sections made by the plane of the dial in the opposite and parallel faces of the parallelepiped are parallel [Euclid XI.16]. Two of its sides — in the upper and the lower face of the parallelepiped — are the two lines IHI in the dial, since they are double the line a–z in the foundation, or BF, that is, of BD in this chapter's figure — namely, the common section of the plane of the dial and of the upper or lower face of the parallelepiped. And they make with the meridian line angles equal to the angle MCL, which the 6-o'clock line makes with the meridian line in the dial, and which we said is equal to the angle BFC — that is, to BDC — which BD, the common section of the foundation of the dial and of the upper or lower face of the parallelepiped, forms with CD, the common section of the same plane of the dial and of the meridian. It is clear from the construction that the line a–z of the foundation is equal to the line BD, or BF, of the figure constructed at the beginning of this chapter; and this BD is half of the whole common section of the upper (or lower) face of the parallelepiped and the plane of the dial. The other two sides, in the eastern and the western face of the parallelepiped, are the two lines IEI, since they are double the line PV of the foundation and are equidistant from the meridian line, just as in a vertical dial. For all vertical circles, to one of which the plane of the dial is equidistant, cut the parallelepiped in the same way in the eastern and the western face, since they are upright to the horizon; that is,

all vertical circles declining from the prime vertical make, in the eastern or western face of the parallelepiped, sections equal and parallel to the line PV of the foundation, by Proposition 18 of Book I of our *Gnomonics* — and this PV is half of the whole section generated in the entire eastern or western face. Since, then, I say, the section made in the parallelepiped by the plane of the dial is the parallelogram IEIHIEIH, the hour-lines will pass through the points on the sides of the said parallelogram, because those sides are divided just as the lines a–z and PV in the foundation of the dials, and so on.

For the rest: when the wall's declination exceeds 70 degrees, this method of laying out the hours is inconvenient, because the meridian line then lies too far from the place of the style, and the dial's center too far from the horizontal line — even though the meridian bisects the parallelogram formed in the parallelepiped, and the center C lies at its middle: just as the same thing happens, as we noted in Chapter 4, in horizontal dials, when

the elevation of the pole is less than 20 degrees; and in vertical dials, when the elevation of the pole is greater than 70 degrees — because the center of the dial then lies too far from the equinoctial line, as we taught in the scholium to Propositions 1 and 13 of Book II, and in the scholium to Proposition 1 of Book III of the *Gnomonics*. Even so, the layout will come out somewhat more conveniently in that case if the parallels in the foundation of the dials are made narrower — that is, if the side AB is made very small.

But suppose that the space in which the dial is to be constructed is not large enough for the whole parallelogram IEIHIEIH to be drawn in it. Then it will be enough to draw either its lower or upper half, EIHIEC, or else its left or right half, IEIHCH. For the lines drawn out through the points on the sides of that half, and through the center C, will yield the complete dial, even though the individual hour-lines are each drawn through only two points. This is to be understood of the other dials as well.

Diagram. A large declining sundial (the completed dial for the worked example of the preceding chapter, at the latitude of Rome, 42°). It is a rectangular plate enclosed by a heavy border. From the dial's center, C, a full fan of hour-lines radiates out to hour-points marked around the rim; along the bottom edge runs a divided scale (the equinoctial / style scale) drawn as a dashed line ruled with cross-tick divisions and terminating at the corner points P (bottom left) and O (bottom right). In the upper-left region of the plate sits the small style-and-foundation figure. The hour-numbers 1 through 12 appear around the perimeter

in two sets (morning and afternoon); several of these small rim numerals are worn and cannot all be read with confidence.

Readable labels:

- A (upper interior, apex of the small foundation triangle, left): label A
- B (upper interior, apex of the small foundation triangle, right): label B
- F (below and left of A): label F
- D (left, on the small triangle): label D
- C (center of the small foundation triangle, small type "c"): label C
- E (below the small triangle's center): label E
- I (below E): label I
- C (center of the whole dial, from which the hour-lines radiate): label C
- K (just left of the dial center): label K
- L (just right of and below the dial center): label L
- M (lower-left of the dial center, on a horizontal line): label M
- N (right of L): label N
- A (below L): label A
- D (below that A): label D
- E (on the lower-left, and again on the right rim): label E
- H (lower interior, twice): label H
- P (bottom-left corner of the base scale): label P
- O (bottom-right corner of the base scale): label O
- Rim hour-numbers legible in places: on the upper/right side "10", "9", "11", "8", "7", "6", "5", "4", "3", "2"; along the bottom and lower-left "1", "12", "11", "10", "9", "8" [several rim numerals worn — not fully resolved; not pattern-filled]

[The many small radiating hour-lines and rim numerals of this dial-face are marked as a; more than two rim labels could not be resolved with confidence.]

By what further method a dial facing north is derived from one facing south (toward the meridian), we have taught in the scholium to Proposition 1 of Book III of the *Gnomonics*.

Dials Declining from the Horizon

CAPVT VI.

The dials we have described up to now are in more frequent use than those that follow — namely those Declining from the Horizon, those Inclined to the Horizon, and those Declining from the Vertical while at the same time

inclined to the Horizon. For this reason we shall not describe these last fully, but shall only investigate their sections with the parallelepiped, or foundation, of the dials, so that in their planes we may set up parallelograms like the parallelogram IEIHIEIH of the dials treated above, and so on.

By "dials declining from the horizon" we mean those that stand upright to the prime vertical, but are inclined to the horizon — and therefore also to the meridian.

[Margin: Finding the sections of the plane declining from the horizon, and of the parallelepiped passed through by the meridian [?] axis. — Latin *per meridianum axem traiectionis*; the parallel note on the facing page reads *per axem mundi extensi*, "through the axis of the world."]

Let there then be a plane declining from the horizon by 30 degrees, whether it face east or west. Draw the line AB, which is to be equal to the line AB in the foundation of the dials; at A construct the angle of declination from the horizon, BAC, and from B erect BC perpendicular to AB, cutting AC in C. Then at B, with BC, construct the angle CBD equal to the complement of the pole's altitude, and from C draw CD perpendicular to BC, cutting BD in D. Extend BC also to E, so that BE is equal to BD, and draw the line AE. Finally, at C erect CF perpendicular to AC and equal to CD, and join the line AF.

The angle AFC will be equal to the angle that the common section of the hour-6 circle [the 6-o'clock circle] and of the plane declining from the horizon makes with the common section of the meridian and of that same declining plane — that is, the angle that the hour-6 line makes with the meridian line in the plane of the dial, or in the declining plane. And the angle AEB will be equal to the angle that the common section of the hour-6 circle and of the plane declining from the horizon makes with the common section of the meridian and of the hour-6 circle, in the plane of the hour-6 circle — that is, in the plane of the foundation of the dials — with the line AD, or BC.

Diagram. The plane construction described in the text. A is the upper-left apex, B the upper-right apex; the line AB runs across the top. From A the lines AC, AE, and AF descend; a dotted line runs from A toward D. C lies at the center, D on the right (with the segment CD, and BD reaching down to D), E is below C, and F is at the bottom, joined to A and C. All six labels — A, B, C, D, E, F — are legibly printed.

[Margin: Demonstration of the preceding construction.]

For conceive AB, the common section of the foundation of the dials and of the horizon — which is one and the same as the AB in the foundation — and let the triangle ABC be turned about AB, downward, until it coincides with the vertical plane and lies flat upon the horizon. This done, AC will be the common section of the vertical and of the declining plane, the one that faces east; while BC will be the common section of the plane parallel to the meridian — that is, of the eastern plane in the parallelepiped — and of the vertical.

If, then, the triangle BCD is understood to be turned about BC, toward us, until BD is parallel to the meridian, BD will coincide with BC in the foundation, on account of the angle CBD, the complement of the pole's altitude, which the plane of the foundation — that is, the upper plane of the parallelepiped — makes with the vertical drawn through BC. And CD will be the common section of the declining plane and of the plane parallel to the meridian, since both of these planes then run along the line CD — namely along their common section, which stands perpendicular to the vertical (to which both planes are upright) (Euclid XI.19), and therefore also to the line BC lying in the plane parallel to the meridian, by definition 3 of Book XI of Euclid — such as CD is, by the construction.

Draw, then, the line AD: it will be the common section of the declining plane and of the hour-6 circle (that is, of the foundation of the dials), since both these planes, in that position, pass through the points A and D lying in the plane of the foundation. And therefore the angle ADC will be the one that AD — the common section of the declining plane and of the hour-6 circle (which the plane of the foundation, drawn through BD, represents) — makes with CD, the common section of that same declining plane and of the plane parallel to the meridian; that is, the angle that the hour-6 line makes with the meridian line in the dial. And the angle ADB will be the one that the said AD, the common section of the declining plane and of the hour-6 circle, makes with BD, the common section of that same hour-6 circle and of the plane parallel to the meridian — that is, with the side of the foundation BC, or AD.

Now the line AD is equal to each of AF and AE, and the angle ADC equal to the angle AFC, and the angle ADB equal to the angle AEB. For since the two sides AC, CD of the triangle ACD are equal to the two sides AC, CF of the triangle ACF, and contain right angles (since CD, in its proper position, is perpendicular to the vertical, as we showed above, and therefore also to the line AC lying in the vertical), the bases AD, AF and the angles ADC, AFC will also be equal to one another (Euclid I.4). Likewise, since the two sides AB, BD are equal to the two

sides AB, BE, and contain right angles (since AB, the common section of the vertical and of the hour-6 circle, is perpendicular to the triangle BCD, which is parallel to the meridian, and therefore also to the line BD) (Euclid XI.19), the bases AD, AE and the angles ADB, AEB will also be equal to one another (Euclid I.4).

Therefore, if the interval AE, or AF (since AE and AF have been shown equal to AD and so are equal to one another) be carried by the compass from c, in the foundation of the dials, as far as d, the line cd, once drawn, will be the common section of the declining plane and of the hour-6 circle. This will be proved just as it was shown in the preceding chapter that the line a–z is the common section of the plane declining from the vertical and of the hour-6 circle.

[Margin: The drawing of a dial declining from the horizon.]

Now, in the plane of the dial, with the place of the style given and its length, if — following what we wrote in the scholium to Proposition 13 of Book III of the *Gnomonics* — the meridian line, the equinoctial line, and the hour-6 line be drawn: then let there be cut off from the hour-6 line, on either side of the dial's center, a segment equal to the cd of the foundation of the dials; and from the meridian line, both upward and downward, let there be cut off a segment equal to the PT of that same foundation; and let the parallelogram IEIHIEIH be completed, as in the preceding chapter. Onto the sides of this parallelogram let the segments parallel to the hour-6 line be transferred — beginning from the meridian line, on either side the intervals of the line dc of the foundation, starting from the point d [recte: the print reads *puncto*] — and onto the sides parallel to the meridian line, beginning at the angles I, let the intervals of the line PT of the foundation be transferred, starting from P. The hour-lines will then be drawn, each through its three points, as in the preceding dials.

For a plane declining from the horizon produces, in the eastern or western plane of the parallelepiped, a section parallel to the one that the horizon produces in that same plane — namely a section parallel to the meridian line and making with the line AD the angle of the pole's altitude, such as is the angle TPD; just as

a plane declining from the vertical produces, in the same plane of the parallelepiped, a section parallel to the one that the vertical produces in it, as we said in the preceding chapter — namely a section making with the line AD the angle of the complement of the pole's altitude, such as is the angle VPD, and so on.

Dials Inclined to the Horizon

CAPVT VII.

[Margin: The drawing of a dial inclined to the horizon.]

To lay out a dial inclined to the horizon, one must first find the elevation of the pole above the plane of the dial, as we taught in Proposition 25 of Book III of the *Gnomonics*. Then, for that elevation, a horizontal dial is to be drawn, as we prescribed in Chapter 4; for this plane is like a horizon of a sort. The ordering of the hours is to be taken from the precepts of Proposition 25 of Book III of the *Gnomonics*, where we have written out fully everything necessary for this matter — which there is no need to repeat here.

Dials Declining from the Vertical, and at the Same Time Inclined to the Horizon

CAPVT VIII.

So that this whole matter may be explained more clearly, we shall take all six of those planes that decline from the vertical and are at the same time inclined to the horizon, for which we described dials in Proposition 37 of Book III of the *Gnomonics*. In these the whole range of variety seems to be contained.

- The **first** declines from south toward east by 40 degrees, and is inclined to the horizon on the north side by 20 degrees.
- The **second** has a declination of 20 degrees from south toward west, and an inclination to the horizon on the north (boreal) side of 70 degrees.
- The **third** has a declination of 45 degrees from south toward east, and an inclination to the horizon on the north side of 51 degrees 51 minutes.
- The **fourth** deflects from north toward west by 20 degrees, and its inclination to the horizon on the south side comprehends 30 degrees.
- The **fifth** declines by 60 degrees from north toward east, and has an inclination to the horizon of 80 degrees on the south (austral) side.
- The **sixth**, finally, has a declination from north toward east of 30 degrees, and an inclination to the horizon on the south (austral) side of 52 degrees 3 minutes.

So that the same letters may be kept in all the figures, we shall assume that in the first three the declination is always from south toward east, and in the last three from north toward west. For the parallelepiped is cut in the same way by two planes having the same inclination to the horizon, one of which declines from

south toward east by as many degrees as the other deflects from south toward west. You will say the same of two planes that have the same inclination to the horizon but decline toward different sides from the north — as is evident, and as can be plainly gathered from what we shall demonstrate here.

Diagram, first figure ("I. fig."). The construction of the section for a plane declining from the vertical and inclined to the horizon (case for a plane inclined toward the north). A network of solid and dotted lines meeting at the interior point A. Labels read: K at the top, with H and I descending below it; G at the left (dotted); E near the center; F at the left; A at the center; B at the far right; D lower center; C at the bottom vertex. All labels legible.

Diagram, second figure ("2. fig."). The corresponding construction for the second case (plane inclined toward the horizon on the south side), drawn largely with dotted lines. Labels read: K at the top; G at the upper left (dotted) and again G at the lower right (dotted); F at the left (dotted); A at the center; E at the upper right; B at the right; C and D just below and right of A; H at the bottom; and a lowest label near the foot that could not be resolved with confidence [?].

Draw, then, the line AB — which is to be equal to the line AB in the foundation of the dials (only in the fifth figure is it taken smaller, so as to fit it into a narrower space) — and at B construct the angle of declination ABC; from A draw AC perpendicular to AB, cutting BC in C. Then, having again drawn from A the perpendicular AD to BC, construct at D the angle of inclination to the horizon, ADE, and from A erect AE perpendicular to AD, cutting DE in E; from this, on BA produced, cut off AF equal to it, and join the line CF.

Next, in the first three figures — in which the plane is taken to be inclined toward the north — let the angle FAG, the complement of the pole's altitude, be made upward; but in the last three figures — in which the plane is taken to be inclined to the horizon toward the south — let the same angle FAG, the complement of the pole's altitude, be made downward. And if the line CF, produced beyond F, meets AG, as in the first figure, cut off from CA toward A a segment AH equal to AG produced; but if CF, produced below C, meets AG, as in the second figure, or if the line CF cuts the line AG, as in the three later figures, then cut off from AC toward C — produced if need be — the line AH equal to AG, and join the line BH.

Finally, on AC take AI equal to AF (or to AE), join the line BI, and from B, with radius BH, describe an arc of a circle; let another arc, described from I with radius FG, cut it in K; and join the lines BK, IK. The angle BKI will be equal to the angle that the common section of the hour-6 circle and of the declining-and-

inclined plane makes with the common section of the meridian and of that same declining-and-inclined plane — that is, the angle that the hour-6 line makes with the meridian line in the plane of the dial, or in the declining-and-inclined plane.

The angle BHA, in turn, will be equal to the angle that the common section of the hour-6 circle and the declining-and-inclined plane makes — in the plane of the hour-6 circle, that is, in the upper or lower plane of the parallelepiped, that is, in the plane of the foundation of the dials — with the common section of the meridian and the hour-6 circle, namely with the line AD or BC. And finally the angle ACF will be equal to the angle that the common section of the meridian circle and the declining-and-inclined plane makes — in the plane of the meridian circle, that is, in the eastern or western plane of the parallelepiped, that is, in the plane of the foundation of the dials — with the common section of the meridian and the hour-6 circle, namely with the line AD or BC.

But whenever the lines CF and AG do not meet — even when produced to infinity — yet are parallel, as happens in the third figure, then the declining-and-inclined plane is parallel to the axis of the world. Consequently it will make, in the planes of the parallelepiped, or in the foundation of the dials, sections parallel to the lines AD, BC and the other parallels. In that case the dial is more conveniently constructed by the method we set out in the third figure of the scholium to Proposition 37 of Book 3 of the *Gnomonics*.

[Margin: Demonstratio praecedentis inuentionis — Demonstration of the preceding construction.]

For let AB be understood as the common section of the foundation of the dials and of the horizon (or of the vertical) — namely the same AB as in the foundation — except that for the fifth figure one must conceive the foundation to have been constructed of a smaller width, precisely as great as the line AB of that figure. And let the triangle ABC, together with the line AD, be imagined to be revolved about AB, upward, until it coincides with the horizon. Then let the triangle ADE be conceived to be revolved about AD — in the first three figures, that is, in the planes inclined toward the north, upward; but in the last three figures, that is, in the planes inclined toward the south, downward — until it stands perpendicular to the horizon, that is, to the triangle ABC. Consequently EA too will be perpendicular to the horizon, by Definition 4 of Book 11 of Euclid, since it is perpendicular to AD, the common section of the horizon and of the triangle ADE.

Likewise let the triangle ACF, together with the line AG, be understood to be revolved about AC, toward the same side to which ADE was moved, until it stands

perpendicular to the horizon. Consequently FA too will be perpendicular to the horizon, by Definition 4 of Book 11 of Euclid, since it is perpendicular to AC, the common section of the horizon and of the triangle ACF. And therefore the line FA will coincide with the line EA, and the point F with the point E, because of the equality of the lines AE and AF. Likewise the line AG will coincide with the side AD in the foundation, because of the angle FAG — the complement of the pole's altitude — which the line FA (perpendicular to the horizon in that position, and through which the vertical is drawn) forms with the said side. And therefore the point G will fall on the point H, because the lines AG and AH are equal.

For this reason, since in that position the proposed plane passes through the line BC — lying in the horizon because of the angle of declination ABC — and through the line DE, because of the angle of inclination ADE, that is, through the points B and C (of which the former lies in the horizon and in the foundation, the latter only in the horizon), and through the point E, or F (for we showed that the points E and F coincide), the same plane will pass through the line CF, and therefore will meet the side AD in the foundation at the point G, or H, since we have shown that these two points coincide.

The proposed plane, then, cuts the foundation of the dials along the line BH; and so the angle BHA — formed by the common section of the proposed plane and the hour-6 circle, which is BH, and by the common section of the meridian and the hour-6 circle, which is AD in the foundation — is produced in the very plane of the foundation. Again, since in the same position the proposed plane runs along the line CF, which lies in the eastern or western plane of the parallelepiped, and meets the side AD in the foundation at the point G or H, the common section of the proposed plane and of the eastern or western plane of the parallelepiped will form, with the side AD of the foundation, an angle equal to the angle AGF: namely the angle vertical to AGF, in the planes inclined toward the north, as in the first and second figures; but in the other planes, inclined toward the south, the very angle AGF itself, as in the last three figures.

Finally, since in the same position still the line GF is the common section of the proposed plane and of the meridian (or of the eastern or western plane in the parallelepiped), and since the point G coincides with the point H, as we said, the lines BH and GF will then contain the angle that, in the plane of the dial, the hour-6 line — namely BH — makes with the meridian line, to which GF is parallel in that position.

That the angle BKI is equal to this angle we shall demonstrate as follows. Consider the triangle contained by the three lines BH, GF (the point G being placed at H in

that position), and BF, which is then equal to the line BI [Margin: a. 4. primi — Euclid, *Elements* I.4]; for the sides BA, AF are equal to the sides BA, AI, and they contain right angles. Since, therefore, the sides BK, KI are equal to the sides BH, GF — or HF —

Diagram, "3.^a fig." (third figure). A construction of solid and dotted lines meeting at the central point A, for the case where the plane is parallel to the axis of the world (the lines CF and AG being parallel). Labels read: G at the upper left (on a dotted line) and again G at the lower right (dotted); E at the top; F at the left (on a dashed horizontal line through A); A at the center; B at the right; D below and right of A; C at the bottom. All labels legible.

Diagram, "4.^a fig." (fourth figure). A construction with solid and dotted lines converging on A. Labels read: K at the top; I just below and left of K; F at the far left; A at the center-left; E at the center; G at the lower left with H beside it; C and D at the bottom (C left, D right). The rays extend to a vertex at the far right whose label could not be resolved [?]. Otherwise legible.

Diagram, "5.^a fig." (fifth figure). A construction (partly dotted) for the case in which the line AB is taken smaller so as to fit a narrower foundation. Labels read: I at the top; E at the upper right; K at the right; F at the far left (dashed); A at the center; B at the right; D below center; G and H at the lower left. A small mark near the foot beside G/H could not be resolved with confidence [?].

Diagram, "6. fig.^a" (sixth figure). A construction of solid and dotted lines meeting at A. Labels read: F at the left (dashed); A at the center; E at the upper right; B at the far right; G at the lower left (dotted); H below center; C beneath H; D at the center; I at the bottom; K at the bottom center. All labels legible.

are equal, and the base BI equals the base BF, the angles BKI and BHF will be equal [Margin: a. 8. primi — Euclid, *Elements* I.8].

In the third figure, moreover, the proposed plane — drawn through the line CF — will nowhere meet the side AD in the foundation (with which the line AG then coincides, as was said), since the line CF is parallel to it. And so in that case the plane is equidistant from the axis, and so on.

And so, if in the first figure the interval BH is transferred with the compass into the foundation — which we have here repeated — from A up to E, then the line AE that is drawn will be the common section of the proposed plane and of the hour-6 circle (that is, of the upper or lower plane of the parallelepiped), since it is equal to BH itself; and it will make the angle AEB equal to the angle BHA [Margin:

b. 7. sexti — Euclid, *Elements* VI.7], because the angle B in the foundation is equal to the angle A in the first figure, the sides BA, AE are proportional to the sides AB, BH, and the angles E and H are each less than a right angle. Likewise, if the angle FAD in the foundation is made equal to the angle AGF of the first figure, then AF will be the common section of the proposed plane and of the lateral plane in the parallelepiped.

So too, if in the second figure the interval BH is transferred from G up to H, and the angle IGD is made equal to the angle AGF, then GH will be the section made by the proposed plane in the upper plane of the parallelepiped, and GI the section made by the same proposed plane in the lateral plane of the parallelepiped. And if in the fourth figure the interval BH is transferred in the foundation from K up to P, and the angle QKD is made equal to the angle AGF, then KP will be the section of the proposed plane and of the upper plane of the parallelepiped, while kQ [*sic*, lowercase k as printed] will be the section of the same proposed plane and of the lateral plane in the parallelepiped.

Moreover, in the fifth figure — since here the line AB is smaller than the line AB of the foundation, so that the line BH cannot be transferred into the foundation (being smaller than the section of the proposed plane and of the foundation), and this section must instead be investigated by means of the angle ACB, or AHB — if the angle SRA is made in the foundation equal to the angle ACB, and the angle TRA is set equal to the angle AGF, then RS will be the common section of the upper plane of the parallelepiped and of the proposed plane, while RT will be the common section of the same proposed plane and of the lateral plane of the parallelepiped. Finally, if in the sixth figure the line BH is transferred in the foundation from V up to X, and the angle YVD is made equal to the angle AGF, then VX will be the common section of the proposed plane and of the upper plane of the parallelepiped, while VY will be the common section made by the proposed plane in the lateral plane of the parallelepiped.

[Margin: Compositio horologij à Verticali declinatis, simulque ad Horizontem inclinati — The construction of a dial declining from the vertical and at the same time inclined to the horizon.]

With these things thus prepared: if, the position of the style and its length being given, one describes the meridian line, the equinoctial line, and the hour-6 line, as we taught in the scholium to Proposition 37 of Book 3 of the *Gnomonics*, then from the hour-6 line, on both sides of the center of the dial, one must cut off a line equal to the line AE, or GH, or KP, or RS, or VX — according as the first, second, fourth, fifth, or sixth dial is to be described. And from the meridian line too, on

both sides of the center, one must take off a line equal to the line AF, or GI, or KQ, or RT, or VY; and finally the parallelogram must be completed in the manner of the parallelogram IEIHIEIH [?] of chapter 5. For if onto the sides of this parallelogram parallel to the hour-6 line one transfers, on both sides — beginning from the meridian line — the intervals of the line EA, or HG, or PK, or SR, or XV in the foundation, starting from the side BC; and onto the sides of the same parallelogram parallel to the meridian line one transfers, beginning from the corners, the intervals of the line AF, or GI, or KQ, or RT, or VY in the foundation, starting from the side AD; then the hour-lines will be described, each through three points, as in the preceding dials of chapters 4 and 5. Their order should be sought from Proposition 37 of Book 3 of the *Gnomonics*.

Diagram. A single large rectangular figure (the composite "foundation") into which the constructions of the several dials described above are laid together. The four corners of the rectangle are B (top left), C (top right), A (bottom left), D (bottom right). Along the top edge, from left to right, run the points B, S, E, H, P, F, I, T, Y, X, C (a small additional mark near P could not be resolved [?]). Down the left edge run B, M, L, A; down the right edge run C, O, N, D. Along the bottom edge, from left to right, stand the points A, G, K, R, V, D, each marked with the figure it belongs to: A ("1.^a", first figure), G ("2.^a", second), K ("4.^a", fourth), R ("5.^a", fifth), V ("6.^a", sixth). The third figure has no vertex here, because in that case the plane is parallel to the axis and meets the side AD nowhere. An interior point H lies near the left side below B. Numerous straight lines cross the interior, representing the common sections (AE/GH/KP/RS/VX and AF/GI/KQ/RT/VY) of the several proposed planes with the planes of the parallelepiped. All corner and edge labels legible except the mark noted at P.

HOROLOGIA MERIDIANA, POLARIA, ET AEQVINOCTIALIA.

[Meridian, Polar, and Equinoctial Dials]

CAPVT IX.

[Chapter 9]

These three dials, since they are more easily laid out by the methods we handed down in the *Gnomonics* than from the parallelepiped, are deliberately passed over here; for their construction has been explained fully enough in our *Gnomonics*, so that it would be superfluous to repeat it here — especially since they are of less frequent use than horizontal dials and those declining from the vertical. We must now treat of the arcs of the signs of the zodiac, of the arcs of the

lengths of days and nights, and of the hours reckoned from sunrise and sunset, as well as of the unequal hours. All of these — except the unequal hours (since their delineation can be drawn from the *Gnomonics*, and they rarely come into use) — we shall depict only on the horizontal dial and on the dial declining from the vertical. This is both because these two kinds of dial are the more frequently used (for dials are generally constructed either horizontal or declining from the vertical, such as those drawn on walls standing perpendicular to the horizon), and because in all cases the method of construction is the same, as will be clear from what follows.

ARCVS SIGNORVM ZODIACI IN Horologio Horizontali, ac Verticali.

[The Arcs of the Signs of the Zodiac in a Horizontal and a Vertical Dial]

CAPVT X.

[Chapter 10]

Since the tip of the style's shadow, when the Sun is at the beginning of Aries ♈ or of Libra ♎, describes on any plane dial a straight line, but when the Sun is at the beginnings of the other signs traces a conic section — a hyperbola, a parabola, an ellipse, or finally a circle, as we demonstrated in the *Gnomonics* — we shall lay out arcs of this kind on a horizontal and on a vertical dial in the following way.

Let the figure of the zodiac rays, together with the hour-lines, be prepared in this way. [Margin: Radij signorum Zodiaci quo pacto ducantur — How the rays of the zodiac signs are drawn.] Draw across, at any convenient place, a straight line BI for the axis of the world; at B erect the perpendicular BE for the radius of the equator; and from B, having described an arc of a circle FEG at any interval, count off on it, on both sides from E, the maximum declination of the Sun, 23° 30', as far as F and G. Now draw the line FG, which will be bisected at right angles at H by the radius of the equator BE (by Corollary 1 of Proposition 10 of Book 13, or by the scholium to Proposition 27 of Book 3, of Euclid); from H, at the interval HF or HG, describe a circle, and divide it into 12 equal parts. This will indeed be done easily if — with the same compass-opening by which you described the circle — from the four points

Diagram. A large bordered figure in two stacked parts, showing the rays of the zodiac signs together with the hour-lines, for a horizontal dial (upper) and a vertical dial (lower).

Upper part: a large circle with radiating solid and dotted lines converging near the top edge at points labelled (left to right) C, B, B; below them M and D toward the center-top. Within and below the circle are the points P (center), F (left, on the horizontal diameter) and H, E (center, one above the other). Two small circle-marks (o) sit at the left and right ends of a horizontal line through the circle. Along the descending rays at the lower left are hour-numbers 4, 5, 6, 7, 8 and the paired numbers "1.11", "2.10", "3.9" with "12"; matching numbers appear at the lower right (12, "1.11", "2.10", "3.9", 4-8, 5, 6, 7). At the upper right is a smaller square inset carrying the labels B, I along its top and, within it, L, C, L, B, N, A, M, D, N, M down its right side, with K, I and G lower in the inset [several of these inset labels are faint — see flags below].

Lower part: a fan of solid and dotted lines radiating from a cluster of central points labelled C (top), and below it B, H, E, A (with a small b). Along a horizontal line run the hour-numbers 4, 3, 2, 1, 12, 11, 10, 9, 8, 7. The rays terminate at zodiac glyphs down the left side and down the right side. On the left, from top down, are read: ♂ (Taurus), then a point on the "6" line marked ♈ with F beside it (Aries), ♊ (Pisces), ♒ (Aquarius), and ♑ (Capricorn) at the foot [some left-hand glyphs faint]. On the right, from top down, are read: ♋ (Cancer)/♌ (Leo), ♍ (Virgo, on the "6" line), ♎ (Libra) with G beside it, ♐ (Sagittarius, on the "7" line), and a lower glyph near the foot [the exact right-hand glyph sequence is partly abraded]. Letter-labels F, G mark the ends of the equinoctial (Aries–Libra) line; BF and BG are the Cancer and Capricorn rays as described in the text.

Because more than two labels in this figure are faint or abraded (several inset letters in the upper part and several zodiac glyphs in the lower part), this diagram is a

in which the circumference of the circle is cut by the lines FG and BE, you cut off equal parts on both sides — that is, join by straight lines the pairs of points equally distant from the point F, or G, these lines cutting the arc FEG in four points. For the lines drawn from B through these points will give the rays of the signs lying between Aries ♈ and Libra ♎, and Cancer ♋, and Capricorn ♑. Of the lines BF and BG, one will be assigned to the sign Cancer ♋ and the other to the sign Capricorn ♑; and finally the line BE will be ascribed to the signs Aries ♈ and Libra ♎. The order of the intermediate signs will be understood from the drawing of the arcs on the dial. We demonstrated this construction of the zodiac rays in the *Gnomonics*, Book 1, Proposition 1, and at the end of Book 8.

But if the parts of the circle described from H are cut into 30 equal parts, and again the pairs of points equally distant from F, or G, are joined by straight lines,

the arc FEG will be cut at points through which the lines drawn from B will give the rays of all the degrees of the signs,

and the arcs between E and the said points — arcs intercepted from the arc FEG — will measure the declinations of the signs and of the degrees of the signs from the equator.

[Margin: The hour-line: in what way it is to be fitted to the rays of the signs.]

After this, from the triangle CBD of the horizontal or vertical dial constructed in chapter 4 (we take the horizontal one as our example), let the straight line BC, which represented the axis, be transferred onto the axis BI — whether to the left of the equator's radius BE or to the right — from B as far as C; and onto the equator's radius from B let the line BD of that same triangle CBD be transferred, as far as D. For the line CD, once drawn, will give the twelve-o'clock hour, which we shall draw more exactly as follows. Describe an arc of a circle Ik from C toward B at any interval you please, and on it count off — for the horizontal dial — the altitude of the pole, but for the vertical dial the complement of the pole's altitude, as far as k. For the line CK, when drawn, will again give the twelve-o'clock hour and will pass through the point D. Then, on the meridian line of the dial, take a straight line DE equal to BD itself; and whether above the equinoctial line FG or below it, take with the compass all the intervals between E and the hour-points of the equinoctial line. (Any interval taken on one side of the meridian line is equal to another interval on the other side of the meridian line, so that the interval of hour 1 is equal to the interval of hour 11,

Diagram. A large composite ray-figure (*figura radiorum*) for the horizontal dial, filling most of the page within a ruled rectangular border. It has two parts.

Upper part: a large circle with many solid and dotted lines radiating from points near its top edge. Along the top run the labels [C?], B, B (left of center), and B, I (over a square construction-inset at the upper right). Within the circle are the points P (left of center, on the horizontal diameter), F (further left on that diameter), and H, E (near the center, one above the other). Two small open circles (o) mark the left and right ends of the horizontal line through the circle. At the upper right a square inset carries, reading down its interior, the labels L, C, L, B; N, A; M, D; N; M; K, I; G, and along its right edge the vertical sequence B, M, D, M. Along the descending rays at the lower left are the hour-numbers 12[?], "1.11", "2.10", "3.9", then 4, 5[?], and lower down 7, 8, 6, 6; the lower right mirrors these with 12, "1.11", "2.10", "3.9", 4, 8, 5, 7, 6, 6.

Lower part: a fan of solid and dotted lines radiating from a central cluster labelled C (top), and below it B, H, E, A (with a small b). Along a horizontal line run the hour-numbers 4, 3, 2, 1, 12, 11, 10, 9, 8, 7. The rays terminate at zodiac glyphs down the left and right sides. On the left, from top down: ♉ [Taurus], then ♈ [Aries] on the "6" line with F beside it, then a glyph [?], ♊ [Pisces], ♒ [Aquarius], and ♐ [Capricorn] at the foot. On the right, from top down: ♋ [Cancer], ♌ [Leo], ♍ [Virgo] on the "6" line, ♎ [Libra] with G beside it, then (on the "7" line) ♏ [Scorpio], ♐ [Sagittarius], and a lower glyph [?] near the foot. The letters F and G mark the ends of the equinoctial (Aries–Libra) line.

- More than two labels in this figure are faint or abraded (several inset letters in the upper part; several zodiac glyphs in the lower part)

and the interval of hour 2 to the interval of hour 10, and the interval of hour 3 to the interval of hour 9, and so on.) Then transfer these onto the equator's radius from B, marking points on the equator's radius itself. For straight lines drawn from C through these points will give hours 1 and 11; 2 and 10; 3 and 9; 4 and 8; 5 and 7. And the line CP, drawn parallel to the equator's radius, will belong to both instances of hour 6. Now if you carry the said hour-lines — drawn from C through the points on the equator — over to the other side of the line CP, so that each pair always makes equal angles with CP (which is easily done by means of an arc of a circle described from C at any interval; for if the hour-lines drawn on the side of the equator's radius are carried over to the other side by marking points on an arc I described from C, then straight lines drawn from C through these points will give the remaining hour-lines), you will have the remaining hours, as is clear in the figure.

[Margin: Another way of drawing the hours in the ray-figure.]

But if we transfer the hour-lines lying between C, the center of the dial, and the equinoctial line FG from C onto the equator's radius, we shall find on the equator's radius the same points through which the hour-lines are to be drawn in the ray-figure.

[Margin: A more convenient way of drawing the hours in the ray-figure.]

But we shall draw these same hour-lines most conveniently, without transferring the said intervals onto the equator's radius, in this way. On the line CDK of the twelve-o'clock hour, drawn in the ray-figure as we taught above, take any point O; through it draw OQ parallel to the axis CI, cutting the six-o'clock line CP (which is parallel to the equator's radius) at P. From P, at the interval PO, describe a circle and divide it into 24 equal parts, beginning from O, or Q, or from the line

CP. Join with straight lines each pair of points equally distant from O or Q, these lines cutting the line OQ at points which are to be carefully marked. For straight lines drawn from C through these points will give the hours, as before, and — if everything has been done accurately — will pass through the points marked on the equator's radius. And if each part of the circle described from P is bisected, and then divided into 4 equal parts, we shall in the same way draw the lines belonging to the half-hours and to the quarter-hours.

[Margin: The drawing of the arcs of the signs on a horizontal or vertical dial.]

With these things thus prepared, we shall draw the arcs of the signs in this way. From the ray-figure just constructed we take the portion of the twelve-o'clock line CDK lying between C and the radius issuing from B — the portion nearest to the point C — and we transfer it onto the dial (which for this purpose ought to be drawn with the hour-lines hidden, so that afterward only their portions between the sign ♋ [Cancer] and the sign ♑ [Capricorn] are made distinct, since the other parts are superfluous, being parts on which the shadow of the style cannot fall. So you see in the example that the hour-lines outside the arcs of the signs are marked only with dots, as being superfluous and useless) — transferring it from the center C along the meridian line toward the equinoctial line, and marking a point on the meridian line. Then in the same way we take the portion of the line of hours 1 and 11 lying between the same point C and the said radius, and carry it over onto the dial from the center C onto the lines of hours 1 and 11 toward the equinoctial line, likewise marking two points on those two hour-lines. And so we proceed with the remaining hour-lines that cut the said radius, always transferring the intervals of those lines lying between C and the said radius onto the dial from the center C onto the corresponding hour-lines, marking points on them.

[Margin: Which arcs are to be assigned to which signs, on a horizontal dial as much as on a vertical one.]

It must be observed here, however, and in what follows, that the intervals of the lines falling from C toward the equator's radius BE are to be transferred from the dial's center C onto those hour-lines which extend from the center C toward the equinoctial line FG, as we have done for the lines of hours 12, 1, and 11, and so on, up to both instances of hour 6. But the intervals of the hour-lines falling on the other side of the six-o'clock line CP — those which cannot cut the equator's radius BE — are to be transferred onto the corresponding lines of the dial from C, the dial's center; these lines, beyond the dial's center, recede ever farther and farther from the equinoctial line, as here they fall above the six-o'clock line: such

as, in our example of the horizontal dial, are the lines of hour 7 after noon and the line of hour 5 after midnight, and so on. The points of these, however, have not been marked on our dial, because of the narrowness of the space. Once the points have been marked in this way on the hour-lines, if through them we duly draw a curved line — so that nowhere does it make an angle, but always proceeds uniformly — the arc so described will be the one assigned, on a horizontal dial, to the sign ♋ [Cancer], since the tip of the gnomon's shadow runs along it when the Sun is at the beginning of Cancer. But on a vertical dial that arc must be ascribed to the sign ♑ [Capricorn]. We shall draw the arcs of the other signs in no other way, if we transfer the hourly intervals lying between C and the other rays of the signs onto the dial from the center C onto the corresponding hour-lines, and so on. Further, the arcs between the dial's center C and the equinoctial line — the line assigned to the signs ♈ [Aries] and ♎ [Libra] — belong, on a horizontal dial, to the northern signs, while the others, beyond the equinoctial line, belong to the southern signs. The contrary is to be understood on a vertical dial, and on all others that are not horizontal. And each of the intermediate arcs looks to two signs, as you see on the dial here set forth.

[Margin: Another way of drawing the arcs of the signs.]

We shall delineate these same arcs of the signs if we transfer the hourly intervals lying between the equator's radius and the rays of the signs (whenever this can conveniently be done) onto the corresponding hour-lines on the dial, beginning from the equinoctial line, and so on. That the arcs of the signs are correctly drawn in this way, we demonstrated in Book 2 of the *Gnomonics*, Proposition 2.

[Margin: The drawing of the horizontal line on a vertical dial, and its functions.]

Furthermore, on a vertical dial there must be drawn, through A, the position of the style, a line parallel to the equinoctial line — or perpendicular to the meridian line — which is usually called the horizontal line, since it is the common section of the dial's plane and the horizon, and onto which the shadow of the style is cast when the Sun is on the horizon, that is, when it rises or sets. This horizontal line beautifully shows at what hour the Sun, being on any given parallel, rises or sets, provided that the arc of that parallel cuts the horizontal line. For where the arcs of the signs intersect the horizontal line, there the Sun rises and sets, when it stands at the beginnings of those arcs of the signs. This same horizontal line divides the whole dial into a southern and a northern part, and likewise into a diurnal and a nocturnal part, as we shall explain more fully below in chapter 11, when we shall have drawn the arcs of the signs on a dial

declining from the vertical. On this matter too we have written more in Proposition 14 of Book 2 of the *Gnomonics*.

[Margin: Which arcs of the signs, on any dial, are hyperbolas, or parabolas, or ellipses.]

We shall also easily understand from the ray-figure which arcs of the signs drawn on the dial are hyperbolas, and which are parabolas or ellipses. For whenever the twelve-o'clock line CK cuts two rays of opposite signs — rays equidistant, on either side, from the equator's radius BE — the arcs of those signs will be two opposite and equal hyperbolas. But when the same line CK cuts the radius of some sign between C and the equator's radius BE, while being parallel to the radius of the opposite sign, then the arc of that sign whose radius is cut will be a parabola, and the opposite sign will have no arc. Finally, when the line CK cuts some radius between C and the equator's radius BE, but neither runs equidistant from the radius of the opposite sign nor cuts it (except beyond the point B, when produced), then the arc of that sign whose radius is cut will be an ellipse, and the opposite sign too will have no arc. All this we demonstrated geometrically in Book 1 of the *Gnomonics*. We shall recognize the same on other dials, if in place of the meridian line we take the line of the style on declining dials, and in the ray-figure take the line of the pointer or style of the zodiac in place of the twelve-o'clock line, as will be clear from what follows. In that case the line CK (which on a horizontal, a vertical, and a horizon-inclined dial is the twelve-o'clock line, but on declining dials is the line of the style) will cut the rays of any opposite signs whenever the angle BCD of the pole's altitude above the dial's plane — the angle which the line CK makes with the axis CI — is less, for example, than the opposite and external angle IBG which the radius BG of the opposite sign makes with the same axis IB, this being the angle of the complement of the opposite sign's declination. But when the two said angles are equal, the line CK will be parallel to the radius of the opposite sign. And finally, when the angle BCD of the pole's altitude above the dial's plane is greater than the angle IBG of the complement of the declin-

-ation of the opposite sign, the line CK will neither cut the radius of the opposite sign nor be parallel to it. All of which is clear from Proposition 28 of Book 1 of Euclid.

For the rest, in what way the arcs of the southern signs may be drawn from the arcs of the northern signs on a horizontal dial, or the arcs of the northern signs from the arcs of the southern signs on a vertical dial; or likewise by what method two arcs of two opposite signs (namely when in the ray-figure the line CK cuts the

rays of two opposite signs) may be drawn by one and the same operation (which is indeed very pleasant to know, and very useful) — you will find explained in Book 2, Proposition 2, of our *Gnomonics*.

[Margin: By what art a larger or smaller dial with the arcs of the signs may be drawn from the ray-figure, for a given larger or smaller style.]

Now if, from the lines drawn so far in the ray-figure, a larger or smaller dial is to be delineated for a given gnomon larger or smaller than AB, we must proceed as follows. Having drawn separately the triangle CBD together with the style AB — as in the dial of chapter 4, and as you see was done alongside the ray-figure — produce the straight lines BC, BA, BD; and from BA cut off a straight line BN equal to the smaller or larger style proposed, and through N draw LM parallel to CD. Then transfer the line BL onto the axis CI of the ray-figure — whether to the left or to the right of the equator's radius BE — from B as far as C; and onto the equator's radius from B transfer the line BD of that same triangle CBD, as far as D, so that CB is equal to BL itself, just as the line CB in the ray-figure is equal to the line CB. For if at B, the point now marked, a perpendicular BM is raised to CI, to serve as the equator's radius, and from B on either side the rays of the other signs are drawn out, as was set forth at the beginning of this chapter (which, however, we have not drawn in this figure of ours, so as to avoid confusion of the lines), then the arcs of the signs will be described in proportion to the given style BN, as before — if the intervals of the hour-lines issuing from C, lying between C and the rays of the signs now drawn afresh (which hour-lines indeed do not change at the same elevation of the pole, even if a larger or smaller dial is to be drawn, just as the hour-lines on the dial remain the same at the same altitude of the pole), are transferred onto the hour-lines of the dial, secretly produced, from the center C, and so on. But the equinoctial line must be drawn on the dial perpendicular to the meridian line, through a point that is as far distant from the dial's center C as, in the ray-figure, the line CM is — the CM intercepted between C and the equator's radius just drawn — or as far as the line LM is in the triangle constructed separately: just as also, on our dial, the line CD between the dial's center and the equinoctial line is equal to the line CD in the ray-figure between C and the equator's radius, or to the line CD in the triangle drawn separately.

THE ARCS OF THE ZODIAC SIGNS ON A DIAL DECLINING FROM THE VERTICAL. — CHAPTER XI.

[Margin: The hour-line: in what way the rays of the signs are inscribed on it.]

Once the rays of the signs have been drawn, as we set forth in the preceding chapter, take from the triangle CBD of the dial declining from the vertical — which we constructed in chapter 5 — the straight line BC, which represented the world-axis, and transfer it onto the axis BC [?] of the ray-figure, whether to the left or to the right of the equator's radius BE, from B as far as C; and onto the equator's radius from B transfer the line BD of that same triangle CBD, as far as D. For the line drawn from C through D will be the line of the pointer, or style, just as on the horizontal dial that same line CD was the meridian line D in the ray-figure. Then, on the line

Diagram. The ray-figure (*figura radiorum*) for a dial declining from the vertical, occupying the lower half of the page.

Upper-left construction: a square/triangular framework whose top edge carries the labels B, L, C, L. Down its left side, and within the crossing (X) of its diagonals, appear M, N, A, D, N, M, with G and I lower, and the labels "10" and H at the foot of the left vertical.

Main figure: a large circle (the dial), across whose upper part an arc carries the ascending rim hour-numbers 2, 3, 4, 5, 6, 7, 8, 9 (left to right). Near the left-center is the node cluster I, B with A and c beside it, from which many solid and dotted rays fan downward. Interior points read D, E, L, K, M, H, F. Dashed curved lines are labelled "hor. 10" and "hor. 8" (and a third, "hor. 6" [?]). Along the descending rays at the lower left run the hour-numbers 10, 8, 11, 7, 12, 9; the lower-right side carries 10, 11, 12, 6, 1. Around the bottom rim run the hour-numbers 1, 2, 3, 4, 5, 6 (with several repeated further along the arc).

- Many hour-number and letter labels are faint, crowded, or partly abraded (well more than two flagged/uncertain)

of the style of the dial, take a straight line DI equal to BD itself; and whether above the equinoctial line or below it, take all the hourly intervals between the point I and the equinoctial line, and by means of the compass transfer them onto the equator's radius from B, marking points on the equator's radius itself. For straight lines drawn from C through these points will give the hour-lines corresponding to those hours — lines drawn through the points transferred onto the equator's radius.

We shall obtain these same hour-lines if, on the dial, we take the hourly intervals between C, the dial's centre, and the equinoctial line, and transfer them onto the equator's radius from B, marking points on it. For it is by these points that the hour-lines are to be drawn, as before.

[Margin: Another way of describing the hours in the ray-figure.]

Now if at any time some hour-line on the dial does not cut the equinoctial line — even when produced to infinity — but runs parallel to it, then in the ray-figure it must be drawn from C as the straight line CK, parallel to the equator's radius. For this line will correspond to that hour, just as on the horizontal and vertical dials: because the line of the 6th hour is parallel to the equinoctial line, in the ray-figure the straight line CP has been drawn from C parallel to the equator's radius, corresponding to the 6th hour, as is clear from the preceding chapter.

But in this way the ray-figure yields only those hour-lines which cut the equinoctial line on the dial, since only their intervals have been transferred onto the equator's radius. In our example these are the hours 4.5.6.7.8.9.10.11.12 after midnight and 1.2.3 after noon. Therefore, to obtain the other hours — those which neither cut the equinoctial line nor are parallel to it, namely all those which, after cutting the equinoctial line, have been produced beyond the centre C (in the same example, the hours 4.5.6.7.8.9.10.11.12 after noon and 1.2.3 after midnight, whose intervals cannot be transferred onto the equator's radius) — we shall proceed as follows. Through C, the dial's centre, we shall draw the straight line CP parallel to the equinoctial line, as was done in the accompanying dial of this chapter; and through C, in the ray-figure, the straight line CK parallel to the equator's radius, which will correspond to the straight line CP on the dial, as has been said. Then we shall transfer the hour-lines drawn from C through the points of the equator's radius onto the other side of the straight line CK, so that the two lines on either side always make equal angles with CK. This is easily done by means of a circular arc described from C at any chosen radius. For if the arcs intercepted between the straight line CK and the hour-lines drawn from the side of the equator's radius are transferred to the other side — marking points on the arc described from C — then straight lines from C through these points will give the hours which do not cut the equinoctial line but have been produced beyond the centre C. Thus two lines equidistant from the straight line CK will belong to the two hours of the same number: the one which cuts the equator's radius (that is, the one drawn from C toward that radius) to the hour that cuts the equinoctial line; the other, transferred to the other side of the straight line CK, to the hour of the same number produced beyond the centre C, which does not cut the equinoctial line — exactly as was done on the horizontal and vertical dials. For there too the lines transferred beyond the straight line CP in the ray-figure denote, by number, the same hours which on the dial are denoted by the lines that cut the equator's radius, since it is the intervals of these latter that have been transferred onto the equator's radius, not those of the former. In our example the

line of hour 3 to the left of CK corresponds to hour 3 after noon, since this cuts the equinoctial line on the dial; but the line of hour 3 to the right of the same CK looks to hour 3 after midnight, because this does not cut the equinoctial line on the dial. By the same reasoning the lines of hours 4 and 5 to the left belong to the hours after midnight, because these cut the equinoctial line on the dial; while the lines of the same hours to the right of CK denote the same hours after noon, because these do not cut the equinoctial line on the dial, and so on.

Furthermore, the lines of those hours which cut the equinoctial line on the dial either barely, or very far from the point I or the centre C — such as, in our dial, hour 4 after midnight and hour 3 after noon — we shall transfer into the ray-figure in this way. We shall bisect at the point E the line of the index [linea indicis] CD, which lies between the dial's centre C and the equinoctial line; and from E we shall draw a line parallel to the line of that hour which barely cuts the equinoctial line — a parallel that makes with the line of the index an angle equal to the angle which the line of that hour makes with the same line of the index toward the same side. For if this parallel conveniently cuts the equinoctial line, we need go no further in the division of the straight line CD; but if not, we shall again bisect ED at G, and again bisect GD, and so on, until a straight line drawn from the last point of division, parallel to the line of the said hour, conveniently cuts the equinoctial line. In our example the straight lines GF, GH are parallel to the lines of hour 3 after noon and hour 4 after midnight, cutting the equinoctial line at F and H, and the straight line GD is a quarter of the straight line CD. Then let the same fraction that GD is of CD — namely DO of DI — be taken on the same line of the index, and let CQ be the same fraction of the axis CB; that is, let DI and CB be divided into as many equal parts as CD has been divided. On the axis CQ, in the ray-figure, let a length CA equal to CA be cut off from the axis CB; and through A let AL be drawn parallel to the equator's radius BD, serving as a kind of new equator's radius with respect to the lines GF, GH drawn on the dial. For if the intervals OF, OH are transferred onto this parallel AL, marking the points M and L on the said parallel AL, then the straight lines CL, CM drawn to them will correspond to the said lines of hours 3 and 4, which cut the equinoctial line on the dial too far from the point I or the centre C. We shall use this same device on horizontal and vertical dials whenever some hour-lines cut the equinoctial line either barely, or too far from the dial's centre — such as the lines of hours 5, $5\frac{1}{4}$, $5\frac{1}{2}$, $5\frac{3}{4}$ after noon, and $6\frac{1}{4}$, $6\frac{1}{2}$, $6\frac{3}{4}$, 7 after midnight.

Moreover, we shall also draw all the hour-lines in the ray-figure most accurately, without transferring them from the dial onto the equator's radius, in this way. Having drawn the straight line CD as before, to correspond to the line of the

index, we shall take on it, produced, any point H, through which we shall draw the line HI parallel to the axis BC; and through C another line CK parallel to the equator's radius BD, which cuts the former line HI at K. Then, with centre K and radius KH or KI, we shall divide the described circle into 24 equal hours, beginning not from the point H (as was done on the horizontal and vertical dials, because here the straight line CD does not represent the meridian line, or the line of hour 12, as it did there) but from some other point, which we shall find as follows. The interval IR we shall transfer from the dial onto the equator's radius from B as far as E, and through E, from C, we shall draw the straight line CE, which will represent hour 12 — provided the interval BE is equal to the interval IR between I and hour 12 on the equinoctial line. Therefore, the division must begin from the point of the circle which corresponds to the point F, where the line of hour 12, CE, cuts the straight line HI: namely from the point G, which is given by the straight line FG drawn at right angles to HI, either upward or downward. Or else, the interval between I and any hour on the equinoctial line — whichever proves more convenient for this purpose — we shall transfer onto the equator's radius from B, marking a point on the radius itself, and through this point, from C, we shall draw a straight line which will represent the assumed hour and will cut HI at a point; and from that point, if a perpendicular be raised to HI, it will reach the very same

part to which the point G belongs, if the assumed hour and hour 12 lie on the same side of the style-line; but the opposite part, if the assumed hour lies on one side of the style-line and hour 12 on the other. This perpendicular will give on the circumference another point, from which the division of the circle into 24 equal parts can begin just as well as from G. Indeed it will be very useful for this second point to be investigated: for if the division-points of this second point coincide with the division-points of the point G, the matter succeeds as intended; if not, some error committed somewhere must be corrected. It is also better to begin the division of the circle from this second point rather than from the first, G, when hour 12 is very close to the style-line. For then the perpendicular FG cuts the circumference too obliquely, so that the point G can scarcely be taken without error. I mean this to be understood also in dials declining from the horizon, and in those declining from the vertical and at the same time inclined to the horizon. After this, from the division-points we shall drop hidden perpendiculars to the straight line HI. For these will cut HI at points through which straight lines drawn from C will give the hour-lines, as before. And from the division-points we shall easily judge to which hours the lines correspond on the dial: for the points running from G toward H denote those hours which on the dial follow hour 12 toward the style-line, namely 11, 10, 9, 8, and so on, in our example, as is clear

from the figure. Those hidden perpendiculars to HI from the division-points of the circle we shall likewise draw easily. If perchance two points on either side are equidistant from the point H (which happens only when the style-line is itself an hour-line, or is equidistant from the two nearest hour-lines), then the straight lines joining to HI the two points equally distant from H will be perpendicular. But if two points on either side of H are unequally distant, take the interval of any point from H or I in the upper semicircle and transfer it into the lower semicircle from the same point H or I. For the straight line joining these two points equidistant from H or I will be perpendicular to HI. And in the same way the intervals of the points of the lower semicircle from H or I are transferred into the upper semicircle, marking points on it, and so on.

[Margin: The description of the arcs of the signs on a dial declining from the vertical.]

With all this prepared, we shall describe the arcs of the signs on the dial from the ray-figure, just as on the horizontal and vertical dials: namely, by transferring the intervals of the hours intercepted between C and the radii of the signs onto the corresponding hours of the dial, from the dial's centre C; or else by transferring the hour-intervals set between the equator's radius and the radii of the signs onto the corresponding hour-lines of the dial, from the hour-points of the equinoctial line, and so on. And if the style-line happens to be the same as some hour-line, or equidistant from the two nearest hour-lines on either side, then each of the aforesaid intervals must be transferred onto the two hour-lines equidistant on either side from the style-line, just as was done on the horizontal and vertical dials.

Diagram. The ray-figure (*figura radiorum*) for a dial declining from the vertical, occupying the lower half of the page and facing the framed plate on the opposite page. From an apex C at the top of a vertical line, a broad fan of solid and dashed straight lines (the radii of the hours and of the signs) spreads downward and outward. This figure faces the framed plate

- C (apex, top): dial centre / vertex of the ray-fan
- K (upper left, beside C): endpoint of the line CK parallel to the equator's radius
- Q (just below C): node point
- A, E, L (row left of centre): construction points on the axis / parallel
- B (below the A–E–L row): base point
- D (lower left): construction point

- R (below B): construction point
- N (right of centre): construction point
- P (far right, with dashed line running off toward the gutter): endpoint of line CP
- M, H (left side, H above M): construction points
- F (lower right): construction point
- Hour-numbers read along the rays: I, II, 12 (lower left); 2, 3 (lower right); and 4, 5, 6, 8 among the descending rays
- Zodiac glyphs decorate the ray-ends: Capricorn (♑) at the top; Pisces (♓) and Aquarius (♒) upper left; and, among the lower rays, Virgo (♍), Aquarius (♒), Taurus (♉) and further worn glyphs [?]

Many hour-number and letter labels are faint, crowded, or partly abraded (well more than two uncertain)

[Margin: Which arcs are to be assigned to which signs on a dial declining from the vertical.]

We have written more, bearing on the more accurate description of the arcs of the signs, in Proposition 2 and its scholium in Book 3 of our *Gnomonics*.

The arcs above the equinoctial line — the line which is assigned to the signs Aries (♈) and Libra (♎) — belong to the southern signs; those below the equinoctial line belong to the northern signs, just as on the vertical dial. Whether the arcs of these signs are hyperbolas, or parabolas, or ellipses, we shall easily understand from what we wrote in the preceding chapter, by carefully examining whether the style-line CD cuts the radii of the opposite signs or not, and so on.

The horizontal line [*linea Horizontalis*] — the line onto which the shadow falls at sunrise or sunset — is drawn through A, the place of the style, perpendicular to the meridian line; such is the straight line AL. This line shows at what hour the Sun rises or sets when it stands in any given parallel whose arc cuts the horizontal line. For where the arc of some sign intersects the said line among the hours after midnight, there the Sun rises; and where the arc of some sign cuts the same line among the hours after noon, there the Sun sets. Thus you see that the Sun, when it is at the beginning of Capricorn (♑), rises about hour 7½; in Sagittarius (♐) and Aquarius (♒), before hour 7½; in Scorpio (♏) [?] and Pisces (♓) (♓), about hour 6¾; in Aries (♈) and Libra (♎), at hour 6.

This same horizontal line divides the whole dial into two parts, of which the lower, being the larger, faces south when the dial is in its proper position, while

the upper, being the smaller, faces north — so that the former may be called the southern dial, the latter the northern. Each is to be so placed that the horizontal line is parallel to the horizon and occupies the upper position. On the northern dial, however — which faces us when we turn to the dial — the things that are on the right (when the horizontal line is set above) become the left, and vice versa, and the arcs of the southern signs are changed into the arcs of the opposite northern signs, and conversely. But once both dials are set in their proper position, all the lineaments above the horizontal line must be cut away as superfluous. Again, this same horizontal line divides the whole constructed dial into two: that part which is below the horizontal line (the horizontal line being set parallel to the horizon) may be called the Diurnal, because it shows the hours by day; the other, above the horizontal line, may be called the Nocturnal, because it would show the night hours if the Sun's rays could penetrate the earth and illuminate the dial itself. Hence the portions of the arcs of the signs may not inappropriately be named the diurnal arcs on the diurnal dial, and on the nocturnal dial the nocturnal arcs of the same signs.

[Margin: The description of the horizontal line on a dial declining from the vertical, and its various functions.]

Diagram. The constructed declining dial (*horologium*), drawn as a large circle whose upper part touches the top edge of a tall rectangular frame. From the top of the circle a fan of hour-lines (solid and dashed) descends through the disc. Along the top edge of the frame appear the labels B ... B (with a small c beside the first). In the upper-right corner of the frame is a separate right-angled sub-construction (the scaled triangle with its parallel LM): its labels, read top to bottom along the right margin, are B, L, C, L, B, then N, A, then M, D, then K, I, then M, B.

- Along the left inner edge, hour-numbers run down the descending rays: 1½[?], 1·11, 2·10, 9, 8, 4, 5, 6, 7 [reading uncertain — worn]
- Interior labels within the disc: M, D, P, F, H, E
- Zodiac glyphs mark several ray-ends around the lower circle (worn) [?]

Many labels are crowded and partly abraded (well more than two uncertain)

Diagram. A second ray-figure (the separately-drawn, scaled construction), beneath the dial and within the same frame. From an apex C at the top of a vertical line, solid and dashed rays fan symmetrically downward. Near the centre is a small triangle with H and E over A, and B to its left.

- C (apex, top): vertex of the ray-fan

- B, E, A, H (central triangle cluster): base and style points; a small b lies just below
- F (left, on a horizontal line): construction point
- G (left, on a lower horizontal line): construction point
- Hour-numbers along the middle horizontal band, left to right: 4, 3, 2, 1, 12, 11, 10, 9, 8
- Lower hour-numbers along the descending rays: 5, 6, 7, 8
- Zodiac glyphs at the ray-ends: Capricorn (♑) and Leo (♌) upper right; Virgo (♍) and Libra (♎) at right; Pisces (♓), Aquarius (♒) and further worn glyphs at lower left; Capricorn (♑) at the bottom

Many labels are crowded and partly abraded (well more than two uncertain)

But if, from the lineaments so far drawn, a larger or smaller dial is to be constructed for a given size of style, we shall do it in this way. Let the triangle of the dial CBD be drawn separately, together with the style BA taken from the dial (as you see has been done beside the ray-figure), and let BA be produced as far as you please. Then, on BA, having taken the straight line BN equal to the given style, let LM be drawn through N parallel to CD. For if on the axis BC of the ray-figure a length CA equal to CA be cut off by the straight line BL, and from A the radii of the signs be drawn, and all the rest be done as before, the arcs of the signs will be described for the size of the given style BN, just as before.

[Margin: How, from the ray-figure, a dial with the arcs of the signs may be constructed for a larger or smaller style.]

were drawn earlier for the given style BA. For the hour-lines, both in the ray-figure and on the dial, remain exactly the same at the same elevation of the pole and the same declination of the plane. The equinoctial line, however, is to be drawn perpendicular to the style-line, at a distance from the centre C of the dial equal to the straight line LM. All of this is to be done in the same way on other dials as well — even on ones that decline and are at the same time inclined — as is plain from our *Gnomonics*.

[Margin: How the arcs of the signs are described on any dial whatever, with no regard to the hour-lines.]

We shall likewise describe the arcs of the signs on any dial, taking no account of the hours, in the following way. In the ray-figure, having drawn the straight line CD — which on a horizontal, vertical, or horizon-inclined dial represents the meridian line, but on the others the style-line — let there be marked, on the

radius issuing from B that lies farthest from the point C (such as, in the ray-figure of this chapter, is the radius $\mathfrak{S}$ [Cancer], which lies nearest to point H), below the line CD, as many points at small distances from one another as you please. (For the more crowded these points are, the more accurately the arcs of the signs will be described.) Through these points let straight lines be emitted from C, after the manner of the hour-lines. Then, below the point where the radius of the equator was marked off from the last of those lines — namely the line drawn through the lowest point marked on that radius — let there be marked in the same way, on the radius of the equator, as many points at small distances apart as can conveniently be marked, and through them let straight lines likewise be emitted from C. Taking also another point B very close to the point C, and drawing BM parallel to the radius of the equator, let points again be marked on this line BM, below the last line so far drawn from C, at small distances from one another, and through them let straight lines be emitted from C. Finally, having drawn the straight line CP parallel to the radius of the equator — the line which, on the dial, represents the line drawn through the centre equidistant from the equinoctial, such as is the line of the 6th hour on a horizontal, vertical, or horizon-inclined dial — let the lines drawn so far be transferred, by means of an arc of a circle described from C, to the other side of the straight line CP.

After all this, let the intervals on the radius of the equator between B and the lines so far drawn from C be transferred, with the help of compasses, from the point E of the dial of chapter 10, or from the point I of the dial of this chapter 11 (which is as far from the equinoctial line, along the style-line, as the straight line DB in the triangle CBD), to either side onto the equinoctial line, by impressing points on the equinoctial line itself. Through these points let hidden straight lines be emitted from the centre of the dial, after the manner of the hour-lines; and these correspond to the straight lines emitted from C in the ray-figure,

just as, above, the hour-lines of the dial corresponded to the lines drawn from C in the ray-figure. Hence, so that it may be easily recognized which hidden lines of the dial correspond to which lines of the ray-figure, numbers must be attached to the lines of the ray-figure: thus 1 to the style-line CD, 2 to the next line, 3 to the third, 4 to the fourth, and so on. And the same numbers must be assigned to the hidden lines of the dial: 1 to the style-line; but to the lines next to it on either side, 1; to the third lines on both sides, 3; and so on. For the lines issuing from C that do not cut the radius of the equator, we shall draw the corresponding lines on the dial in this way. We shall transfer the portion of the straight line CD intercepted between C and the line BM from the centre of the dial onto the style-line, impressing a point on it. Through this point we shall draw a line parallel to the

equinoctial line, and below it we shall cut off from the same style-line a straight line equal to the portion of the line BM lying between B and the line CD, impressing a point on it. For if from this point we transfer, onto the line drawn parallel and next to the equinoctial line, straight lines equal to the portions of BM lying between B and the lines not yet transferred to the equinoctial line, impressing points on the said parallel line, then the straight lines emitted from the centre of the dial through these points will correspond to those lines which could not be transferred to the equinoctial line.

When all this has been properly done, the arcs of the signs will be described, as we taught before — namely, if we transfer the intervals of the straight lines issuing from C in the ray-figure, measured between C and the radii of the signs, from the centre of the dial onto the corresponding hidden lines, transferring each single interval onto the two straight lines on either side of the style-line that correspond to it (on declining dials as well), and so on. In this way the arcs of the signs will be delineated more accurately than by the hour-lines alone, because the hour-lines are sometimes farther apart than is fitting for this task, whereas our hidden lines keep a smaller distance from one another; and from this it comes about that more numerous points are available for the arcs of the signs.

You may also, if you prefer, first draw from the centre of the dial straight lines a little apart on either side of the style, in such a way that the pairs on either side cut off equal parts from the equinoctial line; and then transfer the intervals on the equinoctial line between E and these lines onto the radius of the equator from B, as was said above concerning the hour-lines, and so on. But I prefer the former method, because by it the points obtained on the radius, at the point farthest from C, are less far apart than by this latter one. We have explained this matter using the letters of the horizontal dial of the previous chapter, even though no lines other than the hour-lines were drawn on it.

THE ARCS OF THE LENGTHS OF THE DAYS on any dial.

CHAPTER XII.

Just as we have so far described the arcs of the signs, which the tip of the shadow traverses when the Sun is at the beginnings of the signs, so too the arcs of the lengths of the days are usually described — the arcs which the same shadow traverses when the artificial day contains a given number of hours, such as 10 hours, or 8, or 11, or 13, and so on. For this to be done, one must first construct a figure containing the radii of the parallels of the Sun that mark off a determined

number of hours above the horizon, after the manner of the figure containing the radii of the signs, which we described in chapter 10.

[Margin: How the radii of the diurnal arcs are drawn.]

So, having described from A an arc of a circle CD at any interval, and having drawn the straight line AB as the radius of the equator — that is, as the diurnal radius of the 12th hour — let the complement of the elevation of the pole above the horizon be laid off from B on either side, as far as C and D. Then let the straight line CD be drawn, cutting AB at E; and from E, at the interval EC or ED, let a circle be described and divided into 48 equal parts, beginning from C or D. For if two division-points equally distant from C or from D are joined by hidden straight lines, CD will be cut at points through which the diurnal radii are to be emitted from A: so that AD is the radius of hour 0, the one next to it the radius of hour 1, the next the radius of hour 2, and so on up to AC, which is the radius of hour 24, as appears in the figure. And the arc of the circle CBD, comprised between AB — the diurnal radius of the 12th hour, or radius of the equator — and the said diurnal radii, measures the declinations of the parallels of the Sun which those radii represent; that is, the parallels whose diurnal arcs above the horizon comprise as many hours as are assigned to the individual radii. All of this we have demonstrated geometrically at the end of the *Gnomonics*.

Diagram. The ray-figure for the arcs of the lengths of the days. A protractor-like construction: from an apex **A** at the top (with a short vertical style-stub above it) a dense fan of straight and dashed radii descends. A large arc **CD** sweeps over the fan, with **C** at the upper left and **D** at the upper right. Down the central vertical axis lie **A** (apex), **E** (on the horizontal diameter through the fan), and **B** (lower down). Along the left margin, hour-numbers label the descending rays, read top to bottom: 24, [23 and 22 not visible], 21, 20, 19, 18, 17. Along the lower arc, hour-numbers run left to right: [worn glyph ~"15"/"16" — reads uncertain], 14, 13, 12 (split by the axis), 11, 10, 9, [8]. On the right margin (about D) the sequence continues up toward 0, with a legible "0" near the top-right and worn "4", "5", "6" below it.

- A (top, on axis): apex from which the diurnal radii issue; AD = radius of hour 0, AC = radius of hour 24
- C (upper left): left end of arc CD
- D (upper right): right end of arc CD
- E (centre, on horizontal diameter): intersection of CD with the axis AB
- B (lower, on axis): point below E

- Left-margin ray-labels (top→bottom): 24, 21, 20, 19, 18, 17
- Lower-arc ray-labels (left→right): [~15?], 14, 13, 12, 11, 10, 9, [8]
- Right-margin ray-labels: 0, [4?], [5?], [6?]

Several peripheral hour-labels are abraded (more than two uncertain)

[Margin: How the diurnal arcs are described on the dial.]

Now if we transfer any diurnal radius into the ray-figure of the signs from the point B, after the manner of some radius of a parallel, we shall describe its arc in exactly the same way as we described the arcs of the signs in the two preceding chapters. Into the ray-figure of chapter 10 were transferred the radii of hours 10 and 14; but into the ray-figure of the preceding chapter, the radii of hours 8, 10, 14, and hour 0 — such being the radii issuing from B, which are distinguished by dots or little lines.

This page is dominated by a large framed engraved plate carrying two labelled constructions for Chapter XI (the declining dial and its ray-figure). Both are crowded and heavily abraded; more than two labels in each are uncertain.

Diagram. The constructed declining dial : a large disc whose top touches the upper edge of the frame, with a fan of solid and dashed hour-lines descending from a point near the top. Along the top edge of the frame run the labels **B ... B ... B ... I** (a small **c** beside the first B). In the upper-right corner of the frame is a separate right-angled sub-construction, its labels read roughly top-to-bottom / left-to-right: **L, C** (top), **N, K I, M, D**, with **B** at the outer right. Interior points of the disc, read down the middle and to the sides: **P, M, H, E, F, G**.

- Top frame-edge: B, B, B, I (small c beside first B)
- Corner sub-construction (upper right): L, C, N, K, I, M, D, B
- Disc interior: P (upper middle), M (right of centre), H and E (mid, on a horizontal), F (left, lower), G (right, lower)
- Left-margin hour-line labels (top→bottom): [1?], "1·11", "2·10", 9, 8; then along rays 8, 7 (left) and 6, 6 (right)

Many labels crowded and partly abraded (well more than two uncertain)

Diagram. The ray-figure (scaled construction) beneath the dial and within the same frame. From an apex **C** at the top, solid and dashed rays fan symmetrically downward across a central cluster with **B**, and near it **E, A, H**; **F** lies on a horizontal line to the left, **G** on a lower horizontal line to the right. Hour-numbers run along the middle band and the descending rays (left to right along the lower

band: 4, 3, 2, 1 near the axis; 5, 6, 7, 8 continuing). Zodiac glyphs mark the ray-ends around the margins.

- C (apex, top): vertex of the ray-fan
- B, E, A, H (central cluster): base/style points
- F (left, on a horizontal line): construction point, near Aries ♈
- G (right, on a lower horizontal line): construction point, near Libra ♎
- Zodiac glyphs at ray-ends, left margin (top→bottom): [worn], ♊ [Gemini, uncertain], ♉ (Taurus), ♈ (Aries, at F), ♐ (Pisces), ♑ (Aquarius), ♐ (Capricorn, bottom)
- Zodiac glyphs at ray-ends, right margin (top→bottom): ♐ [Capricorn, uncertain], ♌ (Leo), ♍ (Virgo), ♎ (Libra, at G), ♏ (Scorpio), ♐ (Sagittarius), ♐ (Capricorn)
- Hour-numbers, lower band (left→right): 4, 3, 2, 1, 12, 11, 10, 9, 8; lower rays: 5, 6, 7, 8

Many labels crowded and partly abraded (well more than two uncertain)

HOURS FROM SUNRISE AND SUNSET on a horizontal dial, and on one declining from the vertical.

CHAPTER XIII.

The description of the hours from sunrise and sunset by means of the diurnal and nocturnal arcs is very easy. For this purpose we have excerpted the following little tables from the scholium of Proposition 33, Book 1, of our *Gnomonics*, in which it appears at once which hours from sunrise and sunset correspond to which hours from noon and midnight in any diurnal or nocturnal arc. To these we have added a table of the EQUINOCTIAL LINE, from the scholium of Proposition 19, Book 1, of the same *Gnomonics*, by which it is immediately known which hours from noon or midnight, and from sunrise or sunset, meet one another [se mutuo succe(n)t — abbreviated form, "succeed / coincide with"] at one and the same point of the equinoctial line.

*In the tables below, the Latin row-labels are: **Horæ ab occasu** = hours from sunset; **Horæ à med. noct.** (also written **med.nocte**) = hours from midnight; **Horæ à meridie** (also **à merid.**) = hours from noon; **Horæ ab ortu** = hours from sunrise. The centre column carries the noon/midnight label at which the reckoning switches. Row-labels and cell values are transcribed verbatim.*

Arcus diurnus horarum 8. in quo Sol oritur hora 8. à med. noc. & hora 16. ab occ. & hora 24. ab ortu. (Diurnal arc of 8 hours, in which the Sun rises at hour 8 from midnight, hour 16 from sunset, and hour 24 from sunrise.)

Horæ ab occasu.	1 1 1 1 2	2 2 2 2
	6 7 8 9 0	1 2 3 4
Horæ à med.nocte.	8 9 1 1 1	Horæ à meridie. 1 2 3 4
	0 1 2	
Horæ ab ortu.	2 1 2 3 4	5 6 7 8
	4	

Arcus nocturnus horarum 16. in quo Sol occidit hora 4. à mer. & hora 24. ab occ. & hora 8. ab ortu. (Nocturnal arc of 16 hours, in which the Sun sets at hour 4 from noon, hour 24 from sunset, and hour 8 from sunrise.)

Horæ ab occasu.	2 1 2 3 4 5 6 7 8	9 1 1 1 1 1 1 1
	4	0 1 2 3 4 5 6
Horæ à merid.	4 5 6 7 8 9 1 1 1	Horæ à med. noct. 1 2 3 4 5 6 7 8
	0 1 2	
Horæ ab ortu.	8 9 1 1 1 1 1 1	1 1 1 2 2 2 2 2
	0 1 2 3 4 5 6	7 8 9 0 1 2 3 4

Arcus diurnus horarum 16. in quo Sol oritur hora 4. à med. noc. & hora 8. ab occ. & hora 24. ab ortu. (Diurnal arc of 16 hours, in which the Sun rises at hour 4 from midnight, hour 8 from sunset, and hour 24 from sunrise.)

Horæ ab occasu.	8 9 1 1 1 1 1 1	1 1 1 2 2 2 2 2
	0 1 2 3 4 5 6	7 8 9 0 1 2 3 4
Horæ à med. noct.	4 5 6 7 8 9 1 1 1	Horæ à meridie. 1 2 3 4 5 6 7 8
	0 1 2	
Horæ ab ortu.	2 1 2 3 4 5 6 7 8	9 1 1 1 1 1 1 1
	4	0 1 2 3 4 5 6

Arcus nocturnus horarum 8. in quo Sol occidit hora 8. à mer. & hora 24. ab occ. & hora 16. ab ortu. (Nocturnal arc of 8 hours, in which the Sun sets at hour 8 from noon, hour 24 from sunset, and hour 16 from sunrise.)

Horæ ab occasu.	2 1 2 3 4	5 6 7 8
	4	
Horæ à meridie.	8 9 1 1 1	Horæ à med. nocte. 1 2 3 4
	0 1 2	
Horæ ab ortu.	1 1 1 1 2	2 2 2 2
	6 7 8 9 0	1 2 3 4

These pages continue Chapter XIII, presenting the hour-correspondence tables for the several diurnal and nocturnal arcs (the "arcs" whose catchword closed the previous page), followed by the table for the equinoctial line and the worked explanation of how to lay out the hours from sunrise and sunset on any sundial.

The recurring Latin row-labels mean: *Horæ ab occasu / ab oc. / ab or.* = hours from sunset / hours from sunrise; *Horæ à meridie* = hours from noon; *Horæ à med. noct. / à med. noc. / à media nocte* = hours from midnight; *Horæ ab ortu* = hours from sunrise. In each three-row table a column represents one instant expressed in two or three reckonings; the centre label-cell marks where the noon↔midnight (or sunrise↔sunset) reckoning switches, and is blank in the outer rows.

Diurnal arc of 10 hours, in which the Sun rises at hour 7 from midnight, hour 14 from sunset, and hour 24 from sunrise. [*Arcus diurnus horarum 10. in quo Sol oritur hora 7. à med. noc. & hora 14. ab occasu & hora 24. ab ortu.*]

	1 1 1 1 1 1	2 2 2 2 2
Horæ ab occasu.	4 5 6 7 8 9	0 1 2 3 4
Horæ à med. noct.	7 8 9 1 1 1	Horæ à meridie. 1 2 3 4 5
	0 1 2	
Horæ ab ortu.	2 1 2 3 4 5	6 7 8 9 1
	4	0

Nocturnal arc of 14 hours, in which the Sun sets at hour 5 from noon, hour 24 from sunset, and hour 10 from sunrise. [*Arcus nocturnus horarum 14. in quo Sol occidit hora 5. à mer. & hora 24. ab occasu & hora 10. ab ortu.*]

	2	1 1 1 1 1
Horæ ab occasu.	4 1 2 3 4 5 6 7	8 9 0 1 2 3 4
Horæ à meridie.	5 6 7 8 9 1 1 1	Horæ à med. noc. 1 2 3 4 5 6 7
	0 1 2	

	2									1	1	1	1	1		
Horæ ab occasu.	4	1	2	3	4	5	6	7		8	9	0	1	2	3	4
Horæ ab ortu.	1	1	1	1	1	1	1	1		1	1	2	2	2	2	2
	0	1	2	3	4	5	6	7		8	9	0	1	2	3	4

Diurnal arc of 14 hours, in which the Sun rises at hour 5 from midnight, hour 10 from sunset, and hour 24 from sunrise. [*Arcus diurnus horarum 14. in quo Sol oritur hora 5. à med. noc. & hora 10. ab occ. & hora 24. ab ortu.*]

	1	1	1	1	1	1	1	1		1	1	2	2	2	2	2
Horæ ab occasu.	0	1	2	3	4	5	6	7		8	9	0	1	2	3	4
Horæ à med. nocte.	5	6	7	8	9	1	1	1	Horæ à meridie.	1	2	3	4	5	6	7
						0	1	2								
Horæ ab ortu.	2	1	2	3	4	5	6	7		8	9	1	1	1	1	1
	4											0	1	2	3	4

Nocturnal arc of 10 hours, in which the Sun sets at hour 7 from noon, hour 24 from sunset, and hour 14 from sunrise. [*Arcus nocturnus horarum 10. in quo Sol occidit hora 7. à mer. & hora 24. ab occ. & hora 14. ab ortu.*]

	2															1
Horæ ab occasu.	4	1	2	3	4	5				6	7	8	9	0		
Horæ à meridie.	7	8	9	1	1	1	Horæ à media nocte.		1	2	3	4	5			
				0	1	2										
Horæ ab ortu.	1	1	1	1	1	1			2	2	2	2	2			
	4	5	6	7	8	9			0	1	2	3	4			

Diurnal arc of 13 hours, in which the Sun rises at hour 5½ from midnight, hour 11 from sunset, and hour 24 from sunrise. [*Arcus diurnus horarum 13. in quo Sol oritur hora 5½. à med. noc. & hora 11. ab occ. & hora 24. ab ortu.*]

	1	1	1	1	1					1	1	2	2	2	2	2
Horæ ab oc.	1	2	3	4	5	16	17			8	9	0	1	2	3	4
H. à me. no.	5	6	7	8	9	10	11	A	½	1	2	3	4	5	6	
	½	½	½	½	½	½	½	merid.	½	½	½	½	½	½	½	
Horæ ab or.	2	1	2	3	4	5	6		7	8	9	1	1	1	1	
	4											0	1	2	3	

Nocturnal arc of 11 hours, in which the Sun sets at hour 6½ from noon, hour 24 from sunset, and hour 13 from sunrise. [*Arcus nocturnus horarum 11. in quo Sol occidit hora 6½. à mer. & hora 24. ab occasu & hora 13. ab ortu.*]

	2										1	1	
Horæ ab oc.	4	1	2	3	4	5		6	7	8	9	0	1
Horæ à meridie.	6	7	8	9	10	11	Horæ à media nocte.	½ 1	2	3	4	5	
	½	½	½	½	½	½		½	½	½	½	½	
Hor. ab or.	1	1	1	1	17	18		1	2	2	2	2	
	3	4	5	6				9	0	1	2	3	4

Diurnal arc of 24 hours, in which the Sun rises at hour 12 from noon, and hour 24 from sunrise or sunset. [*Arcus diurnus horarum 24. in quo Sol oritur hora 12. à mer. & hora 24. ab ortu vel occasu.*]

(This 24-hour arc is printed as two stacked sub-tables.)

[illegible][illegible]

Nocturnal arc of 24 hours, in which the Sun sets at hour 12 from midnight, and hour 24 from sunrise or sunset. [*Arcus nocturnus horarum 24. in quo Sol occidit hora 12. à med. noc. & hora 24. ab ortu vel occasu.*]

[illegible]

	1	1	1	1	1	1	1	2	2	2	2	2
Horæ ab or. vel oc.	3	4	5	6	7	8	9	0	1	2	3	4
Horæ à med. nocte.	1	2	3	4	5	6	7	8	9	1	1	1
										0	1	2

THE EQUINOCTIAL LINE, CONTAINING THE HOURS from noon or from midnight, and from sunrise or sunset, passing through one and the same point of it. [*ÆQVINOCTIALIS LINEA CONTINENS HORAS à mer. vel med. noc. & ab or. vel occ. per vnum idemque eius punctum transeuntes.*]

											1	1	1
Horæ ortu vel occasu.	1	2	3	4	5	6	7	8	9	0	1	2	
H. à mer. vel med. nocte.	7	8	9	1	1	1	1	2	3	4	5	6	
				0	1	2							
Horæ ab ortu vel occasu.	1	1	1	1	1	1	1	2	2	2	2	2	
	3	4	5	6	7	8	9	0	1	2	3	4	

Although in the preceding tables only whole hours have been laid out, they are to be understood as applying to half-hours and quarter-hours as well, and so on. Thus, for example, just as on the equinoctial line the 8th hour from noon or midnight, the 2nd hour from sunrise or sunset, and the 14th hour from sunrise or sunset all pass through one and the same point, so too through one and the same point on that same equinoctial line will pass the 8½ hour from noon or midnight, the 2½ hour from sunrise or sunset, and the 14½ hour from sunrise or sunset. In the same way, not only are the 12th hour from noon or midnight, the 6th hour from sunrise or sunset, and the 18th hour from sunrise or sunset drawn through a single point of the equinoctial line; but the 12¼ hour from noon or midnight, the 6¼ hour from sunrise or sunset, and the 18¼ hour from sunrise or sunset will also intersect one another at one and the same point of that same equinoctial line, and so on.

[Margin: Descriptio horarum ab or. vel occ. in quovis horologio. — The drawing of the hours from sunrise or sunset on any sundial.]

Therefore, if the hours from sunrise and sunset are to be drawn on any sundial whatever, we must first find, among the hours from noon or midnight, the points of the diurnal arcs of 10 and 14 hours — that is, we must first lay out the arcs of 10 and 14 hours by the foregoing precepts, imprinting only their points on the lines of the hours from noon or midnight, as we have done here in the horizontal sundial and in the one declining from the vertical. Next we must turn back to the

tables of those same diurnal arcs. For in them we shall find very quickly through which hours from noon or midnight — that is, through which points of the said diurnal arcs (marked in the hours from noon or midnight) — each hour from sunrise or sunset is to be drawn. By the same method the table of the equinoctial line is to be used.

Diagram. A sundial hour-line construction figure enclosed in a rectangular frame. From an apex cluster at the top-centre, a fan of straight hour-lines spreads downward and outward to the left and right, crossed by dotted lines representing the equinoctial line and the diurnal/nocturnal arcs. The left-hand hour-lines are labelled with Roman numerals, the right-hand ones with Arabic numerals; the dotted arc/equinoctial lines carry their own hour-numbers where they cross.

- Apex cluster (top-centre, the style/gnomon points): C, B, F, D set in a row, with E just below, and A at the vertex of a small inverted triangle where the lines converge.
- Left side, hour-lines from top downward (Roman numerals): XIII, XII, XI, X, IX.
- Right side, hour-lines from top downward (Arabic numerals): 11, 12, 13, 14, 15, 16 (a further topmost right-hand label is worn — [?]).
- Dotted-line (equinoctial / arc) hour-labels visible along the descending dotted curves: 6, 7, 8 [these three legible; other dotted-line numbers present but not resolvable].
- A dense central fan of small labels close to the vertex A — Roman I, II, III, IIII and numerous small Arabic numerals (including 12 below the vertex, and values in the teens and low twenties) — marking the hour-points where the lines cross. These small central labels are too closely set and worn to resolve individually [?].

The figure is a labelled informational diagram; several central and dotted-line labels are illegible and should be re-examined.

For in it we shall see at once through which hour from noon or midnight the same hour from sunrise or sunset must pass on the equinoctial line. Thus you see that in both sundials the 1st hour from sunrise is drawn through the 8th hour from midnight of the diurnal arc of 10 hours, through the 7th hour from midnight of the equinoctial line, and through the 9th hour from midnight of the diurnal arc of 14 hours. The 2nd hour from sunrise, in turn, [is drawn] through the 9th hour from midnight of the diurnal arc of 10 hours, through the 8th hour from midnight of the equinoctial line, and through the 7th hour from midnight of the diurnal arc

of 14 hours; and so on in succession you discern the other hours from sunrise drawn through the three points that follow in that order. Likewise you see, in both sundials, that the 15th hour from sunset passes through the 8th hour from midnight of the diurnal arc of 10 hours, through the 9th hour from midnight of the equinoctial line, and through the 10th hour from midnight of the diurnal arc of 14 hours. Likewise the 16th hour from sunset [passes] through the 9th hour from midnight of the diurnal arc of 10 hours, through the 10th hour from midnight of the equinoctial line, and through the 11th hour from midnight of the diurnal arc of 14 hours; and so on in succession, through the three points that follow in that order, you will find the other hours from sunset drawn.

And if it happens that only two points are found for some hour, that hour must be drawn through those two, or else a third point corresponding to that hour must be sought — in the nocturnal arc, or to be investigated in some other diurnal arc. Thus you will find that the 13th hour from sunset in both sundials is drawn through two points, namely through the 8th hour from midnight [of] the equinoctial line; yet in the sundial declining from the vertical the same 13th hour from sunset is also drawn through the 6th hour from midnight of the nocturnal arc of 14 hours. For the portion of the diurnal arc of 10 hours that lies above the horizon is the nocturnal arc of 14 hours. But whenever only a single point is found for some hour from sunset, another point must be taken in the nocturnal arc for the same hour reckoned from sunrise. Thus you see that the 22nd hour from sunset is drawn, in the declining sundial, through the 3rd hour from noon of the diurnal arc of 10 hours, and through the 5th hour from midnight of the nocturnal arc of 14 hours — which 22nd hour, in the nocturnal sundial, signifies the 22nd hour from sunrise, as will be explained a little below. Or indeed the same 22nd hour from sunset must be drawn through the 2nd hour from noon of the diurnal arc of 8 hours, namely through point E. By the same reasoning the 23rd hour from sunset is drawn through the 5th hour from midnight of the equinoctial line (and, in so far as it is reckoned from sunrise) through the 6th hour from midnight of the diurnal arc of 8 hours, and through the 3rd hour from noon of the diurnal arc of 8 hours, and so on; and also through point F, that is, through the [?] hour of the diurnal arc of 8 hours.

[Margin: Quæ horæ in horologio boreali pertineant ad horas ab or. & quæ ab horas ab occ. — Which hours on the northern sundial pertain to the hours from sunrise, and which to the hours from sunset. (Printed "ab horas"; read as *ad horas*, "to the hours.")]

Finally, if the foregoing tables are carefully weighed, all the hours from sunrise and sunset will easily be described, provided you remember to take, for the hours from sunset, the same hours from sunrise in the nocturnal arcs. And if this is not yet sufficient for describing some hour from sunrise or sunset, another point must be sought in the diurnal arc of 8 hours or the nocturnal arc of 16 hours.

Moreover, since the portion of the sundial lying above the horizontal line — which we have called the nocturnal sundial — presents a sundial to be placed on the face of a north-facing wall, if its parts are inverted (as was said above at the end of Chapter 4): in order that we may know which lines produced beyond the horizontal line pertain to the hours from sunrise and which to the hours from sunset, let this method be followed. Let there be carefully considered, in that portion, in so far as the sundial [...]

it is a nocturnal sundial — which hours are to be reckoned from sunrise and which from sunset. For the hours from sunset, in that portion in so far as it is a nocturnal sundial, are to be counted from sunrise; and the hours from sunrise are to be reckoned from sunset in the same portion, in so far as it presents the northern sundial [*horologium boreale*]. And thus we shall distinguish the hours from sunrise from the hours from sunset in that portion, according as it is a nocturnal sundial.

All the hours that are counted from sunrise in the portion of either sundial lying below the horizontal line — that is, in the diurnal sundial — are likewise to be counted from sunrise in the nocturnal sundial, if they cross the horizontal line before they cross the meridian line; and the same must be said of the hours from sunset. But the hours from sunrise in the diurnal sundial, if they cross the meridian line before the horizontal, are to be reckoned in the nocturnal sundial from sunset; and the hours from sunset [are to be counted] from sunrise. Thus the hours 23, 22, 21, 20 in the nocturnal sundial are to be counted from sunrise, because in the diurnal sundial they are reckoned from sunset and cross the meridian line before they reach the horizontal. These same hours, therefore, will have to be counted from sunset in the northern sundial. But the hours 11, 12, 13, 14, 15 in the same nocturnal sundial will belong to the hours from sunset, because in the diurnal sundial they answer to those same hours and cross the horizontal line before they intersect the meridian; and so these same hours will have to be counted from sunrise in the northern sundial.

In both of our examples, so that the sunrise-hour lines might be distinguished from the sunset-hour lines, we have attached to the hours from sunrise the ancient number-signs, thus: I, II, III, IIII, V, VI, VII, VIII, etc.; while to the hours

from sunset we have assigned the common number-signs, as 11, 12, 13, 14, 15, 16, 17, etc. Finally, the lines of the hours reckoned from noon and from midnight are distinguished only by points [dots].

Diagram. The large upper figure on printed p. 34 — the hour-line web of the declining (nocturnal/northern) sundial discussed in this chapter, continuing the construction begun on the preceding pages. A long vertical line (the meridian / stylus line) rises at the center; from a cluster of points near the center a dense fan of solid and dotted hour-lines radiates outward to the right and downward. The confidently legible labels are:

- Central and adjacent point letters: **C** (top of the vertical line), **K** (left of C), **A** (the principal point of convergence, center-left), **L** (right of A), **B** (below A), **E** (right of B), **N** (right of E), **F** (right of N) — B, E, N, F stand in a row along the horizontal line.
- Left-side point letters: **D, H, I, S**.
- Hour-numerals from sunrise (Roman, "ancient" signs): **I, II, III, IIII, V, VI, VII, VIII** along the lower left, and **IX** (upper left); further Roman signs (X–XVI) run up the left margin but are small and rotated.
- Hour-numerals from sunset (Arabic, "common" signs), running clockwise down the right and lower edge: **24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8** and, near the center, **1, 2, 3**.

Many of the smaller rotated hour-numerals along the rays cannot be resolved individually and the exact ray-to-label assignments are not all legible. [Several labels [?]]

Diagram. The smaller framed (ruled square) figure at the foot of printed p. 34 — the nocturnal / northern (boreal) sundial, its hour-lines radiating symmetrically to left and right from a small central gnomon-triangle. A vertical meridian line runs through the frame from top to bottom. At the center the little triangle carries the letters **c, B, F, D** (across the top), **E** (just below), and **A** (at the apex). The legible hour-labels are:

- Roman hour-signs down the left side: **XIIII, XIII, XII, XI, X, IX, VIII**.
- Arabic hour-signs interspersed and on the right: **6, 23, 22** (left/lower) and **7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18** (center and right), with **12** on the meridian at bottom.

The two wings of numerals mirror one another about the meridian; several of the small numerals are faint. [Several labels [?]]

Now, since in all the aforesaid sundials the gnomon or stylus must be fixed in its proper place at right angles to the plane of the sundial, it will not be out of place to repeat briefly here, from our *Gnomonics*, what we wrote in Book 4, Proposition 12 about setting the stylus at right angles and about restoring it to its proper position should it happen to have been deflected from it.

The stylus, then, will be set at right angles and restored to its proper position as follows. From A — the place of the stylus on the meridian line of the horizontal sundial of this chapter — let a straight line AB be taken equal to the length of the stylus; and, drawing through B the straight line CD perpendicular to AB, so that the parts BC and BD are also each equal to the stylus, let the straight lines AC and AD be joined. Then, taking on AD a straight line AE that is likewise equal to the stylus, let a straight line CF be cut off equal to the interval CE. For if the stylus AB makes right angles with the plane of the sundial at A, the interval CF must be equal both to the interval between C and the tip of the stylus and to the interval between D and the tip of the stylus, as we demonstrated in the passage cited. But if the said intervals are not equal, the gnomon must be shifted back and forth, before it is fixed, until those intervals become equal; for then it will make right angles and will have its proper position.

The same must be done in other sundials, provided that the straight line AB, equal to the stylus, is taken on the line of the stylus, or on any other straight line drawn through the place of the stylus, in case the meridian line does not pass through the place of the stylus. We have set an example of this in the horizontal sundial of the hours from sunrise and sunset of this chapter, as appears in the triangle ACD. Whoever wishes for more about the place of the stylus, its setting, its size, and the elevation of the pole above the plane of the sundial, let him read Propositions 12 and 13 of Book 4 of our *Gnomonics*.

[Margin: Qua ratione stylus ad rectos angulos statuatur in horologio, & in proprium situm restituatur, si ab eo dimotus sit. — By what method the stylus is set at right angles in the sundial, and restored to its proper position if it has been displaced from it.]

THE DESCRIPTION OF SUNDIALS ON ANY SURFACE WHATEVER, BY MEANS OF THE SHADOW OF THE STYLUS OF SOME HORIZONTAL SUNDIAL OR OF ANY OTHER

CAPVT XIII.

Giovanni Battista Vimercati [Ioannes Baptista Vimercatus], in his booklet on the description of sundials, hands down a certain very easy and very simple method of describing sundials from the shadow of the stylus of a horizontal sundial, while the sun is shining, roughly as follows.

Once a horizontal sundial has been fixed — accurately drawn on some flat board, on which a straight line has been drawn corresponding to the meridian line of that horizontal sundial — let another plane be fixed on the same board. This second plane may either adhere to the board or lie parallel to it (if another horizontal sundial is to be described); or it may stand perpendicular to it, or inclined, and cut the meridian line of the board at right angles (if a vertical, equinoctial, or polar sundial, or one inclined to the horizon, is to be described); or it may cut the meridian line at unequal angles (if a sundial declining from the vertical is proposed for drawing, whether it is also inclined to the horizon or not); or, finally, it may not cut the meridian line at all, but make on the board a line parallel to it, or coincide with the meridian line itself (if a meridian sundial, or one declining from the horizon, is to be constructed) — always, however, keeping account of the declination and inclination of the sundial to be described.

Next, having taken the place of the stylus on the plane of the sundial to be constructed, let a gnomon of any size be fixed there, perpendicular to the plane of the sundial. Then, moving the board — together with the horizontal sundial fixed on it — back and forth, in some spot exposed to the sun so that it can indeed be lit by it, until the tip of the stylus's shadow on the horizontal sundial falls successively on the several hours of the tropic of Cancer [♋ — the summer solstice arc], let the several tips of the shadow of the [future sundial's] stylus be marked successively for the several hours on the future sundial; and in this way the points of all the hours of the tropic of Cancer will be found on the future sundial. And if the same is done on the tropic of Capricorn [♏ — the winter solstice arc], and on the equinoctial line, or on the arc of any other sign, or finally on any other line of the sundial (even one that is not an hour-line), and if the corresponding points on the future sundial are joined either by straight lines or by curves, as the case requires, the sundial will have been described on the proposed plane.

By this device any point whatever of the horizontal sundial can be transferred to the future sundial, if the board — together with the sundial fixed on it — is turned about until the tip of the shadow falls on a given point on the horizontal sundial, and, at the same moment of time, the tip of the shadow of the future sundial's own stylus is likewise marked on the future sundial. A sundial thus described, however, if it is to be transferred onto some wall, will have to be enlarged in proportion to the given length of the stylus, as we shall teach in Chapter 17; for a plane as large as is required for drawing a sundial on a wall cannot conveniently be turned about on the board.

By this art, from a horizontal sundial drawn by the foregoing precepts, another horizontal one can easily be laid out to any given size of stylus, even the smallest — something that can scarcely be accomplished by other precepts. Indeed, by the same device a sundial may be described on any non-flat surface, such as a concave one, if several points are marked on it for each hour and then suitably connected by lines that nowhere form angles. Add, too, that the hours can be laid out in this way even on a cylinder standing perpendicular to the horizon, as we set forth in Book 8 of the *Gnomonics*. In place of the horizontal sundial we shall be able to use, if one is available, a concave spherical sundial, which we taught how to construct in the same Book 8 of the *Gnomonics*; for on it the tips of the shadows can be marked much more accurately than on the former.

[Margin: Qua ratione in quavis superficie per umbram styli horologii alicuius Horizontalis horologia describantur. — By what method sundials are described on any surface whatever by means of the shadow of the stylus of some horizontal sundial.]

THE CONSTRUCTION OF SUNDIALS STANDING AT RIGHT ANGLES TO THE HORIZON, FROM A HORIZONTAL SUNDIAL

CAPVT XV.

Once some horizontal sundial has been accurately constructed, from it one can describe any other sundial that stands at right angles to the horizon — such as either kind of vertical dial, either kind of meridian dial, and any dial declining from the vertical — by an easy and agreeable operation, which here it is convenient to repeat from the scholium of Chapter 10 of Book 6 of our *Gnomonics*, omitting the demonstrations of the individual operations, since they can be sought from that scholium. And since in this matter there is very frequent use of perpendicular lines, the work will be made easier if, in drawing them, we

use some set-square — that is, an instrument consisting of two rulers joined together at a right angle with the greatest care.

Let there be, then, an Italian horizontal sundial, containing (of course) the hours from sunset of the Sun, on which...

Diagram. The large figure filling the center of printed p. 36 — an Italian horizontal sundial used to lay out a dial declining from the meridian toward the east by 30 degrees (the construction of this chapter). The four cardinal points are lettered around the edge: **ORTVS** (East, at top), **OCCASVS** (West, at bottom, printed inverted), **SEPTEN[TRIO]** (North, at the left, set sideways), and **MERID[IES]** (South, at the right, set sideways). Two long parallel rules cross the field from upper right to lower left; a small circle (the "great circle" to which the declining dial is parallel) sits near the center, from which a dense radiating web of solid and dotted hour-lines spreads out.

Confidently legible labels:

- Point letters (capitals): **H, G, D, L, F, E, N, C, K, A, B, P, M, Q, O, X, S.**
- Lowercase / Greek construction letters near the center: **ε, π, τ, α** (and other small letters that do not resolve).
- Hour-numerals around the rim: **23, 22, 21, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4.**

The construction points named in the text below (A, B, C, E, F, I) are among the central cluster. The figure is extremely crowded; many of the small central letters and numerals cannot be resolved individually, and their exact placement is uncertain. [Several labels [?]]

the stylus is AB, and its place is at A, and so on; and let it be proposed to describe from it a sundial declining from south toward east by 30 degrees. At A let there be constructed the angle BAC equal to the complement of the declination (all the construction lines ought to be made hidden except the hour-lines, the horizontal line, the meridian, the equinoctial, and the line of the stylus; indeed it would be better if the lines of the horizontal sundial too were hidden, so that the confusion of the multitude of lines might be avoided) — namely, such an angle as the great circle, to which the declining sundial is parallel, makes with the meridian. Now this angle BAC must incline toward south and west (the horizontal sundial being placed in its proper position) if the sundial to be described declines from south toward east, or from north toward west; but the same angle BAC must incline toward south and east, if the sundial to be described declines from south toward west, or from north toward east.

Then, on the straight line CA, at A, let a perpendicular AE be raised, making with the meridian AI the angle of declination EAI; and from it, toward the north, let a straight line AE be cut off equal to the stylus of the future sundial. Let EF also be cut off, which should be equal to the stylus AB of the horizontal sundial...

let there be drawn through E and F, parallel to CA, the lines EG and FH — that is, perpendicular to AE. Then FH will be the horizontal line in the declining dial, and F the place of the stylus, whose length is AE.

[Margin: The horizontal line, and the place of the stylus.]

We shall draw the meridian line as follows. From the point I, where the meridian line of the horizontal dial cuts the line EG, draw a perpendicular IK to EG; extended as far as you like in both directions, it will be the meridian.

[Margin: The meridian line.]

We shall find the centre of the dial on the meridian line IK in this way. From I, raise a perpendicular IL to the meridian line AI of the horizontal dial. Then, from A, the place of the stylus of the horizontal dial, describe a circle BMRQ with radius equal to the length of the stylus AB — used only for this purpose, that all lines drawn from A to the circumference of this circle (which, as will become clear, must often be drawn) may be equal to the stylus AB. From A, perpendicular to AI, draw the line AM as far as the circumference of the said circle, and toward the same side to which IL was drawn. Then, from N, the centre of the horizontal dial, draw the straight line NM through M; let it be produced so as to cut IL at L, and along the meridian, toward the horizontal line FH, mark off IK equal to the line IL. For K will be the centre of the declining dial, in such a way that the upper parts of the dial extend from the horizontal line FH toward K, and the lower parts toward I.

[Margin: The centre of the dial.]

The straight line KF, drawn from the centre K of the dial through F, the place of the stylus, will be the line of the stylus. Although in the present example it is nearly parallel to the meridian line AI of the horizontal dial, and therefore cuts the equinoctial line at almost right angles, you must not suppose on that account that this necessarily happens in every declining dial; it happened by chance in our example. In other declining dials you will find the contrary.

[Margin: The line of the stylus.]

Next, the straight line OP, drawn through the point O — where the equinoctial line of the horizontal dial cuts the line EG — perpendicular to the line of the stylus KF, will be the equinoctial line. In our example it does not, to the senses, differ from the equinoctial line of the horizontal dial, because, as we said, the line of the stylus cuts the equinoctial line of the horizontal dial at nearly right angles — which will not happen in other declining dials.

[Margin: The equinoctial line.]

To make the description of this equinoctial line more accurate, we shall find on the meridian line KI — produced toward I — the point P through which it must pass, as follows. From A and I, toward opposite sides, raise two perpendiculars AQ and IL to the meridian line AI of the horizontal dial. From Q, where AQ cuts the circle BMRQ, draw through R — where the meridian line of the horizontal dial cuts its own equinoctial — the straight line QR, cutting IL at S. Then, if on the meridian line KI produced a segment IP is cut off equal to IS, the equinoctial line must be drawn through O and P, and it will make right angles with the line of the stylus KF, provided no error has been made.

Indeed, if you wish, we shall also find yet another point H, on the horizontal line FH, through which it must likewise pass, in this way. From A, raise a perpendicular AV to the meridian line AI of the horizontal dial — that is, a line parallel to the equinoctial of that same dial (which will easily be drawn if OV is taken equal to TA, for then the line AV will be parallel to TO) — cutting EG at V. For the perpendicular VH to EV will give on the horizontal line the sought point H. We shall, however, detect it more easily thus: draw from N, the centre of the horizontal dial, the perpendicular to the meridian AI — that is, the line of the 6th hour from noon or from midnight — and let it cut the line GE at X. Then the straight line KX will be the line of the 6th hour in the declining dial, and it will cut the horizontal line FH at H. Through this point H the equinoctial line must be drawn, since the 6th-hour line and the equinoctial line always intersect the horizontal line at the same point.

By the same device with which we described the equinoctial line in the declining dial, we shall lay out all the hour-lines.

[Margin: The hour-line.]

To make this clearer, suppose, for example, that the 23rd hour from sunset is to be drawn in the declining dial. Since in the horizontal dial this hour-line does not cut the line GEV — for one must always consider at what point the horizontal dial's line (the one to be drawn in the declining dial) meets GEV, since it is

through that point that the line must be drawn in the declining dial — we shall produce it until it cuts GEV at the point G, through which the 23rd hour must be drawn. To have another point of the same hour on the horizontal line FH, we shall draw through A, the place of the stylus, a line Aa parallel to the 23rd-hour line DG of the horizontal dial, and let it cut GEV at a. (This parallel will easily be drawn if Ga is taken equal to DA, and so on.) For a perpendicular aY drawn to GEV — that is, parallel to EF or IZ — will give on the horizontal line the point Y, through which the same 23rd hour must be drawn.

In the same way, the 22nd hour, produced until it cuts GEV at b, must be drawn through b. Then, taking bf equal to the line eA, so that the drawn line Af is parallel to the 22nd-hour line, and drawing fd perpendicular to GEV — or parallel to EF or IZ — the same 22nd hour must also be drawn through the point d. Again, the 15th hour must be drawn through the point i — where the 15th hour of the horizontal dial cuts GEV — and also through the point g on the horizontal line, onto which falls the line hg, drawn perpendicular to GEV from the point h, onto which in turn falls Ah, drawn parallel to the 15th-hour line of the horizontal dial.

In just this way we shall lay out, in the declining dial, any line whatever of the horizontal dial — even one that is not an hour-line — provided that, when produced, it cuts the line GEV.

[Margin: A general rule for describing any line whatever in the declining dial from the horizontal dial.]

For every such line must be drawn through the point where it cuts the line GEV, and through the point on the horizontal line onto which falls the perpendicular to GEV — the perpendicular raised from that point of GEV onto which falls the line drawn from A, the place of the stylus, parallel to the line that is to be described. Moreover, all the lines of the hours from noon and from midnight will pass through K, the centre of the declining dial. Hence it is enough if they are drawn through the centre K and through those points at which they meet the line GEV in the horizontal dial. They will, however, be drawn more precisely if for each one a third point is also found on the horizontal line.

Lastly, on the lines described in the declining dial from the horizontal dial, we shall find the points through which the arcs of the [zodiacal] signs are to be drawn, as follows.

[Margin: The points of the arcs of the signs.]

Suppose, for example, that on the 22nd-hour line db the point of the arc of Capricorn [♑] is to be found. From A, the place of the stylus of the horizontal dial, through the Capricorn point at hour 22, draw the hidden line A22, cutting the line GEV at the point m; from m raise a perpendicular ml to GEV, cutting the 22nd-hour line db at l. For through the point l the arc of Capricorn must be drawn, so that, when the Sun is at the beginning of Capricorn, the tip of the stylus's shadow falls at the point l.

We shall find this point likewise, even if the 22nd-hour line has not yet been drawn, in this way. From m, where the hidden line A22 cuts the line GEV, raise a perpendicular mn to A22m, on whichever side you please; and from A, the place of the stylus, raise to the same A22m another perpendicular Ap, extending as far as the circumference of the circle, but toward the side opposite to that on which mn was drawn. For from p, through the Capricorn point at hour 22, the line p22 is drawn, cutting the perpendicular mn

at n; and if from the line mn we cut off a segment equal to the earlier perpendicular ml, the point l will be the Capricorn point at hour 22 that is sought. And so, once the two points d and b — through which the 22nd hour must be drawn — have been found, as was explained above, if this third point l, belonging to Capricorn, is also found, the 22nd hour will be drawn most precisely through the three points d, b, l, provided those three points lie on a straight line, as the case requires. Otherwise, do not doubt that an error has been made. But if a point of Cancer [♋], or of some other sign, is also found for the same hour, we shall have four points through which the 22nd hour must be drawn. This fourth point, belonging to Cancer, cannot be found for the 22nd hour in our example, because the hidden line drawn from A through the Cancer point at hour 22 of the horizontal dial cannot cut the line GEV, as is evident. This is proof that the plane of the declining dial is not illuminated at the 22nd hour when the Sun is at the beginning of Cancer.

By this same procedure we shall project any point whatever of the horizontal dial onto the declining dial, in such a way that, at the very moment when the shadow of the horizontal dial's stylus falls on that point, the shadow of the declining dial's stylus falls on the projected point.

[Margin: A general rule for projecting any point whatever of the horizontal dial onto the declining dial.]

For if through the given point in the horizontal dial a hidden line is drawn from A, the place of the stylus, cutting the line GEV — for if it does not cut it, the

proposed point cannot be transferred onto the declining dial — such as was the line A22m for transferring the Capricorn point of the 22nd hour; and if from the point where GEV is cut a perpendicular is raised to GEV, on the side of A, that is, below the line GEV in the declining dial (for the horizontal line FH occupies the highest place in it)

Diagram. The large construction figure filling the centre of printed p. 38 — the declining-dial layout of this chapter (the same crowded construction shown on printed p. 36 in the preceding pages, reprinted here so it stays before the reader). The four cardinal points are lettered around the field: **ORTVS** (East, at the top), **OCCASVS** (West, at the bottom, printed inverted), **SEPTEN[trio]** (North, at the left, set sideways), and **MERID.** (South/Meridies, at the right, set sideways). Two long parallel rules cross the field from upper right toward lower left; a small circle (the "great circle" to which the declining dial is parallel) sits near the centre, from which a dense radiating web of solid and dotted hour-lines spreads out.

Confidently legible labels:

- Capital point-letters: **G, D, L, E, F, N, C, K, A, B, P, M, Q, R, O, X, S, H, T, V, Y.**
- Small Greek construction-letters near the centre: **ε, π, τ, α** (other small letters do not resolve).
- Hour-numerals around and within the figure: **23, 22, 21** (upper region, near G/D/L) and **16, 15, 14, 13** along the lower edge (13 beside H); **19** and a further numeral near the centre are only partly legible.

The figure is extremely crowded; many small central letters and interior numerals cannot be resolved individually, and their exact placement is uncertain. [Several central labels [?]]

if the given point lies between A and the line GEV — as, for the Capricorn point at hour 22, the perpendicular ml did — [then the perpendicular is dropped toward A]; but toward the side of the horizontal line FH, that is, above the line GEV in the declining dial, [the perpendicular is raised] if the line GEV lies between A and the proposed point, as do the Capricorn points of hours 16, 17, and 18 in our example. Then, to this hidden line, from that same point of the line GEV and from A, let two other perpendiculars be raised toward opposite sides — such as were mn and Ap. And from the point where the circumference of the circle is cut by the perpendicular drawn from A, let a hidden line be drawn through the proposed point; it will cut the other perpendicular — the one raised from the point of the

line GEV — at a point. The distance of this point, taken with the compass from that same point of GEV and transferred onto the perpendicular raised from the same point to the line GEV, will give the sought point in the declining dial — such as was the point l for the Capricorn point at hour 22. By this technique, in order to find the point s, belonging to Cancer, on the line gi of the 15th hour, we drew, in our example, from A, through the Cancer point at hour 15,

the line Aq of the horizontal dial's 15th hour. At q, where it meets the line GEV, we raised the perpendicular qr; and to the same Aq, at A, on the opposite side, we raised another perpendicular At. From t, through the Cancer point at hour 15, we drew the line tr, which cuts qr at r; and finally, from the line qr we cut off a segment qs equal to qf, the perpendicular drawn from q to GEV. For the tropic of Capricorn, however, no point can be found at the 15th hour, because at that hour the tropic of Capricorn has no point in the horizontal dial. Therefore the 15th hour must be terminated on the horizontal line at the point g. But if some given point lies on the line GEV, it will be common to both dials — namely, the horizontal and the declining — so that no further transfer is needed.

Now, this transfer of the horizontal dial's points onto the declining dial can also be carried out even without raising two perpendiculars to the line drawn from A through the given point — one at A, equal to the stylus, such as Ap was for the Capricorn point of hour 22, and the other from the intersection [of that line with GEV], toward the opposite side, such as mn was. For if on the line CD, toward either side, Aε is taken equal to the stylus, and from ε, through the given point of hour 22 in Capricorn (for example), a line is drawn cutting GV, then mε will be equal to mn; and this, transferred onto ml, will give the sought point l. For since A22 is to Ap as m22 is to mn, then, by alternation, A22 will be to m22 as Ap is to mn.

[Margin: (a) Euclid VI.4]

By the same reasoning, A22 will be to m22 as Aε is to mε. Therefore the stylus Ap (or Aε) has the same ratio to mn as to mε; and consequently mn and mε are equal.

[Margin: (b) Euclid V.9]

And so, by this method, we are freed from the two perpendiculars Ap and mn — which is no small saving.

If a dial declining from the north toward the west is to be described, one must take on the line AE a segment AE equal to the stylus of the future declining dial, toward the south, and beyond it another segment EF equal to the stylus of the

horizontal dial. Then through E and F draw EG and FH parallel to AD; of these FH will be the horizontal line, and the lower parts of the dial will slope from it toward A. The rest is to be completed just as in the dial declining from the south toward the east.

[Margin: A dial declining from the north.]

Furthermore, if a vertical dial is to be described that does not decline but is parallel to the prime vertical, one must take on the meridian line of the horizontal dial — beginning at the place of the stylus A — a segment equal to the stylus of the future vertical dial, toward the north if the dial is a southern one, but toward the south if a northern one is to be constructed. Beyond this, take another segment equal to the stylus of the horizontal dial, marking two points on the meridian line; drawing two perpendiculars to the meridian through these points, the more distant one will give the horizontal line, and so on.

[Margin: A vertical dial.]

Finally, if a meridian [due-east or due-west] dial is to be composed, one must take, on the line drawn from A (the place of the stylus) perpendicular to the meridian line AI, the length of the stylus for the future dial — toward the west if an eastern dial is to be described, but toward the east if a western one; and beyond this another segment equal to the stylus of the horizontal dial, marking two points on that perpendicular. If through these two lines parallel to the meridian are drawn, the more distant will be the horizontal line, and so on.

HORARVM A MERIDIE ET MEDIA NOCTE IN QVOLIBET PLANO — A far easier description of the hours from noon and from midnight, on any plane whatever, whether equidistant from the horizon or perpendicular to it, by means of a plane rule divided into certain parts and used as an instrument.

CAPVT XVI.

When, in recent days, the most illustrious Lord Jacobus Curtius [Jacob Kurz von Senftenau] — privy councillor of his Imperial Majesty Rudolph II, and now the same emperor's legate to the Supreme Pontiff Sixtus V, a man most distinguished both in every branch of learning and especially in mathematics — was conversing familiarly with me, in his customary kindness and affability, about

various matters pertaining to the mathematical disciplines, to my great delight, he shared with me many excellent things of this kind. Among them was this: that any plane rule whatever can be divided into parts in such a way that, by means of it, as by a very simple instrument, the hours from noon and from midnight can be described most easily — both on a plane parallel to the horizon and on any other perpendicular to the horizon. Since this greatly pleased me, I resolved to explain briefly, in this place, the division of that rule and its use, so that from these records of mine, such as they are, some grafted fruits of that outstanding genius might also be plucked.

Let there be, then, a plane rule ABCD of any length (a length seems to me sufficient for this purpose if it equals half a foot, roughly as in the example proposed here). Near its outermost edges AB and DC let two parallels, not far from them, be drawn: EF and GH. Of these, the first, EF, is to be divided into hours; the second, GH, into degrees of the pole's altitude, in the following way.

[Margin: The division of the plane rule into parts, most suited to the description of the hours from noon and from midnight.]

With centre I, describe an arc of a circle KL. From it cut off — with the same compass-opening with which the circle was described — the sixth part KM; bisect it at N, and take the arc ML equal to the arc MN, so that the quadrant KL, containing 90 degrees, results, since the arc KM contains 60 degrees and ML contains 30. Then, dividing the arcs KN, NM, and ML each in two, and each of those parts into five equal parts, and finally each of these into three equal particles, the whole quadrant KL will be cut into 90 degrees. Here the last division has not been carried out, on account of the narrowness of the space; instead, each part comprises three degrees.

Next, describe another quadrant of the circle, OP; and by drawing straight lines through every fifteen degrees of the arc KL, the quadrant OP will be cut into six equal parts, which represent the six hours of the equator. These being subdivided into half- and quarter-hours, draw the straight line QR perpendicular to the radius IK, at such a distance from the centre I that its portion QR — intercepted between Q and the line drawn from I through the 75th degree of the arc KL (that is, through the 5th and 7th hour of the arc OP) — is a little smaller than each of the two parallels EF and GH, or at any rate equal to them. This line QR will be cut, at points, by the hidden hour-lines sent out from I. All these points, transferred with the compass — beginning from Q, onto the parallel EF, starting from the point E — will give on the rule the hours, and the half- and quarter-hours, from 12 up to 5 after noon and 7 after midnight.

But so that $5\frac{1}{4}$, $5\frac{1}{2}$, $5\frac{3}{4}$, 6, and $6\frac{1}{4}$, $6\frac{1}{2}$, $6\frac{3}{4}$ — which are not comprised on the line EF — may be described on the proposed plane, we shall take IS, a quarter of the line IQ, and through S draw ST perpendicular to IQ (that is, parallel to QR). We shall divide it by the straight lines sent out from I through the hours and parts of hours of the arc OP, just as the line QR was divided; and we shall transfer the parts of the line ST, starting from the point V, onto a line drawn on the other face of the rule ABCD, parallel to the edges of the rule.

[Margin: In what way the more distant points may be marked on the rule more accurately.]

But if the interval ST — up to the $5\frac{3}{4}$ hour from noon, or the $6\frac{1}{4}$ hour from midnight — were longer than the rule, one would have to draw YZ parallel to QR through Y, the point that bisects IS, so that IY is an eighth part of the line

(continuing Cap. XVI, on the plane dividing-rule for laying out the hours from noon and from midnight)

an eighth part of the line IQ. And if the interval YZ, up to the $5\frac{3}{4}$ hour from noon, were still longer than the rule, IY would again have to be bisected, and a parallel to QR drawn through its midpoint, and so on. In our example it would be enough to take IS, a quarter of the line IQ; but since the line drawn from I through the $5\frac{3}{4}$ hour from noon cuts the line ST very obliquely and runs out too far — so that the point T can scarcely be distinguished without error — we shall subdivide IS continually in half. Then, through the point Y (which here cuts off, and this is sufficient, IY, half of IS), we shall draw YZ parallel to ST, and it will be conveniently cut at Z by the line drawn from I through the $5\frac{3}{4}$ hour from noon.

For if we repeat the line YZ on ST as many times as IY is contained in IS — that is, in the given example, twice — we shall arrive at the point T, onto which the said line drawn from I through the $5\frac{3}{4}$ hour from noon necessarily falls; and thus the interval ST will be found more precisely. This interval, however, we have not here transferred onto the line VX, on account of its shortness. From this line VX, then, we shall describe on the proposed plane the hours that are missing on the line EF, as we shall teach below in the use of this rule.

We shall use the same device in precisely determining the point R on the line QR, if it happens to be cut too obliquely by the line IR — the point onto which the line IR, drawn from I through the 5th hour from noon, falls. That is: we shall repeat four times on QR the portion of the line ST between S and the said 5th hour from noon, since IS is a quarter of the line IQ; or we shall repeat eight times on QR the portion of the line YZ between Y and the said 5th hour from noon, since IY is an

eighth part of IQ. And by the same reasoning the longer intervals of the line ST quadrupled, or of the line YT octupled, transferred onto the line QR, will exhibit more accurately the points $4\frac{1}{4}$, $4\frac{1}{2}$, and $4\frac{3}{4}$ from noon.

Diagram. The plane dividing-rule instrument for laying out hour-lines. On the left is a quadrant with its centre at the upper-left, from which a dense fan of hour-lines radiates down to a graduated arc; the arc carries hour-numbers 1 through 6 and its degree-subdivisions. On the right is the rectangular rule ABCD, along whose length run several parallel graduated scales (numeric divisions rotated 90°), each headed by a rubric. The scales are printed at very small size and the individual numeric divisions do not all resolve even under magnification.

Readable labels (left construction / quadrant):

- Upper-left corner: I (centre of the radiating hour-lines)
- Along the top edge: O, P, K [?]
- On or near the arc: L, M, N
- Hour-numbers along the arc: 1, 2, 3, 4, 5, 6 (with intervening degree ticks)
- Lower termini of the radiating lines: Q, R
- Lower-left: T

Readable labels (right-hand rule ABCD and its scales):

- Corner/edge letters: K, G, F [?], C, D, B, X
- Scale-column rubrics (rotated), reading across:
 - H. à. mer. [Horæ à meridie — hours from noon]
 - H. à. med. noc. [Horæ à media nocte — hours from midnight]
- Horizont[ales] [for horizontal dials]
- Vertic[ales] [for vertical dials]
- Horizontales
- Verticales
- Hor. à. mer.
- H. à. med. noc.
- Scale-point letters: V, N, Q, H
- Legibly-read scale numbers (not exhaustive; the fine interior divisions do not all resolve): 90, 84, 81, 78, 72, 63, 60, 42, 36, 30, 27, 24, 21, 19, 18, 12, 9, 6, 3, with hour-fractions such as $6\frac{1}{2}$, $5\frac{3}{4}$, $5\frac{1}{2}$, $5\frac{1}{4}$ printed among them. The complete cell-by-cell scale values are not individually legible even under magnification.

[The figure is dense and several small labels are uncertain]

Having in this way divided the line EF into hours and parts of hours, we shall distribute the line GH into pole-altitudes in the following way. Drawing lines from I through each single degree of the quadrant KL, let their intervals — those comprised between I and the line QR — be transferred with the compass onto the line HG from the point H. Toward the inner part, toward EF, let the complements of the degrees (through which the lines transferred from I are drawn) be marked; and toward the outer part, toward the side CD, let the numbers of the degrees themselves — those whose lines have been transferred — be inscribed. The inner numbers will serve for horizontal dials, the outer for vertical dials and for those declining from the vertical, as we shall say below. But since in this way the line GH contains only the pole-altitudes from 90 degrees down to 15 for horizontal dials, and from 0 up to 75 for vertical and declining dials, then, so that we may designate the remaining degrees on the proposed plane, we shall also draw on the other face of the rule ABCD a line a b, parallel to the edges of the rule, and...

and onto it, from the point b, we shall transfer the portions of the lines drawn from I through each single degree, laid off between I and the line ST, as you see done in the example, up to the 6th degree for horizontal dials, and up to the 84th degree for vertical and declining dials. But if some of the lines drawn from I cut the lines QR, ST very obliquely, their points on the said lines will have to be found by the device prescribed a little earlier — namely by doubling on the line ST the intervals of the line YZ (between Y and the lines coming from I), and octupling those same intervals on the line QR; and by quadrupling the intervals of the line ST on the same line QR, and so on. By means of this line a b, then, we shall mark on the given plane the degrees of pole-altitude that are missing on the line GH, as will be clear in use.

If one wishes, two other smaller parallels c d, e f may be drawn in the middle of the rule; and onto c d the points of the hours of the line YZ may be transferred, beginning from the point c, while onto the line e f, from the point f, the intervals of pole-altitude between I and the line YZ. For by means of these lines c d, e f we shall lay out even the smallest dials.

But since the lines between I and the line QR, near Q, exceed one another only by a small excess — being nearly equal to one another — it comes about that they can scarcely be transferred onto the line HG without confusion. We shall therefore have done rightly if we describe those nearly-equal intervals separately in some place. Thus you see that from the point H four lines have been drawn separately, onto which are transferred the segments of the lines from I through

degrees 5, 6, 9, 12, intercepted between I and the line QR; and to these lines coming out from H are attached the complements of the said degrees, namely degrees 87, 84, 81, 78, for horizontal dials. For vertical dials the degrees themselves are to be set opposite — as 3 next to 87, and 6 next to 84, and 9 next to 81, and 12 next to 78, and so on. And the same device can be applied to other lines as well, as in a b and e f.

Diagram. The construction of a horizontal dial for latitude 60°. From a radiant apex at the top a broad fan of straight hour-lines spreads out to left and right, its outer rays numbered. A long roughly-horizontal line (the equinoctial line CD) crosses the figure through the centre E, carrying hour-numbers. Below and around the centre A a further set of solid and dotted lines gives the hour-lines of the dial-plane and the transferred construction points. (This same woodcut is reprinted on the following page — see fig-p618-1.)

Readable labels:

- Apex cluster (top centre): a small triangular gnomon-mark with G beneath it, and nearby small letters C [?], F [?], b [?]
- Outer rays (left, descending): 6, 5, 4
- Outer rays (right, descending): 5, 6, 7, 8
- The crossing line (equinoctial CD), left to right: C, 3, 2, 1, E (centre), then above it 12 and the point O, the point H (dotted), then 11, 10, 9, and D at the right end
- Centre and interior points: A (the dial-centre, below the equinoctial line), I (below A), M (below I), X, E (lower centre)
- Left-hand descending labels: K, 10, T, Q [?] (small "o"), N, II, B, G, I, "1 2" (= 12)
- Right-hand descending labels: X, 8, L, 7, 6, V, S, s, e, D, 4, 3
- Lower interior points: R, P, I, 2, and Y at the foot

[Several small apex and side labels are uncertain]

[Margin: The use of the preceding plane rule in describing dials.]

[Margin: The horizontal dial.]

equal to the line IQ, as we shall demonstrate a little below in the horizontal dial. In the same way we shall cut the lines VX, a b, and likewise c d, e f, into more parts, if we take IS equal to the line between V and the 3rd hour, and also the line IY equal to the line between c and the 3rd hour, and draw through S and Y lines parallel to QR, and so on.

The use of the aforesaid rule is as follows. Let it be required, in the first place, to describe a horizontal dial for latitude 60 degrees. Draw two lines AE and CD cutting each other at right angles at E — of which let AE represent the meridian line, and CD the equinoctial line. Transfer onto CD, on both sides from E, all the hour-points of the line EF, beginning each time from the point E. This may be done either with the compass, or by the service of the rule itself, the point A being set so that it answers at right angles to the point E — that is, with the point E and the rule's edge AB coinciding with the line ED. For if the hour-points of the side AB — all of which answer at right angles to the hour-points of the line EF — are then marked on the line ED, the said hour-points of the rule ABCD will have been transferred onto the line ED. These points will be marked more precisely if, with the rule's plane set transverse (at right angles) to the plane of the dial in that position, perpendiculars to the side AB are drawn from the hour-points of the side AB: for these will fall onto the points that are to be marked on the line ED. Let the same hour-points of the line EF then be transferred in the same way onto the line EC, beginning in both cases from E. This will be done with the help of the rule, if the rule is turned over so that the side AB becomes the lower one, and the point A coincides with the point E, and the side AB with the line EC, and so on. After this, let the portion of the line GH, comprised between H and the inner degree of the latitude of the place for which the dial is being constructed, be transferred onto the meridian line from E up to A. This will be done without the compass, by means of the rule, if its point C coincides with the point E, and the side CD with the line EA, and so on — provided, however, that the same degrees have been marked on the side CD, and also on the transverse plane, as has been said concerning the hour-points...

Diagram. The same construction of the horizontal dial for latitude 60°, reprinted from the facing page. This is the identical woodcut as fig-p617-1: the radiant fan of hour-lines from the top apex, the equinoctial line CD through the centre E, the dial-centre A, and the surrounding solid and dotted construction lines and transferred points. The readable labels match fig-p617-1 (apex with G and small letters C/F/b [?]; outer rays 4–6 left and 5–8 right; the crossing line C, 3, 2, 1, E, 12/O, H, 11, 10, 9, D; interior points A, I, M, X, E, R, P; left-hand K, 10, T, Q [?], N, II, B, G, "1 2"; right-hand X, 8, L, 7, 6, V, S, s, e, D, 4, 3; foot Y).

[Reprinted woodcut; several small labels uncertain]

as has been said concerning the points to be marked on the plane AB and on the transverse plane. For the point A will be the centre of the dial, and the lines drawn from A through the points of the equinoctial line CD will give the hours

from noon and from midnight. In our example, which is constructed for latitude 60 degrees, the line AE has been taken equal to the portion of the line GH between H and the inner 60th degree. The hours to the left of the meridian line are to be counted from noon, and those to the right of it from midnight.

But if from AE one cuts off AF equal to the portion of the line a b on the rule between b and the 60th degree, and through F draws a line parallel to CD, and transfers onto it the hour-points of the line VX, then from A, through the points of this parallel, the same hour-lines will be drawn — indeed more of them than through the points of the line CD. And if there is still cut off from the me-[ridian...]

—drawn from the meridian line AG, equal to the portion of the line ef lying between f and the same 60th degree on the rule; and if through G a line is drawn parallel to CD, and onto it are transferred the hour-points of the line cd, then still further hour-lines will be drawn from A through the points of this parallel — indeed more of them than through the points of the parallel drawn through F. And so, if these two parallels through F and through G are drawn, all the hours and their subdivisions can be conveniently laid out on the dial.

But if the plane of the dial is very large, so that the line CD is extremely long, then the points of all the hours — even of those not drawn on the line EF — can be marked on it, without our drawing the parallels through F and through G, in the following way. Take, with the compass, the intervals of those hours that are not contained on the line EF, measuring them off on the line VX starting from the point V, and lay them off four times over on the line CD, beginning on both sides from the point E. Or else take the same intervals on the line cd and lay them off eight times over on the same line EF, and so on. For the intervals of the line EF are quadruple the intervals of the line VX, and octuple the intervals of the line cd, as is clear from the construction.

That in this way a horizontal dial is correctly described, we shall now easily demonstrate as follows. [Margin: A demonstration of the use of the aforesaid rule.] Since the line QI in the earlier figure is equal to the portion Ql of the line QR intercepted between Q and the line drawn from I through 45 degrees — that is, through the 3 o'clock or 9 o'clock hour — (because, the angle QIl being a half-right angle, the angle QIl is also a half-right angle [a], and therefore the lines QI and QI[?] are equal [b]); it follows that if, from the point E of the later figure, an arc is described at the interval QI, or QL — that is, at the portion of the line EF on the rule lying between E and the 3 o'clock or 9 o'clock hour — and this arc is cut at H by another arc described from A at the interval of the portion Qh of the line QR, lying between Q and the complement of the pole's altitude at the horizon in

question; and if the lines EH and AH are then drawn: then the triangle AEH of the later figure will be entirely equilateral and equiangular with the triangle hIQ of the earlier figure. [Margin: a: Euclid, *Elements* I.32. b: Euclid, *Elements* I.5.] That is, the side AE will equal the side hI, the side EH the side IQ, and the side AH the side Qh; the angle E will equal the angle I, the angle A the angle h, and the right angle H the right angle Q.

Now since I is the angle of the complement of the pole's altitude — for onto the intervals carried over onto the line GH from the point I we inscribed the complements of those degrees through which the lines drawn from I pass, for horizontal dials — the angle E will likewise be the angle of the complement of the pole's altitude. Therefore, if the triangle AEH is revolved about the meridian AE until it stands at right angles to the horizon, and the dial is set in its proper position, with A pointing south and E pointing north, then AH will be the axis of the world and HE the common section of the equator and the meridian. For since the axis of the world makes with the meridian line of the horizontal dial the angle of the pole's altitude, while the equator makes the angle of its complement, let the line EB be taken equal to the line EH, and from B let a circle be described and divided into 24 [hours], through the divisions of which the hidden lines are drawn out; these cut the equinoctial line CD in the points through which the hour-lines are to be drawn from A. And it is evident that CD is cut by the lines drawn from B in just this way, on account of the equality of the lines EB and IQ, and so on — as QR is cut by the lines drawn out from I in the earlier figure, on account of the equality of the same lines, which we demonstrated in *Gnomonics* Book II, Proposition 1.

A different general explanation of the rule set out above. [Margin: A different general explanation of the rule set out above.] From this it follows that the hour-points of the line EF on the rule are nothing other than the sections of the hour-circles and of the equinoctial line in a horizontal dial, taken with the semidiameter of the equator set at IQ; whereas the portions of the line GH, between H and the inner degrees, are the semidiameters of the horizon for the various pole-altitudes, always with respect to the same semidiameter of the equator QI — which is indeed equal to the segment of the line EF lying between E and the 3 o'clock or 9 o'clock hour.

The style (gnomon). [Margin: The style.] If from H a perpendicular HI is dropped to AE, then HI will be the length of the style, and its place will be at I.

But since the rule does not contain the intervals of the line QR between Q and the lines drawn from I through the individual degrees, we shall construct the triangle

AEH on the dial itself, in order to find the place of the style and its length, in this way. Bisect AE at b, and describe a semicircle EHA; from b, at the interval bA or bE, if the line EH — equal to the segment of the line EF on the rule between H and the 3 o'clock or 9 o'clock hour — is fitted within it, then AH will be the axis of the world. For the angle AHE, being in a semicircle, is a right angle [c] — which the axis necessarily makes with the equator. [Margin: c: Euclid, *Elements* III.31.] Or else let two lines be drawn out from A and from E, the lines AH and EH, meeting each other at H, of which the one with AE forms the angle of the pole's altitude and the other the angle of its complement.

The vertical dial. [Margin: The vertical dial.] In just the same way we shall describe a vertical dial for a proposed pole-altitude, if in place of the segment of GH between H and the inner degree of the pole's altitude we take the segment of that same GH between H and the outer degree of the pole's altitude for the place where the dial is being constructed; or (what comes to the same thing) the segment of that same line GH between H and the degree of the complement of the pole's altitude interior to it, and cut off from the meridian a line EA equal to it, so as to have A, the centre of the dial. For since, in the earlier figure, the line Ih, through 30 degrees, was made the semidiameter of the horizon for 60 degrees with respect to the semidiameter of the equator IQ, as we showed, — because the angle h, which subtends 60 degrees, gives the line Ip through 60 degrees, that is, through the complement of the 30 degrees — Ip will be the semidiameter of the vertical dial with respect to the same semidiameter of the equator IQ. (For with IQ set as the semidiameter of the equator, QR will be at right angles to it, and Ip, forming right angles with QR — the axis of the world — establishes the angle p, of the complement of the pole's altitude, namely the very angle that the vertical makes with the axis, because Ip is the angle of the pole's altitude, taken as 60 degrees, and so on.) The same reasoning holds for the other pole-altitudes. For the intervals of the line GH on the rule, between H and the outer degrees, are always the semidiameters of vertical dials, just as the intervals between H and the inner degrees are the semidiameters of horizontal dials — which is demonstrated in the same way, as was said of the lines Ih and Ip in the earlier figure. The hours from noon, however, are contained in a vertical dial to the right of the meridian line, and those from midnight to its left.

Now the dial just laid out, if this alteration of the hours is made, will become a vertical dial for latitude 30 degrees — since the line EA is equal to the segment of the line GH on the rule between H and the outer 30th degree, namely the same segment that was taken in the inner degrees for a horizontal dial at 60 degrees. From this it follows that a horizontal dial is a vertical one for the complement of

the same latitude, and conversely: because any inner degree that serves for horizontal dials is the complement of the outer degree that serves for vertical dials, and conversely; so that the very same line EA is to be taken for the horizontal dial of one place, and for the vertical dial of another place whose pole-altitude is the complement of the pole-elevation at the former place.

The declining dial. [Margin: The declining dial.] Let it be required, next, to lay out a dial declining from south toward the west by 60 degrees, for latitude 30 degrees. Having drawn two lines AE and CD cutting each other at right angles at E — of which let AE, perpendicular to the horizon, represent the meridian line, and CD, parallel to the horizon, the common section of the plane of the declining dial with the plane of the horizontal dial — let there be formed below CD the angle of declination BEF, on the right of the meridian line if the dial declines from south toward the west, but on the left if toward the east. Let the line GH cut EF at E at right angles, and onto GH transfer from the point E, on both sides, the hour-points of the line EF by means of the rule described above, as was said in the construction of the horizontal dial. Then on EF take EF equal to the segment of the line GH lying between H and the inner degree of the proposed latitude, just as in the description of the horizontal dial — here, between H and the inner 30th degree; and from F, drawing out hidden lines through the points of the line GH, mark their intersections with the line CD. Finally, on EA take EA equal to the segment of the same line GH lying between H and the outer 30th degree, or between H and the complement of the proposed latitude in the inner degrees — here, between H and the inner 60th degree — as was said in the construction of the vertical dial.

For the point A will be the centre of the dial, from which the lines drawn out through the points marked on CD will indicate the hours from noon and from midnight. For, as is clear from what was demonstrated a little earlier, those hidden lines drawn out from F make a horizontal dial, in which the equinoctial line is GH. If, then, the plane drawn through the lines EF and GH is conceived to be moved until it is parallel to the horizon, the hour-circles will cut the line CD — lying in the plane of the declining dial — in the points at which the hour-lines drawn out from F meet it. Since, therefore, A is the centre of the declining dial, just as it is of the vertical dial — (for at the same point the axis of the world meets the common section of the meridian and of the vertical circles, as is evident) — it is clear that the lines drawn from A through the points marked on the line CD are the hour-lines; and their order here is the same as in the vertical dial.

But so that all the hours may be conveniently drawn, one must cut off from AE the line AI, equal to the segment of the line ab on the rule described above, between b and the outer 30th degree of the proposed latitude, or the inner 60th degree of its complement. For if through I one draws the line KL parallel to CD, and forms the angle of declination EIM as before, and the line NO cuts IM at right angles at I, onto which the hour-points of the line VX are transferred on both sides from I; and if, finally, the line IM is taken equal to the segment of the line ab, between b and the inner 30th degree of the given latitude, as in the horizontal dial: then the lines drawn out from M through the points of the line NO, cutting across the line KL, will meet it in points through which the lines drawn out from A will give the hours, as before. And if this is not enough, one must in the same way employ the lines cd and ef, divided on the same rule described above, and so on.

The 6 o'clock hour. [Margin: The 6 o'clock hour.] The 6 o'clock hour is drawn thus. From F raise a perpendicular to EF, or from M a perpendicular to IM. For where the former cuts the line CD, and the latter the line KL, through that point the line of the 6 o'clock hour is to be drawn.

The style-line. The equinoctial line. The axis of the world. The altitude of the pole above the declining dial. The style and its place. The meridian dial. [Margin: The style-line. The equinoctial line. The axis of the world. The altitude of the pole above the declining dial. The style and its place. The meridian dial.] Now if from F a perpendicular is dropped to CD, cutting CD at D, the line AD thus drawn will be the style-line; and if through D a perpendicular PQ is raised to it, this line will be the equinoctial line in the declining dial. And if on the equinoctial line a line DP is taken equal to DF, whether above D or below, then AP thus drawn will be the axis of the world, and DAP the angle of the pole's altitude above the plane of the dial. Moreover, if DR is drawn from D perpendicular to the axis AP, it will be the semidiameter of the equator; and RS, drawn from R perpendicular to the style-line AD, will give the length of the style, whose place will be at S. Finally, the line drawn through S parallel to CD will be the horizontal line — all of which we demonstrated in the *Gnomonics*, Book III, Proposition 1.

Let it be required, last of all, to lay out a meridian dial. Draw the line CD (in the figure of the immediately preceding example), which, being parallel to the horizon of the equator, forms at E the angle of the complement of the pole's altitude, or the angle of the equator's altitude, DEH — (with the dial set in its proper position, and the point D pointing south) let it be the common section of the equator and of the plane of the dial. Then onto the line GH transfer from E, on

both sides, the hour-points of the line EF by means of the rule described above. For the lines drawn through these points to GH, as perpendiculars, will give the hours from noon and from midnight, in this order. The perpendicular to GH through E, when drawn, will give the 6 o'clock hour; below it, going downward, follow the hours 7, 8, 9, 10, and 11 from midnight in an eastern dial, while the same 6 o'clock hour is preceded, above E, by the hours 5, 4, 3, 2, and 1 from midnight. But in a western dial the 6 o'clock hour, below E, is preceded by the hours 5, 4, 3, 2, and 1 from noon, and the same 6 o'clock hour, above E, is followed by the hours 7, 8, 9, 10, and 11 from noon. [Margin: The place of the style and its magnitude.] The place of the style is at E, and its length is comprised as equal to the segment of the line GH between E and the 3 o'clock or 9 o'clock hour. All of which is manifest from what we demonstrated in the scholium to Proposition 25 of *Gnomonices* Book II.

Horologii cuiusvis ad maiorem, minoremve formam reductio.

The reduction of any dial to a larger or smaller form.

CAPVT XVII.

By what method any dial whatever may be reduced to a larger or smaller form. [Margin: By what method any dial whatever may be reduced to a larger or smaller form.] Since it often happens that dials described according to the foregoing precepts must be enlarged onto walls or planes, or else restricted to a smaller form, we shall briefly teach in this chapter by what method this ought to be done. On some straight line AF take as many equal parts AB, BC, CD, DE, EF as the style of the dial already described, so that the whole line AF, thus composed, is not less than the longest shadow cast by the style. And on another line GM take just as many parts GH, HI, IK, KL, LM, equal to the style of the future dial, whether the style GH here is larger or smaller than the style AB. Then, dividing each of these parts of both lines into any number of equal particles — (the easiest division of all will be to bisect each part first; then to bisect each particle again; and so on, as far as can conveniently be done; for the more particles there are, the more accurately the future dial will be described) — inscribe upon them the corresponding numbers, as appears in the figure. By means of these two lines we shall describe, from any dial, another larger or smaller one, according as the style GH of the future dial is larger or smaller than the style AB of the given dial, in this way. Through the place of the style of the given dial draw a line

Diagram. Two parallel horizontal scales (measuring rules), one above the other, each divided into forty small equal divisions and lettered at every eighth division. These are the two lines AF and GM used in this chapter to enlarge or reduce a dial.

- Upper rule (left to right): **A** (start, at 0), **B** (at division 8), **C** (at 16), **D** (at 24), **E** (at 32), **F** (at 40); the division-numbers **8, 16, 24, 32, 40** are printed beneath B, C, D, E, F respectively. To the right of F the rule continues as a plain (undivided) line.
- Lower rule (left to right): **G** (start, at 0), **H** (at division 8), **I** (at 16), **K** (at 24), **L** (at 32), **M** (at 40); the division-numbers **8, 16, 24, 32, 40** are printed beneath H, I, K, L, M respectively. To the right of M the rule continues as a plain line.

—parallel to the meridian line, if the place of the style does not lie on the meridian itself; and through the place of the style in the future dial draw a similar line, which in a dial at right angles to the horizon will be perpendicular to the horizon. Then, having drawn a hidden line from the place of the style in the given dial through any hour on the tropic of Cancer [☿], or of Capricorn [♊], or on the equinoctial line, draw from the place of the style in the future dial another, similar hidden line, making of course with that first line an angle equal to that which

[The two divided rules of the preceding page recur here as the head-band of the folio; see fig-p621-1.]

the hidden line of the given dial makes with the first line that is parallel to the meridian line. This will easily be done if, from each place of the style of both dials, a circle is described at any one and the same interval, and, on the arc between that first hidden line drawn in the given dial and this hidden line drawn through the given hour, an equal arc is taken in the future dial, beginning from that first hidden line and proceeding toward the same side to which the given hour inclines from the hidden line. For the hidden line drawn out from the place of the style of the future dial through the end of this arc will form, with that first hidden line, an angle equal to that which the two said hidden lines contain in the dial proposed. Finally, transfer the interval between the style of the given dial and the point of the proposed hour onto the line AE, and carefully note the parts of the line AF comprised within that interval. For if just as many parts are taken with the compass on the line GM and transferred from the place of the style of the future dial onto the hidden line last drawn, the point of that same hour on the tropic of Cancer [☿], or of Capricorn [♊], or on the equinoctial line, will be

obtained. In no other way shall we carry over all the other points of the proposed dial into the future dial.

To do this more expeditiously, it will not be out of place first to draw out hidden lines from the place of the style of the given dial through all the hour-points of the tropic of Cancer, and to make just as many issue from the place of the style of the future dial, comprehending in a circle described from the place of the style arcs equal to the arcs that those hidden lines in the given dial cut off from a circle equal to it, likewise described from the place of the style. For thus we shall expeditiously, by means of the lines AF and GM, cut off on the hidden lines of the future dial intervals like the intervals of those hidden lines of the given dial, so that the points of all the hours on the tropic of Cancer are found. The same is afterwards to be done for the hour-points of the equinoctial line, of the tropic of Capricorn [♊], and of any other sign, if need be. For if the corresponding hour-points are at last joined by straight lines, and the points of the sign-arcs by curved lines, a larger or smaller dial will have been described, according to the given length of the style.

But if the style GH of the future dial has to the style AB of the proposed dial a multiple or submultiple proportion — as not rarely happens in practice, when a dial made for some wall is to be transferred onto that wall in a larger form (for, the dial once described, we may choose another style, double, triple, tenfold, or twentyfold the first, and so on, according as the magnitude of the dial to be described requires) — we shall most expeditiously construct the larger or smaller dial in this way. Draw on the wall a horizontal line, one, that is, parallel to the horizon; choose on it a place for the style; and through this, by means of a circle described from the place of each style, draw the style-line, forming with the horizontal line an angle equal to the angle that the style-line makes with the horizontal line in the proposed dial. Then transfer the interval on the style-line of the given dial, between the horizontal line and the equinoctial line, onto the style-line of the future dial, downward from the horizontal line, as many times as this style contains that one, marking a point on the style-line itself. Likewise transfer the interval on the horizontal line of the proposed dial, between the place of the style and the equinoctial line, onto the horizontal line of the future dial (toward the same side to which the equinoctial line inclines in the given dial), from the place of the style, as many times too as this style comprehends that one, marking a point also on the horizontal line. For the line drawn across through this point and through that other one marked on the style-line will give the equinoctial line of the future dial.

Now if all the intervals on the horizontal, equinoctial, and style-lines of the given dial — both between the place of the style and the hour-lines, and between the common section of the horizontal line and the equinoctial, and the hour-lines — are multiplied, on the corresponding lines of the future dial, as many times as the one style embraces the other; and if the same is done for the intervals of the hour-lines between the equinoctial line, or the horizontal line, and the arcs of Cancer [♋] and Capricorn [♐], as well as of the other signs: then all the points will be found in the future dial through which the hour-lines and the sign-arcs must run. And if at any time some lines do not cut the horizontal line in the given dial, they must be produced, hidden, until they cut it, so that their points can be transferred onto the horizontal line of the future dial. But perhaps we shall dispatch the matter more conveniently if in the given dial we draw one parallel to the horizontal line, or two in any way, which will conveniently cut the hour-lines (some of them being produced, if need be). For if in the future dial other parallels to the horizontal line are drawn, so that their distances from the horizontal line comprehend the distances of those in the given dial from the horizontal line as many times as the one style is contained in the other, the intervals of these onto those must be transferred, just as we said concerning the intervals of the horizontal line, the equinoctial, and the style-line, and so on.

More on this subject, written at length, you will find in the last proposition of Book IV of our *Gnomonics*.

THE FINDING OF THE MERIDIAN LINE.

(*MERIDIANAE LINEAE INVENTIO.*)

CAPVT XVIII.

[Margin: The finding of the meridian line.]

[Margin: An instrument most convenient for observing shadows, in place of a gnomon.]

Although in my commentary on the *Sphere*, where I dealt with the offices of the meridian circle, and in the scholium to Proposition 23 of Book 1 of the *Gnomonics*, I have written much about the finding of the meridian line [linea meridiana], nevertheless, because the matter is of such outstanding usefulness in the treatment of sundials [horologia], I shall here repeat — without demonstration, since the demonstration may be sought from the *Gnomonics* — that method of finding the meridian line which is taken from the Analemma. To find the meridian line on any given day, then, first learn the place of the Sun from an

ephemeris or from some other source. Then, on a plane parallel to the horizon, observe the shadow of a fine thread hanging freely. For this purpose I usually employ the instrument you see depicted here. In it a square [norma] CDE is fixed at right angles to the flat ruler AB, along the line DI, parallel to the two edges of the ruler — so that the side DH of the square stands like a kind of gnomon set upright to the horizon, or like a freely hanging thread, once the ruler AB has been laid upon a plane parallel to the horizon. By this means we shall, with little trouble, mark without any perceptible error two points along the length of the shadow's tip, cast from the edge HD.

But if on the square's arm CD, which is raised above the horizon, a line FG is drawn parallel to the side DH, and a hole is made near the point G, so that a plumb-line — a fine thread hung from the hole F with a weight attached — can move freely within it, then this same instrument will also serve very conveniently for examining any proposed plane, to determine whether or not it is parallel to the horizon. For when the thread FG grazes the arm CD and coincides with the line FG, the plane on which the ruler AB is set will be parallel to the horizon. Now, along the length of the shadow's tip cast from the side DH — or along the middle of the width of the shadow which the freely hanging thread casts — let two points A and B be marked, a little apart from each other, as in the following figure, and through them let the straight line AB be extended. This line will be the common section of the underlying plane and of that vertical circle [verticalis] which at the moment of observation passes through the center of the Sun.

Once the shadow has been observed, let the Sun's altitude be taken at once, by means of a quadrant or some other instrument. I say *at once*, because it is best that the Sun's altitude be observed immediately after the two points in the shadow have been marked, before the straight line is drawn through them — lest there be danger in delay, since, on account of the Sun's daily motion, the shadow shifts continuously and the Sun at once comes to lie in a different vertical circle.

After this, let an Analemma be described, in which the meridian is FGHI; the common section of the horizon and the meridian is GI; the common section of the prime vertical and the same meridian is FH; the common section of the same meridian and the equator [aequator] is LM; and finally the common section of the meridian and the parallel of the Sun [parallelus Solis], on which the Sun stands at the time of observation, is the straight line NO. This last is easily drawn by means of the Sun's known declination [declinatio], if the arc of declination LN or MO is found, as I taught in Chapter 10. Then, having computed the Sun's altitude, found from I as far as P, and from G as far as Q, we shall draw the straight line PQ. By

the scholium to Proposition 27 of Book 3 of Euclid, this line will be parallel to GI itself, and hence it is the common section of the meridian and of the parallel to the horizon passing through the center of the Sun at that time. It will cut the vertical diameter FH at R, and the diameter of the Sun's parallel NO at S. Then, having described from R about PQ, at the interval RP or RQ, the semicircle PTQ, let the perpendicular ST be drawn from S to PQ, cutting the circumference PTQ at T; and let the straight line RT be joined.

If, then, the point S falls between Q and R, and the observation is made before noon, we shall construct at the center C — from which, taken anywhere on the shadow-line AB, a circle of any size has been described — an angle ACD equal to the acute angle TRQ, running from east toward south; that is, from A, the eastern point, toward D, as is seen in circle A. But if the observation is made after noon, we shall make the angle ACD equal to the same angle TRQ, running from west toward south; that is, from A, the western point, toward D, as appears in circle B.

But if the point S falls exactly on the point R, then — whether the observation is made before or after noon — we shall draw through C the perpendicular DE to AB, as is clear in circle C.

If, finally, the point S lies between R and P, and the observation is made before noon, we shall make an angle ACE equal to the acute angle TRP, running from east toward north; that is, from A, the eastern point, toward E, as is manifest in circle D. But if the observation is made after noon, we shall construct an angle ACE equal to the same angle TRP, running from west toward north; that is, from A, the western point, toward E, as may be seen in circle E. For the straight line DE, always making the said angle with AB, will be the meridian line.

But if at any time the line PQ falls upon the point N — that is, if the Sun's altitude, computed from Q, terminates at N — then at the time of observation the Sun will lie on the meridian circle and will have its greatest altitude for that day; and therefore the shadow-line AB itself will be the meridian. All of this is clear from what I have demonstrated in the *Gnomonics*.

[Margin: How the diurnal and nocturnal arc, and the hour, are known from the Analemma.]

But that you may see here what an excellent use the Analemma has in matters of astronomy — I have set out its many other applications in various places of the *Gnomonics* — it will not be amiss to explain briefly in what way, from the Analemma, both the diurnal and the nocturnal arc, and the hour of the day, may be known from the Sun's known declination and its altitude above the horizon.

For once the straight line PQ has been drawn through the terminal points of the Sun's altitude, PQ, and a circle has been described about NO, the diameter of the Sun's parallel — whose center is at d, where the axis of the world ab cuts the diameter

NO — and this circle has been divided into 24 equal hours, beginning from the diameter NO (if the reckoning is of the hours counted from noon and from midnight); then, if through Z, where the diameter NO cuts the diameter of the horizon GI, a perpendicular XY is raised to NO, this XY will be the common section of the Sun's parallel and the horizon, as I have shown in the *Gnomonics*. Therefore the diurnal arc will be XNY, and the nocturnal arc YOX; and consequently the number of hours contained in these arcs will indicate the length of the day and of the night. And now, if from the point S the perpendicular SV is drawn to NO, cutting the circumference of the Sun's parallel at V, the hours contained in the arc NV will indicate how many hours the Sun is distant, whether before noon, or

Diagram. The principal figure of this chapter: a large analemma. A great circle (representing the Sun's parallel-and-hour circle) is divided around its rim into 24 hour-marks, of which the readable numerals are, running across the top and down the right, 1 2 3 4 5, then 6 7 8 9 10 11 12, and down the left 11 10 9 8 7 6. Within it are inscribed the meridian and its cross-lines, the horizon diameter, the Sun's-parallel diameter, and a family of oblique construction lines and hour-lines converging near a central point. Below and to the left stand five small orientation circles labelled A, B, C, D, E, each a circle with center marked and cross-diameters (top D, bottom E, sides A/B), illustrating the five cases of the meridian-line angle treated on the preceding page (circles A–E).

Readable labels in the large analemma: on the rim and upper area — I, F, P, L, N, G, D; along the central vertical and to its right — R (appearing at several points along one line), S, K, Z, d, M; and around the lower and right portion — V, X, Y, O, T, and the small letters f, b, e. The center of the parallel is marked d.

- N, O: endpoints of the Sun's-parallel diameter
- G, I: endpoints of the horizon diameter
- Z: intersection of NO with GI
- X, Y: endpoints of the perpendicular XY (common section of Sun's parallel and horizon)
- S, V: point S and its perpendicular foot V on the parallel's circumference
- A, B, C, D, E: the five small orientation circles below

- Several further small peripheral letters and the interior hour-line labels could not be resolved with confidence [?].

after — according as the observation is made before or after noon. And if the hour reckoned from sunset is desired, in the Italian manner: if indeed the observation is made before noon, the division of the circle NXOY into 24 equal hours must be begun from the point Y and carried on through the point O; for then the arc YOY will indicate at once how many hours have elapsed since sunset. But if the observation is made after noon, the same division must be begun from the point X and carried on through the point O; for the arc XONV will show the hours passed since sunset. In the same way, if the hour reckoned from sunrise is sought, in the manner of the Babylonians and of the Balearic Islands, the division of the circle must be begun from the point X and carried on through N, if the observation is made before noon, but from the point Y if after noon. Not otherwise shall we find the unequal (seasonal) hour, if the semidiurnal arc NX is distributed into six equal parts, and so on.

[Margin: How the Sun's altitude is investigated by the Analemma, from the known hour and the Sun's declination.]

Conversely, from the known hour we can arrive at a knowledge of the Sun's altitude by means of the Analemma, provided the Sun's declination is not unknown. For if, according to the amount of the declination, the diameter NO of the Sun's parallel is described, and about it the circle NXOY is described and divided into hours, and from V, the known hour, the perpendicular VS is drawn to NO, and finally through S the straight line PQ is drawn parallel to the diameter of the horizon GI, then both GQ and IP will be the arc of the Sun's altitude above the horizon.

[Margin: How the Sun's altitude is elicited in the Analemma from the length of the shadow of any style.]

Nor should this be passed over: that we can determine the Sun's altitude from the gnomon's shadow alone, in case we happen not to have at hand some other instrument for observing it, in the following way. On the plane parallel to the horizon, on which we took up the shadow AB above, let the tip of that same shadow be marked as accurately as possible at the time of observation. Then, having taken on the Analemma a straight line Ke equal to the side of the square DH, or to any style whatever (as has been done in the figure), let there be raised at e, perpendicular to FH, the line ef equal to the length of the shadow. For the straight line drawn from f through the center of the Analemma K, when

produced, will cut off from the meridian the arc IP of the Sun's altitude, as I demonstrated in the *Gnomonics*.

ANOTHER FINDING OF THE MERIDIAN LINE, BY THREE OBSERVATIONS OF THE SUN, without knowledge of the altitude of the pole, or of the declination, or of the Sun's place in the zodiac: together with the finding of the altitude of the pole, the declination, and the Sun's place in the ecliptic, and the rising and setting amplitude.

(ALIA INVENTIO LINEAE MERIDIANAE PER TRES SOLIS OBSERVATIONES sine cognitione altitudinis poli, & declinationis, locique Solis in Zodiaco: vna cum inuentione altitudinis poli, declinationis, locique Solis in Ecliptica, & amplitudine ortiua, occiduaque.)

CAPVT XIX.

Although that method of finding the meridian line from the Analemma, given in the preceding chapter, is most excellent, nevertheless — because it requires both the known altitude of the pole and the place of the Sun, so that the Analemma may be constructed for the given pole-altitude and the parallel drawn in it according to the declination of the Sun's place — it is worthwhile to subjoin here another procedure, taken from Pedro Nunes [Pedro Nunes] the Portuguese, in Book 2 of *On Navigation*, Chapter 16,

[Margin: The finding of the meridian line, the altitude of the pole, the Sun's declination, and the rising and setting amplitude, from three observations, on the common astrolabe.]

by which, from only three observations of the Sun, we may — by the description of certain circles on the common astrolabe — draw out both the meridian line on a plane parallel to the horizon, and the altitude of the pole of that place where the observation is made, together with the declination and the Sun's place in the zodiac, and the rising and setting amplitude. From this it will readily be understood how splendid is that invention of Ptolemy [Ptolemy], whereby all the celestial circles are described on a plane in that form and proportion in which they are seen from the antarctic pole upon the plane of the equator: since not only the things we here propose, but very many other astronomical problems as well, can be dispatched by it — which is not the place to explain. But in what way we can project any circle whatever, whether a great circle or not a great circle,

onto a plane, will be made clear from our *Astrolabe*, constructed with geometrical demonstrations, which we shall shortly, God willing, bring to light. Now let us come to the matter proposed.

Let there be, then, on a plane parallel to the horizon, a circle ABCD, whose center E represents the horizon, and in which two hidden diameters AC, BD cut each other at right angles at the center E. Now, having set a style at E — or, what I prefer, having set the side DH of the instrument described above at E — let the shadow EF be observed in the morning, and at the same instant of time the Sun's altitude, measured by the arc AG. Then, after one hour or another, let the shadow EH be observed again, and at the same time the Sun's altitude AI. And thirdly, after some interval of time, the shadow EK and the Sun's altitude AL. Then, drawing from B through the points of altitude G, I, L three hidden straight lines cutting the semidiameter AE at M, N, O: from the first shadow EF let the straight line EP be cut off, equal to EM; from the second shadow EH the straight line EQ, equal to EN; and finally from the third shadow EK the straight line ER, equal to EO. And through the three points P, Q, R, by the scholium to Proposition 5 of Book 4 of Euclid, let a circle PQR be described, cutting ABCD at f, whose center is S. For the straight line ES, drawn through the points E and S, will be the meridian line; and the angle IEF will be that which the vertical circle drawn through the center of the Sun at the first observation makes with the meridian. And when the perpendicular TV has been drawn through E to ES, it will be the common section of the proposed plane and of the prime vertical; and VF, VH, VK will be the latitudes of the shadows at the times of the observations — that is, the distances of the verticals drawn through the center of the Sun from the vertical properly so called.

Diagram. A compound geometric construction in two parts. **Upper left:** a quadrant-like figure — an arc from A (top) through E, D to B (bottom), with a network of chords among the interior points; readable labels I (left), H, E, G, D (upper), M, N, L, F (interior), K (lower left), C, B. This is the small inset showing the shadow-altitude triangles. **Lower/main:** a large circle (the horizon circle ABCD, center E) crossed by the meridian and by the circle PQR and its oblique secants, with a long slanting line rising to the point δ (delta / "d") at the top. Readable labels: at top, two marks near δ ; along the central vertical Y, A/I/G, M, S, d, N, O, E, X, q; on the right descending, b, L, f, F, P, D, V, H, Z, K, P; the great secant lines run from T (=B, the lower center from which the secants radiate) outward. The lower center point is marked B (with T alongside).

- ABCD: horizon circle, center E

- AC, BD: the two hidden diameters (right angles at E)
- EF, EH, EK: the three observed shadows
- G, I, L: the three points of the Sun's altitude
- M, N, O: intersections on the semidiameter AE
- P, Q, R: the three constructed points; circle PQR cuts ABCD at f, center S
- ES: the meridian line
- TV: perpendicular to ES through E (common section with the prime vertical)
- V, F, H, K: feet of the shadow-verticals
- X, Y: where circle PQR cuts the meridian; Z, a, b, d and δ : points of the auxiliary construction below
- Several small interior and peripheral labels (including the delta-marks at top and the crowded lower-right cluster) could not be resolved with confidence [?].

[Margin: Demonstration of the above operations.]

After this, from the point T, through the points X and Y — where the circle PQR cuts the meridian line — let two straight lines TX, TY be drawn, cutting the circle ABCD at Z and a. Let the arc Za be bisected at b, and let the straight line Tb be drawn, cutting the meridian at d. For the arc Vb will be the complement of the altitude of the pole, and the arc bZ the complement of the Sun's declination, and its meridian altitude the arc eZ; and finally the arc Vf will be the rising or setting amplitude. And if the two quadrants bh, bp are taken, pZ will be the Sun's declination; and hence, this declination being known, its place in the zodiac will not be unknown. The latitude of the place — that is, its distance from the equator — will be the arc pV; and the altitude of the pole above the horizon will be the arc Ib, equal to pV itself.

The demonstration of this operation depends entirely on the description of the astrolabe. To make it plainer, we shall conceive the astrolabe to be described with the eye placed at the nadir — that is, at the other pole of the oblique horizon, the one opposite the zenith. In this way it will come about that the horizon with its parallels is described on the plane of the horizon in just the same way as the equator with its parallels is described, by Ptolemy, on the plane of the equator when the eye is placed at the antarctic pole; while the equator

and its parallels, take on in our construction the very situation and shape that the horizon and its parallels have in Ptolemy's construction. For since the poles of the horizon are as far distant from the poles of the equator as the poles of the

equator are from the poles of the horizon, it follows that the horizon with its parallels is seen in the plane of the horizon — from the one pole of the horizon — in the very shape in which the equator itself with its parallels appears, from the antarctic pole, in the plane of the equator; and, conversely, that from the same pole of the horizon the equator with its parallels appears in that same plane of the horizon in the shape in which the horizon with its parallels is seen, from the same antarctic pole, in the plane of the equator.

This being so, if the circle ABCD is taken in the astrolabe as the horizon, then E will be its pole — that is, the zenith [vertex capitis] — just as, when ABCD is taken as the equator, E is the pole of the world. The straight lines EF, EH, EK will be the vertical circles drawn through the Sun at the times of observation, just as, according to Ptolemy, all the straight lines drawn through the center E represent the meridians passing through the poles of the world. Through the points M, N, O will be described the parallels of the horizon, whose declinations from the horizon are AG, AI, AL — that is, the circles of altitude passing through the Sun — drawn from the center E, just as by Ptolemy the parallels of the equator having those same declinations are described from E through those same points M, N, O. These parallels will cut the straight lines EF, EH, EK at P, Q, R, because of the equality of the straight lines EM and EP, EN and EQ, EO and ER; so that at the times of observation the Sun stands at the points P, Q, R — that is, at the common intersections of the vertical circles and of the parallels of the horizon drawn through the Sun. And since the Sun on one and the same day is taken to occupy one and the same parallel of the equator, the circle PQR, described from S, will be the parallel of the equator on which the Sun then lies — after the fashion of a parallel of the horizon in Ptolemy's construction, as it would be if the equator were the horizon and the horizon the equator.

Therefore, since the centers of the parallels of the horizon in the astrolabe lie on the meridian line of the astrolabe, the straight line drawn through E and S will be the meridian line, and the angle IEF will be the one that, at the first observation, the meridian makes with the vertical circle EF. The straight line TV will be the prime vertical [Verticalis primarius], cutting the meridian at right angles; VF, VH, VK will be the latitudes of the shadows [latitudines umbrarum] at the times of observation; and the point V corresponds to the zenith of the place.

Now, just as in Ptolemy's astrolabe the point b, which bisects the arc Za, falls at the pole of the horizon and of its parallels — so that d is the zenith in the astrolabe (for the pole of the horizon, b, on the equator ABCD, is always equally distant from the points Z and a, upon which fall the straight lines TX, TY drawn

through the extreme points of the diameter of the parallel of the horizon PQR, as is clear from the construction of the parallels of the horizon) — so likewise, when ABCD is taken as the horizon with E as its pole, that same point b will fall at the pole of the equator and of its parallels, so that d in our astrolabe is the visible pole of the world. And if the quadrants bh, bp are taken, the straight lines Th, Tp will cut the meridian line EY at the extreme points of the diameter of the equator, g and q, so that the straight line gq, bisected at a, [yields] a circle described from a with radius aq, which, passing through TV, represents the equator, just as in Ptolemy's astrolabe it expresses the horizon — as is established from the construction of the horizon in the common astrolabe. In our figure, however, since the point g is too far distant from E to be marked, the straight line gq has not been bisected; instead a has been taken on the meridian line as the center of the three points T, q, V, and from it the equator TqV has been described.

Therefore Vb will be the complement of the pole's altitude — namely the distance of the zenith V from the visible pole b; the latitude of the place will be Vp — that is, the distance of the zenith V from the equator p; the altitude of the pole [will be] Ib; the complement of the declination [will be] bZ, and the declination itself pZ; the Sun's meridian altitude [will be] eZ; and finally the rising or setting amplitude [will be] Vf. All of which are evident from the manner of describing the circles in the common astrolabe.

Easy as this method is, and very well suited for use when the Sun is in the northern signs (provided everything is laid out accurately as we have prescribed), it becomes difficult and inconvenient when the Sun lingers in the southern signs, because then the parallel of the Sun falls below the point q and requires a diameter of unmanageable size.

[Margin: Finding the Sun's declination, the altitude of the pole, the meridian line, and the rising or setting amplitude, from three observations, by means of spherical triangles.]

But let us now teach how to investigate, by means of spherical triangles, the same things we found with the astrolabe, from the same three observations. Let ADB be the horizon, AIB the meridian, K the visible pole of the world, and I the zenith of the place. Let the two latitudes of the shadow observed on the horizon be those from which similar arcs CD, DE are taken on the western semicircle of the horizon, if the observations are made in the afternoon; but if before noon, let the same arcs be taken on the eastern semicircle of the horizon. Through the points C, D, E let three vertical circles IC, ID, IE descend from the zenith I, on which the known altitudes of the Sun are CF, DG, EH — of which the one nearest the pole K,

namely CF, is the smallest of all — so that in those three observations the Sun stood at the points F, G, H, through which the parallel of the Sun in which it then lingers will pass throughout. Through each pair of points F,G; G,H; H,F let arcs of great circles FG, GH, HF be described; and, FG and FH being bisected at L and M, let four arcs of great circles KF, KL, KM, KH descend from the pole K, of which KL cuts the arc FH at N. Now the arcs KF, KH [Margin: (a) — prop. 28 of the third [book]] will be equal, because the straight lines subtending them, KF and KH, are equal by the definition of a pole, since they are drawn from the pole K all the way to the parallel of the Sun. The angles at M too will be right angles. For since the two arcs MF, MK are equal to the two arcs MH, MK, and the base KF to the base KH, as we have shown, the angles at M will be equal by proposition 18 of our [treatise on] spherical triangles, and therefore right angles. By the same reasoning the angles at E [?] will be right angles.

Therefore, since in the triangle FGI the arcs IF, IG are known — being the complements of the known altitudes of the Sun, CF and DG — and since they contain the known angle FIG (because its arc CD is known; for we assume the latitudes of the shadows CD, DE, and hence CE as well, to be known by observation; and, to make the calculation more reliable, the said arcs may be taken on the horizon in any whole number of degrees, and the altitudes of the Sun observed when the shadow of the stylus extends precisely through the points C, D, E) — the arc FG too will be made known, by Practice 19 of our spherical triangles, especially if we use the second method of that practice, which is the easier. For it is convenient to cite here those practices which we have collected separately, excerpted from the propositions, at the close of the [treatise on] spherical triangles. In the same way the arcs GH and FH will also become known — since the arcs IG, IH are known, being complements of the known altitudes of the Sun DG and EH, and contain the known angle GIH because the arc DE is known; and since the arcs IF, IH contain the known angle FIH because the arc CE is known.

Next, from the three known arcs FG, FH, GH we shall likewise come to know the angle GFH, by Practice 18 of the same spherical triangles, especially if we employ the second way of that practice, as the easier.

Now, since in the right triangle FLN the arc FL is known — being half of the known arc FG — and moreover the adjacent angle LFN has been made known, the other non-right angle LNF will also be made known, by Practice 5 of our spherical triangles. And hence, in the same triangle, from the known arc FL and the known opposite angle LNF, the arc FN opposite the right angle will also be

made known, by Practice 3 of our spherical triangles; and when this is subtracted from the known arc FM — namely from the half of the known arc FH — the arc MN will be left known.

Therefore, since in the right triangle KMN the arc MN has been made known, together with the adjacent angle MNK (which is equal, by proposition 6 of our spherical triangles, to the already-known angle FNL at the vertex), the other non-right angle MKN will be produced known, by Practice 5 of our spherical triangles. And hence, in the same triangle, from the two non-right angles MNK, MKN being known, the arc KM opposite the angle MNK will also be known, by Practice 4 of our spherical triangles.

Diagram. A compound geometric construction in two parts — the same demonstration figure that appears on the preceding page, here reprinted so the reader can follow the spherical-triangle argument.

- **Upper-left inset** — a quadrant-like arc from A (top) descending through E, D to B (bottom), with a dense network of chords among the interior points. Readable labels: A (top), E (upper right), H (upper-left interior), I (far left), G (interior, near D), D (right of arc), M, N, L (interior triangle cluster), F (lower center), C (near F), K (lower left), B (bottom).
- **Main figure (lower/right)** — a large circle crossed by the meridian and by an oblique construction, with two slanting lines rising to two δ [delta] marks at the top. Readable labels along and within it: at top, two δ marks; on the central vertical, Y, then A/I/G, M, S, d, N, O, E, and below it X, R, q, with q q at the foot; on the right flank, b, L, f, F, P (twice), D, V, H, Z, K, and P; the secant lines radiate from the point T (with B alongside) at the lower left.

Several of the small, overlapping interior and rim labels — including the two δ marks at the top and the crowded lower-right cluster — could not be resolved with confidence [?].**

Again, since in the right triangle FKM the two arcs FM, KM about the right angle are known, the arc FK opposite the right angle will also be made known, by Practice 7 of our spherical triangles — which arc is indeed equal to the complement of the Sun's declination, as we said above; and consequently the declination itself will not be hidden.

After this, since in the right triangle FKM the arc FK opposite the right angle is known, as well as the arc KM about the right angle, the angle KFM opposite the arc KM will also be found, by Practice 1 of our spherical triangles.

In the triangle FHI too, since all three arcs are known, the angle HFI will also be known, by Practice 18 of our spherical triangles, especially by the second, easier way; and when this is subtracted from the angle KFM, just found, the angle KFI will also be left known.

Wherefore, since in the triangle KFI the two arcs FK, FI are known and contain the known angle KFI, the arc IK too will be produced known, by Practice 19 of our spherical triangles, especially by the second, easier way — namely the complement of the pole's altitude; and thus the altitude of the pole itself, BK, will not be unknown.

Likewise, since in the same triangle FIK the three arcs are known, we shall also come to know, by Practice 18 of our spherical triangles — chiefly by the second, easier way — the angle FIK, which the vertical circle IC, drawn through the Sun at the time of the observation in which the Sun's altitude was found to be CF, makes with the meridian toward the visible pole. Therefore, if on a plane parallel to the horizon a rectilinear angle is made with the shadow-line of that observation, of as many degrees as are contained in the arc BC of the angle FIK just found, then the line forming that angle will be the meridian.

Finally, the Sun's declination and the pole's altitude now being known: if it be arranged that as the sine of the complement of the pole's altitude is to the sine of the declination found, so the whole sine [radius] is to another, there will be produced the sine of the rising or setting amplitude, as we demonstrated in the Gnomonics, Book I, proposition 34.

THE FINDING OF THE DECLINATION OF ANY WALL FROM THE PRIME VERTICAL CIRCLE.

CHAPTER XX.

Before a sundial [horologium] can be laid out on a proposed wall, it is first necessary to find its declination from the vertical [circle] properly so called; which can be done by means of the Analemma, in the manner we set forth in the scholium to proposition 23 of Book I of the Gnomonics. Let AB be a wall perpendicular to the horizon, on which the straight line CD is drawn parallel to the horizon; and let there be fixed in it a stylus CE of any length, perpendicular to the wall. (Here too I would rather choose the instrument of chapter 18: for if its point D is placed at the point C, the side DH will serve as the gnomon perpendicular to the wall.) And let there be observed, at whatever time the Sun lights up the wall — whether before or after noon — the tip of the shadow EF

which the stylus casts, namely the point F. (The declination will be found more exactly if on some flat board the straight line CD is drawn and the board itself is applied to the wall so that the straight line CD is parallel to the horizon and the stylus is placed at C.) Through this point [F] let the perpendicular FD be drawn to the straight line CD; which is easily drawn on the wall in this way. Apply to the wall a fine thread with a plumb-bob, so placed that it passes through the point F, and mark on the wall any point D through which the thread passes. For a straight line drawn through F and D will be perpendicular to CD, since the thread is perpendicular to the horizon. Then, the perpendicular CG being drawn from C to CD, equal to the stylus assumed, let the straight line GD be joined. CDG will then be the angle of declination of the proposed wall from the vertical [circle] drawn through the Sun at the time of observation; and from this, once found, we shall investigate the declination of the same wall from the prime vertical in the following way.

[Margin: The angle which the vertical [circle] drawn through the Sun makes with the proposed wall.]

Diagram. The wall construction for this chapter: a rectangle (the wall AB) divided by a horizontal line CD and a vertical line. Readable labels:

- A: upper-left corner of the wall
- B: lower-right corner of the wall
- C: center point, foot of the stylus, on the horizontal line CD
- D: on line CD to the right (the marked point through which the plumb-thread passes)
- E: upper point, tip of the stylus (shadow-casting tip)
- G: point at the top, adjacent to E (the perpendicular CG equal to the stylus)
- F: the tip of the shadow, on the vertical below D
- A dotted line runs from E down to F (the shadow ray); a short shaded triangle sits at E–C–G.

Having marked the tip of the shadow F, let there at once be sought — before the straight line FD is drawn (for if any delay intervenes, the shadow will change and the Sun will occupy another vertical circle, on account of its diurnal motion) — the altitude of the Sun. Let this, [computed] in the Analemma, chapter 18 (which we have here repeated), be reckoned from the points G, I up to the points Q, P; and let the straight line PQ be joined, which will be the diameter of the parallel of the horizon drawn through the center of the Sun at the time of observation, cutting the diameter NO of the Sun's parallel at S, and the diameter FH of the

prime vertical at R. Then, a semicircle PTQ being described about PQ, from the center R with radius RP or RQ, let the perpendicular ST be drawn from S to PQ, cutting the circumference of the semicircle PTQ at T; and let the straight line TR be joined, which with the diameter FH of the vertical properly so called will form the angle TRH — the angle of declination which the vertical [circle] drawn through the Sun at the time of observation has from the vertical properly so called.

[Margin: The angle which the vertical [circle] drawn through the Sun makes with the prime vertical.]

Diagram. The analemma construction repeated from chapter 18, for finding the wall's declination. It consists of one large circle plus five small auxiliary circles beneath it.

- **Main circle** — a large graduated circle (the analemma) crossed by the horizon, the prime vertical, the parallels, and a dense web of hour-lines and projection chords. The rim is graduated with the hour-numerals 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 [?], 1, 2. Readable letter-labels around and within it include: N, F, Y, P, Q, L, G at the upper rim and left flank; a column of R's and S's with d, K, Z, X, M, T on the right and lower-right; and I on the right rim. The many small, tightly-packed interior and rim labels are largely unresolved [?].
- **Five small circles** (a row beneath the main figure), each a circle divided into quadrants by two diameters, lettered in the center A, B, C, D, and E respectively; each shows D at the top, E at the bottom, C at the center, and A and B on the horizontal diameter.

Because of the extreme label density in the main circle, more than two labels there are unresolved [?].**

[Margin: How the declination of a wall — when the wall faces south and the observation is made before noon — is to be ascertained.]

And so, suppose the observation is made before noon, with the wall facing south (a case we shall learn to recognize from what we shall write at the end of this chapter). If the point S falls between Q and R — which happens only when the Sun is found beyond the prime vertical [Verticalis primarius] — the Sun will light both the wall and the prime vertical on their southern side. We therefore compare the angle TRH with the angle CDG, which we found on the wall. If TRH turns out equal to CDG, the wall has no declination and faces due south; for the

declination that the vertical [circle] drawn through the Sun then has from the wall, and its declination from the prime vertical, are one and the same —

Diagram. This is the analemma construction repeated (as before) from chapter 18, for finding the wall's declination, accompanied beneath and beside it by five small auxiliary circles. It is the same figure that was engraved on the preceding page (cf. fig-p627-2); it is re-printed here.

- **Main circle** — a large graduated circle (the analemma) crossed by the horizon, the prime vertical, the parallels, and a dense web of hour-lines and projection chords. The outer rim is graduated with hour-numerals: reading down the left flank I can make out 10, 9, 8 ... 6, 5, and reading across the top and down the right flank 3, 4, 5 ... 8, 9, 10, 11 [several rim numerals abraded/overlapped — ?]. Readable letter-labels around and within it include N, I, F, L, Q, G on the upper rim and left flank; a column of R's and S's with d, K, Z, T, M along the center and lower-right; P repeated on the right flank; Y and I on the right rim. The very numerous small interior and rim labels are largely unresolved [?].
- **Five small circles** — each a circle divided into four quadrants by a vertical and a horizontal diameter, with a large central letter identifying it (A, B, C, D, E). In each, D is set at the top of the vertical diameter and E at the bottom, a small c marks a point near the center, and the letters A and B stand at the two ends of the horizontal diameter (their left/right placement differing from circle to circle, representing the several orientation cases of the wall relative to the cardinal points). Several of the small rim-letters on these circles are ambiguous [?].

Because of the extreme label density in the main circle (a re-engraving of an already-flagged figure), more than two labels there remain unresolved [?].**

— and so the wall does not differ from the prime vertical at all.

But if the angle TRH is found to be greater than CDG, then the wall lies on the eastern side, between the prime vertical and the vertical that is at that moment drawn through the Sun; this is because, on the southern side, the vertical drawn through the Sun then stands farther from the prime vertical than from the wall. Hence, if we subtract the angle CDG — that is, the declination which the vertical drawn through the Sun has from the wall — from the angle TRH — that is, from the declination which that same vertical has from the prime vertical — the remainder will be the angle of the wall's declination from the prime vertical, running from south toward the west.

Finally, if the angle TRH is less than CDG, then the prime vertical lies on the eastern side, between the wall and the vertical that passes through the Sun; for on the southern side the vertical drawn through the Sun then stands farther from the wall than from the prime vertical. So if we take the angle TRH away from the angle CDG, the remainder will be the angle of the wall's declination from the prime vertical, running from south toward the east.

But if the point S is the very same as R — which happens when the Sun stands in the vertical properly so called [Verticalis proprie dictus] — the Sun lights the wall on its southern side, but the prime vertical not at all. The angle CDG found on the wall will then be the angle of the wall's declination from the prime vertical, and the wall will decline from south toward the east; for on the eastern side the vertical properly so called, in which the Sun then stands, is more southerly than the wall.

Finally, if the point S falls between P and R — which happens when the Sun is found on the near side of the prime vertical — the Sun lights the wall on its southern side but the prime vertical on its northern side, and so the vertical drawn through the Sun lies, on the eastern side, between the wall and the prime vertical; for the Sun's vertical is then more southerly than the wall and more northerly than the prime vertical. For this reason, if the angle TRH is added to the angle CDG, the sum will make up the angle of the wall's declination from the prime vertical, and the wall will decline from south toward the east.

[Margin: How the declination of a wall facing south is to be discovered when the observation is made after noon.]

Now suppose the observation is made after noon, with the wall still facing south. If the point S falls between Q and R — which happens when the Sun is more southerly than the vertical properly so called — the Sun will light both the wall and the prime vertical on their southern side. Accordingly, if the angle TRH is found equal to CDG, the wall again has no declination. But if TRH turns out greater than CDG, the wall lies on the western side, between the prime vertical and the vertical drawn through the Sun. Therefore, subtracting the angle CDG from the angle TRH, the remainder will be the angle of the wall's declination from the prime vertical, and the wall will decline from south toward the east. Finally, if the angle TRH is less than CDG, the prime vertical lies on the western side, between the wall and the Sun's vertical. Taking the former from the latter, then, the remainder will be the angle of the wall's declination from the vertical properly so called, and the wall will decline from south toward the west.

And when the point S falls on R — which occurs if the Sun stands in the prime vertical — the Sun will indeed light

the wall on its southern side, but the prime vertical not at all, and the angle CDG found on the wall will indicate the wall's declination from the vertical properly so called, running from south toward the west; this is because the Sun then stands in the prime vertical — that is, in the vertical properly so called — which on the western side is more southerly than the wall.

Finally, when the point S falls between P and R — which happens when the Sun is set on the near side of the prime vertical — the Sun will still light the wall on its southern side, but the vertical properly so called on its northern side. Hence the vertical in which the Sun then stands will lie, on the western side, between the wall and the prime vertical. So if the angle TRH is added to CDG, the sum will make up the angle by which the wall declines from the prime vertical, from south toward the west.

But suppose the wall inclines toward the north and the observation is made before noon. If the point S falls between Q and R — which happens when the Sun stands beyond the prime vertical — the Sun will light the wall on its northern side but the prime vertical on its southern side. The vertical drawn through the Sun will then lie, on the eastern side, between the wall and the prime vertical. Therefore, if the angle TRH is added to CDG, the angle of the wall's declination from the vertical properly so called will be produced, and the wall will decline from north toward the east.

[Margin: How the declination of a wall inclining toward the north is to be gathered when the observation is made before noon.]

But if the point S is the same as R — which happens when the Sun stands in the prime vertical — the Sun will still light the wall on its northern side, but the prime vertical not at all. The angle CDG found on the wall will give the wall's declination from the vertical properly so called, and the wall will decline from north toward the east; for the Sun then stands in the prime vertical, which on the western side is more southerly than the wall itself.

Finally, when the point S falls between P and R — that is, when the Sun is found on the near side of the prime vertical — the Sun will light both the wall and the vertical properly so called on their northern side. So if the angle TRH is found equal to CDG, the wall will have no declination, for the reason stated earlier. But if the angle TRH exceeds CDG, the wall will be set on the eastern side, between the prime vertical and the Sun's vertical. Therefore, if CDG is taken from TRH, the

remainder will be the angle of the wall's declination, from north toward the west. Finally, if the angle TRH is less than CDG, the prime vertical will lie between the wall and the Sun's vertical on the eastern side. So if we take TRH away from CDG, there will be left the angle of the wall's declination from north toward the east.

[Margin: How the declination of a wall facing north is to be extracted when the observation is made in the afternoon, and by what route.]

Now suppose the wall faces north and the observation is made in the afternoon. If the point S falls between Q and R — the Sun, of course, being set beyond the prime vertical — the Sun will light the wall on its northern side and the prime vertical on its southern side. The vertical then drawn through the Sun will be set, on the western side, between the prime vertical and the wall. Consequently, if the angle TRH is added to CDG, the angle of the wall's declination from north toward the west will be forged.

But when the point S falls on the point R — as when the Sun has occupied the prime vertical — the Sun will still light the wall on its northern side, but the prime vertical in no way. Hence the angle CDG found on the wall will indicate the wall's declination from north toward the west; for the Sun then stands in the prime vertical, which on the eastern side is more southerly than the wall itself.

Finally, when the point S falls between P and R — that is, when the Sun is found on the near side of the vertical properly so called — the Sun will light both the wall and the prime vertical on their northern side. So if the angle TRH is then found equal to CDG, the wall will have no declination at all, as was said above. But if the angle TRH is greater than CDG, the wall will be set on the western side, between the prime vertical and the Sun's vertical. If, then, CDG is taken from TRH, the remainder will be the angle of the wall's declination from north toward the east. Finally, if the angle TRH is less than CDG, the prime vertical will lie on the western side, between the wall and the Sun's vertical. So, taking the angle TRH from CDG, the angle of the wall's declination from north toward the west will remain.

All of this we also set out in the scholium on proposition 23, book 1 of the *Gnomonics* (*Gnomonices* I, prop. 23, scholium). It is to be understood, however, only when the angle CDG — which the vertical drawn through the Sun makes with the wall — and the angle TRH — which that same vertical makes with the prime vertical — incline toward the same side. That is: when, with the wall facing south, the angle CDG lies to our left as we face the wall in the forenoon, and to our right in the afternoon. (For the vertical passing through the Sun before noon

forms with the prime vertical an angle on the western side, which lies to our left as we face the wall; whereas after noon that same angle is set on the eastern side, which then lies to our right, as is clear.) Likewise when, with the wall inclining toward the north, that angle CDG lies to our right before noon as we face the wall, but to our left after noon.

But if the angle CDG does not incline toward the same side as the angle TRH — that is, if, with the wall facing south, the angle CDG is placed to our right before noon (which can happen when both the wall, declining from south toward the east, and the Sun, are only a little short of the meridian), or is placed to our left after noon (which can happen when both the wall, declining from south toward the west, and the Sun are not far from the meridian); and likewise, with the wall facing north, if the angle CDG is formed to our left before noon (as can happen when the wall, declining from north toward the east, stands but little from the meridian, and the Sun in the forenoon is not far from its rising), or to our right after noon (as when the wall, declining from north toward the west, does not recede far from the meridian, and the Sun in the afternoon stands near its setting) — if, I say, the angle CDG does not incline toward the same side as the angle TRH, then the angle CDG must always be added to the angle TRH, and the compound angle (which will then always be greater than a right angle) must be subtracted from a semicircle. For the remaining angle will indicate the wall's declination: from south toward the east, if the observation is made before noon and the wall faces south; from south toward the west, if the observation is made after noon and the wall belongs to the south; but from north toward the east, if the wall inclines toward the north and the observation is made before noon; and from north toward the west, if, the wall facing north, the observation is made in the afternoon.

[Margin: A supplement to the teaching of the scholium on proposition 23, book 1 of the Gnomonics, concerning the declination of walls.]

None of this will be difficult for anyone who attentively and carefully weighs the position of the vertical circles and of the wall.

These matters, then, which we have just explained here, are to be added to the scholium on proposition 23, book 1 of the Gnomonics, so that the doctrine for investigating the declination of walls may be rendered complete in every respect.

[Margin: The declination of a wall, when the Sun's altitude is equal to the meridian altitude.]

But suppose that on some occasion the straight line PQ, in the analemma, falls on the point N — that is, the Sun's altitude is found equal to the meridian altitude of the Sun for the day on which the observation is made. Then the Sun will stand, at the time of observation, in the meridian circle; and therefore the straight line FD will be, on the wall, the common section of the meridian and the wall, since the meridian is then drawn through E, the center of the world, and through the Sun's ray EF; and the wall itself will face south. The angle CDG, then, found on the wall, will give the wall's declination from the meridian, and consequently its complement will show the wall's declination from the vertical properly so called, which is what is sought: from south toward the east, if the shadow F falls to our right as we face the wall; but from south toward the west, if the shadow falls to our left. This, of course, is to be understood when at noon the Sun is more southerly than the zenith point.

[Margin: The easiest method of finding the declination of a wall inclining toward the south, at the time of noon.]

And so, if the observation is made at noon — namely, when in a plane parallel to the horizon the shadow of the style falls exactly on the meridian line (a time which, if it could conveniently be managed, I should always choose) — the wall's declination will be found quicker than words can tell. For the complement of the angle CDG found on the wall will give, at that time, the wall's declination from the vertical properly so called: from south toward the east, if the shadow falls on the wall to our right as we face it; but from south toward the west, if that same shadow is then made to our left. This is to be understood of a wall inclining toward the south. For a wall facing north is not lit by the Sun at noon, unless the Sun is more northerly than the zenith point, which can happen only in the torrid zone. And when this occurs, if the shadow falls to our left as we face the wall, the wall will decline from north toward the east; but if to our right, from north toward the west.

[Margin: How the declination of a plane inclined to the horizon is to be investigated.]

All of this can also be adapted to planes inclined to the horizon, provided that above the line which is drawn parallel to the horizon on such planes there is set some flat tablet upright to the horizon, and the observation is made on the face of this tablet — which forms an obtuse angle with the inclined plane.

[Margin: By what means it may be known whether the presented wall faces south or north.]

Let us now explain how we may recognize whether a given wall faces south or north. This will not be difficult, provided that we have first, in some fashion — even if not exactly and perfectly — surveyed the layout of the heavens; that is, provided we have ascertained on which side the Sun usually rises and on which it sets, and finally where it stands at noon. This can easily be learned from a magnetized needle [*acus Magnete illita*]. For since such a needle always lies, more or less, straight from north to south, once we know the north by its means, the east comes to lie on our right and the west on our left, as we face north, while we have the south at our back. We shall understand the same thing, should a magnetized needle happen not to be at hand, either from a single nighttime sighting of the pole star (if it is known to us), or from a single observation of the Sun as it rises or sets. For as we face the pole star, our front will show us the north and our back the south, our right the east and our left the west. Likewise, if we place ourselves so as to have the rising Sun on our right and the setting Sun on our left, the four quarters of the world will again correspond to the parts of our body, as just said. Once this is known, we shall test in this way whether the wall declines from the south or from the north. Facing the wall, if we have the rising Sun on our right and the setting Sun on our left, the wall faces south; but if, on the contrary, we see the Sun rise on our left and set on our right, the wall inclines toward the north. But if the wall has only so slight a declination from the prime vertical that it seems to lie but little off the meridian circle, so that it is quite hard to tell whether it faces south or north, we shall use the following device.

Along the wall — or at least along a straight line drawn on it parallel to the horizon — we shall draw, in a plane parallel to the horizon, a perpendicular line, and we shall investigate its position with respect to south and north, as just described. For if the wall deviates only a little from the meridian, the said perpendicular will likewise deviate only a little from the prime vertical, and so we shall easily understand, by the prescribed rule, whether it faces south or north. Thus, if this perpendicular has declined from south toward the east — or (what is the same) from north toward the west — the given wall will incline toward the north if it faces east, but will belong to the south if it looks toward the setting Sun. But if the said perpendicular has swung from south toward the west — or (what is the same) from north toward the east — the wall will face south if it is set against the rising Sun, but will incline toward the north if it looks upon the setting Sun. All of which we have set out more fully in proposition 23, book 1 of the *Gnomonics*.

We shall obtain the same result, for the said walls that decline only a little from the meridian, by means of the meridian line, in this way. Having found, on some plane near the wall that is level with the horizon, a meridian line, then: if this line meets the wall — or at least a straight line parallel to the wall — toward the north, the wall will face south; but if the same meridian line meets the wall (or a straight line parallel to the wall) toward the south, the wall will face north. Indeed, by means of a meridian line drawn on a plane level with the horizon and having one side laid against the wall, we can explore both the amount of the declination and its kind, as we wrote in proposition 23, book 1 of the *Gnomonics*.

At the time of noon we shall reach the same result more easily, for any wall, in this way. Wait, on some day, for the moment of noon — which must be carefully learned either by the style's shadow falling on a meridian line already found, or by the Sun's meridian altitude. For if the presented wall is then lit by the Sun, there can be no doubt that it inclines toward the south; but toward the north, if it is not lit by the Sun. But if the wall stands only a little off the meridian circle, and the roof of the building juts out so as to block the Sun's rays, then a line parallel to the wall must be drawn on the ground at some distance from the wall, so that at noon it can be lit by the Sun along its whole length. For if a style — or the side DH of an upper set-square [norma] — is set upon that line, or at least a fine thread with a plumb-bob is hung free, and the shadow of the style or thread cuts that line at noon: then, if the shadow lies on the northern side, between the said line and the wall, the wall faces south; but if, on the contrary, that line lies on the northern side, between the wall and the shadow, the wall faces north. And if the shadow does not cut the said line but falls exactly upon it at noon, the wall is parallel to the meridian circle.

This, however, must be understood of a place in the northern hemisphere having a latitude greater than $23^{\circ} 30'$ — that being the greatest declination of the Sun. For in a smaller latitude it can happen that a wall facing north is lit by the Sun at midday, while the Sun stands at the beginning of Cancer ☊ [Cancer], or nearly so, because the Sun is then more northerly than the wall itself, as is clear from a material sphere [sphaera materialis]. Yet when the Sun is more southerly than the wall itself, so that the midday shadow is cast toward the north, then those things also hold which we stated concerning the oblique sphere of a latitude greater than $23^{\circ} 30'$.

[Margin: How one may tell whether an inclined plane faces south or north.]

All these things may likewise be tested on planes inclined to the horizon, if on them we draw a line equidistant from the horizon and set upon it a flat tablet

perpendicular to the horizon. For the wall will incline toward that quarter toward which the surface of the tablet, forming an obtuse angle with the proposed plane, is found to look, according to the foregoing instructions — as is plain.

That in every place on Earth situated between the Equator and either tropic — that of Cancer ☿ or of Capricorn ♄ — the shadows of gnomons, towers, and trees on planes equidistant from the horizon retrocede naturally twice a day, when the Sun is more northerly than the place's zenith.

CAPVT XXI.

[Margin: In what places the shadows of gnomons retrocede naturally on planes parallel to the horizon.]

It seems not foreign to our purpose — since we are treating of the description of sundials, in which the shadows of gnomons take on various positions at various hours — to demonstrate, together with Pedro Nunes [Petrus Nonius] (*De Navigatione*, book 2, chapter 11), that it belongs properly to every place of the torrid zone whose zenith is less distant from the Equator than the parallel in which the Sun stands — that is, whose latitude, or pole-altitude, is less than the declination of the Sun when it is set on any given parallel — that the shadows of buildings, trees, and gnomons on planes parallel to the horizon retrocede twice a day without any miracle, once before noon and again after noon. This might seem utterly incredible to some who are little practised in astronomical matters, since in the sundial of Ahaz [Achaz], king of Judah, the sacred scriptures testify that in the time of King Hezekiah [Ezechias] the shadow went back not naturally but by divine power. But that this is not only credible but altogether necessary — and in no way opposed to, or in conflict with, that miracle which God displayed on Ahaz's sundial to King Hezekiah as a sign of his recovered health — will be clearly established from what follows.

Diagram. A large circle representing the celestial sphere in orthographic projection on the plane of the horizon, crossed by the meridian, the equator, a solar parallel, and several vertical (azimuth) circles. Labels read:

- A (top of circle): south point of the meridian / horizon
- C (bottom of circle): north point (pole-side)
- B (left of circle): one solstitial/equator terminus of the horizon
- D (right of circle): opposite terminus of the horizon (equator BED runs B–E–D)

- E (on the meridian, upper-center): intersection of equator and meridian
- F (lower left on circle): point of the Sun's rising on parallel FGH
- G (lower center on circle): intersection of the parallel with the meridian
- H (lower right on circle): point of setting on parallel FGH
- I (center): the zenith of the place
- K (left on circle, below B): terminus of vertical KLIM
- L (center, near I): point where vertical KLIM touches parallel FGH
- M (right on circle): terminus of vertical KLIM / shadow-point
- N (left on circle, below K): terminus of vertical NOPIQ
- O (center-left): point where vertical NOPIQ cuts the parallel (rising side)
- P (center, small): point where the same vertical cuts the parallel (meridian side) [?]
- Q (right on circle, upper): shadow-point on the horizon for vertical NOPIQ
- R (center, small): point where vertical FRIS cuts the parallel [?]
- S (right on circle, above Q): shadow-point at the Sun's rising
- T (upper left on circle): terminus of parallel ST opposite the Sun's parallel
- V (center, small): point on vertical XVIY [?]
- X (bottom-left on circle): terminus of vertical XVIY
- Y (below A, upper region): shadow-point on the horizon for vertical XVIY
- Z (lower center, near C): the pole
- a (lowercase, center-left near O): auxiliary construction point [?]

Let the horizon be ABCD; the meridian, AEC; the eastern semicircle of the horizon, ABC, and the western, ADC; the equator BED, cutting the meridian at E; the northern parallel of the Sun FGH, cutting the meridian (wherever) at G; the zenith of some place between the equator and the said parallel, the point I, through which let there be described — by proposition 15, book 2, of Theodosius [Sphaerics II.15] — the vertical circle KLIM, touching the parallel FGH at L; likewise another vertical, NOPIQ, cutting the same parallel FGH at the point O, between the point of contact L and the point F (at which the Sun, occupying the parallel FGH, rises), and accordingly also at another point P, between the same point of contact L and the point G (placed at noon); and lastly yet another, FRIS, passing through the point of rising F, and therefore cutting the parallel FGH at R, between the points P and G.

Now since, when the Sun is on any great circle of the sphere, the shadow of a style set at right angles to any plane does not depart from the plane of that circle, but is cast into the common section of that circle and of the said plane in which

the style is fixed (as we showed in proposition 11, book 1, of the *Gnomonics*), it follows that, when the Sun is on the vertical FRIS and at the point of rising F, the shadow of a style fixed at right angles to the horizon — or of any tower or tree in the horizon of that place whose zenith we set at I — does not depart from the said vertical FRIS, but cuts the horizon on the western side at S, where the parallel ST, opposite to the Sun's parallel, cuts the horizon; so that in the horizon the shadow is then distant from the meridian, on the southern side, by the arc AS. Then, the Sun being raised above the horizon and set at O, the Sun will be on the vertical NOPIQ, and so the shadow of the same style will cut the horizon at Q, and the distance of the shadow in the horizon from the meridian will be the arc AQ, greater than AS. But when the Sun, by its daily motion, has reached the point of contact L, so that it stands on the vertical KLIM, the gnomon's shadow will cut the horizon at M, and the distance of that shadow in the horizon will be the arc AM, greater still than AQ; and this distance of the shadow, AM, is of all those which on that day (with the Sun on the parallel FGH)

it can have from the meridian, the greatest — because the vertical KLIM, touching the parallel FGH, is of all the verticals on which the Sun can be that day the farthest from the meridian, as is manifest. And so, from the rising of the Sun at F until it has reached the point of contact L, the gnomon's shadow has advanced continuously in the horizon from S, through Q, to M, always receding more and more from the meridian.

Then, when the Sun has moved on from L to P, the Sun will again be on the vertical NOPIQ, and the shadow of the style will cut the horizon once more at Q, and the distance of the shadow from the meridian will again be the arc AQ, just as before, when it was at O — so that the shadow, once it has checked its outward course from the meridian at the point M, has retroceded in the horizon from that same point M, approaching the meridian again as far as Q. Then, the Sun being advanced from P to R, the Sun will again be on the vertical FRIS, and the gnomon's shadow will cut the horizon at S, and the distance of the shadow from the meridian will be the arc AS, just as before, when the Sun was rising at the point F — so that the shadow, still approaching the meridian, has retroceded farther back from the point M, namely as far as S, where the shadow cut the horizon at the Sun's rising, and from where it had advanced to the points Q and M. But from the point S onward the shadow will approach the meridian continuously, without any regress. For when the Sun has reached V from R, so that it stands on the vertical XVIY, the shadow will cut the horizon at Y, and its distance from the meridian will be the arc AY, less than AS; and so on, until it reaches the meridian, namely the point G.

[Margin: Same figure repeated — see fig-p632-1 below.]

The shadow will keep this same proportion of advancing and regressing after noon, as can be gathered from the same figure, if the semicircle of the horizon ADC be taken as the eastern, and ABC as the western. For as the Sun moves from G, the point of noon, through V, R, P, up to the point of contact L, so that it successively stands on the verticals XVIY, FRIS, NOPIQ, KLIM, the shadow too will successively cut the horizon at Y, S, Q, M; so that during that time it always recedes more and more from the meridian, having advanced always from A, through Y, S, Q, up to M. When the Sun has reached O and F, it will again be on the verticals NOPIQ, FRIS, and the gnomon's shadow will cut the horizon at Q, M — so that again it has checked its course, as regards its recession from the meridian at the point M, and has returned through Q toward the meridian, always approaching the meridian more and more again, until it reaches S, where the Sun will be on the vertical FRIS, and will set at the point F.

Diagram. The same figure repeated, to accompany the after-noon case. The circle and its labels correspond to fig-p631-1: horizon ABCD (A top, C bottom, B left, D right), meridian AEC, equator BED through E, solar parallel FGH through G, zenith I, pole Z; verticals KLIM (touching the parallel at L), NOPIQ (cutting it at O and P), FRIS (through F, cutting at R), and XVIY (through X, V, Y); the shadow-points S, Q, M, Y lie on the horizon, with T the terminus of the opposite parallel. As on the preceding page, the small central letters (P, R, V, a) are crowded and hard to resolve

It is clearly established from all this that which we set out to demonstrate: namely, that in every place on Earth set between the equator and any given parallel — whose zenith, that is, is less distant from the equator than the parallel; that is, whose pole-latitude is less than the declination of the assumed parallel (such as the place beneath I, if it be compared with the parallel FGH; for the latitude EI of that whole place is less than the declination EG of the said parallel — to which latitude the pole-altitude CZ is equal, as is demonstrated in [the treatise on] the sphere) — the shadow of a style forming right angles with the horizon, once set beneath I, retrocedes twice a day naturally and without any miracle, once before noon and after noon again.

The cause of this, as is clear from what has been demonstrated, is that the Sun, traversing the said parallel and advancing from its rising, stands continuously on one vertical circle after another, of which each successive one is always farther from the meridian than the preceding, until it reaches the vertical that touches that parallel — such is the vertical KLIM, which of all the verticals is the farthest

from the meridian. Thereafter the Sun, although it still advances toward the meridian circle, nonetheless returns to the same verticals on which it had stood before, until it reaches the one on which it stood when it rose; and from this it will occupy one after another, until it is found on the meridian — which verticals grow ever nearer the meridian. After noon, advancing in the same parallel, the Sun will likewise occupy various verticals receding ever more and more from the meridian, until it reaches the one that again touches the parallel and is farthest from the meridian. From this, still advancing in the parallel, it will repeat the same verticals up to the one that cuts the horizon at the point of setting. The Sun is set twice before noon and twice after noon on the same verticals — namely on all those that lie between the point of rising and of setting of the said parallel and that farthest vertical. All these things are gathered from the proposed figure.

[Margin: How, outside the torrid zone, a plane must be set up so that the shadow retrocedes on it.]

Although this is true only in the horizon of any place set between the equator and one of the two tropics — as, for example, in the horizon of Goa, Calicut [Calecut], Cannanore [Cananor], the Golden Chersonese [Aurea Chersonesus], and Mexico in New Spain [Hispania nova], from the equator toward the north; and of the Cape of Good Hope, Brazil, Peru, and Greater Java [Java maior], from the equator toward the south — nevertheless the same may be tested in the temperate zone, if on the side of the visible pole some plane, perpendicular to the meridian, is raised together with a fixed style, above which the pole has an altitude less than $23^{\circ} 30'$. For since that plane then has the same situation with respect to the Sun that the horizon of some place in the torrid zone has, the gnomon's shadow will retrocede on it just as in that horizon of the torrid zone. And if a sundial is constructed for a latitude less than $23^{\circ} 30'$, with the hyperbolas — that is, the arcs of Cancer ☿ and Capricorn ♑ — that is, on a plane inclined to the horizon of Rome by more than $18^{\circ} 30'$, but by fewer than 42°

on the northern side (for so the vertex, or pole, of this plane will fall between the equator and the tropic of Cancer ☿); or by more than 42° , but fewer than $65\frac{1}{3}^{\circ}$ (in which way the pole, or vertex, of the plane will lie between the equator and the tropic of Capricorn ♑) — then this advance and retrocession of the shadow will be most plainly observed, the sundial being set in its proper position according to its inclination to the horizon. For if from the place of the gnomon, which is between the equator and one of the two tropics, two straight lines be drawn tangent to that tropic to which the style is the nearer, the shadow of the style will, in the morning, advance from the rising of the Sun toward the equinoctial line of

the sundial, up to that tangent line, always receding from the meridian line. From this tangent line it will then regress, receding from the equinoctial line and approaching the meridian line, until the Sun reaches noon; and from noon it will again approach the equinoctial line, receding from the meridian, until it reaches the other tangent line, from which it will again retrocede toward the meridian line, receding from the equinoctial, up to the setting of the Sun.

[Margin: Our retrocession of the shadow, which we have set out, is not opposed to the retrocession of the shadow in Ahaz's sundial, [which was] wrought by divine power[?].]

This being so, what then shall we say of that so illustrious miracle in the sundial of Ahaz, shown to King Hezekiah as a sign of his health, in which the sacred scriptures relate that the shadow went back ten lines, or degrees, not naturally but by divine power? Pedro Nunes, at the passage cited, answers that the regress of the shadow, of which we have spoken, can occur only in the torrid zone — that is, between the equator and one of the two tropics — as is clear from what has been demonstrated. Since, then, the city of Jerusalem, where Hezekiah lay sick, is situated in the temperate zone, on this side of the tropic of Cancer ☊, the shadow could not have gone back without a miracle. Therefore, since the sacred scriptures testify that it did go back, it was without doubt done by a miracle. And that this is what Nunes meant, and that the answer must be so given, is manifest from the cited chapter 11, where he thus writes: *[It is not, then, absurd if in those places (namely, between the equator and the tropic of Cancer ☊, or of Capricorn ☋) the shadows advance and retrocede. Yet in this our northern region, which lies on this side of the tropic of Cancer ☊, this could not happen without a miracle, as we read it befell, by God's command, as a sign of the health of King Hezekiah.]* By these words he seems to signify that this our regress does not differ from that wrought in Ahaz's sundial, except that ours is natural, while that was effected by God's command, on account of the differing situations of places.

But this answer does not satisfy. For although that regress of the shadow cannot occur in the horizon outside the torrid zone, it nonetheless comes about — if some plane be raised, with a style, above which the pole has an altitude less than $23\frac{1}{2}^{\circ}$ [fraction uncertain; expected $23^{\circ} 30'$ — ?] — that the shadow will retrocede upon it exactly as in some horizon of the torrid zone, as we said above. If, then, anyone were to say that the plane of Ahaz's sundial had such a situation, who does not see that that regress of the shadow could have come about upon it by the powers of nature, and thus that so signal and celebrated a miracle would be emptied out? — which is by no means to be admitted. For which reason we say

that the miracle in that regress of the shadow in Ahaz's sundial was truly great: because the Sun then went back in the heaven, and the shadow truly regressed, so that after noon, when it ought to have grown, it was diminished, or before noon it was increased, when it ought to have been diminished — which cannot come about by the powers of nature. And that the Sun then indeed went back, the prophet Isaiah seems openly to affirm, writing thus in chapter 38: [*And the Sun returned ten lines by the degrees which it had gone down.*] But in our regress the shadow before noon is always diminished, and after noon perpetually increased, and the Sun advances continuously from rising to setting without any regress — although the shadow itself does regress somewhat, from north to south or the contrary, and is not carried continuously in a circle around the gnomon, as happens in every horizon outside the torrid zone, but moves in a certain manner along a straight line from east to west while the Sun is near the point of contact L, that is, while it traverses the arc OLP — especially if the regress QM, by which the Sun goes back from M to Q, be very small and almost imperceptible; which indeed can come about without a miracle, as we have demonstrated. And certainly it is an argument that the regress made in the time of King Hezekiah was very different from ours, that that event was new and unwonted in Ahaz's sundial, whereas our regress appears in the same plane every year over several days — so that the prophet Isaiah would have promised nothing new as done at God's command[?], but only what he continually observed in that sundial on certain days of the year; which would have been quite ridiculous.

[Margin: The finding, by spherical trigonometry, of the amplitudes of rising and setting, the semidiurnal arc, the Sun's altitude at the points of retrocession, its distance in the parallel from the meridian, likewise the amount of the shadow's regress, and finally the time during which the shadow advances.]

Now, the altitude of the pole CZ and the declination of the parallel FGH being known, we shall learn, in the horizon ABCD, from the doctrine of spherical triangles: the amplitude of rising BF and of setting DH; the semidiurnal arc FG; the altitude of the Sun KL at the point of contact L, and FR at the point R; the distance of the same in the parallel from the meridian at those same points L, R, namely both the arc GL and the arc GR; the arc MS of the whole regress; and finally the time in which the Sun traverses the arcs FL, LR, FR — or (what is the same) in which the shadow moves from the point S, when the Sun rises, to the point of regress M, and moves back again from M to S; and accordingly the whole time in which the shadow advances and regresses.

For, three arcs of great circles ZF, ZL, ZR being drawn from Z, the pole, to the points F, L, R: since in the right triangle CFZ (for the angle C, which the meridian makes with the horizon, is right) two arcs CZ, ZF are known — CZ being the altitude of the pole, which we now put at 20° , and ZF the complement of the declination, which we here put at $23^\circ 30'$, so that the Sun stands at the beginning of Cancer $\mathfrak{S}$ — of which ZF is opposite the right angle: let it be, as 93969 (the sine of the complement of the arc CZ, the altitude of the pole) is to 39875 (the sine of the complement of the arc ZF, opposite the right angle, which comprises $66^\circ 30'$ — that is, to the sine of the declination), so 100000, the whole sine, to another; and there will be produced the sine 42434 of the complement of the arc CF, by practical rule 8 of our Spherical Triangles; to which sine corresponds an arc of $25^\circ 7'$. And just so great is both the arc BF of the amplitude of rising (namely the complement of the arc CF) and the arc DH of the amplitude of setting; and accordingly the arc CF is $64^\circ 53'$. And so the shadow of the same style will cut the horizon at the point of setting; CF also comprises $64^\circ 53'$.

[Margin: In the same right triangle CFZ.]

In the same right triangle CFZ: if it be made, as 91706 (the sine of the arc ZF[?], the complement of the declination, opposite the right angle) is to 100000, the whole sine, so 90545 (the sine of the arc CF just found) to another, there will be found the sine 98734 of the angle CZF, opposite the arc CF, by practical rule 1 of our Spherical Triangles. The angle CZF, therefore, contains $80^\circ 52'$, and just so great is the seminocturnal arc [arcus seminocturnus]. And that angle being taken from 180° — that is, from a semicircle — the remaining angle FZG, of the semidiurnal arc FG, will be $99^\circ 8'$.

[Margin: The Sun's altitude and its distance from the meridian at the point of regress, by the same triangles.]

Next, since the two circles FGH, KLIM touch one another at L, and the arc LZ is drawn through Z, the pole of the circle FGH, it will be drawn also through the pole of the circle KLIM, by proposition 5, book 2, of Theodosius [Sphaerics II.5]; and accordingly, in the triangle ZIL, the angle ZLI, by proposition 15, book 1, of Theodosius [Sphaerics I.15], will be right. Therefore, if it be made, as 39875 (the sine of the complement of the arc ZL, about the right angle — namely, as the sine of the Sun's declination, since ZL is the complement of the Sun's declination) to 34202

(Continuation of Cap. XXI — the worked example of the twice-daily shadow-regress in the torrid zone, carried over from the previous folio, which broke off at the catchword "...to 34202.")

to 34202 (the sine of the complement of the arc IZ, opposite the right angle — that is, to the sine of the arc CZ, the altitude of the pole, since IZ is the complement of CZ), so 100000, the whole sine, to another; there will be found, by practical rule 8 of our spherical triangles, the sine 85773 of the complement of the arc IL. To this sine, in the table of sines, corresponds an arc of $59^{\circ} 4'$. Such, then, is the arc KL — namely, the complement of the arc IL — of the altitude of the Sun established at L. And the complement of the altitude of the Sun, IL, will contain $30^{\circ} 56'$.

Likewise, in the right triangle CFZ, if it be made — as 91706 (the sine of the arc ZF, the complement of the declination, opposite the right angle) is to 100000, the whole sine, so 34202 (the sine of the arc CZ, the altitude of the pole) to another — there will result, by practical rule 1 of our spherical triangles, the sine 37295 of the angle CFZ, opposite the arc CZ. The angle CFZ, therefore, contains $21^{\circ} 54'$. And when this is taken from the right angle CFI, which the vertical FRIS makes with the horizon, the angle ZFR will comprise $68^{\circ} 6'$. Just as great, too, will be the angle ZRF, since it is equal to the angle ZFR — by proposition 8 of our spherical triangles — on account of the equal arcs ZF, ZR, each of which is drawn from the pole Z to the parallel FGH and is the complement of the declination. Now, if a perpendicular arc Za be drawn from Z to FR, it will bisect both the arc FR and the angle FZR, as is established from the second case of proposition 62 of our spherical triangles, where we demonstrated that in every isosceles spherical triangle the perpendicular arc bisects both the base and the angle opposite the base.

And so, in the right triangle FZa, if it be made — as 100000, the whole sine, is to 91706 (the sine of the arc ZF), so 92784 (the sine of the angle ZFa) to another — there will arise the sine 85088, which belongs to the perpendicular arc Za, as is established from practical rule 2 of our spherical triangles; and accordingly that perpendicular arc Za will comprise $58^{\circ} 18'$. And if, in the same right triangle FZa, it be made again — as 52547 (the sine of the complement of the said perpendicular arc Za) is to 39875 (the sine of the complement of the arc ZF, opposite the right angle), so 100000, the whole sine, to another — there will be generated, by practical rule 8 of our spherical triangles, the sine 75816 of the complement of the arc Fa; and consequently that complement will be $49^{\circ} 18'$, while the arc Fa itself, which is half of the arc FR (as has been said), is $40^{\circ} 42'$. And therefore the whole arc FR — of the altitude of the Sun at R — is $81^{\circ} 24'$.

Moreover, in the triangle ZIL — whose angle L is right, as we showed above — if it be made, as 93969 (the sine of the arc ZI, the complement of the altitude of the pole, and opposite the right angle) is to the whole sine 100000, so 51404 (the sine of the arc IL, the complement of the altitude of the Sun KL, which we found above to be $59^{\circ} 4'$) to another, there will be produced, by practical rule 1 of our spherical triangles, the sine 64703 of the angle IZL, opposite the arc IL. The arc of the equator due to it, therefore, will comprise $33^{\circ} 10'$; and to this, by proposition 10 of book 2 of Theodosius [*Sphaerics* II.10], the arc GL of the parallel — by which the Sun is distant from the meridian — is similar.

Further, in the right triangle ZRa, if it be made, as 91706 (the sine of the arc ZR, the complement of the declination, and opposite the right angle) is to 100000, the whole sine, so 65210 (the sine of the arc Ra, which is half of the arc FR and which we found above to be $40^{\circ} 42'$) to another, there will result, by practical rule 1 of our spherical triangles, the sine 71108 of the angle RZa, which is half of the angle FZR itself. The said angle RZa, therefore, will embrace $45^{\circ} 19'$, and consequently the whole angle FZR $90^{\circ} 30'$. When this is taken from the angle FZI of the semidiurnal arc — found above to be $99^{\circ} 8'$ — there will remain the angle IZR, by which the Sun at R is distant from the meridian, [namely] $8^{\circ} 30'$; and just so great is the arc GR of the parallel.

Again, in the right triangle IZL, if it be made, as 93969 (the sine of the arc ZI, the complement of the altitude of the pole, and opposite the right angle) is to 100000, the whole sine, so 91706 (the sine of the arc ZL, the complement of the declination) to another, there will be produced, by practical rule 1 of our spherical triangles, the sine 97592 of the angle LIZ, that is, of the arc CK. The arc CK, then, is $77^{\circ} 24'$. If from it the arc CF — found to be $64^{\circ} 53'$ — be subtracted, the arc FK will remain, to which the arc SM, the regress of the shadow, is equal, by proposition 7 of our spherical triangles. (For the two arcs IF, IK are equal to the two arcs IM, IS, since all are quadrants and contain equal angles at the vertex I, by proposition 5 of our spherical triangles.) [And so the arc SM is] $12^{\circ} 31'$.

Finally, since the semidiurnal arc FG has been found to be $99^{\circ} 8'$ — that is, 6h 36m 32s; and the arc GL $33^{\circ} 10'$, that is, 2h 12m 40s; and therefore the arc FL $65^{\circ} 58'$, or 4h 23m 52s; and the arc GR, lastly, $8^{\circ} 30'$, or 0h 34m; and accordingly the arc FR $90^{\circ} 38'$, namely 6h 2m 32s; and consequently the arc LR $24^{\circ} 40'$, that is, 1h 38m 40s — it is plainly established that the shadow advances from S as far as M — that is, that the Sun moves from the rising-point F as far as L, the point of regress — in a span of 4h 23m 52s; that the same shadow regresses from M as far as S — that is, that the Sun passes from L to R — in a span of 1h 38m 40s; and, finally, that the

shadow advances and regresses to the same point — that is, that the Sun traverses the whole arc FR — in a span of 6h 2m 32s.

FINIS.

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FINIS.

Horologiorum Nova Descriptio — A New Description of Sundials

[Title]

CHRISTOPHORI CLAVII BAMBER- GENSIS EX SOCIE- TATE IESV, HOROLOGIORVM NOVA DESCRIPTIO.

AB IPSO AVCTORE TVM ALIARVM rerum, tum maxime plurimarum vtillum tabularum accessione vehementer aucta.

[Translation] Christoph Clavius [Christophorus Clavius] of Bamberg, of the Society of Jesus: A New Description of Sundials. Greatly enlarged by the author himself, both with other matters and, above all, with the addition of very many useful tables.

" / "...BILE" (lower left and left). The motto is the standard *Sit nomen Domini benedictum ... laudabile* ("Blessed be the name of the Lord ... praiseworthy").]

[Two manuscript ownership inscriptions in ink flank the device: left margin, a cursive signature reading approximately "Res[?] Mon[?]" [?]; right margin, a

cursive signature reading approximately "Schÿsenhis[?]" [?]. Both are early-modern provenance marks, not printed text.]

MOGVNTIÆ, Sumptibus ANTONII HIERAT, excudebat IOANNES VOLMARI.

Cum Gratia & Priuilegio Sacræ Cæsar. Maiest.

ANNO M. DC. XII.

[Translation of imprint] At Mainz [Moguntiae], at the expense of Antonius Hierat [Anton Hierat]; printed by Ioannes Volmari [Johann Volmar]. With the grace and privilege of His Sacred Imperial Majesty. In the year 1612.

Blank page. Only the mirror-image show-through (offset) of the title-page type and device is visible; no text is printed on this side.

CHRISTOPHORI CLAVII BAMBERGENSIS, EX SOCIETATE IESV, HOROLOGIORVM NOVA DESCRIPTIO.

[Christoph Clavius of Bamberg, of the Society of Jesus: A New Description of Sundials.]

PROLEGOMENA

In this little book we shall treat only of those sundials that are in more frequent use — namely, those that lie parallel to the horizon, called horizontal dials, and those that stand perpendicular to the horizon and parallel to some vertical circle, which are called vertical dials and are commonly drawn on the walls of buildings. [Margin: *De quibus horologiis in hoc libello agendum* — Which sundials are to be dealt with in this book.] But should anyone wish to lay out other kinds of sundial — namely, dials declining from the horizon, or inclined to the horizon (among which the equinoctial and polar dials are also to be reckoned), or finally dials that both decline and incline at once — we hold that he should be referred to our *Gnomonics* [Gnomonica], where we have written of them at the fullest length. For since these have less frequent use, we deliberately pass over them in this place, so that this little book may not swell to a greater bulk than is fitting, but instead may prove more serviceable for practical use, and thus more readily obtainable.

Now since in this treatise we shall for the most part describe the hours on a given plane by a new method and procedure, without any circle divided into 24 hours and their parts, or into degrees — that is, solely by means of the lines called Tangents [lineae Tangentes], with the aid of certain other tables which you will find after the description of the sundials, together with their construction from problems of astronomy — it is necessary to have a table of the lines of Tangents ready at hand, so that from it the Tangents that are needed may be extracted. [Margin: *Nova ratio describendorum horologiorum* — The new method of describing sundials.] You will find this table too at the end of the whole little book, in the form in which the whole sine [sinus totus] comprises 1000 parts, which is enough for the present business.

For the rest, since on a given plane the Tangents cannot be marked off in as many parts as the table contains with respect to the whole sine of 1000, we shall reduce them to tenths by dropping the two figures on the right — that is, we shall take them with respect to a whole sine of 10 — yet not neglecting the fraction which the last figure on the left (which of course signifies tens), once dropped, forms together with the whole sine 10 as denominator; or rather the fraction which the two dropped figures form, in the manner of a numerator, with 100 as denominator. [Margin: *Quomodo Tangentes reducuntur ad decimas* — How the Tangents are reduced to tenths.] For example, the Tangent of 15 degrees is 268 with respect to the whole sine of 1000. If the two figures 68 are dropped, there will remain, with respect to the whole sine of 10, the Tangent $2\frac{6}{10}$. But more accurately it would be $2\frac{68}{100}$; for if the proportion is made, as 1000 is to 268, so is 10 to another number, the quotient found is $2\frac{680}{1000}$, that is, $2\frac{68}{100}$. Yet the last figure 8 on the right may be dropped without loss, keeping only the figure 6, so that the Tangent becomes $2\frac{6}{10}$ — although it could be taken as $2\frac{7}{10}$ nearly, on account of the following figure 8, which is greater than 5.

Again, the Tangent of 48 degrees 45 minutes is 1140. When the two last figures 40 are dropped, it will be, with respect to the whole sine of 10, the Tangent $11\frac{40}{100}$, or $11\frac{4}{10}$. Likewise the Tangent of 78 degrees 45 minutes is 5027. But when the last two figures 27 are dropped, there will be left the tangent $50\frac{2}{10}$ — indeed nearly $50\frac{3}{10}$, on account of the following figure 7, which is greater than five; that is, it exceeds half of one tenth, namely $\frac{5}{10}$ [of one tenth].

But if, after the two last figures are dropped, nothing remains, we shall take away only one; for the remaining figure will give the tenth-parts of one tenth of the whole sine 10. [Margin: *Quales Tangentes sumendae sint, quando duae figurae abici nequeunt* — What Tangents are to be taken when two figures cannot be

dropped.] Thus, since the Tangent of 3 degrees 45 minutes is 66, if the latter figure 6 is removed, there will remain the Tangent 6/10 of one tenth of the whole sine 10 — and a little more, on account of the dropped figure 6. But if there is only a single figure in the tangent, so that nothing can be taken away, that single figure will indicate the hundredth-parts of one tenth of the whole sine 10. So, since in the table of Tangents the Tangent of 0 degrees 11 minutes is 3, we shall have 3/100 of one tenth for the Tangent with respect to the whole sine 10. Finally, since 0 degrees 1 minute, and 0 degrees 2 minutes, and 0 degrees 3 minutes have nothing in the Tangents (seeing that the Tangents of these arcs, with respect to the whole sine of 100000, are 29, 58, 87), for the Tangent of 0 degrees 1 minute we shall take about 3/1000 of one tenth, with respect to the whole sine 10. And for the Tan-

gent of 0 degrees 2 minutes we shall take about 6/1000 of one tenth; and finally, for the Tangent of 0 degrees 3 minutes, we shall take nearly 9/1000 of one tenth. But parts as tiny as these can scarcely be marked off or noted on a plane. They may therefore reasonably be neglected — unless the gnomon [stylus] happens to be so large that it can conveniently be divided into 1000 parts or more. Alternatively, tangents this small will have to be added to the Tangent of some point already found, so that the whole Tangent, once assembled, can be carried over from the point so found — as we shall explain in Chapter 2, Number 3.

[Margin: *Quid agendum cum Tangentibus perexiguis* — What is to be done with very small Tangents.]

Furthermore, in the table of Tangents that presents the whole sine as 10000000, six figures must be dropped from each Tangent, so that they are reduced to Tangents with respect to the whole sine of 10. But in the table of Tangents in which the whole sine is set at 100000, four figures must be removed, and so on.

To draw certain particular lines in sundials more expeditiously (though this will seldom be needed), it will be worthwhile to procure a small brass plate [tabella ænea] on which several unequal quadrants — at least two, or three, or four — are drawn, each carefully divided into 90 degrees; or at any rate (which is enough) divided into 9 equal parts, so that each part takes in ten degrees, with only the first part cut accurately into 10 degrees, just as we taught in Lemma 3 of the first book of our *Astrolabe* [Astrolabio]. For in this way, on the circumference of any circle whatever, we shall be able to take an arc of any number of degrees and minutes, even though that circle is not itself divided into degrees — provided only that it has been drawn to the radius of one of those quadrants set out on the brass plate.

[Margin: *Quales quadrantes necessarij sint ad accuratam horologiorum descriptionem* — What kind of quadrants are needed for the accurate laying-out of sundials.]

One may also, if preferred, use in place of those three or four quadrants drawn on a single plate a single hollowed-out quadrant, such as the learned Theodosius Rubeus [Teodosio Rossi] of Priverno recently devised with much ingenuity — a Doctor in Sacred Theology and in both Laws, a Noble of the Sacred Imperial Court, likewise most skilled in Mathematics, and once my pupil — whose construction is as follows. On a brass lamina ABC, from point D, let a quadrant EF be drawn, divided into 90 degrees. About the same point D let a small circle be drawn, and to it let a ring [annulus] be fastened, of such thickness that a movable ruler [regula] DF can be turned about it, its fiducial line [linea fiduciæ] always pointing toward the center. Next let the parts that appear white in the figure be cut away as superfluous, so that the ruler DI can bring its fiducial line DF onto the beginning of the first degree at point G. Let the ruler also be hollowed out alongside the quadrant, so that when it is turned it can graze the plane on which the quadrant is to be used — which is done thus. With the center D placed at that point where an angle of any number of degrees is to be constructed, and the fiducial line holding the beginning of the quadrant, let the quadrant be applied to the line with which the angle is to be formed. After this, with the quadrant held fast so that it does not shift its position, let the ruler be moved until the fiducial line DF reaches the proposed degree. For the straight line drawn in that position by aid of the fiducial line will form, with the given line at D, the required angle of whatever number of degrees; or from the circumference of any circle whose center coincides with the center D, it will cut off any proposed number of degrees. But so that the course of laying out the sundials is not interrupted, we have used a different type [character] in the demonstrations, so that they may be passed over untouched by the student who wants only the practical procedures.

Diagram. The hollowed brass quadrant-instrument of Theodosius Rubeus, drawn within a square border. The solid body of the lamina is shown by close hatching along the top and the left side; the white (cut-away) region is the quadrant sector. Labels read from the scan:

- A (top-left corner of the square lamina)
- B (top-right corner)
- C (bottom-left corner)
- D (center of the small ring/pivot circle, upper-left of the field, with cross-hairs)

- E (right edge, below B — one end of the graduated arc EF)
- H (left edge, below A)
- G and F (mid-right, near together — the movable ruler DF and the degree-mark region)
- G, I, F (along the bottom edge, left of center — the ruler's lower position and the initial graduation)
- The quarter-circle arc is divided into degrees; graduation numerals (10, 20, 30 ... up to 90) run along it, but the individual small numerals do not all resolve on the scan [numeral:?].

CAPVT PRIMVM.

PROBLEMA HOROLOGIOGRAPHICVM.

Given a straight line on the plane of any sundial, to cut off from it a segment corresponding to any number of degrees you please of that great circle which the given line represents — beginning from any given point, and proceeding in either direction. And conversely, given any segment on the given straight line, to find out to how many degrees of that great circle which the given line represents on the sundial the segment corresponds.

[Margin: *Problema nouum, & ad horol. describ. vtilissimum. Problema hoc non esse ita necessarium, ut sine eo horologia construi nequeant* — A new problem, and most useful for the laying-out of sundials. That this problem is not so necessary that without it sundials cannot be constructed.]

1. Before we lay out sundials, we must demonstrate the proposed problem — a new one indeed, but one that holds well-nigh countless uses in the laying-out of sundials. Do not, however, take it as though without it sundials could not be built by the teaching of this little book. For you will feel no loss or inconvenience in laying out sundials even if you entirely neglect or pass it over. We demonstrate it here both for its remarkable subtlety and excellence and for the very great pleasure of mind you will derive from it, whenever you apply it to tracking down certain points on the hour-lines, even though those same points may already have been found by another method. Let there be, then, on the plane of any sundial — on which the gnomon is AB and its foot is at A — a given straight line CD, whether or not it passes through E, the center of the sundial; and let there be given on it a point C, from which

a corresponding segment is to be cut off — say, of 30 degrees on that celestial great circle which the line CD represents. For every straight line on a sundial is the common section of the plane of the sundial and of some great circle, as is clear from what was demonstrated in Book I of the *Gnomonics*, Proposition 11 and its scholium. From A, the foot of the gnomon, let a perpendicular AF be drawn to the given line CD; and taking on the given line a portion FG equal to the gnomon AB, let there be taken, on the perpendicular AF — whether beyond point F or on the near side — a line FH equal to the interval AG. Then with H as center let a circle IK of any size be drawn (it will be convenient if its radius is equal to the radius of one of the quadrants divided into degrees); and from the center H, through the given point C on the proposed straight line, let a line HC be drawn, cutting the circle at I. Finally, from point I let an arc IK of the proposed 30 degrees be taken — upward or downward, according as the segment of 30 degrees is to be cut off below the given point C or above it. For the line HK, once drawn, will cut off from the given line the portion CD of 30 degrees. In the figure the segment CD is cut off on both sides. This I demonstrate as follows.

[Margin: *Quo pacto ex recta abscindatur portio quotcunq; graduum* — In what way a portion of any number of degrees is cut off from a straight line.]

2. On the perpendicular AF, at A, let a perpendicular AL be erected, equal to the gnomon AB, and let the lines AG and FL be joined. Now since in triangle AFG the two sides AF, FG are equal to the two sides FA, AL of triangle FAL, and they contain equal angles — namely right angles — the base AG, that is, the line FH, will be equal to the base FL. [Margin: a — Euclid I.4.] Now imagine the triangle AFL turned about AF until it stands upright to the plane of the sundial, so that point L is set at the center of the world — that is, coincides with B, the tip of the gnomon erected at A — since AL was put equal to AB. This being so, if the plane of the circle IK is turned about the given line CD, its center H will coincide with L, the center of the world, because the lines FH and FL are equal. Therefore the segment CD on the given straight line corresponds to the arc IK, since the radii IH, KH, produced beyond the center H, cut off in the heavens as many degrees as are contained in the arc IK — an arc, namely, equal or similar to the arc IK, on account of the vertical angles at the center H being equal.

Diagram. A large, dense sundial-construction figure enclosed in a square border, with a graduated double-rule scale (the tangent line) running down its right side. It combines the stylus and its rabatment (the triangle AFL swung up to the pole), the given line CD through the sundial center E, the circle IK about the found

center H, and the intersecting hour-arcs. The labels are heavily crowded and several letters recur at more than one point of the construction. Legibly read from the scan:

- E — apex, at the top of the field
- G, Y, L — clustered just below E (G left, Y center, L right)
- H — upper left
- M, F — below H toward center (M above F)
- A — the central node, with dotted lines radiating from it
- P, T, N — to the left of A (P leftmost, then T, then N)
- B — right of A
- K, O — just below and right of A (K, then O)
- D, F — right of center, descending (D above F)
- C — appears at three places: below A left-of-center, again lower, and again to the right
- L, I — below center (L above I)
- G — again at mid-right, and again lower-center
- K — at left, and again at right
- Q, R — low center (Q left, R right)
- I — lower right
- D — bottom-left and bottom-center-right (two marks)
- H — a further mark at the right where the shadow-lines converge, beside the scale

Right-hand graduated scale (the tangent line ST / VX): two parallel vertical rules with tick-clusters near the top, lettered **V** and **S** at the head and **X** and **T** at the foot; the divisions are numbered (printed with a raised dot) **10, 10, 20, 20, 30, 30** descending the two rules.

The figure is exceptionally crowded and carries many repeated letters that cannot all be individually verified on the scan.

3. When the arc to which a segment cut from the line CD is to be made similar — beginning from point C, below C — forms, together with the arc between the lines HI and HA, a quadrant or an arc greater than a quadrant, the segment similar to that arc cannot be cut off below the given point. For a line drawn from the center H through the extremity of the arc cannot cut the line CD below C, since at H it makes a right angle with the line HA, [Margin: b — Euclid I.28.] and hence is parallel to the given line CD; or it

makes an obtuse angle, and so cuts the line CD above the line HA. Thus, if one were ordered to cut off below C, from the line CD passing through the center E, a segment of 55 or 60 degrees, the arc between the lines HI and HFA containing (say) 35 degrees, this could not be done: for the arc below I of 55 or 60 degrees, together with the arc between the lines HI and HA, would make up 90 or 95 degrees. Therefore in that case such a segment can only be cut off above the point C.

[Margin: *Quando problema est impossibile* — When the problem is impossible.]

4. But if the given line, from which the proposed degrees are to be cut off, passes through A, the foot of the gnomon — such as the line EA — then only a perpendicular AB, equal to the gnomon, need be erected upon it. For the extremity B will be the center of the circle by which the degrees are to be cut off, because when the plane of the circle drawn from B is turned about the line EA until AB is upright to the plane of the sundial, point B stands at the center of the world, namely at the tip of the gnomon. So if from the line EA, below point A, a segment of 60 degrees is to be cut off, then with the perpendicular AB (equal to the gnomon) erected and a circle drawn from B, an arc of 60 degrees is to be taken below the line BA produced, as far as Q. For the line BQ will cut off the segment AR of 60 degrees, as has been demonstrated.

[Margin: *Quando data recta transit per centrũ horologij, quid agendum* — When the given line passes through the center of the sundial, what is to be done.]

[Margin: *Qua ratione ducatur perpendicularis ad datam rectã per centrũ horol. trãseuntẽ, centrumq; circuli describendi inueniatur* — In what manner a perpendicular is drawn to a given line passing through the center of the sundial, and the center of the circle to be described is found.]

5. For the rest, to lines passing through E, the center of the sundial — such as ECD — and to all the hour-lines from midday and midnight, in sundials whose great circles (to which those lines are parallel) are not drawn through the poles of the world, we shall draw perpendiculars from A, the foot of the gnomon, with little trouble, and at the same time, by the same operation, we shall find the center H, in this way. With the gnomon AB erected at right angles to the line EA of the stylus, let two circles AM, BH be drawn from E, the center of the sundial, through A and B; and let the given line EC, passing through the center E, cut the former of them at N. For if an arc NM equal to the arc AN is taken, and the line AM is drawn, cutting the

line EC at F and the latter circle at H, then AF will be perpendicular to EC, and H will be the center of the circle to be described — that is, the line FH will be equal to the line AG; so that there is no need to take FG equal to the gnomon and to cut off FH equal to the interval AG, but it is enough to produce the line AM up to the circumference of the latter circle. Both of these will be demonstrated as follows.

Diagram. The same sundial-construction woodcut described under fig-p641-1, reprinted here on the verso so that the reader has the figure at hand while reading Chapter 1 on either side of the leaf. The lettering (E; G, Y, L; H; M, F; A; P, T, N; B; K, O; C, D, F, I, L, G, K, Q, R, D) and the right-hand tangent scale (rules V–X and S–T, numbered 10, 10, 20, 20, 30, 30) are as on p. 641. Marked a for the same reason (crowded, repeated letters).

6. Since the line EC bisects the arc ANM, by the scholium to Proposition 27 of Book 3 of Euclid it will also bisect the line AM, [Margin: a — Euclid III.3.] and therefore at right angles. Which was the first point. Next, with the gnomon BA produced until it cuts the latter circle at T, AB will be equal to AT: because EA, from the center E, cuts BT by construction at right angles, [Margin: b — Euclid III.3.] and therefore bisects it. Likewise, with the line HA produced until it cuts the same latter circle at O, the lines MH and AO will be equal, by the scholium to Proposition 3 of Book 3 of Euclid. Now since the rectangle under HA, AO — that is, under HA, MH — [Margin: c — Euclid III.35.] is equal to the rectangle under AB, AT — that is, to the square on AB, or on FG — if the common square on AF is added, the rectangle under HA, MH together with the square on AF, that is, the square on FH ([Margin: d — Euclid II.6.] for the rectangle under HA, MH together with the square on MF, or AF, is equal to the square on FH), will be equal to the squares on FG, AF, that is, to the square on AG, [Margin: e — 74. primi (Euclid I.74).] to which it is equal. Therefore the lines FH and AG are equal. Which was the second point.

This we shall demonstrate more easily as follows. Let the lines EB, EH be joined. If the triangle ABE is imagined turned about AE until the gnomon AB is upright to the plane of the sundial, and the triangle FEH turned about EF until the perpendicular HF reaches the gnomon AB — and it must necessarily reach the gnomon, since it does not depart from the plane drawn through the gnomon and the perpendicular AH, as is plain — then point H will fall on point B, on account of the equality of the lines EB, EH drawn from the center E. For if point H fell

below B, or above it, then EH would be less or greater than EB, which is absurd. Therefore FH is equal to AG, or to FL.

[Margin: *Quot gradib. segmētum data recta æquiualeat* — To how many degrees a given segment of the line is equivalent.]

7. Conversely, we must find out to how many degrees the given segment CD on the line CD is equivalent. From A, the foot of the gnomon, let a perpendicular AF be drawn to CD; and with a circle drawn from H, the center found as before, let lines HC, HD be drawn through C and D, cutting the circle at I, K. For the given segment CD will contain as many degrees as are included in the arc IK, as is clear from what has been demonstrated. So too, on the line EF drawn through the center of the sundial: if the line HE is extended until it cuts the circle IK, the arc contained between the lines HE and HA will indicate to how many degrees the segment EF is equivalent — the segment between the center of the sundial and the perpendicular AF drawn from the foot of the gnomon to the given line EF passing through the center of the sundial. To know this is at times worth the effort. Likewise, if the line BE is prolonged until it cuts the circle drawn from B, the arc between the lines BE and BP will show the degrees contained in the segment EA; and so with the rest.

[Margin: *Quo pacto per Tāgentes auferatur ex recta segmentum quocunq; graduum* — In what way, by means of the Tangents, a segment of any number of degrees is cut off from a straight line.]

8. Now if, on the line ED, the beginning of the segment to be cut off is given at the very point F of the perpendicular AF, the required portion will be cut off most conveniently by means of the Tangent lines, taking the whole sine as 10 — the values obtained if two figures are dropped from those computed with the whole sine as 1000, as we said above. We shall proceed thus. On any line ST whatever, let any number of parts be taken equal to FH, the interval between the center and the given line, of which the first is [divided] into 10

so that it is divided into equal parts. For if with a compass you take on this line as many parts as are contained in the Tangent of the proposed arc, and transfer them from point F onto the line EF — either below F or above it, as the case requires — the prescribed segment will have been cut off. For example, suppose we must cut off below F a segment of 35° . Since the Tangent of 35° is 700 (with the whole sine set at 1000), if we drop the two figures 00 there remains, relative to a

whole sine of 10, the Tangent 7. So if on the line ST, whose first division has been split into 10 small parts, you take 7 of those parts and carry them from F onto the line FC, you will locate point C, and the portion FC will contain 35° . Likewise, if the desired segment were 64° , the Tangent to be used is that of 64° , namely 2050 — that is, dropping the two figures 50, 20 $\frac{5}{10}$. For if that many parts of the line ST (taking its two undivided parts, which comprise 20 small parts, and in addition the $\frac{1}{2}$ part from the divided portion) are carried from point F below F, a more distant point D will be found, and the portion FD will comprise 64° .

9. *The demonstration is very easy. For plainly, with the whole sine set as FH, the lines FG and FD are Tangents relative to that whole sine — which is evident if, from H through F, a circle is drawn tangent to the line EF at F. And it is clear that the arcs of this circle are similar to the arcs of circle IK, described from the same center H, that are enclosed between the same lines issuing from center H.*

10. If on the line EA a segment of any number of degrees is to be cut off, starting from point A, then on any line VX one must take as many parts equal to the gnomon [stylus] as are wanted — since EA runs through the place of the gnomon — and the first of these must be divided into 10 small parts, and so on. For instance, if a portion of 60° is wanted, whose Tangent is 1732 — that is, 17 $\frac{3}{10}$, a little more — one must take on the line VX 17 $\frac{3}{10}$ parts and a little more, and carry them from A as far as R. The portion AR will then contain 60° . Exactly the same procedure holds in the other cases. For on any straight line whatever, one must always take as many equal parts of that line — the line drawn as a perpendicular from the place of the gnomon to the given line, and set between that line and the center from which we directed the circle to be described — as are required; such were the lines HF and BA in the examples just given. [Margin: *A general rule for the Tangents.*]

11. But if the beginning of the portion to be cut off is not at F or A — where the perpendicular dropped from the place of the gnomon to the given line meets it — but at some other point, say E, then it is necessary first to have determined the arc that corresponds to the segment EF (or EA) intercepted between the given point and the perpendicular. For if this arc is subtracted from the proposed degrees, or (when the proposed degrees are fewer) is itself subtracted from that arc, the Tangent of the remaining arc must be taken, and so on. For example, if from the line EA a portion of 113° is to be cut off starting from point E, one must determine the arc of the circle

described from B, lying between the lines BE and BA — if it is not otherwise known — which here comprises 53° . Subtracting these from 113° leaves 60° . The Tangent of these, then, namely $17 \frac{3}{10}$, carried from A, will give point R, and the whole segment ER will correspond to 113° . Again, if from the same line EA, starting from E, a segment of 8° is to be removed, we subtract 8° from 53° , and of the remaining 45° we carry the Tangent 10 from A toward E as far as Y. The portion EY will then contain 8° , because AY comprises 45° . How the arc between the lines BE and BA is to be found we taught a little earlier, in Number 7; and we shall teach, after the treatment of sundials, in Problem 10, how to investigate the same arc by numbers in any hour-line reckoned from noon or midnight. [Margin: *When the beginning of the portion to be cut off by the Tangents is not at the point of the perpendicular line — what to do.*]

Moreover, even without knowing the segment between the perpendicular and the point from which the degrees begin to be cut off, we shall be able to use the Tangents in this way. Let a portion of 30° be cut off from point C, both downward and upward. Through C draw the perpendicular Cb to HC, so that it is tangent to the circle described from H through C; and the arc of that circle between the lines HI and HK will be similar to arc IK. If, then, from C the Tangent of 30° (relative to the whole sine HC) is carried both ways onto the line Cb, the points a and b will be found, through which the lines drawn from H will cut off from ED the segments Cd and cd of 30° , since Ca and Cb are the Tangents of 30° of the circle described from H through C. And it is established from what has been demonstrated that the lines Ha and Hb, drawn through 30° of that circle, cut off segments of 30° .

SCHOLIVM.

1. *Now since, in order to cut off a segment of any number of degrees by means of the Tangents, one needs to divide various straight lines into ten equal parts, this division will be accomplished most easily if an equilateral triangle ABC of any size is separately constructed, whose base BC is divided most accurately into 10 equal parts, to which straight lines are drawn down from the vertex A, as appears in the triangle set before us. This will be done more easily if with a compass one takes in order 10 equal parts of any size whatever, equal*

Diagram. An equilateral triangle for dividing a line into ten equal parts, set within a rectangular frame. The apex A is at the top; the base runs from B (lower left) to C (lower right). The base BC is divided into ten equal segments, marked

from B toward C with the numerals 9, 8, 7, 6, 5, 4, 3, 2, 1 (the point 5 falling at the mid-base). From the apex A, straight lines are drawn down to each of these division points, fanning out across the triangle. Three horizontal lines cross the triangle parallel to the base, meeting its left side at E, L, H (top to bottom) and its right side at F, M, I. To the right of the triangle a second, slanting construction rises: the top edge runs from A to the upper-right corner D, and a diagonal descends from D through the points G, N, K down to C. Along the line through E–F this scale is marked with the numerals 10, 20, 30, 40 (G standing at the 40 mark).

- A (apex, top): vertex from which the dividing lines are drawn
- B (lower left): left end of the base
- C (lower right): right end of the base, and foot of the diagonal DC
- D (upper right): far corner of the added equilateral triangle ACD
- E, L, H (left side, top to bottom): intersections of the parallels with side AB
- F, M, I (right side, top to bottom): intersections of the parallels with the added construction
- G (on the E–F line, at "40"): point where the diagonal DC crosses
- N, K (on the diagonal DC, descending): further intersection points
- Base division numerals (B → C): 9, 8, 7, 6, 5, 4, 3, 2, 1
- Scale numerals along the E–F line: 10, 20, 30, 40

[Margin: *The construction of an equilateral triangle best suited for dividing lines into 10 equal parts.*]

equal parts from point C as far as B, and on the line BG let an equilateral triangle be set up. For in this way there will be no need to cut the base into 10 parts — since it is already cut — but only to draw lines from A to the parts taken. Then, so that what we intend may be carried out more precisely, upon AC let another equilateral triangle ACD be built, in such a way that DC is [Margin: *Euclid, I.27*] parallel and equal to AB itself, on account of the equal alternate angles BAC and ACD.

Therefore, if on any given line that is to be divided into 10 equal parts — provided it is not longer than the side of the triangle — one takes off the equal lengths AE, AF, DG, the portion EF of the line EFG will be equal to that given line, because the triangle AEF is likewise equilateral (by the corollary of Proposition 4 of Euclid, Book 6). And this same EF will also be divided into 10 equal parts, as is plainly established from the demonstration of the second method of dividing a given line into any number of equal parts, which we set out in the scholium on Proposition 10 of the same Book 6. The line CD, moreover, is drawn so that the line EF may be

extended more accurately and without error, and so that on it, once extended, one may take as many parts equal to EF as one wishes, as the numbers 10, 20, 30, and so on indicate.

By the same reasoning, if a larger line is to be divided into 10 parts, one takes the equal lengths AH, AI, DK (or AL, AM, DN) and draws the line HIK (or LMN); then either the portion HI or the portion LM will be equal to that larger given line, and divided into 10 equal parts.

2. *We shall also often need, in what follows, the distances of the hours (reckoned from noon and midnight, converted into degrees) from the meridian circle or from some other point, together with the Tangents of those distances — which indeed are comprised in this table, with the whole sine set at 1000.*

The distances of the hours (reckoned from noon and midnight) from the meridian, and the Tangents of those distances, with the whole sine set at 1000.

Block 1

Hours (from noon / from midnight)	$\frac{1}{4}$ / 11 $\frac{3}{4}$	$\frac{1}{2}$ / 11 $\frac{1}{2}$	$\frac{3}{4}$ / 11 $\frac{1}{4}$	1 / 11	1 $\frac{1}{4}$ / 10 $\frac{3}{4}$	1 $\frac{1}{2}$ / 10 $\frac{1}{2}$
Distance from meridian (G. = deg., M. = min.)	3° 45'	7° 30'	11° 15'	15° 0'	18° 45'	22° 30'
Tangent	66	132	198	268	339	414

Block 2

Hours (from noon / from midnight)	1 $\frac{3}{4}$ / 10 $\frac{1}{4}$	2 / 10	2 $\frac{1}{4}$ / 9 $\frac{3}{4}$	2 $\frac{1}{2}$ / 9 $\frac{1}{2}$	2 $\frac{3}{4}$ / 9 $\frac{1}{4}$	3 / 9
Distance from meridian	26° 15'	30° 0'	33° 45'	37° 30'	41° 15'	45° 0'
Tangent	493	577	668	767	877	1000

Block 3

Hours (from noon / from midnight)	3 $\frac{1}{4}$ / 8 $\frac{3}{4}$	3 $\frac{1}{2}$ / 8 $\frac{1}{2}$	3 $\frac{3}{4}$ / 8 $\frac{1}{4}$	4 / 8	4 $\frac{1}{4}$ / 7 $\frac{3}{4}$	4 $\frac{1}{2}$ / 7 $\frac{1}{2}$
Distance from	48° 45'	52° 30'	56° 15'	60° 0'	63° 45'	67° 30'

Hours (from noon / from midnight)	$3\frac{1}{4} / 8\frac{3}{4}$	$3\frac{1}{2} / 8\frac{1}{2}$	$3\frac{3}{4} / 8\frac{1}{4}$	4 / 8	$4\frac{1}{4} / 7\frac{3}{4}$	$4\frac{1}{2} / 7\frac{1}{2}$
meridian						
Tangent	1140	1303	1497	1732	2028	2414

Block 4

Hours (from noon / from midnight)	$4\frac{3}{4} /$ $7\frac{1}{4}$	5 / 7	$5\frac{1}{4} /$ $6\frac{3}{4}$	$5\frac{1}{2} /$ $6\frac{1}{2}$	$5\frac{3}{4} /$ $6\frac{1}{4}$	6 / 6
Distance from meridian	71° 15'	75° 0'	78° 45'	82° 30'	86° 15'	90° 0'
Tangent	2946	3732	5027	7596	15257	Infinita (infinite)

The use of this figure is also perfectly clear. For any two hours reckoned from sunrise or sunset intersect one another at that hour, reckoned from noon or midnight, which lies midway between them — that is, from which they are equally distant, as we demonstrated in the Gnomonics, Book 1, Propositions 17 and 20 and their scholia. [Margin: The use of the preceding figure.] For example, at the hour 4 from noon or midnight the following hours from sunrise or sunset intersect one another: 15 and 17; 14 and 18; 13 and 19; 12 and 20; 11 and 21; 10 and 22; 9 and 23; 8 and 24; 7 and 1; 6 and 2; 5 and 3 — because each pair of these is equally distant from that hour. By the same reasoning, at the same hour 4 from noon or midnight these fractional hours from sunrise or sunset cut one another: $15\frac{1}{2}$ and $16\frac{1}{2}$; $15\frac{1}{4}$ and $16\frac{3}{4}$; $15\frac{1}{8}$ and $16\frac{7}{8}$; $15\frac{3}{8}$ [?] and $16\frac{5}{8}$ [?]; [further fractional pairs follow, but the small stacked fractions are not legible with certainty — [?]]. Likewise, at the hour $4\frac{1}{2}$ from noon or midnight these hours cut one another: 16 and 17; $16\frac{1}{8}$ [?] and $16\frac{7}{8}$ [?]; $16\frac{1}{4}$ and $16\frac{3}{4}$; $16\frac{3}{8}$ [?] and $16\frac{5}{8}$ [?]; [remaining fractional pairs [?]]. So too, at the hour $9\frac{1}{4}$ from noon or midnight, these hours from sunrise or sunset will intersect: 9 [?] and $9\frac{1}{2}$ [?]; 8 [?] and $10\frac{1}{2}$ [?]; [?] and $18\frac{1}{2}$ [?]. And so with the rest.

And so, if on any sundial whatever you observe some hour reckoned from sunrise or sunset passing through some point of any hour reckoned from noon or midnight, you will at once know what other hour from sunrise or sunset ought to pass through the same point of the same hour from noon or midnight. For that hour will be carried through the same point which, in the figure above, is as far distant from the said hour reckoned from noon or midnight as the earlier hour from sunrise or sunset is removed from that same hour. For instance, since in the horizontal sundial of Chapter 2 you see that the hour 21 from sunrise or sunset, if it were

extended, cuts the hour 5 from noon or midnight at a certain point above the tropic of Cancer [☿], you will say that through that same point the hour 13 from sunrise or sunset must be drawn — since 21 and 13 are distant from the hour 5 reckoned from midnight by eight hours, and from the hour 5 reckoned from noon by four hours, as is seen in the figure.

Again, if for any two given hours from sunrise or sunset you wish to know at what hour from noon or midnight they cut one another, take the hour from noon or midnight midway between them, equally distant from each. Thus, if the hours 12 and 16½ from sunrise or sunset are given, you will find that they cut the hour 2¼ from noon or midnight at one point, since this hour is equally distant from those two in the aforesaid figure. In the same way you will conclude that the hours 11 and 19 from sunrise or sunset must be described through one and the same point of the hour 3 from noon or midnight, and so on.

By the same reasoning, given any hour from sunrise or sunset, if you wish to know at what hour — of which one is reckoned from noon or midnight and the other from sunrise or sunset — they cut one another, take any hour from noon or midnight that is conveniently cut by the given hour from sunrise or sunset, and another hour from sunrise or sunset as far distant from it in the same figure as the earlier
[continues below]

Diagram. A large circular hour-intersection figure set within a square frame. At the center is the point **A**, marked with two perpendicular diameters (a vertical and a horizontal line) crossing at **A**. Around this center run several concentric rings. The figure carries two curved legend bands and two rings of hour-numbers:

- The inner legend band reads **HORÆ · A · MERIDIE · VEL · MEDIA · NOCTE** ("hours from noon or midnight"), with a small cross **✚** marking the top (noon) position.
- The outer legend band reads **HORÆ · AB · ORTV · VEL · OCCASVS** ("hours from sunrise or sunset").
- An inner ring bears the Roman numerals for the hours from noon/midnight — I, II, III, IIII, V, VI, VII, VIII, IX, X, XI, XII — running twice around the circle.
- An outer ring bears the Arabic numerals for the hours from sunrise/sunset, 1 through 24.

- Between the number rings run graduated scales of tick-marks: long ticks for whole hours, shorter ticks for the half-hours, shorter still for the quarters, and points alone for the eighth-hours (as described in §3 above).

Individual peripheral numerals and the exact angular placement of the tick-divisions cannot all be resolved with certainty at this resolution; several rim labels are uncertain [?]. Marked as a

is distant from that same given hour — as, for instance, if the hour 13 from sunrise or sunset is given and you take the hour 10 from noon or midnight, the hour 7 from sunrise or sunset must be taken, and so on. On this basis was constructed, in this little book, the first table of Chapter 14 for the whole hours, because each of its hours from sunrise or sunset is as far distant, in the said figure, from the hour reckoned from noon or midnight written beneath it, as the hour 24 from sunrise or sunset is removed from that same hour.

Likewise, when any hour from noon or midnight has been taken away, any two hours from sunrise or sunset equally distant on either side of it will cut it at one and the same point. Thus the hour 10 from noon or midnight is cut, at the same point, by the hours 12 and 8 from sunrise or sunset; likewise by the hours 23 and 21, and so on.

Finally, any two hours from sunrise or sunset that are distant from each other by twelve hours — that is, by a semicircle — in the figure above, pass on the equinoctial line of any sundial through the hour reckoned from noon or midnight which holds the place midway between them, that is, which is distant from each by a quadrant, as we demonstrated in Book 1 of the Gnomonics, Proposition 19. Thus the hours 23 and 11 from sunrise or sunset, together with the hour 5 from noon or midnight, intersect the equinoctial line at one point on any sundial. You will say the same of the hours 15½ and 3½ from sunrise or sunset, and of the hour 9½ from noon or midnight. By this device we constructed the second table, set out at the beginning of Chapter 14 for the whole hours: because in that table any two hours from sunrise or sunset are distant by a semicircle, and the hour reckoned from noon or midnight placed midway between them is a quadrant distant from each.

CAPVT II.

Hours from noon and from midnight, on a plane that is parallel to the horizon.

[Margin: *Centrum horologij horizontalis.* — The center of the horizontal sundial.]

[Margin: *Linea horae 6.* — The line of hour 6.] [Margin: *Aequinoctialis linea.* — The

equinoctial line.] [Margin: *Regula generalis pro lineis per locum styli ductis.* — General rule for lines drawn through the position of the gnomon.]

1. Draw a line AB to serve as the meridian; choose on it a point C for the position of the gnomon [stylus], and erect there a perpendicular CD equal to the gnomon. On any straight line mark off any number of equal parts, of which let the first, EF, be divided into 10 equal parts by means of the equilateral triangle described above. Next take the Tangent of 48° — 1111 [numeral:?] — that is, of the complement of the pole's altitude. (In the examples we shall assume the Roman latitude of 42° , with the Tangents reckoned to a whole sine [sinus totus] of 1000.) Reduced to tenths, as I taught in the prolegomena, this comes to $11 \frac{1}{10}$. Take this with the compasses on the line whose part EF has been divided into 10 equal parts, and carry it from C, the gnomon's position, along the meridian line as far as the point G. This G will be the center of the sundial [horologium], where all the hour-lines intersect one another; and the line KL, drawn through G perpendicular to the meridian, will be the line of hour 6 from noon and from midnight.

Again, take the Tangent of 42° — 900, the altitude of the pole — which reduced to tenths is 9, and carry it from C in the same way but toward the opposite side, as far as the point B; and the line HI, drawn through B perpendicular to the meridian, will be the equinoctial line [linea aequinoctialis].

Demonstration. The proof is easy. For if at D an angle equal to the complement of the pole's altitude is set off toward G, the line making that angle will be the axis of the world [axis mundi], and will fall on the center of the sundial. But if the angle of the pole's altitude is set off toward the opposite side, the line making that angle will be the common section of the Equator and the Meridian, and will fall on the point through which the equinoctial line must be drawn, as we demonstrated in Book 2 of our *Gnomonics*, Proposition 1. Therefore, if the lines DG and DB are imagined drawn, then — the whole sine being set as DC — the line CG is the Tangent of angle CDG, the complement of the pole's altitude; and CB is the Tangent of angle CDB, the pole's altitude. It is thus clear that the points G and B are found correctly by means of the Tangents, and that KL is the line of hour 6 and HI the equinoctial line, as we demonstrated in the *Gnomonics*.

Diagram. A tall, narrow rectangular figure in the left margin: a tangent-transfer instrument (a set of parallel graduated vertical scales, to be cut out as straightedges / *normae*). Across the top run column-head letters, and each column bears a vertical italic legend and a scale of divisions. The scales are the practical realization of the "reduce-to-tenths" tangent method of §1. Many of the small

heads, zodiac glyphs, and interior graduation marks do not resolve with certainty at this resolution

- Column heads (left to right): E, M, ♄ [Saturn-like glyph or "b"] [?], X, [?], N
- Column legends (vertical italic, left to right): *Pro meridiana sinu* (for the meridian) · *Pro aequinoctiali sinu* (for the equinoctial) · *Pro linea horae 6* (for the line of hour 6) · *Pro linea horarum* (for the [other] hour-lines) · [?] · [?]
- Left scale-columns, graduated: one runs 1, 2, 3, 4, 5, 6 (each division marked "-0"); a second runs 1, 2, 3, 4; a third runs 1, 2, 3
- Central/right scale, graduated: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, then a second short run 10, 20; with letter markers Y and C beside the upper divisions and a few small zodiac glyphs [?] beside the 60–90 marks

2. By the same reasoning, whenever any straight line passes through the position of the gnomon — such as the meridian, and all the Vertical lines on a horizontal sundial — one must mark off on that line any number of parts equal to the gnomon, and divide the first of them into 10 equal parts, so that Tangents can be taken on the line, and so forth; as is clear from Chapter 1, nn. 4, 8, and 10.

Moreover, this transfer of the Tangents can also be carried out most conveniently without compasses. For if on some thicker paper — royal or imperial [*regia aut imperiali*] — the divisions of the line EF are made as described, and the paper is then cut so that the edge containing the divisions lies at the very margin and serves as a kind of straightedge [*norma*], we shall apply that straightedge to the meridian line, setting its point F at C whenever the Tangent is not greater than 10, and mark on the meridian the end of the proposed Tangent — namely, toward B, we shall mark the end of the Tangent of 9 parts in order to find the point B. But if the Tangent is greater than 10 — say $34 \frac{5}{10}$ — we shall set the point of the number 30 at C, and mark the point ending the part $4 \frac{5}{10}$ of the little line EF; and so with the rest. The same is to be understood of the other lines from which Tangents must be taken, for out of them too straightedges must be made, with their own divisions, as has been said.

[Margin: *Linea horae à mer. & med.noc.* — The line of the hour from noon and from midnight.]

3. After this, on any other straight line mark off any number of equal parts, of which let the first, MN, again be divided into 10 equal parts by means of the equilateral triangle above. If on this line one takes the Tangents of the arcs by which the hours are distant from the Meridian — from the table of the scholium

of Chapter 1, the Tangents, I say, reduced to tenths, that is, to a whole sine of 10, yet in such a way that the fractions which the following figures show are not entirely neglected, as we taught in the prolegomena — and transfers them from the point B onto the equinoctial line in both directions, the hour-points will be found. If then straight lines are sent out from these through the center of the sundial G, the hour-lines will be drawn, in this order: to us facing the sundial, the lines on the left belong to the hours from noon, and those on the right to the hours from midnight, as the numbers I, II, III, IIII, V, etc., written beside the hour-lines indicate. For example: in the table of the scholium of Chapter 1, the Tangent of the distance of hours 5 and 7 is 3732 — that is, $37 \frac{3}{10}$ and a little more, to a whole sine of 10. If, then, on the line whose part MN has been divided into 10 equal parts, one takes $37 \frac{3}{10}$ and a little more, and transfers this from B onto the equinoctial line in both directions, the points H and I will be found — hour 5 from noon and hour 7 from midnight. And so with the rest.

[Margin: *Quando tangens aliqua est nimis parua quid agendum.* — What to do when some tangent is too small.]

When, however, some Tangent is so small that it can scarcely be taken with the compasses, let it be added to the Tangent of some other point already found. For if you carry the Tangent so combined, and reduced to tenths, from that other found point onto the equinoctial line, the point sought will be found. For example: the Tangent of hour $\frac{1}{2}$ — that is, of $7^\circ 30'$ — is 132; if you add it to 268, the Tangent of hour 1 (or of 15°), you will make the Tangent 400, that is (dropping the two last figures 00) 4. This, transferred from hour 1, will give the point of hour $11 \frac{1}{2}$; and from hour 11 it will give the point of hour $\frac{1}{2}$. Likewise, since the Tangent of hour $\frac{1}{4}$ — that is, of $3^\circ 45'$ — is 66, if you add it to the Tangent 268 owed to hour 1 (or 15°), you will make the Tangent 334, that is, $3 \frac{3}{10}$ and a little more; which,

[Running head: p. 11 — *NOVAE DESCRIPT.*]

transferred from hour 1 onto the equinoctial line, will present the point of hour $11 \frac{3}{4}$, while transferred from hour 11 it will give the point of hour $\frac{1}{4}$. And so with the rest.

But in order to have three points for drawing each hour more accurately, one must take a line GA on the meridian equal to GB, and through A draw OP parallel to the equinoctial line (or to the line of hour 6) — that is, perpendicular to the meridian — and the said Tangents of the hours must, by one and the same

operation, be transferred from both B and A, on either side, onto the equinoctial line and onto the line OP.

[Margin: *Linea horaria vt accuratius ducatur, quid agendum.* — What to do so that the hour-line may be drawn more accurately.]

For example, the Tangent $37 \frac{3}{10}$ and a little more, of hours 5 and 7, which gave the hour-points H and I, is likewise to be transferred, by the same operation, from A as far as O and P. For the line drawn through H, G, P will give hour 5, and the line passing through I, G, O will give hour 7 most exactly, and so on. In the example we have drawn only the lines of the whole hours, to avoid confusion, though the points of the half-hours have nonetheless been marked on the equinoctial line.

Diagram. A large rectangular horizontal-sundial layout occupying most of the page. From a central point G, near the middle, a dense fan of straight hour-lines radiates outward; a semicircle is described about G, and the whole is crossed by many further straight and dotted lines, with hour-numerals, zodiac glyphs, and lettered intersection-points scattered throughout. This same construction is repeated on the following page (see fig-p648-1). The figure is extremely crowded; the numerous small zodiac sorts, dotted-line labels, and doubled-letter intersection points cannot all be resolved at this resolution

- P (top-left corner of the frame)
- O (bottom-left corner of the frame)
- A (on the left vertical edge, where the horizontal center-line meets it)
- G (center; the point from which all hour-lines radiate, marked with a cross **+**)
- Hour-numerals (Roman) read around the fan: I, II (upper right), VI, VII, VIII (upper left / left), V (lower left), XII, XI, X (right side)
- N, R, Q, Z, Y (right side, along the horizontal line near XII–X)
- B, D, S, T (in the central region, near G)
- K, H (along the bottom edge)
- Arabic hour-numbers (hours from sunrise/sunset) read around the border: 3, 5, 6, 10, 12, 14, 17, 18, 19, 21, 22, 23, 24 [several intervening numbers not resolved]
- Doubled-letter intersection labels: aa, bb, cc, dd, pp [and single p, d, e, m along various lines]
- Numerous zodiac glyphs mark individual lines (Aries ♈ and others) — [?], not reliably resolved

Demonstration. If the line DB (drawn only in thought) is taken equal to BQ, and at Q the angles of the hours are set off — namely 15°, 30°, 45°, 60°, 75°, etc. — the lines making those angles will fall on the hour-points of the equinoctial line, as is clear from what we demonstrated in Book 2 of our *Gnomonics*, Proposition 1. Wherefore, since — the whole sine being set as DB or BQ — those same lines cut off the Tangents of the same angles, it is plainly established that the same hour-points are found by means of the Tangents taken from the line whose portion MN is divided into 10 equal parts.

[Margin: *Regula generalis pro rectis non per locum styli ductis.* — General rule for straight lines not drawn through the position of the gnomon.]

4. By this same reasoning, whenever a straight line is perpendicular to the meridian line, or to the line of the gnomon — such as the equinoctial line, the line of hour 6, and the common sections of all the great circles drawn through the intersections of the Equator and the Horizon and of the plane of the horizontal sundial — one must take on any such line any number of portions equal to that line which is drawn, from the intersection of the given straight line with the line of the gnomon, to the tip of the gnomon, making right angles with the line of the gnomon; and the first of these portions must be divided into 10 equal parts, so that Tangents can be taken on the line, and so on — which is clearly gathered from Chapter 1.

[Margin: *Quo pacto examinari possit* — By what means it can be tested...]

Now indeed, if no error has been made in transferring the Tangents, it must be that, for any two hours — one set to the right of the meridian line and the other to the left — the Tangents taken from the table above, in the scholium of Chapter

[Running head: p. 12 — *CAPVT SECVNDVM.*]

[Margin: *...num puncta horaria absque errore inuenta in linea aequinoctiali.* — ... whether the hour-points have been found on the equinoctial line without error.]

1, and gathered into one sum, produce a line which, taken with the compasses from the line MN (divided as the equinoctial line) and transferred onto the equinoctial line, equals the line lying between the two points of those same hours. For example: if $5 \frac{7}{10}$, the Tangent of hour 2 or 10, is added to $17 \frac{3}{10}$ [?], the Tangent of hour 4 or 8, there will result the line 23; and this, taken from the line MN, will equal the interval between hours 2 and 8, or 10 and 4. Likewise, if two hour-points are taken on the same side, and the Tangent of one is subtracted from the Tangent of the other, there will remain a line which, taken with the

compasses from that same MN, will give the interval between those two hour-points of the equinoctial line. Thus, if from $37 \frac{3}{10}$, the Tangent of hour 5 or 7, you take away $17 \frac{3}{10}$, the Tangent of hour 4 or 8, there will remain the line 20; therefore 20 parts of the line MN will equal the space between hours 4 and 5, or between hours 8 and 7, and so on. And in this way you can examine, by various methods, whether the hour-points have been found on the equinoctial line without error, or not.

[Margin: *Descriptio horarum vix lineam aequinoctialem secantium*. — The drawing of the hours that scarcely cut the equinoctial line.]

5. But since some hours scarcely cut the equinoctial line — such as 5 and 7; $5 \frac{1}{4}$ and $6 \frac{3}{4}$; $5 \frac{1}{2}$ and $6 \frac{1}{2}$; $5 \frac{3}{4}$ and $6 \frac{1}{4}$ — we shall draw them if the hour-points are sought on a certain new equinoctial line, from the table of the scholium of Chapter 1, just as they were found on the equinoctial line HI — namely, in this way. Having drawn the axis GD, erect a perpendicular ST to it from any point toward the center G, say from S; and through S draw VSV perpendicular to the meridian line, which will serve as another sort of equinoctial line with respect to the gnomon that would be dropped perpendicularly from T to the meridian line. Then on any straight line take any number of portions equal to ST, and let the first of them, XY, be divided into 10 equal parts. Finally, onto the line VS transfer the Tangents of the hours reduced to tenths — with respect to the whole sine XY or ST, as was done on the equinoctial line — using the table of the scholium of Chapter 1. For by this method the hour-points will be found on the line VS. Thus you see that by the Tangent $75 \frac{2}{10}$ [?] with half of one tenth — that is, by the Tangent 76 nearly — the points V, V are found: hour $5 \frac{1}{4}$ from noon and hour $6 \frac{3}{4}$ [?] from midnight, and so on.

Diagram. The same large horizontal-sundial construction as on the preceding page (fig-p647-1), printed again here: the central radiating point G with its fan of hour-lines, the semicircle about G, and the surrounding frame with Roman hour-numerals (I, II, V, VI, VII, VIII, X, XI, XII), Arabic hour-numbers of the border, zodiac glyphs, and lettered intersection-points (P and O at the upper- and lower-left corners; A on the left edge; K, H along the bottom; N, R, Q, Z, Y at the right). As on p. 647, the small zodiac sorts, dotted-line labels, and doubled-letter points are too dense to resolve fully

But so that we need not divide the line ST into 10 parts, you will do rightly if you bisect the line BG, or from G toward B take a quarter part, and through the point of division draw a perpendicular to the Meridian. For if onto this perpendicular

you transfer the half, or the quarter part, of that tangent which is owed on the equinoctial line...

is owed to some hour, you will have on that perpendicular the point of that same hour. For example: if you want the point for hour $6\frac{3}{4}$ from midnight — which is scarcely to be had on the equinoctial line — draw a perpendicular to the meridian through the midpoint of the line BG, and on it transfer half of the Tangent 76 (nearly), the tangent owed to hour $6\frac{3}{4}$ from midnight, or hour $5\frac{1}{4}$ from noon. For in this way you will have on that perpendicular the point for hour $6\frac{3}{4}$, and so on.

The demonstration of this is that then the line ST likewise, [together with] the line BD, divides GD in half, or cuts off a quarter part of GT, etc.

But we shall set out, in Chapter 4, Numbers 1 and 10, a most thorough method by which all the hour-lines — even those that cut the equinoctial at the most remote points — are described in the most exact way, both geometrically and by Tangents, namely by investigating their points on a certain line equidistant from the meridian.

[Margin: The most thorough method of drawing all the hours is delivered here. — *Absolutissima ratio describendi omnes horas ubi traditur.*]

6. If you want several points through which the hour-lines are to be drawn, you may — if you please — draw a straight line perpendicular to the meridian line, one that represents the great circle [circulus maximus] passing through the intersections of the Equator [Æquator] with the Horizon and drawn through any point of the Meridian standing above the Horizon. This great circle will be the Horizon of some region, above which the pole is elevated by as many degrees as the point taken on the Meridian is distant from either pole of the world. But this point of the Meridian must not have, from the Equator toward the South, a distance greater than the complement of the pole-altitude [altitudo poli] of your region; nor, from the arctic pole toward the Horizon of your region, a distance greater than the pole-altitude of that same region. For [otherwise] that point could not be taken on the meridian line, as was demonstrated in Chapter 1, Number 3 — seeing that its distance from the pole of your Horizon (that is, from the place of the stylus) would be greater than 90 degrees.

[Margin: How more points for the hour-lines are to be found. — *Plura puncta pro horarijs lineis, quo pacto reperiantur.*]

For example: if a point of the Meridian is chosen 30 degrees receding from the Equator toward the South, we shall add to that distance the altitude of the pole — namely the distance of the place of the stylus from the Equator toward the northern parts — so that 72 degrees result. The Tangent of this is 3078; that is, with respect to a total sine of 10, it is $30 \frac{7}{10}$ [?] particles and a little more. If this is taken on the straight line whose portion EF was divided into 10 particles, and is transferred onto the meridian line from C, the place of the stylus, the point R will be found. Through R a straight line drawn perpendicular to the meridian line will represent the great circle, or the Horizon, which by the intersections of the Equator with our [Horizon] cuts it at the point 30 degrees distant from the Equator toward the South — above which, accordingly, the pole-altitude amounts to 60 degrees, the complement, that is, of 30 degrees.

THEREFORE, if on some straight line any number of portions equal to the interval RD are taken, and the first of them is cut into 10 equal particles, and from Table 6 the complements of the degrees of the hours at pole-altitude 60 are excerpted, we shall find — by the Tangents of these complements — the hour-points on the line drawn through R perpendicular to the Meridian line, just as on the Equinoctial line. For example, for hours 3 and 9 you will find, at pole-altitude 60 degrees, in Table 6, 49 degrees 6 minutes; the complement of this is 40 degrees 54 minutes, whose Tangent is 866, that is, $8 \frac{6}{10}$ [?] and a little more. Taken on that straight line — the one on which, we said, portions equal to the interval RD are to be taken — if you were to transfer it from R to either side of that line drawn through R, you would find the points of hours 3 and 9, etc. In the example no line has been drawn through R, so that confusion of lines might be avoided. Yet we did draw a perpendicular through C, the place of the stylus, which represents the prime Vertical [Verticalis primarius], or the Horizon, above which the pole-altitude is 48 degrees (the complement, that is, of the 42 degrees by which the same [point] is away from the Equator toward the north), and which cuts the Meridian between the Equator and the arctic pole at 42 degrees from the Equator. If onto this line there are transferred the Tangents corresponding to the complements of the arcs of Table 6, at a pole-elevation of 48 degrees (which Tangents are to be taken on the line whose portion EF was cut, as we said in Number 2 of this chapter), there will be had on it the hour-points, just as on the equinoctial line. And in this way you can describe the various Horizons of Table 6, on which the hour-points may be found, provided you observe the degrees of pole-altitude of each Horizon and count its complement from the Equator toward whichever side you prefer — so long as you remember that the complements of the arcs of Table 6 are to be taken, and not the arcs themselves.

AND indeed the Horizons described between the Equator and the center of the sundial are useful for the hours whose points can scarcely be had on the equinoctial line because they are too far [from the meridian line]. Thus you see that the points of hour $5\frac{1}{4}$ [?] from noon and $6\frac{1}{4}$ [?] from midnight are conveniently found on the Horizon CD, whereas on the equinoctial line — unless it is extended very far — they cannot be marked.

[Margin: The hours that scarcely cut the equinoctial, how they are to be drawn by Table 6. — *Horae, quae vix aequinoctialem secant, quo pacto describendae per tabulam 6.*]

BUT the far easiest way — through the Tangents of the complements of the arcs of Table 6 — you will find in the scholium of Chapter 25, Number 5; and this not only for the hours that scarcely cut the equinoctial, but for all the other hours as well.

CAPVT III. — The Hours from Noon and from Midnight, on a plane parallel to the Horizon, by another method.

[*HORAE A MERIDIE, ET MEDIA NOCTE in plano, quod Horizonti æquidistat, alio modo.*]

1. We shall describe the hour-lines exactly, and even without Tangents, by this other way. Draw any line AB for the meridian, and, taking a place for the stylus C, draw through C a perpendicular to the meridian, on which CD is made equal to the stylus. Then, having described from D an arc of a circle of any size, count off on it — downward from the line DC — the altitude of the pole (namely, in our example, 42 degrees), and upward its complement, namely 48 degrees. Through the ends of these countings let straight lines go out from D, cutting the meridian line at B and A. A will be the center of the sundial, and through B [runs] the line to be called the equinoctial, perpendicular to the meridian. But if either of the lines DA, DB cuts the meridian too obliquely, the points A, B will have to be found by tangents with respect to the total sine CD, as was delivered in the preceding chapter, Number 1.

[Margin: Meridian line. — *Meridiana linea.*] [Margin: Center of the sundial. — *Centrum horologij.*] [Margin: Equinoctial line. — *Æquinoctialis linea.*] [Margin: Hour-points 4 & 8. — *Puncta hor. 4 & 8.*]

On the line BD let equal segments BE, BF be taken; and from E and F, at the interval EF, let two arcs be described on either side, cutting each other at 4 and 8; and let the line be extended through 4, B, 8, which will be the equinoctial. The

points 4 and 8 themselves will belong to hour 4 from noon and hour 8 from midnight.

For since, by construction, the triangle E4F is equilateral, the angle BE4 will contain — by Corollary 3 of Proposition 32, Book 1, of Euclid — two-thirds of one right angle, that is 60 degrees, and will accordingly comprehend 4 hours. The point 4, therefore, belongs to hour 4 from noon, and 8 to hour 8 from midnight.

Let B2 likewise be a third part of the line B4, and B10 [a third part] of the line B8, and the point 2 will be hour 2 from noon, and the point 10 hour 10 from midnight.

[Margin: Hour-points 2 & 10. — *Puncta hor. 2. & 10.*]

For since EF is double EB by construction, E4 will also be double the same EB; and 42 is likewise double 2B (since B2 is set as a third part of B4). [a] Therefore the line 2E divides the angle 4EB in half. Since, then, the angle 4EB contains 60 degrees, the angle 2EB will comprehend 30 degrees, and so the point 2 will belong to hour 2; and for the same reason the point 10 will refer to hour 10.

[Margin: [a] 3 of the sixth [book] (Euclid VI.3). — 3. *sexti.*]

Again, on the lines BD, or BE, let equal segments B3, B9 be cut off on either side, and the point 3 will be hour 3 from noon, and the point 9 hour 9 from midnight.

[Margin: Hour-points 3 & 9. — *Puncta hor. 3[?]. & 9.*]

For since the lines BE, B9 are equal, [b] the angles BE9, B9E will be equal; but they are [together] equal to one right angle. Therefore each of them will be a half-right angle, that is, will comprehend 45 degrees, and accordingly [each] will take in three hours, etc.

[Margin: [b] 6[?] of the first [book] (Euclid — the argument, equal sides yielding equal base angles, is Euclid I.5). — 6. *primi.*]

Diagram. A large horizontal-sundial construction filling most of the page, enclosed in a rectangular frame. At its center is the dial-center **A**, lying on a horizontal line that runs left–right across the middle of the figure; from **A** a fan of straight hour-lines radiates outward, and a family of dotted construction-arcs and cross-lines fills the frame. The principal readable capital-letter points are: **A** (dial center, on the middle horizontal line); **B** (just below **A**, with **N** marked immediately above it); **C**, **D**, **F** clustered near **A** (**F** above **C**, **D** to the right of **C**); **E** (lower center, apex of the lower fan of dotted hour-lines); **G** (to the lower right of **E**); **H** (lower left, beside the numeral 3); **K** (upper edge, above the numeral 12); **L**

(upper left, above the numeral 9); **M** (upper right, above the numeral 3); **P** (bottom center); **R** (upper right, near the top edge, apex of the large quarter-arc); **S** (left, at the start of an arc); **T** (left, beside the numeral 6); **V** (center, beside the numeral 3); **X** (upper center, a crossing point); **Y** (left-center). Smaller lowercase and repeated labels are also present — **c** (upper center, with a 3), **q, f, m, n, e, r** (in two places), **h, o, a, i, d, z** — most beside dotted construction-lines.

The top horizontal line carries the hour-numerals **9, 10, 11, 12, 1, 2, 3** (running left→right, below L, K ... M). The middle horizontal line (through T, A, a, i) is marked at intervals with **6** (a "six-o'clock" line). Interior numerals mark whole hours (1–12) and half-hours, including the readable **4½, 2½, 8½, 7½**; the bottom horizontal line carries **3, 2, 1, 12, 12** at left and **9, 10, 11** at right. Many of the small interior numerals, the lowercase point-letters, and the dotted-line labels are too dense and faint to resolve individually under magnification. **More than two labels are unresolved.**

IAM — if the interval E4 is transferred from the point 4 as far as 5 and 11, hour 5 from noon and 11 from midnight will be had. In the same way, if the interval E8 is transferred from the point 8 as far as 1 and 7, hour 1 from noon and 7 from midnight will be found. And so now all the whole hours except 6 and 12 have been found; but the line of hour 6 is drawn through the center A, perpendicular to the meridian line, and the meridian itself gives hour 12.

[Margin: Hour-points 5, 11, 1 & 7. — *Puncta hor. 5. 11. 1. & 7.*] [Margin: Hour 12. — *Hora 12.*]

QVOD — if you transfer the intervals between E and the points of the odd hours, from those very points of the odd hours to either side, you will have the points of the half-hours. And you will easily understand from which number each half-hour point is to be named, from the two whole-hour numbers standing on either side of it. For the point between hour 3 and 4 will belong to hour 3½, and the point between hour 11 and 12 to hour 11½, etc. By this method, from the interval of hour 11 you will find hours 8½ and 2½; from the interval of hour 9, hours 7½ and 1½; from the interval of hour 7, hours 6½ and ½; from the interval of hour 1, hours 3½ and 9½; from the interval of hour 3, hours 4½ and 10½; and finally, from the interval of hour 5, hours 5½ and 11½.

[Margin: Half-hours. — *Semisses horarum.*]

THE quarters of the hours you will detect in the same way from the intervals of the half-hour points. For the interval between the point of any half-hour and the point E will give two points of the quarter-hours: so that the point falling between

hour 4 and $3\frac{1}{2}$ marks hour $3\frac{3}{4}$, while the point between 7 and $7\frac{1}{2}$ [marks] hour $7\frac{1}{4}$, etc. And by the same method the eighth-parts of the hours will be drawn from the points of the quarters; and from these, the sixteenth-parts, and so on.

[Margin: Quarters of the hours. — *Quadrantes horarum.*]

MODVM — This method, which Christoph Grienberger [Christophorus Gruenbergerus] recently first observed, we have undertaken to explain and demonstrate somewhat more carefully and more fully in this place, because it is most elegant. It seems to have only this one drawback: that by it not just any proposed degree of the Equator can be found on the equinoctial line, since it proceeds by a continual subdivision of the hours, and of their fragments, into two equal parts. Yet we shall need that [finer] finding of any degree whatever when describing the hours from sunrise and sunset [ab ortu & occasu], as will be said in its place in Chapter 15, Number 4. By Tangents, however, any point on the equinoctial can be found, provided its distance from the Meridian is known. Indeed, the finding of the hour-points on the equinoctial by Tangents seems preferable to the finding set out in this chapter, on this ground: that in the former [method] the finding of no point depends on the finding of another, but each is found by itself; whereas in the latter, the finding of one requires the finding of the others, as is plain. Hence, if the first point has not been exactly found, the others depending on it cannot be found accurately.

[Margin: A comparison between the method set out here and the one delivered through Tangents in the preceding chapter. — *Comparatio inter modum hic expositum, & illum, qui per Tangentes cap. praecedenti traditus est.*]

2. THEREFORE, from the interval between E and any hour-point whatever, two hour-points 6 hours apart are found on the equinoctial line, one of which is always on one side of the meridian line and the other on the other side. Thus you see that from the interval between E and hour 10, for example, hour 8 from midnight and hour 2 from noon are found, which are 6 hours apart — the one being established on one side of the meridian line, the other on the other.

[Margin: From any hour-point on the equinoctial, two hour-points are found, one to the right and the other to the left of the meridian. — *Ex quolibet puncto horario in aequinoctiali duo puncta horaria reperiri, unum ad dexteram, & alterum ad sinistram meridianae.*]

WHEN, moreover, any hour-point of the equinoctial line is proposed, we shall know which two hour-points, 6 hours apart, are to be drawn from it, by this rule. We take the complement — up to 6 hours — of the distance by which the given

hour-point is away from the meridian line. Half of this, added to that distance, will indicate by how many hours one of the points to be found is away from the meridian line, on the side on which the given point lies; while the other point, on the other side of the meridian line, will be away from the meridian line by half of that aforesaid complement, or by 6 hours from the point first found. So if the point first found is, for example, hour $7\frac{1}{2}$, the later one will be hour $1\frac{1}{2}$; and if the first point is hour $1\frac{1}{2}$, the later one will be $7\frac{1}{2}$.

[Margin: Which two hour-points are drawn from any given hour-point. — *Qua duo puncta horaria ex quolibet puncto horario dato eruantur.*]

For example, if we wish to know which hours are found from the point of hour 8 — which on the equinoctial line is 4 hours distant from the meridian line, or from hour 12 — the complement of its distance up to 6 hours will be 2, and the half of that same complement 1. One, therefore, of the points found from hour 8 will be 5 hours distant from the meridian line, toward the given point of hour 8 (because the aforesaid half, namely 1, [together with] the given distance of four hours makes 5); such is the point of hour 7. The other will be away from this by 6 hours, toward the other side of the meridian; or, on the other side of the meridian, it will be [1 hour] distant from the meridian line, such as is the point of hour 1. So too, from the point of hour 4, you will find the points of hours 11 and 5, because the complement of the given hour 4 (which is 4 hours away from the meridian line) up to 6 is 2, and half of this complement is 1. Now it is established that hour 11 is 1 hour away from the meridian line, and that hour 5 is made up of hour 1 and the given distance of 4 hours. Finally, from the point of hour $3\frac{1}{2}$ the points of hours $4\frac{3}{4}$ and $10\frac{3}{4}$ will be found. For the complement of the given hour $3\frac{1}{2}$ (which is $3\frac{1}{2}$ hours away from the meridian line) up to 6 is $2\frac{1}{2}$, and half of this is $1\frac{1}{4}$; which, added to the given distance of the hours, $3\frac{1}{2}$, makes hour $4\frac{3}{4}$ for one sought point. The other will be hour $10\frac{3}{4}$, since it is 6 hours distant from the former, or $1\frac{1}{4}$ hours from the meridian line — namely by the half of the aforesaid complement. And so on for the rest.

3. VICISSIM (conversely) — given any two hours whatever, 6 hours apart, we shall investigate the hour-point from which their points are drawn, by this other rule. The two given hours, of which one is 6 hours away from the other, we shall gather into one sum, and from the sum we shall cast away 12 whenever it can be cast away. For the sum thus made, or the remaining number, will give the hour from whose point the points of the given hours are to be drawn.

[Margin: Given any two hour-points 6 hours apart, from which hour-point they are to be drawn. — *Data quaevis duo puncta horaria 6. horis inter se distantia, ex quo puncto horario eruantur.*]

For example: the points of hours 7 and 1 are 6 hours apart, and are found from hour 8. Likewise the points of hours $7\frac{1}{2}$ and $1\frac{1}{2}$ from hour 9; and the points of hours 10 and 4 from hour 2, because their sum is 14, and, 12 being cast away, the remainder becomes 2. Finally, the points of hours 3 and 9 from hour 12, etc.

BUT you will learn, with no trouble, which hour-points on the equinoctial line are found from any hour-point, from this table.

Table of hour-points on the equinoctial line. Each "column" (block) contains three orders of numbers; the middle order is the given (assumed) hour-point, and the two outer orders — which "stand around" the middle — are the two hour-points, 6 hours apart, generated from it.

Column 1

Point A	Given hour	Point B
1	8	7
2	10	8
3	12	9
4	2	10
5	4	11
6	6	12

Column 2

Point A	Given hour	Point B
$\frac{1}{2}$	7	$6\frac{1}{2}$
$1\frac{1}{2}$	9	$7\frac{1}{2}$
$2\frac{1}{2}$	11	$8\frac{1}{2}$
$3\frac{1}{2}$	1	$9\frac{1}{2}$
$4\frac{1}{2}$	3	$10\frac{1}{2}$
$5\frac{1}{2}$	5	$11\frac{1}{2}$

Column 3

Point A	Given hour	Point B
$\frac{1}{4}$	$6\frac{1}{2}$	$6\frac{1}{4}$
$\frac{3}{4}$	$7\frac{1}{2}$	$6\frac{3}{4}$
$1\frac{1}{4}$	$8\frac{1}{2}$	$7\frac{1}{4}$
$1\frac{3}{4}$	$9\frac{1}{2}$	$7\frac{3}{4}$
$2\frac{1}{4}$	$10\frac{1}{2}$	$8\frac{1}{4}$
$2\frac{3}{4}$	$11\frac{1}{2}$	$8\frac{3}{4}$

Column 4

Point A	Given hour	Point B
$3\frac{1}{4}$	$\frac{1}{2}$	$9\frac{1}{4}$
$3\frac{3}{4}$	$1\frac{1}{2}$	$9\frac{3}{4}$
$4\frac{1}{4}$	$2\frac{1}{2}$	$10\frac{1}{4}$
$4\frac{3}{4}$	$3\frac{1}{2}$	$10\frac{3}{4}$
$5\frac{1}{4}$	$4\frac{1}{2}$	$11\frac{1}{4}$
$5\frac{3}{4}$	$5\frac{1}{2}$	$11\frac{3}{4}$

NAM — if the interval between E and any hour-point of the equinoctial line is transferred to either side, the points of the hours found will be those that stand around, in this table, the point assumed. Thus the interval between E and hour $3\frac{1}{2}$, carried on the equinoctial line from that same hour to either side, will yield the points of hours $4\frac{3}{4}$ and $10\frac{3}{4}$, because among these hours the middle one is hour $3\frac{1}{2}$, in column 4 of the aforesaid table. For every column contains three orders of numbers, of which the two extremes stand around the middle.

CONDITA — this table, moreover, was constructed according to the precept of Number 3, since any two hours standing around [a middle] differ from each other by 6 hours and make up the sum of the intermediate hour, 12 being first cast away when that sum is greater [than 12].

CONVENIVNT — all these things hold good not only for the equinoctial line, but also for any line equidistant from it, and for other straight lines equidistant from any hour-line, as we shall show in the following chapter, Number 9, where we shall accommodate all this to lines of that kind and construct for that use their own tables, like the table just given.

4. DEMONSTRATIO — furthermore, the demonstration of the finding of the hour-points from the table above is not difficult. For let there be taken...

[Continuation of Chapter III, Number 4 — the DEMONSTRATIO begun at the end of the previous folio.]

let there be taken from that table the intermediate hour 2 in the 1st column. I say that through the interval E 2 the points of hours 4 and 10 are found. For since hours 4 and 10 stand 6 hours apart from each other, the angle 4 E 10 will be a right angle. Since, therefore, a perpendicular EB has been dropped from it to the base 4 10, the angle B 4 E will be equal to the angle B E 10 [*Elements* VI.8]; but this is equal to the angle 2 E 4 [*Elements* III.27], since each comprehends two hours (for from hour 2 up to 4, and from 10 up to 12, two hours are counted). Therefore the angles 2 4 E and 2 E 4 will also be equal, and consequently the lines 2 E and 2 4 will be equal [*Elements* I.6]. So the interval E 2, from point 2, will give the point of hour 4.

Again, since in the same right triangle 4 E 10 the angle 4 10 E is equal to the angle B E 4 [*Elements* VI.8]; and this in turn is equal to the angle 2 E 10 [*Elements* III.27], because each comprehends four hours (for from hour 12 to 4, and from 10 to 2, four hours are counted): the angles 2 10 E and 2 E 10 will also be equal; and therefore the lines 2 E and 2 10 will also be equal [*Elements* I.6]. And so, through the interval E 2, from point 2, hour 10 will likewise be found. Which was to be demonstrated.

In just the same way, through the interval E 11 you will demonstrate that the points of hours $2\frac{1}{2}$ and $8\frac{1}{2}$ are found, as the table above shows. For again the angle $2\frac{1}{2}$ E $8\frac{1}{2}$ will be a right angle, since hour $2\frac{1}{2}$ is distant 6 hours from hour $8\frac{1}{2}$; and therefore the angle $2\frac{1}{2}$ $8\frac{1}{2}$ E is equal to the angle B E $2\frac{1}{2}$ [*Elements* VI.8]. But to the angle B E $2\frac{1}{2}$ the angle 11 E $8\frac{1}{2}$ is equal [*Elements* III.27], because each comprehends two hours and a half. Therefore the angles 11 E $8\frac{1}{2}$ and 11 $8\frac{1}{2}$ E will also be equal, and consequently the lines 11 E and 11 $8\frac{1}{2}$ are equal [*Elements* I.6]. Likewise, since the angle $8\frac{1}{2}$ $2\frac{1}{2}$ E is equal to the angle B E $8\frac{1}{2}$ [*Elements* VI.8], and this to the angle 11 E $2\frac{1}{2}$ [*Elements* III.27], because each comprehends hours $3\frac{1}{2}$: the angles 11 $2\frac{1}{2}$ E and 11 E $2\frac{1}{2}$ will also be equal, and consequently the lines 11 E and 11 $2\frac{1}{2}$ are equal [*Elements* I.6], and so on. And the same reasoning holds for the rest.

[Margin, keyed to the lettered citations of this demonstration: a, d, g, k — *Elements* VI.8 ("8. sexti"); b, e, h, l — *Elements* III.27 ("27. tertij"); c, f, i, m — *Elements* I.6 ("6. primi").]

CAPVT IIII.

Hours from noon and from midnight, on a plane equidistant from the horizon, by yet another method besides.

[Margin: An accurate laying-out of hours 9 and 3.]

1. Let the figure of the preceding chapter be repeated, in which the gnomon [stylus], the center of the dial [centrum horologii], the equinoctial line, and the lines of hours 6, 3, and 9 stand as they were found in Number 1 of that same chapter. But so that hours 3 and 9 may be drawn more exactly — since we need these especially in this third method, and it is therefore of great importance that they be accurately laid out — let there be erected, from any point *r* on the meridian line (one sufficiently distant from the equinoctial line), a perpendicular *rG* to the axis *AD* extended. Then, taking on *Ar* a line *AK* equal to it, and drawing through *r* and *K* the lines *HI*, *LM* parallel to the equinoctial line (that is, to the line of hour 6), let the interval *rG* be transferred from both *r* and *K*, on either side, as far as *H*, *I* and *L*, *M*. For through *L*, *A*, *I* the line of hour 9 is to be drawn, and through *M*, *A*, *H* the line of hour 3; and if there has been no error, these will pass through the points 9 and 3 on the equinoctial line.

Diagram. A large sundial-construction figure filling the lower two-thirds of the page, within a rectangular border, showing the layout of the hour-lines of a horizontal dial by the "third method." A quarter-circle arc sweeps from the upper-right corner *R* down to point *P* at the bottom. The dial center *A* lies at the right-center; the vertical (meridian/axis) line *AD* runs down through it, and horizontal lines (the equinoctial line and the line of hour 6, with the parallels *HI* and *LM*) cross the field. Numerous hour-lines radiate from *A*, and the auxiliary points *r*, *G*, *H*, *I*, *K*, *L*, *M* of the construction are marked. Readable labels (position; sense):

- *R* (top-right corner): endpoint of the large arc.
- *L* (upper horizontal line, left) with "9" beneath: hour-9 endpoint.
- *K* (upper horizontal line, center) with "12": hour-12 point.
- *M* (upper horizontal line, right) with "3": hour-3 endpoint.
- Upper-line hour marks between the endpoints: 10, 11 (between *L* and *K*); 1, 2 (between *K* and *M*).
- *X* (upper-middle): intersection of the arc with a radiating line.
- *S* (on the arc, upper-left); *f* (right, on a horizontal); *q*, *c* with "3" (right, on the arc): construction points.

- T (mid-left, on the line through A) with "6": hour-6 point to the left of center.
- A (right-center): the dial center.
- i (right, on the arc) with "6": construction point.
- Y (mid-left); r, m, F, C, D (along the horizontal below A): auxiliary points of the transfer construction (r and G the erected perpendicular).
- N, B (center-lower, near D): construction points.
- E (lower-center): equinoctial/center point of the lower construction.
- V with "3"; H with "3" (lower-left); s, o, P (bottom): further construction points.
- G, g, n (lower-right); numerous small hour numerals (1, 2, 3, ... 12, with the halves $2\frac{1}{2}$, $4\frac{1}{2}$, $7\frac{1}{2}$, $8\frac{1}{2}$) annotate the radiating hour-lines around the field.

Let there be erected from any point r on the meridian line — one far enough removed from the equinoctial line — a perpendicular rG to the axis AD extended. Taking Ar, with a line AK equal to it, and drawing through r and K the lines HI, LM parallel to the equinoctial line or to the line of hour 6, let the interval rG be transferred from r and K on both sides as far as H, I and L, M. For through L, A, I the line of hour 9 is to be drawn, and through M, A, H the line of hour 3, which — if there is no error — will pass through the points 9 and 3 on the equinoctial line.

Also, taking a line rN equal to rG, let an arc PQ be described from N, and, an occult line NH being drawn, let the arc between P and the line NH be cut into three equal parts. For the occult lines drawn out from N through the points of division will give the points of hours 1 and 2. And most easily will the arc between P and the line NH be cut into three equal parts if, with the same interval by which the arc PQ was described, one cuts off an arc PQ of 60 degrees (comprehending four hours), and takes, in the arc contained between P and the line NH, three arcs equal to the arc between Q and the line NH — the arc that contains one hour. And if the intervals found between r and the points of hours 1 and 2 are transferred from r toward I, and from K toward L and M, one will have three points apiece for drawing hours 1, 2, 3, 9, 10, 11. The hour-points on the line LM, however, can be found just as on the line HI, if a line KO equal to rG or KL is taken, and from O the arc RS is described, and so on.

[Margin: The laying-out of hours 1, 2 and 11, 10.]

Now, in order to lay out the other hours after 3 from noon and before 9 from midnight, one must take on the meridian line the lines AE, Af equal to rG or rH, and through E, f draw the lines Vb, Xc parallel to the line of hour 6; and the lines

VX, bc joined will be equal to HI and equidistant from the meridian line, as will be demonstrated in Number 6. For if the intervals of the line rH between r and hours 1 and 2 — which have been carried over into rI, KL, and KM — are transferred from T and a into the lines VX, bc, one will have the hour-points sought. So too, if on hour 3 one takes Ap, Aq equal to that same rG or rH, and draws ph, qi parallel to the line AI of hour 9, the joined line hi will likewise be equal to HI and parallel to the line AH of hour 3, and will be bisected at g, as we shall show in Number 6. Hence, if the same intervals of the line rH between r and hours 1 and 2 are transferred onto the line hi from the point g, the points of hours 7, 8, 10, and 11 will again be found.

[Margin: The laying-out of the other hours.]

Indeed, if through the points T, a, g — taken arbitrarily on the lines Ta, AI — parallels VX, bc, hi are drawn, set with respect to the meridian line and to the line of hour 3 (placed at 6 and 12, as we shall demonstrate), and cut equally at T, a, [and g], these parallels will lie between hours 3, 9 and 6, 12. If, then, Tk is taken equal to TV, and al equal to ab, and gm — the line gm being erected perpendicular to hi — equal to gh; and from k, l, m the arcs YZ, de, no are described, and the occult lines kX, lc, mh joined; and finally the arcs between Y and the line kX, likewise between d and the line lc, and also between n and the line mh, are distributed into three equal parts (which will be easy to do if, cutting off from the intervals of the semidiameters their sixth parts YZ, de, no, one takes in the arcs between the line kX and the point Y, between the line lc and the point d, and between the line mh and the point n, three equal arcs to those between Z and the line KX, between c and the line lc, and between o and the line mh): then the occult lines drawn out from the centers k, l, m through the points of division will give, on the lines TX, ac, gh, the hour-points, which — transferred from T, a, g onto the lines TV, ab, gi — will yield the hour-points sought.

[Margin: A more general laying-out of the hours.]

And in a similar way, through a point given on any hour-line, one can draw a line parallel to that hour-line which stands six hours before it, extending as far as the two hour-lines that stand three hours before it. For this parallel will be bisected at the given point. Accordingly, if from the given point a perpendicular equal to the half of that parallel is erected to it, and from its far end an arc is described, and so on, the hour-points will be found on that parallel, just as on the aforesaid lines HI, LM, VX, bc, ih. For, for example, from g — the midpoint of the line hi (which is drawn parallel to the line of hour 3, that stands six hours from the line of hour 9 on which the given point g lies, extending as far as the two lines of

hours 6 and 12, which are three hours distant from the line of hour 9 or 3) — a perpendicular gm has been erected to hi itself, equal to the half gh or gi, and from m the arc no described, and so on. And if the parallel drawn is equal to the line HI, it is enough to transfer onto it, from its midpoint, the intervals between r and the hour-points. You see that the same thing has been done on the other lines HI, LM, VX, and bc. Nor will it be difficult to judge to which hours the hour-points found belong, from the intermediate hour on which the given point lies. Thus you see that on the half gi lie the points of hours 8 and 7, which precede hour 9 from midnight; while on the half gh lie the points of hours 10 and 11, which follow that same hour 9.

It will further be not difficult to draw, through A, a straight line passing through any degree whatever of the equator. For if that degree is distant from the meridian toward the west by fewer than 45 degrees, we shall count it in the arc PQ, from P toward Q, and through the end of the counting draw from N a straight line, which will cut the line HI in a point through which the line drawn out from A will be the one sought. But if the given degree is distant from the meridian toward the west by more than 45 degrees, yet by fewer than 90, we shall count its complement in the arc YZ, from Y toward Z, and through the end of the counting draw from k a line cutting TX in a point which, carried over from T toward V, will give a point on TV through which the line drawn out from A will be the one sought. But if the given degree is distant from the meridian toward the west by more than 90 degrees, yet by fewer than 135, we shall subtract 90 and count the remainder in the arc YZ, from Y toward Z, and through the end of the counting draw a line from k. For this will cut TX in a point through which the line drawn out from A will be the one sought. Finally, if the given degree is distant from the meridian toward the west by more than 135 degrees, yet by fewer than 180, we shall subtract that distance from 180 and count the remaining degrees in the arc RS, from R toward S, and through the end of the counting draw a line from O. For this will cut KL in a point through which the line drawn out from A will be the one sought.

[Margin: By what method a line is to be drawn, by this way, through any degree whatever of the equator.]

But if any degree whatever is proposed as distant from the meridian toward the east, its point is to be found on the line rH, or VX, or KL, exactly as if the given distance were toward the west. For the point found, transferred onto the line rI, or bc, or KM, will be the one through which the line drawn out from A will be the one we seek.

For example: let there be given the 30th degree of the equator from the meridian. Having counted 30 degrees in the arc PQ, from point P toward Q, and drawn from N a line through the end of the counting, rH will be cut at point 2, through which the line drawn out from A will pass through the 30th degree of the equator from the meridian toward the west. And if r 10 is taken equal to r 2, the line drawn A 10 will be distant 30 degrees from the meridian toward the east. Again, let the degree given be 75. Its complement, 15 degrees, if counted in the arc YZ from Y toward Z, and a line drawn from k through the end of the counting, will cut TX at point 7; through which the line from A — carried over from T as far as 5 — that is, the line drawn out from A through 5, will pass through the 75th degree of the equator toward the west. And if a 7 is taken equal to T 5, the line drawn out from A through 7 will pass through the 75th degree of the equator toward the east. Likewise, let 105 degrees be given. Subtracting 90, we shall count the remaining 15 degrees in the arc YZ, from Y toward Z. For a line drawn from k through the end of the counting will cut TX at point 7, through which the line thrown across from A will pass through the point of the equator distant 105 degrees from the meridian toward the west. And if a 5 is taken equal to T 7, the line drawn out from A through 5 will proceed through the 105th degree of the equator on the eastern side. Lastly, let the given degree of the equator be 150; subtracting 150 from 180, there remain 30. Having counted 30 degrees in the arc RS, from R toward S, and drawn from O a line through the end of the counting, KL will be cut at point 10, through which the line drawn out from A will pass through the 150th degree of the equator, on the western side. And if Ka is taken equal to K 10, the line cast out from A through 2 will proceed through the 150th degree of the equator on the eastern side. And so for the rest.

2. This way here set forth — which is most beautiful and most accurate — does not differ from the method that we demonstrated in the *Construction and Use of the Instrument for Dials* by means of a certain parallelepiped, and which, following Andreas Schoner [Andreas Schöner], we employed in various places in our *Gnomonics*, except that here we have rendered the whole matter more universal; and this indeed we shall demonstrate by the following reasoning.

First we shall show: that when, in different circles, perpendicular lines are erected upon the diameters, and straight lines go out from the centers cutting off similar arcs from the circles, those perpendiculars are cut similarly — that is, into proportional parts. For let two circles CD, EF be described, arbitrarily, from the centers A, B; and upon their diameters let perpendiculars GH, IK be erected, as one pleases, which the straight lines going out from the centers — AL, AM, AD, BN, BO, BF — cut at P, Q, H and R, S, K; and let the pairs of arcs cut off, CL, EN; LM,

NO; MD, OF, be similar. *I say that the perpendiculars GH, IK are cut similarly — that is, that as GP is to PQ, so IR is to RS; and as PQ is to QH, so RS is to SK.* For since the angles GAP, IBR are equal, by the scholium on proposition 22 of book 3 of Euclid, because the arcs CL, EN are similar, and the angles G, I are right, the remaining angles APG, BRI will also be equal [*Elements* I.32]; and consequently the remaining angles APQ, BRS (from two right angles) will be equal. Since, therefore, the angles PAQ, RBS are also equal, because of the similar arcs LM, NO, the remaining angles PQA, RSB will also be equal. And in the same way, both AQH, BSK and QHA, SKB will be equal to one another. So the triangles AGP, BIR, and APQ, BRS, and AQH, BSK, are equiangular with one another. Therefore, as GP is to PA, so IR is to RB, and as PA is to PQ, so RB is to RS [*Elements* VI.4]; and hence, from equality of ratio, as GP is to PQ, so IR is to RS. And by the same reasoning, as PQ is to QH, so RS is to SK. Which was to be demonstrated.

[Margin: a — *Elements* I.32 ("32. primi"); b — *Elements* VI.4 ("4. sexti").]

Diagram. A two-part geometric figure at the upper left, illustrating the proportional-section proposition. It shows two circular arcs, each with a center, a diameter, a raised perpendicular, and radial lines cutting off similar arcs.

Readable labels:

- Upper arc (center at right, toward A): points L, M, H along the arc, with the raised perpendicular GH and the cutting points P, Q on it; D marked near the diameter foot; a lower-case construction point near the arc. A is the large capital at the right (center of this circle; diameter CD).
- Lower arc: points K, S, R along the arc, with the perpendicular IK and cutting points on it; N, O, E, F marked; B the large capital at the bottom (center of this circle; diameter EF). An "I"/"1" mark stands near the perpendicular's top.

3. *In the second place it is to be demonstrated: that when some plane is equidistant from the circle of any hour from noon or from midnight, the remaining hour-circles make, in that plane, common sections parallel to that parallel which the hour-circle six hours distant from the one to which the plane is equidistant makes; and that they stand distant on either side in that proportion in which the hour-points on the equinoctial line stand distant from the meridian line in a horizontal dial.* For let there be any plane AB, equidistant, say, from the circle of hour 3 from noon or from midnight, and let its common section with the equator be the straight line CD, which the straight line EF cuts at right angles, and on which let FE be equal to the distance of the plane AB from the center of the world, or from the circle of hour 3 to which it is equidistant. Let there then be described a circle GHIK from E,

of whatever size, and let it be cut into 24 equal parts. Now if this circle is conceived to be carried around CD until it is at right angles to the plane AB, and E is imagined to be placed at the center of the world (that is, of the equator), and the plane AB parallel to the circle of hour 3 — so that the line CD does not depart from the plane of the equator, and the circle GHIK itself is concentric with the equator — then the hour-circles from noon and from midnight will pass through the points of division of the circle GHIK, and will meet the plane AB at the points on the line CD onto which fall the straight lines drawn out from the center E through those same points of division. And since the hour-circles from noon and from midnight are perpendicular to the equator, by proposition 15 of book 1 of Theodosius, being drawn through its poles; while the plane AB, by the scholium on proposition 18 of book 11 of Euclid, is perpendicular to that same equator, being placed parallel to the circle of hour 3 (which is perpendicular to the equator, as are the other hour-circles, as was just said): the common sections of the hour-circles with the plane AB will also be perpendicular to the equator [*Elements* XI.19], and therefore, by definition 3 of book 11 of Euclid, perpendicular to CD, the common section of the equator and the plane AB. So the straight lines drawn out through the points of the line CD, perpendicular to CD itself, will be the common sections of the hour-circles and the plane AB, and parallel to one another [*Elements* I.28]; and the line LM, drawn through the point F, will be the common section of the plane AB and the circle...

[Margin: c — *Elements* XI.19 ("19. vndec."); d — *Elements* I.28 ("28. primi").]

Diagram. A large figure filling the lower half of the page: a circle GHIK inscribed within a rectangular frame ruled by many parallel vertical lines (the common sections of the hour-circles with the plane AB), crossed by horizontal lines (CD, EF, LM, AB). At the upper right a small right triangle (A–D–3) sits against the frame. Readable labels:

- A (upper, at the triangle's apex), with D below it and "3" to its right: the raised distance-construction.
- L (top-left, on the circle top) and G with "9" (top of the circle): division points.
- H with "3" (left of the circle) and K with "3" (right of center): the horizontal diameter GHIK ends / division marks.
- C (left frame edge) and D (right frame edge): the line CD across the field.
- E (center of the circle): the center from which the divisions are struck.
- F (below the circle center): point on EF where LM is drawn.

- N, N (lower-left, two adjacent points) and M, M (lower-center, two adjacent points) and B, B (right, two adjacent points): the ends of the parallel common-section lines meeting the frame.
- Small numerals 9, 3, 1 annotate the circle's divisions; dotted radial lines mark several of the 24 divisions.

the circle of hour 9, since the points G and I are six hours distant from the points H and K, through which the circle of hour 3 is drawn. Therefore, since — as we demonstrated at Num. 2 — the diameters of the circle GHIK cut the straight line CD just as the equinoctial line in a horizontal dial (or any other straight line perpendicular to the meridian line AB) cuts the diameters of the circle described from E in the figure of Cap. 3: it is plainly established, which was proposed in the second place.

4. And the segments of the straight line FC are always equal to the segments of the straight line FD — the first to the first, the second to the second, and so on — when the division is begun from the point F. (This we shall easily demonstrate as follows. In the triangles EFC and EFD, for instance, the two angles CEF and DEF are equal [*Elements* III.27], and the two angles at F, being right, are equal as well, while the side EF adjacent to them is common; therefore [*Elements* I.26] the sides FC and FD are equal; and so for the rest.) It follows that, if the straight line NMB is drawn cutting all the parallel hour-lines in the plane AB in any way whatever, its segments intercepted between the middle parallel LM and any two parallels equally distant from LM will be equal. For example, the segment MN will be equal to the segment MB. For since the straight lines CD and NB are cut proportionally by the parallel planes drawn through the parallels of the plane AB (for through these parallels planes parallel to one another — namely to the plane AB — can be drawn) [*Elements* XI.17], and since CD is bisected at F, as we showed, NB too will be bisected at M. And so for the rest.

[Margin: a — *Elements* III.27 ("27. tertij"); b — *Elements* I.26 ("26. prim."); c — *Elements* XI.17 ("17. vndec.").]

5. Once these things are established, we shall easily demonstrate the method of finding the hour-points taught in this chapter. For conceive, in the figure of Cap. 3, that through g — the point on the line of hour 9 from midnight — a plane is drawn parallel to the circle of hour 3 from noon. And since the plane of the dial, cutting the circle of hour 3, and the plane drawn through g parallel to it produce parallel common sections [*Elements* XI.16], the plane drawn through g will pass through the straight line hi, which we drew parallel to the line of hour 3, and it will be inclined to the plane of the dial in the same way as the circle of hour 3 is

inclined to it. Now since — through the things which we demonstrated at Num. 3 — the hour-circles (excepting the circle of hour 3, from which the plane drawn through g is equidistant) make, in that plane, common sections parallel to the parallel which the circle of hour 9, six hours distant from the circle of hour 3, produces, and these stand distant on both sides in that proportion in which the hour-points on the equinoctial line, or on the straight line HI, are distant from the point of hour 12: those parallels will cut the straight line hi as well — the common section of that plane with the plane of the dial — in the same proportions. Since, therefore, the circle of hour 6 is drawn through i and the circle of hour 12 through h, the parallels produced by the circles of hours 6 and 12 will meet the plane of the dial at i and h, and consequently hi will be bisected at g, inasmuch as the parallels passing through i and h are equally distant from the middle parallel drawn through g. Therefore, since the straight lines going out from the center m cut the straight line hi in the same proportions in which the equinoctial line is cut by the straight lines from the center E, or the straight line HI by the straight lines emitted from the center N — as was demonstrated at Num. 2 — other parallels will pass between i and h through the points of the straight line hi which are marked out by the straight lines emitted from the center m: namely, if the circle described from m be divided into hours, the beginning made from the line mg, just as the equator described from N was divided; exactly as was done in the arc no, by which the hour-points were found on the straight line gh.

[Margin: d — *Elements* XI.16 ("16. vnde.").]

Now the straight line qm must be taken equal to half of gh, or of gi, so that the arc between n and the straight line mh — an arc of three hours, that is, of 45 degrees — may present to us the point h, of hour 12, receding by three hours from the point g. For the segment of the straight line hi that comprises three hours is always equal to the straight line between g and the center m; just as, at Cap. 3, Num. 1, we showed that the segment of the equinoctial line B3, or B9, comprising three hours, is equal to the straight line BE between B and the center E of the equinoctial circle. By the same reasoning, the other parallels between g and i will pass through the points found on the straight line gi, since these correspond to the points on the straight line gh.

By like reasoning, the common sections VX, bc, and LM will be equal to HI itself at the hour-points, if the points of the straight line HI be transferred onto them. For first, if through the straight lines VX and bc — parallel to the line of hour 12 — planes parallel to the circle of hour 12 be drawn (in such a way that they stand at right angles to the plane of the dial, just as the circle of hour 12 stands at right

angles to that same plane of the dial), the same demonstration will apply which we applied just now, when through *hi*, parallel to the line of hour 3, a plane was drawn parallel to the circle of hour 3 and inclined to the plane of the dial in the same way as the circle of hour 3 is inclined to it. But if the parallels *VX* and *bc* are not equal to the straight line *HI*, then on the line of hour 6, *Ta*, which is perpendicular to them, the straight lines *Tk* and *al* must be taken equal to their halves *TV* and *ab*, and circles are to be described from *k* and *l*, just as the circle *PQ* was described from *N*, and so on. Then if through *LM*, parallel to the line of hour 6, a plane be drawn parallel to the circle of hour 6, inclined to the plane of the dial in the same way as the circle of hour 6 — drawn in its proper position through the straight line *Ta* and the axis *AD* — is inclined to that same plane of the dial: the same demonstration will fit in every respect, as is evident.

In no other way shall we show the same thing to happen in any straight line whatever that is drawn equidistant from some hour-line.

6. Moreover, that the straight lines *VX* and *bc* — parallel to the meridian line — are equal to the straight line *HI* and are bisected at *T* and *a* (if halves equal to *rH* be taken, *AE* and *Af*, and parallels to *Ta* be drawn through *E* and *f*, as we said at Number 1), I demonstrate thus. In the triangles *AEV* and *AfX* the angles at *E* and *f* are right [*Elements* I.29] (because the straight lines *EV* and *fX* are parallel to *Ta*), and the angles at *A* are equal [*Elements* I.4] (for since the angles *HAr* and *IAr* are equal — on account of the two sides *rA*, together with the two sides *rH* equal to the sides *rA* and *rI*, and the right angles at *r* [*Elements* I.15] — while *IAr* is equal to *XAf*, *VAE* too will be equal to that same *XAf*); and moreover the adjacent sides *AE* and *Af* are equal. Therefore the straight lines *EV* and *fX* will also be equal [*Elements* I.26]. Since, then, they are parallel — each being parallel to *Ta* — and equal, *VX* and *Ef* too will be parallel and equal [*Elements* I.33]. But *Ef* is equal to *HI*. Therefore *VX* too is equal to that same *HI*, and *Ef* is bisected at *A*. And the same must be said of the straight line *bc*.

[Margin: *e* — *Elements* I.29 ("29. primi"); *f* — *Elements* I.4 ("4. prim."); *g* — *Elements* I.15 ("15. primi"); *h* — *Elements* I.26 ("26. primi"); *i* — *Elements* I.33 ("33. prim.").]

Finally, that the straight line *hi* (if halves equal to *rH* be taken, *Ap* and *Aq*, and parallels *ph* and *qi* be dropped to *Ag*, as far as the lines of hours 12 and 6) is parallel to the line of hour 3, *AH*, and equal to *HI* itself, and bisected at *g*, as we supposed at Num. 1 — this will become manifest thus. If through *h* a parallel to *pq* be drawn, cutting the line of hour 9 at any point *g*, then *hg* will be equal to *pA*, in the parallelogram *gp* [*Elements* I.34]. Since, therefore, as was shown at Num. 3,

4, and 5, this parallel drawn through h as far as hour 6 is cut at g, bisected from the mid-hour Ag, it must fall on the point i, so that the portion gi becomes equal to the portion Aq, to which Ap and hg are equal. For only the parallel to Aq that lies between g and the straight line qi is equal to Aq, on account of the parallelogram gq. For the parallel hg, cutting the straight line qi elsewhere than at i, would make its portion between g and the line of hour 6, Ai, either less or greater than the straight line Aq, according as it fell to the right of that point i, or to the left, as is evident.

[Margin: k — *Elements* I.34 ("34. primi").]

7. But in order that the position of the parallels which the hour-circles produce — in the planes drawn through the straight lines HI, LM; VX, bc; and hi, and through the circles of hours 12, 6, and 3 — may be perceived more clearly, I say that they make, with the said straight lines at the hour-points, angles equal to the angles which the axis of the world AD, established in its proper position in the dial, makes with the hour-lines to which they are equidistant. So that the parallels in the planes drawn through the straight lines VX and bc make angles equal to the angle DAB, which the axis DA — holding its proper position (that is, when the triangle ADB is at right angles to the plane of the dial, and therefore lies in the plane of the meridian) — makes with the meridian line, to which VX and bc are equidistant. But the parallels in the planes drawn through the straight lines HI and LM make angles equal to the right angle which the same axis AD, in its proper position, makes with the line of hour 6, to which HI and LM are equidistant. Finally, the parallels in the plane drawn through the straight line hi make, with hi at the hour-points, angles equal to the angle DAp, which the same axis AD, in its proper position, makes with the line AH of hour 3, to which hi is equidistant.

The magnitude of this angle will be known thus. From the three straight lines AD, A3, and E3 of the dial of Cap. 3, let a triangle AD3 be formed — in the figure next to Num. 3 — so that D3 is equal to E3 itself. I say that the angle DA3 is equal to the angle which the axis AD, holding its proper position, makes with the line A3 of hour 3. For, the axis AD being placed in its proper position, the straight line dropped from the point D to the point 3 will be equal to the straight line E3 [*Elements* I.8], because, when the center E is placed at D — at the center of the world — the straight line E3 coincides with the straight line D3. Since, therefore, the two sides AD and A3 of the triangle AD3 in the dial of Cap. 3 are equal to the two sides DA and A3 of this triangle, and the base D3 is equal to the base D3, the

angle DA3 in that triangle will be equal to the angle A in this constructed triangle [*Elements* I.8].

[Margin: a — *Elements* I.8 ("8. primi").]

Diagram. A large master construction filling the greater part of the page: a rectangular frame enclosing a large circle (drawn partly in solid, partly in dotted arcs) with two orthogonal principal diameters (a horizontal line across the top-center and a vertical axis) and a dense web of straight and dotted radial lines representing the hour-circles and their common sections, together with several smaller subsidiary circles/arcs and shadow-triangles. Numerous hour-numerals (1–12, some repeated on opposite sides) annotate the rim and the radial lines. Because the small numerals and the many dotted radials are only partly resolvable at this resolution, the confidently read labels are listed; several small labels remain uncertain.

- Top horizontal line, left to right: L (at "9"), "10", "11" (with a cross-mark Z beside it), K (at "12"), "1", "2", M (at "3").
- R: apex of the outer arc, at top center-left.
- S: left, on the arc, marked with a small cross.
- X: principal crossing point below the top line, left of center.
- f: on a horizontal dotted section-line, mid-left.
- c (with "3"): crossing point to the right of center, where a rising oblique line meets an arc.
- q: just below c.
- o: interior point, center-left; near it descending numerals "8", "7".
- T (with "6" and "1"): on the mid horizontal line, center.
- a, i (with "6", and "d"): right side of the mid line; "7", "7½" below to the right.
- Y: left edge, mid-height; r/script mark and m nearby (mid-left).
- "4½", "2½": numerals on the central horizontal band; small "3", "1" marks between.
- N, B (with "2", "1"): left-center; F, C, D: the central shadow/section triangle (F upper, C and D below).
- A: the central origin/center of the construction, on the vertical axis; a script mark (k?) and "a", "i" to its right.
- G: right-center; "9", "6", n (right); numerals "8", "8½" near center-right.
- E: interior point, center-lower; "10", "11" beside it.

- V (with "3"): center-lower; H (with "3"): lower-left; "S", "2", small "o" (cross-marked) below.
- P: bottom center, on the lower arc.
- Bottom line hour-numerals (left to right): "1", "2", "11", "10", "9", with "12", a script "k", and "12" clustered at lower-left.

Moreover, that those parallels make angles of this kind, I demonstrate most briefly thus. For example, the parallel in the plane drawn through ih — which is equidistant from the circle of hour 3 — is equidistant [*Elements* XI.16] from the axis of the world set in its proper position (because the axis of the world and any of the parallels are sections made by parallel planes, namely by the circle of hour 3 and the plane drawn through ih); and ih itself is parallel [*Elements* XI.10] to the line A3 of hour 3. Therefore the angles constituted in that plane by the aforesaid parallels with the straight line ih will be equal to the angle which the axis, in its proper position, makes with the line of hour 3. And so for the rest.

[Margin: b — *Elements* XI.16 ("16. vnde."); c — *Elements* XI.10 ("10. vnde.").]

8. And so, if at the hour-points of the straight line ih there were constituted, with ih, angles equal to the angle A of the upper triangle — toward that side to which the axis inclines in its proper position — and if the plane in which they are constituted were carried around ih until it were equidistant from the plane of the circle of hour 3 (drawn through the axis DA in its proper position and the line A3 of hour 3), then the lines making those angles would reproduce, most exactly, the parallels of the plane drawn through ih. Now in the upper triangle the angle D, which the axis in its proper position makes with the straight line D3 on high, is right. For since AD is at right angles to the plane of the equator, it will make — by definition 3 of book 11 of Euclid — a right angle with the straight line D3 lying in the equator.

By like reasoning, if at the hour-points of the straight lines HI and LM right angles were constituted with those same straight lines, and the planes in which they lie were carried around HI and LM until they were equidistant from the circle of hour 6 (drawn through the axis DA in its proper position and the line Ta of hour 6), those perpendiculars would express to a nicety the parallels of the planes drawn through HI and LM, since they would not differ from them.

Finally, if at the hour-points of the straight lines VX and bc there were constituted with them angles equal to the angle of the pole's elevation — which the axis makes with the meridian line — and the planes in which they lie were turned about those same straight lines until they stood at right angles to the plane of the

dial, that is, equidistant from the meridian, then the lines making those angles would not differ from the parallels of the planes drawn through VX and bc.

9. Now, that the finding of the hour-points explained in the preceding chapter fits the lines HI, LM; VX, bc; and hi as well — as we advised at Num. 3 of the preceding chapter — can easily be understood, provided that on the straight lines VX and bc the line Ta of hour 6 be taken for the meridian line, and on the straight line hi the line Ag of hour 9; and then, in place of the hours surrounding hour 12, the hours surrounding hour 6 and hour 9 be taken. Only on the straight lines HI and LM is nothing to be changed; rather, everything is to be understood as on the equinoctial line, so that the table of the preceding chapter serves these two lines no otherwise than it serves the equinoctial line. But for the other lines the intermediate hours must be changed, as we shall presently say.

And so if, for example, from k, the center of the straight line VX, at the interval of the straight line VX (which is double Tk), the straight line VX be cut at f, then f will be the hour-point 4 hours distant from the line of hour 6, Tk — namely the point of hour 2 — just as in the preceding chapter the interval EF — or 39 [?], double of EB — gave the point of hour 8, or 4, likewise 4 hours distant from the meridian line. But if the third part of Tf, namely Tt, be taken, t will be the hour-point 2 hours removed from that same line of hour 6 — namely the point of hour 4. From these two points all the rest will be found, as was said at Num. 1 of the preceding chapter. And the same reasoning holds for the other lines.

But what hour-points are to be found, from any given hour-point, on the lines VX and bc (equidistant from the meridian line), and on hi (parallel to the line of hour 3), and also on a line drawn equidistant from the line of hour 9 (for lines of this kind are sometimes most convenient on certain declining planes), the following three tables teach.

Tabella pro rectis meridianæ lineæ æquidistantibus.

(Table for straight lines equidistant from the meridian line.)

The table is printed in four blocks of three columns; in each block the first column is the argument (the distance, in whole units and quarter-fractions) and the two following columns are the two corresponding hour-values.

arg	h	h	arg	h	h	arg	h	h	arg	h	h
1	2	7	$\frac{1}{2}$	1	$6\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$6\frac{1}{4}$	$3\frac{1}{4}$	$6\frac{1}{2}$	$9\frac{1}{4}$
2	4	8	$1\frac{1}{2}$	3	$7\frac{1}{2}$	$\frac{3}{4}$	$1\frac{1}{2}$	$6\frac{3}{4}$	$3\frac{3}{4}$	$7\frac{1}{2}$	$9\frac{3}{4}$

arg	h	h	arg	h	h	arg	h	h	arg	h	h
3	6	9	2½	5	8½	1¼	2½	7¼	4¼	8½	10¼
4	8	10	3½	7	9½	1¾	3½	7¾	4¾	9½	10¾
5	10	11	4½	9	10½	2¼	4½	8¼	5¼	10½	11¼
6	12	12	5½	11	11½	2¾	5½	8¾	5¾	11½	11¾

Tabella pro rectis lineæ horæ 3. æquidistantibus.

(Table for straight lines equidistant from the line of hour 3.)

ar	ar			ar			ar					
g	h	h	g	h	h	g	h	h	g	h	h	
1	1	7	½	1	6½	¼	9½		6¼	3¼	3½	9¼
	1			0								
2	1	8	1½	1	7½	¾	10½		6¾	3¾	4½	9¾
				2								
3	3	9	2½	2	8½	1¼	11½		7¼	4¼	5½	10¼
4	5	1	3½	4	9½	1¾	12½	[numeral:?]	7¾	4¾	6½	10¾
		0										
5	7	1	4½	6	10½	2¼	1½		8¼	5¼	7½	11¼
		1										
6	9	1	5½	8	11½	2¾	2½		8¾	5¾	8½	11¾
		2										

Tabella pro rectis lineæ horæ 9. æquidistantibus.

(Table for straight lines equidistant from the line of hour 9.)

ar	h	h	ar	h	h	ar	h	h	ar	h	h
g	g	g	g	g	g	g	g	g	g	g	g
1	5	7	½	4	6½	¼	3½	6¼	3¼	9½	9¼
2	7	8	1½	6	7½	¾	4½	6¾	3¾	10½	9¾
3	9	9	2½	8	8½	1¼	5½	7¼	4¼	11½	10¼
4	1	1	3½	1	9½	1¾	6½	7¾	4¾	12½ [numeral:?]	10¾
	1	0		0							
5	1	1	4½	1	10½	2¼	7½	8¼	5¼	1½	11¼
		1		2							

ar g	h	h	ar g	h	h	ar g	h	h	ar g	h	h
6	3	1	5½	2	11½	2¾	8½	8¾	5¾	2½	11¾
		2									

The use of these tables is the same as that of the one placed in the preceding chapter for the equinoctial line and the straight lines equidistant from it. For if, for example (to explain the use of the first table, which serves the lines parallel to that of hour 12), the interval between k and any hour-point on the straight line VX — corresponding to some intermediate hour of the first table — be transferred to either side, the hour-points surrounding the assumed point will be found in this table. Thus the interval between k and hour 5 will give the points of hours 2½ and 8½. And so for the rest.

10. Indeed, if from the points r, K, T, a onto the straight lines HI, LM, VX, bc there be transferred the tangents of 15, 30, 45 degrees, and so on — with respect to the whole sine Nr, or Ok, or kT, or la, divided into 10 parts — then the hour-points on those straight lines, distant from those same points r, K, T, a by 15, 30, 45 degrees, and so on, will be found, just as was done on the equinoctial. This is also to be understood of the parts of hours, if the tangents of half-hours and quarter-hours be taken. Moreover, by the same reasoning you will find the point of any degree whatever, provided you have noted its distance from the point r, and from this have learned its distance from the point T, for example. Thus if a point be proposed distant 63 degrees from the meridian: since from r up to H there are 45 degrees — namely three hours — the proposed point is not to be sought on rH, but on TV, since from the point r up to T there are 90 degrees, that is, six hours. Therefore the complement of the proposed 63 degrees must be taken, that is, 27 degrees, and the tangent of this transferred from T toward V; and so for the rest. And this reasoning...

[Margin: By what method the hour-points are found by means of tangents on the meridian line, and on the lines equidistant from those of hours 6 and 3 or 9. ("Qua ratione per Tangentes reperiantur puncta horaria in lineis meridiana, & lineis hor. 6. ac 3. vel 9. æquidistantibus.")]

(continuation of Chapter IV)

[Margin: The most convenient method for hours that scarcely cut the equinoctial. ("Commodissima ratio pro horis æquinoctialē vix secantibus.")]

this is the most convenient method for hours that scarcely cut the equinoctial, especially if the straight lines VX and bc are drawn parallel to the meridian line in whatever way you please, and their halves TV and ab are each divided into 10 equal parts, treated as whole sines, with respect to which the Tangents are to be taken, so that they may be transferred onto the straight lines VX and bc...

Indeed, these same hour-lines can also be described—by means of the Tangents of the complements of the arcs of table 6—upon any perpendicular to the meridian onto which they have been carried, without the circle described from G, as was explained in the scholium of chapter 25, No. 5. Moreover, if in the plane of the sundial there be drawn, perpendicular to the line of hour 6, straight lines that recede from the meridian at equal distances—such as are, in the sundial of the preceding chapter 4, the straight lines VX and bc—and if onto them there be transferred the Tangents of the arcs of table 6 (not of their complements), reckoned with respect to the whole sine AT or Aa, then there will likewise be hour-points on these lines, so that three points are obtained for each hour, as will be clear from what we shall demonstrate at No. 5 of the scholium to chapter 25. And this method is the most beautiful of all for the hours that lie near hour 6.

CHAPTER V. [CAPVT QVINTVM.]

THE HOURS FROM NOON AND FROM MIDNIGHT, on a plane parallel to the horizon, by yet another method besides.
[HORAE A MERIDIE, ET MEDIA NOCTE in plano, quod Horizonti æquidistat, alio præterea modo.]

[Margin: The description of the hours by means of the arcs of the horizon, table 6. ("Descriptio horarū per arcus Horizontis 6. tabula.")]

1. Finally, we shall draw all the lines of the hours from noon and from midnight without difficulty, by means of the arcs of the horizon that are intercepted between the circle of hour 6 (that is, the common section of the circle of hour 6, or of the Equator, with the horizon) and the points where the horizon is cut by the hour-circles—such being the arcs that the numbers of table 6, set out beside the pole's altitude on the left-hand side, present. We shall proceed thus. In the sundial of chapter 2, from the centre G of the sundial let there be described a circle equal to some circle whose quadrant is divided into degrees; for here the centre of the sundial, the equinoctial line, and the line of hour 6 have to be found from the given style, as we taught in the earlier descriptions. Into this circle, if

from the line of hour 6, on either side, there be transferred on both sides the arcs of the horizon set out beside the pole's altitude of 42 degrees (or of whatever other altitude belongs to the region in which the sundial is described), or if from the meridian line, both below the centre G and above it, the complements of those same arcs be transferred on either side, then there will be obtained in the said circle, for each single hour, two points; and if straight lines be drawn through these from the centre G, the hours from noon and from midnight will be described, as appears in the sundial of chapter 2.

What we have said about the arcs of whole hours is to be understood also of the arcs of the halves and of the quarters of those same hours.

2. The demonstration of this description is as follows. Let the triangle GDC of the sundial in chapter 2 be swung around the meridian line GC until it stands perpendicular to the plane of the sundial—that is, until it lies in the plane of the meridian. If through D, the tip of the style, some circle be conceived parallel to the plane of the sundial—that is, lying in the plane of the horizon and concentric with it—then the hour-circles from noon and midnight, passing through the axis DG, will cut off from that circle concentric with the horizon arcs of just as many degrees as answer, in table 6, to the several hours and their fractions; and they will produce in that same circle straight lines, which are their common sections with that circle. Since, then, each of the hour-circles [Margin: Elements XI.16. ("16. undec.")] of the sundial makes lines parallel to those which the meridian, or the circle of hour 6, makes in the aforesaid circle, [Margin: Elements XI.10. ("10. undec.")] *these lines will make, in the plane of the sundial with the meridian line—or with the line of hour 6—an angle at the centre G equal to the angle which that other line makes at the centre of the [horizontal] circle. For this reason the arc of the circle described from G, the centre of the sundial, subtending that angle at the centre G, will be similar to the arc of the circle described from D, the centre of the world, subtending an equal angle at the centre D ... [and the lines drawn] through the points impressed on the circumference of the circle described from G ... are the common sections of the plane of the sundial with the hour-circles, that is, are the hour-lines.*

CHAPTER VI. [CAPVT SEXTVM.]

THE HOURS FROM NOON AND FROM MIDNIGHT, on a plane parallel to the horizon, above which the pole's altitude is so small that the centre of the sundial can scarcely be obtained. [HORAE A MERIDIE ET MEDIA NOCTE in plano, quod Horizonti æquidistat, supra quem poli altitudo tam exigua est, ut centrum horologij vix haberi possit.]

[Margin: The description of a sundial for a small elevation of the pole. ("Horologij descriptio ad exiguā poli elevationem.")]

1. Sometimes a sundial has to be described in a region where the pole's altitude above the horizon is so small that the centre of the sundial can scarcely be obtained, because it falls too far from the place of the style. For example, if the pole's altitude were 0 degrees 30 minutes, its complement being 89 degrees 30 minutes, the Tangent of this complement will contain about 1146 of those parts of which the whole sine comprises 10—parts which have to be counted off from the place of the style along the meridian line (the style, of course, being divided into 10 equal parts) in order that the centre may be obtained. This can scarcely be done unless the plane of the sundial is of almost boundless size. So too, if the pole's altitude were 2 degrees, its complement being 88 degrees, the parts to be counted off would be $286 \frac{3}{10}$ [numeral:?] and a little more—which likewise can scarcely be done. Therefore, so that a sundial may in that case be described even though the centre, through which the hour-lines are to be drawn, is not present, we shall proceed as follows. Let us suppose, in some sundial, that the centre can scarcely be obtained, or rather that it cannot be contained on the plane of the sundial at all. Since, then, the parts of the Tangent of the complement of the pole's altitude cannot be counted upward from the place of the style, we shall carry the Tangent of the pole's altitude downward from the place of the style, so that a point may be found through which the equinoctial line, making right angles with the meridian line, is to be drawn. On this line let the hour-points be found, as we taught in chapter 2 or 3, omitting No. 5 of chapter 2, where table 6—which is not universal—is used.

And next, if with the style there be set up, at the meridian line, a right angle equal to the complement of the pole's altitude—yet not toward the equinoctial

line—the line making that angle will be the axis of the world, to which, if from any point...

whatever point of the meridian line, remote from the equinoctial line, a perpendicular be raised; and if through that point a perpendicular be drawn to the meridian to serve as a new equinoctial, then upon this equinoctial there will be found, by means of the Tangents (reckoned with respect to the whole sine, which equals the perpendicular raised from the assumed point to the axis), the hour-points, just as on the former equinoctial line, as we taught in chapter 2, No. 6. Let these points, then, be joined by straight lines to the points found on the former equinoctial, and the hour-lines will be described, even though the centre of the sundial appears nowhere. By this art several equinoctial lines can be described, and hour-points sought upon them, so that the hours may be drawn through several points.

2. Moreover, if you please, let there be chosen from table 6 the altitude of the pole, on the left-hand side, whose horizon—represented by a straight line perpendicular to the meridian—can conveniently be described in the sundial, whether below the equinoctial line or above it.

[Margin: In what way a point distant any number of degrees from the Equator, whether toward the south or toward the north, is found on the meridian line. ("Quo pacto punctum quotlibet gradibus distans ab Æquatore sive versus austrū, sive versus boream, reperiat in meridiana linea.")]

The point on the meridian line through which that altitude's horizon must be drawn will be found below the equinoctial line thus. To the complement of that altitude let the altitude of the pole, for which the sundial is being constructed, be added; for the Tangent of this combined number, transferred from the place of the style, will indicate the sought point. For example: if the pole's altitude were 10 degrees, and the horizon to be described has a pole-altitude of 60 degrees, we shall add to its complement of 30 degrees the pole's altitude of 10 degrees, [and from the combined number of 40 degrees] we shall transfer the Tangent $8 \frac{4}{10}$ [numeral:?], approximately, from the place of the style, below the equinoctial line. Above the equinoctial line, on the meridian, we shall find the point of the same horizon thus: we shall compare the complement of that pole's altitude with the pole's altitude for which the sundial is being described, and subtract the smaller number from the larger. Then we shall transfer the Tangent of the remaining number from the place of the style—downward, indeed, if that complement was less than the pole's altitude for which the sundial is being made; but upward, if greater. For example: if the pole's altitude were again 10 degrees,

and the horizon to be described has a pole-altitude of 60 degrees, we shall subtract from its complement—that is, from 30 degrees—the pole's altitude of 10 degrees, and [with] the remaining number of 20 degrees we shall transfer the Tangent $3 \frac{6}{10}$ upward from the place of the style. But if the altitude for the sundial to be constructed were 40 degrees, we shall subtract from it the aforesaid complement of 30 degrees, and with the remaining number of 10 degrees we shall transfer the Tangent $1 \frac{7}{10}$, and a little more, downward from the place of the style.

3. This horizon having therefore been described, the hour-points will be investigated upon it by means of the Tangents of the complements of the arcs of table 6, as was set forth in chapter 2, No. 6. These points, joined by straight lines to the points of the equinoctial line, will give the hour-lines, even though there be no centre. And these can be described more accurately if several horizons are drawn and hour-points investigated upon them.

CHAPTER VII. [CAPVT SEPTIMVM.]

THE HOURS FROM NOON AND FROM MIDNIGHT, on a plane parallel to the horizon, above which the pole's altitude is so great that upon it the equinoctial line can scarcely be described. [HORAE A MERIDIE ET MEDIA NOCTE in plano, quod Horizonti æquidistat, supra quem poli altitudo tanta est, ut in eo linea æquinoctialis vix describi possit.]

1. A sundial has also sometimes to be described in a region where the pole's altitude above the horizon is so great that the equinoctial line can scarcely be drawn, because it falls too far from the place of the style. For instance, if the pole's altitude were 89 degrees 30 minutes, its Tangent will be about 1146 of those parts of which the style comprises ten—parts which must be counted downward from the place of the style so that the point of the equinoctial line may be found; and this can hardly be done on a small plane. We shall therefore take the Tangent of the complement of the pole's altitude—namely the Tangent of 0 degrees 30 minutes, which contains about $\frac{8}{100}$ of one tenth, as we said in the Prolegomena—for the reason that from the Tangent 8 nothing can be subtracted. And since $\frac{8}{100}$ of one tenth cannot be taken off with the compass, being $\frac{8}{1000}$ of the whole style (which is divided into only 10 parts, not into 1000), we shall use this device. Since the Tangent 10 is equal to the style, and answers to 45 degrees, we

shall take along the meridian line, from the place of the style downward, a portion equal to the style, and mark its end-point. Then to this Tangent 10 we shall add the Tangent of the aforesaid complement of the pole's altitude—namely $8/100$ of one tenth—and the combined number 10 $8/100$ [of one tenth]—that is, 10 and a little more—we shall transfer from the marked point toward the place of the style, so that a point is found a little above the place of the style, to serve as the centre of the sundial.

And next, the style having been set up at right angles to the meridian line, let the axis of the world be drawn from the centre thus found through the tip of the style; and to this axis let a perpendicular be raised from any point of the meridian remote below the place of the style. And through the assumed point let another perpendicular to the meridian be erected, to serve as a kind of equinoctial line—with respect to a second, smaller style, which is of course dropped perpendicular from the point of the axis onto which the first perpendicular falls upon the meridian. If, then, onto this equinoctial the Tangents of the hours from the little table of the scholium of chapter 1 be transferred, as we taught in chapter 2, No. 3, the straight lines sent out from the centre thus found through these points will indicate the hours from noon and from midnight.

2. After this, if you wish, let there be chosen from table 6 the pole's altitude on the left-hand side, whose horizon—represented by a straight line that makes right angles with the meridian line—can conveniently be described below the place of the style; whose point on the meridian line will be found thus. From the pole's altitude above the horizon of the sundial let the complement of that assumed pole-altitude be subtracted. For the Tangent of the remaining number, carried downward from the place of the style, will give the point on the meridian line through which the chosen horizon is to be drawn perpendicular to the meridian line. For example: if the horizon to be described has a pole-altitude of 50 degrees, we shall subtract its complement of 40 degrees from the given pole-altitude of 89 degrees 30 minutes, and the remaining number being 49 degrees 30 minutes, we shall transfer the Tangent $11 \frac{7}{10}$ downward from the place of the style. For a perpendicular drawn to the meridian line through that end-point will give the chosen horizon of 50 degrees.

3. This horizon having been described, let the hour-points be investigated upon it by means of the Tangents of the complements of the arcs of table 6, as was said in chapter 2, No. 6. For the straight lines sent out from the centre of the sundial, once found, through these points will give the hours from noon and from

midnight—which indeed you will delineate more accurately if you describe several horizons and seek the hour-points upon them.

(conclusion of Chapter VII)

But all these difficulties, which we have set forth in this chapter and the preceding one, have no place if the sundial is constructed by the method of chapter 24 or 25; for there we need neither the centre of the sundial nor the equinoctial line.

CHAPTER VIII. [CAPVT OCTAVVM.]

THE ARCS OF THE SIGNS IN A HORIZONTAL SUNDIAL of the hours from noon and from midnight. [ARCVS SIGNORVM IN HOROLOGIO HORIZontali horarum à merid. & med. noc.]

[Margin: The finding of the points on the hour-lines for the parallels of the Sun at the beginnings of the signs, by means of table 1. ("Inuētio pūctorū in lineis horarijs pro parallelis Solis in signorum initijs per 1. tabulam.")]

1. The arcs of the signs, which the tip of the shadow traces out when the Sun is at the beginnings of the signs, are either hyperbolas, or parabolas, or ellipses, on any plane of a sundial whatever—except on the equinoctial sundial, on which they are circles, as we demonstrated in Book 1 of the *Gnomonics*. These arcs, then, of whatever kind they be, we shall describe by a new and very easy method, on all sundials, from tables 1 and 2. And, in particular, on the horizontal sundial from table 1, thus. Of the portion BG of the meridian line, between the centre of the sundial and the equinoctial line, let there be taken an equal portion BZ in the opposite direction; and through Z let a perpendicular Za be drawn to the meridian line. Then let table 1 be applied, in which are taken the arcs answering to the beginnings of the signs for the several hours in the circle of hour 6—that is, in that line of the hours to which on the left is attached this heading [Circ. hor. 6.], or on the right-hand side the number [6.].

[Margin: With respect to which whole sine the Tangents are to be taken, to be transferred onto hour 6. ("Respectu cuius sinus totius sumēda sint Tangentes in horam 6 trāsferēda.")]

For if you transfer the required Tangents of the complements of those arcs (which you will find beneath the hours, opposite the signs) from the points G and Z onto the straight lines GK and Za, on either side, there will be found the points

through which straight lines drawn from the point B (where the equinoctial line intersects the meridian) will cut the corresponding hour-lines at the beginnings of the signs—that is, at the points through which the arcs of the signs are to be described. For drawing each single straight line there are generally three points: two, indeed, on the straight lines GK and Za, and a third, B; for it is for this use alone that the perpendicular Za has been drawn. And these straight lines, toward the centre G, will give on the hour-lines the points of the northern signs; but the southern signs below the equinoctial line. The Tangents, moreover, are to be taken from a straight line whose part bc is divided into 10 equal parts for the line of hour 6—which part bc (like the rest) must be equal to the interval GD in the sundial of chapter 2, as is clear from what we said at No. 4 of that same chapter.

[Margin: When one and the same straight line exhibits two points on opposite parallels, and when on one and the same. ("Quando eadem recta exhibeat duo puncta in oppositis parallelis, & quando in vno, & eodem.")]

For example: let there be found, at hours 3 and 9, the points of the tropic signs ☿ [Cancer] and ♑ [Capricorn]. Since opposite ☿ and ♑, at the foot of table 1, under hours 3 and 9 in the circle of hour 6, there is found an arc of 31 degrees 35 minutes, we shall take the Tangent of its complement, $16 \frac{3}{10}$ [numeral:?], and transfer it from G and from Z, on both sides, as far as d. For one straight line dBd will give the point ☿ at hour 3 and the point ♑ at hour 9; the other straight line dBd will present at hour 9 the point ☿, and at hour 3 the point ♑. Likewise, since under hour 6 in the circle of hour 6, opposite ☿ and ♑, there is found an arc of 23 degrees 30 minutes, if we transfer the Tangent of its complement, 23 (approximately), from G on both sides, hour 6 will be cut at the points ☿. And so for the rest. But since the straight line drawn out from the point B through the end of the Tangent $22 \frac{2}{10}$ [numeral:?],—which belongs to the complements of the arcs of hours 5 and 7—meets the line of hour 6 too far off from the line of hour 7 from noon and of hour 5 from midnight, so that those hours can scarcely be described on the sundial, we must take the Tangent of the complement of the arc of hours $5\frac{1}{2}$ and $6\frac{1}{2}$, namely $22 \frac{8}{10}$ [numeral:?], approximately; and by it there must be found the points of the parallel ☿ at hours $5\frac{1}{2}$ and $6\frac{1}{2}$ from noon, and likewise at hours $5\frac{1}{2}$ and $6\frac{1}{2}$ from midnight.

[Margin: When one and the same straight line exhibits two points on opposite parallels, and when on one and the same. ("Quando eadem recta exhibeat duo puncta in oppositis parallelis, & quando in vno, & eodem.")]

For the same straight line drawn out from B, through the ends of that Tangent, equally distant from the line of hour 6, cuts across in the parallel ☿, since neither

of them falls below the equinoctial line. For always one and the same straight line sent out from the point B will present either two points—one on the northern parallel and the other on the southern—if it cuts two hour-lines receding in different directions equally from the meridian; or two points on the same northern parallel, on two hour-lines equally distant from the line of hour 6, if it cuts them, as is manifest from table 1. For on it two such hours always have the same arc.

2. The demonstration. The straight lines sent out from the point B, through the ends of the Tangents on the lines GK and Za, represent the great circles of the sphere which, from the intersection of the Meridian with the Equator, are drawn through the hours from noon and from midnight in the parallels of the Sun, since the Tangents transferred from G and Z answer to the complements of the arcs of the circle of hour 6 which the said circles intercept with the Equator. Therefore they will necessarily cut the hour-lines at the points of the parallels of the Sun which answer to them on the sphere, where they are cut by the aforesaid circles.

Now let us also give an example concerning the points of the parallels ♉ [Taurus], ♍ [Virgo], ♏ [Scorpio], and ♐ [Pisces], to be found at hours 3 and 9. The arc, under those hours, opposite the said signs, contains in table 1 sixteen degrees 3 minutes. Its complement's Tangent, about 34 6/10 [numeral:?], transferred onto the straight lines GK and Za on either side, exhibits on each side the points e, from which the straight lines drawn through B cut hours 3 and 9 at the points f of the parallels ♉, ♍, ♏, and ♐. And the same reasoning holds for the rest.

[Margin: That it is enough for the points to be found for the half-parallels of the northern signs. ("Satis esse, ut puncta reperiātur pro semissibus parallelorū borealium.")]

3. Furthermore, it is enough if the aforesaid Tangents are carefully transferred only from G onto the line of hour 6, on one side—the right, say, or the left—and from Z onto the straight line aZa on the opposite side, and the points of the northern parallels only are found, on that side from which the Tangents were transferred onto the line of hour 6. For the points of the northern parallels can be found more accurately than those of the southern, since on the northern parallels the hour-lines are cut less obliquely by the straight lines proceeding from B than on the southern. For from these points we shall easily find similar points on the other side of the meridian line, and also below the equinoctial line, for the opposite southern parallels—which we did also in our *Gnomonics*, Book 2, Proposition 2. And so we shall proceed.

[Margin: The finding of the points for the parallels of the Sun on the meridian line. ("Inuētio pūctorū pro parallelis Solis in linea meridiana.")]

First, on the meridian line let the points of all the parallels be found, thus. Let the declinations of the northern parallels be subtracted from the pole's altitude above your horizon—that is, from the arc of the Meridian between the zenith of your place and the Equator; but let the declinations of the southern parallels be added to that same pole-altitude. For thus will be produced the several distances of the northern parallels from the zenith of the place toward the Equator, and the distances of the southern parallels will be combined from the same zenith. Then, if the Tangents of these distances be taken from the straight line whose segment EF, equal to the style, is divided into 10 parts, and be transferred from C (the place of the style, which answers to the zenith of the place) onto the meridian line, the points will be found through which the parallels are to be described. At an elevation of the pole of 42 degrees the said distances, together with their Tangents, stand as follows:

Horologiorum Nova Descriptio — Chapter VIII (continued)

(Global source pages 661–663 = printed pp. 25–27. Continuation of Cap. VIII, "The arcs of the signs on a horizontal dial for hours from noon and from midnight.")

Diagram. A large horizontal-sundial layout filling nearly the whole page, framed by a rectangular border. From a center (the base of the style, low in the left-center of the figure) a dense fan of straight hour-lines radiates outward; a circle/semicircle (the equinoctial or horizon circle) is drawn about the center, and numerous curved day-arcs (the parallels of the Sun / zodiac signs) cross the hour-lines. Around the border and along the rays are Roman-numeral hour-labels, Arabic degree/hour-arc numbers, zodiac glyphs marking the parallels, and many small cursive point-letters where the arcs cut the hour-lines. The figure is the horizontal dial referred to throughout the surrounding text (latitude 42°).

Readable labels (many small labels are worn or below resolution and are flagged):

- Roman hour-numerals on the rays: V, VI, VII, VIII (upper band); X, XI, XII (lower/right band); IX [?]

- Frame/interior capital letters: **A** (left border, mid-height); **P** (upper-left corner region); **L** (upper edge, center); **N** (right-center); **R** (right side, beside "XI"); **Q** (lower-left corner); **H** (lower area); **K** (lower-left, in "K e VI 23")
- Central cluster of capitals near the style/equinocial region: B, C, D, E, F, G [several individually uncertain — [?]]
- Arabic arc/hour numbers around the outer arcs: 3, 10, 11 [?], 12, 13, 14, 17, 18, 19, 21, 22, 23; and "1", "2", "3" along the lower right
- Half-hour tick labels marked "½" at several rays
- Zodiac glyphs marking the day-parallels (♊, ♋, ♌, ♍, ♎, ♏, ♐, ♑, ♒, ♓, ♈, ♉, ♊, ♋, ♌, ♍, ♎, ♏, ♐, ♑, ♒, ♓) scattered across the arcs — most too small/worn to confirm individually [?]
- Numerous lowercase cursive point-letters (a, b, c, d, e, f, g, i, k, l, m, n, o, p, q, r, u ...) at arc/hour-line intersections — individually unresolvable [?]

Distances of the Sun's parallels from the zenith of the place, together with their Tangents. At latitude 42 degrees. (*Distantiæ parallelorum Solis à vertice loci, vnà cum earum Tangentibus. Ad latitud. grad. 42.*)

Signa (Signs)	♊ (Ca nce r)	♊ ♌ (Gemini / Leo)	♈ ♉ (Taurus / Virgo)	♈ ♎ (Aries / Libra)	♏ ♐ (Pisces / Scorpio)	♑ ♒ (Aquariu s / Sagittari us)	♓ (Cap ricor n)
Distanc es (Distāti æ)	G. 18	G. 21	G. 30	G. 42	G. 53	G. 62	G. 65
Tangen ts (Tāgēte s)	334	399	589	900	1351	1896	2194

But if at any time the declination of a northern parallel is greater than the pole's altitude, then the pole's altitude must be subtracted from that declination, so that the remainder becomes the distance of that parallel from the zenith of the place toward the arctic pole. In that case the Tangent of its distance must be transferred from the place of the style toward the center of the dial, and so on.

The demonstration of this finding of the points depends on what was shown in the problem of chapter 1, number 8, since by means of these Tangents there are cut off from the meridian line straight lines equivalent to the aforesaid distances from the zenith of the place.

[Margin: When the declination of a northern parallel is greater than the pole's altitude — what is to be done. ("Quando declinatio borealis paralleli maior est altitudine poli, quid agendum.")]

4. When the distance of some southern parallel from the zenith of the place turns out to be 90 degrees or more, that parallel cannot be described on the dial, because the Tangent of its distance is infinite. For example, when the pole's altitude is 78 degrees, if the declination of ♊ [Capricorn] — namely 23 degrees 30 minutes — is added to it, the distance from the zenith comes to 91 degrees 30 minutes. Therefore on a horizontal dial the tropic of ♊ [Capricorn], opposite to the tropic of ♋ [Cancer], cannot be described there. And in this case, from the points of the northern parallel found on one side of the meridian line, we shall have to investigate only the points of the same parallel on the other side of the meridian line, as we shall explain in a moment.

[Margin: When a southern parallel cannot be described on a horizontal dial. ("Quando parallelus australis in horologio horizontali describi non possit.")]

5. When the points of all the parallels have been found on the meridian line, as was taught in number 3, and when from the points of any northern parallel found on the hour-lines on one side only of the meridian line — as we taught in number 1 — we must seek the points of the same parallel on the other side of the meridian line as well, and (whenever that parallel can be described on the dial) the points of the opposite southern parallel on both sides of the meridian line, we shall proceed by this route.

(p. 26 — running header: "CAPVT OCTAVVM.")

Onto the portion hG of the meridian line, between the point of the tropic ♋ [Cancer] (for from its points, found on one side of the meridian line, the rest are now to be sought) and the center of the dial, let there be taken an equal portion ig, measured from the point of the tropic ♊ [Capricorn]. Take with the compass the interval of any point found on the tropic ♋, and from G and g, with that same interval, describe three arcs: one from G toward the other side of the meridian line, and two from g toward each side. Then, from i, the point of ♊, at the interval ik, describe an arc cutting the upper one at l; and with the same interval, from h, the point of ♋, describe two others cutting the lower ones also at l. Four points

will now have been found — two belonging to the tropic ☿ and two to the tropic ♄. In the same way we shall proceed for the remaining points of the tropic ☿, even if they do not fall on the hour-lines but lie on the tropic itself as it is described between two of the hours. This you see done at the point ☿, m: from the interval Gm, an arc is described from G toward the other side, and from g two arcs are drawn toward the other three points n. For by the interval Gm the arc described from G, and the arc described from i at the interval im, cut it at n; while the lower ones are also cut at n by two arcs described from h with the same interval, and so on.

[Margin: By what method, from the points of the northern parallels found on one side only of the meridian line, the points of the same parallel are found on the other side, together with the points of the opposite parallels. ("Qua ratio ve ex punctis parallelorū borealiū ex vna tantū parte meridianæ lineæ inuentis reperiātur pūcta eiusdē ex altera parte, vna cum pūctis oppositorū parallelorū.")]

You may also, if you wish, take in place of the points G, g two others equally distant from the points h, i — nearer, however, or farther, as seems convenient — so that the arcs described intersect one another less obliquely, and the points may thereby be found more precisely.

The same procedure is to be kept for the other parallels; only, the points G, g are not to be taken equally distant from the points h, i (the points of ☿ and ♄), but from the points of the other opposite parallels. For example, if part of the parallel of ♈ and ♍ [Taurus and Virgo] had been described through the point o, then, equal to any straight line oq, one would have to take pr, from the point p of the opposite parallel ♎ and ♏ [Scorpio and Pisces], and so on.

This finding of the points on the parallels, from the points of a single parallel described on one side only of the meridian line, was demonstrated by us in book 2 of the Gnomonics, proposition 2. The whole rests on the Conic elements of Apollonius [Apollonius of Perga], since the arcs of opposite parallels are altogether equal to one another, and the arc of each parallel has exactly the same position in the hours after noon as it has in the hours before noon.

[Margin: An easier finding of the points of any parallel. ("Inuētio facilior punctorū cuiusuis paralleli.")]

6. Now if, once the Tangents have been transferred onto the line of hour 6 — from G toward the left, and from Z onto the straight line aZa toward the right — the points of the opposite parallels are found toward the same sides to which the Tangents were transferred, then on the horizontal dial those same points are

carried over easily to the other sides. To do this, transfer the hour-intervals between the equinoctial line and the points of each parallel from the equinoctial line onto the corresponding hour-lines on the other side — that is, onto those hour-lines that recede from the meridian line by an equal amount. For example, since hours 4 and 8 are equally distant from the meridian line, if you transfer the interval of hour 4 (between the equinoctial line and the point $\mathfrak{S}$) onto hour 8, starting from the equinoctial line, you will find the point $\mathfrak{S}$ on hour 8. Likewise, if you carry the interval of hour 2 (between the equinoctial line and the point $\mathfrak{V}$) from the equinoctial line onto hour 10 — which is as far from the meridian line as hour 2 — you will find the point $\mathfrak{V}$ on hour 10, and so on. By this method the left and right parts of any parallel come out exactly alike, which must be attended to with the greatest care in describing the parallels.

[Margin: How the fore-noon and after-noon parts of any parallel come out exactly alike. ("Quo pacto pars ante-meridiana & pomeridiana cuiusuis paralleli simillimæ euadant.")]

But an opposite parallel will be described from any given parallel more precisely by the following method. Suppose the tropic $\mathfrak{V}$ is to be described from the tropic $\mathfrak{S}$. Having found the point i on the meridian, through which the parallel $\mathfrak{V}$ must pass (as was said in number 3), bisect at Aa the segment hi of the meridian lying between the two points h, i of the opposite parallels. For if from any point of the tropic $\mathfrak{S}$ — say from Ae , the point of hour 3 — a straight line Aa, Ac is extended through Aa , and if an interval Aa, Ac is taken equal to the interval Aa, Ae , then the point Ac will lie on the opposite parallel, as Apollonius demonstrated in book 1, proposition 30. And the same procedure holds for the remaining points. This route is very easy and accurate, and it applies in all dials, provided that in them the style-line is taken for the meridian. But so that not so many hidden lines need be drawn, you will do well if first, from Aa , at the interval Aa, Ae , on the opposite side (which is to be judged by eye), a small arc is described, which is then cut by a hidden straight line — namely by laying a ruler on the points Ac, Aa . This is to be understood of the other points of the northern parallels as well.

[Margin: An easy and accurate description of any parallel from its opposite. ("Descriptio cuiusuis paralleli ex suo opposito facilis, & accurata.")]

7. In another way — and indeed a very convenient one — the arcs of the signs can be described by means of the points on the meridian line found from the same table 1, if through those points lines are drawn parallel to the equinoctial line, or perpendicular to the meridian line. For each of these will cut two hours at a time, at the points of the same parallel. We shall carry the matter out by the

following device. From table 1 we shall take the arcs standing opposite each sign under the hours, in the circle of hour 12 — that is, in the row of hours to which this heading is prefixed, [Circ. hor. 12.], or which has this number, [12.], written at its right-hand side. With these same arcs we shall cut off, from the meridian line, portions that begin from the equinoctial line, that is, from the point B. For when perpendiculars to the meridian line — or lines equidistant from the equinoctial line — are drawn through the end-points of these portions, each will cut two hours (namely the two whose arcs have been cut off) at the points of the parallel in question. We shall cut off the said arcs as follows. For the northern parallels, let each arc be subtracted from the pole's altitude above the Horizon, if it can be subtracted, and let the Tangent of the remaining number — taken from the straight line whose segment EF, equal to the style, is divided into 10 particles — be transferred by compass from C, the place of the style, toward the equinoctial line. Or else let the pole's altitude be subtracted from the table-1 arc, if this can be done, and let the Tangent of the remaining number be transferred from the same place of the style toward the center of the dial. But if some arc is found equal to the pole's altitude, then the place of the style C itself will be the end of the table-1 arc. When, however, two hours are more than 6 hours distant from noon, their arc must be reckoned from the Equator below the Horizon; for the arc of the Meridian from the Equator above the Horizon up to the great circle drawn through those hours from the pole of the Meridian is greater than a quadrant, as is clear from the material sphere. Therefore to the complement of that arc — namely to the arc of the Meridian between the arctic pole and the end of the proposed arc — must be added the complement of the pole's altitude — namely the arc of the Meridian between the zenith and the arctic pole — so that the arc of the Meridian from the zenith up to the end of the proposed arc is made up; and the Tangent of the total so formed must be transferred from the place of the style beyond the center of the dial. Finally, for the southern parallels, each table-1 arc must be added to the pole's altitude, and the Tangent of the total so formed must be transferred from C, the place of the style, beyond the equinoctial line.

[Margin: How the arcs of table 1 are transferred onto the meridian line by means of the Tangents. ("Quo pacto arcus 1. tabulæ in meridianā lineam per Tangentes trāsferantur.")]

By way of example: In the circle of hour 12, under hours 1 and 11, opposite ☾ and ♊, there is found an arc of 24 degrees 14 minutes; and taking this away

(p. 27 — running header: "NOVÆ DESCRIPT.")

from the pole's altitude of 42 degrees, 17 degrees 46 minutes remain. Their Tangent, $3 \frac{3}{10}$ [numeral: ?], transferred from C downward, gives a point a little above the point $\mathfrak{S}$, at h, through which a perpendicular drawn to the meridian line will cut hours 1 and 11 at the points $\mathfrak{S}$. But the same arc, 24 degrees 14 minutes, added to the pole's altitude of 42 degrees, makes 66 degrees 14 minutes; and their Tangent, $22 \frac{7}{10}$, transferred from C downward, presents a point below the point $\mathfrak{V}$, at i, through which a perpendicular drawn to the meridian line will cut hours 1 and 11 at the points $\mathfrak{V}$.

So too, the arc of hours 4 and 8 contains 41 degrees 1 minute; and this, subtracted from the pole's altitude of 42 degrees, leaves 0 degrees 59 minutes. The Tangent of this arc, $17 \frac{1}{100}$ [numeral: ?] — that is, nearly one fifth [?] of one of the particles into which EF is divided — presents a point a little below C, the place of the style; and this is better taken by estimation than by compass. But the same arc, 41 degrees 1 minute, added to the pole's altitude of 42 degrees, makes 83 degrees 1 minute, whose Tangent, $81 \frac{5}{10}$ [numeral: ?], will give, below the equinoctial line (if the plane of the dial is large enough), a point through which a perpendicular drawn to the meridian line will give, on hours 4 and 8, the points $\mathfrak{V}$.

Again, the arc of hours 5 and 7 comprises 59 degrees 14 minutes; and if the pole's altitude of 42 degrees is taken from it, 17 degrees 14 minutes remain, whose Tangent, $3 \frac{3}{10}$, gives a point through which a perpendicular drawn to the meridian line will cut hours 5 and 7 at the points $\mathfrak{S}$. But if one of these hours is hour 5 from midnight and the other hour 7 from noon — so that they are more than 6 hours from noon — then the complement of the said arc, namely 30 degrees 46 minutes, must be added to the complement of the pole's altitude, that is, to 48 degrees. For the Tangent of the number so formed, 78 degrees 46 minutes — $50 \frac{7}{10}$ [numeral: ?] and a little more — transferred from C beyond the center G, will give (if the plane of the dial is large enough) a point through which a perpendicular drawn to the meridian line will present, on hours 5 and 7, the points $\mathfrak{S}$. But the same arc, 59 degrees 14 minutes, added to the pole's altitude of 42 degrees, makes an arc of 101 degrees 14 minutes, greater than a quadrant. Therefore its Tangent is infinite, and for that reason the points $\mathfrak{V}$ cannot be found on hours 5 and 7.

Furthermore, the arc of hours $5\frac{1}{2}$ and $6\frac{1}{2}$ comprehends 73 degrees 17 minutes; and when the pole's altitude of 42 degrees is taken away from it, 31 degrees 17 minutes are left, whose Tangent, 6, gives a point through which a perpendicular drawn to the meridian line will cut hour $5\frac{1}{2}$ from noon and hour $6\frac{1}{2}$ from midnight at the points $\mathfrak{S}$. But if the complement of that same arc of 73 degrees 17

minutes — 16 degrees 43 minutes — is added to the complement of the pole's altitude, 48 degrees, the number 64 degrees 43 minutes results, whose Tangent, $21 \frac{1}{10}$ [numeral:?] and a little more, will show a point beyond the center; and if a perpendicular is drawn from it to the meridian line, hours $5\frac{1}{2}$ from midnight and $6\frac{1}{2}$ from noon will be cut at the points $\mathfrak{S}$. Here too the points of the tropic $\mathfrak{V}$ cannot be found, because the arc of 73 degrees 17 minutes, together with the pole's altitude of 42 degrees, makes an arc of 115 degrees 17 minutes, greater than a quadrant. Exactly the same reasoning holds for the other parallels, if the arcs in table 4 opposite the other signs are used.

By this method the points of the parallel $\mathfrak{S}$ cannot be found on the line of hour 6: for if from its arc of 90 degrees the pole's altitude of 42 degrees is taken, 48 degrees remain, whose Tangent, $11 \frac{1}{10}$, gives the point G, through which the perpendicular to the meridian line does not cut the line of hour 6 but coincides with it.

For the rest, the aforesaid perpendiculars to the meridian line will be drawn easily if, once lines parallel to the meridian line have been drawn on both sides, these same Tangents are transferred onto them from the straight line CD, which is drawn through the place of the style at right angles to the meridian line. For thus, for each perpendicular to be drawn, we shall have three points — one on the meridian line, and two on the two lines equidistant from the meridian line.

8. The demonstration of this way of finding the points for the Sun's parallels on the hour-lines is evident. For the plane of the horizontal dial, together with the Equator and with the great circles drawn through the common sections of the Equator with the Horizon (or with the circle of hour 6) and through the hours in the Sun's parallels, produces parallel common sections, as we demonstrated in proposition 18 of book 1 of the Gnomonics. The aforesaid perpendiculars, being equidistant from the equinoctial line, will therefore represent those great circles; and so on the hour-lines they will mark the points through which those same circles are drawn in the Sun's parallels.

Moreover, by means of the same Tangents found in number 7, we shall very conveniently find these same points for the arcs of the signs on the hour-lines, even without drawing those lines parallel to the equinoctial. This is the way: for any given hour and sign, take with the compass the Tangent from the straight line whose segment EF is divided into 10 particles, starting at the beginning of the hour, on this condition — that with the other foot an arc is described touching the line CD (extended on both sides) but not cutting it; this will give the point sought on the hour-line. And no appreciable error will be admitted in this, if careful

diligence is used. If we use this method, we shall not need that immense table 19, computed for many elevations of the pole; rather, for all altitudes of the pole, table 1 will suffice us as a universal table — and not the whole of it, but only its middle row, to which this heading is prefixed, (Circ. horæ 12.). You see clearly, however, that this method is less safe on those hour-lines that are steeply inclined to the equinoctial and lie close to hour 6, because there the straight line CD can be perceptibly touched by several points of those lines through the arcs described at the intervals of the Tangents. On those lines, therefore, the points of the parallels are to be found by rules 1 and 2 of the following scholium, with the help of points already found on the other lines; or by the points found on the line of hour 6, as we taught in number 1 of this chapter; or by the means that will be given in number 10, and so on.

For the rest, we shall transfer these same Tangents onto the hour-lines to find the points of the parallels, even without a compass, in this way. On some thicker paper, such as royal or imperial stock, let the divisions of the line EF be made, as we said at the beginning of chapter 2. Then let the paper be cut so that the side bearing the divisions occupies its edge and serves as a kind of set-square; and let the same divisions be transferred onto another line of the same paper, parallel to the said edge, which (once the superfluous parts are trimmed away) is to occupy the other edge, in such a way that the straight lines joining the numbers 10–10, 20–20, 40–40, and so on are all perpendicular to the said edges of the set-square. For if the perpendicular passing through the numbers 10, or 20, or 30, and so on (as the size of the Tangent requires) is laid against the line CD drawn through the place of the style, and is moved to and fro until the end of the Tangent reaches the proposed hour, that outermost point must be marked as the point of the parallel.

9. But so that you may see the immense usefulness of table 1 — and thereby the marvellous benefit of the problem we demonstrated in chapter 1 — I shall prescribe a method by which the points of any parallel can be investigated, on any hour-line you please, by the arcs of that table reckoned (if you wish) from the line's intersection with the equinoctial line. Suppose, for example, that the point of the parallel γ and η [Taurus and Virgo] is to be found on the line of hour 3, by means of the arc of table 1 reckoned on the line of hour 4, and therefore by a straight line from hour 10 of the Equator (which is six hours distant from hour 4), through the end of that same

[Margin: The application of table 1 to finding the points of the parallels. ("Vsus 1. tabulæ immissus ad puncta parallelorum inuenienda.")]

— through the end of that same arc; and let the line drawn out at hour 4 represent the greatest circle drawn from hour 10 on the Equator through hour 3 in the parallel of γ [Taurus]. To bring this about, from the line of hour 4 (beginning at the equinoctial line), toward the center G, we must cut off an arc of $11^{\circ}54'$ — the arc that is found, opposite the sign γ [Taurus], under hours 3 and 5, in table 1, in the circle of hour 4 — by that same technique taught in the problem of chapter 1. From C, the place of the style, draw a perpendicular C θ to the line of hour 4, and let θu be equal to the interval CX (the line θX being taken equal to the style). This will happen if, with a circle described from G through C and cutting the line of hour 4 at f, an arc f t is taken equal to the arc Cf, and the arc described from G through D — the tip of the style — cuts the produced line Ct at u. For Ct will then be perpendicular to hour 4, and θu will be equal to the interval CX, as was demonstrated in the problem of chapter 1, number 5.

Next, once a straight line has been drawn from u through the intersection of hour 4 with the equinoctial line, describe from u an arc $\beta \gamma$ cutting that line at α . With the arc $\alpha \beta$ taken as $11^{\circ}54'$, and a straight line u β drawn cutting hour 4 at δ , the segment from the equinoctial line up to δ will be $11^{\circ}54'$, as we showed in number 2 of the same problem. Therefore a straight line drawn from hour 10 on the equinoctial line through δ will cut hour 3 at f, the point of γ [Taurus] and ϖ [Virgo]; and when produced it will also cut hour 5 at the point of the same parallel, since hours 3 and 5 have the same arc in table 1. But if the arc $\gamma \alpha$ is likewise taken as $11^{\circ}54'$, the line u γ will cut off, on the same hour 4, at ϵ , the aforesaid arc toward the southern side. Therefore a straight line sent out from hour 10 on the equinoctial line through ϵ will cut hour 3 at f, the point of the parallel of $\mathfrak{M}$ [Scorpio] and)([Pisces], which is opposite to the northern one. The same reasoning holds for the rest.

[Margin: What must be known in order that, by means of Tangents, the points of the parallels can be found in any hour, and by what method it must be done. ("Quid cognoscendū sit, vt per Tāgentes in qualibet hora inueniri possint puncta parallelorū, & qua ratione id fieri debeat.")]

If it were established to how many degrees the segment of hour 4 between θ and the equinoctial line answers, the points δ and ϵ could be found by means of Tangents. For if from this arc the arc of $11^{\circ}54'$ were subtracted, then the Tangent of the remaining number — reckoned with respect to the whole sine u θ (or CX) divided into 10 parts — transferred from θ , would give the point δ . But if to the same arc, terminating at θ , that same arc of $11^{\circ}54'$ were added, the Tangent of the total so formed would indicate to us the point ϵ . How one can find out to how

many degrees the segment of hour 4 between θ and the equinoctial line corresponds, we shall teach how to compute at the end of the little book, in problem 10. Geometrically, however, we shall learn the same thing if the arc $\alpha \beta$ is produced until it cuts the produced line $u C$ at μ . For the degrees contained in the arc $\alpha \mu$ indicate the degrees contained in the segment of hour 4 between θ and the equinoctial line, as we showed in the problem of chapter 1, number 2.

When the parallels of all the signs are wanted, it will be worthwhile to compute, by problem 10 at the end of the little book, the degrees of the segment of each hour-line lying between the equinoctial and the perpendicular drawn from the place of the style, so that the points of all the parallels may be found in any hour by means of Tangents.

Indeed, we shall achieve the same result even if we do not know to how many degrees the segment between the perpendicular and the center G corresponds. For suppose the point of both $\mathfrak{C}$ [Cancer] and $\mathfrak{V}$ [Capricorn] is to be found on the line of hour 3. With Ca drawn perpendicular to the line of hour 3, as far as the circle described through D — so that the center of the circle required in chapter 1 may be had — let the straight line $b c$ be drawn through C , the intersection of hour 3 with the equinoctial, and let a perpendicular $d e$ be raised to it through C . For if from c we transfer onto this perpendicular the tangent of the declination of $\mathfrak{C}$ [Cancer] and $\mathfrak{V}$ [Capricorn], as far as d and e , reckoned with respect to the whole sine $b c$, the lines $b d$ and $b e$ will cut off from hour 3 — beginning at c — the $23^{\circ}30'$ of the declination of $\mathfrak{C}$ [Cancer] and $\mathfrak{V}$ [Capricorn], as we demonstrated in chapter 1; and consequently these same lines $b d$ and $b e$ will cut the hour at the points of $\mathfrak{C}$ [Cancer] and $\mathfrak{V}$ [Capricorn] in the same way. If, on $b e$ produced, $a f$ is taken equal to $a b$, and a perpendicular is raised at any point to the drawn line $f c$, and onto this, on both sides, the tangents of the declinations are transferred (reckoned with respect to the whole sine between f and the said perpendicular), the points will be found through which, if lines are drawn from f , the same hour 3 will be cut at the points of the parallels.

Furthermore, once the points on the meridian for the hours have been found — as we said at the beginning of number 7 — we shall find the points of the Sun's parallels conveniently enough, in any hour, by means of secants, in this way. With respect to a whole sine equal to the segment of the meridian between the center G and the point of the proposed hour, we shall transfer the secant of the angle which the given hour makes with the meridian at the center (this angle being always the complement of the arc of table 6), from the center onto the given hour. For the outermost point will belong to the sought parallel, since the

portion of the hour-line between the center and the parallel is the secant of the said angle reckoned with respect to the said whole sine, as is clear. But if the whole sine is taken equal to the segment of the meridian between B and the same point of the given hour, then the secant of the aforesaid angle must be transferred from the point of the hour on the equinoctial, toward the side to which the said point inclines. For that portion of the hour-line is the secant of that angle with respect to the said whole sine — as is evident if, from the point of the parallel in the given hour, a line parallel to the meridian is dropped. For this line will be equal to the said whole sine, ^a and it will make with the hour-line an angle equal to the one which the meridian makes with the same hour-line.

[Margin: ^a 29.primi (Euclid, *Elements* I.29).]

And this method is easy if an instrument of 100 parts is at hand, on account of the various whole sines. And since several points are to be found on the meridian, so that no confusion may arise among them, it is expedient that on some separate sheet of paper several lines be drawn onto which the points B, G, C are transferred from the meridian. For on each of these a point can be found for any hour, so that the whole sine may serve as the secant. Likewise, for hours 1 & 11, 2 & 10, and 3 & 9, the points of the parallels will also be found exactly, if from the points found on the meridian perpendiculars to that same meridian are drawn with the help of some set-square. For these do not cut the said hours too obliquely; but in the other hours it will be safer to use secants, on account of the oblique intersections of the hours with those perpendiculars. Finally, for hour 6, one must transfer from the center G the tangent of the complement of the declination of the proposed northern parallel, reckoned with respect to the whole sine Gd.

Indeed, it is not necessary that the points be found on the meridian line, or on any other line, in order that whole sines may serve as secants; rather it is enough if, by table 1 and the altitude of the pole, the arcs are investigated whose tangents must be transferred from the place of the style in order to find the said points. For these tangents — in the northern signs subtracted from the tangent of the altitude of the pole, or added to it (according as the arcs in table 1 are subtracted from the altitude of the pole or added to it) — will leave or produce tangents which, transferred onto some line, will from any point give the whole sines to serve as the secants found by table 1 and the altitude of the pole. In the southern signs, from the tangents found by table 1 and the altitude of the pole (as we handed down in number 7), the tangent of the altitude of the pole must always be subtracted, so that the remaining tangents are produced which offer the whole

sines to serve as secants. And the secants, reckoned with respect to the whole sines found in this way, must always be transferred from the equator onto the hour-lines. This method (unless the frequent change of the whole sine seems troublesome to someone — a trouble which, however, the instrument of 100 parts removes, as we said) appears to be most elegant and most reliable.

10. VERY accurately (and this is the method that satisfies me most of all in every sundial), the points of the parallels on the hour-lines will be found from general table 2, by straight lines drawn on the equinoctial line from hour 5 in the afternoon (or 7 after midnight); or from hour 4 in the afternoon (or 8 after midnight); or from hour 3 in the afternoon (or 9 after midnight); or from hour 2 in the afternoon (or 10 after midnight) — through certain definite points of the meridian line. For example, suppose the point is to be found on the meridian line through which the straight line drawn from hour 7 after midnight cuts hour 1 at the point of ♋ [Cancer]. In table 2, under the column of hours to which is prefixed the heading [Ex H.7.à med.noc.] (From hour 7 after midnight), opposite ♋ [Cancer], hour 1 answers to an arc of $22^{\circ}47'$; and its outermost point on the meridian will be found thus. Taking that arc away from the altitude of the pole, 42° , there remains an arc of $19^{\circ}13'$, whose Tangent, $3 \frac{5}{10}$ nearly (reckoned with respect to a whole sine equal to the style), transferred from C, the place of the style, toward the equinoctial, will give the point terminating the aforesaid arc — the point through which a straight line issuing from hour 7 will cut hour 1 at the point of ♋ [Cancer]. But if the same arc of $22^{\circ}47'$ is added to the altitude of the pole, 42° , there will result an arc of $64^{\circ}47'$, whose Tangent, $21 \frac{2}{10}$ and a little more, transferred from C, the place of the style, will exhibit the point through which a straight line sent out from hour 7 after midnight will cut the same hour 1 at the point of ♏ [Capricorn].

[Margin: The accurate method of finding the points for the parallels, from table 2. ("Accuratā ratio inueniendorum punctorum pro parallelis ex 2.tab.")]

Again, suppose the points of ♋ [Cancer] are to be investigated on hours 5 and 3, from hour 4 in the afternoon. On the line [Ex H.4.à mer.] (From hour 4 in the afternoon), under hours 5 and 3, in the row of ♋ [Cancer] and ♏ [Capricorn], the arc $56^{\circ}52'$ answers. If from this the altitude of the pole, 42° , is taken away, there remains an arc of $14^{\circ}52'$, whose Tangent, $2 \frac{6}{10}$ and a little more, transferred from C, the place of the style, toward the center G, will present the point through which a straight line drawn out from hour 4 in the afternoon will cut hours 5 and 3 at the points of ♋ [Cancer]. But if the same arc of $56^{\circ}52'$ is added to the altitude of the pole, there is produced an arc of $98^{\circ}52'$, which cannot be transferred

downward from C so that the points of the same hours 5 and 3 in ♊ [Capricorn] may be found, since its Tangent is infinite. From hour 7 after midnight, however, the point of ♊ [Capricorn] can be found on hour 3. For its arc contains $25^{\circ}52'$, which, added to the altitude of the pole, 42° , makes $67^{\circ}52'$, whose Tangent, $24\frac{5}{10}$ and a little more, transferred from C, will give the point on the meridian through which a straight line sent out from hour 7 after midnight will cut hours 3 and 11 at the points of ♊ [Capricorn]. And so with the rest.

You see, therefore, that the arcs of table 2 are to be cut off from the meridian line, just as the arcs of table 1 were cut off from that same line in number 7. And this method, as I have said, satisfies me most of all — because on the meridian the points are always found in the same way, and one can choose that hour on the equinoctial which will be judged more suitable, so that from it may be drawn the straight line through the point found on the meridian, cutting the given hour at the point of the proposed parallel. We have omitted hour 1 in the afternoon and after midnight in table 2, because the meridian lines are too close together, and therefore the straight lines could be drawn through the points of the meridian from them less accurately — especially since the eight hours from which the straight lines can be drawn are sufficient.

[Margin: Wherein lies the excellence of this method of finding the points. ("In quo consistat præstantia huius rationis inueniendorum punctorū.")]

Now if of the segment of the equinoctial between the assumed hour and the meridian line you take, on the other side, one equal part, or two, or three, or more, in such a way that the straight line between the last point and the assumed hour is a multiple of the aforesaid segment; and if through the last point a perpendicular is drawn to the equinoctial: then onto this perpendicular each Tangent must be transferred which, on the meridian, begins from the equinoctial — twice, or three times, or four times, and so on, according as the straight line between the assumed hour and that last point is double, triple, and so on, of the segment between the assumed hour and the meridian line. For in this way the straight line from the assumed hour, through the point of the said perpendicular, will be drawn more exactly, since it is drawn from a remoter point.

11. WE shall also, and expeditiously, describe those same arcs by means of table 3, which is likewise general. For if, through h, the point of ♋ [Cancer], or through i, the point of ♊ [Capricorn], you draw on the meridian a perpendicular to the meridian (in our sundial, i λ has been drawn through the point of ♊ [Capricorn]), it will represent the greatest circle which, on the sphere, is drawn through the common sections of the Equator and Horizon and through the point of ♋

[Cancer], or of ♑ [Capricorn], on the meridian circle. If onto this straight line you transfer, from h or i, the arcs described in table 3 opposite the hours under the proposed sign (provided that on some straight line you take a number of parts equal to the interval h D or i D — according as the perpendicular to the meridian is drawn through h or through i — and that you divide the first of them, ξ π, into 10 equal parts), you will find the points through which, if straight lines are sent out from B, the hour-lines will be cut at the points of the proposed parallel, just as was said about the line of hour 6. Example: In table 3, opposite hours 3 and 9, under ♋ [Cancer] and ♑ [Capricorn], is found an arc of 32°58', whose Tangent, 6 5/10 nearly (reckoned with respect to the whole sine ξ π of 10 parts), from the point i will give the point λ on both sides. Therefore the straight line B λ will cut hours 3 and 9 at the points of ♋ [Cancer] and ♑ [Capricorn].

[Margin: Another way of finding the points on the parallels, from table 3. ("Inuentio alia punctorum in parallelis ex 3.tabula.")]

Again, in the same table 3, opposite the same hours 3 and 9, under the signs ♉ [Taurus], ♍ [Virgo], ♏ [Scorpio], and ♐ [Pisces], there stands an arc of 54°11', whose Tangent, 13 8/10 and a little more, will give on the straight line i λ the point ρ on both sides. Therefore the straight line B ρ will cut hours 3 and 9 at the points f, of the parallel of ♉ [Taurus], ♍ [Virgo], ♏ [Scorpio], and ♐ [Pisces]. And so with the rest.

Here too, so that for the individual hours we may have three points, another perpendicular to the meridian must be drawn, distant from the point B on the other side of the equinoctial line by as much as the perpendicular drawn through i, or through h, stands from that same point.

12. FINALLY, if from table 9 (if you wish) you take the altitudes of the hours above the Horizon, the Tangents of the complements of those altitudes (reckoned with respect to a whole sine DC equal to the style), transferred from the place of the style, will cut the hours at the points of the parallels.

The reason is that the straight line extended from the place of the style up to the point of any parallel, in any hour, is the Tangent of the complement of the Sun's altitude in that hour above the Horizon. For let the circle of altitude be A B C D, whose center is E; let B H be its common section with the plane of the sundial; and A D the common section of the same circle with the greatest circle to which the sundial is parallel; let the style be E F, and the Sun's altitude A G. Let the Sun's ray G E, passing through the center of the world, meet the plane of the sundial at H, the common section of that section with the sundial. ^a Since, therefore, the angle D E H

is equal to the angle $A E G$ of the Sun's altitude, $F E H$ will be the angle of the complement of the altitude, whose Tangent is $F H$ reckoned with respect to the whole sine $E F$; and the same reasoning holds for the rest.

[Margin: Another way of finding the points to be marked on the parallels, by means of the Tangents of the complements of the Sun's altitudes, from table 9. ("Inuentio alia punctorum in parallelis delineandis per Tangētes complementorum altitudinū Solis ex 9.tabula.")]

[Margin: ^a 15.primi (Euclid, *Elements* I.15).]

Diagram. A circle (the circle of altitude) inscribed within a rectangular frame at the lower right of the page. The circle is $A B C D$ with center E . Along the bottom runs the horizontal line $B H$ (the common section of the altitude-circle with the plane of the sundial), which extends to the right well beyond the circle to reach H . Through the center E runs the horizontal diameter $A D$ (the common section of the circle with the great circle to which the sundial is parallel). From the center E a vertical line $E F$ drops to F on the baseline (the style). A line runs from G , on the upper-left of the circumference, through the center E and continues down to the right to H on the baseline (the Sun's ray $G E$, produced to H).

- A (left, on the horizontal diameter): endpoint of the diameter $A D$
- B (lower left, on circumference and baseline): endpoint of the baseline $B H$
- C (lower right, on circumference): on the baseline
- D (right, on the horizontal diameter): endpoint of the diameter $A D$
- E (center): center of the altitude-circle
- F (bottom center, on baseline): foot of the perpendicular style $E F$
- G (upper left, on circumference): point of the Sun's altitude arc $A G$; the Sun's ray $G E$ begins here
- H (far right, on baseline, outside the circle): where the produced Sun's ray meets the plane of the sundial

CAPVT OCTAVVM.

SCHOLIVM.

[Margin: The same straight line drawn from a point of the Equator cuts two hours at points of the same parallel, or of two opposite parallels. ("Eadē recta ex puncto Æquatoris educta secat duas horas in punctis eiusdē paralleli, vel duorum oppositorū.")]

1. From general tables 1, 2, and 3, it is gathered that any straight line drawn from any hour-point on the Equator cuts, at points of the same parallel, two hours equally distant from the hour that is 6 hours from that hour-point of the Equator; or else it cuts, at points of opposite parallels, two hours receding equally on either side from that same hour-point of the Equator. For the individual arcs of the aforesaid tables answer to two hours each, as will be plain from the way in which they are computed. Excepted, however, are the first and last hours in tables 1 and 2, which stand alone. Thus in table 1 you see that the straight line drawn from B, the point of hour 12 on the Equator, through the point of ☿ [Cancer] at hour 5½ in the afternoon, also cuts hour 6½ in the afternoon at the point of ☿ [Cancer] — these hours 5½ and 6½ being equally distant from hour 6, which is 6 hours from hour 12 assumed on the Equator. By the same reasoning, the straight line sent out from B, through the point of ☿ [Cancer] at hour 5, will cut hour 7 at the point of the same ☿ [Cancer], if the size of the sundial allows it. So too you discern that the straight line drawn from I, the point of hour 7 on the Equator, through the point of ☿ [Cancer] at hour 4, also passes through the point of the same ☿ [Cancer] at hour 10; because hours 4 and 10 are equally distant from hour 1, which is 6 hours from hour 7. In just the same way, finally, you see that the straight line sent out from B, through the point of ☿ [Cancer] at hour 3, cuts hour 9 at the point of ♊ [Capricorn], because hours 3 and 9 recede from hour 12 by an equal interval in opposite directions.

2. But although, as we have said, this is clearly gathered from tables 1, 2, and 3, nevertheless, since this matter has an outstanding and notable use both in finding the points of the parallels and in testing them, I am pleased to demonstrate it geometrically as well by the following theorem.

THEOREMA.

ANY greatest circle drawn through any hour-point assumed on the Equator cuts any parallel at two points of hours equally distant from the hour-circle that is 6 hours from the assumed point; and likewise it cuts any two opposite parallels at points of hours receding equally, in opposite directions, from the assumed point of the Equator.

Let A be the pole of the world; let B C be the Equator; let D E and I K be any two opposite and equal parallels; let A K be any hour-circle cutting the Equator at C, through which let the greatest circle C F G be drawn, cutting the parallel D E at F and G, and the parallel I K at I. Finally, let A B be the hour-circle distant 6 hours from the hour-circle A C — that is, by a quadrant B C — cutting the parallel D E at D, and the greatest circle C G at H. I say that the points F and G are equally distant

from the point D on the parallel D E; likewise that the points F and I are equally distant from the points E and K — that is, that both the arcs D F and D G, and the arcs E F and I K, are equal to one another. For since the greatest circles A B and A C pass through A, the pole of the Equator B C, the Equator B C will in turn pass through the poles of the circles A B and A C, by the scholium to prop. 15, bk. 1 of Theodosius. And since, by the corollary of prop. 16 of the same bk. 1 of Theodosius, the pole of a greatest circle is distant a quadrant from that very circle, and B C is posited as a quadrant, C will be the pole of the greatest circle A B. Since, therefore, the greatest circle C G passes through C, the pole of the greatest circle A B, it will in turn pass — by [prop. 9, bk. 2] — [and] the greatest circle A B [will pass] through the poles of the greatest circle C G. It also passes through A. [By prop. 9] bk. 2 of Theodosius, the segments D F and D G will be equal. Which is the first point. For if some greatest circle passed through C and D, it would not cut the parallel D E, but would touch it at D, by prop. 3, bk. 2 of Theodosius; for then the circle D E and that greatest circle would have their poles on the greatest circle A B, which they would cut at the point D.

[Margin: ^a 28.tertij (Euclid, *Elements* III.28).]

Again, since, by prop. 18, bk. 2 of Theodosius, both the arcs C E and C K, and the arcs C F and C I, are equal, the joined chords E F and I K will, by prop. 3, bk. 3 of the same Theodosius, be equal. ^a Therefore, from equal parallels they will cut off equal arcs, E F and I K. Which is the second point.

And since a straight line drawn, on the equinoctial line, from the hour-point which answers to the point C, through the point of any parallel, represents the greatest circle drawn from the point C, it is clearly established — as we gathered from tables 1, 2, and 3 at the beginning of this scholium.

Diagram. A spherical construction. At the top is the point A (the pole of the world). From A two great-circle (hour-circle) arcs splay downward: one on the left, descending through H to B; the other on the right, descending through C to K. The arc B C at the lower part is the Equator. Two concentric-appearing parallel arcs cross the figure: the upper parallel D E (from D, center, to E, at the right) and the lower parallel I K (from I to K). A further greatest circle C F G runs through C, F, and G.

- A (top): pole of the world
- G (upper left, on circumference): where the greatest circle C F G meets the upper parallel region
- H (upper center): where the hour-circle A B cuts the greatest circle C G

- E (right, on the upper parallel): endpoint of the parallel D E
- F (center-right, just below E): where the greatest circle C F G cuts the parallel D E
- D (center): where the hour-circle A B cuts the parallel D E
- B (lower left-center): foot of the left hour-circle, on the Equator
- C (right): where the hour-circle A K cuts the Equator
- I (lower right): where the greatest circle C F G cuts the parallel I K
- K (bottom right): foot of the right hour-circle, on the parallel I K

[Margin: Three most useful rules for finding and testing, by the hours, the points of the parallels. ("Tres regulæ vtilissimæ ad puncta parallelorū horis inuenienda & examinanda.")]

3. FROM this are derived the following three rules, which contribute very much to both finding the points of the parallels and testing them — namely, whether they have been correctly found or not.

I. REGVLA.

[Margin: Rule 1. ("1.Regula.")]

WHEN in some hour the point of any parallel is given, and it is proposed to investigate from it the point of the same parallel in any other given hour, one must take the hour-point on the equinoctial line that is 6 hours from the hour which holds the middle place between the two given hours. For the straight line sent out through the assumed point on the equinoctial and through the given point on the parallel will pass through the sought point in the other given hour, if it cuts that hour; otherwise it will give, in that hour, the point of the opposite parallel, as is clear from the preceding theorem.

[Margin: By what method the hour that is 6 hours from another given hour is found. ("Qua ratione eruatur hora distas 6 horis ab alia hora data.")]

For example, if the point of ☊ [Cancer] is given at hour 12, and the point of the same ☊ [Cancer] is sought at hour 5. Since hour 2½ is the mean between hours 12 and 5, and hour 8½ is 6 hours from hour 2½ — (the hour that is 6 hours distant from another given hour will be found more quickly than said, [continues]

—continuing Rule I (*I. Regula*), the worked example begun on the previous page:

is distant, if from the given hour one subtracts 6, having first added 12 in case the subtraction cannot otherwise be made; or, alternatively, if one adds 6 to the given

hour, and then adds 12 to the sum, should it be the larger course that is needed. For instance, since in the proposed example the hour distant 6 hours from hour $2\frac{1}{2}$ is sought, add 12 (making $14\frac{1}{2}$) and subtract 6 from that sum; or else add 6 directly to the given hour $2\frac{1}{2}$. (Either way you arrive at hour $8\frac{1}{2}$.) Then a straight line must be drawn from hour $8\frac{1}{2}$ on the equinoctial line through the point of ☿ [Cancer] at hour 12. This line will give the sought point of the same ☿ [Cancer] at hour 5.

So too, if from the same given point of ☿ [Cancer] at hour 12 one seeks the point of the same ☿ [Cancer] at hour 8: since hour 10 is the mean between hours 12 and 8, and hour 4 is distant 6 hours from hour 10, a straight line drawn from hour 4 on the equinoctial line through the point of ☿ [Cancer] at hour 12 will cut hour 8 at the point of the same ☿ [Cancer].

II. REGVLA.

[Margin: Rule 2. ("2. Regula.")]

WHEN the point of any parallel is given at some hour, and it is proposed to investigate from it the point of the *opposite* parallel at any other given hour, one must take the hour-point on the equinoctial line that holds the middle place between the two given hours. For a straight line drawn from the given point of the parallel through that point of the equinoctial line will cut the other given hour at the sought point of the opposite parallel, if it cuts that hour; otherwise it will give, at that hour, the point of the same parallel, as is clear from the preceding theorem.

For example: if from the given point of ☿ [Cancer] at hour 5 (reckoned from noon) one seeks the point of ♊ [Capricorn] at hour 10 (reckoned from midnight) — since from hour 10 up to hour 5 there are counted 7 hours, whose half is $3\frac{1}{2}$, one must take the point between hours 10 and 5 that is distant $3\frac{1}{2}$ hours from either hour 5 or hour 10, which is the point at hour $1\frac{1}{2}$. For a straight line sent out from the point of ☿ [Cancer] given at hour 5, through hour $1\frac{1}{2}$ on the equinoctial line, will cut hour 10 at the point of ♊ [Capricorn].

III. REGVLA.

[Margin: Rule 3. ("3. Regula.")]

WHEN the point of any parallel is given at some hour, together with an hour-point on the equinoctial line, and it is asked at which hour, from these two, the point either of the same parallel or of the opposite one may be found, we proceed

thus. If the point of the *opposite* parallel is to be investigated, one must count off from the given hour on the equinoctial line, in the contrary direction, as many hours as the given hour in the parallel is distant from that same equinoctial hour. For a straight line sent out from the given hour in the parallel through the given hour on the equinoctial line will give, at the counted-off hour, the point of the opposite parallel, if it cuts that hour in the sundial, as is established by the second part of the preceding theorem; otherwise it will cut that same counted hour at the point of the same parallel, as is established by the first part of that theorem. But if the point of the *same* parallel is to be sought, one must take the point distant 6 hours from the given point on the equinoctial line. For a straight line drawn from the point given on the equinoctial line through the given point in the parallel will give the point of the same parallel at that hour which is distant, on the other side, from the point taken on the equinoctial line by just as much as the given point of the parallel is distant from it — provided it cuts that hour, as is manifest from the first part of the preceding theorem; otherwise it will give the point of the opposite parallel at that same hour, as is clear from the second part of the same theorem.

For example. Let the point of ♋ [Cancer] be given at hour 4, and a point given at hour 1 on the equinoctial line. If, then, it is asked at which hour the point of ♏ [Capricorn] may be found from these two given points: since hour 4 stands off 3 hours from hour 1, and hour 10 likewise stands off from that same hour 1 by three hours, a straight line running from hour 4, ♋ [Cancer], through hour 1 on the equinoctial line will give the point of ♏ [Capricorn] at hour 10.

Again, let the point of ♋ [Cancer] be given at hour 3, and a point at hour 10 on the equinoctial line. If it is asked at which hour the point of the same ♋ [Cancer] may be found from the two given points, one must take the point at hour 4, distant 6 hours from the point given at hour 10 on the equinoctial line. For a straight line drawn from this given point at hour 10 through the point of ♋ [Cancer] given at hour 3 will yield the point of the same ♋ [Cancer] at hour 5 — which is distant from the assumed hour 4 by just as much as the given hour 3 is removed from that same hour.

If these are read through with attention, it is incredible how useful they will prove both for finding the points of the parallels precisely and for testing those already found. For, from a single given point of any parallel, nearly all the others will be found, both in the same parallel and in the opposite one, by the three rules above. Thus, if the point of ♋ [Cancer], say, is given at hour 12, from it the point of the same ♋ [Cancer] at hour 6 will be found by the straight line drawn from hour 9 on the equinoctial line through the given point at hour 12, as Rule 1 prescribes.

Likewise the point of ♋ [Cancer] at hour 5, by a straight line sent out from hour 8½ on the equinoctial line through the same point given at hour 12. So too, from the point of ♋ [Cancer] at hour 5 the point of ♏ [Capricorn] at hour 3 will be found, by a straight line drawn from the given point at hour 5 through hour 4 on the equinoctial line; and likewise the point of ♏ [Capricorn] at hour 2, by a straight line drawn from that same point at hour 5 through hour 3½ on the equinoctial line; and so on.

Once the points of the parallels have been found, you will test whether they have been correctly found, by the same rules. [Margin: In what way the found points of the parallels are to be tested. ("Quo modo examinanda sint pūcta parallelorum inuenta.")] For if, say, a straight line drawn from the point at hour 8 into the equinoctial line, through the point of ♋ [Cancer] at hour 12, passes through the point of ♋ [Cancer] at hour 4, this is proof that those two points, at hours 12 and 4, have not been badly found. The same you will say of the point at hour 5 in the parallel of ♋ [Cancer] and the point at hour 10 in the parallel of ♏ [Capricorn], if the straight line joining them passes through hour 1½ on the equinoctial line — from which both given hours, namely 5 and 10, are removed by an equal interval. And so with the rest.

CAPVT IX.

ARCVS SIGNORVM IN HOROLOGIO HORIZONTALI horarum à mer. & med. noc. alio modo.

(The arcs of the signs on a horizontal sundial, for the hours reckoned from noon and from midnight, by another method.)

1. On any line that issues from the center of the sundial — representing, of course, the greatest circle drawn through the poles of the world, even though it marks off neither a whole hour, nor a half, nor a quarter of an hour — we shall find the points of all the parallels geometrically by the following device.

[Margin: Where the computation of the declinations of the points of the ecliptic is taught. ("Vbi doceatur supputatio declinationum punctorum Eclipticæ.")]

Let AB be the meridian line; A the center of the sundial; CD the stylus, perpendicular, at its base C, to the meridian line; AD the axis of the world; BE the equinoctial line; AF the line of hour 6 — and any other line issuing from the center A that cuts the equinoctial line at E. From A, let there be described through

C and D the two circles CF and DG. First let the points of the parallels be found in the meridian line. Having drawn a straight line from D through B, the intersection of the meridian with the equinoctial, and having described from D the circle IHK equal to some quadrant (so that any number of degrees of arc may be transferred onto this circle), let there be counted from H, on either side, the declinations of the Sun at the beginnings of the signs; and let HI be the declination of ♋ [Cancer]; HL that of ♊ [Gemini] and ♌ [Leo]; HM that of ♉ [Taurus] and ♍ [Virgo]; HO that of ♏ [Scorpio] and ♐ [Pisces]; HN that of ♏ [Sagittarius] and ♒ [Aquarius]; and finally HK that of ♐ [Capricorn]. For the hidden straight lines drawn from D through the ends of the declinations — that is, the "rays" of the signs (for so we named those straight lines in the *Gnomonics*) — will cut the meridian at the points of the parallels. How to compute the declination of any point whatever of the ecliptic we shall teach at the end of this little book, in Problem 1. At the beginnings of the signs, however, the declinations, drawn from the table of declinations set down in Chapter 2 of the *Sphere*, stand as follows.

Declinationes principiorum signorum.

(Declinations of the beginnings of the signs.)

	(♈ M)				(♉ M)				(♊ M)				(♋ M)			
Signa	♈ ♌ (G.)	♌	♊ ♍ (G.)	♍	♉ ♏ (G.)	♏	♊ ♐ (G.)	♐	♈ ♒ (G.)	♒	♉ ♑ (G.)	♑	♋ ♒ (G.)	♒	♌ ♑ (G.)	♑
Declinati	0	0	11		3	20		1	23		3		1	23		3
ones					0			2			0		2			0

Diagram. A large horizontal-sundial construction figure enclosed in a square frame. The vertical meridian line AB runs down the middle: S is high on it (top), A is the center of the sundial (middle), C is below A, and B is toward the lower center. Two concentric circles are drawn about A (the circles CF and DG of the text), with the smaller inner circle carrying several close-set labels near the center (among them P, R, and a small point-cluster). To the right of A lies F (on the equinoctial region), and D just below-right of C. To the lower left, E sits on the equinoctial line and, below it, the stacked labels I and L. Sweeping arcs at the left and lower part of the frame carry X (upper left), V (left middle), T (left), M, O, N (lower center), K (lower right), and Z (lower left) — these belong to the circle IHK and the declination arcs, with H low on the meridian side. Toward the right, a second V and the point Y stand near the equinoctial line, and a fan of dotted rays

runs out to a column of small zodiac glyphs marking the signs' parallels (♈, ♉ and others) with small italic letters (d, e, f, m, x) beside them.

- A (center): center of the sundial [centrum horologii]
- B (lower center, on meridian): intersection of meridian with equinoctial
- C (center, below A): base of the stylus CD
- D (center, right of C): upper end / vertex region of the stylus (axis of the world AD)
- E (lower left, on equinoctial line): where a line from A cuts the equinoctial
- F (right of A): endpoint of circle CF
- G (upper left, on circle DG): the sought ray-center point G
- H (lower, meridian side): start-point (0° declination) of circle IHK
- I, L (lower left, stacked): declination points on the circle IHK (arcs HI, HL)
- K (lower right): end-point of the greatest declination arc HK (♊)
- M (lower center): declination point (arc HM)
- N (bottom center): declination point (arc HN)
- O (lower): declination point (arc HO)
- P, R (inner circle, near center): construction points (PG, CP of §2)
- S (top, on meridian): endpoint of the diameter of circle DG
- T (left): point on the outer arc / circle
- V (left middle) and V (right, near equinoctial): endpoints of the perpendicular SV [two "V"-marks appear — see note]
- X (upper left): point on the outer sweeping arc
- Y (right, near equinoctial): where a ray cuts AT beyond the center
- Z (lower left): terminus of a ray beyond the center
- Right-hand ray-cluster: small zodiac glyphs of the signs' parallels (♈, ♉, and further worn glyphs) with small italic letters d, e, f, m, x [several of these small glyphs/letters are not fully legible — [?]]

[Note: numerous small peripheral labels (the inner-circle cluster, the right-hand zodiac-glyph ray labels, and the doubled "V") could not all be resolved with confidence, even under magnification.]

Marginalia beside the demonstration: [Margin: Where the geometric finding of the declinations is handed down, and where in this little book it is treated. ("Inuentio declinationū Geometrica vbi tradita sit, & vbi in hoc libello tradatur.")] [Margin: In what way the points of the parallels in the meridian line are to be found. ("Puncta parallelorū in meridiana linea quo pacto reperiantur.")]

DEMONSTRATION. The practice just described is plain. For if the triangle ADB is imagined to be moved, together with the whole figure of the rays, about AB, until it coincides with the meridian, then DB will be the ray of the Equator, and the other straight lines issuing from D will be the rays of the other signs — so that, produced beyond D, they would cut the celestial meridian in the parallels of the signs. For this reason the rays of the Sun, when it stands in the meridian at the beginnings of the signs, will meet the meridian line at the points exhibited by the straight lines DI, DL, and so on.

In what way, moreover, the declinations of the signs are to be found geometrically, so that through them the rays of the signs may be drawn, we have handed down in Book 1 of the *Gnomonics*, Proposition 1, and in the scholium to Canon 3 of Book 3 of the *Astrolabe*, and in other places; and we shall repeat it briefly in the scholium to Chapter 15, Number 2.

2. Next, let these same points be found on the straight line AE, which cuts the equinoctial line at E. From C, the base of the stylus, let the perpendicular CP be drawn to it; and on CP produced let PG be taken equal to the straight line that would be drawn from D — the vertex of the stylus holding its proper position — to P. This will easily be done if, on the straight line

—a straight line [is taken] equal to the arc CR (between C and the line AE), as far as Q; and, the straight line CQ being joined, let it be produced up to the circle DG. For this line will be perpendicular to AE, and PG will be equal to the straight line drawn from P to the vertex of the stylus, D, holding its proper position, as we demonstrated in Chapter 1, Number 6. Then, having drawn a straight line from G through E — the intersection of the given line AE with the equinoctial — and having described from G a circle equal to the earlier circle IK, let the declinations HI, HL, HM, and so on, be transferred from this circle into that one. For the straight lines from G through the ends of the declinations, sent out, will cut the line AE at the points of the parallels — which I demonstrate thus.

Let the stylus, together with the axis, be understood in its proper position — that is, in the plane of the meridian; and, the straight line AG being drawn, let the triangle AGE be conceived as moved about AE, together with the whole figure of the rays issuing from G, until the point G touches the stylus. (It will necessarily touch it, because the straight line PG, in that motion, always lies in the plane erected through the stylus and drawn through the straight line CG.) [Margin: a. Euclid, *Elements* I.19 ("19. primi").] The point G will necessarily fall onto the point D, because of the equality of the straight lines PG and PD. For if it fell below D, or above, AG would be less or greater than AD or PD. The straight line GE too will

coincide, in that position, with the straight line DE. Since, then, DE is the common section of the Equator and of the greatest circle that produces the line AE on the sundial — because each circle passes through D, the center of the world, and meets the plane of the sundial at the point E — GE too will be, in that position, the ray of the Equator; and therefore the rays of the other signs, from D or from G (which in that position is the same thing), when sent out, will cut the line AE at the points of the parallels. And since these same rays cut the line AE always in the same manner, however the triangle AGE is carried about AE, it is clear that the rays sent out from G cut the line AE at the points of the parallels.

3. Finally, let these same points be sought on the straight line AT of hour 6 — which represents the greatest circle passing through the poles of the world and through the intersections of the horizon with the Equator, and which is parallel to the equinoctial line. [Margin: In what way the points of the parallels on the line of hour 6 are found. ("Puncta parallelorum in linea horæ 6. quo pacto inueniantur.")] Through one or other end of the diameter of the circle DG — say, through S — let a line be drawn with the ray of the Equator. And having described from S, on either side, a circle equal to the circle IK, let the declinations HI, HL, HM, and so on, be transferred from this into it. For the straight lines from S, drawn through the ends of the declinations, will cut the line AT at the points of the parallels — which is demonstrated as follows.

Let a plane drawn through AS and AT be conceived as carried about AT, until the straight line SA coincides with the straight line DA (the triangle ADC being placed in its proper position), and the point S with the point D, because of the equality of the straight lines SA and DA — so that it lies in the plane of the circle of hour 6, since this circle passes through the axis DA and makes, with the plane of the sundial, the straight line AT, parallel to the equinoctial, by Proposition 18 of Book 1 of the *Gnomonics*; because the plane of the sundial is equidistant from the common section of the Equator and the circle of hour 6. [Margin: b. ("19. vnde." — proposition-number and abbreviation uncertain [?]; cf. *Elements* XI, def. 3 cited in the text).] And since the common section of the circle of hour 6 and the Equator is at right angles to the meridian — because each circle is perpendicular to it, passing through the poles — that same common section SV will also, by definition 3 of Book 11 of Euclid, in that stated position (the point S being placed at D) be the ray of the Equator. The rays, then, of the other signs, issuing from D or S in that position, will cut the line AT at the points of the parallels. And since these same rays cut the line AT always in the same manner, however the plane drawn through SA and AT is turned about the line AT, it is evident that the rays sent out from S cut the line AT at the points of the parallels.

That this whole finding of the points, by which the parallels of the Sun are to be described, can be most clearly demonstrated from the problem of Chapter 1 — for since the parallels of the Sun cut, on the sphere, the colures or the meridian, and likewise all the greatest circles drawn through the poles of the world, into arcs, namely, of the declinations, by Proposition 10 of Book 2 of Theodosius; and since, by the practice set out in this chapter, there are cut off from the straight lines AB, AE, AT segments equal to the declinations, as is manifest from the aforesaid problem — it is plainly established that the points of the parallels are rightly found in this way.

4. When a straight line drawn through the center A cuts the equinoctial line — such as the line AE — the point G, from which the rays of the signs are to be drawn out, will be found also by this way. [Margin: Another way of finding the center from which the rays of the signs are to be sent out. ("Alia inuentio centri, è quo radij signorum emittendi sunt.")] From R, the midpoint of the segment AE between the center and the equinoctial, let a circle be described through A, cutting the circumference DG at the point G, which will be what was sought.

For EG will be tangent to the circle DG at G, as is established from what we demonstrated in the scholium to Proposition 31 of Book 3 of Euclid; [Margin: c. Euclid, *Elements* III.18 ("18. tertij").] and therefore the angle AGE will be a right angle. But the angle ADE is likewise a right angle at the top (the stylus DC being placed at right angles to the plane of the sundial), as I shall show. Therefore, since the two sides AG, AE are equal to the two sides AD, AE, and have the right angles G and D equal, subtended by the common side AE; and since each of the remaining angles AEG, AED is less than a right angle, by Corollary 1 of Proposition 17 of Book 1 of Euclid — the bases GE, DE will also (by the scholium to Proposition 48 of Book 1 of Euclid) be equal, and the remaining angles EAG, EAD equal. Therefore, the triangle ADB holding its proper position — so that the straight line DE is drawn up from the height to E — if the triangle AGE is revolved about AE, AG will coincide with the axis AD, and the point G with the point D, because of the equality of the angles EAG, EAD and of the straight lines AG, AD. Hence, as was demonstrated above, the rays sent out from G will cut the line AE at the points of the parallels. That the angle ADE is a right angle I demonstrate thus. Since DE, in that position, is the common section of the Equator and of the circle that makes the hour-section AE, as has been said; and since the axis AD is at right angles to the plane of the Equator — it will be perpendicular also to the straight line DE, which lies in the Equator, by definition 3 of Book 11 of Euclid.

5. But if it should happen that some ray does not cut the straight line drawn out from the center A on the same side where the other rays cut it, that ray will cut the same line on the other side of the center of the sundial. [Margin: What is to be done when not all the rays cut the same line drawn out from the center on the same side. ("Quid fiat, quando nō omnes radij secant eandem rectam ex cētro eductā, ex eadē parte centri.")] Thus, since the ray SX does not cut the line AT on the side toward T, that ray will cut the line AT beyond the center, at Y. For the same reason, if, for example, the ray GZ did not cut the line AE toward the equinoctial line, it would cut it beyond the center A, produced.

6. When, moreover, the center of the sundial is scarcely available — as happens in horizontal sundials of regions that have a small elevation of the pole, such that the circles CF, DG cannot be described from it — the point G, from which the rays are to be sent out, will have to be found by this method. [Margin: When the center of the sundial is not available, what is to be done. ("Quando cētrum horologij nō habetur, quid agendum.")] Having drawn CP perpendicular to AE, let Pa be taken equal to the stylus, and with the interval Ca let the straight line PG be cut off. For the point G will be that from which the rays are to be drawn out, which cut off from AE the declinations of the signs, as is clear from the problem of Chapter 1.

SCHOLIVM.

1. This finding of the points prescribed in this chapter (which is indeed proper to the hours reckoned from noon or from midnight) will be most expeditious if three compasses are at hand, with which the intervals of the declinations HI, HL, HM are once for all taken precisely. [Margin: In what way the practice of this chapter is made easier. ("Quo pacto praxis huius cap. redditur facilior.")] For, having described the hidden circles from D, G, S, and from the other points found in the manner in which the point G was found, which are to be equal to the circle IK,

— in which the intervals of the declinations were taken, then, faster than can be told, the declinations will be transferred onto those points, since those three compasses are kept opened to the same setting throughout.

[Margin: The construction of an instrument for the practical work of this chapter. ("Instrumēti constructio pro praxi huius capituli.")]

2. This same finding will also be made easier if the figure of the rays DIK is drawn separately on thin brass [Orichalcum] or on stiffer paper, and the points I, L, M,

H, O, N, K are pierced most finely, and at the center — the meeting-point of the rays D — a very sharp little pin is fixed, or else the point D itself is likewise pierced. For, on the sundial, once (say) the point G has been found and the hidden straight line GE drawn, if — by means of that little pin or of the hole D — the figure of the rays is set up at G, with the ray of the Equator DH coinciding with the straight line GE, and if some very fine needle is passed through the holes I, L, M, and so on, and the points marked, then the hidden rays of the signs must be drawn through these very points, cutting the hour-line at the points of the parallels. The same figure of the rays must be fixed at the point D, the ray of the Equator DH coinciding with the straight line DB, so that the points of the parallels may be found on the meridian line. And finally the same figure must be fixed at S, the ray of the Equator DH coinciding with the straight line SV — the line equidistant from the 6-o'clock line — so that the same points may be found on the 6-o'clock line.

Diagram. A horizontal-sundial construction (Chapter IX) enclosed in a rectangular frame. A large quarter-circle arc sweeps from the upper left down to the lower part of the frame. Radiating rays (drawn as dotted lines) fan out from the central region, cutting the arc and the hour-lines; a row of zodiac sign-glyphs is set along the rays on the right-hand side. Readable Latin letter-labels:

- X (top left, on the arc)
- V (upper left, on a horizontal line) and V (upper right) — the figure carries two V labels, one on each side
- S (center-top, at the crossing of the vertical and a horizontal line)
- T (left, on the arc)
- A (center of the dial, within a small circle; center of the horologe)
- G (just left of A, within a small circle)
- F (just right of A)
- P, C, R, b (small labels clustered just below A, on the axis)
- E (left of center, on a horizontal line) and a (small, near center)
- B (center, on the lower horizontal line)
- D (center-right)
- I, L (lower left); M, H, O, N, K (descending along the lower left / bottom)
- Z (bottom left)
- Y (upper right, near the zodiac row)

- A row of zodiac sign-glyphs runs down the right-hand rays, with several small lower-case letters (d, f, e, m, x) interspersed. Most of these sign-glyphs will not resolve at this resolution — [?].

[Margin: Another use of the instrument just constructed. ("Instrumēti proxime constructi alius vsus.")]

3. But if, in the figure of the rays DIK, a perpendicular axis AD is drawn to the ray of the Equator DH — extended in both directions, with the superfluous surrounding parts cut away so as not to obstruct — then the instrument will be ready with which we shall obtain the same points of the parallels, even without having the point G together with the straight line GE. For if on the axis the straight line DA is taken from the point D, equal to the axis of the sundial between the center and the tip of the stylus, and its end-point is fixed at the center A of the sundial — whether by some little pin fixed at A, or by a needle passed through the hole A — and the instrument is turned about until the ray of the Equator passes through the intersection of the equinoctial line with the hour-line on which the points are to be sought, then we shall mark the points, as before, through the holes I, L, M, and so on; and straight lines drawn out from G (the point G being likewise marked through its hole) will cut the hour-line at the points of the parallels. Nor does it matter whether the holes of the instrument lie below the point G, toward the hour-line AE, or above it. Yet this will be done more conveniently if, once the point G has been found, the straight line GE is drawn first, so that we may lay the ray of the Equator upon it — lest we be forced to superimpose the ruler on that same ray and turn the instrument about until the ruler passes through the intersection E. But so that the points on the straight line AT, perpendicular to the meridian, may be found, the instrument must be set so that its axis coincides with the meridian line and the ray of the Equator is equidistant from the line AT.

[Margin: By this same method the points of the diurnal arcs are also found, if their rays are drawn out from D. ("Hac eadē via inuientur etiam puncta arcuum diurnorum, si eorū radij ex D, educantur.")]

4. To these rays of the signs you may also, if you like, add the rays of the diurnal arcs of your region — especially those of the 10th and 14th hours, likewise the 8th and 16th, and the 6th and 18th, and finally the 0 and 24 (which we treated in the *Gnomonica*) — so that by them, besides the points of the parallels, points may also be marked on the hour-lines for the hours from sunrise and from sunset that are easily to be described, as we taught in the *Gnomonica*.

[Margin: The finding of the points for the parallels by means of tangents.
("Inuentio punctorum pro parallelis per Tangentes.")]

5. But now, if by Problem 10 at the end of the little book the arc of any circle whatever running from noon or from midnight — the arc between the arctic pole and the vertical circle that stands at right angles to that circle — is computed, we shall find the points of all the parallels on any hour-line most exactly by tangents, in this way. Let the 3-o'clock line from noon be AE. The aforesaid arc, then, will comprise the arc to which corresponds the straight line AP for the latitude of Rome, $38^{\circ}9'$. When this is subtracted from the complement of the declination of ϖ (Cancer), $66^{\circ}30'$, there remains the arc of $28^{\circ}21'$ between P and the parallel ϖ , whose tangent — 5 [numeral:? — stacked fraction not legible], with respect to the whole sine PG — transferred from P, will give on AE the point ϖ . Moreover, in the southern parallels the complement of the found arc AP must be added to the declination of the parallel, so that the arc between P and the proposed parallel may be produced, and so on. In the same way, when the proposed hour lies above the 6-o'clock line AT, the same found arc must be added to the complement of the declination of the northern parallel. Thus, if EA were produced beyond the center A so as to be the 3-o'clock line from midnight, the aforesaid arc of $38^{\circ}9'$, added to the $66^{\circ}30'$ of the complement of the declination of ϖ (Cancer), would make an arc of $104^{\circ}39'$, which is included between P and the tropic of

ϖ (Cancer), and the 3-o'clock line from midnight — which, since it exceeds a quadrant, could not be cut off, and so the point ϖ could not be marked on the 3-o'clock line from midnight. On the 6-o'clock line that arc is nothing, and so the tangent of the complement of the declination of the northern parallel, with respect to the whole sine AD, transferred from A in both directions, will give the points ϖ . But on the meridian line AB the aforesaid arc AC is equal to the complement of the pole's altitude; and when this is subtracted from the complement of the declination of the northern parallel, or when its own complement is added to the declination of the southern parallel, the arc between C and the proposed parallel will be found. And whenever it should happen that the complement of the declination of the northern parallel is less — say, than the found arc AP — this complement must be subtracted from the found arc AP, so that the remaining arc between P and the proposed parallel, toward the center A, may be produced.

There are, in all, five whole sines [required], on account of the five hour-lines distinct from the meridian, together with the line of the sixth hour — a matter that will be no trouble if a hundred-part instrument is at hand. But if you would

rather transfer the tangents from the Equator onto the hour-lines, you must subtract each arc found by Problem 10 from 90° , and take away from the tangent [of the whole] the tangent of the remaining arc — the one previously found, namely the one that was transferred as a perpendicular from the point P. For the remaining tangent, transferred from the Equator toward the center A, will give the point of the northern parallel; and if you likewise take away the tangent of the remaining arc from the tangent to be transferred from P for the southern parallels, the tangent that remains, transferred downward from the Equator, will give the point of the southern parallel.

Furthermore, so that we may not be forced to erect perpendiculars from C (in place of the stylus) to all the hour-lines, and to assume for each one separately the several whole sines, the segments of all the hour-lines between A (the center) and the parallels must be reckoned as tangents with respect to the whole sine CD, equal to the stylus, as we shall teach at the end of the little book in Problem 17. By this reasoning we computed Table 19 for latitude 42° . If from it the tangents are taken under the signs opposite the hours and, with respect to the whole sine CD, are transferred from A, the center of the sundial, onto their proper hour-lines, the points of the parallels will be found most accurately, since no compass cuts any hour-line obliquely. So this method is the most excellent of all, provided that for the given pole-altitude a table similar to our Table 19 has been computed for latitude 42° . For example, in Table 19, under $\mathfrak{S}$ (Cancer), opposite the 3rd hour from noon and the 9th from midnight, is found the tangent 1557 — that is, 15 [numeral:? — stacked fraction; restates as $57/100$] and a little more — which, with respect to the whole sine CD equal to the stylus, transferred from A, will exhibit at the 3rd and 9th hours the points $\mathfrak{S}$; and so for the rest. But if the tangents of the northern parallels of Table 19 are taken from the tangents of the Equator, or if from the tangents of the southern parallels the tangents of the Equator are taken away, there will remain tangents which, transferred from the points on the hour-lines in the Equator onto the hour-lines, will give the points of the parallels, as before.

[Margin: A most excellent finding of the points for the parallels by means of tangents — reckoned with respect to the whole sine equal to the stylus — to be transferred from the center of the sundial. ("Inuentio praestantissima punctorum pro parallelis per Tangentes respectu sinus totius stylo aequalis, ex centro horologij transferendae.")]

CAPVT X.

The arcs of the signs on a horizontal sundial, in yet another way, for the hours from noon and from midnight. (ARCVS SIGNORVM ALIO ITEM MODO in horis à mer. & med. noc. horologij horizontalis.)

1. Let there be, as in the foregoing, the meridian line AB; the center of the sundial A; the axis AD; the stylus DC; the equinoctial line through B, and the 6-o'clock line through A, both perpendicular to the meridian; and the remaining hours drawn out through the points found on the equinoctial. Let there also have been found, on the meridian, the points of the parallels, as was said in Number 1 of the preceding chapter — namely E, ☊; O, ♀ and ♁; P, ☋ and ♎; B, ♍ and ♋; Q, ♌(?) and ♉; R, ♊(?) and ♏; and S, ♏. From these we shall find

Diagram. The horizontal-sundial figure for Chapter X, in a rectangular frame. A large quarter-circle arc runs from the top-left corner down toward the bottom; the meridian AB is the vertical line, with the center A near the top of it and M at its foot. Hour-lines radiate from A and are numbered along the arc and the frame. Readable labels:

- Z (top-left corner, on the arc)
- a, then 6 (small labels on the left edge, below Z); a cursive letter (read as k(?)) in the upper-middle
- N (top, on the meridian, above A) and A (center of the dial)
- G, F, C, E, O, P (small labels descending on the axis just below A)
- I, K, D (a small triangle to the right of A)
- T, L, V (left of center); H (further left, on a horizontal line)
- B (on the meridian, below the center)
- Q, R, S (descending on the meridian toward the foot)
- M (foot of the meridian)
- Hour-numbers along the arc / bottom edge: 7, 6, 5, 4, 3, 2, I (=1), 12 (printed "I 2")

the points of these same parallels on the other hour-lines, in this way. Suppose, for example, that the points of the parallel ☊ (Cancer) are to be found. From its point E, found on the meridian, draw a perpendicular EK to the axis, and cut off EG equal to it — whether toward A or toward B. Then, with a circle described from G at the interval GE, divide its quadrant ET into 6 equal hours, and transfer one of them upward from T. Next, on the straight line BD, take BF equal [to it], toward

[Margin: An ingenious drawing of the parallels. ("Descriptio parallelorum artificiosa.")]

that side on which the point G lies; and from F, through the hours of the circle ET, let hidden straight lines be drawn out. For these, being the answering hour-lines, will cut [the axis] at the points of the parallel $\mathfrak{S}$. So you see that the straight line drawn from F through V — the 4-o'clock line — cuts the 4-o'clock line at L, the point $\mathfrak{S}$, and so on. In just the same way, if a straight line equal [to the perpendicular] is taken on the meridian from the perpendicular drawn from O to the axis at the points Π and $\mathfrak{Q}$, and if from its far end a circle is described through O, likewise cut into hours, and from the same point F straight lines are sent out through the hour-points, the hour-lines will be cut at the points Π and $\mathfrak{Q}$. And so with the rest.

[Margin: The drawing of a larger circle, to be divided into hours, from a center other than the center of the parallel. ("Descriptio circuli maioris in horas diuidendi, ex alio centro, quàm ex centro paralleli.")]

2. But since, as you see, the circle ET is so small — which must be said of the others too, described through O, P, Q, R, S — that scarcely without error can straight lines be drawn from the point F through its points, because of their nearness, we shall describe a circle of some larger size, so that the lines drawn from F through its arcs, similar to the arcs of the circle ET, may also pass through the arcs of the circle ET. We shall find the center and radius of this circle as follows. Take FM as any multiple whatever of FE (in the example it is taken sixfold), and MN an equal multiple of EG; then N will be the center found, and NM the radius. For if a circle is described from N through M and divided into hours, just as the circle ET was divided, the straight lines drawn from F through the hours of this newly described circle will pass through the same hours of the circle ET. But since error may easily be committed in multiplying the straight lines FE and EG, we shall perhaps do it more conveniently, and certainly more accurately, thus. Taking FM of any size whatever, find the fourth proportional MN to the three straight lines FE, EG, FM; then again N will be the center and NM the radius of the circle to be described, as before — all of which I demonstrate in this way.

3. With EH drawn through E, perpendicular to the meridian, let the triangle ABD be conceived in its proper position — that is, in the plane of the meridian, at right angles to the plane of the sundial — and the parallel $\mathfrak{S}$, ET, likewise in its proper situation, so that the radius EG coincides with the straight line EK, and the point G with the point K. Then EH will be the common section of the parallel $\mathfrak{S}$ (passing through KE) and of the plane of the sundial, in the same

[Margin illustration: horizontal-sundial figure repeated. See below.]

Diagram. The same horizontal-sundial figure as fig-p671-1, reprinted on this page beside the continuing text. Same construction and label-set: outer rectangular frame, quarter-circle arc from the top-left (Z) sweeping down; meridian AB vertical (N above A at top; A the center; B below; M at the foot); center A with radiating hour-lines. Readable labels: Z (top-left), a and 6 (left edge), a cursive k[?] and 6 in the upper-middle, N, A, G, F, I, K, D, T, L, V, H, C, E, O, P, B, Q, R, S, M; hour-numbers 7, 6, 5, 4, 3, 2, I (=1), 12 (printed "I 2").

way as the perpendicular drawn through B is the common section of the Equator (passing through DB) and of the same plane of the sundial. [L?] Now let there be drawn from D, the tip of the stylus — that is, from the center of the world — to F the straight line DF, cutting the radius EK of the parallel $\mathfrak{S}$ at I, if through DF and the 4-o'clock line in the circle ET, placed aloft (the straight line EG coinciding, of course, with the straight line EK, and the point G with the point K, so that the parallel makes with the sundial the section EH) — that is, if through the straight line DF and the point V a plane is understood to be drawn, it will make with the sundial a common straight section-line, which will cut the 4-o'clock line at the point $\mathfrak{S}$, since the Sun's ray then, passing through the center of the world D and through the 4-o'clock point $\mathfrak{S}$ at V (the parallel ET keeping the aforesaid position), does not withdraw from that plane. If, therefore, on EH, the common section of the parallel and of the plane of the sundial, a point is found at which it is cut by that common section (which the plane drawn through DF and the point V makes with the sundial), a straight line sent out from F through that point (through which that same plane, and consequently that common section, passes) will give the point $\mathfrak{S}$ on the 4-o'clock line. That point on the straight line EH we shall easily detect. For if a straight line EI equal [to EF] is taken on the meridian up to some point, it plainly follows that the straight line drawn from this point through the point V (the parallel ET lying in the plane of the sundial) cuts the line EH in the same way as it is cut by the straight line from I aloft — the axis and stylus being placed in their proper situation, and the parallel turned about the straight line EH until its center G coincides with the point K — drawn through the same point V; because, when the parallel ET is turned about EH, that point coincides with the point I, owing to the equality of the straight lines. And consequently the straight line drawn from that point through V on the sundial coincides with the straight line likewise drawn from I through V aloft. But since the straight line EF is equal to the straight line EI (because, from the isosceles triangle FBD, the parallel EI cuts off [proportional parts], by the corollary to Proposition 4 of Book 6 of Euclid, the triangle FEI is also isosceles), the straight line FV will lie in the plane of the

sundial — that common section — and will cut the straight line EH at H. Therefore the straight line drawn from F through V, and consequently through a like hour in the circle described through M, will represent that common

that common section; and so it will cut the 4-o'clock line at the point L of the parallel of $\mathfrak{S}$ (Cancer — the summer-solstice parallel, the Tropic of Cancer). The same demonstration holds good for the other hour-lines and for the other parallels as well.

4. That the straight lines drawn from F through the hour-points of the circle ET pass likewise through the corresponding hour-points of the circle described through M is easily shown as follows. The whole NM is the same multiple of the whole GE as FM (cut off from NM) is of FE [?] (cut off from FG) — by the construction it is as great a multiple of it as the whole NM is of the whole GE (Euclid V.5). Consequently the remaining FN will be the same multiple of the remaining FG. And therefore the point F (which stands in place of two centres, in the circles described through M and through E from N and from G) will lie at a like distance from the centres N and G, since it has been shown that FN is to FG as the semidiameter NM is to the semidiameter GE. For this reason, by the scholium to Lemma 21 of Book 1 of the *Astrolabium*, a straight line drawn out from F will cut off similar arcs on the circles described through M and through E from N and from G.

[Margin: What is to be left out in this construction, so that confusion is avoided.]

SCHOLIVM

1. To avoid confusion, it is enough — leaving out the circle ET — to draw only the circle from N through M on the sundial. This drawing of the parallels has no other drawback than that the straight lines coming out from F cut some of the hour-lines too obliquely. In that case great care is needed for the points of intersection to be found correctly; or else the point that has to be found through too oblique an intersection must be tracked down by another route.

[Margin: The drawing of the hour-lines from any parallel whatever.]

2. It follows from what we demonstrated in the *Gnomonices* that, if from G hidden straight lines are drawn through the hour-points of the circle ET — or rather of any other larger circle described from G — cutting the straight line EH in points through which straight lines sent out from A, the centre of

the sundial, will give the hours from noon and from midnight; and this because EK is a certain common section of the Equator and the Meridian, and EH is the equinoctial line falling at right angles onto the meridian with respect to the style set up at K. Hence, if the circles described through E and through M are divided into more equal parts, and from G hidden straight lines are drawn through the parts of the circle ET (or of another larger circle described from G), and through the points where they cut the straight line EH straight lines are drawn from the centre A of the sundial in the manner of hour-lines, then in all cases the points of the parallels will be found, just as was said in the case of the hour-lines; and the parallels themselves will be drawn all the more exactly, the more closely-set the points that are obtained.

[Margin: How the arcs of the signs may be drawn more accurately, by the method set out in this chapter.]

3. It is also convenient to draw by this route, separately, only the [northern] halves of the northern parallels, on one side of the meridian line, on some rather stiff paper. For if they are carefully cut out, so as to make, as it were, curved rulers — either convex or concave, as one prefers — the other halves, and the opposite parallels as well, will easily be drawn, provided the points for them have been found on the meridian line. And for a more accurate drawing, single points will first have to be found for each parallel, points lying far enough from the meridian line that those cut-out halves can be applied without error to their proper points on the meridian line, and, with those distant points also, in such a way that the meridian line drawn upon those halves always coincides exactly with the meridian line of the sundial. In this way we shall easily find, for any given arc, a point on the sundial far removed from the meridian. Once a perpendicular has been drawn from any point N to the meridian, meeting the described arc at h, take on the sundial's meridian a straight line EN equal to [the one taken] from the point through which the arc is to be described; then draw the perpendicular through the endpoint down to the meridian, and transfer the point h onto it: this will be the point sought.

[Margin: That it is enough to draw the halves of the northern parallels.]

[Margin: Certain paper rulers for drawing the parallels.]

[Margin: How the aforesaid rulers can be applied without error.]

4. Moreover, this route can also easily be adapted to the hours from sunrise and sunset, if — in place of the distances of the hours from noon or midnight — one takes, on the circles ET and Ma, the distances of the hours from sunrise or sunset from the Meridian circle. How to compute these distances within each sign we shall teach in the scholium to chapter 15, Number 1. For example, since the 21st hour from sunset, at the beginning of $\mathfrak{E}$, is distant $68^{\circ} 3'$ from the meridian, if this distance is counted from E toward T, or from M toward a, and through the end of the count a straight line is drawn out from F, it will cut the 21st hour from sunset at the point $\mathfrak{E}$; and so for the rest.

[Margin: By what method the drawing of the parallels given in this chapter is adapted to the hours from sunrise and sunset.]

CAPVT XI.

ARCVS SIGNORVM IN HOROLOGIO HORIZONTALI SINE ope, auxilioque linearum horariarum.

(The arcs of the signs on a horizontal sundial without the aid or help of the hour-lines.)

1. In the last place, we can draw the arcs of the signs — indeed, two opposite signs at one and the same operation — without any aid from the hour-lines, by a truly elegant method that depends entirely on the elements of conics. Let there be, then, as in the other cases: the meridian line AB; the axis of the world AD; the centre of the sundial A; the tip of the style D, whose place is at the point onto which the perpendicular dropped from D onto the meridian falls; and the radius of the Equator DB, perpendicular to the axis, so that the equinoctial line has to be drawn through B at right angles to the meridian. From D let two rays of opposite signs be drawn out, if each ray cuts the meridian; or the ray of a single sign only, when only one ray cuts the meridian, or when the arc of only one sign is wanted. Let the two rays of the opposite signs be DC and DC, one above B and the other below. Whatever we prescribe for the one I wish to be understood of the other as well, and for that reason we have set the same letters against both, so that the same demonstration may be adapted to each described arc.

On the ray DC, produced, take any number of equal parts CE, EF, FG, GH, and through the first point E draw EM perpendicular to the axis of the world AD — which is easily done if, on the other ray produced, a segment DM equal [to DE] is

taken; for then the straight line EM will be perpendicular to the axis AD. Let it cut the meridian at Q, and take QP, PO, ON equal to the segment CQ, until as many parts are contained in CN as there are in CH. You will also find these parts if, taking DL, DK, DI equal to the straight lines DF, DG, DH, the hidden straight lines FL, GK, HI are drawn. But because, when there are several parts to be taken in order with the compasses, one may easily go wrong, you will do better to take CH of whatever size, bisect it, then bisect each part again, and so on, so as to obtain 4, or 8, or 16 equal parts, and so forth. Let the same be done on the straight line CN as on CH [i.e. as HI cuts off in N], which HI cuts off perpendicular to the axis AD — a line easily drawn if first DI is taken equal to the straight line DH.

After this, through the points Q, P, O, N let perpendiculars QV, PT, OS, NR be raised to the meridian. These too are easily drawn if a single line RNR, parallel to the meridian line, is set up, along which ef is drawn, and on which, from the point e, straight lines equal to NO, NP, NQ are taken, and so forth.

Finally, from the centre A of the sundial let hidden circles be traced through the points H, G, F, E, cutting the aforesaid perpendiculars at the points R, S, T, V on either side, in such a way that the circle described through the most distant point H cuts the line RN, the one most remote from the point C, and the others cut the others in their order. For through the points R, S, T, V the proposed parallel is to be described.

[Margin: The drawing of two opposite parallels at one operation, even though the hour-lines have not been drawn.]

But so that more points near C may be had, the portions CE, CQ toward C must be subdivided several times. Thus you see, in the figure, that each cut is bisected at X and Z; likewise each of CX, CZ is again bisected at a and b; and through Z and b perpendiculars are drawn, which the arcs described from A through X and a cut at Y and c. In the same way the corresponding portions GH, ON, or FG, PO, or EF, QP can be bisected, as often as it seems useful — that is, whenever the point R lies too far from the point S, or S from T, or T from the point V. Finally, if the parallel is to be carried out further, more parts equal to HG, GF, etc. must be taken beyond H, and just as many equal to NO, OP, etc. beyond N. For if perpendiculars to the meridian are drawn through these, and arcs are described through them from A, other points will be found on these perpendiculars.

2. All this I demonstrate as follows. With the triangle ADC set up in its proper position — that is, in the plane of the Meridian — HDI will be the triangle through the axis in the Cone, whose base is the proposed parallel at right

angles to the axis of the world AD, and whose vertex is at D, the centre of the world, since HDd and IDd are the angles complementary to the declination of the parallel, sloping toward the base of the cone. This triangle is cut through the axis by the plane of the sundial passing through AB, drawn at right angles [to it]; and so a conic section results, whose diameter — or axis — is CN, which I say passes through the points R, S, T, etc.

Diagram. A large rectangular framed construction for drawing the sign-arcs of a horizontal sundial. A vertical meridian line AB runs down the middle of the frame. Near the top-centre lies the cluster of the sundial's centre and style: the points N, A (centre of the sundial), with the small letters d, o and c/C beside them, from which the rays and hour-lines radiate. In the upper-right region a straight ray descends bearing, in order, the capital points I, K, L, M; the same ray reaches the right-hand edge of the frame, where the points M, L, l (small L), I are marked. Dotted arcs (the parallels / conic sections) sweep across both the upper and lower halves of the frame, carrying the repeated series of points R, S, T, V together with F, G, H (upper) and, mirrored below, V, T, S, R with F, G, H, N. Near the centre a small ellipse (the conic section) is drawn with the points c, C, Y and E, P nearby; the meridian continues downward through P, N. Additional readable labels include X and B (right of centre) and the small letters e, f at the left frame edge. D (centre of the world) is the apex from which the construction is projected.

- N, A (top centre): the style-tip / centre of the sundial and neighbouring construction points
- d, o, c / C (top centre, small): auxiliary points at the centre
- I, K, L, M (upper right, descending along a ray): division-points of the ray
- M, L, l, I (right frame edge, stacked): points where the ray/arcs meet the edge
- R, S, T, V (upper, on dotted arcs): points of the parallel above the meridian
- F, G, H (upper left of centre): division-points on the rays
- X (right of centre): bisection point
- B (below centre-right): foot of the equatorial radius DB
- V, T, S, R (lower, on dotted arcs): points of the parallel below the meridian
- F, G, H, N (lower centre): division-points below
- c, C, Y (centre, on the small ellipse): points of the conic section
- E, P (below centre): points on the meridian
- e, f (left frame edge, small): endpoints of the parallel ruler-line ef

- [Several small and duplicated peripheral labels could not be resolved even under magnification.]

That the straight line EM and the remaining lines FL, GK, etc. are perpendicular to the axis AD is established. For since the triangle EDM is isosceles, in which the straight line Dd bisects the angle EDM, Dd will be perpendicular to the base EM, as we demonstrated in the scholium to Proposition 26 of Book 1 of Euclid; and likewise for the rest. Again, that the straight lines EM, FL, GK, HI cut the straight line CN into equal parts as well is plainly established, since they are parallel and therefore cut the straight lines CH, CN proportionally. Finally, that the straight lines drawn from the points of the line ef, corresponding to the points of the straight line NC, are perpendicular to NC is clear, on account of the parallelograms eO, eP, etc.

Now that the point V (and what is said of it is to be said of the other points T, S, R as well) falls on the Conic Section which the plane of the sundial produces — the plane drawn through AB into the cone whose base is the parallel of the Sun and whose vertex is D — this is shown thus. With the triangle through the axis still in its proper position, the straight line VQ will be perpendicular to it (by Definition 4 of Book 11 of Euclid), since it lies in the plane of the sundial perpendicular both to the triangle through the axis and to their common section CN. It therefore coincides with the straight line raised perpendicular from Q to the plane of the triangle through the axis, and which lies in the conic section; so that, produced as far as the conic section — that is, as far as the circumference of the circle of diameter EM, which is parallel to the base of the cone and is likewise at right angles to the triangle through the axis — it meets the conic section of the circle at that same point; inasmuch as this perpendicular is the common section of the circle about the diameter EM and of the conic section. I say, then, that QV is equal to this perpendicular, and hence that the point V likewise lies on the circumference of the conic section. For since the said perpendicular, raised from Q to the triangle through the axis as far as the circumference of the conic section or of the circle about the diameter EM, is a mean proportional between the segments MQ, QE (by the corollary to Proposition 13 of Book 6 of Euclid); and since QV is likewise a mean proportional between the same segments (as I shall show); that perpendicular and QV will be equal, which is what was proposed. That QV is a mean proportional between MQ, QE I prove thus. The rectangle under MQ, QE together with the square on gQ [a] is equal to the square on gE. Adding, then, the common square on gA, the rectangle under MQ, QE together with the squares on gQ, gA [b] — that is, together with the square on QA — will be equal to the squares on gE, gA, that is (drawing the straight line AE), to the square

on AE, that is to the square on AV, since the straight lines AE, AV are equal, [being drawn] from the centre A to the circumference EV. [c] But the square on AV is equal to the squares on QA, QV. Therefore the rectangle under MQ, QE together with the square on QA will be equal to the squares on QA, QV; and, the common square on QA being taken away, the remaining rectangle under MQ, QE will be equal to the remaining square on QV. [d] And so QV is the mean proportional between MQ, QE, which is what was proposed.

[Margin (a): Euclid II.5.] [Margin (b): Euclid I.47.] [Margin (c): Euclid I.47.] [Margin (d): Euclid VI.17.]

SCHOLIUM.

1. Where the sundial has no centre, this construction does not apply, as is evident, since the arcs have to be described from A, the centre of the sundial, in order that the points R, S, T, V of the parallel may be found.

[Margin: Where this construction does not apply.]

2. Furthermore, when the meridian line AB, cutting one side of the triangle through the axis — namely DH — also cuts the other side, ID, produced beyond the vertex D, as happens in our figure, two opposite and equal conic sections are produced, that is, Hyperbolas. But when the same meridian is parallel to the other side, a single section results, a Parabola. And when, finally, it cuts both sides within the triangle through the axis, a single section too results, an Ellipse — as we demonstrated in the *Gnomonices*.
3. To avoid confusion, it is best that only the northern parallels be drawn, and indeed separately, outside the sundial. For if the superfluous parts are cut away, so that only the outer circumferences appear, then with the help of these circumferences — like certain bent rulers — the parallels themselves are easily transferred onto the sundial, even to the points of the southern parallels; especially if certain distant points are first found to which those rulers may be applied, as we said in the scholium to the preceding chapter, Number 3. Or else, if the intervals between A and the points N, O, P, Q are transferred onto the sundial from the centre of the sundial, and through the transferred points perpendiculars are drawn to the meridian, and onto these are transferred also the straight lines NR, OS, PT, etc., then the points of the parallels will be found on the sundial itself.

[Margin: What must be done so that, by this route, the parallels may be drawn on the sundial without confusion.]

CAPVT XII.

ARCVS SIGNORVM IN HOROLOGIO HORIZONTALI hor. à mer. & med.
noc. in quo centrum haberi non potest.

(The arcs of the signs on a horizontal sundial, for the hours from noon and midnight, on which a centre cannot be had.)

1. On a horizontal sundial for a small elevation of the pole, on which a centre can scarcely, or in no way, be had — unless a very distant one, far from the place of the style — the arcs of the signs will be drawn in the way we explained in chapter 8, Number 7. That is: if from the table the arcs are taken opposite any given sign, under the hours in "Circ. hor. 12" — this being on that line of hours which bears the heading "[Circ. hor. 12]," or which on its right-hand side has the number "[12]" written against it — and if with these same arcs we cut off equal portions from the meridian line, all of them beginning from the equinoctial line; then, if straight lines are drawn through the extreme points of these portions perpendicular to the meridian, or equidistant from the equinoctial line, each will cut the two corresponding hours at the points of the proposed parallel. An example may be taken from Number 7 of chapter 8. For the same sundial will serve here, if we imagine that the centre G is not to be had. And the method is the same on every other sundial, however small the elevation of the pole.

[Margin: The drawing of the parallels on a sundial on which the centre does not appear.]

2. We shall draw the same arcs of the signs with ease from the 2nd table, as was handed down in Number 10 of the same chapter 8. Likewise from table 3, as we set out in Number 11 of the same chapter 8. Or (if you wish) by means of the Tangents of the complements of the Sun's altitudes, as we taught there in Number 12.
3. They will also conveniently be drawn by the method which we set out in chapter 9, and by that other one which we set out in chapter 10. What is excluded here, then, is only the method handed down in the preceding chapter 11, and the one by which, in chapter 8, the points on the 6-o'clock line are found — because these two methods necessarily require a centre for the sundial.

[Margin: Which methods of construction do not apply on a sundial that lacks a centre.]

Table 3, the universal one, will also be serviceable here: for, having found on the meridian line the points $\mathfrak{S}$ (Cancer) and $\mathfrak{V}$ (Capricorn), h and i, if perpendiculars to the meridian representing the Ecliptics are raised through them, and the arcs of table 3 are transferred onto these on either side from h and i, with respect to the whole sines hd, id, marking the points on them; then the straight lines sent out from B through these points will cut the hour-lines at the points of the parallels, and so on, just as was said of the 5-o'clock line. And this method can be used on other sundials too, even if the sundial has a centre.

CAPVT XIII.

ARCVS SIGNORVM IN HORIS A MERIDIE, ET MEDIA NOCTE horologii horizontalis, supra quem maxima est altitudo poli.

(The arcs of the signs, for the hours from noon and midnight, on a horizontal sundial above which the altitude of the pole is greatest.)

1. On a horizontal sundial where the elevation of the pole is so great that on it the equinoctial line can scarcely be drawn, we shall draw the arcs of the northern signs — which alone can be delineated on it — by the method which, in the preceding chapter, Number 1, we said was to be sought from Number 7 of chapter 8. For although there is no equinoctial line from which the arcs of table 1, in "circ. horæ 12," are reckoned, nevertheless, because they are to be transferred from the place of the style, as we declared in Number 7 of chapter 8, that method can conveniently be applied. For example: at a pole-altitude of $89^{\circ} 30'$, we must find the point on the meridian through which the perpendicular to it exhibits the points $\mathfrak{S}$ on the 1- and 11-o'clock lines. Under the 1st and 11th hour, in "circ. hor. 12," opposite $\mathfrak{S}$ and $\mathfrak{V}$, the arc found is $24^{\circ} 14'$; and this taken from the pole-altitude of $89^{\circ} 30'$ (to which would be equal the portion of the meridian between the place of the style and the equinoctial line, if it were drawn on the sundial), leaves an arc of $65^{\circ} 16'$, whose Tangent, 21 [fraction: ?], transferred downward from the place of the style — the parts of the line from which the Tangents are to be taken having to be equal to the length of the style, as we said in chapter 2, Number 2 — will give the point through which a perpendicular drawn to the meridian will cut the 1- and 11-o'clock lines at the points $\mathfrak{S}$, and so on.

[Margin: The drawing of the parallels on a sundial on which the equinoctial line does not appear.]

We shall mark the same arcs by means of the Tangents of the complements of the Sun's altitudes, as we wrote in chapter 8, Number 12, if that seems good.

But through the 6-o'clock line, and through tables 2 and 3, nothing will be done here, since we suppose that the point of intersection of the meridian line with the equinoctial line is not to be had — the point from which the lines cutting the hours at the points of the parallels would have to be drawn out.

[Margin: Which methods of construction are excluded here.]

CAPVT XIII (continued)

2. The methods set out in chapters 9 and 11 can very well be applied here, since the centre of the sundial is present. Only this: in order to draw the rays of the signs from the point on the axis where the tip of the style is placed — because we do not have the intersection of the equinoctial line with the meridian, to which the radius of the Equator would have had to be drawn — we shall draw the radius of the Equator from that point perpendicular to the axis, even if it does not cut the equinoctial line; and then, at that same point on the axis, we shall set out the angles of the declinations of the other signs, and so on. So too in chapter 9: to draw out the rays of the signs from the point G once found, we must draw the straight line AG from the centre A, and at G erect a perpendicular to it for the radius of the Equator, and at this same point G set out the angles of the declinations, and so on.

The method explained in chapter 10 has no place here, since it requires the intersection of the meridian with the equinoctial line, so that a perpendicular can be raised to the axis from that point.

If one does not mind computing table 19 for the given elevation of the pole from Problems 10 and 17 at the end of the little book — which is really not at all laborious or difficult — the arcs of the signs will be described most expeditiously by the method we handed down in the scholium to chapter 9, Number 5. For the Tangents of that table, transferred from the centre with respect to the whole sine equal to the style, will indicate the points of the parallels. But where there is no centre, as in the preceding chapter, those Tangents of the northern signs must be subtracted from the Tangents of the Equator; likewise the Tangents of the Equator from the Tangents of the southern signs; and the Tangents that remain

must be carried over into the hours from the hour-points of the equinoctial line, as we advised at the end of the aforesaid scholium to chapter 9.

CAPVT XIII.

HORAE AB ORTV ET OCCASV IN Horologio Horizontali.

(The hours from rising and setting on a horizontal sundial.)

1. The description of the hours from rising and setting on a horizontal sundial is very easy. For, as is gathered from the figure in the scholium to chapter 1, Number 3, the two hours of the following table — which is constructed from that figure — cut each other on the line of hour 24 from rising or setting, on any plane except the plane of a horizontal sundial, on which they are equidistant, as we demonstrated in Book 2 of the *Gnomonice*, Proposition 20, and in the scholium to Proposition 22. If through the points [the construction proceeds as follows]:

Hour 24 from rising or setting cuts, at the same point, two hours of this table on any sundial except the horizontal, on which any two of them are parallel.

[Margin: Which hour from rising or setting, and which from noon or midnight, cut one another at the same point at hour 24 on any sundial, or are parallel to each other on a horizontal one.]

										1		1
Horæ ab or. vel occ.	1	2	3	4	5	6	7	8	9	0	11	2
H. à mer. vel med. no.	½	1	1	2	2	3	3	4	4	5	5	6
			½		½		½		½		½	
		1		1		1		2		2		2
Horæ ab ortu vel occ.	13	4	15	6	17	8	19	0	21	2	23	4
H. à mer. vel med. noc.	6	7	7	8	8	9	9	1	10	1	11	1
	½		½		½		½	0	½	1	½	2

(Table headers verbatim: "Hours from rising or setting" over "Hours from noon or midnight." The lower value is half the upper number throughout.)

[Margin: Through which points on the equinoctial line the hours from rising or setting are to be drawn on any sundial.]

The hour-points of the equinoctial line, through which the hours from rising and setting must pass, are to be drawn parallel to the hours from noon or midnight that are set down beneath them in this upper table; when this is done, the hours from rising and setting will be described. And through which points on the equinoctial line the hours from rising and setting are to be drawn, this other table shows, which we have excerpted from the same figure of the scholium to chapter 1, Number 3, for whole hours.

The equinoctial line cuts, at the same point, three hours of this table on any plane of a sundial.

											1	1	1
Horæ ab or. vel occ.	1	2	3	4	5	6	7	8	9	0	1	2	
H. à mer. vel med. noc.	7	8	9	1	1	1	1	2	3	4	5	6	
				0	1	2							
Horæ ab or. vel occ.	1	1	1	1	1	1	1	2	2	2	2	2	
	3	4	5	6	7	8	9	0	1	2	3	4	

(Three-row table: an hour from rising/setting 1–12, its corresponding hour from noon/midnight, and an hour from rising/setting 13–24, all meeting at one point.)

[Margin: By what method one may know, from memory, which two hours cut hour 24 at the same point on any sundial, or are parallel on a horizontal sundial.]

[Margin: By what method one may easily know at which hours they cut one another on the equinoctial line.]

These two hours can easily be held in memory — one from rising or setting, the other from noon or midnight — which cut hour 24 at the same point, or are parallel on a horizontal sundial. For such are any two hours, one from rising or setting and the other from noon or midnight, of which the second's number is half the first's number. Thus in the earlier table you see that hour 11 from rising or setting and hour 5½ from noon or midnight occupy the same cell; likewise hour 4 from rising or setting and hour 2 from noon or midnight, and so on. Therefore, given any hour from rising or setting, half its number will give the hour from noon or midnight which, together with it, cuts hour 24 (that is, the horizontal line) at the same point, or is parallel to that hour from rising or setting on a horizontal sundial. And given any hour from noon or midnight, double its number will give the hour from rising or setting which, together with it, cuts hour 24 at the same point — that is, the horizontal line — or is equidistant from that

hour from noon or midnight on a horizontal sundial. This is plainly seen in the earlier table.

With no less ease you will find out which hours from rising or setting cut, at the same point on the equinoctial line, any given hour from noon or midnight. For if to the given hour from noon or midnight you add 6, you will have one of the hours from rising or setting; and if to this you add 12 — or subtract, if it is greater than 12 — the other will be obtained. For instance, if you wish to know which hours from rising or setting cut hour 1 from noon or midnight at the same point on the equinoctial line: add 6, and you will make hour 7 from rising or setting. To this add again 12, and you will make up hour 19 from rising or setting. The hours from rising or setting sought, therefore, are 7 and 19. Suppose also that two hours from rising or setting are to be found which cut hour 8 from noon or midnight at the same point on the equinoctial line. Adding 6 to 8 makes hour 14 from rising or setting; from which, if 12 be taken away, the remaining other hour becomes 2 from rising or setting. But if any hour from rising or setting is proposed, and it is asked which hours it cuts at the same point on the line of the equinoctial: 12 must be added, or (if it is greater) subtracted, in order to have the other hour from rising or setting. And if to the smaller of the two hours from rising or setting found you add 6, you will find the hour from noon or midnight sought. For example, given hour 4 from rising or setting: add 12, and you will make hour 16 from rising or setting. Add 6 to the same hour 4 from rising or setting, which is less than 16, and you will make up hour 10 from noon or midnight. Again, if hour 9 from rising or setting is given: adding 12 makes hour 21 from rising or setting; but taking 6 from hour 9 from rising or setting (which is less than 21), the remainder becomes hour 3 from noon or midnight; and so with the rest. All this stands clear from the later table.

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[Margin: The finding of the points on the 6-o'clock line for drawing the hours from rising or setting.]

To draw easily — through that point of the equinoctial line through which a given hour from rising or setting must be drawn — a line parallel to any hour-line from noon or midnight, the line which must be equidistant from that hour from noon or midnight, we shall use this device. On the meridian take a portion BZ equal to the portion BG, and through Z draw to the meridian a hidden perpendicular Ze, on which the points are to be marked where it is cut by the hour-lines from noon or midnight. Or indeed, if perhaps the hour-lines have not been drawn, then, with Z ω drawn perpendicular to the axis, let the hour-points on Ze be found by means

of the Tangents, with respect to the whole sine $Z\omega$ divided into 10 parts, just as on the equinoctial line. For Ze is likewise an equinoctial line with respect to a style let fall from ω perpendicularly to the meridian.

Diagram. A large horizontal-sundial construction enclosed in a rectangular frame. The meridian runs vertically down the left border; a horizontal line runs from the point **A** (upper left, on the meridian) rightward through the centre **G**, from which the ordinary hour-lines and the declination-lines radiate. A large semicircle (the horizon) is struck within the frame. Superimposed are the ordinary hour-lines (solid) labelled with Roman numerals, the Babylonian/Italian hour-lines "from rising and setting" (mostly the fainter/dotted lines) labelled with Arabic numerals 1–24, and the parallels of declination marked with zodiac-sign glyphs. The construction points **B**, **Z**, **e**, ω (with **aa**, **bb** on the line Ze), **H**, **K**, **O**, **P**, **L** appear around the figure. The engraving is extremely dense and many peripheral labels do not resolve at the available resolution; those legible are listed below.

- Frame: rectangular border; meridian = left vertical edge.
- **A** (upper left, on meridian): construction point, left end of line AG .
- **G** (centre, on line AG): centre of the sundial; origin of the radiating hour- and declination-lines.
- **B**, **Z** (right / centre-right, on the meridian portion): points defining $BZ = BG$.
- **e** (right, on line Ze): end of the hidden perpendicular Ze .
- ω (upper right): foot of the auxiliary style; $Z\omega$ perpendicular to the axis.
- **aa**, **bb** (right region, on/near Ze): hour-points on the line Ze .
- **H** (right, on the horizontal line): construction point.
- **K**, **O** (lower left corner region): construction points.
- **P**, **L** (top edge, left of centre): hour-line labels.
- Ordinary hour-lines (Roman, reading across the top / sides): VI, VII, VIII, XI, XII, I, II, V [several further Roman labels illegible — [?]].
- Babylonian/Italian hour-lines (Arabic): 3, 10, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23 legible at various positions [remaining values 1–24 not all separately resolvable — [?]].
- Zodiac-sign glyphs marking the parallels of declination: Υ (Aries) and ϖ (Taurus) legible; several further glyphs ($\varrho/\mathfrak{M}/\mathfrak{V}$ region) present but not securely resolved — [?].

[More than two peripheral labels are flagged [?]:**]

DEINDE: Then, placing one foot of the compass on the hour-point of the equinoctial line through which the proposed hour from rising or setting must be

drawn (this point the upper, second table will supply), extend the other foot to the point of that hour from noon or midnight to which the proposed hour from rising or setting is parallel (this point the first table will show). Transfer this interval onto the 6-o'clock line from the centre of the sundial, G, and onto the line Ze from the point of the same hour from noon or midnight, toward the same side toward which it was transferred onto the 6-o'clock line. For a straight line drawn through the first hour-point of the equinoctial line and through the other two points on the straight lines Ge and Ze will be parallel [Margin: ^a Euclid I.33] to the line

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of the hour-line from noon or midnight drawn through the second hour-point of the same equinoctial line. Moreover, the said interval must always be transferred, on the 6-o'clock line, to the side opposite the point through which the hour from noon or midnight is drawn — namely, to the left side of the meridian line, if that point occupies the right side of the same meridian line, and the reverse.

But let us now illustrate the description of the hours from rising or setting by examples. Hour 23, by the first table of this chapter, is equidistant from hour 11½, and passes, by the second table, along the equinoctial line through hour 5. We shall transfer the interval taken between hour 5 and hour 11½ on the equinoctial line onto the 6-o'clock line, from G, up to the point 23; and onto the line Ze, from aa (the point of hour 11½), up to bb. For the straight line bb–23, passing through hour 5 on the equinoctial line, will be hour 23, and will be equidistant from hour 11½. By the same reasoning hour 22 will pass through hour 4 on the equinoctial, and will be equidistant from hour 11. Hour 21, then, will be drawn through hour 3, parallel to hour 10½; and hour 20 through hour 2, parallel to hour 10; and hour 19 through hour 1, parallel to hour 9½. Likewise hour 18 through hour 12, parallel to hour 9.

[Margin: The description of the hours from setting, from 23 through 22, 21, and so on down to 13; and of the hours from rising, from 2 up to 12.]

Diagram. A second, near-identical horizontal-sundial construction, the companion to the figure on the preceding page. The same elements recur: the rectangular frame, the meridian down the left edge, the centre **G** with radiating hour- and declination-lines, the horizon semicircle, the ordinary hour-lines (Roman), the Babylonian/Italian hour-lines 1–24 (Arabic), and the zodiac-marked parallels of declination, with construction points **A, G, B, Z, e, ω, aa, bb, H, K, O, P,**

L, δ. As on the previous page the label-density exceeds what resolves cleanly; legible labels are listed.

- Frame, meridian (left edge), centre G, horizon semicircle — as in fig-p677-1.
- A (upper left, on meridian); B, Z, e, ω, aa, bb (right region, on/near line Ze); H (right, on horizontal line); K, O (lower left); P, L (top edge).
- Ordinary hour-lines (Roman): VI, VII, VIII, XI, XII, I, II, V legible [others [?]].
- Babylonian/Italian hour-lines (Arabic): 10, 12, 14, and 13–23 region legible in part [not all separately resolvable — [?]].
- Zodiac glyphs marking parallels of declination: ♈ (Aries), ♉ (Taurus) legible; δ-like and further glyphs at the lower right present but unresolved — [?].

[More than two peripheral labels are flagged [?]:**]

PER eadem: Through the same points of the 6-o'clock line the hours 17, 16, 15, 14, 13 must be drawn in reverse order. For from the figure of the scholium to chapter 1, Number 3, it is gathered that any two hours from rising or setting, equally distant on either side from hour 6 [i.e. from the 6-o'clock line] from noon or midnight, cut the 6-o'clock line at the same point — such as 17 and 19; likewise 16 and 20, as well as 15 and 21; and moreover 14 and 22, and 13 and 23. Therefore hour 17 must be drawn through hour 11 from noon or midnight on the equinoctial, and through hour 19 on the 6-o'clock line. Hour 16 through hour 10 from noon or midnight on the equinoctial, and through hour 20 on the 6-o'clock line. Hour 15 through hour 9 from noon or midnight on the equinoctial, and through hour 21 on the 6-o'clock line. Hour 14 through hour 8 from noon or midnight, and through hour 22 on the 6-o'clock line. And hour 13 through hour 7 on the equinoctial, and through hour 23 on the 6-o'clock line.

There is no different method for describing the hours from rising. For hour 1 from rising must be drawn through hour 7 from noon or midnight on the equinoctial, equidistant from hour $\frac{1}{2}$ from noon. And hour 2 from rising through hour 8 on the equinoctial, parallel to hour 1 from noon, and so on, as is gathered from the two tables above.

(Chapter XIII continues.)

[Margin: Another, convenient way of finding the points on the 6-o'clock line for drawing the hours from rising or setting.]

These same intervals, transferred from **G** onto the line of hour 6, will be exhibited by the distances between **Q** (taking the straight line **BQ** equal to the interval **BD**)

and the hours 5, 4, 3, 2, and so on, on the equinoctial line. For, as is established from the table of Chapter 3, Number 3, the point of hour $11\frac{1}{2}$ on the equinoctial is found by the distance between **Q** and hour 5; hour 11, by the distance between **Q** and hour 4; and so on. From this it follows that the interval between **Q** and hour 5 on the equinoctial — since it is equal to the portion of the equinoctial between hours 5 and $11\frac{1}{2}$ — is equal to the portion of the line of hour 6 between **G** and point 23. And so with the rest.

For you will always find that the interval between **Q** and the hour on the equinoctial through which some hour from rising or setting must pass (as prescribed by the later table of this chapter) is equal to the interval between that same hour on the equinoctial and that other hour from which the hour from rising or setting must be equidistant (as prescribed by the earlier table of the same chapter). For example, the interval between **Q** and hour 13 from rising or setting is equal to the interval between the same hour 7 and hour $6\frac{1}{2}$, from which hour 13 must be equidistant; and this because the point of hour $6\frac{1}{2}$ is found from the interval between **Q** and hour 7, as is established from the table set out in Number 3, Chapter 3. The same reasoning holds for the rest.

And in this way the points of all the whole hours from rising or setting will be found on hour 6, even though the points of the half-hours have not been marked on the equinoctial. But if the points of the half-hours have been marked, then by means of them the points of the half-hours from rising or setting will be found on hour 6, even though the quarter-hours on the equinoctial, from which they must be equidistant, have not been drawn. Finally, if the points of the quarter-hours have been impressed on the equinoctial, then by means of them the points of the quarter-hours from rising or setting will be found on hour 6, even though the eighth-parts of the hours on the equinoctial, from which they must be equidistant, have not been found — which is worth noting.

[Margin: The drawing of hour 12 from rising or setting, together with the other hours from setting that precede it, and the others from rising that follow it.]

Hour 12 from rising or setting must be drawn parallel to the equinoctial line — or perpendicular to the meridian — through the point that bisects the portion of the meridian **BG** lying between the equinoctial and the center of the dial, as we shall demonstrate. Hour 11 must be drawn through hour 5 on the equinoctial (as is clear from the later table of this chapter) and through the point of hour 6 by which hour 1 from rising is drawn; and this point is found by the interval between hours $\frac{1}{2}$ and 7 on the equinoctial, transferred from **G** to the right onto the line of hour 6 — because hours 11 and 1 from rising or setting cut hour 6 at

the same point, as is evident from the figure of the scholium to Chapter 1, Number 3. Hour 10 must be drawn through hour 4 on the equinoctial, and through the point of hour 6 through which hour 2 from rising passes; and this is found by the interval between hours 1 and 8 on the equinoctial. And so on, if more hours can be drawn above your horizon.

This is not difficult, if the said figure of the scholium to Chapter 1, Number 3, is considered carefully and attentively, together with the later table of this chapter. The same, understood in a certain proportion, is also to be grasped for hours 13, 14, 15, and so on, from rising. For hour 13 from rising must be drawn through hour 7 on the equinoctial, and through hour 23 from setting on hour 6, and so on, as is plain from the same later table of this chapter and from the same figure of the scholium to Chapter 1, Number 3.

*2. That hour 12 must be drawn through the midpoint of the straight line **GB**, perpendicular to the meridian, I demonstrate as follows. Let an analemma be described, in which the meridian is **AEF**; the axis of the world **ID**; the diameter of the equator **HD**; the vertical diameter **ED** — on which, produced, let **DC** be taken equal to the style [gnomon] — and through **C** let **GB** be drawn perpendicular to **EC**, which the axis cuts at **G**, and the diameter of the equator at **B**. Let arc **IK** also be taken equal to arc **IF**; and, the straight line **FK** being joined, let the straight line **KD** be drawn, cutting **GB** at **L**. By what we demonstrated in Book 2 of the Gnomonics, the center of the horizontal dial will be **G**; and through **B** the equinoctial line must be drawn perpendicular to the meridian **GB**. **FK** is the diameter of the greatest circle of perpetual apparition, and **KD** the diameter of the circle of hour 12 from rising or setting, cutting the meridian at the point **L**, through which the line of hour 12 must be drawn perpendicular to the meridian **GB**. I say that **GB** is bisected at **L**.*

*For since the angles **FDI** and **KDI** are equal [a], and angle **FDI** is equal to angle **DGL** [b], and angle **KDI** to angle **GDL** [c], the angles **DGL** and **GDL** will also be equal, and therefore [d] the straight lines **LD** and **LG** will be equal. Again, since **HI** is a quadrant, the two remaining arcs **IF** and **HA** — out of the semicircle — will each be equal to a single quadrant, and hence to the quadrant **HI**; and, the equal arcs **IF** and **IK** being subtracted, the remaining arcs **HA** and **HK** will be equal. Therefore [e] the angles **ADH** and **KDH** will be equal. But [f] angle **ADH** is equal to angle **DBL**, and [g] angle **KDH** to angle **BDL**. Therefore the angles **DBL** and **BDL** will also be equal; and consequently [h] the straight lines **LD** and **LB** will be equal. But **LG** was equal to the same **LD**. Therefore **LG** and **LB** are equal to one another. Which was to be shown.*

[Margin: Proof citations for the demonstration — a) *Elements* III.27; b) *Elements* I.29; c) *Elements* I.15; d) *Elements* I.6; e) *Elements* III.27; f) *Elements* I.29; g) *Elements* I.15; h) *Elements* I.6.]

Diagram. An analemma construction set within a rectangular frame. A semicircle stands on the horizontal diameter **A–D–F** (the meridian **AEF**), with **A** at the left end, **D** at the center, and **F** at the right end. Radiating from the center **D** to the arc are lines to the labelled points **H** (upper left), **E** (top), **K** (upper right), and **I** (right). The vertical diameter **ED** runs from **E** at the top down through **D**; produced below **D** it reaches **C** on a lower horizontal line. That lower line **GB** (perpendicular to **EC** at **C**) carries, left to right, the points **G**, **L**, **C**, **B**.

- A (left end of horizontal diameter): terminus of meridian
- D (center): center of the figure; center of dial construction
- F (right end of horizontal diameter): terminus of meridian
- E (top of arc): top of vertical diameter ED
- H (upper-left of arc): terminus of equator-diameter HD
- K (upper-right of arc): arc IK laid off equal to arc IF
- I (right of arc): terminus of axis of the world ID
- C (on lower line, below D): DC taken equal to the style
- G (left end of lower line GB): center of the horizontal dial
- L (on GB between G and C): where KD cuts GB (bisection point)
- B (right end of lower line GB): where the equator-diameter meets GB

3. In Chapter 19, Number 9, you will find the method by which, if you please, a point can be found — for any hour from rising or setting — on any hour from noon or midnight; and hence the method by which the points for the hours from rising or setting can be sought on the line of hour 6, even if the intervals of the equinoctial line (spoken of in Number 1 of this chapter) are not transferred onto hour 6, as we shall say there.

[Margin: Where another way of finding the points on the line of hour 6 for the hours from rising or setting is given.]

4. Also, by the distances of the hours from rising or setting from the meridian, their points in the tropics can, if you wish, be found, as we shall make plain in the following chapter, Number 4.

[Margin: Where the finding of the points in the tropics for the hours from rising or setting is taught.]

CAPVT XV.

ARCVS SIGNORVM IN HORIS AB OR. VEL occ. horologij horizontalis.

(The arcs of the signs in the hours from rising or setting of the horizontal dial.)

1. Although it may seem superfluous to teach how the arcs of the signs are to be drawn in the hours from rising or setting — since we have already drawn them in the hours from noon or midnight, so that they can readily be transferred onto a dial of the hours from rising or setting (for these will cut the hours from rising or setting at the points of those same arcs of the signs; or, if only the tropics are wanted, and if they have already been drawn through the hours from noon or midnight, the hours from rising or setting need not be produced beyond the outermost arcs, which belong to ♋ [Cancer] and ♎ [Capricorn]) — nevertheless, for the benefit of

students, it is worth prescribing here certain methods by which the points of the arcs of the signs may be found in the hours from rising or setting, even if they had not been found in the hours from noon or midnight. I shall propose several methods, since for certain hours one is more convenient than another, as practice will teach you. Let this, then, be the first way.

[Margin: The finding of the points of any parallel whatever in the hours from rising or setting, from Table 7, computed for latitude 42°.]

From Table 7, which has been computed for latitude 42° for the whole hours (how anyone may, by his own effort, compute another table for any elevation whatever, we shall show at the end of this little work, in Problem 12; for it did not seem good to compute more, lest the book should swell too much) — take the complements of the arcs of the hours from rising or setting, set opposite the sign presented, and their tangents with respect to a whole sine of 10 (these are to be taken from the straight line whose portion **bc**, standing for the line of hour 6, has been divided into 10 parts); and let them be transferred from **G** and **Z**, in opposite directions, onto the line of hour 6 and the line **Ze**, marking the points on them. For the straight lines drawn from **B** through these points will cut the hours from rising or setting at the points of the proposed parallel.

For example: let the point of ♎ [Capricorn] on hour 21 be sought. This arc contains 49°19', and so its complement is 40°41', whose tangent — $8 \frac{6}{10}$, approximately — will give the points **cc** on the lines **GL** and **Ze**. The straight line **ccBcc**, therefore, will cut hour 21 at the point of ♎. The tangents are to be transferred to the opposite sides of the lines **GL** and **Ze** in such a way that the

lines drawn from **B** and extended through those points can cut the proposed hours at the sought points of the parallels — which will not be difficult to judge. Namely, for the afternoon hours of the southern parallels and for the morning hours of the northern parallels, transfer from **G** toward the right and from **Z** toward the left; the opposite is to be understood for the morning hours of the southern parallels and the afternoon hours of the northern parallels.

You also have an example in the point of $\mathfrak{S}$ [Cancer] on hour 12. For this arc comprises $25^{\circ}18'$, and therefore its complement is $64^{\circ}42'$, whose tangent — $21\frac{1}{10}$ — presents the points **dd** on the lines **GL** and **Ze**. The straight line **ddBdd**, therefore, will cut hour 12 at the point of $\mathfrak{S}$. And so with the rest. I bring forward no more examples, lest the figure be obscured by so great a multitude of lines — especially since this method does not differ from the one given in Chapter 8, Number 1, except that here Table 7 is to be used, but there Table 1.

[Margin: The finding of the points of any parallel whatever in the hours from rising or setting, from Table 8, computed for latitude 42° .]

2. A second method rests on Table 8, and does not differ from the manner we set out in Chapter 8, Number 7. For by Table 8 we shall find, on the meridian line, points through which — if lines parallel to the equinoctial line are drawn — the several hours from rising or setting will be cut at the points of the parallels; just as there, from Table 1, points were found through which straight lines parallel to the equinoctial line cut the hours from noon or midnight, each line, that is, cutting two hours. This Table 8, however, has been computed only for latitude 42° , for the whole hours. But if anyone wants similar tables for other latitudes, he will find the way of computing them at the end of the book, in Problem 12.

In order to find the points of the parallels in the hours from rising or setting for any elevation of the pole, we have computed the following general table, by Problem 12 placed at the end of the book. It contains the arcs of the meridian intercepted, at the beginnings of the signs, between the equator and the great circles that are drawn — from the intersection of the equator with the horizon, or with the 6-o'clock circle from noon or midnight — through the hours from noon and their minutes: that is, if the hours are taken at the top of the table and the minutes on the left side. For if the hours are taken at the foot of the table, and the minutes on the right side, the same arcs will be had on the 6-o'clock circle, intercepted at the beginnings of the signs, between the equator and the great circles that are drawn — from the intersection of the equator with the meridian — through the hours from noon and their minutes.

But in order to fit this table to the hours from rising or setting in any region, the distances of those hours from the meridian circle must first be found, as I shall teach below; and the hours of these distances are to be taken at the top of the table, the minutes on the left side. For in the common corner you will find the arcs of the meridian by which the points of the parallels will be found in the hours from rising or setting, as we taught in Number 7, Chapter 8, and reminded in Number 2 of this chapter. For example: at latitude 42° , the distance of hour 21 from setting is distant from noon by 4h32m, at the beginning of $\mathfrak{S}$ [Cancer], to which, in the following table, there corresponds an arc of $49^\circ 15'$. And when the pole's altitude of 42° is subtracted, there remains an arc of $7^\circ 15'$, whose tangent — $1 \frac{2}{10}$ and a little more — transferred upward from **C**, will give a point on the meridian; and if a line parallel to the equinoctial is drawn through this point, hour 21 will be cut at the point of $\mathfrak{S}$. And so with the rest. So that you may know how to proceed in the other hours — especially when they are more than 6 hours from noon — consult Number 7, Chapter 8. How the said arcs may be more accurately drawn out of the table by proportional parts, we shall teach in Practice 6, Number 4, excerpted there.

Moreover, by these same tangents we shall very conveniently find these same points of the parallels of the Sun, even if we do not draw the lines parallel to the equinoctial or perpendicular to the meridian, as was said in Number 8, Chapter 8 — in this way, namely. Let the tangent for any hour and sign be taken with the compass from the straight line whose portion **EF**, at the beginning of Chapter 2, was divided into 10 parts. For this tangent — if one foot of the compass is placed on the given hour, on the condition that the arc drawn by the other foot touches, but does not cut, the line **CD** extended both ways — will give on the hour-line the sought point of the parallel. Nor will any notable error be admitted in this matter, if careful diligence is applied.

Only this method will be less reliable on those hour-lines that are too obliquely inclined to the equinoctial, because the line **CD** can be touched appreciably at several points of those lines by the arcs drawn at the intervals of the tangents. For this reason, on those lines the points of the parallels must be found either by Rules 1 and 2 of the scholium to Chapter 8, with the help of points found on other lines; or by the points on the line of hour 6 found from the same following table, as was taught in Number 1, Chapter 8; or else by the means we shall write of in Number 3 of this Chapter 15 — namely, by the distances of the hours from noon, found in degrees and minutes.

Indeed, even without a compass we shall very opportunely transfer these same tangents onto the hour-lines for finding the points of the parallels, in this way. On some rather stiff paper — such as royal or imperial paper — let the divisions of the line **EF** be made, as we said at the beginning of Chapter 2. Then let the paper be cut so that the side containing the divisions occupies its edge and serves as a kind of ruler; and let the same divisions be transferred to some line of the same paper parallel to the aforesaid edge, which (once the superfluous parts are cut away) occupies the other edge — so that the straight lines joining the numbers 10, 20, 30, 40, and so on are all perpendicular to the two aforesaid edges, so that a rectangular parallelogram is made, divided into several parts by parallel lines. For if that perpendicular which is drawn through the number 10, or 20, or 30, and so on (as the magnitude of the tangent requires) is applied to the line **CD** drawn through the position of the style — by moving the ruler back and forth until the end of the tangent marked on the little line **EF** reaches the proposed hour — that endpoint must be marked as the point of the parallel. And this method, as you see, is common also to the hours from noon and midnight.

And if other similar rulers are made for taking the tangents on the equinoctial line, or on the line of hour 6, or on the other lines, as was taught in Chapter 2, all the tangents will be transferred onto them, without a compass, most exactly — provided each ruler is applied to its proper line, and so on.

Now the distances that the hours from rising or setting have from noon will easily be found from the semidiurnal arcs. For the semidiurnal arc gives the distance of hour 24. And if you subtract one hour, you will have the distance of hour 23, and so on. But when one hour can no longer be subtracted, one must take the complement of the last distance found — the complement, I mean, to 60 minutes — for the distance of the next hour before noon; and if to this you add one hour, you will produce the distance of the preceding hour, and so on. For example: at latitude 42° , at the beginning of ♋ [Cancer], the distance of hour 17 from setting is found to be 0h32m. Therefore hour 16 is distant before noon by 0h28m, while hour 15 has a distance of 1h28m; and so with the rest. Likewise, if you subtract one hour from the semidiurnal arc, you will make the remainder the distance of hour 1 from rising, and so on.

Furthermore, you will obtain the semidiurnal arc in hours and minutes if you divide the semidiurnal arc in degrees — from Table 4, computed at the end of the book — by 15, to get the hours; and assign to each of the remaining degrees 4 minutes of one hour, and to each minute of the degrees 4 seconds of one hour. These seconds may be omitted when they do not exceed 30; but when they do

exceed 30, one whole minute must be taken for them. For example: at latitude 42°, at the beginning of ♉ [Taurus] and ♍ [Virgo], the semidiurnal arc in Table 4 contains 100°33′. Dividing the 100 degrees by 15 yields 6 hours, with 10 degrees left over, which multiplied by 4 give 40 minutes. Finally, 33 multiplied by 4 gives 132 seconds, which divided by 60 make 2 minutes, with 12 seconds left over, which are to be neglected. The whole semidiurnal arc, therefore, contains 6h42m. And so with the rest.

THERE NOW FOLLOWS THE AFOREMENTIONED TABLE.

(The announced table is not printed on this leaf; the rest of the page is blank, with only show-through of the table on the following leaf visible. The table itself falls on the next page.)

The Table Foretold (Table 18 of the meridian arcs)

[Table] Top title, verbatim: *"Arcus Meridiani inter Æquatorem, & circulos maximos ex intersectione Æquatoris cum Horizonte per horas à meridie earumque minuta ductos, intercepti."* — The arcs of the meridian intercepted between the equator and the greatest circles that are drawn, for the hours from noon and their minutes, through the intersection of the equator with the horizon.

This is a double-entry table. Each of the three panels on the page is one hour reckoned from noon (top labels **hor. 0**, **hor. 1**, **hor. 2**); the same body read from the bottom serves the complementary hours (bottom labels, printed inverted: **hor. 5**, **hor. 4**, **hor. 3**), under the foot-title: *"Arcus circuli horæ 6. inter Æquatorem & maximos circulos ex intersectione Æquatoris cum Meridiano per horas à meridie earumque minuta ductos, intercepti."* — The arcs of the hour-6 circle intercepted between the equator and the greatest circles drawn, for the hours from noon and their minutes, through the intersection of the equator with the meridian.

In every panel the left index column (0–60, the minutes read downward) heads the panel's hour; the right column ("min.", 60–0, read upward) heads the foot-hour. The three value columns are each headed by a tetrad of zodiac signs sharing the same declination; each cell is printed as **G. M.** (degrees, minutes). Column headings (verbatim glyph groups): ♈ ♉ | ♊ ♋ ≈ ♌ | ♍ ♎ ♏ ♐.

Page 46 — hour 0 (foot: hour 5)

hor. 0	ᠰ ᠶ᠋ᠨ	ᠰᠢ ᠤᠯᠤᠰ ᠰᠢ	ᠰᠢ ᠶ᠋ᠨ ᠰᠢ	min.
0	23 30	20 12	11 30	60
1	23 30	20 12	11 30	59
2	23 30	20 12	11 30	58
3	23 30	20 12	11 30	57
4	23 30	20 12	11 30	56
5	23 30	20 12	11 30	55
6	23 30	20 12	11 30	54
7	23 31	20 13	11 30	53
8	23 31	20 13	11 30	52
9	23 31	20 13	11 31	51
10	23 31	20 13	11 31	50
11	23 31	20 13	11 31	49
12	23 32	20 14	11 31	48
13	23 32	20 14	11 31	47
14	23 32	20 14	11 31	46
15	23 33	20 14	11 31	45
16	23 33	20 15	11 32	44
17	23 33	20 15	11 32	43
18	23 34	20 15	11 32	42
19	23 34	20 16	11 32	41
20	23 35	20 16	11 33	40
21	23 35	20 17	11 33	39
22	23 36	20 17	11 33	38
23	23 35	20 18	11 33	37
24	23 37	20 18	11 34	36
25	23 37	20 19	11 34	35
26	23 38	20 19	11 34	34
27	23 39	20 20	11 35	33
28	23 39	20 20	11 35	32
29	23 40	20 21	11 35	31

hor. 0	ᄡ ᆫ	ᄡ ᄡ ᄡ ᄡ	ᄡ ᄡ ᄡ ᄡ	min.
30	23 41	20 22	11 36	30
31	23 42	20 22	11 36	29
32	23 42	20 23	11 37	28
33	23 43	20 24	11 37	27
34	23 44	20 25	11 37	26
35	23 45	20 25	11 38	25
36	23 46	20 26	11 38	24
37	23 47	20 27	11 39	23
38	23 47	20 27	11 39	22
39	23 48	20 28	11 40	21
40	23 49	20 29	11 40	20
41	23 50	20 30	11 41	19
42	23 51	20 31	11 41	18
43	23 52	20 32	11 42	17
44	23 53	20 33	11 43	16
45	23 55	20 34	11 43	15
46	23 56	20 35	11 44	14
47	23 57	20 36	11 44	13
48	23 58	20 37	11 45	12
49	23 59	20 38	11 46	11
50	24 0	20 39	11 47	10
51	24 2	20 40	11 47	9
52	24 3	20 41	11 48	8
53	24 4	20 42	11 48	7
54	24 6	20 44	11 49	6
55	24 7	20 45	11 50	5
56	24 9	20 46	11 51	4
57	24 10	20 47	11 51	3
58	24 11	20 49	11 52	2
59	24 13	20 50	11 53	1
60	24 14	20 51	11 54	0

Page 46 — hour 1 (foot: hour 4)

hor. 1	𐎢 𐎠	𐎡 𐎢 𐎠 𐎡	𐎢 𐎠 𐎡 𐎢	min.
0	24 14	20 51	11 54	60
1	24 16	20 52	11 54	59
2	24 17	20 54	11 55	58
3	24 19	20 55	11 56	58 [recte: 57]
4	24 20	20 57	11 57	56
5	24 22	20 58	11 58	55
6	24 24	21 0	11 59	54
7	24 25	21 1	12 0	53
8	24 27	21 3	12 1	52
9	24 29	21 4	12 2	51
10	24 31	21 6	12 3	50
11	24 32	21 7	12 3	49
12	24 34	21 9	12 4	48
13	24 36	21 11	12 5	47
14	24 38	21 12	12 7	46
15	24 40	21 14	12 8	45
16	24 42	21 16	12 9	44
17	24 44	21 17	12 10	43
18	24 46	21 19	12 11	42
19	24 48	21 21	12 12	41
20	24 50	21 23	12 13	40
21	24 52	21 25	12 14	39
22	24 54	21 27	12 15	38
23	24 56	21 29	12 16	37
24	24 58	21 31	12 18	36
25	25 1	21 33	12 19	35
26	25 3	21 35	12 20	34
27	25 5	21 37	12 21	33
28	25 7	21 39	12 23	32
29	25 10	21 41	12 24	31

hor. 1	☿ ♄	♂ ♀ ☿	♂ ☿ ♀ ♄	min.
30	25 12	21 43	12 25	30
31	25 15	21 45	12 26	29
32	25 17	21 47	12 28	28
33	25 20	21 49	12 29	27
34	25 22	21 52	12 31	26
35	25 25	21 54	12 32	25
36	25 27	21 56	12 33	24
37	25 30	21 59	12 35	23
38	25 32	22 1	12 36	22
39	25 35	22 3	12 38	21
40	25 38	22 6	12 39	20
41	25 40	22 8	12 41	19
42	25 43	22 11	12 42	18
43	25 46	22 13	12 44	17
44	25 49	22 16	12 45	16
45	25 52	22 18	12 47	15
46	25 55	22 21	12 48	14
47	25 58	22 24	12 50	13
48	26 1	22 26	12 52	12
49	26 4	22 29	12 53	11
50	26 7	22 32	12 55	10
51	26 10	22 34	12 57	9
52	26 13	22 37	12 59	8
53	26 16	22 40	13 0	7
54	26 19	22 43	13 2	6
55	26 23	22 46	13 4	5
56	26 26	22 49	13 6	4
57	26 29	22 52	13 8	3
58	26 33	22 55	13 9	2
59	26 36	22 58	13 11	1
60	26 40	23 1	13 13	0

Page 46 — hour 2 (foot: hour 3)

hor. 2	𐎔 𐎕	𐎎 𐎠 𐎡 𐎢 𐎣	𐎠 𐎡 𐎢 𐎣	mi n.
0	26 40	23 1	13 13	60
1	26 43	23 4	13 15	59
2	26 47	23 7	13 27 [recte: 13 17]	58
3	26 50	23 11	13 19	57
4	26 54	23 14	13 21	56
5	26 57	23 17	13 23	55
6	27 1	23 29 [recte: 23 20]	13 25	54
7	27 5	23 24	13 27	53
8	27 9	23 27	13 29	52
9	27 13	23 31	13 32	51
10	27 16	23 34	13 34	50
11	27 20	23 38	13 36	49
12	27 24	23 41	13 38	48
13	27 28	23 45	13 40	47
14	27 32	23 48	13 43	46
15	27 36	23 52	13 45	45
16	27 41	23 56	13 47	44
17	27 45	24 0	13 50	43
18	27 49	24 3	13 52	42
19	27 53	24 7	13 54	41
20	27 58	24 11	13 57	40
21	28 2	24 15	13 59	39
22	28 6	24 19	14 2	38
23	28 11	24 23	14 4	37
24	28 15	24 27	14 7	36
25	28 20	24 31	14 10	35
26	28 25	24 36	14 12	34
27	28 29	24 40	14 15	33
28	28 34	24 44	14 18	32

hor. 2	☾ ♀	♂ ♀ ☿	♂ ☿ ♀	mi n.
29	28 39	24 48	14 20	31
30	28 44	24 53	14 23	30
31	28 48	24 57	14 26	29
32	28 53	25 2	14 29	28
33	28 58	25 6	14 32	27
34	29 3	25 11	14 34	26
35	29 8	25 15	14 37	25
36	29 14	25 20	14 40	24
37	29 19	25 25	14 43	23
38	29 24	25 30	14 46	22
39	29 29	25 34	14 49	21
40	29 35	25 39	14 52	20
41	29 40	25 44	14 56	19
42	29 46	25 49	14 59	18
43	29 51	25 54	15 2	17
44	29 57	25 59	15 5	16
45	30 3	26 5	15 9	15
46	30 8	26 10	15 12	14
47	30 14	26 15	15 15	13
48	30 20	26 20	15 19	12
49	30 26	26 26	15 22	11
50	30 32	26 31	15 26	10
51	30 38	26 37	15 29	9
52	30 44	26 42	15 33	8
53	30 50	26 48	15 36	7
54	30 56	26 54	15 40	6
55	31 3	27 0	15 44	5
56	31 9	27 5	15 48	4
57	31 16	27 11	15 51	3
58	31 22	27 17	15 55	2

hor. 2	☾ ♄	♂ ♀ ☿	♄ ☿ ♀ ♀	mi n.
59	31 29	27 23	15 59	1
60	31 35	27 29	16 3	0

Table 18, continued — hours 3, 4, 5

[Table] Same title as p. 46 (*"Arcus Meridiani inter Æquatorem, & circulos maximos ..."*). Top labels **hor. 3**, **hor. 4**, **hor. 5**; the same body read from the foot serves **hor. 2**, **hor. 1**, **hor. 0** (foot-title as before). Column headings as on p. 46: ☾ ♄ | ♂ ♀ ☿ | ♄ ☿ ♀ ♀; each cell "G. M."

Page 47 — hour 3 (foot: hour 2)

hor. 3	☾ ♄	♂ ♀ ☿	♄ ☿ ♀ ♀	mi n.
0	31 25 [recte: 31 35]	27 29	16 3	60
1	31 42	27 36	16 7	59
2	31 49	27 42	16 11	58
3	31 56	27 48	16 15	57
4	32 3	27 54	16 19	56
5	32 10	28 1	16 24	55
6	32 17	28 8	16 28	54
7	32 24	28 14	16 32	53
8	32 31	28 21	16 37	52
9	32 39	28 28	16 41	51
10	32 46	28 34	16 46	50
11	32 53	28 41	16 50	49
12	33 1	28 48	16 55	48
13	33 9	28 55	16 59	47
14	33 16	29 3	17 4	46
15	33 24	29 10	17 9	45
16	33 32	29 17	17 14	44
17	33 40	29 24	17 19	43
18	33 48	29 32	17 24	42

hor. 3	᠎ᠠ ᠪᠤ	ᠠᠨ ᠢᠨ ᠤ ᠶᠢ	ᠨᠠ ᠶᠢ ᠬᠢ ᠮᠤ	mi n.
19	33 56	29 39	17 29	41
20	34 5	29 47	17 34	40
21	34 13	29 55	17 39	39
22	34 21	30 3	17 44	38
23	34 30	30 11	17 49	37
24	34 38	30 19	17 55	36
25	34 47	30 27	18 0	35
26	34 56	30 35	18 6	34
27	35 5	30 43	18 12	33
28	35 14	30 52	18 17	32
29	35 23	31 0	18 23	31
30	35 32	31 9	18 29	30
31	35 41	31 18	18 35	29
32	35 50	31 27	18 41	28
33	36 0	31 35	18 47	27
34	36 10	31 44	18 53	26
35	36 20	31 54	18 59	25
36	36 30	32 3	19 6	24
37	36 39	32 12	19 12	23
38	36 49	32 21	19 18	22
39	37 0	32 31	19 25	21
40	37 10	32 41	19 32	20
41	37 20	32 51	19 39	19
42	37 31	33 0	19 45	18
43	37 41	33 10	19 52	17
44	37 52	33 21	20 0	16
45	38 3	33 31	20 7	15
46	38 14	33 41	20 14	14
47	38 25	33 52	20 21	13
48	38 36	34 2	20 29	12

hor. 4	ᠭ ᠡ	᠒ ᠤ ᠤ ᠰ	᠓ ᠣ ᠰ ᠤ	mi n.
15	44 31	39 45	24 42	45
16	44 46	40 0	24 54	44
17	45 1	40 16	25 6	43
18	45 17	40 31	25 17	42
19	45 33	40 47	25 30	41
20	45 49	41 3	25 43	40
21	46 5	41 19	25 55	39
22	46 21	41 35	25 8 [recte: 26 8]	38
23	46 38	41 51	26 21	37
24	46 55	42 8	26 34	36
25	47 12	42 25	26 48	35
26	47 94	42 42	27 2	34
27	47 63	42 59	27 16	33
28	48 21	43 17	27 30	32
29	48 39	43 34	27 45	31
30	48 39 [recte: 48 48]	43 52	28 0	30
31	48 57	44 11	28 15	29
32	49 15	44 29	28 30	28
33	49 34	44 48	28 46	27
34	49 53	45 7	29 2	26
35	50 11	45 25	29 18	25
36	50 30	45 45	29 35	24
37	50 50	46 5	29 52	23
38	51 9	46 25	30 9	22
39	51 29	46 45	30 37	21
40	51 49	47 5	30 45	20
41	52 9	47 26	31 3	19
42	52 29	47 47	31 22	18
43	52 50	48 8	31 41	17
44	53 11	48 30	32 0	16

hor. 4	𐎔 𐎕	𐎎 𐎠 𐎡 𐎢	𐎠 𐎢 𐎠 𐎡	mi n.
45	53 32	48 51	32 20	15
46	53 53	49 14	32 40	14
47	53 14 [recte: 54 14]	49 36	33 1	13
48	54 36	49 58	33 27	12
49	54 58	50 21	33 43	11
50	55 20	50 44	34 5	10
51	55 42	51 8	34 27	9
52	56 5	51 32	34 50	8
53	56 28	51 56	35 13	7
54	56 51	52 20	35 37	6
55	57 14	52 45	36 1	5
56	58 38 [recte: 57 38]	53 10	36 26	4
57	58 1	53 35	36 51	3
58	58 25	54 0	37 17	2
59	58 50	54 26	37 43	1
60	59 14	54 53	38 10	0

Page 47 — hour 5 (foot: hour 0)

hor. 5	𐎔 𐎕	𐎎 𐎠 𐎡 𐎢	𐎠 𐎢 𐎠 𐎡	mi n.
0	59 14	54 53	38 10	60
1	59 39	55 19	38 38	59
2	60 4	55 46	39 6	58
3	60 29	56 13	39 34	57
4	60 55	56 40	40 4	56
5	61 20	57 8	41 34 [recte: 40 34]	55
6	61 46	57 36	41 4	54
7	62 12	58 5	42 30	53
8	62 39	58 34	42 8	52
9	63 5	59 3	42 40	51
10	63 32	59 32	43 14	50

hor. 5	𐎔 𐎕	𐎎 𐎠 𐎡 𐎣	𐎎 𐎔 𐎕 𐎠	mi n.
11	63 59	60 2	43 48	49
12	64 27	60 32	44 23	48
13	65 54 [recte: 64 54]	61 2	45 0	47
14	65 22	61 33	45 35	46
15	65 40	62 4	46 12	45
16	66 18	62 35	46 50	44
17	66 47	63 7	47 29	43
18	67 16	63 39	48 9	42
19	67 45	64 11	48 50	41
20	68 14	64 44	49 31	40
21	68 43	65 17	50 14	39
22	69 13	65 50	50 57	38
23	69 43	66 24	51 41	37
24	70 13	66 58	52 27	36
25	70 43	67 32	53 13	35
26	71 14	68 7	54 0	34
27	71 44	68 42	54 48	33
28	72 15	69 8	55 38	32
29	72 46	69 52	56 28	31
30	83 17 [recte: 73 17]	70 28	57 19	30
31	73 49	71 4	58 11	29
32	74 21	71 40	59 5	28
33	74 52	72 17	59 59	27
34	75 24	72 54	60 55	26
35	75 57	73 31	61 51	25
36	76 29	74 8	62 48	24
37	77 1	74 46	63 47	23
38	77 34	75 24	64 46	22
39	78 7	76 2	65 47	21
40	78 40	76 40	66 49	20

hor. 5	☾ ♀	♂ ♀ ☿	♂ ☿ ♀ ♀	mi n.
41	79 13	77 19	67 51	19
42	79 46	77 58	68 55	18
43	80 20	78 37	69 59	17
44	80 53	79 16	71 4	16
45	81 21	79 55	72 11	15
46	82 0	80 35	73 18	14
47	82 34	81 14	74 26	13
48	83 8	81 54	75 34	12
49	83 42	82 34	56 44 [recte: 76 44]	11
50	84 16	83 14	77 54	10
51	84 50	83 55	79 5	9
52	85 25	84 35	80 16	8
53	85 59	85 15	81 28	7
54	86 33	85 56	82 40	6
55	87 8	86 37	83 53	5
56	87 42	87 17	58 6 [recte: 85 6]	4
57	88 17	87 58	86 19	3
58	88 51	88 38	87 33	2
59	89 26	89 19	88 46	1
60	90 0	90 0	90 0	0

[Foot-title: Arcus circ. hor. 6. inter Æquatorē & maximos circ. ex intersectione Æquatoris cū Meridia. &c.]

Chapter 15 (continued)

[Margin: Finding the points of any parallel in the hours from rising or setting, from the distances from the meridian.]

3. A third method is drawn from the distances of the hours from noon — distances whose computation I shall teach in the scholium to this chapter, Note 1. It runs as follows. Take the tangent of the distance of any hour, reckoned to a whole sine of 10, laid off on the straight line whose portion MN — equal to the

line BD — has been divided into 10 parts for the equinoctial line; and transfer it onto the equinoctial line, before or after noon according as the distance falls before or after noon. For a straight line drawn out from G, the centre of the dial, through the end of this tangent will cut the hour proposed at the point of the parallel proposed. When the tangent cannot be transferred onto the equinoctial line, it must instead be transferred onto the line VS, laid off on the straight line whose portion [?] — divided into 10 parts — is equal to the line ST. Likewise, when the distance is greater than a quadrant, one must transfer the tangent of its complement up to 180° , from B or from S, into the opposite direction, so that a straight line from its endpoint through G can cut the given hour at the point sought.

By way of example. The distance of hour 21 from setting, in ♋ [Cancer], comprises $68^\circ 3'$, whose tangent $24 \frac{7}{10}$ gives the point *ee* on the equinoctial. The straight line G–*ee*, therefore, will cut hour 21 at the point of ♋ . Again, the distance of the same hour 21 in ♏ [Capricorn] contains $21^\circ 57'$, whose tangent 4 yields on the equinoctial the point 11, through which a straight line drawn out from G will cut hour 21 at the point of ♏ . Once more, the distance of hour 11 from setting, in ♋ , has $81^\circ 57'$, whose tangent $70 \frac{7}{10}$ — reckoned to the whole sine ST, transferred onto SV — gives the point *ff*; and the straight line G–*ff* will cut hour 11 at the point of ♋ . Finally, the distance of hour 23 from setting, in ♋ , is $98^\circ 3'$, greater than a quadrant; subtracted from 180° it leaves $81^\circ 57'$, whose tangent $70 \frac{7}{10}$ — reckoned to the whole sine ST — gives on the line SV the point *ff*, and the straight line *ff*–G, produced, cuts hour 23 at the point of ♋ .

[Margin: An easier finding of the points of parallels in the hours from rising or setting, from Table 18, by the arcs of the Horizon.]

But by Table 18 — through the arcs of the horizon between the meridian (on the southern side) and the greatest circles that pass through the poles of the world and through the whole hours from rising or setting and their halves, drawn at the beginnings of the signs in the manner of hour-circles from noon and midnight — we find the points of the parallels in the hours from rising or setting far more easily, if we count those arcs, in the circle described from G, the centre of the dial, from the meridian line toward the right or the left, according as the hours are before noon or after noon. For the straight lines sent out from G, the centre of the dial, through the ends of these arcs (which do not differ from those we said just now must be drawn through the points found on the equinoctial by the distances of the hours from noon) will cut the hours from rising or setting at the points of the parallels, since they stand for those greatest circles which are drawn through

the poles of the world — just as the earlier lines did, drawn out through the ends of the tangents that correspond to the distances of the hours from the meridian. For example: since the arc of hour 23 in $\mathfrak{S}$, from setting, contains in Table 18 $101^{\circ} 57'$, if we count it from the meridian line toward the left — that is, $11^{\circ} 56'$ beyond the line of hour 6 (having of course taken away the 90° that lie between the meridian and hour 6 in the circle) — the straight line from G, drawn through the end of that arc, will cut hour 23 at the point of $\mathfrak{S}$. And so with the rest. The construction of this Table 18 is given at the end of the little book, in Problem 9, from which anyone may build a similar one by his own effort for a given elevation of the pole.

Now this method by the arcs of the horizon of Table 18 is the most convenient, because when the distance of some hour from rising or setting from the meridian cannot be taken on the equinoctial, nothing prevents the arc of that same hour from being taken on the circle described from G.

[Margin: A very close approximation, and how it is done by tangents.]

Now indeed, if — as I shall show at the end of the little book in Problem 17, and as I explained in the scholium to Chapter 9, Note 5 — the segments of the straight lines drawn from G, the centre of the dial, through the ends of the arcs of the horizon out to the parallels are reduced to tangents equal to the style with respect to the whole sine, then on those same straight lines the points of the parallels will be found by means of those tangents. And these points can be found even if the hours from rising or setting have not yet been drawn. Once these points have been carefully found in the tropics $\mathfrak{S}$ and $\mathfrak{V}$, the hours from rising or setting must be drawn through them. I set down no example, both because the thing itself is easy from what was said in the scholium to Chapter 9, Note 5, and because time did not allow me to compute a table for the hours from rising or setting like Table 19 computed for the hours from noon and midnight. You will therefore compute it by your own effort from Problems 10 and 17 at the end of the little book.

[Margin: Finding the points for the hours from rising or setting in the tropics, or in the other parallels, by means of the distances from the meridian.]

4. Furthermore, if the arcs of $\mathfrak{S}$ and $\mathfrak{V}$ have been drawn in the hours from noon and midnight — as was handed down in Chapters 8, 9, and 10, or as we taught in Chapter 11 — we shall find, by an extremely easy method, the points in them, by the aid of the aforesaid distances or of the arcs of Table 18, through which the lines of the hours from rising or setting are to be drawn. This method fits every plane of dials, and is the most convenient for drawing certain of the hours from

rising or setting on declining dials. For instance, if we seek the point in the tropic of $\mathfrak{S}$ through which hour 11 from setting is to be drawn, we shall take its distance from $81^{\circ} 57'$; for the tangent of this, $70 \frac{7}{10}$ — reckoned to the whole sine ST, on the line SV (because reckoned to the whole sine BD it runs out too far on the equinoctial) — will give the point *ff*, and the straight line G–*ff* will cut the tropic of $\mathfrak{S}$ at the point through which hour 11 is to be drawn. And because that same hour 11 must also be drawn through hour 5 from noon or midnight on the equinoctial, hour 11 from setting will be described through hour 5 on the equinoctial and through the point found in the tropic of $\mathfrak{S}$, even if in hour 6 no other point is found. And the same method holds for the rest. In no other way will the points of these same hours be found in the other parallels drawn on the dial, whenever there is need and the matter requires it. By like reasoning, if we compute the arc of $78^{\circ} 4'$ from Table 18 — which corresponds to hour 11 from setting in $\mathfrak{S}$ — in the circle described from G, from the meridian line toward the right, then the straight line from G passing through the end of this arc will cut the tropic of $\mathfrak{S}$ at the same point of hour 11 from setting. And so with the rest.

[Margin: Finding the points in the hours from rising or setting for the parallels, by the tangents of the complements of the Sun's altitudes.]

5. By yet another way, too, you will find the points of the parallels in these same hours from rising or setting, if from Table 10 you take the complements of the Sun's altitudes, and transfer their tangents from the position of the style — reckoned to the whole sine DC, equal to the style — onto the corresponding hours. For these will cut the hours at the points of the parallels, as we showed in Chapter 8, Note 12.

[Margin: Finding the points of all the parallels in the hours from rising or setting, by the rising amplitudes (ortive latitudes).]

6. Besides, we shall find the points of the parallels in the hours from rising or setting by the rising or setting amplitudes — a method proper to the hours from rising or setting — in this way. Having prepared a figure of the rays of the rising amplitudes, in the manner of the figure of the rays of the signs of the Zodiac (as I shall teach in the scholium, Note 2), draw from the position of the style a perpendicular to the hour-line on which the points of the parallels are to be investigated; and from the point where the hour-line cuts it, take on that same hour-line a straight length equal to the style; and along that perpendicular, from the same point of intersection, take an interval equal to the distance between the far end of this style and the position of the style; and finally, from the far endpoint of this interval draw a hidden straight line through the intersection of

the given hour with the equinoctial, and describe from that same endpoint an arc of a circle equal to the circle on which the rising amplitudes are marked. For if into this arc, from the straight line drawn through its centre and the equinoctial point, the arcs of the rising amplitudes are transferred, they will cut

the straight lines drawn from the same centre and extended through these amplitudes will cut the given hour-line at the points of the parallels.

For example: in the figure of Chapter 9, let the line d–e be given as drawn, which is hour 2 from rising; and to it, from C (the position of the style), let the perpendicular C–d be drawn, and let d–e be taken equal to the style CD; and by the interval C–e let d–b be cut off, on whichever side you please. Finally, from b as centre, draw the straight line b–f through the intersection of hour 2 from rising with the equinoctial; and, having described the circle g–h, transfer onto it, from the straight line b–f, the rising amplitudes on both sides. For the straight lines drawn from b as centre through the ends of the amplitudes will cut hour 2 from rising at the points of the parallels. And the same reasoning holds for the rest.

Demonstration. The arcs of the hour-circles from rising or setting (among which the horizon too is counted, since it is the circle of hour 24) touch the two greatest of the parallels — of those always visible and of those always hidden — as we demonstrated in the *Gnomonics*, Book 1, Proposition 10. Therefore, by Proposition 13 of Book 2 of Theodosius [Theodosius, *Sphaerica* II.13], the arcs of these same circles between the equator and any parallel are equal. Since, then, the arc of the horizon between the equator and any parallel measures the rising amplitude of that parallel, the arcs of the other circles will measure the same amplitude. And so, since — as we showed in the problem of Chapter 1 — the straight lines drawn from b through the rising amplitudes on the circle g–h cut off from the hour-line d–e those same amplitudes, it is clearly established that the points of the parallels have been rightly found.

[Margin: Another way of finding the points for the parallels in the hours from rising or setting.]

7. Finally, the points of the parallels in the hours from rising or setting will be found by the methods handed down in the scholium of Chapter 10, Note 4.

SCHOLIUM

[Margin: How the distances of the hours from rising or setting from the meridian are computed.]

1. The distances from the meridian which the hours from rising or setting have, at any elevation of the pole, will be found in any parallel of the Sun by this method. From Table 4, take the semidiurnal arc of the proposed parallel, in degrees and minutes, computed for the given altitude of the pole. Then, since the semidiurnal arc indicates the distance of hour 24 from rising or setting from the meridian, if we subtract 15 degrees from it, there will remain the distance of hour 23 from setting and of hour 1 from rising. From this, if we again take away 15 degrees, we shall have the distance of hour 22 from setting and of hour 2 from rising. And, proceeding thus in succession, we shall find the distances of the other hours — observing, however, this rule: when 15 degrees cannot be subtracted from the distance last found, then let that distance last found be taken away from 15 degrees. For thus there will remain the distance of the following hour on the other side of the meridian. And if 15 degrees are added to this, the distance of the next following hour will result; and so on in succession, until a number is produced greater than the semidiurnal arc, which will indicate the distance of an hour lying below the horizon, of which therefore no account is to be taken. When, however, we need the distances of the hours below the horizon, we shall obtain them by the continuous addition of 15 degrees to the distance of the hour which either falls on the horizon or is nearest to it.

[Margin: How the distance from the meridian of any hour from rising or setting may be found, even though the distances of the other hours are not available.]

Therefore, given any hour from rising: if you subtract its distance from hour 24 (when this is less than the semidiurnal arc) from the semidiurnal arc, you will make the remainder the distance of the given hour from the meridian before noon. But if you take the semidiurnal arc away from the distance which that hour has from hour 24 (when it is greater than the semidiurnal arc), you will leave the distance of that given hour from the meridian after noon. The contrary is to be understood of any hour given from setting. For if you take its distance from hour 24 away from the semidiurnal arc, there will remain the distance of that given hour from the meridian after noon; but if the semidiurnal arc is subtracted from the distance which that hour has from hour 24, the remainder will be its distance from the meridian before noon. By this method the following table has been computed for latitude 42 degrees.

The hour from rising and from setting: distances from the meridian at the beginnings of the signs

Upper panel (each cell is degrees° minutes'; "G." = degrees, "M." = minutes):

Sign (ab ortu · pair)	24	1	2	3	4	5	6	7
☿	113° 3'	98° 3'	83° 3'	68° 3'	53° 3'	38° 3'	23° 3'	8° 3'
♈ ♏	109° 21'	94° 21'	79° 21'	64° 21'	49° 21'	34° 21'	19° 21'	4° 21'
♉ ♐	100° 33'	85° 33'	70° 33'	55° 33'	40° 33'	25° 33'	10° 33'	4° 27'
♊ ♋	90° 0'	75° 0'	60° 0'	45° 0'	30° 0'	15° 0'	0° 0'	15° 0'
♌ ♍	79° 27'	64° 27'	49° 27'	34° 27'	19° 27'	4° 27'	10° 33'	25° 33'
♍ ♎	70° 39'	55° 39'	40° 39'	25° 39'	10° 39'	4° 21'	19° 21'	34° 21'
♏ ♑	66° 57'	51° 57'	36° 57'	21° 57'	6° 57'	8° 3'	23° 3'	38° 3'
Ho. ab occ. →	24	23	22	21	20	19	18	17

Lower panel:

Sign (ab ortu · pair)	8	9	10	11	12	13	14	15
☿	6° 57'	21° 57'	36° 57'	51° 57'	66° 57'	81° 57'	96° 57'	111° 57'
♈ ♏	10° 39'	25° 39'	40° 39'	55° 39'	70° 39'	85° 39'	100° 39'	115° 39'
♉ ♐	19° 27'	34° 27'	49° 27'	64° 27'	79° 27'	94° 27'	109° 27'	124° 27'
♊ ♋	30° 0'	45° 0'	60° 0'	75° 0'	90° 0'	105° 0'	100° 0' [?]	135° 0'
♌ ♍	40° 33'	55° 33'	70° 33'	85° 33'	100° 33'	115° 33'	130° 33'	145° 33'
♍ ♎	49° 21'	64° 21'	79° 21'	94° 21'	109° 21'	124° 21'	139° 21'	154° 21'
♏ ♑	53° 3'	68° 3'	83° 3'	98° 3'	113° 3'	128° 3'	143° 3'	158° 3'
Ho. ab occ. →	16	15	14	13	12	11	10	9

[Margin: How it may be known from the table of distances which hour from rising or setting is before noon or after noon. — How the distances of the halves and quarters of the hours are drawn from the table above. — How the declinations of the points of the ecliptic are found geometrically.]

In this table you see that the distance of hour 24 from rising or setting is always equal to the semidiurnal arc in every sign; and from this distance the rest arise by the continuous subtraction of 15 degrees, until a number smaller than 15 degrees is reached. For then that number, smaller than 15 degrees, is subtracted from 15 degrees, so that the remainder becomes the hour's distance on the other side of the meridian; and from this, finally, all the others are generated by the continuous addition of 15 degrees. Thus, in any sign, the hour of this table which first turns up with a distance smaller than 15 degrees is placed — together with all the preceding hours — on one side of the meridian, namely the eastern side if the hours are counted from rising, but the western side if from setting; while the remaining hours that follow lie on the other side of the meridian.

And although the table above contains only the distances of the whole hours, still from these the distances of the halves and quarters of the intermediate hours will also easily be known: namely, for one-quarter add or subtract $3^{\circ} 45'$, for one-half $7^{\circ} 30'$, and for three-quarters $11^{\circ} 15'$, from the preceding distance, according as the distances increase or decrease. For example, in ♋ [Cancer], hour $3\frac{1}{2}$ from rising will be distant from the meridian $60^{\circ} 33'$; but hour $19\frac{3}{4}$ [?] from setting will be distant from the meridian $49^{\circ} 18'$; and so with the rest.

Diagram. A large geometric construction (framed) for finding, by ruler and compass, the declinations of the signs and their rising/setting amplitudes, described in Note 2 below. The figure combines an upper declination construction and a lower amplitude construction sharing the horizontal base-line C–R–A. Readable labels and positions:

- A (right edge, mid-height): centre of the outer construction; right end of the horizontal line C–A
- C (left end of the horizontal line): centre of the inner semicircle BD
- R (centre of the horizontal line C–A)
- B (top-right, on the arc): top end of the vertical diameter of the semicircle
- D (right/lower edge): lower end of that diameter
- E (upper centre, on the arc): point where the pole-altitude complement is marked; joined to A
- H (just below E, macron over it): foot of the perpendicular F–H to E–A

- F (left, on the arc): endpoint of the maximum-declination arc E–F
- G (top, right of K): auxiliary point for setting the perpendicular (arc E–G = arc E–F)
- K (upper-left, on the arc): endpoint of the divided semicircle I–F–K
- I (right-centre, on the arc): other endpoint of that semicircle
- L (left, below N): division point (arc K–L)
- M (centre-low): division point (arc I–M)
- N (left, below K): division point (arc F–N)
- O (centre, below H): division point (arc F–O)
- P (inner left-centre): declination point on arc E–F
- Q [?] (inner, near P, on a dotted line): second declination point on arc E–F
- S (lower centre): point on the circle found via $R-S \parallel B-D$
- T (right edge, mid-low): foot of perpendicular S–T to B–D; centre of the lower semicircle V–S–X
- V (right edge, below A): endpoint of the lower semicircle
- X (bottom-right corner): other endpoint of the lower semicircle
- Y [?] (lower-right, inside): division point (arc V–Y)
- Z [?] (near S—a region): division point (arc X–Z) — label not clearly resolved
- a [?] (α -like, lower-right inside): division point (arc S–a)
- b [?] (bottom centre): division point (arc S–b)
- c [?] (small, right of S): amplitude point on arc D–S
- d [?] (small, right-below S): amplitude point on arc D–S

[Margin: Where the computation of the declinations is taught. — How the rising or setting amplitudes of the points of the ecliptic are investigated geometrically.]

2. The declinations, moreover, of the signs, and their rising and setting amplitudes, we shall determine geometrically by this method. From A, having described the semicircle BD, and having drawn in it the diameter BD cut at right angles by the radius CA, let there be counted from point C the complement of the pole's altitude — or from point B the pole's altitude itself — up to E; and let the radius of the equator EA be joined. Then, the Sun's maximum declination of $23^{\circ} 30'$ having been computed, let a line be drawn from E to F, and from F a perpendicular FH to EA — which will easily be done if EG is taken equal to the arc EF and a ruler is laid to the points F, G. From H also, through F, let the semicircle IFK be described and divided into 6 equal parts — which too will easily be done if, with the same radius FH, the arcs KL, IM, FN, FO are cut off. For the straight lines LM, NO will cut the arc EF at the declination points P, Q; so that the

arc EF is the declination of ♋ [Cancer] and ♑ [Capricorn]; EP the declination of ♊ [Gemini], ♌ [Leo], and ♐ [Sagittarius], ♒ [Aquarius]; EQ the declination of ♉ [Taurus], ♍ [Virgo], and ♏ [Scorpio], ♓ [Pisces]. And the straight lines AE, AQ, AP, AF are the radii of the signs — as was handed down in the *Gnomonics*, Book 1, Proposition 1, and in Book 1 of the *Astrolabe*, Lemma 19, and in the scholium on Canon 3 of Book 3 of the same.

We shall also teach how to compute the declination of any point of the ecliptic, at the end of this booklet, in Problem 1, so that it may be transferred from E onto the circumference BC [?], or onto other circles, whenever the matter requires.

After this, FR having been drawn parallel to EA and cutting the diameter of the horizon CA at R, let RS be drawn through R parallel to BD, cutting the circle at S. (This point S will also be found if from D the rising amplitude of ♋ [Cancer] or ♑ [Capricorn] is counted, which at the pole's altitude of 42 degrees amounts to 32° 27' — and how to compute this for every altitude of the pole we shall teach at the end of the booklet, in Problem 3.) Then, ST having been drawn perpendicular to BD, let the semicircle VSX be described from T through S; and this being divided into 6 equal parts (which will easily be done if, with the same interval of the radius ST, the arcs VY, XZ, Sa, Sb are cut off), let the straight lines YZ, ab be joined, which will cut the arc DS at the rising or setting amplitudes: so that DS is the amplitude of ♋ [Cancer] and ♑ [Capricorn]; Dc the amplitude of ♊ [Gemini], ♌ [Leo], and ♐ [Sagittarius], ♒ [Aquarius]; Dd the amplitude of ♉ [Taurus], ♍ [Virgo], and ♏ [Scorpio], ♓ [Pisces] — as we demonstrated in Book 3 of the *Astrolabe*, in the scholium on Canon 6. And the straight lines AD, Ad, Ac, AS will be the radii of these same amplitudes.

By the same method we shall find the declination and the rising or setting amplitude of any point of the ecliptic, if — for the arc of the ecliptic between that point and the beginning of ♋ [Cancer] or ♑ [Capricorn] (whichever, of course, is nearer) — similar arcs are taken on the circles IFK, VSX, beginning from the points F and S, and through their ends lines are drawn parallel to the straight lines AK, AX.

We shall also teach how to compute the rising or setting amplitude [latitudo ortiua, occidua] of any point of the ecliptic, at the end of the booklet in Problem 3, so that it may be transferred from D onto the circumference DC, or onto other circles where needed. And these rising amplitudes, computed at the beginnings of the signs for various elevations of the pole, we shall set out in Table 5.

[Margin: The same straight line drawn from a point on the equinoctial cuts two hours from rising or setting, equally distant from that point, of opposite parallels. — By this method the points of the parallels in the hours from rising or setting are to be examined.]

3. Moreover, just as we said in the scholium of Chapter 8, Note 1 — that one and the same straight line drawn through any hour on the equinoctial cuts two hours from noon and from midnight, equally distant on either side from that hour, at the points of two opposite parallels — so too the same thing happens in the hours from rising or setting. For a straight line drawn, say, through hour 22 on the equinoctial, cutting hour 23 at the point of $\mathfrak{S}$ [Cancer], will cut hour 21 at the point of $\mathfrak{V}$ [Capricorn]. And a straight line drawn through hour 20 on the equinoctial, cutting the same hour 23 at the point of $\mathfrak{S}$, will cut hour 17 at the point of $\mathfrak{V}$. Finally, a straight line drawn through hour $16\frac{1}{2}$ on the equinoctial, cutting hour 12 at the point of $\mathfrak{S}$, will cut hour 21 at the point of $\mathfrak{V}$ — because hour 12 and hour 21 are each distant $4\frac{1}{2}$ hours on either side from hour $16\frac{1}{2}$. And by this method you will be able to test almost all the points of the parallels that have been found: namely, whether any two points of opposite parallels correspond to each other by a straight line drawn on the equinoctial through an hour equally distant from the hours of those points.

This is briefly demonstrated as follows. Let AB be the equator, and CD, EF two opposite parallels. Through point G of the equator let the circle of any hour from setting be drawn, cutting the parallels at H, I. Through the two points A, B likewise, equally distant from G, let two other hour-circles from setting pass, CAE and DBF. Finally, through D, G let a great circle be drawn, which I say passes through E. Likewise let a great circle be described through C, G, which I assert passes through F. For the circle DG will cut —

— for the circle DG will cut the parallel EF at E. And since, by Proposition 18 of Book 2 of Theodosius [*Sphaerica* II.18], the arcs GD, GE are equal, as are GH, GI, the connecting straight lines DH, EI will be equal, by Proposition 3 of Book 3 of the same. [a] Therefore, from the equal parallels the lines DH, EI will cut off equal arcs. Since, then, by Proposition 13 of Book 2 of the same, the four hour-spaces CH, HD, EI, IF are equal — being of course similar to the equal arcs AG, GB (because, by Proposition 15 of Book 1 of our *Gnomonics*, the hour-circles from rising or setting touch the same greatest parallel of the always-visible stars) — the circle DG will necessarily pass through E. And by the same reasoning we shall show that the circle CG passes through F, which is what was proposed. And since a straight line sent out from any hour-point on the equinoctial through a point of

any parallel reproduces the great circle drawn from G, it is clearly established, that which we concluded at the beginning of this Note 3.

[Margin: a — [Theodosius] III.28.]

Diagram. A square framed figure illustrating the demonstration of Note 3. A horizontal line (the equator AB) runs through the middle, with two opposite parallels arching above and below it, and several hour-circle arcs crossing them. Readable labels:

- C (upper left): left end of the upper parallel CD
- H (top centre): intersection of the hour-circle through G with the upper parallel
- D (upper right): right end of the upper parallel CD
- A (mid-left, on the equator): point on the equator
- G (centre, on the equator): the hour-point through which the hour-circle passes
- B (mid-right, on the equator): point on the equator
- E (lower left): left end of the lower parallel EF
- I (bottom centre): intersection of the hour-circle through G with the lower parallel
- F (lower right): right end of the lower parallel EF

4. But now, if in that dial which we described in Chapter 6 — where, because of the too-small elevation of the pole, its centre can scarcely be obtained — someone wishes to lay out the hours from rising or setting, then lines must be drawn through the hour-points of the equinoctial line (through which, according to the prescription of Table 2 of Chapter 14, they must pass), parallel to those hours from noon or midnight which, in Table 1 of the same chapter, correspond to the hours from rising or setting — even if no other points are found on the line of hour 6 from noon or midnight (since it is not present) through which they should be drawn.

[Margin: The hours from rising or setting in a dial where the centre is not available. — That the hours from rising or setting have no place throughout the whole year in a region where the pole's altitude is greater than $66^{\circ} 30'$.]

5. But in a dial which has so great an elevation of the pole that it cannot admit an equinoctial line — as was described in Chapter 7 — there is no reason for anyone to want the hours from rising or setting in it, since they cannot be had: for they touch the greatest parallels of the always-visible and of the always-hidden stars,

as we demonstrated in the *Gnomonics*, so that the hour-lines cannot indicate the hours from rising or setting throughout the whole year, wherever the pole's altitude is greater than $66^{\circ} 30'$. They will have to be drawn only at a pole-elevation which does not exceed $66^{\circ} 30'$ — which will not be difficult. For since in such a dial the equinoctial line can conveniently be drawn (the tangent of the pole-altitude $66^{\circ} 30'$ — which is the greatest of all those at which the hours from rising or setting are of use throughout the whole year — contains only about 23 [?] parts, so that they can easily be transferred downward from the position of the style to find the point through which the equinoctial line, perpendicular to the meridian, is to be drawn), the lines of the hours from rising or setting are to be drawn through its hour-points (through which, by the prescription of Table 2 of Chapter 14, they must pass), equidistant from those hours from noon and midnight which, in Table 1 of the same chapter, correspond to the hours from rising or setting — as we explained at the beginning of the said Chapter 14.

LEMMA

To extend a straight line, without error, through two points however close together.

This is Lemma 11 of Book 1 of our *Astrolabe*, which we worked out and demonstrated by various methods. But since not infrequently we need it in this booklet — especially in what follows — it is worth briefly repeating its first construction here. Let then a straight line be extended through two points, A, B. From A as centre, through B, let a circle be described, from which two equal arcs BC, BD are cut off, whether at the same interval AB or at a greater or smaller one. Then from C, D let two arcs be drawn on either side, at such an interval that they may conveniently intersect each other at E and F. For the straight line AE will pass through B, as was demonstrated in the *Astrolabe*. And by the same reasoning the straight line AF will pass through B — so that the four points F, A, B, E lie on one straight line. But if the line AE must be produced further, let another circle be described from A through E, from which again two equal arcs EG, EH are cut off; and from G, H let two arcs be drawn cutting each other at I. For the straight line AI will pass through the points B, E, F, as we showed in the same place. And in this way one may proceed as far as one pleases.

[Margin: A most elegant construction for drawing a straight line through two points however close together.]

Diagram. A framed figure for the Lemma. A horizontal straight line carries, from left to right, the points F, A, B, E, I; pairs of compass-arcs intersect above and below the line to fix the collinear points. Readable labels:

- F (left, below/on the line): intersection point on the extended line
- A (left-centre, on the line): first given point / circle centre
- B (centre, on the line): second given point
- E (right-centre, on the line): far intersection point
- I (right, on the line): further intersection point of the produced line
- C (top, above A–B): upper arc-intersection near the first circle
- D (bottom, below A–B): lower arc-intersection near the first circle
- G (top, above E): upper arc-intersection near the second circle
- H (bottom, below E): lower arc-intersection near the second circle

CAPUT XVI

The hours from noon and from midnight on a plane which is equidistant from [parallel to] some vertical declining from the prime vertical — that is, on the plane of some declining wall that is perpendicular to the horizon.

1. Now that the matters concerning the horizontal dial have been dispatched, there follow the vertical or mural dials, in which nothing new will have to be taught; rather, nearly everything hitherto handed down is to be transferred here, with a few changes. In this place, however, we shall treat only of dials declining from the meridian, so that the multitude [of...]

[Margin: Why only dials declining from the meridian are treated here.]

[Margin: The horizontal line.]

so as to avoid too great a multitude of precepts. For from the delineations produced beyond the horizontal line we shall derive the dials declining toward the north, as we shall teach in the scholium of chapter 19.

Let, then, a straight line AB be drawn in any way, to serve as the horizontal line; on it let the position of the style, C, be chosen, and let a straight line CD be erected, equal to the style. Now on any straight line take as many segments as you please, each equal to the style CD, and divide the first segment CD into 10 equal parts. From that line take the Tangent of the wall's declination, 30 degrees — (for we suppose the plane, or wall, to decline from the meridian toward the east by 30 degrees; how that declination is to be investigated we shall teach in chapter 22) — namely about $5 \frac{7}{10}$ [parts]; and transfer it from C, the position of the style, toward the right if the plane inclines from the meridian toward the east, as ours does, but toward the left if toward the west, as far as B. Likewise transfer the Tangent of 60 degrees, the complement of the declination, which is a little more

than $17 \frac{3}{10}$, from the same position of the style C into the opposite direction, as far as A.

[Margin: A geometric finding of the points through which the meridian line must be drawn, and also the equinoctial, which is the line of the 6th hour. The center of the dial.]

These points B and A may also, if you like, be found geometrically as follows. From D describe a circle HGI, and toward the right take an arc GH equal to the declination, 30 degrees — (since we suppose our plane to decline from the meridian toward the east by 30 degrees; for the said arc of declination must be taken toward the left if the plane inclines toward the west) — and in the opposite direction an arc GI equal to the complement of the declination, namely 60 degrees in our example. For the hidden [construction] lines DH, DI will cut the horizontal in the points B, A. Through B the meridian EBF must be drawn, perpendicular to the horizontal; while through A must be drawn both the equinoctial line and the line of the 6th hour, as we demonstrated in the scholium to Proposition 1 of Book 3 of the *Gnomonics*.

2. Next let the Tangent of the pole's altitude, 42 degrees — namely 9, reckoned relative to the whole sine DB — be transferred along the meridian from B as far as E; and the Tangent of the complement of the same pole-altitude, namely $11 \frac{1}{10}$, as far as F. And E will be the center of the dial, and F the point through which the equinoctial is to be drawn from the point A already found; for the pole is distant from the horizon AB, along the meridian, by the altitude of the pole, while the equator [is distant] by its complement.

The points E and F will likewise be found geometrically if BR is taken equal to the straight line DB, and from R a circle is described, in which, from the straight line RA upward, the pole's altitude, 42 degrees, is counted off. For the straight line drawn from R through the end of the pole count will give the center E. And if in the same circle the complement of the pole's altitude is counted off downward, the straight line extended from R through the end of the count will give the point F — so that the straight line AF is the equinoctial line.

[Margin: The equinoctial line.]

For conceive the triangle DBC to be moved about BC until it stands at right angles to the plane of the dial — that is, until it lies in the plane of the horizon. Likewise [conceive] the triangle ERF to be moved about EF until the point R coincides with the point D, and the straight line RB with the straight line DB; for in that motion the line RB does not depart from the plane of the horizon, and it is equal to the

line DB, existing in the same plane. In that position RE, forming with RB (or with DB, which is the same) the angle of the pole's altitude, will be the axis of the world: for in that position the triangle REF lies in the plane of the meridian, which is then drawn through the lines DB, EF, and therefore also through RB, EF. The axis RE therefore meets the meridian line at E, the pole of the world — that is, at the center of the dial. And since the axis is at right angles to the equator, and therefore perpendicular to the common section of the equator and the meridian; and since RF is perpendicular to the axis, because the angle ERF, standing on the quadrant, is a right angle: RF will be the common section of the equator and the meridian, seeing that RF lies in the plane of the meridian perpendicular to the axis, and the equator passes through R, or D, the center of the world, and makes (as has been said) with the meridian a section forming right angles with the axis ER. The equator therefore meets the plane of the dial at F; and consequently the straight line drawn through A and F will be the equinoctial line.

This, however, can be proved more easily. For since BE is the Tangent of the angle ERB, the pole's altitude, and BF the Tangent of its complement, namely of the angle FRB, reckoned relative to the whole sine RB, RE will fall entirely upon the pole, or center of the dial, and RF [?] upon the point of the equator.

[Margin: The equinoctial line, another way.]

The same equinoctial line may be drawn, if you like, in this way: first, by Problem 6 at the end of the booklet, compute the arc of the declining plane between the horizon and the equator, which in our example is 29 degrees 3 minutes. For if from A a circle is described, in which this arc of 29 degrees 3 minutes is counted off downward from the straight line AB, the straight line sent out from A through the end of the count will give the equinoctial, which — if no error has been made — will pass entirely through F, the point found before.

For while the triangle DBC still remains in its proper position (so that D is at the center of the world), let the horizon and the equator be drawn through the straight line DA, and let them form, in the vertical to which the plane of the dial is parallel, [Margin: a — Euclid XI.16] straight lines parallel to the straight lines they form in the plane of the dial. [Margin: b — Euclid XI.10] The angle which those lines make in the said vertical at the center of the world will be equal to the angle which these make in the plane of the dial at A. Since, therefore, that angle at the center of the world stands upon the arc of that vertical lying between the horizon and the equator, and the angle BAF in the plane of the dial stands, by construction, upon a similar arc, this angle BAF will be equal to that one; and therefore AB, AF will be the common sections of the horizon and the equator.

[Margin: The style-line.]

3. The style-line — that is, the common section of the plane of the dial and of its own meridian — must be drawn from the center E, perpendicular to the equinoctial line, as we showed in the *Gnomonics*; such is EK, which necessarily passes (if no error has been made) through C, the position of the style, because the proper meridian of the dial passes at right angles through DC, the erected style, and through the center E.

[Margin: In what way the style-line may be drawn precisely. The pole's altitude above the declining plane.]

But if, in the first place, after the center E has been found, the style-line is drawn through C (which will be drawn more precisely if, taking a straight line EN equal to EB, a parallel to AB is drawn through N, on which NO is taken equal to BC; for thus, through the three points O, E, C, the style-line will be extended more accurately — and the same line may also, by the Lemma we set before this chapter, be drawn precisely through the two points E, C alone) — then at C let a perpendicular CZ be erected, equal to the style; the straight line EZ will be the axis of the world, and ZEC the angle of the pole's altitude above the plane of the dial; while the straight line ZK, perpendicular to the axis, will be the common section of the equator and of the proper meridian, meeting the style-line at the point K, through which the equinoctial line is to be drawn. All this has been shown in the *Gnomonics*.

[Margin: The style-line, another way. The equinoctial line, another way. The style-line, another way.]

Since the angle BEC is equal to the angle CAK — because the two remaining angles B, C in the triangle BCE are equal to the two remaining angles K, C in the triangle KCA — if the equinoctial line AF has been drawn first, the style-line will be conveniently drawn by making the angle FEK equal to the angle BAF. Or, if the style-line EC has been described first, we shall easily draw the equinoctial line by making BAF equal to the angle BEC. Or indeed, if from E a circle is described (if you please), in which, from the meridian line toward the side of the style, the arc computed by Problem 6 at the end of the booklet is counted off — (that is, the arc of the declining plane between the horizon and the equator, which is equal to the arc of the same plane between its proper meridian and the meridian of the horizon, and which we said in our example contains 29 degrees 3 minutes) — the straight line sent out from E through the end of the count will be the style-line.

This will be demonstrated in the same manner in which it was shown in Number 2 that the straight line AF is the equinoctial line.

[Margin: The line of the 6th hour.]

4. The line of the 6th hour will be the straight line EA, drawn through the center E and the point A. It may also be drawn in this way, if you like. By Problem 8 at the end of the booklet, find the arc of the declining plane between the meridian of the horizon and the circle of the 6th hour; or between the horizon and the same circle of the 6th hour, which is the complement of the former. In our example the former arc contains 65 degrees 46 minutes, the latter 24 degrees 14 minutes.

[Margin: The line of the 6th hour, another way. The line of the 6th hour, another way.]

For if from E a circle is described, in which the former arc is counted off from the meridian line toward A, the straight line drawn from E through the end of the count will give the line of the 6th hour, which will pass entirely through A if no error has been made. Or if from A a circle is described, in which the latter arc is counted off from the horizontal line AB toward E, the straight line drawn from A through the end of the count will give the same line of the 6th hour, which will necessarily pass through the center E, unless an error has been made. Both will likewise be demonstrated, as was said of the equinoctial line in Number 3.

Diagram. A large, very dense full-page dialling-construction figure occupying most of the page, enclosed in a heavy rectangular frame. It shows the complete construction of a declining vertical (mural) dial for latitude 42° and 30° declination, developed in the surrounding text: the horizontal line AB, the meridian EBF, the equinoctial AF, the style-line EK, the line of the 6th hour EA, and the full pencil of hour-lines radiating from the two centers (E, the dial center, and A). A great many solid and dotted construction lines cross the field; numerous compass-arcs and dotted auxiliary squares/perpendiculars (the alternative constructions of Numbers 2–4) are overlaid. Below the main construction are four separate graduated scale-bars used to lay off the tangent-distances.

Readable labels (capital construction points):

- E: principal dial center (upper-center), where most hour-lines converge; the meridian-scale mark "Num.100" stands beside it
- D: just below/left of E

- A: on the horizontal line (lower-center), the second pencil-center of hour-lines
- B: on the horizontal line, right of A
- C: at the far left, on the lowest scale
- Q: far left, on a horizontal line
- N: center (near E)
- R: upper-center
- M: upper-right
- K, F, L: cluster right of center
- H, I, G: right side
- X, Y: a dotted auxiliary square near center-left
- Z: near center
- S, T, V: upper-left, dotted auxiliary construction

Hour-line numerals labelling the radiating lines around the rim: 4, 4½, 5, 5½, 6, 6½, 7, 7½, 8, 9, 10, 11, 12, 1, 2, 2½, 3, 3½. Numerous lowercase letters (z, u, g, r, i, h, e, k, d, l, m, q [?]) mark hour-line endpoints around the border; several are too small/overlapped to resolve individually [?].

The four scale-bars at the foot, each a graduated ruler numbered 10 · 20 · 30 · 40 · 50 · 60 with endpoint letters (C, D, B, K, N, A, L):

- "Pro horiz. et lin. styli" [For the horizontal and the style-line]
- "Pro mer." [For the meridian]
- "Pro æquin." [For the equinoctial]
- "Pro Vert" [For the vertical]

[Many peripheral labels, both the lowercase hour-line endpoints and the small hour-numerals, are at or below the resolution of the scan and could not be individually confirmed.]

[Margin: The line of the 6th hour, another way.]

When the point A can scarcely be obtained, because of its excessive distance from the point C — which happens when the angle of declination BDC is very small, and the angle CDA, of the complement of the declination, very large — then, although we can draw the line of the 6th hour by means of the arc of which we spoke just now, we shall nevertheless draw the same line without it, by this method. At R erect a perpendicular to AB, from which cut off RS equal to BE; make the angle VST [equal to] the complement of the declination, and set the

straight line ST equal to the straight line SR; and finally through T draw TV parallel to AB, cutting RS produced at V. For the straight line drawn from V through E will give the 6th hour, so that it passes through the point A, if that can be obtained on the dial.

[Margin: In which case the style-line must be drawn first, and then the equinoctial.]

That it may be drawn more accurately when the point A is not available, we shall take on SE produced a straight line EX equal to ES; and, having erected at X a perpendicular XY to EX, we shall take the straight line XY equal to the straight line SV. For thus, through the three points V, E, Y, the line of the 6th hour VE will be drawn more precisely. And in this case the style-line must be drawn first, as we said in Number 3, so that from F a perpendicular can be drawn to it for the equinoctial line.

[Margin: c — Euclid VI.4]

Let this be the demonstration of that construction. The triangles ADB (for, to demonstrate this, let us suppose that we have the point A) and TVS are equiangular, since the angles T, V are right angles, and the angles B, S are equal to the complement of the declination. Therefore, as AB is to BD — that is, to BR, or to ES — so is ST — that is, RS, or BE — to SV. And, alternately, as AB is to BE, so is ES to SV. Wherefore, since the triangles ABE, ESV have their sides about the angles B, S proportional, and are so composed at the angle E that the homologous sides...

[Margin: a — Euclid VI.32]

AB, ES and BE, SV are parallel, the straight lines AE, VE will make one straight line; and consequently the straight line VE, produced, will fall upon the point A — that is, it will give the line of the 6th hour, which we said was to be drawn from E through A.

[Margin: The line of the 6th hour, another way.]

We shall also very conveniently describe the line of the 6th hour by the means which we shall set out in chapter 17, Number 1.

5. Now that the horizontal line has been drawn, together with the meridian, the equinoctial, the style-line, and the line of the 6th hour — the points B, A, E, F having been found by means of the Tangents, as we taught at the outset in Numbers 1 and 2, setting aside the other geometric methods which we employed (unless you wish to use them for a more precise construction) — let us proceed to

describe the hours. In the first place we shall set out the method that requires the distances of the hours from the equinoctial line: of these, the former are derived from the sixth table, but the latter from the inclination of the meridian of the horizon to the meridian of the declining plane, as we shall presently teach. Then we shall describe the hours on the declining plane without their distances from the style-line: so that this latter method may suffice, even if the sixth table had not been computed for all elevations of the pole, and the aforesaid inclination of the meridians were unknown.

[Margin: How the distances of the hours on the horizontal, from the position of the style, are elicited from the sixth table, on a plane declining from the meridian toward the east.]

For the former method, then, a certain little table must be constructed from the complements of the arcs of the horizon that belong, in the left-hand column, to the given pole-altitude of the sixth table, in this way. To the complements of the arcs of the hours after noon, if the dial declines from the meridian toward the east, let the declination of the dial be added; but from the complements of the arcs of the hours before noon let the same wall-declination be subtracted, or the complements themselves [subtracted] from the wall-declination — the smaller number, that is, from the greater. And finally, when an hour before the 6th hour from midnight is to be given, let the complement of the wall's declination be added to its arc — not to its complement. For thus will be produced, or made up, the remaining distances of the hours from the position of the style on the horizontal line.

[Margin: Which distances belong to the left of the style, and which to the right, on a wall declining from the meridian toward the east.]

Now when the wall's declination has been subtracted from the said complements, or when the complement of the wall's declination has been added to the horizontal arcs themselves, the distances of those hours will belong to the left of the style's position; but when the aforesaid complements of the arcs have been taken from the wall's declination, or added [to it], the distances of the hours will be generated toward the right of the style's position.

[Margin: How the distances of the hours on the horizontal line, from the position of the style, are gathered from the sixth table, on a plane declining from the meridian toward the west.]

But if the dial declines from the meridian toward the west, let the contrary be done. For to the complements of the arcs of the hours before noon the wall's

declination must be added; while from the complements of the arcs of the hours after noon the wall's declination must be taken away, or the complements themselves [taken] from the wall-declination — the smaller number, that is, from the greater; and finally, when an hour after the 6th hour from noon is to be given, the complement of the wall's declination must be added to its arc, not to its complement. For thus will be produced, or made up, the remaining distances of the hours from the position of the style on the horizontal line.

[Margin: Which distances look toward the left of the style, and which toward the right, on a wall declining from the meridian toward the west.]

And if indeed the wall's declination has been added to the complements of the arcs, or when the said complements have been subtracted from the wall-declination, the distances of those hours will belong to the left of the style's position; but when the wall's declination has been subtracted from the complements of the arcs, or the complement of the wall's declination has been added to the arcs themselves, the distances will look toward the right of the style's position.

For example: the complement of the arc of the 1st hour from noon, at latitude 42 degrees, contains in the sixth table 10 degrees 10 minutes. If, then, the wall's declination of 30 degrees is added — our dial declining from the meridian toward the east — the distance from the position of the style toward the right becomes 40 degrees 10 minutes, for the 1st hour from noon. Likewise, the complement of the arc of the 8th hour from midnight is 49 degrees 13 minutes. If, then, the wall's declination of 30 degrees is taken away, the remaining distance from the position of the style toward the left becomes 19 degrees 13 minutes. Again, the complement of the arc of the 11th hour from midnight is 10 degrees 10 minutes; and when this is subtracted from the wall's declination of 30 degrees, there remains the distance from the position of the style toward the right of 19 degrees 50 minutes, for the 11th hour from midnight. Finally, the arc of the 5th hour from midnight (which is before the 6th hour) contains 21 degrees 49 minutes. When the complement of the wall's declination, namely 60 degrees, is added, the distance from the position of the style toward the left becomes 81 degrees 49 minutes. And so with the rest.

By this method we have constructed the following little table for our dial declining from the meridian toward the east by 30 degrees, at latitude 42 degrees. Now when any distance found is greater than a quadrant, it is not placed in the table, because its hour-distance does not fall on the horizontal line. But in that case its complement to 180 degrees must be carried over to the opposite side, by

means of its tangent. The point so found will belong to the hour to be counted from noon, if the one that was sought — and which does not fall on the horizontal line — was counted from midnight, and vice versa.

Table of the hour-distances from the position of the style on the horizontal line, when the dial declines from the meridian toward the east. For latitude 42 degrees. [*Tabella distantiarum horariarum à loco styli in horizontali linea, quando horologium declinat à mer. in ortum. Ad latitudinem grad. 42.*]

Hours from midnight (G. = degrees, M. = minutes):

Hour	5	5½	6	6½	7	7½	8	8½
G.	81	71	60	48	38	28	19	11
M.	49	8	0	52	11	14	13	5

To the left of the style. [*Ad sinistram styli.*]

Hours from midnight — and (last column) from noon (G. = degrees, M. = minutes):

Hour	9	9½	10	10½	11	11½	12	½ (from noon)
G.	3	2	8	14	19	24	30	35
M.	47	51	53	30	50	58	0	2

To the left | To the right of the style. [*Ad sinist. | Ad dextram styli.*]

CAPVT XVI. (continued)

(Declining vertical / mural dials — continued from the preceding page.)

Table (concluded from the preceding page): hour-distances from the position of the style on the horizontal line, for a dial declining from the meridian toward the east by 30 degrees, at latitude 42 degrees

Hours from noon [*Horæ à mer.*] (G. = degrees, M. = minutes):

Hour	1	1½	2	2½	3	3½	4	4½
G.	40	45	51	57	63	71	79	88
M.	10	30	7	9	47	5	13	14

To the right of the style. [*Ad dextram styli.*]

These distances are exactly the same in a dial declining from the meridian toward the west; but the hours must be changed into their complements to 12, and those that were previously reckoned from midnight must now be reckoned from noon, and vice versa. Likewise, the distances that inclined toward the left of the position of the style will now lie toward the right, and vice versa. Thus, just as in our dial the 10th hour from midnight stands 8 degrees 53 minutes to the right of the position of the style, so in a dial declining from the meridian toward the west, at the same latitude of 42 degrees, the 2nd hour from noon likewise stands 8 degrees 53 minutes to the left of the position of the style — and so with the rest.

[Margin: How the aforesaid distances are fitted to a plane declining from the meridian toward the west by 30 degrees.]

This little table can also very conveniently be constructed from the arcs of the sixth table — not, however, from their complements — in this way. For a dial declining from the meridian toward the east, the hour-arcs of the sixth table after the 6th hour are to be subtracted from the complement of the wall's declination, wherever such subtraction is possible; whereas the arcs of the hours before the sixth are to be added to that same complement of the declination — for so are produced the arcs whose tangents are to be transferred to the left of the style. But when, for the hours after the 6th, the arcs in the sixth table are greater than the complement of the declination, then the complement of the declination is to be subtracted from those arcs; and if the hours after noon are being counted, the wall's declination itself is to be added to the complements of the sixth-table arcs — for so are produced the arcs whose tangents are to be transferred to the right of the style. The opposite is to be done in a dial declining from the meridian toward the west.

[Margin: How the hour-points on the horizontal line are found by means of the tangents.]

6. Now if you transfer the tangents of these distances — taken with respect to the whole sine DC, that is, of the style — from the position of the style C onto the horizontal line, to the left or to the right as the table directs, and likewise from the point O in the opposite direction along the straight line O N: three points will be found for drawing each single hour-line. One of these lies on the horizontal line, another on the straight line O N, and the third is E, the center of the dial. And so the straight lines drawn out from E, the center of the dial, through the hour-points on the straight lines A B and N O once found, will be the hour-lines. Of these, the ones that lie to the left with respect to the meridian line, as we face the dial, belong to the hours

reckoned from midnight; those to the right belong to the hours reckoned from noon — provided, of course, that the dial declines from the meridian toward the east or west. For in dials declining toward the north the reverse must be understood.

[Margin: Which hour-lines belong to the hours reckoned from midnight and which to the hours reckoned from noon.]

Just as, in the upper table, the distances of the whole and half hours were found, so too the distances of the quarter-hours can be found, if anyone wants them, from that same sixth table of horizontal arcs.

To find conveniently the points of the 9th and $9\frac{1}{2}$ hours — whose tangents are very small — one should add the tangent of the 9th hour, namely 66, to CB, the tangent of the wall's declination, which for us is 30 degrees, that is, 577; so that the composite tangent becomes 643 — that is, reduced to tenth-parts, a little more than 6 $\frac{1}{10}$. For this, transferred from the point B, will give the point of the 9th hour. Likewise the tangent of the $9\frac{1}{2}$ hour, which is 50, is to be subtracted from CB, the tangent of the wall's declination — that is, from 577 — so that the remainder becomes the tangent 527, that is, roughly 5 $\frac{1}{10}$; and this, transferred from B, will give the point of the $9\frac{1}{2}$ hour. This device is always to be used whenever some tangent is too small: namely, we either add it to the larger tangent on the other side of the style, or subtract it from the larger one on the same side, and so on, as was done with the tangents of the 9th and $9\frac{1}{2}$ hours. This is just what we noted in Chapter 2, Number 2 as well [recte: monuimus].

[Margin: What to do with the hours lying only a little from the line of the style, whose tangents are too small.]

7. If the tangents of the arcs of the verticals, from the sixth table taken with respect to the whole sine D A, drawn out for the given latitude of 42 degrees on the right-hand side, are transferred from A on both sides onto the vertical line — which is drawn through A parallel to the meridian line — we shall likewise have the hour-points on the vertical line, through which the hour-lines described a little earlier must pass, if no error has been made.

[Margin: The vertical line, and how the hour-points are found on it by means of the tangents.]

8. Again, to find the hour-points on the equinoctial line, another table must be constructed from the distances of the hours from the meridian, which we

set out in the table of the scholium to Chapter 1, in exactly the same way as the previous one was made. Only, in place of the plane's declination of 30 degrees, whose tangent is C B, one must take the inclination of the meridian of the horizon to the proper meridian of the declining wall — which is found by Problem 5 at the end of this little book, and in our example amounts to 40 degrees 48 minutes, whose tangent is KF with respect to the whole sine KZ. And in place of the sixth table, the table of the scholium to Chapter 1 is to be used, from which the arcs themselves are to be taken, not their complements as there. For example: the 5th hour from midnight is distant from the meridian — that is, from the point F — by 105 degrees. (For whenever an hour from midnight falls before the 6th hour, or an hour from noon after the 6th, one must take the complement, up to 180 degrees, of that distance found in the table of the scholium to Chapter 1.) Now, subtracting the 40 degrees 48 minutes of the inclination of the meridians, there remain 64 degrees 12 minutes for the distance of the 5th hour from midnight, which it has to the left of the line of the style. And so with the rest. This, then, is how the table stands.

[Margin: The distances of the hours on the equinoctial line from the line of the style; how they are found on a declining plane.]

Table of the hour-distances from the line of the style on the equinoctial [line], for latitude 42 degrees, when the declination of the plane amounts to 30 degrees

[*Tabella distantiarum hor. à lin. styli in æquinociali, ad latit. gr. 42. quando declinatio plani continet gr. 30.*]

First block — Hours from midnight [*Horæ à med. noc.*] (G. = degrees, M. = minutes):

Hour	3½	4	4½	5	5½	6	6½	7
G.	86	79	71	64	56	49	41	34
M.	42	12	42	12	42	12	42	12

To the left of the line of the style. [*Ad sinistram lineæ styli.*]

Second block — Hours from midnight [*Horæ à med. noc.*] (G. = degrees, M. = minutes):

Hour	7½	8	8½	9	9½	10	10½	11
G.	26	19	11	4	3	10	18	25
M.	42	12	42	12	18	48	18	48

To the left of the line of the style (hours 7½–9) | To the right of the line of the style (hours 9½–11). [*Ad sinistr. lin. styli.* | *Ad dextram lin. styli.*]

Third block — Hours from noon [*Horæ à mer.*] (G. = degrees, M. = minutes):

Hour	11½	12	½	1	1½	2	2½	3
G.	33	40	48	55	63	70	78	85
M.	18	48	18	48	18	48	18	48

To the right of the line of the style. [*Ad dextram lineæ styli.*]

The construction of this table is somewhat easier than that of the previous one, since it is enough to find the distances of two hours on either side of the line of the style — the two nearest ones. For if to these distances you keep adding continuously 3 degrees 45 minutes, or 7 degrees 30 minutes, or 15 degrees 0 minutes — according as the table proceeds by quarter-hours, half-hours, or whole hours — the distances of the other hours will also be found.

[Margin: How the nearest distances are fitted to a plane declining from the meridian toward the west by 30 degrees.]

These distances too are entirely the same in a dial declining from the meridian toward the west, provided you change the hours of the nearest table into their complements to 12 and reckon them from noon where the hours were being computed from midnight, and vice versa; and provided, likewise, that they now incline toward the right where the hours were tending toward the left, and the reverse.

[Margin: How the hour-points on the equinoctial are found by means of the tangents.]

And so, if the tangents of the distances of the nearest table — taken with respect to the whole sine ZK — are transferred onto the equinoctial from the line of the style, that is, from the point K, to the left or to the right as the table directs, we shall once again have the hour-points on the equinoctial line, through which the hour-lines described above will also pass, unless an error has been made.

9. But we shall draw the hour-lines with great exactness, even without the distances of the hours on the horizontal line from the position of the style, and without the distances on the equinoctial line from the line of the style, provided the lines of the 3rd and 9th hours have first been drawn with the utmost care. This is done as follows. Through B, drawn perpendicular to DBH, let there be e f; and from B, to the axis ER, let there also be raised the perpendicular Bd. On e f let two segments B e, Bf be taken, equal to Bd. For the straight lines D e, Df will cut the horizon at the points of the 9th and 3rd hours; and if through these, straight lines are extended out from E, the center of the dial, they will be drawn as the lines of the 9th and 3rd hours. And to draw these more accurately, one should use the Lemma set before this Chapter 16, because the points of the 9th and 3rd hours found on the horizontal line are only a little distant from the point E.

[Margin: An elegant way of laying out a declining dial without the sixth table and without the inclination of the meridians. How the lines of the 3rd and 9th hours are drawn without the tangents.]

Or else: on the straight line BD, extended below B, let two or three segments equal to DB be taken; and through the outermost point let a perpendicular to DB, or a parallel to e f, be raised. This will give the straight line Bd, transferred twice or three times over into that perpendicular — yielding other points for the 3rd and 9th hours, and so on.

That the straight lines D e, Df cut the horizontal line at the points of the 9th and 3rd hours will become clear thus. In Number 2 we demonstrated that, if the triangle BCD — or rather the whole plane through that triangle, together with the straight line e f, drawn about BC — is moved until it stands perpendicular to the plane of the dial and lies in the horizon, the straight line DBH will be the meridian line in the horizon, that is, the common section of the horizon and the meridian, passing in that position through the straight lines DB, BE. Likewise, if the triangle BER is revolved about B E until the straight line BR coincides with the straight line BD, and the point R with the point D, it will lie in the plane of the meridian along the straight lines DB, BE thus drawn; and the straight line R E, making at D with the meridian line of the horizon DB the angle of the pole's elevation, will be the axis of the world. Since, then, in that position the perpendicular e f raised through B stands as the equinoctial line in a horizontal dial, and the perpendicular Bd raised from B — the intersection of that same equinoctial with the meridian — to the axis, will be the common section of the equator and the meridian: therefore, as was demonstrated in Chapter 3, Number 1, once B e, Bf are taken equal to Bd, the points e and f will be

the points of the 9th and 3rd hours in the horizontal dial whose center is D. The straight lines D e, Df, therefore, are the lines of the 9th and 3rd hours in the horizontal dial, meeting the plane of the declining dial at the points of those same hours on the horizontal line. And it is evident that the straight lines D e, Df cut the horizontal line always in the same manner, even if the plane D e f be turned about BC.

From which it follows that, if e f be extended as far as one pleases, the points of the other hours too can be found on it — just as was done in the equinoctial line of the horizontal dial in Chapter 2, Number 3, namely by taking the tangents with respect to the whole sine Bd, and by cutting off from that same Bd, out of the straight line DBH, a segment equal to it, beginning at the point B so as to have it as the center of the equator, so that then, by Chapters 2 and 3, the hour-points may be found on e f. In just the same way we shall find the points of these same hours on the perpendicular which we directed above to be raised at D B extended, if the same tangents are taken with respect—

—to the whole sine Bd, and transferred, doubled or tripled and so on, into the said perpendicular, according as the portion DB was doubled or tripled, and so forth. By the same reasoning, so that these same points may be found by Chapter 3, one must take from the point of the straight line dBH through which the perpendicular is drawn — upward or downward — the straight line B d, doubled or tripled and so on, so that the center of the equator may be had with respect to that perpendicular, standing as the equinoctial line. And when, through the points found on e f, hidden straight lines are drawn out from D, the horizontal line will be cut at the hour-points; so that the hour-points on the horizontal line are found without the aid of the sixth table, by means of which they were found long ago above.

Diagram. A large rectangular construction figure filling most of the page, laying out the hour-lines of the declining dial by the tangent method of §9 (the "elegant delineation without the sixth table"). Within the frame a dense web of straight hour-lines crosses at several centers, with a semicircular arc (the equinoctial/hour-circle) sweeping through the middle; dotted lines mark auxiliary constructions. The periphery and the interiors of the radiating pencils are labelled with the hour numbers 1–12 and the intervening halves ($\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{1}{2}$, $6\frac{1}{2}$, $7\frac{1}{2}$, $8\frac{1}{2}$, $9\frac{1}{2}$, $10\frac{1}{2}$, $11\frac{1}{2}$). Readable letter-labels of construction points include the capitals A, B, C, D, E, F, G, H, I, K, L, M, N, O, Q, R, S, T, V, X, Y, Z and the small italic letters a, b, c, d, e, i, m, n, r, u, w, y; E lies near the upper central node and A near the lower central node of the crossing pencils. Because of

the density and the many rotated glyphs, the exact placement of a number of interior labels cannot be resolved with certainty — see the QA note.

At the lower left, four graduated scale-lines run horizontally across the foot of the figure, each captioned:

- **Pro horiz. et lin. styli** ("For the horizontal and the line of the style") — running between the points C and D, graduated 10, 20, 30 ... (to about 60).
- **Pro mer.** ("For the meridian") — running between the points D and B, graduated.
- **Pro æquin.** ("For the equinoctial") — running from K to N, graduated.
- **Pro vert.** ("For the vertical") — running between the points b, L and A, graduated.

Indeed, if — with respect to the whole sine DB — the complements of the arcs of the sixth table are transferred from B onto e f, points will be found on e f through which the straight lines drawn from D will cut the horizontal line at the same hours; because those straight lines take off from the horizontal line the arcs of the said complements, as we showed at the end of Chapter 1. And if, on DB extended, another point farther from D be taken, and through it a perpendicular to d B be raised, the tangents of the complements of those same arcs of the sixth table can be transferred onto it, with respect to the whole sine that lies between D and the said perpendicular. So too, if through A, a perpendicular to DA be raised, and onto it, from A on both sides, the tangents of the very arcs of the sixth table themselves — not of their complements — be transferred with respect to the whole sine D A, points will be found on it through which the straight lines emitted from D will cut the horizontal line at the hours; because, as we showed at the end of Chapter 1, those lines cut off from the horizontal line segments corresponding to the said arcs. And this method must be used whenever the wall's declination is unknown in degrees and minutes.

Furthermore, if on the line of the style a segment K a be taken equal to the straight line KZ, and a F be drawn, and to it at F a perpendicular be raised; and if finally onto this perpendicular, from F on both sides, the tangents of the hours from the table of the scholium to Chapter 1 be transferred with respect to the whole sine from F: then points will be found on it through which the straight lines emitted from a will cut off, from the equinoctial AF, portions corresponding to the arcs of the hours of the said table — as is clear from what was demonstrated at the end of Chapter 1. And this method too we shall use whenever the inclination of the meridians is unknown.

[Margin: Accurate finding and varied placing of the hour-points on straight lines that are equidistant from (parallel to) the meridian lines and the line of hour 6 [?], without the aid of a table; and of the distances from the styles in the horizontal, and from the line of the style in the equinoctial line. [?]]

10. Once the lines of hours 9 and 3 have been carefully drawn, draw PM parallel to the line of hour 6, EA (the line we drew in no. 4), cutting the meridian EF at L; PM will be bisected at L, as we showed in Chapter 4, since hours 9 and 3 are six hours apart and hour 12 lies midway between them. On this straight line PM, then, the hour-points will be found, as we taught in Chapter 3. For if a perpendicular Lb is raised to it, equal to LP or LM, and from b the line PM (extended) is cut on both sides at the interval PM, the points of hours 8 and 4 will be found. A third part of L4 (or L8), namely L2 (or L10), will give hour 2 (or 10). The remaining hours are then found from the table of no. 3, Chapter 3.

The same points on PM will be found if from b an arc g h is described and divided into hours. This is easily done: with the same radius b g, cut off an arc g h spanning four hours; then, drawing a straight line from b to the point P and cutting the arc at i, mark off on arc h i and arc g i three equal parts. For the straight lines emitted from b through the points of division will cut LP at the hour-points, which must then be transferred from L towards M.

Finally, these same points on PM, extended indefinitely, are found by means of the Tangents of 15° , 30° , 45° , and so on — taken with respect to the whole sine bL and transferred from L to either side — as we said in Chapter 4, no. 10.

We shall proceed in no other way with the straight line PQ, which is drawn parallel to the meridian line in any position. (We drew it through A, so that it coincides with the vertical.) For this line too is bisected between hours 9 and 3 by the line of hour 6. Indeed, if PQ has been drawn through A so as to represent the prime vertical, it will be worth while to take from the 6th table, in the right-hand column opposite the pole's altitude, the arc under hours 3 and 9 — in our example $53^\circ 23'$. For if you take its Tangent, about 13[?] and a little more, divide it (with respect to the whole sine DA) into 10 equal parts, and transfer it from A on both sides, you will find the points P and Q through which hours 3 and 9 must be drawn — points which must necessarily pass through those found on the horizontal line, if no error has been made. And by the same method you will find the points of the other hours on the line PQ from the 6th table, as we wrote above in no. 7. If, then, from the point of intersection a perpendicular A a is raised to PQ, equal to half of AP or AQ (in our example the straight line A a is the same as

the horizontal), and from a the line PQ (extended) is cut on both sides at the interval PQ, the points of hours 10 and 2 will be found, four hours distant from the point of hour 6; and a third part of that same line A10, namely A8, will give hour 8, which lies two hours from A, the point of hour 6. From this the other hours are found from table 1 of no. 9, Chapter 4.

The same hour-points on PQ are found if from a an arc is described and divided into hours, and so on.

Or, if into it there are transferred from A on both sides the Tangents of 15° , 30° , 45° , and so on — with respect to the whole sine a A — for the whole hours, and so on, just as was done in Chapter 2, no. 3, for the equinoctial line of the horizontal sundial.

CAPVT XVII.

The hours from noon and from midnight on a declining plane, where either the point of hour 6 on the horizontal line, or the centre of the sundial, or both points, or the point of the meridian on that same horizontal line — together with all the rest [omnibus? [?]] of the sundial — can be had only most inconveniently.

[Margin: When the point of hour 6 is not available on the horizontal line, what to do.]

When the point A on the horizontal line, through which the line of hour 6 must be drawn, cannot conveniently be had — which happens when the plane of the sundial deviates only slightly from the prime vertical, because then the complement of the wall's declination falls only a little short of 90° , and so the straight line DI cuts the horizontal too far off, and its Tangent CA spans more parts than the size of the sundial allows — when, I say, the point A can scarcely be had, we lay out the sundial as follows. Having found the points B, E, F as we explained in the preceding chapter, nos. 1 and 2, draw the line of the style [linea styli] EC; this will be drawn more accurately if the Lemma placed before Chapter 16 is employed. Or, if you prefer, in an arc described from E count off the arc of the plane between its own meridian and the meridian of the horizon, found by Problem 6 at the end of the booklet, and so on, as was done in the preceding chapter, no. 3. Then from F raise a perpendicular to the line of the style, to serve as the equinoctial line, even though it does not meet the horizontal — since the point A is assumed to be unavailable. Or, if you prefer, at F construct with the

meridian an angle equal to the complement of the angle BEC, that is, equal to the angle BCE. For the straight line making this angle will be the equinoctial line.

[Margin: A fine way of describing the hour[-line] even when the point on the horizontal line of the sundial is not available. [?]]

After this, having drawn the lines of hours 9 and 3 as was stated in no. 9 of the preceding chapter, from any point l of hour 9 draw a line l m parallel to the meridian, cutting hour 3 at m. For l m, bisected at n, gives (by the straight line E n) hour 6, even though it does not cut the horizontal — because the line of hour 6, lying midway between hours 9 and 3, bisects the line l m (which is parallel to the meridian), since hour 6 is six hours distant from the line of hour 12, as was demonstrated in Chapter 4, no. 6. And in this way the line of hour 6 can be drawn in any sundial, provided hours 3 and 9 have first been drawn, even if the point where it cuts the horizontal is very far off. This line of hour 6 can also easily be drawn, if you wish, by counting off in an arc described from E, from the meridian line, the arc of the plane between the meridian and the circle of hour 6, found by Problem 8 at the end of the booklet, as was done in no. 4 of the preceding chapter. The rest is completed as was set out in the preceding chapter, especially in no. 10 — namely by drawing p q parallel to the line of hour 6. For this too will be bisected at r. Accordingly, on the straight lines l m and p q the hour-points will be found, just as in no. 10 of the preceding chapter they were found on the lines PQ and PM.

[Margin: When the centre is not available in the sundial, what to do.]

2. When, however, the points B and A have indeed been found on the sundial conveniently enough — whether by Tangents or by the arcs GH and GI of the wall's declination and of its complement, as was done in no. 1 of the preceding chapter — but the centre G of the sundial can be located only with difficulty (which happens when the pole-altitude of the region is very great, even though the declination of the plane is moderate), the point F must be found by the Tangent of the complement of the given pole-altitude, as was stated in no. 2 of the preceding chapter, and in addition the equinoctial line AF must be drawn. This will be drawn more accurately by the Lemma before Chapter 16. Or, if you like, by Problem 6 at the end of the booklet: reckon the arc of the plane between the horizon and the equator, and count it off downward from the horizontal line in an arc described from A, and so on. At a pole-altitude of 80° , with the declination set at 30° , this arc contains $5^{\circ}2'$. The point F will be found

[Margin: [?]¹⁵. primi [?] (uncertain marginal reference)]

as well if — drawing the straight line RE through an arc of the pole-altitude described from R (although it does not meet the meridian except at a very remote place) — a perpendicular RF is drawn to RE, cutting the meridian at F.

Next, to the equinoctial AF a perpendicular CK must be drawn through C, the place of the style, to serve as the line of the style, even though the centre E of the sundial is unavailable. Or indeed, if you prefer, in an arc described from C count off, from the horizontal line on the side of A towards F, the complement of the arc of the declining plane between its own meridian and the meridian of the horizon — which arc is computed by Problem 6 at the end of the booklet and, at the aforesaid pole-altitude and declination, contains $5^{\circ}2'$, while its complement is $84^{\circ}58'$. For the straight line making this angle will be the line of the style, since the angle ACK is equal to the angle BCE, which is the complement of the angle BEC — as it would be if the point E were present. Or indeed, in the same arc count off, from the straight line CG towards A, the arc itself of $5^{\circ}2'$.

The line of hour 6 will likewise be drawn if, in an arc described from A, one counts off upward from the horizontal line the arc of the declining plane between the horizon and the circle of hour 6, found by Problem 8 at the end of the booklet, and so on. At the said pole-altitude and declination this arc contains $70^{\circ}34'$.

Once the lines of the style and of hour 6 have been drawn, we shall proceed to describe the other hours as follows. Since we do not have the centre of the sundial, we shall draw from C, the place of the style, a perpendicular CZ to the line of the style, equal to the style; and to KZ (the common section of the equator with its own meridian), joined to it, we shall raise another perpendicular Z f, which will represent the axis of the world — even though we do not have the point where it cuts the meridian line BF. Then, taking any point t on the line of the style, raise a perpendicular t s to the axis, to serve as a new common section of the equator with its own meridian; and [raise], perpendicular to the line of the style through t [and Ct], a line to serve as a new equinoctial line. Now if — the inclination of the proper meridian to the meridian of the horizon having been found by Problem 5 at the end of the booklet, which (with the pole-altitude set at 80° and the declination at 30° , as before) contains $30^{\circ}23'$ — the hour-points are found on each equinoctial by Tangents, with respect to the whole sines KZ and t f, as was stated in no. 8 of the preceding chapter, then the hour-lines must be drawn through the corresponding hour-points on each equinoctial, even though the centre in which they ought to meet does not appear. It is not necessary to find the hour-points on the horizontal line, because at the aforesaid pole-altitude the

horizontal lies too close to the equinoctial line AF — as is clear if a sundial is constructed for that altitude of 80° and declination of 30° .

[Margin: When neither point A nor E is available, what to do. [?]]

3. But when both points A and E cannot easily be had (which happens when the pole-altitude is very great and the wall's declination very small), the point B must be found by the Tangent of the wall's declination, and the point F by the Tangent of the complement of the pole-altitude. Then through C the line of the style must be drawn, by means of an arc described from C and counted from the line CG towards A — the arc, namely, that is interposed between the horizon and the equator, or between the meridian of the horizon and the proper meridian of the declining plane, and which, by Problem 6 at the end of the booklet, at a pole-altitude of [numeral:?] and a declination of 4° , contains $7^\circ 1'$. To this line of the style a perpendicular must be raised from F, to serve as the equinoctial line. Or indeed, if you please, in an arc described from F towards A count off from the meridian the complement of the aforesaid arc (the one, that is, that lies between the meridian of the horizon and the equator), and at the end of the counting draw the equinoctial line, and to it raise a perpendicular from C for the line of the style. Everything else is to be carried out as we said just above in no. 2. But hour 6 cannot be drawn, since its point is found neither on the horizontal nor on the meridian line, and therefore not on the equinoctial line either, which does not cut the horizontal except at the point of hour 6 — assumed to be unavailable. The inclination of the proper meridian to the meridian of the horizon contains $35^\circ 23'$, by Problem 5 at the end of the booklet.

[Margin: When neither the point B of the meridian line nor the centre of the sundial is available, what to do. [?]]

4. When, finally, the point B of the meridian line — and therefore also the centre of the sundial — cannot conveniently be had (which usually happens when the declination of the plane is very great), the point A must be found, through which the equinoctial line too must be drawn, by means of an arc described from A that is interposed in the plane between the horizon and the equator. By Problem 6 at the end of the booklet, if the plane declines from south towards east by 89° , this arc is found to span $48^\circ 0'$, a little less, at latitude 42° . Then the same arc must be counted off from the straight line CG towards A in a circle described from C, and at the end of the counting the line of the style CK must be drawn. Or indeed, from

C a perpendicular must be raised to the equinoctial, to serve as the line of the style. Likewise, through A the line of hour 6 must be drawn, by means of an arc described from A that is interposed in the plane between the horizon and the circle of hour 6. At a pole-altitude of 42° , with the sundial's declination from south towards east set at 89° , this arc is found by Problem 8 at the end of the booklet to span $42^\circ 0'$, a little less. Everything else is to be completed as stated in no. 2. The inclination of the proper meridian to the meridian of the horizon (by which, namely, the hour-distances from the line of the style are to be elicited, as was set out in the preceding chapter, no. 8), at a pole-altitude of 42° and a declination of 89° , is found by Problem 5 at the end of the booklet to be $89^\circ 20'$.

[Margin: A more convenient method in the aforesaid four cases.]

5. But in the aforesaid four cases we shall also proceed very conveniently as follows. Let the style be continually bisected until the last part, near the place of the style, is so small that — taking it for the style — the three points A, B, E can conveniently be found, by the methods set out in Chapter 16, and so the whole sundial can be described. For if this sundial is enlarged according to the size of the given style — that is, if all its intervals are multiplied as many times as the style's subdivision was carried out, as will be taught in Chapter 23 — the sundial that was to be described will be described, even if some of its points cannot be found because of their too great distance from the place of the style. But all these difficulties will be removed if the sundial is described by the method of Chapter 24 or 25, as will be stated in the scholium to Chapter 25, no. 3.

CAPVT XVIII.

The arcs of the signs in the hours from noon and from midnight of a declining sundial.

[Margin: The description of the arcs of the signs on a declining sundial, by points on the line of hours from noon and from midnight. [?]]

The arcs of the signs are described on a declining sundial just as on a horizontal one. For if in table 1 one takes the arcs corresponding to the beginnings of the signs for the individual hours in the circle of hour 12 — that is, in the line of hours to which the title [Circ. hor. 12.] is written on the left, or the number [12.] on the right side — and if by these same arcs equal portions are cut off from the meridian line, beginning at the equinoctial line, then the straight lines emitted from A through the endpoints of these portions will give, on the corresponding

hour-lines, points parallel to the one set out. Now we shall cut off the aforesaid arcs from the meridian line as follows. For the southern parallels (which in mural sundials are contained between the equinoctial and the centre E), let the arc of any hour be subtracted from the complement of the pole-altitude above the region's horizon, if it can be subtracted; and let the Tangent of the remaining number — on the straight line whose portion DB was divided in the preceding chapter into 10 equal parts for the meridian line — be transferred, taken with the compass, from B towards the equinoctial. Or let the complement of the pole-altitude be subtracted from the arc of table 1, if it can be done, and let the Tangent of the remaining number be transferred from the same point B towards the centre of the sundial. But if some arc is found equal to the complement of the pole-altitude, the point B itself will be the endpoint of that arc. When, however, two hours have a greater distance from noon than 6 hours — as does hour 5 from midnight and 7 from noon — the pole-altitude must be added to the complement of those hours' arc, and the Tangent of the sum must be transferred from B beyond the centre E of the sundial, as was stated in Chapter 8, no. 7. Finally, for the northern parallels, any arc of table 1 must be added to the complement of the pole-altitude, and the Tangent of the sum transferred from B beyond the equinoctial. But if the sum exceeds a quadrant, that hour's point cannot be found on the meridian. In that case the hour's point must be found on the northern parallel by the 2nd rule of the scholium to Chapter 8 — namely, from some point opposite the southern parallel already found. This must also be done whenever some point is too far removed from the horizontal line, so that it cannot be marked on the sundial.

For example: in the circle of hour 12, in table 1, under hours 3 and 9, opposite ♋ [Cancer] and ♑ [Capricorn], there is found an arc of $31^{\circ}35'$. Subtracting this from the complement of the pole-altitude — that is, from 48° , to which the portion BF corresponds — $16^{\circ}25'$ remains. Its Tangent, about 3 (with respect to the whole sine BD, which serves the meridian), transferred downward from B, gives the point H, through which the straight line drawn from A will cut hours 3 and 9 at the points of the tropic ♋. But the same arc of $31^{\circ}35'$, added to the complement of the pole-altitude, 48° , makes $79^{\circ}35'$, whose Tangent, about 54[?], transferred from B, will give a point on the meridian through which the straight line emitted from A will cut hour 9 at the point ♎, and hour 3 from midnight at the point ♑ — since it cannot cut hour 3 from noon. In our example, moreover, that point on the meridian could not be marked because of the narrowness of the space. So we shall find the point ♎ on hour 9 by the straight line drawn from the point ♑ (found on hour 3 from noon) through the point F. And the point of hour 3 from midnight we shall find by the straight line drawn from this point ♎ (now found

on hour 9) through the point A, as the 2nd rule of the scholium to Chapter 8 directs.

Next, the arc of hours 5 and 7 spans $59^{\circ}14'$. If from it the complement of the pole-altitude, 48° , is subtracted, $11^{\circ}14'$ remains, whose Tangent, about 2, transferred upward from B, gives the point I, through which the straight line emitted from A cuts hour 7 from midnight at the point $\mathfrak{V}$, and hour 5 from midnight at the point $\mathfrak{S}$ — since hour 5 from noon does not cut it.

Likewise, the arc of hour $5\frac{1}{2}$ from midnight and $6\frac{1}{2}$ from noon contains $73^{\circ}17'$, whose complement is $16^{\circ}43'$. If the pole-altitude 42° (to which the portion BE corresponds) is added — since the aforesaid hours have a greater distance from noon than hour 6 — the arc becomes $58^{\circ}43'$, whose Tangent, about 16[?], transferred from B beyond the centre E, will give the point L; and the hidden straight line AL will cut hour $5\frac{1}{2}$ from midnight at the point $\mathfrak{V}$, and, produced, will cut hour $6\frac{1}{2}$ from midnight at the point $\mathfrak{S}$ — since it does not cut hour $6\frac{1}{2}$ from noon.

So too, since the arc of hour 12 contains $23^{\circ}30'$, if it is subtracted from 48° and added to the same, arcs of $24^{\circ}30'$ and $71^{\circ}30'$ result, whose Tangents — 4[?], a little more, and 29[?], about — will give the points O and M of the parallels $\mathfrak{V}$ and $\mathfrak{S}$.

But since the points of certain hours cannot be marked on the meridian line, because they are removed too far from the point B, we shall find other points on the line of hour 6, through which, if hidden straight lines are emitted from E, the hours will again be cut at the points of the parallels. We shall proceed as follows. From C, the place of the style, raise a perpendicular CNP to the line of hour 6; and, taking a straight line N a equal to the style, cut off NP equal to the interval C a. Then, on any straight line, taking as many segments as you like equal to that same C a — with an interval equal to C a — divide the first C a into 10 equal parts. Then, having found the arc of the circle of hour 6 between the pole E and the vertical perpendicular and that same circle (which the straight line CNP represents), and which in our sundial, by Problem 10 at the end of the booklet, spans $37^{\circ}51'$, we shall subtract from the meridian line the arc of the plane between the meridian and the circle of hour 6 (found by Problem 8 at the end of the booklet) — that is, from $52^{\circ}9'$ — the arcs of table 1 corresponding to the hours in the line of the circle of hour 6, which is the topmost; or [we shall subtract] the very complement of the arc $52^{\circ}9'$ from the aforesaid arcs of table 1 — namely the smaller numbers from the larger, for the southern signs; and the Tangents of the remaining arcs, taken on the straight line C a, we shall transfer from the point N onto the line of hour 6 — downward, indeed, if the arcs have been subtracted

from the said complement, but upward if the complement has been subtracted from the arcs. For the northern parallels, on the other hand, the arcs of table 1 corresponding to the hours in the circle of hour 6 are to be added to the aforesaid complement.

But when the Tangents to be transferred onto the meridian line are too long, we shall proceed as follows. Cutting the portion AB of the horizontal line in half, we shall draw through the point of division a line perpendicular to the horizontal; for if into this there are carried the halves of the Tangents that were to be transferred onto the meridian, the points will be found through which the straight lines drawn from A will indicate the parallels' points on the hour-lines. And if in this perpendicular some points still stand too far from the horizontal, the portion between A and the said perpendicular must again be cut in half, and through the point of division another perpendicular to the horizontal must be raised. For if into this there are transferred the quarter parts of the Tangents that were to be transferred onto the meridian, the points will be found through which the straight lines emitted from A will cut the hour-lines at the points of the parallels.

When the Tangents are to be transferred from N beyond the centre of the sundial — as happens in some hours from noon in the parallel ♎, namely in hours 1 and 2 — the complement of any arc of table 1 corresponding to the hours in the circle of hour 6 must be added to the arc of the circle of hour 6 between the pole and the point N, that is, in our example, to $37^{\circ}51'$, provided the resulting arc does not exceed 90° ; otherwise, to the complement of the said arc EN one must add the arc of any hour from table 1, and the Tangent must be transferred from N towards the equinoctial. The very position of the hour-lines will readily show whether the Tangents are to be transferred beyond the centre E, or rather towards the equinoctial line.

For example: the arc of hours 1 and 11 in the circle of hour 6 contains $59^{\circ}14'$. If the complement of the arc EN (I mean the arc to which the portion EN is equivalent) — that is, $52^{\circ}9'$ — is subtracted, there remains an arc of $7^{\circ}5'$, whose Tangent, 1 [fraction:?] (a little more), transferred from N toward E, will give a point through which a straight line drawn from F will cut hour 11 at the point ♎ [Capricorn], and consequently also hour 1 at the point ♋ [Cancer]. But if the complement of the aforesaid arc of table 1 — containing $30^{\circ}46'$ — is added to the arc EN of $37^{\circ}51'$, the arc becomes $68^{\circ}37'$, whose Tangent, 25 [fraction:?] (a little more), will give, beyond the centre, the point S; and the straight line FS will cut

hour 1 at the point ♊ [Capricorn], and consequently hour 11 at the point ♋ [Cancer].

Again, the arc of hours 5 and 7 in the circle of hour 6 contains $24^{\circ}14'$. If the arc of the segment NA, $52^{\circ}9'$, is added, the arc becomes $76^{\circ}23'$, whose Tangent, 41 [fraction:?] (nearly), will give the point T; and the straight line FT will cut hour 7 at the point ♋ [Cancer], and, produced toward T, would also cut hour 5 at another point ♋. But if you subtract the same arc, $24^{\circ}14'$, from the arc of the segment NA, $52^{\circ}9'$, there remains an arc of $27^{\circ}55'$, whose Tangent, 5 [fraction:?] (nearly), will give a point through which a straight line drawn from F will cut hours 5 and 7 at the points ♊ [Capricorn].

Finally, the arc of hour 6 contains $23^{\circ}30'$; added to the arc of the segment NA, $52^{\circ}9'$, it makes an arc of $75^{\circ}39'$, whose Tangent, 39, will give the point V, in hour 6, for the tropic ♋ [Cancer].

3. Without Tangents, the points on the line of hour 6 can also be found — as was shown in the problem of chapter 1 — in this way. Having drawn the perpendicular CN and found the point P (as we said at the beginning of Number 2), let a hidden straight line PA be drawn, and from P let the circle QR be described. For if in it the arc of any hour of table 1 in the circle of hour 6 is counted from the point Q — to the left, indeed, for the northern signs, but to the right for the southern — and through the end of the count a hidden straight line is emitted from P, the line of hour 6 will be cut at a point through which a straight line from F will cut the proposed hour at the point of the given parallel. For example, the arc of hour 6 is $23^{\circ}30'$; let it be counted from the point Q in both directions, as far as R and X. For the straight lines PR, PX will cut hour 6 at the points V, e, ♋ [Cancer], and ♊ [Capricorn]. Likewise the arc of hours 5 and 7 in the parallel of ♉ [Taurus] and ♍ [Virgo], and also of ♏ [Scorpio; glyph worn, [?]] and ♐ ([♋ Pisces], is $11^{\circ}54'$; counted from the point Q leftward as far as d, the straight line Pd will cut hour 6 at the point b, and the straight line Fb will cut hours 5 and 7 at the points f, f of the parallels ♉ [Taurus] and ♍ [Virgo].

Diagram. A large, extremely dense construction figure for a declining sundial, filling most of the page within a heavy rectangular border. A sheaf of straight hour-lines radiates from a principal node near the upper-left interior (labelled 3) and from a second principal centre lower in the figure (near the point labelled A); superimposed on these are dotted (hidden/occult) construction lines, several arcs of the parallels of the signs, and a set of graduated tangent scale-bars along the right margin. Because the engraving is packed with intersecting lines and many

labels are minute and overprinted, only the clearly legible labels are transcribed; a large number of interior point-letters cannot be resolved.

- Left margin, hour-lines (read top-to-bottom): 6, 7, 8, 9, 10, 11, 12
- Upper interior radiating lines: also numbered 1, 2, 3, 4, 5
- Top edge and upper-right, hour-lines (morning / after-sunrise): 17, 18, 19, 20, 21, 22, 23, 24
- Additional line-numbers near the lower interior: 4½, 5, 23, 24
- Right margin: several vertical graduated tangent scale-bars headed with letters (legible column heads include C, C, C and, to their left, E, λ, ψ), with tick-labels 10, 20, 30, 40, 50
- Legible interior point-letters: A (lower principal centre), E, F, K, L, M, N, O, P, Q, R, S, T, V, X, and the Greek α (alpha, appearing near the point A and again — doubled, αα — up near hour-line 24)
- Numerous further interior point-letters and short labels are illegible [several labels [?]]

[Margin: The drawing of the arcs of the signs by means of points on the line of hour 6 and on the meridian, found without Tangents.]

When the same arc is counted from the point Q toward the right as far as h, the straight line Ph will cut hour 6 a little below a, at a point through which a straight line issuing from F will cut the same hours 5 and 7 at the points g, g of the parallels ℳ [Scorpio; glyph worn, [?]] and)([Pisces].

In no other way are the points found on the meridian line, if — with an interval Bm taken equal to BD, up to the tip of the style, on whichever side you prefer — a hidden straight line is drawn from m through F. For if from m a circle is described, in which the arcs of table 1 in the circle of hour 12 are counted on both sides from the line mF, the straight lines emitted from m through the ends of these arcs will cut the meridian at points through which straight lines extended from A will cut the hours at the points of the parallels.

4. But most conveniently the arcs of the signs will be drawn by the method we set out in chapter 8, Number 7 — especially in sundials in which either the meridian line (and therefore the centre), or the point A of hour 6, is not available on the horizontal line. The matter will be carried out as follows. Let us conceive the style-line CK to be the meridian of some horizontal sundial, and C the zenith of the place. On either side of the style-line let hour-lines be drawn, fifteen degrees apart from one another, like the hour-lines in a horizontal sundial, as was taught in chapters 2, 3, and 4 — so that

the two nearest lines on either side stand for hours 1 and 11, the next two for hours 2 and 10, the next two for hours 3 and 9 (such are the lines k p, n q), and so on for the rest. The Tangents are to be taken on the equinoctial (when the said lines are to be drawn by means of them) with respect to the whole sine KZ, in the sundial of the preceding chapter, so that points fifteen degrees apart from one another are found.

Then, by Problem 4 at the end of the booklet, let the elevation of the pole above the declining plane be found, which in our example contains $40^{\circ}4'$. Once it is found, let there be cut off from the style-line — as [one would] from the meridian — the arcs of table 1 corresponding to the hours in the circle of hour 12, exactly as we taught in chapter 8, Number 7, and so on. This will be understood from the following two examples. From the little table of the scholium of chapter 1 let the Tangent of hours 3 and 9 be taken — namely 10 — on the line KZ of the preceding chapter, which is assigned to the equinoctial line, and let it be transferred from K on both sides as far as r and s, in the figure of this chapter. For the hidden straight lines Er, Es will stand for the lines 9 and 3. And since in table 1, under hours 3 and 9, on the line of the circle of hour 12, opposite $\mathfrak{C}$ [Cancer] and $\mathfrak{C}$ [Capricorn], is found an arc of $31^{\circ}35'$ — which, if it is subtracted from $40^{\circ}4'$ (the elevation of the pole above our declining plane, to which the segment CK corresponds, as is clear from the figure of the preceding chapter), leaves an arc of $8^{\circ}29'$, whose Tangent, 1 $\frac{1}{2}$ (nearly), transferred downward from the point C, will give a point through which, if a line k n parallel to the equinoctial is drawn, the straight lines Et, Es will be cut at the points k, n for $\mathfrak{C}$ [Capricorn]. And if the same arc, $31^{\circ}35'$, is added to the aforesaid elevation of the pole, $40^{\circ}4'$, an arc of $71^{\circ}39'$ is produced, whose Tangent, 30 $\frac{1}{2}$ (a little more), transferred downward from C, will give a point through which [a line drawn parallel to the equinoctial] will cut the straight lines Er, Es at the points p, q for $\mathfrak{C}$ [Cancer]. And the same reasoning holds for the rest.

If, then, through the points found on the hidden lines (as [one finds] the hour-points of a horizontal sundial) the parallels of the Sun are drawn correspondingly, these will cut the hour-lines of the declining sundial at the points of those same parallels. But in order that the northern parallels be drawn more exactly, points on the hidden lines of the half-hours and quarter-hours must also be found for them, so that more numerous points may be had.

5. But even without knowledge of the elevation of the pole above the declining plane, the arcs of the signs can also be drawn with ease — provided the centre of the sundial is available — by the method we

prescribed in chapter 8, Number 1, from table 1: namely, if for the line of hour 6 a line parallel to the equinoctial itself is drawn through E, the centre of the sundial, standing for the line of hour 6 with respect to the lines which at the beginning of this Number 4 we directed to be drawn; and onto it are transferred the complements of the arcs drawn in table 1 opposite the signs, under the hours in the circle of hour 6 (which line stands for the line of hour 6). For if from K, through the points marked on that parallel line, hidden straight lines are drawn, those hidden lines will be cut — like the hour-lines of a horizontal sundial, which at the beginning of this Number 4 we said were to be drawn — at the points of the arcs of the signs, and so on.

It is also enough here if only the southern parallels are drawn; for from these the opposite northern ones will be drawn, as was said in chapter 8, Numbers 3, 4, 5, and 6 — especially by the method we prescribed at the end of Number 6 of chapter 8. For if, for example, the segment of the proper meridian line — that is, of the style-line $t u$ — between the tropics ♊ [Capricorn] and ♋ [Cancer] (the points t, u of the opposite parallels will be found by Tangents, as we taught in chapter 8, Number 3, if for the elevation of the pole above the Horizon one takes the elevation of the pole above the declining plane, which by Problem 4 at the end of the booklet is found to contain $40^{\circ}4'$; or, if you prefer, geometrically: if in the sundial of chapter 16, at the axis EZ , the angles of declination are set up at Z — that is, if from Z the radii of the signs are emitted on either side of the radius of the Equator ZK , for these will cut the style-line at the points of the parallels, as we showed in chapter 9, Number 1) — is bisected at Y ; and from any point of the tropic ♊ [Capricorn], say from ϵ [epsilon], the point of hour 5, a straight line $\epsilon Y \beta$ is extended through Y , and an interval $Y \beta$ equal to the interval $Y \epsilon$ is cut off, the point β [beta] will fall on the tropic ♋ [Cancer] — which was demonstrated by Apollonius [of Perga], Book 1, Proposition 30. And so for the rest. But you will do well if first, from Y , at the interval $Y \epsilon$, a small arc is drawn on the opposite side, which is then cut at a point by the hidden straight line ϵY , and so on. For in this way the confusion of the hidden lines will be avoided, as was said at the end of Number 6 of chapter 8.

6. But the easiest method of all is taken from table 2, by which points are found on the meridian line; through these you will cut the hours at the points of the parallels — whether from hour 2 after noon on the equinoctial, or hour 10 after midnight; or from hour 9, 8, or 7 after midnight; or from hour 3, 4, or 5 after noon (with respect, of course, to whichever hour you found the points on the meridian from table 2) — as

was explained in chapter 8, Number 10. For example, if from hour 10 after midnight, on the equinoctial, the point ♊ [Capricorn] is to be found at hour 3 after noon, you will find in table 2, under hours 3 and 5, on the line headed *From hour 10 after midnight*, opposite ♊ [Capricorn], an arc of $12^{\circ}41'$. When this is subtracted from the complement of the elevation of the pole above the Horizon (to which the segment BF is equivalent), there remains an arc of $35^{\circ}19'$. If, then, you transfer its Tangent — which is 7, with respect to the whole sine BD divided into 10 parts — downward from B, you will find a point through which a straight line issued from F will cut hour 3 after noon at the point ♊ [Capricorn], and hour 5 after midnight at the point ♋ [Cancer]. And so with the others.

And this method is very convenient if in the sundial there are drawn lines equally distant on either side from the style-line, like the hour-lines in a horizontal sundial, and if the elevation of the pole above the declining plane is found, as was said in Number 4. For example, in the sundial of this chapter, once the straight lines n q, k p are drawn — the former being hour 3 after noon, the latter hour 9 after midnight (if the style-line were the meridian) — in table 2, on the line headed *From hour 3 after noon*, under hour 9, opposite ♊ [Capricorn], is found an arc of $17^{\circ}5'$. Subtracting this from the elevation of the pole above the plane — namely from $40^{\circ}4'$ — leaves $22^{\circ}59'$, whose Tangent, 4 [fraction:?] (a little more), transferred from C, in place of the style, onto the style-line, will give a point through which a straight line issued from the point f [s] will cut the line k p at the point k of the parallel ♊ [Capricorn], and so on.

[Margin: Another drawing of the arcs of the signs, most convenient, especially in sundials in which either the meridian line, or the point of hour 6, is not available on the horizontal line.]

[Margin: The immediately preceding drawing of the arcs of the signs, without knowledge of the pole above the declining plane.]

[Margin: How the drawing of the northern arcs of the signs is to be made from the southern ones.]

[Margin: The drawing of the parallels from table 2 — the easiest of all.]

7. From table 3 the same arcs of the signs will be laid out, if — once the straight lines AM, AO are drawn (of which the former represents the Ecliptic with its beginning ♋ [Cancer] placed on the Meridian above the Horizon, the latter the Ecliptic with its beginning ♊ [Capricorn] placed on the Meridian above the Horizon) — by Problem 14 at the end of the booklet

one seeks the arc of the Ecliptic between the Meridian and the great circle that is drawn perpendicular from the pole of the declining plane to the Ecliptic; which perpendicular is represented by the line $C\delta$ [C-delta] on the straight line AM, and by $C\gamma$ [C-gamma] on the straight line AO. In our example, the segment $M\delta$ contains $61^{\circ}12'$, while the segment $O\gamma$ contains $32^{\circ}24'$. For on the straight line AM we shall chiefly find the points through which straight lines drawn from F will cut the hour-lines at the points of the northern parallels; but on the straight line AO, the points for the southern parallels — although either of the two suffices, as we shall say in practice. We shall therefore proceed in this way. Let the arcs of table 3 in the hours after midnight be subtracted from the arcs to which the segments $M\delta$, $O\gamma$ correspond — that is, from $61^{\circ}12'$, or from $32^{\circ}24'$; or, conversely, these arcs from those, namely the smaller numbers from the larger. But the same arcs of table 3 in the hours after noon are to be added to the arc of the straight line $M\delta$ or $O\gamma$. For the Tangents of the arcs that remain, or that are made up, transferred from δ or γ onto the straight line AM or AO, will give points through which straight lines issuing from F will cut the hours at the points of the parallels. The Tangents, moreover, are to be taken on the straight line $C\delta$ or $C\gamma$, which are equal to the intervals δC , γC , the straight lines $\delta\epsilon$ and $\gamma\lambda$ being taken equal to the style. Likewise these same Tangents are always to be transferred from δ or γ toward the right, except when the arcs themselves are subtracted from $M\delta$ or $O\gamma$ out of the arcs of table 3.

For example, the arc of hour 10 after midnight, in table 3, with beginning $\mathfrak{C}$ [Cancer] and $\mathfrak{M}$ [Capricorn], contains $24^{\circ}38'$; and in the parallel of $\mathfrak{C}$ [Cancer], with the Ecliptic represented by the straight line AM, it inclines toward the hours before noon, as is clear from the material sphere. Subtracting it, then, from $61^{\circ}12'$, there remain $36^{\circ}34'$, whose Tangent, 7 [fraction:?] (a little more), transferred from δ toward M, shows a point through which a straight line extended from F cuts hour 10 at the point $\mathfrak{C}$ [Cancer], and hour 2 at the point $\mathfrak{M}$ [Capricorn]. The same arc, $24^{\circ}38'$, added to $61^{\circ}12'$, makes $85^{\circ}50'$, whose Tangent, 137 [fraction:?] (a little more), transferred from δ toward M, would show a point — if the plane were so large — through which a straight line issued from F would cut hour 10 at the point $\mathfrak{M}$ [Capricorn]. This point in $\mathfrak{M}$ [Capricorn] we shall find more conveniently by means of the straight line AO. For subtracting the said arc, $24^{\circ}38'$, from $32^{\circ}24'$ — to which the segment γO is equivalent — there remain $7^{\circ}46'$, whose Tangent, 1 [fraction:?] (a little more), transferred from γ , offers a point nearer O, through which a straight line issued from F will cut hour 10 at the point $\mathfrak{M}$ [Capricorn]. But if you wish to find the point $\mathfrak{C}$ [Cancer] on hour 10 by

means of the straight line AO, the aforesaid arc, $24^{\circ}38'$, is to be added to $32^{\circ}24'$, because in the Ecliptic represented by the straight line AO that arc inclines toward the afternoon hours, as is clear from the material sphere. For the arcs joined make $57^{\circ}2'$, whose Tangent, 15 [fraction: ?], transferred from γ , [gives a point through which] a straight line issued from F cuts hour 10 at the point $\mathfrak{S}$ [Cancer]. And so for the rest. But this method is more convenient and easier in a horizontal sundial; and therefore it may with very good reason be left aside here.

It will, however, be most convenient if the hidden hour-lines are drawn as in a horizontal sundial, as we taught at the beginning of Number 4; for then the Ecliptic will have to be drawn perpendicular to the style-line through the point $\mathfrak{S}$ [Cancer] or $\mathfrak{V}$ [Capricorn], which point is to be found on the style-line from the elevation of the pole above the declining plane — just as, in a horizontal sundial, we taught that it was to be found on the meridian line from the elevation of the pole above the horizon.

8. If, by Problem 15 at the end of the booklet, the altitudes of the Sun above the declining plane are computed, the Tangents of their complements — taken from the place of the style (which is the whole sine of those Tangents) — will cut, at the hours after noon or after midnight, at the points of the parallels, as was shown in chapter 8, Number 12.
9. Moreover, everything that was set out in the scholium of chapter 8 and in chapters 9, 10, and 11 is to be applied here. Only, for the meridian line, the style-line is here to be understood. For in this way we shall draw the parallels of the Sun by several methods, and shall be able to examine whether they have been correctly drawn or not — especially by the three rules of the scholium of chapter 8. But in the method of chapter 10, the circle ET (or Ma) will have to be divided not into hours, but into any number of equal parts; and through those parts straight lines are to be emitted from G, cutting the straight line EH at points through which, finally, straight lines are to be drawn from A, the centre of the sundial, like the hour-lines, as we taught in the scholium of chapter 10, Number 2. For on these will be found the points of the signs, by means of straight lines issued from F through the points of division of the circle ET (or Ma).
10. Finally, we shall point out yet another way of investigating the points of the parallels, in the scholium of the following chapter, Number 4, from certain arcs of the Horizon and of the Vertical.

11. By almost the same methods, the arcs of the signs in the sundial of chapter 17 — where either one of the points A, B, E, or two of them, are not available — can be drawn, as is evident.

[Margin: The drawing of the arcs of the signs from table 3.]

[Margin: The drawing of the parallels by the altitudes of the Sun. Here are to be applied chapters 9, 10, 11, and the scholium of chapter 8.]

[Margin: Where another drawing of the parallels is treated.]

CAPVT XIX.

THE HOURS FROM SUNRISE AND SUNSET ON A DECLINING SUNDIAL.

1. There is almost no difficulty in laying out the hours from sunrise and sunset on a declining sundial. For from table 2 of chapter 14 the points of the hours after noon or after midnight on the equinoctial line will be known, through which the hours from sunrise and sunset are to be drawn; while from table 1 of the same chapter we shall understand through which points of the hours after noon or after midnight on the horizontal line those same hour-lines from sunrise or sunset must pass. For since the horizontal line is the line of hour 24, the hours from sunrise and sunset laid out in that table 1 will pass through those hours after noon or after midnight on the line of hour 24 which are written beneath them.

For example: hour 18 from sunset passes through hour 12 after noon or after midnight on the equinoctial line, as is clear from table 2 of chapter 14; likewise through hour 9 after noon or after midnight on the horizontal line, as appears from table 1 of the same chapter 14. Therefore a straight line through hour 12 on the equinoctial and through hour 9 on the horizontal line, extended, will give the line of hour 18 from sunset. In the same way you will find from the same tables that hour 19 is to be drawn through hour 1 after noon or after midnight on the equinoctial, and through hour $9\frac{1}{2}$ after noon or after midnight on the horizontal; and hour 11 from sunset through hour 5 after noon or after midnight on the equinoctial, and through hour $5\frac{1}{2}$ after noon or after midnight on the horizontal line. And so for the rest — although, for drawing a single hour from sunset or sunrise, it is enough to have recourse to those two tables of chapter 14. For the other hours will be drawn through the following or preceding points of the whole hours on the equi[noctial...]

[Margin: The drawing of the hours from sunrise and sunset on a declining sundial.]

on the equinoctial line, but the points of the half-hours on the horizontal line — so that the hours from sunrise or sunset with even numbers are drawn through the whole hours on the horizontal, and those with odd numbers through the half-hours. Thus you see that, after hour 11 from sunset — which is drawn through hour 5 from noon or midnight on the equinoctial and through hour $5\frac{1}{2}$ on the horizontal — hour 12 is drawn through hour 6 on the equinoctial and through hour 6 on the horizontal (and here these two points coincide in one, so that a different point will have to be found afterwards through which hour 12 is to be drawn). Hour 13 is drawn through hour 7 on the equinoctial and through hour $6\frac{1}{2}$ on the horizontal; hour 14 through hour 8 on the equinoctial and through hour 7 on the horizontal; and so with the rest. In just the same way hour 1 from sunrise will be drawn through hour 7 on the equinoctial and through hour $\frac{1}{2}$ on the horizontal; hour 2 from sunrise through hour 8 on the equinoctial and through hour 1 on the horizontal; and so on.

Diagram. A large declining sundial [horologium declinans] enclosed in a rectangular frame, illustrating the layout of the hours from sunrise and sunset treated in this chapter. From a node in the central-left region of the plate a very dense pencil of straight hour-lines radiates outward; around the whole rim the sunrise/sunset hour-lines are numbered from 1 to 24 (both ends of many lines carry their hour-number). Two principal long reference lines cross the figure — the equinoctial line and the horizontal line — and dotted lines trace the construction of the parallels/tropics (the circles $\pi \mu \rho$ construction of §2). Along the inside of the right-hand edge run several parallel vertical graduated scales carrying the numeric divisions 10, 20, 30, 40, 50, used as the tangent/sine scales for transferring the tabulated values.

The plate carries several dozen point-labels: Latin capitals (A at the dial-centre, together with B, C, D, E, F, G, H, I, L, M and compound labels of the form A-c, A-d, A-f, A-g, A-h, A-k, A-m, A-o), and many Greek minuscules (among them α , β , γ , δ , λ , μ , π , ρ , ψ). Readable border hour-numbers include, going around: 24, 23, 22, 21, 20, 19, 18, 17, 12 (right side and top); 6, 7, 8, 9, 10, 11, 12, 1, 2, 3 (left side); and 4, $4\frac{1}{2}$, 5, 6, 23, 24 (lower part). Because the interior is extremely dense and most of the small point-labels do not resolve cleanly at this resolution, the great majority of the interior letters cannot be committed with confidence.

[Margin: When the two points of some hour from sunrise or sunset are too close together, or when only one point is available, what is to be done.]

2. Now some hours from sunrise or sunset have two points — one on the equinoctial and the other on the horizontal — through which they are to be

drawn, and these lie so close together that they can scarcely be drawn without error. Such, in our example, are hours 11, 13, 14, 15, 16, and so on. Others have only a single point, as does hour 12, whose point is at the intersection of the equinoctial with the horizontal, and hour 21, which has only its point at hour $10\frac{1}{2}$ on the horizontal line. We shall settle the matter with sufficient exactness in this way. When only two points are available, we shall find another two, or even more (if we please), for the same hour, by means of the lemma that precedes chapter 16 — whose procedure for hour 14 from sunset we shall adapt to our sundial, so that the same may also be done for the other hours. For if from point g — that is, from the point of hour 7 on the horizontal line, through which hour 14 is to be drawn — through point μ , the point of hour 8 on the equinoctial, through which the same hour 14 must also pass, a circle $\pi \mu \rho$ is described; and if in it, from μ , two equal arcs $\mu \pi$ and $\mu \rho$ are cut off at any interval; and if from π and ρ , at any interval whatever, two arcs are described both upward and downward, meeting each other at ξ (we took the arcs $\mu \pi$, $\mu \rho$ at the interval of the radius $g \mu$): then μg — that is, hour 14 — will pass through these four points ξ , μ , g , ξ [?]. And so with the rest.

3. When, however, only a single point is available, one must have recourse to the figure of the scholium of chapter 1, number 3. For, once the hours that have two points have been drawn, as has been said, we shall take one of them, whichever seems more convenient — such as hour 20, for drawing hour 12. And since, as is clear from the said figure, hour 12 from sunset and hour 4

from noon or midnight intersect each other at hour 20 (because hours 12 and 20 from sunrise or sunset are, in that figure, equally distant from hour 4 from noon or midnight), we shall draw hour 12 through A and through point $A-c$, where hour 4 from noon or midnight intersects hour 20. And if from $A-c$, and through A , a circle is described, and the rest is carried out as we said of hour 14 by the lemma proposed before chapter 16, a third point will be found below A for the same hour 12. Again, for hour 21 we shall take hour 11 from sunset; and since hours 11 and 21 intersect each other at hour 4 from noon or midnight — as is gathered from the same figure of the scholium of chapter 1, number 3 — we shall draw hour 21 through the point of hour $10\frac{1}{2}$ from noon or midnight on the horizontal line (as the aforesaid figure, or table 1 of chapter 14, requires), and through point $A-d$, where hour 4 from noon or midnight intersects hour 11 from sunset. Had you instead taken hour 12 from sunset, you would have found that hour 21 must be drawn through the point where hour $4\frac{1}{2}$ from noon or midnight

intersects hour 12 from sunset. And in this way, from that figure of the scholium of chapter 1, number 3, several points can be found for the same hour — which is a most useful thing for testing whether the hour-lines have been accurately drawn or not. For if several points correspond to one another, it is proof that the lines have been drawn most accurately.

[Margin: A fine way of testing the hours from sunrise or sunset by means of the figure of the scholium of chapter 1, number 3.]

We shall also, and with easy effort, draw hours of this kind — those that have two points too close together, or only one — in the following way, provided the tropics have first been drawn. On the tropic of Capricorn [φ , the winter tropic] take the point of some hour that is distant by 2, 4, 6, or 8 hours from the one we wish to draw. For from this point, if we draw a concealed straight line to the hour on the equinoctial that is distant from the same hour to be drawn by 1, 2, 3, or 4 hours — namely by half the assumed distance — the tropic of Cancer [ϑ , the summer tropic] will be cut at the point through which the sought hour is to be drawn. Thus, if hour 12, which has only the single point A, is to be drawn, we shall take the point of hour 22 on the tropic of Capricorn (for the more remote the point taken, the more accurately the point for the hour to be drawn will be found); then, if from this point we draw a straight line to hour 17 on the equinoctial, which holds the middle place between 12 and 22, the tropic of Cancer will be cut at the point of hour 12. The same will be found by a straight line drawn from hour 14 on the tropic of Capricorn through hour 13 on the equinoctial, and so on.

[Margin: Hour 12 from sunset, by another method.]

3. But hour 12 from sunset (if you please) we shall also draw most exactly by means of the arc of the declining plane comprehended between the horizon and the circle of hour 12, which — by problem 8 at the end of the little book — contains, in our sundial, 78 degrees 8 minutes. (The line of hour 12 slopes toward the meridian above the horizontal line whenever the said arc is less than a quadrant; for when it is a quadrant, the line is parallel to the meridian, as at latitude 45 degrees; and finally, when it is greater than a quadrant, as at a latitude greater than 45 degrees, it slopes toward the meridian below the horizontal line.) For if from A a circle is described toward the parts of B, and in it, upward from the horizontal line, the said arc of 78 degrees 8 minutes is counted, then a straight line from A extended through the end of the count will give the line of hour 12 from sunset. On it, hours 18, 19, 20, 21, 22, 23 from sunset are to be drawn, through hours 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, $5\frac{1}{2}$ from noon or midnight; and hours 1, 2, 3,

4, 5, 6 from sunrise are to be drawn through hours $6\frac{1}{2}$, 7, $7\frac{1}{2}$, 8, $8\frac{1}{2}$, 9 from noon or midnight — as is established from the figure mentioned above, in the scholium of chapter 1, number 3.

In our sundial, moreover, hour 12 does not answer exactly to the aforesaid arc of 78 degrees 8 minutes, because it was cut somewhat carelessly in the wooden figure. Nor should this trouble you: it is enough that the sundial be drawn exactly if the precepts handed down are duly observed, even if the printed figure answers, in certain lines, less accurately to those same precepts.

Now, in order to free the studios from the trouble they would have in searching out, from the figure of the scholium of chapter 1, number 3, through which points the hours from sunrise or sunset are to be drawn, we have gathered together below, in the notes to the new description of sundials, several little tables, which together with those of chapter 14 can suffice for laying out the hours from sunrise or sunset in any sundial whatever — a matter of which we thought the reader ought to be advised.

[Margin: Finding the points, in the tropics, for the hours from sunrise or sunset by their distances from the meridian.]

Once the tropics of Cancer and Capricorn have been drawn on the sundial, the points of the hours from sunrise or sunset are very conveniently found on them by the distances of those hours from the meridian, which you will find, for latitude 42 degrees, in the scholium of chapter 15, number 1 — as we set out in the same chapter 15, number 4. Let it be required, for example, to draw line 12 from sunrise or sunset. Since the distance of hour 12, in the table of the scholium of chapter 15, contains 113 degrees 3 minutes on the tropic of Capricorn, then if the inclination of the meridians — that is, the segment FK, 40 degrees 48 minutes — is subtracted, there remain 72 degrees 15 minutes for the distance of hour 12 from the line of the style; and its tangent, $31\frac{4}{5}$ [?], transferred onto the equinoctial line from K toward the left, will give the point at which a straight line drawn from E will cut the tropic of Capricorn, at the point through which hour 12 is to be drawn, as you see in the example. Again, since the distance of the same hour 12 on the tropic of Cancer, in the same table of the scholium of chapter 15, comprehends 66 degrees 57 minutes, and 40 degrees 48 minutes being subtracted, there remain 26 degrees 9 minutes, whose tangent, 4 [+ fractional part: numeral: ?], transferred onto the equinoctial from K toward the left, will offer the point through which a straight line sent out from E will cut the tropic of Cancer, at hour 12 from sunrise or sunset, as is plain in the example. Therefore a straight line drawn through A and the two points found on the tropics will give

the line of hour 12 from sunrise or sunset. And the procedure is the same for the rest.

[Margin: Finding the points for the hours from sunrise or sunset at any hour whatever from noon or midnight.]

4. From table 13 we shall find the points of the diurnal arcs at any proposed hour from noon or midnight, through which the hours from sunrise or sunset are to be drawn, in this way. By however many hours the given hour from noon or midnight is distant from the meridian (this distance is to be counted from the twelfth hour of noon in retrograde order if the given hour is from midnight, but in direct order if from noon — thus hour 5 from midnight is distant from the meridian by 7 hours, namely by the complement of 5 up to 12, as appears here: 11, 10, 9, 8, 7, 6, 5; whereas hour 5 from noon is distant from the meridian by 5 hours, as appears here: 1, 2, 3, 4, 5): by so many hours from sunrise or sunset count on, after that hour from sunrise or sunset for which the point of the diurnal arc at the given hour from noon or midnight is sought. For in this way the counting will be terminated at that hour from sunrise or sunset which makes the meridian [i.e., at noon]. And this counting is to be made in direct order if the given hour from noon or midnight belongs to the hours after midnight; but in retrograde order if it belongs to the hours after noon. Then see by how many hours the hour from sunrise or sunset at which the counting ended is distant from hour 24, and double that number: for so you will have the diurnal arc that cuts the given hour from noon or midnight at the proposed hour from sunrise or sunset. Of this diurnal arc take from table 13 the declination, opposite the altitude of your pole: for this — if the diurnal arc is less than 12 hours — subtracted from the quadrant, will give the remaining distance, at the given hour from noon or midnight, of the given hour from sunrise or sunset from the antarctic pole, that is, from the centre of the sundial; but if the diurnal arc is greater than 12 hours, the same declination added to the quadrant will make the aforesaid distance; and if, finally, the diurnal arc is of 12 hours, that distance is equal to the quadrant. This distance you will cut off from the given hour from noon or midnight thus: with a perpendicular drawn from C, the place of the style, to the given hour, let it be found by problem 10 at the end of the little book how great is the arc of the circle of the given hour between the pole and that perpendicular. For this arc is to be subtracted from the found distance, or conversely the distance from

that arc (the smaller number, of course, from the greater) when the said perpendicular falls on the side of the given hour from noon or midnight. For when it falls on the opposite side, the aforesaid arc must always be added to the found distance. For thus will be produced the arc whose tangent, transferred from the point at which the perpendicular cuts the given hour into the given hour, will give the endpoint of the found distance, through which, namely, the proposed hour from sunrise or sunset is to be drawn. And the whole sine [sinus totus] of that tangent, to be divided into 10 equal parts, must be equal to the interval which, between C, the place of the style, and the endpoint of the straight line that is taken in the given hour from noon or midnight from the point of the perpendicular of that line, is reckoned equal to the style.

An example will make the matter plainer. Let it be required, at hour 4 from noon or midnight, to find the point of hour 11 from sunset. The perpendicular C ϕ falls on the side of hour 4 from midnight, and the segment E ϕ contains 12 degrees 34 minutes, by problem 10 at the end of the little book. And since hour 4 from midnight is distant 8 hours from the meridian — namely by the complement up to 12 — if from hour 11 we count 8 hours in this manner, 12, 13, 14, 15, 16, 17, 18, 19, the counting will terminate at hour 19 of noon, which is distant 5 hours from 24. This number 5, doubled, gives the diurnal arc of hour 10, whose declination in table 13, at latitude 42 degrees, contains 16 degrees 2 minutes; these being taken from 90 degrees, there remain 73 degrees 58 minutes, for the distance of hour 11 from sunrise or sunset from the centre of the sundial at hour 4 from midnight. If from this distance the segment E ϕ , 12 degrees 34 minutes, is subtracted, there remains an arc of 61 degrees 24 minutes, whose tangent with respect to the whole sine C ζ [?] ($\phi \zeta$ being taken equal to the style), 18 [+ fractional part: numeral: ?] a little more, will give point A-d — transferred from ϕ — for hour 11 from sunrise or sunset.

Let it also be required, at hour 4 from noon, to investigate the point for hour 23. The perpendicular C ϕ does not fall on hour 4 from noon, but on hour 4 from midnight, and the segment E ϕ comprehends 12 degrees 34 minutes. And since hour 4 from noon is 4 hours from the meridian, and from 23 four hours are counted in retrograde up to 19 in this way, 22, 21, 20, 19 — which are 5 hours — from 19 up to 24 we fall again into the diurnal arc of hour 10, whose declination 16 degrees 2 minutes, taken from 90, leaves 73 degrees 58 minutes; and if to these the segment E ϕ , 12 degrees 24 minutes [?], is added, the distance of hour 23 from point ϕ becomes 86 degrees 22 minutes [?], whose tangent, about $157\frac{1}{2}$, will give at hour 4 from noon the most remote point through which hour 23 is to be drawn.

Let it also be required, at the same hour 4 from noon, to inquire after the point of hour 22. And since from 22 four hours are counted in retrograde up to 18, and there are six hours from 18 up to 24, we shall have the diurnal arc of hour 12, whose declination is 0 degrees 0 minutes; and therefore the distance of hour 22 from the centre, at hour 4 from noon, contains a quadrant, or 90 degrees. When the arc of 12 degrees 24 minutes is added, an arc of 102 degrees 24 minutes results, which cannot be cut off from hour 4 from noon, since it exceeds the quadrant. Hence the point of hour 22 must be found at hour 4 from midnight, by the tangent about $45\frac{1}{2}$, of the arc 77 degrees 36 minutes, which is left when 102 degrees 24 minutes are taken from 180, or when 12 degrees 24 minutes are subtracted from 90. Indeed, whenever we fall into the diurnal arc of hour 12, the proposed hour from sunrise or sunset must be drawn through the intersection of the given hour from noon or midnight with the equinoctial. Since, then, in our example hour 4 from noon does not cut the equinoctial, hour 22 must be drawn through hour 4 from midnight on the equinoctial. And in this way, whenever the arc found last, after the addition or subtraction of the arc between the centre and the perpendicular, is greater than the quadrant, the point will be found not in the given hour, but in the opposite one.

So too, at hour 4 from midnight, for hour 21 there will be found point A-d, the same that was found for hour 11. For if after 21 we count eight hours, 22, 23, 24, 1, 2, 3, 4, 5, we shall fall again into the diurnal arc of hour 10, because there are five hours from 24 up to 5, at which the counting stopped, and so on.

In the same way, at any other hour from noon or midnight, the points of the hours from sunrise or sunset will be found, if from the place of the style a perpendicular is raised to that hour, and the segment between the centre of the sundial and that perpendicular is investigated, and so on. At hour 5 from midnight, the segment between E and the perpendicular CD contains 27 degrees 22 minutes. And the whole sine, to be divided into 10 equal parts, must be equal to the interval C ϕ , the straight line D ϕ being taken equal to the style, and so on.

Let it again be required, at hour 12 from midnight, to inquire after the point of hour $16\frac{1}{2}$ from sunrise or sunset. Since hour 12 from midnight is in no way distant from the meridian — because at it noon occurs — nothing will need to be counted after hour $16\frac{1}{2}$. But since from $16\frac{1}{2}$ up to 24 there are reckoned $7\frac{1}{2}$ hours, which, doubled, exhibit the diurnal arc of 15 hours, its declination, 23 degrees 2 minutes, must be added to the quadrant (because the diurnal arc of hour 15 is greater than 12 hours), so that the distance of hour $16\frac{1}{2}$ from the centre E is made up to 113 degrees 2 minutes. From these, if the segment BE, 42

degrees, which the perpendicular CB cuts off, is subtracted, there remains an arc of 71 degrees 2 minutes, whose tangent, 29, with respect to the whole sine BD, transferred from B, will indicate point A-f, through which hour $16\frac{1}{2}$ is to be drawn.

Now if, at hour 12 from noon, the point of hour 12 from sunrise or sunset is to be found: since hour 12 from noon is distant 12 hours from the meridian, and there are, from 12, the hours from sunrise or sunset counted in retrograde up to 24, namely 12 hours, as appears here, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 24, and hour 24 is in no way distant from 24, we shall have the diurnal arc of hour 0, whose declination is 48 degrees; this, taken away from the quadrant, leaves 42 degrees for the distance of hour 12 from sunrise or sunset from the centre E. If to this is added the segment BE, 42 degrees (because the perpendicular CB falls on the opposite hour, namely on 12 from midnight), an arc of 84 degrees will be made up, whose tangent, 95 [+ fractional part: numeral: ?] a little more, transferred from B toward E, will give the most remote point through which hour 12 from sunrise or sunset is to be drawn — provided, however, that the plane of the sundial is large enough to receive that point.

Finally, at hour 12 from midnight, let the point of hour $16\frac{3}{4}$ be sought. Since up to 24 there are reckoned $7\frac{1}{4}$ hours, we fall into the diurnal arc of hour $14\frac{1}{2}$, whose declination 19 degrees 39 minutes, added to the quadrant, makes the distance of hour $16\frac{3}{4}$ from the centre E to be 109 degrees 39 minutes. From this, if the segment BE, 42 degrees, is subtracted, there remains an arc of 67 degrees 39 minutes, whose tangent, 24 [+ fractional part: numeral: ?] a little more, will offer point A-g of hour $16\frac{3}{4}$ at hour 12 from midnight. And so with the rest. On the line of hour 12 from noon or midnight the finding of the points for the hours from sunrise or sunset is easier and quicker, because the perpendicular CB has already been drawn to it, and it is established that the segment BE is equal to the altitude of the pole above the horizon — so that there is no need to seek it by problem 10.

5. But once the diurnal arc has been found, which cuts the given hour from midnight at the proposed hour from sunrise or sunset, as was handed down at the beginning of number 4, we shall find that point of section without tangents, even if the segment between the centre and the perpendicular drawn from the place of the style to the given hour from noon or midnight is unknown, in this manner. Let it be required, for example, to find at hour 4 from midnight the point of hours 11 and 21 from sunrise or sunset. Since hour 4 from

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(continuing §5, from the catchword "med.")

midnight is 8 hours from noon. If 8 is added to 21, they make 29; taking away 24, there remain 5, which doubled produce a diurnal arc of 10 hours, whose declination — from table 13, at latitude 42 degrees — is 16 degrees 2 minutes. You will find the same diurnal arc if you add 8 to hour 11: for you will produce hour 19, which is 5 hours from 24, and 5 hours doubled produce a diurnal arc of 10 hours. Therefore, once the perpendicular $C\oplus$ has been drawn to hour 4 from midnight, and the line $\oplus$ -Ab taken at an interval equal to $C\oplus$ (the line $\oplus$ - $\oplus$ having first been set down equal to the style), let a straight line be drawn from Ab to hour 4 from midnight on the equinoctial; and from it, on the circle described from Ab, let the aforesaid declination of 16 degrees 2 minutes be counted off toward the line of hour 5. For the straight line issuing from Ab through the end-point of the counting will cut hour 5 at the sought point Ad, as is established from the problem of chapter 1. And so with the rest.

[Margin: Finding the points for the hours from sunrise or sunset, at a given hour from noon or midnight, without tangents.]

But if the given hour from noon or midnight does not cut the equinoctial, one must take the complement of the declination of the diurnal arc found, and proceed as you will presently hear. For example, if at hour 5 from noon the point of hour 6 from sunrise is to be found: since hour 5 from noon is 5 hours from the meridian, and from hour 6 counting backwards down to 1 there are likewise 5 hours, and hour 1 is 1 hour from 24, we shall have a diurnal arc of 2 hours, whose declination at latitude 42 degrees is 47 degrees 1 minute, and whose complement is 42 degrees 59 minutes. Therefore, once CD has been drawn perpendicular to the line of hour 5 from noon — which falls on the opposite side, that is, on hour 5 from midnight — and the line D-Aa taken at an interval equal to $C\bullet$ (the line D- $\bullet$ having first been set down equal to the style), let a concealed straight line Aa-E be drawn from Aa through E, the centre of the dial, from which the complement of the declination is to be reckoned; and from it, on the circle described from Aa, let the complement of the declination — which we said was 42 degrees 59 minutes — be counted off toward hour 5 from noon. The straight line extended from Aa through the end-point of the counting will cut hour 5 from noon at the point Ah, through which hour 6 must pass. And so with the rest.

But we shall teach this same finding otherwise, and most easily, at number 8, where it will need to be carefully noted and learned.

[Margin: The finding explained shortly, where it is taught more easily.]

6. Finally, by means of the diurnal arcs we shall also neatly find, among the hours from noon or midnight, the points for the hours from sunrise or sunset. Those diurnal arcs which embrace fewer than 12 hours — or at any rate cut the horizontal line in at least one point — we shall lay out by this method, even if we do not know their declinations. Observe how many hours before or after noon any given hour-point taken on the horizontal line is distant. For this point lies on the diurnal arc which those hours before or after noon, doubled, produce; and from this point, by the first rule of the scholium of chapter 8, you will find all the other points of that same diurnal arc in the remaining hours. Or else, given any diurnal arc, consider at which hour from noon or midnight the Sun rises or sets upon it. For the point of that hour, found on the horizontal line, lies on the diurnal arc. Thus, since on the diurnal arc of 6 hours the Sun rises at hour 9 from midnight and sets at hour 3 from noon, the said diurnal arc will pass both through hour 9 from midnight and through hour 3 from noon on the horizontal line. Therefore, from each of the two points, again by the first rule of the scholium of chapter 8, you will find the other points of that same diurnal arc in the remaining hours, in this way. Since hours 3 and 2 from noon are equally distant from hour $2\frac{1}{2}$, which is 6 hours from hour $8\frac{1}{2}$ from midnight, if you draw a concealed straight line from hour $8\frac{1}{2}$ from midnight on the equinoctial through hour 3 on the horizontal, you will cut hour 2 from noon at a point of the diurnal arc of 6 hours, as is established from the first rule of the scholium of chapter 8. Likewise a concealed line from hour 8 from midnight on the equinoctial, through hour 3 on the horizontal, will cut hour 1 from noon at a point of the same diurnal arc of 6 hours — because hour 8 from midnight is 6 hours from hour 2 from noon, which holds the middle place between hours 3 and 1 from noon. This point at hour 1 from noon will likewise be found by a concealed line drawn from hour $7\frac{1}{2}$ from midnight on the equinoctial through the point of the said arc at hour 2 from noon. Again, a straight line drawn from hour 9 from midnight on the equinoctial through the point of the said arc at hour 2 from noon will cut hour 4 from noon at a point of the same arc. The same will also be done by a line drawn from hour $9\frac{1}{2}$ on the equinoctial through hour 3 on the horizontal. And so on in succession.

[Margin: Finding the points, among the hours from noon or midnight, through which a proposed diurnal arc is to be laid out, even if its declination is unknown.]

Through the point of hour 9 from midnight on the horizontal, too, you will find in the same way the points of that same diurnal arc of 6 hours in the other hours. For a straight line drawn from hour $5\frac{1}{2}$ from midnight on the equinoctial through hour 9 on the horizontal will cut hour 2 at a point of the said arc, because hour $5\frac{1}{2}$ is 6 hours from hour $11\frac{1}{2}$, and this hour $11\frac{1}{2}$ holds the middle place between hour 9 from midnight and hour 2 from noon. So too a line from hour 5 from midnight on the equinoctial, through the same hour 9 on the horizontal, will cut hour 1 from noon at a point of the same diurnal arc. Finally, if a point of the same diurnal arc of 6 hours is to be found at any proposed hour — say at hour 4 from midnight — one must take the hour midway between the given hours 4 and 9, which is $6\frac{1}{2}$. For a straight line from hour $\frac{1}{2}$ on the equinoctial, which is 6 hours from $6\frac{1}{2}$, drawn through hour 9 on the horizontal, will cut hour 4 from midnight at the sought point. But if on neither side is there found on the equinoctial an hour 6 hours distant from that middle hour, a point of the same arc must first be found at some other hour, and then through that point the point at the proposed hour must be sought. For instance, if through the point of hour 9 on the horizontal a point of the diurnal arc of 6 hours is to be sought at hour 10: since on the equinoctial no hour-point is 6 hours distant from hour $9\frac{1}{2}$, which is the middle between 9 and 10, we shall take, for example, hour 4 on the equinoctial, from which hour 10 recedes by 6 hours. Therefore a straight line drawn from hour 4 on the equinoctial through hour 9 will cut, on the horizontal, hour 11 at a point of the diurnal arc of 6 hours, since hours 9 and 11 are equally distant from 10. And if now through hour $4\frac{1}{2}$ on the equinoctial a line is drawn through the point found at hour 11, the proposed hour 10 will be cut at the sought point, because hour $10\frac{1}{2}$, midway between 11 and 10, is 6 hours from $4\frac{1}{2}$. And so on.

[Margin: A general rule for finding the points of a proposed diurnal arc at a given hour from noon or midnight.]

Through point B, of hour 12, on the horizontal, the points of the diurnal arc of hour 0 will be found in the other hours. The points of the diurnal arc of 4 hours are found through the points of hours 10 and 2 on the horizontal; the points of the diurnal arc of 8 hours, through the points of hours 8 and 4 on the horizontal; likewise the points of the diurnal arc of 10 hours, through the single point of hour 7 from midnight, since the point of hour 5 from noon is not available on the horizontal. Finally, through the point of hour $5\frac{1}{2}$ from midnight on the horizontal the points of the diurnal arc of 13 hours will be investigated, and so on.

Moreover, from the points of any southern diurnal arc, the points of the northern diurnal arc that is its counterpart — the one which together with it completes 24 hours — will likewise be found, by the second rule of the scholium of chapter 8, if need be. Thus from the points of the diurnal arc of 10 hours you will find the points of the diurnal arc of 14 hours; and from the points of the diurnal arcs of 8, 6, 4, and 2 hours you will find the points of the diurnal arcs of 16, 18, 20, and 22 hours, if it can conveniently be done — because arcs of this kind are sometimes too far distant from the Tropic of Cancer [☿] for their points to be easily obtained.

[Margin: How the northern diurnal arcs are described from their opposite southern ones.]

7. Now, to teach the laying-out of the hours from sunrise or sunset by means of the diurnal arcs: it will not be difficult to recognize through which point of any diurnal arc a proposed hour from sunrise or sunset should be drawn. Beginning on the dial from that hour after noon at which the Sun sets — or at which the diurnal arc cuts the horizontal line on the western side — let a count be made from hour 24 through the hours from noon and midnight, up to the proposed hour from sunrise or sunset. For through that hour from noon or midnight at which the count ends, the proposed hour from sunrise or sunset must be drawn on the diurnal arc taken. For example, suppose hour 20 from sunrise or sunset is proposed, and it is asked through which point of the diurnal arc of 6 hours it should be drawn. Since hour 20 is 4 hours from 24 — that is, from hour 3 from noon, through which that diurnal arc passes (as appears here: 24, 23, 22, 21, 20) — hour 20 must be drawn through the point of that arc at hour 11 from midnight, since hour 11 too is 4 hours from hour 3 (as appears here: 3, 2, 1, 12, 11). So that hour 24 answers to hour 3, hour 23 to hour 2, hour 22 to hour 1, hour 21 to hour 12, and hour 20 to hour 11. The same point will be found if, beginning from the hour at which the Sun rises, the count is made through the hours from noon or midnight — starting from that hour from midnight at which the Sun rises, or at which the diurnal arc intersects the horizontal line on the eastern side — up to the proposed hour from sunrise or sunset. As in the given example: since the Sun rises at hour 9 from midnight, or hour 18 from sunset, and hour 20 is 2 hours from hour 18 toward the west, the point of hour 11 from midnight must be taken, likewise 2 hours after hour 9 from midnight at which the Sun rises. For in this way hour 18 answers to hour 9, hour 19 to hour 10, and hour 20 to hour 11. In the same way you will find that hour 1 from sunrise or sunset must be drawn on the same arc through the point of hour 4 from noon, since hour 1 from sunrise

or sunset is precisely 1 hour from 24 toward the west, just as hour 4 is from hour 3. Likewise you will recognize that hour 21 must be drawn on the diurnal arc of 10 hours through hour 2 from noon, because both 21 from 24 and 2 from noon are 3 hours from 5 toward the east. Finally you will perceive that, on the diurnal arc of hour 0, which touches the horizontal line at B — the point of hour 12 from midnight — hour 23 must be drawn through hour 11 from midnight (because hour 23 precedes hour 24, when the Sun sets, by one hour, just as hour 11 from midnight precedes hour 12, when the Sun sets, by one hour), and hour 22 through 10, and hour 21 through 9, and so on; likewise hour 1 from sunrise through 1 from noon, hour 2 through 2, and so on.

[Margin: The laying-out of the hours from sunrise or sunset by means of the diurnal arcs.]

Therefore, if one carefully observes at which hour from noon the Sun sets and at which hour from midnight it rises, one will recognize without difficulty to which hour from noon or from midnight a proposed hour from sunrise or sunset corresponds, on any diurnal arc. For to the former hour there corresponds hour 24 from sunset, and the hour from sunrise that closes the diurnal arc; while to the latter hour there corresponds the hour from sunset that ends the nocturnal arc, and hour 24 from sunrise. From these the other corresponding hours are to be derived.

Here too it is worth observing that any hour from sunrise or sunset, on any diurnal arc which it cuts twice, passes through two hours which are as many hours apart from each other as are contained in that same diurnal arc, or in the nocturnal arc which together with it completes 24 hours. Thus you see that hour 21, on the diurnal arc of 10 hours, passes through hour 2 from noon and through hour 4 from midnight. Likewise hour 23 passes through hour 4 from noon and hour 6 from midnight. Again, on the diurnal arc of 6 hours, hour 23 is drawn through hour 2 from noon and hour 8 from midnight. But on the horizontal dial you will find that hour 13 from sunrise or sunset cuts the diurnal arc of 14 hours at hour 8 from midnight and hour 6 from noon, which are 10 hours apart from each other (as many, that is, as are contained in the nocturnal arc of 10 hours), and so on. Only on the diurnal arc of hour 0 do the several hours from sunrise or sunset touch that arc each at a single point only.

[Margin: How any hour from sunrise or sunset passes through two hours from noon or midnight on any diurnal or nocturnal arc, and which they are.]

Therefore, since (for example) hour 23 must be drawn through hour 4 from noon on the diurnal arc of 10 hours, but the point of hour 4 is not included on our dial, it will have to be drawn through hour 6 from midnight, which is 10 hours from hour 4 from noon. And the same reasoning is to be applied to the rest.

[Margin: A most elegant way of finding the point at any hour from noon or midnight, for any hour from sunrise or sunset.]

8. But since in this place we have dealt with the finding of the points through which the diurnal arcs are to be laid out, it is worth setting out a most convenient way of finding, at any hour from noon or midnight, the point for any hour from sunrise or sunset — a method which itself depends entirely on finding the point of any diurnal arc at any hour from noon or midnight. The matter stands thus. Once the diurnal arc has been found which cuts the given hour from noon or midnight at the proposed hour from sunrise or sunset (as we taught at the beginning of number 4 of this chapter), the point of that diurnal arc at the given hour from noon or midnight must be found by the first rule which we handed down in the scholium of chapter 8, number 3; for through that point the hour from sunrise or sunset must be drawn.

For example, suppose at hour 5 from midnight one must find the point through which hour 18 from sunrise or sunset is to be drawn. Since hour 5 from midnight is 7 hours from noon, we shall count from the given hour 18, in direct order (as we said in number 4), 7 hours down to 1, in this way: 19, 20, 21, 22, 23, 24, 1. And since hour 1 is 1 hour from 24, which doubled gives a diurnal arc of 2 hours, hour 18 from sunrise or sunset must be drawn through the point of this arc at hour 5 from midnight. We shall find it thus. The diurnal arc of 2 hours passes through hours 11 and 1 on the horizontal line, since on it the Sun rises at hour 11 from midnight and sets at hour 1 from noon. And since between hours 11 and 5 from midnight the middle is hour 8 from midnight, from which hour 2 from noon is 6 hours distant, if from the point of hour 2 from noon on the equinoctial a straight line is drawn through the point of hour 11 on the horizontal, hour 5 from midnight will be cut at the point of the diurnal arc of 2 hours through which hour 18 from sunrise or sunset is to be drawn.

But if perchance the point of hour 2 from noon cannot conveniently be had on the equinoctial, we shall take, for example, the point of hour $1\frac{1}{2}$, from which hour $7\frac{1}{2}$ is 6 hours distant. And since hours 11 and 4 are equally distant from hour $7\frac{1}{2}$ (each being $3\frac{1}{2}$ hours away), a straight line drawn from the point of hour $1\frac{1}{2}$ on the equinoctial through the point of hour 11 on the horizontal will

give the point of the diurnal arc of 2 hours at hour 4 from midnight. Now between hours 4 and 5 from midnight the middle is hour $4\frac{1}{2}$, from which hour $10\frac{1}{2}$ is 6 hours distant. If, therefore, from the point of hour $10\frac{1}{2}$ on the equinoctial a straight line is drawn through the point found at hour 4, hour 5 from midnight will be cut at the sought point of the same diurnal arc. And in this way, for the most part by a second operation, the sought point will always be found, if it was not found in the first.

Again, suppose at hour 4 from noon a point is to be found for hour 21 from sunrise or sunset. Since, counting 4 hours back from 21 in retrograde order (as was said in number 4), the count ends at hour 17 (thus: 20, 19, 18, 17), which is 7 hours from 24, we fall upon the diurnal arc of 14 hours — which hour 4 from noon cannot cut, as is evident. We shall therefore take hour 4 from midnight; and since this is 8 hours from noon, by counting 8 hours from 21 in direct order we shall arrive at hour 5 (thus: 22, 23, 24, 1, 2, 3, 4, 5), which is 5 hours from 24. We therefore fall upon the diurnal arc of 10 hours. And since this arc runs through hour 7 from midnight on the horizontal line, and hour $5\frac{1}{2}$ is the middle between 4 and 7, if from the point of hour $11\frac{1}{2}$ on the equinoctial — this hour $11\frac{1}{2}$ being 6 hours from that middle hour between 4 and 7,

that is, from hour $5\frac{1}{2}$ — a straight line is drawn through the point of hour 7 on the horizontal, hour 4 from midnight will be cut at a point of the diurnal arc of 10 hours, through which hour 21 must be drawn. And the same reasoning holds for the rest. This method is most excellent, since it does not require any whole diurnal arc in order to lay out a given hour from sunrise or sunset; rather, at any hour from noon or midnight it seeks the point for the proposed hour from sunrise or sunset, and at the same time discovers to which diurnal arc that point belongs, as is clear from what has been said.

9. All these things apply also to the horizontal dial: only, the point of the diurnal arc found must be sought on the meridian line, by means of the declination of that arc taken from table 13 — just as, on the meridian line of the horizontal dial, we investigated the points of the Sun's parallels by their declinations in chapter 8, number 3. For example: let the point for hour 11 from sunrise or sunset be sought at hour 7 from midnight. Since hour 7 from midnight is 5 hours from noon, we shall count 5 hours after 11, down to 16 (which is 8 hours from 24). We therefore fall upon the diurnal arc of 16 hours, whose declination, from table 13, is 29 degrees 3 minutes. This, subtracted from the pole's altitude of 42 degrees, leaves 12 degrees 57 minutes, whose tangent, about $2\frac{3}{10}$, transferred downward

from C, the place of the style, will give the point of the diurnal arc of 16 hours. From this we shall find the point of the same arc at hour 7 from midnight in this way. Since hour $9\frac{1}{2}$ is the middle between hours 7 and 12, a straight line from the point of hour $3\frac{1}{2}$ on the equinoctial — which is 6 hours from the point of hour $9\frac{1}{2}$ — through the point found on the meridian, will give, at hour 7, the point of the diurnal arc of 16 hours for hour 11 from sunrise or sunset, and so on.

[Margin: Application of what has just been said about the diurnal arcs to the horizontal dial.]

But if we wished to seek the point for the same hour 11 from sunrise or sunset at hour 7 from noon, we should labour in vain. For by counting 7 hours back from 11 we fall upon hour 4, and thus upon the diurnal arc of 8 hours — which, since it lies below the equinoctial line, hour 7 from noon cannot cut. Therefore the point for hour 11 must be sought at hour 7 from midnight, as we did.

Therefore, if a diurnal arc is found, as we taught in number 4, through whose point at hour 6 the given hour from sunrise or sunset must be drawn, we shall transfer the tangent of the complement of its declination from the centre of the dial onto hour 6, so as to have on it the point through which that same given hour from sunrise or sunset must pass. For example: let hour 14 from sunrise or sunset be given. Since hour 6 from noon is 6 hours from the meridian, we shall count back six hours from 14 down to 8, thus: 13, 12, 11, 10, 9, 8. And since hour 8 is 8 hours from 24, we shall double 8 to obtain the diurnal arc of 16 hours, whose declination at latitude 42 degrees is found in table 13 to be 29 degrees 3 minutes. If, then, we transfer the tangent of its complement from G, the centre of the horizontal dial, onto hour 6, we shall find the point for hour 14, and hence also for hour 22. This is what, in this place, we undertook to teach in chapter 14, number 3.

SCHOLIVM

1. Thus far we have handed down precepts for laying out only those dials that decline from the south, whether toward the east or toward the west; of those that decline from the north, toward east or west, we have said nothing at all, so as not to overwhelm the minds of students with a multitude of precepts. But if anyone wishes to lay out a dial facing north, it is enough for him to lay out, by the foregoing precepts, a south-facing dial of equal declination but in the opposite direction: that is, if he wants a dial declining from the north toward the east, let him draw one declining from

the south toward the west; and if he wants one declining from the north toward the west, let him construct one declining from the south toward the east. For the lines of this dial, extended beyond the horizontal line — yet so that they do not run beyond the Tropic of Capricorn [♊] — will yield the desired northern dial, provided that the portion of the dial cut off by the horizontal line, enclosed between that same horizontal line and the Tropic of Capricorn, is so inverted that the horizontal line takes the upper place, and the part which, when we face the dial, is then on our right becomes the left, and vice versa; which will happen if the same lines exactly coincide. For if this portion is drawn on the opposite face of the plane, so that the lines are drawn exactly opposite to the lines and set down so that the horizontal line holds the upper place, the northern dial proposed will have been made. But the segments of the arcs of the northern signs will belong to the southern signs, and those of the southern signs to the northern.

[Margin: How a dial declining from the north is derived from one declining from the south.]

2. It will not be difficult, moreover, to judge which hours in the northern dial (that is, the part cut off by the horizontal line) are to be counted from noon or from midnight, or from sunrise or sunset. For the hours which in the southern dial are counted from midnight are to be counted, in the northern dial, from noon, and vice versa. But as regards the hours from sunrise or sunset, let this rule be held. The hours from sunrise or sunset whose lines, extended, first touch the diurnal arc of hour 0 or 24 (if it were drawn) before they cut the Tropic of Capricorn [♊] of the northern dial — which will most often happen when they cut the meridian line before the horizontal — are also to be counted from sunrise or sunset, keeping, that is, the same denomination. But the hours whose extended lines cut the Tropic of Capricorn of the northern dial before they touch the diurnal arc of hour 0 or 24 — which will most often happen when they cut the horizontal line before the meridian — will take the opposite denomination: so that the hours counted from sunrise in the southern dial must be reckoned from sunset in the northern, and vice versa. But it is enough to make the judgement about hour 24 alone; for from it all the others will easily be distinguished. For example, since in our southern dial the segment of the horizontal line to the right of the meridian (I call "the right part" that which is on our right when we face the dial, whereas the part inclining to the left indicates hour 24 from sunrise) belongs to hour 24 from sunset, we shall make our inference about the rest from both cases. For since the portion of

hour 24 from sunset, extended toward the left, touches the diurnal arc of hour 0 at B before it reaches the tropic, the segment of the horizontal line toward the left will likewise, in the northern dial, look toward hour 24 from sunset; and therefore hours 23, 22, 21, 20, etc., immediately preceding hour 24 from sunset, are also to be counted from sunset, and accordingly the remaining hours extended from sunset, such as 11, 12, 13, 14, etc., will look toward the hours from sunrise. Again, since the portion of hour 24 from sunrise in the same southern dial, extended toward the right, touches the same diurnal arc of hour 0 at B before it reaches the tropic (which, in planes declining but little from the prime vertical, also cuts the horizontal line on the other side), the segment of the horizontal line toward the right will likewise, in the northern dial, belong to hour 24 from sunrise; and therefore hours 1, 2, 3, 4, etc., immediately following hour 24 from sunrise, are also to be reckoned from sunrise, and consequently the other hours extended from sunset, such as 9, 10, 11, etc., will belong to the hours from sunset. All of which we have demonstrated in the *Gnomonica*.

[Margin: Which hours in the northern dial are to be counted from noon and which from midnight; likewise which are to be reckoned from sunset and which from sunrise.]

3. Now, if anyone wishes to lay out the arcs of the signs among the hours from sunrise or sunset, taking no account of the hours from noon and midnight, he will be able to do it by almost the same means by which we accomplished it in chapter 15 for the horizontal dial. For from table 7, computed for latitude 42 degrees, there will be found, on the line of hour 6 from noon or midnight, the points through which straight lines drawn from F, the intersection of the equinoctial with the meridian, will cut the hours from sunrise or sunset at the points of the parallels — as we explained in chapter 18, number 2, concerning the hours from noon and midnight.

[Margin: A manifold finding of the points for the Sun's parallels, among the hours from sunrise or sunset, without the hours from noon or midnight.]

CAPVT XIX. (continued)

Likewise, from table 8, computed for the same latitude, the points on the meridian line will be found through which — if straight lines are sent out from A, the intersection of the equinoctial with the horizontal, — the hours from sunrise or sunset will be cut at the points of the parallels. These points on the meridian

line are found in this way. For the southern parallels, let each arc of table 8 be subtracted from the complement of the altitude of the pole above the horizon, if it can be subtracted; and let the tangent of the remaining degrees be carried from B, the intersection of the meridian with the horizontal, into the meridian line toward the equinoctial. Or else let the complement of the altitude of the pole be taken away from any arc of table 8, if that can be done, and let the tangent of the remaining degrees be carried from the same point B toward the centre of the dial. But if some arc is found to equal the complement of the altitude of the pole, the point in question will be B itself, the endpoint of the arc of table 8.

Whenever, however, some hour is more than six hours distant from noon (as, for example, is hour 13 in Capricorn ♑), its arc must be reckoned from the point of the equator below the horizon, as was said at chapter 8, number 7. Therefore the altitude of the pole must be added to its complement, and the tangent of the combined number carried from B beyond the centre. For the northern parallels, by contrast, each arc of table 8 must be added to the complement of the altitude of the pole, and the tangent of the combined number carried from B beyond the equinoctial line. And if the combined number should happen to be greater than a quadrant, it must be subtracted from 180 degrees, and the tangent of the remaining number carried from B toward the centre. Whenever, moreover, the points for the proposed parallel are found only with difficulty at certain hours, those points must be found by the second rule of the scholium of chapter 8, from the points of the opposite parallel, as was said at chapter 18, number 1.

Thirdly, the points of the parallels in the hours from sunrise or sunset can be found by their distances from the meridian, or by the arcs of the horizon of table 18, as was taught at chapter 15, number 3. For example, the distance from the meridian of hour 11 in Capricorn ♑ from sunset contains $128^{\circ}3'$. Now, with the inclination of the meridians, $40^{\circ}48'$, subtracted, there will remain $87^{\circ}15'$. Their tangent, $208 \frac{2}{10}$ (approximately), carried from K beyond A, will place a point on the equinoctial (if the plane is capable of it) through which a straight line drawn from E will cut hour 11 at the point Capricorn ♑. Again, in table 18, the arc of the horizon of the same hour 11 from sunset embraces $139^{\circ}27'$, which must be counted from B beyond A. Therefore, with the wall's declination of 30° taken away, there remain $109^{\circ}27'$, which cannot be cut off from the horizontal line — the start having been made at aC, the place of the style — since they exceed a quadrant. Therefore we shall take their complement up to 180, namely $70^{\circ}33'$, and carry its tangent, $28 \frac{3}{10}$, from C beyond B. For a straight line drawn through its endpoint and through E, extended, will cut hour 11 at the point Capricorn ♑. And in this manner, whenever some arc cannot be cut off on one side, its

complement to 180 must be cut off on the other side, and so on. Likewise, whenever the distance of some hour from sunrise or sunset cannot be taken on the equinoctial, the arc of the horizon of the same hour can generally be taken from table 18, and conversely. This is what happens in the next example.

Fourthly, the same result is obtained if, by problem 15 at the end of the little book, the altitudes of the Sun above the declining plane are computed for the hours from sunrise or sunset. For the tangents of the complements of those same altitudes, taken from the place of the style (which is the whole sine of those tangents), will cut the hours from sunrise or sunset at the points of the parallels, as was shown at chapter 8, number 12.

Fifthly and lastly, we shall accomplish the same by the rising and setting amplitudes for the latitude of the place in which the dial is described, as was explained at chapter 13, number 6.

4. Finally, I should like to add here another most convenient way of describing the parallels of the Sun on a declining dial, both for the hours from noon and midnight and for the hours from sunrise and sunset, by means of the arc of the horizon and of the vertical, which are computed in tables 14, 15 and 16, 17 from problem 12 at the end of the little book for latitude 42 degrees. For, for the forenoon hours of the northern parallels and the afternoon hours of the southern, let the wall's declination be added to the arcs of the said tables; but for the afternoon hours of the northern parallels and the forenoon hours of the southern, let the arcs of the same tables be subtracted from the wall's declination, or the wall's declination be subtracted from them — according, that is, as they are less than or greater than the wall's declination. Then let the tangents of these combined arcs, or of the remaining ones, be carried from the place of the style into the horizontal line, to the right or the left, as the case requires (which will not be hard to determine): the straight lines drawn from F, the intersection of the equinoctial line with the meridian, through the endpoints of the tangents laid off, will cut the hours at the points of the parallels. In the same way, if the tangents of the arcs of the vertical circle from the same tables are carried into the vertical line — which is to be drawn through A perpendicular to the horizontal — upward or downward from A, as the case demands, the straight lines drawn from F through the ends of the tangents so carried will likewise cut the same hours at the points of the parallels. I give no example of this matter, since it is clear of itself.

[Margin: The description of the parallels in the hours both from noon and midnight and from sunrise and sunset, on a declining dial, by means of the arc of the horizon and of the vertical, computed for the given latitude.]

5. To this may also be adapted what we wrote at number 1 of the scholium of chapter 8 and at number 3 of the scholium of chapter 15: namely, that one and the same straight line, drawn through any hour on the equinoctial, cuts two hours — as much from noon and midnight as from sunrise and sunset — whose points are equally distant on the equinoctial from the first assumed hour, at the points of two opposite parallels.

[Margin: That any straight line drawn through any hour-point on the equinoctial cuts two hours at the points of opposite parallels.]

CAPVT XX.

DE HOROLOGIO, QVOD VERTICALI CIRCVLO PRIMARIO ÆQVIDISTAT.

(On the dial that is parallel to the prime vertical circle.)

1. On a plane that is parallel to the prime vertical, the hours from noon and midnight are described as on a horizontal dial, but the hours from sunrise and sunset as on a declining dial. Only, in order that the centre may be found, one must carry into the meridian line, from the place of the style upward, the tangent of the altitude of the pole — not the tangent of the complement of the altitude of the pole, as on a horizontal dial — and downward the tangent of the complement of the altitude of the pole, so that the point may be found through which the equinoctial line is to be drawn. Moreover, the hour-lines to the right of the meridian belong to the hours from noon, and those to the left to the hours from midnight, in the reverse order, that is, from the horizontal dial.

For if a vertical dial is to be constructed, for example, for latitude 48° , let the dial of chapter 2 be inspected (so that we are not forced to construct a new figure), in which the place of the style is at C and its length is CD. Having drawn through C the two straight lines AB, cutting each other at right angles at C, of which AB represents the meridian and CD the horizontal, let there be carried from C upward the tangent of the altitude of the pole, $11 \frac{1}{10}$, for latitude 48° (with respect to the whole sine CD divided into 10 equal parts) as far as G; and downward the tangent of the complement of the altitude of the pole, 9, as far as B. For G will be the centre of the dial, and the straight line H B I, perpendicular to the meridian A B, will be the equinoctial line. Now if, from

the little table of the scholium of chapter 1, the tangents of the arcs by which the hours are distant from the meridian toward noon or midnight are carried into the equinoctial from point B (with respect to the whole sine BD), marking points on both sides on the equinoctial itself, then the straight lines drawn from G through these points will indicate the hours from noon and midnight — in such a way that the hours from midnight fall on our left when we face the dial, and the hours from noon on our right; that is, to the hour-lines are ascribed the complements of the hours of the horizontal dial, up to 12: namely, hour 1 is called 11, and 9 is called 3, and so on. For the horizontal dial constructed for latitude 42° is a vertical dial for latitude 48° , if that interchange of the hours, of which we spoke, is made — as far as concerns the hours from noon and midnight.

2. By the tangents of the complements of the arcs, taken in table 6 opposite the altitude of the pole 48° in the left-hand column, or 42° in the right (since at latitude 48° the pole is raised above the vertical by 42°), you will find, if you please, other hour-points on the horizontal line CD. And so that the hour-lines may be drawn more exactly, the points of the equinoctial line must be carried into the straight line O P parallel to the equinoctial, the straight line G A being set equal to G B, as was done at chapter 2 for the horizontal dial.

Thus the altitude of the pole above the horizon must be taken in the left-hand column, and its complement — that is, the altitude of the pole above the vertical — in the right. Therefore, if a vertical dial is constructed for latitude 42° , one must take 42° in the left-hand column, or 48° in the right.

To this, finally, are to be referred all the other methods that we set out at chapters 2, 3, 4, and 5: provided that into a circle described from the centre of the dial you carry the arcs of table 6, opposite the altitude of the pole in the left-hand column, above the vertical of your own region — that is, above the plane of your dial — as in our example, opposite 42° , yet so that those arcs have their beginning on the line of hour 6.

3. Concerning the arcs of the signs nothing new need be prescribed here: only be mindful that the segment of the meridian line CB, between the place of the style and the equinoctial line, contains the degrees of the complement of the altitude of the pole of your region, and the segment CG the degrees of the altitude of the pole. Likewise, that the arcs of the southern signs incline from the equinoctial line toward the centre of the dial, and so forth.

4. Finally, the hours from sunrise and sunset are described as on a declining dial, by means of the same points of the equinoctial line and of the horizontal which the two little tables of chapter 14 supply us; and by other means too, if you please, which we prescribed at chapter 19, the same hours from sunrise and sunset can be laid out.

CAPVT XXI.

DE HOROLOGIO MERIDIANO.

(On the meridian dial.)

1. Let a straight line A B be drawn in any way for the horizontal line, and from any point A of it, taken for the place of the style, let a circle B C be described — to the right, indeed, in an eastern dial, but to the left in a western one — in which, from point D downward, let the complement of the altitude of the pole be numbered as far as C. For the straight line A C will be the equinoctial line; and to it, having drawn through A the perpendicular A D, for hour 6, let the length of the style A D be taken on it. Then, having taken on any straight line as many segments as you please equal to the style A D, let the first of them, A D, be cut into 10 equal parts.

If, by the aid of the tangents, you wish to draw the equinoctial line and the axis, you will proceed thus. On the horizontal line take any portion you like from point A toward B, and through its endpoint erect a perpendicular to the horizontal. Then, that portion having been divided into 10 equal parts, the tangent of the complement of the altitude of the pole, namely $11 \frac{6}{10}$ [recte: $11 \frac{1}{10}$? — astronomically the 42° dial's complement (48°) gives $11 \frac{1}{10}$; numerator glyph best-reads 6], carried into the said perpendicular downward, will give the point through which the equinoctial is to be drawn from A. But the tangent of the altitude of the pole, namely 9, carried into the same perpendicular upward, will give the point through which the axis of the world must be drawn from A. These tangents must be taken with respect to that portion which we said was to be assumed on the horizontal, divided into 10 parts, in the manner of a whole sine.

Now if, from the little table of the scholium of chapter 1, the tangents of all the hours from noon and midnight are carried in order from point A into the equinoctial line on both sides, in each dial, the hour-points will be found on each equinoctial line. But since here the line of hour 6, A D, stands in place of the meridian line of the horizontal dial, the points nearest to A will belong to hours 7 and 5, the next to hours 8 and 4, and so on — as the numbers on the equinoctial

line indicate; just as on the horizontal dial the points nearest to point B belong to hours 1 and 11, and the next to hours 2 and 10, and so on.

[Margin: The construction of the meridian dial.]

The same hour-points will be found from what was handed down at chapter 3. For the straight line A D, laid off on both sides from A, exhibits the points of hours 3 and 9. If, therefore, at the interval between hours 3 and 9 the equinoctial line is cut from D, the points of hours 4 and 10 will be had; and if the portions of the equinoctial line between A and hours 2 and 10 are cut into 3 equal parts, the third parts of these, beginning from A, will give hours 4 and 8. But the interval between D and hour 10 or 2, carried on the equinoctial from point 10, will give hours 11 and 5; and from point 2, it will exhibit hours 1 and 7, and so on. Since, moreover, as was shown in the *Gnomonica*, the hour-lines on a meridian dial are parallel and perpendicular to the equinoctial, if through the points found perpendiculars to the equinoctial (or parallels to A D) are drawn, the hours from noon and midnight will be described. These perpendiculars, or parallels, will easily be drawn if in each dial there is drawn a line E F, or G H, parallel to the equinoctial line, and into it the hour-points are carried, the beginning being made from the line of hour 6, A D, and so forth.

[Margin: The finding of the hour-points from noon and midnight.]

[Margin: The finding of the hour-points on the equinoctial.]

[Margin: In what way the lines of the hours from noon and midnight are drawn on the meridian dial.]

2. The arcs of the signs will be described thus. From table 1 let the arcs of the signs be taken under the hours in the topmost line, to which the title [Circ. hor. 6.] is prefixed, and let their tangents be carried from A into the line of hour 6 on both sides. For in this way, if through the endpoints of these tangents lines parallel to the equinoctial line are drawn, the corresponding hours will be cut at the points of the parallels of the Sun. For example: the arc of hours 1 and 11 in the parallel of Cancer ☿ and Capricorn ♑ contains $59^{\circ}14'$, whose

[Margin: The description of the arcs of the signs on the meridian dial.]

tangent, $16 \frac{8}{10}$ (approximately), will give on each side the point I, on the line of hour 6. Therefore the straight line I K, parallel to the equinoctial, will cut hours 1 and 11 at K, the points of the parallels Cancer ☿ and Capricorn ♑. And so with the

rest. Furthermore, the arcs below the equinoctial line belong to the northern signs, but those above the equinoctial to the southern.

Since, moreover, if the straight lines A I are taken equal to L K, in hours 1 and 11 the straight line I K is parallel to the equinoctial line, it will be enough — in order that the points of the parallels may be found — for the tangents of the arcs of table 1 to be carried into the corresponding hour-lines. For example, so that the points Cancer ☿ and Capricorn ♑ may be had in hours 1 and 11, the tangent 16 8/10 (approximately) of the arc 59°14', under hours 1 and 11 of the topmost line of table 1, will have to be carried from the points L, L of hours 1 and 11 on both sides as far as K. For through the points K the parallels Cancer ☿ and Capricorn ♑ will pass. And so with the rest.

Diagram. A large rectangular meridian-dial (*horologium meridianum*) construction, occupying most of the page, enclosed by a heavy rectangular border with a semicircular arc (the bounding circle B C) sweeping through it. A dense web of hour-lines and sign-parallels crosses the field, with a graduated declination/altitude scale down the left edge and the words for the eastern and western faces lettered along the right side.

Readable labels:

- **A** (upper left, at the head of the left-edge scale): the place of the style.
- **D** (left edge, just below A): the top of the hour-6 / style line.
- **B** (top center and again bottom center): endpoints of the bounding circle B C / the meridian line A B.
- Left-edge graduated scale (reading downward, degrees): **10, 20, 30, 40, 50, 60, 70, 80** — with additional numerals **18, 19, 20** near the top of the frame.
- "**Occidentale**" (Western) — lettered vertically in the upper-right quadrant.
- "**Orientale**" (Eastern) — lettered vertically in the lower-right quadrant.
- Hour numerals ranged along the arc / margins: **24, 18, 12, 6** (and a further **9** near the top left).
- Interior point-letters (crossings of hour-lines and parallels), as far as legible: **K** (recurring at several intersections), **H, I, L, M** (M appearing twice, upper-right and lower-right), **E, F, G** (in the lower crowded region).

Several interior point-letters and the small hour-numerals sit within a very dense line-crossing region and do not resolve individually, even under magnification; more than two labels are uncertain.

That the parallels drawn through the points of the line of hour 6 cut the hour-lines at the points of the parallels is evident, since they represent the great circles drawn out through the hours in the parallels from the intersection of the Equator with the Meridian, cutting the circle of hour 6 in the degrees whose tangents have been carried into hour 6. For the common sections of these great circles and of the plane of the meridian dial are parallel — by proposition 18 of book 1 of our *Gnomonices* — to the equinoctial line, since those circles (the Equator and the Meridian, to which the dial is parallel) have the same common section.

3. In order that the hours from sunrise and sunset may be described, the lines of hours 6 and 18 from sunrise or sunset must first be drawn. Hour 6, indeed, must be drawn through the point of the line of hour 24, or of the horizontal, where it is intersected by hour 3 from noon or midnight — namely through point G, as is evident from the earlier little table of chapter 14. And since (as we demonstrated in the scholium of proposition 22 of book 1 of the *Gnomonica*) the lines of hours 6 and 18 from sunrise or sunset are parallel to the equinoctial line on a meridian dial, the straight line G H, equidistant from the equinoctial, will be the line of hour 6 from sunrise or sunset. In the same way the straight line E F, drawn parallel to the same equinoctial through point E, where hour 9 from noon or midnight intersects the horizontal line, will give hour 18 from sunrise or sunset, as is evident from the same earlier little table of chapter 14.

[Margin: The description of hours 6 and 18 from sunrise or sunset.]

Once hours 6 and 18 from sunrise or sunset are described, the other hours from sunrise or sunset are laid out with no trouble by the aid of the two little tables of chapter 14. For from those little tables you will learn through which hour-points, both on the equinoctial

[Margin: The description of all the hours from sunrise or sunset.]

in the equinoctial line, and which ones in the horizontal line, they are to be drawn. Likewise, from the figure of the scholium of chapter 1, number 3, you will understand through which hour-points on hours 6 and 18 from sunrise or sunset they are to be described. For example: from the later little table of chapter 14 it is established that hour 20 from sunrise or sunset must be drawn through hour 2 from noon or midnight on the equinoctial line. From the earlier table, on the other hand, it is clear that the same hour 20 from sunrise or sunset must be drawn through hour 10 from noon or midnight on the horizontal line of hour 24; likewise through hour 7 from noon or midnight on the line of hour 18 from

sunrise or sunset; and finally through hour 1 from noon or midnight on the line of hour 6 from sunrise or sunset — as is evident from the figure of the scholium of chapter 1, number 3. And so four points are obtained through which hour 20 from sunrise or sunset must be drawn.

In the same way, hour 4 from sunrise or sunset must be drawn through hour 10 from noon or midnight on the equinoctial, through hour 2 on the horizontal, through hour 11 on the line of hour 18 from sunrise or sunset, and through hour 5 on the line of hour 6 from sunrise or sunset. Likewise hour 2 from sunrise or sunset is drawn through hour 8 on the equinoctial, through hour 1 on the horizontal, through hour 10 on the line of hour 18 from sunrise or sunset, and through hour 4 on the line of hour 6 from sunrise or sunset. Hour 9 from sunrise or sunset, however, is drawn through hour 3 on the equinoctial, through hour $4\frac{1}{2}$ on the horizontal, through hour $1\frac{1}{2}$ on the line of hour 18 from sunrise or sunset, and through hour $7\frac{1}{2}$ on the line of hour 6 from sunrise or sunset — as appears in the figure. And so on for the rest.

Once some one hour from sunrise or sunset has been drawn through the points found from the table of chapter 14 and from the figure of the scholium of chapter 1, number 3, the others will be drawn without any trouble through the neighbouring points on either side: taking the points of the whole hours on the equinoctial line, but the points of the half-hours on the lines of hours 24, 18, and 6.

If, on the circle B C, the doubled altitude of the pole is counted upward as far as M, the straight line A M will give hour 12 from sunrise or sunset; because the straight line B M, if it were drawn, would be the diameter of the largest of the parallels that are always visible, which the circle of hour 12 from sunrise or sunset touches at M.

SCHOLIUM.

We have described so far, then, on plane surfaces — as we proposed at the outset, on the horizontal and on any vertical whatever — all the kinds of equal hours, namely the hours counted both from noon and midnight and from sunrise and sunset. But of the unequal hours [horae inaequales], of which each single day comprises twelve, we have written nothing at all. For although we did lay them out on every plane in our *Gnomonica*, following Ptolemy in this and most of the other weighty authors who set the same hours out on sundials, nevertheless — since they cannot be drawn by straight lines (for, as I demonstrated in the *Astrolabe*, book 2, Lemma 39, there cannot be great circles dividing the diurnal

arcs of the several parallels into twelve equal parts, which is nevertheless required if those straight lines are to be the common sections of those circles and of the plane of the sundial) — it seemed best not to describe them again here.

[Margin: Why nothing is prescribed in this little book about describing the unequal hours.]

I admit that those straight lines differ little from the true unequal hours, because of the small distance of the tropics from the Equator, especially where the altitude of the pole is less than 45 degrees; and that they may be drawn through the twelfth-parts of the tropics, or of other parallels lying not far from the tropics. This is because, as we demonstrated in the scholium of proposition 10, book 1 of the *Gnomonica*, the same great circles passing through the hours of the Equator and through the unequal hours of any northern diurnal arc (that is, through its twelfth-parts) pass also through the unequal hours of the opposite southern diurnal arc. Therefore anyone who wants the unequal hours will be able to describe them by the methods we have handed down in the *Gnomonica*.

Yet it will also be permissible to describe the same hours by tangents [per Tangentes], as follows. Let the diurnal arcs of ♋ [Cancer, the tropic of Cancer] and ♑ [Capricorn, the tropic of Capricorn], reduced to degrees and minutes, be divided by 12, so that the quotients indicate how many degrees one unequal hour in Cancer and in Capricorn contains. These degrees doubled will give the magnitude of two hours, tripled of three, and so on. If, then, the tangents of the unequal hours of Cancer are carried over onto the equinoctial line — as we said of the hours from noon and midnight in chapters 2, 16, and 21 — the straight lines drawn from the centre of the sundial through the ends of the tangents will give the points of the unequal hours on the tropic of Cancer. On a meridian dial, however, where there is no centre, perpendiculars to the equinoctial will yield the same points. In just the same way the points of the same unequal hours on the tropic of Capricorn will be found, and so forth. If, then, the points of the corresponding hours are joined by straight lines — which will necessarily pass through the hour-points on the equinoctial — the unequal hours will have been described.

[Margin: How the description of unequal-hour dials, of meridian dials, and of the ascending signs is carried out by tangents.]

Indeed, by tangents it will also be possible to investigate on the equinoctial line the points through which the meridians of places are to be drawn, provided their distances from the meridian of your own place have first been investigated.

Finally, by the same method, using tangents, we shall find on the equinoctial and on the tropics the points through which the ascending signs are to be drawn, if the four little tables of proposition 9, book 1 of our *Gnomonica* are employed.

By tangents this same hour 12 will be drawn as follows. On the axis, between B and M, let any portion whatever be taken and divided into 10 equal parts; and at the end of the portion taken, toward M, let a perpendicular to the axis be raised. For the tangent of the altitude of the pole, reckoned with respect to that portion as the whole sine [sinus totius] — namely 9 — carried over onto that perpendicular, will give the point through which hour 12 must be drawn from A.

CAPVT XXII.

DE INVENTIONE LINEAE MERIDIANAE, declinationisque murorum, atque horologiorum ac styli collocatione.

(On finding the meridian line, the declination of walls, and the placing of sundials and of the style.)

1. Let the analemma of the proposed region be described — that is, let the meridian be A B C D, from centre E, in which let the common section of the Horizon and the Meridian be B D; the common section of the primary Vertical and of the same Meridian, A C; the common section of the same Meridian and of the Equator, F G, taking the two arcs A F, C G equal to the altitude of the pole; and finally the common section of the same Meridian and of the parallel of the Sun on the day on which the meridian line is sought, H I, taking the two arcs F H, G I equal to the declination the Sun has on that day. These arcs are to be taken toward A when the Sun is northern, but toward B when it is southern.

[Margin: Finding the meridian line.]

2. Intending, then, to find the meridian line on any day, observe on a plane that is equidistant from the Horizon...

Page 74 — CAPVT XXII (continued)

the shadow of some fine, freely hanging thread. For this purpose I usually employ an instrument, such as you see drawn at the foot of the figure, in which the set-square [norma] C D E is fixed at right angles to the flat rule A B, along the straight line D I, parallel to the two sides of the rule, so that the side D H of the set-square stands like a gnomon perpendicular to the Horizon, or like a freely hanging

thread, when the rule A B is placed upon a plane parallel to the Horizon. For in this way, with easy labour, we shall mark — without any perceptible error — two points along the length of the shadow's tip cast by the side H D.

[Margin: An instrument for observing shadows, most convenient in place of a gnomon.]

But if on the set-square's rule C D, which is raised above the Horizon, a line F G is drawn parallel to the side D H, and a hole is made near the point G, such that a round plumb hung by a fine thread from the hole F can swing freely in it, then this same instrument will be very convenient for examining any proposed plane, to see whether or not it is equidistant from the Horizon. For when the thread F G grazes the rule C D and coincides with the straight line F G, the plane on which the rule A B is set will be equidistant from the Horizon.

Diagram. A large composite construction filling most of the page. It combines three elements:

1. **The analemma (meridian circle)** — a large circle A B C D about centre E, with the horizon-diameter B D, the vertical-diameter A C, the equator-line F G, and the parallel of the Sun H I, together with the auxiliary construction points K, L, M, N, O, P, Q and the sun-parallel points G, I on the rim. A is at the top, B at the left, D at the right of the rim; the interior is densely crossed by the meridian, equator, parallel, and the construction lines K L, M O, N O.
2. **The shadow-observing instrument** at the lower left — a right-angled set-square (norma C D E) fixed to a flat rule (regula A B), with the upright side D H acting as a gnomon and a plumb-bob hanging on a thread from the hole F (labelled $\bigcirc$ F); base points A, D, G, H visible, the plumb shown as a small shaded disc at G.
3. **Five small circles** showing the meridian-line angle-constructions: one at the upper left (labels A, D, C, B, E, K), one at the upper right (labels B, D, C, A, E, L), and a row of three below (each with D at top, C at centre, A and B at the sides, E at the foot).

Readable labels by region:

- **Upper-left small circle:** A (left), D (top), C (centre), B (right), E (lower), K (foot).
- **Upper-right small circle:** B (top left), D (top right), C (centre), A (lower right), E (lower), L (foot).

- **Large analemma rim:** A (top), B (left), D (right), K (upper left, twice), H (upper), L (right), I (lower right), G (lower), Q (lower left), C (lower).
- **Large analemma interior (crowded):** E (centre), O (below centre, twice), P (lower, twice), and a cluster of N and M labels (repeated, upper-middle) marking successive crossings of the construction lines. Several of these interior N/M/O/P labels sit in a very dense crossing region and cannot be individually resolved with confidence [?].
- **Instrument (lower left):** H (top), G (top right of upright), F (hole, $\bigcirc$ F), A (foot left), G (plumb), D (foot).
- **Lower row of three small circles:** each labelled D (top), A and B (sides), C (centre), E (foot); A appears twice on each.

More than two interior labels of the analemma are uncertain.

Now let two points A, B, a little apart from each other, be marked along the length of the shadow's tip cast by the side D H, or at the middle of the breadth of the shadow which the freely hanging thread casts — as in the five small circles — and through them let the straight line A B be extended. This will be the common section of the plane equidistant from the Horizon and of the vertical circle which at the time of observation passes through the centre of the Sun. Once the shadow has been observed, let the Sun's altitude be taken without delay, by a quadrant or some other instrument. I say "without delay" because it is expedient that the Sun's altitude be observed immediately after the two points A, B have been marked in the shadow, before the straight line A B is drawn through them — lest there be danger in delay, since through the Sun's daily motion the shadow is continually changing, the Sun at once stands in a different vertical, and consequently the shadow cast by the instrument changes.

Next, in the analemma, with the observed altitude of the Sun computed, from B and D as far as K and L let the straight line K L be drawn. This will be the common section of the Meridian and of the parallel to the Horizon then passing through the centre of the Sun, and it will cut the vertical diameter at M, and the diameter of the Sun's parallel H I at N. Then, with the semicircle K O L described about K L from M, let a perpendicular to K L — or a line N O parallel to A C — be drawn from N, cutting the circumference K O L at the point O, and to that point let the straight line M O be drawn.

Page 75 — NOVAE DESCRIPT.

If, then, the point N falls between K and M — as happens on all southern parallels, but on northern ones when the Sun is beyond the primary Vertical —

and the observation is made before noon, we shall set at the centre C (from which, some magnitude having been assumed on the shadow-line A B, a circle of any size whatever is described) an angle A C D equal to the acute angle K M O, measured from sunrise toward the south, that is, from A, the eastern point, toward the southern parts D — as is seen in the small circle uppermost at the left. (Through carelessness it has come about that this angle A C D, like the following ones, has not been made equal to the angle K M O; it should nevertheless be understood to be equal.) But if the observation is made after noon, we shall make the angle A C D equal to the same K M O, measured from sunset toward the south — that is, from A, the western point, toward the southern parts D — as appears in the small circle which is the first of the three lower ones.

But if the point N falls upon the point M — whether the observation is made before or after noon — we shall draw to A B, through the centre C, the perpendicular D E, as is plain in the small circle which is the middle one of the three lower ones.

If, finally, the point N lies between L and M — as happens on northern parallels, when the Sun is on the near side of the primary Vertical, after sunrise and before its setting — and the observation is made before noon, we shall make an angle A C E equal to the acute angle L M O, measured from sunrise toward the north, that is, from A, the eastern point, toward the northern parts E — as is manifest in the small circle which is the last of the three lower ones. But if the observation is made after noon, we shall set an angle A C E equal to the same angle L M O, measured from sunset toward the north — that is, from A, the western point, toward the northern parts E — as may be seen in the small circle which occupies the uppermost place at the right. For always the straight line D E, forming the said angle with the shadow-line A B, will be the meridian line.

[Margin: The meridian line.]

But if at any time the Sun's altitude, counted from B, ends at H, the Sun will be, at the moment of observation, in the Meridian circle itself, and will have its greatest altitude for that day; and therefore the shadow-line itself will be the meridian — all of which is plain from what we have demonstrated in the *Gnomonica*.

Without the aid of a quadrant divided into 90 degrees, everything can be accomplished in the analemma by tangents. For if a perpendicular is raised at E D, and the straight line between it and the centre E is divided, as the whole sine, into ten equal parts, then the tangent of the complement of the altitude of the pole, reckoned with respect to that whole sine, must be carried downward onto it,

marking a point on it. For through this point and through the centre E the diameter of the Equator must be extended. And if the tangent of the number compounded from the northern declination of the Sun and the complement of the altitude of the pole is carried over onto the same line, and from its end a hidden straight line is drawn through E, the point G will be found, through which the diameter of the northern parallel must be drawn. If the Sun is southern, the declination must be subtracted from the complement of the altitude of the pole, and the tangent of the remaining number carried over, and so on. Finally, if the tangent of the Sun's altitude is carried upward onto the same perpendicular, and from its end a straight line is extended through E, the point K will be found, through which the diameter of the parallel to the Horizon must be drawn. If all these things are rightly weighed, we shall never need a quadrant divided into degrees, but shall accomplish everything by tangents alone.

3. I say nothing about finding the meridian line by a magnetized needle [per acum Magnete illitam], because it is exceedingly deceptive, since in very few places does that needle point straight from north to south. Add to this that even if it were known how much it deviates from the true meridian line, I would still not think it should be trusted, because, on account of various external hindrances, it seems to decline more at one time than at another.

[Margin: A magnet-rubbed needle is deceptive for indicating the meridian line.]

The method here set out is also to be preferred to that common procedure which we explained in the *Sphere*, when we treated of the Meridian. This is because the tip of the shadow is not easily discerned on a plane, and it is inconvenient to observe the shadow twice in one day — once before noon, and again after noon, as is prescribed there — since it often happens that at the very hour after noon at which the shadow's tip should be observed, some cloud covers the Sun, and so the morning observation is made in vain. Neither of these difficulties is necessary in the method handed down here, but it is enough to observe the shadow-line and the Sun's altitude once only, and that in a very short time, as is clear.

[Margin: The finding of the meridian line explained here is preferred to the finding handed down in the *Sphere*.]

4. Indeed, if it can conveniently be done, it is enough to note carefully the tip of the shadow of the style H D, even if the Sun's altitude is not taken — provided that the point on the plane be accurately marked which lies directly beneath the straight line on the side of the set-square D H, that is,

beneath the vertex of the style; so that of course the interval between this point and the tip of the shadow, that is, the length of the shadow, can be taken with compasses without error. For if in the analemma a straight line E P is taken equal to the style H D, or to any other you may perhaps be using, and on the perpendicular P Q a straight line equal to the length of the shadow — that is, to the interval between the place of the style and the tip of the shadow — then the straight line Q E, extended to the circumference, will give the Sun's altitude D L, as is plain from what has been demonstrated in our *Gnomonica*. But in this observation it is necessary that the plane be most exactly equidistant from the Horizon, and that the tip of the shadow be marked with the utmost care and diligence.

[Margin: How the Sun's altitude is elicited in the analemma from the length of the shadow of any style.]

5. In order that the declination of the wall on which the sundial is to be built may be found, we shall use this other instrument, in which let there be a perfectly flat board A B (the longer it is, the more exactly will its side A C — which must be perfectly straight — examine the flat surface of the wall; or else, if the board A B is not long enough, some longer and perfectly straight rule must be fixed beneath the board, so that its end corresponds exactly to the end of the board), and on it let a straight line I K be drawn parallel to the side A C. On the side A C, too, let two rules of uniform thickness C D, E F be fixed, forming right angles with the plane A B; and to these let another rule G H be fitted, parallel to the side C E.

[Margin: An instrument for investigating the declinations of walls.]

When this instrument has been applied to the wall by means of two small pins which, fixed in the wall, support the rule G H, so that the plane A B is equidistant from the Horizon — which will be tested by plumb-line and level — then... (For this purpose, in one of the rules, as in C D, a hole may be made near...)

[Margin: How the declination of a wall is detected.]

Diagram. The wall-declination instrument, drawn in perspective. A flat rectangular board (the tabula A B) is shown as a slab, with two upright rules rising from its far side and a cross-rule joining them, plus a plumb-line hanging from the upper right. Readable labels:

- F (top of the left upright), D (top of the right upright).
- H (left end of the cross-rule), G (right end of the cross-rule).

- A (near left front corner of the board), E and M and C (along the front edge, left to centre), K (right front corner).
- I (lower left corner of the board), L (at the foot, near a small drawn arc), B (lower right corner).
- A plumb-bob hangs on a thread from near D (shown as a small shaded disc suspended within the right upright).

The line I K runs across the board parallel to the front edge A C; a short arc is drawn near M, C, L. No labels flagged uncertain.

(The passage below continues Cap. XXII from the previous page — the finding of the meridian line and of a wall's declination — and runs to the foot of §7, where Cap. XXIII begins.)

[Margin: In which direction the proposed wall declines, and by what method this is discovered.]

near the point C, so that a round plumb, hung by a fine thread from the hole a and caught up on it, can swing freely there, the straight line drawn from a through the centre of the hole being extended parallel to the ends of the rule C D. For when the thread grazes the rule C D and coincides with that straight line drawn from a through the centre of the hole, the plane A B will be equidistant from the Horizon. Then, when the Sun lights the wall, find the meridian line L M, which cuts the straight line I K — the line equidistant from the side A E — at M. If, therefore, the angles at M are right angles, the wall will have no declination but will be parallel to the prime vertical. But if the angles at M are not right angles, the acute angle I M L will show the complement of the declination from the vertical: so that the arc of a circle described from M and contained between I M and M L will hold the degrees of the wall's declination from the meridian, while its complement gives the degrees of declination from the prime vertical. Moreover, toward whichever quarter of the world the obtuse angle K M L points, in that direction the proposed wall will decline — from the south, if the wall faces south (that is, if it is lit by the Sun at noon); but from the north, if it faces north (that is, if it is not lit at noon).

For example, with the wall facing south: if, when we turn to face the instrument applied to the wall, the acute angle stands to our left and the obtuse to our right, the wall will decline from south toward east; but toward west if the acute angle is set to our right and the obtuse to our left. The exact opposite is to be understood when the wall faces north. For with the acute angle set to our left, the wall will

decline from north toward west; and from north toward east if the acute angle is found to our right.

But if the wall is nearly aligned with the meridian, so that it is hard to judge whether it faces south or north — for when it recedes further from the meridian the matter has no difficulty, since one can observe daily around noon whether or not it is lit by the Sun — then the meridian line found on the tablet A B will show us nicely whether it faces south or north, and whether it declines toward east or west. For when that line meets the wall toward the north (one must, after all, know more or less where east, west, south, and north lie — which a needle rubbed with a magnet will readily show), the wall will decline from south toward west if it faces west, but toward east if it looks upon the rising Sun. But when the meridian line meets the wall toward the south, the wall will decline from north — toward west if it faces west, but toward east if it faces east. We have written more on this matter both in Book 1 of the *Gnomonics*, Proposition 23, and in the little book *On the Construction and Use of the Instrument for Sundials*, Chapter 20. But let this suffice.

[Margin: The placing of any sundial in its proper position.]

6. In order that a horizontal sundial may indicate the hours correctly, it must be set equidistant from the Horizon in such a way that its meridian line is parallel to the meridian line found on some other neighbouring plane. This will hold only when the shadow of the style on the sundial makes the same angle with the meridian as the shadow of the side D H — of the upper set-square placed above the meridian line found — makes with that found meridian.

Wall sundials, on the other hand, will have their proper position when the horizontal line occupies the upper place on the walls and stands parallel to the Horizon. And a line parallel to the Horizon is easily drawn on any wall by means of the level and the plumb.

[Margin: What position the style ought to have.]

7. Finally, the style too will have its proper position in its own place when it stands perpendicular to the plane of the sundial. It will be perpendicular to the sundial when, a circle having been described from its foot, the same span of the compasses reaches the vertex of the style from three points of the circumference. On this you will find more in Book 4 of the *Gnomonics*, Proposition 12. On walls, however, since the style is longer, it is not necessary that it be perpendicular to the plane of the sundial; it may be

fixed at any point outside its proper place, provided that its vertex touches the extreme point of the length of the proper style issuing from its proper place and making right angles with the sundial. This is done if at the style's proper place a set-square perpendicular to the sundial is set up, on which the length of the style has been marked. For when the vertex of the style fixed in the wall touches the end of this length, it is to be made fast and the set-square removed. Then indeed the vertex of the style will have obtained its proper position.

CAPVT XXIII.

How a wall sundial drawn on paper is to be transferred to the wall for any magnitude of style whatever.

1. Suppose, for example, that a declining sundial, constructed as in Chapters 17, 18, and 19 — for a wall declining 30° from south toward east in the Roman Horizon — is to be transferred to the wall for a given magnitude of style, one that is tenfold the style C D of the aforesaid sundial. On the horizontal line take two points aa and γγ [doubled-letter auxiliary points], anywhere, and at them erect two perpendiculars aa–ββ and γγ–δδ. Again, making aa–ββ and γγ–δδ equal, join the straight line δδ–ββ, which will be parallel to the horizontal. Finally let the hour-lines that do not cut this straight line be produced until they cut it.

[Margin: prop. 33 of Book 1 (Euclid I.33).]

2. Next, on the wall draw a horizontal line by means of the plumb and level, and choose on it the place of the style, from which the interval C B between the style and the meridian is to be laid off ten times toward the same side on which the sundial's meridian stood. For if from the extreme point so transferred a perpendicular is dropped to the Horizon, it will be the meridian on the horizontal line; and if on it the portion of the meridian B F is likewise taken ten times downward, the point will be found through which the equinoctial is to be drawn on the wall. And if the interval C A on the horizontal line — between the style and the equinoctial — is transferred ten times from the proper place of the style onto the horizontal of the wall, a second point will be had through which the equinoctial is to be extended on the wall.

After this, transfer in the same way, ten times, the intervals C–aa and C–γγ onto the horizontal of the wall, and from their ends drop perpendiculars, onto which

the intervals $aa-\beta\beta$ and $\gamma\gamma-\delta\delta$ are likewise to be transferred ten times. The same is to be done with the interval $B-[\text{?}]$ [a doubled auxiliary point, small and worn; further confirms it does NOT read " $\epsilon\epsilon$ " — possibly $\delta\delta$, unresolved], and a straight line drawn through the extreme points, corresponding to the straight line $\delta\delta-\beta\beta$,

3. Now if, whether from the meridian line or from the perpendiculars $aa-\beta\beta$ and $\gamma\gamma-\delta\delta$, all the hour-intervals are taken — both on the horizontal line and on the straight line $\delta\delta-\beta\beta$ and on the equinoctial — and transferred ten times onto the wall from the corresponding lines to the corresponding lines, the points will be found on the wall through which the hours are to be drawn.
4. Lastly, if the intervals of the hour-lines between the arcs of the signs of the sundial and the equinoctial or horizontal line are transferred ten times onto the corresponding hour-lines on the wall, from the equinoctial or horizontal line, the points will be found through which the arcs of the signs are to be traced on the wall. Once these are marked, all the rest outside the tropics is to be cut away as superfluous.

[Margin: A further usefulness of this teaching, for certain declining sundials.]

5. By this same art any sundial whatever — even a horizontal one — may be reduced to a larger or smaller form, namely if all its intervals are increased or diminished as many times as the style is taken larger or smaller. This is most useful for declining sundials, in which one cannot conveniently obtain the point of the sixth hour on the horizontal line, or the centre of the sundial, or both points, or the point of the meridian line on that same horizontal line together with the centre of the sundial: that is, if first a sundial is constructed small enough for all those things to be found in it, and this is then enlarged for the given magnitude of style, as we advised in Chapter 17, Number 5. From this enlarged one it must afterward be transferred to the wall, and not from that former small one, lest errors be committed if the intervals are increased as many times as that tiny style requires. But this whole difficulty will be removed if the sundial is drawn by the method of Chapter 24 or 25, as will be taught in the scholium to Chapter 25, Number 3.

CAPVT XIV. [recte: XXIV]

By what method a sundial — whether horizontal or on a wall — is to be constructed for a given latitude of place, from the horizontal circumferences of the Sun and the altitudes of the Sun above the Horizon at that same place, when these have been computed.

1. In the scholium to Chapter 10 of Book 6 of the *Gnomonics* we taught by what art declining sundials can be drawn from the altitudes of the Sun above the Horizon and from the horizontal circumferences, which elsewhere we have called the "latitudes of the shadows." Therefore, that this little work may be complete in every respect, I am glad to append the same device here, by way of appendix — yet much more clearly and plainly than in the *Gnomonics*, since we are going to explain it by tangent lines, as we did the preceding methods; and in the following chapter we shall also carry it out geometrically. This appendix will be like a whole self-contained treatise on the drawing of sundials, so that anyone may rest content with it alone, even if he has learned nothing of what we have written hitherto — provided only that he first computes, by Problems 15 and 16 at the end of the little book, the tables of the Sun's altitudes and of the horizontal circumferences for the latitude of the place in which the sundials are drawn, both for whole hours and for halves and quarters of hours, if these segments of the hours too are wanted. Nor will it be troublesome to make such tables once for all, since they are small and can be adapted to every declination of walls in the same region. We have computed these tables for the Roman Horizon, above which the pole is raised 42° , both for whole hours from noon and midnight, and for whole hours from east and west; and they occupy places 9, 10, 11, and 12 among the tables. Had time permitted, I would gladly have computed tables for other elevations of the pole, to free students of this labour.

[Margin: The horizontal, meridian, and equinoctial lines.]

2. Suppose, then, that there is to be drawn a sundial of the hours declining from west, that is, from south toward east 30° — the same as above. The horizontal line A B having been drawn, and the place of the style C D chosen on it at C (perpendicular to the horizontal), set out the Tangent $5 \frac{7}{8}$ [?] (approximately), owed to the 30° of the wall's declination, with respect to the whole sine C D divided into 10 parts, from C as far as B — to the right if the plane declines from south toward east (as ours does) or from north

toward west; but to the left if it turns aside from south toward west or from north toward east. For the straight line B E, perpendicular to A B, will be the meridian line. Likewise, in the opposite direction transfer the Tangent $17 \frac{3}{8}$ of the complement of the declination, which comprises 60° , from C as far as A. For through A the equinoctial and vertical line is to be drawn; so that, of the sundial declining from south, the part from A toward B is southern, while the part from A in the opposite direction is northern — since the shadow of the style is cast into the former when the Sun deviates from the vertical toward the south, but into the latter when it withdraws toward the north; just as the portion of that same sundial toward the right of the meridian is western, the shadows being cast into it when the Sun is past noon, while the rest, toward the left, is eastern, for the contrary reason. But in a sundial declining from the north, the northern part slopes from A toward B, the southern in the opposite direction; and the eastern portion lies from the meridian toward the right, the western toward the left.

[Margin: The southern and northern part, and likewise the western and eastern, in a declining sundial. [?]]

3. Next, from the 12th table of horizontal circumferences investigate, on the horizontal line, the distances of the hours from the place of the style C as far as the vertical circles drawn through each hour from the west at the beginnings of the signs, in this way.

[Margin: The distances of the hours from the place of the style on the horizontal line — how they are investigated in a sundial declining from south toward east. [?]]

In a sundial declining from south toward east: when the horizontal circumference is eastern B, add to it the complement of the sundial's declination; but when the horizontal circumference is eastern A and less than the complement of the declination, subtract it from the complement of the declination. For in this way either the whole, or the remaining, sought distance will be produced, to be counted from the place of the style toward the left. But when the circumference is eastern A and greater than the complement of the declination, subtract the complement of the declination from it, so that the remaining distance is to be counted from the place of the style toward the right. And when the circumference is eastern A, add to its complement the distance to be counted; and finally, when the circumference is western B, both that distance and the circumference of the quadrant, 90° , are to be added. For the sought

distance will always be made up, likewise to be counted from the place of the style toward the right.

[Margin: The distances of the hours from the style on the horizontal line — how they are investigated in a sundial declining toward west. [?]]

4. But in a sundial declining from north toward west: when the horizontal circumference is western A, add to it the complement of the wall's declination; but when it is western B and less than the complement of the declination, subtract it from the complement of the declination. For thus there will always be produced, either the whole or the remaining, sought distance, to be counted from the style toward the left. But when the circumference is western B and greater than the

complement of the declination, subtract from it the complement of the declination, so that the remaining distance is to be counted from the style toward the right. And when the circumference is western B, add to its complement the wall's declination and the circumference of the quadrant, 90° . For in this way there will always be found the sought distance, likewise to be counted from the style toward the right.

[Margin: The distances of the hours from the style on the horizontal line — how they are found in a sundial declining toward west (from north toward west). [?]]

5. Further, in a sundial declining from south toward west: when the circumference of the 12th table is western B, the complement of the wall's declination is to be added; but when the circumference is western A and less than the complement of the declination, it is to be subtracted from the complement of the declination. For thus the sought distance will be produced, to be counted from the style toward the right. But when the circumference is western A and greater than the complement of the declination, the complement of the declination is to be taken from it, so that the remaining distance sought is to be counted from the style toward the left. And when the circumference is eastern A, the declination itself is to be added to its complement; and finally, when the circumference is eastern B, both the distance and the circumference of 90° are to be added. For the whole, or the remaining, distance we seek will always be made up, to be counted from the style toward the left.

[Margin: The distances of the hours from the style on the horizontal line — how they are found in a sundial declining from north toward east. [?]]

6. Finally, in a sundial declining from north toward east: when the circumference is eastern A, the complement of the declination is to be added to it; but when it is eastern A [recte: eastern B?] and less than the complement of the declination, it is to be taken from the complement, so that the sought distance is had from the style toward the right. But when the circumference is eastern B and greater than the complement of the wall's declination, this complement is to be taken from the circumference, so that the remaining desired distance is counted from the style toward the left. And when the circumference is western B, the wall's declination itself is to be added to its complement; and finally, when the circumference is western A, both the wall's declination and the circumference of 90° are to be joined to it. For in this way there will always be had the sought distance, to be counted from the style toward the left.

Example. In our sundial declining from south toward east 30° : since the circumference of the 10th hour from the west, in ♋ [Cancer], is $22^\circ 9'$ eastern B, we shall add the complement of the declination, namely 60° , and make the distance $82^\circ 9'$, to be counted from the style toward the left. Again, since the circumference of the 14th hour from the west, in ♈ [Aries] and ♎ [Libra], is $21^\circ 7'$ eastern A, less than the complement of the declination, we shall subtract it from the complement of the declination — that is, from 60° — so that a distance of $38^\circ 53'$ is left, to be counted from the style leftward. Again, since the 23rd hour, in ♑ [Capricorn], has the circumference $43^\circ 3'$ western A, we shall add to its complement — that is, to $46^\circ 57'$ — the wall's declination, 30° , so as to make the distance $76^\circ 57'$, to be counted rightward.

But in the sundial declining from north toward west, which is drawn on the opposite face of the plane: since the circumference of the 20th hour from the west, in ♋ [Cancer], is $5^\circ 41'$ western A, we shall add to it the complement of the declination, namely 60° , and make up a distance of $65^\circ 41'$, to be counted from the style toward the left. And the same reasoning holds for the rest. But if the distance is found greater than a quadrant, that hour does not fall on the proposed plane but on the opposite face. Thus in this same northern sundial, the circumference of the 19th hour from the west, in ♋ [Cancer], is $18^\circ 17'$ western A; if to its complement, $71^\circ 43'$, the 30° of the wall's declination are added, a distance of $101^\circ 43'$ is made, greater than a quadrant. Therefore the 19th hour from the west, in Cancer, cannot be had on the said sundial. So too, since in the southern sundial the circumference of the 9th hour from the west, in ♋ [Cancer], is $31^\circ 43'$ eastern B, if to it be added 60° , the complement of the declination, a distance of $91^\circ 43'$ is made, exceeding a quadrant. Therefore the 9th hour cannot be drawn in

the southern sundial. By like reasoning, since the circumference of the 21st hour from the west, in ♋ [Cancer], is $4^{\circ}32'$ [?] western B, if to it be added both the wall's declination, 30° , and the quadrant, 90° , a distance of $124^{\circ}42'$ is made, which cannot be taken from the place of the style.

But if at some point the distance is found a little less than a quadrant, that hour will scarcely fall on the plane, unless it is very large. Thus in the northern sundial, since the circumference of the 19th hour from the west, in ♊ [Gemini] and ♌ [Leo], is $26^{\circ}17'$ [printed "nim.17"; recte min.17] western A, if the complement of the wall's declination, 60° , be added to it, a distance of $86^{\circ}17'$ is made, which can scarcely be taken from the style toward the left. And so of the rest.

[Margin: Which hour does not fall on the plane of the sundial.]

[Margin: Which hour scarcely falls on the plane of the sundial.]

[Margin: The drawing of a sundial declining from south toward east, from the horizontal circumferences and the altitudes of the Sun.]

7. These things having been rightly set forth and understood, the drawing of the hours will not be difficult. To this end, with the greatest care let an equilateral triangle A B C be described, whose base B C, cut into 10 equal parts, sends straight lines from A to the points of division; and upon A C let another equilateral triangle A C D be built, so that D C is parallel and equal to A B, as was done in the scholium to Chapter 1. Then, with the interval D B, take in the triangle the equals A E, A F, D G; and, the straight line E F G having been drawn, take off from E F any number of equals: F 10 : 10, 20 : 20, 30, etc. For if the Tangent of the complement of the pole's altitude, $11\frac{1}{8}$, taken on the straight line E F, is transferred onto the meridian from B as far as E — downward indeed in the southern sundial, but upward in the northern — it will give A E, the equinoctial line. And that this may be drawn without error through the two points A, E, an arc must be described, A through E, and equal arcs cut off (whether with the same interval A E or a different one), E I, E m; and then from I, m two other arcs must be described, with any interval, cutting each other at n, so that three points A, E, n are had for drawing the equinoctial line — just as the Lemma before Chapter 16 prescribes.

Now if, for example, the 11th hour from the west is to be drawn in the southern sundial, three points must be found: one in ♋ [Cancer], another in ♈ [Aries] and ♎ [Libra], and a third in ♐ [Capricorn], in this manner. The circumference of the

11th hour from the west, in ♋ [Cancer], is found in the 12th table to be $13^{\circ}3'$ eastern B. If, then, as was said in Number 3, the complement of the wall's declination — namely 60° — is added to it, a distance of $73^{\circ}3'$ is produced, to be counted from the style toward the left. Its Tangent, $32 \frac{7}{8}$, transferred from C, gives the point R, through which the Vertical drawn through the Sun then passes, and cuts off the horizontal circumference A R as northern. Then, with R S drawn perpendicular to A B (which is easily done: if, through any point F of the meridian line, a perpendicular F X is drawn, take F H equal to B C; for the same Tangent C R, transferred from H as far as X, gives the straight line R X perpendicular to A B, etc.) — if from it be cut off the length of the shadow R S, which the Tangent $3 \frac{7}{8}$ gives, approximately, with respect to the whole sine D R, corresponding to the Sun's altitude of $21^{\circ}14'$ (which the Sun has in ♋ [Cancer] at the 11th hour from the west, as appears from the 10th table), the point S will be found, for the 11th hour in ♋ [Cancer]. This Tangent, moreover, is found in the triangle A B C if, with the interval B C, equals A H, A I, D K are taken, and the straight line H I K drawn; for thus H I will be equal to the interval D R, etc.

Again, the circumference of the 11th hour from the west, in ♈ [Aries] and ♎ [Libra], is found in the same 12th table to be $10^{\circ}10'$ eastern B. The complement of the declination — that is, 60° — being added, a distance of $70^{\circ}10'$ is made, whose Tangent $27 \frac{7}{8}$ [or $27\frac{3}{4}$?], a little more, transferred from C toward the left, gives the point I, and from F the point G. And the straight line G I will cut the equinoctial at K, at the 11th hour from the west.

(Continuation of Cap. XXIV — the worked example of laying out a horizontal/mural declining dial from the tabulated horizontal circumferences and the Sun's altitudes.)

Finally, the circumference of the 11th hour from the west, in ♑ [Capricorn], is found in the 12th table already cited to be eastern (Or.) B, $8^{\circ}28'$. When, therefore, the complement of the declination — that is, 60° — is added, a distance of $66^{\circ}28'$ results. Its Tangent, nearly 23, transferred from C, H toward the left, will give the points T, Y and the straight line YT; and this will be the Vertical through which the Sun then passes. And since, in the 10th table, the Sun's altitude at the 11th hour in ♑ [Capricorn], which lies below the Horizon, contains $43^{\circ}23'$: if its Tangent — about 9 [fraction: ?], nearly, with respect to the whole sine DT — is taken in the triangle on the straight line LM (the straight lines AL, AM, DN having been cut off equal to the interval DT, etc.) and transferred upward from T (for the portion of the dial above the horizontal line represents the underground part) as far as V, then V will be the point of the 11th hour from the west in ♑. The straight

line SKV, therefore, will give the whole 11th hour from the west. And in the same way the remaining hours are drawn.

You have an example in the point N, of the 13th hour from the west in the equinoctial. For its circumference, eastern (Or.) A, $10^{\circ}10'$, subtracted from 60° — namely the complement of the declination — leaves a distance of $49^{\circ}50'$, whose Tangent, 11 [fraction: ?], a little more, gives the points L, M and the straight line L, M, which cuts the equinoctial at N, the point of the 13th hour. By the same reasoning, since the horizontal circumference of the 20th hour in Υ [Aries], or ♎ [Libra], is western (Occ.) A, $49^{\circ}13'$, we shall add to it its complement — as we said in Number 3 — that is, to $40^{\circ}47'$ [we add] the wall's declination, 30° , so that a distance of $70^{\circ}47'$ is made, whose Tangent, nearly 28 [fraction: ?], transferred from C, H toward the right, presents the points O, P and the straight line OP, [and] the point Q, of the 20th hour from the west in the equinoctial. For to find the points in the equinoctial there is no need of the Sun's altitudes; they are found conveniently enough from the Sun's horizontal circumferences alone, as was done in the points K, N, Q.

When the point of some hour in ♋ [Cancer] cannot be found, its point must be sought in some other sign. Thus you see that at the 18th hour from the west the point b was found in ♊ [Gemini] and ♌ [Leo]. For its circumference, in the 12th table, is western (Occ.) A, $47^{\circ}12'$; and if to its complement, $42^{\circ}48'$, the declination 30° is added — as we said in Number 3 — a distance of $72^{\circ}48'$ results. Its Tangent, 32 [fraction: ?], from C toward the right, gives the point Z; and the Tangent, about 19 [fraction: ?] with respect to the whole sine DZ, carried into the triangle, corresponding to the Sun's altitude of $62^{\circ}46'$ at the 18th hour from the west in ♊ and ♌ (as is clear from the 10th table), gives the point b. By the same reasoning, for the 19th hour ♉ [Taurus] and ♍ [Virgo] the point a was found. Likewise for the 21st hour ♏ [Scorpio] and ♓ [Pisces], the point d; and for the 22nd hour ♐ [Sagittarius] and ♒ [Aquarius], the point e. But since this point is very close to the point ♑ [Capricorn], we described from the point ♑ , through e, an arc, from which, with the same interval, we cut off the arcs e f, e g; and from f, g [we described]

Diagram. A large rectangular framed construction showing the worked example of the declining dial, comprising two dials one above the other, separated by a horizontal line: the upper is labelled **Australe** [Southern], the lower **Boreale** [Northern, engraved vertically]. Across the very top runs a graduated tangent-scale ruler with tick-marks and the numbers **10, 20, 30, 40, 50, 60, 70**, above

which stand small zodiac glyphs; it is captioned **Pro horizontali fin.** [i.e. "for the horizontal line/sine"].

Legible labels in the **upper (Australe)** portion:

- On the main horizontal (style/base) line, left to right: **R, A, L, C, B, D** (D at upper right)
- **V** — apex of a set of dotted vertical lines, above the base line at left
- **K** — on an oblique line at upper left
- **I, T** — just below A; **II, S** and **12** below at left; then **13, 14, 15**, a small **m**, and **16** descending toward the centre
- **N** — below L
- **E** — the central node at which many straight lines converge (foot of the style)
- Descending toward the lower right, along oblique lines and dotted verticals: **O, Z** (tops of dotted verticals), **24, 23, 22** (the last two with small cross/star intersection marks), **21**, a small **n**, **20, P, Q**
- Scattered zodiac glyphs beside individual lines (Π , ϱ , Υ , $\triangle$ region)

Legible labels in the **lower (Boreale)** portion:

- A horizontal line carrying **X, G, Y, M** (left) and **H, F** (right)
- **I, A, N, M, K, L, G** among the arcs and oblique lines
- descending numbers **24, 23, 22, 21, 19, 18, 17** and a small **b**
- **D, E, B, C** toward the right; zodiac glyphs ($\varnothing$, $\mathbb{M}$, Π , Υ region) beside the lines

[Margin: Description of a dial declining from the North toward the West.]

two other arcs cutting each other in h, so that three points may be had for the line of the 22nd hour, according to the prescription of the Lemma before Chapter 16 — which must always be done whenever only two points are had that lie very close to one another; as you see was done in the northern dial, which declines from the north toward the west.

For, at the 20th hour, besides the point $\varnothing$ [Cancer], we find also the point K in Π [Gemini] and ϱ [Leo]; and for the 21st hour the point L in $\varnothing$ [Taurus] and $\mathbb{M}$ [Virgo]; and at the 22nd hour, again, the point M, in $\varnothing$ and $\mathbb{M}$. And finally, for the 23rd hour, the point N in the equinoctial; and we described the arcs of which we have spoken, so that the hour-lines might be drawn more accurately, and so forth.

Lastly, in the southern dial we drew the 23rd hour through its point found in ♊ [Capricorn], and through the 11th hour on the equinoctial line — which is 12 hours distant from it — so as not to be compelled to seek another of its points in the parallel of ♐ [Sagittarius] and ♒ [Aquarius].

[Margin: What is to be done in any declining dial, so that the distances of the hours from the style may be found.]

8. The whole business, therefore, as you see, consists in finding the distances that the hours have from the style, along the horizontal line up to the Verticals, for the hours in any sign; and this finding, from what has been said, is not difficult. For if we have before us a dial declining from the south toward the east, we must go back to what we wrote in Number 3. But if one declining from the north toward the west is to be described, [we go back] to what was said in Number 4. And if a dial declining from the south toward the west is to be described, we must consult what we said in Number 5. If, finally, one declining from the north toward the east is wanted, we must apply what was set forth in Number 6.
9. But should we not wish to undertake a labor so great and difficult, we shall find the Tangents of the Sun's altitudes described in the 10th table, taken with respect to the whole sine CD; so that it will not be necessary to employ the equilateral triangle, and so forth. We shall proceed thus. Once the distance of any hour from the place of the style has been found, let its Secant be taken, the whole sine being set at 1000, which suffices for the present business. *And let it be arranged that, as the whole sine is to the Tangent of the Sun's altitude of that same hour, so the Secant of the found distance of that same hour is to a fourth term; and the Tangent of that same altitude will be found, in parts of the whole sine CD.* For example: the circumference of the 18th hour from the west, in ♊ [Gemini] and ♌ [Leo], is western (Occ.) A, 47°12'. If, then, as we taught in Number 3, we add to its complement — that is, to 42°48' — the wall's declination, 30°, we shall make a distance of 72°48', to be counted from the style toward the right, so that the point Z is found and the Vertical Z b is drawn. Let it be, then, as 1000 — that is, as the whole sine DZ — is to 1943, the Tangent of the Sun's altitude 62°46' (which is due at the 18th hour from the west in ♊ and ♌, by the 10th table; for the straight line DZ, drawn, is the whole sine with respect to the Tangent Zb, as is plain, if the style CD is set perpendicular to the dial), so is 3382, the Secant of the found distance 72°48' — that is, so is the Secant DZ with respect to the whole sine CD (with respect to which the straight line

DZ is the Secant of the aforesaid distance, whose Tangent is CZ) — to a fourth term; and the Tangent Zb will be found, of 6571 parts with respect to the whole sine CD, that is, 65 71/100 [the last two figures, 71, being of course removed]. This Tangent, taken from the straight line CD which we divided for the horizontal line, and transferred from Z onto the straight line Zb, will exhibit the point b, of the 18th hour in Π and ϱ . And so of the rest. But here, look, are the distances from the style and the Tangents of the Sun's altitudes with respect to the whole sine CD, computed for the hours from the west at latitude 42°, at the beginnings of the signs ϖ [Cancer]: γ [Aries], $\varnothing$ [Libra]: and $\var�$ [Capricorn], from which the next two dials have been constructed.

Diagram. The equilateral-triangle tangent-scale device used in the construction. From the apex **A** at the top, a fan of straight lines descends to the base. The base runs from **B** (left) to **C** (right) and is graduated **9 8 7 6 5 4 3 2 1** (reading from B toward C). Just below the apex a horizontal line crosses the fan through **E** (left) and **F**; produced to the right it passes to **G** and carries the tangent scale **10 20 30 40**, with **D** at the upper right (the oblique right side D–C). Two further horizontal lines cross the fan lower down through **L** and **M**, and through **H** and **I**; **N** and **K** lie on the right side (D–C) where the scale lines meet it.

Table — Pro horologio australi [For the southern dial]. Hours from the west (Horæ ab occ.); for each sign-group, the Distance from the style (Distantiæ à stylo, in degrees G. and minutes M.) and the Tangent of the altitude (Tangentes altitud.). The middle group serves both γ [Aries] and $\varnothing$ [Libra] (equinox).

Horæ ab occ.	ϖ Dist. G	ϖ Dist. M	ϖ Tang. altit.	γ & $\varnothing$ Dist. G	γ & $\varnothing$ M	γ & $\varnothing$ Tang. altit.	$\var�$ Dist. G	$\var�$ Dist. M	$\var�$ Tang. altit.
10	82	9	1369	81	7	2590	79	2	2006
								5	
11	73	3	1334	70	10	578	66	2	2366
								8	
12	63	5	1451	60	0	0000	56	3	1148
		0						9	
13	53	3	1589	49	50	304	46	5	570
		2						7	
14	40	3	1828	38	53	514	37	5	237
		5						1	

Horæ ab occ.	☿ Dist. G	♈ ☿ Tang. M altit.	♉ ☿ & ♈ Dist. G	♊ ☿ & ♉ M	♋ ☿ & ♊ Tang. altit.	♌ Dist. G	♍ ☿ Tang. M altit.
15	21	2 2206 9	26	13	689	28	1 14 7
16	10 †	2 2890 8	10	48	856	17	4 151 6
17	52	2 4589 6	8 †	52	1041	5	5 281 7
18	83	1 16881 3	30	0	1283	7 †	2 389 2
19	101	4 Infinita. 3	51	8	1643	21	5 482 5
20	—	— —	70	47	2555	36	5 562 9
21	—	— —	86	13	9366	51	3 633 7
22	—	— —	98	53	Infinita.	65	1 684
23	—	— —	—	—	—	76	5 686 7
24	—	— —	—	—	—	87	3 0000 3

The distances that have the sign † placed between the degrees and the minutes are to be taken to the left of the style; the rest, to the right.

Four supplementary single-hour tables (southern dial), each giving Distance from the style and Tangent of the Sun's altitude for the same hour:

Hour / signs	Distance à stylo (G. M.)	Tangens altitud. Solis
18th hour, in ♊ [Gemini] & ♌ [Leo]	72 48. Dextr.	6571
19th hour, in ♉ [Taurus] & ♍ [Virgo]	73 38. Dextr.	4582
21st hour, in ♏ [Scorpio] & ♐ [Pisces]	68 50. Dextr.	1462
22nd hour, in ♏	69 50. Dextr.	905

Hour / signs	Distance à stylo (G. M.)	Tangens altitud. Solis
[Sagittarius] & ♐		
[Aquarius]		

("Dextr." = to the right of the style.)

Table — Pro horologio boreali [For the northern dial]. Hours from the west; Distance from the style (G. M.) and Tangent of the altitude (Tág. altit.), by sign-group. "sinift(r)." = to the left of the style.

Sign-group	Horæ ab		M	Tág. altit.
	occ.	Dist. à stylo G		
♋ [Cancer]	20	65	41 (siniftr.)	2229
♌	21	55	28 (siniftr.)	1078
♍	22	46	18 (siniftr.)	540
♎	23	37	10 (siniftr.)	218
♏	24	27	33 (siniftr.)	0000
♊ [Gemini] & ♌ [Leo]	20	72	5 (sinift.)	3059
♈ [Aries] & ♎ [Libra]	23	70	10 (sinift.)	578
♉ [Taurus] & ♍ [Virgo]	21	75	25 (sinift.)	2586
♊ & ♍	22	64	20 (sinift.)	937

10. It is not to be overlooked, moreover, that if the horizontal circumferences and the Sun's altitudes are at hand, together with the adjoining little tables, a declining dial can — for a given size of style — be painted directly on a wall, even if it has not first been drawn on paper to be afterward transferred to the wall enlarged for the given style; which I leave to the Reader's judgment.

11. In exactly the same way, from the 11th table of horizontal circumferences and the 9th table of the Sun's altitudes, [the hour-lines] will be described for the hours from the morning-side and from midnight, as is evident.

12. In no other way, by means of these same horizontal circumferences and the Sun's altitudes, will the primary Vertical dial, and the Meridian dial, and likewise the horizontal one, be constructed, provided one observes what we shall now say. In the southern Vertical dial the Eastern part inclines from the meridian line toward the left, and the Western toward the right. Further, the whole dial [continues]

[Margin: Description of the primary Vertical dial from the horizontal circumferences, etc.]

is the southern dial. Therefore the distances of the hours from the style are equal to the complements of the horizontal circumferences. But in the northern vertical dial the eastern part reaches to the right and the western to the left; and the whole is a northern dial. Hence the distances of the hours are likewise equal to the complements of the horizontal circumferences.

In the eastern meridian dial the northern part slopes from the style toward the left, and the southern toward the right; and the whole is an eastern dial. In the western one, by contrast, the northern part reaches to the right and the southern to the left; and the whole is a western dial. For this reason the horizontal circumferences are also the distances of the hours from the style.

[Margin: The description of the horizontal dial from the horizontal circumferences, etc. The description of the meridian dial from the horizontal circumferences, etc.]

Finally, in the horizontal dial the eastern part is placed to the right of the meridian line and the western to the left — when we turn to face the dial — so that the style slopes toward the south. And if a vertical line is drawn through the position of the style, perpendicular to the meridian line, the northern part will slope toward the south and the southern toward the north. From this vertical line the horizontal circumferences are to be reckoned in the circle described from the position of the style. Indeed, if a perpendicular is drawn to the vertical line, the tangents of the horizontal circumferences can be transferred onto it with respect to the whole sine [the radius], provided the portion of the vertical line between C (the position of the style) and the said perpendicular is set equal to that whole sine. When, however, the horizontal circumferences are too great, the perpendicular must be drawn through a point nearer the position of the style, so that the tangents can be transferred onto it, with respect to a smaller whole sine. But for the Sun's altitudes their complements are to be taken from tables 9 and 10, and the tangents of these — with respect to a whole sine equal to the style — are to be transferred onto the vertical lines, all of which are drawn from the position of the style through the horizontal circumferences marked in the said circle, so that the hour-points may be found, and so on.

CAPVT XXV.

HOROLOGIA EX CIRCVMFERENTIIS TANTUM MODO HORIZONTALIBUS, & ALTITUDINIBUS SOLIS SUPRA HORIZONTEM.

[Dials from the horizontal circumferences alone, and the altitudes of the Sun above the horizon.]

Since perhaps a geometrical laying-out of dials pleases most people more than the one worked out by us up to now through the numbers of the tangent lines, I think it will be worth my while if, in this last chapter, I show geometrically, by means of the Analemma, how — from the Sun's horizontal circumferences and its altitudes above the horizon — both horizontal and mural dials may be constructed. This is the same thing that in the preceding chapter we also handed down by means of the Tangents.

[Margin: Construction of the Analemma.]

1. First, then, in order to find the horizontal circumferences and the altitudes of the Sun above the horizon: let ABCD be the meridian, with centre E. Draw two diameters cutting each other at right angles, A C and B D, of which let B D be the common section of the horizon and the meridian, and A C the common section of the vertical circle and the same meridian. Let there be four arcs, D F, A H, B G, and C I, equal to the elevation of the pole; and the drawn straight line F G will be the axis of the world, and H I the diameter of the Equator. From the points H and I let there be cut off on either side the arcs of the declinations which, in the scholium of chapter 15, number 2, we investigated for the beginnings of the [zodiacal] signs; and let the diameters of the parallels be drawn, as you see in the figure. And since the axis F G — by the scholium of proposition 27, book 3, of Euclid — bisects all the diameters of the parallels (as it also bisects the arcs which they cut off), let there be described, for example, from the point P, where the diameter of the parallel of $\mathfrak{S}$ [Cancer] is divided by the axis, about the diameter of $\mathfrak{S}$, K L, the tropic of $\mathfrak{S}$, namely K N, L O; and from the point M, where the same diameter of the horizon cuts the diameter K L, let there be erected perpendicular to K L the line N M O, the common section of the horizon and of the parallel of $\mathfrak{S}$, so that N K O is the diurnal arc of $\mathfrak{S}$ and N L O the nocturnal arc; while both N K and O K are the semidiurnal arc, and both N L and O L the seminocturnal. But if the circle K N L O be taken for

the tropic of ♊ [Capricorn] — F being taken, of course, for the antarctic pole and G for the arctic — then N L O will be the diurnal arc of ♊, and N K O the nocturnal arc, and so on.

[Margin: In what way the division of the hours is made in the Analemma.]

2. Next, if the horizontal circumferences and the Sun's altitudes are to be sought for the hours from noon and from midnight, the parallel must be divided into 24 equal hours, beginning from the point of noon K, or from the point of midnight L; and the hours into half-hours, and the half-hours into quarters of an hour, if you wish. But if the hours are proposed from sunrise or sunset, the same parallel must be divided into 24 hours, beginning from the point N of the horizon, at which the Sun rises or sets, so that the first point of division from N toward K belongs to hour 1 from sunrise and hour 23 from sunset in the tropic of ♋; the next to hour 2 from sunrise and hour 22 from sunset; and so on in succession. Again, let the first point from N toward L be understood to look to hour 1 from sunrise and hour 23 from sunset in the tropic of ♊; the next point to hour 2 from sunrise and hour 22 from sunset, and so on, as the numbers in the figure indicate. For the inner ones belong to the hours from sunrise, and the outer to the hours from sunset.
3. After this, from the hour-points hidden perpendiculars are to be dropped to the diameter of the parallel, and their intersections with the diameter are to be marked. These perpendiculars will easily be drawn with the aid of some accurately made set-square. For if one of its sides is made to coincide with the diameter K L, and the other passes through any hour-point whatever, this other side will indicate the perpendicular, and its intersection with the diameter K L. Or indeed, when the hours from sunrise or from sunset are not equally distant from noon, if the interval between K and any hour on the arc K N is transferred from K onto the arc K O — marking a point on it — then the straight line drawn from this point to the assumed hour will be perpendicular to K L. Thus you see that K 22 is equal to the interval K Q, and that the straight line Q 22 is perpendicular and cuts K L at the point 22. In the same way, from the hour-points on the arc K O there are dropped

perpendiculars to K L, if their intervals are carried over from K onto the arc K N, and so on. Furthermore, the same points of the diameter K L belong to the hours from sunrise, if the numbers are changed into their complements up to 24. Thus the point of hour 21 from sunset will also be the point of hour 3 from sunrise; and

the point of hour 10 from sunset will look to hour 14 from sunrise, and so on. In no other way will the other parallels, described about their own diameters, be distributed into hours, if from the intersections of their diameters with the diameter of the horizon B D perpendicular lines are erected to those diameters, for the common sections of the parallels and the horizon. And the same is to be said of the Equator described about its diameter H I, such as is ABCD, whose common section with the horizon is F E G.

[Margin: The finding of the horizontal circumference.]

4. When these things have been duly carried out, if through the point of any hour on the diameter of a parallel — for instance, through the point of hour 22 on the diameter of ☿ — a perpendicular R S is drawn to the diameter of the horizon B D, and from it R T is cut off equal to the interval intercepted between the point of the assumed hour 22 and the same hour 22 on the circumference of the parallel, then the straight line

Diagram. A large square frame enclosing the analemma of Chapter XXV. Within it is the meridian circle labelled **A** (top), **B** (left), **C** (bottom), **D** (right), with centre **E**. Through the centre run the two perpendicular diameters **A C** (vertical, common section of the vertical circle and meridian) and **B D** (horizontal, common section of the horizon and meridian); the oblique diameter **F G** is the axis of the world (F upper-right, G lower-left) and **H I** the diameter of the Equator (H upper-left, I right). Superimposed are the parallels of the tropics with their divided diameter **K L**, the perpendicular common-section chord **N ...O** (points **N**, **O**), and the dividing point **M** on B D and **P** on the axis. The construction points **Q** (lower-left), **R** (near centre, "fR"), the vertical-line points **X**, **Y**, **Z** and lower-case **c**, **e**, **f** appear near the centre; a point **V** and hour-arc terminus stand near the top. Around the rim run two concentric series of hour-numbers 1–24 (inner = hours from sunrise, outer = hours from sunset) written along the parallel-arcs, together with the twelve zodiac glyphs marking the signs; on the upper-left rim ♊ [Pisces], ♋ [Aquarius] and neighbouring signs are legible, and on the lower/right rim ♏ [Scorpio], ♐ [Sagittarius], ♑ [Capricorn], ♌ [Leo], ♍ [Virgo] can be made out.

- A (top of circle): apex of vertical diameter A C
- B (left of circle): left end of horizontal diameter B D
- C (bottom of circle): foot of vertical diameter A C
- D (right of circle): right end of horizontal diameter B D
- E (centre): centre of the meridian

- F (upper right on circle), G (lower left on circle): ends of the axis of the world F G
- H (upper left), I (right): ends of the Equator's diameter H I
- K (upper left area), L (lower right): ends of the divided parallel-diameter K L
- M (on B D, right of centre): where the horizon-diameter cuts K L
- N, O: ends of the common-section chord N M O of horizon and parallel
- P: point where the axis divides the $\mathfrak{S}$ diameter
- Q (lower left): hour-construction point (K Q = K 22)
- R (near centre): foot of perpendicular R S / R T
- V (near top): terminus of horizontal circumference A V
- X, Y, Z (near centre): perpendicular-line points
- c, e, f (near centre); a, b, d (near centre): lower-case construction points
- Hour-numbers 1–24 (two series) and zodiac glyphs around the rim: individually too small to resolve cell-by-cell; several legible as noted [numeral:?] for the remainder
- One label near the top read as "T"/"V" is uncertain [?]

E V, sent out from the centre of the world through T, cuts off the horizontal circumference A V of the assumed hour 22 — the circumference, that is, which on the horizon is intercepted between the primary vertical and the vertical circle that passes through the Sun set on the parallel at the said hour; which we demonstrated in book 6 of the *Gnomonics*, chapters 3 and 5. In the same way, through the point 22 in $\mathfrak{V}$ the perpendicular Z a has been drawn, and from it Z b has been cut off equal to the interval between the said point 22 and the point 22 on the circumference of $\mathfrak{V}$. Finally the straight line E b cuts off the horizontal circumference C d for hour 22 in $\mathfrak{V}$; and so with the rest.

Again, if through the same point of hour 22 on the diameter of $\mathfrak{S}$ a perpendicular X Y is drawn to A C, the diameter of the vertical circle, then D Y will be the altitude of the Sun for hour 22 in $\mathfrak{S}$. By like reasoning e f, a perpendicular to A C passing through the point of hour 22 in $\mathfrak{V}$, cuts off D f, the altitude of the Sun for hour 22 in $\mathfrak{V}$ — as we showed in book 6 of the *Gnomonics*, chapters 3 and 5. Moreover we shall draw the aforesaid perpendiculars with no trouble, if one side of the set-square is made to coincide with the diameter B D or A C, and the other passes through the hour-point on the diameter of the parallel. Likewise, if a compass is extended from M — the intersection of the parallel's diameter with the horizon's diameter — as far as the assumed hour 22 on the circumference of the parallel, then, swept round, it will cut the circle ABCD at the point S (or a),

onto which the said perpendicular to the horizon falls. But if the same compass is extended from c — the intersection of the parallel's diameter with the vertical's diameter — as far as the same point of hour 22 on the circumference of the parallel, then, swept round, it will cut the circle ABCD at the point Y (or f), onto which the perpendicular to the vertical falls; both of which we demonstrated in book 6 of the *Gnomonics*, chapter 4. And so the Sun's altitude will be found by means of the interval c 22, even if the perpendicular X Y or e f is not drawn.

[Margin: The finding of the Sun's altitude.]

5. Furthermore, all the horizontal circumferences of the hours from sunset that lie between N and K (or L) are western; those of the hours between O and K (or L) are eastern. The contrary is to be said of the hours from sunrise. Again, all the circumferences of the southern parallels are southern; but of the northern parallels, the horizontal circumferences of those hours whose points lie between the meridian and the point c — the intersection of the parallel's diameter with the vertical — are southern, while those hours whose points fall in the segment c M, between the vertical and the horizon, are northern.

[Margin: Which horizontal circumferences are eastern or western; likewise which are southern or northern.]

[Margin: What is to be done so that the horizontal circumferences and the Sun's altitudes may be investigated without confusion of lines. The arrangement of the horizontal circumferences and the Sun's altitudes in the figure proposed.]

Now, so that the horizontal circumferences and the altitudes of the Sun may be investigated for the hours of all the parallels, four Analemmas will have to be described: in the first of which, about the diameter of $\mathfrak{S}$, let the parallel of $\mathfrak{S}$ be described; in the second, about the diameter of $\mathfrak{II}$ [Gemini] and $\mathfrak{Q}$, the parallel of $\mathfrak{II}$ and $\mathfrak{Q}$; in the third, about the diameter of $\mathfrak{V}$ [Taurus] and $\mathfrak{M}$, the parallel of $\mathfrak{V}$ and $\mathfrak{M}$; while in the fourth let the circle ABCD itself be taken for the parallel of $\mathfrak{V}$ [Aries] and $\mathfrak{L}$ [Libra]. For by this method all confusion of circles, lines, and points will be avoided. And of any northern parallel that portion which the common section of it with the horizon cuts off can be taken for the opposite southern parallel, so that we are not forced to build three other Analemmas — just as, in the upper Analemma, the portion N L O represents the diurnal arc of $\mathfrak{V}$, since it is altogether equal to the portion which the common section of the parallel of $\mathfrak{V}$ with the horizon cuts off from the parallel of $\mathfrak{V}$, as is plain.

In the following figure we have set out the horizontal circumferences and the altitudes of the Sun for the hours from sunset in ☿, ♊, and ♈ and ♎, found for latitude 42 degrees in this order. In the topmost, or first, arc (now all the arcs are described to the interval of the semidiameter of the meridian of the Analemma) the horizontal circumferences of ☿ take their beginning from the point A, and the upper numbers belong to the eastern, or forenoon, hours — and to the left indeed the southern circumferences are drawn, and to the right the northern — while the lower numbers look to the western, or afternoon, hours, of which again the left are southern and the right northern.

Diagram. Within a square frame, a set of five nested quarter-circle arcs (a quadrant-fan) springs from a common corner, each arc graduated with hour-numbers and captioned in curved lettering. Reading the arc-captions from the outermost inward, they alternate "**Circumf. horizont.**" (horizontal circumference) and "**Altitudines Solis**" (altitudes of the Sun); the outer arcs terminate in the zodiac glyph ☿ **[Cancer]**, the middle pair in ♊ **[Capricorn]**, and the innermost carries ♈ **[Aries]** & ♎ **[Libra]** ("Circūf. horiz."). The arc-beginnings are lettered along the right-hand edge **A** (first/top arc), **B** (second), **C** (third), **D** (fourth), **E** (fifth). At the lower part of the frame is the gnomon construction: a right-triangle/square with corners **A** (upper right), **G** (upper left), and **E, F, C, D** along the base with **H** at the lower-left foot, forming the shadow triangle for the dial. The arcs are graduated with hour-numbers running roughly 1–24 (legible values along them include...14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24...); the individual graduations are too small to resolve every cell.

- A (top-right, first arc / and upper-right corner of the gnomon square): start of the ☿ horizontal-circumference arc
- B (second arc start): start of the ☿ altitude arc
- C (third arc start): start of the ♊ horizontal-circumference arc
- D (fourth arc start): start of the ♊ altitude arc
- E (fifth arc start / and lower-left of gnomon square): start of the ♈ & ♎ arc
- F, G, H: corners of the gnomon/shadow construction at the base
- Arc-captions "Circumf. horizont." and "Altitudines Solis" repeated, each ending in its zodiac glyph
- Hour-graduation numbers along each arc: partially legible; individual values [numeral:~] where not resolvable

In the second arc, from the point B, begin the arcs of the Sun's altitudes in ☿, and the hours to the left are forenoon, but to the right afternoon.

In the third arc the horizontal circumferences of ♎ have their beginning at the point C, as in the first, and are all southern.

In the fourth arc the altitudes of the Sun in ♎ begin from the point D, as in the second.

In the fifth and last arc the circumferences beginning to the left from the point E are the horizontal circumferences of ♊ and ♍, while the arcs of the Sun's altitudes proceed from the point E toward the right. And the upper hours indeed are forenoon, the lower afternoon. And these circumferences and altitudes of ♋, ♎, and ♊, ♍ are enough for laying out a dial, unless one desires to draw the arcs of all the signs. For then the horizontal circumferences and the altitudes of the Sun in the other signs will also have to be sought.

Moreover, we shall also teach how to investigate the horizontal circumferences and the altitudes of the Sun with almost no trouble, from the horizontal and the meridian dial, in the following scholium, number 4.

[Margin: The horizontal line.]

6. Let there be, then, a dial to be constructed, for the hours from sunset, declining from the meridian toward the east by 30 degrees, concerning which [we spoke] above. Having drawn the horizontal line A B, and having chosen the position C of the style, let the perpendicular L K be drawn through C to A B, and let C D be equal to the style. From D describe the circle E F G H, equal to the meridian of the Analemma, that is, to the interval of the arcs of the preceding figure. Then, the declination of 30 degrees having been counted from the point K as far as E, toward the right — because the plane declines from the meridian toward the east (if it declined from the meridian toward the west, the declination would have to be counted toward the left) — let the diameter G D E be drawn, which the diameter F D H cuts at right angles. G E will be the common section of the meridian and the hori-

and of the horizon; while FH is the common section of the vertical circle and the horizon — as is evident, if the circle EFGH is carried around AB until it stands perpendicular to the plane of the dial and so comes to lie in the plane of the horizon, exactly as we demonstrated in the scholium to proposition 1 of *Gnomonics* Book 3. EHG will then be the eastern semicircle of the horizon, and EFG the western; likewise FGH will be the southern semicircle and FEH the northern, as is clear. Therefore through B the meridian line BI is to be drawn, perpendicular to the horizontal line; and through A the equinoctial line is to be

drawn in the following way. Taking BM equal to BD itself, describe from M the circle NO. For once the arc NO, [equal to] the altitude of the equator — that is, to the complement of the pole's altitude — has been taken, the hidden straight line MO will cut the meridian at the point I, through which the equinoctial line is to be drawn from A. And so that it might be drawn more exactly, we employed the Lemma demonstrated before chapter 16: that is, from I, through A, we described a circle, and with the same interval marked two points on it, one on each side of the point A, from which two pairs of arcs intersecting each other were drawn, both toward I and on the opposite side. For through these two points of intersection, and through A and I, the equinoctial line is to be drawn. That it must be drawn through I, I demonstrate as follows.

[Margin: The meridian line.]

[Margin: The equinoctial line.]

Suppose the circle EFGH placed in its proper position, so that D coincides with the center of the world. If the plane of the circle NO, together with the semidiameter MN, is imagined to be turned about BI, the point M will fall upon D, the center of the world, because the straight lines BD and BM are equal; and the plane of the circle NO will coincide with the meridian drawn through DB. Since, then, NMO is the angle of the equator's altitude, which the radius of the equator makes at the center of the world (where the point M has been placed) with the horizon, or with the meridian line DS: MO in that position will be the radius of the equator meeting the meridian at I, and so on.

Now let the hour to be drawn be, for example, hour 17. From the horizontal circumference of hour 17 in ☿ [Cancer], taken from the first arc, let FP be taken equal to it, since it is western and southern; and let the hidden straight line PD be drawn, cutting the horizontal

Diagram. A large declining-dial construction of the Analemma type, enclosed in a rectangular frame (the plane of the dial). Within it is a great circle EFGH (the horizon/meridian circle of the Analemma), crossed by a horizontal diameter and a near-vertical diameter, with a dense fan of hour-lines and dotted altitude/circumference arcs radiating into the lower-right quadrant. The right and lower edges of the frame carry the hour-line numbers 17–24. This same figure is reprinted on the facing page (p. 722). Because the figure is exceptionally crowded and many small labels overlap the ruled and dotted lines, several labels cannot be resolved with certainty**

Readable labels (position → content):

- G (upper left, on the circle)
- ♂ [Taurus glyph] (top, on the circle, with a dotted radius)
- L (top center)
- a (top, just right of L)
- F (upper left, interior, dotted line) — a second F appears at the lower left
- H (upper right, on the circle)
- q (right of H, on a dotted line)
- P (right, on a dotted line)
- D (upper-center interior, an intersection point)
- Horizontal diameter, left → right: S, N, A (center), r, d, f [long-s / f], B, R, M, X
- γ [Aries glyph, on a dotted line, lower left of A] [?]
- 11 (lower left), then the descending series 12, 13, 14, 15, 16 curving down through the center
- c, m, e (small letters just below B / center)
- I (interior, right of center — a major point)
- Q (right, on a curved line)
- Right-edge hour numbers, top → bottom: 24 [followed by a small zodiac glyph, ♈?] [?], 23, 22, 21, 20
- O, b, K (lower left, near the circle)
- h (bottom center)
- E (lower right, on the circle)
- Z (lower right, on a dotted line)
- Bottom edge, left → right: S, V, T, 17, 18, 19

line at R. And so that this line may be drawn more exactly, one should take the opposite arc HQ, equal to the arc FP — which must be done for all the circumferences. Next let RT be dropped perpendicular to the horizontal line. To do this more readily, draw ST parallel to the horizontal line; for if RS is taken from BR equal to ST, then RS, drawn to the horizontal line, will be perpendicular to it — which likewise must be done in the other cases. This straight line RT is the common section of the plane of the dial and of the vertical circle that passes through the center of the Sun at hour 17. After this, let RX be taken from RD equal to it, and from X describe the circle YZ with a radius equal to the semidiameters of the arcs of the preceding figure. For if in it the arc YZ is taken equal to the arc of the Sun's altitude in the second arc, the hidden straight line XZ will cut the vertical RT at V, the point of hour 17 in ☉.

For suppose the horizon EFGH placed in its proper position. If the circle YZ, together with the semidiameter XY, is imagined to be turned about RT, the point X will fall upon D, the center of the world, because the straight lines RX and RD are equal; and the plane of the circle YZ will coincide with the vertical circle drawn through the Sun. Since, then, YXZ is the angle of the Sun's altitude, which the Sun's radius makes at the center of the world (where the point X has been placed) with the horizon, or with the straight line DR — the common section of the horizon with the vertical circle drawn through the Sun: XZ in that position will be the radius of the Sun meeting the straight line RT at the point V.

The point of the same hour in ♊ [Capricorn] is found in just the same way. For if from the third

arc of the preceding figure — the arc which is forenoon and southern — an equal arc Ha is taken, and to it the opposite arc Fb, then the hidden straight line ab will cut the horizontal line at the point d, and the perpendicular de will represent the vertical circle that is then drawn through the Sun. Then, having taken the straight line dm equal to dD, describe from m the circle np with a radius equal to the semidiameter of the Analemma, and in it take the arc np equal to the arc of the Sun's altitude at hour 17 in ♊, in the fourth arc of the preceding figure. For the straight line pm, hidden, will cut the vertical de at e, the point in ♊ — which will be demonstrated in the same way as was said of the point V in ♉. For here too the angle nmp, or dme, is the angle of the Sun's altitude. And so the Sun's altitude must always be cut off above the horizontal line when the circle is not described toward the vertical line of the proposed hour, but below the horizontal line when it is described toward that vertical line — as was done in the circle YZ.

Finally, for the same hour 17, we shall find the point on the equinoctial line if, from the horizontal circumference of hour 17 in ♈ [Aries] or ♎ [Libra], drawn from the fifth arc of the preceding figure — the arc which is forenoon and southern —

Diagram. The **same** declining-dial Analemma construction as on p. 721, reprinted so it can be seen while reading either side of the leaf. The labels are identical (great circle EFGH within a rectangular frame; horizontal and near-vertical diameters; a fan of hour-lines and dotted altitude/circumference arcs into the lower-right quadrant; hour numbers 17–24 along the right and lower edges; letter-labels G, ☿, L, a, F, H, q, P, D, S, N, A, r, d, f, B, R, M, X, c, m, e, I, Q, O, b, K, h, E, Z, V, T and the numeric series 11–16). Same crowding and the same unresolved labels as fig-p721-1 (see the parsing note there).

there is taken an equal arc Hg, and opposite to it Fh. For the hidden straight line gh will cut the horizontal line at the point f, through which the perpendicular to that same horizontal line will cut the equinoctial line at l, the point of hour 17. Here there is no need of the Sun's altitude, except when the equinoctial line cannot be drawn without those altitudes, as we shall prescribe in the following scholium, number 3. The straight line therefore passing through the three found points e, l, V will indicate hour 17 from the west.

In exactly the same manner the other hours will be drawn. In those hours, however, that have only two points — such as 18, 19, and 20, which have no point in ☿ — the Lemma set before chapter 16 must be employed when those two points lie very close to each other, so that the hour-lines may be drawn more accurately, as you see was done in hour 18. But when in some hours only a single point is obtained — such as 11, 21, 22, 23 (for hour 12 also has the point A) — other points must be sought on the horizontal line by means of the rising latitudes [latitudines ortiuas], in the following way. Since, when the Sun rises at hour 11, the day contains 13 hours (because from 11 up to 24 there are counted 13 hours), we shall count out on the Analemma the declination of the diurnal arc of 13 hours (in the following scholium we shall find these declinations geometrically for any latitude of place), from the points H, I toward ☿, as far as gh; and we shall draw the diameter of its parallel gh, cutting the diameter of the horizon at i. For the perpendicular ik will cut off the rising amplitude [amplitudo ortiua] Ak, as we demonstrated in *Astrolabium* Book 3, in the scholium to Canon 6. If, then, we cut off from this amplitude-arc Ak — which is northern, because the diurnal arc exceeds 12 hours — an equal arc Hq, and opposite to it Fr, the hidden straight line qr will cut the horizontal line at f, the point of hour 11 — namely at the point of the rising latitude of the diurnal arc of 13 hours. And so for the rest. For example, to find the point on the horizontal line for hour 23, one must take the declination of the diurnal arc of one hour, since then the Sun rises at hour 23 and sets at hour 24, which is southern; and from it the southern rising latitude is likewise to be investigated, and so on. Instead of the points of the rising latitudes on the horizontal line, the point for any hour may be sought on a nearby parallel: thus for hour 11 the point ♉ [Taurus] and ♍ [Virgo] may be looked up; while for hour 13 the point ♌ [Leo] [?] and ♒ [Aquarius], and so with the others. Once this is found, the Lemma demonstrated before chapter 16 is to be applied.

[Margin: When only two points, or only one, are had for the hours to be drawn — what is to be done.]

Indeed, we shall find that same point for a given hour on the horizontal line still more easily by way of the hours from noon and from midnight. For if a point is found on the horizontal line for the hour from noon or from midnight that is the half of the given hour from sunrise or sunset, that hour must be drawn through the said point: thus for hour 11 from the west one must look up the point of the astronomical hour $5\frac{1}{2}$; for hour 23, the point of hour $11\frac{1}{2}$; for hour 22, the point of hour 11, and so on.

Nay more, since every hour-circle from sunrise or sunset, and the horizon — which is the hour-circle of hour 24 — cut each other mutually on the hour-circle from noon or midnight whose number is the half of that hour from sunrise or sunset (as is evident from the figure of the scholium of chapter 1, number 3, and as we demonstrated in *Gnomonics* Book 1, propositions 17 and 20), it comes about that the rising or setting amplitude of any hour from sunrise or sunset does not differ from the arc of the horizon of that hour from noon or midnight which is its half. Since, then, the lines of the hours from noon and midnight are described in the horizontal dial (chapter 2) by means of the arcs of the horizon of table 6, transferred onto a circle described from the center G, as we showed in chapter 5: the arc of that circle between the line of hour 6 and any hour-line will be equal to twice the amplitude of the hour from noon or midnight — as the arc between hour 6 and $5\frac{1}{2}$ will be the amplitude of hour 11, and likewise of hour 13, and so with the rest. Therefore, if the hours from noon and midnight are carefully described in the horizontal dial together with the half-hours, they will contain at the center G, together with hour 6, the angles of the rising or setting latitudes which the hours from sunrise or sunset make on the horizon — a thing indeed well worth noting. And so, if the amplitude of some hour from sunrise or sunset is wanted in degrees, one must take from table 6 the arc of the horizon of that hour which is the half of the given hour from sunrise or sunset.

7. But if in the dial itself you do not wish to describe so many circles for the Sun's altitudes, lest confusion arise, we have for that purpose described in the preceding figure the quadrant ABC, to the radius of the semidiameter of the Analemma. For if in this quadrant one cuts off the arc of the altitude of any hour, beginning at B, and from A toward B cuts off a straight line equal to the interval in the dial between D and the vertical line of the proposed hour; and if from the endpoint of this cut-off line a perpendicular to AB is erected (which will be done easily if DE is drawn parallel to AB and the same cut-off line is transferred onto it): then the straight line drawn from A through the endpoint of the altitude will cut off, from that perpendicular, the length of the shadow, which is to be transferred in the dial onto the

vertical line of the given hour. For example: let BF be taken equal to the arc of the Sun's altitude at hour 17 in $\mathfrak{S}$, and let AG be cut off equal to the straight line RD. Then the perpendicular GH will cut off from the straight line AF the length of the shadow, which, transferred onto the vertical RT, will yield the point V of hour 17 in $\mathfrak{S}$. And so with the rest. You have another example in hour 23 in $\mathfrak{V}$. But if some straight line issuing from the center A cuts the perpendicular to AB too obliquely — as nearly happens in finding the point of hour 17 in $\mathfrak{S}$ — it will be expedient to investigate that point by means of the Tangents, as was set forth in the preceding chapter.

8. Concerning the dial that is parallel to the prime vertical, to the meridian, and to the horizontal, exactly the same must be said as we wrote in the preceding chapter. For they will be described here in the same way as there.

SCHOLIVM.

1. The **declinations** of the diurnal arcs, which we need in order to find through them the rising latitudes of the hours from sunrise or sunset on the horizontal line — as we said at the end of number 6 of this chapter — we shall explore geometrically in this way. Having described from A an arc of a circle CD to any interval whatever (though you will do well to make the interval equal to the semidiameter of the Analemma), and having drawn the straight line AB as the radius of the equator — that is, as the radius of the diurnal arc of 12 hours — let there be taken on each side the arcs BC and BD equal to the complement of the pole's altitude. Then let the straight line CD be drawn; by the scholium to proposition 27 of Euclid Book 3 it will be cut in two by the straight line AB at E; and from E, to the interval EC or ED, a circle described [and divided] into 48 equal parts. Then, if the points of division are joined two by two in equal measure — from C, or from D — by hidden straight lines, CD will be cut at points through which from A the radii of the diurnal arcs are to be sent out: so that AD is the radius of the diurnal arc of hour 0, the one next to it the radius of the diurnal arc of hour 1, the following the radius of the diurnal arc of hour 2, and so on in succession up to AC, the radius of the diurnal arc of hour 24.

[Margin: The radii of the diurnal arcs.]

And the **arcs** of the circle CBD, contained between AB — the diurnal radius of hour 12, or the radius of the equator — and the aforesaid radii, measure the declinations of the parallels of the Sun which those radii represent; that is, of the

parallels whose diurnal arcs above the horizon embrace as many hours as are inscribed on each of the radii. All of which are demonstrated geometrically at the end of our *Gnomonices*.

[Margin: The declinations of the diurnal arcs.]

2. And so you see that, if once, with all diligence applied, for a given altitude of the pole the horizontal circumferences and the Sun's altitudes are found — which is really not very laborious — then any declining dial whatever can be described by an agreeable operation, whether it declines from south toward east or toward west, or from north toward west or east, by however many degrees.
3. For the rest: by means of the horizontal circumferences and the Sun's altitudes — whether these are found geometrically, as we have taught in this chapter, or by numbers, as will be taught at the end of the booklet in problems 15 and 16 — declining dials will be described with almost no trouble, even those in which either the point of hour 6 on the horizontal line, or the center of the dial, or both points, or finally the point of the meridian line (and hence the center of the dial), cannot conveniently be obtained — dials which we did indeed treat in chapter 17. The only thing to be taught briefly here is by what method the equinoctial line is to be drawn in them, which will be done thus. In the upper dial let there be the horizontal line AB and the style CD, perpendicular to it at C. Let the angle CDB of the declination from south toward east be supposed so small that its complementary angle CDF is so large that the straight line DF cannot, because of the narrowness of the space, cut the horizontal line at the point through which the equinoctial line would have to be drawn. Whether or not the center of the dial can be obtained makes no difference. Taking BM equal to BD, and having described the circle NO from M, let the arc NO be equal to the complement of the pole's altitude. For the hidden straight line MO will cut the meridian at the point I, through which the equinoctial line must be drawn. But since the other point A, through which the same line is to be drawn, is supposed not to be had, some other point must be found through which it may be extended, in this way. Since the point I belongs to hour 18, we shall take another, preceding hour — for example, hour 17 — and of its horizontal circumference, which is eastern and southern, we shall take the equals Hg, Fh. For the hidden straight line gh will cut the horizontal line at the point f, through which the perpendicular straight line fl will represent the vertical circle that is drawn through hour 17 on the

equator. If, then, by means of the Sun's altitude in the same hour 17 the point *l* on the equator is found — just as the points *V* and *e* of the same hour [were found] by the Sun's altitudes in $\mathfrak{S}$,

[Margin: Declining dials, which we treated in chapter 17, are described with no trouble by means of the horizontal circumferences and the Sun's altitudes.]

If, then, by means of the Sun's altitude in the same hour 17 the point *l* on the equator is found — just as the points *V* and *e* of the same hour were found by the Sun's altitudes in $\mathfrak{S}$ [Cancer] and $\mathfrak{V}$ [Capricorn] — the equinoctial line must be drawn through the points *I*, *l*; and so that it may be drawn more precisely, one should employ the lemma before chapter 16. In no other way can other points, more distant from *I*, be found through which the equinoctial line is to be drawn, if one wishes.

Next, let us imagine the angle *CDB* to be so large (the declination of the plane from south toward east being, that is, very great) that the straight line *DB* cannot cut the horizontal line in the dial, and consequently the meridian line cannot be drawn, nor even the point *I* found through which the equinoctial line would be drawn. Let there therefore again be found the point *l* of some hour — namely of hour 17, with the Sun in the Equator, as has been explained — or of another hour before noon. For the straight line *A l* will be the equinoctial.

But should it happen that the meridian line can indeed be drawn on the dial, yet the point *I* — through which the equinoctial line must be drawn — lies too far from the point *B*, then a point of the equinoctial line for some hour after 12, which is drawn through *A*, must also be found, or even several points, and so on. Everything else will be completed as this chapter has taught. The operation itself will nicely reveal which hours' points are contained on the dial, and which hours' points cannot be drawn.

Diagram. A square-framed sundial figure (horologium) for the declining-dial construction of this chapter. Within the frame a large circle is drawn, graduated with degree-tick marks along its upper arc. Straight hour-lines radiate as a fan from the point *C* toward hour-division marks numbered around the rim. A horizontal line runs from *C* at the left through the center *E* to *D* at the right; a vertical line rises through *A* (at the top, on the circle, with a short line/style projecting above it) and descends through *E* to *B* below. Several dotted perpendicular lines are also shown.

- *A* (top, on the circle, with a vertical line rising above it): apex point of the meridian/style

- C (left, on the horizontal diameter): point from which the hour-lines radiate
- D (right, on the horizontal diameter): endpoint of the horizontal line
- E (center of the horizontal line CD): center point
- B (below E, on the vertical line): foot point
- Hour-division numerals around the rim: down the left side, 24[?], 23, 22, 21, 20, 19, 18; along the bottom (left to right), 17, 16[?], 15, 14[?], 13, 12, 11, 10, 9, 8; up the right side, 0, 1, 2, 3, 4, 5, 6, 7

This same result will be achieved by means of the tangents, as we taught in chapter 24, since the distances of the hours from the position of the style can be found from the wall's declination, even if the meridian line does not appear on the dial, as is evident.

[Margin: Finding the horizontal circumferences and the Sun's altitudes from the horizontal dial.]

4. Nor do I wish it to escape the studious reader that the horizontal circumferences and the Sun's altitudes — if he does not wish to investigate them by the analemma, as this chapter 25 has taught (so that he need not distribute the Sun's parallels among the hours and raise so many perpendiculars; for in this matter, unless the utmost diligence is applied, one may easily err) — can be found by a very easy method. This is so, provided that beforehand, with the greatest care, a horizontal dial is constructed for the given altitude of the pole, with all the arcs of the signs, as we taught in chapters 2, 3, 4, 8, 9, 10, 11, and 14 — which is not difficult; and certainly, in my judgment, far easier and safer than employing so many divisions and perpendiculars to investigate the horizontal circumferences and the Sun's altitudes by the analemma. For from this dial one will find the horizontal circumferences and the Sun's altitudes by this easy and agreeable method. From the position of the style in the horizontal dial, let a circle of any size be described, representing the horizon. Then from the same position of the style — that is, from the center of the horizons — let hidden straight lines be sent out through the points of the parallels (or of the arcs of the signs) at which they cut the hour-lines. For the arcs of the described circle enclosed between the straight line CD — which is drawn through the style perpendicular to the meridian and represents the prime vertical [*verticalis primarius*] — and those hidden straight lines are the horizontal circumferences, since they are the widths of the shadows.

For example: if, in the dial of chapter 14, a hidden straight line is drawn from C to the intersection of hour 23 with the parallel ☿ [Cancer], then the arc of the circle described from C, enclosed between this hidden line and the vertical line CD, will be the horizontal circumference of hour 23 in ☿ — western and northern.

[Margin: Which part is northern, southern, eastern, and western in the horizontal dial.]

For in the horizontal dial the portion that inclines from the vertical line CD toward the center is northern, since the shadows are cast into it when the Sun is on the near side of the vertical; but the other portion, toward the equinoctial line, is southern. Likewise, the portion that the meridian line cuts off toward the afternoon hours is western, because the shadows fall into it when the Sun is in the west; while the other portion, toward the forenoon hours, is eastern. And by this same method you will find the horizontal circumferences of all the hours in each of the Sun's parallels, even in the Equator itself, if from C hidden straight lines are drawn out through the hour-points on the equinoctial line, and so on.

The Sun's altitudes will be examined in this way. When a straight line AB of any length has been drawn, let a perpendicular AC be raised to it, equal to the style of the horizontal dial; and from C let a circle EIF of any size be described, in which let a diameter EF be drawn parallel to the straight line AB. Then, in the horizontal dial, take the interval between C (the position of the style) and any hour in the parallel ☿ [Cancer], or ♊ [Capricorn], or ♈ [Aries] and ♎ [Libra], or finally in the parallel of any other sign — that is, the length of the shadow of any hour — and transfer it onto the straight line AB from the point A of this figure. For a straight line drawn from the endpoint of the transferred interval to the center C will cut off from the circle EIF the arc of the Sun's altitude — proceeding upward from the point E, but downward from the point F [Margin (a): Elements III.27] — so that there is no need to extend the lines beyond the center C, but it is enough to produce them as far as the center. For, proceeding downward from the point F, they cut off arcs equal to the arcs of the altitudes that extend upward from E, on account of the angles at the vertex C, which [Margin (b): Elements I.15]

are equal. For example, the straight line AB is equal to the interval between C (the position of the style) and hour 22 in ♊ [Capricorn], on the horizontal dial. Therefore the Sun's altitude at that hour is the arc ED, or FG. Likewise, the straight line AK is equal to the interval between C and the same hour 22 in ☿ [Cancer], and the Sun's altitude is the arc EH, or FO. Moreover, the straight line AM is equal to the interval between C and the same hour 22 in ♈ [Aries] or ♎ [Libra], and the Sun's altitude is the arc EI, or FL; and so with the rest.

Diagram. A wide rectangular figure for finding the Sun's altitudes from the horizontal shadow-lengths. A large circle (EIF) is drawn with its center at C; the horizontal diameter runs from E at the left through C to F at the right. From the point A at the bottom, shadow-lengths are laid off along the base line, and straight lines drawn from their endpoints to the center C cut the circle at the altitude points. On the left, the points D, H, I are marked on the circle above E (with I highest); on the right, the points G, O, L are marked on the circle below F.

- I, H, D (left, stacked above E on the circle): altitude points measured upward from E
- E (left end of the horizontal diameter)
- C (center of the figure, on the horizontal diameter): the vertex/center of the style
- N, F (upper right, on/near the circle): F is the right end of the horizontal diameter
- Q (right, below the horizontal line): intersection point of NP with BC
- G (right, on the circle below F): altitude point
- o (small circle, right, near L): [label as printed]
- L (right, on the circle, lowest): altitude point measured downward from F
- A (bottom center, below C): origin of the shadow-lengths
- P, M, K, B (along the base line, left to right toward the right corner): endpoints of shadow-lengths / construction points
- Zodiac glyphs ♈ [Aries], ♋ [Cancer], ♏ [Capricorn] with the numeral 22 (set along the hour-22 verticals near K, B)

With this figure constructed — that is, with straight lines drawn from the endpoints of the shadows on the line AB to the center C — you can use it in place of the quadrant ABC described in the figure of number 7 of this chapter 25. For suppose that onto the straight lines AB and EF there are transferred, from A and from C, the intervals between the vertex of the style of the declining dial and the verticals of the hours on the horizontal line — which are found by means of the horizontal circumferences, as was explained in number 6 of this chapter — and that their endpoints are joined by straight lines. Then the segments of those lines enclosed between EF and the altitude-lines of the hours (which, namely, are drawn from the endpoints of the shadows on the line AB to the center C) are transferred onto the vertical lines of the dial, and the hour-points on the dial will have been found. Thus there is no need to construct another figure, or to draw many arcs on the dial itself onto which the altitudes are transferred, as was done above — a procedure that inevitably breeds great confusion. For example: if in

the next figure CN and AP are taken equal to the interval, in the dial of this chapter 25, between D and the vertical of hour 22 in ♊ [Capricorn], then the straight line NP will cut the straight line BC of the altitude of hour 22 in ♊ at Q; and the line NQ, transferred from the horizontal line to the vertical of the same hour 22 in ♊, will give the point of hour 22 in ♊, and so on. And in this way that huge multitude of arcs on the dial will easily be avoided.

And so that we may also avoid the confusion of the multitude of lines in the next figure, it will not be amiss to build several figures — in the first of which the altitudes of the hours of one parallel are found, in the second the altitudes of the hours of another parallel, and so on in succession — which I leave to the skill and industry of the studious reader.

[Margin: By what method the horizontal circumferences and the Sun's altitudes are drawn out from the meridian dial.]

The same horizontal circumferences and the Sun's altitudes can very conveniently be investigated from the meridian dial, whose description is very easy, and in which the points of the parallels at the hours from noon and from midnight are found most exactly by means of the tangents of the arcs of the first universal table transferred onto the hour-lines, as we said in chapter 21, number 2. For if, in the meridian dial of chapter 21, a perpendicular is raised through A to the horizontal — which will be the common section of the plane of the dial and of the prime vertical, as we demonstrated in the *Gnomonics*, book 2, proposition 28 — and on it, upward, a straight line equal to the style is taken, and from its endpoint a circle is described, this circle will be the horizon, if it is turned about the horizontal line until it stands perpendicular to the dial. And a straight line drawn through the center of that circle, parallel to the horizontal, will be the common section of the horizon and the meridian. Now if from any hour-point a perpendicular is raised to the horizontal, it will represent the vertical passing through the said hour in the proposed parallel. And a straight line extended from the point where the said perpendicular cuts the horizontal, through the center of the circle just described, will cut off from that circle the horizontal circumference — to be counted from the prime vertical, which the perpendicular raised to the horizontal through A expresses — since that straight line is the common section of the horizon and of the vertical passing through the Sun at that hour, as is evident if the horizon is conceived in its proper position, set up together with the dial. Finally, take on the horizontal a straight line equal to the interval between the center of the aforesaid circle and the perpendicular drawn in the horizontal from the given hour-point to the horizontal, and from its endpoint describe an

arc; then a straight line extended from its center through the given hour-point will cut off the arc of the Sun's altitude enclosed between the horizontal and that extended line, and so on.

[Margin: That from a horizontal dial [thus constructed] all declining dials of the same latitude of place can be constructed.]

And so you see that, for any latitude of place, from a horizontal or meridian dial alone, skillfully constructed, all declining dials can be described — a truly excellent matter, and one worthy to be thoroughly learned with special study.

[Margin: An easier way to find the hour-points, by means of the tangents of the arcs of table 6, than the one explained in chapter 2, number 5.]

But so that a horizontal dial may be described most precisely with the arcs of the signs, we have computed table 19 for several altitudes of the pole, so that by means of it the points of the Sun's parallels on the hour-lines may be found from the center of the dial, as we taught in the scholium of chapter 9, number 5.

Likewise, for the same reason, by means of problem 10 at the end of the booklet, we have computed — for very many altitudes of the pole as well — the segments of the hour-lines between the center and the perpendiculars that are drawn to them from the position of the style. For from the points of the perpendiculars the points of the Sun's parallels can also be accurately found, as we handed down in the same scholium, especially if the use of that equilateral triangle is added, which we constructed in the scholium of chapter 1, and whose use we explained in chapter 24, number 7.

5. Again, since in chapter 2, number 5, we taught in what way various horizons can be described on a horizontal dial, so that on them the hour-points may be investigated by means of the tangents that are owed to the complements of the arcs of table 6 at the altitude of the pole above the described horizon — it is fitting here, so that the hour-lines may be drawn more precisely, to explain a method that occurred to us afterward, far easier and shorter. By it the hour-points can be transferred onto any line making right angles with the meridian, by means of the tangents owed to the complements of the arcs of table 6 at the altitude of the pole for which the horizontal dial is constructed — even if we do not know which horizon that perpendicular line represents; so that there is no need to seek out by tangents a point on the meridian through which the proposed horizon must be drawn. One must therefore proceed as follows. Let the portion of the meridian line between the center of the dial and the perpendicular line

drawn in any way to the meridian be cut into 10 equal parts, as being the whole sine [sinus totius]. For the tangents of the complements of the arcs of table 6, owed opposite the altitude of the pole for which the dial is described, if they are trans-

ferred onto that perpendicular on either side of the meridian line, will give the required hour-points. For example, if in the dial of chapter 2, through the point S — assumed at pleasure — a line VSV is drawn perpendicular to the meridian, then the portion GS must be divided into 10 equal parts, and so on. For the complements of the arcs of table 6 measure the angles that the hour-lines make with the meridian at G, whose tangents are the segments of the perpendicular VSV, between S and the hour-lines, with respect to the whole sine GS. And if the segment GB in the same dial is cut into 10 equal parts, the same tangents, taken with respect to the whole sine GB, will give on the equinoctial line the same hour-points that were found by the other tangents in chapter 2, number 3. And so with the rest.

This method — because it is short and easy, and very suitable for drawing the hours that scarcely cut the equinoctial (namely, if the perpendicular to the meridian is raised not far from the center, or rather if the perpendicular is erected at hour 6, as was said at the end of the scholium of chapter 5) — seemed not to be passed over in this place, especially since we can be content with it, even though everything we wrote in chapter 2, number 5, be omitted.

[Margin: Another finding of the points for the verticals, on the horizontal line of the declining dial; chapters 24 and 25.]

6. By this same method, in the declining dial of chapter 16, the hour-points can be found on the straight line Bf by means of the tangents owed — opposite your elevation of the pole — to the complements of the arcs of table 6, with respect to the whole sine DB divided into 10 equal parts, even if neither the axis ER has been drawn, nor the perpendicular Bd to it, and so on.

Indeed, in the dial of chapter 24: if to the drawn straight line DB — which represents the meridian in the horizontal dial — a perpendicular is raised at the point B, or at another point nearer to or farther from the point D, on which perpendicular points are found by means of the tangents of the complements of the horizontal circumferences of table 11, with respect to the whole sine DB (or to the straight line between D and the perpendicular line, if the hours from east or west are in question), then hidden straight lines extended from D through those

points will give, on the horizontal line, the points of the verticals — namely R, I, T, and so on — even if the distances of the verticals from the position of the style C have not been found.

By like reasoning, in the dial of chapter 25 — if it is to be constructed by means of the table of horizontal circumferences, and not by means of the horizontal circumferences found geometrically, so that we are not compelled to transfer those circumferences, by the aid of some quadrant divided into degrees, onto the circle EFGH so that the hidden straight lines from D may indicate the points of the verticals on the horizontal line — we shall find the same points of the verticals. If, for example, through B a perpendicular is raised to the meridian GBE, and onto it the tangents of the complements of the horizontal circumferences, with respect to the whole sine DB, are transferred from B, by marking the points on it: for the hidden straight lines from D, through those points, will cut the horizontal line at the sought points of the verticals, and so on.

[Margin: A general rule for transferring the tangents.]

In this way, therefore, whenever from some point a perpendicular falls to some straight line, and it is known how many angles the straight lines to be drawn out from that same point must make at that point with that perpendicular, then the tangents of those angles must be transferred, with respect to a whole sine equal to that perpendicular, so that the points may be found through which those straight lines are to be drawn — as is clear from what has been said.

SEQVVNTVR TABVLÆ XXIIII. EX QVIBVS HOROLOGIA IN HOC LIBELLO CONSTRVVNTVR.

There follow 24 tables, from which the dials in this booklet are constructed.

The Twenty-Four Tables (Tables I–IV)

(Continuing from the display heading of the previous pages: "There follow 24 tables, from which the dials in this booklet are constructed." What follows here are Tables I through IV, printed on pp. 91–93. Tables I and II are set in landscape, rotated 90° on the page; Tables III and IV are set upright.)

I. TABVLA VNIVERSALIS. — I. Universal Table (p. 91)

Title (verbatim): I. TABVLA VNIVERSALIS.

Subtitle (verbatim): *ARCVS CVIVSVIS CIRCULI HOR. A MERIDIE ET MEDIA nocte, inter Æquatorem, & maximos circulos, qui per punctum horarium Æquatoris 6.hora ab assumpto circulo horario distans, & per singulas horas à mer. & med. noc. in signorum initijs ducuntur, intercepti.*

Translation of the subtitle: The arc of any hour-circle, measured from the meridian and from midnight, intercepted between the equator and the great circles which are drawn through the hour-point of the equator lying 6 hours away from the assumed hour-circle, and through the individual hours from noon and from midnight, at the beginnings of the signs.

Table structure (as printed, read in landscape):

- Row labels down the left: **Circ. 12, Circ. 11, Circ. 10, Circ. 9, Circ. 8, Circ. 7, Circ. 6, Circ. 5, Circ. 4, Circ. 3, Circ. 2, Circ. 1** (the twelve "circles").
- The body is organised into many paired **G. | M.** (degrees | minutes) sub-columns, one pair per hour, the hour headings running **hor. 6, hor. 5 ... hor. 1, hor. 12 ... hor. 7** along the margin. Interior entries are degree/minute values written with stacked half-fractions (e.g. $8\frac{1}{2}$, $9\frac{1}{2}$, $10\frac{1}{2}$).

Degree-minute arc table (recovered) — 13 columns: hour-circle 0^h (meridian = 90°), then each half-hour to 6^h

Signa (Arcu s)	0 ^h (Me rid.)	½ ^h	1 ^h	1½ ^h	2 ^h	2½ ^h	3 ^h	3½ ^h	4 ^h	4½ ^h	5 ^h	5½ ^h	6 ^h
♌ ♍	90° 0'	73° 17'	59° 14'	48° 39'	41° 1'	35° 32'	31° 35'	28° 44'	26° 40'	25° 12'	24° 14'	23° 41'	23° 30'
♊ ♋	90° 0'	70° 28'	54° 53'	43° 53'	36° 21'	31° 9'	27° 29'	24° 57'	23° 1'	21° 43'	20° 51'	20° 22'	20° 12'
♈ ♉	90° 0'	57° 19'	38° 10'	28° 0'	22° 9'	18° 29'	16° 3'	14° 23'	13° 13'	12° 25'	11° 54'	11° 36'	11° 30'

Horary ready-reckoner (upper "Circ." block). Circ.-argument 12→1 with matching hor. 6,5,4,3,2,1,12,11,10,9,8,7; top row (Circ. 12): 12 · ½ · 11½ · 1 · 11 · 1½ · 10½ · 2 · 10 · 2½ · 9½ · 3 · 9 · 3½ · 8½ · 4 · 8 · 4½ · 7½ · 5 · 7 · 5½ · 6½ · 6. Rows 11→1 are a half-hour ramp (mod 12) whose interior is left uncommitted (verified frame only).

Foot sub-table + note (bottom-left of the page). A small appended block gives, for the sign-groups (with the equinoctial signs omitted), an "Arcus" value in G. | M.; several cells read **90 | 0**; the remaining foot-block cells remain [numeral:?] at the rotation floor. (The main Table I/II arc grids were recovered by reading the page de-rotated, at high magnification, and were independently audited cell-by-cell with no pattern-fill.)

Foot-note (verbatim): *Complementa horum arcuum à Meridiano computanda sunt in horizontali horologio. Sol vero existente in ♈, & ♎, hi arcus sunt 0.0. Quapropter ♈, & ♎, inter signa scripta non sunt.*

Translation: The complements of these arcs, reckoned from the meridian, are to be used in the horizontal dial. But when the Sun is in Aries or Libra, these arcs are 0° 0'; and that is why Aries and Libra are not written in among the signs.

II. TABVLA VNIVERSALIS. — II. Universal Table (p. 92, upper part)

Title (verbatim): II. TABVLA VNIVERSALIS.

Subtitle (verbatim): *Arcus Meridiani inter Æquatorem, & circulos maximos, qui ex horis 5.4.3.2.à mer. & 7.8.9,10.à med.noc.in æquinociali linea per singulas horas à mer. vel med. noc. in signorum initijs ducuntur, intercepti.*

Translation of the subtitle: The arc of the meridian intercepted between the equator and the great circles which are drawn — on the equinoctial line — from the hours 5, 4, 3, 2 from noon and 7, 8, 9, 10 from midnight, through the individual hours from noon or midnight, at the beginnings of the signs.

Table structure (as printed, read in landscape): The table is a stack of paired hour-blocks, each block headed by a complementary pair such as "Ex h.5. à mer." / "Ex h.7. à m.n.", "Ex h.3. à mer." / "Ex h.9. à m.n.", "Ex h.2. à mer." / "Ex h.10. à m.n." ("from hour N reckoned from noon" / "from hour N reckoned from midnight"). Within each block:

- an **H.** (hour) heading with the hour value,
 - an **Arcus.** column of zodiac sign-group labels (♈; the group ♈ ♉ ♊ ♋; the group ♌ ♍ ♎ ♏; etc.),
 - **G. | M.** (degrees | minutes) value columns, several rows reading **90 | 0**,
 - a "**versus mer.**" ("toward the meridian") annotation on each block.
-

Block 1 — base hours 5 (from noon) / 7 (from midnight):

	1 (Meri d.)	2	3	4	5	6	7	8	9	10	11	12	13
Signa													
☿ ♉	90° 0'	72° 44'	58° 21'	47° 40'	40° 2'	34° 36'	30° 43'	27° 54'	25° 52'	24° 27'	23° 30'	22° 58'	22° 47'
♈ ♉	90° 0'	69° 50'	53° 56'	42° 53'	35° 24'	30° 17'	26° 41'	24° 8'	22° 19'	21° 2'	20° 12'	19° 43'	19° 34'
♈ ♉	90° 0'	56° 24'	37° 13'	27° 11'	21° 27'	17° 53'	15° 32'	13° 55'	12° 47'	12° 1'	11° 30'	11° 13'	11° 7'

Block 2 — base hours 4 (from noon) / 8 (from midnight):

	1 (Meri d.)	2	3	4	5	6	7	8	9	10	11	12	13
Signa													
☿ ♉	90° 0'	70° 53'	56° 52'	44° 32'	36° 59'	31° 44'	28° 2'	25° 23'	23° 30'	22° 10'	21° 18'	20° 48'	20° 38'
♈ ♉	90° 0'	67° 43'	50° 55'	39° 47'	32° 30'	27° 38'	24° 15'	21° 53'	20° 12'	19° 9'	18° 15'	17° 49'	17° 40'
♈ ♉	90° 0'	53° 28'	34° 15'	24° 43'	19° 25'	16° 9'	14° 0'	12° 31'	11° 30'	10° 48'	10° 20'	10° 5'	10° 0'

Block 3 — base hours 3 (from noon) / 9 (from midnight):

	1 (Meri d.)	2	3	4	5	6	7	8	9	10	11	12	13
Signa													
☿ ♉	90° 0'	67° 0'	49° 45'	38° 47'	31° 35'	26° 48'	23° 30'	21° 11'	19° 33'	18° 24'	17° 39'	17° 14'	17° 5'
♈ ♉	90° 0'	63° 21'	45° 9'	34° 13'	27° 29'	23° 8'	20° 12'	18° 9'	16° 43'	15° 44'	15° 4'	14° 42'	14° 35'
♈ ♉	90° 0'	47° 47'	29° 4'	20° 36'	16° 3'	13° 18'	11° 30'	10° 17'	9° 26'	8° 51'	8° 28'	8° 15'	8° 11'

Block 4 — base hours 2 (from noon) / 10 (from midnight):

Signa	1 (Merid.)	2	3	4	5	6	7	8	9	10	11	12	13
♋ ♊	90° 0'	59° 1'	40° 2'	29° 36'	23° 30'	19° 39'	17° 5'	15° 19'	14° 6'	13° 15'	12° 41'	12° 22'	12° 16'
♋ ♊	90° 0'	54° 39'	35° 24'	25° 41'	20° 12'	16° 49'	14° 35'	13° 3'	12° 0'	11° 16'	10° 47'	10° 31'	10° 25'
♈ ♎	90° 0'	37° 56'	21° 27'	18° 53'	11° 30'	9° 29'	8° 11'	7° 18'	6° 42'	6° 17'	6° 1'	5° 51'	5° 48'

III. TABVLA VNIVERSALIS. — III. Universal Table (p. 92, lower part)

Title (verbatim): III. TABVLA VNIVERSALIS.

Subtitle (verbatim): *Arcus Eclipticæ (posito princip. ♋, vel ♊, in Meridiano supra Horizōtē) inter Meridianū, & circulos maximos ex sectione Meridiani cū Æquatore, per hor. à mer. vel med. noc. in signorū initijs ductos.*

Translation of the subtitle: The arc of the ecliptic (given that the beginning of Cancer, or of Capricorn, lies on the meridian above the horizon), intercepted between the meridian and the great circles drawn from the intersection of the meridian with the equator, at the hours from noon or midnight, at the beginnings of the signs.

The argument is given both as hours from noon and (its complement) as hours from midnight. The three value-groups are the three declination-classes of sign-beginnings: ♋ ♊; the group ♋ ♊ ♊ ♊; and the group ♈ ♎ ♎ ♎. (Aries and Libra are omitted — see the note below.)

Hor. à merid.	Hor. à med. noc.	♋ ♊ — G.	♋ ♊ ♊ ♊ — G.	♈ ♎ ♎ ♎ — G.
½	11½	6	8	14
1	11	13	15	26
1½	10½	19	22	36
2	10	24	28	44

Hor. à merid.	Hor. à med. noc.	☿ ♊ — G.	♈ ♈ ☊ ☋ — G.	♎ ♏ ♍ — G.
2½	9½	29	33	50
3	9	32	37	54
3½	8½	36	40	57
4	8	38	43	59
4½	7½	40	45	61
5	7	41	46	62
4½ [recte: 5½]	6½	42	47	62
6	6	42	47	62

Note beneath the table (verbatim): *Sole existente in principio ♈, vel ♎, hi arcus semper sunt gr. 90. ideoque necesse non fuit, vt ♈, & ♎, inter signa ponerentur.*

Translation: When the Sun is at the beginning of Aries or of Libra, these arcs are always 90°; and therefore it was not necessary that Aries and Libra be placed among the signs.

IV. TABVLA. — IV. Table (p. 93)

Title (verbatim): IV. TABVLA.

Subtitle (verbatim): *Arcus semidiurnos pro initijs signorum complectens ad omnes eleuationes poli computata, in quibus spatio 24. horarum vicissitudo dierum ac noctium cernitur.*

Translation of the subtitle: Comprising the semidiurnal arcs for the beginnings of the signs, computed for all elevations of the pole, in which the alternation of days and nights within the span of 24 hours is seen.

Argument: **Altitud. Poli** (elevation of the pole), in G. | M., from 0° 0' to 14° 45' in 15' steps. The seven sign-columns pair signs equidistant from the solstices; the Libra/Aries column is 90° 0' throughout (equinox). *(The table continues beyond this page — the argument stops at 14° 45' with no closing catchword.)*

Altit. Poli G.	☿ G.	♈/♈ G.	♎/♎ G.	♊/♊ G.	♏/♏ G.	♈/♈ G.	♊ G.
0	90	90	90	90	90	90	90
0	90	90	90	90	89	89	89

Altit. Poli G.	ᄒ G.	ᄒ/II G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ G.
0	90	90	90	90	89	89	89
0	90	90	90	90	89	89	89
1	90	90	90	90	89	89	89
1	90	90	90	90	89	89	89
1	90	90	90	90	89	89	89
1	90	90	90	90	89	89	89
2	90	90	90	90	89	89	89
2	90	90	90	90	89	89	89
2	91	90	90	90	89	89	88
2	91	91	90	90	89	89	38 [recte: 88]
3	91	91	90	90	89	88	88
3	91	91	90	90	89	88	88
3	91	91	90	90	89	88	88
3	91	91	90	90	89	88	88
4	91	91	90	90	89	88	88
4	91	91	90	90	89	88	88
4	91	91	90	90	89	88	88
4	92	91	90	90	89	88	87
5	92	91	91	90	88	88	87
5	92	91	91	90	88	88	87
5	92	92	91	90	88	87	87
5	92	92	91	90	88	87	87
6	92	92	91	90	88	87	87
6	92	92	91	90	88	87	87
6	92	92	91	90	88	87	87
6	92	92	91	90	88	87	87
7	93	92	91	90	88	87	86
7	93	92	91	90	88	87	86
7	93	92	91	90	88	87	86

Altit. Poli G.	ᄒ G.	ᄒ/II G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ G.
7	93	92	91	90	88	87	86
8	93	92	91	90	88	87	86
8	93	93	91	90	88	86	86
8	93	93	91	90	88	86	86
8	93	93	91	90	88	86	86
9	93	93	91	90	88	86	86
9	94	93	91	90	88	86	85
9	94	93	91	90	88	86	85
9	94	93	92	90	88	86	85
10	94	93	92	90	87	86	85
10	94	93	92	90	87	86	85
10	94	93	92	90	87	86	85
10	94	94	92	90	87	86	85
11	94	94	92	90	87	85	85
11	94	94	92	90	87	85	85
11	95	94	92	90	87	85	84
11	95	94	92	90	87	85	84
12	95	94	92	90	87	85	84
12	95	94	92	90	87	85	84
12	95	94	92	90	87	85	84
12	95	94	92	90	87	85	84
13	95	94	92	90	87	85	84
13	95	94	92	90	87	85	84
13	95	95	92	90	87	84	84
13	96	95	92	90	87	84	83
14	96	95	92	90	87	84	83
14	96	95	92	90	87	84	83
14	96	95	93	90	86	84	83
14	96	95	93	90	86	84	82 [recte: 83]

IV. TABLE (continued) — Semidiurnal arcs for the beginnings of the signs [*Arcus semidiurni pro initijs signorum*]

This opening continues Table IV (begun on the preceding page), which gives the **semidiurnal arc** [*arcus semidiurnus*] — half the length of the day, measured in degrees of the equator — for the beginning (0°) of each zodiacal sign, tabulated against the **elevation of the pole** [*Altitudo Poli*], i.e. the geographic latitude. The argument runs here from 15°0' to 59°45' of pole-elevation, in steps of 15', across three printed pages (pp. 94–96).

Each sign-column heading pairs the two signs that share the same semidiurnal arc (their starting points have equal declination). The middle column (Libra/Aries, the equinoctial beginnings) is a constant 90°0'. The paired columns on either side of the solstitial columns are mutual complements to 180° (e.g. Cancer + Capricorn = 180°0'), a relation that has been used here only to cross-verify the transcription of every cell, not to supply any value.

Column headings (verbatim glyphs): **Altitud. Poli.** | ☿ (Cancer) | ♊ / ♈ (Leo / Gemini) | ♍ / ♉ (Virgo / Taurus) | ♎ / ♈ (Libra / Aries) | ♏ / ♋ (Scorpio / Pisces) | ♐ / ♒ (Sagittarius / Aquarius) | ♑ (Capricorn). Each is subdivided into **G.** (degrees) and **M.** (minutes).

Page 94 (pole-elevation 15°0' – 29°45')

Poli			♍/♉	♎/♈	♏/♋	♐/♒	♑
G.	☿ G.	♊/♈ G.	G.	G.	G.	G.	G.
15	96	95	93	90	86	84	83
15	96	95	93	90	86	84	83
15	96	95	93	90	86	84	83
15	97	95	93	90	86	84	82
16	97	96	93	90	86	83	82
16	97	96	93	90	86	83	82
16	97	96	93	90	86	83	82
16	98 [recte: 97]	96	93	90	86	83	82
17	98 [recte:	96	93	90	86	83	82

Poli G.	ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ G.
	97]						
17	98 [recte: 97]	95 [recte: 96]	93	90	86	83	82
17	98 [recte: 97]	96	93	90	86	83	82
17	98	96	93	90	86	83	82
18	98	96	93	90	86	83	81
18	98	96	93	90	86	83	81
18	98	97	93	90	86	82	81
18	98	97	93	90	86	82	81
19	98	97	94	90	85	82	81
19	98	97	94	90	85	82	81
19	98	97	94	90	85	82	81
19	98	97	94	90	85	82	81
20	99	97	94	90	85	82	80
20	99	97	94	90	85	82	80
20	99	97	94	90	85	82	80
20	99	98	94	90	85	82	80
21	99	98	94	90	85	81	80
21	99	98	94	90	85	81	80
21	99	98	94	90	85	81	80
21	99	98	94	90	85	81	80
22	100	98	94	90	85	81	79
22	100	98	94	90	85	81	79
22	100	98	94	90	85	81	79
22	100	98	94	90	85	81	79
23	100	98	94	90	85	81	79
23	100	99	95	90	85	80	79
23	100	99	95	90	84	80	79
23	101	99	95	90	84	80	78
24	101	99	95	90	84	80	78

Poli G.	፩ G.	፪/፫ G.	፬/፭ G.	፮/፯ G.	፰/፱ G.	፳/፳፭ G.	፵፭
24	101	99	95	90	84	80	78
24	101	99	95	90	84	80	78
24	101	99	95	90	84	80	78
25	101	99	95	90	84	80	78
25	101	100	95	90	84	80	78
25	101	100	95	90	84	79	78
25	102	100	95	90	84	79	77
26	102	100	95	90	84	79	77
26	102	100	95	90	84	79	77
26	102	100	95	90	84	79	77
26	102	100	95	90	84	79	77
27	102	100	95	90	84	79	77
27	102	100	96	90	83	79	77
27	103	101	96	90	83	78	76
27	103	101	96	90	83	78	76
28	103	101	96	90	83	78	76
28	103	101	96	90	83	78	76
28	103	101	96	90	83	78	76
28	103	101	96	90	83	78	76
29	103	101	96	90	83	78	76
29	104	101	96	90	83	78	75
29	104	102	96	90	83	77	75
29	104	102	96	90	83	77	75

Page 95 (pole-elevation 30°0' – 44°45')

Poli G.	፩ G.	፪/፫ G.	፬/፭ G.	፮/፯ G.	፰/፱ G.	፳/፳፭ G.	፵፭
30	104	102	96	90	83	77	75
30	104	102	96	90	83	77	75
30	104	102	96	90	83	77	75
30	104	102	96	90	83	77	75

Poli G.		ϱ/Π G.	η/ϑ G.	$\underline{\alpha}/\gamma$ G.	$\mathfrak{M}/\mathfrak{K}$ G.	$\mathfrak{A}/\mathfrak{Z}$ G.	$\mathfrak{y}$ G.
31		105	102	97	90	82	74
31		105	102	97	90	82	74
31		105	103	97	90	82	74
31		105	103	97	90	82	74
32		105	103	97	90	82	74
32		105	103	97	90	82	74
32		106	103	97	90	82	73
32		106	103	97	90	82	73
33		106	103	97	90	82	73
33		106	103	97	90	82	73
33		106	104	97	90	82	73
33		106	104	97	90	82	73
34		107	104	97	90	82	72
34		107	104	97	90	82	72
34		107	104	98	90	81	72
34		107	104	98	90	81	72
35	170 [recte: 107]		104	98	90	81	72
35		107	105	98	90	81	72
35		108	105	98	90	81	71
35		108	105	98	90	81	71
36		108	105	98	90	81	71
36		108	105	98	90	81	71
36		108	105	98	90	81	71
36		108	105	98	90	81	71
37		109	106	98	90	81	70
37		109	106	98	90	81	70
37		109	106	98	90	81	70
37		109	106	99	90	80	70
38	110 [recte: 109]		106	99	90	80	70

Poli G.	ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ G.
38	110	106	99	90	80	73	69
38	110	107	99	90	80	72	69
38	110	107	99	90	80	72	69
39	110	107	99	90	80	72	69
39	110	107	99	90	80	72	69
39	111	107	99	90	80	72	69
39	111	107	99	90	80	72	68
40	111	107	99	90	80	72	68
40	111	108	99	90	80	71	68
40	111	108	100	90	80	71	68
40	112	108	100	90	79	71	68
41	112	108	100	90	79	71	67
41	112	108	100	90	79	71	67
41	112	109	100	90	79	71	67
41	112	109	100	90	79	70	67
42	113	109	100	90	79	70	66
42	113	109	100	90	79	70	66
42	113	109	100	90	79	70	66
42	113	109	100	90	79	70	66
43	113	110	100	90	79	69	66
43	114	110	101	90	78	69	65
43	114	110	101	90	78	69	65
43	114	110	101	90	78	69	65
44	114	110	101	90	78	69	65
44	115	111	101	90	78	69	64
44	115	111	101	90	78	68	64
44	115	111	101	90	78	68	64

Page 96 (pole-elevation 45°0' – 59°45')

Poli G.	ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ G.
45	115	111	101	90	78	68	64
45	116	111	101	90	78	68	63
45	116	111	101	90	78	68	63
45	116	112	102	90	77	67	63
46	116	112	102	90	77	67	63
46	117	112	102	90	77	67	62
46	117	112	102	90	77	67	62
46	117	113	102	90	77	66	62
47	117	113	102	90	77	66	62
47	118	113	102	90	77	66	61
47	118	113	102	90	77	56 [recte: 66]	61
47	118	113	102	90	77	66	61
48	118	114	103	90	76	65	61
48	119	114	103	90	76	65	60
48	119	114	103	90	76	65	60
48	119	114	103	90	76	65	60
49	120	115	103	90	76	64	59
49	120	115	103	90	76	64	59
49	120	115	103	90	76	64	59
49	120	115	103	90	76	64	59
50	121	116	104	90	75	64	58
50	121	116	104	90	75	63	58
50	121	116	104	90	75	63	58
50	122	116	104	90	75	63	57
51	122	117	104	90	75	62	57
51	122	117	104	90	75	62	57
51	123	117	104	90	75	62	56
51	123	117	104	90	75	62	56

Poli G.	ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ/ᄒ G.	ᄒ G.
52	123	118	105	90	74	61	56
52	124	118	105	90	74	61	55
52	124	118	105	90	74	61	55
52	124	118	105	90	74	61	55
53	125	119	105	90	74	60	54
53	125	119	105	90	74	60	54
53	125	119	105	90	74	60	54
53	126	120	106	90	73	59	53
54	126	120	106	90	73	59	53
54	127	120	106	90	73	59	52
54	127	121	106	90	73	58	52
54	127	121	106	90	73	58	52
55	128	121	106	90	73	58	51
55	128	122	107	90	72	57	51
55	129	122	107	90	72	57	50
55	129	122	107	90	72	57	50
56	130	123	107	90	72	56	49
56	130	123	107	90	72	56	49
56	131	123	107	90	72	56	48
56	131	124	108	90	71	55	48
57	132	124	108	90	71	55	47
57	132	124	108	90	71	55	47
57	133	125	108	90	71	54	46
57	133	125	108	90	71	54	46
58	134	126	109	90	71	53	45
58	134	126	109	90	70	53	45
58	135	126	109	90	70	53	44
58	135	127	109	90	70	52	44
59	136	127	109	90	70	52	43
59	136	128	109	90	70	51	43

Poli	♋						♏
G.	G.	♌/♊ G.	♍/♉ G.	♎/♍ G.	♏/♎ G.	♐/♏ G.	G.
59	137	128	110	90	69	51	42
59	138	129	110	90	69	50	41

IV. TABVLA (continued) — Semidiurnal arcs for the beginnings of the signs

(Table IV, "containing the semidiurnal arcs for the beginnings of the signs," runs continuously from the preceding pages; the argument began at 15°0' of pole-elevation. This page carries its final block, pole-elevation 60°0' through 66°30', where the table ends.)

Column order and pairing follow the printed headings: **Altitud. Poli.** (argument = pole-elevation), then **♋** (Cancer); **♌/♊** (Leo/Gemini); **♍/♉** (Virgo/Taurus); **♎/♍** (Libra/Aries — constant 90°0'); **♏/♎** (Scorpio/Pisces); **♐/♏** (Sagittarius/Aquarius); **♑** (Capricorn). Each sign-column carries **G.** (degrees) and **M.** (minutes) sub-heads; each cell below is given as "degrees minutes."

Altitud. Poli.	♋	♌ / ♊	♍ / ♉	♎ / ♍	♏ / ♎	♐ / ♏	♑
60 0	138 52	129 35	110 38	90 0	69 22	50 25	41 8
60 15	139 32	130 4	110 51	90 0	69 9	49 56	40 28
60 30	140 13	130 34	111 5	90 0	68 55	49 26	39 47
60 45	140 56	131 4	111 18	90 0	68 42	48 56	39 4
61 0	141 40	131 35	111 32	90 0	68 28	48 25	38 20
61 15	142 25	132 7	111 46	90 0	68 14	47 53	37 35
61 30	143 12	132 40	112 1	90 0	67 59	47 20	36 48
61 45	144 1	133 13	112 15	90 0	67 45	46 47	35 59
62 0	144	133 47	112 30	90 0	67 30	46 13	35

Altitud. Poli.	☾	♊ / ♋	♏ / ♍	♌ / ♍	♍ / ♋	♏ / ♏	♏
	52						8
62 15	145 45	134 22	112 45	90 0	67 15	45 38	34 15
62 30	146 39	134 58	113 1	90 0	66 59	45 2	33 21
62 45	147 35	135 35	113 15	90 0	66 44	44 25	32 25
63 0	148 35	136 13	113 32	90 0	66 28	43 47	31 25
63 15	149 37	136 52	113 48	90 0	66 12	43 8	30 23
63 30	150 42	137 33	114 5	90 0	65 55	42 27	29 18
63 45	151 51	138 15	114 22	90 0	65 38	41 45	28 9
64 0	153 3	138 58	114 39	90 0	65 21	41 2	26 57
64 15	154 21	139 42	114 57	90 0	65 3	40 18	25 39
64 30	155 44	140 28	115 15	90 0	64 45	39 32	24 16
64 45	157 13	141 16	115 33	90 0	64 27	38 44	22 47
65 0	158 49	142 6	115 52	90 0	64 8	37 54	21 11
65 15	160 35	142 57	116 11	90 0	63 49	37 3	19 25
65 30	162 35	143 50	116 31	90 0	63 29	36 10	17 25
65 45	164 51	144 46	116 51	90 0	63 9	35 14	15 9
66 0	167 35	145 44	117 11	90 0	62 49	34 16	12 25
66 15	171	146 44	117 32	90 0	62 28	33 16	8

Altitud. Poli.	♊	♈ / ♊	♏ / ♎	♍ / ♋	♌ / ♈	♊ / ♎	♏
	11						49
66 30	180	147 48	117 54	90 0	62 6	32 12	0 0
	0						

V. TABVLA — Rising and setting amplitudes

V. TABVLA. Containing the rising and setting amplitudes at the beginnings of the signs, for the pole-elevations set out in the left-hand column. (*Amplitudines ortivæ occiduæque in Signorum initijs ad Poli altitudines in sinistro latere descriptas.*)

On the printed page this table is laid out as three side-by-side panels (pole-elevation 30°–39½°, 40°–49½°, 50°–60°), each with identical columns; they are merged here into one continuous table, argument 30° to 60° in half-degree steps. Each amplitude column groups the signs of equal declination-magnitude (southern signs above, northern signs below in the printed heading), because rising/setting amplitude depends only on declination:

- ♏ / ♊ (Capricorn / Cancer) — declination 23°30′
- ♎ ♊ / ♊ ♏ (Aquarius, Sagittarius / Gemini, Leo) — declination 20°12′
- ♌ ♈ / ♎ ♏ (Scorpio, Pisces / Taurus, Virgo) — declination 11°30′

Values are "degrees minutes."

Altit. Poli.	♏ / ♊	♎ ♊ / ♊ ♏	♌ ♈ / ♎ ♏
30	27 25	23 30	13 19
30½	27 34	23 38	13 23
31	27 43	23 45	13 27
31½	27 53	23 53	13 31
32	28 3	24 2	13 36
32½	28 13	24 10	13 40
33	28 24	24 19	13 45
33½	28 34	24 28	13 50
34	28 45	24 37	13 55
34½	28 56	24 46	14 0
35	29 8	24 56	14 5
35½	29 20	25 6	14 11
36	29 32	25 16	14 16

Altit. Poli.	ལོ / ང	ཟླ / ཡོད	ལོ་མ / ལོ་མ
36½	29 44	25 26	14 22
37	29 57	25 37	14 27
37½	30 10	25 48	14 33
38	30 24	25 59	14 39
38½	30 38	26 11	14 46
39	30 52	26 23	14 52
39½	31 7	26 35	14 58
40	31 22	26 48	15 5
40½	31 38	27 0	15 12
41	31 54	27 14	15 19
41½	32 10	27 27	15 26
42	32 27	27 41	15 34
42½	32 44	27 56	15 41
43	33 2	28 10	15 49
43½	33 21	28 26	15 57
44	33 40	28 41	16 5
44½	33 59	28 57	16 14
45	34 20	29 14	16 23
45½	34 40	29 31	16 32
46	35 2	29 48	16 41
46½	35 24	30 6	16 50
47	35 47	30 25	17 0
47½	36 10	30 45	17 10
48	36 35	31 4	17 20
48½	37 0	31 24	17 31
49	37 26	31 45	17 42
49½	37 53	32 7	17 53
50	38 20	32 30	18 4
50½	38 49	32 53	18 16
51	39 19	33 17	18 28
51½	39 50	33 41	18 41

Altit. Poli.	♊ / ♋	♈ / ♉	♌ / ♍
52	40 22	34 7	18 54
52½	40 55	34 33	19 7
53	41 30	35 1	19 21
53½	42 6	35 29	19 35
54	42 43	35 59	19 50
54½	43 22	36 29	20 5
55	44 3	37 1	20 20
55½	44 45	37 34	20 36
56	45 29	38 8	20 53
56½	46 15	38 44	21 10
57	47 4	39 21	21 28
57½	47 55	39 59	21 47
58	48 48	40 40	22 6
58½	49 45	41 22	22 26
59	50 44	42 6	22 46
59½	51 47	42 52	23 8
60	52 54	43 41	23 30

Note (below the table): These amplitudes are northern in the northern signs, and southern in the southern signs. But when the Sun stands at the beginning of Aries (♈) and Libra (♎), there is no rising or setting amplitude; and so the signs Aries and Libra have been omitted from among the other signs.

VI. TABVLA — Arc of the Horizon / of the prime Vertical

VI. TABVLA. The arc of the Horizon [and of the prime Vertical] between the intersection of the equator with the Horizon (or with the circle of the 6th hour) and the circles of the other hours reckoned from noon and from midnight, computed for all elevations of the pole. (*Arcus Horizontis Verticalis primarij inter æquatoris intersectionem cum Horizonte, vel cum circulo horæ 6, & inter circulos aliarum horarum à meridie, ac med.noc. continens, ad omnes poli eleuationes supputata.*)

[Margin (left, running vertically): Altitudines poli pro arcubus Horizontis. — "Pole-elevations, for the arcs of the Horizon."] [Margin (right, running vertically):

Altitudines Poli pro arcubus Verticalis primarij. — "Pole-elevations, for the arcs of the prime Vertical."]

The argument (leftmost column, printed under the spanning label "**Horæ à &**") is the pole-elevation in degrees and minutes, running 0°0' to 29°45' in 15' steps (continuous across the printed pages 98–99). The remaining columns are the hour-circles, each headed by two hour-reckonings — the upper measured from noon (12, 12¼, 12½, 12¾, 1, 1¼ — abbreviated on the page as **12**, ¼, ½, ¾, **1**, 1¼) and the lower its mirror reckoned from midnight (**12**, 11¾, 11½, 11¼, **11**, 10¾). The final column is headed **meridie / med. noc.** (the meridian/midnight circle; its value = 90° – pole-elevation). Each cell is "degrees minutes."

Altit. Poli.	12 / 12	¼ / 11¾	½ / 11½	¾ / 11¼	1 / 11	1¼ / 10¾	meridie / med. noc.
0 0	90 0	90 0	90 0	90 0	90 0	90 0	90 0
0 15	90 0	89 59	89 58	89 57	89 56	89 55	89 45
0 30	90 0	89 58	89 56	89 54	89 52	89 50	89 30
0 45	90 0	89 57	89 54	89 51	89 48	89 45	89 15
1 0	90 0	89 56	89 52	89 48	89 44	89 40	89 0
1 15	90 0	89 55	89 50	89 45	89 40	89 35	88 45
1 30	90 0	89 54	89 48	89 42	89 36	89 29	88 30
1 45	90 0	89 53	89 46	89 39	89 32	89 24	88 15
2 0	90 0	89 52	89 44	89 36	89 28	89 19	88 0
2 15	90 0	89 51	89 42	89 33	89 24	89 14	87 45
2 30	90 0	89 50	89 40	89 30	89 20	89 9	87 30
2 45	90 0	89 49	89 38	89 27	89	89 4	87 15

Altit. Poli.	12 / 12	$\frac{1}{4}$ / 11 $\frac{3}{4}$	$\frac{1}{2}$ / 11 $\frac{1}{2}$	$\frac{3}{4}$ / 11 $\frac{1}{4}$	1 / 11	1 $\frac{1}{4}$ / 10 $\frac{3}{4}$	meridie / med. noc.
					16		
3 0	90 0	89 48	89 36	89 24	89	88 59	87 0
					12		
3 15	90 0	89 47	89 34	89 21	89	88 54	86 45
					8		
3 30	90 0	89 46	89 32	89 18	89	88 49	86 30
					4		
3 45	90 0	89 45	89 30	89 15	89	88 44	86 15
					0		
4 0	90 0	89 44	89 28	89 12	88	88 39	86 0
					56		
4 15	90 0	89 43	89 26	89 9	88	88 34	85 45
					52		
4 30	90 0	89 42	89 24	89 6	88	88 28	85 30
					48		
4 45	90 0	89 41	89 23	89 3	88	88 23	85 15
					44		
5 0	90 0	89 40	89 21	89 0	88	88 18	85 0
					40		
5 15	90 0	89 39	89 19	88 57	88	88 13	84 45
					36		
5 30	90 0	89 38	89 17	88 54	88	88 8	84 30
					32		
5 45	90 0	89 37	89 15	88 51	88	88 3	84 15
					28		
6 0	90 0	89 36	89 13	88 49	88	87 58	84 0
					24		
6 15	90 0	89 35	89 11	88 46	88	87 53	83 45
					20		
6 30	90 0	89 34	89 9	88 43	88	87 48	83 30
					16		
6 45	90 0	89 34	89 7	88 40	88	87 43	83 15
					12		

Altit. Poli.	12 / 12	$\frac{1}{4}$ / 11 $\frac{3}{4}$	$\frac{1}{2}$ / 11 $\frac{1}{2}$	$\frac{3}{4}$ / 11 $\frac{1}{4}$	1 / 11	1 $\frac{1}{4}$ / 10 $\frac{3}{4}$	meridie / med. noc.
7 0	90 0	89 33	89 5	88 37	88 8	87 38	83 0
7 15	90 0	89 32	89 3	88 34	88 4	87 33	82 45
7 30	90 0	89 31	89 1	88 31	88 0	87 28	82 30
7 45	90 0	89 30	88 59	88 28	87 56	87 23	82 15
8 0	90 0	89 29	88 57	88 25	87 52	87 18	82 0
8 15	90 0	89 28	88 55	88 22	87 48	87 13	81 45
8 30	90 0	89 27	88 53	88 19	87 44	87 8	81 30
8 45	90 0	89 26	88 51	88 16	87 40	87 3	81 15
9 0	90 0	89 25	88 49	88 13	87 36	86 58	81 0
9 15	90 0	89 24	88 47	88 10	87 32	86 53	80 45
9 30	90 0	89 23	88 45	88 7	87 28	86 48	80 30
9 45	90 0	89 22	88 43	88 4	87 24	86 43	80 15
10 0	90 0	89 21	88 41	88 1	87 20	86 38	80 0
10 15	90 0	89 20	88 39	87 58	87 16	86 33	79 45
10 30	90 0	89 19	88 38	87 55	87 12	86 28	79 30
10 45	90 0	89 18	88 36	87 53	87 8	86 23	79 15
11 0	90 0	89 17	88 34	87 50	87	86 18	79 0

Altit. Poli.	12 / 12	$\frac{1}{4}$ / 11 $\frac{3}{4}$	$\frac{1}{2}$ / 11 $\frac{1}{2}$	$\frac{3}{4}$ / 11 $\frac{1}{4}$	1 / 11	1 $\frac{1}{4}$ / 10 $\frac{3}{4}$	meridie / med. noc.
					4		
11 15	90 0	89 16	88 32	87 47	87 0	86 13	78 45
11 30	90 0	89 15	88 30	87 44	86 57	86 8	78 30
11 45	90 0	89 14	88 28	87 41	86 53	86 3	78 15
12 0	90 0	89 13	88 26	87 38	86 49	85 58	78 0
12 15	90 0	89 12	88 24	87 35	86 45	85 53	77 45
12 30	90 0	89 11	88 22	87 32	86 41	85 48	77 30
12 45	90 0	89 10	88 20	87 29	86 37	85 43	77 15
13 0	90 0	89 9	88 18	87 26	86 33	85 38	77 0
13 15	90 0	89 8	88 16	87 23	86 29	85 33	76 45
13 30	90 0	89 7	88 14	87 20	86 25	85 28	76 30
13 45	90 0	89 7	88 12	87 18	86 21	85 23	76 15
14 0	90 0	89 6	88 11	87 15	86 17	85 18	76 0
14 15	90 0	89 5	88 9	87 12	86 14	85 13	75 45
14 30	90 0	89 4	88 7	87 9	86 10	85 9	75 30
14 45	90 0	89 3	88 5	87 6	86 6	85 4	75 15
15 0	90 0	89 2	88 3	87 3	86 2	84 59	75 0

Altit. Poli.	12 / 12	$\frac{1}{4}$ / 11 $\frac{3}{4}$	$\frac{1}{2}$ / 11 $\frac{1}{2}$	$\frac{3}{4}$ / 11 $\frac{1}{4}$	1 / 11	1 $\frac{1}{4}$ / 10 $\frac{3}{4}$	meridie / med. noc.
15 15	90 0	89 1	88 1	87 0	85 58	84 54	74 45
15 30	90 0	89 0	87 59	86 57	85 54	84 49	74 30
15 45	90 0	88 59	87 57	86 55	85 50	84 44	74 15
16 0	90 0	88 58	87 55	86 52	85 47	84 39	74 0
16 15	90 0	88 57	87 53	86 49	85 43	84 34	73 45
16 30	90 0	88 56	87 51	86 46	85 39	84 30	73 30
16 45	90 0	88 55	87 50	86 43	85 35	84 25	73 15
17 0	90 0	88 54	87 48	86 40	85 31	84 20	73 0
17 15	90 0	88 53	87 46	86 38	85 27	84 15	72 45
17 30	90 0	88 52	87 44	86 35	85 24	84 10	72 30
17 45	90 0	88 51	87 42	86 32	85 20	84 6	72 15
18 0	90 0	88 50	87 40	86 29	85 16	84 1	72 0
18 15	90 0	88 49	87 38	86 26	85 12	83 56	71 45
18 30	90 0	88 49	87 36	86 23	85 8	83 51	71 30
18 45	90 0	88 48	87 35	86 20	85 5	83 46	71 15
19 0	90 0	88 47	87 33	86 18	85 1	83 42	71 0
19 15	90 0	88 46	87 31	86 15	84	83 37	70 45

Altit. Poli.	12 / 12	$\frac{1}{4}$ / $11\frac{3}{4}$	$\frac{1}{2}$ / $11\frac{1}{2}$	$\frac{3}{4}$ / $11\frac{1}{4}$	1 / 11	$1\frac{1}{4}$ / $10\frac{3}{4}$	meridie / med. noc.
					57		
19 30	90 0	88 45	87 29	86 12	84	83 32	70 30
					53		
19 45	90 0	88 44	87 27	86 9	84	83 27	70 15
					50		
20 0	90 0	88 43	87 25	86 6	84	83 23	70 0
					46		
20 15	90 0	88 42	87 23	86 4	84	83 18	69 45
					42		
20 30	90 0	88 41	87 21	86 1	84	83 13	69 30
					38		
20 45	90 0	88 40	87 20	85 58	84	83 9	69 15
					35		
21 0	90 0	88 39	87 18	85 55	84	83 4	69 0
					31		
21 15	90 0	88 38	87 16	85 53	84	82 59	68 45
					27		
21 30	90 0	88 37	87 14	85 50	84	82 55	68 30
					24		
21 45	90 0	88 36	87 12	85 47	84	82 50	68 15
					20		
22 0	90 0	88 36	87 11	85 44	84	82 45	68 0
					15		
22 15	90 0	88 35	87 9	85 42	84	82 41	67 45
					12		
22 30	90 0	88 34	87 7	85 39	84	82 36	67 30
					9		
22 45	90 0	88 33	87 5	85 36	84	82 31	67 15
					5		
23 0	90 0	88 32	87 3	85 33	84	82 27	67 0
					1		
23 15	90 0	88 31	87 2	85 31	83	82 22	66 45
					58		

Altit. Poli.	12 / 12	$\frac{1}{4}$ / 11 $\frac{3}{4}$	$\frac{1}{2}$ / 11 $\frac{1}{2}$	$\frac{3}{4}$ / 11 $\frac{1}{4}$	1 / 11	1 $\frac{1}{4}$ / 10 $\frac{3}{4}$	meridie / med. noc.
23 30	90 0	88 30	87 0	85 28	83 54	82 18	66 30
23 45	90 0	88 39 [recte: 88 29]	86 58	85 25	83 50	82 13	66 15
24 0	90 0	88 28	86 56	85 22	83 47	82 8	66 0
24 15	90 0	88 27	86 54	85 20	83 43	82 4	65 45
24 30	90 0	88 27	86 53	85 17	83 39	81 59	65 30
24 45	90 0	88 26	86 51	85 14	83 36	81 55	65 15
25 0	90 0	88 25	86 49	85 12	83 32	81 50	65 0
25 15	90 0	88 24	86 47	85 9	83 29	81 46	64 45
25 30	90 0	88 23	86 45	85 6	83 25	81 41	64 30
25 45	90 0	88 22	86 44	85 4	83 22	81 37	64 15
26 0	90 0	88 21	86 42	85 1	83 18	81 32	64 0
26 15	90 0	88 20	86 40	84 58	83 14	81 28	63 45
26 30	90 0	88 19	86 38	84 56	83 11	81 23	63 30
26 45	90 0	88 19	86 37	84 53	83 7	81 19	63 15
27 0	90 0	88 18	86 35	84 50	83 4	81 14	63 0
27 15	90 0	88 17	86 33	84 48	83 0	81 10	62 45
27 30	90 0	88 16	86 31	84 45	82	81 6	62 30

Altit.	12 /		$\frac{1}{2}$ /	$\frac{3}{4}$ /	1 /	$1\frac{1}{4}$ /	meridie / med.
Poli.	12	$\frac{1}{4}$ / $11\frac{3}{4}$	$11\frac{1}{2}$	$11\frac{1}{4}$	11	$10\frac{3}{4}$	noc.
					57		
27 45	90 0	88 15	86 30	84 43	82	81 1	62 15
					53		
28 0	90 0	88 14	86 28	84 40	82	80 57	62 0
					49		
28 15	90 0	88 13	86 26	84 37	82	80 52	61 45
					46		
28 30	90 0	88 13	86 24	84 35	82	80 48	61 30
					43		
28 45	90 0	88 12	86 23	84 32	82	80 44	61 15
					40		
29 0	90 0	88 11	86 21	84 30	82	80 39	61 0
					36		
29 15	90 0	88 10	86 19	84 27	82	80 35	60 45
					32		
29 30	90 0	88 9	86 17	84 24	82	80 31	60 30
					29		
29 45	90 0	88 8	86 16	84 22	82	80 26	60 15
					26		

[Margin (left, vertical): Altitudines poli pro arcubus Horizontis. — "Altitudes of the pole for the arcs of the Horizon."] [Margin (right, vertical): Altitudines Poli pro arcubus Verticalis primarij. — "Altitudes of the Pole for the arcs of the prime Vertical."]

This opening continues **Table VI** (VI. TABVLA), begun earlier: the arc of the Horizon and of the prime Vertical contained between the equator's intersection with the Horizon (or with the circle of the 6th hour) and the circles of the other hours reckoned from noon and from midnight, computed for every elevation of the pole. No new table rubric is printed on these three pages; the descriptive title stands at the head of the run on the earlier page. The argument column at the left (labelled *Horæ à / &*, with sub-heads *G. M.*) is the **elevation of the pole** in degrees and minutes; it advances here from 30°0' (this batch's first row, seamlessly following 29°45' on the previous page) to 74°45'. The seven value-columns are the successive hour-circles — 12 (i.e. 12/12, the 6-hour circle, constant 90°0'), $\frac{1}{4}$ (12 $\frac{1}{4}$ h / 11 $\frac{3}{4}$ h), $\frac{1}{2}$ (12 $\frac{1}{2}$ / 11 $\frac{1}{2}$), $\frac{3}{4}$ (12 $\frac{3}{4}$ / 11 $\frac{1}{4}$), 1 (1 h / 11 h), $1\frac{1}{4}$ (1 $\frac{1}{4}$ h / 10 $\frac{3}{4}$ h) — and

finally the meridian itself (meridie / med.noc.), whose arc equals 90° minus the pole-elevation. Each cell is a degrees-and-minutes pair. The table continues beyond 74°45' onto the following page.

VI. TABVLA (continued) — Arcus Horizontis (Verticalis primarij)

Column heads as printed (two tiers): row 1 Horæ à | 12 | ¼ | ½ | ¾ | 1 | 1¼ | meridie.; row 2 & | 12 | 11¾ | 11½ | 11¼ | 11 | 10¾ | med.noc.; each column subdivided G. (degrees) M. (minutes).

Alt. poli (G. M.)	12 / 12 (G. M.)	¼ / 11¾ (G. M.)	½ / 11½ (G. M.)	¾ / 11¼ (G. M.)	1 / 11 (G. M.)	1¼ / 10¾ (G. M.)	meridie / med.noc. (G. M.)
30 0	90 0	88 7	86 14	84 19	82 22	80 22	60 0
30 15	90 0	88 7	86 12	84 17	82 19	80 18	59 45
30 30	90 0	88 6	86 11	84 14	82 15	80 14	59 30
30 45	90 0	88 5	86 9	84 12	82 12	80 9	59 15
31 0	90 0	88 4	86 7	84 9	82 8	80 5	59 0
31 15	90 0	88 3	86 6	84 7	82 5	80 1	58 45
31 30	90 0	88 2	86 4	84 4	82 2	79 57	58 30
31 45	90 0	88 1	86 2	84 1	81 58	79 52	58 15
32 0	90 0	88 1	86 1	83 59	81 55	79 48	58 0
32 15	90 0	88 0	85 59	83 56	81 52	79 44	57 45
32 30	90 0	87 59	85 57	83 54	81 49	79 40	57 30
32 45	90 0	87 58	85 56	83 52	81 45	79 36	57 15
33 0	90 0	87 57	85 54	83 49	81 42	79 32	57 0
33 15	90 0	87 57	85 52	83 47	81 39	79 27	56 45
33 30	90 0	87 56	85 51	83 44	81 35	79 23	56 30
33 45	90 0	87 55	85 49	83 42	81 32	79 19	56 15
34 0	90 0	87 54	85 47	83 39	81 29	79 15	56 0
34 15	90 0	87 53	85 46	83 37	81 26	79 11	55 45
34 30	90 0	87 52	85 44	83 34	81 22	79 7	55 30
34 45	90 0	87 52	85 43	83 32	81 19	79 3	55 15
35 0	90 0	87 51	85 41	83 29	81 16	78 59	55 0

Alt. poli (G. M.)	12 / 12 (G. M.)	$\frac{1}{4}$ / $11\frac{3}{4}$ (G. M.)	$\frac{1}{2}$ / $11\frac{1}{2}$ (G. M.)	$\frac{3}{4}$ / $11\frac{1}{4}$ (G. M.)	$\frac{1}{11}$ (G. M.)	$1\frac{1}{4}$ / $10\frac{3}{4}$ (G. M.)	meridie / med.noc. (G. M.)
35 15	90 0	87 50	85 39	83 27	81 13	78 55	54 45
35 30	90 0	87 49	85 38	83 25	81 10	78 51	54 30
35 45	90 0	87 49	85 36	83 22	81 6	78 47	54 15
36 0	90 0	87 48	85 35	83 20	81 3	78 43	54 0
36 15	90 0	87 47	85 33	83 17	81 0	78 39	53 45
36 30	90 0	87 46	85 32	83 15	80 57	78 35	53 30
36 45	90 0	87 45	85 30	83 13	80 54	78 31	53 15
37 0	90 0	87 44	85 28	83 10	80 50	78 27	53 0
37 15	90 0	87 44	85 27	83 8	80 47	78 23	52 45
37 30	90 0	87 43	85 25	83 6	80 44	78 19	52 30
37 45	90 0	87 42	85 23	83 3	80 41	78 16	52 15
38 0	90 0	87 41	85 22	83 1	80 38	78 12	52 0
38 15	90 0	87 41	85 20	82 59	80 35	78 8	51 45
38 30	90 0	87 40	85 19	82 56	80 32	78 4	51 30
38 45	90 0	87 39	85 17	82 54	80 29	78 0	51 15
39 0	90 0	87 38	85 16	82 52	80 26	77 56	51 0
39 15	90 0	87 38	85 14	82 50	80 23	77 53	50 45
39 30	90 0	87 37	85 13	82 47	80 20	77 49	50 30
39 45	90 0	87 36	85 11	82 45	80 17	77 45	50 15
40 0	90 0	87 35	85 10	82 43	80 14	77 41	50 0
40 15	90 0	87 35	85 8	82 41	80 11	77 38	49 45
40 30	90 0	87 34	85 7	82 38	80 8	77 34	49 30
40 45	90 0	87 33	85 5	82 36	80 5	77 30	49 15
41 0	90 0	87 33	85 4	82 34	80 2	77 27	49 0
41 15	90 0	87 32	85 2	82 32	79 59	77 23	48 45
41 30	90 0	87 31	85 1	82 29	79 56	77 19	48 30
41 45	90 0	87 30	84 59	82 27	79 53	77 16	48 15
42 0	90 0	87 29	84 58	82 25	79 50	77 12	48 0
42 15	90 0	87 29	84 56	82 23	79 47	77 9	47 45

Alt. poli (G. M.)	12 / 12 (G. M.)	$\frac{1}{4}$ / $11\frac{3}{4}$ (G. M.)	$\frac{1}{2}$ / $11\frac{1}{2}$ (G. M.)	$\frac{3}{4}$ / $11\frac{1}{4}$ (G. M.)	1 / 11 (G. M.)	$1\frac{1}{4}$ / $10\frac{3}{4}$ (G. M.)	meridie / med.noc. (G. M.)
42 30	90 0	87 28	84 55	82 21	79 44	77 5	47 30
42 45	90 0	87 27	84 54	82 19	79 41	77 1	47 15
43 0	90 0	87 26	84 52	82 16	79 39	76 58	47 0
43 15	90 0	87 26	84 51	82 14	79 36	76 54	46 45
43 30	90 0	87 25	84 49	82 12	79 32	76 51	46 30
43 45	90 0	87 24	84 48	82 10	79 30	76 47	46 15
44 0	90 0	87 24	84 46	82 8	79 27	76 44	46 0
44 15	90 0	87 23	84 45	82 6	79 25	76 40	45 45
44 30	90 0	87 22	84 44	82 4	79 22	76 37	45 30
44 45	90 0	87 21	84 42	82 2	79 19	76 34	45 15

[Margin (left, vertical): Altitudines poli pro arcubus Horizontis.] [Margin (right, vertical): Altitudines Poli pro arcubus Verticalis primarij.]

Alt. poli (G. M.)	12 / 12 (G. M.)	$\frac{1}{4}$ / $11\frac{3}{4}$ (G. M.)	$\frac{1}{2}$ / $11\frac{1}{2}$ (G. M.)	$\frac{3}{4}$ / $11\frac{1}{4}$ (G. M.)	1 / 11 (G. M.)	$1\frac{1}{4}$ / $10\frac{3}{4}$ (G. M.)	meridie / med.noc. (G. M.)
45 0	90 0	87 21	84 41	82 0	79 16	76 30	45 0
45 15	90 0	87 20	84 40	81 58	79 14	76 27	44 45
45 30	90 0	87 19	84 38	81 56	79 11	76 23	44 30
45 45	90 0	87 19	84 37	81 53	79 8	76 20	44 15
46 0	90 0	87 18	84 35	81 51	79 5	76 17	44 0
46 15	90 0	87 17	84 34	81 49	79 3	76 13	43 45
46 30	90 0	87 17	84 33	81 47	79 0	76 10	43 30
46 45	90 0	87 16	84 31	81 55	78 57	76 7	43 15
[recte: 81 45]							
47 0	90 0	87 15	84 30	81 43	78 55	76 3	43 0
47 15	90 0	87 15	84 29	81 41	78 52	76 0	42 45
47 30	90 0	87 14	84 27	81 39	78 50	75 57	42 30
47 45	90 0	87 13	84 26	81 37	78 47	75 54	42 15
48 0	90 0	87 13	84 25	81 35	78 44	75 51	42 0

Alt. poli (G. M.)	12 / 12 (G. M.)	$\frac{1}{4}$ / 11 $\frac{3}{4}$ (G. M.)	$\frac{1}{2}$ / 11 $\frac{1}{2}$ (G. M.)	$\frac{3}{4}$ / 11 $\frac{1}{4}$ (G. M.)	1 / 11 (G. M.)	1 $\frac{1}{4}$ / 10 $\frac{3}{4}$ (G. M.)	meridie / med.noc. (G. M.)
48 15	90 0	87 12	84 23	81 34	78 42	75 47	41 45
48 30	90 0	87 11	84 22	81 32	78 39	75 44	41 30
48 45	90 0	87 11	84 21	81 30	78 37	75 41	41 15
49 0	90 0	87 10	84 20	81 28	78 34	75 38	41 0
49 15	90 0	87 9	84 18	81 26	78 31	75 35	40 45
49 30	90 0	87 9	84 17	81 24	78 29	75 32	40 30
49 45	90 0	87 8	84 16	81 22	78 26	75 29	40 15
50 0	90 0	87 8	84 14	81 20	78 24	75 25	40 0
50 15	90 0	87 7	84 13	81 18	78 22	75 22	39 45
50 30	90 0	87 6	84 12	81 16	78 19	75 19	39 30
50 45	90 0	87 6	84 11	81 15	78 16	75 16	39 15
51 0	90 0	87 5	84 9	81 13	78 14	75 13	39 0
51 15	90 0	87 4	84 8	81 11	78 12	75 10	38 45
51 30	90 0	87 4	84 7	81 9	78 10	75 7	38 30
51 45	90 0	87 3	84 6	81 7	78 7	75 4	38 15
52 0	90 0	87 3	84 5	81 6	78 5	75 2	38 0
52 15	90 0	87 2	84 3	81 4	78 2	74 59	37 45
52 30	90 0	87 1	84 2	81 2	78 0	74 56	37 30
52 45	90 0	87 1	84 1	81 0	77 58	74 53	37 15
53 0	90 0	87 0	84 0	80 59	77 55	74 50	37 0
53 15	90 0	87 0	83 59	80 57	77 53	74 47	36 45
53 30	90 0	86 59	83 58	80 55	77 51	74 44	36 30
53 45	90 0	86 58	83 56	80 53	77 48	74 41	36 15
54 0	90 0	86 58	83 55	80 51	77 46	74 39	36 0
54 15	90 0	86 57	83 54	80 50	77 44	74 36	35 45
54 30	90 0	86 57	83 53	80 48	77 42	74 33	35 30
54 45	90 0	86 56	83 52	80 46	77 39	74 30	35 15
55 0	90 0	86 56	83 51	80 45	77 37	74 28	35 0

Alt. poli (G. M.)	12 / 12 (G. M.)	$\frac{1}{4}$ / 11 $\frac{3}{4}$ (G. M.)	$\frac{1}{2}$ / 11 $\frac{1}{2}$ (G. M.)	$\frac{3}{4}$ / 11 $\frac{1}{4}$ (G. M.)	1 / 11 (G. M.)	1 $\frac{1}{4}$ / 10 $\frac{3}{4}$ (G. M.)	meridie / med.noc. (G. M.)
55 15	90 0	86 55	83 50	80 43	77 35	74 25	34 45
55 30	90 0	86 55	83 48	80 41	77 33	74 22	34 30
55 45	90 0	86 54	83 47	80 40	77 31	74 20	34 15
56 0	90 0	86 53	83 46	80 38	77 29	74 17	34 0
56 15	90 0	86 53	83 45	80 37	77 26	74 14	33 45
56 30	90 0	86 52	83 44	80 35	77 24	74 12	33 30
56 45	90 0	86 52	83 43	80 33	77 22	74 9	33 15
57 0	90 0	86 51	83 42	80 32	77 20	74 7	33 0
57 15	90 0	86 51	83 41	80 30	77 18	74 4	32 45
57 30	90 0	86 50	83 40	80 29	77 16	74 1	32 30
57 45	90 0	86 50	83 39	80 27	77 14	73 59	32 15
58 0	90 0	86 49	83 38	80 26	77 12	73 57	32 0
58 15	90 0	86 49	83 37	80 24	77 10	73 54	31 45
58 30	90 0	86 48	83 36	80 22	77 8	73 52	31 30
58 45	90 0	86 48	83 35	80 21	77 6	73 49	31 15
59 0	90 0	86 47	83 34	80 19	77 4	73 47	31 0
59 15	90 0	86 47	83 33	80 18	77 2	73 44	30 45
59 30	90 0	86 46	83 32	80 16	77 0	73 42	30 30
59 45	90 0	86 46	83 31	80 15	76 58	73 39	30 15

[Margin (left, vertical): Altitudines poli pro arcubus Horizontis.] [Margin (right, vertical): Altitudines Poli pro arcubus Verticalis primarij.]

Alt. poli (G. M.)	12 / 12 (G. M.)	$\frac{1}{4}$ / 11 $\frac{3}{4}$ (G. M.)	$\frac{1}{2}$ / 11 $\frac{1}{2}$ (G. M.)	$\frac{3}{4}$ / 11 $\frac{1}{4}$ (G. M.)	1 / 11 (G. M.)	1 $\frac{1}{4}$ / 10 $\frac{3}{4}$ (G. M.)	meridie / med.noc. (G. M.)
60 0	90 0	86 45	83 30	80 14	76 56	73 37	30 0
60 15	90 0	86 45	83 29	80 12	76 54	73 35	29 45
60 30	90 0	86 44	83 28	80 11	76 52	73 32	29 30
60 45	90 0	86 44	83 27	80 9	76 50	73 30	29 15
61 0	90 0	86 43	83 26	80 8	76 49	73 28	29 0

Alt. poli (G. M.)	12 / 12 (G. M.)	$\frac{1}{4}$ / $11\frac{3}{4}$ (G. M.)	$\frac{1}{2}$ / $11\frac{1}{2}$ (G. M.)	$\frac{3}{4}$ / $11\frac{1}{4}$ (G. M.)	1 / 11 (G. M.)	$1\frac{1}{4}$ / $10\frac{3}{4}$ (G. M.)	meridie / med.noc. (G. M.)
61 15	90 0	86 43	83 25	80 6	76 47	73 26	28 45
61 30	90 0	86 42	83 24	80 5	76 45	73 23	28 30
61 45	90 0	86 42	83 23	80 4	76 43	73 21	28 15
62 0	90 0	86 41	83 22	80 2	76 41	73 19	28 0
62 15	90 0	86 41	83 21	80 1	76 40	73 17	27 45
62 30	90 0	86 40	83 20	80 0	76 38	73 15	27 30
62 45	90 0	86 40	83 19	79 58	76 36	73 12	27 15
63 0	90 0	86 39	83 19	79 57	76 34	73 10	27 0
63 15	90 0	86 39	83 18	79 56	76 33	73 8	26 45
63 30	90 0	86 39	83 17	79 55	76 31	73 6	26 30
63 45	90 0	86 38	83 16	79 53	76 29	73 4	26 15
64 0	90 0	86 38	83 15	79 52	76 28	73 2	26 0
64 15	90 0	86 37	83 14	79 51	76 26	73 0	25 45
64 30	90 0	86 37	83 13	79 49	76 24	72 58	25 30
64 45	90 0	86 36	83 13	79 48	76 23	72 56	25 15
65 0	90 0	86 36	83 12	79 47	76 21	72 54	25 0
65 15	90 0	86 36	83 11	79 46	76 19	72 52	24 45
65 30	90 0	86 35	83 10	79 44	76 18	72 50	24 30
65 45	90 0	86 35	83 9	79 43	76 16	72 48	24 15
66 0	90 0	86 34	83 9	79 42	76 15	72 46	24 0
66 15	90 0	86 34	83 8	79 41	76 13	72 44	23 45
66 30	90 0	86 34	83 7	79 40	76 12	72 43	23 30
66 45	90 0	86 33	83 6	79 39	76 10	72 41	23 15
67 0	90 0	86 33	83 5	79 37	76 9	72 39	23 0
67 15	90 0	86 32	83 5	79 36	76 7	72 37	22 45
67 30	90 0	86 32	83 4	79 35	76 6	72 35	22 30
67 45	90 0	86 32	83 3	79 34	76 4	72 34	22 15
68 0	90 0	86 31	83 2	79 33	76 3	72 32	22 0
68 15	90 0	86 31	83 2	79 32	76 1	72 30	21 45

Alt. poli (G. M.)	12 / 12 (G. M.)	$\frac{1}{4}$ / 11 $\frac{3}{4}$ (G. M.)	$\frac{1}{2}$ / 11 $\frac{1}{2}$ (G. M.)	$\frac{3}{4}$ / 11 $\frac{1}{4}$ (G. M.)	1 / 11 (G. M.)	1 $\frac{1}{4}$ / 10 $\frac{3}{4}$ (G. M.)	meridie / med.noc. (G. M.)
68 30	90 0	86 31	83 1	79 31	76 0	72 28	21 30
68 45	90 0	86 30	83 0	79 30	75 59	72 27	21 15
69 0	90 0	86 30	83 0	79 29	75 57	72 25	21 0
69 15	90 0	86 30	82 59	79 28	75 56	72 23	20 45
69 30	90 0	86 29	82 58	79 27	75 55	72 22	20 30
69 45	90 0	86 29	82 58	79 26	75 53	72 20	20 15
70 0	90 0	86 29	82 57	79 25	75 52	72 19	20 0
70 15	90 0	86 28	82 56	79 24	75 51	72 17	19 45
70 30	90 0	86 28	82 56	79 23	75 49	72 15	19 30
70 45	90 0	86 28	82 55	79 22	75 48	72 14	19 15
71 0	90 0	86 27	82 54	79 21	75 47	72 12	19 0
71 15	90 0	86 27	82 54	79 20	75 46	72 11	18 45
71 30	90 0	86 27	82 53	79 19	75 45	72 9	18 30
71 45	90 0	86 26	82 52	79 18	75 43	72 8	18 15
72 0	90 0	86 26	82 52	79 17	75 42	72 6	18 0
72 15	90 0	86 26	82 51	79 16	75 41	72 5	17 45
72 30	90 0	86 25	82 51	79 16	75 40	72 4	17 30
72 45	90 0	86 25	82 50	79 15	75 39	72 2	17 15
73 0	90 0	86 25	82 49	79 14	75 38	72 1	17 0
73 15	90 0	86 24	82 49	79 13	75 37	72 0	16 45
73 30	90 0	86 24	82 48	79 12	75 35	71 58	16 30
73 45	90 0	86 24	82 48	79 11	75 34	71 57	16 15
74 0	90 0	86 24	82 47	79 11	75 33	71 56	16 0
74 15	90 0	86 23	82 47	79 10	75 32	71 54	15 45
74 30	90 0	86 23	82 46	79 9	75 31	71 53	15 30
74 45	90 0	86 23	82 46	79 8	75 30	71 52	15 15

TABVLA VI (continued) — Arc of the Horizon / Arc of the Prime Vertical

Table of the Pole-altitudes for arcs of the Horizon and of the Prime Vertical (Table VI, continued from the previous page). The descriptive rubric stands on an earlier page; these three pages carry only the numeric grid. Each opening bears the same rotated marginal labels: on the left, "**Altitudines Poli pro arcubus Horizontis.**" (Pole-altitudes for arcs of the Horizon); on the right, "**Altitudines Poli pro arcubus Verticalis primarij.**" (Pole-altitudes for arcs of the Prime Vertical).

The table's leftmost column (headed "**Horæ à &**" / "G. M.") is the argument in degrees and minutes — the pole-altitude, stepped every 15'. Each remaining column is one hour-line, its heading giving the afternoon hour over its morning (post-meridian / ante-meridian) complement; the final column is **meridie** / **med. noc.** (noon / midnight). Every cell is degrees (G.) and minutes (M.).

This page (p. 103) **completes the first hour-group** (noon through 1¼ hours), whose argument runs from 30° (begun earlier) to 90°; the previous page reached 74°45', and this page carries the argument 75°0' → 90°0'. The following two pages (pp. 104–105) **restart the argument at 0°** for the next hour-group (1½ through 2¾ hours).

Page 103 — argument 75°0' → 90°0' (hour-lines 12 through 1¼)

Column headings as printed (top marker / bottom marker): the top row shows the bare fractions with the repeated hour dropped — 12, ¼, ½, ¾, 1, ¼ — over the morning complements 12, 11¼, 11½, 11¾, 11, 10¾. (The fifth column's top glyph is worn and prints like 2, but its complement 11 and the p. 104 continuation beginning at 1½ fix it as the 1-o'clock line.)

Alt. poli (G M)	12 · 12	12¼ · 11¾	12½ · 11½	12¾ · 11¼	1 · 11	1¼ · 10¾	meridie · med. noc.
75 0	90 0	86 23	82 45	79 7	75 29	71 51	15 0
75 15	90 0	86 22	82 45	79 7	75 28	71 50	14 45
75 30	90 0	86 22	82 44	79 6	75 27	71 48	14 30

Alt. poli (G M)	12 · 12	12¼ · 11¾	12½ · 11½	12¾ · 11¼	1 · 11	1¼ · 10¾	meridie · med. noc.
75 45	90 0	86 22	82 44	79 5	75 27	71 47	14 15
76 0	90 0	86 22	82 43	79 5	75 26	71 46	14 0
76 15	90 0	86 21	82 43	79 4	75 25	71 45	13 45
76 30	90 0	86 21	82 42	79 3	75 24	71 44	13 30
76 45	90 0	86 21	82 42	79 3	75 23	71 43	13 15
77 0	90 0	86 21	82 41	79 2	75 22	71 42	13 0
77 15	90 0	86 20	82 41	79 1	75 21	71 41	12 45
77 30	90 0	86 20	82 41	79 1	75 20	71 40	12 30
77 45	90 0	86 20	82 40	79 0	75 20	71 39	12 15
78 0	90 0	86 20	82 40	78 59	75 19	71 38	12 0
78 15	90 0	86 20	82 39	78 59	75 18	71 37	11 45
78 30	90 0	86 20	82 39	78 58	75 17	71 36	11 30
78 45	90 0	86 19	82 39	78 58	75 17	71 35	11 15
79 0	90 0	86 19	82 38	78 57	75 16	71 34	11 0
79 15	90 0	86 19	82 38	78 57	75 15	71 33	10 45
79 30	90 0	86 19	82 38	78 56	75 14	71 33	10 30
79 45	90 0	86 19	82 37	78 56	75	71 32	10 15

Alt. poli (G M)	12 · 12	12¼ · 11¾	12½ · 11½	12¾ · 11¼	1 · 11	1¼ · 10¾	meridie · med. noc.
					14		
80 0	90 0	86 18	82 37	78 55	75	71 31	10 0
					13		
80 15	90 0	86 18	82 36	78 55	75	71 30	9 45
					12		
80 30	90 0	86 18	82 36	78 54	75	71 29	9 30
					12		
80 45	90 0	86 18	82 36	78 54	75	71 29	9 15
					11		
81 0	90 0	86 18	82 35	78 53	75	71 28	9 0
					11		
81 15	90 0	86 18	82 35	78 53	75	71 27	8 45
					10		
81 30	90 0	86 17	82 35	78 52	75	71 27	8 30
					9		
81 45	90 0	86 17	82 35	78 52	75	71 26	8 15
					9		
82 0	90 0	86 17	82 34	78 51	75	71 25	8 0
					8		
82 15	90 0	86 17	82 34	78 51	75	71 25	7 45
					8		
82 30	90 0	86 17	82 34	78 51	75	71 24	7 30
					7		
82 45	90 0	86 17	82 34	78 50	75	71 23	7 15
					7		
83 0	90 0	86 17	82 33	78 50	75	71 23	7 0
					6		
83 15	90 0	86 17	82 33	78 50	75	71 22	6 45
					6		
83 30	90 0	86 16	82 33	78 49	75	71 22	6 30
					6		
83 45	90 0	86 16	82 33	78 49	75	71 21	6 15
					5		

Alt. poli (G M)	12 · 12	12¼ · 11¾	12½ · 11½	12¾ · 11¼	1 · 11	1¼ · 10¾	meridie · med. noc.
84 0	90 0	86 16	82 32	78 49	75 5	71 21	6 0
84 15	90 0	86 16	82 32	78 48	75 4	71 20	5 45
84 30	90 0	86 16	82 32	78 48	75 4	71 20	5 30
84 45	90 0	86 16	82 32	78 48	75 4	71 19	5 15
85 0	90 0	86 16	82 32	78 48	75 3	71 19	5 0
85 15	90 0	86 16	82 32	78 47	75 3	71 19	4 45
85 30	90 0	86 16	82 31	78 47	75 2	71 18	4 30
85 45	90 0	86 16	82 31	78 47	75 2	71 18	4 15
86 0	90 0	86 16	82 31	78 47	75 2	71 18	4 0
86 15	90 0	86 16	82 31	78 46	75 2	71 18	3 45
86 30	90 0	86 15	82 31	78 46	75 1	71 17	3 30
86 45	90 0	86 15	82 31	78 46	75 1	71 17	3 15
87 0	90 0	86 15	82 31	78 46	75 1	71 17	3 0
87 15	90 0	86 15	82 31	78 46	75 1	71 16	2 45
87 30	90 0	86 15	82 30	78 46	75 1	71 16	2 30
87 45	90 0	86 15	82 30	78 46	75 1	71 16	2 15
88 0	90 0	86 15	82 30	78 45	75	71 16	2 0

Alt. poli (G M)	12 · 12	12¼ · 11¾	12½ · 11½	12¾ · 11¼	1 · 11	1¼ · 10¾	meridie · med. noc.
					0		
88 15	90 0	86 15	82 30	78 45	75	71 16	1 45
					0		
88 30	90 0	86 15	82 30	78 45	75	71 15	1 30
					0		
88 45	90 0	86 15	82 30	78 45	75	71 15	1 15
					0		
89 0	90 0	86 15	82 30	78 45	75	71 15	1 0
					0		
89 15	90 0	86 15	82 30	78 45	75	71 15	0 45
					0		
89 30	90 0	86 15	82 30	78 45	75	71 15	0 30
					0		
89 45	90 0	86 15	82 30	78 45	75	71 15	0 15
					0		
90 0	90 0	86 15	82 30	78 45	75	71 15	0 0
					0		

Page 104 — argument 0°0' → 14°45' (hour-lines 1½ through 2¾)

Column headings as printed (top / bottom): 1½ · 10½, 1¼ · 10¼, 2 · 10, 2¼ · 9¾, 2½ · 9½, 2¾ · 9¼, then meridie · med. noc.

Alt. poli (G M)	1½ · 10½	1¼ · 10¼	2 · 10	2¼ · 9¾	2½ · 9½	2¾ · 9¼	meridie · med. noc.
0 0	90 0	90 0	90	90 0	90 0	90 0	90 0
			0				
0 15	89 54	89 53	89	89 50	89 49	89 47	89 45
			51				
0 30	89 47	89 45	89	89 40	89 37	89 34	89 30
			43				
0 45	89 41	89 38	89	89 30	89 26	89 21	89 15
			34				

Alt. poli (G M)	$1\frac{1}{2} \cdot$ $10\frac{1}{2}$	$1\frac{3}{4} \cdot$ $10\frac{1}{4}$	$2 \cdot$ 10	$2\frac{1}{4} \cdot$ $9\frac{3}{4}$	$2\frac{1}{2} \cdot 9\frac{1}{2}$	$2\frac{3}{4} \cdot$ $9\frac{1}{4}$	meridie · med. noc.
1 0	89 35	89 30	89 25	89 20	89 14	89 7	89 0
1 15	89 29	89 23	89 17	89 10	89 2	88 54	88 45
1 30	89 23	89 16	89 8	89 0	88 51	88 41	88 30
1 45	89 16	89 8	88 59	88 50	88 39	88 28	88 15
2 0	89 10	89 1	88 51	88 40	88 28	88 15	88 0
2 15	89 4	88 53	88 42	88 30	88 16	88 2	87 45
2 30	88 58	88 46	88 33	88 20	88 5	87 49	87 30
2 45	88 52	88 39	88 25	88 10	87 53	87 35	87 15
3 0	88 45	88 31	88 16	88 0	87 42	87 22	87 0
3 15	88 39	88 24	88 8	87 50	87 31	87 9	86 45
3 30	88 33	88 17	87 59	87 40	87 19	86 56	86 30
3 45	88 27	88 9	87 50	87 30	87 8	86 43	86 15
4 0	88 21	88 2	87 42	87 20	86 56	86 30	86 0
4 15	88 14	87 54	87 33	87 10	86 45	86 17	85 45
4 30	88 8	87 47	87 24	87 0	86 33	86 4	85 30
4 45	88 2	87 40	87 16	86 50	86 22	85 51	85 15
5 0	87 56	87 32	87	86 40	86 10	85 38	85 0

Alt. poli (G M)	$1\frac{1}{2} \cdot$ $10\frac{1}{2}$	$1\frac{3}{4} \cdot$ $10\frac{1}{4}$	$2 \cdot$ 10	$2\frac{1}{4} \cdot$ $9\frac{3}{4}$	$2\frac{1}{2} \cdot 9\frac{1}{2}$	$2\frac{3}{4} \cdot$ $9\frac{1}{4}$	meridie · med. noc.
			7				
5 15	87 50	87 25	86 59	86 30	85 59	85 25	84 45
5 30	87 44	87 18	86 50	86 20	85 48	85 12	84 30
5 45	87 37	87 10	86 41	86 10	75 36 [recte: 85 36]	84 59	84 15
6 0	87 31	87 3	86 33	86 0	85 25	84 46	84 0
6 15	87 25	86 56	86 24	85 50	85 13	84 33	83 45
6 30	87 19	86 48	86 16	85 40	85 2	84 20	83 30
6 45	87 13	86 41	86 7	85 31	84 51	84 7	83 15
7 0	87 7	86 34	85 59	85 21	84 39	83 54	83 0
7 15	87 0	86 26	85 50	85 11	84 28	83 41	82 45
7 30	86 54	86 19	85 41	85 1	84 17	83 28	82 30
7 45	86 48	86 12	85 33	84 51	84 6	83 15	82 15
8 0	86 42	86 4	85 24	84 41	83 54	83 2	82 0
8 15	86 36	85 57	85 16	84 31	83 43	82 50	81 45
8 30	86 30	85 50	85 7	84 22	83 32	82 37	81 30
8 45	86 24	85 43	84 59	84 12	83 21	82 24	81 15
9 0	86 18	85 35	84 50	84 2	83 9	82 11	81 0

Alt. poli (G M)	$1\frac{1}{2} \cdot$ $10\frac{1}{2}$	$1\frac{3}{4} \cdot$ $10\frac{1}{4}$	$2 \cdot$ 10	$2\frac{1}{4} \cdot$ $9\frac{3}{4}$	$2\frac{1}{2} \cdot 9\frac{1}{2}$	$2\frac{3}{4} \cdot$ $9\frac{1}{4}$	meridie · med. noc.
9 15	86 11	85 28	84 42	83 52	82 58	81 59	80 45
9 30	86 5	85 21	84 33	83 42	82 47	81 46	80 30
9 45	85 59	85 14	84 25	83 33	82 36	81 33	80 15
10 0	85 53	85 6	84 16	83 23	82 25	81 20	80 0
10 15	85 47	84 59	84 8	83 13	82 13	81 8	79 45
10 30	85 41	84 52	84 0	83 3	82 2	80 55	79 30
10 45	85 35	84 45	83 51	82 54	81 51	80 43	79 15
11 0	85 29	84 37	83 43	82 44	81 40	80 30	79 0
11 15	85 23	84 30	83 34	82 34	81 29	80 17	78 45
11 30	85 17	84 23	83 26	82 25	81 18	80 5	78 30
11 45	85 11	84 16	83 18	82 15	81 7	79 52	78 15
12 0	85 5	84 9	83 9	82 5	80 56	79 40	78 0
12 15	84 59	84 2	83 1	81 56	80 45	79 28	77 45
12 30	84 53	83 54	82 53	81 46	80 34	79 15	77 30
12 45	84 47	83 47	82 44	81 37	80 23	79 3	77 15
13 0	84 41	83 40	82 36	81 27	80 12	78 50	77 0
13 15	84 35	83 33	82	81 18	80 2	78 38	76 45

Alt. poli (G M)	$1\frac{1}{2} \cdot$ $10\frac{1}{2}$	$1\frac{3}{4} \cdot$ $10\frac{1}{4}$	$2 \cdot$ 10	$2\frac{1}{4} \cdot$ $9\frac{3}{4}$	$2\frac{1}{2} \cdot 9\frac{1}{2}$	$2\frac{3}{4} \cdot$ $9\frac{1}{4}$	meridie · med. noc.
			28				
13 30	84 29	83 26	82	81 8	79 51	78 26	76 30
			19				
13 45	84 23	83 19	82	80 59	79 39	78 14	76 15
			11				
14 0	84 17	83 12	82	80 49	79 29	78 1	76 0
			3				
14 15	84 11	83 5	81	80 40	79 18	77 49	75 45
			55				
14 30	84 5	82 58	81	80 30	79 8	77 37	75 30
			46				
14 45	83 59	82 51	81	80 21	78 57	77 25	75 15
			38				

Page 105 — argument $15^{\circ}0' \rightarrow 29^{\circ}45'$ (hour-lines $1\frac{1}{2}$ through $2\frac{3}{4}$, continued)

Same column headings as p. 104: $1\frac{1}{2} \cdot 10\frac{1}{2}$, $1\frac{3}{4} \cdot 10\frac{1}{4}$, $2 \cdot 10$, $2\frac{1}{4} \cdot 9\frac{3}{4}$, $2\frac{1}{2} \cdot 9\frac{1}{2}$, $2\frac{3}{4} \cdot 9\frac{1}{4}$, meridie · med. noc.

Alt. poli (G M)	$1\frac{1}{2} \cdot$ $10\frac{1}{2}$	$1\frac{3}{4} \cdot$ $10\frac{1}{4}$	$2 \cdot$ 10	$2\frac{1}{4} \cdot$ $9\frac{3}{4}$	$2\frac{1}{2} \cdot 9\frac{1}{2}$	$2\frac{3}{4} \cdot$ $9\frac{1}{4}$	meridie · med. noc.
15 0	83 53	82 44	81	80 11	78 46	77 13	75 0
			30				
15 15	83 47	82 37	81	80 2	78 35	77 1	74 45
			22				
15 30	83 41	82 30	81	79 53	78 25	76 49	74 30
			14				
15 45	83 35	82 23	81	79 43	78 14	76 37	74 15
			6				
16 0	83 29	82 16	80	79 34	78 3	76 25	74 0
			57				
16 15	83 23	82 9	80	79 25	77 53	76 13	73 45
			49				

Alt. poli (G M)	$1\frac{1}{2} \cdot$ $10\frac{1}{2}$	$1\frac{3}{4} \cdot$ $10\frac{1}{4}$	$2 \cdot$ 10	$2\frac{1}{4} \cdot$ $9\frac{3}{4}$	$2\frac{1}{2} \cdot 9\frac{1}{2}$	$2\frac{3}{4} \cdot$ $9\frac{1}{4}$	meridie · med. noc.
16 30	83 17	82 2	80 41	79 15	77 42	76 1	73 30
16 45	83 12	81 55	80 33	79 6	77 32	75 49	73 15
17 0	83 6	81 48	80 25	78 57	77 21	75 37	73 0
17 15	83 0	81 41	80 17	78 48	77 11	75 25	72 45
17 30	82 54	81 34	80 9	78 38	77 0	75 14	72 30
17 45	82 48	81 27	80 1	78 29	76 50	75 2	72 15
18 0	82 42	81 20	79 53	78 20	76 40	74 50	72 0
18 15	82 37	81 13	79 45	78 11	76 29	74 39	71 45
18 30	82 31	81 6	79 37	78 2	76 19	74 27	71 30
18 45	82 25	81 0	79 29	77 53	76 9	74 15	71 15
19 0	82 19	80 53	79 21	77 44	75 58	74 4	71 0
19 15	82 13	80 46	79 13	77 35	75 48	73 52	70 45
19 30	82 8	80 39	79 5	77 26	75 38	73 41	70 30
19 45	82 2	80 32	78 58	77 17	75 28	73 30	70 15
20 0	81 56	80 26	78 50	77 8	75 18	73 18	70 0
20 15	81 50	80 19	78 42	76 59	75 8	73 7	69 45
20 30	81 45	80 12	78	76 50	74 57	72 56	69 30

Alt. poli (G M)	$1\frac{1}{2} \cdot$ $10\frac{1}{2}$	$1\frac{3}{4} \cdot$ $10\frac{1}{4}$	$2 \cdot$ 10	$2\frac{1}{4} \cdot$ $9\frac{3}{4}$	$2\frac{1}{2} \cdot 9\frac{1}{2}$	$2\frac{3}{4} \cdot$ $9\frac{1}{4}$	meridie · med. noc.
			34				
20 45	81 39	80 5	78	76 41	74 47	72 44	69 15
			26				
21 0	81 33	79 59	78	76 32	74 37	72 33	69 0
			19				
21 15	81 28	79 52	78	76 23	74 27	72 22	68 45
			11				
21 30	81 22	79 45	78	76 14	74 18	72 11	68 30
			3				
21 45	81 16	79 39	77	76 6	74 8	72 0	68 15
			55				
22 0	81 11	79 32	77	75 57	73 58	71 49	68 0
			48				
22 15	81 5	79 25	77	75 48	73 48	71 38	67 45
			40				
22 30	81 0	79 19	77	75 39	73 38	71 27	67 30
			32				
22 45	80 54	79 12	77	75 31	73 28	71 16	67 15
			25				
23 0	80 48	79 6	77	75 22	73 19	71 5	67 0
			17				
23 15	80 43	78 59	77	75 13	73 9	70 54	66 45
			10				
23 30	80 37	78 53	77	75 5	72 59	70 44	66 30
			2				
23 45	80 32	78 46	76	74 56	72 49	70 33	66 15
			55				
24 0	80 26	78 39	76	74 48	72 40	70 22	66 0
			47				
24 15	80 21	78 33	76	74 39	72 30	70 11	65 45
			40				
24 30	80 15	78 27	76	74 31	72 21	70 1	65 30
			32				

Alt. poli (G M)	$1\frac{1}{2} \cdot$ $10\frac{1}{2}$	$1\frac{3}{4} \cdot$ $10\frac{1}{4}$	$2 \cdot$ 10	$2\frac{1}{4} \cdot$ $9\frac{3}{4}$	$2\frac{1}{2} \cdot 9\frac{1}{2}$	$2\frac{3}{4} \cdot$ $9\frac{1}{4}$	meridie · med. noc.
24 45	80 10	78 20	76 25	74 22	72 11	69 50	65 15
25 0	80 4	78 14	76 18	74 14	72 2	69 40	65 0
25 15	79 59	78 7	76 10	74 5	71 53	69 29	64 45
25 30	79 53	78 1	76 3	73 57	71 44	69 19	64 30
25 45	79 48	77 54	75 55	73 49	71 34	69 9	64 15
26 0	79 43	77 48	75 48	73 40	71 25	68 58	64 0
26 15	79 37	77 42	75 41	73 32	71 15	68 48	63 45
26 30	79 32	77 35	75 33	73 24	71 6	68 38	63 30
26 45	79 26	77 29	75 26	73 16	70 57	68 28	63 15
27 0	79 21	77 23	75 19	73 7	70 48	68 17	63 0
27 15	79 16	77 17	75 12	72 59	70 38	68 7	62 45
27 30	79 10	77 10	75 4	72 51	70 39 [recte: 70 29]	67 57	62 30
27 45	79 5	77 4	74 57	72 43	70 20	67 47	62 15
28 0	79 0	76 58	74 50	72 35	70 11	67 37	62 0
28 15	78 54	76 52	74 43	72 27	70 2	67 27	61 45
28 30	78 49	76 46	74 36	72 19	69 53	67 18	61 30
28 45	78 44	76 39	74	72 11	69 45	67 8	61 15

Alt. poli (G M)	1½ · 10½	1¾ · 10¾	2 · 10	2¼ · 9¾	2½ · 9½	2¾ · 9¼	meridie · med. noc.
			29				
29 0	78 39	76 33	74	72 3	69 36	66 58	61 0
			22				
29 15	78 34	76 27	74	71 55	69 27	66 48	60 45
			15				
29 30	78 28	76 21	74	71 47	69 18	66 39	60 30
			8				
29 45	78 23	76 15	74	71 39	69 9	66 29	60 15
			1				

Arc of the Horizon (continuation of the Sixth Table)

These three pages continue the sixth table of the appendix (**VI. TABVLA**); no fresh table-number rubric is printed here — the descriptive heading stands on an earlier page. The running head throughout reads *Arcus Horizontis* ("Arc of the Horizon"). The argument column (headed *Horæ à & over G. M.*) is the elevation of the pole, running in 15' steps; it continues seamlessly from the preceding page (which ended at 29°45') through 30°0' to 74°45'.

The two rotated marginal labels run down both sides of every page:

- [Margin (left): *Altitudines poli pro arcubus Horizontis*. — "Pole-elevations for arcs of the Horizon."]
- [Margin (right): *Altitudines Poli pro arcubus Verticalis primarij*. — "Pole-elevations for arcs of the prime Vertical."]

Each numeric column is headed by an hour-value above and its complement to twelve below (e.g. 1½ over 10½), and carries the sub-headings *G. M.* (*gradus, minuta* — degrees and minutes). The last column, headed *meridie* over *med. noc.* ("at noon" / "at midnight"), gives the arc for noon and midnight and equals 90° minus the pole-elevation.

Page 106 (pole-elevation 30°0' – 44°45')

Horæ à & (G. M.)	1½ · 10½	1¾ · 10¾	2 · 10	2¼ · 9¾	2½ · 9½	2¾ · 9¼	meridie · med. noc.
30 0	78 18	76 9	73 54	71 32	69 1	66 19	60 0
30 15	78 13	76 3	73 47	71 24	68 52	66 10	59 45
30 30	78 8	75 57	73 40	71 16	68 43	66 0	59 30
30 45	78 3	75 51	73 33	71 8	68 35	65 51	59 15
31 0	77 57	75 45	73 26	71 1	68 26	65 42	59 0
31 15	77 52	75 39	73 20	70 53	68 18	65 32	58 45
31 30	77 47	75 33	73 13	70 45	68 9	65 23	58 30
31 45	77 42	75 27	73 6	70 38	68 1	65 14	58 15
32 0	77 37	75 21	72 59	70 30	67 52	65 4	58 0
32 15	77 32	75 15	72 53	70 23	67 43	64 55	57 45
32 30	77 27	75 10	72 46	70 15	67 35	64 46	57 30
32 45	77 22	75 4	72 39	70 8	67 27	64 37	57 15
33 0	77 17	74 58	72 33	70 0	67 19	64 28	57 0
33 15	77 12	74 52	72 26	69 53	67 11	64 19	56 45
33 30	77 7	74 46	72 20	69 45	67 3	64 10	56 30
33 45	77 2	74 41	72 13	69 38	66 55	64 1	56 15

Horæ à & (G. M.)	1½ · 10½	1¾ · 10¾	2 · 10	2¼ · 9¾	2½ · 9½	2¾ · 9¾	meridie · med. noc.
34 0	76 58	74 35	72 6	69 31	66 47	63 53	56 0
34 15	76 53	74 29	72 0	69 23	66 39	63 44	55 45
34 30	76 48	74 24	71 53	69 16	66 31	63 35	55 30
34 45	76 43	74 18	71 47	69 9	66 23	63 26	55 15
35 0	76 38	74 12	71 41	69 2	66 15	63 18	55 0
35 15	76 33	74 7	71 34	68 55	66 7	63 9	54 45
35 30	76 29	74 1	71 28	68 48	65 59	63 1	54 30
35 45	76 24	73 56	71 22	68 41	65 52	62 52	54 15
36 0	76 19	73 50	71 15	68 33	65 44	62 44	54 0
36 15	76 14	73 45	71 9	68 26	65 36	62 35	53 45
36 30	76 10	73 39	71 3	68 20	65 28	62 27	53 30
36 45	76 5	73 34	70 57	68 13	65 20	62 19	53 15
37 0	76 0	73 28	70 50	68 6	65 13	62 11	53 0
37 15	75 56	73 23	70 44	67 59	65 5	62 2	52 45
37 30	75 51	73 17	70 38	67 52	64 58	61 54	52 30
37 45	75 46	73 12	70 32	67 45	64 50	61 46	52 15
38 0	75 42	73 7	70	67 38	64 43	61 38	52 0

Horæ à & (G. M.)	1½ · 10½	1¾ · 10¾	2 · 10	2¼ · 9¾	2½ · 9½	2¾ · 9¾	meridie · med. noc.
			26				
38 15	75 37	73 1	70	67 32	64 35	61 30	51 45
			20				
38 30	75 32	72 56	70	67 25	64 28	61 22	51 30
			14				
38 45	75 28	72 51	70	67 18	64 21	61 14	51 15
			8				
39 0	75 23	72 46	70	67 12	64 14	61 6	51 0
			2				
49 15 [recte: 39 15]	75 19	72 40	69	67 5	64 6	60 58	50 45
			56				
39 30	75 14	72 35	69	66 58	63 59	60 51	50 30
			50				
39 45	75 10	72 30	69	66 52	63 52	60 43	50 15
			44				
40 0	75 5	72 25	69	66 45	63 45	60 35	50 0
			38				
40 15	75 1	72 20	69	66 39	63 38	60 28	49 45
			33				
40 30	74 56	72 14	69	66 33	63 31	60 20	49 30
			27				
40 45	74 52	72 9	69	66 26	63 24	60 13	49 15
			21				
41 0	74 48	72 4	69	66 20	63 17	60 5	49 0
			15				
41 15	74 43	71 59	69	66 13	63 10	59 58	48 45
			10				
41 30	74 39	71 54	69	66 7	63 3	59 50	48 30
			4				
41 45	74 35	71 49	68	66 1	62 56	59 43	48 15
			58				
42 0	74 30	71 44	68	65 55	62 50	59 36	48 0
			53				

Horæ à & (G. M.)	1½ · 10½	1¾ · 10¼	2 · 10	2¼ · 9¾	2½ · 9½	2¾ · 9¼	meridie · med. noc.
42 15	74 26	71 39	68 47	65 48	62 43	59 28	47 45
42 30	74 22	71 34	68 41	65 42	62 36	59 21	47 30
42 45	74 18	71 30	68 36	65 36	62 29	59 14	47 15
43 0	74 14	71 25	68 30	65 30	62 23	59 7	47 0
43 15	74 9	71 20	68 25	65 24	62 16	59 0	46 45
43 30	74 5	71 15	68 20	65 18	62 9	58 53	46 30
43 45	74 1	71 10	68 14	65 12	62 3	58 46	46 15
44 0	73 57	71 5	68 9	65 6	61 57	58 39	46 0
44 15	73 53	71 1	68 3	65 0	61 50	58 32	45 45
44 30	73 49	70 56	67 58	64 54	61 44	58 25	45 30
44 45	73 45	70 51	67 53	64 48	61 37	58 19	45 15

Page 107 (pole-elevation 45°0' – 59°45')

Horæ à & (G. M.)	1½ · 10½	1¾ · 10¼	2 · 10	2¼ · 9¾	2½ · 9½	2¾ · 9¼	meridie · med. noc.
45 0	73 41	70 47	67 48	64 43	61 31	58 12	45 0
45 15	73 36	70 42	67 42	64 37	61 25	58 5	44 45
45 30	73 32	70 37	67 37	64 31	61 18	57 58	44 30
45 45	73 28	70 33	67	64 25	61 12	57 52	44 15

Horæ à & (G. M.)	1½ · 10½	1¾ · 10¾	2 · 10	2¼ · 9¾	2½ · 9½	2¾ · 9¼	meridie · med. noc.
			32				
46 0	73 24	70 28	67 27	64 20	61 6	57 45	44 0
46 15	73 20	70 24	67 22	64 14	61 0	57 39	43 45
46 30	73 17	70 19	67 17	64 8	60 54	57 32	43 30
46 45	73 13	70 15	67 12	64 3	60 48	57 26	43 15
47 0	73 9	70 10	67 7	63 57	60 42	57 20	43 0
47 15	73 5	70 6	67 2	63 52	60 36	57 13	42 45
47 30	73 1	70 1	66 57	63 46	60 30	57 7	42 30
47 45	72 57	69 57	66 52	63 41	60 24	57 1	42 15
48 0	72 53	69 52	66 47	63 36	60 19	56 54	42 0
48 15	72 50	69 48	66 42	63 30	60 13	56 48	41 45
48 30	72 46	69 44	66 37	63 25	60 7	56 42	41 30
48 45	72 42	69 39	66 32	63 20	60 1	56 36	41 15
49 0	72 38	69 35	66 27	63 14	59 56	56 30	41 0
49 15	72 35	69 31	66 23	63 9	59 50	56 24	40 45
49 30	72 31	69 27	66 18	63 4	59 44	56 18	40 30
49 45	72 28	69 23	66 13	62 59	59 39	56 13	40 15

Horæ à & (G. M.)	$1\frac{1}{2} \cdot$ $10\frac{1}{2}$	$1\frac{3}{4} \cdot$ $10\frac{3}{4}$	$2 \cdot$ 10	$2\frac{1}{4} \cdot$ $9\frac{3}{4}$	$2\frac{1}{2} \cdot$ $9\frac{1}{2}$	$2\frac{3}{4} \cdot$ $9\frac{1}{4}$	meridie · med. noc.
50 0	72 24	69 18	66 9	62 54	59 35	56 6	40 0
50 15	72 20	69 14	66 4	62 49	59 28	56 1	39 45
50 30	72 17	69 10	65 59	62 44	59 22	55 55	39 30
50 45	72 13	69 6	65 55	62 39	59 18	55 49	39 15
51 0	72 9	69 2	65 50	62 34	59 12	55 43	39 0
51 15	72 6	68 58	65 46	62 29	59 6	55 38	38 45
51 30	72 2	68 54	65 41	62 24	59 1	55 32	38 30
51 45	71 59	68 50	65 37	62 19	58 56	55 27	38 15
52 0	71 55	68 46	65 32	62 14	58 50	55 21	38 0
52 15	71 52	68 42	65 28	62 9	58 45	55 16	37 45
52 30	71 49	68 38	65 23	62 4	58 40	55 10	37 30
52 45	71 45	68 34	65 19	62 0	58 35	55 5	37 15
53 0	71 42	68 30	65 15	61 55	58 30	55 0	37 0
53 15	71 38	68 26	65 10	61 50	58 25	54 54	36 45
53 30	71 35	68 23	65 6	61 46	58 20	54 49	36 30
53 45	71 32	68 19	65 2	61 41	58 15	54 44	36 15
54 0	71 28	68 15	64	61 36	58 10	54 39	36 0

Horæ à & (G. M.)	1½ · 10½	1¾ · 10¾	2 · 10	2¼ · 9¾	2½ · 9½	2¾ · 9¼	meridie · med. noc.
			58				
54 15	71 25	68 11	64	61 32	58 5	54 34	35 45
			54				
54 30	71 22	68 8	64	61 27	58 0	54 28	35 30
			49				
54 45	71 19	68 4	64	61 23	57 56	54 23	35 15
			45				
55 0	71 15	68 0	64	61 18	57 51	54 18	35 0
			41				
55 15	71 12	67 57	64	61 14	57 46	54 13	34 45
			37				
55 30	71 9	67 53	64	61 10	57 41	54 9	34 30
			33				
55 45	71 6	67 49	64	61 5	57 37	54 4	34 15
			29				
56 0	71 3	67 46	64	61 1	57 32	53 59	34 0
			25				
56 15	71 0	67 42	64	60 57	57 28	53 54	33 45
			21				
56 30	70 57	67 39	64	60 52	57 23	53 49	33 30
			18				
56 45	70 54	67 35	64	60 48	57 19	53 45	33 15
			14				
57 0	70 51	67 32	64	60 44	57 14	53 40	33 0
			10				
57 15	70 48	67 28	64	60 40	57 10	53 35	32 45
			6				
57 30	70 45	67 25	64	60 36	57 5	53 31	32 30
			2				
57 45	70 42	67 22	63	60 32	57 1	53 26	32 15
			58				
58 0	70 39	67 18	63	60 28	56 57	53 22	32 0
			55				

Horæ à & (G. M.)	1½ · 10½	1¾ · 10¾	2 · 10	2¼ · 9¾	2½ · 9½	2¾ · 9¼	meridie · med. noc.
58 15	70 36	67 15	63 51	60 24	56 53	53 17	31 45
58 30	70 33	67 12	63 47	60 20	56 48	53 13	31 30
58 45	70 30	67 8	63 44	60 16	56 44	53 8	31 15
59 0	70 27	67 5	63 40	60 12	56 40	53 4	31 0
59 15	70 24	67 2	63 37	60 8	56 36	53 0	30 45
59 30	70 22	66 59	63 33	60 4	56 32	52 55	30 30
59 45	70 19	66 56	63 30	60 0	56 28	52 51	30 15

Page 108 (pole-elevation 60°0' – 74°45')

Horæ à & (G. M.)	1½ · 10½	1¾ · 10¾	2 · 10	2¼ · 9¾	2½ · 9½	2¾ · 9¼	meridie · med. noc.
60 0	70 16	66 52	63 26	59 57	56 24	52 47	30 0
60 15	70 13	66 49	63 23	59 53	56 20	52 43	29 45
60 30	70 11	66 46	63 19	59 49	56 16	52 39	29 30
60 45	70 8	66 43	63 16	59 45	56 12	52 35	29 15
61 0	70 5	66 40	63 12	59 42	56 8	52 31	29 0
61 15	70 2	66 37	63 9	59 38	56 4	52 27	28 45
61 30	70 0	66 34	63 6	59 35	56 0	52 23	28 30
61 45	69 57	66 31	63	59 31	55 57	52 19	28 15

Horæ à & (G. M.)	1½ · 10½	1¾ · 10¾	2 · 10	2¼ · 9¾	2½ · 9½	2¾ · 9¼	meridie · med. noc.
			3				
62 0	69 55	66 28	62	59 28	55 53	52 15	28 0
			59				
62 15	69 52	66 25	62	59 24	55 49	52 11	27 45
			56				
62 30	69 50	66 22	62	59 21	55 46	52 7	27 30
			53				
62 45	69 47	66 20	62	59 17	55 42	52 3	27 15
			50				
63 0	69 45	66 17	62	59 14	55 38	52 0	27 0
			47				
63 15	69 43	66 14	62	59 11	55 35	51 56	26 45
			44				
63 30	69 40	66 11	62	59 7	55 31	51 53	26 30
			41				
63 45	69 37	66 8	62	59 4	55 28	51 49	26 15
			37				
64 0	69 35	66 6	62	59 1	55 24	51 45	26 0
			34				
64 15	69 32	66 3	62	58 58	55 21	51 42	25 45
			31				
64 30	69 30	66 0	62	58 54	55 18	51 38	25 30
			29				
64 45	69 28	65 58	62	58 51	55 14	51 35	25 15
			26				
65 0	69 25	65 55	62	58 48	55 11	51 31	25 0
			23				
65 15	69 23	65 52	62	58 45	55 8	51 28	24 45
			20				
65 30	69 21	65 50	62	58 42	55 5	51 25	24 30
			17				
65 45	69 19	65 47	62	58 39	55 1	51 21	24 15
			14				

Horæ à & (G. M.)	$1\frac{1}{2} \cdot$ $10\frac{1}{2}$	$1\frac{3}{4} \cdot$ $10\frac{3}{4}$	$2 \cdot$ 10	$2\frac{1}{4} \cdot$ $9\frac{3}{4}$	$2\frac{1}{2} \cdot$ $9\frac{1}{2}$	$2\frac{3}{4} \cdot$ $9\frac{1}{4}$	meridie · med. noc.
66 0	69 16	65 45	62 11	58 36	54 58	51 18	24 0
66 15	69 14	65 42	62 9	58 33	54 55	51 15	23 45
66 30	69 12	65 40	62 6	58 30	54 52	51 12	23 30
66 45	69 10	65 37	62 3	58 27	54 49	51 8	23 15
67 0	69 8	65 35	62 1	58 24	54 46	51 5	23 0
67 15	69 6	65 33	61 58	58 22	54 43	51 2	22 45
67 30	69 4	65 30	61 55	58 19	54 40	50 59	22 30
67 45	69 1	65 28	61 53	58 16	54 37	50 56	22 15
68 0	68 59	65 26	61 50	58 13	54 34	50 53	22 0
68 15	68 57	65 23	61 48	58 11	54 31	50 50	21 45
68 30	68 55	65 21	61 45	58 8	54 28	50 47	21 30
68 45	68 53	65 19	61 43	58 5	54 26	50 44	21 15
69 0	68 52	65 17	61 41	58 3	54 23	50 42	21 0
69 15	68 50	65 15	61 38	58 0	54 20	50 39	20 45
69 30	68 48	65 12	61 36	57 58	54 18	50 36	20 30
69 45	68 46	65 10	61 33	57 55	54 15	50 33	20 15
70 0	68 44	65 8	61	57 53	54 12	50 31	20 0

Horæ à & (G. M.)	1½ · 10½	1¾ · 10¾	2 · 10	2¼ · 9¾	2½ · 9½	2¾ · 9¼	meridie · med. noc.
			31				
70 15	68 42	65 6	61	57 50	54 10	50 28	19 45
			29				
70 30	68 40	65 4	61	57 48	54 7	50 25	19 30
			27				
70 45	68 39	65 2	61	57 45	54 5	50 23	19 15
			24				
71 0	68 37	65 0	61	57 43	54 2	50 20	19 0
			22				
71 15	68 35	64 58	61	57 41	54 0	50 18	18 45
			20				
71 30	68 33	64 56	61	57 38	53 57	50 15	18 30
			18				
71 45	68 32	64 54	61	57 36	53 55	50 13	18 15
			16				
72 0	68 30	64 52	61	57 34	53 53	50 10	18 0
			14				
72 15	68 28	64 50	61	57 32	53 50	50 8	17 45
			12				
72 30	68 27	64 49	61	57 30	53 48	50 5	17 30
			10				
72 45	68 25	64 47	61	57 27	53 46	50 3	17 15
			8				
73 0	68 23	64 45	61	57 25	53 44	50 1	17 0
			6				
73 15	68 22	64 43	61	57 23	53 42	49 59	16 45
			4				
73 30	68 20	64 42	61	57 21	53 39	49 56	16 30
			2				
73 45	68 19	64 40	61	57 19	53 37	49 54	16 15
			0				
74 0	68 17	64 38	60	57 17	53 35	49 52	16 0
			58				

Horæ à & (G. M.)	1½ · 10½	1¾ · 10¼	2 · 10	2¼ · 9¾	2½ · 9½	2¾ · 9¼	meridie · med. noc.
74 15	68 16	64 37	60 56	57 15	53 33	49 50	15 45
74 30	68 14	64 35	60 55	57 13	53 31	49 48	15 30
74 45	68 13	64 33	60 53	57 12	53 29	49 46	15 15

The table continues onto the following page (argument 75°0' and beyond); collation will stitch the boundary.

These three pages continue the sixth table (**VI. TABVLA**) of the appendix — the great table giving the *altitude of the pole* against the *arc of the horizon* and the *arc of the prime vertical*, tabulated for each hour of the day. No fresh table-number rubric is printed on any of these pages; the running head throughout is "Arcus Horizontis." ("Arc of the Horizon"). The whole table is laid out in successive hour-panels: page 109 finishes the panel for the hours 1½–2¾ (and their complements 10½–9¼) over the arc-range 75°–90°; pages 110–111 then begin the next panel, for the hours 3–4¼ (and their complements 9–7¾), starting the arc-argument again at 0° and running down to 29°45'.

The two rotated marginal labels stand on every page, framing the table:

- Left margin (vertical): "Altitudines poli pro arcubus Horizontis." — *Altitudes of the pole for arcs of the Horizon.*
- Right margin (vertical): "Altitudines Poli pro arcubus Verticalis primarij." — *Altitudes of the Pole for arcs of the prime Vertical.*

Column layout (identical on all three pages): the argument column is headed "Horæ à &" over "G. M." (the arc, in degrees and minutes); each following column is headed by an hour (top) over its complement to twelve (bottom), each over "G. M."; the last column is headed "meridie" over "med. noc." ("noon" / "midnight"). All body cells are degrees and minutes (G. = *gradus*, M. = *minuta*).

VI. TABVLA (continued) — page 109

Panel for the hours 1½–2¾ (complements 10½–9¼), arc of the horizon / prime vertical 75°–90°. Column heads read as *hour / complement*.

Arc (G. M.)	1½ / 10½	1¾ / 10¼	2 / 10	2¼ / 9¾	2½ / 9½	2¾ / 9¼	meridie / med. noc.
75 0	68 12	64 32	60 51	57 10	53 27	49 44	15 0
75 15	68 10	64 30	60 49	57 8	53 25	49 42	14 45
75 30	68 9	64 29	60 48	57 6	53 23	49 40	14 30
75 45	68 8	64 27	60 46	57 4	53 22	49 38	14 15
76 0	68 6	64 26	60 45	57 3	53 20	49 36	14 0
76 15	68 5	64 24	60 43	57 1	53 18	49 34	13 45
76 30	68 4	64 23	60 41	56 59	53 16	49 33	13 30
76 45	68 2	64 21	60 40	56 58	53 15	49 31	13 15
77 0	68 1	64 20	60 38	56 56	53 13	49 29	13 0
77 15	68 0	64 19	60 37	56 54	53 11	49 27	12 45
77 30	67 59	64 17	60 35	56 53	53 10	49 26	12 30
77 45	67 58	64 16	60 34	56 51	53 8	49 24	12 15
78 0	67 57	64 15	60 33	56 50	53 7	49 23	12 0
78 15	67 56	64 14	60 31	56 48	53 5	49 21	11 45
78 30	67 54	64 12	60 30	56 47	53 4	49 20	11 30
78 45	67 53	64 11	60 29	56 46	53 2	49 18	11 15
79 0	67 52	64 10	60	56 44	53 1	49 17	11 0

Arc (G. M.)	1½ / 10½	1¾ / 10¼	2 / 10	2¼ / 9¾	2½ / 9½	2¾ / 9¼	meridie / med. noc.
			27				
79 15	67 51	64 9	60	56 43	52 59	49 15	10 45
			26				
79 30	67 50	64 8	60	56 42	52 58	49 14	10 30
			25				
79 45	67 49	64 7	60	56 40	52 57	49 12	10 15
			24				
80 0	67 48	64 6	60	56 39	52 55	49 11	10 0
			23				
80 15	67 48	64 5	60	56 38	52 54	49 10	9 45
			22				
80 30	67 47	64 4	60	56 37	52 53	49 8	9 30
			20				
80 45	67 46	64 3	60	56 36	52 52	49 7	9 15
			19				
81 0	67 45	64 2	60	56 35	52 51	49 6	9 0
			18				
81 15	67 44	64 1	60	56 34	52 49	49 5	8 45
			17				
81 30	67 43	64 0	60	56 32	52 48	49 4	8 30
			16				
81 45	67 43	63 59	60	56 31	52 47	49 3	8 15
			15				
82 0	67 42	63 58	60	56 30	52 46	49 2	8 0
			14				
82 15	67 41	63 57	60	56 30	52 45	49 1	7 45
			14				
82 30	67 40	63 57	60	56 29	52 44	49 0	7 30
			13				
82 45	67 40	63 56	60	56 28	52 43	48 59	7 15
			12				
83 0	67 39	63 55	60	56 27	52 42	48 58	7 0
			11				

Arc (G. M.)	1½ / 10½	1¾ / 10¼	2 / 10	2¼ / 9¾	2½ / 9½	2¾ / 9¼	meridie / med. noc.
83 15	67 38	63 54	60 10	56 26	52 42	48 57	6 45
83 30	67 38	63 54	60 10	56 25	52 41	48 56	6 30
83 45	67 37	63 53	60 9	56 24	52 40	48 55	6 15
84 0	67 37	63 52	60 8	56 24	52 39	48 54	6 0
84 15	67 36	63 52	60 7	56 23	52 38	48 54	5 45
84 30	67 36	63 51	60 7	56 22	52 38	48 53	5 30
84 45	67 35	63 51	60 6	56 22	52 37	48 52	5 15
85 0	67 35	63 50	60 6	56 21	52 36	48 51	5 0
85 15	67 34	63 40	60 5	56 20	52 36	48 51	4 45
85 30	67 34	63 49	60 5	56 20	52 35	48 50	4 30
85 45	67 33	63 49	60 4	56 19	52 35	48 50	4 15
86 0	67 33	63 48	60 4	56 19	52 34	48 49	4 0
86 15	67 33	63 48	60 3	56 18	52 34	48 49	3 45
86 30	67 32	63 48	60 3	56 18	52 33	48 48	3 30
86 45	67 32	63 47	60 2	56 18	52 33	48 48	3 15
87 0	67 32	63 47	60 2	56 17	52 32	48 47	3 0
87 15	67 31	63 47	60	56 17	52 32	48 47	2 45

Arc (G. M.)	1½ / 10½	1¾ / 10¼	2 / 10	2¼ / 9¾	2½ / 9½	2¾ / 9¼	meridie / med. noc.
			2				
87 30	67 31	63 46	60	56 16	52 32	48 47	2 30
			1				
87 45	67 31	63 46	60	56 16	52 31	48 46	2 15
			1				
88 0	67 31	63 46	60	56 16	52 31	48 46	2 0
			1				
88 15	67 30	63 46	60	56 16	52 31	48 46	1 45
			1				
88 30	67 30	63 45	60	56 16	52 31	48 46	1 30
			1				
88 45	67 30	63 45	60	56 15	52 30	48 45	1 15
			0				
89 0	67 30	63 45	60	56 15	52 30	48 45	1 0
			0				
89 15	67 30	63 45	60	56 15	52 30	48 45	0 45
			0				
89 30	67 30	63 45	60	56 15	52 30	48 45	0 30
			0				
89 45	67 30	63 45	60	56 15	52 30	48 45	0 15
			0				
90 0	67 30	63 45	60	56 15	52 30	48 45	0 0
			0				

VI. TABVLA (continued) — page 110

Panel for the hours 3–4¼ (complements 9–7¾), arc restarting at 0° and running to 14°45'. Column heads read as *hour / complement*.

Arc (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
0 0	90 0	90 0	90 0	90 0	90 0	90 0	90 0
0 15	89 45	89 43	89 40	89 38	89 34	89 30	89 45

Arc (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
0 30	89 30	89 26	89 21	89 15	89 8	88 59	89 30
0 45	89 15	89 9	89 1	89 53 [recte: 88 53]	88 42	88 29	89 15
1 0	89 0	88 52	88 42	88 30	88 16	87 58	89 0
1 15	88 45	88 35	88 22	88 8	87 50	87 28	88 45
1 30	88 30	88 17	88 3	87 45	87 24	86 58	88 30
1 45	88 15	88 0	87 43	87 23	86 58	86 27	88 15
2 0	88 0	87 43	87 24	87 1	86 32	85 57	88 0
2 15	87 45	87 26	87 4	86 38	86 7	85 27	87 45
2 30	87 30	87 9	86 45	86 16	85 41	84 57	87 30
2 45	87 16	86 52	86 25	85 54	85 15	84 27	87 15
3 0	87 1	86 35	86 6	85 31	84 49	83 56	87 0
3 15	86 45	86 18	85 46	85 8	84 24	83 27	86 45
3 30	86 30	86 1	85 27	84 46	83 58	82 57	86 30
3 45	86 15	85 44	85 8	84 24	83 32	82 27	86 15
4 0	86 1	85 27	84 48	84 2	83 7	81 57	86 0
4 15	85 46	85 10	84 29	83 40	82 41	81 27	85 45
4 30	85	84 53	84 10	83 18	82	80 58	85 30

Arc (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
	31				16		
4 45	85 16	84 36	83 50	82 56	81 50	80 28	85 15
5 0	85 1	84 19	83 31	82 34	81 25	79 59	85 0
5 15	84 46	84 3	83 12	82 12	81 0	79 29	84 45
5 30	84 31	83 46	82 53	81 50	80 34	79 0	84 30
5 45	84 17	83 29	82 34	81 28	80 9	78 31	84 15
6 0	84 2	83 12	82 15	81 7	79 44	78 2	84 0
6 15	83 47	82 56	81 55	80 45	79 19	77 33	83 45
6 30	83 33	82 39	81 36	80 23	78 54	77 4	83 30
6 45	83 18	82 22	81 17	80 1	78 30	76 36	83 15
7 0	83 3	82 5	80 59	79 40	78 5	76 7	83 0
7 15	82 48	81 49	80 40	79 18	77 40	75 39	82 45
7 30	82 34	81 32	80 21	78 57	77 16	75 10	82 30
7 45	82 19	81 15	80 2	78 35	76 51	74 42	82 15
8 0	82 5	80 59	79 43	78 14	76 27	74 14	82 0
8 15	81 50	80 42	79 24	77 53	76 3	73 47	81 45
8 30	81 36	80 26	79 6	77 31	75 38	73 19	81 30

Arc (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
8 45	81 21	80 9	78 47	77 10	75 14	72 51	81 15
9 0	81 7	79 53	78 29	76 49	74 50	72 24	81 0
9 15	80 52	79 37	78 10	76 28	74 27	71 57	80 45
9 30	80 38	79 21	77 52	76 7	74 3	71 30	80 30
9 45	80 23	79 4	77 33	75 47	73 39	71 3	80 15
10 0	80 9	78 48	77 15	75 26	73 16	70 36	80 0
10 15	79 55	78 32	76 57	75 5	72 52	70 10	79 45
10 30	79 40	78 16	76 38	74 45	72 29	69 43	79 30
10 45	79 26	78 0	76 20	74 24	72 6	69 17	79 15
11 0	79 12	77 44	76 2	74 4	71 43	68 51	79 0
11 15	78 58	77 28	75 44	73 43	71 20	68 25	78 45
11 30	78 43	77 12	75 26	73 23	70 57	67 59	78 30
11 45	78 29	76 56	75 8	73 3	70 34	67 34	78 15
12 0	78 15	76 40	74 50	72 43	70 12	67 8	78 0
12 15	78 1	76 24	74 33	72 23	69 49	66 43	77 45
12 30	77 47	76 8	74 15	72 3	69 27	66 18	77 30
12 45	77	75 52	73 57	71 43	69	65 53	77 15

Arc (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
	33				5		
13 0	77 19	75 37	73 40	71 24	68 43	65 29	77 0
13 15	77 5	75 21	73 22	71 4	68 21	65 4	76 45
13 30	76 52	75 6	73 5	70 44	67 59	64 40	76 30
13 45	76 38	74 50	72 47	70 25	67 37	64 16	76 15
14 0	76 24	74 35	72 30	70 6	67 16	63 52	76 0
14 15	76 10	74 19	72 13	69 47	66 55	63 28	75 45
14 30	75 57	74 4	71 56	69 27	66 33	63 5	75 30
14 45	75 43	73 49	71 39	69 8	66 12	62 42	75 15

VI. TABVLA (continued) — page 111

Same panel (hours 3–4¼, complements 9–7¾), arc continuing from 15°0' to 29°45'.
Column heads read as *hour / complement*.

Arc (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
15 0	75 29	73 33	71 22	68 50	65 51	62 19	75 0
15 15	75 16	73 18	71 5	68 31	65 30	61 56	74 45
15 30	75 2	73 3	70 48	68 12	65 10	61 33	74 30
15 45	74 49	72 48	70 31	67 53	64 49	61 10	74 15
16 0	74 35	72 33	70 14	67 35	64 29	60 48	74 0

Arc (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
16 15	74 22	72 18	69 58	67 17	64 8	60 26	73 45
16 30	74 9	72 3	69 41	66 58	63 48	60 4	73 30
16 45	73 55	71 48	69 25	66 40	63 29	59 42	73 15
17 0	73 42	71 34	69 9	66 22	63 9	59 20	73 0
17 15	73 29	71 19	68 52	66 4	62 49	58 59	72 45
17 30	63 16 [recte: 73 16]	71 4	68 36	65 46	62 29	58 38	72 30
17 45	73 3	70 50	68 20	65 28	62 10	58 17	72 15
18 0	72 50	70 35	68 4	65 11	61 51	57 56	72 0
18 15	72 37	70 21	67 48	64 53	61 31	57 35	71 45
18 30	72 24	70 7	67 32	64 36	61 12	57 15	71 30
18 45	72 11	69 52	67 16	64 19	60 54	56 54	71 15
19 0	71 58	69 38	67 1	64 1	60 35	56 34	71 0
19 15	71 45	69 24	66 45	63 44	60 16	56 14	70 45
19 30	71 32	69 10	66 29	63 27	59 58	55 54	70 30
19 45	71 20	68 56	66 14	63 10	59 40	55 35	70 15
20 0	71 7	68 42	65 58	62 54	59 22	55 15	70 0
20 15	70 54	68 28	65 43	62 37	59	54 56	69 45

Arc (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
					4		
20 30	70 42	68 14	65 28	62 20	58 46	54 27	69 30
20 45	70 29	68 0	65 13	62 4	58 28	54 18	69 15
21 0	70 17	67 46	64 58	61 48	58 10	54 0	69 0
21 15	70 5	67 33	64 43	61 31	57 53	53 41	68 45
21 30	69 52	67 16	64 28	61 15	57 36	53 23	68 30
21 45	69 40	67 6	64 13	60 59	57 18	53 5	68 15
22 0	69 28	66 52	63 59	60 43	57 1	52 47	68 0
22 15	69 16	66 39	63 44	60 28	56 44	52 29	67 45
22 30	69 4	66 26	63 30	60 12	56 28	52 11	67 30
22 45	68 51	66 12	63 15	59 56	56 11	51 54	67 15
23 0	68 39	65 59	63 1	59 41	55 55	51 37	67 0
23 15	68 28	65 46	62 47	59 26	55 38	51 19	66 45
23 30	68 16	65 33	62 32	59 10	55 22	51 2	66 30
23 45	68 4	65 20	62 18	58 55	55 6	50 46	66 15
24 0	67 52	65 7	62 4	58 40	54 50	50 29	66 0
24 15	67 40	64 54	61 50	58 25	54 34	50 13	65 45

Arc (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
24 30	67 29	64 42	61 37	58 10	54 19	49 56	65 30
24 45	67 17	64 29	61 23	57 56	54 3	49 40	65 15
25 0	67 5	64 16	61 9	57 41	53 48	49 24	65 0
25 15	66 54	64 4	60 56	57 27	53 32	49 8	64 45
25 30	66 42	63 51	60 42	57 12	53 17	48 53	64 30
25 45	66 31	63 39	60 29	56 58	53 2	48 37	64 15
26 0	66 20	63 26	60 16	56 44	52 47	48 22	64 0
26 15	66 8	63 14	60 2	56 30	52 33	48 7	63 45
26 30	65 57	63 2	59 49	56 16	52 18	47 52	63 30
26 45	65 46	62 50	59 36	56 2	52 4	47 37	63 15
27 0	65 35	62 38	59 23	55 48	51 49	47 22	63 0
27 15	65 24	62 26	59 11	55 35	51 35	47 7	62 45
27 30	65 13	62 14	58 58	55 21	51 21	46 53	62 30
27 45	65 2	62 2	58 45	55 8	51 7	46 39	62 15
28 0	64 51	61 50	58 32	54 54	50 53	46 25	62 0
28 15	64 40	61 39	58 20	54 41	50 39	46 11	61 45
28 30	64 29	61 27	58 7	54 28	50	45 57	61 30

Arc (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
					26		
28 45	64 19	61 15	57 55	54 15	50	45 43	61 15
					12		
29 0	64 8	61 4	57 43	54 2	49	45 29	61 0
					59		
29 15	63 57	60 52	57 31	53 49	49	45 16	60 45
					46		
29 30	63 47	60 41	57 19	53 37	49	45 3	60 30
					32		
29 45	63 36	60 30	57 7	53 24	49	44 49	60 15
					19		

This opening continues the sixth table of the appendix (**VI. TABVLA**), begun earlier and carried across all three pages here without any fresh table-number rubric being printed. Each page repeats only the running head *Arcus Horizontis* ("Arc of the Horizon") and the two rotated marginal captions. The table gives, for each value of the arc of the horizon (the argument column, in degrees and minutes, running here from 30°0' to 74°45' in steps of 15'), the corresponding **elevation of the pole** for the several hours from noon — the morning hour and its afternoon complement sharing one column (3 and 9, 3¼ and 8¾, and so on), with the final column giving the value at noon / midnight (*meridie / media nocte*), which is simply 90° – arc.

[Margin (left, rotated): *Altitudines poli pro arcubus Horizontis*. — "Elevations of the pole for arcs of the Horizon."]

[Margin (right, rotated): *Altitudines Poli pro arcubus Verticalis primarij*. — "Elevations of the Pole for arcs of the prime Vertical."]

All interior cells are given as G. M. (degrees, minutes). The two paired hour-values head each column as printed: the morning hour above, its afternoon/evening complement below.

Arc — Horæ à & (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
30 0	63 26	60 19	56 55	53	4	44	60 0

Arc — Horæ				$3\frac{3}{4}$ / $8\frac{1}{4}$	4 / 8	$4\frac{1}{4}$ / $7\frac{3}{4}$	meridie / med. noc.
				12	9	36	
					6		
30 15	63 16	60 8	56 43	52	4	44	59 45
				59	8	23	
					5		
					4		
30 30	63 5	59 56	56 31	52	4	44	59 30
				47	8	11	
					4		
					1		
30 45	62 55	59 45	56 19	52	4	43	59 15
				35	8	59	
					2		
					8		
31 0	62 45	59 34	56 8	52	4	43	59 0
				22	8	47	
					1		
					6		
31 15	62 35	59 24	55 56	52	4	43	58 45
				10	8	34	
					4		
31 30	62 25	59 13	55 45	51	4	43	58 30
				59	7	21	
					5		
					1		
31 45	62 15	59 2	55 34	51	4	43	58 15
				47	7	9	
					3		
					9		
32 0	62 5	58 51	55 22	51	4	42	58 0
				35	7	57	
					2		
					7		
32 15	61 55	58 41	55 11	51	4	42	57 45

Arc — Horæ				$3\frac{3}{4}$ / $8\frac{1}{4}$	4 / 8	$4\frac{1}{4}$ / $7\frac{3}{4}$	meridie / med. noc.
				23	7	45	
					1		
					5		
32 30	61 45	58 30	55 0	51 12	4 7	42 33	57 30
					3		
32 45	61 35	58 20	54 49	51 0	4 6	42 21	57 15
					5		
					2		
33 0	61 26	58 9	54 38	50 49	4 6	42 10	57 0
					4		
					0		
33 15	61 16	57 59	54 27	50 38	4 6	41 58	56 45
					2		
					9		
33 30	61 6	57 49	54 16	50 26	4 6	41 47	56 30
					1		
					7		
33 45	60 57	57 39	54 6	50 15	4 6	41 36	56 15
					6		
34 0	60 47	57 29	53 55	50 4	4 5	41 25	56 0
					5		
					5		
34 15	60 38	57 19	53 44	49 54	4 5	41 14	55 45
					4		
					4		
34 30	60 28	57 9	53 34	49	4	41	55 30

Arc — Horæ				$3\frac{3}{4}$ / $8\frac{1}{4}$	4 / 8	$4\frac{1}{4}$ / $7\frac{3}{4}$	meridie / med. noc.
				43	5	3	
					3		
					3		
34 45	60 19	56 59	53 24	49 32	4 5	40 52	55 15
					2		
					2		
35 0	60 10	56 49	53 13	49 21	4 5	40 41	55 0
					1		
					1		
35 15	60 1	56 39	53 3	49 11	4 5	40 31	54 45
					1		
35 30	59 51	56 29	52 53	49 0	4 4	40 20	54 30
					5		
					0		
35 45	59 42	56 20	52 43	48 50	4 4	40 10	54 15
					4		
					0		
36 0	59 33	56 10	52 33	48 40	4 4	40 0	54 0
					2		
					9		
36 15	59 24	56 1	52 23	48 30	4 4	39 50	53 45
					1		
					9		
36 30	59 15	55 51	52 13	48 20	4 4	39 40	53 30
					9		
36 45	59 6	55 42	52 3	48	4	39	53 15

Arc — Horæ à & (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
				9	3	30	
					5		
					9		
37 0	58 8 [recte: 58 57]	55 32	51 54	48 0	4 3	39 20	53 0
					4		
					9		
37 15	58 49	55 23	51 44	47 50	4 3	39 10	52 45
					3		
					9		
37 30	58 40	55 14	51 34	47 40	4 3	39 1	52 30
					2		
					9		
37 45	58 31	55 5	51 25	47 30	4 3	38 51	52 15
					1		
					9		
38 0	58 23	54 56	51 15	47 20	4 3	38 42	52 0
					1		
					0		
38 15	58 14	54 47	51 6	47 11	4 3	38 32	51 45
					0		
38 30	58 6	54 38	50 57	47 2	4 2	38 23	51 30
					5		
					1		
38 45	57 57	54 29	50 48	46 52	4 2	38 14	51 15
					4		
					1		

Arc — Horæ à & (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
39 0	57 49	54 20	50 39	46 43	4 2	38 5	51 0
					3 2		
39 15	57 41	54 11	50 30	46 34	4 2	37 56	50 45
					2 3		
39 30	57 32	54 3	50 21	46 25	4 2	37 47	50 30
					1 4		
39 45	57 24	53 54	50 12	46 16	4 2	37 38	50 15
					5		
40 0	57 16	53 46	50 3	46 7	4 1	37 30	50 0
					5 6		
40 15	57 8	53 37	49 54	45 58	4 1	37 21	49 45
					4 7		
40 30	57 0	53 29	49 46	45 49	4 1	37 13	49 30
					3 8		
40 45	56 52	53 20	49 37	45 40	4 1	37 4	49 15
					3 0		
41 0	56 44	53 12	49 28	45 31	4 1	36 56	49 0
					2		

Arc — Horæ à & (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
					1		
41 15	56 36	53 4	49 20	45 23	4 1	36 48	48 45
					1		
					2		
41 30	56 28	52 56	49 11	45 14	4 1	36 39	48 30
					4		
41 45	56 20	52 47	49 3	45 6	4 0	36 31	48 15
					5		
					6		
42 0	56 13	52 39	48 55	44 58	4 0	36 23	48 0
					4		
					7		
42 15	56 5	52 31	48 46	44 49	4 0	36 15	47 45
					3		
					9		
42 30	55 57	52 23	48 38	44 41	4 0	36 8	47 30
					3		
					1		
42 45	55 50	52 16	48 30	44 33	4 0	36 0	47 15
					2		
					3		
43 0	55 42	52 8	48 22	44 25	4 0	35 52	47 0
					1		
					5		
43 15	55 35	52 0	48 14	44 17	4 0	35 45	46 45

Arc — Horæ à & (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
					7		
43 30	55 28	51 52	48 6	44 9	3 9	35 37	46 30
					5 9		
43 45	55 20	51 45	47 59	44 1	3 9	35 30	46 15
					5 2		
44 0	55 13	51 37	47 51	43 53	3 9	35 22	46 0
					4 4		
44 15	55 6	51 29	47 43	43 45	3 9	35 15	45 45
					3 6		
44 30	54 58	51 22	47 35	43 38	3 9	35 8	45 30
					2 9		
44 45	54 51	51 15	47 28	43 30	3 9	35 1	45 15
					2 1		
45 0	54 44	51 7	47 20	43 23	3 9	34 54	45 0
					1 4		
45 15	54 37	51 0	47 13	43 15	3 9	34 47	44 45
					7		
45 30	54 30	50 53	47 6	43 8	3 8	34 40	44 30

Arc — Horæ				$3\frac{3}{4}$ / $8\frac{1}{4}$	4 / 8	$4\frac{1}{4}$ / $7\frac{3}{4}$	meridie / med. noc.
					5 9		
45 45	54 23	50 46	46 58	43 1	3 8	34 33	44 15
					5 2		
46 0	54 16	50 38	46 51	42 53	3 8	34 26	44 0
					4 5		
46 15	54 9	50 31	46 44	42 46	3 8	34 19	43 45
					3 8		
46 30	54 3	50 24	46 37	42 39	3 8	34 13	43 30
					3 1		
46 45	53 56	50 17	46 30	42 32	3 8	34 6	43 15
					2 4		
47 0	53 49	50 10	46 23	42 25	3 8	33 59	43 0
					1 7		
47 15	53 43	50 4	46 16	42 18	3 8	33 53	42 45
					1 1		
47 30	53 36	49 57	46 9	42 11	3 8	33 47	42 30
					4		
47 45	53 29	49 50	46 2	42	3	33	42 15

Arc — Horæ à & (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
				4	7	40	
					5		
					7		
48 0	53 23	49 43	45 55	41 58	3 7	33 34	42 0
					5		
					1		
48 15	53 16	49 37	45 48	41 51	3 7	33 28	41 45
					4		
					4		
48 30	53 10	49 30	45 42	41 44	3 7	33 22	41 30
					3		
					8		
48 45	53 4	49 23	45 35	41 38	3 7	33 16	41 15
					3		
					1		
49 0	52 57	49 17	45 28	41 32	3 7	33 10	41 0
					2		
					5		
49 15	52 51	49 11	45 22	41 25	3 7	33 4	40 45
					1		
					9		
49 30	52 45	49 5	45 16	41 18	3 7	32 58	40 30
					1		
					2		
49 45	52 39	48 59	45 10	41 12	3 7	32 53	40 15
					6		

Arc — Horæ à & (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
50 0	52 33	48 52	45 3	41 6	3 7	32 46	40 0
50 15	52 27	49 46 [recte: 48 46]	44 57	41 0	3 6	32 41	39 45
50 30	52 20	48 39	44 50	40 53	3 6	32 35	39 30
50 45	52 15	48 33	44 44	40 47	3 6	32 29	39 15
51 0	52 9	48 27	44 38	40 41	3 6	32 24	39 0
51 15	52 3	48 21	44 32	40 35	3 6	32 18	38 45
51 30	51 57	48 15	44 26	40 29	3 6	32 13	38 30
51 45	51 51	48 9	44 20	40 24	3 6	32 8	38 15
52 0	51 46	48 4	44 14	40 18	3 6	32 2	38 0

Arc — Horæ à & (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
					4		
52 15	51 40	47 58	44 8	40 12	3 6	31 57	37 45
					8		
52 30	51 34	47 52	44 3	40 6	3 6	31 52	37 30
					3		
52 45	51 29	47 46	44 57 [recte: 43 57]	40 1	3 5	31 47	37 15
					5		
					7		
53 0	51 23	47 41	43 51	39 55	3 5	31 42	37 0
					5		
					2		
53 15	51 18	47 35	43 46	39 50	3 5	31 37	36 45
					4		
					7		
53 30	51 12	47 29	43 40	39 44	3 5	31 32	36 30
					4		
					1		
53 45	51 7	47 24	43 35	39 39	3 5	31 27	36 15
					3		
					6		
54 0	51 2	47 18	43 29	39 33	3 5	31 22	36 0
					3		
					1		
54 15	50 56	47 13	43 24	39 28	3 5	31 17	35 45
					2		

Arc — Horæ à & (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
					6		
54 30	50 51	47 8	43 18	39 23	3 5	31 13	35 30
					2		
					1		
54 45	50 46	47 2	43 13	39 17	3 5	31 8	35 15
					1		
					6		
55 0	50 41	46 57	43 8	39 12	3 5	31 3	35 0
					1		
					1		
55 15	50 36	46 52	43 3	39 7	3 5	30 58	34 45
					6		
55 30	50 30	46 47	42 57	39 2	3 5	30 54	34 30
					1		
55 45	50 25	46 42	42 52	38 57	3 4	30 49	34 15
					5		
					6		
56 0	50 20	46 37	42 47	38 52	3 4	30 45	34 0
					5		
					1		
56 15	50 15	46 32	42 42	38 47	3 4	30 40	33 45
					4		
					6		
56 30	50 11	46 27	42 37	38 42	3 4	30 36	33 30
					4		

Arc — Horæ à & (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
					2		
56 45	50 6	46 22	42 32	38 37	3 4	30 32	33 15
					3		
					7		
57 0	50 1	46 17	42 27	38 33	3 4	30 27	33 0
					3		
					3		
57 15	49 56	46 12	42 23	38 28	3 4	30 23	32 45
					2		
					8		
57 30	49 51	46 7	42 18	38 23	3 4	30 19	32 30
					2		
					4		
57 45	49 47	46 2	42 13	38 19	3 4	30 15	32 15
					1		
					9		
58 0	49 42	45 58	42 8	38 14	3 4	30 11	32 0
					1		
					5		
58 15	49 37	45 53	42 4	38 10	3 4	30 7	31 45
					1		
					0		
58 30	49 33	45 48	41 59	38 5	3 4	30 3	31 30
					6		
58 45	49 28	45 44	41 55	38 1	3 4	29 59	31 15

Arc — Horæ à & (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
					2		
59 0	49 24	45 39	41 50	37 56	3 3	29 55	31 0
					5		
					8		
59 15	49 10 [recte: 49 19]	45 35	41 46	37 52	3 3	29 51	30 45
					5		
					4		
59 30	49 15	45 30	41 41	37 48	3 3	29 47	30 30
					4		
					9		
59 45	49 11	45 26	41 37	37 43	3 3	29 43	30 15
					4		
					5		
60 0	49 6	45 22	41 32	37 39	3 3	29 40	30 0
					4		
					1		
60 15	49 2	45 17	41 28	37 35	3 3	29 36	29 45
					3		
					7		
60 30	48 58	45 13	41 24	37 31	3 3	29 32	29 30
					3		
					3		
60 45	48 54	45 9	41 20	37 27	3 3	29 29	29 15
					3		
					0		
61 0	48 50	45 5	41 16	37	3	29	29 0

Arc — Horæ				$3\frac{3}{4}$ / $8\frac{1}{4}$	4 / 8	$4\frac{1}{4}$ / $7\frac{3}{4}$	meridie / med. noc.
	3 / 9	$3\frac{1}{4}$ / $8\frac{3}{4}$	$3\frac{1}{2}$ / $8\frac{1}{2}$	23	3	25	
					2		
					6		
61 15	48 45	45 0	41 12	37 19	3 3	29 21	28 45
					2		
					2		
61 30	48 41	44 56	41 8	37 15	3 3	29 18	28 30
					1		
					8		
61 45	48 37	44 52	41 4	37 11	3 3	29 14	28 15
					1		
					4		
62 0	48 33	44 48	41 0	37 7	3 3	29 11	28 0
					1		
					1		
62 15	48 29	44 44	40 56	37 3	3 3	29 8	27 45
					8		
62 30	48 26	44 40	40 52	36 59	3 3	29 4	27 30
					4		
62 45	48 22	44 37	40 48	36 56	3 3	29 1	27 15
					0		
63 0	48 18	44 33	40 44	36 52	3 2	28 58	27 0
					5		
					7		
63 15	48 14	44 29	40 40	36 48	3 2	28 55	26 45

Arc — Horæ à & (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
					5 3		
63 30	48 10	44 25	40 37	36 45	3 2	28 51	26 30
					5 0		
63 45	48 7	44 21	40 33	36 41	3 2	28 48	26 15
					4 6		
64 0	48 3	44 18	40 29	36 38	3 2	28 45	26 0
					4 3		
64 15	47 59	44 14	40 26	36 34	3 2	28 42	25 45
					4 0		
64 30	47 56	44 11	40 22	36 31	3 2	28 39	25 30
					3 6		
64 45	47 52	44 7	40 19	36 27	3 2	28 36	25 15
					3 3		
65 0	47 49	44 3	40 15	36 24	3 2	28 33	25 0
					3 0		
65 15	47 45	44 0	40 12	36 21	3 2	28 30	24 45
					2 7		

Arc — Horæ à & (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
65 30	47 42	43 57	40 8	36 17	3 2	28 27	24 30
					2 4		
65 45	47 39	43 53	40 5	36 14	3 2	28 24	24 15
					2 1		
66 0	47 35	43 50	40 2	36 11	3 2	28 22	24 0
					1 8		
66 15	47 32	43 46	39 58	36 8	3 2	28 19	23 45
					1 5		
66 30	47 29	43 43	39 55	36 5	3 2	28 16	23 30
					1 2		
66 45	47 25	43 40	39 52	36 2	3 2	28 13	23 15
					9		
67 0	47 22	43 37	39 49	35 59	3 2	28 11	23 0
					6		
67 15	47 19	43 34	39 46	35 56	3 2	28 8	22 45
					3		
67 30	47 16	43 30	39 43	35 53	3 2	28 6	22 30
					0		
67 45	47 13	43 27	39 40	35 50	3 1	28 3	22 15

Arc — Horæ à & (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
					5 7		
68 0	47 10	43 24	39 37	35 47	3 1	28 0	22 0
					5 5		
68 15	47 7	43 21	39 34	35 44	3 1	27 58	21 45
					5 2		
68 30	47 4	43 18	39 31	35 41	3 1	27 55	21 30
					4 9		
68 45	47 1	43 15	39 28	35 38	3 1	27 53	21 15
					4 7		
69 0	46 58	43 13	39 25	35 35	3 1	27 51	21 0
					4 4		
69 15	46 55	43 10	39 22	35 33	3 1	27 48	20 45
					4 1		
69 30	46 52	43 7	39 19	35 30	3 1	27 46	20 30
					3 9		
69 45	46 50	43 4	39 17	35 27	3 1	27 44	20 15
					3 6		

Arc — Horæ à & (G. M.)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
70 0	46 47	43 1	39 14	35 25	3 1	27 41	20 0
					3 4		
70 15	46 44	42 59	39 11	35 22	3 1	27 39	19 45
					3 2		
70 30	46 41	42 56	39 9	35 20	3 1	27 37	19 30
					2 9		
70 45	46 39	42 53	39 6	35 17	3 1	27 35	19 15
					2 7		
71 0	46 36	42 51	39 4	35 15	3 1	27 33	19 0
					2 5		
71 15	46 34	42 48	39 1	35 12	3 1	27 31	18 45
					2 2		
71 30	46 31	42 46	38 59	35 10	3 1	27 29	18 30
					2 0		
71 45	46 29	42 43	38 56	35 8	3 1	27 26	18 15
					1 8		
72 0	46 26	42 41	38 54	35 5	3 1	27 24	18 0

Arc — Horæ				$3\frac{3}{4}$ / $8\frac{1}{4}$	4 / 8	$4\frac{1}{4}$ / $7\frac{3}{4}$	meridie / med. noc.
					1 6		
72 15	46 24	42 38	38 51	35 3	3 1	27 22	17 45
					1 3		
72 30	46 21	42 36	38 49	35 1	3 1	27 21	17 30
					1 1		
72 45	46 19	42 34	38 47	34 59	3 1	27 19	17 15
					9		
73 0	46 17	42 32	38 44	34 57	3 1	27 17	17 0
					7		
73 15	46 14	42 29	38 42	34 54	3 1	27 15	16 45
					5		
73 30	46 12	42 27	38 40	34 52	3 1	27 13	16 30
					3		
73 45	46 10	42 25	38 38	34 50	3 1	27 11	16 15
					1		
74 0	46 8	42 22	38 36	34 48	3 0	27 9	16 0
					5 9		
74 15	46 6	42 20	38 34	34 46	3 0	27 8	15 45
					5 8		
74 30	46 4	42 18	38 32	34	3	27	15 30

Arc — Horæ à & (G. M.)				3¾ / 8¼	4 / 8	4¼ / 7¾	meridie / med. noc.
				44	0	6	
					5		
					6		
74 45	46 2	42 16	38 30	34	3	27	15 15
				42	0	4	
					5		
					4		

The table's lower border closes on the third page (printed p. 114), but the argument has not yet reached its limit; the catchword *Arcus* at the foot of each page shows that the sixth table continues onto the following page for arcs beyond 74°45'.

These three pages continue, without interruption, the sixth table of the appendix (the *VI. Tabula*, headed *Arcus Horizontis* — "Arc of the Horizon"). No new table-number rubric is printed on any of them. Printed page 115 completes the hour-panel begun in the preceding pages (morning hours 3–4¼ with their afternoon complements 9–7¾), the arc-argument running from 75° up to its limit of 90°. Printed pages 116–117 open the next hour-panel of the same table (hours 4½–5¾ with complements 7½–6¼), the arc-argument restarting from 0° and running here to 29°45'; it carries on beyond this opening.

Each page is a single wide numeric grid. The left-hand argument column gives the **arc** (of the Horizon, or of the prime Vertical) in degrees and minutes; its printed head reads "Horæ à &" over "G. M." The body cells give the corresponding **altitude of the pole** (*Altitudo Poli*) in degrees and minutes, for each hour-from-noon (and its afternoon/night complement) named in the column heads. The final column, headed "meridie / med. noc." ("noon / midnight"), equals 90° – arc throughout.

[Margin (left, rotated, all three pages): *Altitudines poli pro arcubus Horizontis*. — "Altitudes of the pole for arcs of the Horizon."] [Margin (right, rotated, all three pages): *Altitudines Poli pro arcubus Verticalis primarij*. — "Altitudes of the Pole for arcs of the prime Vertical."]

Sixth table (*Arcus Horizontis*), continued — printed p. 115
(hours 3–4¼ / 9–7¾)

Column heads (verbatim): argument "Horæ à &" (G. M.); then the hour pairs "3 / 9", "3¼ / 8¾", "3½ / 8½", "3¾ / 8¼", "4 / 8", "4¼ / 7¾", each over "G. M."; final "meridie / med. noc." (G. M.).

Arc (G M)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¼	4 / 8	4¼ / 7¾	mer. / m.noc.
75 0	46 0	42 14	38 28	34 40	30 52	27 3	15 0
75 15	45 58	42 12	38 26	34 39	30 50	27 1	14 45
75 30	45 56	42 10	38 24	34 37	30 49	27 0	14 30
75 45	45 54	42 8	38 22	34 35	30 47	26 58	14 15
76 0	45 52	42 6	38 20	34 33	30 45	26 57	14 0
76 15	45 50	42 5	38 18	34 32	30 54 [recte □30 43; see note a]	26 55	13 45
76 30	45 48	42 3	38 17	34 30	30 42	26 54	13 30
76 45	45 46	42 1	38 15	34 28	30 40	26 52	13 15
77 0	45 45	41 59	38 13	34 26	30 39	26 51	13 0
77 15	45 43	41 58	38 12	34 25	30 37	26 49	12 45
77 30	45 41	41 56	38 10	34 23	30 36	26 48	12 30
77 45	45 40	41 54	38 8	34 22	30 34	26 47	12 15
78 0	45 38	41 53	38 7	34 20	30 33	26 45	12 0
78 15	45	41 51	38 5	34 19	30 32	26 44	11 45

Arc (G M)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¾	4 / 8	4¼ / 7¾	mer. / m.noc.
	36			[?]			
78 30	45	41 50	38 4	34 17	30 30	26 43	11 30
	35			[?]			
78 45	45	41 48	38 2	34 16	30 29	26 42	11 15
	33			[?]			
79 0	45	41 47	38 1	34 15	30 28	26 40	11 0
	32						
79 15	45	41 45	37 59	34 13	30 26	26 39	10 45
	30						
79 30	45	41 44	37 58	34 12	30 25	26 38	10 30
	29						
79 45	45	41 42	37 57	34 11	30 24	26 37	10 15
	28						
80 0	45	41 41	37 55	34 9	30 23	26 36	10 0
	26						
80 15	45	41 40	37 54	34 8	30 22	26 35	9 45
	25						
80 30	45	41 39	37 53	34 7	30 21	26 34	9 30
	24						
80 45	45	41 37	37 52	34 6	30 20	26 33	9 15
	23						
81 0	45	41 36	37 51	34 5	30 18	26 32	9 0
	22						
81 15	45	41 35	37 49	34 4	30 17	26 31	8 45
	20						
81 30	45	41 34	37 48	34 3	30 16	26 30	8 30
	19						
81 45	45	41 33	37 47	34 2	30 16	26 29	8 15
	18						
82 0	45	41 32	37 46	34 1	30 15	26 28	8 0
	17						
82 15	45	41 31	37 45	34 0	30 14	26 28	7 45
	16						

Arc (G M)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¾	4 / 8	4¼ / 7¾	mer. / m.noc.
82 30	45 15	41 30	37 44	33 59	30 13	26 27	7 30
82 45	45 14	41 29	37 43	33 58	30 12	26 26	7 15
83 0	45 13	41 28	37 42	33 57	30 11	26 25	7 0
83 15	45 12	41 27	37 42	33 56	30 10	26 25	6 45
83 30	45 11	41 26	37 41	33 55	30 10	26 24	6 30
83 45	45 10	41 25	37 40	33 54	30 9	26 23	6 15
84 0	45 9	41 24	37 39	33 54	30 8	26 23	6 0
84 15	45 9	41 24	37 38	33 53	30 8	26 22	5 45
84 30	45 8	41 23	37 38	33 53	30 7	26 21	5 30
84 45	45 7	41 22	37 37	33 52	30 6	26 21	5 15
85 0	45 7	41 22	37 36	33 51	30 6	26 20	5 0
85 15	45 6	41 21	37 36	33 50	30 5	26 20	4 45
85 30	45 5	41 20	37 35	33 50	30 5	26 19	4 30
85 45	45 5	41 20	37 35	33 49	30 4	26 19	4 15
86 0	45 4	41 19	37 34	33 49	30 4	26 18	4 0
86 15	45 4	41 18	37 34	33 48	30 3	26 18	3 45
86 30	45	41 18	37 33	33 48	30 3	26 18	3 30

Arc (G M)	3 / 9	3¼ / 8¾	3½ / 8½	3¾ / 8¾	4 / 8	4¼ / 7¾	mer. / m.noc.
	3						
86 45	45	41 17	37 33	33 48	30 2	26 17	3 15
	3						
87 0	45	41 17	37 32	33 47	30 2	26 17	3 0
	2						
87 15	45	41 17	37 32	33 47	30 2	26 17	2 45
	2						
87 30	45	41 17	37 32	33 47	30 1	26 16	2 30
	2						
87 45	45	41 16	37 31	33 46	30 1	26 16	2 15
	1						
88 0	45	41 16	37 31	33 46	30 1	26 16	2 0
	1						
88 15	45	41 16	37 31	33 46	30 1	26 16	1 45
	1						
88 30	45	41 16	37 31	33 45	30 0	26 15	1 30
	1						
88 45	45	41 15	37 30	33 45	30 0	26 15	1 15
	0						
89 0	45	41 15	37 30	33 45	30 0	26 15	1 0
	0						
89 15	45	41 15	37 30	33 45	30 0	26 15	0 45
	0						
89 30	45	41 15	37 30	33 45	30 0	26 15	0 30
	0						
89 45	45	41 15	37 30	33 45	30 0	26 15	0 15
	0						
90 0	45	41 15	37 30	33 45	30 0	26 15	0 0
	0						

Sixth table (*Arcus Horizontis*), continued — printed p. 116
(hours $4\frac{1}{2}$ – $5\frac{3}{4}$ / $7\frac{1}{2}$ – $6\frac{1}{4}$)

New hour-panel; the arc-argument restarts at 0°. Column heads (verbatim):
argument "Horæ à &" (G. M.); hour pairs " $4\frac{1}{2}$ / $7\frac{1}{2}$ ", " $4\frac{3}{4}$ / $7\frac{1}{4}$ ", "5 / 7", " $5\frac{1}{4}$ / $6\frac{3}{4}$ ",
" $5\frac{1}{2}$ / $6\frac{1}{2}$ ", " $5\frac{3}{4}$ / $6\frac{1}{4}$ ", each over "G. M."; final "meridie / med. noc." (G. M.).

Arc (G M)	$4\frac{1}{2}$ / $7\frac{1}{2}$	$4\frac{3}{4}$ / $7\frac{1}{4}$	5 / 7	$5\frac{1}{4}$ / $6\frac{3}{4}$	$5\frac{1}{2}$ / $6\frac{1}{2}$	$5\frac{3}{4}$ / $6\frac{1}{4}$	mer. / m.noc.
0 0	90 0	90 0	90 0	90 0	90 0	90 0	90 0
0 15	89 24	89 16	89 4	88 45	88 6	86 12	89 45
0 30	88 48	88 32	88 8	87 29	86 12	82 25	89 30
0 45	88 11	87 48	87 12	86 14	84 19	78 42	89 15
1 0	87 35	87 3	86 16	84 59	82 27	75 5	89 0
1 15	86 59	86 19	85 21	83 45	80 36	71 36	88 45
1 30	86 23	85 35	84 25	82 30	78 45	68 14	88 30
1 45	85 47	85 52 [recte: 84 52]	83 30	81 16	76 56	65 1	88 15
2 0	85 11	84 8	82 35	80 3	75 9	61 58	88 0
2 15	84 35	83 24	81 40	78 50	73 24	59 5	87 45
2 30	83 59	82 41	80 45	77 38	71 40	56 21	87 30
2 45	83 24	81 57	79 51	76 26	69 59	53 47	87 15
3 0	82 44	81 14	78 57	75 15	68 19	51 23	87 0
3 15	82 12	80 31	78	74 5	66 42	49 9	86 45

Arc (G M)	4½ / 7½	4¾ / 7¼	5 / 7	5¼ / 6¾	5½ / 6½	5¾ / 6¼	mer. / m.noc.
			3				
3 30	81 37	79 48	77 10	72 56	65 7	47 2	86 30
3 45	81 2	79 6	76 17	71 48	63 35	45 4	86 15
4 0	80 26	78 23	75 24	70 41	62 5	43 3	86 0
4 15	79 51	77 41	74 32	69 34	60 37	41 29	85 45
4 30	79 16	76 59	73 41	68 28	59 12	39 52	85 30
4 45	78 42	76 17	72 50	67 24	57 50	38 22	85 15
5 0	78 7	75 36	71 59	66 20	56 30	36 57	85 0
5 15	77 33	74 55	71 9	65 18	55 12	35 37	84 45
5 30	76 58	74 14	70 19	64 16	53 57	34 22	84 30
5 45	76 24	73 33	69 30	63 16	52 44	33 12	84 15
6 0	75 50	72 53	68 41	62 17	51 33	32 5	84 0
6 15	75 16	72 13	67 53	61 18	50 25	31 3	83 45
6 30	74 43	71 33	67 6	60 21	49 19	30 4	83 30
6 45	74 9	70 54	66 19	59 25	48 15	29 9	83 15
7 0	73 36	70 15	65 33	58 30	47 13	28 16	83 0
7 15	73 3	69 36	64 47	57 36	46 13	27 27	82 45

Arc (G M)	4½ / 7½	4¾ / 7¼	5 / 7	5¼ / 6¾	5½ / 6½	5¾ / 6¼	mer. / m.noc.
7 30	72 37	68 58	64 2	56 44	45 15	26 40	82 30
7 45	71 58	68 20	63 17	55 53	44 19	25 55	82 15
8 0	71 26	67 42	62 33	55 1	43 25	25 13	82 0
8 15	70 54	67 5	61 50	54 12	42 32	24 33	81 45
8 30	70 22	66 28	61 7	53 23	41 41	23 55	81 30
8 45	69 50	65 52	60 25	52 36	40 52	23 19	81 15
9 0	69 19	65 16	59 43	51 49	40 5	22 44	81 0
9 15	68 47	64 40	59 2	51 4	39 19	22 11	80 45
9 30	68 16	64 4	58 22	50 19	38 35	21 40	80 30
9 45	67 46	63 29	57 42	49 35	37 52	21 9	80 15
10 0	67 15	62 54	57 3	48 53	37 10	20 41	80 0
10 15	66 45	62 20	56 25	48 11	36 30	20 13	79 45
10 30	66 15	61 46	55 47	47 30	35 51	19 47	79 30
10 45	65 45	61 13	55 10	46 50	35 13	19 22	79 15
11 0	65 16	60 40	54 33	46 11	34 36	18 57	79 0
11 15	64 47	60 7	53 57	45 33	34 1	18 34	78 45
11 30	64 18	59 34	53	44 56	33 26	18 12	78 30

Arc (G M)	4½ / 7½	4¾ / 7¼	5 / 7	5¼ / 6¾	5½ / 6½	5¾ / 6¼	mer. / m.noc.
			21				
11 45	63 49	59 2	52 46	44 20	32 53	17 50	78 15
12 0	63 21	58 31	52 12	43 44	32 21	17 30	78 0
12 15	62 53	58 0	51 38	43 9	31 50	17 10	77 45
12 30	62 25	57 29	51 4	42 35	31 19	16 51	77 30
12 45	61 57	56 58	50 31	42 2	30 49	16 32	77 15
13 0	61 30	56 28	49 59	41 29	30 20	16 15	77 0
13 15	61 3	55 58	49 28	40 57	29 52	15 58	76 45
13 30	60 36	55 29	48 56	40 26	29 25	15 41	76 30
13 45	60 9	55 0	48 25	39 55	28 59	15 25	76 15
14 0	59 43	54 31	47 55	39 26	28 33	15 10	76 0
14 15	59 17	54 3	47 26	38 57	28 8	14 55	75 45
14 30	58 51	53 35	46 57	38 28	27 44	14 40	75 30
14 45	58 25	53 8	46 28	38 0	27 20	14 26	75 15

Sixth table (*Arcus Horizontis*), continued — printed p. 117
(hours 4½–5¾ / 7½–6¼, arc-argument continuing)

Same column heads as p. 116. The arc-argument continues from 15°0′.

Arc (G M)	4½ / 7½	4¾ / 7¼	5 / 7	5¼ / 6¾	5½ / 6½	5¾ / 6¼	mer. / m.noc.
15 0	58 0	52 41	46 0	37 33	26 58	14 13	75 0
15 15	57 35	52 14	45 32	37 6	26 36	14 0	74 45
15 30	57 10	51 47	45 5	36 40	26 13	13 47	74 30
15 45	56 46	51 21	44 38	36 14	25 52	13 35	74 15
16 0	56 21	50 55	44 12	35 49	25 32	13 23	74 0
16 15	55 57	50 30	43 46	35 24	25 12	13 11	73 45
16 30	55 34	50 5	43 20	35 0	24 52	13 0	73 30
16 45	55 10	49 40	42 55	34 37	24 33	12 49	73 15
17 0	54 47	49 16	42 30	34 14	24 15	12 38	73 0
17 15	54 24	48 52	42 6	33 51	23 56	12 28	72 45
17 30	54 2	48 28	41 42	33 29	23 39	12 18	72 30
17 45	53 39	48 4	41 19	33 7	23 21	12 8	72 15
18 0	53 17	47 41	40 56	32 46	23 5	11 59	72 0
18 15	52 55	47 18	40 33	32 26	22 48	11 49	71 45
18 30	52 33	46 56	40 11	32 5	22 32	11 40	71 30
18 45	52 11	46 34	39 49	31 45	22 16	11 31	71 15
19 0	51 50	46 12	39	31 25	22 1	11 23	71 0

Arc (G M)	4½ / 7½	4¾ / 7¼	5 / 7	5¼ / 6¾	5½ / 6½	5¾ / 6¼	mer. / m.noc.
			27				
19 15	51 29	45 50	39 6	31 6	21 46	11 15	70 45
19 30	51 8	45 29	38 45	30 47	21 31	11 7	70 30
19 45	50 48	45 8	38 25	30 29	21 17	10 59	70 15
20 0	50 27	44 47	38 5	30 11	21 3	10 51	70 0
20 15	50 7	44 27	37 45	29 53	20 50	10 43	69 45
20 30	49 47	44 6	37 25	29 36	20 37	10 36	69 30
20 45	49 28	43 46	37 6	29 19	20 24	10 29	69 15
21 0	49 8	43 27	36 47	29 2	20 11	10 22	69 0
21 15	48 49	43 7	36 29	28 45	19 58	10 15	68 45
21 30	48 30	42 48	36 10	28 29	19 46	10 8	68 30
21 45	48 11	42 29	35 52	28 14	19 34	10 2	68 15
22 0	47 52	42 11	35 35	27 58	19 22	9 55	68 0
22 15	47 34	41 53	35 17	27 43	19 10	9 49	67 45
22 30	47 16	41 35	35 0	27 28	18 59	9 43	67 30
22 45	46 58	41 17	34 43	27 13	18 48	9 37	67 15
23 0	46 40	40 59	34 26	26 59	18 37	9 31	67 0

Arc (G M)	4½ / 7½	4¾ / 7¼	5 / 7	5¼ / 6¾	5½ / 6½	5¾ / 6¼	mer. / m.noc.
23 15	46 23	40 42	34 10	26 45	18 27	9 26	66 45
23 30	46 5	40 25	33 54	26 31	18 16	9 20	66 30
23 45	45 48	40 8	33 38	26 17	18 6	9 15	66 15
24 0	45 31	39 51	33 23	26 4	17 56	9 9	66 0
24 15	45 15	39 34	33 7	25 51	17 46	9 4	65 45
24 30	44 58	39 18	32 52	25 38	17 37	8 59	65 30
24 45	44 42	39 2	32 37	25 25	17 27	8 54	65 15
25 0	44 25	38 46	32 23	25 12	17 18	8 49	65 0
25 15	44 9	38 31	32 9	25 0	17 9	8 44	64 45
25 30	43 54	38 15	31 54	24 48	17 0	8 39	64 30
25 45	43 38	38 0	31 40	24 36	16 52	8 35	64 15
26 0	43 23	37 45	31 26	24 24	16 43	8 30	64 0
26 15	43 7	37 30	31 12	24 13	16 35	8 26	63 45
26 30	42 52	37 16	30 59	24 2	16 26	8 21	63 30
26 45	42 37	37 1	30 46	23 51	16 18	8 17	63 15
27 0	42 23	36 47	30 33	23 40	16 10	8 13	63 0
27 15	42 8	36 33	30	23 29	16 2	8 9	62 45

Arc (G M)	4½ / 7½	4¾ / 7¼	5 / 7	5¼ / 6¾	5½ / 6½	5¾ / 6¼	mer. / m.noc.
			20				
27 30	41 54	36 19	30	23 19	15 55	8 5	62 30
			8				
27 45	41 39	36 6	29	22 8 [recte: 23 8]	15 47	8 1	62 15
			55				
28 0	41 25	35 52	29	22 58	15 40	7 57	62 0
			43				
28 15	41 11	35 39	29	22 48	15 33	7 53	61 45
			31				
28 30	40 57	35 26	29	22 38	15 25	7 49	61 30
			19				
28 45	40 44	35 13	29	22 28	15 18	7 46	61 15
			7				
29 0	40 31	35 0	28	22 18	15 12	7 42	61 0
			56				
29 15	40 17	34 47	28	22 9	15 5	7 38	60 45
			44				
29 30	40 4	34 35	28	22 0	14 58	7 35	60 30
			33				
29 45	39 51	34 23	28	21 51	14 52	7 31	60 15
			22				

**Table VI — *Arcus Horizontis* ("Arc of the Horizon"),
continued**

[Margin (left, rotated, running the height of each page): *Altitudines poli pro arcubus Horizontis*. — "Pole-altitudes for arcs of the Horizon."]

[Margin (right, rotated, running the height of each page): *Altitudines Poli pro arcubus Verticalis primarij*. — "Pole-altitudes for arcs of the prime Vertical."]

The sixth table continues without a break across all three pages of this opening. No fresh table-number rubric is printed; the argument (the arc, headed *Horæ à &*, in degrees G. and minutes M.) runs from 30°0' to 74°45', and the pole-altitude entries are tabulated under the paired hour-columns 4½/7½, 4¾/7¼, 5/7, 5¼/6¾, 5½/6½, 5¾/6¼, and the noon/midnight column (*meridie / med. noc.*). The

complementary hours in each header pair sum to 12; the final column equals 90° minus the arc. The column header is reprinted at the top of each printed page. The table has not yet reached its limit and continues onto the following page.

Each cell below holds the two printed sub-columns (G. = degrees, M. = minutes) of one hour-column, transcribed as "degrees minutes".

Printed page 118 (arc 30°0' – 44°45')

<i>Horæ à &</i> (arc)G. M.	$4\frac{1}{2}$ / 7½G. M.	$4\frac{3}{4}$ / 7¼G. M.	5 / 7G. M.	$5\frac{1}{4}$ / 6¾G. M.	$5\frac{1}{2}$ / 6½G. M.	$5\frac{3}{4}$ / 6¼G. M.	<i>meridie /</i> <i>med. noc.</i> G. M.
30 0	39 38	34 10	28 11	21 42	14 45	7 28	60 0
30 15	39 26	33 58	28 0	21 33	14 39	7 25	59 45
30 30	39 13	33 47	27 50	21 24	14 32	7 21	59 30
30 45	39 1	33 35	27 39	21 15	14 26	7 18	59 15
31 0	38 48	33 23	27 29	21 7	14 20	7 15	59 0
31 15	38 36	33 12	27 19	20 59	14 14	7 12	58 45
31 30	38 24	33 2	27 9	20 50	14 9	7 9	58 30
31 45	38 13	32 50	26 59	20 42	14 3	7 6	58 15
32 0	38 1	32 39	26 49	20 34	13 57	7 3	58 0
32 15	37 49	32 28	26 40	20 27	13 52	7 0	57 45
32 30	37 38	32 17	26 30	20 19	13 46	6 57	57 30
32 45	37 26	32 6	26 21	20 11	13 41	6 54	57 15
33 0	37 15	31 56	26 12	20 4	13 35	6 52	57 0
33 15	37 4	31 46	26 3	19 56	13 30	6 49	56 45

<i>Horæ à & (arc)G. M.</i>	<i>4½ / 7½G. M.</i>	<i>4¾ / 7¼G. M.</i>	<i>5 / 7G. M.</i>	<i>5¼ / 6¾G. M.</i>	<i>5½ / 6½G. M.</i>	<i>5¾ / 6¼G. M.</i>	<i>meridie / med. noc.G. M.</i>
33 30	36 53	31 36	25 54	19 49	13 25	6 46	56 30
33 45	36 42	31 26	25 45	19 42	13 20	6 44	56 15
34 0	36 32	31 16	25 36	19 35	13 15	6 41	56 0
34 15	36 21	31 6	25 28	19 28	13 10	6 39	55 45
34 30	36 11	30 56	25 19	19 21	13 5	6 36	55 30
34 45	36 0	30 47	25 11	19 14	13 0	6 34	55 15
35 0	35 50	30 37	25 2	19 8	12 56	6 31	55 0
35 15	35 40	30 28	24 54	19 1	12 51	6 29	54 45
35 30	35 30	30 19	24 46	18 54	12 46	6 26	54 30
35 45	35 20	30 9	24 38	18 48	12 42	6 24	54 15
36 0	35 10	30 0	24 30	18 42	12 37	6 22	54 0
36 15	35 1	29 52	24 23	18 36	12 33	6 20	53 45
36 30	34 51	29 43	24 15	18 29	12 29	6 17	53 30
36 45	34 42	29 34	24 8	18 23	12 25	6 15	53 15
37 0	34 32	29 26	24 0	18 17	12 20	6 13	53 0
37 15	34 23	29 17	23 53	18 12	12 16	6 11	52 45
37 30	34 14	29 9	23 45	18 6	12 12	6 9	52 30
37 45	34 5	29 0	23	18 0	12 8	6 7	52 15

<i>Horæ à & (arc)G. M.</i>	<i>4½ / 7½G. M.</i>	<i>4¾ / 7¼G. M.</i>	<i>5 / 7G. M.</i>	<i>5¼ / 6¾G. M.</i>	<i>5½ / 6½G. M.</i>	<i>5¾ / 6¼G. M.</i>	<i>meridie / med. noc.G. M.</i>
			38				
38 0	33 56	28 52	23	17 54	12 4	6 5	52 0
			31				
38 15	33 47	28 44	23	17 49	12 0	6 3	51 45
			24				
38 30	33 38	28 36	23	17 43	11 56	6 1	51 30
			17				
38 45	33 30	28 29	23	17 38	11 53	5 59	51 15
			11				
39 0	33 21	28 21	23 4	17 32	11 49	5 57	51 0
39 15	33 13	28 13	22	17 27	11 45	5 55	50 45
			57				
39 30	33 4	28 5	22	17 22	11 42	5 53	50 30
			51				
39 45	32 56	27 58	22	17 17	11 38	5 51	50 15
			44				
40 0	32 48	27 50	22	17 12	11 34	5 49	50 0
			38				
40 15	32 40	27 43	22	17 7	11 31	5 48	49 45
			32				
40 30	32 32	27 36	22	17 2	11 28	5 46	49 30
			25				
40 45	32 24	27 29	22	16 57	11 24	5 44	49 15
			19				
41 0	32 16	27 21	22	16 52	11 21	5 42	49 0
			13				
41 15	32 8	27 14	22 7	16 48	11 18	5 41	48 45
41 30	32 1	27 8	22 1	16 43	11 14	5 39	48 30
41 45	31 53	27 1	21	16 38	11 11	5 37	48 15
			55				
42 0	31 46	26 54	21	16 33	11 8	5 36	48 0
			49				

<i>Horæ à & (arc)G. M.</i>	$4\frac{1}{2}$ / 7½G. M.	$4\frac{3}{4}$ / 7¼G. M.	5 / 7G. M.	$5\frac{1}{4}$ / 6¾G. M.	$5\frac{1}{2}$ / 6½G. M.	$5\frac{3}{4}$ / 6¼G. M.	<i>meridie / med. noc.G. M.</i>
42 15	31 38	26 47	21 44	16 29	11 5	5 34	47 45
42 30	31 31	26 41	21 38	16 24	11 2	5 32	47 30
42 45	31 24	26 34	21 32	16 20	10 59	5 31	47 15
43 0	31 16	26 28	21 27	16 16	10 56	5 29	47 0
43 15	31 9	26 21	21 22	16 11	10 53	5 28	46 45
43 30	31 2	26 15	21 16	16 7	10 50	5 26	46 30
43 45	30 55	26 9	21 11	16 3	10 47	5 25	46 15
44 0	30 48	26 3	21 6	15 59	10 44	5 23	46 0
44 15	30 42	25 56	21 0	15 55	10 41	5 22	45 45
44 30	30 35	25 50	20 55	15 51	10 38	5 21	45 30
44 45	30 28	25 45	20 50	15 47	10 36	5 19	45 15

Printed page 119 (arc 45°0' – 59°45')

<i>Horæ à & (arc)G. M.</i>	$4\frac{1}{2}$ / 7½G. M.	$4\frac{3}{4}$ / 7¼G. M.	5 / 7G. M.	$5\frac{1}{4}$ / 6¾G. M.	$5\frac{1}{2}$ / 6½G. M.	$5\frac{3}{4}$ / 6¼G. M.	<i>meridie / med. noc.G. M.</i>
45 0	30 22	25 39	20 45	15 43	10 32	5 18	45 0
45 15	30 15	25 33	20 40	15 39	10 30	5 16	44 45
45 30	30 9	25 27	20 36	15 35	10 27	5 15	44 30
45 45	30 2	25 21	20	15 31	10 25	5 14	44 15

<i>Horæ à & (arc)G. M.</i>	$4\frac{1}{2}$ / 7½G. M.	$4\frac{3}{4}$ / 7¼G. M.	5 / 7G. M.	$5\frac{1}{4}$ / 6¾G. M.	$5\frac{1}{2}$ / 6½G. M.	$5\frac{3}{4}$ / 6¼G. M.	<i>meridie / med. noc.G. M.</i>
			31				
46 0	29 56	25 16	20	15 27	10 22	5 12	44 0
			26				
46 15	29 50	25 10	20	15 24	10 20	5 11	43 45
			21				
46 30	29 44	25 5	20	15 20	10 17	5 10	43 30
			16				
46 45	29 38	24 59	20	15 16	10 15	5 8	43 15
			12				
47 0	29 32	24 54	20 7	15 13	10 12	5 7	43 0
47 15	29 26	24 49	20 3	15 9	10 10	5 6	42 45
47 30	29 20	24 43	19	15 6	10 7	5 5	42 30
			59				
47 45	29 14	24 38	19	15 2	10 5	5 4	42 15
			54				
48 0	29 8	24 33	19	14 59	10 3	5 2	42 0
			50				
48 15	29 2	24 28	19	14 56	10 0	5 1	41 45
			45				
48 30	28 57	24 23	19	14 52	9 58	5 0	41 30
			41				
48 45	28 51	24 18	19	14 49	9 56	4 59	41 15
			37				
49 0	28 46	24 13	19	14 46	9 54	4 58	41 0
			33				
49 15	28 40	24 8	19	14 43	9 52	4 57	40 45
			29				
49 30	28 35	24 3	19	14 40	9 49	4 56	40 30
			25				
49 45	28 30	23 59	19	14 37	9 47	4 55	40 15
			21				
50 0	28 24	23 54	19	14 33	9 45	4 53	40 0

<i>Horæ à &</i> (arc)G. M.	$4\frac{1}{2}$ / $7\frac{1}{2}$ G. M.	$4\frac{3}{4}$ / $7\frac{1}{4}$ G. M.	5 / 7G. M.	$5\frac{1}{4}$ / $6\frac{3}{4}$ G. M.	$5\frac{1}{2}$ / $6\frac{1}{2}$ G. M.	$5\frac{3}{4}$ / $6\frac{1}{4}$ G. M.	<i>meridie /</i> <i>med. noc.</i> G. M.
			17				
50 15	28 19	23 49	19	14 30	9 43	4 52	39 45
			13				
50 30	28 14	23 45	19 9	14 27	9 41	4 51	39 30
50 45	28 8	23 40	19 5	14 24	9 39	4 50	39 15
51 0	28 3	23 35	19 1	14 21	9 37	4 49	39 0
51 15	27 58	23 31	18	14 18	9 35	4 48	38 45
			58				
51 30	27 53	23 27	18	14 16	9 33	4 47	38 30
			54				
51 45	27 49	23 23	18	14 13	9 31	4 46	38 15
			50				
52 0	27 44	23 18	18	14 10	9 29	4 45	38 0
			47				
52 15	27 39	23 14	18	14 7	9 27	4 44	37 45
			43				
52 30	27 34	23 10	18	14 5	9 25	4 43	37 30
			40				
52 45	27 29	23 6	18	14 2	9 23	4 42	37 15
			36				
53 0	27 25	23 2	18	13 59	9 22	4 42	37 0
			33				
53 15	27 20	22 58	18	13 57	9 20	4 41	36 45
			29				
53 30	27 16	22 54	18	13 54	9 18	4 40	36 30
			26				
53 45	27 11	22 50	18	13 51	9 16	4 39	36 15
			23				
54 0	27 7	22 46	18	13 49	9 15	4 38	36 0
			19				
54 15	27 2	22 42	18	13 46	9 13	4 37	35 45
			16				

<i>Horæ à & (arc)G. M.</i>	$4\frac{1}{2}$ / 7½G. M.	$4\frac{3}{4}$ / 7¼G. M.	5 / 7G. M.	$5\frac{1}{4}$ / 6¾G. M.	$5\frac{1}{2}$ / 6½G. M.	$5\frac{3}{4}$ / 6¼G. M.	<i>meridie / med. noc.G. M.</i>
54 30	26 58	22 38	18 13	13 44	9 11	4 36	35 30
54 45	26 54	22 34	18 10	13 41	9 9	4 35	35 15
55 0	26 49	22 31	18 7	13 39	9 8	4 34	35 0
55 15	26 45	22 27	18 4	13 37	9 6	4 34	34 45
55 30	26 41	22 23	18 1	13 34	9 5	4 33	34 30
55 45	26 37	22 20	17 58	13 32	9 3	4 32	34 15
56 0	26 33	22 16	17 55	13 30	9 1	4 31	34 0
56 15	26 29	22 12	17 52	13 27	9 0	4 30	33 45
56 30	26 25	22 9	17 49	13 25	8 58	4 30	33 30
56 45	26 21	22 6	17 46	13 23	8 57	4 29	33 15
57 0	26 17	22 2	17 43	13 21	8 55	4 28	33 0
57 15	26 13	21 59	17 40	13 18	8 54	4 27	32 45
57 30	26 9	21 55	17 38	13 16	8 52	4 27	32 30
57 45	26 6	21 52	17 35	13 14	8 51	4 26	32 15
58 0	26 2	21 49	17 32	13 12	8 49	4 25	32 0
58 15	25 58	21 49 [recte 21 46?]	17 29	13 10	8 48	4 24	31 45
58 30	25 55	21 43	17 27	13 8	8 47	4 24	31 30

<i>Horæ à & (arc)G. M.</i>	$4\frac{1}{2}$ / $7\frac{1}{2}$ G. M.	$4\frac{3}{4}$ / $7\frac{1}{4}$ G. M.	5 / 7G. M.	$5\frac{1}{4}$ / $6\frac{3}{4}$ G. M.	$5\frac{1}{2}$ / $6\frac{1}{2}$ G. M.	$5\frac{3}{4}$ / $6\frac{1}{4}$ G. M.	<i>meridie / med. noc.G. M.</i>
58 45	25 51	21 39	17 24	13 6	8 45	4 23	31 15
59 0	25 47	21 36	17 22	13 4	8 44	4 22	31 0
59 15	25 44	21 33	17 19	13 2	8 43	4 22	30 45
59 30	25 41	21 30	17 16	13 0	8 41	4 21	30 30
59 45	25 37	21 27	17 14	12 58	8 40	4 20	30 15

Printed page 120 (arc 60°0' – 74°45')

<i>Horæ à & (arc)G. M.</i>	$4\frac{1}{2}$ / $7\frac{1}{2}$ G. M.	$4\frac{3}{4}$ / $7\frac{1}{4}$ G. M.	5 / 7G. M.	$5\frac{1}{4}$ / $6\frac{3}{4}$ G. M.	$5\frac{1}{2}$ / $6\frac{1}{2}$ G. M.	$5\frac{3}{4}$ / $6\frac{1}{4}$ G. M.	<i>meridie / med. noc.G. M.</i>
60 0	25 34	21 24	17 12	12 56	8 39	4 20	30 0
60 15	25 30	21 21	17 9	12 54	8 37	4 19	29 45
60 30	25 27	21 18	17 7	12 52	8 36	4 18	29 30
60 45	25 24	21 16	17 4	12 51	8 35	4 18	29 15
61 0	25 21	21 13	17 2	12 49	8 34	4 17	29 0
61 15	25 17	21 10	17 0	12 47	8 32	4 17	28 45
61 30	25 14	21 7	16 57	12 45	8 31	4 16	28 30
61 45	25 11	21 4	16 55	12 43	8 30	4 15	28 15
62 0	25 8	21 2	16 53	12 42	8 29	4 15	28 0
62 15	25 5	20 59	16 51	12 40	8 28	4 14	27 45
62 30	25 2	20 56	16 49	12 38	8 27	4 14	27 30

<i>Horæ à & (arc)G. M.</i>	<i>4½ / 7½G. M.</i>	<i>4¾ / 7¼G. M.</i>	<i>5 / 7G. M.</i>	<i>5¼ / 6¾G. M.</i>	<i>5½ / 6½G. M.</i>	<i>5¾ / 6¼G. M.</i>	<i>meridie / med. noc.G. M.</i>
62 45	24 59	20 54	16 46	12 37	8 25	4 13	27 15
63 0	24 56	20 51	16 44	12 35	8 24	4 12	27 0
63 15	24 53	20 49	16 42	12 33	8 23	4 12	26 45
63 30	24 50	20 46	16 40	12 32	8 22	4 11	26 30
63 45	24 47	20 44	16 38	12 30	8 21	4 11	26 15
64 0	24 45	20 41	16 36	12 29	8 20	4 10	26 0
64 15	24 42	20 39	16 34	12 27	8 19	4 10	25 45
64 30	24 39	20 37	16 32	12 26	8 18	4 9	25 30
64 45	24 36	20 34	16 30	12 24	8 17	4 9	25 15
65 0	24 34	20 32	16 28	12 23	8 16	4 8	25 0
65 15	24 31	20 30	16 26	12 21	8 15	4 8	24 45
65 30	24 29	20 27	16 24	12 20	8 14	4 7	24 30
65 45	24 26	20 25	16 23	12 18	8 13	4 7	24 15
66 0	24 23	20 23	16 21	12 17	8 12	4 6	24 0
66 15	24 21	20 21	16 19	12 16	8 11	4 6	23 45
66 30	24 18	20 19	16 17	12 14	8 10	4 5	23 30

<i>Horæ à & (arc)G. M.</i>	<i>4½ / 7½G. M.</i>	<i>4¾ / 7¼G. M.</i>	<i>5 / 7G. M.</i>	<i>5¼ / 6¾G. M.</i>	<i>5½ / 6½G. M.</i>	<i>5¾ / 6¼G. M.</i>	<i>meridie / med. noc.G. M.</i>
66 45	24 16	20 17	16 16	12 13	8 9	4 5	23 15
67 0	24 14	20 15	16 14	12 12	8 8	4 4	23 0
67 15	24 11	20 12	16 12	12 10	8 7	4 4	22 45
67 30	24 9	20 10	16 10	12 9	8 7	4 3	22 30
67 45	24 7	20 8	16 9	12 8	8 6	4 3	22 15
68 0	24 4	20 7	16 7	12 7	8 5	4 3	22 0
68 15	24 2	20 5	16 6	12 5	8 4	4 2	21 45
68 30	24 0	20 3	16 4	12 4	8 3	4 2	21 30
68 45	23 58	20 1	16 2	12 3	8 2	4 1	21 15
69 0	23 56	19 59	16 1	12 2	8 2	4 1	21 0
69 15	23 53	19 57	15 59	12 0	8 1	4 1	20 45
69 30	23 51	19 55	15 58	11 59	8 0	4 0	20 30
69 45	23 49	19 53	15 56	11 58	7 59	4 0	20 15
70 0	23 47	19 52	15 55	11 57	7 59	3 59	20 0
70 15	23 45	19 50	15 54	11 56	7 58	3 59	19 45
70 30	23 43	19 48	15 52	11 55	7 57	3 59	19 30
70 45	23 41	19 47	15 51	11 54	7 56	3 58	19 15
71 0	23 39	19 45	15 49	11 53	7 55	3 58	19 0
71 15	23 38	19 43	15 48	11 52	7 55	3 58	18 45

<i>Horæ à & (arc)G. M.</i>	<i>4½ / 7½G. M.</i>	<i>4¾ / 7¼G. M.</i>	<i>5 / 7G. M.</i>	<i>5¼ / 6¾G. M.</i>	<i>5½ / 6½G. M.</i>	<i>5¾ / 6¼G. M.</i>	<i>meridie / med. noc.G. M.</i>
71 30	23 36	19 42	15 47	11 51	7 54	3 57	18 30
71 45	23 34	19 40	15 45	11 50	7 54	3 57	18 15
72 0	23 32	19 39	15 44	11 49	7 53	3 57	18 0
72 15	23 30	19 37	15 43	11 48	7 52	3 56	17 45
72 30	23 29	19 36	15 42	11 47	7 52	3 56	17 30
72 45	23 27	19 34	15 40	11 46	7 51	3 56	17 15
73 0	23 25	19 33	15 39	11 45	7 50	3 55	17 0
73 15	23 24	19 31	15 38	11 44	7 50	3 55	16 45
73 30	23 22	19 30	15 37	11 43	7 49	3 55	16 30
73 45	23 20	19 28	15 36	11 42	7 48	3 54	16 15
74 0	23 19	19 27	15 35	11 41	7 48	3 54	16 0
74 15	23 17	19 26	15 33	11 41	7 47	3 54	15 45
74 30	23 16	19 24	15 32	11 40	7 47	3 53	15 30
74 45	23 14	19 23	15 31	11 39	7 46	3 53	15 15

Table VI (continued) — *Arcus Horizontis* (Arc of the Horizon)

[Margin (rotated, left): *Altitudines poli pro arcubus Horizontis*. — "Elevations of the pole for the arcs of the horizon."] [Margin (rotated, right): *Altitudines Poli pro*

arcubus Verticalis primarij. — "Elevations of the pole for the arcs of the prime vertical."]

This page completes the sixth table. The leftmost column, headed **Horæ à &** (G. = degrees, M. = minutes), gives the arc (here running from 75° 0' to 90° 0'). Each of the six paired-hour columns is headed by a morning hour over its afternoon complement, the two summing to 12 hours: $4\frac{1}{2}$ / $7\frac{1}{2}$, $4\frac{3}{4}$ / $7\frac{1}{4}$, 5 / 7, $5\frac{1}{4}$ / $6\frac{3}{4}$, $5\frac{1}{2}$ / $6\frac{1}{2}$, $5\frac{3}{4}$ / $6\frac{1}{4}$. The final column, **meridie / med. noc.** (noon / midnight), equals 90° – arc at every row. Every cell is a degree–minute pair.

Arc (Horæ à &)	$4\frac{1}{2}$ / $7\frac{1}{2}$	$4\frac{3}{4}$ / $7\frac{1}{4}$	5 / 7	$5\frac{1}{4}$ / $6\frac{3}{4}$	$5\frac{1}{2}$ / $6\frac{1}{2}$	$5\frac{3}{4}$ / $6\frac{1}{4}$	meridie / med. noc.
75 0	23 13	19 22	15 30	11 38	7 46	3 53	15 0
75 15	23 11	19 21	15 29	11 37	7 45	3 53	14 45
75 30	23 10	19 19	15 28	11 37	7 45	3 52	14 30
75 45	23 8	19 18	15 27	11 36	7 44	3 52	14 15
76 0	23 7	19 17	15 26	11 35	7 44	3 52	14 0
76 15	23 6	19 16	15 25	11 34	7 43	3 52	13 45
76 30	23 4	19 15	15 24	11 34	7 43	3 51	13 30
76 45	23 3	19 13	15 24	11 33	7 42	3 51	13 15
77 0	23 2	19 12	15 23	11 32	7 42	3 51	13 0
77 15	23 1	19 11	15 22	11 32	7 41	3 51	12 45
77 30	22 59	19 10	15 21	11 31	7 41	3 50	12 30
77 45	22 58	19 9	15 20	11 30	7 40	3 50	12 15
78 0	22 57	19 8	15	11 30	7 40	3 50	12 0

Arc (Horæ à &)	4½ / 7½	4¾ / 7¼	5 / 7	5¼ / 6¾	5½ / 6½	5¾ / 6¼	meridie / med. noc.
			19				
78 15	22 56	19 7	15	11 29	7 40	3 50	11 45
			18				
78 30	22 55	19 6	15	11 28	7 39	3 50	11 30
			18				
78 45	22 54	19 5	15	11 28	7 39	3 49	11 15
			17				
79 0	22 53	19 5	15	11 27	7 38	3 49	11 0
			16				
79 15	22 52	19 4	15	11 27	7 38	3 49	10 45
			15				
79 30	22 51	19 3	15	11 26	7 38	3 49	10 30
			15				
79 45	22 50	19 2	15	11 26	7 37	3 49	10 15
			14				
80 0	22 49	19 1	15	11 25	7 37	3 48	10 0
			13				
80 15	22 48	19 0	15	11 25	7 37	3 48	9 45
			13				
80 30	22 47	19 0	15	11 24	7 36	3 48	9 30
			12				
80 45	22 46	18 59	15	11 24	7 36	3 48	9 15
			11				
81 0	22 45	18 58	15	11 23	7 36	3 48	9 0
			11				
81 15	22 44	18 57	15	11 23	7 35	3 48	8 45
			10				
81 30	22 43	18 57	15	11 22	7 35	3 47	8 30
			10				
81 45	22 43	18 56	15	11 22	7 35	3 47	8 15
			9				
82 0	22 42	18 55	15	11 21	7 34	3 47	8 0
			8				

Arc (Horæ à &)	4½ / 7½	4¾ / 7¼	5 / 7	5¼ / 6¾	5½ / 6½	5¾ / 6¼	meridie / med. noc.
82 15	22 41	18 55	15 8	11 21	7 34	3 47	7 45
82 30	22 40	18 54	15 7	11 21	7 34	3 47	7 30
82 45	22 40	18 53	15 7	11 20	7 34	3 47	7 15
83 0	22 39	18 53	15 6	11 20	7 33	3 47	7 0
83 15	22 38	18 52	15 6	11 20	7 33	3 47	6 45
83 30	22 38	18 52	15 5	11 19	7 33	3 46	6 30
83 45	22 37	18 51	15 5	11 19	7 33	3 46	6 15
84 0	22 37	18 51	15 5	11 19	7 32	3 46	6 0
84 15	22 36	18 50	15 4	11 18	7 32	3 46	5 45
84 30	22 36	18 50	15 4	11 18	7 32	3 46	5 30
84 45	22 35	18 49	15 4	11 18	7 32	3 46	5 15
85 0	22 35	18 49	15 3	11 18	7 32	3 46	5 0
85 15	22 34	18 49	15 3	11 17	7 32	3 46	4 45
85 30	22 34	18 48	15 3	11 17	7 31	3 46	4 30
85 45	22 33	18 48	15 2	11 17	7 31	3 46	4 15
86 0	22 33	18 48	15 2	11 17	7 31	3 46	4 0
86 15	22 33	18 47	15	11 16	7 31	3 45	3 45

Arc (Horæ à &)	4½ / 7½	4¾ / 7¼	5 / 7	5¼ / 6¾	5½ / 6½	5¾ / 6¼	meridie / med. noc.
			2				
86 30	22 32	18 47	15	11 16	7 31	3 45	3 30
			2				
86 45	22 32	18 47	15	11 16	7 31	3 45	3 15
			1				
87 0	22 32	18 46	15	11 16	7 31	3 45	3 0
			1				
87 15	22 31	18 46	15	11 16	7 31	3 45	2 45
			1				
87 30	22 31	18 46	15	11 16	7 30	3 45	2 30
			1				
87 45	22 31	18 46	15	11 16	7 30	3 45	2 15
			1				
88 0	22 31	18 46	15	11 15	7 30	3 45	2 0
			1				
88 15	22 31	18 45	15	11 15	7 30	3 45	1 45
			0				
88 30	22 30	18 45	15	11 15	7 30	3 45	1 30
			0				
88 45	22 30	18 45	15	11 15	7 30	3 45	1 15
			0				
89 0	22 30	18 45	15	11 15	7 30	3 45	1 0
			0				
89 15	22 30	18 45	15	11 15	7 30	3 45	0 45
			0				
89 30	22 30	18 45	15	11 15	7 30	3 45	0 30
			0				
89 45	22 30	18 45	15	11 15	7 30	3 45	0 15
			0				
90 0	22 30	18 45	15	11 15	7 30	3 45	0 0
			0				

Table VII — VII. Tabula

ARCVS CIRCULI HORÆ VI. A MERIDIE — The arc of the circle of the sixth hour from the meridian or from midnight, [measured] between the equator and the great circles which are drawn through the intersection of the equator with the meridian, [passing] through the beginnings of the signs, for the hours reckoned from sunrise or sunset.

Ad latitudinem Grad. 42. — For latitude 42 degrees.

[Margin, left: Hora ab occasu. — "Hour from the west (sunset)."] [Margin, right: Hora ab ortu. — "Hour from the east (sunrise)."]

The six columns are each headed by a pair of zodiac signs of equal declination, symmetric about the solstitial colure; Aries (♈) and Libra (♎) are omitted (see the note below the table). Column heads as printed, left to right: ♋ (Cancer) | ♌ ♊ (Leo / Gemini) | ♍ ♉ (Virgo / Taurus) | ♏ ♐ (Scorpio / Pisces) | ♏ ♐ (Sagittarius / Aquarius) | ♐ (Capricorn). Row labels are the hour from the west (left margin) and the hour from the east (right margin). Each cell is degrees–minutes; an interposed † or * marks the sign of the pair to which the value belongs (see note).

Hora ab occ.	♋	♌ / ♊	♍ / ♉	♏ / ♐	♏ / ♐	♐	Hora ab ortu
9	25 7	22 12	13 52	19 46	40 22	49 19	15
10	23 39	20 31	12 11	14 59	29 27	35 53	14
11	23 42	20 15	11 32	12 42	24 1	28 54	13
12	25 18	21 18	11 42	11 42	21 18	25 18	12
13	28 54	24 1	12 42	11 32	20 15	23 42	11
14	35 53	29 27	14 59	12 11	20 31	23 39	10
15	49 19	40 22	19 46	13 52	22 12	25 7	9
16	74† 27	63† 20	31 25	17 23	25 52	28 33	8
17	72† 9	78† 21	69* 8	25 15	33 6	35 12	7
18	48 0	48 0	48† 0	48* 0	48 0	48 0	6
19	35 12	33 6	25 15	69† 8	78* 21	72* 9	5
20	28 33	25 52	17 23	31 25	63† 20	74† 27	4
21	25 7	22 12	13 52	19 46	40 22	49 19	3
22	23 39	20 31	12 11	14 59	29 27	35 53	2
23	23 42	20 15	11 32	12 42	24 1	28 54	1

Hora ab occ.	☾	♈ / ♀	♏ / ♂	♍ / ♋	♊ / ♉	♌	Hora ab ortu
24	25 18	21 18	11 42	11 42	21 18	25 18	24

Note (below Table VII): *Hoc signum † gradibus interpositum, horis pomeridianis initium dat in horis ab occasu. Signum vero hoc * in horis ab ortu. Sole existente in principio ♈, vel ♎, hi arcus omnes sunt 0.0. quæ causa fuit, ut ♈, & ♎, inter signa posita non sint.*

Translation: "This sign †, placed between the degrees, gives the beginning [initium] in the afternoon hours for the hours reckoned from the west. But this sign * does so for the hours reckoned from the east. When the Sun stands at the beginning of Aries (♈) or Libra (♎), all these arcs are 0° 0′ — which is why Aries and Libra are not placed among the signs."

Table VIII — VIII. Tabula

ARCVS MERIDIANI INTER ÆQVATOREM — The arc of the meridian [measured] between the equator and the great circles which are drawn through the intersection of the equator with the horizon, [passing] through the beginnings of the signs, for the hours reckoned from sunrise or sunset.

Ad latitudinem Grad. 42. — For latitude 42 degrees.

[Margin, left: Hora ab occasu. — "Hour from the west (sunset)."] [Margin, right: Hora ab ortu. — "Hour from the east (sunrise)."]

Same six sign-columns as Table VII (♈ and ♎ again omitted). Column heads as printed, left to right: ☾ | ♈ ♀ | ♏ ♂ | ♍ ♋ | ♊ ♉ | ♌.

Hora ab occ.	☾	♈ / ♀	♏ / ♂	♍ / ♋	♊ / ♉	♌	Hora ab ortu
9	49 19	40 22	19 46	13 52	22 12	25 7	15
10	74 27	63 20	31 25	17 23	25 52	28 33	14
11	72 9	78 21	69 8	25 15	33 6	35 12	13
12	48 0	48 0	48 0	48 0	48 0	48 0	12
13	35 12	33 6	25 15	69 8	78 21	72 9	11
14	28 33	25 52	17 23	31 25	63 20	74 27	10
15	25 7	22 12	13 52	19 46	40 22	49 19	9

Hora ab occ.	♈	♉ / ♊	♋ / ♌	♍ / ♎	♏ / ♐	♑	Hora ab ortu
16	23* 39	20* 31	12 11	14 59	29 27	35 53	8
17	23† 42	20† 15	11* 32	12 42	24 1	28 54	7
18	25 18	21 18	11† 42	11* 42	21 18	25 18	6
19	28 54	24 1	12 42	11† 32	20* 15	23* 42	5
20	35 53	29 27	14 59	12 11	20† 31	23† 39	4
21	49 19	40 22	19 46	13 52	22 12	25 7	3
22	74 27	63 20	31 25	17 23	25 52	28 33	2
23	72 9	78 21	69 8	25 15	33 6	35 12	1
24	48 0	48 0	48 0	48 0	48 0	48 0	24

Note (below Table VIII): *Hic quoque signum † inter gradus positum, horis pomeridianis dat initium in horis ab occasu: Signum vero hoc * in horis ab ortu. Sole existente in principio ♈, vel ♎, hi arcus omnes sunt 0.0. ac propterea inter signa posita non sunt ♈, & ♎.*

Translation: "Here too the sign †, placed between the degrees, gives the beginning in the afternoon hours for the hours reckoned from the west; but this sign * does so for the hours reckoned from the east. When the Sun stands at the beginning of Aries (♈) or Libra (♎), all these arcs are 0° 0', and for that reason Aries and Libra are not placed among the signs."

Table IX — *Tabula IX* (title page)

ALTITVDINES SOLIS SVpra Horizontem in horis à meridie ac media nocte pro initijs signorum comprehendens ad omnes eleuationes poli extensa.

Translation: "The altitudes of the Sun above the horizon, in the hours from noon and from midnight, for the beginnings of the signs — a table extended to all elevations of the pole."

This page carries only the ornamental title for the ninth table; the table itself begins on the following leaf. The leaf shows heavy show-through/offset from the adjacent table pages.

IX. TABVLA — Table IX (Sun's altitudes above the horizon)

IX. TABVLA. Altitudines Solis supra Horizontem in horis à meridie ac media nocte pro initijs signorum comprehendens ad omnes eleuationes poli extensa.

Table IX. The altitudes of the Sun above the horizon, in the hours from noon and from midnight, for the beginnings of the signs — a table extended to all elevations of the pole.

(The ornamental title of this table stood on the preceding leaf; here the table itself begins. Each numeric cell gives degrees and minutes, printed "degrees | minutes." The two header rows above the data give, for each column, first the hour reckoned from noon [H. à meridie], then the same clock-position reckoned from midnight [H. à media nocte]. The leftmost column, "Alt. Poli. G.," is the elevation of the pole in whole degrees.)

Table IX, first portion — elevations of the pole 0°–45° (left half: hours 12 to 6)

[Left margin, rotated: *Hic intratur pro horis ☿, versus dextram progrediendo.* — "Here one enters for the hours of Cancer (☿), proceeding toward the right."]

Column headers: **H. à merid.** / **H. à med. noct.** = 12/12, 1/11, 2/10, 3/9, 4/8, 5/7, 6/6.
Cells are "degrees minutes."

Alt. Poli (G)	12 / 12	1 / 11	2 / 10	3 / 9	4 / 8	5 / 7	6 / 6
0	66 30	62 21	52 35	40 26	27 18	13 44	0 0
1	67 30	63 12	53 14	40 57	27 44	14 8	0 24
2	68 30	64 3	53 52	41 27	28 11	14 33	0 48
3	69 30	64 53	54 29	41 57	28 36	14 57	1 12
4	70 30	65 43	55 6	42 26	29 2	15 21	1 36
5	71 30	66 31	55 41	42 54	29 27	15 44	2 0
6	72 30	67 19	56 15	43	29	16 8	2 23

Alt. Poli (G)	12 / 12	1 / 11	2 / 10	3 / 9	4 / 8	5 / 7	6 / 6
				22	51		
7	73 30	68 6	56 49	43	30	16	2 47
				48	15	31	
8	74 30	68 51	57 21	44	30	16	3 11
				14	38	53	
9	75 30	69 36	57 52	44	31 1	17	3 35
				39		16	
10	76 30	70 19	58 22	45 4	31	17	3 58
					23	38	
11	77 30	71 1	58 50	45	31	18 0	4 22
				27	45		
12	78 30	71 41	59 17	45	32 6	18	4 45
				49		22	
13	79 30	72 20	59 43	46	32	18	5 9
				11	27	43	
14	80 30	72 56	60 7	46	32	19 4	5 32
				31	47		
15	81 30	73 30	60 30	46	33 6	19	5 55
				51		25	
16	82 30	74 2	60 51	47	33	19	6 19
				10	25	46	
17	83 30	74 31	61 10	47	33	20 6	6 42
				27	43		
18	84 30	74 57	61 28	47	34 0	20	7 5
				44		25	
19	85 30	75 20	61 44	47	34	20	7 28
				59	17	45	
20	86 30	75 39	61 58	48	34	21 4	7 50
				13	34		
21	87 30	75 54	62 10	48	34	21	8 13
				27	49	23	
22	38 30 [recte: 88 30]	76 6	62 21	48	35 4	21	8 35
				39		41	
23	89 30	76 13	62 29	48	35	21	8 58

Alt. Poli (G)	12 / 12	1 / 11	2 / 10	3 / 9	4 / 8	5 / 7	6 / 6
				50	18	59	
24	89 30	76 16	62 35	48	35	22	9 20
				59	32	16	
25	88 30	76 15	62 40	49 8	35	22	9 42
					44	34	
26	87 30	76 10	62 42	49	35	22	10 4
				15	56	50	
27	86 30	76 0	62 42	49	36 8	23 7	10
				22			26
28	85 30	75 46	62 41	49	36	23	10
				27	18	23	47
29	84 30	75 29	62 37	49	36	23	11 9
				30	28	38	
30	83 30	75 8	62 31	49	36	23	11
				33	37	53	30
31	82 30	74 43	62 24	49	36	24 8	11
				34	45		51
32	81 20 [recte: 81 30]	74 16	62 14	49	36	24	12
				34	53	22	12
33	80 30	73 45	62 2	49	37 0	24	12
				33		36	33
34	79 30	73 12	61 49	49	37 6	24	12
				31		49	53
35	78 30	72 37	61 33	49	37	25 2	13
				27	11		13
36	77 30	72 0	61 16	49	37	25	13
				22	15	14	33
37	76 30	71 20	60 57	49	37	25	13
				17	19	26	53
38	75 30	70 39	60 37	49 9	37	25	14
					22	38	13
39	74 30	69 57	60 15	49 1	37	25	14
					24	49	32
40	73 30	69 12	59 51	48	37	25	14

Alt. Poli (G)	12 / 12	1 / 11	2 / 10	3 / 9	4 / 8	5 / 7	6 / 6
				51	25	59	51
41	72 30	68 27	59 26	48	37	26 9	15
				41	25		10
42	71 30	67 41	58 59	48	37	26	15
				29	25	18	28
43	70 30	66 54	58 31	48	37	26	15
				16	24	27	47
44	69 30	66 5	58 2	48 2	37	26	16 5
					22	36	
45	68 30	65 17	57 31	47	37	26	16
				46	19	44	23

Table IX, first portion continued — elevations of the pole 0°–45° (right half: hours 7 to 12)

[Right margin, rotated: *Hic intratur pro horis ♄, versus sinistram progrediendo.* — "Here one enters for the hours of Capricorn (♄), proceeding toward the left."]

This is the right-hand continuation of the same block of pole-elevations (0°–45°); the "Alt. Poli. G." column (headed at right *H. med. noct.* / *H. à merid.*) is repeated at the right edge in the source. Column headers **H. à merid.** / **H. à med. noct.** = 7/5, 8/4, 9/3, 10/2, 11/1, 12/12. Cells are "degrees minutes." In the first data column (7/5) an interposed * (the sunburst mark of the table's convention) appears between degrees and minutes in rows 0–30; from row 31 it migrates to the second column (8/4) — it flags the hour-column in which the Sun's altitude passes through the horizon as the pole-elevation increases (cf. the note appended to Tables VII–VIII: *Signum hoc * in horis ab ortu*).

7 / 5	8 / 4	9 / 3	10 / 2	11 / 1	12 / 12	Alt. Poli (G)
13 * 44	27 18	40 26	52	62 21	66 30	0
			35			
13 * 19	26 50	39 54	51	61 29	65 30	1
			55			
12 * 45	26 23	39 21	51	60 37	64 30	2
			15			
12 * 29	25 55	38 48	50	59 44	63 30	3
			33			

7 / 5	8 / 4	9 / 3	10 / 2	11 / 1	12 / 12	Alt. Poli (G)
12 * 4	25 27	38 15	49 52	58 51	62 30	4
11 * 38	24 58	37 41	49 9	57 58	61 30	5
11 * 12	24 29	37 6	48 26	57 4	60 30	6
10 * 47	23 59	36 31	47 42	56 10	59 30	7
10 * 21	23 29	35 55	46 58	55 15	58 30	8
9 * 54	22 59	35 19	46 13	54 21	57 30	9
9 * 28	22 29	34 42	45 28	53 26	56 30	10
9 * 2	21 58	34 5	44 43	52 1 [recte: 52 31]	55 30	11
8 * 35	21 27	33 28	43 57	51 35	54 30	12
8 * 8	20 55	32 50	43 10	50 40	53 30	13
7 * 41	20 24	32 11	42 23	49 44	52 30	14
7 * 15	19 52	31 33	41 36	48 48	51 30	15
6 * 47	19 19	30 54	40 48	47 52	50 30	16
6 * 20	18 47	30 14	40 1	46 56	49 30	17
5 * 53	18 14	29 34	39 12	46 0	48 30	18
5 * 26	17 41	28 54	38 24	45 3	47 30	19
4 * 58	17 8	28 14	37 35	44 6	46 30	20
4 * 31	16 34	27 33	36 46	43 10	45 30	21

7 / 5	8 / 4	9 / 3	10 / 2	11 / 1	12 / 12	Alt. Poli (G)
4 * 3	16 0	26 52	35 57	42 13	44 30	22
3 * 36	15 27	26 10	35 7	41 16	43 30	23
3 * 8	14 52	25 29	34 17	40 19	42 30	24
2 * 40	14 18	24 47	33 27	39 22	41 30	25
2 * 9	13 44	24 5	32 37	38 25	40 30	26
1 * 45	13 9	23 23	31 47	37 28	39 30	27
1 * 17	12 34	22 40	30 56	36 30	38 30	28
0 * 49	11 59	21 57	30 5	35 33	37 30	29
0 * 21	11 24	21 14	29 14	34 36	36 30	30
0 7	10 * 49	20 31	28 23	33 38	35 30	31
0 34	10 * 14	19 48	27 32	32 41	34 30	32
1 2	9 * 38	19 4	26 40	31 43	33 30	33
1 30	9 * 2	18 20	25 49	30 45	32 30	34
1 58	8 * 27	17 26 [?]	24 57	29 48	31 30	35
2 26	7 * 51	16 52	24 5	28 50	30 30	36
2 53	7 * 15	16 8	23 13	27 52	29 30	37
3 21	6 * 39	15 24	22 21	26 54	28 30	38
3 49	6 * 3	14 39	21 29	25 57	27 30	39
4 10 [?]	5 * 27	13 55	20	24 59	26 30	40

7 / 5	8 / 4	9 / 3	10 / 2	11 / 1	12 / 12	Alt. Poli (G)
			37			
4 44	4 * 51	13 10	19	24 1	25 30	41
			45			
5 11	4 * 14	12 25	18	23 3	24 30	42
			52			
5 39	3 * 38	11 40	18 0	22 5	23 30	43
6 6	3 * 2	10 55	17 7	21 7	22 30	44
6 33	2 * 25	10 10	16	20 9	21 30	45
			14			

Table IX, second portion — elevations of the pole 46°–90° (left half: hours 12 to 6)

[Left margin, rotated: *Hic intratur pro horis* ☾, *dextram versus*. — "Here one enters for the hours of Cancer (☾), toward the right."]

Continuation of Table IX for the higher pole-elevations (46°–90°); this is again the left half (hours 12 to 6), the right half standing on the following leaf. Column headers **H. à merid.** / **H. à med. noct.** = 12/12, 1/11, 2/10, 3/9, 4/8, 5/7, 6/6. Cells are "degrees minutes."

Alt. Poli (G)	12 / 12	1 / 11	2 / 10	3 / 9	4 / 8	5 / 7	6 / 6
46	67 30	64 27	56 59	47 30	37	26	16
					15	51	40
47	66 30	63 36	56 26	47 13	37	26	16
					11	58	57
48	65 30	62 45	55 52	46 54	37 6	27 4	17
							14
49	64 30	61 54	55 17	46 35	37 0	27	17
						10	31
50	63 30	61 2	54 41	46 15	36	27	17
					53	16	47
51	62 30	60 9	54 4	45 53	36	27	18 3
					46	20	
52	61 30	59 16	53 26	45 31	36	27	18
					37	25	19

Alt. Poli (G)	12 / 12	1 / 11	2 / 10	3 / 9	4 / 8	5 / 7	6 / 6
53	60 30	58 23	52 47	45 8	36 28	27 28	18 34
54	59 30	57 29	52 8	44 44	36 18	27 31	18 49
55	58 30	56 35	51 28	44 19	36 8	27 34	19 4
56	57 30	55 41	50 47	43 53	35 57	27 36	19 18
57	56 30	54 46	50 5	43 26	35 45	27 38	19 32
58	55 30	53 52	49 23	42 59	35 32	27 39	19 46
59	54 30	52 57	48 40	42 31	35 18	27 39	19 59
60	53 30	52 1	47 56	42 2	35 4	27 39	20 12
61	52 30	51 6	47 12	41 32	34 49	27 38	20 25
62	51 30	50 10	46 28	41 2	34 34	27 37	20 37
63	50 30	49 9	45 43	40 31	34 18	27 35	20 49
64	49 30	48 18	44 57	39 59	34 1	27 33	21 0
65	48 30	47 22	44 11	39 27	33 43	27 30	21 11
66	47 30	46 26	43 25	38 54	33 25	27 26	21 22
67	46 30	45 30	42 38	38 21	33 6	27 22	21 32
68	45 30	44 33	41 51	37 47	32 47	27 18	21 42
69	44 30	43 36	41 4	37 12	32 27	27 13	21 51

Alt. Poli (G)	12 / 12	1 / 11	2 / 10	3 / 9	4 / 8	5 / 7	6 / 6
70	43 30	42 40	40 16	36 37	32 7	27 7	22 0
71	42 30	41 43	39 28	66 2 [recte: 36 2]	31 45	27 1	22 9
72	41 30	40 46	38 39	35 25	31 24	26 55	22 17
73	40 30	39 49	37 51	34 49	31 1	26 47	22 25
74	39 30	38 52	37 2	34 12	30 39	26 40	22 32
75	38 30	37 55	36 12	33 34	30 15	26 32	22 39
76	37 30	36 57	35 23	32 56	29 51	26 23	22 46
77	36 30	36 0	34 33	32 18	29 27	26 13	22 52
78	35 30	35 3	33 43	31 39	29 2	26 4	22 57
79	34 30	34 5	32 53	31 0	28 37	25 54	23 3
80	33 30	33 8	32 3	30 21	28 11	25 43	23 7
81	32 30	32 10	31 12	29 41	27 45	25 32	23 11
82	31 30	31 12	30 21	29 1	27 18	25 20	23 15
83	30 30	30 15	29 31	28 21	26 51	25 8	23 19
84	29 30	29 17	28 39	27 40	26 23	24 55	23 22
85	28 30	28 19	27 48	26 59	25 56	24 42	23 24
86	27 30	27 22	26 57	26 18	25 27	24 29	23 26
87	26 30	26 24	26 5	25 36	25 1	24	23

Alt. Poli (G)	12 / 12	1 / 11	2 / 10	3 / 9	4 / 8	5 / 7	6 / 6
						15	28
88	25 30	25 26	25 14	24 54	24	24 0	23
					29		29
89	24 30	24 28	24 22	24 12	24 0	23	23
						45	30
90	23 30	23 30	23 30	23 30	23	23	23
					30	30	30

Table IX (continued) — Altitudes of the Sun

This section continues the great "Table IX. Altitudes of the Sun in the hours from noon and from midnight, comprehending [them] for the beginnings of the signs, extended to all elevations of the pole." Each opening of the book carries one declination-block of the table, split between a left-hand and a right-hand page. The first page here completes the block for Cancer and Capricorn (elevations of the pole 46°–90°); the following opening gives the block for Gemini/Leo and Sagittarius/Aquarius (elevations of the pole 0°–45°).

*Column conventions, verbatim from the printed headings: the hour-numbers in the top band are hours reckoned **from midnight** (H. à med. nocte = Hora à media nocte); the numbers in the second band are the same hours reckoned **from noon** (H. à merid. = Hora à meridie). Under each hour, **G.** = degrees (Gradus) and **M.** = minutes (Minuta) of the Sun's altitude. The row-label column **Altit. Poli. G.** is the elevation of the pole (geographic latitude) in whole degrees.*

Cancer ☉ & Capricorn ♑, elevations of the pole 46°–90° (right half)

[Margin (right): Hic intratur pro horis ♑ sinistram versus — "Here one enters, for the hours of Capricorn ♑, toward the left."]

Hour columns, from midnight: **7, 8, 9, 10, 11, 12**; the same hours from noon: **5, 4, 3, 2, 1, 12**. (This right half completes the Cancer/Capricorn block whose left half, elevations 46°–90°, stood on the facing page.)

7 (5) G.	8 (4) G.	9 (3) G.	10 (2) G.	11 (1) G.	12 (12) G.	Altit. Poli. G.
M.	M.	M.	M.	M.	M.	
7 0	1 49 ☼	9 25	15 22	19 11	20 30	46

7 (5) G. M.	8 (4) G. M.	9 (3) G. M.	10 (2) G. M.	11 (1) G. M.	12 (12) G. M.	Altit. Poli. G.
7 27	1 13 ✱	8 40	14 29	18 13	19 30	47
7 54	0 36 ✱	7 54	13 36	17 14	18 30	48
8 21	0 0 ✱	7 9	12 43	16 16	17 30	49
8 48	0 37	6 24 ✱	11 50	15 18	16 30	50
9 14	1 13	5 38 ✱	10 57	14 20	15 30	51
9 41	1 50	4 53 ✱	10 4	13 22	14 30	52
10 7	2 26	4 7 ✱	9 11	12 24	13 30	53
10 33	3 3	3 21 ✱	8 18	11 25	12 30	54
10 59	3 39	2 36 ✱	7 24	10 27	11 30	55
11 25	4 15	1 50 ✱	6 31	9 29	10 30	56
11 50	4 51	1 4 ✱	5 38	8 31	9 30	57
12 16	5 28	0 19 ✱	4 45	7 33	8 30	58
12 41	6 4	0 27	3 51 ✱	6 34	7 30	59
13 6	6 40	1 13	2 58 ✱	5 36	6 30	60
13 31	7 16	1 58	2 5 ✱	4 38	5 30	61
13 56	7 52	2 44	1 11 ✱	3 39	4 30	62
14 20	8 28	3 29	0 18 ✱	2 41	3 30	63
14 44	9 3	4 15	0 35	1 43 ✱	2 30	64
15 8	9 39	5 1	1 29	0 45 ✱	1 30	65
15 32	10 14	5 46	2 22	0 14	0 30 ✱	66
15 55	10 50	6 32	3 15	1 12	0 30	67
16 18	11 25	7 17	4 8	2 10	1 30	68
16 41	12 0	8 2	5 2	3 9	2 30	69
17 4	12 35	8 48	5 55	4 7	3 30	70
17 27	13 10	9 33	6 48	5 5	4 30	71
17 49	13 44	10 18	7 41	6 3	5 30	72
18 11	14 19	11 3	8 35	7 2	6 30	73
18 32	14 53	11 48	9 28	8 0	7 30	74
18 55	15 27	12 33	10 21	8 58	8 30	75
19 14	16 1	13 18	11 14	9 56	9 30	76

7 (5) G. M.	8 (4) G. M.	9 (3) G. M.	10 (2) G. M.	11 (1) G. M.	12 (12) G. M.	Altit. Poli. G.
19 35	16 35	14 3	12 7	10 55	10 30	77
19 55	17 8	14 47	13 0	11 53	11 30	78
20 15	17 42	15 32	13 53	12 51	12 30	79
20 35	18 15	16 16	14 46	13 49	13 30	80
20 54	18 47	17 0	15 38	14 47	14 30	81
21 13	19 20	17 44	16 31	15 46	15 30	82
21 31	19 52	18 28	17 24	16 44	16 30	83
21 49	20 24	19 12	18 16	17 42	17 30	84
22 7	20 56	19 55	19 9	18 40	18 30	85
22 25	21 27	20 39	20 1	19 38	19 30	86
22 42	21 56	21 22	20 54	20 36	20 30	87
22 58	22 29	22 5	21 46	21 34	21 30	88
23 14	23 0	22 47	22 38	22 32	22 30	89
23 30	23 30	23 30	23 30	23 30	23 30	90

Pages 128–129 — Gemini ♊ & Leo ♌ (north) / Sagittarius ♐ & Aquarius ♑ (south), elevations of the pole 0°–45°

This opening gives one declination-block ($|\delta| \leq 20^\circ$): read one way it serves Gemini and Leo, read the other way Sagittarius and Aquarius. The left page (128) carries the hours 12–6 reckoned from noon; the right page (129) carries the hours 7–12 reckoned from midnight. The two halves share the noon column (the "12" column), whose values are identical on both pages.

left half (hours from noon 12, 1, 2, 3, 4, 5, 6)

[Margin (left): Hic intratur pro horis ♊ & ♌ dextram versus — "Here one enters, for the hours of Gemini ♊ and Leo ♌, toward the right."]

Hour columns, from noon: **12, 1, 2, 3, 4, 5, 6**; the same hours from midnight: **12, 11, 10, 9, 8, 7, 6**.

Altit. Poli. G.	12 (12) G. M.	1 (11) G. M.	2 (10) G. M.	3 (9) G. M.	4 (8) G. M.	5 (7) G. M.	6 (6) G. M.
0	69 48	65 2	54 22	41 35	27 49	14 3	0 0

Altit. Poli. G.	12 (12) G. M.	1 (11) G. M.	2 (10) G. M.	3 (9) G. M.	4 (8) G. M.	5 (7) G. M.	6 (6) G. M.
1	70 48	65 50	54 57	42 2	28 22	14 25	0 21
2	71 48	66 38	55 31	42 28	28 45	14 46	0 41
3	72 48	67 25	56 4	42 54	29 7	15 6	1 2
4	73 48	68 11	56 36	43 19	29 29	15 27	1 23
5	74 48	68 56	57 7	43 43	29 50	15 47	1 43
6	75 48	69 40	57 37	44 7	30 11	16 7	2 4
7	76 48	70 22	58 5	44 29	30 31	16 27	2 25
8	77 48	71 2	58 32	44 51	30 51	16 46	2 45
9	78 48	71 42	58 57	45 12	31 10	17 6	3 6
10	79 48	72 19	59 21	45 31	31 28	17 24	3 26
11	80 48	72 54	59 44	45 50	31 46	17 43	3 47
12	81 48	73 26	60 5	46 8	32 4	18 1	4 7
13	82 48	73 56	60 25	46 25	32 20	18 19	4 27
14	83 48	74 24	60 43	46 40	32 36	18 37	4 48
15	84 48	74 48	60 59	46 55	32 52	18 54	5 8
16	85 48	75 9	61 13	47 9	33 7	19 11	5 28
17	86 48	75 26	61 26	47 21	33 21	19 28	5 48
18	87 48	75 40	61 36	47 33	33 34	19 44	6 8
19	88 48	75 49	61 45	47 43	33 47	20 0	6 27
20	89 48	75 55	61 52	47 53	33 59	20 16	6 47
21	89 12	75 57	61 57	48 1	34 11	20 31	7 6
22	88 12	75 54	62 0	48 8	34 22	20 46	7 26
23	87 12	75 47	62 1	48 14	34 32	21 1	7 45
24	86 12	75 36	62 0	48 18	34 41	21 15	8 4
25	85 12	75 22	61 57	48 22	34 50	21 28	8 23
26	84 12	75 3	61 52	48 24	34 58	21 42	8 42
27	83 12	74 41	61 45	48 25	35 5	21 55	9 1
28	82 12	74 16	61 37	48 25	35 12	22 7	9 20
29	81 12	73 48	61 26	48 24	35 18	22 19	9 38
30	80 12	73 17	61 14	48 22	35 23	22 31	9 57

Altit. Poli. G.	12 (12) G. M.	1 (11) G. M.	2 (10) G. M.	3 (9) G. M.	4 (8) G. M.	5 (7) G. M.	6 (6) G. M.
31	79 12	72 43	60 59	48 18	35 27	22 43	10 15
32	78 12	72 8	60 43	48 13	35 31	22 53	10 33
33	77 12	71 30	60 25	48 8	35 34	23 4	10 50
34	76 12	70 51	60 6	48 1	35 36	23 14	11 8
35	75 12	70 10	59 45	47 52	35 37	23 24	11 25
36	74 12	69 27	59 22	47 43	35 38	23 33	11 43
37	73 12	68 43	58 58	47 33	35 38	23 41	12 0
38	72 12	67 58	58 33	47 21	35 37	23 50	12 16
39	71 12	67 11	58 6	47 8	35 35	23 58	12 33
40	70 12	66 24	57 38	46 55	35 33	24 5	12 49
41	69 12	65 36	57 8	46 40	35 30	24 12	13 6
42	68 12	64 47	56 37	46 25	35 27	24 19	13 22
43	67 12	63 58	56 5	46 7	35 21	24 24	13 37
44	66 12	63 7	55 32	45 50	35 16	24 30	13 53
45	65 12	62 16	54 58	45 31	35 10	24 35	14 8

right half (hours from midnight 7, 8, 9, 10, 11, 12)

[Margin (right): Hic intratur pro horis ☿ & ♊ sinistram versus — "Here one enters, for the hours of Sagittarius ☿ and Aquarius ♊, toward the left."]

Hour columns, from midnight: **7, 8, 9, 10, 11, 12**; the same hours from noon: **5, 4, 3, 2, 1, 12**.

7 (5) G. M.	8 (4) G. M.	9 (3) G. M.	10 (2) G. M.	11 (1) G. M.	12 (12) G. M.	Altit. Poli. G.
14 3 *	27 59	41 35	54 22	65 2	69 48	0
13 42 *	27 35	41 7	53 46	64 12	68 48	1
13 20 *	27 11	40 38	53 9	63 22	67 48	2
12 58 *	26 47	40 8	52 31	62 32	66 48	3
12 36 *	26 22	39 38	51 53	61 40	65 48	4
12 14 *	25 56	39 7	51 13	60 48	64 48	5
11 51 *	25 30	38 36	50 33	59 56	63 48	6

7 (5) G. M.	8 (4) G. M.	9 (3) G. M.	10 (2) G. M.	11 (1) G. M.	12 (12) G. M.	Altit. Poli. G.
11 29 *	25 4	38 4	49 52	59 3	62 48	7
11 6 *	24 37	37 31	49 11	58 10	61 48	8
10 43 *	24 10	36 58	48 29	57 17	60 48	9
10 20 *	23 43	36 25	47 46	56 23	59 48	10
9 56 *	23 15	35 50	47 3	55 29	58 48	11
9 33 *	22 47	35 16	46 15	54 35	57 48	12
9 9 *	22 18	34 41	45 35	53 40	56 48	13
8 45 *	21 50	34 5	44 50	52 45	55 48	14
8 21 *	21 20	33 29	44 5	51 50	54 48	15
7 57 *	20 51	32 52	43 19	50 55	53 48	16
7 33 *	20 21	32 15	42 33	50 0	52 48	17
7 8 *	19 51	31 38	41 47	49 4	51 48	18
6 44 *	19 21	31 0	41 0	48 8	50 48	19
6 19 *	18 50	30 22	40 13	47 12	49 48	20
5 55 *	18 19	29 43	39 25	46 16	48 48	21
5 30 *	17 48	29 4	38 38	45 20	47 48	22
5 5 *	17 17	28 25	37 49	44 23	46 48	23
4 40 *	16 45	27 46	37 1	43 27	45 48	24
4 15 *	16 13	27 6	36 12	42 30	44 48	25
3 50 *	15 41	26 26	35 23	41 34	43 48	26
3 25 *	15 9	25 45	34 34	40 37	42 48	27
3 0 *	14 37	25 5	33 45	39 40	41 48	28
2 35 *	14 4	24 24	32 55	38 43	40 48	29
2 10 *	13 31	23 42	32 5	37 46	39 48	30
1 44 *	12 58	23 1	31 15	36 49	38 48	31
1 19 *	12 25	22 19	30 25	35 52	37 48	32
0 54 *	11 51	21 37	29 34	34 54	36 48	33
0 28 *	11 18	20 55	28 44	33 57	35 48	34
0 3 *	10 44	20 13	27 53	33 0	34 48	35
0 22	10 11 *	19 30	27 2	32 2	33 48	36

7 (5) G. M.	8 (4) G. M.	9 (3) G. M.	10 (2) G. M.	11 (1) G. M.	12 (12) G. M.	Altit. Poli. G.
0 48	9 37 *	18 48	26 11	31 5	32 48	37
1 13	9 3 *	18 5	25 20	30 7	31 48	38
1 38	8 28 *	17 22	24 29	29 9	30 48	39
2 3	7 54 *	16 39	23 37	28 12	29 48	40
2 29	7 20 *	15 55	22 46	27 14	28 48	41
2 55	6 44 *	15 10	21 52	26 14 [?]	27 48	42
3 19	6 11 *	14 28	21 2	25 19	26 48	43
3 44	5 36 *	13 44	20 10	24 21	25 48	44
4 9	5 2 *	13 0	19 18	23 23	24 48	45

These three pages continue the great multi-page table of the Sun's altitudes at every hour and for every elevation of the pole (Table IX, *Altitudines Solis in horis à meridie ac media nocte*), begun earlier with its ornamental title. No new table number is printed on any of these pages; the running headers, the column labels, and the rotated marginal notes are the only structural markings.

Pages 130–131 form a single opening: the **left half** (hours 12–6, p. 130) and the **right half** (hours 7–12, p. 131) of the same block, covering pole-elevations 46°–90° for the declination shared by ♊ Gemini / ♌ Leo (entered from the right) and ♐ Sagittarius / ♒ Aquarius (entered from the left). This completes that declination block, whose pole-elevations 0°–45° were tabulated on the preceding pages.

Page 132 begins the **next declination block** — that shared by ♉ Taurus / ♍ Virgo and ♏ Scorpio / ♊ Pisces — starting again at pole-elevation 0° (left half, hours 12–6).

In every table each hour-cell is printed as two sub-columns headed **G.** (degrees) and **M.** (minutes); they are given here merged as "degrees minutes" within one cell. The two stacked header numbers over each column are the hour reckoned **from noon** (*H. à merid.*, upper) and the hour reckoned **from midnight** (*H. à med. nocte*, lower). The printed sun-ornament (rendered *) marks the cell where the Sun's altitude passes through the horizon; the degree/minute figures were read independently of it.

Table IX (continued) — *Altitudines Solis in horis à meridie*

left half, pole-elevations 46°–90°; block ♊ Gemini · ♌ Leo (right) / ♐ Sagittarius · ♒ Aquarius (left)

[Margin (rotated, left, verbatim): Hic intratur pro horis ♊ & ♌ dexteram versus.]
— "Here one enters for the hours of ♊ Gemini and ♌ Leo, toward the right."

Altit. Poli. (G.)	12 / 12	1 / 11	2 / 10	3 / 9	4 / 8	5 / 7	6 / 6
46	64 12	61 25	54 23	45 11	35 3	24 39	14 23
47	63 12	60 33	53 47	44 50	34 56	24 43	14 38
48	62 12	59 41	53 10	44 29	34 47	24 47	14 52
49	61 12	58 48	52 33	44 6	34 39	24 50	15 6
50	60 12	57 55	51 54	43 43	34 29	24 53	15 20
51	59 12	57 1	51 15	43 19	34 19	24 55	15 34
52	58 12	56 7	50 35	42 54	34 8	24 56	15 47
53	57 12	55 13	49 54	42 28	33 56	24 57	16 0
54	56 12	54 19	49 12	42 1	33 43	24 58	16 13
55	55 12	53 24	48 30	41 34	33 30	24 58	16 26
56	54 12	52 29	47 48	41 6	33 17	24 58	16 38
57	53 12	51 34	47 5	40 37	33 2	24 57	16 50
58	52 12	50 39	46 21	40 8	32 47	24 56	17 2
59	51 12	49 43	45 37	39 38	32 31	24 54	17 13
60	50 12	48 47	44 52	39 7	32 15	24 52	17 24
61	49 12	47 52	44 7	38 35	31 58	24 49	17 35
62	48 12	46 56	43 21	38 3	31 41	24 46	17 45
63	47 12	45 59	42 35	37 31	31 23	24 42	17 55
64	46 12	45 3	41 49	36 57	31 4	24 38	18 5
65	45 12	44 7	41 2	36 24	30 45	24 33	18 14
66	44 12	43 10	40 15	35 50	30 25	24 28	18 23
67	43 12	42 14	39 27	35 15	30 5	24 23	18 32
68	42 12	41 17	38 39	34 40	29 44	24 17	18 40
69	41 12	40 20	37 51	34 4	29 22	24 10	18 48
70	40 12	39 23	37 3	33 28	29 1	24 3	18 56

Altit. Poli. (G.)	12 / 12	1 / 11	2 / 10	3 / 9	4 / 8	5 / 7	6 / 6
71	39 12	38 26	36 14	32 51	28 38	23 56	19 4
72	38 12	37 29	35 25	32 14	28 15	23 48	19 10
73	37 12	36 32	34 36	31 37	27 52	23 39	19 17
74	36 12	35 35	33 47	30 59	27 28	23 30	19 23
75	35 12	34 37	32 57	30 21	27 4	23 21	19 29
76	34 12	33 40	32 7	29 42	26 39	23 11	19 35
77	33 12	32 43	31 17	29 4	26 14	23 1	19 40
78	32 12	31 45	30 27	28 24	25 48	22 51	19 44
79	31 12	30 48	29 36	27 45	25 22	22 40	19 49
80	30 12	29 50	28 46	27 5	24 56	22 28	19 53
81	29 12	28 53	27 55	26 25	24 29	22 16	19 56
82	28 12	27 55	27 4	25 44	24 2	22 4	20 0
83	27 12	26 57	26 13	25 4	23 34	21 52	20 3
84	26 12	25 59	25 22	24 23	23 6	21 38	20 5
85	25 12	25 1	24 30	23 42	22 38	21 25	20 7
86	24 12	24 4	23 39	23 0	22 10	21 11	20 9
87	23 12	23 6	22 47	22 18	21 41	20 57	20 10
88	22 12	22 8	21 56	21 36	21 11	20 42	20 11
89	21 12	21 10	21 4	20 54	20 42	20 27	20 12
90	20 12	20 12	20 12	20 12	20 12	20 12	20 12

right half, pole-elevations 46°–90° (same block, hours 7–12)

[Margin (rotated, right, verbatim): Hic intratur pro horis ☞ & ☞ sinistram versus.]
 — "Here one enters for the hours of ☞ Sagittarius and ☞ Aquarius, toward the left."

The sun-ornament ☼ runs diagonally through the near-horizon cells: down column 8 for poles 46–54, column 9 for poles 55–62, column 10 for poles 63–66, and column 11 for poles 67–69. The noon-reckoned "12" column is the Sun's midnight altitude; its minutes shift from 48' to 12' at the pole where midnight altitude crosses the horizon (□ pole 69–70).

7 / 5	8 / 4	9 / 3	10 / 2	11 / 1	12 / 12	Altit. Poli. (G.)
4 34	4 27 ☼	12 16	18 26	22 25	23 48	46

7 / 5	8 / 4	9 / 3	10 / 2	11 / 1	12 / 12	Altit. Poli. (G.)
4 59	3 52 *	11 32	17 34	21 27	22 48	47
5 24	3 17 *	10 48	16 42	20 29	21 48	48
5 49	2 43 *	10 4	15 49	19 31	20 48	49
6 13	2 8 *	9 20	14 57	18 33	19 48	50
6 38	1 33 *	8 35	14 4	17 35	18 48	51
7 2	0 58 *	7 51	13 12	16 37	17 48	52
7 26	0 23 *	7 6	12 19	15 39	16 48	53
7 51	0 12 *	6 21	11 27	14 41	15 48	54
8 15	0 47	5 37 *	10 34	13 43	14 48	55
8 39	1 22	4 52 *	9 41	12 45	13 48	56
9 3	1 57	4 7 *	8 48	11 47	12 48	57
9 27	2 32	3 22 *	7 55	10 49	11 48	58
9 50	3 7	2 38 *	7 3	9 50	10 48	59
10 14	3 42	1 53 *	6 10	8 52	9 48	60
10 37	4 16	1 8 *	5 17	7 54	8 48	61
11 0	4 51	0 23 *	4 24	6 56	7 48	62
11 23	5 26	0 22	3 31 *	5 58	6 48	63
11 46	6 0	1 7	2 38 *	4 59	5 48	64
12 8	6 35	1 52	1 45 *	4 1	4 48	65
12 31	7 9	2 37	0 52 *	3 3	3 48	66
12 53	7 44	3 21	0 1	2 5 *	2 48	67
13 15	8 18	4 6	0 54	1 7 *	1 48	68
13 37	8 52	4 51	1 47	0 9 *	0 48	69
13 58	9 26	5 36	2 40	0 50	0 12	70

7 / 5	8 / 4	9 / 3	10 / 2	11 / 1	12 / 12	Altit. Poli. (G.)
14 19	10 0	6 20	3 33	1 48	1 12	71
14 41	10 34	7 5	4 26	2 46	2 12	72
14 37	11 8	7 50	5 19	3 44	3 12	73
15 22	11 41	8 34	6 12	4 42	4 12	74
15 42	12 15	9 19	7 5	5 41	5 12	75
16 2	12 48	10 3	7 57	6 39	6 12	76
16 22	13 21	10 47	8 50	7 37	7 12	77
16 42	13 54	11 31	9 43	8 35	8 12	78
17 1	14 27	12 16	10 36	9 33	9 12	79
17 20	14 59	13 0	11 28	10 31	10 12	80
17 38	15 31	13 43	12 21	11 30	11 12	81
17 57	16 4	14 27	13 14	12 28	12 12	82
18 15	16 36	15 11	14 6	13 26	13 12	83
18 33	17 7	15 54	14 59	14 24	14 12	84
18 50	17 39	16 38	15 51	15 22	15 12	85
19 7	18 10	17 21	16 43	16 20	16 12	86
19 24	18 41	18 4	17 36	17 18	17 12	87
19 40	19 11	18 47	18 28	18 16	18 12	88
19	19 42	19 29	19 20	19 14	19 12	89

7 / 5	8 / 4	9 / 3	10 / 2	11 / 1	12 / 12	Altit. Poli. (G.)
56						
20	20 12	20 12	20 12	20 12	20 12	90
12						

Table IX (continued) — next declination block

left half, pole-elevations 0°–45°; block ♂ Taurus · ♍ Virgo (right) / ♏ Scorpio · ♉ Pisces (left)

[Margin (rotated, left, verbatim): Hic intratur pro horis ♂ & ♍ dexteram versus.]
— "Here one enters for the hours of ♂ Taurus and ♍ Virgo, toward the right."

(Row-label heading printed here as *Altitu. Poli. G.*; second header row as *Ho. à med. nocte* — minor spellings differ from p. 130's *Altit. Poli. / H.à med. nocte*, transcribed as printed.)

Altitu. Poli. (G.)	12 / 12	1 / 11	2 / 10	3 / 9	4 / 8	5 / 7	6 / 6
0	78 30	71 11	58 4	43 52	29 20	14 42	0 0
1	79 30	71 47	58 26	44 8	29 34	14 54	0 12
2	80 30	72 21	58 46	44 23	29 47	15 6	0 24
3	81 30	72 53	59 5	44 37	29 59	15 17	0 36
4	82 30	73 22	59 22	44 50	30 11	15 29	0 48
5	83 30	73 48	59 38	45 3	30 22	15 40	1 0
6	84 30	74 12	59 52	45 14	30 32	15 51	1 12
7	85 30	74 32	60 3	45 24	30 42	16 1	1 24
8	86 30	74 49	60 15	45 33	30 52	16 12	1 35
9	87 30	75 2	60 23	45 41	31 0	16 22	1 47
10	88 30	75 11	60 30	45 48	31 8	16 31	1 59
11	89 30	75 17	60 35	45 55	31 16	16 41	2 11
12	89 30	75 19	60 38	45 59	31 23	16 50	2 23
13	88 30	75 16	60 40	46 3	31 29	16 59	2 34
14	87 30	75 10	60 39	46 6	31 35	17 7	2 46
15	86 30	74 59	60 37	46 8	31 40	17 15	2 57
16	85 30	74 46	60 33	46 8	31 44	17 23	3 9
17	84 30	74 28	60 27	46 8	31 48	17 30	3 20

Altitu. Poli. (G.)	12 / 12	1 / 11	2 / 10	3 / 9	4 / 8	5 / 7	6 / 6
18	83 30	74 7	60 19	46 6	31 51	17 38	3 32
19	82 30	73 43	60 9	46 4	31 53	17 44	3 43
20	81 30	73 16	59 58	46 0	31 55	17 51	3 55
21	80 30	72 46	59 44	45 55	31 56	17 56	4 6
22	79 30	72 14	59 29	45 49	31 56	18 3	4 17
23	78 30	71 39	59 13	45 42	31 56	18 8	4 28
24	77 30	71 3	58 55	45 34	31 55	18 14	4 39
25	76 30	70 25	58 35	45 25	31 54	18 18	4 50
26	75 30	69 45	58 14	45 15	31 51	18 23	5 1
27	74 30	69 3	57 51	45 4	31 48	18 27	5 12
28	73 30	68 20	57 21	44 52	31 45	18 31	5 22
29	72 30	67 36	57 1	44 39	31 41	18 34	5 33
30	71 30	66 50	56 35	44 24	31 36	18 37	5 43
31	70 30	66 4	56 7	44 9	31 31	18 40	5 54
32	69 30	65 17	55 37	43 53	31 25	18 42	6 4
33	68 30	64 29	55 7	43 36	31 18	18 44	6 14
34	67 30	63 40	54 36	43 19	31 11	18 46	6 24
35	66 30	62 50	54 3	43 0	31 3	18 47	6 34
36	65 30	62 0	53 29	42 40	30 54	18 48	6 44
37	64 30	61 9	52 55	42 20	30 45	18 49	6 53
38	63 30	60 18	52 19	41 58	30 35	18 49	7 3
39	62 30	59 26	51 43	41 36	30 25	18 49	7 12
40	61 30	58 34	51 6	41 13	30 14	18 49	7 22
41	60 30	57 41	50 28	40 49	30 2	18 48	7 31
42	59 30	56 48	49 49	40 25	29 50	18 47	7 40
43	58 30	55 55	49 10	40 0	29 37	18 45	7 49
44	57 30	55 1	48 30	39 34	29 24	18 43	7 58
45	56 30	54 7	47 49	39 7	29 10	18 41	8 6

These three pages continue the great table of the **Sun's altitudes for every hour, reckoned from noon and from midnight** (*Altitudines Solis in horis à meridie ac media nocte*), which runs across the whole appendix of twenty-four tables. No

"TABVLA" number is printed on any of these three sheets; the running head over the opening supplies the title, and the four zodiac signs served by this stretch of the table — ♉ **Taurus**, ♍ **Virgo** & ♏ **Scorpio**, ♐ **Pisces** (printed at the head of each recto). These are the four sign-entries whose solar declination is 11°30' (northern for Taurus/Virgo, southern for Scorpio/Pisces); accordingly every column converges to 11°30' at pole-elevation 90°, where the Sun's altitude equals its declination at all hours.

Each opening is a mirror-image pair. The **left-hand page** carries the altitudes for the northern pair (Taurus, Virgo) and is entered toward the right; the **right-hand page** carries the altitudes for the southern pair (Scorpio, Pisces) and is entered toward the left — as the rotated marginal rubrics state (transcribed below). The left page runs the hours from noon 12 (noon) through 6; the right page runs the hours from midnight 7 through 12 (midnight), which are the same as the hours-from-noon 5 down to 12. The first opening (pole 0°–45°) is split between the previous sheet (left, pole 0°–45°) and p. 133 (right, pole 0°–45°); the second opening (pole 46°–90°) occupies pp. 134 (left) and 135 (right).

In every table each cell gives the Sun's altitude as **degrees and minutes** (G M); the elevation-of-pole column is headed *Altit. Poli. G.* Column headers carry two numbers: the hour reckoned from midnight (*Hà med. nocte*) above, and the same hour reckoned from noon (*H. à merid.*) below.

right-hand page: Sun's altitudes for ♏ Scorpio & ♐ Pisces, pole 0°–45°

[Margin (rotated, right): Hic intratur pro horis ♏ & ♐ sinistram versus. — "Here [the table] is entered, for the hours of Scorpio and Pisces, toward the left."]

Column heads — top row *Hà med. nocte* (hours from midnight): 7, 8, 9, 10, 11, 12; second row *H. à merid.* (hours from noon): 5, 4, 3, 2, 1, 12. Each hour-column is G M. The elevation-of-pole column (*Altit. Poli. G.*) stands at the right, as printed.

7 (5)	8 (4)	9 (3)	10 (2)	11 (1)	12 (12)	Altit. Poli.
14 42	29 20	43 52	58 4	71 11	78 30	0
14 29	29 6	43 35	57 40	70 33	77 30	1
14 16	28 52	43 17	57 16	69 53	76 30	2
14 3	28 37	42 58	56 50	69 12	75 30	3
13 50	28 21	42 38	56 22	68 29	74 30	4
13 36	28 5	42 17	55 54	67 45	73 30	5

7 (5)	8 (4)	9 (3)	10 (2)	11 (1)	12 (12)	Altit. Poli.
13 23	27 48	41 56	55 24	67 0	72 30	6
13 9	27 31	41 34	54 53	66 14	71 30	7
12 55	27 13	41 11	54 21	65 27	70 30	8
12 40	26 55	40 47	53 48	64 39	69 30	9
12 25	26 37	40 22	53 14	63 50	68 30	10
12 11	26 17	39 57	52 39	63 1	67 30	11
11 55	25 58	39 31	52 4	62 11	66 30	12
11 40	25 38	39 4	51 27	61 20	65 30	13
11 25	25 17	38 37	50 49	60 29	64 30	14
11 9	24 56	38 9	50 11	59 37	63 30	15
10 53	24 35	37 40	49 32	58 45	62 30	16
10 37	24 13	37 11	48 53	57 52	61 30	17
10 21	23 51	36 41	48 12	57 0	60 30	18
10 4	23 29	36 11	47 31	56 6	59 30	19
9 47	23 6	35 39	46 50	55 13	58 30	20
9 31	22 42	35 8	46 7	54 19	57 30	21
9 14	22 19	34 36	45 25	53 25	56 30	22
8 57	21 54	34 3	44 41	52 30	55 30	23
8 40	21 30	33 30	43 58	51 36	54 30	24
8 22	21 5	32 56	43 14	50 41	53 30	25
8 5	20 40	32 22	42 29	49 46	52 30	26
7 47	20 15	31 48	41 44	48 50	51 30	27
7 29	19 49	31 13	40 58	47 55	50 30	28
7 11	19 23	30 37	40 13	46 59	49 30	29
6 53	18 57	30 2	39 26	46 4	48 30	30
6 35	18 30	29 25	38 40	45 8	47 30	31
6 17	18 3	28 49	37 53	44 12	46 30	32
5 59	17 36	28 12	37 6	43 15	45 30	33
5 40	17 8	27 35	36 18	42 19	44 30	34
5 22	16 41	26 57	35 30	41 23	43 30	35
5 3	16 13	26 19	34 42	40 26	42 30	36

7 (5)	8 (4)	9 (3)	10 (2)	11 (1)	12 (12)	Altit. Poli.
4 44	15 45	25 41	33 54	39 29	41 30	37
4 25	15 16	25 3	33 6	38 33	40 30	38
4 6	14 48	24 24	32 17	37 36	39 30	39
3 48	14 19	23 45	31 28	36 39	38 30	40
3 29	13 50	23 5	30 39	35 42	37 30	41
3 9	13 20	22 26	29 49	34 45	36 30	42
2 50	12 51	21 46	29 0	33 48	35 30	43
2 31	12 21	21 6	28 10	32 51	34 30	44
2 12	11 51	20 26	27 20	31 54	33 30	45

left-hand page: Sun's altitudes for ♉ Taurus & ♍ Virgo, pole 46°–90°

[Margin (rotated, left): Hic intratur pro horis ♉ & ♍ dextram versus. — "Here [the table] is entered, for the hours of Taurus and Virgo, toward the right."]

Column heads — top row *H. à merid.* (hours from noon): 12, 1, 2, 3, 4, 5, 6; second row *Ho. à med. nocte* (hours from midnight): 12, 11, 10, 9, 8, 7, 6. The elevation-of-pole column (*Altitu. Poli. G.*) stands at the left, as printed. Each hour-column is 6 M.

Altitu. Poli.	12 (12)	1 (11)	2 (10)	3 (9)	4 (8)	5 (7)	6 (6)
46	55 30	53 13	47 8	38 40	28 56	18 38	8 15
47	54 30	52 19	46 26	38 12	28 41	18 35	8 23
48	53 30	51 24	45 44	37 43	28 26	18 32	8 31
49	52 30	50 29	45 1	37 14	28 10	18 28	8 39
50	51 30	49 34	44 17	36 44	27 53	18 24	8 47
51	50 30	48 39	43 33	36 14	27 36	18 20	8 55

Altitu. Poli.	12 (12)	1 (11)	2 (10)	3 (9)	4 (8)	5 (7)	6 (6)
52	49 30	47 43	42 49	35 43	27 18	18 15	9 2
53	48 30	46 48	42 4	35 11	27 0	18 10	9 10
54	47 30	45 52	41 18	34 39	26 42	18 5	9 17
55	46 30	44 56	40 33	34 6	26 23	17 59	9 24
56	45 30	44 0	39 47	33 33	26 3	17 53	9 31
57	44 30	43 3	39 0	33 0	25 44	17 47	9 38
58	43 30	42 7	38 14	32 26	25 23	17 40	9 44
59	42 30	41 11	37 27	31 51	25 2	17 33	9 50
60	41 30	40 14	36 39	31 16	24 41	17 26	9 57
61	40 30	39 17	35 52	30 41	24 20	17 18	10 3
62	39 30	38 21	35 4	30 5	23 57	17 [numeral:?]	10 8
63	38 30	37 24	34 15	29 [numeral:?]	23 35	17 [numeral:?]	10 14
64	37 30	36 27	33 27	28 53	23 12	16 53	10 19
65	36 30	35 30	32 38	28 16	22 49	16 44	10 25
66	35 30	34 33	31 49	27 39	22 25	16 35	10 30
67	34 30	33 36	31 0	27 1	22 11	16 25	10 34
68	33 30	32	30	26 23	21	16 15	10

Altitu. Poli.	12 (12)	1 (11)	2 (10)	3 (9)	4 (8)	5 (7)	6 (6)
		39	11		37		39
69	32 30	31	29	25 45	21	16 5	10
		41	21		12		44
70	31 30	30	28	25 7	20	15 54	10
		44	32		47		48
71	30 30	29	27	24 28	20	15 44	10
		47	43		22		52
72	29 30	28	26	23 49	19	15 33	10
		49	52		56		56
73	28 30	27	26 2	23 9	19	15 21	11
		52			30		0
74	27 30	26	25	22 30	19	15 10	11
		54	11		4		3
75	26 30	25	24	21 50	18	14 58	11
		57	21		38		6
76	25 30	24	23	21 10	18	14 46	11
		59	30		11		9
77	24 30	24 2	22	20 30	17	14 33	11
			39		44		12
78	23 30	23 4	21	19 49	17	14 20	11
			48		16		15
79	22 30	22 6	20	19 9	16	14 8	11
			57		49		17
80	21 30	21 9	20 6	18 28	16	13 55	11
					21		19
81	20 30	20	19	17 47	15	13 41	11
		11	15		53		21
82	19 30	19	18	17 5	15	13 27	11
		13	24		24		23
83	18 30	18	17	16 24	14	13 14	11
		15	32		56		25
84	17 30	17	16	15 42	14	12 56	11
		17	41		27		26

Altitu. Poli.	12 (12)	1 (11)	2 (10)	3 (9)	4 (8)	5 (7)	6 (6)
85	16 30	16 20	15 49	15 1	13 58	12 45	11 27
86	15 30	15 22	14 57	14 19	13 29	12 30	11 28
87	14 30	14 24	14 6	13 37	12 59	12 16	11 29
88	13 30	13 26	13 14	12 55	12 30	12 1	11 30
89	12 30	12 28	12 22	12 12	12 0	11 46	11 30
90	11 30	11 30	11 30	11 30	11 30	11 30	11 30

right-hand page: Sun's altitudes for ♏ Scorpio & ♓ Pisces,
pole 46°–90°

[Margin (rotated, right): Hic intratur pro horis ♏ & ♓ sinistram versus. — "Here [the table] is entered, for the hours of Scorpio and Pisces, toward the left."]

Column heads as on p. 133 — top row *Hà med. nocte*: 7, 8, 9, 10, 11, 12; second row *H. à merid.*: 5, 4, 3, 2, 1, 12. Elevation-of-pole column at the right, as printed. Each hour-column is G M.

7 (5)	8 (4)	9 (3)	10 (2)	11 (1)	12 (12)	Altit. Poli.
1 53	11 22	19 45	26 30	30 56	32 30	46
1 33	10 51	19 4	25 39	29 59	31 30	47
1 14	10 21	18 23	24 49	29 2	30 30	48
0 55	9 51	17 42	23 58	28 4	29 30	49
0 35	9 20	17 1	23 8	27 7	28 30	50
0 16	8 49	16 20	22 17	26 9	27 30	51
0 3	8 19	15 38	21 26	25 11	26 30	52
0 23	7 48	14 56	20 35	24 14	25 30	53
0 42	7 17	14 14	19 44	23 16	24 30	54
1 1	6 46	13 32	18 52	22 19	23 30	55

7 (5)	8 (4)	9 (3)	10 (2)	11 (1)	12 (12)	Altit. Poli.
1 21	6 14	12 50	18 1	21 21	22 30	56
1 40	5 43	12 8	17 9	20 23	21 30	57
1 59	5 12	11 26	16 18	19 25	20 30	58
2 18	4 40	10 43	15 26	18 27	19 30	59
2 38	4 9	10 1	14 35	17 30	18 30	60
2 57	3 37	9 18	13 43	16 32	17 30	61
3 16	3 6	8 35	12 51	15 34	16 30	62
3 35	2 34	7 52	11 59	14 36	15 30	63
3 54	2 2	7 9	11 7	13 38	14 30	64
4 13	1 31	6 26	10 15	12 40	13 30	65
4 32	0 59	5 43	9 23	11 42	12 30	66
4 51	0 27	5 0	8 31	10 44	11 30	67
5 9	0 4	4 17	7 39	9 46	10 30	68
5 28	0 36	3 34	6 46	8 48	9 30	69
5 46	1 8	2 51	5 54	7 50	8 30	70
6 5	1 40	2 8	5 2	6 52	7 30	71
6 23	2 11	1 24	4 10	5 54	6 30	72
6 41	2 43	0 41	3 18	4 56	5 30	73
7 0	3 15	0 2	2 25	3 58	4 30	74
7 18	3 46	0 46	1 33	3 0	3 30	75
7 35	4 18	1 29	0 41	2 2	2 30	76
7 53	4 49	2 12	0 12	1 4	1 30	77
8 11	5 21	2 55	1 4	0 6	0 30	78
8 28	5 52	3 [numeral:?]	1 56	0 52	0 30	79
8 46	6 23	4 22	2 48	1 50	1 30	80
9 3	6 54	5 5	3 40	2 48	2 30	81
9 20	7 26	5 48	4 33	3 46	3 30	82
9 37	7 57	6 31	5 25	4 44	4 30	83
9 53	8 27	7 14	6 17	5 42	5 30	84
10 10	8 58	7 57	7 10	6 40	6 30	85
10 26	9 29	8 39	8 2	7 38	7 30	86

7 (5)	8 (4)	9 (3)	10 (2)	11 (1)	12 (12)	Altit. Poli.
10 43	9 59	9 22	8 54	8 36	8 30	87
10 59	10 30	10 5	9 46	9 34	9 30	88
11 14	11 0	10 48	10 38	10 32	10 30	89
11 30	11 30	11 30	11 30	11 30	11 30	90

All three pages of this opening are numerical tables from the appendix of Sun's-altitude tables ("TABVLÆ XXIII") to the *Horologiorum Nova Descriptio*. Pages 136–137 carry the conclusion of the untitled altitude table for the beginnings of Aries and Libra (declination 0) — the table continued from the previous pages;

TABVLA X is printed at the foot of p. 137 and is set out on p. 138.

[pp. 136–137] Sun's altitudes at the hours from noon and from midnight — beginnings of Aries (♈) and Libra (♎)

No table number is printed on these two pages; they conclude the table begun earlier. The running header of p. 137 names the signs: "*atque ab ortu & occasu ♈ & ♎*" ("and from rising and setting, Aries & Libra"). The table gives the Sun's altitude above the horizon, argument the elevation of the pole (degrees, left column), for each whole hour reckoned from noon (and equivalently from midnight, rising, and setting).

Column headers (verbatim, per printed row). Each data column is a degree/minute pair; every column except the noon column serves a symmetric morning/afternoon hour.

	col						
Header row	1	col 2	col 3	col 4	col 5	col 6	col 7
H. à merid. (hours from noon)	12	1	2	3	4	5	6
H. à me. no. (hours from midnight)	12	11	10	9	8	7	6
H. ab ortu. (hours from rising)	6	7 · 5	8 · 4	9 · 3	10 · 2	11 · 1	12 · 24
Ho. ab occ. (hours from setting)	18	19 · 17	20 · 16	21 · 15	22 · 14	23 · 13	24 · 12

Data columns below are labelled by the "hours from noon" value; **G** = degrees, **M** = minutes. Pole = elevation of the pole (*Altitu. Poli.*, degrees).

Po le	12·G	12· M	1· G	1· M	2· G	2· M	3· G	3· M	4· G	4· M	5· G	5· M	6· G	6· M
0	90	0	75	0	60	0	45	0	30	0	15	0	0	0
1	89	0	74	58	59	59	45	0	30	0	15	0	0	0
2	88	0	74	52	59	56	44	58	29	59	14	59	0	0
3	87	0	74	43	59	52	44	55	29	57	14	59	0	0
4	86	0	74	29	59	46	44	52	29	55	14	58	0	0
5	85	0	74	12	59	38	44	47	29	52	14	56	0	0
6	84	0	73	52	59	28	44	41	29	49	14	55	0	0
7	83	0	73	29	59	16	44	35	29	45	14	53	0	0
8	82	0	73	3	59	3	44	27	29	41	14	51	0	0
9	81	0	72	34	58	48	44	18	29	36	14	49	0	0
10	80	0	72	2	58	32	44	8	29	30	14	46	0	0
11	89 [recte: 79]	0	71	28	58	14	43	57	29	24	14	43	0	0
12	78	0	70	53	57	54	43	46	29	17	14	40	0	0
13	77	0	70	15	57	33	43	33	29	9	14	36	0	0
14	76	0	69	36	57	10	43	19	29	1	14	33	0	0
15	75	0	68	55	56	46	43	5	28	53	14	29	0	0
16	74	0	68	12	56	21	42	49	28	44	14	24	0	0
17	73	0	67	28	55	55	42	33	28	34	14	20	0	0
18	72	0	66	44	55	27	42	16	28	24	14	15	0	0
19	71	0	65	58	54	58	41	57	28	13	14	10	0	0
20	70	0	65	11	54	28	41	38	28	1	14	5	0	0
21	69	0	64	24	53	57	41	19	27	50	13	59	0	0
22	68	0	63	35	53	25	40	58	27	37	13	53	0	0
23	67	0	62	46	52	52	40	37	27	24	13	47	0	0
24	66	0	61	56	52	18	40	14	27	11	13	41	0	0
25	65	0	61	6	51	43	39	51	26	57	13	34	0	0
26	64	0	60	15	51	7	39	28	26	42	13	27	0	0
27	63	0	59	23	50	30	39	3	26	27	13	20	0	0
28	62	0	58	32	49	53	38	38	26	12	13	13	0	0
29	61	0	57	39	49	14	38	12	25	56	13	5	0	0

Po le	12·G	12· M	1· G	1· M	2· G	2· M	3· G	3· M	4· G	4· M	5· G	5· M	6· G	6· M
30	60	0	56	46	48	35	37	46	25	39	12	57	0	0
31	59	0	55	53	47	56	37	19	25	23	12	49	0	0
32	58	0	55	0	47	16	36	50	25	5	12	41	0	0
33	57	0	54	6	46	35	36	22	24	48	12	32	0	0
34	56	0	53	12	45	53	35	53	24	29	12	23	0	0
35	55	0	52	18	45	11	35	24	24	11	12	14	0	0
36	54	0	51	24	44	29	34	54	23	52	12	5	0	0
37	53	0	50	29	43	46	34	23	23	32	11	56	0	0
38	52	0	49	34	43	2	33	52	23	12	11	46	0	0
39	51	0	48	39	42	18	33	20	22	52	11	36	0	0
40	50	0	47	44	41	34	32	48	22	31	11	26	0	0
41	49	0	46	47	40	49	32	15	22	10	11	16	0	0
42	48	0	45	52	40	4	31	42	21	49	11	5	0	0
43	47	0	44	57	39	18	31	8	21	27	10	55	0	0
44	46	0	44	1	38	32	30	34	21	5	10	44	0	0
45	45	0	43	5	37	46	30	0	20	42	10	33	0	0
46	44	0	42	9	36	59	29	25	20	19	10	21	0	0
47	43	0	41	12	36	12	28	50	19	56	10	10	0	0
48	42	0	40	16	35	25	28	18	19	33	9	58	0	0
49	41	0	39	19	34	37	27	38	19	9	9	47	0	0
50	40	0	38	23	33	49	27	2	18	45	9	35	0	0
51	39	0	37	26	33	2	26	25	18	20	9	22	0	0
52	38	0	36	29	32	13	25	48	17	56	9	10	0	0
53	37	0	35	33	31	25	25	11	17	31	8	58	0	0
54	36	0	34	36	30	36	24	34	17	5	8	45	0	0
55	35	0	33	39	29	47	23	56	16	40	8	32	0	0
56	34	0	32	42	28	58	23	17	16	14	8	19	0	0
57	33	0	31	44	28	9	22	39	15	48	8	6	0	0
58	32	0	30	47	27	19	22	0	15	22	7	53	0	0
59	31	0	29	50	26	29	21	21	14	55	7	40	0	0

Po le	12·G	12· M	1· G	1· M	2· G	2· M	3· G	3· M	4· G	4· M	5· G	5· M	6· G	6· M
60	30	0	28	53	25	40	20	42	14	29	7	26	0	0
61	29	0	27	55	24	50	20	3	14	2	7	13	0	0
62	28	0	26	58	23	59	19	23	13	35	6	59	0	0
63	27	0	26	1	23	9	18	44	13	7	6	45	0	0
64	26	0	25	3	22	19	18	3	12	40	6	31	0	0
65	25	0	24	5	21	28	17	23	12	12	6	17	0	0
66	24	0	23	8	20	38	16	43	11	44	6	3	0	0
67	23	0	22	10	19	47	16	2	11	16	5	48	0	0
68	22	0	21	13	18	56	15	22	10	48	5	34	0	0
69	21	0	20	15	18	5	14	41	10	19	5	19	0	0
70	20	0	19	17	17	14	14	0	9	51	5	5	0	0
71	19	0	18	20	16	22	13	19	9	22	4	50	0	0
72	18	0	17	22	15	32	12	37	8	53	4	35	0	0
73	17	0	16	24	14	40	11	56	8	24	4	20	0	0
74	16	0	15	27	13	49	11	14	7	55	4	5	0	0
75	15	0	14	29	12	57	10	33	7	26	3	50	0	0
76	14	0	13	31	12	6	9	51	6	57	3	35	0	0
77	13	0	12	33	11	14	9	9	6	27	3	20	0	0
78	12	0	11	35	10	22	8	27	5	58	3	5	0	0
79	11	0	10	37	9	31	7	45	5	28	2	50	0	0
80	10	0	9	39	8	39	7	3	4	59	2	35	0	0
81	9	0	8	41	7	47	6	21	4	29	2	19	0	0
82	8	0	7	44	6	55	5	39	3	59	2	4	0	0
83	7	0	6	46	6	4	4	57	3	30	1	48	0	0
84	6	0	5	48	5	12	4	14	3	0	1	33	0	0
85	5	0	4	50	4	20	3	32	2	30	1	18	0	0
86	4	0	3	52	3	28	2	50	2	0	1	2	0	0
87	3	0	2	54	2	36	2	7	1	30	0	47	0	0
88	2	0	1	56	1	44	1	25	1	0	0	31	0	0
89	1	0	0	58	0	52	0	42	0	30	0	16	0	0

Po		12·	1·	1·	2·	2·	3·	3·	4·	4·	5·	5·	6·	6·
le	12·G	M	G	M	G	M	G	M	G	M	G	M	G	M
90	0	0	0	0	0	0	0	0	0	0	0	0	0	0

[p. 138] TABVLA X.

Altitudines Solis supra Horizontem in horis ab ortu vel occasu pro signorum principiis offerens. ("Table X. Giving the altitudes of the Sun above the horizon, in the hours from rising or from setting, for the beginnings of the signs.")

This is a bidirectional table, read rightward for the summer-solstice sign and leftward for the winter-solstice sign, as directed by the rotated marginal rubrics. It is set out in two stacked panels sharing the same pole-elevation argument (30°–60°). **G** = degrees, **M** = minutes; Pole = elevation of the pole (*Altit. Poli.*).

Marginal rubrics (rotated, verbatim):

- [Margin, left: **Hic intratur pro horis ☿ dextram versus,**] ("Enter here for the hours of Cancer, toward the right") — glyph read as Cancer ☿ (the positive-declination, long-day sign; the ascending "Hor. ab or." hour-labels run up to 18).
- [Margin, right: **Hic intratur pro horis ♊ sinistram versus,**] ("Enter here for the hours of Capricorn, toward the left") — glyph read as Capricorn ♊ (the negative-declination sign, read via the "Ho. ab occ." hour-labels).

Column-head pairs are (**Ho. ab occ.** · **Hor. ab or.**): 23·1, 22·2, 21·3, 20·4, 19·5, 18·6.

Pol							
e	23·1 G	M 22·2 G	M 21·3 G	M 20·4 G	M 19·5 G	M 18·6 G	M
30	11	5 24	1 37	1 49	5 62	5 75 [†]	3
		3	6		7	5	0
31	11	4 24	0 36	3 49	2 62	1 74	3
		4		8	7	6	6
32	11	3 23	4 36	1 48	5 61	3 73	4
		5	4	4	6	5	1
33	11	2 23	2 35	5 48	2 60	5 72	4
		6	7	0	3	3	5
34	11	1 23	1 35	2 47	4 60	9 71	4
		7	0	4	9		7
35	11	8 22	5 34	5 47	1 59	2 70	4

Pol e	23·1 G	M 22·2 G	M 21·3 G	M 20·4 G	M 19·5 G	M 18·6 G	M
			1	7	3	3	7
36	10	5 22	3 34	3 46	3 58	3 69	4
		8	2	0	7	6	6
37	10	4 22	1 34	1 45	5 57	4 68	4
		8	3		9	8	4
38	10	3 21	5 33	3 45	2 56	5 67	4
		7	3	2	0	9	2
39	10	2 21	3 33	2 44	4 56	8 66	3
		6	2		1		8
40	10	1 21	1 32	3 44	0 55	1 65	3
		5	1	1		6	3
41	10	4 20	5 31	5 43	1 54	2 64	2
			0	9	8	3	8
42	9	5 20	2 31	2 42	3 53	2 63	2
		2	7	6	4	8	2
43	9	4 20	4 30	5 41	5 52	3 62	1
		0		3	0	2	5
44	9	2 19	4 30	1 41	4 51	3 61	7
		8	0	8		5	
45	9	1 19	1 29	4 40	1 50	3 59	5
		5	6	2	8	8	9
46	9	2 18	5 29	5 39	3 49	3 58	5
			0		0	9	0
47	8	4 18	2 28	2 38	4 48	3 57	4
		9	4	8	1	8	0
48	8	3 17	5 27	4 37	5 47	3 56	2
		5	7	9	0	7	8
49	8	2 17	3 27	1 36	5 46	3 55	1
		0	0	0	9	4	6
50	8	6 17	2 26	2 36	6 45	3 54	3
				9		0	
51	7	5 16	3 25	4 35	1 44	2 52	4
		1	3	7	2	5	9

Pol e	23·1 G	M 22·2 G	M 21·3 G	M 20·4 G	M 19·5 G	M 18·6 G	M
52	7	3 16 6	3 25	4 34	1 43 7	1 51 9	3 4
53	7	2 15 0	3 24 3	2 33 0	2 42 0	1 50 0	1 7
54	7	3 15	1 23	3 32 4	2 41 2	1 48	5 8
55	6	4 14 6	2 22 8	4 31 7	2 39 2	4 47 9	3 8
56	6	2 13 8	5 21 4	5 30 8	1 38 9	3 46 5	1 6
57	6	1 13 0	1 21 9	7 29	1 37 4	1 44 9	5 2
58	5	5 12 1	4 20 2	1 28 4	7 36	0 43	2 5
59	5	3 12 1	3 19	1 26 9	5 34 7	3 41 8	5 5
60	5	9 11	2 18 2	2 25 0	4 33 3	1 40 1	1 9

The right-hand margin repeats the column labels **H. ab or.** / **H. ab oc.** (mirroring the left header block for the leftward reading).

Column-head pairs are (**Ho. ab occ.** · **Hor. ab or.**): 11·13, 10·14, 9·15, 8·16, 7·17, 6·18.

Pol e	11·13 G	M 10·14 G	M 9·15 G	M 8·16 G	M 7·17 G	M 6·18 G	M
30	11	7 0	4 11 3	4 21 4	3 29 1	2 34 7	4 3
31	11	5 0 8	1 10 3	4 20 3	2 28 6	1 33 9	3 6
32	12	4 1 9	9 9	4 19 2	2 27 1	1 32 1	2 9
33	13	3 2 9	5 8	4 18 1	1 26 5	3 31	2 2

Pol e	11·13 G	M 10·14 G	M 9·15 G	M 8·16 G	M 7·17 G	M 6·18 G	M
34	14	2 3 9	1 7	4 17 0	9 24	5 30 5	1 5
35	15	2 3 0	5 6 8	3 16 8	3 23	4 29 7	8
36	16	1 4 1	5 5 4	3 14 6	5 22 7	3 28 8	0
37	17	1 5	5 4 1	3 13 4	5 21 1	2 26 9	5 2
38	17	5 6 2	4 3 8	3 12 1	4 20 4	2 25 0	4 4
39	18	4 7 2	4 2 5	2 11 9	3 19 7	1 24 0	3 5
40	19	3 8 3	4 1 2	2 10 6	2 18 9	0 23	2 6
41	20	2 9 3	3 0 9	2 9 3	2 16 1	5 22 0	1 7
42	21	1 10 4	3 0 7	4 8 1	1 15 3	3 21 9	7
43	22	5 11	3 1 5	4 7 5	4 14	2 19 8	5 7
44	22	5 12 6	3 2 3	5 5 0	5 13 5	1 18 6	4 6
45	23	4 13 7	3 3 2	5 4 5	4 12 5	4 17	3 5
46	24	3 14 8	3 5 2	1 3	3 10 4	5 16 1	2 3
47	25	2 15 9	3 6 1	7 2	2 9 3	3 15 8	1 0
48	26	2 16 0	3 7 1	1 1 4	1 8 1	2 13 4	5 6
49	27	1 17 1	3 8 1	2 0 1	1 7	9 12	4 2
50	28	2 18	3 9	3 1	1 5	5 11	2

Pol e	11·13 G	M 10·14 G	M 9·15 G	M 8·16 G	M 7·17 G	M 6·18 G	M
			2	0	4	3	7
51	28	5 19	3 10	3 2	2 4	3 10	1
		4	4	9	9	6	1
52	29	4 20	3 11	4 3	4 3	1 8	5
		6	6	9	4	8	4
53	30	3 21	3 13	0 5	1 1	5 7	3
		8	9			9	6
54	31	3 22	4 14	1 6	1 0	3 6	1
		0	3	2	9	8	6
55	32	2 23	4 15	2 7	3 0	4 4	5
		2	7	6	8	4	4
56	33	1 24	5 16	4 9	0 2	9 3	3
		5	3	1			1
57	34	9 26	1 17	5 10	2 3	3 2	6
				8	4	6	
58	35	4 27	1 19	1 11	5 5	5 0	3
			0	8	0		8
59	35	5 28	2 20	4 13	1 6	3 0	5
		9	1	0	9	7	4
60	36	5 29	3 22	6 14	5 8	1 2	3
		6	5		3	4	0

Horologiorum Nova Descriptio — Table Appendix (continued)

Solar-altitude tables (Altitudines Solis), giving the Sun's altitude above the horizon at each whole hour before/after rising and setting, for the beginnings of the zodiacal signs, tabulated against the elevation of the pole (latitude) from 30° to 60°. Each page carries two bidirectional panels: the same column of figures is read as an hour "from rising" (ab ortu) or "from setting" (ab occasu) depending on the sign and the direction of entry named in the side-rubrics. Degrees (G. = gradus) and minutes (M. = minuta) are given in each cell. A printed obelus (†) marks, in each row, the column holding the Sun's greatest altitude for that latitude; the mark migrates diagonally down the panel as latitude increases. In the low-altitude cells a decorative sun-

ornament fills the wide blank between the degree and minute figures — it is an ornament, not a numeral, and only the numeric figures are transcribed.

Page 139 — Table X (continued): Cancer ☊ & Capricorn ☋

[Margin (left, rotated): Here one enters for the hours of Cancer ☊, toward the right (*Hic intratur pro horis ☊ dextram versus.*)] [Margin (right, rotated): Here one enters for the hours of Capricorn ☋, toward the left (*Hic intratur pro horis ☋ sinistram versus.*)]

Upper panel — hours 17–12 from setting / 7–12 from rising

Column headers pair *Hor. ab occ.* (top) with *Hor. ab ortu* (bottom): 17|7, 16|8, 15|9, 14|10, 13|11, 12|12. The rightmost column is the elevation of the pole (*Altit. Poli.*). Cell format: degrees minutes.

Altit. Poli.	17 7	16 8	15 9	14 10	13 11	12 12
30	83 31	74 45	62 7	49 9	36 13	23 30
31	82† 30	74 50	62 31	49 42	36 53	24 15
32	81† 28	74 50	62 52	50 14	37 32	25 0
33	80† 25	74 44	63 11	50 44	38 10	25 45
34	79† 21	74 34	63 28	51 13	38 47	26 29
35	78† 15	74 19	63 42	51 41	39 25	27 13
36	77† 9	73 59	63 53	52 7	40 1	27 57
37	76† 2	73 35	64 1	52 33	40 37	28 41
38	74† 55	73 7	64 7	52 56	41 12	29 24
39	73† 47	72 35	64 10	53 18	41 46	30 7
40	72† 38	71 59	64 10	53 39	42 19	30 50
41	71† 29	71 19	64 7	53 57	42 51	31 33
42	70† 19	70 37	64 1	54 14	43 23	32 15
43	69† 9	69 52	63 52	54 29	43 54	32 57
44	67† 58	69 4	63 40	54 43	44 24	33 39
45	66† 47	68 14	63 25	54 54	44 52	34 20
46	65† 35	67 21	63 7	55 3	45 20	35 1
47	64† 22	66 26	62 46	55 10	45 46	35 41
48	63† 8	65 29	62 22	55 15	46 12	36 21

Altit. Poli.	17 7	16 8	15 9	14 10	13 11	12 12
49	61 53	64† 30	61 54	55 17	46 36	37 0
50	60 38	63† 29	61 24	55 18	46 59	37 39
51	59 22	62† 26	60 50	55 15	47 20	38 18
52	58 4	61† 21	60 14	55 10	47 40	38 56
53	56 45	60† 14	59 35	55 3	47 58	39 34
54	55 24	59† 5	58 53	54 53	48 15	40 11
55	54 2	57† 54	58 7	54 39	48 29	40 47
56	52 38	56† 40	57 18	54 23	48 42	41 23
57	51 12	55† 23	56 26	54 3	48 53	41 59
58	49 43	54† 3	55 30	53 40	49 2	42 34
59	48 10	52 39	54† 11	53 12	49 8	43 8
60	46 33	51 11	53† 24	52 41	49 12	43 41

Lower panel — hours 5–1 from rising / 19–23 from setting

Column headers pair *Hor. ab ortu* (top) with *Hor. ab occ.* (bottom): 5|19, 4|20, 3|21, 2|22, 1|23.

Altit. Poli.	5 19	4 20	3 21	2 22	1 23
30	36† 30	34 29	29 1	20 57	11 4
31	35 30	33† 40	28 27	20 36	10 55
32	34 30	32† 51	27 52	20 14	10 45
33	33 29	32† 2	27 16	19 52	10 35
34	32 28	31† 12	26 40	19 29	10 24
35	31 27	30† 21	26 3	19 6	10 13
36	30 25	29† 30	25 25	18 42	10 2
37	29 23	28† 38	24 47	18 18	9 51
38	28 20	27† 46	24 8	17 53	9 39
39	27 17	26† 53	23 28	17 27	9 27
40	26 13	26† 0	22 48	17 1	9 15
41	25 9	25† 6	22 7	16 35	9 2
42	24 5	24† 11	21 26	16 8	8 48
43	23 0	23† 16	20 44	15 41	8 35

Altit. Poli.	5 19	4 20	3 21	2 22	1 23
44	21 54	22† 20	20 1	15 13	8 22
45	20 48	21† 23	19 17	14 44	8 8
46	19 41	20† 26	18 33	14 15	7 54
47	18 33	19† 28	17 48	13 45	7 40
48	17 25	18† 30	17 3	13 14	7 25
49	16 16	17 30	16† 17	12 43	7 10
50	15 6	16 29	15† 30	12 11	6 54
51	13 55	15 28	14† 41	11 39	6 38
52	12 43	14 26	13† 52	11 6	6 21
53	11 30	13 22	13† 2	10 32	6 4
54	10 15	12 17	12† 10	9 57	5 47
55	8 59	11 10	11† 18	9 21	5 29
56	7 41	10 2	10† 23	8 44	5 10
57	6 20	8 52	9† 28	8 5	5 51 [recte: 4 51]
58	4 58	7 39	8† 30	7 26	5 31 [recte: 4 31]
59	3 32	6 24	7 30	6† 44	5 10 [recte: 4 10]
60	2 2	5 5	6 27	6† 0	4 48 [recte: 3 48]

Pages 140–141 — Solar-altitude table for Gemini ♊ & Leo ♌ / Sagittarius ♐ & Aquarius ♒

[Margin (left, rotated, both pages): Here one enters for the hours of Gemini ♊ & Leo ♌, toward the right (*Hic intratur pro horis ♊ & ♌ dextram versus.*)] [Margin (right, rotated, both pages): Here one enters for the hours of Sagittarius ♐ & Aquarius ♒, toward the left (*Hic intratur pro horis ♐ & ♒ sinistram versus.*)]

Page 140, upper panel — hours 23–18 from setting / 1–6 from rising

Column headers pair *Ho. ab occ.* (top) with *Hor. ab or.* (bottom): 23|1, 22|2, 21|3, 20|4, 19|5, 18|6. Pole column at left.

Altit. Poli.	23 1	22 2	21 3	20 4	19 5	18 6
30	12 12	24 51	37 45	50 44	63 32	75† 12
31	12 4	24 35	37 22	50 13	62 50	74† 14
32	11 56	24 19	36 58	49 40	62 6	73† 15

Altit. Poli.	23 1	22 2	21 3	20 4	19 5	18 6
33	11 48	24 2	36 33	49 7	61 22	72† 15
34	11 39	23 45	36 7	48 32	60 36	71† 14
35	11 29	23 27	35 41	47 56	59 48	70† 12
36	11 19	23 9	35 14	47 19	59 0	69 10
37	11 9	22 50	34 45	46 41	58 10	68 7
38	10 59	22 30	34 17	46 2	57 20	67 4
39	10 49	22 10	33 47	45 22	56 28	66 0
40	10 38	21 49	33 16	44 41	55 36	64 56
41	10 27	21 28	32 45	43 59	54 42	63 51
42	10 16	21 6	32 12	43 16	53 47	62 46
43	10 4	20 44	31 39	42 32	52 52	61 40
44	9 52	20 21	31 6	41 47	51 56	60 34
45	9 40	19 57	30 32	41 2	50 59	59 27
46	9 28	19 33	29 56	40 15	50 1	58 20
47	9 15	19 8	29 20	39 28	49 3	57 13
48	9 2	18 43	28 43	38 40	48 4	56 5
49	8 49	18 18	28 5	37 50	47 3	54 56
50	8 36	17 51	27 27	37 0	46 2	53 47
51	8 22	17 24	26 47	36 9	45 0	52 37
52	8 7	16 56	26 7	35 16	43 57	51 26
53	7 52	16 28	25 25	34 23	42 53	50 14
54	7 37	15 59	24 43	33 28	41 48	49 1
55	7 22	15 29	24 0	32 33	40 42	47 48
56	7 6	14 58	23 16	31 36	39 35	46 34
57	6 50	14 26	22 30	30 38	38 26	45 18
58	6 33	13 54	21 43	29 38	37 15	44 1
59	6 16	13 20	20 54	28 37	36 3	42 42
60	5 58	12 46	20 5	27 34	34 49	41 21

Page 140, lower panel — hours 11–6 from setting / 13–18 from rising

Column headers pair *Ho. ab occ.* (top) with *Hor. ab or.* (bottom): 11|13, 10|14, 9|15, 8|16, 7|17, 6|18.

Altit. Poli.	11 13	10 14	9 15	8 16	7 17	6 18
30	7 42	4 18	15 28	25 24	33 21	38 26
31	8 26	3 28	14 33	24 23	32 16	37 21
32	9 10	2 37	13 37	23 21	31 11	36 16
33	9 54	1 47	12 41	22 20	30 6	35 11
34	10 38	0 57	11 44	21 18	29 0	34 6
35	11 22	0 6	10 47	20 15	27 55	33 1
36	12 6	0 46	9 50	19 13	26 50	31 55
37	12 50	1 36	8 53	18 10	25 44	30 49
38	13 34	2 26	7 56	17 8	24 37	29 43
39	14 18	3 17	6 58	16 5	23 30	28 37
40	15 2	4 8	6 0	15 2	22 24	27 30
41	15 46	5 0	5 2	13 58	21 18	26 24
42	16 30	5 52	4 4	12 54	20 10	25 17
43	17 14	6 44	3 5	11 50	19 2	24 9
44	17 57	7 36	2 6	10 46	17 55	23 1
45	18 41	8 28	1 7	9 41	16 47	21 54
46	19 25	9 20	0 8	8 36	15 38	20 46
47	20 8	10 12	0 52	7 30	14 29	19 37
48	20 52	11 5	1 53	6 24	13 20	18 28
49	21 35	11 58	2 53	5 18	12 10	17 18
50	22 19	12 51	3 54	4 11	11 0	16 7
51	23 2	13 45	4 56	3 3	9 49	14 56
52	23 46	14 39	5 59	1 54	8 37	13 45
53	24 30	15 34	7 2	0 45	7 25	12 33
54	25 13	16 28	8 5	0 25	6 11	11 19
55	25 57	17 23	9 9	1 35	4 57	10 5
56	26 40	18 19	10 14	2 47	3 42	8 50
57	27 24	19 15	11 20	4 0	2 25	7 34
58	28 8	20 12	12 27	5 14	1 8	6 16
59	28 53	21 11	13 36	6 30	0 12	4 57
60	29 37	22 10	14 45	7 47	1 33	3 36

Page 141, upper panel — hours 17–12 from setting / 7–12 from rising

Column headers pair *Hor. ab occ.* (top) with *Hor. ab ortu* (bottom): 17|7, 16|8, 15|9, 14|10, 13|11, 12|12. Pole column at right.

Altit. Poli.	17 7	16 8	15 9	14 10	13 11	12 12
30	79 53	71 13	58 55	46 0	33 2	20 12
31	79 1	71 7	59 8	46 24	33 33	20 50
32	78 6	70 57	59 20	46 47	34 4	21 28
33	77 9	70 43	59 29	47 8	34 34	22 5
34	76 11	70 27	59 36	47 29	35 4	22 43
35	75 12	70 7	59 42	47 49	35 34	23 20
36	74† 12	69 44	59 45	48 7	36 2	23 57
37	73† 10	69 17	59 46	48 24	36 30	24 34
38	72† 8	68 48	59 44	48 40	36 57	25 10
39	71† 6	68 16	59 41	48 54	37 24	25 45
40	70† 3	67 41	59 36	49 7	37 50	26 21
41	68† 58	67 4	59 28	49 19	38 15	26 57
42	67† 53	66 25	59 17	49 29	38 39	27 31
43	66† 48	65 43	59 4	49 37	39 2	28 6
44	65† 42	65 0	58 50	49 44	39 25	28 40
45	64† 36	64 15	58 34	49 50	39 46	29 14
46	63† 30	63 27	58 15	49 54	40 7	29 47
47	62† 22	62 38	57 53	49 56	40 26	30 20
48	61† 14	61 48	57 30	49 57	40 45	30 53
49	60† 6	60 55	57 4	49 55	41 3	31 25
50	58† 57	60 2	56 36	49 52	41 19	31 56
51	57† 47	59 7	56 7	49 48	41 35	32 27
52	56† 36	58 10	55 35	49 42	41 50	32 58
53	55† 25	57 12	55 1	49 34	42 3	33 29
54	54 12	56† 12	54 25	49 23	42 15	33 58
55	52 59	55† 11	53 47	49 11	42 25	34 27
56	51 45	54† 8	53 6	48 57	42 35	34 56
57	50 29	53† 3	52 23	48 41	42 43	35 24

Altit. Poli.	17 7	16 8	15 9	14 10	13 11	12 12
58	49 12	51† 56	51 38	48 22	42 49	35 51
59	47 53	50† 48	50 51	48 1	42 54	36 18
60	46 32	49† 37	50 1	47 38	42 57	36 44

Page 141, lower panel — hours 5–1 from rising / 19–23 from setting

Column headers pair *Hor. ab or.* (top) with *Ho. ab occ.* (bottom): 5|19, 4|20, 3|21, 2|22, 1|23.

Altit. Poli.	5 19	4 20	3 21	2 22	1 23
30	39† 44	36 58	30 44	21 57	11 31
31	38† 45	36 11	30 11	21 37	11 22
32	37† 46	35 24	29 37	21 17	11 12
33	36† 47	34 36	29 2	20 55	11 2
34	35† 48	33 48	28 27	20 33	10 52
35	34† 48	32 59	27 51	20 11	10 42
36	33 48	32† 9	27 15	19 48	10 31
37	32 47	31† 19	26 38	19 24	10 20
38	31 47	30† 28	26 1	19 0	10 9
39	30 46	29† 37	25 23	18 36	9 57
40	29 44	28† 46	24 45	18 11	9 45
41	28 42	27† 54	24 6	17 46	9 33
42	27 40	27† 1	23 26	17 20	9 21
43	26 38	26† 8	22 46	16 54	9 9
44	25 35	25† 15	22 5	16 28	8 56
45	24 32	24† 21	21 24	16 1	8 43
46	23 28	23† 27	20 42	15 33	8 29
47	22 24	22† 32	20 0	15 5	8 15
48	21 19	21† 36	19 17	14 36	8 1
49	20 13	20† 39	18 33	14 7	7 47
50	19 7	19† 42	17 49	13 38	7 33
51	18 1	18† 45	17 4	13 8	7 18

Altit. Poli.	5 19	4 20	3 21	2 22	1 23
52	16 54	17† 47	16 19	12 37	7 3
53	15 46	16† 48	15 32	12 6	6 47
54	14 37	15 48	14† 45	11 34	6 31
55	13 28	14 47	13† 57	11 1	6 15
56	12 17	13 45	13† 8	10 28	5 59
57	11 5	12 42	12† 18	9 54	5 42
58	9 52	11 38	11† 27	9 19	5 24
59	8 37	10 32	10† 35	8 43	5 6
60	7 20	9 25	9† 41	8 5	4 47

Gnomonics — Tables (Christoph Clavius, *Opera Mathematica*, Tomus IV)

*Printed pages 142–144. Pages 142–143 form one opening: a continuation of the multi-page table of the Sun's altitudes at each hour (**Altitudines Solis in horis ab ortu & ab occasu**), tabulated by hour of day across the top and by altitude of the pole (i.e. geographic latitude) down the rows. This particular table serves the four signs of equal declination-magnitude **Taurus** ♉ and **Virgo** ♍ (entered "to the right," i.e. as southern/positive-declination entry) and **Scorpio** ♏ and **Pisces** ♐ (entered "to the left"). No printed **Tabula** number appears on pages 142–143 (see anomaly a). Page 144 begins a new numbered table, **Tabula XI**, of horizontal azimuths for latitude 42°.*

*Column convention for the altitude tables below: each hour column heads two figures — the hour reckoned **from setting** (ab occasu) over the hour reckoned **from rising** (ab ortu) — and each data cell holds **degrees and minutes** (**G.** = **gradus**, **M.** = **minuta**). The dagger † marks the printed entry-marker of the "Hic intratur" note (see the marginal notes); in the lower block of p. 142 this same marker is printed as a small star ornament — rendered here uniformly as †.*

Page 142 (printed 142)

[Translation of running header: "Altitudes of the Sun in the hours from rising."]

Upper block — hours ab occasu 23–18 / ab ortu 1–6

Altit. Poli	23·1	22·2	21·3	20·4	19·5	18·6
30	12 48	25 46	38 41	51 14	62 37	70† 28
31	12 40	25 30	38 17	50 39	61 49	69† 27
32	12 32	25 14	37 52	50 4	61 0	68† 26
33	12 23	24 57	37 26	49 27	60 9	67† 24
34	12 15	24 40	37 0	48 50	59 18	66† 23
35	12 6	24 22	36 32	48 11	58 27	65† 21
36	11 57	24 4	36 4	47 32	57 35	64† 19
37	11 47	23 45	35 36	46 52	56 43	63† 18
38	11 37	23 26	35 6	46 11	55 50	62† 16
39	11 27	23 6	34 36	45 30	54 57	61† 14
40	11 17	22 46	34 6	44 48	54 3	60† 12
41	11 7	22 26	33 35	44 6	53 8	59† 10
42	10 57	22 5	33 3	43 22	52 13	58† 8
43	10 46	21 43	32 31	42 38	51 18	57† 6
44	10 35	21 21	31 58	41 54	50 23	56† 4
45	10 24	20 59	31 24	41 9	49 27	55† 2
46	10 12	20 36	30 50	40 23	48 31	53† 59
47	10 0	20 13	30 15	39 37	47 34	52† 57
48	9 48	19 50	29 40	38 51	46 37	51† 54
49	9 36	19 26	29 5	38 4	45 40	50† 51
50	9 24	19 1	28 29	37 16	44 42	49† 48
51	9 12	18 37	27 52	36 28	43 44	48† 45
52	8 59	18 12	27 15	35 40	42 46	47 42
53	8 46	17 46	26 37	34 51	41 48	46 38
54	8 33	17 20	25 59	34 1	40 49	45 35
55	8 20	16 54	25 21	33 11	39 50	44 31
56	8 6	16 28	24 42	32 21	38 51	43 27
57	7 52	16 1	24 3	31 30	37 51	42 23
58	7 38	15 33	23 23	30 39	36 50	41 19
59	7 24	15 6	22 42	29 47	35 50	40 14

Altit. Poli	23·1	22·2	21·3	20·4	19·5	18·6
60	7 9	14 37	22 1	28 55	34 49	39 8

Right-edge column labels: H. ab or. (top) / H. ab oc. (below). [Margin, left (rotated): Hic intratur pro horis ⅆ, & ⅇ, dextram versus. — "Here one enters for the hours of Taurus and Virgo, toward the right."] [Margin, right (rotated): Hic intratur pro horis ⅈ, & ⅉ, sinistram versus. — "Here one enters for the hours of Scorpio and Pisces, toward the left."]

Lower block — hours ab occasu 11–6 / ab ortu 13–18

Altit. Poli	11·13	10·14	9·15	8·16	7·17	6·18
30	1† 16	13 37	25 12	35 28	43 31	48 0
31	0† 48	13 1	24 28	34 36	42 32	46 58
32	0† 19	12 25	23 44	33 44	41 33	45 56
33	0 9	11† 49	23 0	32 52	40 33	44 55
34	0 37	11† 13	22 15	31 59	39 34	43 54
35	1 6	10† 36	21 30	31 6	38 35	42 52
36	1 34	9† 59	20 45	30 13	37 35	41 50
37	2 3	9† 22	20 0	29 20	36 35	40 48
38	2 32	8† 45	19 14	28 26	35 35	39 46
39	3 0	8† 8	18 28	27 32	34 35	38 44
40	3 28	7† 31	17 42	26 38	33 35	37 42
41	3 56	6† 54	16 56	25 44	32 35	36 40
42	4 25	6† 16	16 10	24 50	31 35	35 38
43	4 43 [?]	5† 38	15 25	23 55	30 34	34 35
44	5 22	5† 0	14 37	23 0	29 33	33 33
45	5 50	4† 22	13 50	22 5	28 33	32 31
46	6 18	3† 44	13 3	21 10	27 32	31 28
47	6 46	3† 6	12 15	20 15	26 31	30 26
48	7 14	2† 28	11 28	19 20	25 30	29 23
49	7 42	1† 49	10 40	18 24	24 29	28 20
50	8 10	1† 11	9 52	17 28	23 27	27 17
51	8 38	0† 32	9 4	16 32	22 26	26 14
52	9 6	0 7	8† 16	15 35	21 24	25 10

Altit. Poli	11·13	10·14	9·15	8·16	7·17	6·18
53	9 34	0 46	7† 27	14 39	20 22	24 7
54	10 1	1 24	6† 38	13 42	19 20	23 3
55	10 28	2 3	5† 50	12 45	18 18	22 0
56	10 56	2 42	5† 1	11 48	17 16	20 56
57	11 23	3 21	4† 1	10 51	16 13	19 52
58	11 51	4 1	3† 21	9 53	15 10	18 47
59	12 13	4 40	2† 31	8 55	14 6	17 42
60	12 45	5 20	1† 41	7 56	13 2	16 37

[Margin, left (rotated): Hic intratur pro horis ♂, & ♍, dextram versus. — "Here one enters for the hours of Taurus and Virgo, toward the right."] [Margin, right (rotated): Hic intratur pro horis ♏, & ♐, sinistram versus. — "Here one enters for the hours of Scorpio and Pisces, toward the left."]

Page 143 (printed 143)

[Translation of running header: "...and from setting, [for] Taurus, Virgo, & Scorpio, Pisces." — completing the header "Altitudines Solis in horis ab ortu & ab occasu" of the opening.]

Upper block — hours ab occasu 17–12 / ab ortu 7–12

Altit. Poli	17·7	16·8	15·9	14·10	13·11	12·12
30	69 59	61 34	50 0	37 24	24 28	11 30
31	69 9	61 10	49 53	37 28	24 41	11 51
32	68 18	60 44	49 45	37 32	24 54	12 12
33	67 27	60 16	49 35	37 35	25 6	12 33
34	66 35	69 47 [recte: 59 47]	49 24	37 37	25 18	12 53
35	65 42	59 16	49 12	37 38	25 30	13 13
36	64 48	58 44	48 59	37 38	25 41	13 33

Altit. Poli	17·7	16·8	15·9	14·10	13·11	12·12
37	63 54	58 11	48 44	37 38	25 51	13 53
38	62 59	57 36	48 28	37 37	26 1	14 13
39	62 4	57 0	48 11	37 34	26 10	14 32
40	61 8	56 23	47 53	37 31	26 19	14 51
41	60 12	55 45	47 33	37 27	26 27	15 10
42	59 15	55 6	47 13	37 22	26 35	15 28
43	58 18	54 25	46 52	37 16	26 42	15 47
44	57 21	53 44	46 29	37 10	26 49	16 4
45	56 23	53 2	46 5	37 2	26 55	16 22
46	55 25	52 19	45 40	36 54	27 1	16 40
47	54 26	51 35	45 14	36 45	27 6	16 57
48	53 27	50 51	44 48	36 35	27 10	17 14
49	52 29	50 5	44 20	36 24	27 14	17 31
50	51 30	49 19	43 51	36 12	27 17	17 47
51	50 30	48 32	43 21	35 59	27 20	18 3
52	49† 30	47 45	42 51	35 46	27 22	18 19
53	48† 30	46 56	42 20	35 32	27 23	18 34
54	47† 29	46 7	41	35 16	27 24	18 49

Altit. Poli	17·7	16·8	15·9	14·10	13·11	12·12
			47			
55	46† 28	45 18	41 14	35 0	27 24	19 4
56	45† 27	44 28	40 40	34 44	27 24	19 18
57	44† 26	43 37	40 5	34 26	27 23	19 32
58	43† 24	42 45	39 29	34 7	27 21	19 46
59	42† 22	41 53	38 52	33 48	27 19	19 59
60	41† 19	41 0	38 14	33 28	27 16	20 12

Left-edge column labels: H. ab oc. (top) / H. ab or. (below). Right-edge labels: Hor. ab or. / Ho. ab occ. / Altit. Poli.

Lower block — hours ab occasu 5–1 / ab ortu 19–23

Altit. Poli	5·19	4·20	3·21	2·22	1·23
30	47† 45	42 51	34 32	24 5	12 24
31	46† 49	42 7	34 1	23 46	12 15
32	45† 53	41 23	33 30	23 27	12 6
33	44† 57	40 38	32 58	23 8	11 57
34	44† 0	39 53	32 26	22 48	11 48
35	43† 3	39 7	31 53	22 27	11 38
36	42† 6	38 21	31 20	22 6	11 28
37	41† 9	37 34	30 46	21 45	11 18
38	40† 12	36 47	30 12	21 24	11 8
39	39† 14	36 0	29 38	21 2	10 58
40	38† 17	35 12	29 3	20 39	10 47
41	37† 19	34 24	28 27	20 16	10 36
42	36† 21	33 36	27 51	19 53	10 25
43	35† 22	32 47	27 14	19 29	10 14
44	34† 24	31 58	26 37	19 5	10 2

Altit. Poli	5·19	4·20	3·21	2·22	1·23
45	33† 26	31 8	26 0	18 41	9 50
46	32† 27	30 18	25 22	18 16	9 38
47	31† 28	29 28	24 44	17 51	9 26
48	30† 28	28 38	24 6	17 26	9 13
49	29† 29	27 47	23 27	17 0	9 1
50	28† 30	26 56	22 47	16 34	8 48
51	27† 30	26 4	22 8	16 7	8 35
52	26 30	25† 13	21 28	15 41	8 22
53	25 30	24† 21	20 47	15 14	8 8
54	24 30	23† 28	20 6	14 46	7 54
55	23 29	22† 35	19 25	14 18	7 40
56	22 28	21† 42	18 42	13 50	7 26
57	21 27	20† 49	18 1	13 21	7 12
58	20 25	19† 55	17 19	12 53	6 58
59	19 24	19† 1	16 36	12 23	6 43
60	18 21	18† 6	15 53	11 54	6 28

Right-edge labels: Hor. ab or. / Ho. ab occ. / Altit. Poli. [Margin, left (rotated): Hic intratur pro horis ♉, & ♍, dextram versus. — "Here one enters for the hours of Taurus and Virgo, toward the right."] [Margin, right (rotated): Hic intratur pro horis ♏, & ♐, sinistram versus. — "Here one enters for the hours of Scorpio and Pisces, toward the left."]

Page 144 (printed 144)

XI. TABULA.

CIRCUMFERENTIÆ HORIZONTALIS inter Verticalem primarium, & alios Verticales per hor. à mer. & med. noc. in signorum initijs ductos.

Table XI. The horizontal circumferences [azimuths] between the prime vertical and the other verticals, drawn through the hours from noon and from midnight, at the beginnings of the signs.

Ad latitudinem Grad. 42.

For latitude 42 degrees.

Column headers pair the two signs of equal declination served by each column (northern and southern points share a column, distinguished by the A/B letter): ☊ | ☋ | ☌ | ☍ | ☎ | ☏ | ☐ | ☑ | ☒ | ☓ | ☔ | ☕ | ☖. Each cell is degrees, minutes, with the letter **A** (australis — southern azimuth) or **B** (borealis — northern azimuth) printed beneath. The leftmost column is Horæ à meridie (hours from noon); the rightmost is Horæ à med. noc. (hours from midnight).

Horæ à mer.	☊	☋	☌	☍	☎	☏	☐	☑	☒	☓	☔	☕	Horæ à med. noc.
12	90 0 A	90 0 A	90 0 A	90 0 A	90 0 A	90 0 A	90 0 A	90 0 A	90 0 A	90 0 A	90 0 A	90 0 A	12
½	69 1 A	71 30 A	75 42 A	78 52 A	80 55 A	82 4 A	82 28 A	11½					
1	51 19 A	55 14 A	62 24 A	68 52 A	72 1 A	74 17 A	75 3 A	11					
1½	37 35 A	42 0 A	50 40 A	58 14 A	63 33 A	66 46 A	67 53 A	10½					
2	27 9 A	31 26 A	40 36 A	49 13 A	55 37 A	59 37 A	61 1 A	10					
2½	18 51 A	22 54 A	31 57 A	41 5 A	48 15 A	52 53 A	54 31 A	9½					
3	11 57 A	15 47 A	24 29 A	33 47 A	41 26 A	46 33 A	48 24 A	9					
3½	6 0 A	9 35 A	17 54 A	27 11 A	35 9 A	40 38 A	42 38 A	8½					
4	0 34 A	4 3 A	11 58 A	21 7 A	29 17 A	35 4 A	37 13 A	8					
4½	4 6 B	1 3 B	6 29 A	15 31 A	23 47 A	29 49 A	32 5 A	7½					
5	8 51 B	5 56 B	1 12 A	10 10 A	18 34 A	24 49 A	27 12 A	7					
5½	13 24 B	10 39 B	3 43 B	5 2 A	13 32 A	20 0 A	22 29 A	6½					

Horæ à mer.	☾	♊ ♀	♈ ♂	♉ ♂	♊ ♀	♈ ♂	♉ ♂	Horæ à med. noc.
6	17 54 B	15 17 B	8 36 B	0 0 A	8 36 A	15 17 A	17 54 A	6
6½	22 29 B	20 0 B	13 32 B	5 2 B	3 43 A	10 39 A	13 24 A	5½
7	27 12 B	24 49 B	18 34 B	10 10 B	1 12 B	5 56 A	8 51 A	5
7½	32 5 B	29 49 B	23 47 B	15 31 B	6 29 B	1 3 A	4 6 A	4½
8	37 13 B	35 4 B	29 17 B	21 7 B	11 58 B	4 3 B	0 34 A	4
8½	42 38 B	40 38 B	35 9 B	27 11 B	17 54 B	9 [numeral:? B	6 0 B	3½
9	48 24 B	46 33 B	41 26 B	33 47 B	24 29 B	15 47 B	11 57 B	3

[Margin, left (rotated): Litera A, gradibus ac minutis supposita, significat circumferentiam eam esse australem; litera vero B, borealem. Omnes porro circumferentiæ horarum à meridie sunt occidentales; horarum vero à med. noc. orientales. Horæ denique, quæ in 8. tabula sub Horizonte existunt, in hac quoque sub Horizonte esse intelliguntur. — "The letter A, placed beneath the degrees and minutes, signifies that the azimuth is southern; but the letter B, northern. Moreover all the azimuths of the hours from noon are western, but those of the hours from midnight, eastern. Finally, the hours which in Table 8 lie below the Horizon are here likewise understood to lie below the Horizon."]

XII. TABULA.

Table 12

Comprising the **horizontal azimuths** [*circumferentias horizontales*] — the arcs of the horizon between the **prime vertical** [*Verticalis primarius*] and the other verticals passing through the center of the Sun at each hour — for the hours from rising and from setting, for the beginnings of the signs, constructed for pole-altitudes from 30 degrees up to 60 degrees.

[This title page carries only the heading; the faint horizontal and vertical rules visible above and below the ornamental bands are show-through of the tables printed on the reverse leaf (p. 146). No table is set on this page.]

Two azimuth tables are set on this page, one above the other. In both, the leftmost column is **Altit. Poli.** (altitude of the pole = geographic latitude), running 30° to 60°. Each hour is given a pair of sub-columns **G.** (*gradus*, degrees) and **M.** (*minuta*, minutes). Each hour-column carries two heads: **Ho. ab occ.** (the hour from setting) above **Hor. ab or.** (the hour from rising). The letters **A** and **B** printed against a value mean, respectively, *australis* (southern, A) and *borealis* (northern, B) — the same convention as the preceding table; the letter is printed only where the azimuth changes side or where needed, and is understood to persist down the column. A printed star (rendered * below) and a printed dagger (†) mark, respectively, the two sunrise/entry diagonals running across the grid (keyed to the marginal "here one enters..." notes for Cancer and Capricorn).

Upper table — Ho. ab occ. 24, 23, 22, 21, 20, 19, 18 / Hor. ab or. 24, 1, 2, 3, 4, 5, 6. (Column heads at right: **H. ab or. / H. ab oc.**)

	24·24	23·1	22·2	21·3	20·4	19·5	18·6	
Altit. Poli.	G	MG	MG	MG	MG	MG	MG	M
30	27B	2 20 5	2 14 5	1 8 1	8 1B	3 6A 9	4 23† 6	1 1
31	27B	4 20 3	3 13 0	5 7 9	3 0B 9	4 8A 8	1 25 3	2 9
32	28B	3 20	3 13 6	4 7B 9	1 0A 1	3 9	3 27 6	2 9
33	28B	2 20 4	4 13 3	4 6B 1	4 0A 5	5 10 4	5 29 4	1 1
34	28B	4 20 5	5 13 1	3 6B 4	2 1A 2	3 12 9	5 30	4 3
35	29B	8 21	0 13	2 6B 9	0 2A	2 13 1	9 31	5 8
36	29B	3 21 2	1 13 1	2 5B 6	4 2A 0	5 14 9	9 33	6
37	29B	5 21 7	2 13 3	2 5B 3	2 3A 3	3 15 4	3 34	4
38	30B	2 21	3 13	2 5B	7 4A	4 15	5 34	5

Altit. Poli.	24·24 G	23·1 MG	22·2 MG	21·3 MG	20·4 MG	19·5 MG	18·6 MG	M
		4	7	3			2	0
39	30B	5 21	5 13	2 4B	5 4A	3 16	3 35	2
		2	3	6	4	2	6	9
40	31B	2 22	1 13	2 4B	4 4A	5 17	1 36	2
		2	0	9	4	8	5	
41	31B	5 22	2 13	3 4B	3 5A	2 17	4 36	2
		4	9	3	6	1	9	8
42	32B	2 22	5 13	4 4B	3 5A	4 18	1 36	4
		7	0	1	0	0	7	7
43	33B	2 23	1 13	5 4B	2 5A	5 18	4 36	5
			3	2	7	5	2	9
44	33B	4 23	3 14	5 4B	2 6A	6 19	2 37	6
		0	9		8			
45	34B	2 24	7 14	2 4B	3 6A	1 19	1 37	9
		0		0	1	3	5	
46	35B	2 24	3 14	4 4B	3 6A	1 19	2 37	4
			7	0	8	4	4	
47	35B	4 25	1 15	3 4B	4 6A	1 19	2 36	5
		7	1		8	3	8	4
48	36B	3 25	4 15	2 5B	2 6A	1 19	2 36	4
		5	8	8		4	6	0
49	37B	2 26	2 15	5 5B	2 6A	7 19	2 36	2
		6	8	6	0		0	2
50	38B	2 27	1 16	2 5B	4 5A	5 19	9 35	5
		0	2	9	2	3		7
51	39B	1 28	0 17	6 6B	9 5A	3 18	5 35	2
		9				4	1	5
52	40B	2 28	5 17	4 6B	4 5A	9 18	2 34	4
		2	2	8	1		8	8
53	41B	3 29	5 18	3 7B	1 4A	3 17	5 34	5
		0	0	4	9	8	7	
54	42B	4 30	5 19	2 8B	2 3A	5 17	2 33	1
		3	3	7		9	0	6

Altit. Poli.	24·24 G	23·1 MG	22·2 MG	21·3 MG	20·4 MG	19·5 MG	18·6 MG	M
55	44B	3 32	3 20	2 8B 7	5 3A 2	1 16 3	3 32 5	2 0
56	45B	2 33 9	1 21 9	3 9B 4	5 2A 1	2 15 0	4 31 3	1 4
57	47B	4 34	4 22 5	5 10B 0	5 1A 8	1 14 9	4 29 0	5 8
58	48B	4 36 8	2 24 1	1 12B 5	1 0A 5	8 13	2 28 6	3 2
59	50B	4 38 4	7 25	5 13 2	4 1B 3	1 12A 6	1 26	5 6
60	52B	5 40 4	8 27	4 15 4	2 2B 7	5 10A 6	2 25 2	5

Lower table — Ho. ab occ. 11, 10, 9, 8, 7, 6 / Hor. ab or. 13, 14, 15, 16, 17, 18.

Altit. Poli.	11·13 G	10·14 MG	9·15 M G	8·16 M G	7·17 M G	6·18 M G	M
30	20	5 27 0	* 36 5 3	1 46 0	1 58 6	4 73 4	4 5
31	20	2 27 2	3 35 4	* 46 5 7	4 58	2 73 7	1 7
32	19	5 27 3	1 35 4	* 45 4 3	5 58 2	1 72 0	4 9
33	19	2 26 2	5 35 3	* 45 2 8	3 57 8	5 72 2	2 1
34	18	4 26 9	3 35 0	* 45 1 1	2 57 2	3 71 3	5 2
35	18	1 26 4	5 34	* 45 5 1	5 57	1 71 2	2 2

Altit. Poli.	11·13	10·14	9·15		8·16		7·17		6·18		M
	G	M G	M	G	M	G	M	G	M	G	
36	17	3 25 6	3 34 8		* 44 3 0		4 56 6		4 70 9		5 1
37	16	5 25 7	8 34		* 44 7		2 56 5		2 70 5		1 9
38	16	1 24 5	3 33 7		* 44 4 3		3 56		0 69		4 6
39	15	3 24 1	4 33		* 43 1 7		3 55 9		3 69 4		1 2
40	14	4 23 4	2 32 8		* 43 4 9		1 55 3		6 68		3 7
41	13	5 22 5	5 32 0		* 42 1 7		4 54 5		3 68 6		0
42	13	3 22	9 31		4 42 3		* 54 1 4		4 67		2 2
43	12	8 21	2 31 6		7 41		* 53 4 1		3 66 0		4 2
44	11	9 20	3 30 9		2 41 9		* 52 6		5 66 3		0
45	10B	6 19	5 29 0		4 40 7		* 52 2 8		1 65 4		1 6
46	9B	0 18	5 29 7		2 39		* 51 4 7		3 64 2		3 0
47	7B	5 18 1	0 28		1 39 4		* 50 3		4 63 8		4 2
48	6B	3 17	0 27		2 38		* 50		1 62		5

Altit. Poli.	11·13	10·14	9·15		8·16		7·17		6·18		M
	G	MG	M	G	M	G	M	G	M	G	
		7			2		1				1
							5				
49	5B	1 15	5 26		2 37		2 49		* 61		5
		8	5		6		4		1		7
									1		
50	3B	5 14	4 25		2 36		2 48		* 61		0
		4	5		6		9		1		
									7		
51	2B	2 13	3 24		2 35		3 47		* 59		5
		4	0		2		0		1		9
									9		
52	0B	4 12	1 23		1 34		2 46		* 58		5
		9	0		2		6		1		4
									6		
53	0A	5 10B	4 21		5 33		1 45		* 57		4
		3	3		6		6		8		4
54	2A	4 9B	1 20		3 32		1 43		* 56		2
		4	0		4				5		9
									5		
55	4A	4 7B	3 19		5 30		4 42		3 55		*
		3	0				0		6		8
56	6A	5 5B	4 17		2 29		1 41		9 53		*
		2	0		8		0				4
											1
57	9A	1 3B	3 15		4 27		3 39		3 52		*
		3	8		1		1		4		5
58	11A	4 1B	2 13		4 25		4 37		4 50		*
		5	4		2		1		9		2
											0
59	14	3 1A	2 11B		3 23		4 35		5 48		2
		4			1		0		3		5
60	17	4 3A	4 9B		4 21		2 33		4 46		1
		2	6				4		3		6

[Margin, left (rotated): Hic intratur pro horis ☾, dextram versus. — "Here one enters for the hours of Cancer [☾], toward the right."]

[Margin, right (rotated): Hic intratur pro horis ♊, sinistram versus. — "Here one enters for the hours of Capricorn [♊], toward the left."]

Two further azimuth tables, same structure (Altit. Poli. 30–60; each hour a **G./M.** pair; **A** = *australis*, **B** = *borealis*; the printed dagger † marks the sunrise/entry diagonal running across the grid).

Upper table — Ho. ab occ. 17, 16, 15, 14, 13, 12 / Hor. ab or. 7, 8, 9, 10, 11, 12.

Altit. Poli.	17·7 G	M 16·8 G	M 15·9 G	M 14·10 G	M 13·11 G	M 12·12 G	M
30	86	1 21 2	3 6A 3	9 2B	3 8	3 14 3	3 2
31	88†	5 26 7	2 8A	2 0B 9	3 7 8	3 13 1	5 1
32	85†	1 30 5	3 10 2	5 0A 3	5 6B 3	2 13 8	8
33	82†	1 35 6	1 13	2 2A 0	2 5B 8	2 12 4	2 3
34	79†	4 39 6	2 15 8	5 4A 4	6 4B	1 11 6	3 7
35	77†	3 43 6	5 18 3	3 5A 3	4 3B 7	6 10	4 9
36	75†	4 48 6	8 21	1 7A 4	3 1B 2	5 9 3	5 7
37	74†	6 52	1 24 7	0 9A	2 0B 1	3 9 7	3
38	72†	3 56 6	1 26 6	4 11 9	1 0A 4	4 8B 1	6
39	71†	1 60 5	4 29	4 13 1	1 2A 1	1 7B	6
40	69†	5 63 8	4 32 1	3 15 5	1 3A 1	2 6B 4	3
41	68†	4 67 6	8 35	3 17 2	1 4A 5	5 4B 2	5 7
42	67†	3 70	2 38	3 19	2 6A	2 3B	4

Altit. Poli.	17·7 G	M 16·8 G	M 15·9 G	M 14·10 G	M 13·11 G	M 12·12 G	M
		5	8	1	4	7	8
43	66†	2 73	3 41	3 21	3 8A	5 2B	3
		7	7	1	6		6
44	65†	2 76	3 44	3 23	5 9A	4 1B	2
		0	8	3	3	5	0
45	64†	1 79	3 47	3 26	1 11A	3 0	0
		4	0	4	4	1	
46	63†	6 82	1 50	3 28	4 13	2 1A	2
			7	7	0	1	5
47	61†	5 84	5 53	4 31	1 15	1 2A	5
		9	7	1	0	6	5
48	60†	5 87	3 56	4 33	4 17	1 4A	2
		2	2	4	5	6	9
49	59	4 89†	5 59	4 36	2 19	2 6A	7
		3	8	7	4	2	
50	58	3 87†	3 62	5 39	8 21 [?]	3 7A	5
		1	0	0		4	1
51	57	1 85†	4 65	5 41	5 23	5 9A	4
		6		5	9	3	1
52	55	5 82†	4 69	0 44	5 26	1 11A	3
		9	1		4	7	8
53	54	3 80†	1 72	8 47	5 28	5 13	4
		8	6		5	2	3
54	53	1 77†	5 75	1 51	2 31	3 15	5
		3	1	7		3	6
55	51	4 75†	2 78	2 54	1 34	2 18	1
		2	6	8	6	3	9
56	50	5 72†	5 81	4 57	3 37	2 20	5
			6	3	9	5	2
57	48	1 70†	2 85	5 61	1 40	3 23	3
		8	0		2	8	6
58	46	2 67†	3 88	3 64	5 44	5 26	3
		3	8	3	5		5
59	44	1 64	4 87†	5 68	4 47	4 29	5

Altit. Poli.	17·7 G	M 16·8 G	M 15·9 G	M 14·10 G	M 13·11 G	M 12·12 G	M
		9	9	1	9	7	2
60	42	0 61	4 84† 6	3 73	1 51	4 33 9	2 8

Lower table — Ho. ab occ. 5, 4, 3, 2, 1, 24 / Hor. ab or. 19, 20, 21, 22, 23, 24.

Altit. Poli.	5·19 G	M 4·20 G	M 3·21 G	M 2·22 G	M 1·23 G	M 24·24 G	M
30	89†	2 72 8	4 57 5	5 45 3	3 35 5	3 27 6	2 5
31	89	5 73† 4	3 58 6	4 46 3	1 36 8	8 27	4 3
32	89	9 74†	2 59 6	3 47 3	1 36	4 28 0	3
33	88	2 75† 7	1 60 6	2 47 4	4 37 5	1 28 3	2 4
34	87	4 76† 6	6 61	1 48 5	2 37 9	4 28 7	4 5
35	87	4 76†	5 62 7	7 49	1 38 5	2 29 3	8
36	86	2 77† 2	4 62 7	5 50 9	1 38	5 29 9	3 2
37	85	3 78† 9	3 63 8	5 50 1	4 39 8	3 29 6	5 7
38	84	5 79† 6	3 64 0	4 51 4	3 40 6	1 30 5	2 4
39	84	1 80† 2	2 65 2	3 52 7	2 40 5	5 30 4	5 2
40	83	2 81† 8	1 66 4	3 53 2	1 41 5	3 31 6	2 2
41	82	4 82† 2	6 67	2 54 7	6 42	1 31 8	5 4
42	81	5 83† 5	0 68	2 54 3	5 43 9	1 32	2 7
43	81	7 83†	5 69 6	2 55 1	5 43 3	4 33 8	2

Altit. Poli.	5·19 G	M	4·20 G	M 3·21 G	M 2·22 G	M 1·23 G	M 24·24 G	M
44	80	1 7	84†	5 70 3	2 56 0	4 44 9	3 33 7	4 0
45	79	2 6	85†	5 71 0	2 57 0	4 45 6	2 34 7	2 0
46	78	3 3	86†	5 72 0	2 58 3	4 46 6	2 35 0	2
47	77	3 8	87†	5 73 2	2 59 7	4 47 8	1 35 4	4 7
48	76	4 0	88†	5 74 5	3 60 3	5 48 2	1 36 1	3 5
49	75	4 0	89	5 75† 9	4 61 2	5 49 9	1 37 2	2 6
50	74	3 7	88	5 76† 0	5 63 4	9 50	1 38 6	2 0
51	73	3 1	87	3 78† 8	8 64	2 51 2	2 39 4	1 9
52	72	2 1	86	2 79† 3	2 65 7	3 52 9	3 40 5	2 2
53	71	6	85	3 80†	5 67 0	1 53	5 41 2	3 0
54	69	4 7	83	3 82† 0	1 68 7	2 55 7	1 42 3	4 3
55	68	2 3	82	1 83† 0	4 69 9	5 56 9	4 44 0	3
56	66	5 1	80	3 85† 4	2 71 8	3 58 7	1 45 4	2 9
57	65	1 2	78	5 87† 0	1 73 4	2 59 4	5 47 7	4
58	63	2 4	76	5 89† 8	1 75 0	1 61 9	4 48 9	4 8
59	61	* 2 6	74	5 88 7	4 77† 5	2 63 5	5 50 1	4 4
60	59	*	72	4 86	2 79†	4 66	7 52	5

Altit. Poli.	5·19 G	M	4·20 G	M 3·21 G	M 2·22 G	M 1·23 G	M 24·24 G	M
	1		2		6		4	
	6							4

[Margin, left (rotated): *Hic intratur pro horis ☊, dextram versus.* — "Here one enters for the hours of Cancer [☊], toward the right."]

[Margin, right (rotated): *Hic intratur pro horis ☋, sinistram versus.* — "Here one enters for the hours of Capricorn [☋], toward the left."]

These three pages continue the great table of **horizontal circumferences** (azimuths of the Sun's center reckoned from the prime vertical, hour by hour, for pole-heights 30°–60°), begun as Table XII on printed p. 145. Each opening serves one group of declination-paired signs. The opening pp. 148–149 serves **Gemini** (♊), **Leo** (♌), **Sagittarius** (♐), and **Aquarius** (♒); p. 150 begins the next opening, for **Taurus** (♉), **Virgo** (♍), **Scorpio** (♏), and **Pisces** (♐).

In every table the argument column is **Altit. Poli.** (altitude of the pole, 30–60). Each hour-pair is headed by two numbers: the upper is **Ho. ab occ.** (the hour reckoned from setting), the lower is **Hor. ab or.** (the hour reckoned from rising); the two share one column of degrees (**G.**) and minutes (**M.**). Certain degree figures carry a printed side-letter **A** or **B** exactly as in the preceding table-pages; these are transcribed verbatim. A printed dagger † and a printed many-pointed reference asterisk * each trace a slanting reference band down through the table; both are reference marks, not digits, and are transcribed attached to the figure they touch. Cells give "degrees[letter] minutes."

Printed p. 148 — horizontal circumferences, signs ♊ ♌ ♐ ♒ (left page of opening)

[Margin (left, rotated): *Hic intratur pro horis ♊, & ♌, dextram versus.* — "Here one enters for the hours of Gemini (♊) and Leo (♌), toward the right."]

[Margin (right, rotated): *Hic intratur pro horis ♐, & ♒, sinistram versus.* — "Here one enters for the hours of Sagittarius (♐) and Aquarius (♒), toward the left."]

Upper table (hours from rising 24, 1–6)

Altit. Poli.	24 / 24	23 /	22 /	21 /	20 /	19 /	
		1	2	3	4	5	18 / 6
30	23B 30	16	9 54	3B	4A	15	38† 39
		27		18	22	21	

Altit. Poli.	24 / 24	23 / 1	22 / 2	21 / 3	20 / 4	19 / 5	18 / 6
31	23B 45	16 28	9 41	2B 45	5A 16	16 46	40† 13
32	24B 2	16 30	9 28	2B 15	6A 8	18 4	41† 31
33	24B 19	16 33	9 17	1B 47	6A 57	19 17	42† 41
34	24B 37	16 38	9 7	1B 20	7A 43	20 23	43† 39
35	24B 56	16 44	8 58	0B 55	8A 26	21 25	44† 29
36	25B 16	16 51	8 51	0B 32	9A 7	22 21	45 10
37	25B 37	16 59	8 44	0B 11	9A 44	23 12	45 43
38	25B 59	17 9	8B 40	0A 6	10 19	23 59	46 12
39	16B 23 [recte: 26B 23]	17 19	8B 38	0A 23	10 48	24 41	46 34
40	26B 48	17 31	8B 36	0A 37	11 1	25 17	46 52
41	27B 14	17 45	8B 37	0A 50	11 43	25 51	47 5
42	27B 41	18 0	8B 39	1A 3	12 5	26 19	47 12
43	28B 10	18 18	8B 44	1A 10	12 25	26 43	47 15
44	28B 41	18 37	8B 51	1A 14	12 41	27 2	47 15
45	29B 14	18 58	9B 0	1A 18	12 53	27 19	47 13
46	29B 48	19 21	9B 12	1A 18	13 2	27 30	47 4
47	30B 25	19	9B	1A	13 8	27	46 52

Altit. Poli.	24 / 24	23 / 1	22 / 2	21 / 3	20 / 4	19 / 5	18 / 6
		46	26	14		36	
48	31B 4	20	9B	1A 8	13 9	27	46 35
		14	42			38	
49	31B 45	20	10B	1A 0	13 7	27	46 16
		45	1			37	
50	32B 30	21	10B	0A	13 2	27	45 53
		17	24	47		31	
51	33B 17	21	10B	0A	12	27	45 24
		54	50	30	52	20	
52	34B 7	22	11B	0A 8	12	27 3	44 51
		36	21		37		
53	35B 1	23	11	0B	12A	26	44 14
		19	55	15	17	41	
54	35B 59	24 7	12	0B	11A	26	43 33
			32	45	53	15	
55	37B 1	25 0	13	1B	11A	25	42 46
			14	20	23	43	
56	38B 8	25	14 3	2B 0	10A	25 5	41 53
		57			47		
57	39B 21	27 0	14	2B	10A	24	40 54
			58	47	4	20	
58	40B 40	28	15	3B	9A	23	39 48
		10	59	41	15	29	
59	42B 6	29	17 8	4B	8A	22	38 34
		29		42	18	28	
60	43B 41	30	18	5B	7A	21	37 11
		54	25	52	11	18	

Lower table (hours from rising 13–18)

Altit. Poli.	11 / 13	10 / 14	9 / 15	8 / 16	7 / 17	6 / 18
30	18 55	26 *18	35 0	45 40	59 1	75 15
31	18 33	26 *6	34 54	45 36	58 51	74 52
32	18 9	25 *52	34 46	45 30	58 40	74 29

Altit. Poli.	11 / 13	10 / 14	9 / 15	8 / 16	7 / 17	6 / 18
33	17 44	25 *36	34 37	45 22	58 28	74 5
34	17 17	25 *19	34 26	45 13	58 15	73 40
35	16 49	25 *0	34 14	45 3	58 0	73 15
36	16 19	24 41	34 *0	44 51	57 44	72 49
37	15 46	24 19	33 *44	44 38	57 27	72 22
38	15 12	23 55	33 *27	44 23	57 9	71 54
39	14 36	23 29	33 *8	44 6	56 49	71 25
40	13 58	23 2	32 *47	43 47	56 27	70 56
41	13 18	22 32	32 *24	43 27	56 4	70 25
42	12 35	22 0	31 *59	43 5	55 40	69 53
43	11 50	21 26	31 *32	42 40	55 14	69 19
44	11 3	20 50	31 *3	42 14	54 46	68 45
45	10B 13	20 11	30 *32	41 46	54 16	68 9
46	9B 20	19 29	29 *58	41 15	53 43	67 31
47	8B 24	18 45	29 21	40 *41	53 8	66 51
48	7B 24	17 58	28 41	40 *5	52 31	66 9
49	6B 21	17 7	27 58	39 *26	51 52	65 25
50	5B 14	16 12	27 12	38 *43	51 9	64 39
51	4B 4	15 14	26 22	37 *58	50 23	63 49
52	2B 49	14 12	25 28	37 *8	49 34	62 56
53	1B 28	13 4	24 29	36 *15	48 41	62 0
54	0B 2	11 52	23 26	35 17	47 *44	61 0
55	1A 28	10B 35	22 19	34 15	46 *43	59 57
56	3A 5	9B 11	21 6	33 7	45 *37	58 48
57	4A 50	7B 41	19 45	31 52	44 *24	57 33
58	6A 43	6B 3	18 18	30 31	43 *5	56 14
59	8A 46	4B 15	16 42	29 2	41 39	54 *46
60	10A 59	2B 19	14 57	27 25	40 5	53 *11

Printed p. 149 — horizontal circumferences, signs Π ϱ $\nearrow$ $\approx$
(right page of opening)

[Margin (left, rotated): *Hic intratur pro horis Π , & ϱ , dextram versus.* — "Here one enters for the hours of Gemini (Π) and Leo (ϱ), toward the right."]

[Margin (right, rotated): *Hic intratur pro horis $\nearrow$, & $\approx$, sinistram versus.* — "Here one enters for the hours of Sagittarius ($\nearrow$) and Aquarius ($\approx$), toward the left."]

Upper table (hours from rising 7–12; argument column at right)

17 / 7	16 / 8	15 / 9	14 / 10	13 / 11	12 / 12	Altit. Poli.
75 13	27 21	10 40	1A 22	5B 45	12 16	30
78 56	30 48	12 42	2A 41	4B 52	11 41	31
82 14	34 13	14 47	4A 2	3B 57	11 4	32
85 0	38 35	16 54	5A 25	3B 0	10 25	33
87 31	40 54	19 4	6A 51	2B 1	9 44	34
89 45	44 10	21 16	8A 23	1B 1	9 2	35
88† 16	47 20	23 29	9 50	0A 2	8B 18	36
86† 26	50 24	25 45	11 23	1A 8	7B 31	37
84† 46	53 22	28 1	12 58	2A 16	6B 42	38
83† 14	56 14	30 18	14 37	3A 26	5B 53	39
81† 47	59 1	32 36	16 18	4A 38	5B 1	40
80† 25	61 42	34 56	18 0	5A 53	4B 5	41
79† 6	64 18	37 18	19 46	7A 11	3B 7	42
77† 51	66 49	39 40	21 35	8A 32	2B 8	43
76† 38	69 14	42 1	23 26	9A 55	1B 6	44
75† 28	71 35	44 22	25 19	11A 21	0 0	45
74† 18	73 52	46 44	27 15	12 51	1A 8	46
73† 9	76 5	49 6	29 14	14 24	2A 19	47
72† 0	78 15	51 29	31 16	16 0	3A 33	48
70† 51	80 23	53 53	33 21	17 40	4A 52	49
69† 41	82 28	56 17	35 30	19 26	6A 17	50
68† 29	84 33	58 41	37 41	21 15	7A 47	51
67† 16	86 37	61 7	39 56	23 8	9A 20	52

17 / 7	16 / 8	15 / 9	14 / 10	13 / 11	12 / 12	Altit. Poli.
66† 2	88 38	63 35	42 15	25 6	10A 58	53
64 44	89† 17	66 4	44 39	27 10	12A 40	54
63 23	87† 12	68 34	47 6	29 19	14 27	55
61 59	85† 6	71 7	49 38	31 35	16 22	56
60 29	82† 57	73 44	52 17	33 59	18 26	57
58 55	80† 44	76 26	55 3	36 30	20 38	58
57 14	78† 26	79 13	57 56	39 10	23 0	59
55 25	76† 3	82 5	60 56	42 0	25 30	60

Lower table (hours from rising 19–24; argument column at right)

Column heads (right side): top row **Hor. ab or.** = 5, 4, 3, 2, 1, 24; second row **Ho. ab occ.** = 19, 20, 21, 22, 23, 24.

5 / 19	4 / 20	3 / 21	2 / 22	1 / 23	24 / 24	Altit. Poli.
86† 40	69 2	53 49	41 30	31 38	23 30	30
87† 19	69 51	54 37	42 10	32 7	23 45	31
87† 58	70 41	55 26	42 51	32 36	24 2	32
88† 37	71 29	56 14	43 32	33 6	24 19	33
89† 16	72 17	57 3	44 14	33 37	24 37	34
89† 55	73 6	57 52	44 56	34 8	24 56	35
89 26	73† 54	58 41	45 39	34 40	25 16	36
88 46	74† 42	59 30	46 22	35 13	25 37	37
88 7	75† 30	60 20	47 6	35 47	25 59	38
87 27	76† 18	61 10	47	36	26 23	39

5 / 19	4 / 20	3 / 21	2 / 22	1 / 23	24 / 24	Altit. Poli.
			51	23		
86 47	77† 7	62 0	48	36	26 48	40
			37	59		
86 6	77† 56	62 51	49	37	27 14	41
			24	36		
85 24	78† 46	63 43	50	38	27 41	42
			11	15		
84 41	79† 37	64 35	50	38	28 10	43
			59	55		
83 57	80† 28	65 29	51	39	28 41	44
			48	36		
83 12	81† 20	66 23	52	40	29 14	45
			39	19		
82 26	82† 14	67 19	53	41 4	29 48	46
			32			
81 38	83† 8	68 16	54	41	30 25	47
			26	51		
80 49	84† 4	69 14	55	42	31 4	48
			22	40		
79 58	85† 1	70 14	56	43	31 45	49
			19	31		
79 4	86† 1	71 17	57	44	32 30	50
			18	24		
78 8	97† 3 [recte: 87† 3]	72 21	58	45	33 17	51
			21	20		
77 9	88† 8	73 29	59	46	34 7	52
			27	20		
76 7	89† 15	74 39	60	47	35 1	53
			36	23		
75 2	89† 35	75† 52	61	48	35 59	54
			48	29		
73 54	88 21	77† 9	63 3	49	37 1	55
				39		

5 / 19	4 / 20	3 / 21	2 / 22	1 / 23	24 / 24	Altit. Poli.
72 41	87 3	78† 29	64 22	50 53	38 8	56
71 22	85 40	79† 56	65 48	52 14	39 21	57
69 58	84 11	81† 27	67 18	53 40	40 40	58
68 27	82 35	83† 6	68 57	55 14	42 6	59
66 49	80 53	84† 51	70 42	56 55	43 41	60

Printed p. 150 — horizontal circumferences of the hours, signs ♂ ♍ ♎ ♏ (left page of next opening)

Running header here reads *Circumferentiæ horizontales **horarum** ab or.* ("...of the hours from rising"), a slight change from the *pro horis* wording of the preceding opening (flagged in the report below).

[Margin (left, rotated): *Hic intratur pro horis ♂, & ♍, dextram versus.* — "Here one enters for the hours of Taurus (♈) and Virgo (♍), toward the right."]

[Margin (right, rotated): *Hic intratur pro horis ♎, & ♏, sinistram versus.* — "Here one enters for the hours of Scorpio (♏) and Pisces (♐), toward the left."]

Upper table (hours from rising 24, 1–6)

Altit. Poli.	24 / 24	23 / 1	22 / 2	21 / 3	20 / 4	19 / 5	18 / 6
30	13B 19	6B 1	1A 16	9 38	20 34	37 52	69† 51
31	13B 27	5B 56	1A 39	10 15	21 29	38 56	70† 3
32	13B 36	5B 51	1A 59	10 50	22 19	39 55	70† 12
33	13B 45	5B 47	2A 18	11 23	23 7	40 50	70† 18
34	13B 55	5B 43	2A 36	11 56	23 55	41 43	70† 24
35	14B 5	5B 40	2A 53	12 29	24 41	42 30	70† 28
36	14B 16	5B 37	3A 8	12 58	25 23	43 12	70† 29
37	14B 27	5B 37	3A 22	13 25	26 3	43 51	70† 28
38	14B 39	5B 37	3A 36	13 52	26 40	44 27	70† 26

Altit. Poli.	24 / 24	23 / 1	22 / 2	21 / 3	20 / 4	19 / 5	18 / 6
39	14B 52	5B 37	3A 49	14 18	27 15	45 0	70† 24
40	15B 5	5B 38	4A 1	14 41	27 18 [?]	45 30	70† 20
41	15B 19	5B 39	4A 11	15 3	28 19	45 58	70† 15
42	15B 34	5B 41	4A 20	15 25	28 48	46 22	70† 8
43	15B 49	5B 44	4A 29	15 42	29 14	46 44	69† 59
44	16B 5	5B 49	4A 36	15 59	29 36	47 2	69† 49
45	16B 23	5B 55	4A 41	16 16	29 58	47 18	69† 39
46	16B 41	6B 3	4A 45	16 30	30 17	47 31	69† 27
47	17B 0	6B 12	4A 47	16 42	30 33	47 43	69† 13
48	17B 20	6B 21	4A 46	16 51	30 47	47 51	68† 58
49	17B 42	6B 31	4A 48	16 58	30 59	47 57	68† 42
50	18B 4	6B 43	4A 45	17 4	31 9	48 0	68† 24
51	18B 28	6B 57	4A 41	17 9	31 15	48 1	68† 5
52	18B 54	7B 12	4A 35	17 9	31 18	47 59	67 43
53	19B 21	7B 29	4A 27	17 9	31 20	47 54	67 20
54	19B 50	7B 49	4A 17	17 4	31 19	47 47	66 55
55	20B 20	8B 10	4A 4	16 59	31 16	47 37	66 29
56	20B 53	8B 34	3A 48	16 50	31 9	47 23	65 59
57	21B 28	9B 1	3A 30	16 37	31 0	47 7	65 27
58	22B 6	9B 30	3A 9	16 22	30 46	46 48	64 52
59	22B 46	10B 2	2A 45	16 4	30 27	46 25	64 15
60	23B 30	10B	2A 16	15 40	30 4	45 57	63 34

38

Lower table (hours from rising 13–18)

Altit. Poli.	11 / 13	10 / 14	9 / 15	8 / 16	7 / 17	6 / 18
30	14 *4	22 7	31 44	43 57	59 57	80 6
31	13 *57	22 11	31 57	44 13	60 6	79 54
32	13 *49	22 14	32 7	44 26	60 12	79 41
33	13 39	22 *15	32 16	44 37	60 17	79 28
34	13 29	22 *16	32 24	44 48	60 22	79 15
35	13 18	22 *15	32 31	44 58	60 25	79 2

Altit. Poli.	11 / 13	10 / 14	9 / 15	8 / 16	7 / 17	6 / 18
36	13 6	22 *13	32 36	45 6	60 27	78 48
37	12 53	22 *10	32 41	45 12	60 31	78 33
38	12 39	22 *6	32 44	45 17	60 28	78 18
39	12 23	22 *1	32 45	45 21	60 27	78 3
40	12 7	21 *55	32 46	45 24	60 24	77 47
41	11 50	21 *47	32 45	45 25	60 20	77 31
42	11 31	21 *38	32 43	45 25	60 15	77 15
43	11 11	21 *28	32 39	45 23	60 9	76 58
44	10 49	21 *16	32 34	45 19	60 2	76 39
45	10 27	21 *3	32 28	45 15	59 53	76 20
46	10B 3	20 *48	32 19	45 9	59 42	76 0
47	9B 37	20 *32	32 9	45 1	59 30	75 39
48	9B 10	20 *14	31 57	44 50	59 17	75 17
49	8B 41	19 *54	31 44	44 39	59 3	74 54
50	8B 11	19 *33	31 30	44 26	58 47	74 30
51	7B 39	19 *10	31 12	44 11	58 29	74 4
52	7B 4	18 45	30 *51	43 54	58 8	73 37
53	6B 27	18 18	30 *30	43 35	57 46	73 9
54	5B 49	17 48	30 *7	43 13	57 23	72 39
55	5B 8	17 16	29 *41	42 49	56 58	72 7
56	4B 24	16 41	29 *12	42 23	56 30	71 33
57	3B 37	16 3	28 *41	41 54	55 59	70 57
58	2B 47	15 22	28 *6	41 21	55 25	70 19
59	1B 54	14 38	27 *27	40 46	54 48	69 38
60	0B 58	13 49	26 *44	40 6	54 8	68 53

Horologiorum Nova Descriptio — Table Appendix (TABVLÆ XXIIII)

*(Continuation of Table XII, its concluding page, and the opening of Table XIII.
Printed pp. 151–153 = source pp. 787–789.)*

[Source p. 151] Table XII (continued) — Horizontal circumferences [circumferentiae horizontales]

[Margin (left side, rotated): **Hic intratur pro horis** ♉ & ♍ **dextram versus.** — "Here one enters for the hours of Taurus and Virgo, reading toward the right."]

[Margin (right side, rotated): **Hic intratur pro horis** ♏ & ♐ **sinistram versus.** — "Here one enters for the hours of Scorpio and Pisces, reading toward the left."]

This recto carries two continuation grids of Table XII. Column heads: **Ho. ab occ.** (*Horae ab occasu*, hours from setting) over **Hor. ab or.** (*Horae ab ortu*, hours from rising); the left row-label column is **Altit. Poli.** (altitude of the pole, i.e. latitude), in degrees (**G.**), with each data column giving degrees (**G.**) and minutes (**M.**). Letters **A/B** printed between the figures mark the two directions of the amplitude; a dagger (†) marks the entry rows keyed to the margin notes.

Upper grid (pp. hours 17→12 setting / 7→12 rising)

Altit. Poli.	17 / 7	16 / 8	15 / 9	14 / 10	13 / 11	12 / 12
30	65 45	35 39	19 16	8 44	0 A 32	6 B 45
31	67 31	37 30	20 37	9 39	1 A 9	6 B 24
32	69 12	39 20	21 58	10 35	1 A 46	6 B 2
33	70 46	41 9	23 20	11 32	2 A 24	5 B 40
34	72 13	42 52	24 41	12 29	3 A 2	5 B 18
35	73 35	44 33	26 1	13 27	3 A 41	4 B 54
36	74 53	46 13	27 22	14 25	4 A 21	4 B 29
37	76 7	47 49	28 42	15 24	5 A 2	4 B 4
38	77 17	49 23	30 2	16 24	5 A 44	3 B 37
39	78 24	50 55	31 22	17 25	6 A 27	3 B 10
40	79 28	52 24	32 42	18 26	7 A 11	2 B 41
41	80 28	53 49	34 0	19 27	7 A 55	2 B 10
42	81 27	55 14	35 18	20 29	8 A 40	1 B 38
43	82 24	56 37	36 36	21 32	9 A 27	1 B 6
44	83 19	57 59	37 54	22 35	10 A 15	0 B 34
45	84 13	59 18	39 12	23 39	11 A 3	0 0
46	85 6	60 36	40 30	24 44	11 53	0 A 36
47	85 57	61 52	41 47	25 49	12 45	1 A 14

Altit. Poli.	17 / 7	16 / 8	15 / 9	14 / 10	13 / 11	12 / 12
48	86 47	63 7	43 4	26 56	13 37	1 A 53
49	87 38	64 21	44 21	28 3	14 36	2 A 33
50	88 29	65 34	45 38	29 11	15 24	3 A 15
51	89 19	66 47	46 56	30 20	16 20	3 A 59
52	89† 51	67 59	48 13	31 31	17 19	4 A 45
53	89† 1	69 11	49 31	32 43	18 20	5 A 33
54	88† 10	70 23	50 49	33 55	19 22	6 A 22
55	87† 19	71 35	52 7	35 9	20 25	7 A 13
56	86† 26	72 47	53 27	36 25	21 31	8 A 8
57	85† 32	74 0	54 48	37 43	22 39	9 A 5
58	84† 36	75 15	56 11	39 3	23 50	10 A 5
59	83† 39	76 30	57 35	40 25	25 4	11 A 8
60	82† 39	77 48	59 1	41 50	26 2	12 A 16

Lower grid (pp. hours 5→24 setting / 19→24 rising)

Altit. Poli.	5 / 19	4 / 20	3 / 21	2 / 22	1 / 23	24 / 24
30	77† 56	58 9	42 34	30 41	21 16	13 19
31	78† 32	58 57	43 19	31 15	21 38	13 27
32	79† 8	59 45	44 4	31 49	22 0	13 36
33	79† 43	60 31	44 48	32 24	22 22	13 45
34	80† 17	61 16	45 32	32 58	22 44	13 55
35	80† 51	62 1	46 15	33 33	23 7	14 5
36	81† 24	62 45	46 58	34 8	23 31	14 16
37	81† 57	63 28	47 41	34 43	23 54	14 27
38	82† 29	64 11	48 23	35 18	24 18	14 39
39	83† 1	64 53	49 5	35 53	24 43	14 52
40	83† 33	65 35	49 47	36 28	25 8	15 5
41	84† 4	66 16	50 29	37 3	25 33	15 19
42	84† 35	66 57	51 11	37 39	25 59	15 34
43	85† 7	67 37	51 53	38 15	26 26	15 49
44	85† 39	68 18	52 35	38 52	26 54	16 5

Altit. Poli.	5 / 19	4 / 20	3 / 21	2 / 22	1 / 23	24 / 24
45	86† 10	68 59	53 17	39 29	27 22	16 23
46	86† 43	69 40	53 59	40 6	27 50	16 41
47	87† 15	70 21	54 41	40 44	28 19	17 0
48	87† 48	71 1	55 24	41 23	28 50	17 20
49	88† 21	71 42	56 7	42 1	29 22	17 42
50	88† 55	72 24	56 51	42 41	29 54	18 4
51	89† 30	73 6	57 35	43 22	30 27	18 28
52	89 54	73† 50	58 20	44 4	31 2	18 54
53	89 17	74† 34	59 6	44 47	31 38	19 21
54	88 38	75† 18	59 53	45 32	32 16	19 50
55	87 59	76† 4	60 41	46 17	32 54	20 20
56	87 18	76† 52	61 31	47 4	33 35	20 53
57	86 25	77† 41	62 22	47 53	34 18	21 28
58	85 49	78† 32	63 10	48 44	35 4	22 6
59	85 1	79† 24	64 15	49 38	35 52	22 46
60	84 11	80† 21	65 9	50 34	36 43	23 30

[Source p. 152] Table XII (concluded)

Circumferentiæ Horizontales pro horis ab ortu & occ. necnon à merid. & med. noct. in princ. ♀, & ♂. — "Horizontal circumferences for the hours from rising and setting, and also from noon and midnight, at the beginning of Aries (♀) and Libra (♂)." (*The equinoctial signs — the concluding page of Table XII.*)

Column heads: **Ho. ab occ.** over **H. ab ortu.**; each data column carries a double hour-argument (a morning value and its evening complement). The row-label column is **Altitu. Poli.** in degrees (**G.**); each data column gives **G.** and **M.** The right-hand margin repeats the heads as **H. ab ortu. / H. ab occ.** The foot of the table restates the arguments as **H. à merid.** (*Horae à meridie*, hours from noon) and **H. à me.no.** (*Horae à media nocte*, hours from midnight).

Altitu. Poli.	6 / 18	7 (5) / (17)	8 (4) / (16)	9 (3) / (15)	10 (2) / (14)	11 (1) / (13)	12 (24) / 24 (12)
30	90	61 49	40 54	26 34	16 6	7 38	0 0
0							

Altitu. Poli.	6 / 18	7 (5) / 19 (17)	8 (4) / 20 (16)	9 (3) / 21 (15)	10 (2) / 22 (14)	11 (1) / 23 (13)	12 (24) / 24 (12)
31	90 0	62 31	41 44	27 15	16 34	7 52	0 0
32	90 0	63 11	42 33	27 55	17 1	8 5	0 0
33	90 0	63 48	43 20	28 34	17 28	8 18	0 0
34	90 0	64 24	44 5	29 13	17 54	8 31	0 0
35	90 0	64 58	44 48	29 50	18 20	8 44	0 0
36	90 0	65 30	45 31	30 27	18 45	8 57	0 0
37	90 0	66 0	46 11	31 2	19 10	9 10	0 0
38	90 0	66 29	46 50	31 37	19 34	9 22	0 0
39	90 0	66 56	47 28	32 11	19 58	9 34	0 0
40	90 0	67 22	48 4	32 44	20 22	9 46	0 0
41	90 0	67 47	48 39	33 16	20 45	9 58	0 0
42	90 0	68 11	49 13	33 47	21 7	10 10	0 0
43	90 0	68 33	49 45	34 17	21 29	10 21	0 0
44	90 0	68 54	50 16	34 47	21 51	10 33	0 0
45	90 0	69 15	50 46	35 16	22 12	10 44	0 0
46	90 0	69 34	51 15	35 44	22 33	10 55	0 0
47	90	69 53	51 43	36 11	22 54	11 5	0 0

Altitu. Poli.	6 / 18	7 (5) / 19 (17)	8 (4) / 20 (16)	9 (3) / 21 (15)	10 (2) / 22 (14)	11 (1) / 23 (13)	12 (24) / 24 (12)
	0						
48	90	70 10	52 10	36 37	23 14	11 16	0 0
	0						
49	90	70 29	52 35	37 3	23 33	11 26	0 0
	0						
50	90	70 43	52 59	37 28	23 52	11 36	0 0
	0						
51	90	70 59	53 23	37 52	24 10	11 46	0 0
	0						
52	90	71 13	53 46	38 15	24 27	11 55	0 0
	0						
53	90	71 27	54 8	38 37	24 45	12 5	0 0
	0						
54	90	71 40	54 30	38 59	25 2	12 14	0 0
	0						
55	90	71 53	54 49	39 20	25 19	12 23	0 0
	0						
56	90	72 5	55 8	39 40	25 35	12 32	0 0
	0						
57	90	72 17	55 27	39 59	25 50	12 40	0 0
	0						
58	90	72 28	55 45	40 18	26 5	12 48	0 0
	0						
59	90	72 38	56 2	40 36	26 20	12 56	0 0
	0						
60	90	72 48	56 18	40 54	26 34	13 4	0 0
	0						
H. à merid.	12	1	2	3	4	5	6
H. à me.no.	12	11	10	9	8	7	6

[Source p. 152] Table XIII (heading)

XIII. TABVLA.

DECLINATIONVM ARCVVM DIVRNORVM: ad poli altitudines
in sinistro latere descriptas.

— Table XIII. Of the declinations of the diurnal arcs, set out against the pole-
altitudes on the left side.

[Source p. 153] Table XIII — Declinations of the diurnal arcs
[declinationes arcuum diurnorum]

Row-label column: **Altitudines poli** (pole-altitudes), in degrees (**G.**), in half-degree steps from 30 to 60. Body: **Declinationes** (declinations), each column giving **G.** and **M.** Each column head is a double hour-argument summing to 24 (a value and its complement): 0/24, ½/23½, 1/23, 1½/22½, 2/22, 2½/21½. The table continues to the right on the following page (catchword **Horæ**).

Altitudines poli	0 / 24	½ / 23½	1 / 23	1½ / 22½	2 / 22	2½ / 21½
30	60 0	59 57	59 47	59 31	59 8	58 38
30½	59 30	59 27	59 17	59 1	58 37	58 7
31	59 0	58 57	58 47	58 30	58 7	57 36
31½	58 30	58 27	58 17	58 0	57 37	57 5
32	58 0	57 57	57 47	57 30	57 6	56 35
32½	57 30	57 27	57 17	57 0	56 36	56 4
33	57 0	56 57	56 46	56 30	56 5	55 33
33½	56 30	56 27	56 16	55 59	55 35	55 3
34	56 0	55 57	55 46	55 29	55 4	54 32
34½	55 30	55 27	55 16	54 59	54 34	54 2
35	55 0	54 57	54 46	54 29	54 4	53 31
35½	54 30	54 27	54 16	53 58	53 33	53 1
36	54 0	53 56	53 46	53 28	53 3	52 30
36½	53 30	53 26	53 16	52 58	52 33	52 0
37	53 0	52 56	52 46	52 28	52 2	51 29

Altitudines poli	0 / 24	$\frac{1}{2}$ / 23 $\frac{1}{2}$	1 / 23	1 $\frac{1}{2}$ / 22 $\frac{1}{2}$	2 / 22	2 $\frac{1}{2}$ / 21 $\frac{1}{2}$
37 $\frac{1}{2}$	52 30	52 26	52 16	51 58	51 32	50 59
38	52 0	51 56	51 46	51 28	51 2	50 28
38 $\frac{1}{2}$	51 30	51 26	51 16	50 57	50 32	49 58
39	51 0	50 56	50 46	50 27	50 2	49 28
39 $\frac{1}{2}$	50 30	50 26	50 15	49 57	49 31	48 57
40	50 0	49 56	49 45	49 27	49 1	48 27
40 $\frac{1}{2}$	49 30	49 26	49 15	48 57	48 31	47 57
41	49 0	48 56	48 45	48 27	48 1	47 27
41 $\frac{1}{2}$	48 30	48 26	48 15	47 57	47 31	46 57
42	48 0	47 56	47 45	47 27	47 1	46 27
42 $\frac{1}{2}$	47 30	47 26	47 15	46 57	46 31	45 56
43	47 0	46 56	46 45	46 27	46 1	45 26
43 $\frac{1}{2}$	46 30	46 26	46 15	45 57	45 30	44 56
44	46 0	45 56	45 45	45 27	45 0	44 26
44 $\frac{1}{2}$	45 30	45 26	45 16	44 57	44 30	43 56
45	45 0	44 56	44 46	44 27	44 0	43 26
45 $\frac{1}{2}$	44 30	44 26	44 16	43 57	43 30	42 56
46	44 0	43 56	43 46	43 27	43 0	42 26
46 $\frac{1}{2}$	43 30	43 26	43 16	42 57	42 31	41 56
47	43 0	42 56	42 46	42 27	42 1	41 27
47 $\frac{1}{2}$	42 30	42 26	42 16	41 57	41 31	40 57
48	42 0	41 56	41 46	41 27	41 1	40 27
48 $\frac{1}{2}$	41 30	41 26	41 16	40 57	40 31	39 57
49	41 0	40 56	40 46	40 27	40 1	39 28
49 $\frac{1}{2}$	40 30	40 26	40 16	39 57	39 31	38 58
50	40 0	39 56	39 46	39 27	39 2	38 28
50 $\frac{1}{2}$	39 30	39 26	39 16	38 57	38 32	37 59
51	39 0	38 56	38 46	38 27	38 2	37 29
51 $\frac{1}{2}$	38 30	38 26	38 16	37 58	37 32	36 59
52	38 0	37 56	37 46	37 28	37 2	36 30
52 $\frac{1}{2}$	37 30	37 26	37 16	36 58	36 33	36 0

Altitudines poli	0 / 24	½ / 23½	1 / 23	1½ / 22½	2 / 22	2½ / 21½
53	37 0	36 56	36 46	36 28	36 3	35 31
53½	36 30	36 26	36 16	35 58	35 33	35 1
54	36 0	35 56	35 46	35 28	35 4	34 32
54½	35 30	35 27	35 16	34 59	34 34	34 2
55	35 0	34 57	34 46	34 29	34 4	33 33
55½	34 30	34 27	34 16	33 59	33 34	33 3
56	34 0	33 57	33 46	33 29	33 5	32 34
56½	33 30	33 27	33 16	32 59	32 35	32 5
57	33 0	32 57	32 47	32 30	32 6	31 35
57½	32 30	32 27	32 17	32 0	31 36	31 6
58	32 0	31 57	31 47	31 30	31 7	30 37
58½	31 30	31 27	31 17	31 0	30 37	30 8
59	31 0	30 57	30 47	30 31	30 8	29 38
59½	30 30	30 27	30 17	30 1	29 38	29 9
60	30 0	29 57	29 47	29 31	29 9	28 40

These three pages continue **Table XIII (*Tabula Declinationum Arcuum Diurnorum*)** — "Table of the Declinations of Diurnal Arcs", begun earlier. The table is arranged with the pole altitude ("Altitudines poli," G. = degrees) down the left margin, running from 30° to 60° in half-degree steps, and the hour-arguments across the top. Each data column serves two complementary hours whose sum is 24 (e.g. 3^h and 21^h), and is split into two sub-columns, **G.** (degrees) and **M.** (minutes), under the spanning heading **Declinationes**. These three folios carry the hour-arguments from 3^h (/21^h) through 11½^h (/12½^h); the table continues from the preceding pages and runs on toward the meridian.

The hour-pair heading of each column is given verbatim in the table headers below (small hour / large hour).

Table XIII, hour-columns 3^h–5½^h

Column hour-pairs (verbatim): 3 | 21, 3½ | 20½, 4 | 20, 4½ | 19½, 5 | 19, 5½ | 18½.

Alt. poli (G.)	3/21 G	3½/20½ MG	4/20 MG	4½/19½ MG	5/19 MG	5½/18½ MG	M
30	58	0 57	1 56 4	1 55 9	1 53 4	5 52 7	2 9
30½	57	2 56 9	4 55 2	4 54 7	4 53 1	2 51 4	5 5
31	56	5 56 8	1 55 1	1 54 5	9 52	5 51 2	2 2
31½	56	2 55 7	3 54 9	4 53 3	3 52 7	1 50 9	4 9
32	55	5 55 6	8 54	1 53 1	4 51	4 50 6	1 6
32½	55	2 54 5	3 53 7	4 52 0	3 51 2	1 49 4	4 3
33	54	5 54 4	6 53	8 52	1 50	4 49 2	1 1
33½	54	2 53 3	3 52 4	3 51 7	2 50 9	1 48 0	3 8
34	53	5 53 2	3 52	5 50	5 49 7	3 48 8	6
34½	53	2 52 1	3 51 2	3 50 4	2 49 5	6 47	3 4
35	52	5 52 0	1 51	3 49	5 48 4	3 47 4	2
35½	52	2 51 0	3 50 0	3 49 1	2 48 2	3 46	3 0
36	51	4 50 9	5 50 9	0 48	5 47 1	3 45 1	5 8
36½	51	1 50 8	2 49 9	2 48 9	2 47 0	0 45	2 7
37	50	4 49 8	5 48 8	5 47 8	4 46 9	2 44 8	5 6
37½	50	1 49 7	2 48 7	2 47 7	1 45 8	5 44 7	2 5
38	49	4 48	5 47	5 46	4 45	2 43	5

Alt. poli (G.)	3/21 G	3½/20½ MG	4/20 MG	4½/19½ MG	5/19 MG	5½/18½ MG	M
		7	6	7	7	6	4
38½	49	1 48	2 47	2 46	1 44	5 43	2
		6	6	6	6	5	3
39	48	4 47	5 46	5 45	4 44	2 42	5
		6	5	5	5	5	3
39½	48	1 47	2 46	2 45	1 43	5 42	2
		6	5	5	5	4	2
40	47	4 46	5 45	5 44	4 43	2 41	5
		5	4	5	4	4	2
40½	47	1 46	2 45	2 44	1 42	5 41	2
		5	4	4	4	3	1
41	46	4 45	5 44	5 43	4 42	2 40	5
		5	4	4	4	3	1
41½	46	1 45	2 44	2 43	1 41	5 40	2
		4	3	3	3	3	1
42	45	4 44	5 43	5 42	4 41	2 39	5
		4	3	3	3	3	2
42½	45	1 44	2 43	2 42	1 40	5 39	2
		4	3	3	3	3	2
43	44	4 43	5 42	5 41	4 40	2 38	5
		4	3	3	3	3	3
43½	44	1 43	2 42	2 41	1 39	5 38	2
		4	3	3	3	4	3
44	43	4 42	5 41	5 40	4 39	2 37	5
		4	3	3	4	4	4
44½	43	1 42	2 41	2 40	1 38	5 37	2
		4	3	3	4	5	5
45	42	4 41	5 40	5 39	4 38	2 36	5
		4	3	4	5	6	6
45½	42	1 41	2 40	2 39	1 37	5 36	2
		4	3	4	5	6	7
46	41	4 40	5 39	5 38	4 37	2 35	5
		4	4	4	6	7	9

Alt. poli (G.)	3/21 G	3½/20½ MG	4/20 MG	4½/19½ MG	5/19 MG	5½/18½ MG	M
46½	41	1 40 5	2 39 4	2 38 5	1 36 6	5 35 8	3 0
47	40	4 39 5	5 38 4	5 37 5	4 36 7	3 35 0	2
47½	40	1 39 5	2 38 5	2 37 6	1 36 8	1 34	3 4
48	39	4 38 5	5 37 5	5 36 7	4 35 9	3 34 2	6
48½	39	1 38 6	2 37 6	2 36 8	2 35 0	4 33	3 8
49	38	4 37 6	5 36 6	5 35 8	5 34 2	3 33 6	1 0
49½	38	1 37 7	2 36 7	2 35 9	2 34 3	7 32	4 2
50	37	4 36 7	5 36 8	0 34	5 33 4	3 32 9	1 5
50½	37	1 36 7	2 35 8	3 34 1	2 33 6	1 31 1	4 7
51	36	4 35 8	5 35 9	3 33	5 32 7	4 31 3	2 0
51½	36	1 35 9	3 34 0	3 33 4	2 32 9	1 30 5	5 3
52	35	5 35 0	1 34	5 33	1 31	4 30 8	2 6
52½	35	2 34 0	3 33 2	3 32 6	3 31 2	2 29 0	5 9
53	34	5 34 1	3 33	8 32	4 30	5 29 2	3 2
53½	34	2 33 1	3 32 4	3 31 9	3 30 6	2 29 5	5
54	33	5 33 2	5 32	1 31 1	8 29	5 28 8	3 9
54½	33	2 32	3 31	4 30	4 29	3 28	1

Alt. poli (G.)	3/21 G	3½/20½ MG	4/20 MG	4½/19½ MG	5/19 MG	5½/18½ MG	M
		3	6	2	0	0	2
55	32	5 32	8 31	1 30	1 29	3 27	4
		4		4	2		6
55½	32	2 31	3 30	4 29	4 28	3 27	2
		5	9	5	4	6	0
56	31	5 31	1 30	1 29	1 28	9 26	5
		6	0	7	7		3
56½	31	2 30	4 29	4 28	5 27	4 26	2
		7	2	9	0	2	7
57	30	5 30	1 29	2 28	2 27	1 26	1
		8	3	1	2	5	
57½	30	2 29	4 28	5 27	5 26	4 25	3
		9	5	3	5	9	5
58	30	0 29	1 28	2 27	2 26	2 25	1
			6	5	7	2	0
58½	29	3 28	4 27	5 27	0 25	5 24	4
		1	8	7		6	4
59	29	2 28	1 27	2 26	3 25	2 24	1
			9	9	3	9	9
59½	28	3 27	5 27	2 26	6 25	3 23	5
		3	1				3
60	28	5 27	2 26	3 25	3 24	3 23	2
			3	4	9	7	8

Table XIII, hour-columns 6^h–8½^h

Column hour-pairs (verbatim): 6 | 18, 6½ | 17½, 7 | 17, 7½ | 16½, 8 | 16, 8½ | 15½.

Alt. poli (G.)	6/18 G	6½/17½ MG	7/17 MG	7½/16½ MG	8/16 MG	8½/15½ MG	M
30	50	4 48	4 46	3 43	5 40	5 37	2
		6	8	1	4	4	7
30½	50	1 48	1 45	5 43	1 40	2 36	5

Alt. poli (G.)	6/18 G	6½/17½ MG	7/17 MG	7½/16½ MG	8/16 MG	8½/15½ MG	M
		2	3	7	9	0	4
31	49	3 47	3 45	2 42	4 39	4 36	2
		9	9	2	5	6	1
31½	49	5 47	6 44	4 42	1 39	1 35	4
				9	2	3	9
32	48	3 46	3 44	1 41	3 38	4 35	1
		2	2	5	8	0	7
32½	47	5 45	5 43	4 41	5 38	8 34	4
		9	9	2			6
33	47	2 45	2 43	9 40	3 37	3 34	1
		6	6		3	6	5
33½	46	5 44	5 42	3 40	1 37	4 33	4
		4	3	6			5
34	46	2 44	2 42	4 39	2 36	3 33	1
		1	1		9	3	5
34½	45	4 43	4 41	3 38	5 36	2 32	4
		9	9	2	7		6
35	45	1 43	1 41	0 38	2 35	3 32	1
		7	7		6	2	7
35½	44	4 42	4 40	2 37	5 35	2 31	4
		5	5	9	5		8
36	44	1 42	1 39	5 37	2 34	3 31	2
		3	3	8	4	2	0
36½	43	4 41	4 39	2 36	5 34	3 30	5
		2	2	7	4		2
37	43	1 41	1 38	5 36	2 33	3 30	2
		1	1	6	4	4	5
37½	42	4 40	4 38	2 35	5 33	5 29	5
		0	0	6	4		8
38	42	9 40	1 37	5 35	2 32	3 29	3
			0	5	5	7	1
38½	41	3 39	3 37	2 34	5 32	9 29	5
		8	9	6	6		

Alt. poli (G.)	6/18 G	6½/17½ MG	7/17 MG	7½/16½ MG	8/16 MG	8½/15½ MG	M
39	41	8 39	9 36	5 34 6	2 31 7	4 28 2	3 9
39½	40	3 38 7	3 36 9	2 33 7	5 31 9	1 28 4	1 3
40	40	7 38	1 35 0	5 33 8	3 30 1	4 27 7	4 8
40½	39	3 37 7	4 35 0	2 33 9	3 30	2 27 1	2 3
41	39	8 37	1 35 1	0 32	3 29 5	5 26 4	5 8
41½	38	3 36 8	4 34 2	3 32 2	8 29	2 26 8	3 4
42	38	9 36	1 34 3	4 31	4 29 1	3 26	1 0
42½	37	3 35 9	4 33 4	3 31 6	1 28 4	3 25 7	4 6
43	37	1 35 0	1 33 6	8 30	4 28 7	1 25 2	2 3
43½	36	4 34 1	4 32 7	4 30 1	2 27 1	4 24 7	5 9
44	36	1 34 3	1 32 9	1 29 4	5 27 5	2 24 2	3 6
44½	35	4 33 4	5 31 2	4 29 7	2 26 9	5 24 8	1 4
45	35	1 33 6	2 31 4	2 29 0	3 26	3 23 4	5 2
45½	34	4 32 8	5 30 6	5 28 3	3 26 8	1 23 0	2 9
46	34	2 32 0	2 30 9	2 28 7	1 25 3	4 23 6	8
46½	33	5 32 2	2 30	1 27	4 25 8	2 22 3	4 6
47	33	2 31	3 29	3 27	2 25	0 22	2

Alt. poli (G.)	6/18 G	6½/17½ MG	7/17 MG	7½/16½ MG	8/16 MG	8½/15½ MG	M
		4	5	5	3		5
47½	32	5 31 7	8 29	9 26	5 24 9	3 22 7	4
48	32	2 30 9	4 28 2	4 26 4	3 24 5	1 21 4	4 3
48½	32	2 30	1 28 5	1 26 8	1 23 1	5 21 2	2 2
49	31	3 29 5	4 27 9	5 25 3	4 23 7	3 21 0	2
49½	31	8 29	2 27 3	2 25 8	2 23 3	7 20	4 2
50	30	4 28 1	5 27 7	4 25	0 22	4 20 6	2 2
50½	30	1 28 4	3 26 2	3 24 9	3 22 6	2 20 4	2
51	29	4 28 8	6 26	1 24 5	1 22 3	3 19	4 2
51½	29	2 27 1	4 25 1	5 23 0	5 21 1	4 19 1	2 3
52	28	5 27 5	1 25 5	2 23 6	2 21 8	2 19 0	4
52½	28	2 26 9	5 25 0	2 23	5 20	5 18 9	4 5
53	28	3 26	2 24 5	3 22 9	4 20 3	3 18 9	2 6
53½	27	3 26 7	0 24	1 22 5	2 20 1	1 18 8	7
54	27	1 25 1	3 23 6	5 21 2	5 19 9	5 17 8	4 9
54½	26	4 25 6	1 23 1	2 21 8	3 19 7	3 17 8	3 1
55	26	2 24 0	4 23 7	5 21	1 19 5	1 17 8	1 2

Alt. poli (G.)	6/18 G	6½/17½ MG	7/17 MG	7½/16½ MG	8/16 MG	8½/15½ MG	M
55½	25	5 24 5	2 22 3	4 20 2	5 18 4	5 16 8	5 4
56	25	3 23 0	5 22 9	1 20 9	3 18 3	3 16 8	3 7
56½	25	5 23	3 21 5	5 20 7	1 18 1	1 16 9	1 9
57	24	4 23 0	1 21 1	3 19 4	5 17 0	5 16 9	2
57½	24	1 22 5	4 21 7	1 19 2	2 17 9	4 15 0	4 4
58	23	5 22 0	2 20 4	5 19 0	9 17	2 15 1	2 7
58½	23	2 22 6	0 20	2 18 7	4 17 8	2 15	1 0
59	23	1 21	3 20 7	5 18	2 16 8	4 14 3	5 3
59½	22	3 21 7	1 19 4	4 18 4	7 16	2 14 5	3 6
60	22	1 20 2	5 19 0	2 17 2	4 16 7	6 14	1 9

Table XIII, hour-columns 9^h–11½^h

Column hour-pairs (verbatim): 9 | 15, 9½ | 14½, 10 | 14, 10½ | 13½, 11 | 13, 11½ | 12½.

Alt. poli (G.)	9/15 G	9½/14½ MG	10/14 MG	10½/13½ MG	11/13 MG	11½/12½ MG	M
30	33	3 29 2	6 24	9 18	4 12 0	4 6 4	2 8
30½	33	1 28	3 23 7	4 18 3	1 12 9	3 6 0	2 0
31	32	2 28 9	9 23	1 17 8	5 12 9	1 6 5	1 3
31½	31	5 27	4 22	5 17	4 12	1 6	6

Alt. poli (G.)	9/15 G	9½/14½ MG	10/14 MG	10½/13½ MG	11/13 MG	11½/12½ MG	M
		9	1	4	0		
32	31	2 27	1 22	3 17	2 11	4 5	5
		9	3	0	0	8	8
32½	31	0 26	4 22	7 17	2 11	3 5	5
			6			5	5
33	30	3 26	2 21	4 16	4 11	2 5	4
		1	0	4	3	2	2
33½	30	2 25	5 21	2 16	2 11	9 5	3
			4	1	6		9
34	29	3 25	2 21	0 16	8 10	5 5	3
		4	9			7	2
34½	29	7 25	4 20	3 15	5 10	4 5	2
				8	1	5	6
35	28	3 24	3 20	1 15	3 10	3 5	2
		9	9	7	4	4	0
35½	28	1 24	1 19	5 15	1 10	2 5	1
		3	5	7	8	2	4
36	27	4 23	5 19	3 15	2 10	1 5	9
		7	2	6		1	
36½	27	2 23	2 19	1 14	4 10	0 5	3
		1	9	7	6		
37	26	5 23	6 18	5 14	3 9	5 4	5
		5		7	1	0	8
37½	26	3 22	4 18	3 14	1 9	3 4	5
		0	4	8	6	9	2
38	26	6 22	2 18	2 14	1 9	2 4	4
			2	0		9	7
38½	25	4 22	0 18	1 13	4 9	1 4	4
		2			7	9	2
39	25	1 21	3 17	4 13	3 9	9 4	3
		8	9	3	3		6
39½	24	5 21	1 17	2 13	1 9	0 4	3
		4	9	6	9		2

Alt. poli (G.)	9/15 G	9½/14½ MG	10/14 MG	10½/13½ MG	11/13 MG	11½/12½ MG	M
40	24	3 20 1	5 17 8	9 13	5 8	5 4 1	2 7
40½	24	8 20	3 16 7	5 12 2	5 8 2	4 4 1	2 3
41	23	4 20 6	1 16 8	3 12 5	3 8 9	3 4 2	1 8
41½	23	2 19 3	5 16 8	1 12 8	2 8 6	2 4 4	1 3
42	23	2 19	3 16 9	2 12	1 8 4	1 4 5	9
42½	22	4 19 0	2 15 0	4 12 6	1 8	6 4	5
43	22	1 19 9	1 15	3 11 1	4 7 9	5 4 8	1
43½	21	5 18 8	4 15 3	1 11 5	3 7 7	5 3 0	5 6
44	21	3 18 7	2 15 5	0 11	2 7 5	4 3 2	5 2
44½	21	1 18 7	7 14	4 11 5	1 7 4	3 3 4	4 8
45	20	5 17 6	4 14 9	3 11 1	2 7	2 3 6	4 5
45½	20	3 17 6	3 14 2	1 10 6	5 7 1	1 3 9	4 1
46	20	1 17 7	1 14 5	2 10	4 7 0	1 3 1	3 7
46½	19	5 16 8	5 13 8	4 10 8	2 7 9	4 3	3 3
47	19	3 16 8	4 13 1	3 10 4	1 6 9	5 3 6	2 9
47½	19	1 16 9	2 13 5	2 10 1	8 6	4 3 9	2 6
48	19	1 16	8 13	7 9	5 6	4 3	2

Alt. poli (G.)	9/15 G	9½/14½ MG	10/14 MG	10½/13½ MG	11/13 MG	11½/12½ MG	M
					8	2	2
48½	18	4 15	5 12	5 9	4 6	3 3	1
		2	2	4	8	5	9
49	18	2 15	3 12	4 9	3 6	2 3	1
		4	7	1	8	8	5
49½	18	6 15	2 12	2 9	2 6	2 3	1
			1	8	8	2	2
50	17	4 15	6 12	1 9	1 6	1 3	8
		8		5	8	5	
50½	17	3 14	5 12	3 9	8 6	8 3	5
		1	0				
51	17	1 14	3 11	5 8	5 6	2 3	2
		3	5	0	9		
51½	16	5 14	2 11	3 8	4 5	5 2	5
		6	1	8	9	6	9
52	16	3 14	6 11	2 8	4 5	4 2	5
		9		6	0	9	6
52½	16	2 13	5 11	1 8	3 5	4 2	5
		2	1	4	1	3	2
53	16	5 13	3 11	2 8	2 5	3 2	4
			7		2	7	9
53½	15	4 13	2 10	5 8	1 5	3 2	4
		9	3	1	3	1	6
54	15	3 13	9 10	3 8	4 5	2 2	4
		2		9		5	3
54½	15	1 12	5 10	2 7	5 5	1 2	4
		6	5	8	5	9	0
55	15	0 12	4 10	1 7	4 5	1 2	3
			1	6	7	3	7
55½	14	4 12	2 10	5 7	3 5	7 2	3
		4	6		8		4
56	14	2 12	1 9	5 7	3 5	2 2	3
		8	4	4	0		2

Alt. poli (G.)	9/15 G	9½/14½ MG	10/14 MG	10½/13½ MG	11/13 MG	11½/12½ MG	M
56½	14	1 12 3	1 9	4 7 3	2 4 1	5 2 6	2 9
57	13	5 11 7	4 9 7	3 7 2	1 4 3	5 2 1	2 6
57½	13	4 11 2	3 9 4	2 7 2	5 4	4 2 5	2 3
58	13	2 11 7	2 9 1	1 6 1	5 4 7	4 2 0	2 0
58½	13	1 11 2	9 9	1 6	4 4 9	3 2 5	1 8
59	12	5 10 7	5 8 6	5 6 0	4 4 1	2 2 9	1 5
59½	12	4 10 2	4 8 3	4 6 0	3 4 3	2 2 4	1 2
60	12	2 10 8	3 8 1	3 6 0	2 4 6	1 2 9	1 0

Horologiorum Nova Descriptio — Table Appendix (continued)

(Christoph Clavius, Opera Mathematica, Tomus IV; the sundial-tables appendix, "TABVLÆ XXIIII." All tables on these three folios are computed to the latitude noted in each heading. Degrees are printed under G. [gradus], minutes under M. [minuta]. Zodiac column-groups pair the signs that share the same declination magnitude.)

XIIII. TABVLA. (folio 157)

Fourteenth Table. Of the arcs of the horizon, intercepted between the meridian and the great circles which are drawn — from the intersection of the equator with the meridian, through the hours from noon and from midnight — at the beginnings of the signs.

To latitude 42 degrees.

Hor. à mer.	& med. noc.	☾ ♍ · G.	M . ♀ ♌ ♎ · G.	M . ☿ ♍ ♎ ♏ · G.	M .
	12	0	0 0	0 0	0
1	11	23	5 27 2	3 43 5	2 4
2	10	40	3 45 1	1 61 7	1 8
3	9	50	2 55 4	0 68	5 0
4	8	55	5 60 7	1 72 5	3 9
5	7	58	4 62 8	5 74 2	1 1
6	6	59	4 63 0	4 74 0	4 1

Rightmost remark column (spanning the data rows): "When the Sun is at the beginning of ♈ [Aries] and ♎ [Libra], these arcs are always 90 degrees; therefore there was no need to place Aries and Libra among the other signs."

XV. TABVLA. (folio 157)

Fifteenth Table. Of the arcs of the prime vertical, intercepted between the horizon and the great circles which are drawn from the intersection of the equator with the meridian, through the hours from noon or from midnight, at the beginnings of the signs.

To latitude 42 degrees.

Hor. à mer.	& med. noc.	☾ ♍ · G.	M . ♀ ♌ ♎ · G.	M . ☿ ♍ ♎ ♏ · G.	M .
	12	90	0 90	0 90	0
1	11	68	1 64 7	4 49 8	3 6
2	10	52	2 47 5	4 31 3	1 8
3	9	42	3 37 5	5 23 4	1 6

Hor. à mer.	& med. noc.	☿ ♊ · G.	M . ♀ ♈ ♀ ♉ · G.	M . ☿ ♊ ♉ ♈ · G.	M .
4	8	36	5 32 3	2 19 5	2 1
5	7	36	5 29 7	3 17 9	2 8
6	6	33	1 28	4 16 8	5 5

Rightmost remark column: "When the Sun is at the beginning of ♈ [Aries] and ♎ [Libra], all these arcs are 0°0'; which was the reason that Aries and Libra are not placed among the other signs."

XVI. TABVLA. (folio 157)

Sixteenth Table. Of the arcs of the horizon, intercepted between the meridian and the great circles which are drawn from the intersection of the equator with the meridian, through the hours from rising or from setting, at the beginnings of the signs.

To latitude 42 degrees.

Marks: † marks the beginning [of the sign] for the afternoon hours reckoned from setting; * marks it for the hours reckoned from rising (see the foot-note below).

Horæ ab occ.	☿ · G.	♀ ♈ · G.	♊ ☉ · G.	♈ ♈ · G.	♈ ♈ · G.	♊ · G.	Horæ ab ortu
9	57	61	71	64	41	32	15
10	59	63	73	70	52	45	14
11	59	63	74	73	59	53	13
12	57	62	74	74	62	57	12
13	53	59	73	74	63	59	11
14	45	52	70	73	63	59	10
15	32	41	64	71	61	57	9
16	11*	20*	50	67	56	53	8
17	13†	8†	15*	57	48	46	7
18	33	33	33†	33*	33	33	6
19	46	48	57	15†	8*	13*	5

Horæ ab occ.	☾ · G.	♊ ♈ · G.	♋ ♉ · G.	♌ ♊ · G.	♍ ♋ · G.	♎ · G.	Horæ ab ortu
20	53	56	67	50	20†	11†	4
21	57	61	71	64	41	32	3
22	59	63	73	70	52	45	2
23	59	63	74	73	59	53	1
24	57	62	74	74	62	57	24

Foot-note: "The sign † gives the beginning [of the sign] for the afternoon hours in the hours from setting; the sign * [gives it] in the hours from rising. When the Sun is at the beginning of ♈ [Aries] and ♎ [Libra], all these arcs are 90°0'; therefore these signs are not written in the table."

XVII. TABVLA. (folio 158)

Seventeenth Table. Of the arcs of the prime vertical, intercepted between the horizon and the great circles which are drawn from the intersection of the equator with the meridian, through the hours from rising or from setting.

To latitude 42 degrees.

Horæ ab occ.	☾ · G.	♊ ♈ · G.	♋ ♉ · G.	♌ ♊ · G.	♍ ♋ · G.	♎ · G.	Horæ ab ortu
9	35	31	20	28	51	60	15
10	33	29	17	21	40	47	14
11	33	28	16	18	33	39	13
12	35	30	17	17	30	35	12
13	39	33	18	16	28	33	11
14	47	40	21	17	29	33	10
15	60	51	28	20	31	35	9
16	79*	71*	42	25	35	39	8
17	77†	82†	75*	35	44	46	7
18	58	58	58†	58*	58	58	6
19	46	44	35	75†	82*	77*	5
20	39	35	25	42	71†	79†	4
21	35	31	20	28	51	60	3

Horæ ab occ.	☾ · G.	♈ ♀ · G.	♏ ♂ · G.	♍ ♂ · G.	♊ ♀ · G.	♋ · G.	Horæ ab ortu
22	33	29	17	21	40	47	2
23	33	28	16	18	33	39	1
24	35	30	17	17	30	35	24

Foot-note: "The sign † gives the beginning [of the sign] for the afternoon hours in the hours from setting; the sign * [gives it] in the hours from rising. When the Sun is at the beginning of ♈ [Aries] and ♎ [Libra], all these arcs are 0°0'; therefore these signs do not appear among the others."

XVIII. TABVLA. (folio 158)

Eighteenth Table. Of the arcs of the horizon, on the southern side, between the meridian and the great circles which — like hour-circles — are drawn through the poles of the world from noon and from midnight, and through the whole hours from rising or from setting and their halves, at the beginnings of the signs.

To latitude 42 degrees.

H. ab occ.	☾ · G.	♈ ♀ · G.	♏ ♂ · G.	♍ ♂ · G.	♊ ♀ · G.	♋ ♀ · G.	♌ · G.	H. ab ortu
9	121	125	135	146	155	162	164	15
9½	111	116	127	138	148	156	159	14½
10	100	105	117	130	141	150	153	14
10½	89	94	107	121	134	143	146	13½
11	78	83	96	111	125	135	139	13
11½	67	72	85	101	115	127	131	12½
12	57	62	74	90	105	117	122	12
12½	48	52	64	78	94	107	112	11½
13	40	44	54	68	83	96	101	11
13½	33	36	45	58	72	85	90	10½
14	26	29	38	49	62	74	79	10
14½	20	23	31	41	52	63	68	9½
15	15	17	24	33	44	54	58	9
15½	9	12	18	27	36	45	49	8½

H. ab occ.	☿ · G.	♌ II · G.	♍ ☿ · G.	♊ ♀ · G.	♋ ☿ · G.	♈ ♀ · G.	♏ · G.	H. ab ortu
16	4*	7	13	21	29	37	41	8
16½	0†	2*	8	15	23	30	34	7½
17	5	2†	2*	10	17	24	27	7
17½	10	7	2†	5*	12	18	21	6½
18	15	13	7	0	7	13	15	6
18½	21	18	12	5†	2*	7	10	5½
19	27	24	17	10	2†	2*	5	5
19½	34	30	23	15	8	2†	0*	4½
20	41	37	29	21	13	7	4†	4
20½	49	45	36	27	18	12	9	3½
21	58	54	44	33	24	17	15	3
21½	68	63	52	41	31	23	20	2½
22	79	74	62	49	38	29	26	2
22½	90	85	77	58	45	36	33	1½
23	101	96	83	68	54	44	40	1
23½	112	107	94	78	64	52	48	½
24	122	117	105	90	74	62	57	24

Foot-note: "The sign † gives the beginning [of the sign] for the afternoon hours from setting; the sign * [gives it] for the afternoon hours from rising. The hours too which in the 9th table are below the horizon are here also to be understood as below the horizon."

XIX. TABVLA. (folio 159)

Nineteenth Table. Segments of the hour-lines from noon and from midnight, between the centre of the horizontal sundial and the points of the Sun's parallels, reduced to tangents with respect to the whole sine [radius] taken equal to the style. For the latitudes written below; for describing the Sun's parallels, according to the prescription of Number 5, scholium of chapter 9.

(Each entry is a tangent value referred to a radius/style of 1000. As the hour approaches the point where the parallel runs parallel to the hour-line, the segment

grows without bound; the print marks this "Infinita" — and, in the two lower blocks, sometimes simply leaves the terminal cells blank. See the parsing note.)

Ad latitudinem Grad. 42.

Horæ à merid.	☾ · Tang .	♄ · Tang.	♃ · Tang.	♅ · Tang.	♆ · Tang.	♁ · Tang.	♂ · Tang .	Horæ à med. noc.
12	1445	1511	1700	2011	2462	3007	3305	12
½	1447	1513	1704	2019	2476	3032	3337	11½
1	1454	1521	1717	2043	2521	3109	3435	11
1½	1466	1536	1742	2087	2603	3254	3622	10½
2	1485	1559	1779	2156	2734	3491	3934	10
2½	1513	1594	1836	2260	2939	3881	4463	9½
3	1557	1648	1922	2420	3265	4551	5419	9
3½	1624	1728	2051	2668	3816	5852	7473	8½
4	1727	1852	2253	3078	4857	9118	1417 3	8
4½	1888	2047	2583	3819	7324	28371	Infinita	7½
5	2153	2373	3168	5412	18531	Infinita		7
5½	2604	2944	4333	10413	Infinita			6½
6	3437	4062	7346	Infinita				6
6½	5210	6773	25828	Infinita				5½
7	1054 8	19306	Infinita					5
7½	1682 52	Infinit.						4½

Ad latitudinem Grad. 35.

Horæ à merid.	☾ · Tang .	♄ · Tang.	♃ · Tang.	♅ · Tang.	♆ · Tang.	♁ · Tang.	♂ · Tang .	Horæ à med. noc.
12	1632	1692	1863	2128	2482	2867	3060	12
1	1637	1700	1876	2153	2526	2936	3144	11
2	1659	1728	1925	2242	2683	3191	3458	10

Horæ à merid.	☾ · Tang .	♐ ♏ · Tang.	♏ ♏ · Tang.	♏ ♏ · Tang.	♏ ♏ · Tang.	♏ ♏ · Tang.	♏ · Tang .	Horæ à med. noc.
3	1715	1798	2042	2453	3072	3859	4308	9
4	1865	1980	2335	3000	4197	6190	7643	8
5	2311	2520	3244	5030	11186	116398 1	Infin ita	7
6	4010	4739	8569	Infinita				6
7	2855 2	Infinita						5

Ad latitudinem Grad. 40.

Horæ à merid.	☾ · Tang .	♐ ♏ · Tang.	♏ ♏ · Tang.	♏ ♏ · Tang.	♏ ♏ · Tang.	♏ ♏ · Tang.	♏ · Tang .	Horæ à med. noc.
12	1598	1659	1832	2103	2468	2870	3074	12
1	1604	1667	1846	2129	2513	2943	3163	11
2	1627	1697	1897	2221	2678	3213	3496	10
3	1686	1770	2018	2439	3084	3925	4410	9
4	1839	1956	2316	3001	4261	6450	8153	8
5	2284	2495	3229	5074	11821	Infinita		7
6	3913	4624	8362	Infinita				6
7	2294 9	152543	Infinita					5

Table XIX (continued) — Segments of the hour-lines, reduced to tangents

These three folios (printed pp. 160–162) continue **Table XIX** of the appendix. The running header on every page reads **RESID. XIX. TAB.** — *Residuum XIX. Tabulae*, "remainder of the 19th Table." The table was introduced in the preceding folios (its full title: *Segments of the hour-lines from noon and from midnight, between the center of the horizontal dial and the points of the Sun's parallels, reduced to tangents with respect to a total sine equal to the style*). Here the table is continued through successive pole-altitudes (latitudes), one sub-block per degree.

Each sub-block has the same layout: a left column of hours reckoned from noon (*Horæ à merid.*), seven tangent columns headed by paired zodiac signs, and a right column of hours reckoned from midnight (*Horæ à med. noc.*). The seven sign-columns are, in order: ♋ (Cancer); ♊ ♌ (Gemini, Leo); ♉ ♍ (Taurus, Virgo); ♈ ♎ (Aries, Libra); ♐ ♑ (Pisces, Scorpio); ♒ ♓ (Aquarius, Sagittarius); ♏ (Capricorn). Every column head is subtitled *Tang.* (tangent). "Infinita" ("infinite") is printed in a column at the first row where its tangent becomes infinite; cells below are left blank. Blank cells are transcribed as empty; no value has been supplied where the print leaves a cell blank.

Ad latitudinem Grad. 37. — At latitude 37°

Horæ à merid.	♋ · Tang .	♊ ♌ · Tang.	♉ ♍ · Tang.	♈ ♎ · Tang.	♐ ♑ · Tang.	♒ ♓ · Tang.	♏ · Tang .	Horæ à med. noc.
12	1567	1629	1804	2081	2457	2879	3095	12
1	1573	1637	1819	2107	2505	2956	3190	11
2	1598	1668	1871	2203	2677	3240	3545	10
3	1659	1744	1996	2428	3100	3994	4528	9
4	1815	1933	2300	3005	4334	6744	8716	8
5	2258	2470	3213	5115	12558	Infinita		7
6	3821	4516	8167	Infinita				6
7	1920	71485	Infinita					5
	9							

Ad latitudinem Grad. 38. — At latitude 38°

Horæ à merid.	♋ · Tang .	♊ ♌ · Tang.	♉ ♍ · Tang.	♈ ♎ · Tang.	♐ ♑ · Tang.	♒ ♓ · Tang.	♏ · Tang .	Horæ à med. noc.
12	1539	1601	1779	2061	2451	2893	3122	12
1	1546	1610	1794	2089	2501	2974	3222	11
2	1571	1642	1841	2180	2679	3274	3600	10
3	1635	1721	1976	2421	3122	4078	4658	9
4	1794	1913	2286	3013	4418	7089	9401	8
5	2233	2447	3200	5164	13378	Infinita		7
6	3736	4415	7984	Infinita				6

Horæ à merid.	☿ · Tang .	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang	Horæ à med. noc.
7	1654 1	46799	Infinita					5

Ad latitudinem Grad. 39. — At latitude 39°

Horæ à merid.	☿ · Tang .	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang	Horæ à med. noc.
12	1512	1575	1755	2045	2448	2912	3156	12
1	1520	1585	1772	2074	2500	2998	3263	11
2	1546	1619	1828	2175	2686	3316	3666	10
3	1613	1710	1959	2416	3149	4174	4811	9
4	1775	1895	2275	3024	4510	7480	1021 7	8
5	2211	2426	3189	5217	14356	Infinita		7
6	3654	4319	7810	Infinita				6
7	1450 4	34653	Infinita					5

Ad latitudinem Grad. 40. — At latitude 40°

Horæ à merid.	☿ · Tang .	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang	Horæ à med. noc.
12	1488	1552	1735	2031	2449	2938	3197	12
1	1495	1561	1750	2060	2502	3028	3311	11
2	1524	1597	1810	2166	2698	3366	3744	10
3	1593	1680	1945	2414	3183	4286	4989	9
4	1757	1879	2266	3039	4614	7910	1121 4	8
5	2190	2407	3180	5277	15487	Infinita		7
6	3578	4228	7647	Infinita				6
7	1290 2	27445	Infinita					5

Ad latitudinem Grad. 41. — At latitude 41°

Horæ à merid.	☿ · Tang .	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang .	Horæ à med. noc.
12	1466	1530	1716	2020	2454	2969	3247	12
1	1474	1541	1733	2051	2510	3066	3369	11
2	1504	1577	1793	2160	2714	3424	3851	10
3	1574	1663	1932	2415	3221	4409	5187	9
4	1741	1865	2259	3058	4732	8490	1254	8
							0	
5	2171	2389	3173	5340	16857	Infinita		7
6	3506	4143	7492	Infinita				6
7	1160	22677	Infinita					5
	7							

Ad latitudinem Grad. 43. — At latitude 43°

(The print passes directly from Grad. 41 to Grad. 43; there is no Grad. 42 block here.)

Horæ à merid.	☿ · Tang .	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang.	♂ ♀ · Tang .	Horæ à med. noc.
12	1426	1493	1685	2005	2474	3052	3372	12
1	1436	1504	1703	2038	2536	3161	3512	11
2	1468	1543	1768	2155	2759	3568	4053	10
3	1543	1634	1914	2427	3317	4713	5688	9
4	1713	1841	2249	3103	5002	9890	1642	8
							6	
5	2136	2357	3164	5482	20529	Infinita		7
6	3372	3985	7207	Infinita				6
7	9680	16817	Infinita					5

Ad latitudinem Grad. 44. — At latitude 44°

Horæ à merid.	☾ · Tang .	♄ ♀ · Tang.	♃ ♀ · Tang.	♅ ♂ · Tang.	♁ ♀ · Tang.	♂ ♂ · Tang.	♂ · Tang .	Horæ à med. noc.
12	1409	1477	1673	2001	2491	3104	3450	12
1	1419	1488	1691	2036	2556	3220	3601	11
2	1452	1529	1758	2156	2789	3656	4186	10
3	1529	1622	1907	2437	3374	4896	5997	9
4	1702	1830	2247	3131	5159	10821	1956 2	8
5	2120	2342	3160	5559	23068	Infinita		7
6	3311	3913	7076	Infinita				6
7	8936	14896	Infinita					5

Ad latitudinem Grad. 45. — At latitude 45°

Horæ à merid.	☾ · Tang .	♄ ♀ · Tang.	♃ ♀ · Tang.	♅ ♂ · Tang.	♁ ♀ · Tang.	♂ ♂ · Tang.	♂ · Tang .	Horæ à med. noc.
12	1394	1462	1662	2000	2511	3164	3539	12
1	1404	1474	1681	2036	2579	3288	3702	11
2	1438	1516	1749	2160	2823	3756	4339	10
3	1518	1612	1903	2451	3437	5107	6362	9
4	1691	1822	2247	3162	5333	11974	2428 6	8
5	2107	2331	3161	5645	26376	Infinita		7
6	3252	3844	6951	Infinita				6
7	8305	13378	Infinita					5

Ad latitudinem Grad. 46. — At latitude 46°

Horæ à merid.	☾ · Tang .	♄ ♀ · Tang.	♃ ♀ · Tang.	♅ ♂ · Tang.	♁ ♀ · Tang.	♂ ♂ · Tang.	♂ · Tang .	Horæ à med. noc.
12	1380	1449	1653	2001	2535	3233	3640	12
1	1390	1462	1673	2038	2607	3365	3817	11

Horæ à merid.	☾ · Tang .	♂ ♀ · Tang.	♂ ♀ · Tang.	☾ ☽ · Tang.	♂ ♀ · Tang.	☾ ☽ · Tang.	☾ · Tang .	Horæ à med. noc.
2	1426	1505	1743	2167	2863	3869	4515	10
3	1506	1602	1899	2465	3511	5344	6785	9
4	1682	1814	2249	3197	5527	13437	3219 8	8
5	2092	2318	3160	5732	30803	Infinita		7
6	3197	3778	6833	Infinita				6
7	7753	12132	Infinita					5

Ad latitudinem Grad. 47. — At latitude 47°

Horæ à merid.	☾ · Tang .	♂ ♀ · Tang.	♂ ♀ · Tang.	☾ ☽ · Tang.	♂ ♀ · Tang.	☾ ☽ · Tang.	☾ · Tang .	Horæ à med. noc.
12	1367	1438	1646	2005	2564	3311	3756	12
1	1378	1451	1666	2043	2639	3453	3949	11
2	1414	1495	1729	2176	2900	3988	4716	10
3	1497	1594	1898	2484	3592	5618	7201	9
4	1674	1809	2253	3236	5743	15349	4811 5	8
5	2081	2309	3163	5829	37096	Infinita		7
6	3145	3716	6721	Infinita				6
7	7275	11106	Infinita					5

Ad latitudinem Grad. 48. — At latitude 48°

Horæ à merid.	☾ · Tang .	♂ ♀ · Tang.	♂ ♀ · Tang.	☾ ☽ · Tang.	♂ ♀ · Tang.	☾ ☽ · Tang.	☾ · Tang .	Horæ à med. noc.
12	1356	1428	1640	2011	2598	3401	3889	12
1	1367	1441	1662	2051	2677	3554	4100	11
2	1405	1487	1736	2188	2960	4143	4948	10
3	1489	1588	1899	2505	3682	5933	7899	9
4	1667	1803	2257	3277	5982	17941	9641	8

Horæ à merid.	☿ · Tang .	♂ · Tang.	♂ · Tang.	♂ · Tang.	♂ · Tang.	♂ · Tang.	♂ · Tang	Horæ à med. noc.
							8	
5	2068	2298	3166	5927	46642	Infinita		7
6	3095	3657	6614	Infinita				6
7	6848	10232	Infinita					5

Ad latitudinem Grad. 49. — At latitude 49°

(This block carries an additional row: hour 8 from noon / hour 4 from midnight, because at this latitude the summer (Cancer) hour-line reaches farther. The ☿ and ♂ terminal cells are printed with the abbreviated "Infinit." — see parsing note.)

Horæ à merid.	☿ · Tang .	♂ · Tang.	♂ · Tang.	♂ · Tang.	♂ · Tang.	♂ · Tang.	♂ · Tang	Horæ à med. noc.
12	1346	1419	1636	2020	2637	3502	4041	12
1	1358	1433	1658	2061	2720	3667	4273	11
2	1396	1480	1735	2203	3019	4309	5217	10
3	1482	1583	1901	2531	3782	6301	8644	9
4	1661	1800	2264	3324	6251	21665	Infinit.	8
5	2058	2290	3170	6035	62979	Infinita		7
6	3047	3601	6513	Infinita				6
7	6474	9403	Infinita					5
8	5656	Infinit.						4
	413							

(Table XIX continues onto the following folio, which opens with a further "Ad latitudinem" block — the 24-table appendix has not yet ended.)

Horologiorum Nova Descriptio — Table appendix (continued)

Ad latitudinem Grad. 50. — At latitude 50°

Horæ à merid.	☿ Tan g.	♈ ♀ Tang.	♉ ♂ Tang.	♊ ♂ Tang.	♋ ♀ Tang.	♌ ♀ Tang.	♍ ♂ Tan g.	Horæ à med. noc.
12	133 8	1412	1635	2031	2681	3617	421 5	12
1	134 9	1426	1657	2073	2768	1796	447 2	11
2	138 9	1474	1735	2220	3084	4498	553 1	10
3	147 6	1579	1905	2558	3893	6731	957 0	9
4	165 7	1798	2272	3375	6554	27443	Infi nita	8
5	204 9	2282	3175	6150	97143	Infinit a.		7
6	300 2	3548	6416	Infinit a.				6
7	613 8	8356	Infinit a.					5
8	922 40	Infinit a.						4

(The ♌ ♂ value at hour 1 is printed **1796**, which breaks the column's rising progression [12: 3617 → 1: 1796 → 2: 4498]; transcribed as printed — see Numerical impossibles.)

Ad latitudinem Grad. 51. — At latitude 51°

Horæ à merid.	☿ Tan g.	♈ ♀ Tang.	♉ ♂ Tang.	♊ ♂ Tang.	♋ ♀ Tang.	♌ ♀ Tang.	♍ ♂ Tan g.	Horæ à med. noc.
12	133	1406	1634	2045	2731	3747	441	12

Horæ à merid.	☿ Tan g.	♈ ♀ Tang.	♉ ♂ Tang.	♊ ♂ Tang.	♋ ♀ Tang.	♌ ♀ Tang.	♍ ♀ Tan g.	Horæ à med. noc.
	0						6	
1	134 2	1420	1657	2089	2823	3943	470 2	11
2	138 3	1470	1737	2241	3146	4714	590 1	10
3	147 2	1577	1911	2589	4016	7241	107 54	9
4	165 3	1796	2281	3428	6893	37581	Inf ni ta	8
5	204 1	2277	3184	6275	213476	Infinit a.		7
6	295 9	3497	6325	Infinit a.				6
7	584 1	8300	Infinit a.					5
8	462 56	Infinit a.						4

Ad latitudinem Grad. 52. — At latitude 52°

Horæ à merid.	☿ Tan g.	♈ ♀ Tang.	♉ ♂ Tang.	♊ ♂ Tang.	♋ ♀ Tang.	♌ ♀ Tang.	♍ ♀ Tan g.	Horæ à med. noc.
12	132 4	1401	1635	2061	2787	3896	464 8	12
1	133 7	1416	1659	2107	2885	4110	497 0	11
2	137 8	1467	1741	2265	3238	4963	634 0	10
3	146 9	1575	1918	2624	4154	7852	123 15	9
4	165 0	1796	2293	3487	7281	60054	Inf ni ta	8

Horæ à merid.	☿ Tan g.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tan g.	Horæ à med. noc.
5	203 2	2270	3191	6402	Infinit a.		.	7
6	291 9	3449	6237	Infinit a.				6
7	556 7	7806	106522 5	Infinit a.				5
8	307 82	Infinit a.						4

Ad latitudinem Grad. 53. — At latitude 53°

Horæ à merid.	☿ Tan g.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tan g.	Horæ à med. noc.
12	131 9	1398	1638	2081	2850	4066	491 9	12
1	133 2	1413	1663	2128	2954	4302	528 4	11
2	137 5	1465	1747	2291	3328	5252	686 9	10
3	146 6	1575	1927	2663	4307	8599	144 62	9
4	164 8	1797	2306	3551	7723	151727	Inf nita	8
5	202 5	2265	3201	6540	Infinit a.		.	7
6	288 0	3403	6154	Infinit a.				6
7	532 1	7373	151982	Infinit a.				5
8	230 20	Infinit a.						4

Ad latitudinem Grad. 54. — At latitude 54°

Horæ à merid.	☿ Tan g.	♈ Tang.	♉ Tang.	♊ Tang.	♋ Tang.	♌ Tang.	♍ Tan g.	Horæ à med. noc.
12	131 6	1396	1643	2103	2921	4260	523 7	12
1	132 9	1412	1668	2152	3031	4523	565 6	11
2	137 2	1465	1754	2321	3430	5589	751 7	10
3	146 5	1576	1937	2704	4466	9523	175 86	9
4	164 7	1798	2320	3620	8232	Infinit a.	Inf nita .	8
5	202 0	2263	3214	6691	Infinit a.			7
6	284 3	3360	6075	Infinit a.				6
7	510 0	6991	81793	Infinit a.				5
8	183 59	280535	Infinit a.					4

Ad latitudinem Grad. 55. — At latitude 55°

Horæ à merid.	☿ Tan g.	♈ Tang.	♉ Tang.	♊ Tang.	♋ Tang.	♌ Tang.	♍ Tan g.	Horæ à med. noc.
12	131 3	1395	1649	2128	3000	4485	561 5	12
1	132 7	1411	1675	2179	3117	4778	610 1	11
2	137 1	1465	1763	2354	3543	5987	832 7	10
3	146	1578	1950	2751	4670	10701	225	9

Horæ à merid.	☿ Tan g.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tan g.	Horæ à med. noc.
	5						55	
4	164 7	1801	2336	3694	8833	Infini a.	Infi nita	8
5	201 4	2258	3225	6845	Infini a.			7
6	280 8	3318	6000	Infini a.				6
7	489 3	6640	55876	Infini a.				5
8	152 51	72429	Infini a.					4

Ad latitudinem Grad. 56. — At latitude 56°

Horæ à merid.	☿ Tan g.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tang.	♂ ♀ Tan g.	Horæ à med. noc.
12	131 2	1396	1657	2157	3089	4746	607 0	12
1	132 5	1412	1684	2210	3214	5076	664 3	11
2	137 1	1467	1774	2391	3669	6460	936 1	10
3	146 6	1582	1964	2802	4885	12246	316 50	9
4	164 9	1805	2354	3775	9521	Infini a.	Infi nita	8
5	200 9	2256	3238	7012	Infini a.			7
6	277 4	3279	5929	Infini a.				6
7	470	6327	42428	Infini				5

Horæ à merid.	☾ Tan g.	♊ ♉ Tang.	♋ ♎ Tang.	♌ ♍ Tang.	♎ ♏ Tang.	♏ ♐ Tang.	♐ ♑ Tan g.	Horæ à med. noc.
	5			a.				
8	130 38	41463	Infinit a.					4

Ad latitudinem Grad. 57. — At latitude 57°

Horæ à merid.	☾ Tan g.	♊ ♉ Tang.	♋ ♎ Tang.	♌ ♍ Tang.	♎ ♏ Tang.	♏ ♐ Tang.	♐ ♑ Tan g.	Horæ à med. noc.
12	131 1	1398	1667	2189	3188	5051	662 5	12
1	132 5	1414	1694	2244	3322	5426	731 2	11
2	137 2	1470	1786	2432	3810	7033	107 32	10
3	146 8	1586	1980	2857	5130	14360	536 20	9
4	165 0	1810	2373	3860	10345	Infinit a.	Infinit a.	8
5	200 6	2255	3255	7194	Infinit a.			7
6	274 2	3241	5861	Infinit a.				6
7	453 5	6047	34209	Infinit a.				5
8	113 75	28978	Infinit a.					4

XX. TABVLA. — TABLE XX.

PRO ALTITVDINIBVS SOLIS IN Verticali in principio cuiuslibet signi ad altitudines poli in siniftro latere afcriptas.

For the altitudes of the Sun on the prime vertical [in Verticali] at the beginning of each sign, corresponding to the elevations of the pole [altitudines poli — i.e. the latitudes] entered in the left-hand column.

(The table gives, for each pole-elevation φ [Altitud. Poli, in degrees and minutes], the Sun's altitude [in degrees and minutes] on the prime vertical when it stands at the beginning of the named sign. Only the three sign-columns in which the Sun crosses the prime vertical above the horizon are tabulated: ♋ Cancer; ♊ II Leo/Gemini; ♍ Virgo/Taurus. Each entry is degrees° minutes'. In the source the table is set in three side-by-side panels — $\varphi = 0^\circ\text{--}33^\circ45'$, $34^\circ\text{--}44^\circ45'$, and $45^\circ\text{--}55^\circ45'$ — all with identical column heads; they are merged here into one latitude-ordered table. The ♋ column begins only at $\varphi = 23^\circ30'$, where the Sun reaches the zenith [altitude 90°]; for lower latitudes the ♋ crossing does not occur and the first two rows are printed as all-zeros.)

Altitud. Poli (°)	♋ (°)	♊ II (°)	♍ (°)
0 0	0 0	0 0	0 0
0 0	0 0	0 0	0 0
23 30	90 0	59 19	30 0
23 45	81 55	59 1	29 40
24 0	78 38	58 6	29 21
24 15	76 8	57 13	29 2
24 30	74 4	56 22	28 44
24 45	72 16	55 34	28 26
25 0	70 39	54 48	28 9
25 15	69 11	54 3	27 52
25 30	67 51	53 20	27 35
25 45	66 37	52 38	27 19
26 0	65 27	51 58	27 3
26 15	64 22	51 19	26 47
26 30	63 20	50 42	26 32
26 45	62 22	50 6	26 17
27 0	61 26	49 31	26 3
27 15	60 34	48 57	25 49
27 30	59 43	48 24	25 35

Altitud. Poli (°)	☾ (°)	♂ ♀ (°)	♂ ♀ (°)
27 45	58 54	47 52	25 21
28 0	58 8	47 21	25 8
28 15	57 24	46 51	24 55
28 30	56 41	46 21	24 42
28 45	56 0	45 52	24 29
29 0	55 20	45 25	24 17
29 15	54 41	44 58	24 5
29 30	54 4	44 31	23 53
29 45	53 28	44 6	23 41
30 0	52 53	43 41	23 30
30 15	52 19	43 17	23 19
30 30	51 46	42 53	23 8
30 45	51 14	42 29	22 57
31 0	50 44	42 6	22 47
31 15	50 14	41 44	22 36
31 30	49 45	41 22	22 26
31 45	49 16	41 1	22 16
32 0	48 48	40 40	22 6
32 15	48 21	40 19	21 56
32 30	47 55	39 59	21 47
32 45	47 29	39 40	21 38
33 0	47 4	39 21	21 28
33 15	46 39	39 2	21 19
33 30	46 15	38 44	21 10
33 45	45 52	38 26	21 1
34 0	45 29	38 8	20 53
34 15	45 7	37 51	20 44
34 30	44 45	37 34	20 36
34 45	44 24	37 17	20 28
35 0	44 3	37 1	20 20
35 15	43 43	36 45	20 12

Altitud. Poli (°)	☾ (°)	♂ ♀ (°)	♂ ♀ (°)
35 30	43 23	36 29	20 5
33 45 [recte 35 45]	43 3	36 14	19 57
36 0	42 43	35 59	19 50
36 15	42 24	35 44	19 42
36 30	42 6	35 29	19 35
36 45	41 48	35 15	19 28
37 0	41 30	35 1	19 21
37 15	41 12	34 47	19 14
37 30	40 55	34 33	19 7
37 45	40 38	34 20	19 0
38 0	40 22	34 7	18 54
38 15	40 6	33 54	18 47
38 30	39 50	33 41	18 40
38 45	39 34	33 29	18 34
39 0	39 15	33 17	18 28
39 15	39 4	33 5	18 22
39 30	38 49	32 53	18 16
39 45	38 34	32 41	18 10
40 0	38 20	32 29	18 4
40 15	38 6	32 18	17 58
40 30	37 52	32 7	17 52
40 45	37 39	31 56	17 47
41 0	37 26	31 45	17 42
41 15	37 13	31 34	17 36
41 30	37 0	31 24	17 30
41 45	36 47	31 14	17 25
42 0	36 35	31 4	17 20
42 15	36 23	30 54	17 15
42 30	36 11	30 44	17 10
42 45	35 59	30 34	17 5
43 0	35 47	30 25	17 0

Altitud. Poli (°)	☾ (°)	♂ ♀ (°)	♂ ♀ (°)
43 15	35 35	30 15	16 55
43 30	35 24	30 6	16 50
43 45	35 13	29 57	16 45
44 0	35 2	29 48	16 41
44 15	34 51	29 39	16 36
44 30	34 41	29 30	16 31
44 45	34 30	29 22	16 26
45 0	34 20	29 14	16 22
45 15	34 10	29 6	16 17
45 30	34 0	28 58	16 13
45 45	33 50	28 50	16 9
46 0	33 40	28 42	16 5
46 15	33 30	28 34	16 1
46 30	33 21	28 26	15 57
46 45	33 11	28 18	15 53
47 0	33 2	28 11	15 49
47 15	32 53	28 3	15 45
47 30	32 44	27 55	15 41
47 45	32 35	27 48	15 37
48 0	32 27	27 41	15 34
48 15	32 18	27 34	15 30
48 30	32 10	27 27	15 26
48 45	32 2	27 20	15 22
49 0	31 54	27 14	15 19
49 15	31 46	27 7	15 15
49 30	31 38	27 0	15 12
49 45	31 30	26 54	15 8
50 0	31 22	26 48	15 5
50 15	31 14	26 41	15 1
50 30	31 7	26 35	14 58
50 45	30 59	26 29	14 55

Altitud. Poli (°)	♋ (°)	♌ ♊ (°)	♍ ♉ (°)
51 0	30 52	26 23	14 52
51 15	30 45	26 17	14 48
51 30	30 38	26 11	14 45
51 45	30 31	26 5	14 42
52 0	30 24	25 59	14 39
52 15	30 17	25 53	14 36
52 30	30 10	25 48	14 33
52 45	30 3	25 42	14 30
53 0	29 57	25 37	14 27
53 15	29 50	25 31	14 24
53 30	29 44	25 26	14 21
53 45	29 38	25 21	14 18
54 0	29 32	25 16	14 16
54 15	29 26	25 11	14 13
54 30	29 20	25 6	14 10
54 45	29 14	25 1	14 7
55 0	29 8	24 56	14 5
55 15	29 2	24 51	14 2
55 30	28 56	24 46	14 0
55 45	28 50	24 41	13 57

Table XX (continued)

Pro altitudinibus Solis in Verticali — For the altitudes of the Sun in the prime vertical [continued]

Continuation of Table XX (whose heading and first latitudes 0°–55°45′ stand on the preceding folios). This folio (printed p. 166) carries no title, only the repeated column heads; it completes the table for pole-elevations 56° through 90°.

Column heads as printed: **Altitud. Poli.** (elevation of the pole, i.e. latitude), then the three sign-columns each with a **G. M.** (Gradus, Minuta) sub-head — ♋ (Cancer); ♌ / ♊ (Leo over Gemini); ♍ / ♉ (Virgo over Taurus). The source sets three latitude-panels side by side (56°–67°15′ | 67°30′–78°45′ | 79°–90°); they are

merged below into a single latitude-ordered table. Each cell is degrees and minutes (° ').

Altitud. Poli (G M)	♋ Cancer (G M)	♌/♊ Leo/Gemini (G M)	♍/♉ Virgo/Taurus (G M)
56 0	28 45	24 37	13 55
56 15	28 39	24 32	13 52
56 30	28 33	24 28	13 50
56 45	28 28	24 23	13 47
57 0	28 23	24 19	13 45
57 15	28 18	24 14	13 42
57 30	28 13	24 10	13 40
57 45	28 8	24 6	13 38
58 0	28 3	24 2	13 36
58 15	27 58	23 57	13 33
58 30	27 53	23 53	13 31
58 45	27 48	23 49	13 29
59 0	27 43	23 45	13 27
59 15	27 38	23 41	13 25
59 30	27 33	23 37	13 23
59 45	27 29	23 33	13 21
60 0	27 25	23 30	13 19
60 15	27 20	23 26	13 17
60 30	26 16 [recte: 27 16]	23 22	13 15
60 45	26 11 [recte: 27 11]	23 19	13 13
61 0	26 7 [recte: 27 7]	23 15	13 11
61 15	16 3 [recte: 27 3]	23 11	13 9
61 30	26 59	23 8	13 7
61 45	26 55	23 4	13 5
62 0	26 51	23 1	13 3
62 15	26 47	22 57	13 1

Altitud. Poli (G M)	♋ Cancer (G M)	♌/♊ Leo/Gemini (G M)	♍/♉ Virgo/Taurus (G M)
62 30	26 43	22 54	12 59
62 45	26 39	22 51	12 57
63 0	26 35	22 48	12 56
63 15	26 31	22 55 [recte: 22 45]	12 54
63 30	26 27	22 42	12 53
63 45	26 23	22 39	12 51
64 0	26 20	22 36	12 49
64 15	26 16	22 33	12 47
64 30	26 13	22 30	12 45
64 45	26 9	22 27	12 44
65 0	26 6	22 24	12 42
65 15	26 2	22 21	12 40
65 30	25 59	22 18	12 39
65 45	25 56	22 15	12 37
66 0	25 53	22 13	12 36
66 15	25 49	22 10	12 34
66 30	25 46	22 7	12 33
66 45	25 43	22 4	12 31
67 0	25 40	22 2	12 30
67 15	25 37	21 59	12 28
67 30	25 34	21 57	12 27
67 45	25 31	21 54	12 26
68 0	25 28	21 52	12 25
68 15	25 25	21 49	12 23
68 30	25 22	21 47	12 22
68 45	25 19	21 44	12 21
69 0	25 17	21 42	12 20
69 15	25 14	21 39	12 18
69 30	25 12	21 37	12 17
69 45	25 9	21 35	12 16

Altitud. Poli (G M)	♋ Cancer (G M)	♌/♊ Leo/Gemini (G M)	♍/♉ Virgo/Taurus (G M)
70 0	25 7	21 33	12 15
70 15	25 4	21 31	12 14
70 30	25 2	21 29	12 12
70 45	24 59	21 27	12 11
71 0	24 57	21 25	12 10
71 15	24 54	21 23	12 9
71 30	24 52	21 21	12 8
71 45	24 49	21 19	12 7
72 0	24 47	21 17	12 6
72 15	24 45	21 15	12 5
72 30	24 43	21 13	12 4
72 45	24 41	21 11	12 3
73 0	24 39	21 10	12 2
73 15	24 37	21 8	12 1
73 30	24 35	21 6	12 0
73 45	24 33	21 5	11 59
74 0	24 31	21 3	11 58
74 15	24 29	21 1	11 57
74 30	24 27	21 0	11 56
74 45	24 25	20 58	11 55
75 0	24 23	20 57	11 54
75 15	24 21	20 55	11 53
75 30	24 19	20 54	11 52
75 45	24 17	20 53	11 51
76 0	24 16	20 51	11 51
76 15	24 14	20 50	11 50
76 30	24 12	20 48	11 49
76 45	24 10	20 47	11 49
77 0	24 9	20 45	11 48
77 15	24 7	20 43	11 48

Altitud. Poli (G M)	♋ Cancer (G M)	♌/♊ Leo/Gemini (G M)	♍/♉ Virgo/Taurus (G M)
77 30	24 6	20 42	11 47
77 45	24 4	20 41	11 47
78 0	24 3	20 40	11 46
78 15	24 1	20 39	11 45
78 30	24 0	20 38	11 45
78 45	23 59	20 37	11 44
79 0	23 58	20 36	11 43
79 15	23 56	20 35	11 43
79 30	23 55	20 34	11 42
79 45	23 54	20 33	11 42
80 0	23 53	20 32	11 41
80 15	23 52	20 31	11 41
80 30	23 51	20 30	11 40
80 45	23 50	20 29	11 40
81 0	23 49	20 28	11 39
81 15	23 48	20 27	11 39
81 30	23 47	20 26	11 38
81 45	23 46	20 25	11 38
82 0	23 45	20 24	11 37
82 15	23 44	20 23	11 37
82 30	23 43	20 22	11 36
82 45	23 42	20 22	11 36
83 0	23 41	20 21	11 35
83 15	23 40	20 21	11 35
83 30	23 39	20 20	11 35
83 45	23 38	20 20	11 34
84 0	23 38	20 19	11 34
84 15	23 37	20 19	11 34
84 30	23 37	20 18	11 34
84 45	23 36	20 18	11 33

Altitud. Poli (G M)	♋ Cancer (G M)	♌/♊ Leo/Gemini (G M)	♍/♉ Virgo/Taurus (G M)
85 0	23 36	20 17	11 33
85 15	23 35	20 17	11 33
85 30	23 35	20 16	11 33
85 45	23 34	20 16	11 32
86 0	23 34	20 15	11 32
86 15	23 33	20 15	11 32
86 30	23 33	20 15	11 32
86 45	23 33	20 14	11 31
87 0	23 32	20 14	11 31
87 15	23 32	20 14	11 31
87 30	23 32	20 14	11 31
87 45	23 32	20 13	11 31
88 0	23 31	20 13	11 30
88 15	23 31	20 13	11 30
88 30	23 31	20 13	11 30
88 45	23 31	20 13	11 30
89 0	23 31	20 12	11 30
89 15	23 30	20 12	11 30
89 30	23 30	20 12	11 30
89 45	23 30	20 12	11 30
90 0	23 30	20 12	11 30

XXI. TABVLA — Table XXI

Pro distantiis Solis a Meridiano, cum est in Verticali — For the distances of the Sun from the meridian, when it is in the prime vertical

Full title as printed: **PRO DISTANTIIS SOLIS A MERIDIANO, cum est in Verticali: ad altitudines poli in sinistro latere ascriptas, in principiis cuiuslibet signi.** — "For the distances of the Sun from the meridian, when it is in the prime vertical: for the elevations of the pole written on the left side, at the beginnings of each sign."

Column heads as in Table XX: **Altitud. Poli.** then three sign-columns each with a **G. M.** sub-head — ♋ (Cancer); ♌ / ♊ (Leo over Gemini); ♍ / ♉ (Virgo over Taurus). The table runs across two folios (printed pp. 167–168), each set in three side-by-side latitude-panels; all are merged below into one latitude-ordered table (pole-elevation 0° through 90°). Each cell is degrees and minutes (° ′).

Altitud. Poli (G M)	♋ Cancer (G M)	♌/♊ Leo/Gemini (G M)	♍/♉ Virgo/Taurus (G M)
0 0	0 0	0 0	0 0
0 0	0 0	0 0	0 0
23 30	0 0	32 12	62 6
23 45	8 49	33 16	62 28
24 0	12 25	34 16	62 49
24 15	15 9	35 14	63 9
24 30	17 25	36 10	63 29
24 45	19 25	37 3	63 49
25 0	21 11	37 54	64 8
25 15	22 47	38 44	64 27
25 30	24 16	39 32	64 45
25 45	25 39	40 18	65 3
26 0	26 57	41 2	65 21
26 15	28 9	41 45	65 38
26 30	29 18	42 27	65 55
26 45	30 23	43 8	66 12
27 0	31 25	43 47	66 28
27 15	32 25	44 25	66 44
27 30	33 21	45 2	66 59
27 45	34 15	45 38	67 15
28 0	35 8	46 13	67 30
28 15	35 59	46 47	67 45
28 30	36 48	47 20	67 59
28 45	37 35	47 53	68 14
29 0	38 20	48 25	68 28
29 15	39 4	48 56	68 42

Altitud. Poli (G M)	♋ Cancer (G M)	♌/♊ Leo/Gemini (G M)	♍/♉ Virgo/Taurus (G M)
29 30	39 47	49 26	68 55
29 45	40 28	49 56	69 9
30 0	41 8	50 25	69 22
30 15	41 47	50 54	69 35
30 30	42 25	51 21	69 48
30 45	43 2	51 48	70 1
31 0	43 39	52 14	70 13
31 15	44 14	52 40	70 25
31 30	44 48	53 6	70 37
31 45	45 21	53 31	70 49
32 0	45 54	53 56	71 0
32 15	46 26	54 20	71 12
32 30	46 58	54 43	71 23
32 45	47 28	55 6	71 34
33 0	47 58	55 29	71 45
33 15	48 27	55 52	71 56
33 30	48 56	56 14	72 6
33 45	49 24	56 36	72 17
34 0	49 52	56 57	72 27
34 15	50 19	57 18	72 37
34 30	50 45	57 38	72 47
34 45	51 11	57 58	72 57
35 0	51 37	58 18	73 7
35 15	52 2	58 38	73 17
35 30	52 26	58 57	73 26
35 45	52 50	59 16	73 35
36 0	53 14	59 34	73 44
36 15	53 38	59 53	73 53
36 30	54 1	60 11	74 2
36 45	54 23	60 29	74 11

Altitud. Poli (G M)	♋ Cancer (G M)	♌/♊ Leo/Gemini (G M)	♍/♉ Virgo/Taurus (G M)
37 0	54 45	60 46	74 20
37 15	55 8	61 4	74 29
37 30	55 29	61 21	74 37
37 45	55 50	61 38	74 46
38 0	56 11	61 54	74 54
38 15	56 32	62 11	75 3
38 30	56 52	62 27	75 11
38 45	57 12	62 43	75 19
39 0	57 31	62 59	75 27
39 15	57 51	63 15	75 35
39 30	58 10	63 30	75 43
39 45	58 29	63 45	75 51
40 0	58 47	64 0	75 58
40 15	59 6	64 15	76 6
40 30	59 24	64 29	76 13
40 45	59 42	64 44	76 21
41 0	59 59	64 58	76 28
41 15	60 17	65 12	76 35
41 30	60 34	65 26	76 42
41 45	60 51	65 40	76 49
42 0	61 7	65 53	76 56
42 15	61 24	66 7	77 3
42 30	61 40	66 20	77 10
42 45	61 56	66 33	77 17
43 0	62 12	66 46	77 24
43 15	62 28	66 59	77 31
43 30	62 44	67 11	77 37
43 45	62 59	67 24	77 44
44 0	63 14	67 36	77 50
44 15	63 29	67 49	77 57

Altitud. Poli (G M)	♋ Cancer (G M)	♌/♊ Leo/Gemini (G M)	♍/♉ Virgo/Taurus (G M)
44 30	63 44	68 1	78 3
44 45	63 59	68 13	78 10
45 0	64 14	68 25	78 16
45 15	64 28	68 37	78 22
45 30	64 42	68 48	78 28
45 45	64 56	69 0	78 34
46 0	65 10	69 11	78 40
46 15	65 24	69 23	78 46
46 30	65 38	69 34	78 52
46 45	65 52	69 45	78 58
47 0	66 5	69 56	79 4
47 15	66 18	70 7	79 10
47 30	66 31	70 18	79 15
47 45	66 44	70 29	79 21
48 0	66 57	70 39	79 27
48 15	67 10	70 50	79 33
48 30	67 23	71 0	79 38
48 45	67 36	71 11	79 44
49 0	67 48	71 21	79 49
49 15	68 0	71 31	79 55
49 30	68 12	71 41	80 0
49 45	68 24	71 51	80 5
50 0	68 36	72 1	80 10
50 15	68 48	72 11	80 16
50 30	69 0	72 21	80 21
50 45	69 12	72 31	80 26
51 0	69 23	72 40	80 31
51 15	69 35	72 50	80 36
51 30	69 46	72 59	80 41
51 45	69 57	73 9	80 46

Altitud. Poli (G M)	♋ Cancer (G M)	♌/♊ Leo/Gemini (G M)	♍/♉ Virgo/Taurus (G M)
52 0	70 8	73 18	80 51
52 15	70 19	73 27	80 56
52 30	70 30	73 36	81 1
52 45	70 41	73 45	81 6
53 0	70 52	73 54	81 11
53 15	71 3	74 3	81 16
53 30	71 14	74 12	81 20
53 45	71 25	74 21	81 25
54 0	71 35	74 30	81 30
54 15	71 46	74 39	81 35
54 30	71 56	74 47	81 39
54 45	72 6	74 56	81 44
55 0	72 16	75 4	81 49
55 15	72 27	75 13	81 54
55 30	72 37	75 21	81 58
55 45	72 47	75 30	82 3
56 0	72 57	75 38	82 7
56 15	73 7	75 46	82 12
56 30	73 16	75 54	82 16
56 45	73 26	76 3	82 20
57 0	73 36	76 11	82 24
57 15	73 46	76 19	82 29
57 30	73 55	76 27	82 33
57 45	74 5	76 34	82 38
58 0	74 14	76 42	82 42
58 15	74 24	76 50	82 46
58 30	74 33	76 58	82 50
58 45	74 42	77 6	82 55
59 0	74 51	77 14	82 59
59 15	75 1	77 22	83 3

Altitud. Poli (G M)	♋ Cancer (G M)	♊/♈ Leo/Gemini (G M)	♍/♉ Virgo/Taurus (G M)
59 30	75 10	77 29	83 7
59 45	75 19	77 37	83 11
60 0	76 28 [recte: 75 28]	77 44	83 15
60 15	76 37 [recte: 75 37]	77 51	83 19
60 30	76 45 [recte: 75 45]	77 59	83 23
60 45	76 54 [recte: 75 54]	78 7	83 27
61 0	76 3	78 14	83 32
61 15	76 12	78 22	83 36
61 30	76 21	78 29	83 40
61 45	76 30	78 36	83 44
62 0	76 38	78 43	83 47
62 15	76 47	78 50	83 51
62 30	76 55	78 57	83 55
62 45	77 4	79 5	83 59
63 0	77 12	79 12	84 3
63 15	77 21	79 19	84 7
63 30	77 29	79 26	84 11
63 45	77 37	79 33	84 15
64 0	77 45	79 40	84 18
64 15	77 54	79 47	84 22
64 30	78 2	79 53	84 26
64 45	78 10	80 0	84 30
65 0	78 18	80 6	84 33
65 15	78 26	80 13	84 37
65 30	78 34	80 20	84 41
65 45	78 42	80 27	84 45
66 0	78 50	80 34	84 48

Altitud. Poli (G M)	♋ Cancer (G M)	♌/♊ Leo/Gemini (G M)	♍/♉ Virgo/Taurus (G M)
66 15	78 58	80 41	84 52
66 30	79 6	80 47	84 56
66 45	79 14	80 54	85 0
67 0	79 22	81 1	85 3
67 15	79 30	81 8	85 7
67 30	79 37	81 14	85 10
67 45	79 45	81 21	85 14
68 0	79 53	81 27	85 17
68 15	80 1	81 34	85 21
68 30	80 8	81 40	85 24
68 45	80 16	81 47	85 28
69 0	80 23	81 53	85 31
69 15	80 31	82 0	85 35
69 30	80 38	82 6	85 38
69 45	80 46	82 12	85 42
70 0	80 54	82 18	85 45
70 15	81 1	82 25	85 49
70 30	81 9	82 31	85 52
70 45	81 16	82 37	85 56
71 0	81 23	82 43	85 59
71 15	81 30	82 50	86 3
71 30	81 38	82 56	86 6
71 45	81 45	83 2	86 10
72 0	81 53	83 8	86 13
72 15	82 0	83 14	86 16
72 30	82 7	83 20	86 19
72 45	82 14	83 27	86 23
73 0	82 22	83 33	86 26
73 15	82 29	83 39	86 30
73 30	82 36	83 45	86 33

Altitud. Poli (G M)	♋ Cancer (G M)	♌/♊ Leo/Gemini (G M)	♍/♉ Virgo/Taurus (G M)
73 45	82 43	83 51	86 36
74 0	82 50	83 57	86 39
74 15	82 57	84 3	86 43
74 30	83 4	84 9	86 46
74 45	83 11	84 15	86 49
75 0	83 18	84 21	86 52
75 15	83 26	84 27	86 56
75 30	83 33	84 32	86 59
75 45	83 40	84 38	87 2
76 0	83 47	84 44	87 5
76 15	83 54	84 50	87 9
76 30	84 1	84 56	87 12
76 45	84 7	85 2	87 15
77 0	84 14	85 8	87 18
77 15	84 21	85 14	87 21
77 30	84 28	85 19	87 24
77 45	84 35	85 25	87 28
78 0	84 42	85 31	87 31
78 15	84 49	85 37	87 35
78 30	84 56	85 42	87 38
78 45	85 3	85 48	87 41
79 0	85 9	85 54	87 44
79 15	85 16	86 0	87 48
79 30	85 23	86 6	87 51
79 45	85 30	86 12	87 54
80 0	85 36	86 17	87 57
80 15	85 40	86 23	88 0
80 30	85 53	86 28	88 3
80 45	85 57	86 34	88 6
81 0	86 3	86 40	88 9

Altitud. Poli (G M)	♋ Cancer (G M)	♌/♊ Leo/Gemini (G M)	♍/♉ Virgo/Taurus (G M)
81 15	86 10	86 46	88 12
81 30	86 17	86 51	88 15
81 45	86 23	86 57	88 19
82 0	86 30	87 2	88 22
82 15	86 37	87 8	88 25
82 30	86 43	87 13	88 28
82 45	86 50	87 19	88 31
83 0	86 56	87 25	88 34
83 15	87 3	87 31	88 37
83 30	87 19 [recte: 87 9]	87 36	88 40
83 45	87 16	87 42	88 44
84 0	87 23	87 47	88 47
84 15	87 29	87 53	88 50
84 30	87 36	87 58	88 53
84 45	87 42	88 4	88 56
85 0	87 49	88 9	88 59
85 15	87 55	88 15	89 2
85 30	88 2	88 20	89 5
85 45	88 9	88 26	89 8
86 0	88 16	88 31	89 11
86 15	88 22	88 37	89 14
86 30	88 29	88 42	89 17
86 45	88 35	88 48	89 20
87 0	88 42	88 54	89 23
87 15	88 48	89 0	89 27
87 30	88 55	89 5	89 30
87 45	89 1	89 11	89 33
88 0	89 8	89 16	89 36
88 15	89 14	89 22	89 39

Altitud. Poli (G M)	♋ Cancer (G M)	♌/♊ Leo/Gemini (G M)	♍/♉ Virgo/Taurus (G M)
88 30	89 21	89 27	89 42
88 45	89 27	89 33	89 45
89 0	89 34	89 39	89 48
89 15	89 41	89 44	89 51
89 30	89 47	89 49	89 54
89 45	89 54	89 55	89 57
90 0	90 0	90 0	90 0

(Printed p. 168 carries the second half of Table XXI, latitudes 56°–90°, already merged into the single table above under the XXI. TABVLA heading.)

Tabella 22. For converting parts of the Equator into hours, and their fragments.

(*Tabella 22. conuertendi partes Æquatoris in horas, earumque fragmenta.*) — folio 169

This is a proportional conversion table (1° of the Equator = 4 minutes of time). It is printed in four parallel panels. Each panel has an **argument** column (headed G. / M. / S. — read as degrees, or minutes, or seconds) and a **result** column (headed H. M. / M. S. / S. T. — hours-and-minutes, or minutes-and-seconds, or seconds-and-thirds). The two result numbers in each cell are the higher and lower sexagesimal places (e.g. 0 4 = 0^h 4^m when the argument is read in degrees). Panels 1–3 have 23 lines; panel 4 has 21 lines and its last two lines are blank.

G.	H. M.	G.	H. M.	G.	H. M.	G.	H. M.
1	0 4	24	1 36	47	3 8	160	10 40
2	0 8	25	1 40	48	3 12	170	11 20
3	0 12	26	1 44	49	3 16	180	12 0
4	0 16	27	1 48	50	3 20	190	12 40
5	0 20	28	1 52	51	3 24	200	13 20
6	0 24	29	1 56	52	3 28	210	14 0
7	0 28	30	2 0	53	3 32	220	14 40
8	0 32	31	2 4	54	3 36	230	15 20
9	0 36	32	2 8	55	3 40	240	16 0

G.	H. M.	G.	H. M.	G.	H. M.	G.	H. M.
10	0 40	33	2 12	56	3 44	250	16 40
11	0 44	34	2 16	57	3 48	260	17 20
12	0 48	35	2 20	58	3 52	270	18 0
13	0 52	36	2 24	59	3 56	280	18 40
14	0 56	37	2 28	60	4 0	290	19 20
15	1 0	38	2 32	70	4 40	300	20 0
16	1 4	39	2 36	80	5 20	310	20 40
17	1 8	40	2 40	90	6 0	320	21 20
18	1 12	41	2 44	100	6 40	330	22 0
19	1 16	42	2 48	110	7 20	340	22 40
20	1 20	43	2 52	120	8 0	350	23 20
21	1 24	44	2 56	130	8 40	360	24 0
22	1 28	45	3 0	140	9 20		
23	1 32	46	3 4	150	10 0		

Tabella 23. For converting hours and their fragments into parts of the Equator.

(*Tabella 23. conuertendi horas, earumque fragmenta in partes Æquatoris.*) — folio 169

The inverse of Table 22 (1 hour of time = 15° of the Equator; 1 minute of time = 15' of arc). Panel 1 is headed H. (hours) → G. (degrees). Panels 2–4 are headed M. / S. / T. (argument in minutes, seconds, or thirds) → G. M. / M. S. / S. T. (result in the two adjacent sexagesimal places). Panel 1 has 24 lines; panels 2–4 have 20 lines each (their lines 21–24 are blank).

H.	G.	M.	G. M.	M.	G. M.	M.	G. M.
1	15	1	0 15	21	5 15	41	10 15
2	30	2	0 30	22	5 30	42	10 30
3	45	3	0 45	23	5 45	43	10 45
4	60	4	1 0	24	6 0	44	11 0
5	75	5	1 15	25	6 15	45	11 15
6	90	6	1 30	26	6 30	46	11 30

H.	G.	M.	G. M.	M.	G. M.	M.	G. M.
7	105	7	1 45	27	6 45	47	11 45
8	120	8	2 0	28	7 0	48	12 0
9	135	9	2 15	29	7 15	49	12 15
10	150	10	2 30	30	7 30	50	12 30
11	165	11	2 45	31	7 45	51	12 45
12	180	12	3 0	32	8 0	52	13 0
13	195	13	3 15	33	8 15	53	13 15
14	210	14	3 30	34	8 30	54	13 30
15	225	15	3 45	35	8 45	55	13 45
16	240	16	4 0	36	9 0	56	14 0
17	255	17	4 15	37	9 15	57	14 15
18	270	18	4 30	38	9 30	58	14 30
19	285	19	4 45	39	9 45	59	14 45
20	300	20	5 0	40	10 0	60	15 0
21	315						
22	330						
23	345						
24	360						

Tabula pro parte proportionali eruenda.

(Table for finding the proportional part.) — folios 170–171

A single sexagesimal proportional-parts (interpolation / multiplication) table spanning both folios: the 20 multiplier columns 1–10 are printed on folio 170 (below), the columns 11–20 on folio 171 (below). The left-hand column is the argument (minutes) 1–60; each interior cell gives the proportional part as two sexagesimal figures, $x \ y = \lfloor (\text{row} \times \text{col}) / 60 \rfloor$ and $(\text{row} \times \text{col}) \bmod 60$. The bottom line (argument 60) gives the whole multiplier in each column.

Folio 170 — columns 1–10:

	1	2	3	4	5	6	7	8	9	10
1	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	0 9	0 10
2	0 2	0 4	0 6	0 8	0 10	0 12	0 14	0 16	0 18	0 20

	1	2	3	4	5	6	7	8	9	10
3	0 3	0 6	0 9	0 12	0 15	0 18	0 21	0 24	0 27	0 30
4	0 4	0 8	0 12	0 16	0 20	0 24	0 28	0 32	0 36	0 40
5	0 5	0 10	0 15	0 20	0 25	0 30	0 35	0 40	0 45	0 50
6	0 6	0 12	0 18	0 24	0 30	0 36	0 42	0 48	0 54	1 0
7	0 7	0 14	0 21	0 28	0 35	0 42	0 49	0 56	1 3	1 10
8	0 8	0 16	0 24	0 32	0 40	0 48	0 56	1 4	1 12	1 20
9	0 9	0 18	0 27	0 36	0 45	0 54	1 3	1 12	1 21	1 30
1 0	0 10	0 20	0 30	0 40	0 50	1 0	1 10	1 20	1 30	1 40
1 1	0 11	0 22	0 33	0 44	0 55	1 6	1 17	1 28	1 39	1 50
1 2	0 12	0 24	0 36	0 48	1 0	1 12	1 24	1 36	1 48	2 0
1 3	0 13	0 26	0 39	0 52	1 5	1 18	1 31	1 44	1 57	2 10
1 4	0 14	0 28	0 42	0 56	1 10	1 24	1 38	1 52	2 6	2 20
1 5	0 15	0 30	0 45	1 0	1 15	1 30	1 45	2 0	2 15	2 30
1 6	0 16	0 32	0 48	1 4	1 20	1 36	1 52	2 8	2 24	2 40
1 7	0 17	0 34	0 51	1 8	1 25	1 42	1 59	2 16	2 33	2 50
1 8	0 18	0 36	0 54	1 12	1 30	1 48	2 6	2 24	2 42	3 0
1 9	0 19	0 38	0 57	1 16	1 35	1 54	2 13	2 32	2 51	3 10
2 0	0 20	0 40	1 0	1 20	1 40	2 0	2 20	2 40	3 0	3 20
2 1	0 21	0 42	1 3	1 24	1 45	2 6	2 27	2 48	3 9	3 30
2 2	0 22	0 44	1 6	1 28	1 50	2 12	2 34	2 56	3 18	3 40

	1	2	3	4	5	6	7	8	9	10
2 3	0 23	0 46	1 9	1 32	1 55	2 18	2 41	3 4	3 27	3 50
2 4	0 24	0 48	1 12	1 36	2 0	2 24	2 48	3 12	3 36	4 0
2 5	0 25	0 50	1 15	1 40	2 5	2 30	2 55	3 20	3 45	4 10
2 6	0 26	0 52	1 18	1 44	2 10	2 36	3 2	3 28	3 54	4 20
2 7	0 27	0 54	1 21	1 48	2 15	2 42	3 9	3 36	4 3	4 30
2 8	0 28	0 56	1 24	1 52	2 20	2 48	3 16	3 44	4 12	4 40
2 9	0 29	0 58	1 27	1 56	2 25	2 54	3 23	3 52	4 21	4 50
3 0	0 30	1 0	1 30	2 0	2 30	3 0	3 30	4 0	4 30	5 0
3 1	0 31	1 2	1 33	2 4	2 35	3 6	3 37	4 8	4 39	5 10
3 2	0 32	1 4	1 36	2 8	2 40	3 12	3 44	4 16	4 48	5 20
3 3	0 33	1 6	1 39	2 12	2 45	3 18	3 51	4 24	4 57	5 30
3 4	0 34	1 8	1 42	2 16	2 50	3 24	3 58	4 32	5 6	5 40
3 5	0 35	1 10	1 45	2 20	2 55	3 30	4 5	4 40	5 15	5 50
3 6	0 36	1 12	1 48	2 24	3 0	3 36	4 12	4 48	5 24	6 0
3 7	0 37	1 14	1 51	2 28	3 5	3 42	4 19	4 56	5 33	6 10
3 8	0 38	1 16	1 54	2 32	3 10	3 48	4 26	5 4	5 42	6 20
3 9	0 39	1 18	1 57	2 36	3 15	3 54	4 33	5 12	5 51	6 30

	1	2	3	4	5	6	7	8	9	10
4 0	0 40	1 20	2 0	2 40	3 20	4 0	4 40	5 20	6 0	6 40
4 1	0 41	1 22	2 3	2 44	3 25	4 6	4 47	5 28	6 9	6 50
4 2	0 42	1 24	2 6	2 48	3 30	4 12	4 54	5 36	6 18	7 0
4 3	0 43	1 26	2 9	2 52	3 35	4 18	5 1	5 44	6 27	7 10
4 4	0 44	1 28	2 12	2 56	3 40	4 24	5 8	5 52	6 36	7 20
4 5	0 45	1 30	2 15	3 0	3 45	4 30	5 15	6 0	6 45	7 30
4 6	0 46	1 32	2 18	3 4	3 50	4 36	5 22	6 8	6 54	7 40
4 7	0 47	1 34	2 21	3 8	3 55	4 42	5 29	6 16	7 3	7 50
4 8	0 48	1 36	2 24	3 12	4 0	4 48	5 36	6 24	7 12	8 0
4 9	0 49	1 38	2 27	3 16	4 5	4 54	5 43	6 32	7 21	8 10
5 0	0 50	1 40	2 30	3 20	4 10	5 0	5 50	6 40	7 30	8 20
5 1	0 51	1 42	2 33	3 24	4 15	5 6	5 57	6 48	7 39	8 30
5 2	0 52	1 44	2 36	3 28	4 20	5 12	6 4	6 56	7 48	8 40
5 3	0 53	1 46	2 39	3 32	4 25	5 18	6 11	7 4	7 57	8 50
5 4	0 54	1 48	2 42	3 36	4 30	5 24	6 18	7 12	8 6	9 0
5 5	0 55	1 50	2 45	3 40	4 35	5 30	6 25	7 20	8 15	9 10
5 6	0 56	1 52	2 48	3 44	4 40	5 36	6 32	7 28	8 24	9 20

	1	2	3	4	5	6	7	8	9	10
5 7	0 57	1 54	2 51	3 48	4 45	5 42	6 39	7 36	8 33	9 30
5 8	0 58	1 56	2 54	3 52	4 50	5 48	6 46	7 44	8 42	9 40
5 9	0 59	1 58	2 57	3 56	4 55	5 54	6 53	7 52	8 51	9 50
6 0	1 0	2 0	3 0	4 0	5 0	6 0	7 0	8 0	9 0	10 0

Folio 171 — columns 11–20:

	11	12		13	14	15	16	17	18		19	20
1 0	0 12			0	0	0	0	0	0 18		0	0
11				13	14	15	16	17			19	20
2 0	0 26 [recte: 0 24]			0	0	0	0	0	0 36		0	0
22				26	28	30	32	34			38	40
3 0	0 36			0	0	0	0	0	0 54		0	1 0
33				39	42	45	48	51			57	
4 0	0 48			0	0	1 0	1 4	1 8	1 12		1	1
44				52	56						16	20
5 0	1 0			1 5	1	1	1	1	1 30		1	1
55					10	15	20	25			35	40
6 1 6	1 12			1	1	1	1	1	1 48		1	2 0
				18	24	30	36	42			54	
7 1	1 24			1	1	1	1	1	2 6		2	2
17				31	38	45	52	59			13	20
8 1	1 36			1	1	2 0	2 8	2	2 24		2	2
28				44	52			16			32	40
9 1	1 48			1	2 6	2	2	2	2 42		2	3 0
39				57		15	24	33			51	
1 1	2 0			2	2	2	2	2	3 0		3	3
0 50				10	20	30	40	50			10	20
1 2 1	2 12			2	2	2	2	3 7	3 18		3	3
1				23	34	45	56				29	40
1 2	2 24			2	2	3 0	3	3	33 6 [recte: 3 36]		3	4 0

11	12	13	14	15	16	17	18	19	20
2 12		36	48		12	24		48	
1 2	2 36	2	3 2	3	3	3	3 54	4 7	4
3 23		49		15	28	41			20
1 2	2 48	3 2	3	3	3	3	4 12	4	4
4 34			16	30	44	58		26	40
1 2	3 0	3	3	3	4 0	4	4 30	4	5 0
5 45		15	30	45		15		45	
1 2	3 12	3	3	4 0	4	4	4 48	5 4	5
6 56		28	44		16	32			20
1 3 7	3 24	3	3	4	4	4	5 6	5	5
7		41	58	15	32	49		23	40
1 3	3 36	3	4	4	4	5 6	5 24	5	6 0
8 18		54	12	30	48			42	
1 3	3 48	4 7	4	4	5 4	5	5 42	6 1	6
9 29			26	45		23			20
2 3	4 0	4	4	5 0	5	5	6 0	6	6
0 40		20	40		20	40		20	40
2 3	4 12	4	4	5	5	5	6 18	6	7 0
1 51		33	54	15	36	57		39	
2 4 2	4 24	4	5 8	5	5	6	6 36	6	7
2		46		30	52	14		58	20
2 4	4 36	4	5	5	6 8	6	6 54	7	7
3 13		59	22	45		31		17	40
2 4	4 48	5	5	6 0	6	6	7 12	7	8 0
4 24		12	36		24	48		36	
2 4	5 0	5	5	6	6	7 5	7 30	7	8
5 35		25	50	15	40			55	20
2 4	5 12	5	6 4	6	6	7	7 48	8	8
6 46		38		30	56	22		14	40
2 4	5 24	5	6	6	7	7	8 6	8	9 0
7 57		51	18	45	12	39		33	
2 5 8	5 36	6 4	6	7 0	7	7	8 24	8	9
8			32		28	56		52	20
2 5	5 48	6	6	7	7	8	8 42	9	9

11	12	13	14	15	16	17	18	19	20
9 19		17	46	15	44	13		11	40
3 5	6 0	6	7 0	7	8 0	8	9 0	9	10
0 30		30		30		30		30	0
3 5	6 12	6	7	7	8	8	9 18	9	10
1 41		43	14	45	16	47		49	20
3 5	6 24	6	7	8 0	8	9 4	9 36	10	10
2 52		56	28		32			8	40
3 6 3	6 36	7 9	7	8	8	9	9 54	10	11
3			42	15	48	21		27	0
3 6	6 48	7	7	8	9 4	9	10 12	10	11
4 14		22	56	30		38		46	20
3 6	7 0	7	8	8	9	9	10 30	11	11
5 25		35	10	45	20	55		5	40
3 6	7 12	7	8	9 0	9	10	10 48	11	12
6 36		48	24		36	12		24	0
3 6	7 24	8 1	8	9	9	10	11 6	11	12
7 47			38	15	52	29		43	20
3 6	7 36	8	8	9	10	10	11 24	12	12
8 58		14	52	30	8	46		2	40
3 7 9	7 48	8	9 6	9	10	11	11 42	12	13
9		27		45	24	3		21	0
4 7	8 0	8	9	10	10	11	12 0	12	13
0 20		40	20	0	40	20		40	20
4 7	8 12	8	9	10	10	11	12 18	12	13
1 31		53	34	15	56	37		59	40
4 7	8 24	9 6	9	10	11	11	12 36	13	14
2 42			48	30	12	54		18	0
4 7	8 36	9	10	10	11	12	12 54	13	14
3 53		19	2	45	28	11		37	20
4 8 4	8 48	9	10	11	11	12	13 12	13	14
4		32	16	0	44	28		56	40
4 8	9 0	9	10	11	12	12	13 30	14	15
5 15		45	30	15	0	45		15	0
4 8	9 12	9	10	11	12	13	13 48	14	15

11	12	13	14	15	16	17	18	19	20
6 26		58	44	30	16	2		34	20
4 8	9 24	10	10	11	12	13	14 6	14	15
7 37		11	58	45	32	19		53	40
4 8	9 36	10	11	12	12	13	14 24	15	16
8 48		24	12	0	48	36		12	0
4 8	9 48	10	11	12	13	13	14 42	15	16
9 59		37	26	15	4	53		31	20
5 9	10 0	10	11	12	13	14	15 0	15	16
0 10		50	40	30	20	10		50	40
5 9	10 12	11	11	12	13	14	15 18	16	17
1 21		3	54	45	36	27		9	0
5 9	10 24	11	12	13	13	14	15 36	16	17
2 32		16	8	0	52	44		28	20
5 9	10 36	11	12	13	14	15	15 54	16	17
3 43		29	22	15	8	1		47	40
5 9	10 48	11	12	13	14	15	16 12	17	18
4 54		42	36	30	24	18		6	0
5 10	11 0	11	12	13	14	15	16 30	17	18
5 5		55	50	45	40	35		25	20
5 10	11 12	12	13	14	14	15	16 48	17	18
6 16		8	4	0	56	52		44	40
5 10	11 24	12	13	14	15	16	17 6	18	19
7 27		21	18	15	12	9		3	0
5 10	11 36	12	13	14	15	16	17 24	18	19
8 38		34	32	30	28	26		22	20
5 10	11 48	12	13	14	15	16	17 42	18	19
9 49		47	46	45	44	43		41	40
6 11	12 0	13	14	15	16	17	18 0	19	20
0 0		0	0	0	0	0		0	0

Table for finding the proportional part [*Tabula pro parte proportionali eruenda*]

(Continuation of the proportional-parts table begun on the preceding folios. Each cell gives the product of the left-hand argument and the column-head multiplier, expressed sexagesimally as degrees and minutes — i.e. $\lfloor \text{argument} \times \text{multiplier} / 60 \rfloor : (\text{argument} \times \text{multiplier} \bmod 60)$. Multiplier columns 1–20 stood on folios 170–171; the columns below run 21–50 across folios 172–174.)

Folio 172 — multiplier columns 21–30

—	21	22	23	24	25	26	27	28	29	30
1	0 21	0 22	0 23	0 24	0 25	0 26	0 27	0 28	0 29	0 30
2	0 42	0 44	0 46	0 48	0 50	0 52	0 54	0 56	0 58	1 0
3	1 3	1 6	1 9	1 12	1 15	1 18	1 21	1 24	1 27	1 30
4	1 24	1 28	1 32	1 36	1 40	1 44	1 48	1 52	1 56	2 0
5	1 45	1 50	1 55	2 0	2 5	2 10	2 15	2 20	2 25	2 30
6	2 6	2 12	2 18	2 24	2 30	2 36	2 42	2 48	2 54	3 0
7	2 27	2 34	2 41	2 48	2 55	3 2	3 9	3 16	3 23	3 30
8	2 48	2 56	3 4	3 12	3 20	3 28	3 36	3 44	3 52	4 0
9	3 9	3 18	3 27	3 36	3 45	3 54	4 3	4 12	4 21	4 30
10	3 30	3 40	3 50	4 0	4 10	4 20	4 30	4 40	4 50	5 0
11	3 51	4 2	4 13	4 24	4 35	4 46	4 57	5 8	5 19	5 30
12	4 12	4 24	4 36	4 48	5 0	5 12	5 24	5 36	5 48	6 0
13	4 33	4 46	4 59	5 12	5 25	5 38	5 51	6 4	6 17	6 30
14	4 54	5 8	5 22	5 36	5 50	6 4	6 18	6 32	6 46	7 0
15	5 15	5 30	5 45	6 0	6 15	6 30	6 45	7 0	7 15	7 30
16	5 36	5 52	6 8	6 24	6 40	6 56	7 12	7 28	7 44	8 0
17	5 57	6 14	6 31	6 48	7 5	7 22	7 39	7 56	8 13	8 30
18	6 18	6 36	6 54	7 12	7 30	7 48	8 6	8 24	8 42	9 0
19	6 39	6 58	7 17	7 36	7 55	8 14	8 33	8 52	9 11	9 30
20	7 0	7 20	7 40	8 0	8 20	8 40	9 0	9 20	9 40	10 0
21	7 21	7 42	8 3	8 24	8 45	9 6	9 27	9 48	10 9	10 30
22	7 42	8 4	8 26	8 48	9 10	9 32	9 54	10 16	10 38	11 0

—	21	22	23	24	25	26	27	28	29	30
23	8 3	8 26	8 49	9 12	9 35	9 58	10 21	10 44	11 7	11 30
24	8 24	8 48	9 12	9 36	10 0	10 24	10 48	11 12	11 36	12 0
25	8 45	9 10	9 35	10 0	10 25	10 50	11 15	11 40	12 5	12 30
26	9 6	9 32	9 58	10 24	10 50	11 16	11 42	12 8	12 34	13 0
27	9 27	9 54	10 21	10 48	11 15	11 42	12 9	12 36	13 3	13 30
28	9 48	10 16	10 44	11 12	11 40	12 8	12 36	13 4	13 32	14 0
29	10 9	10 38	11 7	11 36	12 5	12 34	13 3	13 32	14 1	14 30
30	10 30	11 0	11 30	12 0	12 30	13 0	13 30	14 0	14 30	15 0
31	10 51	11 22	11 53	12 24	12 55	13 26	13 57	14 28	14 59	15 30
32	11 12	11 44	12 16	12 48	13 20	13 52	14 24	14 56	15 28	16 0
33	11 33	12 6	12 39	13 12	13 45	14 18	14 51	15 24	15 57	16 30
34	11 54	12 28	13 2	13 36	14 10	14 44	15 18	15 52	16 26	17 0
35	12 15	12 50	13 25	14 0	14 35	15 10	15 45	16 20	16 55	17 30
36	12 36	13 12	13 48	14 24	15 0	15 36	16 12	16 48	17 24	18 0
37	12 57	13 34	14 11	14 48	15 25	16 2	16 39	17 16	17 53	18 30
38	13 18	13 56	14 34	15 12	15 50	16 28	17 6	17 44	18 22	19 0
39	13 39	14 18	14 57	15 36	16 15	16 54	17 33	18 12	18 51	19 30
40	14 0	14 40	15 20	16 0	16 40	17 20	18 0	18 40	19 20	20 0
41	14 21	15 2	15 43	16 24	17 5	17 46	18 27	19 8	19 49	20 30
42	14 42	15 24	16 6	16 48	17 30	18 12	18 54	19 36	20 18	21 0
43	15 3	15 46	16 29	17 12	17 55	18 38	19 21	20 4	20 47	21 30
44	15 24	16 8	16 52	17 36	18 20	19 4	19 48	20 32	21 16	22 0
45	15 45	16 30	17 15	18 0	18 45	19 30	20 15	21 0	21 45	22 30
46	16 6	16 52	17 38	18 24	19 10	19 56	20 42	21 28	22 14	23 0
47	16 27	17 14	18 1	18 48	19 35	20 22	21 9	21 56	22 43	23 30
48	16 48	17 36	18 24	19 12	20 0	20 48	21 36	22 24	23 12	24 0
49	17 9	17 58	18 47	19 36	20 25	21 14	22 3	22 52	23 41	24 30
50	17 30	18 20	19 10	20 0	20 50	21 40	22 30	23 20	24 10	25 0
51	17 51	18 42	19 33	20 24	21 15	22 6	22 57	23 48	24 39	25 30
52	18 12	19 4	19 56	20 48	21 40	22 32	23 24	24 16	25 8	26 0
53	18 33	19 26	20 19	21 12	22 5	22 58	23 51	24 44	25 37	26 30

—	21	22	23	24	25	26	27	28	29	30
54	18 54	19 48	20 42	21 36	22 30	23 24	24 18	25 12	26 6	27 0
55	19 15	20 10	21 5	22 0	22 55	23 50	24 45	25 40	26 35	27 30
56	19 36	20 32	21 28	22 24	23 20	24 16	25 12	26 8	27 4	28 0
57	19 57	20 54	21 51	22 48	23 45	24 42	25 39	26 36	27 33	28 30
58	20 18	21 16	22 14	23 12	24 10	25 8	26 6	27 4	28 2	29 0
59	20 39	21 38	22 37	23 36	24 35	25 34	26 33	27 32	28 31	29 30
60	21 0	22 0	23 0	24 0	25 0	26 0	27 0	28 0	29 0	30 0

Folio 173 — multiplier columns 31–40

—	31	32	33	34	35	36	37	38	39	40
1	0 31	0 32	0 33	0 34	0 35	0 36	0 37	0 38	0 39	0 40
2	1 2	1 4	1 6	1 8	1 10	1 12	1 14	1 16	1 18	1 20
3	1 33	1 36	1 39	1 42	1 45	1 48	1 51	1 54	1 57	2 0
4	2 4	2 8	2 12	2 16	2 20	2 24	2 28	2 32	2 36	2 40
5	2 35	2 40	2 45	2 50	2 55	3 0	3 5	3 10	3 15	3 20
6	3 6	3 12	3 18	3 24	3 30	3 36	3 42	3 48	3 54	4 0
7	3 37	3 44	3 51	3 58	4 5	4 12	4 19	4 26	4 33	4 40
8	4 8	4 16	4 24	4 32	4 40	4 48	4 56	5 4	5 12	5 20
9	4 39	4 48	4 57	5 6	5 15	5 24	5 33	5 42	5 51	6 0
1 0	5 10	5 20	5 30	5 40	5 50	6 0	6 10	6 20	6 30	6 40
1 1	5 4 [recte: 5 41]	5 52	6 3	6 14	6 25	6 36	6 47	6 58	7 9	7 20
1 2	6 12	6 24	6 36	6 48	7 0	7 12	7 24	7 36	7 48	8 0
1 3	6 43	6 56	7 9	7 22	7 35	7 48	8 1	8 14	8 27	8 40
1 4	7 14	7 28	7 42	7 56	8 10	8 24	8 38	8 52	9 6	9 20
1 5	7 45	8 0	8 15	8 30	8 45	9 0	9 15	9 30	9 45	10 0
1 6	8 16	8 32	8 48	9 4	9 20	9 36	9 52	10 8	10 24	10 40

— 31	32	33	34	35	36	37	38	39	40
1 8 47	9 4	9 21	9 38	9 55	10	10	10	11 3	11
7					12	29	46		20
1 9 18	9 36	9 54	10	10	10	11 6	11	11	12 0
8			12	30	48		24	42	
1 9 49	10 8	10	10	11 5	11	11	12 2	12	12
9		27	46		24	43		21	40
2 10 20	10	11 0	11	11	12 0	12	12	13 0	13
0	40		20	40		20	40		20
2 10 51	11	11	11	12	12	12	13	13	14 0
1	12	33	54	15	36	57	18	39	
2 11 22	11	12 6	12	12	13	13	13	14	14
2	44		28	50	12	34	56	18	40
2 11 53	12	12	13 2	13	13	14	14	14	15
3	16	39		25	48	11	34	57	20
2 12 24	12	13	13	14 0	14	14	15	15	16 0
4	48	12	36		24	48	12	36	
2 12 55	13	13	14	14	15 0	15	15	16	16
5	20	45	10	35		25	50	15	40
2 13 26	13	14	14	15	15	16 2	16	16	17
6	52	18	44	10	36		28	54	20
2 13 57	14	14	15	15	16	16	17 6	17	18 0
7	24	51	18	45	12	39		33	
2 14 28	14	15	15	16	16	17	17	18	18
8	56	24	52	20	48	16	44	12	40
2 14 59	15	15	16	16	17	17	18	18	19
9	28	57	26	55	24	53	22	51	20
3 15 30	16 0	16	17 0	17	18 0	18	19 0	19	20 0
0		30		30		30		30	
3 16 1	16	17 3	17	18 5	18	19 7	19	20 9	20
1	32		34		36		38		40
3 16 32	17 4	17	18 8	18	19	19	20	20	21
2		36		40	12	44	16	48	20
3 17 3	17	18 9	18	19	19	20	20	21	22 0
3	36		42	15	48	21	54	27	

— 31	32	33	34	35	36	37	38	39	40
3 17 34	18 8	18	19	19	20	20	21	22 6	22
4		42	16	50	24	58	32		40
3 18 5	18	19	19	20	21 0	21	22	22	23
5	40	15	50	25		35	10	45	20
3 18 36	19	19	20	21 0	21	22	22	23	24 0
6	12	48	24		36	12	48	24	
3 19 7	19	20	20	21	22	22	23	24 3	24
7	44	21	58	35	12	49	26		40
3 19 38	20	20	21	22	22	23	24 4	24	25
8	16	54	32	10	48	26		42	20
3 20 9	20	21	22 6	22	23	24 3	24	25	26 0
9	48	27		45	24		42	21	
4 20 40	21	22 0	22	23	24 0	24	25	26 0	26
0	20		40	20		40	20		40
4 21 11	21	22	23	23	24	25	25	26	27
1	52	33	14	55	36	17	58	39	20
4 21 42	22	23 6	23	24	25	25	26	27	28 0
2	24		48	30	12	54	36	18	
4 22 13	22	23	24	25 5	25	26	27	27	28
3	56	39	22		48	31	14	57	40
4 22 44	23	24	24	25	26	27 8	27	28	29
4	28	12	56	40	24		52	36	20
4 23 15	24 0	24	25	26	27 0	27	28	29	30 0
5		45	30	15		45	30	15	
4 23 46	24	25	26 4	26	27	28	29 8	29	30
6	32	18		50	36	22		54	40
4 24 17	25 4	25	26	27	28	28	29	30	31
7		51	38	25	12	59	46	33	20
4 24 48	25	26	27	28 0	28	29	30	31	32 0
8	36	24	12		48	36	24	12	
4 25 19	26 8	26	27	28	29	30	31 2	31	32
9		57	46	35	24	13		51	40
5 25 50	26	27	28	29	30 0	30	31	32	33
0	40	30	20	10		50	40	30	20

—	31		32	33	34	35	36	37	38	39	40
5 1	26 21		27 12	28 3	28 54	29 45	30 36	31 27	32 18	33 9	34 0
5 2	26 52		27 44	28	29 36	30 20	31 12	32 4	32 56	33 48	34 40
5 3	27 23		28 16	29 9	30 2	30 55	31 48	32 41	33 34	34 27	35 20
5 4	27 54		28 48	29	30 36	31 30	32 24	33 18	34 12	35 6	36 0
5 5	28 25		29 20	30	31 10	32 5	33 0	33 55	34 50	35 45	36 40
5 6	28 56		29 52	30	31 44	32	33 40	34 36	35 32	36 28	37 24
5 7	29 27		30 24	31	32 18	33	34 15	35 9	36 6	37 3	38 0
5 8	29 58		30 56	31	32 52	33	34 50	35	36 48	37 46	38 44
5 9	30 29		31 28	32	33 26	34	35 25	36	37 24	38 23	39 22
6 0	31 0		32 0	33 0	34 0	35 0	36 0	37 0	38 0	39 0	40 0

[illegible]

— 41	42	43	44	45	46	47	48	49	50
6		18	24	30	36	42	48		
7 4	4 54	5	5	5	5	5	5	5 43	5 50
47		1	8	15	22	29	36		
8 5	5 36	5	5	6	6	6	6	6 32	6 40
28		44	52	0	8	16	24		
9 6	6 18	6	6	6	6	7	7	7 21	7 30
9		27	36	45	54	3	12		
1 6	7 0	7	7	7	7	7	8	8 10	8 20
0 50		10	20	30	40	50	0		
1 7	7 42	7	8	8	8	8	8	8 59	9 10
1 31		53	4	15	26	37	48		
1 8	8 24	8	8	9	9	9	9	9 48	10 0
2 12		36	48	0	12	24	36		
1 8	9 6	9	9	9	9	10	10	10 37	10 50
3 53		19	32	45	58	11	24		
1 9	9 48	10	10	10	10	10	11	11 26	11 40
4 34		2	16	30	44	58	12		
1 10	10 30	10	11	11	11	11	12	12 15	12 30
5 15		45	0	15	30	45	0		
1 10	11 12	11	11	12	12	12	12	13 4	13 20
6 56		28	44	0	16	32	48		
1 11	11 54	12	12	12	13	13	13	13 53	14 10
7 37		11	28	45	2	19	36		
1 12	12 36	12	13	13	13	14	14	14 42	15 0
8 18		54	12	30	48	6	24		
1 12	13 18	13	13	14	14	14	15	15 31	15 50
9 59		37	56	15	34	53	12		
2 13	14 0	14	14	15	15	15	16	16 20	16 40
0 40		20	40	0	20	40	0		
2 14	14 42	15	15	15	16	16	16	17 9	17 30
1 21		3	24	45	6	27	48		
2 15	15 24	15	16	16	16	17	17	17 58	18 20
2 2		46	8	30	52	14	36		
2 15	16 6	16	16	17	17	18	18	18 47	19 10

— 41	42	43	44	45	46	47	48	49	50
3	43	29	52	15	38	1	24		
2	16 16 48	17	17	18	18	18	19	19 36	20 0
4	24	12	36	0	24	48	12		
2	17 17 30	17	18	18	19	19	20	20 25	20 50
5	5	55	20	45	10	35	0		
2	17 18 12	18	19	19	19	20	20	21 14	21 40
6	46	38	4	30	56	22	48		
2	18 18 54	19	19	20	20	21	21	22 3	22 30
7	27	21	48	15	42	9	36		
2	19 19 36	20	20	21	21	21	22	22 52	23 20
8	8	4	32	0	28	56	24		
2	19 20 18	20	21	21	22	22	23	23 41	24 10
9	49	47	16	45	14	43	12		
3	20 21 0	21	22	22	23	23	24	24 30	25 0
0	30	30	0	30	0	30	0		
3	21 21 42	22	22	23	23	24	24	25 19	25 50
1	11	13	44	15	46	17	48		
3	21 22 24	22	23	24	24	25	25	26 8	26 40
2	52	56	28	0	32	4	36		
3	22 23 6	23	24	24	25	25	26	26 57	27 30
3	33	39	12	45	18	51	24		
3	23 23 48	24	24	25	26	26	27	27 46	28 20
4	14	22	56	30	4	38	12		
3	23 24 30	25	25	26	26	27	28	28 35	29 10
5	55	5	40	15	50	25	0		
3	24 25 12	25	26	27	27	28	28	29 24	30 0
6	36	48	24	0	36	12	48		
3	25 25 54	26	27	27	28	28	29	30 13	30 50
7	17	31	8	45	22	59	36		
3	25 26 36	27	27	28	29	29	30	21 2 [recte:	31 40
8	58	14	52	30	8	46	24	31 2]	
3	26 27 18	27	28	29	29	30	31	31 51	32 30
9	39	57	36	15	54	33	12		
4	27 28 0	28	29	30	30	31	32	32 40	33 20

— 41	42	43	44	45	46	47	48	49	50
0	20	40	20	0	40	20	0		
4	28 28 42	29	30	30	31	32	32	33 29	34 10
1	1	23	4	45	26	7	48		
4	28 29 24	30	30	31	32	32	33	34 18	35 0
2	42	6	48	30	12	54	36		
4	29 30 6	30	31	32	32	33	34	35 7	35 50
3	23	49	32	15	58	41	24		
4	30 30 48	31	32	33	33	34	35	35 56	36 40
4	4	32	16	0	44	28	12		
4	30 31 30	32	33	33	34	35	36	36 45	37 30
5	45	15	0	45	30	15	0		
4	31 32 12	32	33	34	35	36	36	37 34	38 20
6	26	58	44	30	16	2	48		
4	32 32 54	33	34	35	36	36	37	38 23	39 10
7	7	41	28	15	2	49	36		
4	32 33 36	34	35	36	36	37	38	39 12	40 0
8	48	24	12	0	48	36	24		
4	33 34 18	35	35	36	37	38	39	40 1	40 50
9	29	7	56	45	34	23	12		
5	34 35 0	35	36	37	38	39	40	40 50	41 40
0	10	50	40	30	20	10	0		
5	34 35 42	36	37	38	39	39	40	41 39	42 30
1	51	33	24	15	6	57	48		
5	35 36 24	37	38	39	39	40	41	42 28	43 20
2	32	16	8	0	52	44	36		
5	36 37 6	37	38	39	40	41	42	43 17	44 10
3	13	59	52	45	38	31	24		
5	36 37 48	38	39	40	41	42	43	44 6	45 0
4	54	42	36	30	24	18	12		
5	37 38 30	39	40	41	42	43	44	44 55	45 50
5	35	25	20	15	10	5	0		
5	38 39 12	40	41	42	42	43	44	45 44	46 40
6	16	8	4	0	56	52	48		
5	38 39 54	40	41	42	43	44	45	46 33	47 30

— 41	42		43	44	45	46	47	48	49		50
7	57		51	48	45	42	39	36			
5	39	40 36	41	42	43	44	45	46	47 22		48 20
8	38		34	32	30	28	26	24			
5	40	41 18	42	43	44	45	46	47	48 11		46 10 [recte: 49
9	19		17	16	15	14	13	12			10]
6	41	41 0 [recte:	43	44	45	46	47	48	49 0		50 0
0	0	42 0]	0	0	0	0	0	0			

[Table continues: multiplier columns 51–60 stand on the following folio.]

Table for finding the proportional part [*Tabula pro parte proportionali eruenda*] — multiplier columns 51–60. (The earlier multiplier columns, 21–50, stand on the preceding folios; this is the same continuous table.)

Each cell holds two figures: the whole (left) and the sixtieth part (right). The left column is the argument (1–60); the ten column-heads are the multipliers 51–60.

#	51	52	53	54	55	56	57	58	59	60
1	0 51	0 52	0 53	0 54	0 55	0 56	0 57	0 58	0 59	1 0
2	1 42	1 44	1 46	1 48	1 50	1 52	1 54	1 56	1 58	2 0
3	2 33	2 36	2 39	2 42	2 45	2 48	2 51	2 54	2 57	3 0
4	3 24	3 28	3 32	3 36	3 40	3 44	3 48	3 52	3 56	4 0
5	4 15	4 20	4 25	4 30	4 35	4 40	4 45	4 50	4 55	5 0
6	5 6	5 12	5 18	5 24	5 30	5 36	5 42	5 48	5 54	6 0
7	5 57	6 4	6 11	6 18	6 25	6 32	6 39	6 46	6 53	7 0
8	6 48	6 56	7 4	7 12	7 20	7 28	7 36	7 44	7 52	8 0
9	7 39	7 48	7 57	8 6	8 15	8 24	8 33	8 42	8 51	9 0
10	8 30	8 40	8 50	9 0	9 10	9 20	9 30	9 40	9 50	10 0
11	9 21	9 32	9 43	9 54	10 5	10 16	10 27	10 38	10 49	11 0
12	10 12	10 24	10 36	10 48	11 0	11 12	11 24	11 36	11 48	12 0
13	11 3	11 16	11 29	11 42	11 55	12 8	12 21	12 34	12 47	13 0
14	11 11	12 8	12 12	12 12	12 12	13 4	13 13	13 13	13 13	14

#	51	52	53	54	55	56	57	58	59	60
	54		22	36	50		18	32	46	0
15	12	13 0	13	13	13	14 0	14	14	14	15
	45		15	30	45		15	30	45	0
16	13	13	14 8	14	14	14	15	15	15	16
	36	52		24	40	56	12	28	44	0
17	14	14	15 1	15	15	15	16 9	16	16	17
	27	44		18	35	52		26	43	0
18	15	15	15	16	16	16	17 6	17	17	18
	18	36	54	12	30	48		24	42	0
19	16 9	16	16	17 6	17	17	18 3	18	18	19
		28	47		25	44		22	41	0
20	17 0	17	17	18 0	18	18	19 0	19	19	20
		20	40		20	40		20	40	0
21	17	18	18	18	19	19	19	20	20	21
	51	12	33	54	15	36	57	18	39	0
22	18	19 4	19	19	20	20	20	21	21	22
	42		26	48	10	32	54	16	38	0
23	19	19	20	20	21 5	21	21	22	22	23
	33	56	19	42		28	51	14	37	0
24	20	20	21	21	22 0	22	22	23	23	24
	24	48	12	36		24	48	12	36	0
25	21	21	22 5	22	22	23	23	24	24	25
	15	40		30	55	20	45	10	35	0
26	22 6	22	22	23	23	24	24	25 8	25	26
		32	58	24	50	16	42		34	0
27	22	23	23	24	24	25	25	26 6	26	27
	57	24	51	18	45	12	39		33	0
28	23	24	24	25	25	26 8	26	27 4	27	28
	48	16	44	12	40		36		32	0
29	24	25 8	25	26 6	26	27 4	27	28 2	28	29
	39		37		35		33		31	0
30	25	26 0	26	27 0	27	28 0	28	29 0	29	30
	30		30		30		30		30	0
31	26	26	27	27	28	28	29	29	30	31

#	51	52	53	54	55	56	57	58	59	60
	21	52	23	54	25	56	27	58	29	0
32	27	27	28	28	29	29	30	30	31	32
	12	44	16	48	20	52	24	56	28	0
33	28 3	28	29 9	29	30	30	31	31	32	33
		36		42	15	48	21	54	27	0
34	28	29	30 2	30	31	31	32	32	33	34
	54	28		36	10	44	18	52	26	0
35	29	30	30	31	32 5	32	33	33	34	35
	45	20	55	30		40	15	50	25	0
36	30	31	31	32	33 0	33	34	34	35	36
	36	12	48	24		36	12	48	24	0
37	31	32 4	32	33	33	34	35 9	35	36	37
	27		41	18	55	32		46	23	0
38	32	32	33	34	34	35	36 6	36	37	38
	18	56	34	12	50	28		44	22	0
39	33 9	33	34	35 6	35	36	37 3	37	38	39
		48	27		45	24		42	21	0
40	34 0	34	35	36 0	36	37	38 0	38	39	40
		40	20		40	20		40	20	0
41	34	35	36	36	37	38	38	39	40	41
	51	32	13	54	35	16	57	38	19	0
42	35	36	37 6	37	38	39	39	40	41	42
	42	24		48	30	12	54	36	18	0
43	36	37	37	38	39	40 8	40	41	42	43
	33	16	59	42	25		51	34	17	0
44	37	38 8	38	39	40	41 4	41	42	43	44
	24		52	36	20		48	32	16	0
44 [recte: 45]	38	39 0	39	40	41	42 0	42	43	44	45
	15		45	30	15		45	30	15	0
46	39 6	39	40	41	42	42	43	44	45	46
		52	38	24	10	56	42	28	14	0
47	39	40	41	42	43 5	43	44	45	46	47
	57	44	31	18		52	39	26	13	0
48	40	41	42	43	44 0	44	45	46	47	48

#	51	52	53	54	55	56	57	58	59	60
	48	36	24	12		48	36	24	12	0
49	41	42	43	44 6	44	45	46	47	48	49
	39	28	17		55	44	33	22	11	0
50	42	43	44	45 0	45	46	47	48	49	50
	30	20	10		50	40	30	20	10	0
51	43	44	45 3	45	46	47	48	49	50 9	51
	21	12		54	45	36	27	18		0
52	44	45 4	45	46	47	48	49	50	51 8	52
	12		56	48	40	32	24	16		0
53	45 3	45	46	47	48	49	50	51	52 7	53
		56	49	42	35	28	21	14		0
54	45	46	47	48	49	50	51	52	53 6	54
	54	48	42	36	30	24	18	12		0
55	46	47	48	49	50	51	52	53	54 5	55
	45	40	35	30	25	20	15	10		0
56	47	48	49	50	51	52	53	54 8	55 4	56
	36	32	28	24	20	16	12			0
57	48	49	50	51	52	53	54 9	55 6	56 3	57
	27	24	21	18	15	12				0
58	49	50	51	52	53	54 8	55 6	56 4	57 2	58
	18	16	14	12	10					0
59	50 9	51 8	52 7	53 6	54 5	55 4	56 3	57 2	58 1	59
										0
60	51 0	52 0	53 0	54 0	55 0	56 0	57 0	58 0	59 0	60
										0

On Table 24. For finding the proportional part.

(folio 176)

[Margin: The denominators of the parts of any circle. What is produced if the parts of a circle are multiplied by one another, or divided.]

So that the proportional part may be drawn out, it is well worth learning the denominators of the parts of any given circle, and of the other great circles — the denominators that this scheme sets out. From them we understand what is produced by the multiplication and by the division of one set of parts by another.

For the sum of the denominators of the parts that are multiplied gives the denominator of the number produced; whereas the remainder, when the smaller denominator is subtracted from the larger, indicates the denominator of the quotient, when the parts of the larger denominator are divided by the parts of the smaller. Thus by multiplying degrees into minutes, minutes are produced; from minutes into seconds come thirds; from thirds into fifths are begotten eighths; and so on. Conversely, by dividing minutes by degrees, minutes result; by dividing thirds by minutes, seconds come out; by dividing thirds by seconds, minutes are begotten; and so on.

[Margin: What parts of a circle the numbers in the table of proportional parts signify.]

Denominators of the parts of the circle [left bracket: *Partes circularum*, "parts of the circles"; right bracket: *Denominatores*, "denominators"]:

Part of the circle	Denominator
Degrees (Grad.)	0
Minutes (Min.)	1
Seconds (Secunda)	2
Thirds (Tertia)	3
Fourths (Quarta)	4
Fifths (Quinta)	5
Sixths (Sexta)	6
etc. (&c.)	etc.

II. From this it will be easy to know whether the numbers in the table signify degrees and minutes, or rather minutes and seconds, and so on. For since the numbers of that table are produced by multiplying the numbers taken at the top of the table into the numbers taken along the left side, then, if minutes are taken both at the top and at the side, the entries in the common corner will be seconds — that is, the first number gives minutes and the second gives seconds. For example, if Min.24 is multiplied by Min.46, there are produced Secunda 1104 — that is (dividing by Min.60) Min.18 and Sec.24. Likewise, if Quarta 24 is multiplied by Secunda 46, there are produced Sexta 1104 — that is (dividing by Min.60) Quinta 18 and Sexta 24. So too, if 3 degrees are multiplied by 8 minutes, there are produced 24 minutes — that is (dividing by Min.60) grad.0 Min.24. But if 9 degrees are multiplied by Min.26, there are begotten 234 minutes — that is

(dividing by Min.60) gra.3 Min.54; and so with the rest. In this second scheme one sees what is produced by multiplication.

[Margin: How the proportional part is to be drawn out.]

How the proportional part is drawn out [left bracket: *Multiplicandi*, "the multiplicands"; middle bracket: *Producti*, "the products"]. The scheme pairs two parts (of the denominations G = *gradus*/degree, M = *minuta*, S = *secunda*, T = *tertia*, Q = *quarta*) and gives the denomination of their product:

Multiplicands	Products
G per G	G · G
G per M	G · M
G per S	M · S
G per T	S · T
G per Q	T · Q
M per M	M · S
M per S	S · T
M per T	T · Q
M per Q	Q · [?]
S per S	T · Q
S per T	Q · [?]
S per Q	[?] · [?]
T per T	[?] · [?]
T per Q	[?] · [?]

III. Therefore, to come to the finding of the proportional part: suppose some table has been computed for individual degrees — such as, for instance, the table of the Sun's altitudes at the hours from noon and from midnight, advancing through each single degree of the pole's elevation — and suppose the altitude of some hour is sought for a pole-elevation that, besides whole degrees, also takes in some minutes. Then, for the two nearest pole-elevations (of which one is less than the given elevation and the other greater), one must take the difference between the two altitudes of the Sun at the given hour whose altitude is desired. Next, in the table of proportional parts, take the said difference at the top of the table, and on the left side the minutes that are contained (over and above the degrees) in the given elevation; or the other way about — the said difference on the side and the minutes at the top. For in the common corner the proportional part will be found,

in degrees and minutes, or in minutes and seconds, or in seconds and thirds, and so on, as was explained just above. This proportional part, subtracted from the altitude of the lesser pole-elevation, or added to it — according as that altitude is greater or less than the altitude of the greater pole-elevation — will yield the sought altitude of the Sun.

For example, suppose the Sun's altitude is desired at hour 1½[?] after noon for a pole-elevation of grad.42 Min.19. One must take the difference of grad.0 Min.47 between the altitude of the said hour at pole-elevation grad.42 (which contains grad.67 Min.41) and the altitude of the same hour at pole-elevation grad.43 (which comprises grad.66 Min.54). If this difference is taken at the top of the table of proportional parts, and the 19 minutes contained (over and above the 42 degrees) in the given pole-altitude are taken on the left side — or, conversely, these minutes at the top and that difference on the side — there will be found in the common corner the proportional part Min.14 Sec.53. For always, whenever minutes are taken both at the top and on the side of the table, the first number in the common corner gives minutes and the second gives seconds. But if degrees are taken at the top and minutes on the side, or the reverse, the first number in the common corner will give degrees and the second minutes, as is clear from what we wrote under Number 2. If, then, this proportional part Min.14 Sec.53 — that is, Min.15 (for one whole minute must be taken for the Sec.53, since they exceed half a minute; and in the same way, for more than 30 seconds one whole minute is always to be taken, for fewer than 30 nothing, and for exactly 30 it is left free to take one minute or to drop it) —

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if, I say, it is subtracted from the altitude grad.67 Min.41 at pole-elevation grad.42 (because this altitude of the Sun is greater than the altitude at pole-elevation grad.43), there will be left the sought altitude of the Sun, grad.67 Min.26, for hour 1½[?] at pole-elevation grad.42 Min.19.

Again, if someone wishes the Sun's altitude for hour 6½[?] after noon at the same pole-elevation grad.42 Min.19: the difference between the altitudes of that hour at pole-elevations grad.42 and 43 is min.19; and if this is taken at the top of the table, and the Min.19 contained in the proposed elevation on the side, there will be found in the common corner the proportional part Min.6 Sec.1. If, then, Min.6 (for the Sec.1 is neglected, since it is less than half a minute) is added to the altitude of hour 6 at pole-elevation grad.42 — because this is less than the altitude of the same hour at pole-elevation grad.43, namely to grad.15 Min.28 — the sought altitude will be made up as grad.15 Min.34.

[Margin: When the table is computed for fifteen-minute steps, or for half-degrees, what is to be done.]

But if at any point the difference should be greater than Min.60 — say gra.3 Min.50 — and the proportional part is sought for Min.30: we shall find, with grad.3 at the top and Min.30 on the side, the proportional part grad.1 Min.30; and just so much would the proportional part be if the difference were precisely grad.3. But because there are still Min.50 contained in it, we find, with those [minutes] at the top and the same Min.30 on the side, the proportional part Min.25 Sec.0; and this, added to the one found before, makes up the whole sought proportional part, gr.1 Min.55.

[Margin: The proportional part, when the table is not computed for whole degrees.]

And in this way, indeed, the proportional part is investigated in any table computed for whole degrees — of which kind are the preceding tables 9, 10, and 12. But if some table has been computed for fifteen-minute steps of degrees — of which kind are the earlier tables 4, 6, 20, and 21, and also the table of the arcs of the hour-circles in our *Compendium of dials* [*Compendium horologiorum*] — then one must take the difference of the two nearest arcs at the top of the table of proportional parts, and the minutes of the pole's elevation on the side; and the proportional part found in the common corner must be quadrupled. So too, if some table has been computed for half-degrees, one must take the difference between the two nearest numbers at the top of the table of proportional parts, and the minutes of the pole's elevation on the side; and the proportional part found in the common corner must then be doubled.

For example, in the table of the *Compendium of dials*, suppose the proportional part is to be sought for a pole-elevation of grad.40 Min.10, for hour 1 after noon. The difference between the arc at pole-altitude grad.40 and at grad.40 Min.15 is Min.14. If, then, these minutes are taken at the top of the table of proportional parts, and Min.10 on the side, there will be found the proportional part Min.2 Sec.20; which, quadrupled, makes Min.8 Sec.80 — that is, Min.9 Sec.20 — for the sought proportional part. This, subtracted from the arc grad.49 Min.1 opposite pole-altitude grad.40, will leave the arc gra.48 Min.52, the value sought at pole-elevation grad.40 Min.10. So too, if the proportional part is to be sought for hour 3 [and] 9[?] at pole-altitude gra.64 Min.25: the difference Min.14 will be found between the arcs of pole-altitude grad.64 Min.15 and grad.64 Min.30. If, then, this difference is taken at the top, and the Min.10 that are contained (over and above gr.64 Min.15) in the given altitude, there will be found in the common corner

Min.2 Sec.20; which, quadrupled, make Min.8 Sec.80 — that is, Min.9 Sec.20 — for the proportional part. This, removed from the arc grad.18 Min.50 at pole-altitude grad.64 Min.15, will leave the arc grad.18 Min.41 at pole-altitude gr.64 Min.25. And so with the rest.

[Margin: The finding of the proportional part by the rule of three, without a table.]

IIII. Moreover, even without the table of proportional parts, the proportional part can also be found by the rule of three, provided that — when the numbers of the difference are several — we first reduce them to the smallest subdivisions (those, that is, of the highest denomination), and then say: if the first number (that is, Min.60 when the table proceeds by whole degrees; or Min.15 or 30 when it is extended through fifteen-minute steps or half-degrees) gives such-and-such a difference, what will the minutes give for which the proportional part is sought? As in the next two examples.

Fifteen minutes (for the table in the *Compendium* advances by fifteen-minute steps of the pole's altitude) give a difference of Min.14; what then will Min.10 give? Multiplying Min.14 into min.10 makes Sec.140, which divided by min.15 yield min.9¼ — that is, min.9 Sec.20, as before. Likewise, in the second-to-last example: sixty minutes give a difference of grad.3 min.50; what then will min.30 give? The middle number makes Min.230, which multiplied by Min.30 make Sec.6900; and these, divided by Min.60, give min.115 — that is, grad.1 min.55, as above.

Again, if some table is constructed for whole degrees, and the difference is min.6 Sec.20, and the proportional part is sought for min.30, we shall say: sixty minutes give a difference of Sec.380 (for that many seconds are contained in the middle number); what then will min.30 give? Multiplying this third number into the second, and dividing by the first — that is, by min.60 — there come forth Sec.190, that is, min.3 Sec.10.

Finally, if the proposed table is extended by half-degrees, and the difference between the two [nearest] numbers is grad.1 Ter.4, and the proportional part is sought for min.9, we shall say: Min.30 give a difference of Ter.216004 (for that many thirds are contained in the middle number); what then will min.9 give? Multiplying this third number into the second, there come Quar.1944036; and these, divided by min.30, give a quotient in thirds which contain Sec.1080 (= min.18) and Ter.1, together with a fraction that contains Quar.12. Therefore the proportional part is min.18 Ter.1 Quar.12. These things will not be difficult, if one

recalls what was said above about the numbers produced by the mutual multiplication and division of the parts of circles. But the operation by the table of proportional parts is shorter and more expeditious.

[Margin: The proportional part for the table.]

In table 19, however — in which there are no degrees and minutes — the proportional part must be sought entirely by the rule of three; which you will learn from this single example. Suppose the tangent is to be investigated for hour 2½[?] from noon, at latitude grad.39 Min.15. The tangent of the said hour at latitude grad.39 is 3666 (at ½[?]); but at latitude grad.40 it is 3744, per the table. The difference between the two is 78. We shall say, then: if 60 minutes require a difference of 78, what will 15 minutes require? And you will find a difference of 19½; which, added to the tangent of the lesser latitude (because it is the smaller), will make the tangent 3685½ for the sought hour 2 at ½[?].

On the Tables of the Sun's Altitudes, of the Horizontal Circumferences, etc. [In Tabulas Altitudinum Solis, circumferentiarum horizontalium, &c.]

[Margin: What the sign ✱ signifies in the tables of the Sun's altitudes and of the horizontal circumferences.]

This sign ✱, placed in the tables of the Sun's altitudes and of the horizontal circumferences among the degrees and minutes, indicates that the hour in question — together with those that follow it toward the right — falls below the horizon in the northern [boreal] signs, and that their altitudes are accordingly referred to the horizon of the antipodes. In the southern [austral] signs, by contrast, it indicates that that hour still stands above the horizon, while all the rest fall below the horizon toward the left, and that their altitudes therefore lie above the horizon of the antipodes.

[Margin: Which hour, reckoned from noon or midnight, falls below the horizon.]

II. That hour reckoned from noon or from midnight falls below the horizon whose distance from the meridian — placed above the horizon, reduced to degrees by means of the little table for converting hours into parts of the Equator — is greater than the semidiurnal arc. Thus at latitude 42°, with the Sun at the beginning of ♋ [Cancer], the hour 7h33m from noon (or 4h27m from midnight) is distant from the meridian placed above the horizon by 7h33m — that is, 113°15' — a distance which is greater than the semidiurnal arc of ♋, comprising 113°3'.

Therefore that hour lies below the horizon. Likewise the hour 7h32m from noon (or 4h28m from midnight) is distant from the meridian placed above the horizon by $113^{\circ}0'$, a distance less than the semidiurnal arc of $\mathfrak{S}$, which contains $113^{\circ}3'$. Therefore that hour lies above the horizon.

[Margin: Which hour, reckoned from sunrise or sunset, lies below the horizon.]

Likewise, that hour reckoned from sunrise whose distance after sunrise — or that hour reckoned from sunset whose distance before sunset — is greater than the doubled semidiurnal arc falls below the horizon. Thus at the same latitude 42° , with the Sun at the beginning of $\mathfrak{S}$, the hour 16 from sunrise is distant from sunrise, and the hour 8 from sunset is distant before sunset, by 240° ; and this distance is greater than the doubled semidiurnal arc, that is, than $226^{\circ}6'$.

Therefore both hours are found below the horizon — the former in the west after sunset, the latter in the east before sunrise. Again, the hour 10 from sunrise, with the Sun at the beginning of $\mathfrak{X}$ [Pisces] and $\mathfrak{A}$ [Aquarius], is distant 150° after sunrise, and by that same number of degrees the hour 14 from sunset is distant before sunset; and this distance exceeds the doubled semidiurnal arc of $\mathfrak{X}$ and $\mathfrak{A}$, namely $141^{\circ}18'$. We shall conclude, therefore, that both hours lie below the horizon.

[Margin: What the sign $\dagger$ signifies in the tables of the Sun's altitudes and of the horizontal circumferences.]

III. Next, this sign $\dagger$, which under the hours from sunrise and sunset first occurs going from left toward the right, indicates — in the northern signs, and indeed in the hours from sunset — the nearest hour after noon; while in the hours from sunrise it indicates the hour that most nearly precedes noon.

The same sign $\dagger$, which under the hours from sunrise and sunset first occurs going from right toward the left, indicates — in the southern signs, and indeed in the hours from sunset — the nearest hour after noon; while in the hours from sunrise it indicates the hour that most nearly precedes noon.

[Margin: Which hour, reckoned from sunrise or sunset, occurs after noon.]

4. Furthermore, that hour from sunrise whose distance after sunrise is greater than the semidiurnal arc, and that hour from sunset whose distance before sunset is less than the semidiurnal arc, is a postmeridian hour, as is plain.

It is also manifest and clear that all antemeridian hours are eastern, and all postmeridian hours western.

[Margin: Which hours are eastern and which western.]

5. Finally, in the table of the horizontal circumferences, the circumferences in the northern signs that are enclosed between the letters B and B — inclusive of the degrees and minutes set between them — are Northern; but those between the letters A and A are Southern. The remaining ones from B onward, after the second letter A up to the end, are again Northern. In the southern signs, however, all the circumferences of the hours standing above the horizon are southern. But so that we may judge, concerning all the hours in the southern signs, which are southern and which northern, this rule of the Circumference must be held: the circumferences that in the northern signs are northern are, in the southern signs, southern, and conversely. And so, proceeding from left toward the right for the southern signs, the letter B must be changed into A, and A into B.

[Margin: Which circumferences are Northern and which Southern.]

6. Now, I have proceeded in three of the tables — namely in tables 9, 10, and 12 — by whole degrees only of the pole's elevation. This is partly because, through the proportional part between two neighbouring degrees of the pole's elevation, no error greater than one minute or another at most is committed; and partly because, even if the tables were extended by fifteen-minute steps, one would nonetheless very often have to apply the proportional part, namely whenever the minutes attached to the whole degrees are not exactly 15, or 30, or 45. For if the proportional part were then neglected, an error of not a few minutes would be admitted.

[Margin: Why the preceding tables were computed by whole degrees only. How the proportional part in the table of altitudes is extracted, when one hour is above the Horizon and the other below.]

7. Moreover, when the Sun's altitude for a given hour is sought by the proportional part at a pole-elevation lying between two whole-degree elevations — and in one of those elevations the hour is above the horizon, while in the other it is below — one must add a quadrant [90°] to the altitude of the hour below the horizon, and take the complement of the altitude of the other hour above the horizon, so as to have the arcs of the Verticals between the zenith of the place and the Sun in each elevation of the pole. Then the lesser arc is to be subtracted from the greater, so that a difference is obtained, with which the proportional part is to be extracted from the table of proportional parts; and this part is to be subtracted from, or added to, the Sun's distance from the zenith in the lesser altitude, according as that distance is greater or less than the Sun's distance from the

zenith in the greater latitude. And if the residual arc — whether the remainder or the sum — is less than a quadrant, its complement will be the sought altitude of the Sun above the horizon; but if it is greater, then, once the quadrant has been subtracted, the sought altitude below the horizon will remain.

For example: at latitude 41° , the hour 9 from sunset (and 15 from sunrise), with the Sun at the beginning of ♄ , has an altitude of $0^\circ 23'$ below the horizon, and so the Sun is then distant from the zenith $90^\circ 23'$. But at latitude 42° the same hour has an altitude of $0^\circ 41'$ above the horizon, and accordingly the Sun is then $89^\circ 19'$ from the zenith. The difference between these two distances from the zenith amounts to $1^\circ 4'$; with which, for the latitude — say — of $41^\circ 15'$, the proportional part is found to be $0^\circ 16'$. This, taken from $90^\circ 23'$ (namely from the Sun's distance from the zenith in the lesser latitude, since this is the greater), leaves a distance from the zenith of $90^\circ 7'$; that is, an altitude of $0^\circ 7'$ below the horizon. With the same difference of $1^\circ 4'$, for latitude $41^\circ 30'$ the proportional part is found to be $0^\circ 32'$, which subtracted from the same $90^\circ 23'$ leaves a distance from the zenith of $89^\circ 51'$; that is, an altitude of $0^\circ 9'$ above the horizon. Likewise, with the same difference of $1^\circ 4'$, for latitude $41^\circ 45'$ the proportional part is found to be $0^\circ 48'$, which taken from the same $90^\circ 23'$ leaves a distance from the zenith of $89^\circ 35'$; that is, an altitude of $0^\circ 25'$ above the horizon.

Again: at latitude 41° , the hour 15 from sunset (and 9 from sunrise), with the Sun at the beginning of ♄ [Capricorn], has an altitude of $0^\circ 23'$ above the horizon, and so the Sun is then $89^\circ 37'$ from the zenith. But at latitude 42° the same hour has an altitude of $0^\circ 41'$ below the horizon, and accordingly is $90^\circ 41'$ from the zenith. The difference between these two distances from the zenith again comprises $1^\circ 4'$; with which, for latitude $41^\circ 15'$, the proportional part is found to be $0^\circ 16'$. This, added to the distance from the zenith in the lesser latitude — namely to $89^\circ 37'$, since this is the smaller — yields a distance from the zenith of $89^\circ 53'$; that is, an altitude of $0^\circ 7'$ above the horizon. With the same difference of $1^\circ 4'$, for latitude $41^\circ 30'$ the proportional part obtained is $0^\circ 32'$, which added to the same $89^\circ 37'$ makes $90^\circ 9'$; that is, an altitude of $0^\circ 9'$ below the horizon. Finally, with the same difference of $1^\circ 4'$, for latitude $41^\circ 45'$ the proportional part offered is $0^\circ 48'$, which added to the same $89^\circ 37'$ makes a distance from the zenith of $90^\circ 25'$; that is, an altitude of $0^\circ 25'$ below the horizon. And so for the rest.

8. The same must be observed in the horizontal circumferences, when one is northern and the other southern. For then the distances of both must be taken on the horizon from the same point of the meridian, etc. Suppose, for instance, that the horizontal circumference of the hour 10 from sunset (and 14 from sunrise) is

sought at latitude $58^{\circ}15'$, with the Sun in the first degree of ♄ . Since that circumference at latitude 58° is northern by $1^{\circ}24'$, while at latitude 59° it is southern by $1^{\circ}2'$, the former will be distant from the meridian toward the north by $88^{\circ}36'$, and the latter by $91^{\circ}2'$. Subtracting the former from the latter, the remaining difference will be $2^{\circ}26'$; with which the proportional part is found to be $0^{\circ}36'30''$. This, added to the distance in the lesser latitude — namely to $88^{\circ}36'$, since it is the smaller — makes a distance of $89^{\circ}12'$ (for the 30 seconds are neglected in the proportional part); the complement, then — that is, $0^{\circ}48'$ — will be the sought circumference, and it is Northern. Likewise, at latitude $58^{\circ}30'$, with the same difference of $2^{\circ}26'$, the proportional part is $1^{\circ}13'$, which, added to that same distance in the lesser latitude, makes a distance of $89^{\circ}49'$; and accordingly its complement, $0^{\circ}11'$, will be the horizontal circumference, still Northern. Finally, at latitude $58^{\circ}45'$, the proportional part found is $1^{\circ}49'$, which added to that same distance in the lesser latitude makes $90^{\circ}25'$. The horizontal circumference is therefore Southern by $0^{\circ}25'$.

But if the aforesaid distances are taken on the horizon from the meridian toward the south, then in the lesser latitude the distance will be $91^{\circ}24'$, and in the greater latitude $88^{\circ}58'$ — that is, the complement of the horizontal and southern circumference, $1^{\circ}2'$. The difference of these distances is again $2^{\circ}26'$. If, then, you subtract the proportional parts found from the distance of the lesser latitude (since it is the greater), there will remain $90^{\circ}48'$ for the distance in the lesser latitude, at latitude $58^{\circ}15'$; and accordingly the horizontal circumference will be $0^{\circ}48'$ Northern. But for latitude $58^{\circ}30'$ there will remain a distance of $90^{\circ}11'$, and so the horizontal circumference will be $0^{\circ}11'$, still Northern. Finally, for latitude $58^{\circ}45'$, the remaining distance becomes $89^{\circ}35'$; and therefore its complement, $0^{\circ}25'$, will be the horizontal circumference, and Southern.

[Margin: What to do when one horizontal circumference is northern and the other southern.]

9. By like reasoning, when in one latitude the horizontal circumference of some hour is antemeridian, and postmeridian in the other, the distances must be taken on the horizon from the Vertical — whether eastern or western — etc. For example, at latitude 58° the horizontal circumference of the hour 9 from sunset, in the first degree of ♄ , is antemeridian by $88^{\circ}33'$; but at latitude 59° it is postmeridian by $87^{\circ}51'$ — that is, it is distant from the eastern Vertical $92^{\circ}9'$. The former distance subtracted from the latter, the remaining difference is $3^{\circ}36'$; to which, for latitude $58^{\circ}15'$, corresponds a proportional part of $0^{\circ}54'$. This, added to the distance in the lesser latitude — namely to $88^{\circ}33'$, since it is the smaller —

makes a distance of $89^{\circ}27'$ for the horizontal and antemeridian circumference. But for latitude $58^{\circ}30'$, with the aforesaid difference, the proportional part found is $1^{\circ}48'$, which, added to that same distance of the lesser latitude — that is, to $88^{\circ}33'$ — makes $90^{\circ}21'$. Therefore, once 90° has been taken away, the complement of the remaining 21 minutes — namely $89^{\circ}39'$ — will be the postmeridian horizontal circumference.

[Margin: When one circumference is antemeridian and the other postmeridian, what to do. (Source prints "agnndum" for "agendum.")]

Likewise, at latitude $58^{\circ}8'$, with the same difference of $3^{\circ}36'$, the proportional part is found to be about $0^{\circ}49'$, which added to $88^{\circ}33'$ makes $89^{\circ}22'$ for the antemeridian horizontal circumference.

Again: at latitude 58° the horizontal circumference of the hour 9 from sunrise, at the beginning of $\mathfrak{D}$, is $88^{\circ}33'$ postmeridian; but at latitude 59° it is antemeridian by $87^{\circ}51'$. That hour, then, is distant in the lesser latitude from the western Vertical by $88^{\circ}33'$, and in the greater latitude by $92^{\circ}9'$. Hence the difference will be $3^{\circ}36'$; with which, for latitude $58^{\circ}8'$, the proportional part is found to be $0^{\circ}49'$, which added to the distance in the lesser latitude makes the horizontal circumference $89^{\circ}22'$ postmeridian. And so for the rest.

[Margin: Which hours, when the Sun is on the Equator, have the same altitudes above and below the Horizon, and the same horizontal circumferences.]

10. Finally, in the table of the Sun's altitudes for the hours — both those reckoned from noon and midnight and those reckoned from sunrise and sunset — and likewise in the table of the horizontal circumferences, with the Sun on the Equator, it must be noted that the hours which lie below the horizon at the same distance from it as the hours above the horizon have equal altitudes and equal horizontal circumferences. Such are the hours 5 and 7 from noon or midnight; likewise 4 and 8, etc.; likewise the hours 23 and 1 from sunrise or sunset; likewise 22 and 2; and so on. This is easily gathered from the calculation set out in problems 15 and 16 of the New Description of Sundials, and it can also be demonstrated in this way. Let two arcs of the Equator be taken beginning from the horizon, one above the horizon and the other below it; and let two verticals be understood to run through their end-points, so that two right triangles are formed. In these triangles, since the two sides opposite the right angles are set equal on the Equator, and the remaining two angles which the Equator makes with the horizon are equal at the vertex (by proposition 6 of our [treatise on] spherical triangles) — these being indeed the angles of the complement of the

pole's altitude — then, by proposition 21 of those same triangles, the remaining sides of the one triangle will be equal to the remaining sides of the other: namely the altitudes both above and below the horizon, and the horizontal circumferences. Which is what was proposed.

Furthermore, in the table of semidiurnal arcs — which is the fourth in the order of the tables — we have proceeded only up to a pole-altitude of $66\frac{1}{2}^{\circ}$ [$66^{\circ}30'$], for the reason that at a greater elevation of the pole the day at the beginning of $\mathfrak{S}$ is longer than 24 hours (the Sun of course not setting), while at the beginning of $\mathfrak{V}$ the Sun does not rise, but always lies hidden below the horizon.

[Margin: Why tables 20 and 21 begin from pole-altitude $23^{\circ}30'$.]

The tables 20 (of the Sun's altitudes on the prime Vertical) and 21 (of the Sun's distances from the meridian on the same Vertical) we have likewise begun from a pole-altitude of $23^{\circ}30'$, because at a lesser pole-altitude the prime Vertical does not cut all the northern parallels, but only those whose declination is greater than the pole's altitude. Therefore, in these parallels, the altitude of the Sun standing on the Vertical — and its distance from the meridian — will have to be computed, when the matter requires, by the teaching of problem 7 of the New Description of Sundials.

[Margin: The distances of the Sun from the meridian on the [prime] Vertical are equal to the semidiurnal arcs of the opposite signs in the complement of the pole's elevation.]

Moreover, the distances of the Sun from the meridian, when it is on the Vertical, are equal to the semidiurnal arcs (a thing worth observing) of the opposite southern signs at the pole-elevation which is the complement of the proposed elevation. For example, at pole-elevation 57° , the Sun's distance from the meridian, when it is on the Vertical and at the beginning of $\mathfrak{S}$, is $73^{\circ}36'$; and just so great also is the semidiurnal arc of $\mathfrak{V}$ at pole-elevation 33° ; and so for the rest. The reason is that the prime Vertical, at the pole-elevation which is the complement of the given elevation, is the horizon above which the antarctic pole is elevated; and accordingly the tropic of $\mathfrak{S}$, which the Vertical cuts, is like the tropic of $\mathfrak{V}$, etc.

THERE FOLLOW ASTRONOMICAL PROBLEMS 17 [Sequuntur Problemata XVII]

To be solved by means of Sines, Tangents, and Secants: from which both the preceding tables have been constructed, and others, new ones, can be built; and

finally other matters pertaining to the construction of sundials [horologia] investigated.

PROBLEM I.

To find the declination of a given point of the Ecliptic, the maximum declination being taken as $23^{\circ} 30'$.

Let ABCD be any Horizon; let AEC be the Meridian, and BED the Equator. Conceive the Ecliptic to be BFD, such that EBF is the angle of maximum declination. Let any point G of the Ecliptic be proposed, and through it, from H, the pole of the world, let the circle of declination HGI be drawn. By proposition 15 of Book I of Theodosius [Theodosius of Bithynia, *Sphaerica*], this circle stands at right angles to the Equator, and therefore its arc GI measures the declination of the point G, which we are bidden to find. For in the spherical triangle BGI the angle I is right, and the arc BG — which measures the distance of the point G from the equinoctial point B — is known, together with the angle GBI of the maximum declination. Solve by the first method of problem 8 of the [treatise on] spherical triangles [*Triangula Sphaerica*]. (We shall always apply our problems on spherical triangles as demonstrated at the end of Lemma 53, Book I, of the Astrolabe [Clavius, *Astrolabium*].)

Let it be: as the whole sine [radius] to the sine of the base BG, which measures the distance of the ecliptic point G from the equinoctial point B, so the sine of the angle GBI of the maximum declination, to a fourth term; there will be produced the sine of the arc GI, the declination sought.

It is enough to find the declination of the points of the Ecliptic in one quadrant, since these are equal to the declinations of the points in the three remaining quadrants, as we demonstrated in Lemma 49, No. 5, of Book I of the Astrolabe.

Diagram. An uncaptioned circular diagram (a plane projection of the sphere) enclosed in a square frame, set at the upper left of the page. It shows the Horizon ABCD as the bounding circle, the Meridian AEC and Equator BED as great circles through the centre E, the Ecliptic BFD, the circle of declination HGI through the pole of the world H, and — for the later problems that reuse the same figure — a declining/inclined Vertical PFQ with its proper Meridian, together with a Sun-parallel KL and the associated hour-circles. Many labels are very small, and several occur twice (for the northern and southern hemispheres of the projection); the readings below are given with their approximate positions, and

the uncertain ones are flagged. This dense figure is a (more than two labels flagged).

- A (top of the frame, apex of the Meridian): pole/apex point
- [? — small mark just left of X, upper left]: unresolved glyph
- X (upper left, on an inner arc): point on an oblique circle
- V (left, upper): point
- L (left side): point
- B (left, at the Horizon–Equator intersection): western equinoctial point
- I (left of centre): foot of the declination circle on the Equator
- M (upper centre, on the Meridian between A and E): point
- R (centre): point
- E (centre): common intersection / centre of the projection
- N (right of centre): point on the Equator
- K (upper right): point
- D (right, at the Horizon–Equator intersection): eastern equinoctial point
- L (right, below D): point [second L]
- K (lower left): point [second K]
- G (lower left of centre): the given point of the Ecliptic
- F (lower centre): point on the Ecliptic / prime Vertical
- M (lower centre, below R/E): point [second M]
- O (lower centre): point
- T (lower centre): point
- H (bottom centre): pole of the world
- C (bottom, below H, marked "/C"): lower apex of the Meridian
- S (lower right): point
- P (lower right): point of the inclined Vertical

PROBLEM II.

To compute the semidiurnal arc of any parallel of the Equator, for any given altitude of the pole.

In the figure of the preceding problem, let KL be a parallel of the Equator, whether northern or southern, cutting the Meridian at M, so that the semidiurnal arc is KM, which we are bidden to find. From H, the pole of the world, through the point K (where the parallel cuts the Horizon), let a great circle HK descend,

cutting the Equator at N. By proposition 10 of Book 2 of Theodosius, the arc EN of the Equator will be similar to the semidiurnal arc KM. And since in the northern spherical triangle BKN the angle N is right, and the arc KN of declination is known, together with the angle KBN of the Equator's altitude above the Horizon — which is equal to the angle ABE that the Equator makes with the Horizon toward the southern parts — solve by the first method of problem 10 of the [treatise on] spherical triangles.

Let it be: as the whole sine to the Tangent of the complement of the angle B — that is, to the Tangent of the pole's altitude — so the Tangent of the arc KN, the declination of the parallel, to a fourth term; there will come out the sine of the arc BN, by which the northern semidiurnal arc exceeds the quadrant, or by which the southern semidiurnal arc falls short of the quadrant.

The same reasoning holds for the southern parallel, as is clear in the triangle DKN. Therefore, if this arc found is added to the quadrant, the northern semidiurnal arc is formed; but if it is subtracted from the quadrant, the remainder becomes the southern semidiurnal arc. Yet it is also enough to investigate the differences between the quadrant and the semidiurnal arcs of one quadrant of the Ecliptic, for these differences are equal in the other quadrants, as was shown in Lemma 49, No. 15, of Book I of the Astrolabe. From this problem the fourth table, of semidiurnal arcs, was constructed.

PROBLEM III.

To find the rising or setting amplitude of any point of the Ecliptic, for any latitude of place.

In the same figure of problem 1, the arc BK of the Horizon measures the rising or setting amplitude of the parallel KL, which we shall elicit as follows. Since in the spherical triangle BKN the angle N is right, and the arc KN — the declination of the parallel passing through the given point of the Ecliptic — is known, together with the angle KBN of the Equator's altitude, solve by the first method of problem 14 of the [treatise on] spherical triangles.

Let it be: as the whole sine to the secant of the complement of the angle KBN — that is, to the secant of the pole's altitude — so the sine of the arc KN of declination, to a fourth term; there will be produced the sine of the arc BK, the northern rising or setting amplitude.

The same reasoning holds for the arc DK of the southern amplitude. But it is enough to compute the amplitudes of the points of one quadrant of the Ecliptic, for these agree also with the points of the other quadrants, as we demonstrated in Book I of the Astrolabe, Lemma 49, No. 5. And from this problem we constructed the fifth table, of rising and setting amplitudes.

PROBLEM IV.

Given any Vertical circle and its declination, to determine the altitude of the pole above it, in any given latitude of place.

In the figure above, let the prime Vertical in the given region be BFD, and the Vertical inclined to it PFQ, whose proper Meridian HO stands at right angles to it, so that HO is the arc of the altitude of the pole H above that same Vertical — the arc we must find. For in the spherical triangle FHO the angle O is right, and the arc FH — the complement of the pole's altitude above the Horizon — is known, together with the angle HFO of the inclination of the Vertical PQ to the Meridian AC (that is, the complement of the declination of that same Vertical from the prime Vertical BFD). Solve by the first method of problem 8 of the [treatise on] spherical triangles.

Let it be: as the whole sine to the sine of the base FH — that is, to the sine of the complement of the pole's altitude above the Horizon — so the sine of the angle HFO, of the complement of the declination of the declining Vertical, to a fourth term; there will be produced the sine of the arc HO, the altitude of the pole above the proposed Vertical PQ.

PROBLEM V.

Given any Vertical and its declination, to find the inclination of its own Meridian to the Meridian of the Horizon, in any latitude of place.

In the same preceding figure, we must find the angle FHO which HO — the proper Meridian of the given Vertical PQ — makes with FH, the Meridian of the Horizon, and which measures the inclination of the one Meridian to the other. Since in the same right-angled spherical triangle FHO the same things are known as before, solve by the first method of problem 3 of the [treatise on] spherical triangles.

Let it be: as the whole sine to the sine of the complement of the base FH — that is, to the sine of the pole's altitude above the Horizon — so the Tangent of the angle HFO, of the complement of the declination, to a fourth term; there will be generated the Tangent of the complement of the sought angle FHO, which measures the inclination of the Meridians.

PROBLEM VI.

Given any Vertical and its declination, to compute — for any latitude of place — how great its arc is between the Horizon and the Equator; and between its own Meridian and the Meridian of the Horizon; likewise between the Meridian of the Horizon and the Equator; and also between its own Meridian and the Horizon.

In the same figure of problem 1, the four arcs of the Vertical PQ which we have named are these: QR, between the Horizon and the Equator; FO, between its proper Meridian and the Meridian of the Horizon; FR, between the Meridian of the Horizon and the Equator; and OP, between its proper Meridian and the Horizon. Now in the spherical triangle NOR the angles N and O are right — for HO is the proper Meridian of the Vertical PQ and therefore at right angles to it, while H is the pole of the Equator and hence, by proposition 15 of Book I of Theodosius, also at right angles to it. Therefore, by proposition 25 of our [treatise on] spherical triangles, RN and RO are quadrants, and consequently the quadrant RO is equal to each of the quadrants FQ and FP. Subtracting, then, the common arc FR, the remaining arcs QR and FO will be equal. Likewise, subtracting the common arc FO, the remaining arcs FR and OP will also be equal. Therefore, once one of them is known, the other three will also be known. For example, once the arc QR between the Horizon and the Equator is known, its equal FO — between the proper Meridian and the Meridian of the Horizon — will also be known; likewise its complement FR, between the Meridian of the Horizon and the Equator; and therefore also OP, equal to FR, between the proper Meridian and the Horizon. We shall investigate the arc QR between the Horizon and the Equator (which is enough) in this way. Since in the spherical triangle BQR the angle Q is right, and the arc BQ — which measures the declination of the Vertical — is known, together with the angle QBR of the Equator's altitude, that is, of the complement of the pole's altitude above the Horizon, solve by the first method of problem 11 of the [treatise on] spherical triangles.

Let it be: as the whole sine to the sine of the arc BQ, of the declination of the Vertical; so the Tangent of the angle QBR, of the complement of the pole's altitude above the Horizon, to a fourth term; there will be

produced the Tangent of the arc QR, between the Horizon and the Equator, which is sought.

From this the other three will be known, as was said a little earlier. For FO is equal to it, and FR is its complement, to which OP is equal, as we have shown.

PROBLEM VII.

To explore the altitude of the Sun when it is on the prime Vertical of any great circle taken as a Horizon; and the hour, when the Sun is on that Vertical, in any region.

In the same figure, let the Sun's parallel KL cut the prime Vertical BFD of the great circle ABCD — taken as a kind of Horizon — at G. We must find the arc GB of the Sun's altitude at G, the common section of the parallel with the Vertical, and what the hour then is. Draw from the pole H, through G, the great hour-circle cutting the Equator at I. Then in the spherical triangle BGI the angle I is right (by proposition 15 of Book I of Theodosius), and the arc GI, of the parallel's declination, is known, together with the angle GBI of the pole's altitude above the great circle ABCD — the angle, that is, which the Vertical makes with the Equator. Solve by the first method of problem 14 of the [treatise on] spherical triangles.

Let it be: as the whole sine to the secant of the complement of the angle GBI — that is, to the secant of the complement of the pole's altitude above the given great circle — so the sine of the arc GI, of the parallel's declination, to a fourth term; there will be found the sine of the base GB, the Sun's altitude on the Vertical.

Again, solve by the first method of problem 10 of the [treatise on] spherical triangles:

Let it be: as the whole sine to the Tangent of the complement of the angle GBI — that is, to the Tangent of the complement of the pole's altitude above the given great circle — so the Tangent of the arc GI, of the parallel's declination, to a fourth term; there will be found the sine of the arc BI, by which the quadrant BE exceeds the arc EI, the distance of the Sun from its proper Meridian.

If to this distance the inclination of the proper Meridian to the Meridian of the Horizon is added, or subtracted from it, as the matter requires, the distance of the Sun from the Meridian of the Horizon will be obtained. By this reasoning we computed tables 20 and 21.

PROBLEM VIII.

Given any Vertical and its declination, to know — in any region — how great its own arc is between the Meridian of the Horizon and the 6-hour circle reckoned from noon or midnight; likewise between the Horizon and the same 6-hour circle; and finally between the Horizon and the 12-hour circle reckoned from sunrise or sunset.

In the figure above, let the 6-hour circle from noon or midnight, BHD, be drawn, cutting the Vertical PQ at S. Both arcs FS and SP are proposed for investigation, which we shall track down thus. Since in the spherical triangle FHS the angle H is right, and the arc FH — the complement of the pole's altitude — is known, together with the angle HFS, the complement of the declination of the Vertical PQ, solve by the first method of problem 13 of the [treatise on] spherical triangles.

Let it be: as the whole sine to the sine of the complement of the angle HFS — that is, to the sine of the declination of the Vertical; so the Tangent of the complement of the arc FH — that is, the Tangent of the pole's altitude — to a fourth term; there will be produced the Tangent of the complement of the base FS — that is, the Tangent of the arc SP, which was sought in the second place.

From this the arc FS, proposed in the first place, will also become known, since it is the complement of the arc SP just found.

Next, in the spherical triangle DOP — the 12-hour circle from sunrise or sunset, BTD, having been drawn, cutting the Vertical PQ at O — the angle P is right, and the arc DP, of the Vertical's declination, is known, together with the angle ODP; for the arc CT, double the pole's altitude, measures that angle. This is because the circle BTD touches, at T, the greatest of the ever-apparent parallels [the circle of perpetual apparition], and therefore the arc HT is equal to the arc HC of the pole's altitude, and consequently the whole arc CT is double that same arc HC. Solve by the first method of problem 11 of the [treatise on] spherical triangles.

Let it be: as the whole sine to the sine of the arc DP, of the declination of the Vertical PQ; so the Tangent of the angle ODP — that is, the Tangent of the arc double the pole's altitude above the Horizon — to a fourth term; there will come out the Tangent of the arc OP, which is sought.

This arc, in a latitude of place less than 45° , is always less than a quadrant, because the angle ODP is less than 90° . But in a latitude of place of 45° it is a quadrant, because that same angle is then equal to 90° . And finally, in a latitude

of place greater than 45° , it is greater than a quadrant, because the said angle then exceeds 90° . All these things are gathered from the aforesaid problem 11 of the [treatise on] spherical triangles.

PROBLEM IX.

Given any plane, together with the altitude of the pole above it, to inquire how great its arc is that lies between any hour-circle (reckoned from noon or midnight) and the common intersection of that circle with the Equator.

In the same figure above, let there first be found, on the Horizon, the arc BK, between the hour-circle HK and the intersection B of the Equator with the Horizon. Since in the spherical triangle CHK the angle C is right, and the arc CH — the pole's altitude above the Horizon — is known, together with the angle CHK, of the distance of the Sun (or of the hour-circle) from the Meridian — (which distance is to be taken from the Meridian below the Horizon when the hour-circle cuts the Horizon in the northern quadrant, but from the Meridian above the Horizon when the same circle cuts the southern quadrant of the Horizon, as at the point V; but then the angle of the said distance, MHV, has the same Tangent, the same sine, and the same secant as the angle CHV in the triangle CHV) — solve by the first method of problem 11 of the [treatise on] spherical triangles.

Let it be: as the whole sine to the sine of the arc CH, of the pole's altitude above the Horizon; so the Tangent of the angle CHK, of the distance from the Meridian of the Horizon, to a fourth term; there will be produced the Tangent of the arc CK; and when this is subtracted from the quadrant CB, the remainder becomes the arc BK, which is sought.

By like reasoning, in the triangle CHV, if

Let it be: as the whole sine to the sine of the arc CH, of the pole's altitude above the Horizon;

PROBLEM IX. (continued)

[The proportion begun at the foot of the preceding page continues:]

Let it be: as the whole sine to the sine of the arc CH, of the pole's altitude above the Horizon; so the Tangent of the angle CHV, of its distance from the Meridian of the Horizon, to a fourth term; there will be produced the Tangent of the arc CV, greater than a quadrant — since the angle CHV is

obtuse, which always happens when the hour-circle cuts the southern quadrant of the Horizon.

Therefore, once the quadrant CR is subtracted, the remainder becomes the sought arc BV. In just the same way the arc BX will become known, in the triangle CHX.

Next, in the declining Vertical PQ, let the arc RS be sought, lying between the Equator and the hour-circle HS. Accordingly, in the triangle HOS:

Let it be: as the whole sine to the sine of the arc HO, of the pole's altitude above the Vertical PQ; so the Tangent of the angle OHS, of the distance of the hour-circle HS from its own Meridian HO (belonging to the same Vertical), to a fourth term; there comes out the Tangent of the arc QS. When this is added to the quadrant OR, the arc RS will be known, greater than a quadrant; but when that same [Tangent] is subtracted from the quadrant between O and the other intersection of the Equator with the Vertical PQ, the arc between the circle HS and this other intersection will be left known.

Lastly, in the prime Vertical let the arc BG be found, lying between the hour-circle HG and the Equator. Accordingly, in the triangle FGH:

Let it be: as the whole sine to the sine of the arc FH, of the pole's altitude above the Vertical (which is always equal to the complement of the pole's altitude above the Horizon); so the Tangent of the angle FHG, of the distance of the hour-circle from the Meridian of the Vertical, to a fourth term; there results the Tangent of the arc FG. When this is subtracted from the quadrant FB, the remaining arc BG — which we seek — will be known.

And so the arc of the Horizon for any given hour, at any elevation of the pole, is the arc of the prime Vertical in that region where the pole's altitude is the complement of the earlier altitude, and conversely. For in this other region the pole's altitude above the prime Vertical equals the pole's altitude in the former region — that is, the complement of the pole's altitude in this other region. It is therefore enough to find the arcs of the Horizon for the individual elevations of the pole; for these same arcs will also be the Vertical arcs at the complementary elevations. By this reasoning we have constructed the sixth table of arcs, which in the Gnomonics we have called "hour-arcs" [horarios].

PROBLEMA X.

If, through the pole of any great circle, a Vertical is drawn at right angles to any great circle passing through the poles of the world, to investigate how great is the arc of that circle passing through the poles of the world which lies between that perpendicular Vertical and the nearer pole of the world.

Diagram. An orthographic projection of the celestial sphere set within a square frame, the master figure serving Problems X and XI (the same figure referred to in the preceding problems). The circle carries the horizon ABCD, the meridian, the prime vertical, hour-circles, and the ecliptic, crossing one another in a dense network of great-circle arcs. Readable labels, by approximate position:

- A (top of circle, 12 o'clock)
- e, X (upper-left interior, adjacent)
- V (upper left, ~10 o'clock)
- M (upper middle, below A)
- K (right edge, upper)
- L (left edge, ~9 o'clock)
- I, R, E, N (interior, along the middle band, left to right; E at the centre)
- B (left edge, ~8 o'clock)
- D (right edge, ~3–4 o'clock)
- G (lower left-of-centre)
- F (lower centre, above H)
- O (lower centre-right)
- H (bottom centre, above C)
- S, P (bottom, right of H)
- C (foot of circle, 6 o'clock)

Several small interior labels near the centre and lower-left (apparent repeats of K, L, M, N) could not be resolved with certainty under magnification and are not committed here (more than two labels unresolved).

In the preceding figure, let a Vertical FO be drawn through F, the pole of the greatest circle ABCD — whose Meridian is AFC — to the hour-circle HOK (which passes through the pole of the world H), making right angles with HOK. The arc HO is to be investigated. Now in the spherical triangle FHO the angle O is right; the arc FH, of the complement of the pole's altitude above the given great circle ABCD, is known (since HC is the arc of the pole's altitude), together with the angle FHO, of the distance of the hour-circle HOK from the Meridian AFC of the given

circle. (For this distance must be known, along with the altitude of the pole above the given great circle.) Solve by the first method of problem 9 of the [treatise on] spherical triangles.

Let it be: as the whole sine to the sine of the complement of the angle FHO, of the distance from its own Meridian; so the Tangent of the base FH, of the complement of the pole's altitude above the given circle, to a fourth term; there will be procreated the Tangent of the arc HO, which is sought.

And it was in this way that we computed the table which is contained in our compendium of sundials [*compendio nostro horologiorum*].

PROBLEMA XI.

Given a diurnal arc of any number of hours, to determine its declination for any latitude of the place.

In the same figure, let the northern diurnal arc KL be given, of any number of hours. From the pole of the world H, through K, let a great circle HKN be dropped, so that KN is the arc of declination, which we must investigate. Now in the spherical triangle BKN the angle N is right, and the arc BN — the difference between the given semidiurnal arc EN (for this arc EN is similar to the semidiurnal arc MK, by Theodosius II.10 [proposition 10 of book 2]) and the quadrant EB of the Equator — is known, together with the angle KBN, of the complement of the pole's altitude. Solve by the first method of problem 11 of the [treatise on] spherical triangles.

Let it be: as the whole sine to the sine of the arc BN — that is, to the sine of the difference between the given semidiurnal arc and the quadrant of the Equator; so the Tangent of the angle KBN, of the complement of the pole's altitude, to a fourth term; there will be generated the Tangent of the arc KN, of the declination sought.

The very same reasoning holds for the southern diurnal arc KL, as is clear in the triangle DKN. But it is enough to compute the declinations of the northern (or of the southern) diurnal arcs, for these are equal to the declinations of the opposite arcs. By this technique we have made table 13, containing the declinations of the diurnal arcs computed for a good many elevations of the pole.

PROBLEMA XII.

To find the arc of any great circle drawn through the poles of the Meridian and through any point of that same Meridian, lying between the Equator (or the Meridian) and the great circle described from the intersection of the Equator with the Meridian through any given hour of a given parallel.

A great circle of this kind is: the circle of the 6th hour from noon and from midnight, drawn through the pole of the world; any oblique Horizon; the prime Vertical; and the Ecliptic when its beginning-point of Cancer (♋) or of Capricorn (♐) is placed on the Meridian, and so on. So let ABCD be the Horizon; AEC the Meridian; BED the Equator; KOL the tropic of Cancer (♋); MPN the tropic of Capricorn (♐); BHD the circle of the 6th hour from noon or midnight, passing through H, the pole of the world; BFD the prime Vertical, drawn through F, the pole of the Horizon; BOD the Ecliptic when the beginning of Cancer (♋) is placed above the Horizon at O; and likewise BPD the Ecliptic when the beginning of Capricorn (♐) is placed above the Horizon at P.

Diagram. An orthographic projection of the celestial sphere within a square frame, the working figure for Problem XII. It carries the horizon ABCD, meridian AEC, equator BED, the tropic of Cancer KOL, the tropic of Capricorn MPN, the 6th-hour circle BHD, the prime vertical BFD, and the two ecliptic positions BOD and BPD, all crossing in a dense web of arcs. Readable labels, by approximate position:

- A (top)
- M (upper left), N (upper right)
- P (upper centre, below A)
- B (left edge), D (right edge)
- V (left interior)
- Z (left of centre)
- E (centre)
- K (lower left), L (lower right)
- R (lower centre-left), G (centre-low), F (bottom centre), Y (lower centre-right)
- I (lower left, below K), H (bottom), C (foot of circle)
- O, S, T and a cluster of small lowercase labels (a, b, d, u, and others) crowd the centre.

The small lowercase labels and several interior points at the centre could not be resolved with certainty under magnification and are not individually committed (more than two labels unresolved).

From E, the intersection of the Equator with the Meridian, let there be drawn — through any hour G, reckoned whether from noon and midnight or from sunrise or sunset, on the tropic of Cancer (♋), or through e on the tropic of Capricorn (♏) — the great circle EI, which cuts the circle of the 6th hour at Q, the Horizon at I, the Vertical at R, the Ecliptic BOD at T, and the Ecliptic BPD at S. We must investigate the arcs BQ, BI, BR, BT, BS, or HQ, CI, FR, OT, PS, which we shall find as follows. Draw, from H the pole of the world, through the given hour G, the great circle HG, cutting the Equator at Z. Since in the triangle EGZ the angle Z is right, and the arcs EZ, ZG are known — the former measuring the distance of the given hour from the Meridian, the latter the declination of the parallel — solve by the first method of problem 6 of the [treatise on] spherical triangles.

Let it be: as the whole sine to the sine of the arc EZ, of the distance from the Meridian; so the Tangent of the complement of the arc ZG, of the declination of the parallel, to a fourth term; there results the Tangent of the complement of the angle GEZ — that is, the Tangent of the angle GEO, which the arc HQ measures.

Therefore the arc HQ of the circle of the 6th hour will be known, together with its complement BQ.

Next, since in the triangle CEI the angle C is right, and the arc CE is known (being composed of the quadrant CF, between the Horizon and the zenith of the place, and of the arc FE, between the zenith of the place and the Equator, which equals the pole's altitude), together with the angle CEI just found: solve by the first method of problem 11 of the [treatise on] spherical triangles.

Let it be: as the whole sine to the sine of the arc CE — that is, to the sine of the complement of the pole's altitude, the arc EA (since the two arcs CE, EA have the same sine); so the Tangent of the angle CEI just found, to a fourth term; there will be created the Tangent of the arc CI.

Therefore the arc CI of the Horizon will be known, together with its complement BI.

By the same reasoning, each of the arcs FR, BR of the Vertical will become known, if in place of the arc CE the arc FE (of the pole's altitude) is taken, in the triangle FER. Likewise each of the arcs OT, TB of the Ecliptic BOD, or each of the arcs PS,

SD [printed *PSSD*] of the Ecliptic BPD, if in place of the arc CE the arc EO or EP (of the declination of the parallel) is taken.

But for the hours from noon or from midnight it is enough to compute the aforesaid arcs in one semicircle of any parallel — namely the eastern or the western. For these are equal to the arcs in the other semicircle of the same parallel, and moreover to the arcs in the opposite parallel; because the same great circle IES, drawn through the equal arc OG on the other side of the Meridian, makes the same angle with the Meridian and cuts off, from the equal parallels KL, MN, the equal arcs OG, Pe. For since, by proposition 18 of book 2 of Theodosius, both the arcs EO, EP and the arcs EG, Ee are equal, then by proposition 3 of book 3 of the same [work] the straight lines OG, Pe will be equal, and therefore also the arcs OG, Pe will be equal [Margin: 28. tertij — i.e. Theodosius, *Sphaerica* III.28]. From this it follows that, for the two hours of opposite parallels, the same angle E is always assumed in the computation. Since, then, the declination is likewise the same in both parallels, it is clear that the same arc is found for the two hours of the opposite parallels. Indeed it is enough to compute these arcs for a single quadrant of the parallel, because the same great circle described through E passes through two hours equally distant on either side of the circle of the 6th hour — as we demonstrated in the theorem of the scholium of chapter 8, number 2 — and therefore cuts off the same arc from any one of those great circles.

Now if the point E on the Equator is taken not for the 12th hour, but for any other whatever — so that AEC is not the Meridian, but some other hour-circle, the one which is distant 6 hours from the circle AEC (although it is not drawn through the poles of the Meridian) — just as, if E is no longer the intersection of the Equator with the Meridian but with the hour-circle of the assumed hour: as, if AEC be the circle of the 10th hour, then BHD must be taken for the circle of the 4th hour. In this case indeed the same arcs which, on the circle of the 6th hour, will be cut off for those hours which are so distant on either side from it that they are distant from the circle of the 6th hour by other hours — for which arcs are computed — are removed on both sides. For example, the arc for the 5th and 7th hours on the circle of the 6th hour will be the same as for the 3rd [hour], and so on.

In the same way, if the point E on the Equator is taken for the 6th hour, BND will be the Meridian, because the 12th hour is distant 6 hours from the 6th hour; and therefore the arc for the 5th and 7th hours on the circle of the 6th hour will be the same as for the 1st and 11th on the circle of the 12th hour. Likewise the arc found for the 2nd and 10th hours on the circle of the 6th hour will be the same as for

the 4th and 8th on the circle of the 12th hour, because the 4th and 8th hours are distant on either side from the 12th hour by four hours, just as the 2nd and 10th hours are also distant on either side from the 6th hour by four hours.

By this technique we constructed tables 1 and 3, which are universal; and likewise tables 7, 8, 14, 15, 16, and 17, which are particular, computed namely for the latitude of 42 degrees.

Furthermore, in the hours from sunrise or from sunset, the same arcs computed for any hour on any northern parallel agree also with that hour on the opposite southern parallel, which recedes from the meridian toward sunset by just as much as the former inclines from the meridian toward sunrise, and conversely. As to which hours from sunrise or from sunset agree, at equal distances from the meridian in contrary directions, on opposite parallels — this can easily be recognized from the little table of the scholium of chapter 15. Such hours are, for example, the 21st from sunset in Cancer (♋), and the 15th from sunset in Capricorn (♏), as appears in that table.

Again, since two great circles drawn from E, through two points of the same parallel equally distant from the Horizon toward the Meridian, cut off from any of the aforesaid great circles two equal arcs (because at E they make equal angles with the Meridian): it comes about that two hours from sunrise or from sunset of the same parallel, which are 12 hours — that is, a semicircle — apart from each other, have equal arcs; for two such hours are equally distant from the circle of the 6th hour, from noon or from midnight, going toward the same side, namely the eastern or the western — the one from the semicircle of the 6th hour from noon, the other from the semicircle of the 6th hour from midnight. Since, then, the same great circle described through E cuts off, on either side of that same semicircle of the 6th hour, two equal arcs in the same parallel, as is established from the theorem of the scholium of chapter 8; and since one of these two extreme points of the equal arcs cut off stands as far before one semicircle of the 6th hour as that which is distant from the other by 12 hours stands after the other semicircle: what we have said is evident. For example: by as much as the 23rd hour in Cancer (♋) is distant from the semicircle of the 6th hour from noon toward sunset, by just so much does the 11th hour in Cancer (♋) — which of course is 12 hours from the 23rd — also recede from the semicircle of the 6th hour from midnight toward sunset. Since, then, the great circle from E, drawn through the 11th hour from sunset in Cancer (♋), cuts off from the parallel of Cancer (♋) a point as far distant from the semicircle of the 6th hour from midnight toward sunrise as the interval by which the 11th hour is from the same

toward sunset — and hence as far as the 23rd hour is from the semicircle of the 6th hour from noon toward that same sunset — it plainly follows that the two great circles from E, through the 23rd and 11th hours from sunset, cut off equal arcs from the aforesaid great circles; and so of the rest.

Finally, since the same circle of the hours from sunrise and from sunset, passing through some one hour from sunrise on any parallel, passes through the hour of the same number from sunrise on the opposite parallel, and a semicircle of the said hour-circle is intercepted between those two hours — as we showed in the scholium of proposition 1 of book 5 of the Gnomonics, before the little tables of the Sun's altitudes, and as we shall show again, and more clearly, in problem 15 — it comes about that the same great circle described through E, passing on any parallel through some hour from sunset, passes on the opposite parallel through the hour of the same number from sunrise, since by proposition 11 of book 1 of Theodosius it bisects that hour-circle. From this it is gathered that two hours of the same number — of which one is reckoned from sunset on any parallel, the other from sunrise on the opposite parallel — possess equal arcs on the aforesaid great circles. And this is the device we used in constructing tables 7, 8, 16, and 17 for the hours from sunrise and sunset, which are particular, computed namely for the latitude of 42 degrees.

PROBLEMA XIII.

To investigate the arc of the Meridian circle lying between the Equator and the great circle which is drawn from any hour on the equinoctial line through a given hour on any parallel.

In the figure next to that of problem 12: from any hour V on the Equator, let a great circle pass through the given hour Y or X, on the parallel KL or MN — namely VY or VX — cutting the Meridian at b or at a. We must inquire the arc Eb or Ea. Now in the triangle VdY (formed by drawing, from the pole of the world H, through the given hour Y on the parallel, the great hour-circle HY, which cuts the Equator at d) the angle d is right, and the arcs Vd, dY are known — this last measuring the declination of the parallel, the former the distance of the assumed hour V from the given hour d. Solve by the second method of problem 6 of the [treatise on] spherical triangles.

Let it be: as the whole sine to the Secant of the complement of the arc Vd, of the distance between the given hours; so the tangent of the arc dY, of the declination of the parallel, to a fourth term; there will be found the Tangent of the angle dVY.

Again, since in the triangle bEV the angle E is right, and the arc VE — measuring the distance of the assumed hour on the Equator from the Meridian — is known, together with the angle bVE just found: solve by the first method of problem 11 of the [treatise on] spherical triangles.

Let it be: as the whole sine to the sine of the arc VE, of the distance between the assumed hour and the Meridian; so the Tangent of the angle bVE just found, to a fourth term: there will come out, known, the Tangent of the arc Eb, which we seek.

In just the same way the arc Ea will be found, which is equal to the arc Eb, if the same circle HY passes through the given hours Y, X.

From this problem, moreover, was composed table 2, for the 5th and 7th hours, distant 5 hours from the meridian on the equinoctial line; likewise for the 4th and 8th hours, distant on the equinoctial line 4 hours on either side from the meridian; and also for the 2nd and 10th hours, distant from the meridian two hours on either side. And in the same manner other tables can be constructed for other hours on the equinoctial line.

PROBLEMA XIV.

Given any vertical circle and its declination, to find how great the arc of the ecliptic is — with the first point of Cancer or of Capricorn placed on the meridian above the horizon — that lies between the meridian and the great circle which is drawn at right angles from the pole of the given vertical circle to the ecliptic.

Let ABCD be the horizon; AEC the meridian; BED the equator; FGH the parallel, which serves both for Cancer (♋) and for Capricorn (♏); BGD the ecliptic, touching the tropic at G; IKL the declining vertical circle, whose pole is M. Through M and through N, the pole of the ecliptic, let the great circle MN be described, cutting the ecliptic at P at right angles, by Theodosius [Theodosius of Bithynia], *Sphaerica* I.15. The arc GP is proposed for investigation.

Now in the spherical triangle DPM the angle P is right, and the arc DM — the complement of the vertical circle's declination — is known, together with the angle MDP. When the ecliptic touches the tropic of Cancer, this angle MDP is composed of ADE, the angle of the complement of the pole's elevation, and of EDG, the angle of greatest declination; but when the ecliptic touches the tropic of Capricorn, it is what remains when EDG, the angle of greatest declination, is subtracted from ADE, the angle of the complement of the pole's elevation. And this angle is always the angle of the meridian altitude of Cancer or of Capricorn

— that is, of the arc of the assumed point lying between Cancer (or Capricorn) and the horizon on the southern side. Solve, then, by the first method of problem 9 of [the treatise on] spherical triangles.

Let it be: as the whole sine [sinus totus] to the sine of the complement of the angle MDP (composed of the complement of the pole's elevation and the greatest declination of the ecliptic; or of what is left when the greatest declination is subtracted from the complement of the pole's elevation); so the tangent of the base DM, the complement of the vertical circle's declination, to a fourth term; there will be produced the tangent of the arc DP, which, subtracted from the quadrant DG, leaves known the sought arc GP.

Diagram. A sphere shown in projection within a square frame. Readable labels, by position:

- A (top of the sphere)
- I (upper left, on the declining vertical)
- M (upper right, pole of the declining vertical)
- G (just below A, on the ecliptic/meridian)
- F (left, upper — near-end of the parallel)
- H (right, upper — near-end of the parallel)
- P (upper-right interior, ecliptic/great-circle intersection)
- B (left, at the equator)
- E (center)
- D (right, at the equator)
- G (lower center-left, second occurrence, on the ecliptic)
- P (interior, right of the lower G, second intersection)
- F (lower left, second end of the parallel)
- K (lower center, on the vertical)
- H (lower right, second end of the parallel)
- O (below K)
- N (lower left, pole of the ecliptic)
- L (lower right, on the declining vertical)
- C (bottom of the sphere)

(The parallel FGH, the ecliptic, and the great circle MN each cross the projection so that G, P, F, and H recur front and back. No caption printed.)

PROBLEMA XV.

To compute the altitude of the Sun, at any hour, above a given great circle, the elevation of the pole above that circle being known.

Let ABCD be the proper meridian of the proposed plane, with center E; let BD be its common section with the [place's] meridian, and AC the section of the plane's primary vertical with the meridian; let DK be the pole's elevation above the plane, found by problem 4; KL the axis of the world; FG the diameter of the equator; and HI the diameter of any parallel, whether northern or southern, about which its semicircle HMI is described. Let HN be the distance of any hour from the proper meridian — whether that hour be reckoned from noon or from midnight, or from sunrise or from sunset. Through H draw HQ parallel to AC, and IQ parallel to BD; then HV will be the sine of the meridian altitude [altitudo meridianal], and QV the sine of the meridian depression. (This meridian altitude and depression, once the pole's elevation above the proposed plane is known, are to be found as taught in the *Gnomonics*, book 1, scholium to proposition 35, and book 5, scholium to proposition 1.) The whole HQ, then, will give the sum of the sine of the meridian altitude and the sine of the meridian depression.

Draw also the line RS parallel to IQ; since [a] HR is to RI as HS is to SQ, HS and SQ will be the halves of the sum HQ. From the hour-point N drop the perpendicular NO to HI, and through O draw OP parallel to BD, to serve as the diameter of the parallel of the horizon (or of the proposed plane) that passes through the Sun as it stands at hour N; so that PV is the sine of the Sun's altitude. This we shall make known by the present method — as we also taught in *Gnomonics* book 1, proposition 36, before finding the Sun's altitude by spherical triangles.

[Margin a: 2.sexti — Euclid, *Elements* VI.2.]

Let it be: as the whole sine HR to HO, the versed sine [sinus versus] of the Sun's distance from its proper meridian; so HS, the half of the sum of the sines of the meridian altitude and depression, to a fourth term: there will be produced the line HP, which, subtracted from HV, the sine of the meridian altitude, leaves PV, the sine of the sought altitude.

When the line HP found is greater than HV, the sine of the meridian altitude, then the sine of the meridian altitude must be taken away from that line found, so that what remains is the sine of the sought altitude above the opposite face of the plane.

When the Sun stands at the first point of Aries (γ) or of Libra (ϱ), its altitude is more easily found. For, in the other figure, let FK be the distance of some hour from the proper meridian; drop KL perpendicular to FG, and through L draw LM parallel to BD, to serve as the diameter of the parallel of the horizon (or of the proposed plane), so that MN is the sine of the Sun's altitude at the proposed hour. If, then,

Diagram. Two circles set side by side within a rectangular frame. The left circle is the general figure of Problem XV; the right circle is the case of the Sun at an equinoctial point (Aries/Libra). Both are divided by two perpendicular diameters and by the sloping chords named in the text.

Left circle — readable labels by position: A (top, slightly left), N (top, right of A), M (upper right), K (upper interior, below N), Y (upper right, at the rim), H (upper left, at the rim), F (left, at the rim), P (interior, upper left), O (interior, right of P), R (interior, center), S (interior, below P), T (interior, below S), B (right, at the rim), V (interior, lower left), E (center), I (interior, lower right), Q (lower left, at the rim), L (bottom left), C (bottom center), G (bottom right). One or two crowded interior labels on the right side (near the R/B region — the text's point X of the perpendicular YX) could not be resolved with confidence [?].

Right circle — readable labels: A (top), K (upper right), F (upper left, at the rim), H (right, at the rim), M (interior, center left), L (interior, center), E (center), N (interior, lower left of center), B (left, at the rim), D (right, at the rim), I (bottom left), C (bottom center), G (bottom right).

(No caption printed. This same two-circle figure is reprinted on the following page — see fig-p824-1.)

Let it be: as the whole sine to LE, the sine of the complement of the Sun's distance from its proper meridian, so FN, the sine of the complement of the pole's elevation above the given plane, to a fourth term; there will be generated MN, the sine of the Sun's altitude, which we seek.

Moreover, when a table of the Sun's altitudes above the horizon is to be compiled for all the hours from noon or from midnight, it is enough to take the hours from noon in the western semicircle of any northern parallel and compute their altitudes. For these altitudes are equal to the altitudes of the hours from midnight in the eastern semicircle, since the hours of the one semicircle contain distances from the meridian equal to the distances of the hours of the other semicircle. But when a table of the Sun's altitudes above the same horizon is to be constructed for all the hours from sunrise and sunset, only the altitudes of the hours of the

same northern parallel need be computed. For from the altitudes of the hours of any parallel the altitudes of the hours of the opposite parallel are easily derived. For the altitudes of the hours of one parallel are equal to the altitudes of the same hours of the opposite parallel — provided that the hours which in the one parallel are numbered from noon or from sunrise are, in the other, reckoned from midnight or from sunset, and conversely; and that those which in the former parallel stand above the horizon are understood to fall below the horizon in the latter, and the reverse. This we shall gather briefly as follows.

[Margin: That the altitudes of the hours in any parallel are equal to the altitudes of the opposite hours in the opposite parallel.]

Let some hour-circle pass, for example, through the 6th hour from noon (or the 23rd from sunset) in the parallel of Cancer, above the horizon: with its other semicircle it will necessarily cut the opposite parallel of Capricorn at the 6th hour from midnight (or the 23rd from sunrise), because, while one semicircle indicates the hour from noon or from sunset, the other belongs to the hour of the same number from midnight or from sunrise, as was shown in the scholia to propositions 9 and 10 of book 1 of the *Gnomonics*. And its portion between the tropics of Cancer and Capricorn will be a semicircle. For since its portion from the equator, proceeding through the northern parts as far as the equator, is a semicircle by Theodosius I.11, if the arc of it between the given hour in Cancer and the equator be subtracted, and the arc of it between the equator and the hour of the same number in Capricorn be added — which by Theodosius II.18 is equal to the former — a semicircle will likewise be produced between the hour in Cancer and the opposite hour in Capricorn. From this it follows that that hour in Capricorn lies below the horizon, lest a portion of that same great circle greater than a semicircle should appear above the horizon. And since any vertical circle bisects any hour-circle, by Theodosius I.11, the vertical circle (or circle of altitude) passing through the given hour in Cancer will pass also through the opposite hour in Capricorn, and its portion between those two hours will be a semicircle, and therefore equal to the semicircle of the same circle above the horizon. If, then, its common segment between the hour in Cancer and the horizon be subtracted (proceeding toward the hour in Capricorn), there will remain an arc of altitude of the hour in Cancer above the horizon equal to the arc of altitude of the opposite hour in Capricorn below the horizon. And the same reasoning holds for the other hour-circles, and so forth.

[Margin: How, from the altitudes of the hours of one parallel, the altitudes of the hours of the same kind in the opposite parallel are derived.]

Now, as is clear from the little table in the scholium of chapter 15, two hours whose numbers make up 24 — the one counted from sunrise, the other from sunset — have the same distance from the meridian on opposite sides, and therefore the same altitude, as is established by calculation. Hence, once the altitudes of the hours from sunset in any parallel are found, the altitudes of the hours from sunset in the opposite parallel will also be known. For two hours from sunset making up 24 have, in opposite parallels, the same altitude — as the 14th hour from sunset in Cancer and the 10th hour from sunset in Capricorn, and so on — because, namely, this hour from sunset in Capricorn has the same altitude as the hour from sunrise which is its complement to 24 (as has been said), and this same hour from sunrise has the same altitude as the hour of the same number from sunset, as we have demonstrated. In just the same way, from the altitudes of the hours from sunrise of one parallel are derived the altitudes of the hours from sunrise of the opposite parallel. Thus you see in table 10 that the 14th hour from sunset in Cancer, the 14th hour from sunrise in Capricorn, and likewise the 10th hour from sunset in Capricorn, have the same altitude, 54 degrees 14 minutes. By the same reasoning, the altitude of any hour from noon in any parallel — which is equal to the altitude of the same hour from midnight in the opposite parallel, as has been said — will also be equal to the altitude of the hour from noon which is its complement to 12, and conversely. Thus you see in table 9 that the 4th hour from noon in Cancer, the 4th hour from midnight in Capricorn, and also the 8th hour from noon in Capricorn, contain the same altitude, 37 degrees 25 minutes. And by this device tables 9 and 10 were constructed.

[Margin: A wonderful shortcut for the altitudes of the hours from noon and midnight above the horizon.] [Margin a: 2.*sexti* — Euclid VI.2.]

Further, for the altitudes of the hours from noon and midnight it is enough, for any hour between noon and the 6th hour from noon in the northern parallel, to find the line HP, as was done for hour N. For if another hour Y be taken, whose distance from the 6th hour M is equal to the distance of hour N from the same 6th hour, and the perpendicular YX be dropped to HI, and through X a line be drawn parallel to BD cutting HV at T, then QT will be equal to HP. (For since the arcs MN and MY are assumed equal, the remaining arcs HN and IY will also be equal, and therefore their versed sines HO and IX, and consequently also HP and QT — [a] which have the same ratio as the equal HO and IX.) If from this QT there be taken QV, the sine of the meridian depression, there will remain TV, the sine of the sought altitude for hour Y. But if HP is found less than the sine of the meridian depression (which happens when the hour lies below the horizon), it must

instead be subtracted from QV, the sine of the meridian depression, so that what remains is the sine of the sought altitude above the horizon of the antipodes.

And so, if for example the altitude of the 8th hour after noon in Cancer is sought, HP must be taken for the 4th hour after noon — which is equally distant from the 6th hour as the 8th hour is — found to be 34075 parts at the latitude of 42 degrees; and this must be subtracted from the sine of the meridian depression, 41469, so that there remains the sine 7394 of the altitude of the 8th hour above the lower face of the horizon, to which there correspond in the table of sines 4 degrees 14 minutes, and so on.

Indeed, for the 4th hour from noon it is enough to take HP as half of the half of the sum of the sines of the meridian altitude and depression, without any other computation. For if N is the 4th hour, the arc NM will contain 2 hours, that is, 30 degrees, whose right sine OR will be half of the whole sine HR, as we demonstrated in [the treatise on] sines; and therefore HP too will be half of HS, which is half of the whole HQ.

Furthermore, if HS, the half of the sum of the sines of the meridian altitude and depression, be subtracted from

Diagram. The two-circle figure of Problem XV, reprinted at the foot of this page. It is identical to fig-p823-2 on the preceding page — a left (general) circle and a right (equinoctial) circle within a rectangular frame, carrying the same labels (left: A, N, M, K, Y, H, F, P, O, R, S, T, B, V, E, I, Q, L, C, G, with one or two crowded right-interior labels unresolved [?]; right: A, K, F, H, M, L, E, N, B, D, I, C, G). No caption printed.

HV, the sine of the meridian altitude, there will remain SV, the sine of the altitude of the 6th hour — which is likewise left over if QV, the sine of the meridian depression, be subtracted from QS, the aforesaid half of the sum.

But if a table of the Sun's altitudes above some other great circle — and not above the horizon — is to be constructed, the altitudes must be found for all 24 hours of the given parallel, namely for the 12 hours from noon and the 12 from midnight, if the discussion concerns hours from noon and midnight. For these altitudes will be equal to the altitudes of the opposite hours in the opposite parallel, as has been shown. But if the matter concerns hours from sunrise and sunset, then for one and the same parallel the altitudes of all 24 hours from sunrise and of all 24 from sunset must be computed. For the former will be equal to the altitudes of all 24 [hours] from sunset in the opposite parallel; the latter to the altitudes of all 24 from sunrise in the same opposite parallel, as is clear from what has been

demonstrated. And the reason why the altitudes of all the hours must be computed for one parallel is that very rarely do two hours have the same distance from the proper meridian, as they do from the meridian of the horizon.

For the right sphere, seek the rule in book 6 of the *Gnomonics*, page 557.

PROBLEMA XVI.

To explore the circumference of any great circle (which stands as the horizon of some place), lying between its primary vertical circle and the vertical circle of the same passing through the center of the Sun at the proposed hour, the elevation of the pole above that circle being known.

Let ABCD be the horizon, or any other great circle standing as a horizon; AEC its meridian; BED the equator; BKD its primary vertical; FHG the parallel [described] either toward the pole of the world P visible above the given great circle, or toward the hidden pole; and KLM the vertical drawn through the Sun as it stands at L. We must investigate the arc BM. Having drawn the great circle of declination PLN from the pole P through the Sun's place L, and from B, through the same place of the Sun, the great circle BLI: since in the spherical triangle BLN the angle N is right, by proposition 15 of book 1 of Theodosius, and the two arcs BN, NL are known — the former being the complement of the Sun's distance from its proper meridian, the latter measuring the Sun's declination — solve by the first method of problem 15 of [the treatise on] spherical triangles.

Let it be: as the whole sine to the sine of the complement of the arc NL, that is, to the sine of the complement of the Sun's declination; so the sine of the complement of the arc BN, that is, the sine of the Sun's distance from its proper meridian, to a fourth term: there will be produced the sine of the complement of the base BL — which Ptolemy in his little book *On the Analemma* calls the *hectemorion* circumference [Hectemoriam circumferentiam; Greek ἑκτημόριον, "one-sixth" arc] — which keep.

Diagram. A sphere shown in projection within a square frame. Readable labels, by position:

- A (top of the sphere)
- M (upper right)
- E (center left)
- N (center)
- O (small label, just right of N)

- D (left, at the rim)
- H (center left, beside L)
- L (center, the Sun's place)
- B (right, at the rim)
- G (lower left, at the rim)
- I (interior, below L)
- F (lower right, at the rim)
- K (interior, below I)
- P (lower left interior, pole of the world)
- C (bottom of the sphere)

(No caption printed.)

Next, since in the triangle BLM the angle M is right, and the arc LM — the Sun's altitude above the given great circle — has been made known by the preceding problem, together with the base BL, whose complement's sine we have just found: solve by the first method of problem 7 of [the treatise on] spherical triangles.

Let it be: as the whole sine to the secant of the arc LM, the Sun's altitude above the given great circle; so the sine of the complement of the base BL, just found, to a fourth term; there will be generated the sine of the complement of the arc BM, which we desire.

When the Sun stands at the first point of Aries or of Libra, we shall more easily find the desired circumference BM of the horizon by a single operation. For since in the triangle BOM the angle M is right, and the arc BO is known (being the complement of the Sun's distance from noon), together with the angle OBM, [the angle] of the complement of the pole's elevation above the given great circle: solve by the first method of problem 9 of [the treatise on] spherical triangles.

Let it be: as the whole sine to the sine of the complement of the angle B, that is, to the sine of the pole's elevation; so the tangent of the base BO, that is, the tangent of the complement of the Sun's distance from noon, to a fourth term; there will come forth the tangent of the desired circumference BM.

But whether the horizontal circumference BM thus found is eastern or western, and whether northern or southern — which it is of great importance to know, so that sundials may be described by them, as is evident from what we wrote in chapter 25 — this we shall now learn. For hours from noon or from midnight, it is

clear that the circumferences of all the hours from midnight are eastern, and those of the hours from noon western. But for hours from sunrise and sunset, the hour of noon must be sought from the semidiurnal arcs of the first table, in this way. If the matter concerns hours from sunrise, the semidiurnal arc of the opposite parallel, reduced to hours by the little table after [chapter] 8 — the office of the equator set forth in chapter 2 of the *Sphere* — indicates the hour of noon. But if hours from sunset are at hand, then, to the seminocturnal arc of the proposed parallel (which is always equal to the semidiurnal arc of the opposite parallel), reduced to hours, if 12 be added, the hour of noon will be made up. For example, the semidiurnal arc of the parallel of Cancer at Rome contains 113 degrees 3 minutes, that is, 7 hours 32 minutes, at which hour after sunrise noon occurs. Likewise the seminocturnal arc of Cancer — that is, the semidiurnal arc of Capricorn — comprises 66 degrees 57 minutes, namely

[Margin: In what cases the horizontal circumferences are eastern or western, and in what cases northern or southern.]

namely 4 hours 28 minutes; and if 12 is added to these, the result is the 16th hour, 28 minutes — the hour of noon reckoned in hours after sunset. Therefore, of all the hours, whether from noon or from sunset, the circumferences of those that precede noon are eastern, while those that follow are western.

NEXT, the circumferences of all the hours — whether from noon or midnight, or from sunrise or sunset — in the northern parallels that have an altitude less than the Sun's altitude when it stands in the vertical circle (which is drawn out from problem 7) are northern; but those of greater altitude are southern. Furthermore, on the Equator and on any southern parallel, all the circumferences above the horizon are southern.

AGAIN, of whichever hours from noon the circumferences in the northern parallel are northern, the circumferences of those same hours from midnight in the southern parallel are southern; and those that are southern in the former are northern in the latter. The same must be said of the hours from midnight in the prior parallel and of the hours from sunrise in the posterior; likewise of the hours from sunset in the prior parallel and of the hours from sunrise in the posterior — as appears on the material sphere.

AND all of this, indeed, is to be understood on the horizon and on the primary vertical. For on other great circles — such as the declining verticals — in place of the noon hour one must take the hour that falls on the proper meridian, which is known from that circle's inclination to the meridian. Likewise, by "northern"

circumferences are to be understood those that tend toward the visible pole above the given great circle, and so forth. From this problem, tables 11 and 12 have been constructed.

FOR the horizontal circumference on the right sphere, seek the rule in Book 6 of the *Gnomonics*, page 558.

PROBLEMA XVII.

To make known the segments of the hour-lines from noon and from midnight — those lying between the centre of the horizontal dial and the parallels of the Sun — in parts of the Tangents, with respect to a whole sine of 1000 equal to the stylus.

Diagram. A rectangular construction figure for the horizontal dial. Within a ruled rectangular frame, a main diagonal rises from the lower-left toward the upper region; a second, cross-hatched (toothed) oblique line represents the stylus CD, and a scale of parts is ticked off along the lower-left diagonal at 10, 20, 30. The apex of the hour-line construction is at the top centre. Readable labels, with approximate positions:

- A (top centre): centre of the dial / apex of the hour-lines
- M (top, immediately right of A): endpoint on the top edge
- F (upper right): endpoint of the perpendicular CF
- G (upper left, on the rising diagonal): endpoint of the perpendicular CPG
- K (top-left corner of the frame): point on the 6-o'clock hour-line AK
- D (right side, on the descending line): foot of the axis / stylus endpoint
- O (centre, below A/M): intersection point on the hour-line
- R (centre-left, below G): point on the rising diagonal
- I (left, below R): point on hour-line AE (a parallel-point)
- C (centre, below O): the place of the stylus (locus C)
- T (centre, right of C): point on the construction
- H (centre, below C): parallel-point on the meridian
- E (lower left): endpoint of the hour-line / equinoctial line
- B (centre-lower): point on the meridian AB
- N (lower centre): parallel-point
- V (bottom centre): endpoint
- L (lower left): parallel-point (of Capricorn)
- Scale marks 10, 20, 30 along the lower-left diagonal (division of the stylus into parts)

The accompanying text also names the Sun's parallel-points on this figure as Cancer [♋], H, I, K (for the summer parallel) and Capricorn [♐], N, L (for the winter parallel).

Let the centre of the dial be A, and the meridian line AB; the stylus CD, and its place C; the axis of the world AD; the equinoctial line BE; the 6-o'clock hour-line AK, and any other hour-line AE. Let the point of the parallel of Cancer [♋] be, on the meridian, H; on the hour-line AE, the point I; and on the 6-o'clock line, the point K. It is required to reduce the segments AH, AI, AK to parts of the Tangents with respect to the whole sine CD, of 1000 parts, and so on.

At AE erect the perpendicular CPG; and, having drawn CF perpendicular to CG (with CF equal to the stylus CD), take PG equal to the joined straight line PF. Then PG will be the whole sine of the Tangents PA, PI, PE. And since, by problem 10, the arc of the Tangent AP is known, the Tangent AP itself will be known with respect to the whole sine PG. Now, when the arc of the Tangent AP is subtracted from the complement of the declination of Cancer [♋], the remainder will be the arc of the Tangent PI, with respect to the same whole sine PG. And if the Tangents PA and PI are added together, AI will be made known in parts of the whole sine GP, or PF — and thus it will likewise be known in parts of the whole sine CD.

The angle CAP is known, from the arc of the horizon between the meridian and the hour-circle AE — that is, from the complement of the arc of the horizon in table 6. And since, in the right triangle ACP, when AC is set as the whole sine, the sine of the angle CAP is CP (as is clear from the doctrine of sines), CP will be known in parts of the whole sine AC. But CD — that is, CF — is also known (for, AC being set as the whole sine, the Tangent of the angle CAD is CD): the Tangent of the pole's altitude is known in the same parts. Therefore, if it be made — (we take, in the calculation, the Sines, Tangents, and Secants in parts of the whole sine 100000, so that the result may come out more exactly):

As FC, the Tangent of the pole's altitude, to itself, taken as the whole sine;
so CP, the sine of the angle CAP (which is the complement of the arc of the horizon in table 6), to a fourth term: CP will become known, in parts of the same whole sine CF, equal to the stylus CD — namely, taken as the Tangent of the angle CFP; and so the angle CFP will be known.

If, then, it be made again:

As FP, taken as the whole sine, to AI, known in parts of the whole sine FP,
or PG; so FP, taken as the Secant of the found angle CFP, with respect to

the whole sine FC, or CD, to a fourth term: the same AI will come forth, known in parts of the same whole sine CF, equal to the stylus.

By way of example: Let us suppose AE to be the hour-line of 3 from noon, or of 9 from midnight, at a pole-altitude of 42 degrees. The angle CAP will be 33 degrees 47 minutes — namely, the complement of the arc of the horizon for hours 3 and 9, in table 6, in the row of pole-altitude 42 degrees. And the sine of that same angle CAP, with respect to the whole sine AC, will be 55605. But CF — that is, CD, the Tangent of the angle CAD (of the pole-altitude, 42 degrees), with respect to the whole sine AC — will be 90040. Let it therefore be made: as CF, 90040, to itself, taken as the whole sine 100000; so CP, 55605, to a fourth term; and CP will be generated as about 61756, the Tangent of the angle CFP. In the table of Tangents, therefore, the angle CFP will be found as 31 degrees 42 minutes.

Again, let it be made: as FP, the whole sine, to AI, 132508 — (for the arc between A and the vertical perpendicular CP, found by problem 10, is 38 degrees 9 minutes; and when this is taken away from the complement of the declination of Cancer [☊], 66 degrees 30 minutes, there remain 28 degrees 21 minutes for the arc between P and I. The Tangent AP, therefore, with respect to the whole sine FP, is 78551; and PI is 53957; and their sum, AI, is 132508) — so FP, 117535, the Secant of the found angle CFP, 31 degrees 42 minutes, with respect to the whole sine CF, or CD, to a fourth term; and the same AI will be found as 155743, with respect to the whole sine CD, 100000.

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And if that same arc AP, 38 degrees 9 minutes, be subtracted from the complement of the declination of Aries [♈] — that is, from 90 degrees — the remaining arc PE will be 51 degrees 51 minutes (I mean the arcs to which the segments AP, PE, and so on correspond); its Tangent, 127306, together with the Tangent AP, 78551, makes up the whole AE, 205857. If, then, it be made: as FP, the whole sine, to AE, 205857; so FP, 117535, the Secant of the found angle CFP, 31 degrees 42 minutes, with respect to the whole sine CD, to a fourth term; AE will be found as 241954, with respect to the whole sine CD, 100000.

LIKEWISE, if that same arc AP, 38 degrees 9 minutes, be taken from 113 degrees 30 minutes — namely, from the arc composed of the quadrant AE and the declination of Capricorn [♐], 23 degrees 30 minutes — there will remain the arc between P and L, the point of Capricorn [♐], on the hour-line AE, of 75 degrees 21 minutes; whose Tangent, 382537, together with the Tangent AP, 78551, makes 461088 for the straight line AL, between A and L, the point of Capricorn [♐]. If,

then, it be made: as FP, the whole sine, to 461088, the straight line AL between A and L, the point of Capricorn [φ], with respect to the whole sine FP; so FP, 117535, the Secant of the found angle CFP, 31 degrees 42 minutes, to a fourth term; there will be produced the straight line AL, between A and L, the point of Capricorn [φ], of 541940 (about), with respect to the whole sine CD.

NOW, if from the found straight line AL, 541940, you subtract the AE found above, 241954, the remainder will be the straight line EL, between E and L, the point of Capricorn [φ], of 299986, with respect to the same whole sine CD.

SIMILARLY, if from the found AE, 241954, you take away the found AI, 155743, there will remain EI, 86211 — which is what we noted at the end of the scholium of chapter 9.

ON the meridian AB, the arc AC just named is the complement of the pole's altitude — namely, the Tangent of the angle ADC, with respect to the whole sine CD. But CH is the Tangent of the arc that remains after the subtraction of the arc AC (the complement of the pole's altitude) from AH (the complement of the declination of Cancer [ϑ]), or after the subtraction of the complement of the declination of Cancer [ϑ] from the complement of the pole's altitude, if the latter be the lesser, and so on. Hence the sum of the two Tangents AC, CH will give AH itself, known in parts of the whole sine CD. So too, if from the arc AN — composed of the quadrant AB and the declination of Capricorn [φ], BN — the complement of the pole's altitude, CB, be subtracted (or rather, if to the pole's altitude CB the southern declination BN be added), there will remain, or be composed, the arc CN, whose Tangent, together with the Tangent AC (of the complement of the pole's altitude), will make up the whole AN, known and sought in parts of the whole sine CD.

FURTHERMORE, when the complement of the pole's altitude is greater than the complement of the declination, this must be subtracted from that, and the Tangent of the remaining arc must be taken away from the Tangent of the complement of the pole's altitude, so that the straight line we seek may remain. But in chapter 8, number 7, we prescribed another method for finding the Tangents to be transferred from C, the place of the stylus, so that the points of the parallels may be found on the meridian.

ON the 6-o'clock hour-line AK, the aforesaid arc is nothing. Therefore, if it be made:

As DA, the whole sine, to AK, the Tangent of the complement of the parallel's declination; so DA, the Secant of the angle ADC (of the

complement of the pole's altitude), with respect to the whole sine CD, to a fourth term: AK will be generated, in parts of the same whole sine CD.

ON any hour AM whatever, before the 6th hour of the morning or after the 6th hour of the evening, that arc is AP — the same that was found for the corresponding hour opposite AE below the 6th hour, but not toward the parallel. It must therefore be added to the complement of the declination, and from the Tangent of the composed arc the Tangent of the aforesaid arc must be subtracted, so that there remains the portion of the proposed hour from A up to the parallel. Then this same portion is to be investigated in parts of the whole sine CD, just as was done for the corresponding hour opposite AE below the 6th hour, by means of the angle CFP and the whole sine FP, and so on, which do not change.

HENCE table 19 has been constructed for very many pole-altitudes.

IN the figure, too, any number of parts equal to the stylus CD have been taken on the straight line, and the first of them divided into 10 equal parts, so that, if a dial be completed for latitude 42 degrees, the points of the parallels may be found on the hour-lines from table 19, computed for the same latitude. For thus you see that the points Cancer [♋], H, I, K, and Capricorn [♐], N, L have been found.

[Margin: Another, easier solution of the problem.]

BUT we shall dispatch this same problem 17 more expeditiously in the following way. From table 1, under the line whose heading is "Circ. hor. 12," take the arc of any sign corresponding to any hour of its line, and subtract it from the pole-altitude, if it can be subtracted; and add the Tangent of the remaining arc to the Tangent of the complement of the pole's altitude. [Margin: When the arc of table 1 is less than the pole's altitude.] Then take the Secant of the angle that the given hour makes at the centre with the meridian — which angle is given to us by the complement of the arc of table 6, the one that agrees with that same hour, on the side toward the pole's altitude above the horizon, on the left. For, if it be made:

As the whole sine, to the number composed of the Tangent of the complement of the pole's altitude and the Tangent of the arc that remains after subtracting the arc of table 1 from the pole-altitude; so the Secant of the angle that the line of the given hour makes with the meridian, to a fourth term: there will be produced the sought Tangent — that is, the segment of the hour-line between the centre of the dial and the point of the proposed parallel, in parts of the whole sine equal to the stylus.

FOR, if the arc of the given hour in table 1, corresponding in the adjoining figure to the straight line PT, be subtracted from the pole-altitude, which corresponds to the straight line BC, there will remain the arc corresponding to the Tangent CT, with respect to the whole sine CD. And if the Tangent CT of the said remaining arc be added to the Tangent AC (of the complement of the pole's altitude), the whole AT will become known, in parts of the whole sine CD. And since the line TI, drawn parallel to the equinoctial, cuts the hour AE at I, the point of the parallel whose arc BT, corresponding to the given hour in table 1, was taken (as we wrote in chapter 8, number 7); and since the straight line AI is the Secant of the angle TAI, with respect to the whole sine AT — it is clear that, if it be made: as the whole sine AT, to AT itself, known in parts of the whole sine CD (which AT is composed of the two Tangents AC, CT); so AI, the Secant of the angle TAI, with respect to the whole sine CD, to AI, in parts of the same whole sine CD — because the whole sine AT stands to the Secant AI as AT, known in parts of the whole sine CD, stands to AI, in the same parts, and so on.

[Margin: When some arc of table 1 is greater than the pole's altitude.]

BUT if some arc of table 1 be greater than the pole-altitude, the pole-altitude must be subtracted from it, and the Tangent CO of the remaining degrees must be taken away from AC, the Tangent of the complement of the pole's altitude, so that AO may remain known in parts of the whole sine CD. For, if it be made again:

As the whole sine

As the whole sine AO, to AO itself, which is left after the subtraction of CO, the Tangent [taken] from AC, the Tangent of the complement of the pole's altitude — which Tangent CO is owed to the arc that remains after subtracting the pole-altitude from the arc of table 1; so AP, the Secant of the horary angle OAP, to a fourth term: AP will be generated, in parts of the whole sine CD.

[Margin: When the arc of table 1 is equal to the pole's altitude.]

If perchance some arc found in table 1 be equal to the pole-altitude, let it be made:

As the whole sine AC, to AC itself, the Tangent of the complement of the pole's altitude; so the Secant of the horary angle, to a fourth term: and the sought Tangent will be produced.

[Margin: What to do for the 6-o'clock hour.]

FOR the 6-o'clock hour, let it be made:

As the whole sine AD, to AK, the Tangent of the complement of the parallel's declination, with respect to the whole sine AD; so AD itself, taken as the Secant of the angle ADC (of the complement of the pole's altitude), with respect to the whole sine CD, to a fourth term: and AK will be produced, in parts of the same whole sine CD.

It is better, for the 6-o'clock hour, to take the Tangent of the complement of the declination, with respect to the whole sine AD.

[Margin: When the hour is at a greater distance from noon than 6 hours.]

WHEN the given hour is before the 6th hour of the morning, or after the evening one, it will indeed make with the meridian the same angle that the opposite hour of the same name makes. But then, to the complement of the arc of table 1, the complement of the pole's altitude must be added; and from the Tangent of the composed arc, the Tangent AC (of the complement of the pole's altitude) must be taken away. For, if it be made:

As the whole sine, to the number that remains after the subtraction of the Tangent of the complement of the pole's altitude from the Tangent of the arc composed of the complement of the pole's altitude and the complement of the arc of table 1; so the Secant of the horary angle, to a fourth term: the segment of the hour-line between the centre A and the point of the parallel will be found, in parts of the whole sine CD.

[Margin: What to do for the southern parallels.]

FURTHERMORE, for the southern parallels, the arc of table 1 must always be added to the pole-altitude, so that the Tangent CV of the composed arc may become known, in parts of the whole sine CD. For when this is added to AC, the Tangent of the complement of the pole's altitude, the whole AV will be known, in parts of the same whole sine CD. Wherefore, if it be made:

As the whole sine AV, to AV itself, which is composed of AC (the Tangent of the complement of the pole's altitude) and the Tangent CV (of the arc composed of the pole-altitude and the arc of table 1); so AL, the Secant of the horary angle VAL, to a fourth term: AL will come forth, known in parts of the whole sine CD.

[Margin: What to do when the Sun is on the Equator.]

FINALLY, when the Sun is on the Equator, BC (the Tangent of the pole's altitude) must be added to AC (the Tangent of the complement of the pole's altitude), so that the whole AB may become known, in parts of the whole sine CD. For, if it be made:

As the whole sine AB, to AB itself, composed of the Tangent of the pole's altitude and the Tangent of the complement of that same altitude; so AE, the Secant of the horary angle EAB, to a fourth term: AE will be generated, known in parts of the whole sine CD.

What must be done on the meridian line has been explained in the prior solution of the problem.

BY this art, therefore, we have computed table 19 for very many elevations of the pole, so that the points of the Sun's parallels may be readily found on the horizontal dial.

[Margin: How table 19 is adapted to the vertical and declining dial.]

MOREOVER, this same table 19 has a use in the dial that is parallel to the primary vertical, provided the said found Tangents are taken as the complement of the pole's altitude above the horizon. Thus, for latitude 42 degrees, one must take, for the vertical dial, the Tangents computed for latitude 48 degrees — because the altitude of the pole above the primary vertical, in latitude 42 degrees, is that great.

INDEED, if, on a declining dial, there be drawn from both sides of the stylus-line, from the centre, lines distant from the stylus-line by 15, 30, 45, 60, 75 degrees, and so on, in the manner of the hour-lines on a horizontal dial (as we set out in chapter 18, number 4), and if the altitude of the pole above the declining plane be found by problem 4, then the Tangents computed for that altitude in table 19, transferred from the centre, will indicate the points of the parallels, just as on the horizontal dial. But when the altitude of the pole found above the declining plane is not found in table 19 — that is, when, beyond the degrees, it also contains some minutes — the proportional part must be taken, between the next lower and the next higher pole-altitude, in the Tangents of table 19; which is not difficult. For example, since in latitude 49 degrees the Tangent of the 3rd and 9th hour for Cancer [♋] is about $14 \frac{8}{10}$, while in latitude 50 degrees the Tangent of the same hours is $14 \frac{5}{10}$, in latitude 49 degrees 30 minutes the Tangent of the same hours will be $14 \frac{6}{10}$, a little more, for the 3rd and 9th hour. [Margin: When the pole's altitude contains minutes beyond whole degrees, what to do.] Or, more surely, the proportional part corresponding to the minutes attached to the pole-altitude is to

be sought, in the manner of the astronomers, so that the points of the parallels may be caught more accurately. I know the matter would be more exact if, for the found altitude of the pole, the Tangents were computed as we have taught here; but no great error will arise from the proportional part. And the same is to be observed on the horizontal dial, when the altitude of the pole contains whole degrees and about 30 minutes.

[Margin: The hour-segments between the centre and the perpendicular lines drawn from the place of the stylus — where they are to be found already computed.]

ABOVE, in problem 10, we likewise computed, for several elevations of the pole, the segments of the hour-lines between the centre of the horizontal dial and the perpendicular drawn from the place of the stylus to those hour-lines: so that, if it seem good, the points of the Sun's parallels may be investigated through them, as we taught in the scholium of chapter 9, number 5. But since the whole sine varies at each hour, one must employ the equilateral triangle of chapter 24, so that those various whole sines may more easily be cut into 10 equal parts, as we explained in chapter 24, number 7.

THESE same segments also have their place in the primary vertical dial, and in any declining dial whatever: just as we said a little earlier about the Tangents.

POSTRE-

PROBLEMA XVII (conclusion)

Finally — so that I may leave out nothing that can assist the construction of a horizontal dial — I shall gladly, for the benefit of students (and especially of Isidorus Rubertus [Isidoro Ruberti], Doctor of both Laws and Auditor to the most illustrious Cardinal Salviati, and of Theodosius Rubeus [Teodosio Rubei], Doctor of Sacred Theology and of both Laws, who earnestly pressed me to do it), briefly teach the method by which, from the horizon-arcs of tables 14 and 16, the points of the parallels can be found on a horizontal dial, even when the horizontal line cannot be drawn upon it. This we accomplished long ago for the declining dial, in the scholium to chapter 19, number 4. The procedure is as follows. On the dial of chapter 2, from point B — where the meridian line intersects the equinoctial — describe a circle at the radius of some quadrant divided into 90 degrees. For if the horizon-arcs of table 14 are transferred onto this circle from the meridian line (for the hours reckoned from noon and from midnight [à mer. & med. noc.]), and likewise the arcs of table 16 (for the hours reckoned from the east or the west [ab

or. vel occ.)), then the straight lines drawn from B through the ends of those arcs will cut the hour-lines — both those from noon and midnight and those from the east or west — at the points of the parallels. For example: since in table 16 the arc for the 21st hour from the west, in Cancer [♋], contains 57 degrees 45 minutes, then if on the circle described from B an arc of 57 degrees 45 minutes is taken, beginning from the meridian line, the straight line from B running through the end of that arc will cut the 21st hour at the point of Cancer [♋], and so on.

[Margin: The method by which, from the horizon-arcs of tables 14 and 16, the points of the parallels are found on a horizontal dial.]

DEMONSTRATION. The proof of this matter is exactly the same as that of the hour-lines drawn from centre G — through the horizon-arcs of table 6, in the circle described from G — in chapter 5. For the great circles that cut off the arcs of tables 14 and 16 all pass through points D and B (the gnomon C D being set upright to the dial), just as the hour-circles are drawn through points D and G, and so on.

But if one prefers to carry out the same thing by Tangents, without describing a circle from B, then the Tangents of the arcs of tables 14 and 16 must be transferred onto the line of the 6th hour K G L from point G, and onto the line R a from point R, in the opposite direction — the Tangents, I say, reckoned with respect to the whole sine B G divided into 10 equal parts, just as the Tangents belonging to the complements of the arcs of table 6, transferred onto the equinoctial line with respect to the same whole sine G B, mark out the points of the hours, as is clear from what we wrote in the scholium to chapter 25, number 5.

Nor indeed shall I pass over this point either: that the points of the parallels can be found still more precisely from table 2 (whose use I set forth in chapter 8, number 10, and chapter 18, number 6), should the hour-point assumed on the equinoctial happen to lie too close to the meridian line. The method is this. On the equinoctial, beyond the meridian line, take a portion equal to the distance between the assumed hour-point and the meridian line — or two such portions, or more — and through its end draw a line parallel to the meridian. Then, on the horizontal dial, from the point assumed on the equinoctial through the place of the stylus C — but on the declining dial through point B, where the meridian intersects the horizontal — draw a straight line cutting the aforesaid parallel at a point, which is to be marked. For if from this point, whether upward or downward, one transfers onto that parallel double, triple, and so on, the intervals of the Tangents which are to be carried from the place of the stylus C on the

horizontal dial (or from B on the declining dial) into the meridian — according to the prescription of chapter 8, number 10, and chapter 18, number 6 — that is, in proportion as the distance between the point assumed on the equinoctial and the aforesaid parallel is double, or triple, and so on, of the distance between the same assumed point and the meridian line — then straight lines must be drawn from that assumed point through the points on the meridian and on that parallel, so that for each straight line three points are obtained. This practice will be demonstrated from proposition 4 of book 6. For example: suppose the operation is to be carried out from the 2nd hour on the equinoctial. Since the arc of Cancer [♋] for the 5th and 11th hours, in table 2, contains 17 degrees 5 minutes, when this is subtracted from the altitude of the pole on the horizontal dial there remain 24 degrees 55 minutes. The Tangent of these, $4\frac{6}{10}$, transferred from C downward onto that parallel, gives the point through which the straight line from the 2nd hour on the equinoctial cuts the 11th and 5th hours at the points of Cancer [♋], as was said in chapter 8, number 10. But because the 2nd hour lies close to the meridian, take the distance between that hour and the meridian twice over beyond the meridian, and through its end draw a line parallel to the meridian, to be cut by the straight line drawn from the 2nd hour through C. For if from the point of intersection one transfers downward the aforesaid Tangent $4\frac{6}{10}$ tripled, another point will be found on that parallel, and so on. On the declining dial, this same arc of 17 degrees 5 minutes must be subtracted from the complement of the pole's altitude, and so on, as was said in chapter 18, number 6.

[Margin: How the use of table 2 may be made more precise for finding the points of the parallels.]

TABLE OF TANGENTS, THE WHOLE SINE BEING SET AT 1000

(TABVLA TANGENTIVM POSITO SINV TOTO 1000.)

Table of Tangents — Degrees of the quadrant 0–9

Column headers 0–9 are **degrees** (Gradus Quadrantis); the far-left column is minutes of the degrees of the quadrant (Minuta Graduum Quadrantis, read downward 0→60); the far-right column is minutes of the degrees of the complements (Minuta Graduum Complementorum, read upward). The degree labels along the foot (Gradus Complementorum) are 89 88 87 86 85 84 83 82 81 80.

													Min.
Min. (Quadr.)	0		1	2	3	4	5		6	7	8	9	(Compl.)
	0	0	17	3	5	7	8		105	1	1	1	60

Min. (Quadr.)	0	1	2	3	4	5	6	7	8	9	Min. (Compl.)
			5	2	0	7		2	4	5	
								3	1	8	
1	0	18	3	5	7	8	105	1	1	1	59
			5	3	0	8		2	4	5	
								3	1	9	
2	1	18	3	5	7	8	106	1	1	1	58
			6	3	1	8		2	4	5	
								3	1	9	
3	1	18	3	5	7	8	106	1	1	1	57
			6	3	1	8		2	4	5	
								4	1	9	
4	1	19	3	5	7	8	106	1	1	1	56
			6	4	1	9		2	4	6	
								4	2	0	
5	1	19	3	5	7	8	107	1	1	1	55
			6	4	1	9		2	4	6	
								4	2	0	
6	2	19	3	5	7	8	107	1	1	1	54
			7	4	2	9		2	4	6	
								5	2	0	
7	2	19	3	5	7	9	107	1	1	1	53
			7	4	2	0		2	4	6	
								5	3	0	
8	2	20	3	5	7	9	107	1	1	1	52
			7	5	2	0		2	4	6	
								5	3	1	
9	3	20	3	5	7	9	108	1	1	1	51
			8	5	3	0		2	4	6	
								5	3	1	
10	3	20	3	5	7	9	118 [recte: 108]	1	1	1	50
			8	5	3	0		2	4	6	
								6	4	1	
11	3	21	3	5	7	9	108	1	1	1	49
			8	6	3	1		2	4	6	

Min. (Quadr.)	0	1	2	3	4	5	6	7	8	9	Min. (Compl.)
								6	4	2	
12	3	21	3	5	7	9	109	1	1	1	48
			8	6	3	1		2	4	6	
								6	4	2	
13	4	21	3	5	7	9	109	1	1	1	47
			9	6	4	1		2	4	6	
								7	4	2	
14	4	22	3	5	7	9	109	1	1	1	46
			9	6	4	2		2	4	6	
								7	5	3	
15	4	22	3	5	7	9	110	1	1	1	45
			9	7	4	2		2	4	6	
								7	5	3	
16	5	22	4	5	7	9	110	1	1	1	44
			0	7	5	2		2	4	6	
								8	5	3	
17	5	22	4	5	7	9	110	1	1	1	43
			0	7	5	2		2	4	6	
								8	6	3	
18	5	23	4	5	7	9	110	1	1	1	42
			0	8	5	3		2	4	6	
								8	6	4	
19	6	23	4	5	7	9	111	1	1	1	41
			0	8	5	3		2	4	6	
								8	6	4	
20	6	23	4	5	7	9	111	1	1	1	40
			1	8	6	3		2	4	6	
								9	6	4	
21	6	24	4	5	7	9	111	1	1	1	39
			1	9	6	4		2	4	6	
								9	7	5	
22	6	24	4	5	7	9	112	1	1	1	38
			1	9	6	4		2	4	6	
								9	7	5	

Min. (Quadr.)	0	1	2	3	4	5	6	7	8	9	Min. (Compl.)
23	7	24	4	5	7	9	112	1	1	1	37
			2	9	7	4		3	4	6	
								0	7	5	
24	7	24	4	5	7	9	112	1	1	1	36
			2	9	7	5		3	4	6	
								0	8	6	
25	7	25	4	6	7	9	112	1	1	1	35
			2	0	7	5		3	4	6	
								0	8	6	
26	8	25	4	6	7	9	113	1	1	1	34
			2	0	8	5		3	4	6	
								0	8	6	
27	8	25	4	6	7	9	113	1	1	1	33
			3	0	8	5		3	4	6	
								1	9	6	
28	8	26	4	6	7	9	113	1	1	1	32
			3	1	8	6		3	4	6	
								1	9	7	
29	8	26	4	6	7	9	114	1	1	1	31
			3	1	8	6		3	4	6	
								1	9	7	
30	9	25 [recte: 26]	4	6	7	9	114	1	1	1	30
			4	1	9	6		3	4	6	
								2	9	7	
31	9	26	4	6	7	9	114	1	1	1	29
			4	1	9	7		3	5	6	
								2	0	8	
32	9	27	4	6	7	9	115	1	1	1	28
			4	2	9	7		3	5	6	
								2	0	8	
33	1	27	4	6	8	9	115	1	1	1	27
	0		5	2	0	7		3	5	6	
								3	0	8	
34	1	27	4	6	8	9	115	1	1	1	26

Min. (Quadr.)	0	1	2	3	4	5	6	7	8	9	Min. (Compl.)
	0		5	2	0	7		3	5	6	
								3	1	9	
35	1	28	4	6	8	9	115	1	1	1	25
	0		5	3	0	8		3	5	6	
								3	1	9	
36	1	28	4	6	8	9	116	1	1	1	24
	0		5	3	0	8		3	5	6	
								3	1	9	
37	1	28	4	6	8	9	116	1	1	1	23
	1		6	3	1	8		3	5	6	
								4	2	9	
38	1	29	4	6	8	9	116	1	1	1	22
	1		6	3	1	9		3	5	7	
								4	2	0	
39	1	29	4	6	8	9	117	1	1	1	21
	1		6	4	1	9		3	5	7	
								4	2	0	
40	1	29	4	6	8	9	117	1	1	1	20
	2		7	4	2	9		3	5	7	
								5	2	0	
41	1	29	4	6	8	1	117	1	1	1	19
	2		7	4	2	0		3	5	7	
						0		5	3	1	
42	1	30	4	6	8	1	117	1	1	1	18
	2		7	5	2	0		3	5	7	
						0		5	3	1	
43	1	30	4	6	8	1	118	1	1	1	17
	3		7	5	3	0		3	5	7	
						0		6	3	1	
44	1	30	4	6	8	1	118	1	1	1	16
	3		8	5	3	0		3	5	7	
						0		6	4	2	
45	1	31	4	6	8	1	118	1	1	1	15
	3		8	6	3	0		3	5	7	

Min. (Quadr.)	0	1	2	3	4	5	6	7	8	9	Min. (Compl.)
						1		6	4	2	
46	1	31	4	6	8	1	119	1	1	1	14
	3		8	6	3	0		3	5	7	
						1		6	4	2	
47	1	31	4	6	8	1	119	1	1	1	13
	4		9	6	4	0		3	5	7	
						1		7	5	2	
48	1	31	4	6	8	1	119	1	1	1	12
	4		9	6	4	0		3	5	7	
						2		7	5	3	
49	1	32	4	6	8	1	120	1	1	1	11
	4		9	7	4	0		3	5	7	
						2		7	5	3	
50	1	32	4	6	8	1	120	1	1	1	10
	5		9	7	5	0		3	5	7	
						2		8	5	3	
51	1	32	5	6	8	1	120	1	1	1	9
	5		0	7	5	0		3	5	7	
						2		8	6	4	
52	1	33	5	6	8	1	120	1	1	1	8
	5		0	8	5	0		3	5	7	
						3		8	6	4	
53	1	33	5	6	8	1	121	1	1	1	7
	5		0	8	5	0		3	5	7	
						3		8	6	4	
54	1	33	5	6	8	1	121	1	1	1	6
	6		1	8	6	0		3	5	7	
						3		9	7	5	
55	1	33	5	6	8	1	121	1	1	1	5
	6		1	8	6	0		3	5	7	
						4		9	7	5	
56	1	34	5	6	8	1	122	1	1	1	4
	6		1	9	6	0		3	5	7	
						4		9	7	5	

Min. (Quadr.)	0	1	2	3	4	5	6	7	8	9	Min. (Compl.)
57	1	34	5	6	8	1	122	1	1	1	3
	7		2	9	7	0		4	5	7	
					4			0	7	5	
58	1	34	5	6	8	1	122	1	1	1	2
	7		2	9	7	0		4	5	7	
					5			0	8	6	
59	1	35	5	7	8	1	122	1	1	1	1
	7		2	0	7	0		4	5	7	
					5			1	8	6	
60	1	35	5	7	8	1	123	1	1	1	0
	7		2	0	7	0		4	5	7	
					5			1	8	6	
Gradus	8	88	8	8	8	8	83	8	8	8	
Compl. →	9		7	6	5	4		2	1	0	

Table of Tangents — Degrees of the quadrant 10–19

Same layout; degree headers 10–19 (Gradus Quadrantis), complementary degree labels at the foot (Gradus Complementorum) 79 78 77 76 75 74 73 72 71 70.

Min. (Quadr.)	0	1	1	1	1	1	1	1	1	1	Min. (Compl.)
0	1	1	2	2	2	2	2	3	3	3	60
	7	9	1	3	4	6	8	0	2	4	
	6	4	3	1	9	8	7	6	5	4	
1	1	1	2	2	2	2	2	3	3	3	59
	7	9	1	3	5	6	8	0	2	4	
	7	5	3	1	0	8	7	6	5	5	
2	1	1	2	2	2	2	2	3	3	3	58
	7	9	1	3	5	6	8	0	2	4	
	7	5	3	1	0	9	7	6	6	5	
3	1	1	2	2	2	2	2	3	3	3	57
	7	9	1	3	5	6	8	0	2	4	
	7	5	3	2	0	9	8	7	6	5	
4	1	1	2	2	2	2	2	3	3	3	56
	7	9	1	3	5	6	8	0	2	4	

	1	1	1	1	1	1	1	1	1	1	
Min. (Quadr.)	0	1	2	3	4	5	6	7	8	9	Min. (Compl.)
5	8	6	4	2	1	9	8	7	6	6	55
	1	1	2	2	2	2	2	3	3	3	
	7	9	1	3	5	7	8	0	2	4	
6	8	6	4	2	1	0	8	7	7	6	54
	1	1	2	2	2	2	2	3	3	3	
	7	9	1	3	5	7	8	0	2	4	
7	8	6	4	3	1	0	9	8	7	6	53
	1	1	2	2	2	2	2	3	3	3	
	7	9	1	3	5	7	8	0	2	4	
8	8	6	5	3	1	0	9	8	7	7	52
	1	1	2	2	2	2	2	3	3	3	
	7	9	1	3	5	7	8	0	2	4	
9	9	7	5	3	2	0	9	8	7	7	51
	1	1	2	2	2	2	2	3	3	3	
	7	9	1	3	5	7	9	0	2	4	
10	9	7	5	4	2	1	0	9	7	7	50
	1	1	2	2	2	2	2	3	3	3	
	7	9	1	3	5	7	9	0	2	4	
11	9	7	6	4	2	1	0	9	8	8	49
	1	1	2	2	2	2	2	3	3	3	
	8	9	1	3	5	7	9	0	2	4	
12	0	8	6	4	3	1	0	9	8	8	48
	1	1	2	2	2	2	2	3	3	3	
	8	9	1	3	5	7	9	1	2	4	
13	0	8	6	5	3	2	1	0	9	8	47
	1	1	2	2	2	2	2	3	3	3	
	8	9	1	3	5	7	9	1	2	4	
14	0	8	7	5	3	2	1	0	9	9	46
	1	1	2	2	2	2	2	3	3	3	
	8	9	1	3	5	7	9	1	2	4	
15	1	9	7	5	4	2	1	0	9	9	45
	1	1	2	2	2	2	2	3	3	3	
	8	9	1	3	5	7	9	1	3	4	
	1	9	7	5	4	3	1	1	0	9	

		1	1	1	1	1	1	1	1	1		
	Min. (Quadr.)	0	1	2	3	4	5	6	7	8	9	Min. (Compl.)
	16	1	1	2	2	2	2	2	3	3	3	44
		8	9	1	3	5	7	9	1	3	5	
		1	9	7	6	4	3	2	1	0	0	
	17	1	2	2	2	2	2	2	3	3	3	43
		8	0	1	3	5	7	9	1	3	5	
		1	0	8	6	5	3	2	1	0	0	
	18	1	2	2	2	2	2	2	3	3	3	42
		8	0	1	3	5	7	9	1	3	5	
		2	0	8	6	5	4	2	1	1	0	
	19	1	2	2	2	2	2	2	3	3	3	41
		8	0	1	3	5	7	9	1	3	5	
		2	0	8	7	5	4	3	2	1	1	
	20	1	2	2	2	2	2	2	3	3	3	40
		8	0	1	3	5	7	9	1	3	5	
		2	0	9	7	6	4	3	2	1	1	
	21	1	2	2	2	2	2	2	3	3	3	39
		8	0	1	3	5	7	9	1	3	5	
		3	1	9	7	6	5	3	2	2	1	
	22	1	2	2	2	2	2	2	3	3	3	38
		8	0	1	3	5	7	9	1	3	5	
		3	1	9	8	6	5	4	3	2	2	
	23	1	2	2	2	2	2	2	3	3	3	37
		8	0	2	3	5	7	9	1	3	5	
		3	1	0	8	6	5	4	3	2	2	
	24	1	2	2	2	2	2	2	3	3	3	36
		8	0	2	3	5	7	9	1	3	5	
		4	2	0	8	7	5	4	3	3	2	
	25	1	2	2	2	2	2	2	3	3	3	35
		8	0	2	3	5	7	9	1	3	5	
		4	2	0	9	7	6	5	4	3	2	
	26	1	2	2	2	2	2	2	3	3	3	34
		8	0	2	3	5	7	9	1	3	5	
		4	2	0	9	7	6	5	4	3	3	
	27	1	2	2	2	2	2	2	3	3	3	33

	1	1	1	1	1	1	1	1	1	1	
Min. (Quadr.)	0	1	2	3	4	5	6	7	8	9	Min. (Compl.)
	8	0	2	3	5	7	9	1	3	5	
	4	3	1	9	8	6	5	4	4	3	
28	1	2	2	2	2	2	2	3	3	3	32
	8	0	2	3	5	7	9	1	3	5	
	5	3	1	9	8	7	6	5	4	3	
29	1	2	2	2	2	2	2	3	3	3	31
	8	0	2	4	5	7	9	1	3	5	
	5	3	1	0	8	7	6	5	4	4	
30	1	2	2	2	2	2	2	3	3	3	30
	8	0	2	4	5	7	9	1	3	5	
	5	3	2	0	9	7	6	5	5	4	
31	1	2	2	2	2	2	2	3	3	3	29
	8	0	2	4	5	7	9	1	3	5	
	6	4	2	0	9	8	7	6	5	4	
32	1	2	2	2	2	2	2	3	3	3	28
	8	0	2	4	5	7	9	1	3	5	
	6	4	2	1	9	8	7	6	5	5	
33	1	2	2	2	2	2	2	3	3	3	27
	8	0	2	4	6	7	9	1	3	5	
	6	4	3	1	0	8	7	6	6	5	
34	1	2	2	2	2	2	2	3	3	3	26
	8	0	2	4	6	7	9	1	3	5	
	7	5	3	1	0	9	7	7	6	5	
35	1	2	2	2	2	2	2	3	3	3	25
	8	0	2	4	6	7	9	1	3	5	
	7	5	3	2	0	9	8	7	6	6	
36	1	2	2	2	2	2	2	3	3	3	24
	8	0	2	4	6	7	9	1	3	5	
	7	5	4	2	1	9	8	7	7	6	
37	1	2	2	2	2	2	2	3	3	3	23
	8	0	2	4	6	8	9	1	3	5	
	7	6	4	2	1	0	8	8	7	6	
38	1	2	2	2	2	2	2	3	3	3	22
	8	0	2	4	6	8	9	1	3	5	

	1	1	1	1	1	1	1	1	1	1	
Min. (Quadr.)	0	1	2	3	4	5	6	7	8	9	Min. (Compl.)
	8	6	4	3	1	0	9	8	7	7	
39	1	2	2	2	2	2	2	3	3	3	21
	8	0	2	4	6	8	9	1	3	5	
	8	6	4	3	1	0	9	8	8	7	
40	1	2	2	2	2	2	2	3	3	3	20
	8	0	2	4	6	8	9	1	3	5	
	8	6	5	3	2	0	9	8	8	7	
41	1	2	2	2	2	2	3	3	3	3	19
	8	0	2	4	6	8	0	1	3	5	
	9	7	5	3	2	1	0	9	8	8	
42	1	2	2	2	2	2	3	3	3	3	18
	8	0	2	4	6	8	0	1	3	5	
	9	7	5	4	2	1	0	9	8	8	
43	1	2	2	2	2	2	3	3	3	3	17
	8	0	2	4	6	8	0	1	3	5	
	9	7	6	4	3	1	0	9	9	8	
44	1	2	2	2	2	2	3	3	3	3	16
	9	0	2	4	6	8	0	2	3	5	
	0	8	6	4	3	2	1	0	9	9	
45	1	2	2	2	2	2	3	3	3	3	15
	9	0	2	4	6	8	0	2	3	5	
	0	8	6	5	3	2	1	0	9	9	
46	1	2	2	2	2	2	3	3	3	3	14
	9	0	2	4	6	8	0	2	4	5	
	0	8	7	5	4	2	1	0	0	9	
47	1	2	2	2	2	2	3	3	3	3	13
	9	0	2	4	6	8	0	2	4	6	
	0	9	7	5	4	3	2	1	0	0	
48	1	2	2	2	2	2	3	3	3	3	12
	9	0	2	4	6	8	0	2	4	6	
	1	9	7	6	4	3	2	1	0	0	
49	1	2	2	2	2	2	3	3	3	3	11
	9	0	2	4	6	8	0	2	4	6	
	1	9	8	6	5	3	2	1	1	0	

[illegible]

	1	1	1	1	1	1	1	1	1	1	
Min. (Quadr.)	0	1	2	3	4	5	6	7	8	9	Min. (Compl.)
	9	8	7	6	5	4	3	2	1	0	

Table of Tangents (whole sine 1000) — continued

Appended table of tangents to Clavius's Horologiorum Nova Descriptio (the third work in the volume), whole sine (sinus totus) = 1000. Degrees of the quadrant run across the top; minutes of a degree (0–60) down the left margin (Minuta Graduum Quadrantis); the complementary degrees run across the foot (Gradus Complementorum) and complementary minutes down the right margin (Minuta Graduum Complementorum), so a single grid serves a degree read from the top and its complement read from the bottom. This portion covers degrees 20–49, continuing the table begun earlier (degrees 0–19).

degrees 20–29

Top degree-headers (verbatim): **20 21 22 23 24 25 26 27 28 29** Foot
complementary-degree headers (verbatim): **69 68 67 66 65 64 63 62 61 60**

M in .		20°	2°	2°	2°	2°	2°	2°		28°	2°	Min. (compl.)
0	364	3	4	4	4	4	4	5		532	5	60
		8	0	2	4	6	8	1			5	
		4	4	4	5	6	8	0			4	
1	364	3	4	4	4	4	4	5		532	5	59
		8	0	2	4	6	8	1			5	
		4	4	5	6	7	8	0			5	
2	365	3	4	4	4	4	4	5		532	5	58
		8	0	2	4	6	8	1			5	
		5	5	5	6	7	8	0			5	
3	365	3	4	4	4	4	4	5		533	5	57
		8	0	2	4	6	8	1			5	
		5	5	6	6	7	9	1			5	
4	365	3	4	4	4	4	4	5		533	5	56
		8	0	2	4	6	8	1			5	
		5	5	6	7	8	9	1			6	

M in .		20°	2 1°	2 2°	2 3°	2 4°	2 5°	2 6°	2 7°		2 28°	2 9°	Min. (compl.)
5		366	3 8 6	4 0 6	4 2 6	4 4 7	4 6 8	4 9 0	5 1 1		534	5 5 6	55
6	356 [recte: 366]		3 8 6	4 0 6	4 2 7	4 4 7	4 6 8	4 9 0	5 1 2		534	5 5 7	54
7		366	3 8 6	4 0 6	4 2 7	4 4 8	4 6 9	4 9 0	5 1 2		534	5 5 7	53
8		367	3 8 7	4 0 7	4 2 7	4 4 8	4 6 9	4 9 1	5 1 2		535	5 5 7	52
9		367	3 8 7	4 0 7	4 2 8	4 4 8	4 6 9	4 9 1	5 1 3		535	5 5 8	51
1 0		367	3 8 7	4 0 7	4 2 8	4 4 9	4 7 0	4 9 1	5 1 3		535	5 5 8	50
1 1		368	3 8 8	4 0 8	4 2 8	4 4 9	4 7 0	4 9 2	5 1 4		536	5 5 8	49
1 2		368	3 8 8	4 0 8	4 2 9	4 4 9	4 7 1	4 9 2	5 1 4		536	5 5 9	48
1 3		368	3 8 8	4 0 8	4 2 9	4 4 0	4 7 1	4 9 2	5 1 4		537	5 5 9	47
1 4	299 [recte: 369]		3 8 9	4 0 9	4 2 9	4 4 0	4 7 1	4 9 3	5 1 5		537	5 6 0	46
1 5		369	3 8 9	4 0 9	4 3 0	4 5 0	4 7 2	4 9 3	5 1 5		537	5 6 0	45

M in .	20°	2 1°	2 2°	2 3°	2 4°	2 5°	2 6°	2 7°		2 28°	2 9°	Min. (compl.)
1 6	369	3 8 9	4 0 9	4 3 0	4 5 1	4 7 2	4 9 3	5 1 5		538	5 6 0	44
1 7	370	3 9 0	4 1 0	4 3 0	4 5 1	4 7 2	4 9 4	5 1 6		538	5 6 1	43
1 8	370	3 9 0	4 1 0	4 3 1	4 5 1	4 7 3	4 9 4	5 1 6		538	5 6 1	42
1 9	370	3 9 0	4 1 0	4 3 1	4 5 2	4 7 3	4 9 5	5 1 7	439 [recte: 539]	5 6 2		41
2 0	371	3 9 1	4 1 1	4 3 1	4 5 2	4 7 4	4 9 5	5 1 7		539	5 6 2	40
2 1	371	3 9 1	4 1 1	4 3 2	4 5 2	4 7 4	4 9 5	5 1 7		540	5 6 2	39
2 2	371	3 9 1	4 1 1	4 3 2	4 5 3	4 7 4	4 9 6	5 1 8		540	5 6 3	38
2 3	372	3 9 2	4 1 2	4 3 2	4 5 3	4 7 5	4 9 6	5 1 8		540	5 6 3	37
2 4	372	3 9 2	4 1 2	4 3 3	4 5 3	4 7 5	4 9 6	5 1 8		541	5 6 3	36
2 5	372	3 9 2	4 1 3	4 3 3	4 5 4	4 7 6	4 9 7	5 1 9		541	5 6 4	35
2 6	373	3 9 3	4 1 3	4 3 3	4 5 4	4 7 6	4 9 7	5 1 9		541	5 6 4	34

M in .		20°	2 1°	2 2°	2 3°	2 4°	2 5°	2 6°	2 7°		2 28°	2 9°	Min. (compl.)
2 7	273 [recte: 373]		3 9 3	4 1 3	4 3 4	4 5 4	4 7 6	4 9 7	5 1 9		542	5 6 5	33
2 8	373		3 9 3	4 1 4	4 3 4	4 5 5	4 7 7	4 9 8	5 2 0		542	5 6 5	32
2 9	373		3 9 4	4 1 4	4 3 4	4 5 5	4 7 7	4 9 8	5 2 0		543	5 6 5	31
3 0	374		3 9 4	4 1 4	4 3 5	4 5 5	4 7 7	4 9 9	5 2 1		543	5 6 6	30
3 1	374		3 9 4	4 1 5	4 3 5	4 5 6	4 7 7	4 9 9	5 2 1		543	5 6 6	29
3 2	375		3 9 5	4 1 5	4 3 6	4 5 6	4 7 8	4 9 9	5 2 1		544	5 6 7	28
3 3	375		3 9 5	4 1 5	4 3 6	4 5 7	4 7 8	5 0 0	5 2 2		544	5 6 7	27
3 4	375		3 9 5	4 1 6	4 3 6	4 5 7	4 7 8	5 0 0	5 2 2		544	5 6 7	26
3 5	376		3 9 6	4 1 6	4 3 7	4 5 7	4 7 9	5 0 1	5 2 2		545	5 6 8	25
3 6	376		3 9 6	4 1 6	4 3 7	4 5 8	4 7 9	5 0 1	5 2 3		545	5 6 8	24
3 7	376		3 9 6	4 1 7	4 3 7	4 5 8	4 7 9	5 0 1	5 2 3		546	5 6 8	23

M in .	20°	2 1°	2 2°	2 3°	2 4°	2 5°	2 6°	2 7°	28°	2 9°	Min. (compl.)
3 8	377	3 9 7	4 1 7	4 3 8	4 5 9	4 8 0	5 0 2	5 2 4	546	5 6 9	22
3 9	377	3 9 7	4 1 7	4 3 8	4 5 9	4 8 0	5 0 2	5 2 4	546	5 6 9	21
4 0	377	3 9 7	4 1 8	4 3 8	4 5 9	4 8 1	5 0 3	5 2 4	547	5 7 0	20
4 1	378	3 9 8	4 1 8	4 3 9	4 6 0	4 8 1	5 0 3	5 2 5	547	5 7 0	19
4 2	378	3 9 8	4 1 8	4 3 9	4 6 0	4 8 1	5 0 3	5 2 5	547	5 7 0	18
4 3	378	3 9 8	4 1 9	4 3 9	4 6 0	4 8 2	5 0 4	5 2 5	548	5 7 1	17
4 4	379	3 9 9	4 1 9	4 4 0	4 6 1	4 8 2	5 0 4	5 2 6	548	5 7 1	16
4 5	379	3 9 9	4 1 9	4 4 0	4 6 1	4 8 3	5 0 5	5 2 6	549	5 7 2	15
4 6	379	3 9 9	4 2 0	4 4 0	4 6 1	4 8 3	5 0 5	5 2 6	549	5 7 2	14
4 7	380	4 0 0	4 2 0	4 4 1	4 6 2	4 8 3	5 0 5	5 2 7	549	5 7 2	13
4 8	380	4 0 0	4 2 0	4 4 1	4 6 2	4 8 3	5 0 6	5 2 7	550	5 7 3	12

M in .	20°	2 1°	2 2°	2 3°	2 4°	2 5°	2 6°	2 7°	28°	2 9°	Min. (compl.)
4 9	380	4 0 0	4 2 1	4 4 1	4 6 2	4 8 4	5 0 6	5 2 7	550	5 7 3	11
5 0	381	4 0 1	4 2 1	4 4 2	4 6 3	4 8 4	5 0 6	5 2 8	551	5 7 3	10
5 1	381	4 0 1	4 2 1	4 4 2	4 6 3	4 8 4	5 0 7	5 2 8	551	5 7 4	9
5 2	381	4 0 1	4 2 2	4 4 2	4 6 3	4 8 5	5 0 7	5 2 8	551	5 7 4	8
5 3	382	4 0 2	4 2 2	4 4 3	4 6 4	4 8 5	5 0 8	5 2 9	552	5 7 4	7
5 4	382	4 0 2	4 2 2	4 4 3	4 6 4	4 8 6	5 0 8	5 2 9	552	5 7 5	6
5 5	382	4 0 2	4 2 3	4 4 3	4 6 5	4 8 6	5 0 8	5 2 0	552	5 7 5	5
5 6	383	4 0 3	4 2 3	4 4 4	4 6 5	4 8 6	5 0 9	5 2 0	553	5 7 5	4
5 7	383	4 0 3	4 2 3	4 4 4	4 6 5	4 8 7	5 0 9	5 2 1	553	5 7 6	3
5 8	383	4 0 3	4 2 4	4 4 5	4 6 6	4 8 7	5 1 0	5 2 1	554	5 7 6	2
5 9	384	4 0 4	4 2 4	4 4 5	4 6 6	4 8 7	5 1 0	5 2 1	554	5 7 7	1

M in .	20°	2 1°	2 2°	2 3°	2 4°	2 5°	2 6°	2 7°		2 28°	2 9°	Min. (compl.)
6	384	4	4	4	4	4	5	5		554	5	0
0		0	2	4	6	8	1	3			7	
		4	4	5	6	8	0	2			7	

Foot: **Gradus Complementorum.** — complementary degrees **69 68 67 66 65 64 63 62 61 60.**

degrees 30–39

Top degree-headers (verbatim): **30 31 32 33 34 35 36 37 28 39** — the ninth header is printed **28** where the sequence requires **38** [recte: 38]; the column's values (781...810) are those of tangent 38°, so the header digit is a compositor error. Foot complementary-degree headers (verbatim): **59 58 57 56 55 54 53 52 51 50**

M in .	30°		31°	3 2°	3 3°	3 4°	3 5°	3 6°	3 7°	3 8°	3 9°	Min. (compl.)
0	577		601	6	6	6	7	7	7	7	8	60
				2	4	7	0	2	5	8	1	
				5	9	5	0	7	4	1	0	
1	578		601	6	6	6	7	7	7	7	8	59
				2	5	7	0	2	5	8	1	
				5	0	5	1	7	4	2	0	
2	578		602	6	6	6	7	7	7	7	8	58
				2	5	7	0	2	5	8	1	
				6	0	5	1	7	4	2	1	
3	579		602	6	6	6	7	7	7	7	8	57
				2	5	7	0	2	5	8	1	
				6	1	6	2	8	5	3	1	
4	579		602	6	6	6	7	7	7	7	8	56
				2	5	7	0	2	5	8	1	
				7	1	6	2	8	5	3	2	
5	579		603	6	6	6	7	7	7	7	8	55
				2	5	7	0	2	5	8	1	
				7	1	7	2	9	6	4	2	

M in .	30°	31°	3 2°	3 3°	3 4°	3 5°	3 6°	3 7°	3 8°	3 9°	Min. (compl.)
6	580	603	6 2 8	6 5 2	6 7 3	7 0 3	7 2 9	7 5 6	7 8 4	8 1 3	54
7	580	604	6 2 8	6 5 2	6 7 3	7 0 3	7 3 0	7 5 7	7 8 5	8 1 3	53
8	580	604	6 2 8	6 5 3	6 7 8	7 0 4	7 3 0	7 5 7	7 8 5	8 1 4	52
9	581	604	6 2 9	6 5 3	6 7 8	7 0 4	7 3 1	7 5 8	7 8 6	8 1 4	51
1 0	581	605	6 2 9	6 5 4	6 7 9	7 0 5	7 3 1	7 5 8	7 8 6	8 1 5	50
1 1	582	605	6 3 0	6 5 4	6 7 9	7 0 5	7 3 1	7 5 9	7 8 6	8 1 5	49
1 2	582	606	6 3 0	6 5 4	6 7 0	7 0 5	7 3 2	7 5 9	7 8 7	8 1 6	48
1 3	582	606	6 3 0	6 5 5	6 7 0	7 0 6	7 3 2	7 5 9	7 8 7	8 1 6	47
1 4	583	606	6 3 1	6 5 5	6 7 0	7 0 6	7 3 3	7 6 0	7 8 8	8 1 7	46
1 5	583	607	6 3 1	6 5 6	6 7 1	7 0 7	7 3 3	7 6 0	7 8 8	8 1 7	45
1 6	584	607	6 3 1	6 5 6	6 7 1	7 0 7	7 4 1	7 6 1	7 8 9	8 1 8	44

M in .	30°	31°	3 2°	3 3°	3 4°	3 5°	3 6°	3 7°	3 8°	3 9°	Min. (compl.)
1 7	584	608	6 3 2	6 5 6	6 8 2	7 0 8	7 3 4	7 6 1	7 8 9	8 1 8	43
1 8	584	608	6 3 2	6 5 7	6 8 2	7 0 8	7 3 5	7 6 2	7 9 0	8 1 8	42
1 9	585	608	6 3 3	6 5 7	6 8 2	7 0 8	7 3 5	7 6 2	7 9 0	8 1 9	41
2 0	585	609	6 3 3	6 5 8	6 8 3	7 0 9	7 3 6	7 6 3	7 9 0	8 1 9	40
2 1	586	609	6 3 3	6 5 8	6 8 3	7 0 9	7 3 6	7 6 3	7 9 1	8 2 0	39
2 2	586	610	6 3 4	6 5 9	6 8 4	7 1 0	7 3 7	7 6 4	7 9 1	8 2 0	38
2 3	586	610	6 3 4	6 5 9	6 8 4	7 1 0	7 3 7	7 6 4	7 9 2	8 2 1	37
2 4	587	610	6 3 5	6 5 9	6 8 5	7 1 1	7 3 8	7 6 5	7 9 2	8 2 1	36
2 5	587	611	6 3 5	6 6 0	6 8 5	7 1 1	7 3 8	7 6 5	7 9 3	8 2 2	35
2 6	587	611	6 3 5	6 6 0	6 8 5	7 1 2	7 3 8	7 6 5	7 9 3	8 2 2	34
2 7	588	612	6 3 6	6 6 1	6 8 6	7 1 2	7 3 9	7 6 6	7 9 4	8 2 3	33

M in .		30°	31°	3 2°	3 3°	3 4°	3 5°	3 6°	3 7°	3 8°	3 9°	Min. (compl.)
2 8	488 [recte: 588]		612	6 3 6	6 6 1	6 8 6	7 1 2	7 3 9	7 6 6	7 9 4	8 2 3	32
2 9	589		612	6 3 7	6 6 1	6 8 7	7 1 3	7 4 0	7 6 7	7 9 5	8 2 4	31
3 0	589		613	6 3 7	6 6 2	6 8 7	7 1 3	7 4 0	7 6 7	7 9 5	8 2 4	30
3 1	589		613	6 3 7	6 6 2	6 8 8	7 1 4	7 4 0	7 6 8	7 9 6	8 2 5	29
3 2	590		614	6 3 8	6 6 3	6 8 8	7 1 4	7 4 1	7 6 8	7 9 6	8 2 5	28
3 3	590		614	6 3 9	6 6 3	6 8 9	7 1 5	7 4 1	7 6 9	7 9 7	8 2 6	27
3 4	591		614	6 3 9	6 6 4	6 8 9	7 1 5	7 4 2	7 6 9	7 9 7	8 2 6	26
3 5	591		615	6 3 9	6 6 4	6 8 9	7 1 5	7 4 2	7 7 0	7 9 8	8 2 6	25
3 6	591		615	6 4 0	6 6 4	6 9 0	7 1 6	7 4 3	7 7 0	7 9 8	8 2 7	24
3 7	592		616	6 4 0	6 6 5	6 9 0	7 1 6	7 4 3	7 7 1	7 9 9	8 2 7	23
3 8	592		616	6 4 0	6 6 5	6 9 1	7 1 7	7 4 4	7 7 1	7 9 9	8 2 8	22

M in .	30°	31°	3 2°	3 3°	3 4°	3 5°	3 6°	3 7°	3 8°	3 9°	Min. (compl.)
3 9	593	616	6 4 1	6 6 6	6 9 1	7 1 7	7 4 4	7 7 1	8 0 0	8 2 8	21
4 0	593	617	6 4 1	6 6 6	6 9 2	7 1 8	7 4 4	7 7 2	8 0 0	8 2 9	20
4 1	593	617	6 4 2	6 6 6	6 9 2	7 1 8	7 4 5	7 7 2	8 0 1	8 2 9	19
4 2	594	618	6 4 2	6 6 7	6 9 2	7 1 9	7 4 5	7 7 3	8 0 1	8 3 0	18
4 3	594	618	6 4 2	6 6 7	6 9 3	7 1 9	7 4 6	7 7 3	8 0 2	8 3 0	17
4 4	595	618	6 4 3	6 6 8	6 9 3	7 1 9	7 4 6	7 7 4	8 0 2	8 3 1	16
4 5	595	619	6 4 3	6 6 8	6 9 4	7 2 0	7 4 6	7 7 4	8 0 3	8 3 1	15
4 6	596	619	6 4 4	6 6 9	6 9 4	7 2 0	7 4 7	7 7 5	8 0 3	8 3 2	14
4 7	596	620	6 4 4	6 6 9	6 9 5	7 2 1	7 4 7	7 7 5	8 0 4	8 3 2	13
4 8	596	620	6 4 4	6 6 9	6 9 5	7 2 1	7 4 8	7 7 6	8 0 4	8 3 3	12
4 9	597	620	6 4 5	6 7 0	6 9 5	7 2 2	7 4 8	7 7 6	8 0 4	8 3 3	11

M in .	30°	31°	3 2°	3 3°	3 4°	3 5°	3 6°	3 7°	3 8°	3 9°	Min. (compl.)
5 0	597	621	6 4 5	6 7 0	6 9 6	7 2 2	7 4 9	7 7 7	8 0 5	8 3 4	10
5 1	597	621	6 4 6	6 7 1	6 9 6	7 2 3	7 4 9	7 7 7	8 0 5	8 3 4	9
5 2	598	622	6 4 6	6 7 1	6 9 7	7 2 3	7 5 0	7 7 8	8 0 6	8 3 5	8
5 3	598	622	6 4 7	6 7 2	6 9 7	7 2 3	7 5 0	7 7 8	8 0 6	8 3 5	7
5 4	598	622	6 4 7	6 7 2	6 8 8	7 2 4	7 5 1	7 7 8	8 0 7	8 3 6	6
5 5	599	623	6 4 8	6 7 2	6 8 8	7 2 4	7 5 1	7 7 9	8 0 7	8 3 6	5
5 6	599	623	6 4 8	6 7 3	6 8 8	7 2 5	7 5 2	7 7 9	8 0 8	8 3 7	4
5 7	600	624	6 4 9	6 7 3	6 9 9	7 2 5	7 8 2	7 7 0	8 0 8	8 3 7	3
5 8	600	624	6 4 9	6 7 4	6 9 9	7 2 6	7 8 3	7 7 0	8 0 9	8 3 8	2
5 9	600	524 [recte: 624]	6 4 9	6 7 4	7 0 0	7 2 6	7 5 3	7 8 1	8 0 9	8 3 8	1
6 0	601	625	6 4 9	6 7 5	7 0 0	7 2 7	7 5 4	7 7 1	8 1 0	8 3 9	0

Foot: **Gradus Complementorum.** — complementary degrees **59 58 57 56 55 54 53 52 51 50.**

degrees 40–49

Top degree-headers (verbatim): **40 41 42 43 44 45 46 47 48 49** Foot

complementary-degree headers (verbatim): **49 48 47 46 45 44 43 42 41 40**

Note: at 44°60' (= 45°0') the tangent equals the whole sine, 1000; from degree 45 onward the tabulated tangents exceed 1000, as expected.

Mi	40	41		43	44	45	46	47	48	49	
n.	°	°	42°	°	°	°	°	°	°	°	Min. (compl.)
0	83	86	900	93	96	10	10	10	11	11	60
	9	9		3	6	00	36	72	11	50	
1	84	87	901	93	96	10	10	10	11	11	59
	0	0		3	6	01	36	73	11	51	
2	84	87	901	93	96	10	10	10	11	11	58
	0	0		4	7	01	37	74	12	52	
3	84	87	902	93	96	10	10	10	11	11	57
	1	1		4	7	02	37	74	13	52	
4	84	87	902	93	96	10	10	10	11	11	56
	1	1		5	8	02	38	75	13	53	
5	84	87	903	93	96	10	10	10	11	11	55
	2	2		5	9	03	39	76	14	54	
6	84	87	904	93	96	10	10	10	11	11	54
	2	2		6	9	03	39	76	15	54	
7	84	87	904	93	97	10	10	10	11	11	53
	3	3		6	0	04	40	77	15	55	
8	84	87	905	93	97	10	10	10	11	11	52
	3	3		7	0	05	40	77	16	56	
9	84	87	905	93	97	10	10	10	11	11	51
	4	4		7	1	05	41	78	16	56	
10	84	87	906	93	97	10	10	10	11	11	50
	4	4		8	1	06	42	79	17	57	
11	84	87	906	93	97	10	10	10	11	11	49
	5	5		9	2	06	42	79	18	58	
12	84	87	907	93	97	10	10	10	11	11	48

Mi n.	40 °	41 °		42°	43 °	44 °	45 °	46 °	47 °	48 °	49 °	Min. (compl.)
	5	5			9	2	07	43	80	18	59	
13	84	87		907	94	97	10	10	10	11	11	47
	6	6			0	3	08	43	81	19	59	
14	84	87	808 [recte: 908]		94	97	10	10	10	11	11	46
	6	6			0	4	08	44	81	20	60	
15	84	87		908	94	97	10	10	10	11	11	45
	7	7			1	4	09	45	82	20	61	
16	84	87		909	94	97	10	10	10	11	11	44
	7	7			1	5	09	45	82	21	61	
17	84	87		909	94	97	10	10	10	11	11	43
	8	8			2	5	10	46	83	22	62	
18	84	87		910	94	97	10	10	10	11	11	42
	8	9			2	6	11	46	84	22	63	
19	84	87		910	94	97	10	10	10	11	11	41
	9	9			3	6	11	47	84	23	63	
20	84	88		911	94	97	10	10	10	11	11	40
	9	0			3	7	12	48	85	24	64	
21	85	88		912	94	97	10	10	10	11	11	39
	0	0			4	8	12	48	86	24	65	
22	85	88		912	94	97	10	10	10	11	11	38
	0	1			5	8	13	49	86	25	65	
23	85	88		913	94	97	10	10	10	11	11	37
	1	1			5	9	13	49	87	26	66	
24	85	88		913	94	97	10	10	10	11	11	36
	1	2			6	9	14	50	87	26	67	
25	85	88		914	94	98	10	10	10	11	11	35
	2	2			6	0	15	51	88	27	67	
26	85	88		914	94	98	10	10	10	11	11	34
	2	3			7	0	15	51	89	28	68	
27	85	88		915	94	98	10	10	10	11	11	33
	3	3			7	1	16	52	89	28	69	
28	85	88		915	94	98	10	10	10	11	11	32
	3	4			8	2	16	53	90	29	69	

Mi n.	40 °	41 °	42°	43 °	44 °	45 °	46 °	47 °	48 °	49 °	Min. (compl.)
29	85 4	88 4	916	94 8	98 2	10 17	10 53	10 91	11 30	11 70	31
30	85 4	88 5	916	94 9	98 3	10 18	10 54	10 91	11 30	11 71	30
31	85 5	88 5	917	95 0	98 3	10 18	10 54	10 92	11 31	11 72	29
32	85 5	88 6	917	95 0	98 4	10 19	10 55	10 93	11 32	11 72	28
33	85 6	88 6	918	95 1	98 4	10 19	10 56	10 93	11 32	11 73	27
34	85 6	88 7	918	95 1	98 5	10 20	10 56	10 94	11 33	11 74	26
35	85 7	88 7	919	95 2	98 5	10 21	10 57	10 95	11 34	11 74	25
36	85 7	88 8	919	95 2	98 6	10 21	10 57	10 96	11 34	11 75	24
37	85 8	88 8	920	95 3	98 7	10 22	10 58	10 96	11 35	11 76	23
38	85 8	88 9	920	95 3	98 7	10 22	10 59	10 96	11 36	11 76	22
39	85 9	88 9	921	95 4	98 8	10 23	10 59	10 97	11 36	11 77	21
40	85 9	89 0	922	95 5	98 8	10 24	10 60	10 98	11 37	11 78	20
41	86 0	89 0	922	95 5	98 9	10 24	10 61	10 98	11 38	11 78	19
42	86 0	89 1	923	95 6	99 0	10 25	10 61	10 99	11 38	11 79	18
43	86 1	89 1	923	95 6	99 0	10 25	10 62	11 00	11 39	11 80	17
44	86 1	89 2	924	95 7	99 1	10 26	10 62	11 00	11 40	11 81	16
45	86	89	924	95	99	10	10	11	11	11	15

Mi n.	40 °	41 °	42°	43 °	44 °	45 °	46 °	47 °	48 °	49 °	Min. (compl.)
	2	3		7	1	27	63	01	40	81	
46	86	89	925	95	99	10	10	11	11	11	14
	2	3		8	2	27	64	02	41	82	
47	86	89	925	95	99	10	10	11	11	11	13
	3	4		8	2	28	64	02	42	83	
48	86	89	926	95	99	10	10	11	11	11	12
	3	4		9	3	28	65	03	42	83	
49	86	89	927	96	99	10	10	11	11	11	11
	4	5		0	4	29	66	03	43	84	
50	86	89	927	96	99	10	10	11	11	11	10
	4	5		0	4	30	66	04	44	85	
51	86	89	928	96	99	10	10	11	11	11	9
	5	6		1	5	30	67	05	44	85	
52	86	89	928	96	99	10	10	11	11	11	8
	5	6		1	5	31	67	05	45	86	
53	86	89	929	96	99	10	10	11	11	11	7
	6	7		2	6	31	68	06	46	87	
54	86	89	929	96	99	10	10	11	11	11	6
	6	7		2	7	32	69	07	46	88	
55	86	89	930	96	99	10	10	11	11	11	5
	7	8		3	7	33	69	07	47	88	
56	86	89	930	96	99	10	10	11	11	11	4
	7	8		3	8	33	70	08	48	89	
57	86	89	931	96	99	10	10	11	11	11	3
	8	9		4	8	34	70	09	48	89	
58	86	89	931	96	99	10	10	11	11	11	2
	8	9		5	9	34	71	09	49	90	
59	86	90	932	96	99	10	10	11	11	11	1
	9	0		5	9	35	72	10	50	91	
60	86	90	933	96	10	10	10	11	11	11	0
	9	0		6	00	36	72	11	50	92	

Foot: **Gradus Complementorum.** — complementary degrees **49 48 47 46 45 44 43 42 41 40.**

The table of tangents ends here at degree 49 (its complement, 40, is reached at the foot). No further table (e.g. secants), colophon, or FINIS appears within these three pages; the tangent table continues on the following opening for the remaining degrees.

Table of Tangents [Tabula Tangentium] — Degrees of the Quadrant [Gradus Quadrantis]

Continuation of the appended Table of Tangents (whole sine = 1000). These three pages cover degrees 50–79. The layout of each page is unchanged: the left-hand column gives the minutes of a degree of the quadrant [Minuta Graduum Quadrantis], read downward from 0 at the top; the head-row gives the whole degree; the right-hand column gives the minutes of the complementary degree [Minuta Graduum Complementorum], read upward; and the foot-row gives the complementary degree [Gradus Complementorum]. Each degree column runs from its own minute 0 (top) to minute 60 (bottom), where minute 60 of one degree equals minute 0 of the next.

Every cell below is transcribed individually from the scan at high magnification. Values that break the smooth progression of their column are transcribed as printed, with the corrected reading given in [recte: ...].

Degrees 50–59

Head-row degrees: 50 51 52 53 54 55 56 57 58 59.

M i n .	5		5		5		5	5	5	5	5	Compl. min.
	50	1	52	3	54	5	6	7	8	9		
0	1192	1	1280	1	1376	1	1	1	1	1		60
		2		3		4	4	5	6	6		
		3		2		2	8	4	0	6		
		5		7		8	3	0	0	4		
1	1192	1	1281	1	1377	1	1	1	1	1		59
		2		3		4	4	5	6	6		
		3		2		2	8	4	0	6		
		6		8		9	3	1	1	5		
2	1193	1	1281	1	1378	1	1	1	1	1		58
		2		3		4	4	5	6	6		

M i n .		5 50	1	5 52	3		5 54	5	5 6	5 7	5 8	5 9	Compl. min.
			3		2			3	8	4	0	6	
			6		9			0	4	2	2	6	
3	1194	1		1282	1	1379	1	1	1	1	1		57
		2			3		4	4	5	6	6		
		3			2		3	8	4	0	6		
		7			9		1	5	3	3	8		
4	1195	1		1283	1	1380	1	1	1	1	1		56
		2			3		4	4	5	6	6		
		3			3		3	8	4	0	6		
		8			0		2	6	4	4	9		
5	1195	1		1284	1	1381	1	1	1	1	1		55
		2			3		4	4	5	6	6		
		3			3		3	8	4	0	7		
		9			1		3	7	5	6	0		
6	1196	1		1285	1	1381	1	1	1	1	1		54
		2			3		4	4	5	6	6		
		3			3		3	8	4	0	7		
		9			2		3	8	6	7	1		
7	1197	1		1285	1	1282 [recte: 1382]	1	1	1	1	1		53
		2			3		4	4	5	6	6		
		4			3		3	8	4	0	7		
		0			3		4	9	7	8	2		
8	1197	1		1286	1	1383	1	1	1	1	1		52
		2			3		4	4	5	6	6		
		4			3		3	9	4	0	7		
		1			3		5	0	8	9	3		
9	1198	1		1287	1	1384	1	1	1	1	1		51
		2			3		4	4	5	6	6		
		4			3		3	9	4	1	7		
		2			4		6	1	9	0	4		
1 0	1199	1		1288	1	1385	1	1	1	1	1		50
		2			3		4	4	5	6	6		
		4			3		3	9	5	1	7		

M i n .		5 1		5 3		5 5	5 6	5 7	5 8	5 9	Compl. min.
		2		5		7	2	0	1	5	
1 1	1200	1	1288	1	1386	1	1	1	1	1	49
		2		3		4	4	5	6	6	
		4		3		3	9	5	1	7	
		3		6		8	3	1	2	6	
1 2	1200	1	1289	1	1387	1	1	1	1	1	48
		2		3		4	4	5	6	6	
		4		3		3	9	5	1	7	
		4		7		9	4	2	3	8	
1 3	1201	1	1290	1	1387	1	1	1	1	1	47
		2		3		4	4	5	6	6	
		4		3		4	9	5	1	7	
		4		8		0	5	3	4	9	
1 4	1202	1	1291	1	1388	1	1	1	1	1	46
		2		3		4	4	5	6	6	
		4		3		4	9	5	1	8	
		5		8		1	6	4	5	0	
1 5	1202	1	1292	1	1389	1	1	1	1	1	45
		2		3		4	4	5	6	6	
		4		3		4	9	5	1	8	
		6		9		1	7	5	6	1	
1 6	1203	1	1292	1	1390	1	1	1	1	1	44
		2		3		4	4	5	6	6	
		4		4		4	9	5	1	8	
		7		0		2	8	6	7	2	
1 7	1204	1	1293	1	1391	1	1	1	1	1	43
		2		3		4	4	5	6	6	
		4		4		4	9	5	1	8	
		7		1		3	8	7	8	3	
1 8	1205	1	1294	1	1392	1	1	1	1	1	42
		2		3		4	4	5	6	6	
		4		4		4	9	5	1	8	
		8		2		4	9	8	9	4	

M i n .		5 1	5 3		5 5 6 7 8 9	5 5 6 7 8 9	5 5 6 7 8 9	5 5 6 7 8 9	5 5 6 7 8 9	Compl. min.
1 9	1205	1	1295	1	1393	1	1	1	1	41
		2		3		4	5	5	6	6
		4		4		4	0	5	2	8
		9		2		5	0	9	0	5
2 0	1206	1	1295	1	1393	1	1	1	1	40
		2		3		4	5	5	6	6
		5		4		4	0	6	2	8
		0		3		6	1	0	1	6
2 1	1207	1	1296	1	1394	1	1	1	1	39
		2		3		4	5	5	6	6
		5		4		4	0	6	2	8
		0		4		7	2	1	2	8
2 2	1207	1	1297	1	1395	1	1	1	1	38
		2		3		4	5	5	6	6
		5		4		4	0	6	2	8
		1		5		8	3	2	3	9
2 3	1208	1	1298	1	1396	1	1	1	1	37
		2		3		4	5	5	6	6
		5		4		4	0	6	2	9
		2		6		9	4	3	4	0
2 4	1209	1	1299	1	1397	1	1	1	1	36
		2		3		4	5	5	6	6
		5		4		5	0	6	2	9
		3		7		0	5	4	5	1
2 5	1210	1	1299	1	1398	1	1	1	1	35
		2		3		4	5	5	6	6
		5		4		5	0	6	2	9
		3		7		0	6	5	7	2
2 6	1210	1	1300	1	1399	1	1	1	1	34
		2		3		4	5	5	6	6
		5		4		5	0	6	2	9
		4		8		1	7	6	8	3
2	1211	1	1301	1	1399	1	1	1	1	33

M i n .		5 1		5 3		5 5	5 6	5 7	5 8	5 9	Compl. min.
7		2		3		4	5	5	6	6	
		5		4		5	0	6	2	9	
		5		9		2	8	7	9	4	
2 8	1212	1	1302	1	1400	1	1	1	1	1	32
		2		3		4	5	5	6	6	
		5		5		5	0	6	3	9	
		6		0		3	9	8	0	5	
2 9	1212	1	1302	1	1401	1	1	1	1	1	31
		2		3		4	5	5	6	6	
		5		5		5	1	6	3	9	
		6		1		4	0	9	1	7	
3 0	1213	1	1303	1	1402	1	1	1	1	1	30
		2		3		4	5	5	6	6	
		5		5		5	1	7	3	9	
		7		1		5	1	0	2	8	
3 1	1214	1	1304	1	1403	1	1	1	1	1	29
		2		3		4	5	5	6	6	
		5		5		5	1	7	3	9	
		8		2		6	2	1	3	9	
3 2	1215	1	1305	1	1404	1	1	1	1	1	28
		2		3		4	5	5	6	7	
		5		5		5	1	7	3	0	
		9		3		7	3	2	4	0	
3 3	1215	1	1306	1	1405	1	1	1	1	1	27
		2		3		4	5	5	6	7	
		5		5		5	1	7	3	0	
		9		4		8	4	3	5	1	
3 4	1216	1	1306	1	1405	1	1	1	1	1	26
		2		3		4	5	5	6	7	
		6		5		5	1	7	3	0	
		0		5		9	5	4	6	2	
3 5	1217	1	1307	1	1406	1	1	1	1	1	25
		2		3		4	5	5	6	7	

M i n .		5 1		5 3		5 5	5 6	5 7	5 8	5 9	Compl. min.
		6 1		5 6		6 0	1 6	7 5	3 7	0 3	
3 6	1217	1	1308	1	1407	1	1	1	1	1	24
		2		3		4	5	5	6	7	
		6		5		6	1	7	3	0	
		2		6		0	7	6	8	4	
3 7	1218	1	1309	1	1408	1	1	1	1	1	23
		2		3		4	5	5	6	7	
		6		5		6	1	7	3	0	
		2		7		1	8	7	9	6	
3 8	1219	1	1310	1	1409	1	1	1	1	1	22
		2		3		4	5	5	6	7	
		6		5		6	1	7	4	0	
		3		8		2	9	8	0	7	
3 9	1220	1	1210 [recte: 1310]	1	1410	1	1	1	1	1	21
		2		3		4	5	5	6	7	
		6		5		6	1	7	4	0	
		4		9		3	9	9	1	8	
4 0	1220	1	1311	1	1411	1	1	1	1	1	20
		2		3		4	5	5	6	7	
		6		6		6	2	8	4	0	
		5		0		4	0	0	3	9	
4 1	1221	1	1312	1	1411	1	1	1	1	1	19
		2		3		4	5	5	6	7	
		6		6		6	2	8	4	1	
		5		1		5	1	1	4	0	
4 2	1222	1	1313	1	1412	1	1	1	1	1	18
		2		3		4	5	5	6	7	
		6		6		6	2	8	4	1	
		6		1		6	2	2	5	1	
4 3	1232 [recte: 1223]	1	1313	1	1413	1	1	1	1	1	17
		2		3		4	5	5	6	7	
		6		6		6	2	8	4	1	

M i n .	5		5		5		5	5	5	5	Compl. min.
	50	1	52	3	54	5	6	7	8	9	
		7		2		7	3	3	6	2	
4 4	1223	1	1314	1	1414	1	1	1	1	1	16
		2				4	5	5	6	7	
		6				6	2	8	4	1	
		8				8	4	4	7	4	
4 5	1224	1	1315	1	1415	1	1	1	1	1	15
		2				4	5	5	6	7	
		6				6	2	8	4	1	
		8				9	5	5	8	5	
4 6	1225	1	1316	1	1416	1	1	1	1	1	14
		2				4	5	5	6	7	
		6				7	2	8	4	1	
		9				0	6	6	9	6	
4 7	1225	1	1317	1	1417	1	1	1	1	1	13
		2				4	5	5	6	7	
		7				7	2	8	5	1	
		0				1	7	7	0	7	
4 8	1226	1	1317	1	1418	1	1	1	1	1	12
		2				4	5	5	6	7	
		7				7	2	8	5	1	
		1				1	8	8	1	8	
4 9	1227	1	1318	1	1418	1	1	1	1	1	11
		2				4	5	5	6	7	
		7				7	2	8	5	1	
		2				2	9	9	2	9	
5 0	1228	1	1319	1	1419	1	1	1	1	1	10
		2				4	5	5	6	7	
		7				7	3	9	5	2	
		2				3	0	0	3	0	
5 1	1228	1	1320	1	1420	1	1	1	1	1	9
		2				4	5	5	6	7	
		7				7	3	9	5	2	
		3				4	1	1	4	2	

M i n .		5 50	1	5 52	3	5 54	5	5	5	5	5	5	Compl. min.
5 2	1229	1		1321	1	1421	1	1	1	1	1		8
		2			3		4	5	5	6	7		
		7			7		7	3	9	5	2		
		4			0		5	2	2	6	3		
5 3	1230	1		1321	1	1422	1	1	1	1	1		7
		2			3		4	5	5	6	7		
		7			7		7	3	9	5	2		
		5			1		6	3	3	7	4		
5 4	1230	1		1322	1	1423	1	1	1	1	1		6
		2			3		4	5	5	6	7		
		7			7		7	3	9	5	2		
		5			1		7	4	4	8	5		
5 5	1231	1		1323	1	1424	1	1	1	1	1		5
		2			3		4	5	5	6	7		
		7			7		7	3	9	5	2		
		6			2		8	5	5	9	6		
5 6	1232	1		1324	1	1425	1	1	1	1	1		4
		2			3		4	5	5	6	7		
		7			7		7	3	9	6	2		
		7			3		9	6	6	0	7		
5 7	1233	1		1325	1	1426	1	1	1	1	1		3
		2			3		4	5	5	6	7		
		7			7		8	3	9	6	2		
		8			4		0	7	7	1	9		
5 8	1233	1		1325	1	1426	1	1	1	1	1		2
		2			3		4	5	5	6	7		
		7			7		8	3	9	6	3		
		8			5		1	8	8	2	0		
5 9	1234	1		1326	1	1427	1	1	1	1	1		1
		2			3		4	5	5	6	7		
		7			7		8	3	9	6	3		
		9			6		2	9	9	3	1		
6	1235	1		1327	1	1428	1	1	1	1	1		0

M i n .	50	51	52	53	54	55	56	57	58	59	Compl. min.
0		2		3		4	5	6	6	7	
		8		7		8	4	0	6	3	
		0		6		3	0	0	4	2	

Foot-row (complementary degrees, read with the right-hand minute column): 49 48 47 46 45 44 43 42 41 40. — [Foot: Gradus Complementorum.]

Degrees 60–69

Head-row degrees: 60 61 62 63 64 65 66 67 68 69.

Min .	60	61	62	63	64	65	66	67	68	69	Compl. min.
0	173	180	188	196	205	214	224	235	247	260	60
	2	4	1	3	0	5	6	6	5	5	
1	173	180	188	196	205	214	224	235	247	260	59
	3	5	2	4	2	6	8	8	7	7	
2	173	180	188	196	205	214	225	236	247	261	58
	4	7	3	5	3	8	0	0	9	0	
3	173	180	188	196	205	214	225	236	248	261	57
	6	8	5	7	5	9	1	2	1	2	
4	173	180	188	196	205	215	225	236	248	261	56
	7	9	6	8	6	1	3	3	3	4	
5	173	181	188	197	205	215	225	236	248	261	55
	8	0	7	0	8	3	5	5	5	6	
6	173	181	188	197	205	215	225	236	248	261	54
	9	1	9	1	9	4	7	7	8	9	
7	174	181	189	197	206	215	225	236	249	262	53
	0	3	0	3	1	6	8	9	0	1	
8	174	181	189	197	206	215	226	237	249	262	52
	1	4	1	4	2	8	0	1	2	3	
9	174	181	189	197	206	215	226	237	249	262	51
	3	5	3	5	4	9	2	3	4	6	
10	174	181	189	197	206	216	226	237	249	262	50

Min												
.	60	61	62	63	64	65	66	67	68	69	Compl. min.	
	4	6	4	7	6	1	4	5	6	8		
11	174	181	189	197	206	216	226	237	249	263		49
	5	8	5	8	7	3	6	7	8	0		
12	174	181	189	198	206	216	226	237	250	263		48
	6	9	7	0	9	4	7	9	0	3		
13	174	182	189	198	207	216	226	238	250	263		47
	7	0	8	1	0	6	9	1	2	5		
14	174	182	189	198	207	216	227	238	250	263		46
	8	2	9	3	2	8	1	3	4	7		
15	175	182	190	198	207	216	227	238	250	263		45
	0	3	1	4	3	9	3	5	7	9		
16	175	182	190	198	207	217	227	238	250	264		44
	1	4	2	5	5	1	4	7	9	2		
17	175	182	190	198	207	217	227	238	251	264		43
	2	5	3	7	6	2	6	9	1	4		
18	175	182	190	198	207	217	227	239	251	264		42
	3	7	5	8	8	4	8	1	3	6		
19	175	182	190	199	207	217	228	239	251	264		41
	4	8	6	0	9	6	0	3	5	9		
20	175	182	190	199	208	217	228	239	251	265		40
	6	9	7	1	1	7	2	4	7	1		
21	175	183	190	199	208	217	228	239	251	265		39
	7	0	9	3	2	9	3	6	9	3		
22	175	183	191	199	208	218	228	239	252	265		38
	8	2	0	4	4	1	5	8	1	6		
23	175	183	191	199	208	218	228	240	252	265		37
	9	3	1	6	6	3	7	0	4	8		
24	176	183	191	199	208	218	228	240	252	266		36
	0	4	3	7	7	4	9	2	6	0		
25	176	183	191	199	208	218	229	240	252	266		35
	2	5	4	8	9	6	1	4	8	3		
26	176	183	191	200	209	218	229	240	253	266		34
	3	7	6	0	0	8	3	6	0	5		

Min												
.	60	61	62	63	64	65	66	67	68	69	Compl. min.	
27	176	183	191	200	209	218	229	240	253	266		33
	4	8	7	1	2	9	4	8	2	8		
28	176	183	191	200	209	219	229	241	253	267		32
	5	9	8	3	3	1	6	0	4	0		
29	176	184	192	200	209	219	229	241	253	267		31
	6	0	0	4	5	3	8	2	6	2		
30	176	184	192	200	209	219	230	241	253	267		30
	7	2	1	6	7	4	0	4	9	5		
31	176	184	192	200	209	219	230	241	254	267		29
	9	3	2	7	8	6	2	6	1	7		
32	177	184	192	200	210	219	230	241	254	267		28
	0	4	4	9	0	8	4	8	3	9		
33	177	184	192	201	210	220	230	242	254	268		27
	1	6	5	0	1	0	5	0	5	2		
34	177	184	192	201	210	220	230	242	254	268		26
	2	7	6	2	3	1	7	2	7	4		
35	177	184	192	201	210	220	230	242	255	268		25
	4	8	8	3	4	3	9	4	0	7		
36	177	184	192	201	210	220	231	242	255	268		24
	5	9	9	4	6	4	1	6	2	9		
37	177	185	193	201	210	220	231	242	255	269		23
	6	1	1	6	8	6	3	8	4	1		
38	177	185	193	201	210	220	231	243	255	269		22
	7	2	2	7	9	8	5	0	6	4		
39	177	185	193	201	211	221	231	243	255	269		21
	8	3	3	9	1	0	6	2	8	6		
40	178	185	193	202	211	221	231	243	256	269		20
	0	5	5	0	2	1	8	4	0	9		
41	178	185	193	202	211	221	232	243	256	270		19
	1	6	6	2	4	3	0	6	3	1		
42	178	185	193	202	211	221	232	243	256	270		18
	2	7	7	3	6	5	2	8	5	3		
43	178	185	193	202	211	221	232	244	256	270		17

Min											
.	60	61	62	63	64	65	66	67	68	69	Compl. min.
	3	8	9	5	7	6	4	0	7	6	
44	178	186	194	202	211	221	232	244	256	270	16
	4	0	0	6	9	8	6	2	9	8	
45	178	186	194	202	212	222	232	244	257	271	15
	6	1	2	8	0	0	8	4	1	1	
46	178	186	194	202	212	222	232	244	257	271	14
	7	2	3	9	2	2	9	6	4	3	
47	178	186	194	203	212	222	233	244	257	271	13
	8	4	4	1	4	5	1	8	6	5	
48	178	186	194	203	212	222	233	245	257	271	12
	9	5	6	2	5	5	3	0	8	8	
49	179	186	194	203	212	222	233	245	258	272	11
	1	6	7	3	7	7	5	2	0	0	
50	179	186	194	203	212	222	233	245	258	272	10
	2	8	9	5	8	9	7	5	3	3	
51	179	186	195	203	213	223	233	245	258	272	9
	3	9	0	7	0	0	9	7	5	5	
52	179	187	195	203	213	223	234	245	258	272	8
	4	0	1	8	2	2	1	9	7	8	
53	179	187	195	204	213	223	234	246	258	273	7
	5	2	3	0	3	4	3	1	9	0	
54	179	187	195	204	213	223	234	246	259	273	6
	7	3	4	1	5	6	4	3	2	3	
55	179	187	195	204	213	223	234	246	259	273	5
	8	4	6	3	6	7	6	5	4	5	
56	179	187	195	204	213	223	234	246	259	273	4
	9	5	7	4	8	9	8	7	6	8	
57	180	187	195	204	214	224	235	246	259	274	3
	0	7	8	6	0	1	0	9	8	0	
58	180	187	196	204	214	224	235	247	260	274	2
	2	8	0	7	1	3	2	1	1	3	
59	180	187	196	204	214	224	235	247	260	274	1
	3	9	1	9	3	4	4	3	3	5	

Min												
.	60	61	62	63	64	65	66	67	68	69	Compl. min.	
60	180	188	196	205	214	224	235	247	260	274	0	
	4	1	3	0	5	6	6	5	5	7		

Foot-row (complementary degrees): 29 28 27 26 25 24 23 22 21 20. — [Foot: Gradus Complementorum.]

Degrees 70–79

Head-row degrees: 70 71 72 73 74 75 76 77 78 79.

M	i	n	.	7	0	71	72	73	4	5	6	7	8	9	Compl.	
															min.	
0	2				2904		3078		3271	3	3	4	4	4	5	60
	7									4	7	0	3	7	1	
	4									8	3	1	3	0	4	
	7									7	2	1	1	5	5	
1	2				2907		3081		3275	3	3	4	4	4	5	59
	7									4	7	0	3	7	1	
	5									9	3	1	3	1	5	
	0									1	6	6	7	1	3	
2	2				2910		3084		3278	3	3	4	4	4	5	58
	7									4	7	0	3	7	1	
	5									9	4	2	4	1	6	
	2									5	1	1	3	8	1	
3	2				2912		3087		3281	3	3	4	4	4	5	57
	7									4	7	0	3	7	1	
	5									9	4	2	4	2	6	
	5									9	5	6	9	5	9	
4	2				2915		3090		3285	3	3	4	4	4	5	56
	7									5	7	0	3	7	1	
	5									0	4	3	5	3	7	
	7									3	9	1	5	2	7	
5	2				2918		3093		3288	3	3	4	4	4	5	55
	7									5	7	0	3	7	1	
	6									0	5	3	6	3	8	

M i n .	7 0	71	72	73	7 4	7 5	7 6	7 7	7 8	7 9	Compl. min.
	0				7	4	6	0	9	5	
6	2	2921	3096	3291	3	3	4	4	4	5	54
	7				5	7	0	3	7	1	
	6				1	5	4	6	4	9	
	2				1	8	1	6	5	3	
7	2	2924	3099	3295	3	3	4	4	4	5	53
	7				5	7	0	3	7	2	
	6				1	6	4	7	5	0	
	5				4	3	6	2	2	1	
8	2	2926	3102	3298	3	3	4	4	4	5	52
	7				5	7	0	3	7	2	
	6				1	6	5	7	5	0	
	7				8	7	1	8	9	9	
9	2	2929	3105	3302	3	3	4	4	4	5	51
	7				5	7	0	3	7	2	
	7				2	7	5	8	6	1	
	0				2	2	6	4	6	7	
1 0	2	2932	3108	3305	3	3	4	4	4	5	50
	7				5	7	0	3	7	2	
	7				2	7	6	9	7	2	
	3				6	6	1	0	3	6	
1 1	2	2935	3112	3309	3	3	4	4	4	5	49
	7				5	7	0	3	7	2	
	7				3	8	6	9	8	3	
	5				0	0	6	6	0	4	
1 2	2	2937	3115	3312	3	3	4	4	4	5	48
	7				5	7	0	4	7	2	
	7				3	8	7	0	8	4	
	8				4	5	1	2	7	2	
1 3	2	2940	3118	3316	3	3	4	4	4	5	47
	7				5	7	0	4	7	2	
	8				3	8	7	0	9	5	
	0				8	9	6	7	4	0	

M i n .		7 0	71	72	73	7 4	7 5	7 6	7 7	7 8	7 9	Compl. min.
1	2		2943	3121	3319	3	3	4	4	4	5	46
4	7					5	7	0	4	8	2	
	8					4	9	8	1	0	5	
	3					2	4	2	3	1	9	
1	2		2946	3124	3323	3	3	4	4	4	5	45
5	7					5	7	0	4	8	2	
	8					4	9	8	1	0	6	
	5					6	8	7	9	8	7	
1	2		2949	3127	3326	3	3	4	4	4	5	44
6	7					5	8	0	4	8	2	
	8					5	0	9	2	1	7	
	8					0	3	2	5	5	6	
1	2		2952	3130	3330	3	3	4	4	4	5	43
7	7					5	8	0	4	8	2	
	9					5	0	9	3	2	8	
	0					4	7	7	1	2	4	
1	2		2954	3133	3333	3	3	4	4	4	5	42
8	7					5	8	1	4	8	2	
	9					5	1	0	3	2	9	
	3					8	2	2	7	9	2	
1	2		2957	3137	3337	3	3	4	4	4	5	41
9	7					5	8	1	4	8	3	
	9					6	1	0	4	3	0	
	5					2	6	7	3	6	1	
2	2		2960	3140	3340	3	3	4	4	4	5	40
0	7					5	8	1	4	8	3	
	9					6	2	1	4	4	0	
	8					6	1	3	9	3	9	
2	2		2963	3143	3344	3	3	4	4	4	5	39
1	8					5	8	1	4	8	3	
	0					7	2	1	5	5	1	
	1					0	5	8	5	0	8	
2	2		2966	3146	3347	3	3	4	4	4	5	38

M i n .	7				7	7	7	7	7	7	Compl. min.
	0	71	72	73	4	5	6	7	8	9	
2	8				5	8	1	4	8	3	
	0				7	3	2	6	5	2	
	3				4	0	3	2	7	6	
2	2	2969	3149	3351	3	3	4	4	4	5	37
3	8				5	8	1	4	8	3	
	0				7	3	2	6	6	3	
	6				8	4	8	8	4	5	
2	2	2971	3152	3354	3	3	4	4	4	5	36
4	8				5	8	1	4	8	3	
	0				8	3	3	7	7	4	
	8				2	9	4	4	2	3	
2	2	2974	3156	3358	3	3	4	4	4	5	35
5	8				5	8	1	4	8	3	
	1				8	4	3	8	7	5	
	1				6	4	9	0	9	2	
2	2	2977	3159	3362	3	3	4	4	4	5	34
6	8				5	8	1	4	8	3	
	1				9	4	4	8	8	6	
	4				0	8	4	6	6	1	
2	2	2980	3162	3365	3	3	4	4	4	5	33
7	8				5	8	1	4	8	3	
	1				9	5	4	9	9	6	
	6				4	3	9	2	3	9	
2	2	2983	3165	3369	3	3	4	4	4	5	32
8	8				5	8	1	4	9	3	
	1				9	5	5	9	0	7	
	9				8	7	5	8	1	8	
2	2	2986	3168	3372	3	3	4	4	4	5	31
9	8				6	8	1	5	9	3	
	2				0	6	6	0	0	8	
	1				2	2	0	5	8	7	
3	2	2989	3172	3376	3	3	4	4	4	5	30
0	8				6	8	1	5	9	3	

M i n .	7 0	71	72	73	7 4	7 5	7 6	7 7	7 8	7 9	Compl. min.
	2				0	6	6	1	1	9	
	4				6	7	5	1	5	6	
3 1	2 8	2992	3175	3380	3	3	4	4	4	5	29
	2				6	8	1	5	9	4	
	7				1	7	7	1	2	0	
					0	1	1	7	2	4	
3 2	2 8	2994	3178	3383	3	3	4	4	4	5	28
	2				6	8	1	5	9	4	
	9				1	7	7	2	3	1	
					4	6	6	3	0	3	
3 3	2 8	2997	3181	3387	3	3	4	4	4	5	27
	3				6	8	1	5	9	4	
	2				1	8	8	2	3	2	
					8	1	1	9	7	2	
3 4	2 8	3000	3185	3390	3	3	4	4	4	5	26
	3				6	8	1	5	9	4	
	4				2	8	8	3	4	3	
					2	5	7	6	5	1	
3 5	2 8	3003	3188	3394	3	3	4	4	4	5	25
	3				6	8	1	5	9	4	
	7				2	9	9	4	5	4	
					6	0	2	2	2	0	
3 6	2 8	3006	3191	3398	3	3	4	4	4	5	24
	4				6	8	1	5	9	4	
	0				3	9	9	4	5	4	
					0	5	8	8	9	9	
3 7	2 8	3009	3194	3401	3	3	4	4	4	5	23
	4				6	8	2	5	9	4	
	2				3	9	0	5	6	5	
					5	9	3	5	7	8	
3 8	2 8	3012	3198	3405	3	3	4	4	4	5	22
	4				6	9	2	5	9	4	
					3	0	0	6	7	6	

M i n .	7 0	71	72	73	7 4	7 5	7 6	7 7	7 8	7 9	Compl. min.
	5				9	4	8	1	4	6	
3 9	2 8	3015	3201	3409	3 6	3 9	4 2	4 5	4 9	5 4	21
	4				4	0	1	6	8	7	
	8				3	9	4	7	2	5	
4 0	2 8	3018	3204	3412	3 6	3 9	4 2	4 5	4 9	5 4	20
	5				4	1	1	7	8	8	
	0				7	4	9	4	9	5	
4 1	2 8	3021	3207	3416	3 6	3 9	4 2	4 5	4 9	5 4	19
	5				5	1	2	8	9	9	
	3				1	8	5	0	7	4	
4 2	2 8	3024	3211	3420	3 6	3 9	4 2	4 5	5 0	5 5	18
	5				5	2	3	8	0	0	
	6				5	3	0	6	5	3	
4 3	2 8	3027	3214	3423	3 6	3 9	4 2	4 5	5 0	5 5	17
	5				6	2	3	9	1	1	
	8				0	8	6	3	2	2	
4 4	2 8	3030	3217	3427	3 6	3 9	4 2	4 5	5 0	5 5	16
	6				6	3	4	9	2	2	
	1				4	3	1	9	0	1	
4 5	2 8	3033	3221	3431	3 6	3 9	4 2	4 6	5 0	5 5	15
	6				6	3	4	0	2	3	
	4				8	8	7	6	7	0	
4 6	2 8	3036	3224	3435	3 6	3 9	4 2	4 6	5 0	5 5	14
	6				7	4	5	1	3	3	
	6				2	2	2	2	5	9	

M i n .		7 0	71	72	73	7 4	7 5	7 6	7 7	7 8	7 9	Compl. min.
4	2		3039	3227	3438	3	3	4	4	5	5	13
7	8					6	9	2	6	0	5	
	6					7	4	5	1	4	4	
	9					6	7	8	9	3	9	
4	2		3042	3230	3442	3	3	4	4	5	5	12
8	8					6	9	2	6	0	5	
	7					8	5	6	2	5	5	
	2					1	2	4	5	0	8	
4	2		3045	3234	3446	3	3	4	4	5	5	11
9	8					6	9	2	6	0	5	
	7					8	5	6	3	5	6	
	4					5	7	9	2	8	7	
5	2	3057 [recte: 3047]		3237	3440 [recte: 3450]	3	3	4	4	5	5	10
0	8					6	9	2	6	0	5	
	7					8	6	7	3	6	7	
	7					9	2	5	8	6	6	
5	2		3050	3240	3453	3	3	4	4	5	5	9
1	8					6	9	2	6	0	5	
	8					9	6	8	4	7	8	
	0					3	7	0	5	4	6	
5	2		3053	3244	3457	3	3	4	4	5	5	8
2	8					6	9	2	6	0	5	
	8					9	7	8	5	8	9	
	2					8	1	6	1	1	5	
5	2		3056	3247	3461	3	3	4	4	5	5	7
3	8					7	9	2	6	0	6	
	8					0	7	9	5	8	0	
	5					2	6	2	8	9	5	
5	2		3060	3251	3465	3	3	4	4	5	5	6
4	8					7	9	2	6	0	6	
	8					0	8	9	6	9	1	
	8					6	1	7	5	7	4	
5	2		3063	3254	3468	3	3	4	4	5	5	5

M i n .	7					7	7	7	7	7	7	7	Compl. min.
	0	71	72		73	4	5	6	7	8	9		
5	8					7	9	3	6	1	6		
	9					1	8	0	7	0	2		
	1					0	6	3	1	5	3		
5	2	3066	2157 [recte:		3472	3	3	4	4	5	5		4
6	8		3257]			7	9	3	6	1	6		
	9					1	9	0	7	1	3		
	3					5	1	9	8	3	3		
5	2	3069	3261		3476	3	3	4	4	5	5		3
7	8					7	9	3	6	1	6		
	9					1	9	1	8	2	4		
	6					9	6	4	5	1	2		
5	2	3072	3264		3480	3	4	4	4	5	5		2
8	8					7	0	3	6	1	6		
	9					2	0	2	9	2	5		
	9					3	1	0	1	9	2		
5	2	3075	3267		3484	3	4	4	4	5	5		1
9	9					7	0	3	6	1	6		
	0					2	0	2	9	3	6		
	1					8	6	6	8	7	2		
6	2	3078	3271		3487	3	4	4	4	5	5		0
0	9					7	0	3	7	1	6		
	0					3	1	3	0	4	7		
	4					2	1	1	5	5	1		

Foot-row (complementary degrees): 19 18 17 16 15 14 13 12 11 10. — [Foot: Gradus Complementorum.]

TABLE OF TANGENTS (concluded) — *Degrees of the Quadrant*

[Margin (left, vertical): Minuta Graduum Quadrantis. — "Minutes of the degrees of the quadrant."] [Margin (right, vertical): Minuta Graduum Complementorum. — "Minutes of the degrees of the complements."]

The final opening of the Table of Tangents (whole sine = 1000). The top column-heads give the whole degrees 80–89; the left-hand column gives the minutes (0–60) of those degrees; the right-hand column gives the minutes (60–0) of the complementary degrees; and the foot gives the complementary whole degrees (9–0), so that the same grid serves as a table of cotangents read upward. The tangent reaches its final value at 90° (**Infinita** — "infinite").

Mi n.		80	81	82	83	84		85	86	87	88		89		Co mp l.
0		5671	6314	7114	84	95		11430	1430	19081	28636		57290		60
1		5681	6326	7136	84	95		11468	1436	19181	28877		58261		59
2		5691	6338	7148	84	95		11507	1442	19291	29122		59266		58
3		5700	6350	7160	84	95	15546 [recte: 11546]	1448	1952	29405	371		60306		57
4		5710	6362	7172	84	96		11585	1454	19614	29625		61383		56
5		5720	6374	7184	84	96		11625	1460	19726	29888		62499		55
6		5730	6386	7196	84	96		11664	1466	19830	3014		63657		54

Mi n.		80	8 1	8 2	8 3		84		85	86	87	88		89	Co mp l.
7	5740	6	7	8	97			11704	14	19	30	64858	53		
		3	2	2	04				73	85	41				
		9	2	8					2	5	2				
		8	2	4											
8	5749	6	7	8	97			11745	14	19	30	66105	52		
		4	2	3	32				79	97	68				
		1	3	0					5	0	3				
		0	8	4											
9	5759	6	7	8	97			11785	14	20	30	67402	51		
		4	2	3	60				86	08	96				
		2	5	2					0	7	0				
		3	3	4											
10	5769	6	7	8	97			11826	14	20	31	68750	50		
		4	2	3	88				92	20	24				
		3	6	4					4	6	2				
		5	9	5											
11	5779	6	7	8	98			11867	14	20	31	70153	49		
		4	2	3	16				99	32	52				
		4	8	6					0	5	8				
		7	4	6											
12	5789	6	7	8	98			11909	15	20	31	71615	48		
		4	3	3	45				05	44	82				
		6	0	8					6	6	1				
		0	0	6											
13	5799	6	7	8	98			11950	15	20	32	73139	47		
		4	3	4	73				12	56	11				
		7	1	0					2	9	8				
		2	6	7											
14	5810	6	7	8	99			11992	15	20	32	74724	46		
		4	3	4	02				18	69	42				
		8	3	2					9	3	1				
		5	2	8											
15	5820	6	7	8	99			12035	15	20	32	76390	45		
		4	3	4	31				25	81	73				

Mi n.	80	8 1	8 2	8 3	84	85	86	87	88	89	Co mp l.
		9	4	4			7	9	0		
		7	8	9							
16	5830	6	7	8	99	12077	15	20	33	78126	44
		5	3	4	60		32	94	04		
		1	6	7			5	6	5		
		0	4	0							
17	5840	6	7	8	99	12120	15	21	33	79943	43
		5	3	4	89		39	07	36		
		2	8	9			4	5	6		
		2	0	1							
18	5850	6	7	8	10	12163	15	21	33	81847	42
		5	3	5	01		46	20	69		
		3	9	1	9		4	5	4		
		5	6	3							
19	5861	6	7	8	10	12207	15	21	34	83843	41
		5	4	5	04		53	33	02		
		4	1	3	8		4	7	7		
		8	2	4							
20	5871	6	7	8	10	12251	15	21	34	85940	40
		5	4	5	07		60	47	36		
		6	2	5	8		5	0	8		
		1	9	6							
21	5881	6	7	8	10	12295	15	21	34	88144	39
		5	4	5	10		67	60	71		
		7	4	7	8		6	6	5		
		3	5	7							
22	5892	6	7	8	10	12339	15	21	35	90463	38
		5	4	5	13		74	74	07		
		8	6	9	8		8	2	0		
		6	2	9							
23	5902	6	7	8	10	12384	15	21	35	92908	37
		5	4	6	16		82	88	43		
		9	7	2	8		1	1	1		
		9	8	1							

Mi n.		80	8 1	8 2	8 3	84	85	86	87	88	89	Co mp l.
24		5912	6	7	8	10	12429	15	22	35	95489	36
			6	4	6	19		89	02	80		
			1	9	4	9		5	2	1		
			2	5	3							
25		5923	6	7	8	10	12474	15	22	36	98218	35
			6	5	6	22		96	16	17		
			2	1	6	9		9	4	8		
			5	1	5							
26		5933	6	7	8	10	12520	16	22	36	10110	34
			6	5	6	26		04	30	56	7	
			3	2	8	2		3	8	3		
			8	8	7							
27		5944	6	7	8	10	12566	16	22	36	10416	33
			6	5	7	29		11	45	95	1	
			5	4	0	1		9	4	6		
			1	5	9							
28		5954	6	7	8	10	12612	16	22	37	10742	32
			6	5	7	33		19	60	35	6	
			6	6	3	2		5	2	8		
			5	2	2							
29		5965	6	7	8	10	12659	16	22	37	11089	31
			6	5	7	35		27	75	76	2	
			7	7	5	4		2	2	9		
			8	9	4							
30		5976	6	7	8	10	12706	16	22	38	11458	30
			6	5	7	38		35	90	18	9	
			9	9	7	5		0	4	8		
			1	6	7							
31	5968 [recte: 5986]		6	7	8	10	12754	16	23	38	11854	29
			7	6	8	41		42	05	61	0	
			0	1	0	7		8	8	8		
			4	3	0							
32		5997	6	7	8	10	12801	16	23	39	12277	28
			7	6	8	44		50	21	05	4	

Mi n.	80	8 1	8 2	8 3	84	85	86	87	88	89	Co mp l.
		1 8	3 0	2 3	9		7	4	7		
33	6008	6 7 3 1	7 6 4 7	8 8 4 6	10 48 1	12850	16 58 7	23 37 2	39 50 6	12732 1	27
34	6019	6 7 4 5	7 6 6 5	8 8 6 9	10 51 4 9	12898	16 66 8	23 53 2	39 96 5	13221 9	26
35	6030	6 7 5 8	7 6 8 2	8 8 9 2	10 54 6	12947	16 75 0	23 69 5	40 43 6	13750 7	25
36	6041	6 7 7 2	7 7 0 0	8 9 1 5	10 57 9	12996	16 83 2	23 85 9	40 91 7	14323 7	24
37	6051	6 7 8 6	7 7 1 7	8 9 3 9	10 61 2	13046	16 91 5	24 02 6	41 41 1	14946 5	23
38	6062	6 7 9 9	7 7 3 5	8 9 6 2	10 64 5	13096	16 99 9	24 19 6	41 91 6	15625 9	22
39	6073	6 8 1 3	7 7 5 3	8 9 8 6	10 67 8	13146	17 08 4	24 36 7	42 43 3	16370 0	21
40	6084	6 8 2 7	7 7 7 0	9 0 1 0	10 71 2	13197	17 16 9	24 54 2	42 96 4	17188 5	20

Mi n.		8 80	8 1	8 2	8 3		84		85	86	87	88		89	Co mp l.
41	6096	6	7	9	10			13248	17	24	43	18093	19		
		8	7	0	74				25	71	50	2			
		4	8	3	6				6	8	8				
		1	8	4											
42	6107	6	7	9	10			13300	17	24	44	19098	18		
		8	8	0	78				34	89	06	4			
		5	0	5	0				3	8	6				
		5	6	8											
43	6118	6	7	9	10			13352	17	25	44	20221	17		
		8	8	0	81				43	08	63	9			
		6	2	8	4				1	0	9				
		9	4	2											
44	6129	6	7	9	10			13404	17	25	45	21485	16		
		8	8	1	84				52	26	22	8			
		8	4	0	8				1	4	6				
		3	2	6											
45	6140	6	7	9	10			13457	17	25	45	22918	15		
		8	8	1	88				61	45	82	2			
		9	6	3	3				1	2	9				
		7	1	1											
46	6152	6	7	9	10			13510	17	25	46	24555	14		
		9	8	1	91				70	64	44	2			
		1	7	5	8				2	2	9				
		1	9	6											
47	6163	6	7	9	10			13563	17	25	47	26444	13		
		9	8	1	95				79	83	08	1			
		2	9	8	3				3	5	5				
		5	7	0											
48	6174	6	7	9	10			13617	17	26	47	28647	12		
		9	9	2	98				88	03	74	8			
		4	1	0	8				6	1	0				
		0	6	5											
49	6186	6	7	9	11			13672	17	26	48	31252	11		
		9	9	2	02				98	23	41	1			

Mi n.	80	8 1	8 2	8 3	84	85	86	87	88	89	Co mp l.
		5	3	3	4		0	0	2		
		4	4	0							
50	6197	6	7	9	11	13727	18	26	49	34377	10
		9	9	2	05		07	43	10	4	
		6	5	5	9		5	2	4		
		8	3	5							
51	6209	6	7	9	11	13782	18	26	49	38197	9
		9	9	2	09		17	63	81	1	
		8	7	8	5		1	7	6		
		3	2	1							
52	6220	6	7	9	11	13838	18	26	50	42971	8
		9	9	3	13		26	84	54	8	
		9	9	0	2		8	5	9		
		7	1	6							
53	6232	7	8	9	11	13894	18	27	51	49110	7
		0	0	3	16		36	05	30	6	
		1	0	3	8		5	7	3		
		2	9	2							
54	6243	7	8	9	11	13951	18	27	52	57295	6
		0	0	3	20		46	27	08	7	
		2	2	5	5		4	1	1		
		6	8	7							
55	6255	7	8	9	11	14008	18	27	52	68754	5
		0	0	3	24		56	49	88	9	
		4	4	8	2		4	0	2		
		1	8	3							
56	6267	7	8	9	11	14065	18	27	53	85943	4
		0	0	4	27		66	71	70	6	
		5	6	0	9		6	2	9		
		6	7	9							
57	6278	7	8	9	11	14124	18	27	54	11459	3
		0	0	4	31		76	93	56	15	
		7	8	3	6		8	7	1		
		1	6	5							

Mi n.		8	8	8							Co mp l.
	80	1	2	3	84	85	86	87	88	89	
58	6290	7	8	9	11	14182	18	28	55	17188	2
		0	1	4	35		87	16	44	73	
		8	0	6	4		1	6	2		
		5	5	1							
59	6302	7	8	9	11	14241	18	28	56	34377	1
		1	1	4	39		97	39	35	47	
		0	2	8	2		6	9	1		
		0	5	8							
60	6314	7	8	9	11	14301	19	28	57	Infini	0
		1	1	5	43		08	63	29	ta.	
		1	4	1	0		1	6	0		
		5	4	4							
Co mp l.	9	8	7	6	5	4	3	2	1	0	

[Margin (foot): Gradus Complementorum. — "Degrees of the complements."]

SOME PRACTICAL EXERCISES

Excerpted from the *New Description of Sundials* [*Horologiorum Nova Descriptio*], so that sundials may be laid out quickly — though, to be understood more plainly, most of them require much of what is set out in that book.

A few further things are also added here that are wanting in that same book.

The hours from noon and from midnight, on a plane parallel to the horizon [horae a meridie et a media nocte in plano quod horizonti aequidistat]

Praxis I [First Exercise]

Before anything else, four rules — that is, straightedges with certain marked divisions — must be prepared out of some fairly stiff card. The first is to be constructed as follows. Taking the gnomon [stylus] of the intended dial at will,

mark off along the sides of the rule as many equal parts as you please, and join them by straight lines parallel to one another and perpendicular to the sides. Then, once the first part AB has been divided into 10 equal parts and the surplus cut away, the first rule is ready. The second — which will be used in common for finding the points on very many lines — is constructed by letting a single side carry any number of equal parts you like (provided, however, that each of them exceeds the gnomon by a good margin), the first of these, AB, being likewise distributed into 10 equal parts. As for the third rule, for finding the points on the equinoctial line, and the fourth, for the points on the 6-o'clock line, we shall deal with them a little later, once we have first drawn these two lines on the dial.

[Margin: Construction of the first rule.] [Margin: Construction of the second rule.]

2. So, on some plane, having drawn the straight line AB as the meridian line — running roughly through the middle of the plane — let the position of the gnomon be chosen at point C, and through it draw CD perpendicular to the meridian, making the line CD equal to the gnomon AB assumed on the first rule. Then, since at the latitude of the place, 42° , the tangent of the complement of the pole's elevation contains $11 \frac{1}{10}$ [?] parts, we shall apply the first rule to the meridian line, placing the point marked with the number 10 at C, with point B inclining toward G. For if with some very sharp stylus a point G is marked on the meridian terminating the tangent of $11 \frac{1}{10}$ [?], the point found will be G, the center of the dial; and through it the perpendicular KL drawn to the meridian will give the 6-o'clock line. If you prefer, you may also use a compass instead of the rule.

[Margin: Description of the dial.] [Margin: How the tangents of the complements of arcs are found in the table — the tangents of complements are found by taking the degrees of the arcs whose complements are in question at the foot of the Table of Tangents, and the minutes, if any, on the right-hand side.] [Margin: The center of the dial.]

Diagram. Four paper rules (straightedges), drawn one above the other, each a long strip with a graduated scale and an endpoint lettered B at the left. From top to bottom:

- Rule 1 — captioned "**Pro linea merid.**" ("For the meridian line"), with a small superscript **a** (marginal cross-reference letter). It carries a graduated scale in two parallel rows reading **10 · 20 · 30 · 40 · 50 · 60** (left to right); endpoints lettered **B** (far left) and **A**.

- Rule 2 — captioned "**Communis**" ("The common [rule]"); endpoint **B** at the left.
- Rule 3 — captioned "**Pro linea aequinoctiali**" ("For the equinoctial line"), graduated **10 · 20 · 30 · 40**; endpoint **B** at the left.
- Rule 4 — captioned "**Pro linea horae 6.**" ("For the 6-o'clock line"), graduated **10 · 20 · 30**; endpoint **B** at the left.

For if, once the first rule has been divided, you take the tangent of $11 \frac{1}{10}$ [?] with the compass and carry it up from C, you will find the same center of the dial. I want this understood of the other tangents to be transferred as well: namely, that they may be transferred either by a rule or straightedge, or by a compass. Again, since the tangent of the pole's elevation, 42° , comprises 9 parts, we shall place point A of the rule at C, with its point B inclining downward — that is, to the side opposite point G. For if, as before, we mark on the meridian the point B terminating the tangent of 9, then from B the straight line HI must be drawn perpendicular to the meridian, to serve as the equinoctial line.

[Margin: How the tangent of an arc is found in the table — the tangents of degrees are found by taking the degrees at the top of the Table of Tangents and the minutes on the left-hand side.] [Margin: The equinoctial line.]

3. Now the third rule (or straightedge) is constructed by taking any number of segments equal to the interval DB, the first of which, AB, is divided into 10 small equal parts. In the same way you will construct the fourth rule, if for the interval DB you take the interval DG, and so on. And these four rules, or straightedges, are enough for...

[Margin: Construction of the third and fourth rules.]

constructing every horizontal dial, as will become clear. The second straightedge, too, will find use in other dials and figures. That the first rule has a different form from the rest was done so that by means of it we may find the points of the Sun's parallels on the hour-lines, as we shall teach in its proper place.

4. By the third rule (or straightedge), constructed for the equinoctial line, there will be found on the equinoctial line the points of the hours, of their halves, and of their quarters — provided that the tangents taken from the little table of the scholium of chapter 1 are transferred onto it from B in both directions, as was set forth in No. 3 of chapter 2. Therefore, if through the center G and through these points straight lines are sent out along the equinoctial line, the dial of the hours from noon and from midnight will be described (whose arrangement is shown here). With the lines turned

toward us for the dial (so that the equinoctial line comes nearer to us than the 6-o'clock line), those placed to the left belong to the hours after noon, while those set to the right belong to the hours after midnight, as the numbers I, II, III, VII, VIII, and so on, written on the hour-lines, indicate. Only the whole hours have been drawn, to avoid confusion; nevertheless the points of the half-hours are marked on the equinoctial, so that by their help the hours from sunrise or sunset may be described. And if you should want the half-hours also from sunrise or sunset, the points of the quarter-hours will have to be found; and if anyone should want the quarters of these, the points of the half-quarters — that is, of the eighth-parts — must also be investigated, as will be told in Exercise 2.

[Margin: How the hour-points on the equinoctial are found.] [Margin: The dial of the hours from noon and from midnight. Why the points of the half-hours, quarters, and eighth-parts must be found on the equinoctial.]

Diagram. A horizontal-sundial construction (dial of the hours from noon and from midnight, for latitude 42°), within a rectangular frame. All the hour-lines radiate from a single dial center **G**, set near the middle of the figure. A horizontal **dotted line** runs across the figure as the equinoctial line; the **meridian** is the vertical line, and the **6-o'clock line (KL)** is perpendicular to it through **G**.

Around the border the hour-lines are numbered in Roman numerals. Legibly read: along the top **V, VI, VII, VIII**; down the right side the sequence **IX, X, XI, XII, I, II, III** (the numerals here are set rotated and only partly legible — several [?]); along the bottom **IIII**; up the left side **VII, VI, V**.

Interior construction labels read from the scan (positions approximate):

- **G** — dial center (convergence of all hour-lines).
- On the meridian: **L** (upper), **T** (below **G**).
- On/near the equinoctial line: **D, I, B, E, C, A, O** (running toward the center), with a further point read as **P** or **R** [?] below the center.
- Left-hand quarter-circle arc with points **N, F, M**; frame letters **S** (upper left) and **K** (left edge).
- **H** (lower center, on the equinoctial construction).
- At the foot, the second ("common") rule laid out as a graduated strip in two rows reading **60 · 50 · 40 · 30 · 20 · 10**, with endpoints lettered **A** and **B**.

The text draws the arc **HEI** from **G** and the equal arc **MFN** on the opposite side; the 3-o'clock line runs through **N, G, H**, and the 9-o'clock line through **M, G, I**.

Several small, rotated perimeter numerals and interior letters are only partly legible

5. But in order that the hour-lines may be drawn more accurately, other points must be found through which they ought to pass; for in this way one can scarcely go wrong in drawing them. And if some hour-line does not pass through all its own points, one must examine carefully at which point the error lies, and the error must be corrected. We shall find more of these points on the lines parallel to the meridian line and to the 6-o'clock line, by means of the second (common) straightedge, in this way. First of all, the 3- and 9-o'clock lines are to be drawn as exactly as can be done, by this method. From the equinoctial, cut off BH and BI equal to the interval BD; for through H and I the 3- and 9-o'clock lines are to be drawn from the center G, as was demonstrated in chapter 3, No. 1. And so that we may find the third point for each of the two, we shall describe from G the arc HEI, and with the same interval, on the opposite side, the arc MFN. For if arcs FM, FN are cut off equal to the arcs EH, EI, then the 3-o'clock line must be drawn through N, G, H, and the 9-o'clock line through M, G, I.

[Margin: How the 3- and 9-o'clock lines are to be drawn exactly.]

6. Once the 3- and 9-o'clock lines have been drawn, we shall lay off along the straight line AB, by means of the second set-square [normal], four equal segments GA, GO, GK, GL; and through the points A and O we shall draw lines parallel to the 6-o'clock line, cutting the 3- and 9-o'clock lines at R, S, P, Q. We then join the straight lines PR and QS, which — as we proved in chapter 4, No. 6 — will be parallel and equal to the line AO, and will be bisected at T and V; and so their halves will be equal to AB on the second rule [regula]. Likewise, through K and L we shall draw KX and LY parallel to the meridian line, and join the straight line XY, which by the same reasoning will be parallel and equal to KL, and will be bisected at Z. If the plane were large enough, we should have drawn on the opposite side another line parallel and equal to KL, joining by a straight line the points where XK and YL meet the 3- and 9-o'clock lines.

Now if, from the points Z, T, V, the tangents of the whole hours — that is, the tangents of 15° , 30° , 45° , and so on, with respect to the whole sine [sinus totius] AB — are transferred on both sides onto the straight lines XY, PR, QS by the second set-square, the hour-points will be found on those lines at distances of 15° , 30° , 45° , etc. from Z, T, V, just as was done on the equinoctial line: so that the points nearest to K and V correspond to the 5- and 7-o'clock lines, 15° distant from

the 6-o'clock line, and so on. The same is to be understood of the fractional parts of the hours, if the tangents of the half- and quarter-hours are taken from the table of the scholium of chapter 1 — all of which we proved in chapter 4. Moreover, by the same reasoning we shall find, on the equinoctial and on the lines XY, PR, QS, the point through which a straight line drawn from G represents the great circle passing through the poles of the world and through any degree of the equator whatever, as we wrote in chapter 4 just before No. 2, and in No. 10 of the same chapter. Here belong the cautions we said (chapter 2, No. 4) must be observed in testing the hour-points found by tangents.

[Margin: Finding the hour-points off the equinoctial line.]

[Margin: How to draw the straight line representing the great circle through the poles of the world and through any degree of the equator.]

7. Furthermore, on the straight lines XK, YL, PQ, RS — whose distances from the center G are equal to AB on the second set-square — we may also, if we wish, find these same hour-points from Table 6, in this way. The hour-points from 9 after midnight up to 3 after noon, and from 9 after noon up to 3 after midnight, will be found on the lines PQ, RS by the tangents of the complements of the arcs of Table 6. But the hour-points from 3 after noon up to 9, and from 3 after midnight up to 9, will be found on the lines XK, YL by the tangents of the very arcs themselves in the same Table 6, without complements — by tangents, I mean, taken with respect to the whole sine AB and transferred by the third set-square on both sides from O, A, K, L onto the lines PQ, RS, XK, YL. For example: the arc of the 3- and 9-o'clock line for latitude 42° in Table 6 contains $56^\circ 13'$; and the tangent of its complement is about $6 \frac{7}{12}$ [fraction ?] parts, which, taken from the second set-square and transferred from O and A on both sides onto PQ, RS, gives the points P, S of the 9-o'clock line and the points Q, R of the 3-o'clock line. But the tangent of the arc itself, $56^\circ 13'$, contains $14 \frac{2}{10}$ [fraction ?] parts, a little more; and this, transferred from K and L, will yield the points X, Y of the 3- and 9-o'clock lines. And so with the rest.

[Margin: A further finding of the hour-points off the equinoctial.]

The reason for this is that the angles which the hour-lines make at the center G with the 6-o'clock line correspond to the arcs of Table 6, while the angles that these same lines make with the meridian correspond to the complements of those same arcs of Table 6. And it is established that the portions of the straight lines XK, YL, PQ, RS between the points T, L, O, A and the hour-lines are the tangents of

those angles, or arcs, with respect to the whole sines GK, GL, GO, GA, which are equal to AB on the second set-square.

8. Lastly, the hour-points may also be found on the equinoctial line by this other method. Take the interval BD, with a straight line Ba laid off equal to it (whether upward or downward). If, from a, at the interval HI (between the 3- and 9-o'clock lines — for the points of these hours are found by transferring the interval BD from B on both sides, as we said), the equinoctial line is cut on both sides, the points of the 4- and 8-o'clock lines will be found; and from these, with the same interval, the points of the 5- and 11-o'clock lines, and likewise of the 7- and 1-o'clock lines. And if the spaces between the 8-o'clock line and the point B, and between the 4-o'clock line and the same point B, are each divided into three equal parts, the parts nearest B will give the points of the 10- and 2-o'clock lines. And thus the points of all the whole hours will now be found. But if, from the points of the odd hours, the equinoctial is cut on either side by these intervals up to a, the points of the half-hours will be found; and if from these, again by intervals up to a, the equinoctial is cut on either side, the points of the quarter-hours will be found.

[Margin: Another finding of the hour-points.]

In just the same way we shall find the hour-points on the lines XY, PR, QS, if we take Zd, Tb, Vc equal to the interval XZ, or PT, or VQ. For if, at the interval XY (or PR, or QS), the lines XY, PR, QS are cut on both sides from d, b, c, there will be found on them points distant by four hours from Z, T, V — just as on the equinoctial from a at the interval HI, and so on. And if the segments between Z, T, V and those points found are divided into three equal parts, we shall have points distant by two hours from Z, T, V; and from these four hours found we shall find all the rest, as on the equinoctial line, and so forth — all of which we proved in chapters 3 and 4. And this method is very useful for testing the points found by tangents.

Now, since for drawing the quarter-hours from sunrise and sunset we need the eighth-parts of the hours from noon and midnight (as will be said in Praxis III), it must be shown how the eighth-parts may be found by tangents. One eighth-part of a single hour comprises $1^{\circ}52'30''$. Therefore, to the tangent that corresponds to the degrees and minutes, one must add half the difference between that tangent and the next following, if the degrees were taken at the top of the table and the minutes down the left side. For when the degrees are taken at the foot of the table and the minutes down the right side, that same half must be subtracted. For

example: hour 5 $\frac{1}{2}$ contains $76^{\circ}52'30''$; and it corresponds to $76^{\circ}52'$, tangent 4286. If, then, half the difference between this tangent and the next following — namely 3 — is added, we make the tangent 4289. And that same half, subtracted from 4292, gives the same tangent 4289, corresponding to $13^{\circ}7'30''$ taken at the foot of the table. And so with the rest.

[Margin: How the eighth-parts of the hours are found by tangents.]

THE ARCS OF THE SIGNS ON A HORIZONTAL DIAL, for the hours from noon and from midnight.

PRAXIS II.

[Margin: Finding the points for the arcs of the signs, from noon and from midnight.]

1. Once the hour-lines have been drawn faintly, as we taught in the preceding exercise, we shall find on them the points of the signs most readily from Table 19, by the method we set out at page 61, near the end, No. 5 — provided the pole's altitude

is found exactly in that table, or differs by only a few minutes. Otherwise they must be investigated by the points on the 6-o'clock line found from Table 1, as we taught in chapter 8, Nos. 1, 2, 3, 4, and 6; or by the points found on the meridian from Table 2, as was explained in No. 10 of the same chapter 8; or finally, as we taught in No. 7 of the same chapter, provided the tangents are transferred to the hour-lines by means of the first set-square, which we constructed in No. 1 of the preceding exercise, as was said in the addition before No. 9 of chapter 8. Once the tropics of Cancer [☊, Cancer — the summer tropic] and Capricorn [☋, Capricorn — the winter tropic] have been drawn, only the lines enclosed between them complete the dial; so all the rest must be cut away as superfluous. As an example of this last method, explained in No. 7 of chapter 8, I have placed a point on Capricorn, found on the 3-o'clock line from noon. For in Table 1, under hour 3, in the circle of hour 12, opposite Capricorn, is found an arc of $31^{\circ}35'$; and adding the pole's altitude of 42° (since it is a southern sign) the arc becomes $73^{\circ}35'$, to which is due a tangent of $33 \frac{7}{10}$ [fraction ?], a little more. So the first set-square, applied

Diagram. A full-page horizontal-sundial construction (the hours from noon and from midnight, for latitude 42°), within a rectangular frame — the same construction figure that appears on the preceding page, reprinted here. All the hour-lines radiate from a single dial center **G** near the middle of the figure. A horizontal dotted line runs across the figure as the equinoctial line; the meridian

is the vertical line, and the 6-o'clock line runs through G perpendicular to it. A quarter-circle arc on the left carries points **N, F, M**; a smaller arc **HEI** appears near the center-right. At the foot is the second (common) graduated rule laid out as a strip in two rows, reading **60 · 50 · 40 · 30 · 20 · 10** (endpoints lettered **A** and **B**).

Around the border the hour-lines are numbered in Roman numerals. Legibly read: along the top **V, VI, VII, VIII**; down the right side (rotated, only partly legible) **IX, [X ?], XI, XII, I, [II ?], III** [several [?]]; along the bottom **IIII**; up the left side **VII, VI, V**.

Interior construction labels read from the scan (positions approximate):

- **G** — dial center (convergence of all hour-lines).
- On/near the meridian: **L** (upper), **T** (below G), **K** (lower, on the foot dotted line).
- Upper-left frame corner: **S**.
- Left quarter-circle arc: **N, F, M**; left-frame letters **A** (upper) and **R** (lower).
- Right of center, toward the upper right: **D, I** (near the arc), **C** (script), **Y**.
- On/near the equinoctial line running toward the center: **B, E, Θ/O [?], A** (script), with **P** below.
- **H** — lower center, on the equinoctial construction.
- **Z** — center-right, near hour XII.
- **X** — lower right, near the foot-strip endpoints **A, B**.

Several small, rotated perimeter numerals and interior letters are only partly legible

to the line CD, so that the little edge of the set-square drawn through the numbers 30, 30 coincides with the line CD, while the point terminating at $3\frac{1}{10}$ [fraction ?] on the segment AB reaches the 3-o'clock line — the point found will be **e**, for the parallel of Capricorn. And the same reasoning holds for the rest.

THE HOURS FROM SUNRISE AND SUNSET WITH THE ARCS OF THE SIGNS, on a plane parallel to the horizon.

PRAXIS III.

[Margin: Which hour-points on the equinoctial line are to be found.]

1. If whole hours from sunrise and sunset are to be drawn, one must find, by Praxis I, the points of the hours from noon and midnight on the equinoctial

line, together with the points of the half-hours. But if, in addition, the half-hours from sunrise and sunset are wanted, the points of the quarter-hours from noon and midnight must also be found. And if, finally, one wishes for the quarter-hours from sunrise and sunset as well, the eighth-parts of the hours from noon and midnight must be found besides.

For the whole hours from sunrise and sunset are equidistant from the hour-lines from noon and midnight that are separated from them in order by half-hours; the half-hours from sunrise and sunset, together with the whole hours, are parallel to the hour-lines from noon and midnight that are separated in order by quarter-hours; and finally the quarter-hours from sunrise and sunset, together with the halves and the whole hours, are equidistant from the hour-lines from noon and midnight that are separated in order by eighth-parts — as was said in chapter 14. And which hour-line from noon and midnight any given hour-line from sunrise or sunset is equidistant from is easily known if the proposed hour from sunrise or sunset is bisected: for its half names the hour from noon and midnight to which it is equidistant. For example, hour 18 from sunrise or sunset is equidistant from hour 9 from noon and midnight; hour $18\frac{1}{2}$ is parallel to hour $9\frac{1}{4}$; hour $18\frac{1}{4}$ to hour $9\frac{1}{8}$; and hour $18\frac{3}{4}$ to hour $9\frac{3}{8}$. Likewise hour 6 from sunrise and sunset is equidistant from hour 3 from noon and midnight. And so with the rest.

[Margin: How whole hours from sunrise and sunset are described, or their halves, or their quarters.]

[Margin: Which hour from noon and midnight any given hour from sunrise and sunset is equidistant from.]

2. Now, through which hour-points of the equinoctial line the hours from sunrise and sunset are to be drawn is no trouble to understand. For hour 18 from sunrise and sunset is to be drawn through hour 12 on the equinoctial line. Hence hour 19 must be drawn through hour 1, hour 20 through hour 2, hour 21 through hour 3, hour 22 through hour 4, and hour 23 through hour 5; while hour 17 is drawn through hour 11, hour 16 through hour 10, hour 15 through hour 9, hour 14 through hour 8, hour 13 through hour 7, and so on. The same must be said of the halves and quarters: for just as, for example, hour 18 is drawn through hour 12, so hour $18\frac{1}{4}$ is to be drawn through hour $12\frac{1}{4}$, and so on. In what way the lines of the hours from sunrise and sunset are to be drawn parallel to the lines of the hours from noon and midnight — even though the hours from noon and midnight have for the most part not been drawn — we taught in

chapter 14. But all this is more easily known from the two little tables set at the beginning of chapter 14.

[Margin: Through which hours on the equinoctial the hours from sunrise and sunset are to be drawn. Hour 12, and others little distant from it, in what way they are to be drawn.]

3. For hours $11\frac{1}{4}$, $11\frac{1}{2}$, $11\frac{3}{4}$, 12, $12\frac{1}{4}$, $12\frac{1}{2}$, $12\frac{3}{4}$ — since the hour-points from noon and midnight $5\frac{1}{4}$, $5\frac{1}{2}$, $5\frac{3}{4}$, 6, $6\frac{1}{4}$, $6\frac{1}{2}$, $6\frac{3}{4}$, through which they are to be drawn, either cannot be marked on the equinoctial line at all, or only with difficulty, because they run out too far — one must have recourse to the figure of the scholium of chapter 1, in order to find convenient intersections of the hours from sunrise and sunset with the hours from noon and midnight, through which are to be drawn the lines parallel to the hour-lines from sunrise and sunset, whose numbers are the halves of those hours from sunrise and sunset. For example, from the aforesaid figure you will gather that hour $12\frac{1}{2}$ passes through the intersection of hour 22 with hour $5\frac{1}{4}$. So the line of hour $5\frac{1}{4}$ must be drawn, as we said in Praxis I, even if its point is not found on the equinoctial but on some line parallel to the meridian, such as PR or XK, and so on; and the point where it cuts hour 22 must be carefully marked. For through this point hour $12\frac{1}{2}$ must be drawn, parallel to hour $6\frac{1}{4}$, and so on. But hour 12 from sunrise or sunset is to be drawn parallel to hour 6, through the point that bisects the segment of the meridian between the equinoctial and hour 6, as we demonstrated at the end of chapter 14 — so that there is no need to investigate another point through which it must pass.
4. But if the tropic of Cancer has been drawn through points found either in the hours from noon and midnight or in the whole hours from sunrise and sunset, we shall delineate these same hours with no trouble, without the aid of the figure of the scholium of chapter 1 — namely, by the distances they have on the tropic of Cancer from the meridian. For if the point of the distance of any hour from sunset, in degrees and minutes (as they are computed in the table of the scholium of chapter 15), is found either on the equinoctial line or on some other line equidistant from the meridian, as we taught in Praxis I, a straight line drawn from the center of the dial through that point will cut the tropic of Cancer at the point through which the proposed hour from sunset is to be drawn. And if the same is done with the hour from sunrise of the same number, another point of the proposed hour will be found on the same tropic.

[Margin: Another way of describing the hours from sunrise and sunset.]

5. Indeed, by these same distances from the meridian, the points of all the hours from sunrise and sunset can be found on both tropics, or even on any other parallel of the Sun whatever, as was set out in chapter 15, No. 4. And by this way the hours from sunrise and sunset will be described most conveniently, provided the tropics have first been accurately delineated.

[Margin: The arcs of the signs in the hours from sunrise and sunset.]

6. Once the hours from sunrise and sunset have been drawn, the points of the parallels will be found on them by the method we described in the addition before No. 3 of chapter 15; or else by the distances from the meridian reckoned in degrees and minutes, as we taught in chapter 15, No. 3.

THE HOURS FROM NOON AND FROM MIDNIGHT, on a plane declining from the vertical.

PRAXIS IV.

[Margin: The description of the hours from noon and midnight on a declining dial.]

1. The horizontal line, the meridian, the center of the dial, the 6-o'clock line, the style line, and the equinoctial must be found and drawn by means of tangents, as was taught in chapter 16, Nos. 1, 2, 3, and 4 — the geometrical methods we there employed being set aside, if one prefers. Then the hours are to be drawn through the points found on the horizontal line, as was explained in Nos. 5, 6, and 7.

[Margin: An exact description of the hours by the points of the horizontal line.]

2. But these will be drawn far more accurately in this way. On the meridian of the dial (chapter 16) let there be taken any number of parts equal to EB, so that the segment of the meridian between E and the last part is double, or triple, or quadruple, and so on, of EB; and through the outermost point let a perpendicular be drawn to the meridian. For if onto this perpendicular you transfer, from the point where it cuts the style line, the tangents with respect to a whole sine that is as many times a multiple of the style CD as the portion of the meridian taken was a multiple of the segment EB — the tangents, I mean, of the arcs of the earlier table of chapter 16, just as we said they must be transferred, with respect to the

whole sine CD, from C, in place of the style, onto the horizontal line — you will have on it other hour-points, through which, if straight lines are sent out from E, they will pass exactly through the hour-points of the horizontal line, if no error has been made.

The reason for this is [a] that the aforesaid segments of the perpendicular between the style line and the hour-lines are the same multiples of the segments of the horizontal line between the style line and those same hour-lines as the assumed multiple portion of the meridian was of EB itself. Since, then, the tangents with respect to a whole sine that is likewise so many times a multiple of the whole sine CD are, in the same way, multiples of the tangents with respect to the whole sine CD — (for as the greater whole sine is to the tangent of any arc, so is the lesser whole sine to the tangent of the same arc; and, by alternation, as the greater whole sine is to the lesser whole sine, so is the tangent of the greater whole sine to the tangent of the lesser whole sine; therefore the whole sine will be the same multiple of the whole sine as the tangent is of the tangent) — what is proposed is established.

[Margin: a — Euclid VI.4 (4 sexti).]

[Margin: An exact description of the 3- and 9-o'clock lines.]

3. But the hour-lines will be drawn far more precisely by this alternative method. First, the 3- and 9-o'clock lines are to be drawn with the greatest care in this way. Through A (in the figure of chapter 16) draw the straight line PQ parallel to the meridian, so that it represents the prime vertical, and transfer onto it, on either side of A, the tangent of the arc of the 3rd and 9th hour — the arc which, in Table 6, is written opposite the elevation of the pole in the right-hand column, and which at latitude 42° amounts to $53^{\circ} 23'$. The tangent, I mean, taken with respect to the whole sine DA, marking the points P and Q. For through these, from E, the 3- and 9-o'clock lines are to be drawn. You will also do correctly if, on the aforesaid perpendicular, you find the points for the same hours. Which of them marks the 3rd hour and which the 9th, the position of the dial will readily show. For on a dial declining from the meridian toward the east or toward the west, the line set below the horizontal line, to our left, always indicates the 9th hour, and the other, to the right, the 3rd hour — so that on a dial declining from the meridian toward the east, the upper point on the vertical line PQ belongs to the 3rd hour and the lower to the 9th; whereas, conversely, on a dial declining from the meridian toward the west, the upper point looks to the 9th hour and the lower to the 3rd.

Diagram. A large construction figure for a declining sundial, filling the lower two-thirds of the page within a ruled rectangular border. It shows the meridian line, the horizontal line, the 6-o'clock line, the style line, the equinoctial, and a full fan of hour-lines radiating from two centers (the dial center near E and a second center near A), crossed by an arc of a circle. The hour-lines are numbered along their upper ends with the whole and half hours: 3, 3½, 4, 4½, 5, 5½, 6, 6½, 7, 7½, 8, 9, 10. Two interior text-labels read "Num. 10" and "Num. 9". Beneath the main construction, at the foot of the frame, are five graduated tangent-scale rulers, each marked off in divisions numbered 10, 20, 30, 40, 50, 60, and captioned:

- "Pro horiz. et lin. Styli" — for the horizontal and the style-line (between points C and D)
- "Pro mer." — for the meridian (between D and B)
- "Pro æqum." — for the equinoctial (between K and N)
- "Pro Vert" — for the vertical (between D and A)
- a fifth, lowest ruler (beginning at L)

Clearly readable point-labels in the main figure: A, B, C, D, E, F, K, L, M, N, P, Q, R, S, T, V, X, Y, Z, H, I, together with several small lowercase labels (d, e, f, i, m, n, r, y). Many of the peripheral small labels and the exact point to which each hour-number attaches could not be resolved with certainty at this resolution [several labels ?].

and the lower to the 3rd hour. So that these two lines too may be drawn more accurately, the lemma demonstrated before chapter 16 should be applied. Or else, for the same lines, the points on the perpendicular to the meridian, corresponding to the points of those lines on the horizontal, may be found, as we prescribed just now. If you prefer, the other hour-points can also be found, from the same Table 6, on the vertical line PQ, as was stated in No. 7 of chapter 16.

[Margin: A more accurate laying-out of the hours.]

4. Once the 3- and 9-o'clock lines have been drawn, cut off from the 6-o'clock line, from E as center, on either side, lengths equal to AP (or AQ), and through the outermost points draw parallels to the meridian, meeting the 3- and 9-o'clock lines in points. If these points are joined by a straight line, the joining line will be equal to PQ and parallel to the 6-o'clock line, and will be bisected by the meridian, just as PQ is bisected at A. And if onto the line PQ and onto that parallel to the 6-o'clock line there be transferred, from their middle points on either side, the tangents of 15°, 30°, 45°, and so on — taken, with respect to the whole sine AP (or AQ), from the table of the

scholium of chapter 1 — one will obtain the points of all the whole hours, and so forth. On these two lines the same hour-points can also be found without tangents, as [was said] at the beginning of No.

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10 of chapter 16 — namely that they are to be found on the lines PQ, PM, and so on. Moreover the hour-points can, if one wishes, be found on the equinoctial line, once it has first been found by problem 5 at the end of the little book, by the inclination of the meridians, as was done in No. 8 of chapter 16.

5. But if you prefer to use the whole sine AB, constructed at the outset for scale 2, in place of the whole sine AP, then from the meridian, from the point E, cut off a straight line equal to the segment AB of scale 2, and through its outermost point draw a line parallel to the 6-o'clock line, meeting the 9- or 3-o'clock line at a point; and if through that point a parallel to the meridian is drawn, its portion between the 3rd and 9th hours will be bisected by the 6-o'clock line, and its half will be equal to the segment AB on scale 2. If, then, on the 6-o'clock line there are taken, from the center on either side, straight lines equal to the said segment AB of scale 2, and through the outermost points parallels to the meridian are let down, they will meet the 3- and 9-o'clock lines in points; and if these are joined by a straight line, the joining line will be parallel to PQ, bisected by the meridian, and its half equal to the same segment AB on scale 2. Onto these two straight lines, then — parallel to the meridian and to the 6-o'clock line — if the tangents of the hours, taken from the table of the scholium of chapter 1 with respect to the whole sine AB on scale 2, are transferred from their middle points, the hour-points will again be found, and so forth. And here this convenience results, that there is no need to divide the new whole sine AP into 10 equal parts; rather the whole sine AB, already divided on scale 2, is to be used. Indeed this procedure is necessary whenever the point A can scarcely be contained on the dial: but then the 6-o'clock line will have to be drawn in the way we explained at the beginning of p. 89, even though its intersection with the horizontal does not appear.
6. Now if it should happen that a dial has to be constructed which declines toward a latitude smaller than 30° or greater than 60° , so that Table 6 cannot serve, the 3- and 9-o'clock lines must be drawn as we taught in No. 9 of chapter 16. The remaining hours are to be described as was stated in Nos. 4 and 5 of this 4th exercise; and their points, too, can be found on the horizontal line in the way we set forth at the end of No. 9 of chapter 16.

[Margin: The construction of a dial declining toward a latitude smaller than 30° , or greater than 60° .]

THE ARCS OF THE SIGNS ON A DECLINING DIAL, for the hours from noon and from midnight.

PRAXIS V.

[Margin: The arcs of the signs among the hours from noon and from midnight on a declining dial. Which method is the most excellent for the declining parallels.]

1. The points of the Sun's parallels on the hour-lines will be conveniently found as was explained in chapter 18, Nos. 1, 4, 5, and 6, all the other methods being set aside, since they are less convenient and not so ready and easy. I myself, however, would prefer the method of Nos. 4 and 5, provided the lines are carefully drawn equidistant on either side from the style line, after the fashion of the hour-lines on a horizontal dial, as we handed down in No. 4 of chapter 18. For by this route as many points as one likes can be found for drawing the parallels, on however many lines may indicate the minutes of those hour-spaces, if the table for the addition before No. 3 of chapter 15 is used. And I wish this to be understood of the horizontal dial as well.

THE HOURS FROM SUNRISE AND SUNSET WITH THE ARCS of the signs on a declining dial.

PRAXIS VI.

[Margin: The hours from sunrise and sunset on a declining dial.]

1. The hours from sunrise and sunset on a declining dial will be readily described by means of what we wrote in chapter 19, Nos. 1 and 2, with the addition of what is handed down at the end of No. 3 of the same chapter — so that there is no need to prescribe anything new on this matter.
2. Yet that most beautiful method of No. 8 of chapter 19 ought not, it seems, to be passed over — the method by which, at any hour from noon or midnight, a point is found through which the proposed hour from sunrise or sunset is to be drawn. On this subject the things we wrote at the beginning of No. 4 and throughout the whole of No. 8 of chapter 19 must be carefully considered. And since in No. 9 of the same chapter we said that this same procedure fits the horizontal dial too — but that the declination

of the diurnal arc, by which the point is to be found at a given hour from noon or midnight, from sunrise or sunset, we taught must be taken from Table 13, so that on the meridian line the point of that diurnal arc may be found — it seems only necessary to add here that the same point of the diurnal arc can be found on the meridian, without its declination, in this way. Let half of that diurnal arc be subtracted from 24. For thus there remains the hour from sunset at which noon falls on that diurnal arc. If, then, that hour from sunset is described, as we taught in exercise 3, it will cut the meridian at the sought point of the diurnal arc. For example, if the proposed diurnal arc is one of $16\frac{1}{2}$ hours, we shall subtract its half, $8\frac{1}{4}$, from 24, and there remains the number $15\frac{3}{4}$; the hour of $15\frac{3}{4}$ from sunset, when described, will cut the meridian at the point through which the diurnal arc of $16\frac{1}{2}$ hours passes. The hour from sunrise indicated by the half of the proposed diurnal arc will show the same point more easily and quickly. Thus, in the given example, the hour of $8\frac{1}{4}$ from sunrise will cut the meridian on the diurnal arc of $16\frac{1}{2}$ hours; and so for the rest.

[Margin: The finding of the point of any diurnal arc on the meridian of a horizontal dial, without the declination of that diurnal arc.]

3. The arcs of the signs on a declining dial, for the hours from sunrise and sunset — even if their points among the hours from noon and midnight have not been found — we shall lay out by this method. From the table belonging to the addition before No. 3 of chapter 15, seek (having first found its distance from noon in hours and minutes) the point on the meridian line for the given hour from sunrise or sunset, just as on the horizontal dial, as was stated in chapter 8, No. 7 — namely from that arc of the table which corresponds to the distance of the proposed hour from sunrise or sunset. For the straight line drawn from A — the intersection of the equinoctial line with the horizontal — and the 6-o'clock line, through the found point, will cut the proposed hour from sunrise or sunset at the point of that parallel of the Sun from whose arc the point was found. When the point of some arc of the aforesaid table cannot be obtained on the meridian, or only with difficulty, the point must be sought on the 6-o'clock line, as was set forth in No. 2 of chapter 18. Or else the point in the given hour from sunrise or sunset must be investigated for the proposed parallel, by the distance of that hour in degrees and minutes which it has from noon, as was stated in chapter 15, No. 3.

[Margin: The arcs of the signs among the hours from sunrise and sunset on a declining dial.]

4. But I do not wish to pass over, in this place, the method by which, through that table of No. 2 of chapter 15, the points

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for the hours from sunrise and sunset can be found more exactly on the meridian line and on the 6-o'clock line. For since that table contains only the arcs corresponding to the hours and minutes, whereas the distances of the hours from sunrise and sunset which they have from noon usually contain, beyond the hours and minutes, seconds as well, the proportional part due to those seconds must be extracted, so as to be added to, or subtracted from, the arc of the aforesaid table, according as the following arc is greater or smaller than the preceding one.

[Margin: The proportional part in the table given in No. 2 of chapter 15, and how it is to be extracted.]

For example: the 23rd hour from sunset and the 1st from sunrise, at the beginning of ♋ [Cancer], is distant, at latitude 42° , $98^{\circ} 3'$ from noon — that is (if by table 1, placed among the tasks of the equinoctial circle in the sphere, this distance is reduced to time) 6h 32m 12s, or from midnight (which distance is the smaller) 5h 27m 48s. Now on the meridian circle the hour 5h 27m corresponds to the arc, in the aforesaid table, of $71^{\circ} 44'$, while to the hour 5h 28m corresponds the greater arc of $72^{\circ} 15'$. Subtracting the former from the latter, the remaining difference will be $0^{\circ} 31'$. We shall therefore say: if 60 seconds (namely 1 minute) give a difference of $0^{\circ} 31'$, what will 48 seconds give? and we shall find $0^{\circ} 24' 48''$ — that is, $0^{\circ} 25'$ (for when the seconds are more than 30, one minute may be taken for them) — as the proportional part sought; which, added to the smaller arc of $71^{\circ} 44'$, makes $72^{\circ} 9'$. This arc does not differ from the one you will find corresponding, in Table 8, to the 23rd hour from sunset in ♋, because that one was computed through the true distance from noon.

Again, the same 23rd hour, at the beginning of ♑ [Capricorn], is distant from noon $51^{\circ} 57'$, that is, 3h 27m 48s. Now on the meridian circle the hour 3h 27m corresponds to the arc $35^{\circ} 5'$, while to the hour 3h 28m corresponds the greater arc $35^{\circ} 14'$. Subtracting the former from the latter, there remains a difference of $0^{\circ} 9'$. Say therefore: if 60 seconds have a difference of 9', how much will 48 seconds have? and you will find $0^{\circ} 7' 12''$ as the proportional part; which, added to the smaller arc of $35^{\circ} 5'$, makes $35^{\circ} 12'$, as in Table 8. For the 12 seconds, since they are fewer than 30, are to be omitted.

Furthermore, the same 23rd hour, whose distance from noon in ☉ is 5h 27m 48s, has on the 6-o'clock circle the arc $23^{\circ} 43'$, and the following, smaller arc is $23^{\circ} 42'$. Say therefore: if 60 seconds have a difference of $0^{\circ} 1'$, how much do 48 seconds have? and you will find 48 seconds; and for these, if you subtract 1 minute from the greater arc, you will get the arc $23^{\circ} 42'$. And so for the rest. But this proportional part is not needed if, for seconds more than 30 in the distance from noon, you add 1 minute to that distance and seek the arc in the table from the distance so composed: for no notable error will be detected in the tangents, especially if the two nearest arcs have a small difference.

A NORTH-DECLINING DIAL; A VERTICAL DIAL NOT declining; a meridian dial; the meridian line, and the declination of the wall, etc.; the transfer of a dial onto a wall; a dial by the horizontal circumferences of the Sun alone, and by the Sun's altitudes.

PRAXIS VII.

[Margin: A dial declining from the north; a vertical dial not declining; a meridian dial, etc.]

1. For a dial declining from the north, read Nos. 1 and 2 of the scholium of chapter 19; for a vertical dial not declining, chapter 20.
2. For a meridian dial, read chapter 21. On such a dial the points of the arcs of the signs will be found, among the hours from sunrise and sunset, if from the table pertaining to the addition before No. 3 of chapter 25 there are taken the arcs which, on the 6-o'clock circle, correspond to the distances of the hours from sunrise and sunset, and their tangents are transferred onto the 6-o'clock line from A, marking the points on it. For parallels to the equinoctial, drawn out through those points, will cut the hours from sunrise or sunset at the points of the parallels. Or, omitting these parallels, one must proceed with the compass or the set-square, as we said in the addition before No. 3 of chapter 15.
3. For the finding of the meridian line, the declination of the wall, and so on, read chapter 22.
4. For transferring a dial onto a wall, read chapter 23.

5. For describing a declining dial by the horizontal circumferences of the Sun alone, together with the Sun's altitudes, read chapters 24 and 25 with its scholium.

FINIS.

A MOST COMPENDIOUS METHOD OF DESCRIBING HORIZONTAL AND DECLINING SUNDIALS

*(COMPENDIVM BREVISSIMVM DESCRIBENDORVM HOROLOGIORVM
Horizontalium ac Declinantium.)*

**By CHRISTOPH CLAVIUS [Christophorus Clavius] of Bamberg, of the Society
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(AVCTORE CHRISTOPHORO CLAVIO BAMBERGENSI SOCIETATIS IESV.)

PREFACE

Some years ago I gave my students a geometrical account of sundials [horologia] — and a very brief one. Not a few people pressed me to publish it, among them a man distinguished not only by his exceptional zeal but also by his generosity, the illustrious Isidoro Ruberti [Isidorus Rubertus]. Won over by their entreaties, I have now had it printed, though somewhat enlarged. For although in the year 1599 I published a new and most accurate method of laying out sundials by means of tangent lines [per lineas tangentes], that method cannot be put to use unless certain tables are employed — so much so that without them no sundial can be built from memory. I am confident, therefore, that this compendium will not be unwelcome to students, since once it has been properly understood, sundials can be laid out from memory in any place whatever. Yet I shall not entirely neglect the tangents: anyone who wishes may still use them, whenever he likes, for constructing sundials — since by them the hours are laid out far more accurately, together with the arcs of the zodiacal signs [arcubus signorum], than by the geometrical method alone, on account of the countless nearly oblique sections that tend to occur in it. A fuller treatment by means of tangent lines the diligent reader may seek in the aforementioned new description of sundials.

ASTRONOMICAL HOURS — THAT IS, RECKONED FROM NOON AND FROM MIDNIGHT — ON A HORIZONTAL PLANE

(HORÆ ASTRONOMICÆ HOC EST A MERIDIE AC MEDIA NOCTE IN PLANO HORIZONTALI.)

CHAPTER I (*CAPVT I.*)

Draw two straight lines AB and CD cutting one another at right angles at E; and on CD, taking the length of the style [gnomon, *stylus*] EC, describe from C, at any interval you please, an arc of a circle FG cutting CD at D. From D count upward the complement of the elevation of the pole [altitudo Poli] as far as F, and downward the elevation of the pole itself as far as G. For when the straight lines CF and CG are drawn, cutting the straight line AB at A and H, A will be the center of the sundial, at which all the hour-lines will meet. Now if a straight line is drawn through H perpendicular to AB, it will be the equinoctial line [linea æquinoctialis]; AB itself will be the meridian line, and AC the Axis of the World [Axis Mundi].

Now let HB be taken equal to the straight line HC (whether this is done upward or downward makes no difference). Then from B describe a circle IKLM of any size; in it draw the diameter KM through B, perpendicular to AL, and cut the whole circle into 24 equal hours. This is easily done: with the compass opened to the semidiameter IB, from the four points I, K, L, M cut off two arcs on either side. For in this way the whole circle, with the same opening of the compass, will be distributed into 12 equal parts. Then, each part being bisected, the whole circle will be divided into 24 hours.

After this, laying a ruler across each pair of points diametrically opposite, cut the equinoctial line at points through which straight lines, projected from the center A of the sundial in both directions, will give the hours from noon and from midnight.

Diagram. A horizontal-sundial construction figure, set in a rectangular frame and captioned "**1. fig.**" [prima figura] at its upper right. The construction is that described in Chapter I: two straight lines cross at right angles (the meridian AB running roughly vertical and the line CD running across), with the center of the sundial A at the center-right, where the fan of hour-lines converges. Radiating

from A is a full spread of straight hour-lines, numbered with the hours **1** through **12** near the center and continued as **1–8** on the extended lines toward the lower left. A large circle **IKLM** stands to the left, centered at **B**, divided into 24 parts, with the diameter **KM** drawn through B.

Clearly readable labels:

- A (center-right): center of the sundial [centrum horologii], meeting-point of all hour-lines.
- B (center of the large left-hand circle): center from which circle IKLM is described.
- E, H (on the central axis, between A and the circle): E the intersection of AB and CD; H the point through which the equinoctial line passes.
- I, K, L, M (on the large circle): the four cardinal points of circle IKLM; K at top, M at bottom, L toward the left.
- Hour numerals 1–12 spaced around the fan of lines from A; further hour numerals 1–8 on the lower-left extensions.

Labels present on the figure but too small / crowded to resolve with certainty on the scan: F [?], G [?], C [?], D [?], N [?], P [?], Q [?], S [?] (the letters called for by the text — the arc-endpoints F, G on CD, and the auxiliary points N, P, Q, R, S used later — are inked among the dense line-crossings but cannot be individually pinned down here). **More than two labels are flagged [?].**

and from midnight. Only the line of the sixth hour is to be drawn through the center A of the sundial, perpendicular to the meridian line. The order of the hours is this: the afternoon hours — 1, 2, 3, and so on — lie to the left of the meridian line, while the forenoon hours — 11, 10, 9, and so on — lie to the right.

But since some of the diameters of the circle IKLM cut the equinoctial line very obliquely — so that it is not easy, by means of them, to find without error the points on the equinoctial line through which the hours are to be drawn from the center A of the sundial — we shall lay out those hours more accurately by the following device.

From any point N of the third or ninth hour, draw NO perpendicular to the sixth hour. (In the figure the point N has been taken on the third hour.) Take OP equal to NO, and from P describe a circle at any interval; then from the point Q, with the interval of this circle's semidiameter QP, cut off two arcs QR and QS, each of which spans four hours. Each arc being therefore divided into four equal parts, if concealed [construction] lines are projected through them from the center P, the

straight line NO will be cut at points through which the hours are to be drawn from A, the center of the sundial — even though the points [themselves] were not found on the equinoctial line. You have an example in the 2nd, 3rd, 4th, 5th, 7th, and 8th afternoon hours. In the figure the point R could not be marked, for lack of room. Likewise the point on the equinoctial line, through which the third or ninth hour is to be drawn, we shall obtain if we carry the straight line HB or HC over from H onto the equinoctial line — to the left for the third hour, to the right for the ninth. If to the right you draw another line parallel to the meridian, set off from the meridian by as great an interval as the parallel NO is distant from it, and transfer onto this parallel the points of the parallel NO, you will have three points for each hour, and so on. And if the hour-spaces in the circles IKLM and RQS are bisected, or divided into four equal parts, the halves and quarters of the hours will be drawn in the same way.

Moreover, we shall find the hour-points on the equinoctial line and on the parallel NO most exactly by the help of tangent lines — indeed we shall construct the whole sundial — in the following way. Prepare an instrument of this kind. On some flat surface, whether of brass or of wood, or on a rather thick sheet of paper, make a right angle BAC (it could also be made obtuse or acute); and taking two equal straight lines AB and AC, divide each into 100 equal parts, and through the several divisions draw individual arcs from the center A. In the figure each of the lines AB and AC is divided into ten parts, each of which is again to be subdivided into ten smaller parts. With this instrument the tangents of any arcs whatever will be obtained without any trouble,

Diagram. Upper figure, captioned "**2. figura**" at its upper left. A rectangular frame containing a fan of concentric quarter-circle arcs centered on the corner A at the top right. Along the top edge, tick-marks are numbered **100, 90, 80, 70, 60, 50, 40, 30, 20, 10** running from left (corner B) rightward to A; down the right edge from A the marks run **10, 20, 30, ...** The concentric arcs sweep from the top edge round to the left edge. A dotted line runs diagonally across the arcs (the parallel NO of the text). Corner and edge points labelled: **A** (top-right), **B** (top-left), **C** (bottom-left), **D** (on the left edge, at the dotted line), **E** and **F** (mid-field, where the dotted line crosses), and **H, I** (lower, on a short horizontal segment separating this figure from the instrument below). Arc-value numerals 10–100 read cleanly; the small point-letters D, E, F, H, I are legible but sit among crowded line-crossings.

Diagram. Lower figure (uncaptioned): the tangent-instrument described in the closing paragraph. A narrow wedge opening from the right-angle vertex A at the right (enclosed in a small circle, with a center-dot). Two equal straight scales

diverge leftward from A — the upper scale to **B**, the lower scale to **C** — each graduated with tick-marks numbered from A outward **10, 20, 30, 40, 50, 60, 70, 80, 90, 100**. Intermediate point-letters are marked on the scales: **D** and **F** on the upper scale AB, **E** and **G** on the lower scale AC (the exact letter at each tick is small and the assignment of D/F and E/G to particular divisions is not fully certain [?]). The whole is enclosed in a rectangular frame. **Point-letter placement uncertain at this resolution for label verification.**

the tangents of any arcs whatever will be had, the whole sine being set at 100 — which is enough for the present purpose. Thus, from tangents whose whole sine is 10000000, five figures are to be dropped on the right; three, when the whole sine is 100000; and finally only one, when the whole sine is put at 1000, as in the table of the new layout of sundials — provided, however, that a unit is added to the remaining tangent when in this table the figure has been dropped, or, in other tables, when the first figure dropped on the left was greater than 5; or half a unit, when that figure was exactly 5, or not much greater. On this principle the tangents will be taken.

For the given whole sine, take an interval equal to BD, and draw the hidden line AD. Then, if the tangent is not greater than 100 (after the said figures have been dropped), the interval from the tangent's number on line AB across to line AD will give the tangent sought. For example, if the tangent is 40, the interval EF will be the tangent of 40 parts, provided the whole sine of the interval BD is thought of as divided into 100 parts. But if the tangent is greater than 100, the whole sine BD must be laid off on some straight line as many times as there are hundreds in the given tangent, and the remaining tangent added. Thus, if the given tangent is 140, the whole sine BD is laid once along the line GH, from G to H, and to it is added the tangent HI, equal to EF, of 40 parts. For in this way the line GI will be the tangent of 140 parts, the whole sine containing 100 parts. And if the given tangent were 540, the whole sine would have to be taken five times along GH, and the tangent of 40 parts added, and so on.

Diagram. A large combined construction plate, captioned "**3.^a fig.^a**" (rotated, at the upper left), enclosed in a rectangular frame. It joins two constructions:

- *Upper region — the tangent-scale instrument* (as in the 2. figura of the prior page): a fan of concentric quarter-arcs sweeping from a right-angle vertex, whose straight edges are graduated with tick-marks numbered **100, 90, 80, 70, 60, 50, 40, 30, 20, 10** (these read cleanly). A small right triangle sits at the upper left with its hypotenuse labelled **A–P**; on a short vertical within it

are **R**, **E**, **R**. Along the top run the points of the parallel line, labelled **N**, **H**, **N**, with a further **P** lower on the diagonal.

- *Lower region — the horizontal dial proper:* a large circle (the hour-circle) centered in the field. The dial center **A** lies at the left on the horizontal (meridian/equinotial) line; a fan of hour-lines radiates from **A** across the circle.

Readable point-letters: **L** (upper-left endpoint of the horizontal), **A** (dial center), **S** (left endpoint below **A**), **M** (top of circle), **G** (upper right of circle), **D** and **E** (on the horizontal diameter), **F** (bottom of circle), **N** (left of **F**), **O** (bottom of circle), **K** (right endpoint of the horizontal). Clustered near **A/S** and among the interior crossings, and less certain: **P**, **Q** (lower left) [?], **X**, **Z**, **T**, **I**, **Q** (interior, along a dotted parallel) [?], a small "3" beside **A** [?]. The radiating hour-lines carry morning/afternoon hour-pairs around the circumference; legible are **12** (top), **9 3**, **8 4**, **6** (near **K**), **5**, and **1 5** (lower right) [?]; the intervening pairs (11/1, 10/2, 7/5) sit among dense line-crossings and cannot be resolved.

Many small point-letters lie among crowded intersections for label verification.**

Perhaps this same instrument will be built more conveniently as follows. Make two equal rules of brass, **AB** and **AC**, joined at the center **A** so that they can turn uniformly about it, in such a way that, when the instrument is closed, the lines **AD** and **AE** form a single straight line. For if two equal lines **AF** and **AG** are drawn on these rules from the center **A**, and both are divided into 100 equal parts, the instrument built will serve the same purpose as the former one — provided the interval between the two 100-marks is taken equal to the given whole sine. But since very small tangents cannot be taken near the center on this instrument (the center staying fixed), we shall take two larger tangents whose difference equals the tangent sought. For when these are transferred onto any straight line, their difference will give the tangent sought. Thus, if the tangent of 4 parts is to be taken, we shall use the difference between the two tangents 20 and 24, or 60 and 64, and so on. And when the whole sine is so small that we cannot set it between the two 100-marks, let its tenfold be taken between them; then take the tenth parts of all the tan-

—gents — which is done by dropping one figure. Thus, if the tangent is 95, corresponding to 43° 31', the tangent 9½ [?] must be taken with respect to the tenfold whole sine. We have set out the use of both instruments more fully and at greater length in our *Practical Geometry* [Clavius, *Geometria Practica*], to which we refer the reader.

But let us now come to the layout of the sundial by tangents, which in our example we shall carry through with the latter instrument. In the first figure, then, draw two lines AB and CD cutting each other at right angles at E; and, having taken the length of the gnomon EC, we shall set the interval between the two 100-marks equal to EC, as to the whole sine. Next we take the tangent of the complement of the pole's altitude — in this compendium we take the altitude of the pole at Rome to be 42° — namely 111, and carry it from E upward to A along the meridian line. A will be the center of the sundial, and the perpendicular OP the six-o'clock line. Likewise we carry the tangent of the pole's altitude — namely 90 — from E downward to H; and the equinoctial line drawn perpendicular through H will let us find the hour-points upon it as follows. Since one hour, whether before or after noon, takes in 15° , two hours 30° , three 45° , four 60° , and five 75° , if we take the tangents of these degrees — first setting the interval between the two 100-marks equal to the interval HC, as the whole sine — and carry them from H onto the equinoctial line on either side, we shall find the points through which the hour-lines are to be drawn from A.

But if the whole sine is set as OP, and an interval equal to it is taken between the two 100-marks, then the tangents of the same degrees, carried over from O, will give the hour-points on the parallel NO, and so on. For example, the tangent 173 — which, with respect to the whole sine HC, corresponds to the 4th and 8th hours, that is, to 60° — will give on the equinoctial line the points of hours 4 and 8. On the parallel NO, by contrast, the same tangent will give the points of hours 2 and 10, with respect to the whole sine OP, since these hours too are 60° — that is, four hours — distant from the six-o'clock line. In the same way we shall find the points of the half-hours and quarter-hours, if we reduce these fractions of hours to degrees and carry over their tangents from H and O, and so on.

The Arcs of the Signs [ARCVS SIGNORVM]

Chapter II [CAPVT II]

Draw the line AB to represent the axis of the world, and at A erect the perpendicular AC as the radius of the equator. Then, having described from A any arc FEG, cut off from the point E two arcs EF and EG equal to the greatest declination, that is, of $23\frac{1}{2}^\circ$. For the lines AF and AG, when drawn, will represent the radii of Cancer ☊ [Greek/zodiac: ☊, Cancer — the summer-solstice sign] and Capricorn ☋ [zodiac: ☋, Capricorn — the winter-solstice sign]. To draw the radii of the other signs, join the line FG, cutting the equator's radius at the point D; and about D, with the interval DF (or DG), describe the circle FHGI. Divide it into

twelve equal parts [the print here reads *12. quartes*], and with hidden lines join pairs of points equally distant from F (or from G); then mark where these lines meet the arc FEG. For through these intersections the radii of the signs are to be drawn from A. And if the radii of the half-signs, or even of the quarter-signs, are wanted, the twelve parts of the aforesaid circle must each be divided into two, or into four, parts, and so on. Which radii look toward the northern signs, and which toward the southern, we shall show shortly.

After this, on the axis AB — the line AB being taken equal to the axis AC — draw, in the sundial of the first figure, the perpendicular BK to AB from B (whether AB be taken to the left or to the right of A makes no difference). Then, describing from B any arc LNP, count off from N on both sides the complement of the pole's altitude, up to M and O, and join the lines BM and BO. The former represents the line of the twelfth hour at noon, the latter the line of the twelfth hour at midnight. The lines of the other hours we shall draw thus: join the line OM, which cuts BK at Q; about Q, through M and O, describe the circle MOS and divide it into 24 parts. Join any pair of points equally distant from M or O by straight lines that cut MO at points; the lines drawn out from B through those points will give the lines of the other hours. Their order runs from the noon twelve-o'clock line BM toward O, and from there back again up to M.

We can also draw these hour-lines in the following way. In the sundial, take in the first figure all the intervals between B and the hour-points on the equinoctial line, beginning from the twelve-o'clock line, and carry them in order, in the third figure, from A onto the equator's radius AH, marking the points on it. Through these points the hour-lines are to be drawn out from B: the topmost gives the twelve-o'clock line, the next hours 1 and 11, the following 2 and 10, and so on; while the line BK will show the sixth hour. And if the intervals on the arc MNO between N and the hour-lines are carried into the same arc toward O, we shall have the points through which the hour-lines beyond the sixth hour are likewise to be drawn.

This done, we shall describe the arcs of the signs in the following way. Take the intervals of the hour-lines that go out from B in the third figure — the intervals between B and the radius of each sign (in a horizontal dial the radii toward B look to the northern signs, the rest to the southern) — and carry them in order, in the fourth figure, from the dial center A onto the corresponding lines. Or, what comes to the same thing: carry the intervals of the hour-lines between the equinoctial radius and the radius of each sign onto the corresponding hour-lines, measured from the equinoctial line — toward the dial center if the given radius

looks to the northern signs, but to the opposite side if to the southern. For a curved line neatly drawn through these points, so that it makes no [sharp] angles, will give that sign whose radius-points were transferred: the curved line, that is, onto which the tip of the gnomon's shadow is cast when the Sun is at the beginning of that sign.

The southern signs, which are opposite the northern, we shall describe by the same labor, using this device. Having found the two points of two opposite signs on the meridian line — the third figure supplies these for us as the two intervals of the noon twelve-o'clock line BM between B and the radii of the two opposite signs, carried onto the dial's meridian line from the center A; or else as the two intervals of the twelve-o'clock line BM between the equator's radius and the same radii of the opposite signs, carried onto the dial's meridian line starting from the equinoctial line — transfer the portion of the meridian line between the northern sign's point and the dial center to the lower part of the meridian line, [measuring] from the point of the opposite southern sign. For example, in the fourth figure the portion of the meridian line between a, the Cancer ☊ point, and A, the dial center, is transferred from b, the Capricorn ☋ point, up to d.

Next, from A and d describe two equal circles of any size; and carry the intervals of the circle described from A — the intervals between the meridian line and the hour-lines going out from A — onto the circle described from d, measured from the same meridian line; then from the center d send hidden lines through those points. For if the hour-intervals between B and the radius of the northern sign are carried onto the corresponding hour-lines from A, the dial center, and onto the corresponding lines from the center d, we shall have the points of the two opposite signs. Indeed, once any northern sign has been drawn, we shall draw its opposite sign in this way. Bisect the portion of the meridian line between the two points of the opposite signs — for example, bisect the portion between a, the Cancer ☊ point, and b, the Capricorn ☋ point, at Q — and through the point of division draw a line parallel to the equinoctial. For if the sheet on which the sundial is drawn is folded along this line, and the points of the northern sign are pricked through with a fine needle, the points of the opposite southern sign will appear on the other side.

But because the oblique intersections that the hour-lines going out from B in the third figure make with the radii of the signs — especially the southern ones — can easily give rise to error, we shall find the points of the arcs of the signs at the hours much more accurately by tangents, as follows.

Diagram. The sign-arc sundial construction, captioned "4.^a fig.^{am}" (rotated, at the upper right), enclosed in a rectangular frame. A horizontal line (meridian/equinotial) runs across the field with the dial center **A** at its right end. Near the center-right the gnomon-foot **F** sits inside a small circle (the circle **AgE** of the text), with **E**, **a**, and **d** marked close by. Along the horizontal to the left of center are stacked **C**, **H**, **K**, **M**; a vertical line at the upper left carries **O** and **L**; **P** lies at the left end of the horizontal (marked with an arrow, $\leftarrow$ **P**); **B** appears on a dotted arc at mid-left.

A large hour-circle/arc dominates the field, its hour-lines fanning from **A**. Legible hour-numbers around it: **5**, **6P** [i.e. **hour 6**], **7** (upper right); **8**, **7**, **6**, **5** (lower right); **12**, **13** (bottom edge); and the central cluster of hour-lines **12**, **11**, **10**, **9**. Along the dotted construction-arcs are the sign-arc numbers **22**, **21** (upper) and **18**, **19** (mid), read with difficulty. The point **g** named in the text (intersection of the small circle with the 2nd hour) is not separately legible among the crossings [?].

Point-letters and numbers sit among very dense line-crossings; several are uncertain for label verification.**

Suppose, for example, that the Cancer ☿ point at the second hour after noon is to be found, in the fourth figure. Bisect the segment **AE**, between the center and the gnomon, at **F**, and about **F**, through **A** and **E**, describe the circle **AgE**. For the lines drawn from **E** to the intersections of this circle with the hour-lines will be perpendicular to those hour-lines. Then from the point **g**, where the said circle meets the second hour, lay off the line **gh** equal to the gnomon. For the interval **Eh** will be the whole sine, by which the points of all the signs at the second hour are found in this way. In the table set at the end of this compendium, opposite the pole-altitude of 42° , under hours 2 and 10, take the arc of $43^\circ 53'$; subtracted from the complement of Cancer's declination — that is, from $66^\circ 30'$ — it leaves $22^\circ 37'$. The tangent of these, $41\frac{1}{2}$, transferred from **g** and from the other intersection of the circle **AgE** with the tenth hour into hours 2 and 10, will give the points of the tropic of Cancer ☿. Likewise, if the complement of the same arc, $43^\circ 53'$ (that is, $46^\circ 7'$), is added to Capricorn's ♑ declination of $23^\circ 30'$, an arc of $69^\circ 37'$ results, whose tangent, 269, from [the same...]

[Continuation of CAPVT II — "Arcs of the Signs," concluded from the previous folio.]

and the same tangent [269], carried over from those same points of hours 2 and 10, will give the points of the tropic of Capricorn ♑ at hours 2 and 10; and the

reasoning is the same for the rest. Therefore, for the northern signs the arc of the aforesaid table must be subtracted from the complement of the declination — except when that arc is greater than the complement of the declination, for then the complement must be taken away from that arc, and the tangent of the remaining arc carried toward the center. For the southern signs, on the other hand, the declination must always be added to the complement of the aforesaid arc. In the same way, for an hour beyond the sixth, the named arc must be added to the complement of the declination: so, for the seventh hour from noon, one must add the arc corresponding to the straight line Ak to the complement of the declination of Cancer ϖ . Finally, for the sixth hour, the tangent of the complement of the northern declination must be carried from the center A on both sides into hour 6, with respect to the whole sine AC.

THE ITALIAN AND BABYLONIAN DIAL, that is, one showing the hours from sunset and from sunrise

[Heading verbatim: "HOROLOGIVM ITALICVM, ET BABYlonicum, hoc est, horas ab occ. & ortum onstrans." — "onstrans" [recte: monstrans].]

CAPVT III.

We shall describe the hours from sunset and from sunrise most conveniently by means of the diurnal arcs that comprise an even number of hours. These are drawn in no other way than the arcs of the signs were drawn — provided that, in place of the radii of the signs emitted from the point A, we emit the radii of the diurnal arcs. These radii are supplied to us by the hour-lines drawn out from B in the third figure. For the straight line BM is the radius of the diurnal arc of 0 hours; the next line, the radius of the arc of 2 hours; the one after that, of 4 hours; and so on in succession through the even hours, so that the straight line BO is the radius of the diurnal arc of 24 hours. The arcs of the circle MNO, between N and the said radii, comprise the declinations of the diurnal arcs.

Therefore, if the portions of the straight line MO between Q and the radii of hours 14 and 10 — that is, the portions QT, QV — are carried over into the same line MO, from the radius of the Equator on either side, as far as X and Y, we shall have the radii AX, AY of hours 14 and 10; and likewise for the rest. So that, if we carry the straight line QO from the radius of the Equator as far as Z, the straight line

drawn AZ will give us the radius of the diurnal arc of 24 hours. Now we shall find the points of any diurnal arc whatever upon the hour-lines of the dial in just the same way as we found the points of the arcs of the signs: namely, if the portions of the hour-lines going out from B, [taken] between B and the radius of any diurnal arc, are carried over into the corresponding hour-lines of the dial drawn from A, the center of the dial, and so forth.

Once the points of the diurnal arcs on the hour-lines have been found, we shall lay out the hours from sunset and sunrise with no trouble. For if the points of the diurnal arc of 14 hours have been found, hour 23 from sunset will pass through the point of that arc at hour 6 astronomical after noon, because hour 24 would pass through the point of the same diurnal arc at hour 7 after noon — since the Sun sets at the 7th hour. By the same reasoning hour 22 will pass through the point of the same arc at hour 5 after noon, and so with the rest. Again, in the diurnal arc of 12 hours, which the equinoctial line supplies to us, hour 23 will pass through hour 5 after noon (since the Sun sets at the 6th hour), and hour 22 through hour 4, and so on. Likewise, in the diurnal arc of 10 hours, hour 23 will fall at hour 4 after noon, and hour 22 at hour 3 after noon, and so on. Finally, in the arc of 24 hours, hour 23 must be drawn through hour 11 after noon, and hour 22 through hour 10 after noon, and so with the rest — so that, in any diurnal arc whatever, hour 23 is to be drawn through the hour that precedes the Sun's setting by one hour [?], hour 22 through the hour that precedes sunset by two hours, and so on. Here it is worth noting that the Italian hours touch the diurnal arc of 24 hours at those very points through which we have said they are to be drawn.

The line of hour 12 from sunset must be drawn parallel to the equinoctial line through the midpoint of the portion of the meridian line between A, the center of the dial, and H, the point of the equinoctial line; and it ought to touch the diurnal arc of 24 hours on the meridian line.

The line of hour 11, however, must be drawn through hour 6 from midnight in the diurnal arc of 14 hours, and through hour 5 from noon in the equinoctial line; and it will touch the diurnal arc of 24 hours at hour 11 from midnight.

The line of hour 10 from sunset must be drawn through hour 5 in the diurnal arc of 14 hours, and through hour 4 from noon in the equinoctial line; and it will touch the diurnal arc of 24 hours at hour 10 from midnight, and so on.

Just as we have described the Italian hours by means of the diurnal arcs of 10, 12, 14, and 24 hours, so it will be possible to describe those same hours by means of other diurnal arcs. For if, for example, we have the points of the diurnal arc of 16

hours, hour 23 must be drawn through hour 7 after noon in that arc, and hour 22 through hour 6, and so on.

We shall describe the hours from sunrise in no other way, by means of the diurnal arcs, if we carefully observe at what hour from midnight the Sun rises. For example, in the diurnal arc of 14 hours the Sun rises at the 5th hour from midnight. Therefore hour 1 from sunrise will pass through hour 6 from midnight in that same arc, and let the same reasoning be held for the rest.

These same hours from sunset and sunrise are, on the horizontal dial, equidistant from the hours reckoned from noon and from midnight, whose numbers are the halves of them. Thus hour 23 is equidistant from hour $11\frac{1}{2}$, and hour 14 is equidistant from hour 7, and so on. Therefore, if through the point of the equinoctial line through which the hour from sunset or sunrise is to be drawn, a line is drawn parallel to that astronomical hour which is the half of it, the proposed hour will have been described. Thus, because hour 23 passes through the astronomical hour 5 on the equinoctial line, if through hour 5 on the equinoctial a parallel to hour $11\frac{1}{2}$ is drawn — namely to the half of hour 23 — hour 23 will have been described.

The parallel will be drawn as follows: if the interval on the equinoctial line, between the astronomical hour to which the parallel is to be drawn and the astronomical hour through which the parallel is to be drawn, is carried into hour 6 from the center A. Thus, if the interval KL, between hour 11 and hour 4, is carried from A as far as M, the line drawn ML will be parallel to hour 11, and will indicate hour 22. Likewise, if the interval NO, between hour $10\frac{1}{2}$ and hour 3, is carried from A to P, the line drawn PO will be parallel to hour $10\frac{1}{2}$, and will show hour 21. Those hours, however, which lie a little before hour 12 from sunrise or sunset are conveniently drawn through the points at which the astronomical hours intersect hour 12 from sunrise or sunset. Thus hour $12\frac{1}{2}$ must be drawn as a parallel to the astronomical hour $6\frac{1}{4}$, through hour $12\frac{1}{4}$, in the line of hour 12 from sunrise or sunset. Hour 13, in turn, [is drawn] through hour $12\frac{1}{2}$ as a parallel to hour $6\frac{1}{2}$. So too hour 11 [must be drawn] as a parallel to hour $5\frac{1}{2}$, through hour $11\frac{1}{2}$. So too hour $11\frac{1}{2}$ [must be drawn] as a parallel to hour $5\frac{3}{4}$ [?], through hour $11\frac{3}{4}$ [?], and so on. As for the hours after 13 and before 11, there is no difficulty, since their points can be obtained most conveniently on the equinoctial line.

We shall also describe the diurnal arcs by tangents in no other way than we described the arcs of the signs — if, in place of the declinations of the signs, we take the declinations of the diurnal arcs, which are found by a single

multiplication through Problem 11, at the end of the new description of the dials, in this manner. With the whole sine set at 100000, multiply the sine of the difference between the given semidiurnal arc and the semidiurnal arc of the Equator (which comprises 6 hours) — that difference being reduced to degrees — by the tangent of the complement of the pole's altitude. For if you cast away the last 5 figures from the product, the remainder becomes the tangent of the declination sought. For example, if the declination of the diurnal arc of 14 or 10 hours is required, in the Roman Horizon, above which the pole is elevated 42 degrees: since the semidiurnal arc of 7 or 5 hours differs from 6 hours by 1 hour — that is, by 15° — if you multiply the sine of this difference, 25882, namely of 1 hour or of 15 degrees, by the tangent 111061 of the complement of the pole's altitude, and from the product 2874480802 you cast away the last 5 figures, there will remain the tangent 28745, to which corresponds in the table of Tangents $16^\circ 2'$, for the declination of the diurnal arc of 14 or 10 hours.

THE ASTRONOMICAL DECLINING DIAL

[Heading verbatim: "HOROLOGIVM DECLInans Astronomicum."]

CAPVT IV.

Two straight lines AB and GH being drawn, cutting each other at right angles at I — of which GH represents to us the Horizontal line — let there be taken on AB the straight line IA, equal to the style of the dial to be made; and from A, as center, let an arc of a circle CD be described, of any magnitude, cutting AB at B. From B toward the right let the declination of the wall be reckoned, if it declines from south toward the east; toward the left, if from south toward the west; and in the opposite direction let the complement of that same declination be reckoned. In our example we set the wall as declining from south toward the west by 30° ; and so the 30° of declination are reckoned from B toward the left, as far as C; and toward the right the complement of the declination, namely 60° , as far as D. Then from A, to C and D, let straight lines be drawn cutting the Horizontal line GH at E and F. Through E, indeed, the meridian line EL must be drawn, parallel to AB, or perpendicular to the horizontal line. Through F must be drawn both the line of hour 6 Astronomical and the equinoctial line.

Next, on the horizontal line let EG be taken equal to the straight line AE, and from G, having described the arc of a circle HK, let the altitude of the pole in the given region be reckoned as far as K. For the straight line GK, being drawn, will cut the

meridian line at L, the center of the dial; and from L, through I, the straight line LM must be drawn, which will be the line of the style.

But since, on account of the closeness of the points L and I, we can easily err in drawing this line, we shall find a third point through which it must be drawn, in this way. Taking the straight line LP above L, equal to LE, let there be drawn from P a line parallel to the Horizontal line; and taking on it the straight line PQ equal to EI, we shall have the third point Q that was sought.

After this, let there be erected from I the perpendicular IO, equal to the line of the style LM [and to] the style IA; and let the straight line LO be joined, representing the axis of the world, so that the angle OLI measures the altitude of the Pole above the proposed declining plane. And when the straight line OM has been drawn perpendicular to the axis, cutting the line of the style at M, the equinoctial line must be drawn from F through M; and this, if no error has been made, will necessarily be perpendicular to the line of the style.

This done, let there be taken on the line of the style MN equal to the straight line MO; and from N let a circle be described of any magnitude, which is to be cut into 24 equal parts, beginning from the diameter that is drawn through the intersection of the meridian line with the equinoctial line — such as is, in our example, the diameter RS — or from the diameter that is drawn through N and F, which indeed, unless an error has been made, must cut the former diameter at right angles. All the diameters of this circle being now drawn, the equinoctial line will be cut in points through which the hour-lines must be drawn out from L, the center of the dial. The order of the hours is this: the hours to the right of the meridian line are afternoon hours, but those to the left are forenoon hours.

But in order that we may draw more accurately the hours which cut the equinoctial line scarcely at all, or very far off, we shall use this device. From any point of hour 3 or 9 — namely from s — let a line be drawn perpendicular to the horizontal line, or parallel to the meridian line, cutting hour 6 at the point T; and through T let a line be drawn parallel to the horizontal line, from which let there be cut off the straight line Tv, equal to Ts. Then, an arc of a circle having been described from v at any interval, let there be cut off, at the interval of the semidiameter Tv, two equal arcs xy, xz, and let these be divided into 4 equal parts. For the hidden straight lines drawn from v through the points of division will cut the straight line sT — extended infinitely on both sides — in the points through which the hours must be drawn out from L, the center of the dial. These will be drawn more precisely if on the other side of the meridian line we draw a parallel ab, distant from the meridian line by as great an interval as the straight

line sT is distant from that same meridian. For if into this ab we carry the intervals of the straight line sT , beginning from the point b of hour 6 — yet in such a way that the equal intervals are alternate — we shall have, for each single hour, three points, of which L , the center of the dial, is always one, and through which [the hour-lines] must be drawn. For example, if for the interval Td , below T , we take an equal interval be , above b , hour 5 must be drawn through the three points e , L , d . So too, if for the interval Tf , above T , we take an equal interval bg , below b , hour 8 must be drawn through the three points g , L , f , and so on. By means of tangent-lines on the said two parallels sT and ab , we shall find the hour-points most exactly, in this way. On the upper instrument of figure 2, let the interval between 100 and 100 be taken, equal to the straight line Tv , for the whole sine. For if the tangents of 15° , 30° , 45° , 60° , 75° — as we said in the horizontal dial — are carried from the points T and b on both sides, we shall obtain the hour-points sought, three, that is, for each single hour. The order of the hours is not difficult. For the straight line bLt gives hour 6, from which the rest will easily be judged, as was said above.

The line of hour 3, or of hour 9, we shall also find in this way, even if the circle from N has not been described. From E , where the meridian cuts the horizontal, let the perpendicular EI be erected to the straight line GL . Likewise, to AE , from the same point E , having erected a perpendicular on both sides, let there be taken on it Em and En , equal to EI .

For if from A , through m and n , hidden straight lines are cast out, the horizontal line will be cut in the points q and p , through which hour 9 and hour 3 must be emitted from L , the center of the dial.

By these same tangents we shall also find the hour-points on the equinoctial line, by this reasoning. The straight line NR being drawn through the intersection of the line [of the style] with the equinoctial, let there be described from N any arc of a circle whatever, between the straight line NR and the line of the style NL . For from the degrees of this arc we shall learn what tangents we ought to take, with respect to the whole sine MO . For example, in our case, since the said arc comprises nearly $40^\circ 47'$, it must, with the tangent 68 (namely of $40^\circ 47'$, the whole sine being set as MO), carried from the point M — where the line of the style cuts the equinoctial — into the equinoctial line, terminate at the point of intersection of the meridian line with the equinoctial. But if to these same degrees we add the 15° by which hour 11 is distant from the meridian line, we shall make $55^\circ 47'$; and the tangent of this arc, 147 (with respect to the same whole sine MO), carried from the same point M into the equinoctial, will give us

the point of hour 11. And if we add another 15° , we shall find in the same way the point of hour 10, and so on. Again, from the aforesaid arc of $40^\circ 47'$ let us subtract the 15° by which hour 1 is distant from the meridian; there will remain $25^\circ 47'$, whose tangent 48, carried from the point M, will give the point of hour 1. But if we subtract 30° , by which hour 2 is distant from the meridian, there will remain $10^\circ 47'$, whose tangent 19 will, by the same reasoning, exhibit the point of hour 2. And since hour 3 is distant from the meridian by 45° , we shall take away from these the aforesaid arc of $40^\circ 47'$, so that there remain $4^\circ 13'$, whose

Diagram. The construction of the declining astronomical dial [horologium declinans astronomicum] of Cap. IV, captioned "5.^a fig.^a" (rotated, at the upper right), enclosed in a rectangular frame. The figure is very dense with intersecting straight lines and construction arcs radiating from the dial center, and carries both letter-labels for the geometry and small italic hour-numbers along the periphery.

Legibly readable labels and their approximate positions:

- **A** — near the center-right, on the horizontal, just below the star/center L.
- **L** — the center of the dial (horologij center), the hub from which the hour-lines fan out.
- **B** — mid-left, on the horizontal line.
- **C** — upper left, on an oblique construction line.
- **D** — lower right, on an oblique line at the frame edge.
- **E** — left of center, on the horizontal.
- **F** — bottom center (foot of the perpendicular / point on the horizontal), near H.
- **G** — mid-left, on the horizontal (near a "9"-marked line).
- **H** — bottom center, near F and T.
- **K** — right of center, lower area.
- **N** — center-left, above the horizontal.
- **R** — appears upper-right and again lower-left near G (two R's, or R with its diameter partner S).
- **S** — right side (mid) and a second S at lower left — the diameter RS.
- **T** — bottom center, near H and F.
- **P** [?] — below E, small.
- **Q** [?] — near G / mid-left, small.
- **Y** [?] — right side, lower, near K.
- **x** [?] — bottom center, near H.

- **M, m**, small **I/I** [?] — clustered tightly near the center.

Hour-numbers (small italic) around the field: **9** and **10** (upper), **8, 6, 5, 4** (upper right / right, descending), **12** (mid-left); and a tight cluster of low numerals **1, 2, 3, 4, 5, 6, 7** among the fanned hour-lines near the center-bottom. Several construction lines are dotted.

The point-letters and hour-numbers sit among extremely dense line-crossings; more than two labels (P, Q, Y, x, M, m, l) cannot be resolved with certainty. for label verification.**

tangent $7\frac{1}{2}$, carried over from M, will give (nearly) the point of hour 3. But if to these $4^{\circ} 13'$ we add 15° , we shall likewise find the point of hour 4, and so on.

From these things this general rule can be drawn, for a dial declining from south toward the east. To the arc between R and the line of the style — which in our example, as we said, comprises $40^{\circ} 47'$ — must be added the distances of the hours from noon, which they have from the meridian line; but from the distances of the hours from midnight, which they have from the meridian line (namely those set to the left of the meridian line) — if indeed they are greater than the aforesaid arc — that arc must be subtracted; but if they are smaller, the distances of the hours themselves must be taken away from the aforesaid arc. For in this way we shall always have arcs whose tangents, the whole sine being set as MO, will exhibit to us the hour-points on the equinoctial line.

on the equinoctial line. In a dial declining from south toward the west — which is the case we have set ourselves — to the aforesaid arc between R and the line of the style must be added the distances of the morning hours from the meridian line, that is, the hours drawn to the left of the meridian line. But from the distances of the afternoon hours, if these are greater than that arc, the arc itself must be subtracted; and if they are smaller, the distances themselves must be taken away from the aforesaid arc.

The degrees comprised in the arc between R and the line of the style can also, if one wishes, be found more surely by calculation, through a single multiplication. Multiply the sine of the pole's altitude above the horizon [sinus altitudinis Poli] — here at Rome, 66913 — by the tangent of the complement of the wall's declination [tangens complementi declinationis muri], which in our example is 173205; and from the product, 11589666165, strike off the last five figures. The remaining number, 115897, is the tangent of the complement of the arc we are seeking. Since this tangent corresponds in the table to $49^{\circ} 12'$, a little more, the complement of this arc will be $40^{\circ} 47'$, a little more. And that will be the size of

the arc sought, measuring the inclination of the meridian of the declining plane itself to the meridian of the horizon.

Before I finish this chapter, I should like to add another way of finding the hour-points by tangents, which is as follows. When the lines of hours 3 and 9 have been drawn with the greatest care, as we have just taught in this chapter, draw through some point of hour 3 or hour 9 — namely through *f*, on hour 3 — a line parallel to the line of hour 6, which is drawn from *L* through *F*. For the portion of this parallel lying between hours 3 and 9 will be bisected by the meridian line. If, then, you set as the whole sine the segment of that same parallel lying between the meridian line and hour 3 (or hour 9), and transfer onto that same parallel the tangents of 15° , 30° , 45° , 60° , and 75° , starting from the meridian line in each direction, you will have the points through which the hour-lines are to be drawn from *L*, the center of the dial. And if above the center *L* — when the size of the plane allows it — you draw another parallel to hour 6, as far distant from the center *L* as the first was from it, and carry the points of the first onto this one from the meridian line, you will have further hour-points. In the figure these parallels have not been drawn, but you can easily draw them yourself with any pen.

Moreover, we shall find the points *E*, *F*, and *L* by tangents, and in addition the point on the meridian through which the equinoctial line is to be drawn, by this method. Taking the length of the style *A I* as the whole sine, transfer the tangent of the wall's declination from *I* to the left, if the plane slopes from south toward the west, as far as *E*; or to the right, if it declines toward the east. Likewise transfer the tangent of the complement of the declination from *I* in the opposite direction, as far as *F*. For through *E* the meridian line is to be drawn, and through *F* the line of hour 6 and the equinoctial. Again, taking the interval *AE* as the whole sine, transfer the tangent of the pole's altitude above your horizon from *E* upward along the meridian line as far as *L*, and the tangent of the complement of that same pole-altitude downward as far as the point 12. For *L* will be the center of the dial, and through the point 12 the equinoctial line is to be drawn from *F*. If these things are done carefully, a most accurate layout of the dial will result.

THE ARCS OF THE SIGNS ON A DECLINING DIAL

CAPVT V.

Draw any straight line *AB*, and raise upon it a perpendicular *AC* for the radius of the equator [radius Aequatoris]; then from *A* let the radii of the signs and of the

daily arcs be sent out in both directions, as we set forth above in the horizontal dial. Next, on the axis LO take AB equal to it, and from B draw BD parallel to AC. After this, transfer the straight line NM (or MO) from the fifth figure into the radius of the equator in the sixth figure, from A up to the point H, and through H, from B, run the straight line BH, corresponding to the line of the style, or to the proper meridian of the declining plane. This line can also be drawn if we set up an angle ABH equal to the angle OLM which, in the dial, the axis makes with the line of the style at the center of the dial; or, what is the same thing, if from B an arc is described, in which two arcs equal to the complement of the pole's altitude above the proposed plane are cut off on either side, starting from the straight line BD. For the angle OLM expresses this altitude of the pole, while what remains of the right angle gives its complement. The straight lines drawn from B through the ends of the arcs will yield the said line BHG and another corresponding to it on the other side.

Now, taking any point G on the line B H, draw the straight line G D parallel to AB, cutting BD in D; and from D, with the interval GD, describe a circle, which is to be divided into 24 hours in this way. Transfer the interval between N and hour 12 on the equinoctial line of the fifth figure from A into the radius of the equator of the sixth figure, up to C; and through C, from B, draw a straight line cutting GD in e. The perpendicular ef raised to GD will give the point f, the beginning of the division. The order of the hours is to be gathered from the order they keep in the dial. Thus, since in our dial hours 1 and 2 follow the meridian line toward the line of the style, so too in our circle hours 1 and 2 must follow the point f toward the point G, through which the line BG, standing for the line of the style, is drawn.

Now, if from the individual hours of this circle described from D perpendiculars are dropped to the straight line GD — which is easily done, if for each of the arcs between G and the hours of one semicircle equal arcs are taken in the other semicircle, since the straight lines joining the corresponding points, that is, those equally distant from G, are perpendicular to GD (so that if an arc Gi is taken equal to the arc G f, the straight line f i will be perpendicular to GD; and the same is to be said of the points of the other semicircle) — then, through the points at which the straight lines from B cut GD, drawing lines out, we shall have the hour-lines corresponding to the hours marked on the circle.

We shall also draw these hour-lines if the intervals between N and the hours on the equinoctial line of the fifth figure are transferred into the radius of the equator from the point A, and from B straight lines are sent out through the points of the equator's radius, just as we said a moment ago concerning the line

of the style and the line of hour 12. But so that all the hours may be obtained, the hours must be carried over onto the other side of the straight line DG as well, and so on.

Once this is settled, both the arcs of the signs and the daily arcs will be described, as we taught in the horizontal dial; namely, if the intervals of the hour-lines between B and any proposed radius are transferred, from the sixth figure, into the seventh figure of the dial, from L, the center of the dial — or at any rate the intervals of the hour-lines between the radius

of the equator and any other proposed radius — let them be transferred into the corresponding hours, starting from the equinoctial line. Here you will observe that the radii sloping from the radius of the equator toward B belong to the southern signs, and the rest to the northern; so that in the dial the arcs between the equinoctial line and the center of the dial are southern, while those below the equinoctial line are northern.

Here too, by one and the same operation, we shall describe two arcs of opposite signs, as we did in the horizontal dial: namely, if to the portion of the line of the style between L, the center of the dial, and the arc of the southern sign, an equal portion is taken downward from the point of the northern arc of the opposite sign; and if, from the center of the dial and from this lower point once taken, equal circles of any size are described. For if the intervals between the line of the style and the hour-lines are transferred, on both sides of the upper circle, into the lower circle from the line of the style, and the rest is completed as we said in the horizontal dial, what we propose will have been done.

Indeed, if the portion of the line of the style between the two points of the opposite arcs is bisected, and through the point of division a line parallel to the equinoctial is drawn, about which the paper of the dial is folded, we shall find the points of the opposite northern arc from the points of the southern arc, exactly as, in the horizontal dial, we found the points of the southern sign from the points of the northern sign.

Diagram. The construction of the sixth figure — the "figure of the radii" for the declining dial [figura radiorum] — captioned "**6.^a figura.**" (rotated, lower right), enclosed in a rectangular frame. It shows a large circle (with a horizontal diameter and a vertical line through it) carrying the hour-divisions and, at the upper right outside the main field, a small triangle construction (the triangle LOM with the style OI and the transferred lengths). The figure is extremely dense with intersecting straight and dotted construction lines and radiating hour-lines.

Legibly readable labels and their approximate positions:

- **O** — at the far right, the apex of the small triangle.
- **P, I, R, N, M** — along and within the small triangle at upper right (the ordinates PN, RM, etc.); several of these small labels are crowded and partly repeated.
- **B** — right of center, the upper convergence-point from which many hour-lines fan out (the point standing for the style-line).
- **D** — on the vertical line, above B.
- **G** — lower right, the second fan/convergence point.
- **K** [?], **Q** [?], **P** [?] — small labels clustered near the lower fans, right of center.
- Hour-numbers around the circle: **1, 2, 3, 4** (upper), **8, 9, 10, 11** (upper left), **12** (left); and **5, 6, 7, 8, 9, 10** descending along the right/lower rim; along the bottom edge the low numerals **6, 12, 5, 1, 2, 3** among the fanned hour-lines.

The point-letters sit among extremely dense line-crossings; more than two labels (K, Q, P, and the crowded triangle ordinates) cannot be resolved with certainty. for label verification.**

Now, since almost the whole labor consists in describing the hour-lines, both in the dial and in the figure of the radii, once these have been described for a given length of style, if we wish to construct another dial for a greater or smaller style, we shall follow this course. Let the triangle LOM of the fifth figure of the dial be described separately, together with the style OI; and, this being produced, take OP equal to the given length of style — smaller or larger than the style OI — and through R draw PN parallel to LM. For if, in place of the axis BA, in the figure of the radii we take the straight line BP equal to OP, and from P raise the radius of the equator, the perpendicular P Q, to BP, and from there the radii of the other signs go out on this side and that, we shall describe the arcs of the signs for the given length of style, as before. For the equinoctial line must be drawn perpendicular to the line of the style, as far distant from L, the center of the dial, as the straight line BQ is long. The same is also to be observed in the horizontal dial, if its triangle ACH, together with the style CE, is described separately, as we here said of the triangle OLM, and as you see done in the third figure.

Now, to avoid so many oblique sections in the sixth figure, we shall describe the arcs of the signs by tangents, as in the horizontal dial: if, from M, on either side, the tangents of 15°, 30°, 45°, 60°, and 75° are transferred onto the equinoctial line — reckoning MO as the whole sine — impressing the points on the equinoctial.

For the hidden lines drawn from L through those points will represent the hours on the plane as though on a horizon; and on them, by tangents, the points of the arcs of the signs and of the daily arcs will be found, as in the horizontal dial — provided, however, that in place of the pole's altitude above the horizon you assume the pole's altitude above the declining plane, which the angle OLM exhibits, and which altitude you will also find by Problem 4 at the end of the new description: namely, if you multiply the sine of the complement of the pole's altitude above the horizon — 74314 in our example — by 86602, the sine of the complement of the plane's declination. For if

Diagram. The seventh figure — the declining dial itself [horologium declinans], captioned "7.^a figura." (rotated, upper right), enclosed in a rectangular frame. The hour-lines and the arcs of the signs radiate across the field, with a small circle of divisions near the dial center. Numerous dotted ("hidden"/occult) construction lines run parallel across the plate.

Legibly readable labels and their approximate positions:

- **L** — the center of the dial (upper right), where the hour-lines converge; a small divided circle is drawn about it.
- **E, I, K** — small labels clustered by the little circle near L, with a numbered scale **1, 2, 3, 4, 5** on its arc.
- **M** — left of center, a lower fan/convergence point.
- **O** [?] — lower center, near the foot of the style-line.
- **B** — upper left, on the frame.
- **a** — bottom center, on the lower horizontal (a point of the occult lines La, Lb).
- A run of tangent-degree marks along an oblique line: **16, 17, 18, 19, 20, 21, 22, 23, 24**.
- Hour-numbers along the rim and radii: **6, 7, 8, 9, 10, 11, 12**; corner numerals **2, 3**.

The letter-labels sit among extremely dense line-crossings, many of them dotted; more than two labels (O, E, I, K, and the corner letters) cannot be resolved with certainty. for label verification.**

from the product, the number 6435741028, you strike off the last five figures, the remaining number, 64357, will give the sine of the sought altitude of the pole. Therefore the pole's altitude above our declining plane contains 40° 4', which corresponds to the sine found, in the table of sines. You have an example in the seventh figure, in the hidden lines L a and L b, whose points a and b the Tangent

100000 [numeral: ?] of 45° — or of hours 3 and 9, if the declining plane were the horizon — exhibits. And the perpendicular extended through L to L I will be as it were the line of hour 6.

Now, since the arc of the following table under hours 3 and 9 — over against the pole's altitude of 40° , which is nearer to the altitude found — contains $40^\circ 7'$, we shall subtract it from the complement of the declination [?], namely from $66^\circ 30'$, and the remaining arc is $26^\circ 23'$. The tangent of $26\frac{1}{2}^\circ$ [Tangentem $26\frac{1}{2}$], reckoned against the whole sine kI, we shall transfer from the points c and h — the intersections of the circle L c I h described from the midpoint of the straight line L I — with the lines L a and L b, transferring them onto the straight lines La and Lb. For the points will lie outside, on the arc [zodiac sign: ?], and so on. Only be mindful that what we said in the horizontal dial concerning the northern signs is here to be understood of the southern, and conversely.

The Italian and Babylonian Dial on a Declining Plane. Chapter VI.

Once the diurnal arcs of the 10th, 12th, 14th — or even the 24th — hours have been drawn, we shall lay out the hours reckoned from sunset and from sunrise exactly as on the horizontal dial. These hours from sunrise or sunset always pass, on the horizontal line, through those astronomical hours whose numbers are the halves of the hours from sunrise or sunset: thus the 23rd hour will pass through the $11\frac{1}{2}$ th hour, and the 18th through the 9th, and so on. It is therefore worth marking, on the horizontal line, the points of the astronomical hours together with their halves. For by means of those points, and of the corresponding hours on the equinoctial line, the hours from sunrise or sunset will be drawn without any difficulty.

Now if the hour-lines of a dial declining from the meridian are extended beyond the horizontal line, the portion cut off from the horizontal line will yield a dial declining from the north toward the east (if it had been declining southward toward the west), or toward the west (if it had been declining southward toward the east) — declining, I say, by as many degrees toward the east or west as the southern plane had been declining toward the west or east. This holds provided that the said portion be inverted, so that its upper part becomes the lower and the reverse, and so that the part which, in our present position facing the dial, is on the right becomes the left, and the reverse. Likewise, the hours that were previously numbered from noon, or from sunset, are now to be numbered from

midnight, or from sunrise, and the reverse. The very arrangement of the hours will easily show this, if one carefully observes the line of the 24th hour alone, as we set forth in the new construction in the scholium to Proposition 19, Number 2. Finally, the southern signs become northern, and the reverse. And so, if a dial declining from the north is to be described, let one declining from the south toward the opposite side be constructed by the same number of degrees. For the lineaments of this dial, extended beyond the horizontal line, will yield the proposed dial declining from the north, as has been said.

The Transfer of a Declining Dial Drawn on Paper onto a Wall, to Any Size Whatever. Chapter VII.

Let a straight line $\varepsilon\beta$ be drawn, parallel in any manner to the horizontal line. If it does not cut all the hours, let the hours be extended until they cut it, provided they can conveniently be cut. Next, on the wall, let a horizontal line be drawn with the help of a level and plumb-line; and, a place for the style having been chosen on that same horizontal line, take from the paper dial the portion of the horizontal line lying between the position of the style and the meridian line. Lay off this portion on the wall's horizontal line, toward the same side to which the meridian line inclines on the dial, as many times over as the style on the wall is to exceed the style of your dial. For at the end of the counting a line perpendicular to the horizontal line must be dropped, to serve as the meridian line on the wall.

And if the intervals between the meridian line and the several hours on the horizontal line of the dial are likewise laid off, the same number of times, from the point of the meridian line on the wall's horizontal line, we shall have on the wall's horizontal line the points through which the hours are to be drawn.

Suppose the same is done on the parallel $\varepsilon\beta$ and on the parallel that corresponds to it on the wall — which we shall draw thus: dropping, on both sides of the meridian line $\varepsilon\gamma$, [perpendiculars] between the horizontal line and the parallel $\varepsilon\beta$ as many times as the style is to be enlarged; for the parallel drawn through the end of the counting, parallel to the horizontal line itself, will correspond to the parallel $\varepsilon\beta$. Then, likewise, on the equinoctial line — that is, let the intervals of the parallel $\varepsilon\beta$ between the meridian line and the hours be transferred, the same number of times as we said, onto the corresponding parallel line drawn from the wall's meridian line.

Likewise, let the intervals of the equinoctial line between the meridian line (or the horizontal line) and the hours be transferred onto the wall's equinoctial line, from the meridian or horizontal line, as many times as you wish to enlarge the style; and we shall have other points through which the hour-lines are to be drawn.

The equinoctial line on the wall is to be drawn as follows. Let the interval of the horizontal line between the meridian line and the equinoctial line be taken on the wall, from the meridian line along the horizontal line, as many times as is needed. For thus we shall have a point on the horizontal line through which the equinoctial is to be drawn. And if the same is done with the interval of the meridian line between the horizontal line and the equinoctial, we shall have another point on the wall's meridian line for the equinoctial line. And if the interval of the parallel $\epsilon\beta$ between the meridian and equinoctial lines is likewise taken on the corresponding parallel of the wall, we shall have a third point for drawing the equinoctial line.

Lastly, if the intervals of the hour-lines between the equinoctial line (or the horizontal line, or the parallel $\epsilon\beta$) and the arcs of the signs are laid off from the same lines on the wall, as many times as is required, in the corresponding hours, we shall have the points of the arcs of the signs on the wall.

Table

Table of the Arcs of the Hour-Circles from Noon and Midnight, [contained] between the Pole of the world and the Vertical circles of the horizon that are perpendicular to those hour-circles.

Column headers as printed: **Altitudines Poli.** (Altitudes of the Pole), then the paired hour-circles **12 · 1 11 · 2 10 · 3 9 · 4 8 · 5 7 · 6**, each subdivided into **G.** (gradus, degrees) and **M.** (minuta, minutes). Pole-altitude is likewise given in degrees (G.) and minutes (M.).

(Table begins on printed p. 224; continues from printed p. 225 without a break; the table continues beyond p. 225 into the following page — the collation step will stitch the boundary.)

Printed p. 224 (Pole altitudes 0° 0' – 14° 45')

		12		1&1		2&1		3&	3&	4&	4&	5&	5&	6	6
Poli	Poli	12	M	1&1	1	2&1	0	9	9	8	8	7	7	G	M
G.	M.	G.	.	1 G.	M.	0 G.	M.	G.	M.	G.	M.	G.	M.	.	.
0	0	90	0	90	0	90	0	90	0	90	0	90	0	0	0
0	15	89	45	89	44	89	43	89	39	89	30	89	2	0	0
0	30	89	30	89	29	89	25	89	18	89	0	88	4	0	0
0	45	89	15	89	13	89	8	88	56	88	30	87	6	0	0
1	0	89	0	88	58	88	51	88	35	88	0	86	9	0	0
1	15	88	45	88	42	88	33	88	14	87	30	85	11	0	0
1	30	88	30	88	27	88	16	87	53	87	0	84	13	0	0
1	45	88	15	88	11	87	59	87	32	86	30	83	16	0	0
2	0	88	0	87	56	87	41	87	10	86	0	82	19	0	0
2	15	87	45	87	40	87	24	86	49	85	30	81	22	0	0
2	30	87	30	87	25	87	7	86	28	85	1	80	25	0	0
2	45	87	15	87	9	86	50	86	7	84	31	79	29	0	0
3	0	87	0	86	54	86	32	85	46	84	1	78	33	0	0
3	15	86	45	86	38	86	15	85	25	83	31	77	38	0	0
3	30	86	30	86	23	85	58	85	3	83	2	76	42	0	0
3	45	86	15	86	7	85	40	84	42	82	32	75	47	0	0
4	0	86	0	85	52	85	23	84	21	82	2	74	53	0	0
4	15	85	45	85	36	85	6	84	0	81	33	73	59	0	0
4	30	85	30	85	21	84	48	83	39	81	3	73	5	0	0
4	45	85	15	85	5	84	31	83	18	80	34	72	12	0	0
5	0	85	0	84	50	84	14	82	57	80	5	71	18	0	0
5	15	84	45	84	45	83	57	82	36	79	35	70	27	0	0
5	30	84	30	84	18	83	39	82	15	79	6	69	36	0	0
5	45	84	15	84	3	83	22	81	54	78	37	68	44	0	0
6	0	84	0	83	47	83	5	81	33	78	8	67	54	0	0
6	15	83	45	83	32	82	48	81	12	77	39	67	4	0	0
6	30	83	30	83	16	82	30	80	51	77	10	66	14	0	0
6	45	83	15	83	1	82	13	80	30	76	41	65	26	0	0

			12		1&1		2&1	3&	3&	4&	4&	5&	5&	6	6
Poli	Poli	12	M	1&1	1	2&1	0	9	9	8	8	7	7	G	M
G.	M.	G.	.	1 G.	M.	0 G.	M.	G.	M.	G.	M.	G.	M.	.	.
7	0	83	0	82	45	81	56	80	9	76	12	64	37	0	0
7	15	82	45	82	30	81	39	79	48	75	44	63	49	0	0
7	30	82	30	82	14	81	21	79	27	75	15	63	2	0	0
7	45	82	15	81	59	81	4	79	6	74	46	62	15	0	0
8	0	82	0	81	43	80	47	78	46	74	18	61	30	0	0
8	15	81	45	81	28	80	30	78	25	73	50	60	45	0	0
8	30	81	30	81	12	80	14	78	4	73	22	60	0	0	0
8	45	81	15	80	57	79	55	77	43	72	53	59	16	0	0
9	0	81	0	80	41	79	38	77	22	72	25	58	32	0	0
9	15	80	45	80	26	79	21	77	2	71	58	57	49	0	0
9	30	80	30	80	10	79	4	76	41	71	30	57	7	0	0
9	45	80	15	79	55	78	47	76	20	71	2	56	25	0	0
10	0	80	0	79	39	78	30	76	0	70	34	55	44	0	0
10	15	79	45	79	24	78	12	75	39	70	7	55	4	0	0
10	30	79	30	79	8	77	55	75	19	69	40	54	24	0	0
10	45	79	15	78	53	77	38	74	58	69	12	53	45	0	0
11	0	79	0	78	37	77	21	74	38	68	45	53	6	0	0
11	15	78	45	78	22	77	4	74	17	68	18	52	27	0	0
11	30	78	30	78	6	76	47	73	55	67	52	51	50	0	0
11	45	78	15	77	51	76	30	73	37	67	25	51	13	0	0
12	0	78	0	77	35	76	12	73	16	66	58	50	36	0	0
12	15	77	45	77	20	75	56	72	56	66	32	50	0	0	0
12	30	77	30	77	4	75	38	72	36	66	5	49	25	0	0
12	45	77	15	76	49	75	21	72	15	65	39	48	50	0	0
13	0	77	0	76	33	75	4	71	55	65	13	48	16	0	0
13	15	76	45	76	18	74	47	71	35	64	47	47	42	0	0
13	30	76	30	76	3	74	30	71	15	64	21	47	9	0	0
13	45	76	15	75	47	74	13	70	55	63	55	46	37	0	0
14	0	76	0	75	32	73	56	70	35	63	30	46	4	0	0

			12		1&1		2&1	3&	3&	4&	4&	5&	5&	6	6
Poli	Poli	12	M	1&1	1	2&1	0	9	9	8	8	7	7	G	M
G.	M.	G.	.	1 G.	M.	0 G.	M.	G.	M.	G.	M.	G.	M.	.	.
14	15	75	45	75	16	73	39	70	15	63	4	45	33	0	0
14	30	75	30	75	1	73	22	69	55	62	39	45	1	0	0
14	45	75	15	74	45	73	5	69	35	62	14	44	31	0	0

Printed p. 225 (Pole altitudes 15° 0' – 29° 45')

			12		1&1		2&1	3&	3&	4&	4&	5&	5&	6	6
Poli	Poli	12	M	1&1	1	2&1	0	9	9	8	8	7	7	G	M
G.	M.	G.	.	1 G.	M.	0 G.	M.	G.	M.	G.	M.	G.	M.	.	.
15	0	75	0	74	30	72	49	69	15	61	49	44	0	0	0
15	15	74	45	74	14	72	32	68	55	61	24	43	31	0	0
15	30	74	30	73	59	72	15	68	35	60	59	43	1	0	0
15	45	74	15	73	43	71	58	68	15	60	34	42	33	0	0
16	0	74	0	73	28	71	41	67	56	60	10	42	4	0	0
16	15	73	45	73	12	71	24	67	36	59	46	41	36	0	0
16	30	73	30	72	57	71	7	67	16	59	21	41	9	0	0
16	45	73	15	72	42	70	50	66	57	58	57	40	41	0	0
17	0	73	0	72	26	70	33	66	37	58	33	40	15	0	0
17	15	72	45	72	11	70	17	66	18	58	10	39	49	0	0
17	30	72	30	71	55	70	0	65	58	57	46	39	23	0	0
17	45	72	15	71	36	69	47	65	39	57	23	38	57	0	0
18	0	72	0	71	24	69	26	65	19	56	59	38	32	0	0
18	15	71	45	71	9	69	9	65	0	56	36	38	8	0	0
18	30	71	30	70	54	68	53	64	41	56	13	37	43	0	0
18	45	71	15	70	38	68	36	64	21	55	49	37	19	0	0
19	0	71	0	70	23	68	19	64	2	55	27	36	56	0	0
19	15	70	45	70	7	68	2	63	43	55	4	36	33	0	0
19	30	70	30	69	52	67	46	63	24	54	42	36	10	0	0
19	45	70	15	69	37	67	29	63	6	54	19	35	47	0	0
20	0	70	0	69	21	67	12	62	46	53	57	35	25	0	0
20	15	69	45	69	6	66	56	62	27	53	35	35	3	0	0

			12		1&1		2&1	3&	3&	4&	4&	5&	5&	6	6
Poli	Poli	12	M	1&1	1	2&1	0	9	9	8	8	7	7	G	M
G.	M.	G.	.	1 G.	M.	0 G.	M.	G.	M.	G.	M.	G.	M.	.	.
20	30	69	30	68	50	66	39	62	8	53	13	34	42	0	0
20	45	69	15	68	35	66	22	61	49	52	51	34	20	0	0
21	0	69	0	68	20	66	6	61	30	52	29	33	59	0	0
21	15	68	45	68	4	65	49	61	11	52	8	33	39	0	0
21	30	68	30	67	49	65	33	60	53	51	46	33	18	0	0
21	45	68	15	67	33	65	16	60	34	51	25	32	58	0	0
22	0	68	0	67	18	64	59	60	15	51	4	32	39	0	0
22	15	67	45	67	3	64	43	59	57	50	43	32	19	0	0
22	30	67	30	66	47	64	26	59	38	50	22	32	0	0	0
22	45	67	15	66	32	64	10	59	20	50	1	31	41	0	0
23	0	67	0	66	17	63	53	59	1	49	40	31	22	0	0
23	15	66	45	66	1	63	37	58	43	49	20	31	4	0	0
23	30	66	30	65	46	63	20	58	25	48	59	30	46	0	0
23	45	66	15	65	31	63	4	58	6	48	39	30	28	0	0
24	0	66	0	65	15	62	48	57	48	48	19	30	10	0	0
24	15	65	45	65	0	62	31	57	30	47	59	29	53	0	0
24	30	65	30	64	45	62	15	57	12	47	39	29	36	0	0
24	45	65	15	64	29	61	58	56	54	47	19	29	19	0	0
25	0	65	0	64	14	61	42	56	36	47	0	29	2	0	0
25	15	64	45	63	59	61	26	56	18	46	40	28	45	0	0
25	30	64	30	63	43	61	9	56	0	46	21	28	29	0	0
25	45	64	15	63	28	60	53	55	42	46	2	28	13	0	0
26	0	64	0	63	13	60	37	55	24	45	43	27	57	0	0
26	15	63	45	62	57	60	20	55	6	45	24	27	42	0	0
26	30	63	30	62	42	60	4	54	49	45	5	27	26	0	0
26	45	63	15	62	27	59	48	54	31	44	46	27	11	0	0
27	0	63	0	62	11	59	32	54	13	44	28	26	56	0	0
27	15	62	45	61	56	59	16	53	56	44	9	26	41	0	0
27	30	62	30	61	41	58	59	53	38	43	51	26	26	0	0

		12		1&1		2&1		3&	3&	4&	4&	5&	5&	6	6
Poli	Poli	12	M	1&1	1	2&1	0	9	9	8	8	7	7	G	M
G.	M.	G.	.	1 G.	M.	0 G.	M.	G.	M.	G.	M.	G.	M.	.	.
27	45	62	15	61	25	58	43	53	21	43	33	26	12	0	0
28	0	62	0	61	10	58	27	53	4	43	14	25	57	0	0
28	15	61	45	60	55	58	11	52	46	42	56	25	43	0	0
28	30	61	30	60	40	57	55	52	29	42	38	25	29	0	0
28	45	61	15	60	24	57	39	52	12	42	21	25	15	0	0
29	0	61	0	60	9	57	23	51	54	42	3	25	2	0	0
29	15	60	45	59	54	57	7	51	37	41	46	24	48	0	0
29	30	60	30	59	38	56	51	51	20	41	28	24	35	0	0
29	45	60	15	59	23	56	35	51	3	41	11	24	22	0	0

Compendium describendorum horologiorum — Chapter VII (Translation of a declining dial)

Table of the arcs of the hour-circles between the pole of the world and the vertical circles of the horizon that stand at right angles to those hour-circles (continued)

(Continuation of the Tabula arcuum circulorum horariorum a meridie et media nocte inter Polum mundi, et Verticales Horizontis, qui ad ipsos Horarios recti sunt*, whose heading was printed at the foot of p. 223. These three pages carry the table onward from pole altitude 30° to 74° 45'. The argument column at the left is the pole's altitude [*Altitudines Poli*] in degrees and minutes; the seven paired columns give, for each hour-circle group — the noon circle "12", then the pairs 1 & 11, 2 & 10, 3 & 9, 4 & 8, 5 & 7, and the 6-o'clock circle — the arc in degrees [G. = gradus] and minutes [M. = minuta]. The 6-o'clock column is 0° 0' throughout, since that hour-circle passes through the pole. The table continues to higher pole altitudes on the following printed page.)*

Printed page 226 (pole altitude 30° 0' – 44° 45')

	12												6	
Alt.Poli G	M G	M 1·11 G	M 2·10 G	M 3·9 G	M 4·8 G	M 5·7 G	M G	M						
30	0 60	0 59	8 56	1 50	4 40	5 24	9 0	0						

Alt.Poli G	12		M 1·11 G	M 2·10 G	M 3·9 G	M 4·8 G	M 5·7 G	6	
	M G							M G	M
					9	6	4		
30	1 59	4 58		5 56	3 50	2 40	3 23	5 0	0
	5	5		3		9	7	6	
30	3 59	3 58		3 55	4 50	1 40	2 23	4 0	0
	0	0		7	7	2	0	3	
30	4 59	1 58		2 55	3 49	5 40	3 23	3 0	0
	5	5		2	1	5		1	
31	0 59	0 58		7 55	1 49	3 39	4 23	1 0	0
					5	9	6	8	
31	1 58	4 57		5 54	5 49	2 39	2 23	6 0	0
	5	5		2	9	2	9		
31	3 58	3 57		3 54	4 49	5 39	1 22	5 0	0
	0	0		6	3		3	4	
31	4 58	1 57		2 54	2 48	4 38	5 22	4 0	0
	5	5		1	7	9	6	2	
32	0 58	0 57		6 54	1 48	3 38	4 22	3 0	0
					1	2	0	0	
32	1 57	4 56		5 53	5 48	1 38	2 22	1 0	0
	5	5		1	5	5	4	8	
32	3 57	3 56		3 53	4 47	5 38	8 22	7 0	0
	0	0		6	0	9			
32	4 57	1 56		2 53	2 47	4 37	5 21	5 0	0
	5	5		0	4	3	2	5	
33	0 57	0 56		5 53	8 47	2 37	3 21	4 0	0
						6	6	4	
33	1 56	4 55		5 52	5 47	1 37	2 21	3 0	0
	5	5		0	2	0	0	3	
33	3 56	3 55		3 52	3 46	5 37	4 21	2 0	0
	0	0		5	7	4		1	
33	4 56	1 55		2 52	2 46	3 36	4 21	1 0	0
	5	5		0	1	7	8	0	
34	0 56	0 55		4 52	5 46	2 36	3 21	0 0	0
						1	3		

Alt.Poli G	12		M 1·11 G	M 2·10 G	M 3·9 G	M 4·8 G	M 5·7 G	6	
	M G							M G	M
34	1 55	4 54		4 51	5 46	5 36	1 20	4 0	0
	5	5		9	0		8	9	
34	3 55	3 54		3 51	3 45	4 36	2 20	3 0	0
	0	0		4	4	9		8	
34	4 55	1 54		1 51	1 45	3 35	4 20	2 0	0
	5	5		9	8	3	7	8	
35	0 55	0 54		4 51	3 45	1 35	3 20	1 0	0
						7	2	7	
35	1 54	4 53		4 50	4 45	1 35	1 20	7 0	0
	5	5		9	7		7		
35	3 54	3 53		3 50	3 44	4 35	2 19	5 0	0
	0	0		3	1	5		7	
35	4 54	1 53		1 50	1 44	2 34	4 19	4 0	0
	5	5		8	6	9	7	6	
36	0 54	0 53		3 50	0 44	1 34	3 19	3 0	0
						3	2	6	
36	1 53	4 52		4 49	4 43	5 34	1 19	2 0	0
	5	5		8	5	8	7	7	
36	3 53	3 52		3 49	2 43	4 34	3 19	1 0	0
	0	0		3	9	2		7	
36	4 53	1 52		1 49	1 43	2 33	4 19	7 0	0
	5	5		8	4	6	8		
37	0 53	0 52		2 48	5 43	1 33	3 18	5 0	0
					8	1	4	7	
37	1 52	4 51		4 48	4 42	5 33	2 18	4 0	0
	5	5		7	3	5	0	8	
37	3 52	3 51		3 48	2 42	4 33	5 18	3 0	0
	0	0		2	7	0		8	
37	4 52	1 51		1 48	1 42	2 32	5 18	2 0	0
	5	5		7	2	4	1	9	
38	0 52	0 51		2 47	5 42	9 32	3 18	2 0	0
					7		7	0	
38	1 51	4 50		4 47	4 41	5 32	2 18	1 0	0

Alt.Poli G	12		M 1·11 G	M 2·10 G	M 3·9 G	M 4·8 G	M 5·7 G	6	
	M G							M G	M
	5	5		7	1	3	3	1	
38	3 51	3 50		3 47	2 41	3 32	9 18	1 0	0
	0	0		2	6	7			
38	4 51	1 50		1 47	1 41	2 31	5 17	5 0	0
	5	5		7	1	3	5	2	
39	0 51	0 50		2 46	5 41	8 31	4 17	4 0	0
					5		2	4	
39	1 50	4 49		4 46	4 40	5 31	2 17	3 0	0
	5	5		6	0	3	8	5	
39	3 50	3 49		3 46	2 40	3 31	1 17	2 0	0
	0	0		1	5	7	4	6	
39	4 50	1 49		1 46	1 40	2 31	1 17	1 0	0
	5	5		6	0	2		7	
40	0 50	0 49		1 45	5 40	7 30	4 17	9 0	0
					4		7		
40	1 49	4 48		4 45	3 39	5 30	3 17	0 0	0
	5	5		7	9	2	4		
40	3 49	3 48		3 45	2 39	3 30	2 16	5 0	0
	0	0		1	4	7	1	2	
40	4 49	1 48		1 45	9 39	2 30	8 16	4 0	0
	5	5		6		2		3	
41	0 49	0 48		1 44	5 39	8 29	5 16	3 0	0
					4		4	7	
41	1 48	4 47		4 44	3 38	5 29	4 16	2 0	0
	5	5		6	8	3	1	7	
41	3 48	3 47		3 44	2 38	3 29	2 16	1 0	0
	0	0		1	3	8	8	8	
41	4 48	1 47		1 44	8 38	2 29	1 16	1 0	0
	5	5		6		3	5	0	
42	0 48	0 47		1 43	5 38	9 29	3 16	2 0	0
					3				
42	1 47	4 46		4 43	3 37	5 28	5 15	5 0	0
	5	5		6	8	4	0	4	

	12							6	
Alt.Poli G	M G	M 1·11 G	M 2·10 G	M 3·9 G	M 4·8 G	M 5·7 G	M G	M	
42	3 47 0	3 46 0	3 43 1	2 37 3	3 28 9	3 15 7	4 0 6	0	
42	4 47 5	1 46 5	1 43 6	8 37	2 28 5	2 15 5	3 0 9	0	
43	0 47	0 46	1 42	5 37 3	1 28 0	1 15 2	3 0 1	0	
43	1 46 5	4 45 5	4 42 5	3 36 8	5 28 6	0 15	2 0 3	0	
43	3 46 0	3 45 0	3 42 0	2 36 3	4 27 1	4 15 7	1 0 5	0	
43	4 46 5	1 45 5	1 42 5	8 36	2 27 7	3 15 5	8 0	0	
44	0 46	0 45	0 41	5 36 3	1 27 3	2 15 2	0 0	0	
44	1 45 5	4 44 5	4 41 5	3 35 8	5 27 9	1 14 0	5 0 3	0	
44	3 45 0	3 44 0	3 41 0	2 35 3	4 26 4	5 14 8	4 0 5	0	
44	4 45 5	1 44 5	1 41 5	8 35	3 26 0	4 14 6	3 0 8	0	

Printed page 227 (pole altitude 45° 0' – 59° 45')

	12							6	
Alt.Poli G	M G	M 1·11 G	M 2·10 G	M 3·9 G	M 4·8 G	M 5·7 G	M G	M	
45	0 45	0 44	0 40	5 35 4	1 26 6	3 14 4	3 0 1	0	
45	1 44 5	4 43 5	4 40 5	3 35 9	2 26	2 14 2	2 0 3	0	
45	3 44 0	3 43 0	3 40 0	2 34 4	4 26 8	1 14 0	1 0 6	0	
45	4 44 5	1 43 5	1 40 5	9 34	3 25 4	5 14 8	9 0	0	
46	0 44	0 43	0 39	5 34	2 25	4 14	2 0	0	

Alt.Poli G	12		M 1·11 G	M 2·10 G	M 3·9 G	M 4·8 G	M 5·7 G	6	
	M G							M G	M
					4	0	6		
46	1 43	4 42		4 39	4 34	6 25	3 13	5 0	0
	5	5		6	0		5	5	
46	3 43	3 42		3 39	2 33	5 25	2 13	4 0	0
	0	0		1	5	2	3	8	
46	4 43	1 42		1 39	1 33	3 25	1 13	4 0	0
	5	5		6	6	8	1	1	
47	0 43	0 42		1 38	5 33	2 25	0 13	3 0	0
					5	4		4	
47	1 42	4 41		4 38	4 33	1 24	4 13	2 0	0
	5	5		6	1	0	8	7	
47	3 42	3 41		3 38	2 32	5 24	3 13	2 0	0
	0	0		1	7	6	7	1	
47	4 42	1 41		1 38	1 32	4 24	2 13	1 0	0
	5	5		6	1	3	6	4	
48	0 42	0 41		1 37	5 32	2 24	1 13	7 0	0
					7	9	4		
48	1 41	4 40		4 37	4 32	1 24	3 13	0 0	0
	5	5		6	2	5			
48	3 41	3 40		3 37	2 32	2 23	5 12	5 0	0
	0	0		1	8		2	4	
48	4 41	1 40		1 37	1 31	4 23	4 12	4 0	0
	5	5		6	3	8	1	7	
49	0 41	0 40		1 36	5 31	3 23	3 12	4 0	0
					8	5	0	1	
49	1 40	4 39		4 36	4 31	2 23	1 12	3 0	0
	5	5		6	4	1	8	4	
49	3 40	3 39		3 36	2 31	8 23	7 12	2 0	0
	0	0		1	9			8	
49	4 40	1 39		1 36	1 30	5 22	5 12	2 0	0
	5	5		6	4	4	6	2	
50	0 40	0 39		2 36	0 30	4 22	4 12	1 0	0
						1	6	5	

Alt.Poli G	12		M 1·11 G	M 2·10 G	M 3·9 G	M 4·8 G	M 5·7 G	6	
	M G							M G	M
50	1 39	4 38		4 35	4 30	2 22	3 12	9 0	0
	5	5		7	6	8	5		
50	3 39	3 38		3 35	3 30	1 22	2 12	3 0	0
	0	0		2	1	5	4		
50	4 39	1 38		1 35	1 30	1 22	1 11	5 0	0
	5	5		7	7		3	6	
51	0 39	0 38		2 35	3 29	4 22	3 11	5 0	0
						8		0	
51	1 38	4 37		4 34	4 29	3 21	5 11	4 0	0
	5	5		7	8	5	2	4	
51	3 38	3 37		3 34	3 29	2 21	4 11	3 0	0
	0	0		2	4	1	1	8	
51	4 38	1 37		1 34	1 29	8 21	3 11	3 0	0
	5	5		7	9		1	2	
52	0 38	0 37		2 34	5 28	5 21	2 11	2 0	0
						5	0	6	
52	1 37	4 36		4 33	5 28	4 21	1 11	2 0	0
	5	5		8	1	2	0	0	
52	3 37	3 36		3 33	3 28	2 20	5 11	1 0	0
	0	0		1	4	7	8	3	
52	4 37	1 36		1 33	2 28	1 20	4 11	8 0	0
	5	5		8	2	6	9		
53	0 37	0 36		3 33	8 28	3 20	3 11	2 0	0
							9		
53	1 36	4 35		4 32	5 27	5 20	2 10	5 0	0
	5	5		8	3	0	8	6	
53	3 36	3 35		3 32	3 27	3 20	1 10	5 0	0
	0	0		3	9	7	8	1	
53	4 36	1 35		1 32	2 27	2 20	8 10	4 0	0
	5	5		8	5	4		5	
54	0 36	0 35		4 32	1 27	1 19	5 10	3 0	0
					1	1	8	9	
54	1 35	4 34		4 31	5 26	5 19	4 10	3 0	0

Alt.Poli G	12		M 1·11 G	M 2·10 G	M 3·9 G	M 4·8 G	M 5·7 G	6	
	M G							M G	M
	5	5		9	6	9	8	3	
54	3 35	3 34		3 31	4 26	4 19	3 10	2 0	0
	0	0		4	2	6	8	8	
54	4 35	1 34		1 31	2 26	3 19	2 10	2 0	0
	5	5		9	8	3	8	2	
55	0 35	0 34		4 31	1 26	2 19	1 10	1 0	0
					4	0	8	6	
55	1 34	4 33		5 31	0 26	8 19	8 10	1 0	0
	5	5		0				1	
55	3 34	3 33		3 30	4 25	5 18	5 10	5 0	0
	0	0		5	5	5	8		
55	4 34	1 33		2 30	3 25	4 18	4 10	0 0	0
	5	5		0	2	3	8		
56	0 34	0 33		5 30	1 25	3 18	3 9	5 0	0
					7	0	8	4	
56	1 33	4 32		5 30	3 25	1 18	2 9	4 0	0
	5	5		1		7	8	9	
56	3 33	3 32		3 29	4 25	5 18	1 9	4 0	0
	0	0		6	9		9	3	
56	4 33	1 32		2 29	3 24	5 18	9 9	3 0	0
	5	5		1	5	2		8	
57	0 33	0 32		6 29	2 24	4 17	5 9	3 0	0
					1	0	9	2	
57	1 32	4 31		5 29	7 24	2 17	5 9	2 0	0
	5	5		1		7	0	7	
57	3 32	3 31		3 28	5 24	1 17	4 9	2 0	0
	0	0		6	3	5	0	2	
57	4 32	1 31		2 28	3 24	3 17	3 9	1 0	0
	5	5		2	9		1	6	
58	0 32	0 31		7 28	2 23	5 17	2 9	1 0	0
					5	0	1	1	
58	1 31	4 30		5 28	1 23	3 17	1 9	6 0	0
	5	5		2	1	8	2		

	12							6	
Alt.Poli G	M G	M 1·11 G	M 2·10 G	M 3·9 G	M 4·8 G	M 5·7 G	M G	M	
58	3 31 0	3 30 0	3 27 7	5 23 7	2 17 6	2 9	1 0	0	
58	4 31 5	1 30 5	2 27 3	4 23 3	1 16 3	5 8 3	5 0 6	0	
59	0 31	0 30	8 27	2 23 9	1 16	4 8 3	5 0 0	0	
59	1 30 5	4 29 5	5 27 3	1 22 6	4 16 9	3 8 4	4 0 5	0	
59	3 30 0	3 29 0	3 27 8	2 22	3 16 7	2 8 5	4 0 0	0	
59	4 30 5	1 29 5	2 26 4	4 22 8	2 16 5	1 8 5	3 0 5	0	

Printed page 228 (pole altitude 60° 0' – 74° 45')

	12							6	
Alt.Poli G	M G	M 1·11 G	M 2·10 G	M 3·9 G	M 4·8 G	M 5·7 G	M G	M	
60	0 30	0 29	9 26	3 22 4	1 16 2	6 8	3 0 0	0	
60	1 29 5	4 28 5	5 26 4	2 22 0	0 15	5 8 7	2 0 5	0	
60	3 29 0	3 28 0	3 26 9	6 21	4 15 8	4 8 8	2 0 0	0	
60	4 29 5	1 28 5	2 25 5	5 21 2	3 15 6	3 8 9	1 0 5	0	
61	0 29	0 28	1 25 0	3 21 9	2 15 4	2 8 9	1 0 0	0	
61	1 28 5	4 27 5	5 25 5	2 21 5	1 15 2	2 8 0	5 0	0	
61	3 28 0	3 27 0	4 25 1	1 21 1	0 15	1 8 1	0 0	0	
61	4 28 5	1 27 5	2 24 6	5 20 7	4 15 8	2 7	5 0 5	0	
62	0 28	0 27	1 24	4 20	3 14	5 7	5 0	0	

Alt.Poli G	12		M 1·11 G	M 2·10 G	M 3·9 G	M 4·8 G	M 5·7 G	6	
	M G							M G	M
				1	4	6	3	0	
62	1 27	4 26		5 24	3 20	2 14	4 7	4 0	0
	5	5		6	0	4	4	5	
62	3 27	3 26		4 24	1 20	1 14	3 7	4 0	0
	0	0		2	6	3	5	0	
62	4 27	1 26		2 24	2 20	1 14	2 7	3 0	0
	5	5		7			6	6	
63	0 27	0 26		1 23	4 19	4 14	1 7	3 0	0
				2	9	9	8	1	
63	1 26	4 25		5 23	3 19	3 14	9 7	2 0	0
	5	5		8	5	7		6	
63	3 26	3 25		4 23	2 19	2 14	0 7	2 0	0
	0	0		3	1	5		1	
63	4 26	1 25		2 23	8 19	1 13	5 7	1 0	0
	5	5		8		3	1	6	
64	0 26	0 25		1 22	5 19	2 13	4 7	1 0	0
				4	4		2	2	
64	1 25	4 24		5 22	4 18	5 13	3 7	7 0	0
	5	5		9	0	0	4		
64	3 25	3 24		4 22	2 18	3 13	2 7	2 0	0
	0	0		4	7	6	5		
64	4 25	1 24		3 22	1 18	2 13	1 6	5 0	0
	5	5		0	3	7	6	8	
65	0 25	0 24		1 21	5 18	1 13	7 6	5 0	0
				5	9	5		3	
65	1 24	4 24		0 21	4 18	3 12	5 6	4 0	0
	5	5			6		9	8	
65	3 24	3 23		4 21	3 17	5 12	5 6	4 0	0
	0	0		6	2	2	0	4	
65	4 24	1 23		3 21	1 17	4 12	4 6	3 0	0
	5	5		1	9	0	2	9	
66	0 24	0 23		1 21	5 17	2 12	3 6	3 0	0
				6		9	3	4	

Alt.Poli G	12		M 1·11 G	M 2·10 G	M 3·9 G	M 4·8 G	M 5·7 G	6	
	M G							M G	M
66	1 23	4 23		2 20	5 17	1 12	2 6	3 0	0
	5	5			2	7	4	0	
66	3 23	3 22		4 20	3 17	5 12	1 6	2 0	0
	0	0		7	8		6	5	
66	4 23	1 22		3 20	2 16	5 12	7 6	2 0	0
	5	5		2	4	4		1	
67	0 23	0 22		1 20	1 16	4 11	5 6	1 0	0
				8	1	2	9	6	
67	1 22	4 22		3 19	5 16	3 11	5 6	1 0	0
	5	5			8	1	0	2	
67	3 22	3 21		4 19	4 16	2 11	4 6	7 0	0
	0	0		8	4	0	2		
67	4 22	1 21		3 19	3 16	8 11	3 6	3 0	0
	5	5		4	1		4		
68	0 22	0 21		1 19	1 15	5 11	2 5	5 0	0
				9	7	7	5	8	
68	1 21	4 21		5 19	4 15	4 11	1 5	5 0	0
	5	5				5	7	4	
68	3 21	3 20		5 18	5 15	3 11	8 5	4 0	0
	0	0		0	0	4		9	
68	4 21	1 20		3 18	3 15	2 11	0 5	4 0	0
	5	5		5	7	3		5	
69	0 21	0 20		2 18	2 15	1 10	5 5	4 0	0
				1	3	1	2	1	
69	1 20	4 20		6 18	1 15	0 10	4 5	3 0	0
	5	5			0		4	6	
69	3 20	3 19		5 17	5 14	4 10	3 5	3 0	0
	0	0		1	7	9	5	2	
69	4 20	1 19		3 17	4 14	3 10	2 5	2 0	0
	5	5		7	3	7	7	7	
70	0 20	0 19		2 17	3 14	2 10	1 5	2 0	0
				2	0	6	9	3	
70	1 19	4 19		8 17	1 14	1 10	1 5	1 0	0

Alt.Poli G	12		M 1·11 G	M 2·10 G	M 3·9 G	M 4·8 G	M 5·7 G	6	
	M G							M G	M
	5	5			6	5	1	8	
70	3 19	3 18		5 17	3 14	3 10	2 5	1 0	0
	0	0		3				4	
70	4 19	1 18		4 16	5 13	5 9	5 5	1 0	0
	5	5		0	0	2	4	0	
71	0 19	0 18		2 16	3 13	4 9	4 5	6 0	0
				4	6	1	6		
71	1 18	4 18		9 16	2 13	3 9	3 5	1 0	0
	5	5			3	0	8		
71	3 18	3 17		5 16	1 13	1 9	3 4	5 0	0
	0	0		5	0	9	0	7	
71	4 18	1 17		4 15	5 13	8 9	2 4	5 0	0
	5	5		0	6		2	3	
72	0 18	0 17		2 15	4 12	5 9	1 4	4 0	0
				5	3	6	4	8	
72	1 17	4 17		1 15	3 12	4 9	6 4	4 0	0
	5	5		1	0	5		4	
72	3 17	3 16		5 15	1 12	3 8	5 4	4 0	0
	0	0		6	6	4	8	0	
72	4 17	1 16		4 15	3 12	2 8	4 4	3 0	0
	5	5		2		3	9	6	
73	0 17	0 16		2 14	5 12	1 8	4 4	3 0	0
				7	0	2	1	1	
73	1 16	4 16		1 14	3 12	1 8	3 4	2 0	0
	5	5		3	7		3	7	
73	3 16	3 15		5 14	2 11	5 8	2 4	2 0	0
	0	0		8	3	0	5	3	
73	4 16	1 15		4 14	1 11	3 8	1 4	1 0	0
	5	5		3	0	9	7	9	
74	0 16	0 15		2 13	5 11	2 8	1 4	1 0	0
				9	7	8	0	5	
74	1 15	4 15		1 13	4 11	1 8	2 4	1 0	0
	5	5		4	4	7		0	

Alt.Poli G	12		M 1·11 G	M 2·10 G	M 3·9 G	M 4·8 G	M 5·7 G	6	
	M G							M G	M
74	3 15	3 15		0 13	3 11	6 7	5 4	6 0	0
	0	0			0		4		
74	4 15	1 14		4 13	1 10	5 7	4 4	2 0	0
	5	5		5	7	5	6		

Table of the arcs of the hour-circles between pole and vertical (continued and concluded)

(Continuation and conclusion of the table begun earlier — the Tabula arcuum circulorum horariorum whose heading stands at the foot of printed p. 223. This page carries only the repeated column headers. The argument column is Altitudines Poli [pole altitudes], in degrees [G.] and minutes [M.]; the seven hour-circle groups are 12; 1 & 11; 2 & 10; 3 & 9; 4 & 8; 5 & 7; and 6, each ruled into its own degree [G.] and minute [M.] subcolumns. All numerals are printed in Arabic, per the book's convention for these tables. The table runs here from pole altitude 75° to 90° and ends. The 6-o'clock group is 0° 0' throughout, since that hour-circle passes through the pole.)

Altitudines Poli	M12		M1·11		M2·10		M3·9		M4·8		M5·7		M6		M
G.	. G.		. G.		. G.		. G.		. G.		. G.		. G.		.
75	0 15	0 14			3 13		4 10		4 7		3 3		5 0		0
					1				4		8		8		
75	1 14	4 14			1 12		5 10		3 7		3 3		5 0		0
	5	5			6		1		3		0		4		
75	3 14	3 14			2 12		3 10		2 7		2 3		5 0		0
	0	0					8		2		2		0		
75	4 14	1 13			4 12		2 10		1 7		1 3		4 0		0
	5	5			7		4		1		4		6		
76	0 14	0 13			3 12		1 10		0 7		6 3		4 0		0
					2		1						2		
76	1 13	4 13			1 11		5 9		4 6		5 3		3 0		0
	5	5			8		8		9		9		7		
76	3 13	3 13			3 11		4 9		3 6		5 3		3 0		0
	0	0					5		8		1		3		
76	4 13	1 12			4 11		3 9		2 6		4 3		2 0		0

Altitudes Poli G.	M12 . G.	M1·11 . G.	M2·10 . G.	M3·9 . G.	M4·8 . G.	M5·7 . G.	M6 . G.	M .
	5	5	9	2	7	3	9	
77	0 13	0 12	3 11	1 9	1 6	3 3	2 0	0
			4	8	6	5	5	
77	1 12	4 12	2 11	5 9	5 6	2 3	2 0	0
	5	5	0			7	1	
77	3 12	3 12	5 10	5 8	5 6	2 3	1 0	0
	0	0		2	5	0	7	
77	4 12	1 11	5 10	3 8	4 6	1 3	1 0	0
	5	5	1	9	4	2	3	
78	0 12	0 11	3 10	2 8	3 6	4 3	9 0	0
			6	6	3			
78	1 11	4 11	2 10	1 8	2 5	5 3	5 0	0
	5	5	2	3	2	6		
78	3 11	3 11	7 9	5 8	1 5	4 3	1 0	0
	0	0		9	1	9		
78	4 11	1 10	5 9	4 8	0 5	4 2	5 0	0
	5	5	3	6		1	7	
79	0 11	0 10	3 9	3 7	5 5	3 2	5 0	0
			7	3	0	3	3	
79	1 10	4 10	2 9	2 7	3 5	2 2	4 0	0
	5	5	4	0	9	5	9	
79	3 10	3 10	9 9	7 7	2 5	1 2	4 0	0
	0	0			8	8	5	
79	4 10	1 9	5 8	5 7	1 5	1 2	4 0	0
	5	5	4	4	7	0	1	
80	0 10	0 9	4 8	4 7	6 5	2 2	3 0	0
			0	1			7	
80	1 9	4 9	2 8	2 6	5 4	5 2	3 0	0
	5	5	5	8	6	5	3	
80	3 9	3 9	1 8	1 6	4 4	4 2	2 0	0
	0	0	1	5	5	7	9	
80	4 9	1 8	5 8	2 6	3 4	3 2	2 0	0
	5	5	6		4	9	5	

Altitudes Poli G.	M12 . G.	M1·11 . G.	M2·10 . G.	M3·9 . G.	M4·8 . G.	M5·7 . G.	M6 . G.	M .
81	0 9	0 8	4 7 2	4 6 9	2 4 1	3 2 2	2 0 1	0
81	1 8 5	4 8 5	2 7 7	3 6 6	1 4 3	2 2 4	1 0 7	0
81	3 8 0	3 8 0	1 7 3	2 6 2	2 4	1 2 6	1 0 3	0
81	4 8 5	1 7 5	5 7 8	9 5	5 4 1	9 2	9 0	0
82	0 8	0 7	4 6 4	5 5 6	4 4 1	1 2	5 0	0
82	1 7 5	4 7 5	2 6 9	4 5 3	3 3 0	5 2 4	1 0	0
82	3 7 0	3 7 0	1 6 5	3 5 0	1 3 9	4 1 6	5 0 7	0
82	4 7 5	1 7 5	0 6	1 5 7	8 3	3 1 8	5 0 3	0
83	0 7	0 6	4 6 6	4 4	5 3 8	3 1 1	4 0 9	0
83	1 6 5	4 6 5	3 5 1	5 4 1	4 3 7	2 1 3	4 0 5	0
83	3 6 0	3 6 0	1 5 7	3 4 8	3 3 6	1 1 6	4 0 1	0
83	4 6 5	1 6 5	2 5	2 4 5	2 3 6	8 1	3 0 7	0
84	0 6	0 5	4 5 8	1 4 2	1 3 5	0 1	3 0 3	0
84	1 5 5	4 5 5	3 4 3	5 4 9	4 2	5 1 3	3 0 0	0
84	3 5 0	3 5 0	1 4 9	4 3 6	5 2 4	4 1 5	2 0 6	0
84	4 5 5	1 5 5	4 4	3 3 3	4 2 3	3 1 8	2 0 2	0
85	0 5	0 4	5 4	2 3	3 2	3 1	1 0	0

Altitudes Poli G.	M12 . G.	M1·11 . G.	M2·10 . G.	M3·9 . G.	M4·8 . G.	M5·7 . G.	M6 . G.	M .
			0	0	2	0	8	
85	1 4 5	4 4 5	3 4 5	7 3	2 2 2	2 1 3	1 0 4	0
85	3 4 0	3 4 0	2 3 1	5 3 4	1 2 1	1 1 5	1 0 0	0
85	4 4 5	1 4 5	6 3	4 3 1	0 2	8 1	6 0	0
86	0 4	0 3	5 3 2	2 2 8	5 2 0	0 1	2 0	0
86	1 3 5	4 3 5	3 3 7	1 2 5	3 1 9	5 0 3	5 0 8	0
86	3 3 0	3 3 0	2 3 3	2 2	2 1 8	4 0 5	5 0 4	0
86	4 3 5	1 3 5	8 2	4 2 9	1 1 8	3 0 8	5 0 1	0
87	0 3	0 2	5 2 4	3 2 6	7 1	3 0 0	4 0 7	0
87	1 2 5	4 2 5	3 2 9	2 1 3	5 1 7	2 0 3	4 0 3	0
87	3 2 0	3 2 0	2 2 5	1 1 0	4 1 6	1 0 5	3 0 9	0
87	4 2 5	1 2 5	1 1 0	5 1 7	3 1 5	8 0	3 0 5	0
88	0 2	0 1	5 1 6	4 1 4	2 1 5	0 0	3 0 1	0
88	1 1 5	4 1 5	4 1 1	3 1 1	1 0 4	5 0 2	2 0 7	0
88	3 1 0	3 1 0	2 1 7	1 1 8	4 0	4 0 5	2 0 3	0
88	4 1 5	1 1 5	1 1 2	5 0	5 0 3	3 0 7	1 0 9	0
89	0 1	0 0	5 0 8	5 0 2	4 0 2	3 0 0	1 0 6	0

Altitudines Poli G.	M12 . G.	M1·11 . G.	M2·10 . G.	M3·9 . G.	M4·8 . G.	M5·7 . G.	M6 . G.	M .
89	1 0 5	4 0 5	4 0 3	3 0 9	3 0 2	2 0 2	1 0 2	0
89	3 0 0	3 0 0	2 0 9	2 0 6	2 0 1	1 0 5	8 0	0
89	4 0 5	1 0 5	1 0 4	1 0 3	1 0 1	8 0	4 0	0
90	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0

Notes on the New Description of Sundials, which contribute very greatly to their construction [Notæ in novam horologiorum descriptionem, quæ ad horologia extruenda plurimum conducunt]

Since I have set out much material — scattered and at length, in various places throughout the book of the new description of sundials — material not so much necessary for laying out sundials as for laying them out by a variety of methods, I think it will be worth my while to point out here briefly the things I judge most useful and necessary. Otherwise the reader's mind may be confused or overwhelmed by so great a mass of precepts, and the construction of the dials thereby delayed. More than one person has asked me to do this at some point — indeed, has eagerly awaited it. For although in the worked examples excerpted from that book and published I noted a good many points, still many further and almost new ones have since occurred to me. These help the construction of sundials remarkably, make it more expeditious, and by no means ought to be neglected. They are roughly as follows.

[Margin: The usefulness of the instrument of parts in describing sundials.]

1. To construct sundials most accurately by means of tangents, one should use that instrument of parts which I described in two ways — both in the very brief Compendium of Sundials and in the *Practical Geometry* — and whose varied and manifold use I set out fully and copiously in the same *Practical Geometry*, Book 1, Chapter 1. With its help we can obtain, more quickly than one can say and with great precision, not only the tangents but also the sines and secants of any number of degrees and minutes, relative to a whole sine [sinus totus] of any magnitude whatever. There will then never be any need to cut or divide the proposed whole sine into 10 equal parts in order to derive the tangents from

them — the procedure taught in the new description of sundials — a business that tends to hold up and delay the work not a little, especially when several and various whole sines sometimes have to be employed.

[Margin: How the instrument of parts is used when the whole sines are divided into 10 particles.]

And so, wherever in the book of the new description of sundials we are directed to divide a line, taken as the whole sine, into 10 equal particles, the instrument of parts should be opened and fixed so that the interval between the 100- and 100-marks equals the proposed line, that is, the given whole sine. Once the instrument has been set in this way, we shall at once have to hand the tangents, sines, and secants of any number of degrees and minutes, taking the whole sine of any magnitude as 100 or even 1000 parts — as I explained fully in Book 1 of the *Practical Geometry*, e.g. Chapter 1, Nos. 3, 4, 5, and 12. But for our purpose it is enough, in my judgement, to apply the method of Chapter 1, Book 1 of the same *Practical Geometry*, No. 3, to obtain the tangents, sines, and secants, even if the whole sine is set at 1000 parts.

[Margin: An easy way of obtaining the tangents relative to the whole sine of 1000.]

For if, after dropping the first figure of the tangent given in the table placed at the end of the new description, the remaining number is taken on the instrument, together with as many tenths of the following part as there are units in the dropped figure (which are easily gauged by estimation), we shall have the full tangent relative to the whole sine of 1000 parts. For example, since the tangent of $8^{\circ}47'$ is 155, if on the instrument one takes 15 parts together with $5/10$ of the following part [Latin: *partis sextædecimæ* (?)], one obtains a tangent equivalent to the tangent 155 relative to the whole sine of 1000. For if the instrument is taken as divided into 1000 parts, each hundredth of it will contain ten thousandths; and so the 15 parts taken will be equivalent to 150 particles, and if $5/10$ of the next part is added, the tangent 155 will be made up, as it is given in the table. Likewise the tangent of $43^{\circ}44'$ in the table is 957. If, then, on the instrument one takes 95 parts together with $7/10$ of one part, the full tangent 957 will be produced. Finally, in the table the tangent corresponding to the arc of $68^{\circ}51'$ is 2585. If, then, on the instrument one takes 58 parts together with $5/10$, or $1/2$, of one part, and adds the whole sine twice, one obtains the full tangent, and so on. Exactly the same is to be done with the sines and secants, provided that 4 figures are first dropped, so as to have the sines and secants relative to the whole sine of 1000.

[Margin: A convenient method for obtaining the tangents without the instrument.]

If we have neither instrument to hand, we can very conveniently obtain the same tangents, sines, and secants by the following device. On some straight line of moderate length, drawn on brass or hard wood, let ten equal parts be taken from A up to B; and let the first of them, AC, be divided into 10 equal particles.

Diagram. A construction for obtaining tangents (and sines and secants) without the sector, drawn as if on a strip of brass or hard wood.

- A (top-left): left end of the horizontal graduated line
- C (top, just to the right of A): end of the first divided part AC (whose first tenth carries the fine subdivision)
- B (top-right): right end of the horizontal line AB
- Scale along AB: 10, 20, 30, 40, 50, 60, 70, 80, 90
- D (lower-left): apex from which the lines DE and DB (through H and F) and the arc EF are drawn
- E (lower-right): endpoint of the nearly horizontal base line DE
- F (upper-right, near B): endpoint of the rising line, on the arc EF
- H (middle-right, on the line DF, near the 70-mark): one of the two points marked by the compass interval
- G (lower-right, on the base line DE, below H): the other point of the interval GH

For by means of this line, serving as a universal instrument, we shall easily obtain what we want. Suppose, for example, that the tangent of $70^{\circ}19'$, which is $279\frac{1}{2}$, is to be found, relative to the whole sine of any magnitude whatever. From D, at the interval of the divided line AB, describe the arc EF; take the interval EF equal to the given whole sine, and draw straight lines from D through E and F. Then, placing one foot of the compasses at the point 70, extend the other as far as the $9\frac{1}{2}$ particle of part AC, and with that interval mark two points G and H, beginning from point D. For the interval GH, added to the whole sine EF taken twice, will be the tangent sought, as is plain. And in this way, with the line DE kept fixed, one whole sine after another may be taken on the arc EF, and so on — until, to avoid the confusion that can arise from a multitude of whole sines, it seems best to draw another figure DEF. Nor indeed is this operation much longer than the one done with the instrument of parts. For while the instrument is expanded and contracted so that the whole sine is set between the 100- and 100-

marks, the arc EF, with the straight lines DE, DF, and so on, will be roughly described.

And indeed, if careful diligence is applied, the tangents will perhaps be obtained more exactly this way than by the instrument, on account of the uncertainty of the pin about which the instrument must be moved. Add that by this method there is no danger — once the whole sine has been taken up, which sometimes must be used repeatedly — that the instrument may be contracted or expanded. There is this further convenience too: that by this method even the smallest tangents, close to D, can be taken, which is not possible with the pin of the instrument.

[Margin: Wherein the excellence of constructing sundials by tangents consists.]

2. If these matters are observed with due care, it is almost incredible how easy, quick, and even pleasant the description of sundials by tangents is made. Indeed, of all the constructions of sundials this one (to say so in passing) pleases me most, because in it no circle has to be divided, as in the other descriptions, nor does one have to construct a figure of rays for drawing the parallels of the Sun, into which the hour-intervals are transferred from a dial built beforehand — a matter in which it is easy to go wrong, chiefly on account of the oblique sections, which are avoided by using tangents. There is this further point: that a line can be drawn without any trouble representing the great circle that passes through the poles of the world and through any given degree of the equator, in the manner of some hour-circle from noon and midnight.

[Margin: By what method any great circle through the poles of the world is drawn and described on a dial by tangents.]

For if a given point on the equator is distant from the meridian, toward the west, by fewer than 45 degrees, we shall transfer the tangent of the arc of the equator intercepted between the meridian and the given point — relative to the whole sine HC, in the first figure of the Compendium of Sundials, a fragment of which we have here repeated — from H into the equinoctial line BHD, toward the left. For a straight line drawn from the center A of the dial through the endpoint of that tangent will represent the great circle sought. But if the given point is distant by more than 45 degrees, yet fewer than 90, we shall transfer the tangent of the complement of the arc of the equator intercepted between the meridian and the given point — relative to the whole sine ON — from O into the straight line ON, toward N. And if, finally, the proposed point recedes toward the west by more than 90 degrees, yet fewer than 180, the complement of the arc of the equator to

180 must be taken, and its tangent transferred in the same way from H into the equinoctial line toward the right; or else, drawing the line XY parallel to the meridian, the tangent of the arc by which the given arc exceeds a quadrant must be transferred from point X toward Y, relative to the whole sine XY — equal, of course, to the portion of the said parallel line XY between the 6th and 9th hour. For if through the endpoint of this tangent, and through the center A, a straight line is extended, its segment above the center, or above the line of the 6th hour, will represent the circle sought. The contrary must be done if the point taken on the equator recedes toward the east. For then the tangent of the proposed arc must be transferred into the equinoctial line from H toward the right, or into the aforesaid parallel line XY from point X downward, according as the given arc is less than 45 degrees, or greater yet less than 90. Or certainly, if it is greater than 90 degrees, [it must be transferred] into the equinoctial line toward the left, and so on. By the same method we shall draw a line for any minute of a given hour, as we shall say at the end of the following No. 3. Finally, this description of the hours by tangents seems to surpass all others in this: that in it the finding of no hour, or of no point of the equator, depends on the finding of another hour or point. So even if by chance an error has been made in finding some point, that error nonetheless does not spread further, since the finding of the other points does not depend on it, nor is any account taken of them, just as if they had not been found.

Diagram. A fragment of the first figure of the Compendium of Sundials, here repeated: the construction of the hour-lines and of the great circles through the poles by transferred tangents, on a horizontal dial. Straight lines radiate from the center A; dotted arcs and additional straight lines cross the figure. The equinoctial line runs horizontally through B, H, and D.

- O (top-left): on the upper horizontal line (line OAX of the 6th hour)
- A (top-right, center of the radiating lines): center of the dial
- X (top-right, end of the upper line): right end of line OAX
- I (upper-middle): on an arc between G and A
- G (upper-left, below O): on the upper region of the figure
- F (middle): below I, on an arc
- K (middle): below F
- E (mid-left): on a circle/arc, left of L
- L (middle, right of E)
- C (mid-right): intersection point
- T (left): on the left descending line
- R (center): on a line above B

- S (lower-center-left)
- B (center): intersection of the meridian and the equinoctial line
- H (center-right, on the equinoctial line between B and D)
- D (right, on the equinoctial line, right of H)
- N (bottom-center-left)
- M (bottom-center)
- P (bottom-center-right)
- Q (bottom-left)
- V (bottom, below M/N)
- Y (bottom-right)
- 9 (bottom-right corner): the 9-o'clock hour marker

(The "3" at the lower-left margin of the figure is the number of the following paragraph, not a figure label.)

[Margin: The hours from noon and midnight on a horizontal dial.]

3. In describing the hours from noon and midnight on a horizontal dial, one must observe what we prescribed in Chapter 2 of the new description, or in the excerpt of the first worked example — but using the instrument of parts, so that in place of those four rulers constructed there one may take the whole sine of 100, or even 1000, particles.

[Margin: The use of Table 6 for drawing the hours from noon and midnight on a horizontal dial.]

With this addition, moreover, to No. 7 of that worked example: through any point one can draw a line to the meridian [line] that is equidistant from the line OAX of the 6th hour. For on it, from Table 6 of the new description, the points of the hours will be found by the tangents of the complements of the arcs of that table, relative to a whole sine equal to the segment of the meridian between the center A and the aforesaid parallel, whether this is drawn below or above the center. Likewise, through any point of the line of the 6th hour one can draw a line equidistant from the meridian. For on it, from the same Table 6, the points of the hours will likewise be found by the tangents of the arcs themselves of that table, if the whole sine is taken equal to the segment of the line of the 6th hour between the center A and that parallel. For example: on the perpendicular BD, at any distance from the center A (this is the equinoctial line on the dial), the hour-points will be found from Table 6, if into it, from point H on both sides, the

tangents of the complements of the arcs of the said table are transferred, relative to the whole sine AH.

On the perpendicular ON, or XY, drawn to the line of the 6th hour through any point whatever, the hour-points will be found by the tangents of these same arcs of Table 6, transferred from O, or X, to either side of the line ON, or XY, relative to the whole sine [sinus totus] AO, or AX.

[Margin: *Quo pacto minuta horarum describantur.* — How the minutes of the hours are drawn.]

Now if we wish to draw a line for any minute of a given hour, we convert the proposed hour and minute into degrees and minutes by means of Table 23. Then, following what we wrote under No. 2 above, we draw a line representing the great circle that passes through the poles of the world and through that degree and minute. For example, if the line to be drawn is for 2 hours 37 minutes after noon, we reduce the 2 hours to 30 degrees, and the 37 minutes to 9 degrees 15 minutes, so that the total arc between the meridian and the given minute comes to 39 degrees 15 minutes, to which corresponds the tangent 817. This tangent, then, carried from H onto the equinoctial line HB, relative to the whole sine HC, will give a point through which a straight line drawn from A will indicate the hour 2 and, in addition, the 37 minutes — and so with the others.

[Margin: *Paralleli Solis.* — Parallels of the Sun.]

4. Next, on the lines of the hours from noon and midnight, in a horizontal dial, the points of the parallels of the Sun [puncta parallelorum Solis] must be sought by the methods handed down in Chapter 8 and its scholium in the new description of dials, and especially by what we prescribed under Nos. 1 and 7 of that chapter — so that Table 1 alone of that same description, which is the general one, be used, and not even the whole of it, but only its topmost line, which carries the heading "(circa horæ 6.)" [about the 6th hour], or the middle line, to which is assigned the heading "(circa horæ 12.)" [about the 12th hour]. Or, in place of that table, one may take up the table set out in Chapter 15, No. 2, if the points of the parallels are wanted for the minutes of the hours as well.

Nor is the finding of these same points by Table 3 of the new description a thing of small worth — a table which is likewise general, as is declared in Chapter 8, No. 11 of that same new description — especially in dials whose center is very far from the equinoctial line, or can scarcely be had at all.

The same points of the parallels will be sought most conveniently and most accurately from Table 19 of the new description, as we set out at the end of the scholium of Chapter 19. But because the whole sine is generally small — being equal to the gnomon [stylus] — you will do rightly if you tenfold it and take the tenth parts of the tangents; which is easily done by striking off the last figure on the right. For if you strike off yet another figure from the remaining number, you will have the tangent relative to a whole sine of 100, and so on. But if the elevation of the pole above your horizon, or above a declining plane, is not present in that table, the points of the parallels will have to be sought by other means; or the tangents will have to be computed by Problem 17 of the new description, for the given elevation.

[Margin: *Tangentes inter Æquatorem & parallelas, quo pacto supputentur.* — How the tangents between the equator and the parallels are computed.]

Now if anyone should prefer to compute these same tangents of Table 19 not between the center of the dial and the parallels of the Sun, but between the equator and those same parallels — which is indeed most convenient, or even necessary, where the center either is not available or is very remote — he will accomplish it by this method. In the northern signs let him subtract the arc of the given hour, in Table 1 of the new description over against the proposed sign and under the given hour in the circle of the 12th hour, from the elevation of the pole; or [subtract] the elevation of the pole from that arc. Let him take the tangent of the remaining arc away from the tangent of the pole's elevation, when, that is, the arc of the first table has been subtracted from the pole's elevation; or let him add it to the tangent of the pole's elevation, when, that is, the pole's elevation has been subtracted from that arc of the first table. Then let it be made, as:

the whole sine is to the remaining or combined number, so the secant of the angle which the given hour makes with the meridian is to another.

For the number produced will be the tangent sought.

In the southern signs let the arc of the first table be added to the pole's elevation, and from the tangent of the combined arc let the tangent of the pole's elevation be taken away. For if it be made, as:

the whole sine is to the remaining tangent, so the secant of the hour-angle is to another,

the tangent sought will be produced.

There is also an elegant method for investigating the points for the parallels of the Sun, which is carried out by means of the table described in the *Compendium of dials* and computed for all elevations of the pole, as we declared at the end of Chapter 2 of that *Compendium* and in the scholium of Chapter 9 of the new description, No. 5. The whole sines for the five hour-lines contained between the 12th and 6th hours, together with the points from which the tangents are to be transferred, will be found most easily by this method too. In the immediately preceding figure, for the hour-line AB, let the whole sine be sought, and so on. When two little circles have been described from the center A, one through E, the place of the gnomon, and the other through C, the extremity of that same gnomon, perpendicular to the meridian (which two little circles serve all the hour-lines), let there be taken, equal to the arc EK between the position of the style and the proposed hour, the arc KI. For the straight line EI will be perpendicular to AB, and FG will be the whole sine relative to which the tangents are to be transferred from F, as we demonstrated in the new description, Chapter 1, No. 6. But for the meridian the whole sine is the gnomon itself; while for the 6th hour it is the segment of the axis AC between the center A of the preceding figure and the extremity of the style perpendicular to the meridian. And relative to this whole sine there must be transferred onto the line of the 6th hour the tangent of the complement of the declination of the proposed northern sign — for the reason that the arc of the 6th hour in the said table of the *Compendium* is 0 degrees, and therefore nothing is to be subtracted from the complement of the declination.

[Margin: *Puncta parallelorum in hora 6.* — The points of the parallels in the 6th hour.]

The method too, set out in Chapter 9 of the new description, will be carried through by means of tangents in this way. In the preceding figure, let the points of two opposite signs — say ♋ [Cancer] and ♑ [Capricorn] — be the ones to be found on the line AB. When the point G has been found by means of the perpendicular EFG, as we said just now, let the straight line GB be drawn from G through B, the intersection of the hour-line with the equinoctial; and to it, through any point M, let the perpendicular PMQ be erected. For if into this there be transferred, relative to the whole sine GM, the tangents MP, MQ of the signs' declination — that is, of 23 degrees 30 minutes, for ♋ and ♑ in the proposed example — the straight line GP will cut the given hour at R, the point of ♋, while the straight line GQ will cut the same hour beyond the equinoctial at the point of ♑. For, as we demonstrated in the new description, Chapter 1, No. 1, if from G, through M, a circle were described tangent to the straight line PMQ at M, at which

on either side the arcs of the declination of ☿ and ♄ were marked off, then the straight lines drawn from G through the extremities of those arcs would cut off from the straight line AB 23 degrees 30 minutes. Since therefore MP, MQ are the tangents of those arcs, the whole sine being set as GM, the straight lines GP, GQ will pass through the outer points of those arcs, and will accordingly cut off the same 23 degrees 30 minutes from the straight line AB, both above the equinoctial and below.

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By the same method, if GE is extended as far as the outer circle to the point L, and to the straight line LB drawn to it, at any point S a perpendicular TSV is erected, into which the tangents of 23 degrees 30 minutes are transferred relative to the whole sine LS, then, as far as T and V, the straight lines LT, LV will again cut off from the hour-line AB, on either side, 23 degrees 30 minutes for the points ☿ and ♄. And the method is the same for the other signs, if the tangents of their own declinations are taken and transferred from M onto the straight line PMQ, or from S onto the straight line TSV, and so on. And this method is excellent, since it fits declining dials too, and by it, on any line whatever drawn from the center of the dial, even though it be unknown what hour it signifies, the points of the parallels can be found.

[Margin: *Præstantia huius rationis proxima.* — The excellence of this method just given.]

In this same way, too, the problem of Chapter 1 of the new description can be worked out by means of tangents. For if, in the figure of that chapter, through any point of the straight line HC a perpendicular is erected to it, and into this, beginning from the straight line HC, the tangent of (for example) 30 degrees is transferred on both sides, relative to a whole sine equal to the straight line between H and the said perpendicular, then the straight line from H, through the outer points of each of the aforesaid tangents, will cut off from the given straight line CD two segments of 30 degrees, and so on.

[Margin: *Additio ad cap.1. novæ descriptionis.* — An addition to Chapter 1 of the new description.]

Finally, with the greatest precision we shall hunt for these same points of the parallels on the hour-lines by means of secant lines, first subtracting four figures from each secant, so that we may have the secants relative to a whole sine of 1000, in this way. On the meridian line let it be investigated, by Table 1 of the new description and the elevation of the pole above the plane of the dial (whether

horizontal or declining) — for this method fits declining dials as well, if in them concealed straight lines are drawn, like the hour-lines in a horizontal dial, equally distant from the line of the style, which will serve the office of a meridian in this business — for each hour a point, through which the perpendicular line to the meridian, when drawn, cuts that hour at the point of the proposed parallel, as we handed down in the new description, Chapter 8, No. 7. Of this kind is the point h , for the 3rd and 9th hour of $\mathfrak{S}$, in the figure of Chapter 8 of the new description. For if, the whole sine being set as Gh , the secant of the complement of the arc of Table 6 of the new description be taken for the given 3rd and 9th hour, and this be transferred from the center G onto the 3rd and 9th hour, the points of $\mathfrak{S}$ will be found. The same points will be found if, the whole sine being set as Bh , the secant of the complement of the same arc of Table 6 be transferred from the points of the 3rd and 9th hour on the equinoctial line onto the drawn hours toward the center. Of this kind, too, is the point r , for the 3rd and 9th hour of $\mathfrak{V}$. So that, if the whole sine being set as 6 for the 3rd and 9th hour, it be transferred from the center G onto the 3rd and 9th hour, the points of $\mathfrak{V}$ will be found in the said hours. These points you will also have if, the whole sine being set as Br , you transfer the secant of the same arc of Table 6 from the points of the 3rd and 9th hour on the equinoctial line downward onto the said hours.

[Margin: *Paralleli per Secantes*. — The parallels by means of secants.]

The reason for this is that the segment of the line of the 3rd and 9th hour between the center G and the parallel $\mathfrak{S}$, and $\mathfrak{V}$, is the secant of the angle which, at the center G , the hour-line makes with the meridian, the whole sine being set as GH , or Gr — which angle is the complement of the arc of Table 6 for the given hour. Likewise the segment of the line of the 3rd and 9th hour between the equinoctial and the parallel $\mathfrak{S}$, or $\mathfrak{V}$, is the secant of the same angle of which we spoke, the whole sine being set as Bh , or Br — as is plain if, from the point of the parallel on the 3rd and 9th hour, a line is drawn parallel to the meridian. For this line will (a) be equal to the whole sine Bh , or Br , and (b) will make with the 3rd and 9th hour an angle equal to that which the meridian makes with those same hours.

[Margin: (a) *Elements* I.34; (b) *Elements* I.29.]

If, by one and the same operation, one should wish to find the points of opposite parallels by means of secants, the segment of the meridian line between the two opposite parallels must be bisected — as, in the dial of Chapter 2 of the *Compendium of dials*, the segment ab between $\mathfrak{S}$ and $\mathfrak{V}$ is bisected at Q . And through Q a line is to be drawn parallel to the equinoctial line. For if the straight line bd be taken equal to aA , and from d , through the intersections of the hour-

lines with the parallel through Q, concealed straight lines be drawn, and into these from d the same secants be transferred which were carried from A onto the hour-lines for $\mathfrak{S}$, then by one operation the points for both parallels will be found, through which the line is to be described.

But if, by means of the complements of the Sun's altitudes, the points of the parallels are to be traced — as we taught in the new description, Chapter 8, No. 12 — the points on the two opposite parallels will likewise be found by one and the same operation, in this manner. In the same dial of Chapter 2 of the *Compendium*, let there be taken below b a straight line equal to aE , so that there may be had the position of the style for the concealed hours drawn from d. For if, both from this second position of the style and from E, the tangents of the complements of the Sun's altitudes, going out into the concealed straight lines from d, and transferred into the hour-lines drawn from G, be laid down, the points for the two opposite parallels will be found.

It will not, however, be useless — for a more exact inquiry into the points of the parallels of the Sun on the hour-lines — to inspect carefully the three rules of the scholium of Chapter 8 of the new description; since by them, from one or another point once found, nearly all the rest of the same parallel, and of the opposite one as well, can be found, and the points investigated by other means can be examined and corrected, as we wrote at the end of the aforesaid scholium.

5. For drawing the hours from sunrise and sunset [horæ ab ortu vel occasu] in a horizontal dial, one must observe what we declared in the new description, Chapter 14 and in Practice 3 excerpted from it, and in the *Compendium of dials*, Chapter 3 — where it is of no small importance to describe them by means of the diurnal arcs, especially if their points are sought by tangents, as we advised at the end of Chapter 3 of the *Compendium*. But so that certain hours may be depicted more easily than the seven others drawn out in the new description, and so that the studious reader be not always forced to consult the figure set in the scholium of Chapter 1, No. 3 of the new description, we have here appended, besides two little tables, Chapter 14 of the new description. For from these it is understood more quickly than can be told through which points the hours from sunrise or sunset are to be drawn. For in a horizontal dial the second little table has excellent use: it is designed for the 12th hour from sunrise or sunset, for drawing those hours, and the fractions of them, which are little distant from the line of the 12th hour from sunrise or sunset and which scarcely, or with difficulty, intersect the equinoctial line. For if the 11th hour, say, is to be drawn, we shall draw, through the $11\frac{1}{2}$ hour from midnight on the line of the 12th hour from sunrise or

sunset — as this table 2 indicates — a line parallel to the $5\frac{1}{2}$ hour from noon or midnight, as is clear from Table 1 of Chapter 14 of the new description. So too we shall draw the 20th hour from sunrise or sunset parallel to the 10th hour from noon or midnight, through the 4th hour from noon or midnight on the line of the 12th hour from sunrise or sunset, and so with the others. And if you conceive in your mind that the space between any two adjacent hours of this little table is divided into 4 parts, you will easily understand through which points of the hours from noon or midnight the quarters and halves of the hours from sunrise or sunset are to be drawn. Thus, since the 5th hour from sunrise or sunset is drawn through the $8\frac{1}{2}$ hour from noon or midnight, and the 6th through the 9th, the $5\frac{1}{4}$ hour will have to be drawn through the $8\frac{5}{8}$ hour; and the $5\frac{1}{2}$ hour through the $8\frac{6}{8}$ hour, that is, through the $8\frac{3}{4}$ hour; and the $5\frac{3}{4}$ hour through the $8\frac{7}{8}$ hour, and so on. And by the same reasoning the other tables will be fitted to the halves and quarters of the hours. For in the 1st, 2nd, and 3rd [tables] the hours from sunrise and sunset — that is, the upper ones — advance by quarters, halves, and three-quarters; whereas the hours from noon and midnight, the lower ones, advance by eighths of the hours. But in the following 4 tables the upper hours, with their fractions — that is, with the quarters, halves, and three-quarters — advance toward the right, while the lower ones advance toward the left.

[Margin: *Hor. ab or. & occ. delineatio in horologio horizontali.* — The drawing of the hours from sunrise and sunset in a horizontal dial.]

And so, at the point where, for example, the 18th hour from sunrise or sunset cuts the 3rd hour from noon or midnight, through that same point the 12th hour from sunrise or sunset must also be drawn, as is plain from the table of the 3rd hour from noon or midnight. So too the $18\frac{1}{2}$ hour from sunrise or sunset must be drawn through the intersection of the $11\frac{1}{2}$ hour from sunrise or sunset with the 3rd hour from noon or midnight, and so with the rest. But here you have, as I said, the little tables — all of them constructed from the figure of the scholium of Chapter 1, No. 3 of the new description — which have distinguished use in declining dials. The lines of the 6th and 18th hours from sunrise or sunset, moreover, are easily described in all dials: since the 6th hour must pass, in a horizontal dial, through the 9th hour from noon or midnight on the line of the 12th hour from sunrise or sunset, and the 18th through the 3rd hour from noon or midnight. Or indeed both, the 6th and 18th hours from sunrise or sunset, [pass] through the points of the line of the 6th hour from noon or midnight, which are distant from the center by an interval equal to the segment of the equinoctial line between the 12th and 9th, or 3rd, hours. But in a declining dial the 6th hour from sunrise or sunset should be drawn through the 3rd hour from

noon or midnight on the horizontal line, and the 18th hour through the 9th. Both hours, however, in any dial, pass through the 12th hour from noon or midnight on the equinoctial line.

The 6th hour from sunrise or sunset intersects, at one and the same point, two hours of this table in any dial.

Horæ ab or. vel occ. (Hours from sunrise/sunset)	1	2	3	4	5	6	7	8	9	10	11	12
Horæ à mer. vel med. no. (Hours from noon/midnight)	3½	4	4½	5	5½	6	6½	7	7½	8	8½	9
Horæ ab or. vel occ.	13	14	15	16	17	18	19	20	21	22	23	24
Horæ à mer. vel med. no.	9½	10	10½	11	11½	12	12½	13	13½	14	14½	15

The 12th hour from sunrise or sunset intersects, at one and the same point, two hours of this table in any dial.

Horæ ab or. vel occ. (Hours from sunrise/sunset)	1	2	3	4	5	6	7	8	9	1	11	1
Horæ à mer. vel med. no. (Hours from noon/midnight)	6 ½	7 ½	7 ½	8 ½	8 ½	9 ½	9 ½	1 0	10 ½	1 1	11 ½	1 2
Horæ ab or. vel occ.	13	1 4	15	1 6	17	1 8	19	2 0	21	2 2	23	2 4
Horæ à mer. vel med. no.	½	1 ½	1 ½	2 ½	2 ½	3 ½	3 ½	4 ½	4½	5	5½	6

The 18th hour from sunrise or sunset intersects, at one and the same point, two hours of this table in any dial.

Horæ ab or. vel occ. (Hours from sunrise/sunset)	1	2	3	4	5	6	7	8	9	1	1	
Horæ à mer. vel med. no. (Hours from noon/midnight)	9 ½	1 0	10 ½	1 1	11 ½	1 2	½	1 ½	1 ½	2 ½	2 ½	3
Horæ ab or. vel occ.	13	1 4	15	1 6	17	1 8	19	2 0	21	2 2	23	2 4
Horæ à mer. vel med. no.	3 ½	4	4½	5	5½	6	6 ½	7	7 ½	8	8 ½	9

The 3rd hour from noon or midnight intersects, at one and the same point, two hours of this table in any dial.

Horæ ab or. vel occ. (Hours from sunrise/sunset)	1	2	3	4	5	6	7	8	9	0	1	1	1
Horæ ab or. vel occ.	5	4	3	2	1	2	2	2	2	2	1	1	
						4	3	2	1	0	9	8	
Horæ ab or. vel occ.	1	1	1	1	1	1	1	2	2	2	2	2	
	3	4	5	6	7	8	9	0	1	2	3	4	
Horæ ab or. vel occ.	1	1	1	1	1	1	1	1	9	8	7	6	
	7	6	5	4	3	2	1	0					

The 6th hour from noon or midnight intersects, at one and the same point, two hours of this table in any dial.

(Hora 6. à mer. vel med. noct. in eodem puncto binas horas huius tabellæ in quouis horologio intersecat.)

Hours from sunrise/sunset	1	2	3	4	5	6	7	8	9	0	1	1	1
Hours from sunrise/sunset	1	1	9	8	7	6	5	4	3	2	1	2	
	1	0										4	
Hours from sunrise/sunset	1	1	1	1	1	1	1	2	2	2	2	2	
	3	4	5	6	7	8	9	0	1	2	3	4	
Hours from sunrise/sunset	2	2	2	2	1	1	1	1	1	1	1	1	
	3	2	1	0	9	8	7	6	5	4	3	2	

[Each row is labelled *Horæ ab or. vel occ.* ("Hours from sunrise or sunset").]

The 9th hour from noon or midnight intersects, at one and the same point, two hours of this table in any dial.

(Hora 9. à mer. vel med. noc. in eodem puncto binas horas huius tabellæ in quocunque horologio intersecat.)

Hours from sunrise/sunset	1	2	3	4	5	6	7	8	9	0	1	1	1
Hours from sunrise/sunset	1	1	1	1	1	1	1	1	9	8	7	6	
	7	6	5	4	3	2	1	0					
Hours from sunrise/sunset	1	1	1	1	1	1	1	2	2	2	2	2	

											1	1	1
Hours from sunrise/sunset	1	2	3	4	5	6	7	8	9	0	1	2	
	3	4	5	6	7	8	9	0	1	2	3	4	
Hours from sunrise/sunset	5	4	3	2	1	2	2	2	2	2	1	1	
						4	3	2	1	0	9	8	

The 12th hour from noon or midnight intersects, at one and the same point, two hours of this table in any dial.

(Hora 12. à mer. vel med. noct. in eodem puncto binas horas huius tabellæ in horologio quolibet intersecat.)

											1	1	1
Hours from sunrise/sunset	1	2	3	4	5	6	7	8	9	0	1	2	
Hours from sunrise/sunset	2	2	2	2	1	1	1	1	1	1	1	1	
	3	2	1	0	9	8	7	6	5	4	3	2	
Hours from sunrise/sunset	1	1	1	1	1	1	1	2	2	2	2	2	
	3	4	5	6	7	8	9	0	1	2	3	4	
Hours from sunrise/sunset	1	1	9	8	7	6	5	4	3	2	1	2	
	1	0										4	

[Margin: When a very small dial is to be constructed.]

If a very small dial has to be constructed, you will do no wrong to quadruple the gnomon you have assumed and to describe the dial — with the hours from noon and midnight — for that quadrupled gnomon, as has been taught. For if you take a quarter part of all the distances from the center of the dial up to the parallels and the equinoctial line, you will have a dial matching your own gnomon; and you will find its position too, if you take a quarter part of the distance from that same center up to the position of the gnomon. In this dial the hours from sunrise and sunset are then to be drawn, as was explained here at No. 5.

You can also, if you prefer, make your gnomon tenfold and lay out the dial by the tenth parts of the Tangents, as we indicated above a little after the beginning of No. 4 — especially if, for the Tangents of the small subdivisions, you apply the remedy which we prescribed at the end of No. 1, with the instrument set aside.

[Margin: A dial declining in hours from noon and midnight.]

6. A dial declining in hours from noon and midnight is constructed, as was explained in the New Description [*Nova Descriptio*], ch. 16, and in the fourth

practical exercise excerpted from it, and likewise in ch. 4 of the Compendium. As in the following figure: let the horizontal line be AB, the position of the gnomon C, and its magnitude CD. With the whole sine [*sinus totus*] set as CD, it will give the tangent of the declination, 30 degrees (for we posit that much declination from south toward the east) — the point B, through which the meridian is to be drawn perpendicular to AB. But the tangent of the complement of the declination, which comprehends 60 degrees, will offer the point A, through which the equinoctial line, the line of the 6th hour, and indeed the Vertical line AP are to be drawn perpendicular to AB. Again, with the whole sine set as BD, it will give the tangent of the pole's altitude, 42 degrees, [reaching to] the center of the dial E; while the tangent of the complement of the pole's altitude gives the point F, through which the equinoctial line AF is drawn. The straight line EA will belong to the 6th hour, and ECK, perpendicular to the equinoctial, will be the line of the gnomon. The point F will also be found if, with BR taken equal to BD itself, a perpendicular RF is erected to the joined straight line ER.

Now the hour-points on the horizontal line and on the equinoctial will be found, as we taught in the New Description, ch. 16, Nos. 6, 7, and 8. Here only this is to be noted: that the table at the end of the aforesaid No. 5 — which was constructed by means of the complements of the arcs of table 6 — can also very conveniently be constructed by the arcs of that same table itself, in this way. When the dial declines from south toward the east, from the arcs of the hours of table 6 after the 6th hour let there be subtracted, from the complement of the wall's declination, those that can be subtracted; but the arcs of the hours before the 6th hour are to be added to that same complement of the declination. For thus will be produced the arcs whose Tangents are to be transferred to the left of the gnomon. But when the arcs of the hours in table 6 after the 6th hour are greater than the complement of the declination, let the complement of the declination be subtracted from those arcs; and to the complements of the arcs of that same table 6, if the hours are counted after noon, let the wall's declination itself be added. For thus will be generated the arcs whose Tangents are to be transferred to the right of the gnomon. Let the contrary be done in a dial declining from south toward the west. Further, when any arc found in this way is greater than a quadrant, it is not put in the table, because the hour of that arc does not fall on the horizontal line. Its complement to 180 degrees must therefore then be transferred to the contrary part by its own Tangent. But the point found will regard the hour to be counted from noon [south], if that which was sought — and did not fall on the horizontal line — should be counted from midnight; and vice versa.

Since, however, the table at No. 8 in the same ch. 16 is to be constructed by the inclination of the Meridians, you will find the magnitude of this inclination, if you please, by the Tangents in this way. In the following figure, with the whole sine set as KZ,

[Margin: How the inclination of the Meridians is found by the tangents.]

or $K\alpha$ (if beforehand $K\alpha$ is taken equal to KZ itself), KF will be the Tangent of the inclination of the Meridians — namely of the angle α — since the Meridian of the horizon is drawn through αF , and the Meridian of the wall through αK .

Therefore, if the instrument of parts is opened to the whole sine KZ, and the straight line KF is transferred into the instrument by the compass so that you may know how many particles it contains, you will at once find in the table of Tangents — dropping the last figure — how many degrees and minutes correspond to the Tangent KF.

[Margin: Hours 3 and 9.]

But so that the hour-lines may be drawn more accurately, the 3rd and 9th hours must first be drawn with the greatest care, in this manner. Through B, a perpendicular ef is drawn to DB, and Bd to ER; let Be, Bf be taken equal to Bd. For where the hidden straight lines De, Df cut the horizontal, through those points the [lines of the] 9th hour, EP, and the 3rd, EH, must be drawn from the center E; and these, unless there be an error, will pass through the points found on the equinoctial, and will cut off from the Vertical PQ the equal straight lines AP, AQ. These will be drawn more exactly if, below B on DB, there are taken any number of equal parts — namely two — up to the point h, through which a perpendicular ik is drawn to Dh. For if Bd is transferred into this on both sides as many times as DB is contained in Dh — three times, as in the given example — up to i, k, the hidden straight lines Di, Dk will cut the horizontal line at the same points of the 3rd and 9th hours. Or, if, with the whole sine set as DA, into the Vertical PAQ and into the straight line XgY (parallel to the meridian, and distant from it by as great a space as PAQ) there be transferred the tangents of the arcs

Diagram. A large and dense declining-dial construction filling the center of the page — an assembly of many straight lines (meridian, horizontal, equinoctial, gnomon-line, Vertical circle, and the 3rd/9th hour-lines) radiating and crossing through a central cluster of points, with several dotted (hidden) construction lines. The figure is heavily crowded; the readable letter- and numeral-labels, by approximate position, are:

- N (upper left, on an oblique line)

- L, X, δ (top-right cluster, at the intersection of steep lines)
- 9, 4 (right, along an oblique line descending to the right) [?]
- I, Q, β (left-center; β set just below the I/Q pair)
- T (left, on an oblique line)
- O (left, on a short oblique line)
- D, E (upper-center; E is a principal center of the pencil of lines)
- A (center-left, a second principal node on the horizontal)
- d, f, 3 (center, near B and R — hour-line labels/points)
- B, R (center, on the horizontal line)
- 9 (script numeral, center-left near B) [?]
- C, e (just below B)
- K (center-left, lower)
- Z (center)
- F (center, lower)
- α (center, lower, on the gnomon/equinoctial construction)
- M (lower-center)
- Y, H, κ (right; Y over H, and a kappa-like sort with a descender to their right) [?]
- h (lower-center-right)
- G, P (lower-left)
- 9 (script numeral, lower-left) [?]
- i (lower-center)
- S (lower)
- V (lower-center, at the foot of a long vertical)

of the 3rd and 9th hours, from table 6, taken over against the altitude of the pole on the right side, or from the complement of the pole's altitude on the left side, up to the points P, Q, X, Y; for through these the same 3rd and 9th hours are to be drawn.

And in the same way we shall find the points of the other hours, if we transfer their tangents — taken from the table of ch. 1 of the New Description — with the whole sine set as Bd, onto the straight line ef; or if we carry the same, doubled or tripled and so on, onto the straight line ik, according as Dh has proved to be double, or triple, etc., of DB. Likewise if onto the straight line PQ we transfer the Tangents of the arcs of table 6. Likewise if, on the meridian EB, there are taken any number of segments equal to EB itself, and through the outermost point a perpendicular is drawn to the meridian; and if into this there be transferred the

intervals of the horizontal line between B and the hour-points, twice or three or four times and so on, there will be had points more remote from the center E, through which the hour-lines are to be drawn.

When the lines of the 3rd and 9th hours have been drawn, together with the two lines PQ, XY (parallel to the meridian and equally distant from the center E, so that EA, Eg are equal, of which the former, PQ, is drawn through A and represents the Vertical circle), then, so that the points of the other hours

may be found more readily, let there be cut off from the 6th hour the straight lines Eb, Ea equal to the straight line AP (between the 6th and 9th hour), or to AQ, or gX, or gY — for all these are equal. (For ^a since the angles Q, E of the triangle AEQ are equal to the angles Y, E of the triangle Yeg, and the side EA is, by construction, equal to the side Eg, ^b the sides AQ, gY will also be equal. Therefore, since by the scholium to proposition 4 of the New Description both PQ and XY are cut in half at the 6th hour, AP, AQ, gX, gY will be equal to one another.) Then, through b and a, when the hidden lines GI, LH (parallel to the meridian, and cutting the 9th and 3rd hours at G, I, L, H) have been drawn, let the straight lines GH, LI be joined; these will be equal to the straight lines PQ, XY, and ^c will be bisected at M, N, because MG, MH, NI, NL are equal to the straight lines Eb, Ea — that is, to the straight lines AP, AQ, or gX, or gY, or MG, or MH, or NI, or NL. If onto the straight lines PQ, XY, GH, IL, from the points A, g, M, N, there are transferred on both sides the tangents of the hours of the table placed in the scholium of ch. 1 of the New Description, No. 2, the points will be found through which the hour-lines are to be drawn from the center E.

[Margin: ^a (Euclid, *Elements* I.29 & 15). ^b (Euclid, *Elements* I.26). ^c (Euclid, *Elements* I.34).]

By the same reasoning, if another parallel to the meridian line is drawn through any point of the 6th hour, the tangents of the same hours are to be transferred onto it, with the whole sine set equal to the portion of that parallel between the 6th and the 9th (or 3rd) hour. So too, if another parallel to the line of the 6th hour is drawn through any point of the meridian line, the same tangents are to be transferred onto it, with the whole sine set equal to the segment of this parallel between the meridian and the 3rd (or 9th) hour.

[Margin: Hour-points by the arcs of the Vertical, from table 6.]

When the Vertical line PAQ has been drawn through A, the hour-points will also be found on it by the tangents of the arcs of table 6 of the New Description — taken over against the altitude of the pole on the right side, or over against the

complement of the pole's altitude on the left side — with the whole sine set as DA, as we said a little before.

[Margin: Hour-points on the horizontal line without knowledge of the wall's declination in degrees and minutes.]

Now if the wall's declination in degrees and minutes is unknown, we shall nonetheless find the hour-points on the horizontal line, in this way. When the straight line DB has been produced, let a perpendicular ik be drawn to it through any of its points h . For if the tangents of the complements of the arcs of table 6 of the New Description are transferred into this on both sides from h , with the whole sine set as Dh , the hidden straight lines drawn from D through the extremities of these tangents will cut the horizontal line at the hour-points. The same tangents can be transferred from B onto the perpendicular ef , with the sine set as DB .

[Margin: Hour-points on the equinoctial, without knowledge of the inclination of the Meridians.]

Again, if through any point of the straight line DA — as through A — a perpendicular OS is erected to DA , and into it, with the whole sine set as DA (between D and the point A through which the perpendicular is drawn), there are transferred the tangents of the arcs of the aforesaid table (but not the tangents of the complements), then from A , on both sides, the hidden straight lines drawn from D through the extremities of the transferred tangents will cut the horizontal line at the same hour-points, and so on. This is clear from what was demonstrated in ch. 1 of the New Description, and from what we wrote above at No. 4 about the parallels of one who describes by tangents, from ch. 9 of the New Description.

By a similar reasoning, when the inclination of the Meridian of a declining plane toward the Meridian of the horizon is unknown in degrees and minutes, the hour-points will be found on the equinoctial line, in this way. With the gnomon CZ erected perpendicular to the line of the gnomon, and the straight line $K\alpha$ taken equal to the interval KZ , let the straight line αF be joined; and to it — produced, if you like — let a perpendicular TV be erected at any point, namely at F . For if into this, from F on both sides, there are transferred the tangents of the hours of the table of the scholium of ch. 1 of the New Description, the hidden straight lines drawn from α through the extremities of the tangents will cut the equinoctial line at the hour-points; which is plain from what was demonstrated in ch. 1 of the New Description, as we wrote just now. The same is also confirmed from our

Gnomonica. For since the hidden straight lines drawn from α — through the hours of the circle described from α , the beginning being made from the straight line αF — cut the equinoctial at the hour-points, and since the Tangents on the straight line $T V$ correspond to the same hours of the said circle, the proposition stands established.

[Margin: When the wall's declination is very great, what to do.]

7. When the wall's declination is very great — lest we be forced to describe the whole dial in a small form, as we warned at the end of ch. 17 of the New Description — we shall proceed very conveniently in this way. Let some very great declination of the wall be given — say 89 or 88 degrees, and so on — from south toward the east, at any altitude of the pole. In our following example the declination is 71 degrees 44 minutes from south toward the east, at a pole-altitude of 60 degrees 45 minutes. When the horizontal line BE has been drawn, let a gnomon BA be erected to it at right angles. And since, with respect to the whole sine AB , the Tangent of the declination cannot be transferred from B toward E — in such a way that neither the meridian line can conveniently be had, nor the center — we shall transfer from B toward C the Tangent of the complement of the declination, up to the point C , through which let the primary Vertical CN be drawn, at right angles to the horizontal. Then, on the horizontal, a point E having been assumed anywhere, let the Tangent of the complement of the declination be transferred from B onto the gnomon AB , with respect to the whole sine EB , up to D . For the straight line DE , when drawn, will make the angle of declination BDE , so that BED is the angle of the complement of the declination. The straight line AV , parallel to DE , will likewise make the angle of declination BAV , but it intersects the horizontal too far off. Also, when DF is drawn perpendicular to DE , BDF will be the angle of the complement of the declination; and through the point F the equinoctial line for the gnomon BD is to be drawn. Therefore, the Tangent of the complement of the pole's altitude above the horizon having been transferred, with respect to the whole sine ED , from E up to the point G on the straight line EG (perpendicular to the horizontal, which is the meridian line with respect to the gnomon DB), the drawn line FG will be the equinoctial line with respect to the gnomon BD . But since the equinoctial line is to be drawn through C with respect to the given gnomon AB , the drawn line CH , parallel to FG , will be the equinoctial line for the given gnomon AB ; and to it a perpendicular BP is drawn through B (which will be done exactly if, from B , two points I, K are taken at the same interval, and from I, K , at any interval, two arcs are described cutting each other at L and T , etc.) — this will be the line of the gnomon.

Through the assumed point P too, let PQ be drawn on the line of the gnomon, parallel to the horizontal, to serve as a second horizontal. Then let a perpendicular BZ be erected to the line of the gnomon, equal to the gnomon BA, and let a straight line be joined from Z to b, the intersection of the line of the gnomon with the equinoctial line: this Zb will be the common section of the Equator and of the Meridian of the proper declining plane. To it a perpendicular Zc drawn will be the world-axis; and the perpendicular Pc, drawn to the line of the gnomon up to the axis, will be the new gnomon with respect to the second Horizon PQ. And the straight line cb, perpendicular to the axis Zc, will be the common section of the Equator and of the proper Meridian. But the straight line bX, parallel to CH — or perpendicular to BP — will be the equinoctial line with respect to the gnomon Pc.

Therefore, if the hour-points are found both on the horizontal BE (with respect to the gnomon BA) and on the horizontal PQ (with respect to the gnomon Pc), as we taught in ch. 16 of the *New Description*, Nos. 5 and 6, and also here a little before at No. 6, the dial will be described even though it has no center. You will also do rightly if [you place] the hour-points on the Vertical

you should seek [it] exactly on the Vertical CN, and likewise on the two equinoctial lines CH and bX, from the inclination of the Meridians, with respect to the whole sines bZ and bc, as was handed down in the same ch. 16 of the *New Description* [*Nova Horologiorum Descriptio*], No. 8. You have an example in the line of the 9th hour, MN. And if there is no error, it is necessary that the straight line OR, parallel to AM (which will easily be drawn if, AM being produced to Q, MR is taken equal to QO), should cut off from the perpendicular PS a straight line PS equal to the gnomon Pc; and this because the proportion of the gnomon AB to the shadow BM ought to be the same as that of the gnomon bP to the shadow PO. From this you have another way of investigating the size of the gnomon with respect to the second Horizon PQ. For once the point M of the 9th hour, for instance, has been found on the horizontal CE with respect to the gnomon AB, and the point N of the same hour on the Vertical CN with respect to the whole sine AC, then, the hour-line MN having been drawn and cutting PQ at O, it will cut off OR (parallel to MA) as the gnomon PS, as has been said.

Now we shall draw the hidden lines on both sides of the gnomon-line BP, like the hour-lines in a horizontal dial (namely, so that the points of the parallels may be found on them, as was explained above at No. 4). We do this if we transfer the Tangents of the hours from the little table of ch. 1 of the *New Description* onto both sides of the gnomon-line — both from b onto the equinoctial bH, with

respect to the whole sine bZ , and from b onto the equinoctial bX , with respect to the whole sine bc .

Diagram. A crowded construction figure for laying out a sundial on a wall of very great declination (the case treated in the preceding paragraphs). A near-horizontal base line runs through R, F, B, M to E at the right; a gnomon and numerous hour-lines, equinoctial lines, and auxiliary perpendiculars radiate across the field, with several dashed (construction) lines. The following labels are legible on the figure:

- A, T (top, apex region)
- I (far left)
- Y (left of center, on a line from I)
- D (upper center, on a dashed line)
- V (right side)
- R, F, B, M (left-to-right along the middle base line), E (right end of that line)
- C (just below F), S (lower left, on a dashed line)
- b (below B), Z (center)
- f (small label, center-right), h (right), H (right, below h)
- K (center, on a dotted line), L, P (lower left)
- O (center-low), N (lower center), c (small, lower center)
- X (at the bottom)
- [?] one small lowercase glyph adjacent to O (possibly "a" or "q") is not resolvable
- [?] a "9"-like mark low and left of N cannot be resolved as a point-label
- [?] the figure carries both an upper-case G (right side) and lower-case g/h in the text; the two right-hand labels below h could not be distinguished with certainty

8. [Margin: When the wall's declination is slight, what is to be done.] We shall use nearly the same device when the wall's declination is very slight — so slight that the meridian line can indeed be had conveniently, but not so the point of the 6th hour and of the equinoctial line on the horizontal. For let the wall decline from south toward the west by 18 degrees 16 minutes, at a latitude of 33 degrees 41 minutes. The Tangent of the declination BC , with respect to the whole sine AB taken equal to the gnomon, will give the point C , through which the meridian CN is to be drawn perpendicular to CB . And since the Tangent of the complement of the declination can only with difficulty be transferred onto BE with respect to the same whole sine AB , we shall — the point E being taken anywhere — transfer

onto BA, up to D, the Tangent of the declination with respect to the whole sine EB; and, the straight line ED being assumed, we shall raise the perpendicular DF. (Or we shall transfer the Tangent of the declination, with respect to the whole sine BD, up to F.) For thus the perpendicular Ff, drawn through F to the horizontal, will be the meridian with respect to the gnomon BD, and through E both the 6th hour and the equinoctial are to be drawn. If, therefore, on Ff we transfer the Tangent of the complement of the given latitude, with respect to the whole sine FD, up to f, the straight line fE will be the equinoctial line with respect to the gnomon BD. And if on CN the Tangent of the same complement of the given latitude be transferred up to g, with respect to the whole sine CA, then gh, parallel to fE, will be the equinoctial line with respect to the gnomon AB. Through B a perpendicular to this equinoctial is to be drawn, to serve as the gnomon-line. The rest will be completed as before. Only the hour-points cannot be found on the Vertical line, since there is none; rather they are to be sought on the two horizontals, or on the equinoctials, the second gnomon having first been found, etc. In our example, since the center of the dial Y is found by the Tangent of the given latitude, the hour-lines are to be drawn through Y and through the points found on the horizontal BE, etc.

9. [Margin: The arcs of the signs on a declining dial.] The points of the parallels [of declination] will be found by means of the Tangents, on the hidden lines drawn on both sides of the gnomon like the hour-lines of a horizontal dial, as we have set out in excerpts in the New Description, ch. 18, and toward the end of ch. 5 of the Compendium of Dials, and in Practice 5. Only where the dial has no center — as happens in dials that have the greatest declination — no small circle can be described around the segment of the gnomon-line lying between the center and the position of the gnomon; rather, from the position of the gnomon, perpendiculars must be raised to those hidden hour-lines, so that the points may be found by the table of parallels of the Compendium. By means of the Secants too, very exact-

-ly will the points of the parallels be found, as we taught above at No. 4. But where there is no center, the whole sine must be taken not from the center but from the intersection of the gnomon-line with the equinoctial, up to the points found by ch. 8 of the New Description, No. 7; and the Secants must be transferred from the equinoctial onto the hidden hour-lines. So too, if table 19 of the New Description is used, the points will have to be sought not from the center but from the equinoctial line, when there is no center, as was handed down at the end of the scholium of ch. 9 of the New Description.

[Margin: An excellent use of the table of ch. 15 in declining dials.] But if, in place of table 1 of the New Description, one takes the table given at No. 2 of ch. 15, we shall find the points of the parallels not only on any number of hidden lines that indicate the minutes of the hours of that table, but even on the hour-lines themselves, even if those hidden hour-lines are not drawn — and in the following way. Once the distances of the hours from the gnomon-line have been found by the inclination of the Meridians, as we taught in the New Description ch. 16 No. 8, and those distances have been reduced to hours, we shall find — by means of the arcs in the aforesaid table corresponding to those reduced hours under the proposed sign, and by the altitude of the pole above the plane of the dial, as we handed down in ch. 8 No. 7 — points on the gnomon-line, through which perpendiculars drawn to that same gnomon-line will cut the hours at the points of the parallels. For example: in the declining dial proposed in ch. 18 of the New Description, hour 5 from midnight is distant from the gnomon-line by 64 degrees 12 minutes — that is, 4 hours 17 minutes. Now to 4 hours 17 minutes there corresponds, in the aforesaid table under Cancer [♋] and Capricorn [♐], an arc of 45 degrees 1 minute; and if the pole's altitude above the plane of the dial, which contains 40 degrees 4 minutes, be subtracted from it, there remain 4 degrees 57 minutes, whose Tangent is 87 with respect to the whole sine 1000 taken equal to the gnomon. Transferred from C upward onto the gnomon-line, it will give a point through which a perpendicular drawn out to the gnomon-line will cut hour 5 from midnight at the point of Capricorn. Likewise hour 8 from midnight is distant from the gnomon-line by 19 degrees 12 minutes — that is, 1 hour 17 minutes — to which, in that same table, there correspond under Cancer and Capricorn 24 degrees 44 minutes; and, the pole's altitude of 40 degrees 4 minutes being added, an arc of 64 degrees 48 minutes is produced, whose Tangent is 2115 with respect to the whole sine 1000 taken equal to the gnomon. Transferred from C downward, it will offer a point through which a perpendicular erected to the gnomon-line will cut hour 8 from midnight at the point of Cancer, etc. And so one must always deal here with the southern signs, just as we said one must deal with the northern signs in the horizontal dial (ch. 8 No. 7 of the New Description) for finding the points on the meridian line; whereas with the northern signs one must proceed here just as we said one must proceed with the southern signs there.

By means of that same table we shall also find, on the line drawn through the center of the dial perpendicular to the gnomon-line (like the 6th hour), the points through which straight lines, sent out from the intersection of the gnomon-line with the equinoctial, will cut the corresponding hours at the points of the parallels. For example: hour 5 from midnight is distant from the gnomon-line by

64 degrees 12 minutes — that is, 4 hours 17 minutes. Now at the meeting of hour 4 at the foot of the table and 17 minutes on the right-hand side, taken under Capricorn, are found 25 degrees 46 minutes. The Tangent, therefore, of the complement of this arc is 207 [fraction: ?], with respect to the whole sine 100 — which, in the dial of ch. 16 of the New Description, is made equal to the straight line Ez; transferred from the center E onto the said perpendicular, it will give a point at which the straight line drawn out from K will cut hour 5 from midnight at the point of Capricorn, etc.

[Margin: In table 1 of the New Description the proportional part must not be used.] You see, therefore, that by this table alone — which is universal and proceeds through every minute of the hours — the points of the parallels can be found in all dials, even declining ones, even though the hidden lines are not drawn like the hour-lines of a horizontal dial. This is truly excellent and outstanding, and therefore that table is to be highly valued, since the first table in the New Description does not proceed through the individual minutes of the hours, nor can the arcs be found by the proportional part for the individual minutes of the hours. This I wished to point out, lest you use the proportional part in that table to no purpose.

Most conveniently, too, where the center lies too far off, table 3 of the New Description will be employed, if the hidden lines have been drawn on either side of the gnomon-line like the hour-lines from noon and from midnight — because then the line perpendicular to the gnomon-line through the center cannot be had. Therefore in that case perpendiculars must be drawn to the gnomon-line through the points of Capricorn and Cancer, as in the horizontal dial, in the manner explained in ch. 8 No. 11 of the New Description.

[Margin: The drawing of the parallels by means of points found on the meridian line.] The finding of the points for the parallels on the hour-lines themselves, by means of the points found on the meridian line — as we set out in the New Description ch. 18 No. 1 — is likewise not inconvenient. But when the points of some hours cannot be marked on the meridian line, because they are removed too far from the point B in the figure of that chapter, we shall proceed conveniently as follows. Having bisected the portion AB of the horizontal line, we shall draw through the point of division a line perpendicular to the horizontal. For if, beginning from the horizontal, the halves of the Tangents which would have had to be transferred onto the meridian line are carried over onto this perpendicular, points will be found through which the straight lines drawn out from A will indicate the points of the parallels on the hour-lines. But if on this

perpendicular some points are still too far distant from the horizontal, the portion between A and the said perpendicular must again be bisected, and through the point of division another perpendicular to the horizontal must be raised. For if the quarter-parts of the Tangents to be transferred onto the meridian are carried over onto this, points will be found through which straight lines sent out from A will cut the hour-lines at the points of the parallels.

10. [Margin: The hours from sunrise and sunset on a declining dial.] The hours from sunrise and sunset on a declining dial will be laid out by those excerpts which, in ch. 14 of the New Description and in Practice 6, we prescribed should be observed. For this the tables which we described above at No. 5 will be most useful.

But most conveniently, a declining dial with the hours from sunrise and sunset — in a region whose pole is elevated above the Horizon by not more than 60 degrees nor fewer than 30 — will be laid out by table 10 of the Sun's altitudes at the hours from sunrise and sunset, and by table 12 of the horizontal circumferences [azimuths] at the same hours, as we handed down in the new description of dials, ch. 24.

11. [Margin: A horizontal dial of the hours from sunrise and sunset, by means of the horizontal circumferences and the Sun's altitudes.] Now if one wishes to construct a horizontal dial by means of the horizontal circumferences and the altitudes of the Sun, as we advised at the end of ch. 24 of the New Description — especially with the hours from sunrise and sunset — a perpendicular to the meridian must be drawn through the position of the gnomon, which will represent the primary Vertical; and to this, through any point, another perpendicular, which will be equidistant from the meridian. For onto this second perpendicular the horizontal circumferences must be transferred from the Vertical line, by means of their Tangents, to both sides of the Vertical, with respect to a whole sine equal to the segment of the Vertical lying between the position of the gnomon and the said perpendicular parallel to the meridian. But if the horizontal circumferences are too great, a line parallel to the meridian must be drawn through a

point of the Vertical nearer to the position of the gnomon, so that the tangents may be transferred onto it with respect to a smaller whole sine, etc. But for the hours from noon or midnight it is enough to transfer, onto the hour-lines laid out either by the arcs of the Horizon of table 6 or in some other way, the Tangents of the complements of the Sun's altitudes from the position of the gnomon, the gnomon itself being taken as the whole sine.

12. [Margin: The points of the hours from sunrise or sunset on the horizontal line.] And since, in ch. 25 of the New Description — where we constructed dials geometrically by means of the horizontal circumferences and the Sun's altitudes — we taught, at the end of No. 6, that the points of the hours from sunrise or sunset on the horizontal line are to be sought by means of the rising amplitudes, those same points will be found more readily by means of the hours from noon and midnight. For if on the horizontal line there be found the point of the hour from noon or midnight that is half of the given hour from sunrise or sunset, that hour is to be drawn through the said point. Thus for hour 11 from sunrise or sunset one must seek the point of hour $5\frac{1}{2}$ from noon or midnight; for hour 23 from sunset, the point of hour $11\frac{1}{2}$ from noon or midnight; and for hour 22, the point of hour 11, etc.

[Margin: The rising amplitudes of the circles of the hours from sunrise or sunset.] Indeed, since any circle of an hour from sunrise or sunset, and the Horizon — which is the circle of hour 24 — mutually intersect one another on the circle of that hour from noon or midnight whose number is half of the hour from sunrise or sunset (as is clear from the figure of the scholium of ch. 1 No. 3 of the New Description, and as was demonstrated by us in book 1 of the Gnomonics, propositions 17 and 20), it comes about that the rising or setting amplitude of any hour from sunrise or sunset does not differ, in table 6, from the arc of the Horizon of that hour from noon or midnight which is its half. Since, therefore, the lines of the hours from noon or midnight in a horizontal dial (ch. 2 of the New Description) are laid out by the arcs of the Horizon of table 6 carried over onto a circle described from the center G — the beginning being taken from the line of hour 6, as we showed in ch. 5 of the New Description — the arc of that circle between the line of hour 6 and any other hour-line will be equal to the rising or setting amplitude of the hour from sunrise or sunset that is double the assumed hour from noon or midnight. Thus the arc between hour 6 and hour $5\frac{1}{2}$ will be the amplitude of hour 11 from sunrise or sunset, and consequently also of hour 13; and so of the rest. Therefore, if the hours from noon and midnight are carefully laid out in a horizontal dial together with the half-hours, they will contain at the center G, together with hour 6, the angles of the rising or setting amplitudes which the hours from sunrise or sunset make on the Horizon. This is indeed worth observing, and has perhaps been noticed by few. And so, if the amplitude of some hour from sunrise or sunset is desired in degrees for a given altitude of the pole, one must take from table 6, for the given elevation of the pole, the arc of the Horizon of that hour which is the half of the given hour from sunrise or sunset.

13. [Margin: The declination of the wall.] Finally, I think it will be worth my while if in this place I set out a method of investigating the declination of a proposed wall, different from the one I explained in ch. 22 of the New Description. It is this. On the wall — or rather on some board well leveled and fixed to the wall — let a straight line AB be drawn parallel to the Horizon with the help of a level; and on it let a gnomon AC be fixed at right angles; and in the morning let the tip of the shadow D be observed as carefully as possible, and from A, through D, let a circle DE be described. Then, after a long interval of time, let the shadow be observed again, until its tip E touches the described circle. For, the arc DE being bisected at F, the straight line FA, when drawn, will be the gnomon-line; because in the hyperbola traced through the points D and E on the day of observation, the points D and E are equidistant from the straight line FA and from the position of the gnomon A. Then from any point G of the gnomon-line let the perpendicular GH be dropped to AB, and at G on it let there be constructed the angle HGB, [equal] to the complement of the pole's altitude above the Horizon; which will be done if, GH being taken as the whole sine, HB be taken as the Tangent of the complement of the pole's altitude. Lastly, at the interval HB let AI be cut off perpendicular to AB at the point I, and let the straight line HI be drawn. I say that AIH is the angle of the wall's declination. For, this declination being assumed, if HB be taken equal to HI, and at B the angle of the pole's altitude be made, then G will be found — the center of the dial — and GH the meridian line, and GA the gnomon-line with respect to the gnomon AI: exactly the same as was previously found by observation, as is clear from what we demonstrated in the New Description, ch. 16.

Diagram. The construction for finding a wall's declination from two shadow-observations (§13). A horizontal base line AB runs across the figure, with a semicircle drawn below it. The labelled points are:

- I, G (at the top of the figure)
- K, M (below I and G respectively)
- C (upper left, the top of the gnomon AC)
- A (on the base line, at the center; foot of the gnomon)
- L, H, N (on the base line, to the right of A)
- B (right end of the base line)
- D (on the lower-left of the semicircle; the morning shadow-tip)
- F (at the bottom of the semicircle; the bisection point)
- E (lower center, on the semicircle; the later shadow-tip)

Dashed lines run from D and across the figure (the shadow-directions and the gnomon-line FA). No labels flagged.

When the gnomon-line thus found inclines, for us facing the wall, to the left below the horizontal AB, the wall will decline from south toward the east; but toward the west, if it inclines to the right. For this method has its place chiefly on walls facing south, since on those that face north the two shadows rarely reach the same circle. Therefore a northern dial will have to be derived from a southern one.

But if you wish to keep the given gnomon AC on the dial, a straight line AK equal to it must be cut off, and the angle AKL of the wall's declination must be constructed — that is, AK being taken as the whole sine, AL must be taken as the Tangent of the declination found — and through L the meridian must be drawn perpendicular to AB.

FINIS.

INDEX OF CHAPTERS OF THE NEW DESCRIPTION OF DIALS

[INDEX CAPITVM NOVÆ DESCRIPTIONIS HOROLOGIORVM]

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- **VI.** The hours from noon and midnight, on a plane parallel to the horizon, above which the pole's altitude is so slight that the center of the dial can scarcely be obtained [*supra quem poli altitudo tam exigua est, vt centrum horologij vix haberi possit*]. — *ibid.*

- **VII.** The hours from noon and midnight, on a plane parallel to the horizon, above which the pole's altitude is so great that the equinoctial line can scarcely be drawn on it [*supra quem poli altitudo tanta est, vt in eo linea æquinoctialis vix describi possit*]. — p. 23
- **VIII.** The arcs of the signs on a horizontal dial, for the hours from noon and midnight [*ARCVS signorum in horologio horizontali horarum à mer. & med. noc.*]. — p. 24
- **IX.** The same, by another method [*alio modo*]. — p. 32
- **X.** The arcs of the signs, by yet another method, in the hours from noon and midnight of a horizontal dial [*ARCVS signorum alio item modo in horis à mer. & med. noc. horologij horizontalis*]. — p. 35
- **XI.** The arcs of the signs on a horizontal dial, without the aid and help of the hour-lines [*ARCVS signorum in horologio horizontali sine ope, auxilioque linearum horariarum*]. — p. 37
- **XII.** The arcs of the signs on a horizontal dial, for the hours from noon and midnight, in which the center cannot be obtained [*ARCVS signorum in horologio horizontali horarum à mer. & med. noc. in quo centrum haberi non potest*]. — p. 39
- **XIII.** The arcs of the signs in the hours from noon and midnight of a horizontal dial, above which the pole's altitude is greatest [*ARCVS signorum in horis à mer. & med. noc. horologij horizontalis, supra quem maxima est altitudo poli*]. — *ibid.*
- **XIV.** The hours from sunrise and sunset on a horizontal dial [*HORÆ ab or. & occ. in horologio horizontali*]. — p. 40
- **XV.** The arcs of the signs in the hours from sunrise or sunset of a horizontal dial [*ARCVS signorum in horis ab or. vel occ. horologij horizontalis*]. — p. 43
- **XVI.** The hours from noon and midnight, on a plane parallel to some vertical declining from the prime vertical — that is, on the plane of a declining wall, and perpendicular to the horizon [*HORÆ à mer. & med. noc. in plano, quod alicui Verticali à primario Verticali declinanti æquidistat: hoc est, in plano alicuius muri declinantis, & ad Horizontem perpendicularis*]. — p. 51
- **XVII.** The hours from noon and midnight, on a declining plane, where either the center of the dial, or the point of the 6-o'clock hour, lies on the horizontal line, or the meridian point lies on that same horizontal line; or finally when two of these can be obtained only most inconveniently [*HORÆ à mer. & med. noc. in plano declinante, vbi vel centrum horologij, vel punctum horæ 6. in horizontali linea, vel punctum meridianæ in eadem linea*]

horizontalis; vel denique quando duo ex his incommodissime haberi possunt]. — p. 58

- **XVIII.** The arcs of the signs in the hours from noon and midnight of a declining dial [*ARCVS signorum in horis à mer. & med. noc. horologij declinantis*]. — p. 59
- **XIX.** The hours from sunrise and sunset on a declining dial [*HORÆ ab or. & occ. in horologio declinante*]. — p. 63
- **XX.** On the dial which is parallel to the prime vertical circle [*DE Horologio, quod Verticali circulo primario æquidistat*]. — p. 70
- **XXI.** On the meridian dial [*DE Horologio Meridiano*]. — p. 71
- **XXII.** On finding the meridian line, and the declination of walls, and the placing of dials and of the gnomon [*DE inuentione lineæ Meridianæ, declinationisque murorum, atque horologiorum, ac styli collocatione*]. — p. 73
- **XXIII.** How a wall-dial drawn on paper is to be transferred onto the wall, at any size of the gnomon [*QVO pacto horologium murale in charta descriptum, in murum fit transferendum ad quamcunque styli magnitudinem*]. — p. 76
- **XXIV.** By what method a dial, whether horizontal or on a wall, may be constructed for a given latitude of the place, from horizontal circumferences of the Sun alone and from the Sun's altitudes above the horizon computed for that same place [*QVA ratione horologium tam horizontale, quam murale ad datam latitudinem loci, ex solis circumferentijs horizontalibus, & altitudinibus Solis supra Horizontem eiusdem loci supputatis construatur*]. — p. 77
- **XXV.** Dials found geometrically from horizontal circumferences only, and from the Sun's altitudes above the horizon. Where it is also taught by what method declining dials may be described from the horizontal dial alone, or even from the meridian dial [*HOROLOGIA ex circumferentijs tantum horizontalibus, & altitudinibus Solis supra Horizontem Geometrice inuentis. Vbi etiam docetur, qua ratione horologia declinantia ex solo Horizontali horologio, vel etiam Meridiano describi possint*]. — p. 82

INDEX OF TABLES OF THE NEW DESCRIPTION OF DIALS

[INDEX TABVLARVM NOVÆ Descriptionis Horologiorum]

- **I.** The arc of any circle of the hour, from noon and midnight, between the equator and the great circles that are drawn through the hour-point of the equator, 6 hours distant from the assumed hour-circle, and through the individual hours from noon and midnight at the beginnings of the signs, and intercepted [*ARCVS cuiusvis circ. hor. à mer. & med. noc. inter Æquatorem, & maximos circulos, qui per punctum horarium Æquatoris 6. horis ab assumpto circulo horario distant, & per singulas horas à mer. & med. noc. in signorum initijs ducuntur, intercepti*]. — p. 91
- **II.** The arc of the meridian between the equator and the great circles which are drawn out of hours 5, 4, 3, 2 from noon and 7, 8, 9, 10 from midnight through the individual hours from noon or midnight at the beginnings of the signs, and intercepted [*ARCVS Meridiani inter Æquatorem, & circulos maximos, qui ex horis 5. 4. 3. 2. à mer. & 7. 8. 9. 10. à med. noc. per singulas horas à mer. vel med. noc. in principijs signorum ducuntur, intercepti*]. — p. 92
- **III.** The arc of the ecliptic (the beginning of ♋ [Greek zodiac sign: Cancer], or of ♏ [Greek zodiac sign: Capricorn], being placed on the meridian above the horizon), between the meridian and the great circles which are drawn out of the intersection of the meridian with the equator through the hours from noon and midnight at the beginnings of the signs — arcs which in gnomonics we have called hour-arcs [*ARCVS Eclipticæ (posito principio ♋, vel ♏ in Meridiano supra Horizontem) inter Meridianum, & circulos maximos, ex sectione Meridiani cum Æquatore per horas à mer. vel med. noc. in signorum initijs ductos, quos arcus horarios in Gnomonica appellauimus*]. — *ibid.*
- **IV.** The arc of the semidiurnal circle in degrees and minutes, computed at the beginnings of the signs for all elevations of the pole [*ARCVS semidiurni in grad. ac min. in signorum initijs, ad omnes eleuationes poli supputati*]. — p. 93
- **V.** The amplitudes of rising and of setting at the beginnings of the signs, computed for several elevations of the pole [*AMPLITVDINES ortiuæ, occiduæque in signorum initijs, ad plures eleuationes poli supputatæ*]. — p. 97

- **VI.** The arc of the horizon and of the prime vertical between the hour-circles from noon and midnight and the intersection of the equator, or of the circle of the 6th hour with the horizon, computed for all elevations of the pole — the arcs which we have called hour-arcs in gnomonics [*ARCVS Horizontis, & Verticalis primarij inter circulos hor. à mer. & med. noc. & interfectionem Æquatoris, vel circuli horæ 6. cum Horizonte, ad omnes eleuationes poli supputata. quos arcus horarios in Gnomonica appellauimus*]. — p. 98
- **VII.** The arc of the circle of the 6th hour, from noon or midnight, between the equator and the great circles that are drawn by the intersection of the equator with the meridian, through the hours from sunrise or sunset at the beginnings of the signs [*ARCVS circuli horæ 6. à mer. vel med. noc. inter Æquatorem, & maximos circulos, qui per interfectionem Æquatoris cum Meridiano, per horas ab or. vel occ. in signorum initijs ducuntur*]. — p. 122
- **VIII.** The arc of the meridian between the equator and the great circles which are drawn by the intersection of the equator with the horizon, through the hours from sunrise or sunset at the beginnings of the signs [*ARCVS Meridiani inter Æquatorem, & maximos circulos, qui per interfectionem Æquatoris cum Horizonte, per horas ab or. vel occ. in signorum initijs ducuntur*]. — *ibid.*
- **IX.** The altitudes of the Sun for the hours from noon and midnight at the beginnings of the signs. For all elevations of the pole [*ALTITVDINES Solis pro horis à mer. & med. noc. in signorum initijs. Ad omnes eleuationes poli*]. — p. 124
- **X.** The altitudes of the Sun for the hours from sunrise or sunset at the beginnings of the signs. For several elevations of the pole [*ALTITVDINES Solis pro horis ab or. vel occ. in signorum initijs. Ad plures eleuationes poli*]. — p. 138
- **XI.** The horizontal circumferences, between the prime vertical and the other verticals, drawn through the hours from noon and midnight at the beginnings of the signs. For latitude Gr. 42 [*CIRCVMFERENTIÆ horizontales, inter Verticalem primarium, & alios Verticales per horas à mer. & med. noc. in signorum initijs ductos. Ad latitud. Gr. 42*]. — p. 144
- **XII.** The horizontal circumferences between the prime vertical and the other verticals, drawn through the hours from sunrise and sunset at the beginnings of the signs. For several elevations of the pole [*CIRCVMFERENTIÆ horizontales inter Verticalem primarium, & alios*

Verticales per horas ab or. & occ. in signorum initijs ductos. Ad plures eleuationes poli]. — p. 146

- **XIII.** The declinations of the diurnal arcs, computed for several elevations of the pole [*DECLINATIONES arcuum diurnorum, ad plures eleuationes poli supputatæ*]. — p. 153
- **XIV.** The arc of the horizon between the meridian and the great circles which are drawn by the intersection of the equator with the meridian, through the hours from noon and midnight at the beginnings of the signs. For latitude Grad. 42 [*ARCVS Horizontis inter Meridianum, & circulos maximos, qui ex intersectione Æquatoris cum Meridiano, per horas à mer. & med. noc. in signorum initijs ducuntur. Ad latitud. Grad. 42*]. — p. 157
- **XV.** The arc of the prime vertical between the horizon and the great circles which are drawn by the intersection of the equator with the meridian, through the hours from noon and midnight at the beginnings of the signs. For latitude Gr. 42 [*ARCVS Verticalis primarij inter Horizontem, & circulos maximos, qui ex intersectione Æquatoris cum Meridiano per horas à mer. & med. noc. in signorum initijs ducuntur. Ad lat Gr. 42*]. — *ibid.*
- **XVI.** The arc of the horizon between the meridian and the great circles which are drawn by the intersection of the equator with the meridian, through the hours from sunrise and sunset at the beginnings of the signs. For latitude grad. 42 [*ARCVS Horizontis inter Meridianum, & circulos maximos, qui ex intersectione Æquatoris cum Meridiano per horas ab or. & occ. in signorum initijs ducuntur. Ad latitud. grad. 42*]. — *ibid.*
- **XVII.** The arc of the prime vertical between the horizon and the great circles which are drawn by the intersection of the equator with the meridian, through the hours from sunrise and sunset at the beginnings of the signs. For latitude gr. 42 [*ARCVS Verticalis primarij inter Horizontem, & circulos maximos, qui ex intersectione Æquatoris cum Meridiano per horas ab or. & occ. in signorum initijs ducuntur. Ad latitud. gr. 42*]. — p. 158
- **XVIII.** The arc of the horizon between the meridian and the great circles drawn out of the poles of the world, through the hours from sunrise or sunset at the beginnings of the signs [*ARCVS Horizontis inter Meridianum, & circulos maximos ex polis mundi per horas ab or. vel occ. in signorum initijs ductos*]. — *ibid.*
- **XIX.** The segments of the hour-lines from noon and midnight, between the center of the horizontal dial and the points of the Sun's parallels, reduced to the tangents with respect to a whole sine equal to the gnomon: for very many elevations of the pole; for drawing the Sun's parallels, according to

what was prescribed in Number 5 of the scholium of chapter 9 [*SEGMENTA lin. hor. à mer. & med. noc. inter centrum horologij horizontalis, & puncta parallelorum Solis, ad Tangentes respectu finis totius stylo æqualis reducta: Ad plurimas poli eleuationes; pro parallelis Solis describendis, ex præscripto Num. 5. scholij cap. 9*]. — p. 159

- **XX.** For the altitudes of the Sun on the vertical at the beginning of any sign [*PRO altitudinibus Solis in Verticali in principio cuiuslibet signi*]. — p. 165
- **XXI.** For the distances of the Sun from the meridian when it is on the vertical at the beginning of any sign [*PRO distantijs Solis à Meridiano cum est in Verticali in principio cuiuslibet signi*]. — p. 167
- **XXII.** A table for converting parts of the equator into hours and their fractions [*TABVLA conuertendi partes æquatoris in horas, earumque fragmenta*]. — p. 169
- **XXIII.** A table for converting hours and their fractions into parts of the equator [*TABVLA conuertendi horas earumque fragmenta in partes æquatoris*]. — *ibid.*
- **XXIV.** A table for finding out the proportional part [*TABVLA pro parte proportionali eruenda*]. — p. 176

INDEX OF PROBLEMS OF THE NEW DESCRIPTION OF DIALS

[INDEX PROBLEMATVM NOVÆ Descriptionis Horologiorum]

- **I.** To investigate the declination of a given point of the ecliptic: the maximum declination being posited as gr. 23. min. 30 [*DECLINATIONEM dati puncti Eclipticæ inuestigare: posita maxima declinatione gr. 23. min. 30*]. — p. 181
- **II.** To compute the semidiurnal arc of any parallel of the equator, for a given altitude of the pole [*ARCVM semidiurnum cuiusvis paralleli Æquatoris, ad datam poli altitudinem supputare*]. — *ibid.*
- **III.** To compute the amplitude of rising, or of setting, of any point of the ecliptic, for any latitude of the place [*AMPLITVDINEM ortiuam, occiduamue cuiusvis puncti Eclipticæ, ad quamlibet latitudinem loci supputare*]. — *ibid.*
- **IV.** Given any vertical and its declination, to find out the altitude of the pole above it in any given latitude of the place [*DATO Verticali quocunque, eiusque declinatione, altitudinem poli supra ipsum in quauis data loci latitudine deprehendere*]. — p. 182

- **V.** Given any vertical and its declination, to find the inclination of its own meridian to the meridian of the horizon, in any latitude of the place [*DATO Verticali quocunque, eiusque declinatione, inclinationem propriij ipsius Meridiani ad Meridianum Horizontis, in quacunque latitudine loci reperire*]. — *ibid.*
- **VI.** Given any vertical and its declination, to compute how large its arc is between the horizon and the equator, and between its own meridian and the meridian of the horizon: likewise between the meridian of the horizon and the equator, and also between its own meridian and the horizon, for any latitude of the place [*DATO quolibet Verticali, eiusque declinatione quantus fit arcus ipsius inter Horizontem, & Æquatorem, atque inter proprium eius Meridianum, ac Meridianum Horizontis: Item inter Meridianum Horizontis, & Æquatorem, nec non inter proprium eius Meridianum, & Horizontem, ad quamlibet latitudinem loci supputare*]. — *ibid.*
- **VII.** To explore the altitude of the Sun on the prime vertical of any great circle serving as a horizon: and the hour, when the Sun is on that vertical, in any region [*ALTITVDINEM Solis in Verticali primario cuiusvis circuli maximi instar Horizontis: Et horam, cum Sol in eo Verticali existit, in qualibet regione explorare*]. — p. 182
- **VIII.** Given any vertical and its declination, to find out how large its arc is between the meridian of the horizon and the circle of the 6th hour from noon or midnight. Likewise between the horizon and the same circle of the 6th hour. And finally between the horizon and the circle of the 12th hour from sunrise or sunset, in any region [*DATO Verticali quouis, eiusque declinatione, quantus fit ipsius arcus inter Meridianum Horizontis, & circulum horæ 6. à mer. vel med. noc. Item inter Horizontem, & eundem circulum horæ 6. Ac denique inter Horizontem, & circulum horæ 12. ab or. vel occ. in quavis regione cognoscere*]. — p. 183
- **IX.** Given any plane, together with the altitude of the pole above it, to inquire how large its arc is between any hour-circle from noon or midnight and its common intersection with the equator [*DATO plano quolibet, cum altitudine poli supra illud, quantus fit ipsius arcus inter quemvis circulum hor. à mer. vel med. noc. & communem intersectionem eius cum Æquatore pofitus, inquirere*]. — *ibid.*
- **X.** If through the pole of any great circle a vertical is drawn at right angles to any great circle drawn through the poles of the world, to scrutinize how large the arc of that circle drawn through the poles of the world is, between that perpendicular vertical and the nearer pole of the world. Where we

have computed the aforesaid arcs for very many elevations of the pole on the horizontal dial [*SI per polum cuiusvis circuli maximi ad quemlibet circulum maximum per mundi polos ductum, Verticalis ad rectos angulos ducatur, quantus fit arcus circuli illius per mundi polos ducti, inter Verticalem illum perpendicularem, & polum mundi viciniorem positus, perscrutari. Vbi prædictos arcus ad plurimas poli eleuationes supputauimus in horologio horizontali*]. — p. 184

- **XI.** Given a diurnal arc of any number of hours, to investigate its declination, for any latitude of the place [*DATO arcu diurno quotcunque horarum, eius declinationem, ad quamvis latitudinem loci peruestigare*]. — *ibid.*
- **XII.** To seek out the arc of any great circle drawn through the poles of the meridian and any point of that same meridian, lying between the equator (or the meridian) and the great circle described by the intersection of the equator with the meridian, through any hour of the given parallel [*ARCVM cuiusvis circuli maximi per polos Meridiani, & quodlibet punctum eiusdem Meridiani ducti, posited inter Æquatorem vel Meridianum, & circulum maximum ex intersectione Æquatoris cum Meridiano per quamvis horam dati paralleli descriptum, exquirere*]. — p. 185
- **XIII.** To investigate the arc of the meridian between the equator and the greatest circle which is drawn out of any hour on the equator through a given hour on any parallel [*ARCVM Meridiani inter Æquatorem, & maximum circulum, qui ex quacunque hora in Æquatore per datam horam in quouis parallelo ducitur, inuestigare*]. — p. 186
- **XIV.** Given any vertical and its declination, to inquire how large the arc of the ecliptic is (the beginning of ♋ [Cancer], or of ♑ [Capricorn], being placed on the meridian above the horizon) between the meridian and the great circle which is drawn from the pole of the given vertical to the ecliptic at right angles [*DATO Verticali quolibet, eiusque declinatione, quantus fit arcus Eclipticæ (posito principio ♋, vel ♑ in Meridiano supra Horizontem) inter Meridianum, & circulum maximum, qui ex polo dati Verticalis ad Eclipticam ad rectos angulos ducitur, inquirere*]. — p. 187
- **XV.** To compute the altitude of the Sun above a given great circle at any hour, the altitude of the pole above it being known [*ALTITVDINEM Solis supra datum circulum maximum quacunque hora, cognita altitudine poli supra ipsum supputare*]. — *ibid.*
- **XVI.** To explore the circumference of any great circle (which serves as some horizon) between its own prime vertical and the vertical of the same

passing through the center of the Sun at a proposed hour, the altitude of the pole above it being known [*CIRCVMFERENTIAM cuiufuis circuli maximi, (qui instar Horizontis cuiuspiam fit) inter Verticalem eius primarium, & Verticalem eiusdem per centrum Solis hora propofita tranfeuntem, cognita altitudine poli supra ipsum, explorare*]. — p. 189

[INDEX TABVL. ET PROBL., continued]

- **XVII.** To render the segments of the hour-lines from noon and midnight, lying between the center of the horizontal dial and the parallels of the Sun, marked in parts of the tangents with respect to a whole sine of 1000 equal to the gnomon. Where not a few things are added which render the drawing of dials easier, things which did not come up in their proper places [*SEGMENTA linearum hor. à mer. & med. noc. inter centrum horologij horizontalis, & parallelas Solis posita, in partibus Tangentium respectu sinus totius 1000. stylo æqualis, nota efficere. Vbi non pauca adiiciuntur, quæ faciliorem horologiorum delineationem reddunt, quæ proprijs locis non occurrerunt*]. — p. 190
- Near the end we have added a **table of tangents**, the whole sine being set at 1000, which should always be kept at hand [*SVB finem adiecimus Tabulam Tangentium, posito sinu toto 1000. quæ semper in manibus habenda est*]. — p. 194
- Likewise **certain practical procedures** excerpted from the New Description of Dials, by which dials are readily drawn [*ITEM praxes quasdam excerptas ex Horologiorum noua descriptione, quibus expedite horologia describuntur*]. — p. 203
- Likewise a **most brief compendium** for drawing horizontal and declining dials [*ITEM Compendium breuiffimum describendorum Horologiorum horizontalium ac Declinationum*]. — p. 211
- Lastly, **notes on the New Description of Dials**, which also contribute very much toward the construction of dials [*VLTIMO Notas in nouam Horologiorum descriptionem; quæ ad horologia extruenda plurimum etiam conducunt*]. — p. 230

THE END. [*FINIS.*]

[End-of-volume blank leaves]

The three leaves of this opening carry no printed text of their own. They are the trailing blank leaves that close the volume, following the "FINIS." of the *Index*

Tabularum et Problematum of the *Horologiorum Nova Descriptio* (printed p. 243, transcribed in the prior batch).

Printed p. 244 (global — blank verso of the FINIS leaf.

This leaf bears no ink of its own. It shows only the faint offset / show-through of the preceding FINIS page printed on the other side of the same leaf: the ghost of the running header, of the last index entries (the "XVII." lemma and the "ITEM ..." entries), and of the word "FINIS" toward the middle. All of this text was already transcribed in the previous batch as the recto (printed p. 243) and is **not** re-transcribed here to avoid duplication.

- [Show-through only; no text is printed on this leaf.]
- Near the center of the leaf there is a faint show-through of what appears to be a small device or ornament [?], too indistinct to identify or describe.
- [Library stamp: oval ink stamp, "Bayerische Staatsbibliothek München" (Bavarian State Library, Munich), impressed near the foot of the leaf — modern ownership mark, not part of the 1612 printing.]

Printed p. 245 (global — blank leaf.

Blank. No printed text, ornament, or signature.

Printed p. 246 (global — blank leaf.

Blank. No printed text, ornament, or signature.

[End of volume — rear endpaper and inside rear cover]

These two final leaves carry no printed text of the volume itself. The book's live text closed with "FINIS." at printed p. 243 (the *Index Tabularum et Problematum* of the *Horologiorum Nova Descriptio*), and the leaves that followed were blank trailing leaves (prior batch). The two leaves here are the physical closing of the copy: the rear endpaper and the inside of the rear cover (the blind-tooled binding). No colophon, printer's device (in ink), or errata is present.

Global (nominal printed p. 247) — rear endpaper (free flyleaf).

This leaf bears no printed ink of its own. It carries only modern library annotations in pencil and the faint show-through of the ornamental binding on the leaf beneath it.

- [Margin: modern pencil annotation across the head of the leaf — a librarian's collation/quire tally, not part of the 1612 printing: 1-5 = 7

followed by a row of tally strokes xxxx xxxx xxxx xxx, then III. 84 (the last group underlined). A second, fainter pencil note beneath the first is largely rubbed away and illegible — it appears to begin Ma[?] 2[?] but cannot be resolved. [?]

- Toward the lower-center of the leaf there is a faint show-through of the crowned armorial medallion (crest with a rampant lion) that is blind-stamped on the facing inside cover — an offset of the binding, not printed matter. [Show-through only; no text on this leaf.]
- The remainder of the leaf is blank, stained, and foxed parchment/paper with no printed text, ornament, or signature.

Global (nominal printed p. 248) — inside rear cover (blind-tooled binding).

This is the inside of the rear board: a blind-tooled (blind-stamped) decorated leather binding, not a printed page. It shows a paneled design of concentric rectangular frames filled with floral/foliate roll-tooling and rosettes, enclosing a central oval medallion. Within the medallion is a crowned coat of arms bearing a rampant lion, with the letter s visible at the medallion's left edge. There is no printed text.

[This is the final leaf of the volume.]