Cheshbon Mahalechot ha-Kochavim

חשבון מהלכות הכוכבים

[The Reckoning of the Courses of the Stars]

(c. 1136)

by R. Avraham bar Ḥiyya ha-Nasi (אברהם בן חייא הנשיא, "Savasorda"), c. 1070–1145

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Upright reason dictates that the recipients of the good, of whatever type and level of beneficence it may be, must show gratitude and blessing to the beneficent person in every way possible, commensurate with the value of the beneficence. And one who has benefited all the people of the world — for example, one who invented a new instrument for the good of the world, or a good book — it is fitting for every discerning person, out of the obligation of love of fellow beings, to at least purchase it, so that the man will profit and his heart will be encouraged thereby to invent yet more good instruments in the world. And similarly, all other wise-hearted people will likewise strive and exert themselves to invent good things and instruments needed for the repair of the world and its perfection.

And therefore, whoever says: "What do I need this new instrument for?" — he does not act well towards the world. For if not for the man who invented it, where would you be? And what would your city do? And more than this, if the man did not exert himself, what would the world do?

— R. Pinchas Eliyahu Hurwitz, *Sefer HaBris HaShalem* (1797)

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A note on the work

A medieval Hebrew treatise on mathematical astronomy and the computation of the Jewish calendar. It sets out the structure of the heavens and the courses of the sun, moon, and planets, then applies them to the molad (conjunction), the tekufot (seasonal points), the nineteen-year cycle, and the rules of intercalation, organized throughout around astronomical and calendrical tables and step-by-step reckonings. It stands at the head of the Hebrew tradition of computational astronomy that later authors such as Gersonides would extend. This volume reproduces a 1960 printing.

A note on the translation process

This translation was produced with the translation-pipeline. See the linked repository for details on the process.

This text is the product of a highly refined automated translation process. It has undergone only cursory human review and should not be treated as an authoritative or scholarly rendering. It is offered solely as a reference and reading aid — a way to make an otherwise inaccessible work approachable — and any passage of real consequence should be checked against the original source.

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Title Page

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R' Avraham, son of R' Chiya, ha-Bargeloni [Abraham bar Ḥiyya ha-Nasi of Barcelona]

Sefer Cheshbon Mahalechot ha-Kochavim [The Book of the Reckoning of the Courses of the Stars]

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The Book of the Reckoning of the Courses of the Stars

I begin to write the Book of the Reckoning of the Courses, by the divine sage Rabbi Avraham bar Ḥiyya the Spaniard, may his memory be for the life of the World to Come.

[Author's Preface]

Said Avraham bar Ḥiyya the Spaniard, may his soul rest in Eden: I begin in the name of the Lord who sits enthroned upon the cherubim, to set forth the order of the reckoning of the courses of the stars. I open with it, declaring: may the name of my Rock be praised, who has been my help — He who has aided and assisted me to complete the account of the form of the earth and the configuration of the heavens, of that which is exalted, and of those that revolve about it; the order of the courses of the stars set within them, and all the matters expounded in the first book, the *Composition on the Science of Observation* [*Ḥibbur Ḥokhmat ha-Ḥizzayon*]. And now, with the help of my Creator and my Maker, I come to this second book, to fulfill the condition I imposed upon myself: to set forth the order of the reckoning of all the kinds of courses mentioned in the first book, according to their varieties. I pour out my supplication before the Master of mercies, that He sustain me and uphold me, to complete this account of the reckoning in its proper manner, set out in order, through the greatness of His abundant mercy.

This book is divided into twenty great gates [*she'arim*], and each and every gate is divided into sections [*petaḥim*], according to the matters and the arrangements that the given gate comprises. At the head of each gate I mention its topics in their order, so that they may be ready at hand for anyone who seeks them; and afterward I proceed to expound all the matters of each and every gate in its order, explaining all the reckonings that depend upon that gate, as best I can, with the help of Heaven, blessed be He.

A Listing of the Gates of This Book — They Are Twenty Gates, As Mentioned Forthwith

The First Gate: on the number of the degrees [*ma'alot*] of the orb [*galgal*] and their fractions; the fractions of each degree among them, and the fractions of their fractions; the manner of gathering [summing] the degrees and the fractions and their addition; and the manner of multiplying the degrees and the fractions according to their kinds;

[continues:] and the manner of dividing each of them by the second among them [i.e., one by the other]; the manner of extracting the roots [*yesodei*] of the numbers gathered from the degrees, or from the fractions, or from both kinds of them; and all that depends upon this matter, which I find it necessary to set forth first and to mention, with the help of God.

The Second Gate: on the value of the parts of the central chord within the orb — which is called the diameter [*kuṭr*] of the orb (and it is the chord of half the orb) — and the explanation of all chords of every arc among them, which is called the chord *meḥutsah* [the chord subtending an arc]; the manner of their reckoning from the tables [*luḥot*] made for them; the explanation of the straight subtending chord, and the divided subtending chord which is called the arrow [*ḥets*, the sagitta]; and the manner of extracting each of them from the second.

The Third Gate: on the value of the arc of declination [*nemikhut*, "lowness"] — which is the arc grasped between the plane [*ofen*] of the equator [*ofen ha-mishor*] and the plane of the zodiac, from the great plane inclined upon the two poles of the two planes, passing over the head of the sign Capricorn and over the head of the sign Cancer. And the value of the arc grasped between these two planes — whether to the north or to the south — at each and every degree of the degrees of the plane of the zodiac, which is called the declination of that degree; the manner of reckoning this declination from the chords and the arcs, and the manner of extracting it from the tables made for the reckoning, with the help of God.

The Fourth Gate: on the value of the arc that rises from the plane of the equator with the ascending of each and every sign of the plane of the zodiac, upon the planes of the separating circle [*ofnei ha-mafrish*, the equator/horizon dividing-circles] in the equinoctial line [*be-kav ha-shaveh*], and also upon the plane of the meridian [*ofen ḥatsi ha-shamayim*, "the plane of mid-heaven"], in all the dwelling-places of the earth.

The Fifth Gate: on the reckoning of the ascensions [*mitsadei*, "risings"] of the signs upon the planes of the separating circle in the rest of the dwellings of the earth that face northward; the manner of their reckoning by means of the subtending chords, and their reckoning from the tables made for them, from the first climate [*iqlim*] to the seventh climate; and how one knows from the tables the arc of the day and of the night and the parts [fractions] of their hours, and all that depends upon this matter, with the help of God.

The Sixth Gate: to make known the number of the degrees by which your location inclines in latitude toward the northern quarter from the plane of the equator. If you know the number of the additional hours on the longest day at your location relative to the equinoctial day, then conversely you may know the number of the additional hours from the value of the latitude. And to know the breadth [span] of each and every sign — and of each and every part of the signs from the plane of the zodiac — when it ascends upon the plane of the separating circle in all the dwellings of the earth: how far it will be distant from the eastern point upon that separating circle, whether to the north or to the south; and the value of [continues into the Seventh Gate's description] —

— the degree of each and every part of the plane of the zodiac when it is at mid-heaven, which is the arc between it and the plane of the separating circle at the time of its coming to mid-heaven, which is called the altitude [*govah*] of the degree; the value of the degrees at which the stars sit at mid-heaven; and the explanation of all that is known of the course of their degrees, which is their altitude.

The Seventh Gate: on the explanation of the shadow of the vertical gnomon [*tsel ha-kav ha-nitsav*] — which is called the extended shadow [*tsel nifrash*], the plain shadow [*tsel pashuṭ*] — and the shadow of the reclining line [*tsel ha-kav ha-shoḥeh*], which is called the standing shadow [*tsel omed*] and the bent shadow [*tsel me'uqqal*]; to know the value of these two shadows from the degrees of the sun which is its altitude upon the plane of the separating circle; to know the degrees of the sun from the shadows; to know the hours that have come of the day, or those remaining out of the degrees of the sun; to know the degrees from the hours in every place, by the reckoning of the chords and the arcs.

The Eighth Gate: on the number of the days of the year, according to the view upon which we base the courses of the stars in this book; how we are to set right these years according to the days of the order of the world [Creation] upon which we count, and according to the days of the years of Edom [Christendom] and Ishmael [Islam]; how each and every one of these dates is known from the year; and how we can extract them from the tables made for them, with the help of God.

The Ninth Gate: on the reckoning of the difference between the days and nights reckoned for the whole world, and the equal days and nights upon which the courses of the stars are founded; and how we can convert the variable [unequal] days to the equal, and the equal to the variable.

The Tenth Gate: on the reckoning of the course of the sun and the moon, the course of the other planets, and the course of the head of the Dragon [*rosh ha-teli*] and its tail [*zanav*] — which is its mean course relative to the variable course — and the form of the tables made for this reckoning.

The Eleventh Gate: on the reckoning of the course of the latitude of the moon from the plane of the zodiac to the north and to the south, and the latitude of the five wandering planets [*kokhvei ha-nevukhah*]; and the tables made for this reckoning.

The Twelfth Gate: on the explanation of the reckoning of the eclipse of the moon from the tables made for its reckoning, and the form of the eclipses.

The Thirteenth Gate: on the explanation of the reckoning of the difference [*ḥilluf*, parallax] found in the place of the moon between what is seen by the inhabitants of the settled earth and its true place in longitude and latitude; and how you can reckon the difference by means of the chords and the arcs, and the manner of extracting it from the tables made for it.

The Fourteenth Gate: on the reckoning of the eclipse of the sun from the tables made for reckoning it, and the form of the eclipse.

The Fifteenth Gate: on the reckoning of the sighting of the new moon at the end of each month, at the western horizon, in all the inhabited places.

The Sixteenth Gate: on the reckoning of the rising of each one of the wandering stars [planets] from beneath the light of the sun, and their becoming visible to the eyes of the settled world, and the time of their entry into the brightness [of the sun] and their concealment from the eyes of the inhabitants of the settled world.

The Seventeenth Gate: on the reckoning of the course of the fixed stars and the course of the apogee points [*rumei ha-rum*] and the heads [nodes] of the dragons [*teninim*] for the five wandering stars [planets], and [the reckoning of] their positions and these points, which are the root of the reckoning of this book; and the names of the fixed stars that are in the first magnitude and in the second magnitude, and their latitude — by the configuration of the zodiac and the configuration of the equator.

The Eighteenth Gate: on the zodiacal sign rising upon the configuration of the separating [orb — the colure/horizon] that stands in the middle of the heavens, from among the hours that come from the day or from the night; and the manner of dividing the orb into twelve houses, for every single time and for every latitude, from the portion rising upon it [the horizon]; and the latitude.

The Nineteenth Gate: on the [reckoning of the] heads of the years for the sun — that is, the time when it returns to the actual point of the fracture [*shever*], and the degree from the zodiacal sign which was at the head of the year, or at the beginning of the conjunction [*molad*] or of the [astronomical] event; this is what is called the seasonal point [*tekufah*] of the years.

The Twentieth Gate: on the reckoning of the place of the rays [*ziqot*] that issue from the brilliance of the stars upon an arc of a third, or a sixth, or a quarter [of the circle], toward the eastern and western quarters from their station in the firmament; and the order of the courses of the stars and all the parts of the firmament; and according to the order of the degrees of the houses in which they [the stars] stand, from [their place in] the orb.

These, then, are the general headings of the gates into which this book is divided. I shall now come to explain each gate one by one, dividing it into the topics into which it is itself subdivided.

Now if, in studying these matters, you find explanations and reasons that were already set out in the First Book, do not be surprised by them, and do not let it enter your mind that I intend to explain one and the same thing twice without need. Rather, know that I have not repeated them except in order to set out upon them the order of the reckoning that I am about to teach in that [particular] matter. I have seen fit to mention this reason at the outset of my words, so as to remove this puzzlement from the heart. With the help of the Creator — may His name be blessed, exalted, and uplifted — I now begin to explain the First Gate.

The First Gate

On the parts of the firmament; the order of their combination [addition], their multiplication, their division, and the extraction of their root.

You already know that the whole firmament is divided into twelve zodiacal signs, which total 360° [*shesh me'ot shishim*; lit. "360"], 21,600 minutes [*mem-`ayin-kof* = 21,600], 1,296,000 [*gimmel-dalet-dalet* (גד״ד), abbreviated]; [numeral: ?]. Each individual sign is divided into thirty parts, which are called degrees [*ma'alot*]. Thus the whole firmament is divided into 360° [*lamed-shin-samekh* (לש״ס) = 360] degrees, and each single degree into sixty parts, which are called minutes [*shevarim*]. Each minute is divided into sixty seconds [*sheniyim*], and each one of the seconds is divided into sixty thirds [*shelishiyim*]; each one of the thirds is divided into sixty fourths [*revi'iyim*]; and likewise the fourths into fifths, the fifths into sixths, and so on up to the tenths and beyond.

However, in dividing the fractions, we go only as far as is needed to make the reckoning precise by this measure — to the seconds or to the thirds — in the particulars of the positions. That is sufficient for us to reckon the courses of the stars and the other matters that we compute, and we need not be more precise in the division than this; for the approximation that comes from the parts above the seconds in the reckoning does no harm to the reckoning that could give any cause for concern.

Likewise you know that the firmaments encompassing the earth are eight. The first of them, the one nearest to the earth, is — by the view of all — the firmament of the Moon. Above it, the second, is the firmament of the star called by most of the world "Mercury" [*kochav*]. The third is the firmament of Venus [*Nogah*]. The fourth is the firmament of the Sun [*Ḥammah*]. The fifth is the firmament of Mars [*Ma'adim*]. The sixth is the firmament of Jupiter [*Tzedek*]. The seventh is the firmament of Saturn [*Shabtai*]. These are the firmaments of the stars called the "serving stars" [*shimshei ha-raqia*, i.e. the planets].

The eighth firmament, which canopies them all, is the firmament of the zodiac, which revolves around the earth from east to west and carries with it all the firmaments beneath it, as has been explained in many places. Each one of the seven serving firmaments revolves in its own firmament from east to west, following the principal motion [*rov ha-mahalach*, the diurnal motion]; but the [proper] motion of each one of them is not equal in its relation to the second [proper] motion [i.e. their own east-to-west proper motions differ]. In this book we are going to set out their courses according to their measure in years, in months, in days, and in the fractions of the day, so that we may be able to determine the place of each one of them

upon the orb at any time and moment that we wish. For this reason, we shall need to explain the manner of combining [adding] the degrees, their minutes, and their seconds, and the fractions of their fractions, in the reckoning — by the method of subtracting [extracting] one number from another and multiplying each one by the other (such as the multiplication of degrees by degrees, and of the fractions [minutes] by the fractions, and of the seconds by the seconds) — and to explain the manner of dividing each one of them by the other, and how we can extract the root of a given number, whether of the degrees or of the fractions, in a case where we need to know its root. We have found it necessary to set down all these matters at the opening of our words, so that we may be ready to bring them into practice at the time of need, with the help of Heaven.

The opening of the combining [addition] of degrees and fractions, and the method of separating [subtracting] them.

When you combine [add] degrees, fractions, and seconds that lie before you, in order to know the total of the resulting number: begin by combining the seconds. If the sum combined from them is less than sixty, we keep it in hand, and we retain it, for it is the number resulting from the seconds. But if the number combined from them is in excess of sixty, we take out sixties from it, until there remains in hand a number that is less than sixty; and we retain that, for it is the number resulting from the seconds.

For each sixty that we have taken out from the combining of the seconds, we add one minute [*shever*] to the minutes that we are about to combine. If the total of what is combined from them, together with what has been added to them, is less than sixty, we keep it together with the [other] minutes remaining in hand, and it is the number resulting from the minutes. But if it is in excess of sixty, we take out sixties from it until what remains is less than sixty; and that is the number resulting from the minutes.

For each sixty that we have taken out from the combining of the minutes, we add one degree [*ma'alah*] to the degrees that we are about to combine. If, in their total together with what has been added to them, it is less than 360° [*mem-shin-samekh* = 360], that is the number resulting from the degrees; you hold it fast together with the minutes and the seconds that have been combined for you. There will then come out in your hand the total of the degrees, the minutes, and the seconds combined from the number that was in your hand. And if there are in your hand, among the degrees combined, more than 360 [*yoter mi-mem-shin-samekh* = more than 360], take out 360 [*shin-samekh* = 360] from them, and do with the remainder — [keeping it] less than sixty — as I have commanded you. There will then come out in your hand the total that you sought to know its number.

The Method of Subtracting One Number from Another

[Body heading, p. 10: דרך הוצאת חשבון מחשבון]

If you have in hand degrees, fractions, and seconds, and you wish to subtract from them one number, so as to know the remainder that is left of that number after the subtraction — begin with the smallest of the numbers you have. If the degrees you have gathered come to more than 360, take 360 out of them, and work with the remainder as I have instructed you; the total whose value you sought will then come out in your hand.

If the number you have is seconds, begin with them. If, on the other hand, the number you have is of the first kind — and you wish to subtract from it the second number, made up of seconds — and the seconds that you have are equal in value to the seconds you have in hand, or exceed them, then subtract the second from the first. If the two are equal, nothing remains in your hand; but if the first has a surplus left over, a number less than sixty remains in your hand. Keep that in hand, for it is the number left over from the seconds after the subtraction.

If the seconds in the first number are fewer than the seconds in the second number, take one fraction from the fractions in the first number, add sixty seconds to the seconds in the first number, and subtract from them the seconds in the second number. There will remain in your hand the seconds you ought to keep, which are the seconds remaining after the subtraction. After that, proceed to the fractions and treat them by the procedure you followed for the seconds. If the fractions of the second number are equal to the fractions of the first, or fewer than them, you either subtract them or add to them: if they are equal, nothing remains in your hand; if there are fractions to subtract, subtract the second from the first and keep the remainder. But if the fractions to be subtracted are more than those available, take one degree from the degrees in the first number, add sixty fractions to the fractions in the first number, subtract from them the fractions of the second number, and keep the remainder.

Do likewise with the degrees. If they are equal, nothing remains in your hand. If the degrees of the second number are fewer than the degrees of the first, you subtract from the first, and there comes out in your hand the degrees and fractions — and the seconds — that remain from those numbers after their separation. But if the degrees of the second number exceed the degrees of the first, you add to the degrees of the first 360, which is the total of the degrees of the orb, and subtract from them the degrees of the second number; the number remaining in your hand from the first, after subtracting the second from it, is left to you. Here, then, is the method of combining and separating degrees and fractions set out for you. Study it and keep it, so that you may be skilled in it when you have need of it.

The Order of Multiplying Degrees and Fractions

[Body heading, p. 10: סדר כפילת המעלות והשברים]

If you multiply a number of degrees by a number of fractions, the number gathered from that product will be fractions. If you multiply a number of degrees by a number of seconds, the number gathered from that product will be seconds. And likewise every number that you multiply by a number of degrees — the number gathered from that product will be of the same kind as the number multiplied: if degrees, they will be degrees; if fractions, fractions; if seconds, seconds; if thirds, and so on upward from them, without end.

If you multiply fractions by fractions, what you obtain from that product will be seconds. If you multiply seconds by seconds, the product will be thirds. If you multiply thirds [by thirds], it will be sixths (ששיים) [Stage-5: the tile reads ששיים = sixths here, not רביעיים = fourths — a ש/ר misread; and thirds×thirds → sixths is the correct rank]; and if you multiply by fourths, they will be sevenths (שביעיים, so printed — this last rank does not follow cleanly from the algebra; retained as in the source). Understand from this that every number you multiply by fractions, or by fractions of fractions, its product will be removed from its own rank by as much as the fraction by which it is multiplied is distant from the rank of the degrees of the orb.

To make this easier for you, I have drawn up for you one table, and inscribed upon it the degrees and fractions in their orders, along the length of the tables and across their breadth. I wrote along the side of the length "the first" [א'], and along the side of the breadth "the second" [ב']. When you wish to multiply one number by another — whether of one kind or of two different kinds among all those marked on that table — place the one kind on the first side and the other on the second side. Pass your finger across one of them along its side and along its facing line, until you arrive at [the cell] facing the second; and keep the fraction at which you arrive, for it is the fraction obtained from the product of the two numbers. For example, if one wishes to multiply two seconds by fourths, he enters by one of the two sides — the breadth or the length — at the rank of the seconds, and moves his finger across that side; along its facing line, until he reaches the cell facing the rank of the fourths on the second side, and his finger touches the rank of the sixths. It then becomes known that what is obtained from the product of seconds by fourths is sixths. Do likewise with all the numbers multiplied.

As for any number gathered from the product of degrees by degrees — degrees — you ought to keep it and hold to it, for it is degrees, of the very same kind as the numbers multiplied; yet it certainly descends from their kinds [i.e., it may carry down into lower ranks]. The procedure in all those multiplications is this: if that product is less than sixty, hold to it and count it from the rank of the kind that comes out of that product. But if it exceeds sixty — whether the surplus be large or small — you divide that number by sixty, and you count each sixty, and the sixties that come out of the division, as one unit in the rank of the kind that is one [step] above that kind, as though you had multiplied fifty seconds

by fifty seconds: the product would be two thousand five hundred fourths. You take this number and divide it by sixty; it is divided by them forty-one times, which are thirds, and there is a remainder over them of forty fourths. You then find yourself saying that the product of fifty seconds by fifty seconds is forty-one thirds and forty fourths.

Now I have drawn up tables for you, to let you reach this reckoning without toil. The plan and design of these tables was on this order. I divided the first table across its breadth into fifteen columns, in which I inscribed the numbers from 1 to 15; and as I divided it, so I divided the second table and inscribed in it the numbers from 16 to 30; and likewise in the third and the fourth, until I had completed four tables in breadth, reaching the number 60. Then I divided the length of each of them into sixty columns. Had the length been able to hold them [all], these tables — four or more — would have had the numbers inscribed upon them across their breadth, 15, 16 to 30, and along their length from 1 to 60; but if not, I divide it according to its length and inscribe upon it the numbers from 1 to 60 (ס').

After that I came to each table of them and divided all the middle columns in it — those between the length and the breadth — into two columns, and wrote upon them "first" and "second." In these middle columns I inscribed the value of the number gathered from the product of each and every number that is inscribed along the length, multiplied by each and every number among those inscribed across the breadth. If the multiplied number is less than sixty, I inscribe it in the second of the middle columns, the one in which it is fitting to inscribe it; but if it exceeds sixty, I divide it by sixty and write the division in the first of the divisions of the middle columns, and the remainder after the division I write in the second column.

When you come to find the product of two numbers among the fractions, the one of which is below the other on the table — place your finger upon the one, facing the second, in the middle of the table, and you will find in the cell you reach the sum of the number gathered from that product, in its order — whether of one rank or of two ranks. For example, were you wishing to multiply 24 by 24, and you inscribe them upon these tables, you would find their counterparts in two columns, namely 9 in the first and 36 in the second. And you know that, if your two numbers — whose rank, when multiplied, is fourths — you would then say that the number gathered in the tables is 9 thirds and 36 fourths. Were your two numbers thirds, the number you found would be 9 fifths and 36 sixths [in the cell] of table 9, 36 sixths. This explanation suffices you for the multiplication of degrees and their fractions, and the fractions of their fractions.

The Procedure for Dividing Degrees and Fractions, Each by the Other

If you divide one number by another number of its own kind, what you obtain from the division will be degrees — and likewise for all the remaining fractions down to the last of their kinds — provided you are dividing them by their own kind: whatever results from the division will be of the same denomination [as the dividend]. This is the first rule, and it lies at the heart of the matter. Here is a second rule: if you divide one kind among the fractions by a kind that stands one rank below it, you raise the larger [number] to the rank of the kind that stands below the one you intended to divide it by; and when you then divide it by that [lower kind], what comes out of the division for you is a number of degrees.

An illustration of this rule: if you divide degrees by fractions [minutes], you multiply the degrees by sixty, and what results from this multiplication will be fractions. If you then go on and divide those fractions — the ones to which you reduced the degrees — by the fractions you prepared to divide them by, you obtain degrees from the division. Another illustration of this rule: suppose you divide degrees by seconds. You multiply the degrees by sixty, and they become fractions; then you again multiply the fractions by sixty, and you obtain seconds — these being the seconds to which the degrees were reduced. Divide them by the seconds you have prepared, and you obtain degrees. So too in every case like this.

A third rule: if you divide one kind by a kind larger than it, the result of the division will be of the rank of the fraction by which you multiplied [the dividend] when you raised it to the larger kind and carried it over to the second kind by which you are dividing. Thus, were you dividing thirds by first-order fractions [minutes], the result of the division would be seconds — for the kind by which you multiplied [it], carrying it into first-order fractions, was [minutes], and you carry [the result] over to thirds. From this rule you obtain the following: every fraction that you divide by degrees yields a result of the division belonging to the same denomination as that fraction. This is because any fraction you multiply by a degree took on, in the multiplication, the very denomination of that fraction. And in all the cases of this division rule, if there remains to you from the division a number smaller than the number by which you are dividing, double that remainder by sixty and divide it by that [same] number; you then obtain a number that descends one rank below the first division. Do likewise with the remainder from this second division, if you mean

to refine your reckoning thoroughly — as though you were dividing fifty fractions [minutes] by seven fractions [minutes]. The division yields seven degrees, and there remains to you one out of the fifty; double it by sixty, and you get sixty seconds; divide these by the seven fractions [minutes] by which you divided them the first time, and you obtain eight (8) fractions [seconds], with a remainder of four (4) out of the sixty. Double these by sixty, and you get two hundred forty (240) thirds; divide them by the seven fractions [minutes], and you get thirty-four (34) seconds [thirds], with a remainder of two (2) thirds. Double these by sixty, and they become a hundred twenty (120) fourths; divide them by the seven fractions [minutes], and you get seventeen (17) thirds [fourths], with a remainder, from the divided number, of one quarter, which you can go on doubling and dividing if you wish to delve still deeper in refining the division. This exposition is sufficient for all who are skilled in reckoning.

The Procedure for Extracting the Root of Numbers

The number called a "root" — and which the Ishmaelites call *aṣl* [Arabic: al-aṣl, "root, base"], whose meaning is "that which is multiplied by itself" — is one thing; while the number assembled from its products is called a number "founded" upon the number that is its root. As to the matter of the simple [number], it is fitting that every number have a root, since every assembled number is the product of [whole] units, or of units and fractions of units. The difference is that the multiplied number may have a fraction added to it that is well known and that the mouth can pronounce — in which case that [resulting] number is called "founded in deed," and its root is plain and known; or this fraction of the multiplied number may be closed and sealed, such that the mouth cannot pronounce it — in which case that number is called "founded in potentiality," and its root is called mute. The configuration of the whole numbers grants that the root of a number be of the kind founded by its own definition, whereas the configuration of the fractions of a number and the fractions of its fractions does not behave so.

From here, then, comes the method in this craft: if you set out to extract the root of a number of degrees, you extract it by the method for extracting the roots of a whole number, and its root will be degrees of the number, just as it [the original] was.

But if you set out to extract the roots of a whole number of fractions, you must first know that fractions divide into two kinds. Some of them are "founded in deed" — such as the seconds, whose root is first-order fractions [minutes], because their being multiplied by themselves [as minutes] carries them up to the rank of seconds; and likewise the fourths are founded in deed, their root being seconds. And some of them are not founded — such as the first-order fractions [minutes], which have no well-known root, since you can find no fraction whose multiplication by its own kind [reaches up] to fractions [minutes]. And likewise the thirds,

and the fifths, and many others like them, whose root is mute and sealed. From here, then, came the method in extracting the roots of fractions: if the number of fractions whose root you seek is of the kind whose root is known and well attested, you extract that root by the method for extracting [the root] of a whole number, and you assign its root to the kind from which the multiplication of that fraction is assembled. As though you were setting out to extract the root of thirty-six (36) seconds: you know that the root of 36, as of every whole number, is six wholes; and here you say that these six, which are the roots of the seconds in your hand, are six first-order fractions [minutes] — because the multiplication of a fraction by [its] fractions, carrying it up to the seconds, is [the operation of finding] the kind of fraction whose root you seek. So too is the practice with every "founded" fraction.

And if all the fractions are of the kind whose root is sealed [and] has no root — that fraction whose root is unknown — you convert that fraction to the fraction nearest to it whose root is known, and you extract the root of the number, assigning it to the kind of the founded fraction to which you converted it. As though you wished to extract the root of fifteen (15) thirds, a kind for which there is no known root: you convert it to the kind nearest to it which is founded — namely the fourths. If you multiply fifteen (15) thirds by sixty, you obtain nine hundred (900) fourths; and the root of the number 900, as a whole number, is thirty (30) — for the multiplication of seconds [by themselves], carrying them up to fourths [is what gives this]. And you find yourself saying: the root of fifteen (15) thirds is thirty (30) seconds. So be it in all the kinds that are not founded. This suffices for extracting the root of any number among the degrees, the fractions, and the fractions of fractions.

The Second Gate: On the Reckoning of Arcs, Their Chords, and All That Depends on Them.

A chord of an arc is a straight line that issues within a circle from one point set upon the curved line enclosing the circle, to another point opposite it on that enclosing line. This straight line, drawn between these two points, divides the circle into two parts — either into two equal parts or into two unequal parts. The line that divides the circle into two equal parts is called the diameter of the circle; the line that divides it into two unequal parts is called a chord of each of the two unequal parts.

Now once you know the chords of the orb from one degree of it up to the chord of 180 degrees — which are the chords of half the orb — you thereby know, from those same chords, the chords of the second half of the orb, from 181 up to 360. For the chord of the arc that falls short of the half is the very same as the chord of the arc that, opposite it, exceeds the half. This is the method of the full chords [i.e. chords of the full arc], which craftsmen need for some of their reckonings.

But for most of the work of their reckoning, or nearly all of it, they use the bisected chords. The bisected chord is the chord of half the arc whose [full] chord is the one bisected for it. For this reason, one computes these bisected chords from one degree of the orb up to ninety degrees, which is a quarter of the degrees of the orb. It suffices for you, in order to know the bisected chords for all the degrees of the orb, [to compute these]; for the chords of a quarter of the orb are themselves the chords of its remaining quarter together with what remains of the first quarter. For the bisected chord for one degree is the bisected chord for 179 degrees; the bisected chord for 89 degrees is the bisected chord for 91 degrees; and we find

that whoever knows the bisected chords for 90 degrees knows all the chords of the orb. Therefore I have drawn out for you in this book the bisected chords from one degree up to 90 degrees, and I have divided the diameter of the circle into 120 parts. The bisected chord — from the reckoning of an arc of 360 degrees in the enclosing line of the orb — for 90 degrees came out to be 60 degrees [parts], in the row of the straight line called the diameter line of the orb, by the reckoning of 120 degrees [parts]. Before each and every degree I have written the chord, the one proper to it, [corresponding] to the degrees of the arc.

Now whoever wishes to know the bisected chord for whichever arc he wishes to learn the chord of, by means of this table: if that arc is from 1 up to 180, let him bring the degrees of that arc to the two rows of the count of the arcs in the table, and take what stands opposite them, from the degrees and the fractions, in the rows of the chords — and that will be the chord of that arc. And if the arc exceeds 180, let him bring the remainder, in hand, to the two rows of the arcs and take what stands opposite it in the rows of the chords — and that will be the chord of that arc. For we find that, with these tables, we need not concern ourselves with the arc that exceeds 180.

And if there are, in the arcs, fractions that exceed the [whole] degrees that are in that arc, then he proceeds thus: he finds the chord proper to the whole degrees of that arc and keeps it; then he takes also the chord that adds one degree to the degrees of the first arc, and learns the excess between the two chords. From that excess he takes, in proportion to the fractions that are in that arc — whether a sixtieth (60), or a sixth, or a quarter, or more of these or less — and adds that proportion to the first chord, if the second chord exceeds the first; or he subtracts it from it, if the second falls short of the first. What results in our hands after the addition or the subtraction is the corrected chord for that arc.

And he is able to make the correction by another way as well. It is this: he doubles the excess between the two chords by the number of the fractions exceeding [the whole degrees] in the arc, and the sum he divides by sixty (60); and what comes from the division he adds to the first chord if the second exceeds [it], or subtracts it if the second falls short — and the corrected chord for that arc comes up in his hand.

And if a straight bisected chord is in his hand, and he wishes to know the arc to which that chord is proper, let him search for that chord in the table of arcs and chords, and look for it in the rows of that chord. He takes the arc that stands opposite it in the first of the two rows of the arcs and the chords — and that will be the arc proper to that chord. But if he does not find, in the rows of the chords, a number

equal to the number of the chord that is in his hand, let him come to the chord nearest to it, the one that is less than it. He takes what stands opposite it in the first of the rows of the arcs, keeps its arc, and extracts the chord — that is, the lesser one — from the chord that is in his hand. There will then be in his hand the known excess remaining from the first chord, after he has extracted from it the chord found in the table. He multiplies that excess by thirty (30), and the sum from the multiplication he divides by the excess between the lesser chord — the one in the table whose arc he extracted — and the chord that adds one degree to it. What comes up in his hand he adds to the first [arc], and finds the arc that corrects that chord.

He can correct the arc by another way as well: he finds the proportion of the parts of the remaining chord — when he compares it with the excess between the two chords — what proportion it is, whether a quarter, or a third, or whatever proportion it is. He takes, in that proportion, from sixty (60), and adds it to the first arc, and finds the arc that corrects that chord.

And you know, from the principles set down at the head of this gate, that this chord has another arc proper to it — namely the arc remaining from 180 degrees. If you subtract them [the degrees of the known arc] from it [from 180], the corrected arc comes up in your hand.

And if you wish to know, from the arc, how great is the proportion of the chord cut off for that arc — the one called the arrow [Hebrew: חץ, the versed sine] — it is a straight line cut off from the diameter of the circle, standing at a right angle upon the bisected chord at its beginning, together with the end of the chord within the circle, and reaching along its length to the curved line enclosing the circle, facing the arc of the bisected chord.

And whoever wishes to know the cut-off chord [the versed sine] for each and every arc: if that arc is from one degree up to 90 degrees, he subtracts it from 90, and the remainder in his hand is the known bisected chord [of the complement]; he then subtracts it from sixty (60) degrees, which are half the diameter, and what remains of them after this subtraction is the cut-off chord for that arc that falls short of 90 degrees. And if that arc is a full 90 degrees, with no addition and no subtraction, its bisected chord equals its cut-off chord. And if the arc exceeds 90 degrees, he takes off 90 from it, and what remains in his hand is less than 90; he finds its bisected chord and adds it to sixty (60), which are half the diameter, and finds the cut-off chord for that arc that exceeds 90 degrees. And if there is in his hand a cut-off chord, and he wishes to know the arc of that cut-off chord: if the chord is less than sixty (60), he takes it out of sixty, and the remainder he knows its arc by the manner of learning arcs from chords

and the arc you have in hand will come out at less than ninety (90) degrees; and the chord of that cut-off arc, you take it out of sixty (60), and from the remainder you learn its arc by the manner of learning arcs from chords. And if [the chord] exceeds sixty (60), he takes sixty (60) out of it, and the chord he adds — and the arc it yields he adds onto ninety (90) degrees, and so he finds the arc proper to that cut-off chord, which exceeds sixty (60) by that surplus. This is sufficient for you regarding cut-off chords and segmented chords.

If you have an arc in hand and you wish to know its full chord: take half of that arc, find the chord that subtends it, and double it; the doubled value will be the full chord of that arc.

And if you have a full chord in hand and you wish to know its arc from these tables: take half of that chord and derive its arc by the way in which you derive the arcs for the cut-off chords; then double that arc, and the doubled value will be the arc of the full chord you have in hand. This is sufficient for you regarding arcs and chords.

The Third Gate — on the explanation of the arc that lies between the two surfaces, the surface of the equator and the surface of the zodiac, which is called the arc of declination; and the value of this declination for each and every degree, and the direction of the declination

This arc is marked out and made known from within the course of the sun, which encompasses the orb along the obliquity of the surface of the zodiac. From this, the ancients investigated the distance between the position of the sun when it is at the head of the sign of Cancer or of Capricorn — on the inclined surface, at the two solstitial points of the course — and its position on that surface when it is at the head of the sign opposite it, the second [solstice]. From this distance, which lies between the two points of the [solsticial] reversal on the surface that bisects the heavens above the earth, they came to know the value of the declination arc — that is, when they divided this distance into two equal parts, as is explained in the science of observation [astronomy]. This declination, according to the view of Ptolemy [בטלמיוס = Ptolemy], was found to be 23 degrees [כ״ג], 51 minutes [נ״א]; and it is upon that value that we reckon it in this book.

And if you wish to compute this declination from one degree up to ninety (90) degrees — which are the limit of the declination, namely 23[°] 51[′] [כ״ג נ״א] — first know the cut-off chord for all those degrees from one degree up to ninety (90) degrees, and keep them: these are the chords from the first degree of the sign of Aries up to the thirtieth degree of the sign of Gemini.

When you come to determine the declination from these chords, find the chord of the arc whose declination you seek, and multiply it by the chord of the whole declination arc; the product is then divided by sixty (60), and from the quotient determine the value of its arc, as I taught you in the reckoning of the chords and the arcs. That arc will be the declination of the arc whose declination you were seeking. By this procedure you will be able to determine the declination of each of the ninety degrees whose chords you derived; and there will come out in your hand the declination of ninety (90) degrees of those signs, from the head of Aries to the end

of Gemini, arranged in the order of the straightforward counting from 1 to 90 [צ׳]. And from the declination of ninety (90) degrees you will know the declination of the rest of the signs; for the declination of the ninety (90) degrees that come after them, from 91 [צ״א] to 180 [ק״פ], is the declination of the first ninety (90) [degrees] arranged in reverse, from 90 [צ׳] back to 1 [א׳]. The declination of the ninety (90) degrees that come from 181 [קפ״א] to 270 [ר״ע] is the declination of the first ninety (90) degrees in their order, from 1 [א׳] up to 90 [צ׳]; and the declination of the remaining ninety (90) degrees, which are from 271 [רע״א] to 360 [ש״ס], is the declination of the second ninety (90) degrees arranged in reverse, from 90 [צ׳] back to 1 [א׳].

For this reason, the columns of the count in the declination table were divided into four sectors [סטרין]. The first of them is for the degrees of the signs Aries–Taurus–Gemini [טש״ת — i.e. טלה, שור, תאומים], written in it in their order from 1 [א׳] to 30 [ל׳] for each and every sign. The second is for the degrees of the signs Cancer–Leo–Virgo [סא״ב — i.e. סרטן, אריה, בתולה], written in reverse from 29 [כ״ט] to 1 [א׳]. These two sectors are for the signs of the northern declination, their declination being northern. The two remaining sectors: the third is for the degrees of the signs Libra–Scorpio–Sagittarius [מע״ק — i.e. מאזנים, עקרב, קשת], written in order; and the fourth of them is for the degrees of the signs Capricorn–Aquarius–Pisces [גד״ד — i.e. גדי, דלי, דגים], written in reverse. These two sectors are for the signs of the southern declination, their declination being southern. Opposite these four sectors, the portions of the declination were written in these tables.

One who wishes to know the value of the declination from these tables takes the degrees he has in hand to the tables, and looks them up in the number-columns under the sector at whose head is the name of the sign whose degrees' declination he is seeking, and takes what he finds opposite them in the declination columns and their fractions — that will be the declination of the whole degrees of that sign he has in hand. And if, together with those whole degrees, there are fractions [shevarim, minutes] that have not completed a degree, he determines the value of their declination from the surplus that lies between the declination of the [last] complete degree and the declination of the degree after it: he takes, from the portion of those fractions out of sixty (60), the surplus, and adds that portion onto the declination of the complete degree — if the declination of the second degree is greater — or subtracts it, if the declination is the lesser, just as you did in the correction of the arcs and chords explained in the Second Gate.

And if this declination belongs to the first sector — that is, the sectors of the count that are the signs Aries–Taurus–Gemini [טש״ת] — the declination is ascending in the north and increasing. If it is in the second sector — the sectors of the count that are the signs Cancer–Leo–Virgo [סא״ב] — the declination is descending in the north and decreasing. If it is in the third sector — that is, the signs Libra–Scorpio–Sagittarius [מע״ק] — the declination is descending in them in the south and increasing. And if it is in the fourth sector — that is, the signs Capricorn–Aquarius –Pisces [גד״ד] — the declination is ascending in the south and decreasing.

The Fourth Gate: Explaining the Ascensions of the Zodiacal Signs upon the Equator for the Inhabitants of the Line of Equality, and upon the Semicircle of the Heavens for All the Earth's Inhabitants

The author said: Because the circumference of the orb rests upon the earth toward both poles of the equator, we do not reckon the ascension of the zodiacal signs from the circuit of that very orb [i.e., the ecliptic], but rather we take the corresponding value that rises along with it from the equator. We call this rising of theirs the "passage" (מצעד) of the signs, and this portion that advances along the equator — the passage of the signs across the earth — does not follow one single rule nor stand by one fixed measure in all the inhabited regions. In this fourth gate we explain its rule as it comes about at the equator (קו השוה, "the line of equality"): the place where the equator forever circles directly opposite the heads of those who dwell upon it, and this line bends away from the equatorial circle neither to the north nor to the south. From this it follows that it has no latitude at all, and all the days of the year are equal there — no day or night exceeds or falls short of another — and for this reason it is called the "equator" (the line of equality). The value of the passage of the signs upon the equatorial circle, for the equator, is the value of their crossing over the portion of the semicircle of the heavens that marks the half-day for the equator.

One who wishes to know the value of the passages of the signs upon these two circles — the equator and the arc of the half-day for the whole inhabited world — should extract the chord subtended at 23 degrees, 51 [numeral: כ״ג מעלות נ״א] fractions (minutes), which constitute the full degrees of the declination [the maximum declination, 23°51′]; let him call it the "chord of the total declination" and keep it. Likewise he should extract the chord subtended at 66 degrees, 9 [numeral: ס״ו מעלות ט׳] fractions — these being the 90 [צ׳] in excess of the total declination [i.e., 90° − 23°51′ = 66°9′] — and let him call this chord the "chord of the remainder of the declination," and keep it together with the chord of the total declination. After this he will know the declination of the degrees counted from the head of Aries up to the degree whose passage he wishes to know. Let him extract the chord of the declination of those degrees, and subtract the arc of the declination of those degrees

from 90 [numeral: from תשעים, ninety]; for what remains in his hand let him extract its chord — this being the chord of the remainder of the declination of the degree. Then let him multiply the chord of the declination of the degree by the chord of the remainder of the total declination, and the product gathered from this multiplication let him divide by the chord of the total declination; and the result of the division let him multiply by the half of the diameter, which was 60 [ששים, sixty] degrees. The sum gathered from this product let him divide by the chord of the remainder of the declination of the degree, and the result of the division — let him find its arc by the method through which arcs are known from chords. The arc that comes out is the sum of the degrees that advance along the equator in the passage that was reckoned for them, from the circle of the signs at the equator and the circle dividing [the day] for the equator, or in their crossing over the half of the heavens in all the inhabited places.

If the sum of the degrees reckoned for them is thirty, the arc that comes out in his hand will be the sum of the degrees of the passage of the sign Aries upon the equator and upon the circle of the half-day, which is the arc of the half-heaven. And if the degrees reckoned for them are 60 [ס׳], the arc that comes out in your hand will be the sum of the degrees reckoned for the passage of Aries and Taurus together; subtract from them the passage of Aries, and the passage of the sign Taurus will remain for you. You then extract the passages of Aries and Taurus together from the 90 [צ׳] which are the passages of 3 [ג׳] signs of degrees, and there will remain in your hand the passages of the sign Gemini. And from the passages of these three signs you will know the sum of the passages of all 12 [י״ב] signs, and their arrangement. For the passages of the degrees of Aries, which are in the [forward] order, are the passages of the degrees of Libra in the [forward] order; and these [same values] are the passages of Virgo and Pisces in reverse. Likewise the passages of Taurus in the [forward] order are the passages of Scorpio, and these are the passages of Aquarius and Leo in reverse. And likewise the passages of Gemini in the [forward] order are the passages of Sagittarius, and these are the passages of Capricorn and Cancer in reverse — upon the equator and the arc of the half-heaven. According to this arrangement the passages of the signs in the right [sphere / direct ascension] (אופן הישר) were reckoned, and they were inscribed in the table of passages beginning from the head of the sign Capricorn, so that they might be ready and prepared to know from them the portion of the half-heaven — that is, the portion that rises upon the earth in all the inhabited world.

One who wishes to know the passages of the signs in the right sphere from the tables will know the sum of the degrees from the sign reckoned for him, and look them up in the tables of passage in the number-column of that sign which is written at its head; and he will find opposite them the passages counted from the head of the sign Capricorn up to the passage of the signs he is seeking. And if he has in his hand passages reckoned upon the right sphere, and he wishes to know from which sign the degrees of those passages are, let him bring those passages to the columns of passage that are in the table, and find opposite them, in the number-columns, the name of the sign for which those passages are reckoned,

and the total of the number of degrees from that sign. And if, together with the passages of the complete degrees that he brought to this table, there are fractions (minutes) that have not completed a degree, let him correct his portion; and it is fitting for them [to be derived] from the excess found between the passages of the complete degree he has in hand and the passages of the degree that follows after it — as was explained to you in the preceding gate, in the reckoning of arcs and chords.

Gate Five — On Computing the Ascensions of the Zodiacal Signs on the Dividing Circle [the equator] in the Remaining Inhabited Latitudes

Gate Five: on computing the ascensions of the zodiacal signs upon the dividing circle [equator] in the remaining inhabited latitudes that lie toward the north; and the method of computing them by means of the calculated chords, and their computation from the tables drawn up for them, from the first climate up to the seventh [climate]; and how one may know from the tables the lengths of day and night and the fractions (the parts) of their hours, and all that depends upon this matter.

It was already explained, in Gate Four, how to derive the ascensions of the zodiacal signs upon the equator and upon the arc of the half-heaven [the meridian] in all the inhabited regions. In this fifth gate we now explain the method for the ascensions of the zodiacal signs upon the remaining dividing circles [oblique horizons] of all the inhabited regions that lie toward the north of the equator.

I say that the ascensions of these signs differ from their ascension upon the equator. For some of them add to the ascension upon the equator, and some subtract from it. Every individual sign that adds to its ascension upon the oblique sphere, beyond its ascension upon the equator, will have its opposite sign — the seventh [sign] from it — subtract upon the oblique sphere from its ascension upon the equator, by the same amount that the opposite sign added over them [over those degrees]. Consequently, the ascension of each individual sign as it rises upon the oblique sphere, over the earth in the east, will equal the ascension of the sign opposite it — the seventh from it — at its setting beneath the earth in the west.

These ascensions are known from the chords of the degrees of the latitude of the sphere, which is the ratio of its equinoctial [equal] distance, or from the chord of the increase of the longest day over the equinoctial day at that place. I will explain to you the procedure for computing the latitude and the day-increase in Gate Six. In this fifth gate, I place first the procedure for computing the ascensions of the zodiacal signs upon the oblique spheres, since it is fitting to arrange it after computing their ascensions upon the equator, which was explained in Gate Four.

The procedure for computing the ascension on the oblique spheres. One who wishes to know the ascensions of the zodiacal signs on the oblique spheres should seek out, with regard to that particular sign — or those particular degrees whose ascensions he is seeking — how much their ascension was upon the equator, from the beginning of the head of Aries [Taleh] up to the marching-point [the position] of that degree. He should already know the chord of the degrees of that ascension; let him double it by the chord of half the day-increase of the longest day over the equinoctial day at that place. The product that is gathered from the doubling, let him divide by half the diameter. From the division what comes out

— let him know its arc; and that arc will be the measure of the increase of the longest day over the equinoctial day at that place.

You can also know the measure of the increase by another method, namely: take the double of the chord of the latitude of that place by the chord of the declination of the degree. The product that is gathered from the doubling, divide it by the chord of the remainder of the breadth — which is the chord of the excess between 90 [צ׳] degrees and the latitude of that place. What comes out from the division, double it by half the diameter, and the product gathered from the doubling, divide by the chord of the remainder of the declination of the degree — which is the chord of the excess between 90 [צ׳] degrees and the declination of the degree. From the division, what comes out — know its arc; and that arc will be the measure of that increase at that degree. The two procedures come out, in the computation, to one and the same result.

And when you know the measure of the increase by one of these two methods, you subtract that measure due to those degrees from the number of their ascensions upon the equator, from the beginning of the sign of Aries [Taleh] up to that degree. What remains in your hand from the ascensions, after this subtraction, will be the number of the ascensions from the beginning of Aries up to that degree upon the oblique sphere that you are computing. Know that the ascensions of Pisces [Dagim] equal the ascensions of Aries [Taleh], and the ascensions of Aquarius [Deli] equal the ascensions of Taurus [Shor]; likewise the ascensions of Capricorn [Gedi] equal the ascensions of Gemini [Te'omim], and likewise the ascensions of Virgo [Betulah] equal the ascensions of Libra [Moznayim], and the ascensions of Leo [Aryeh] equal the ascensions of Scorpio [Akrav], and the ascensions of Cancer [Sartan] equal the ascensions of Sagittarius [Keshet]. From this you see that it suffices for you to derive the increase-measures for 90 [צ׳] degrees of any oblique sphere you are computing; and out of the measures of the 90 [הצ׳] degrees you will know the day [the ascensions of all the degrees], the ascensions of the degrees of all twelve [י״ב] signs on that sphere.

And this is the method of computing it: first compute the day-increase measures for one degree upon that sphere, and likewise for two degrees, and likewise for three degrees and for four, and so on up to 90 [צ׳] degrees, so that the measures of the variation of the day-increase will be preserved for you. Then take the ascension of one degree from the sign of Aries [Taleh]

upon the right sphere, and write it down in two places. Make it less, at the one place, by the increase-measure for one degree, and add it onto the second place. What remains of the ascensions after the subtraction will be the ascension of one degree from Aries; expand it out from [the full] 360 [מש״ס] degrees, and what remains will be the ascension of 359 [שנ״ט] degrees from the beginning of the sign. [That is, the ascension reckoned from the head of Aries up to 29 (כ״ט) degrees in the sign of Pisces (Dagim).] So too you continue adding the increase-measure for one degree, up to the ascension of one degree from Aries [Taleh] up to 29 [כ״ט] degrees from the sign of Pisces [Dagim]. The degree that you set down in the second place yields in your hand the ascension of one degree from the sign of Libra [Moznayim]; add it onto 180 [ק״פ], and you thereby obtain [find] the ascensions of the [great] circle [galgal] from the head of the sign of Aries up to the first degree from the sign of Libra [Moznayim]. Subtract it from 180 [ק״פ], and what remains will be the ascensions of the [great] circle from the head of the sign of Aries up to 29 [כ״ט] degrees from the sign of Virgo [Betulah]. And so you find, known from the increase-measure for the one degree, that you have computed the ascensions of four degrees — namely the first degree from Aries [Taleh] and from Libra [Moznayim], and the last degree from Pisces [Dagim] and from Virgo [Betulah].

And if you proceed in this way with the increase-measure for two degrees, and for three and for four, and so on up to 90 [צ׳] degrees, you will know, from the measure of each degree, the ascensions of four degrees; and at 90 [ולצ׳] degrees there will be completed for you the ascensions of [the full] 360 [ש״ס] degrees, which are the degrees of the [great] circle [galgal].

We have engraved [recorded], in the tables marked out in this book, the ascensions of the zodiacal signs upon the right sphere, arranged degree by degree, so that they may be suitable for computing by their means the ascensions of the signs upon the oblique spheres, degree by degree, for anyone who wishes to compute.

We have engraved [recorded] the ascensions of the zodiacal signs upon the oblique spheres arranged in intervals of ten and ten degrees, because the variation found between the ascensions, between [each] ten degrees and the ten degrees that follow after it, is not great enough to do any harm — there is, in their computation, [only] a slight discrepancy [of] something it [need not] worry about. We arranged the tables for the oblique spheres in [terms of] the length of day [as it grows] in half-hour and half-hour [increments], from the equator up to the oblique sphere whose longest day is 18 [י״ח] hours. From this sphere onward [northward], they grow by [whole] hour and hour [increments], up to a day of 24 [כ״ד] hours, because of the shortness of the arc between these spheres, owing to their proximity to the [celestial] pole, and because the variation between their ascensions has no consequence great enough for one hour [to register]. In these tables we have transmitted the ascensions of the zodiacal signs in all of the inhabited region, from the equator up to a latitude of 66 [ס״ו] degrees from it toward the north, which is the end of the inhabited region.

And one who wishes to know the chords of the ascensions of the zodiacal signs from these tables should seek out, with regard to the degree…

[The text continues; the procedure for reading the chord-values from the tables breaks off at the foot of the page.]

If one wishes to know the ascensions of a degree from these tables, by the number of degrees that that degree holds within its sign, he should seek it out in the ascension-table of the sphere in which it stands — whether the right sphere or another sphere. After he has determined what corresponds to that degree in the ascension-columns of that sphere, then: if your inquiry was made in the tables of ascension on the right sphere, that number will be the sum of the ascensions on the right sphere from the head of the sign of Capricorn (גדי) up to that degree. But if your inquiry was made in the tables of the oblique spheres, the number you found set against the degree will be the sum of the ascensions in that sphere from the head of the sign of Aries (טלה) up to that degree.

And if you have fractions that have not completed a degree, added on beyond the completed degree whose ascension you are seeking: know the ratio of those fractions to sixty (ששים). Take, from the excess between the ascensions of the completed degree and the ascensions of the completed degree that follows it, by the ratio of those fractions to sixty, and add that ratio onto the ascensions of the completed degree. Then you will have found the ascensions of that degree together with the fractions belonging to it.

This procedure holds for every one of the tables arranged degree-by-degree — such as the ascension-table on the equator (אופן המישור), and every table resembling it that is arranged degree-by-degree. But if your inquiry is in tables arranged in tens of degrees — such as the ascension-tables in the oblique spheres — and you have parts added on beyond the completed tens: determine the ratio of those parts to ten (עשרה), and you will take, by that ratio, from the excess between the ascensions of the completed ten in your hand and the ascensions of the ten that follows it; and you will add it onto the ascensions of the completed tens, so that the ascensions of the degrees you were investigating come up in your hand, at the proper time for them, in that sphere.

And if you wish, conversely, to know the degrees from their ascensions — this is called "correcting the arc of ascension" (תיקון קשת המצעד) — investigate, by the ascension-number you have, in the ascension-columns of the table of ascensions of the right sphere, or in the table of ascensions of the other spheres, in whichever of them you wish to investigate. When you find the number, or a number close to it that is slightly less than it, take what is found against it in the columns of degrees of that number — that is, the degrees of the signs — and that will be the number of ascending degrees corresponding to the given ascensions in your hand. And if anything remains in your hand of the given ascensions, because you did not find their equivalent — a number equal to them in the table: if the table set before you is arranged degree-by-degree, multiply that remainder by sixty (ששים); or if the table is arranged in tens of degrees, multiply it by six hundred (שש מאות). And from the product, the sum gathered: divide it by the excess that lies between the ascensions of the degrees [continues p. 28]

that you took and the degrees of the second sector adjacent before it. And the quotient of the division — add it onto the first degree that you took, and the degrees corrected to those ascensions will be delivered to you: whether for stepping them off on the equator (אופן המפריש), or for passing them across the meridian (חצי השמים).

And you are able to correct the remainder in this way by another method, which is this: take, from the excess between the two adjacent entries in your hand among the ascensions, by the ratio of the remainder — when you compare it to sixty, if you are computing in the tables arranged degree-by-degree, or comparing it to six hundred, if you are computing in the tables arranged ten by ten degrees. Then you will be adding that ratio onto the completed degrees that you took first.

And if you wish to know, from the ascension-tables, the arc of the day (קשת היום) and the arc of the night (קשת הלילה), at any time and any place you wish: this is the ratio of the stepping-arc of the sphere, from the time of the sun's rising over the earth in the east up to the time of its setting from over the earth in the west — that is the arc of the day; or from the time of its setting in the west up to the time of its rising over the earth — that is the arc of the night. Know the degree in which the sun (החמה) stands, within the sign in which it stands on that day; and seek out that degree in the ascension-table in which the hours of the long day are equal to the hours of the place you are in, or the table closest to it. Take what you find set against that degree among the ascensions in that table, and subtract from it the ascension of the degree before that one in the seventh sign from the sun

(השביעי לחמה). The remainder of those ascensions, after this subtraction, is the arc of the day that you are investigating.

And if the ascensions before the sun's degree are greater in their count than the ascensions before the degree opposite it in the seventh sign — and you cannot subtract the greater from the lesser — then add 360 (ש״ס) onto the ascensions of the degree opposite it, and bring out the total from the ascensions of the sun's degree, and the remainder will be the arc of the day you are seeking. And if you bring out that arc of the day from 360 (מש״ס) — which are the degrees of the sphere — the remainder will be the arc of the night. Likewise, if you bring out the arc of the night from 360 (ש״ס), the remainder will be the arc of the day.

And you are able to know the arc of the day by another method, which is this: first know the ascensions of the degree in which the sun stands, in the ascension-table of the right sphere; and bring out from that ascension 90 (צ׳) degrees, and keep the remainder of it that exceeds 90 (צ׳). Then afterward, determine the ascensions of that degree in the oblique sphere you are in, and take the excess between the ascensions of the degree on the right sphere and its ascensions on the oblique sphere — that is, the rule of the difference between the days for that degree. If the ascensions of the right sphere are greater than the ascensions of the oblique sphere, add the excess onto 90 (צ׳); and if those ascensions are the deficient ones, bring them out from 90 (מצ׳). And the result in your hand, after the addition or the subtraction, will be half the arc of the day; double [continues p. 29]

it one time, and you will have the arc of the day. And if you subtract it from 360 (מש״ס), you obtain the arc of the night.

And by another method: first investigate the ascensions of the sun's degrees in the oblique sphere, and subtract from them, by their ratio, the ascensions of the degree on the right sphere as found in the table; and the result in your hand, after this subtraction, will be half of that arc of the day, and its double is the whole arc of the day. The matter is one in all these computations.

And when you wish to know the number of equal hours (שעות הישרות) of the day or the equal hours of the night: divide the arc of the day or the arc of the night — whichever of them you wish to know its equal hours — by 15 (ט״ו) degrees, which are the time-units of one equal hour. The quotient of the division is the number of equal hours for the time you are reckoning, whether day or night. Bring those out from 24 (כ״ד), and the remainder will be the number of equal hours of the time opposite it, whether day or night.

And if you wish to know the count of time-units of the seasonal hours (שעות העקלות) of the day or the seasonal hours of the night — these being the hours reckoned as 12 (י״ב) for each of them, day and night: divide the arc of the day or the arc of the night — whichever of them you wish to know its seasonal hours' time-units — by 12 (י״ב); and the quotient of the division is the time-units of one of the hours of the time you reckoned for it. Subtract those from thirty (משלושים), and the remainder will be the time-units of an hour of the time opposite it, the seasonal one. If you divided the arc of the day, the remainder will be the time-units of the seasonal hours of the night; and if you divided the arc of the night, the remainder will be the time-units of the seasonal hours of the day.

The root of this computation is this: the time-units of one seasonal hour by day, together with the time-units of one seasonal hour by the night joined to it, are the time-units of two equal hours. From this it follows that the excess of the time-units of one [seasonal] hour over the time-units of an equal hour is the deficiency of the second from the equal time-units.

And if you wish, conversely, to derive the time-units of the seasonal hour by another method: determine the rule of the difference of the days for the degree in which the sun stands, as I instructed you earlier in the computation of the arc of the day, and take a sixth of that mentioned rule of difference. If the degree in which the sun stands is in the six northern signs (ששת מזלות צפון), add that sixth onto 15 (ט״ו) degrees, and you will have the time-units of the seasonal hours of the day for that degree. And if the degree is in the six southern signs (ששת מזלות דרום), bring out that sixth from 15 (ט״ו) degrees, and the remainder will be the time-units of the seasonal hours of the day for that degree. Bring each one out from thirty (משלושים), and the remainder will be the time-units of the seasonal hour for the night opposite that day.

And if you wish to know the time-units of the hours of the day or of the hours of the night from the tables made for the passage [ascension] of the oblique spheres: take the degree in which the sun stands, or the degree you are seeking, to the tables of the oblique sphere in which you are; and take the value found opposite it. The number of the time-units of its daytime hours [is] in the tables of the oblique sphere in which you are — the time-units of the hour [together] with its parts and fractions; these will be the time-units of the hour for that day. And if you bring those time-units out from 30 degrees, there will remain for you the time-units of the hours of that night.

And if you wish to know the time-units of the hours of the night: take the value found opposite the degree of the sign which is the seventh from the position of the sun, or the seventh from the degree whose night hours' time-units you are seeking, and that will be their time-units. And if you subtract them from 30 degrees, there will remain the time-units of the hours of the day.

And if you desire to know the arc of the day or the arc of the night from their hours' time-units: multiply the time-units of whichever of the hours whose arc you wish to know by 12, and the sum gathered from the multiplication will be the arc of whichever of them you doubled their time-units — if [it was] day, [then] day; and if [it was] night, [then] night.

And if you wish to convert equal hours into seasonal: multiply the equal hours that you have by 15, and divide the product by the time-units of the seasonal hour that correspond to those parts, and you [have] the number of seasonal hours. And if your desire is to convert seasonal hours into equal: you multiply the seasonal hours by their time-units — if you are reckoning for the day, by the time-units of its hours, and if for the night, by the time-units of its hours — and you divide the sum gathered from the multiplication by the time-units of one equal [hour], which are 15 degrees; and the result of the division will be the number of equal hours corresponding to the seasonal hours that you multiplied.

The Sixth Gate: On the latitude of places inclining northward from the equator [קו השוה]; and how the latitude is known from the increase of the longer day or the deficiency of the shorter day; and how the breadth of degrees of the firmament [is known] at their rising upon the earth from the point of the eastern midheaven, upon which the passage [ascension] of the head of Aries [ראש טלה] passes; and [how] the head of Aries balances upon the earth, whether to the north or to the south; and the value of the height of every degree, [degree] by degree, above the [horizon] when it is at midheaven; and the degrees of the fixed stars [כוכבי שבת] at midheaven; and all that depends on this matter.

The latitude here mentioned is the value of the arc held upon the circle [אופן] inclined upon the poles [קטבי] of the [diurnal] motion, between the plane circle [אופן המישור] and the point opposite the zenith [נוכח נקודת הראש] at that place — and it is what is called the latitude of the place from the equator. This arc is equal to the arc between the northern pole of the [diurnal] motion and the dividing circle [אופן המפריש, the equator] at that place, and it is called the height of the pole [גובה הקוטב].

And you can know this arc from the value of the sun's height at midheaven; and you come to know this latitude in practice. And if you set yourself to know the value of the sun's height above the earth when it is at midheaven on whatever day chances to come for you, and you scrutinize the computation by every means you can scrutinize it by, so that there be no approximation [imprecision] in it — whether from the instruments made for this purpose, such as the quarter-circle [רביע העיגול] and the astrolabe [האצטרולב], or by the standing shadow [צל העומד]: when you know the height of the sun at midheaven, reckon the place of the sun as it appears to the inhabitants of the world, [namely] the sign in which it stands, as you will see from the computation of its place in this book. And know the declination [נמיכות] of the degree in which the sun stands, by the way set out in the Third Gate. If that declination is to the northern side, subtract it from the sun's height at midheaven on that day; and if the declination

is to the southern side, add it to the sun's height at midheaven on that day. And the result in your hand after this subtraction or that addition will be the height of the sun above the [horizon] at midheaven. When it is at the head of the sign Aries [ראש מזל טלה] or at the head of the sign Libra [ראש מזל מאזנים], subtract this height from 90 [90 = מצ׳] degrees, and what remains in your hand is the breadth of that place from the equator [מקו השוה] — and it is the value of the height of the northern pole above the earth at that place.

And you can know the height of the sun at midheaven on every day in all the inhabited places, if you know the height of the pole at that place — that is, that you know the declination of the degree in which the sun stands, or [of the degree] whose height of meridian-passage you are seeking. If that declination is to the northern side, you subtract it from the height of the pole; and if the declination is to the southern side, you add it to the height of the pole. And that which comes to be, the height of the pole after the addition or the subtraction — you subtract it from 90 [90 = מצ׳]; and what remains in your hand, from the 90 [90 = הצ׳], is the height of the pole for that degree at midheaven, which is the height of the sun at midheaven when it is at that degree.

And there are places in which it may chance that the value of the declination, for some of the degrees on the northern side, exceeds the value of the height of the pole at that place, and one will not be able to bring out the declination from the height. In that case you add 90 [90 = תשעים] to the value of the height, and you bring out from the total the declination of that degree; and what remains in your hand will be the height of that degree at midheaven, upon the dividing circle [the equator], to the northern side. This matter will chance to occur in places whose latitude [רחבן] from the northern equator reaches 24 [כ״ד] degrees, which is the whole of the declination [כל הנמיכות]. And you can compute it by another method: namely, that you subtract the height of the pole from 90 [90 = צ׳], and the remainder is the height of the degree at midheaven at the head of the sign Aries or at the head of the sign Libra; add to it the declination of the degree you are reckoning for — if the declination is to the northern side; or subtract from it, if the declination is to the southern side — and that will be the height of the degree at midheaven. And if it be the height of the head of the sign Aries, when you add to it the declination of the degree, exceeding 90 [90 = צ׳], bring it out from 180 [180 = מק״פ], and that remainder will be the height of the degree at midheaven above the dividing circle [the equator] to the northern side at that place.

And if you wish to know the number of the hours by which the longer day exceeds the equal day [היום השוה], or the number of the hours by which the shorter day falls short of the equal day, in all the places inclining from the equator [מקו השוה]

inclining toward the north, then find the altitude of the pole over the earth at that place. Draw out its chord, and likewise determine the chord of the full declination together with the chord of the declination's remainder when you subtract that [arc] from 90. Multiply the chord of the altitude of the pole by the chord of the full declination. Divide the product of this multiplication by half the diameter, and divide the result by the chord of the declination's remainder; the quotient of that division you then halve. So too, from the product, when you subtract that [arc], divide by the chord of the remainder of the altitude of the pole.

From this division find the arc, in the manner in which you find the arcs from chords. That arc will be half the excess of the longer day; and likewise it is half the deficit of the day shortfall from twelve equal hours. Double that number once and divide it by 15, and you will have in hand the number of hours of difference between the days and the nights. Add it to 12, and you will have the hours of the longer day at that place — that is, when the sun is at the head of the constellation Cancer. Likewise, if you subtract the number of hours of difference from 12, what remains for you will be the number of hours of the shorter day — that is, the day when the sun is at the head of the constellation Capricorn.

If you wish to know the number of the day's hours for any other of the degrees of the firmament, take the declination of that degree and proceed by doubling and dividing it according to the same procedure you followed for the declination at the head of the constellation Cancer and at the head of the constellation Capricorn, which is the full declination. By this operation you will find the total difference [in hours] for that degree in days. If the declination of the degree is toward the northern side, you add the difference to 12, and that gives you the hours of the day for that degree. If the declination of the degree is toward the southern side, you subtract the difference from 12, and there remain in hand the hours of the day for that degree. And in this entire operation, if you subtract the hours of the day from 24 hours, there remain for you the hours of the night that follows it. And once the difference between the days [and nights] is known and established for you, you have it as known and correct.

Now if you wish to know the eastern azimuth [breadth of rising] of the degrees at that place, take half the excess of the longer day and add it to 90; this will be half the arc of the longer day. Determine its chord and multiply it by the chord of the remainder of the declination — that is, when you subtract that [declination] from 90. Divide the product of this multiplication by 60, which is half the diameter, and from the result of the division find its arc; the arc that remains is the arc of the eastern azimuth of the head of the constellation Cancer from the point of the east of Aries on the dividing circle [the equator] at that place, toward the northern side. And it is itself also the eastern azimuth of the head of the constellation Capricorn from the east of Aries, toward the south. And if you carry out this operation

with half the arc of the shorter day, there will come up in hand for you the eastern azimuth of the rising of the head of the constellation Capricorn toward the south; and its equivalent is the eastern azimuth of the rising of the head of Cancer toward the north — there is no difference between the two.

If you wish to know the eastern azimuth of another degree from the remaining degrees of the constellations, take the declination of that degree and subtract it from 90; determine the chord of the remainder — this is the chord of the declination's remainder. Multiply it by the chord of half the arc of the day for that degree; divide the product of this multiplication by half the diameter; from the result of the division find its arc, and subtract that [arc] from 90. There remains in hand for you the arc of the eastern azimuth of the rising of that degree on the dividing circle. If the declination is toward the north, the azimuth is north of the point of the east of Aries; and if the declination is toward the south, the azimuth is south.

You are able to derive the eastern azimuth for any degree you wish. If you wish to know the eastern azimuth by another method, [proceed as follows:] if you know the altitude of the pole over the earth at that place and subtract it from 90, determine the chord of the remainder and keep it. Then, if you wish to derive the eastern azimuth of whatever degree you want, determine the declination of that degree and multiply it by half the diameter, which is sixty; divide the product of this multiplication by the chord of the remainder of the altitude of the pole, which you kept in hand; from the result of the division find its arc, and that will be the arc of the eastern azimuth of the rising of that degree. If the declination is toward the north, its rising turns toward the north from the point of the east of the head of Aries by the measure of that arc; and if the declination is toward the south, [its rising] turns toward the south.

Know that every chord I mention to you in this gate — indeed, in all the gates of this book — is the external chord [chord of arc]. If in one of the computations I have need of the full chord [diameter chord], wherever it is a full chord, I announce it to you by its name; but you, keep this rule firmly held in hand: that all the chords mentioned are external chords.

Now if you wish to know the position [stations] of the rest of the planets and the fixed stars when they are at mid-heaven; and to know the hours of the night from their altitude; and to know the order of their ascensions over the earth and the arc of their standing above the earth and below the earth; and to know the degree that rises with them and that which sets in their setting — together with all that depends upon these matters that are fitting to be expounded in this gate — I am coming to set them out, with the help of Heaven. And I say:

All the stars that lie upon the belt of the circle of the constellations [the ecliptic], inclining from it neither to the north nor to the south, follow in their revolution over the earth the course of the sun in all its respects. But those inclining from the belt of the circle of the constellations, whether to the north or to the south, do not follow its course, on account of the difference found between

their distance from the circle of the level [the equator] and the declination of the degree upon which they lie, which causes them to come to mid-heaven sometimes together with the degree on which they lie, sometimes before it, and sometimes after it. The same rule [holds] for their rising over the earth and for their setting beneath it.

We first explain how you may know their distance from the circle of the level. The one who wishes to compute this distance must know the value of the star's latitude [breadth] from the circle of the constellations, by means of the latitude table in the Star Tables, and the side of the latitude; and the value of the declination of the degree upon which it lies, and the side of the declination. If the two of them come from the same side, join them together — that is the star's latitude on that side. But if they do not come from the same side, subtract the lesser from the greater, and what remains is the star's latitude on the side of the greater. With the result you obtain after the addition or the subtraction, which is the corrected latitude for that star, determine its chord and multiply it by the chord of the remainder of the full declination; divide the product of this multiplication by the chord of the remainder of the declination of the degree upon which the star lies, and from the result of the division find its arc — that will be the arc of the star's distance from the circle of the level.

If you wish to know the degree that crosses mid-heaven together with the star, first determine the star's distance from the head of the constellation Capricorn or from the head of the constellation Cancer, whether before them or after them, and reckon it according to this procedure: determine the ascension [passage] of the degree upon which the star lies, from the circle of the constellations into the ascensions of the right sphere. If the ascensions are fewer than 90 degrees, the rule for them is that they are the star's distance from the head of Capricorn, before it. If they are from 270 up to 360, you subtract them from 360, and the remainder is its distance from the head of Capricorn, after it. If the ascensions are from 90 up to 180, subtract them from 180, and the remainder in hand is the star's distance from the head of the constellation Cancer, before it. If the ascensions are from 180 up to 270, subtract them from 270, and the remainder is the star's distance from the head of the constellation Cancer, after it.

When you know the distance, determine its chord and keep it. Then, when you come to know the degree that crosses mid-heaven with the star, double the chord of the star's latitude from the circle of the constellations by the chord of the full declination; divide the product of this multiplication by the chord of the remainder of the star's distance from the circle of the level — that is, when you subtract them [the relevant arc] from 90 — and the result of the division, multiply it by the chord of its distance from the circle of the constellations, which I instructed you to keep. That is its distance from the head of Capricorn or Cancer. Divide the product of this multiplication by half the diameter, and from the result of the division find its arc; that arc will be the value of the difference between the degree on which the star lies and the degree with which it always crosses mid-heaven — in every place, whether before it or after it.

and you will be able to compute the value of this difference by a second method, which is the method that Ptolemy explained in his book. Take the chord of the remaining declination, double it, and from the product subtract the chord of 90 [degrees]. When you bring out the star's distance from the equator by this same procedure, multiply the gathered result by that portion and the result of the multiplication and gathering [is divided] by half the diameter; and from the result of the division find its arc. That arc will be the value of the difference between the place of the star and the degree with which it crosses mid-heaven.

If the star lies in the arc from the head of Cancer to the end of [its] sign, and its latitude inclines toward the north, add the value of the difference to the ascensions of the star's house in the right circle [equator]; and if its latitude is toward the south, you subtract the arc of the difference from the ascensions of the house in the right circle. And if the star lies from the head of Capricorn to the end of Gemini, reverse the operation: if the star spreads toward the north, you subtract the arc of the difference from the ascensions of the house in the right circle, and if it spreads toward the south, you add the arc of the difference to the ascensions of the house. What comes up to you from the ascensions after this addition or subtraction is the ascensions of the degree with which that star crosses mid-heaven. Bring those ascensions to the tables of ascension in the right circle, and you will find opposite them the degree that crosses the arc of mid-heaven at the star's crossing. That [degree] is the same for it in all inhabited places; there is no difference at all.

There are two methods, which I have mentioned, in the reckoning of the difference, that lead to one number, or nearly one — both [arising] from one [principle], and the more correct of them is the method of Ptolemy.

These matters concern the arrangement of the planets and the fixed stars, which are hidden from the [computation of the] circle of the zodiacal signs. In their motion and their station in the firmament they do not differ in all inhabited places. But there are matters in which they do differ in their arrangement, for each and every star, according to the inhabited places: namely, their altitude above the earth when they are at mid-heaven does not follow a single rule in all inhabited places.

If you reckon a circle at the equator, you may know the arc of the star's distance from the equator [in the manner I have informed you] readily, and subtract that from 90 [degrees]; what remains in your hand, less than 90, is the altitude of that star at mid-heaven for the dwellers of the equator. If the arc of distance is toward the north, the altitude will be toward the north, opposite their zenith; and if the arc of distance is toward the south, the altitude will be toward the south, opposite their zenith.

And if you seek the value of the altitude of these stars at mid-heaven in the rest of the places that incline from the equator toward the north, come to the arc of the star's distance from the equator. If the arc is toward the south, subtract it from the altitude of the head of the sign Aries and the head of the sign Libra — which are the points of equity [equinoctial points] when they are at mid-heaven in that place — and the remainder, after this subtraction, from the altitude of the points of equity, will be the altitude of that star at mid-heaven in that place, toward the south, opposite their zenith. And if the arc of this distance is added to the value of the altitude of the points of equity at mid-heaven in that place — and you cannot bring out the greater from the smaller number — it will be clear to you that this star always circles beneath the earth in that place, and does not come out from beneath it, and is never seen there in any way. And if the arc of the star's distance is toward the north, you add that arc to the altitude of the head of the sign Aries and the head of the sign Libra in that place. If the number gathered from them does not reach 90 degrees, that number is the value of the altitude of the star at mid-heaven in that place, toward the south, opposite the zenith of its dwellers. And if the number gathered from them is exactly 90 degrees, neither less nor more, the star passes over the zenith point of the people of that place when it crosses the arc of mid-heaven. And if the number gathered from them exceeds 90, take it out of 180 [ק״פ], because the circle from its beginning to its end is 180; and the remainder in your hand, if it exceeds twice the altitude of the pole in that place, is the value of the altitude of the star at mid-heaven in that place, toward the north, opposite their zenith, and above the point of the pole of the northern course. And if the remainder from 180 equals twice the altitude of the pole in that place, or is less than it, it will be clear to you that this star always stands above the [horizon]-circle and never comes beneath it in any way; and these stars, which have no setting beneath the [horizon]-circle, are seen twice a day in the arc of mid-heaven in all the places where they are always visible: the one time they are seen above the pole, and the second time they are seen beneath [the pole]. And when you investigate their altitude above the pole and join it to their altitude below the pole, there comes up in your hand twice the altitude of the pole in that place; divide that in two, and you obtain the altitude of the pole in that place. Among the matters that differ for the fixed stars according to the variation of places on the earth is the manner of their rising above the [horizon]-circle and their setting beneath it.

And you will see from our words that all the planets and the fixed stars that lie upon the circle of the zodiacal signs itself and incline from the circle, whether toward the north or toward the south — if you investigate the order of their travel along the equator, you will find that all of them rise above the earth and set beneath the earth, and the measure of their rising above it is as the measure of their setting beneath it; no measure is added to the second one or subtracted from it. But if you investigate the order of their revolution along the circles that incline from the equator, whether toward the north or toward the south, you do not find the measure of their rising above the [horizon]-circle equal to the measure of their setting beneath it. Rather, they divide into three classes. There are stars that are not hidden beneath the earth all the days of the world in some of the places, and there are stars among them that are not seen above the earth all the days of the world in some of the places. As for these two classes of stars, there is no need to concern oneself with them when you are an investigator of the measure of their rising or the measure of their setting. The third class is the stars that are seen above the earth and set beneath the earth in most of the inhabited places; and concerning these stars it is fitting for us to investigate the measure of their appearance and the measure of their concealment in those places. If these stars lie upon the curve of the circle of the zodiacal signs itself, not inclining from it either toward the north or toward the south, the measure of their rising above the [horizon]-circle will be as the measure of the day-arc of the degree on which they lie above the equator, and the measure of their setting beneath it as the measure of the night-arc of that degree. And you have already learned the procedure for the reckoning of the day-arc and the night-arc for every degree of the zodiacal signs; and likewise you proceed in reckoning the value of the standing of these stars above the [horizon]-circle and the value of their concealment beneath it.

And if these stars spread out from the circle of the zodiacal signs, whether toward the north or toward the south, you proceed in reckoning their rising and their setting according to this procedure that I explain to you: Double the chord of the arc of the star's distance from the equator by the chord of the altitude of the pole in that place, and the result gathered from the product divide by the rest of the arc of the star's distance from the equator — when you bring it out of 90 [degrees] — and double the result of the division by half the diameter. The result gathered from the product, divide it by the chord of the remainder of the altitude of the pole — when you bring it out of 90 [degrees] — and the result of the division make into an arc, that is, find its arc. That arc will be the value of the difference between 90 [degrees] and the half of the star's standing above the [horizon]-circle. If the star's distance from the equator is toward the north, you add the value of the difference to 90 degrees, and what is joined from them will be the half of the arc with which that star encircles above the earth, from the time of its rising over it until it reaches mid-heaven.

And if the star's distance from the equator is toward the south, subtract the value of the difference from 90 degrees, and what remains from them will be the half of the said arc. And when you know this half-arc, which marks the measure of the half of the star's standing above the earth, double it, and you will have the arc of its standing above the [horizon]-circle from its rising in the east until its setting in the west. Take out this arc...[continues p.39]

from 360, and the remainder will be the arc of that star's standing beneath the earth, from the moment of its setting in the west until its rising in the east.

Now if you wish to know, from this arc, which degree of the ecliptic this star rises with in the east, and which degree it sets with in the west, proceed as follows. Determine how many ascensions in the right sphere [equator] correspond to the ascensions of the degree through which that star passes across mid-heaven in its course. From these subtract half the arc of standing above the earth — that is, from the star's rising until it reaches mid-heaven. Take the ascensions that remain after this subtraction to the tables of ascensions for the clime in which you are reckoning; the degree of the ecliptic that corresponds to those ascensions is the degree that rises on the horizon at that star's rising. Likewise, add half the arc of its standing above the horizon to the ascensions of the degrees through which it passed across mid-heaven in the right sphere. To those ascensions, after the addition, take them to the tables of ascensions for the clime; the degree of the ecliptic that corresponds to them is the degree that rises on the horizon when that star sets. And the degree opposite it, in the seventh sign, is the degree that sets in the west at its setting.

And if you wish to know the altitude of the star above the earth — toward the eastern side or toward the western side — at whatever time happens for you, and from that altitude you wish to know the degree rising over the earth at that time, proceed as follows. First determine the chord drawn [computed] for the half-arc by which the star stands above the earth; you have already learned the reckoning of drawn chords in the second gate of this book. Multiply that drawn chord by the right [perpendicular] chord that corresponds to the altitude of that star which you found — whether toward the east or toward the west. Divide this product by the chord of the star's altitude at mid-heaven; subtract the quotient from the drawn chord that corresponds to the half-arc of standing. From the remainder of the drawn chord that corresponds to the half-arc of standing, after this subtraction, you will know its drawn arc — for it is the arc taken from the orb.

If the altitude you found for that star is toward the eastern side, you subtract this resulting arc from the ascensions of the right sphere belonging to the degree through which that star passes across mid-heaven; the remainder will be the ascensions proper to the altitude that you found. But if the altitude found is toward the western side, you take the resulting arc upon the ascensions

of the degree that passes with the star across mid-heaven; the sum of these will be the ascensions proper to the altitude that you found. Take those ascensions to the tables of ascensions for the clime in which you are reckoning, and you will find, corresponding to them, the degree rising over the earth at that time. Likewise you take them to the tables of ascensions in the right sphere, and you will find, corresponding to them, the degree standing at mid-heaven at that time.

All these computations suffice you for the fixed stars and the planets, with the sole exception of the Moon. For with the Moon you must add to these computations the reckoning of the variation [parallax] in the Moon's place — in latitude, in longitude, and between what is seen by the inhabitants of the world and its true place — as will be explained to you in the thirteenth gate, God willing.

[Gate Seven]

The seventh gate, on the exposition of the shadow found from the gnomon standing erect upon the earth, called the cast [horizontal] shadow; and the shadow of the line inclined upon another line, called the prone [vertical] shadow; and how each one of these two shadows is known from the altitude; and how the altitude is known from them by computation and from the tables made for them; and how the hours of the day are known from the altitude; and all that depends on this matter.

Know that the instruments which the practitioners of this science use in the reckoning of the shadow are two instruments. The one they set upon the earth at a right angle, and upon this line — called the cast shadow — [they measure]; and the second is a line inclined upon an erect pillar, such as a peg that is fixed at its top, and they call its shadow the prone shadow. The shadow of each one of these two lines they divided into twelve parts [and] called [them] *patzlot* [units/digits]. They assigned to them digits, so as to gauge from them the divisions of the shadow that exceed the length of the pillar, or [fall] short of it; and the subdivisions of the digit into minutes, and minutes of minutes in their operations, are like the degrees of the orb.

Now if you know the altitude of the sun and you seek to know the cast shadow proper to that altitude, proceed as follows. Determine the chord of the altitude, and also the chord of the remainder of the altitude when you subtract it from 90. Take the chord of the remainder of the altitude — which is its sine — and multiply by 12, which are the digits of the length of the pillar; divide the product by the chord of the altitude, and the quotient is the value of the cast shadow proper to that altitude.

And if you wish to know the prone shadow proper to that altitude, you reverse the procedure of the computation. Multiply the chord of the altitude by 12, which are the digits of the length of the pillar; divide the product by the chord of the remainder of the altitude, when you subtract it [the altitude] from 90; and the quotient is the value of the prone shadow proper to that altitude. And if you wish to know the shadow of the pillar called the prone shadow,

and you will be seeking to know the height [of the sun] that corresponds to that shadow. Square the digits of that shadow itself, and to that square add 144 — which is the square of 12, the length of the gnomon. Then from the sum of these two squares extract its root, as you learned in the First Gate from the reckoning of the roots of any number. Always divide the resulting root into 720, which is twice the length of the gnomon, taken into [i.e. multiplied by] the half of the diameter. The quotient of that division — take its arc, and that arc will be the height corresponding to that cast shadow.

If you wish to know the height from the prone shadow, then know that the method is the one you followed with the cast shadow: the height that comes out in your hand, take it out of 90, and what remains is the height of the sun that corresponds to that prone shadow.

And if you know the cast shadow and wish to know the prone shadow: divide 144, which is the square of the upright line [the gnomon], by the cast shadow that you know, and the quotient is the prone shadow corresponding to the height for which the upright [cast] shadow was found. And if you know the prone shadow and wish [to find the cast shadow]: divide 144, which is the square of the [gnomon] line, by the prone shadow, and the quotient is the upright [cast] shadow corresponding to the height for which the prone shadow was found.

This is the procedure for reckoning the shadow by means of the chords.

Now, I have inclined to reckon the shadow by means of tables ruled across in length over 90 rows — these being the full extent of the height above the horizon [lit. "above the earth"] — and I divided them into three columns. In the first of them I inscribed, along the whole length of the tables, the degrees of height from 1 to 90, and opposite to them, in the two remaining columns, I inscribed the digits of the shadow and their fractions [minutes] proper to each and every degree of the height.

If you wish to find the cast shadow or the prone shadow from the tables by way of the height: if you are seeking the cast shadow, enter with the height in your hand into the column of heights in these tables, take what is found opposite it in the two columns of the [cast] shadow, and take what is found opposite it in the two columns of the shadow's digits and their fractions — and those are the cast shadow for that height. And if the height has fractions [minutes] added on top of the [whole] degrees, you will adjust them by adjusting the fractions from the surplus between the two rows, just as you learned in the adjusting of the fractions of declination.

If you seek the prone shadow by way of the height: take that height out of 90, bring the remainder to the tables, and take what is found opposite it among the [entries for] the shadow — that is the prone shadow corresponding to the height given you.

And if you wish to know the height from these tables by way of the shadow given you: if that shadow is a cast shadow, you bring it to the tables, or to the columns of the shadow, and take what is found before it [opposite it] in the first column, upon which are inscribed the degrees of height. And if there are degrees

of height proper to that cast shadow [you have your answer]; but if the [given] shadow has, together with it, fractions [minutes] of digits added on, you must adjust [interpolate]: seek what is found in that row and subtract it from the row below it, and adjust the fractions. Look for a shadow that is less than your reckoning and one greater than it, and know how great the difference between the [two] shadows is; then divide by 60, and take the proportion of [your] distance over the difference, so that you may know how far you are from whichever of the two you wish. The proportion of the distance over [the difference of] the shadows — that is the fractions [minutes] of a degree — and there will rise in your hand the adjusted degrees of height proper to that cast shadow. And if that shadow is a prone shadow, you subtract the adjusted degrees of the shadow that come out from this whole operation from 90, and the remainder is the degrees of height proper to that prone shadow.

And now I seal for you this Gate, in the order of the reckonings that depend on the height of the sun found by the shadow.

If you know the height of the sun from the shadow, or from some other thing, and you wish to know the hours elapsed in that day: you must [first] know the beginning of the degree [of the zodiac] in which the sun is lodged, on the wheel of the zodiac on that day, and after it the height of the sun at midday [lit. at the middle of the heavens] in the clime in which you are inquiring, and the arc of the half-day [semidiurnal arc], as was explained in the previous Gate. Then take out the cut [sine] chord of the half-arc of the day, and the upright [versed-sine?] chord for the height of the sun at the middle of the heavens. When these two chords — the cut chord and the upright chord — are in keeping with you [available to you], you know the upright chord for the given height. Multiply it by the cut chord of the half-arc of the day, and divide the product [lit. the gathered total of the doubling/multiplication] by the upright chord for the height of the sun at the middle of the heavens. Subtract the quotient from the cut chord of the half-arc of the day, take the half of the upright arc for the half-arc of the day, and from the remainder make an arc by the method of extracting arcs from the cut chords. The arc that rises in your hand is the arc between the position of the sun at the given height on that day and the half-arc of the day. The remainder will be the arc that rises from the wheel of the equator, from the rising of the sun to that moment.

If the given height is on the western side, join that arc to the half-arc of the day; their sum will be the arc that rises from the wheel of the equator, from the rising of the sun to that moment. Divide this arc by the time-units of the hours [the variable length of the hour] on that day, and the quotient is the number of seasonal hours elapsed in that day. And if you divide that arc by 15, you will have the number of equal hours elapsed in that day.

If you wish to know the zodiac sign rising over the earth at that moment: come to the arc that rises from the wheel of the equator which you divided into hours, and add to it the ascensions of the degrees of

the sun in the clime in which you are inquiring, and the total that rises from both of them — bring it to the tables of ascension in that clime, take what is found before [opposite] them in the column of the number, and there will rise the number of the degrees ascending over the earth from the rising sign at that moment.

And if you bring that total to the table of ascension in the upright sphere [sphaera recta], you will find opposite it the number of degrees lodged upon the curve of the half-heavens, [reckoned] from the sign lodged upon it [the meridian] at that moment. And from these you can know the setting sign and the sign lodged at the [opposite] boundary, [the place of] the feet beneath the earth [the nadir/IMC], and the rest of the houses of the firmament, as you see explained in their place by this number [method].

You can also reckon this matter according to the practice of the people of India, and it is a reckoning lighter [simpler] than this one, except that there is in it a little approximation [imprecision]. It is as follows: you double the chord of the given height — in sixties, which are the half of the diameter — and divide the product by the chord of the height of the sun at the middle of the heavens. Make an arc of the quotient, take its arc, and keep it. If the sun is to the east, that arc is the value [proportion] of the arc rising over the earth from the wheel of the equator, from the rising of the sun to that moment. Divide it by the time-units of the hours at that moment, and you will have the seasonal hours elapsed from the rising of the sun to that moment.

And if the height of the sun is on the western side, you take these seasonal hours out of 12, and the remainder is the sum of the hours elapsed in the day from the rising of the sun to that moment. And if you come to know the rising sign from the hours: you double the hours by the time-units of one seasonal hour, and there rises in your hand the arc that ascends over the earth from the wheel of the equator from the rising of the sun to that moment; and proceed in extracting the rising sign from this arc in the practice you have already been following.

If you have known hours of the day and wish to know from them the height of the sun at that moment: you double those hours by the time-units of the seasonal hour, if they are seasonal, or by 15 if they are equal — at [the rate of] 15 degrees [per hour] — and the product is the arc that ascends from the wheel of the equator to that moment. If the arc is short of [less than] the half-arc of the day, subtract it from the half-arc of the day, and the remainder is the arc fit to be held [retained]. But if that arc exceeds the half-arc of the day, subtract from it the arc of the half-day, and the remainder is the arc fit to be held.

When you have found the arc fit to be held, know that this arc is the arc of the distance of the sun from the half of the heavens [the meridian]. Take the cut chord of the half-arc of the day, and out of it take the cut chord of the arc that is held; the remainder in your hand, multiply it by the chord of the height of the sun at the middle of the heavens, and divide the product by the cut chord of the half-arc of the day. And the quotient [continues p.46]

and the quotient that comes from this division—reckon it in the manner of extracting arcs from straight chords—and this will be the arc that is the measure of the sun's altitude at that moment, by the method that determines it for that moment. If the hours in question were before midday, the altitude will be on the eastern side; and if they fell after midday, the altitude will be on the western side.

Now if you know the rising sign for that day, and the degree in which the sun is lodged within the sphere of the zodiac, and you wish to know the hours that have elapsed of the day at that moment: you know the ascensions of the signs that rise in the clime you are in. Subtract from them the ascensions of the sun's degree in that clime, and what remains in your hand is the arc that has risen from the equator since the sun's rising up to that moment. Divide it by the duration of the seasonal hours of the day, or by the duration of an equal hour, and there will come up for you the hours that have elapsed of that day—the seasonal hours, if you divided the arc by the duration of a seasonal one; the equal hours, if you divided it by the duration of an equal one.

Now if at night you know the altitude of one of the stars among the planets, and you wish to know from that altitude the hours that have elapsed of the night: you know the rising sign from within that star's altitude, just as you saw in the gate that has passed; and from it you will be able to know the degree in which the sun is lodged within the sphere of the zodiac on that night. From that degree you know the degree opposite it, in the seventh sign from the sign of the sun's encampment, and that is the rising degree. Take the ascensions of that rising degree in the ascensions of the clime in which you stand, and subtract from those ascensions the ascensions of the degree opposite the sun's encampment in that clime. What remains will be the arc that has risen from the equator since the sun's setting up to that moment. If you divide this arc by 15 [ט״ו], you will have equal hours that have elapsed of the night; and if degrees remain for you that did not complete 15 [ט״ו], divide them by four, and the quotient that comes from the division is the fractions [minutes] of an equal hour exceeding the equal hours that have elapsed of the night. And if you divide that arc by the duration of a seasonal hour, the seasonal hours that have elapsed of that night will come up for you; and if degrees remain for you that did not complete the duration of a seasonal hour, multiply them by 60 [ס׳], and divide the sum from the multiplication by the duration of a seasonal hour on that night, and the quotient that comes from the division is the fractions [minutes] of a seasonal hour exceeding the complete seasonal hours that have elapsed of that night—at the reckoning of 60 [ס׳] fractions [minutes] for each and every hour.

Gate Eight — On the Number of the Days of the Solar Year and the Days of the Lunar Year

[Gate Eight: on the number of the days of the solar year and the days of the lunar year,] according to the view by which we reckon upon it the mean motions in this book; and the explanation of the foundation of the reckoning from which we begin to compute, and of the tables made for knowing its value at any time we wish; and how we are to know, from these tables, the remaining eras of the nations near to us, should we need them; and an explanation of the name of the era mentioned in this gate, which is hewn from the principle of "the long," as if one said "the beginning of the length of the days" by which they count—and its root is in the language of the Hagarites [Arabic].

We have already explained in the preceding gates every matter of how you must inquire, in your reckoning, into the courses of the stars, and so forth. From this gate onward we come to set out the order of the computation of the mean motions and all that depends upon it. And I say that the foundation of the mean motion for all the stars is built upon the mean motion of the sun and depends upon it. For this reason all the ancients used to set out, at the head of all their words, the number of the days of the solar year, in order to know the value of its course, and to correct and order upon that course the value of the course of the other planets. We rely, for the number of the days of the year, upon the view of Ptolemy [בטלמיוס], the chief of the sages of the astronomers, because his view in most of his words follows the view of our Rabbis, of blessed memory, and relies, in the reckoning of the days of the solar year, upon the words of Rav Adda bar Ahavah [רב אדא בר אהבה], upon which the secret of the intercalation [סוד העבור] is built. So too his view in the lunar course follows the aforementioned Rav Adda, which is the view of our Rabbis, of blessed memory.

We have found that all the nations are divided, in the counting of the days of their years, into these two divisions: some of them count for the sun, and some of them count for the moon. And because we have needed to set out the mean motions according to the number of the year that would be useful to all who count by these two divisions, we first set out an explanation of the custom of the nations in their reckoning.

We say that those who reckon by the count of the sun are divided into two parts. Most of them give its years one fixed number, equal in every year and in all the days of the world—these are the ancient Egyptians and Babylonians, who count for the solar year 365 [שס״ה] full days, no less and no more, all the days of the world. The second part do not count the days of their years by one fixed number every year; rather, they count three years one after another at 365 [שס״ה] days in each year, and they call each of these years a "common" year [שנה פשוטה]; and the fourth year at 366 [שס״ו] days, which they call an "intercalated" year [שנה כבושה]. These are most of the nations that count for the sun.

Likewise those who count for the moon's reckoning are divided into two parts, according to what is fitting to divide them. It falls to me to explain in this book the one who counts for the moon alone and is not concerned with the count of the sun, and who gives the number of the days of his years two values: in some years he counts 354 [שנ״ד], and calls each one of them a "common" year [שנה פשוטה]; and in some years he counts 355 [שנ״ה] days, and calls them "intercalated" years [שנים כבושות]—which are 11 [י״א] years in every 30 [ל׳] years. The second part of those who count for the moon does not rely upon the lunar year alone; rather, he counts some of his years for the moon and some of them for the sun, in order to bring agreement between sun and moon in the counting of his years. For this reason he counts his months in some of the years as 12 [י״ב] months and in some of them as 13 [י״ג] months. These are all the exiles of holy Israel, and all who follow their custom from the ancients of Kedar and the Hagarites [Arabs].

You find that all the nations of the world, prior to this division, do not order the days of their years upon one fixed order in all the years—except the ancient Egyptians alone. We need to correct the order of the reckoning of the courses of the stars by an order equal in days and years, so that the reckoning not become confused for those who engage in it. For this reason we have resolved to compute all the reckonings—counting the years of the nations upon them according to the view of the Egyptians, who count their years at 365 [שס״ה] full days, no less and no more—so as to order upon them the courses of the stars, that they may be sound for everyone who wishes to compute them over the years of all the nations. We set out the manner of this procedure, beginning with the lunar years counted according to our view, with the help of Heaven.

Know that all the exiles of Israel count their months for the moon and their years for the sun, and bring agreement between sun and moon every 19 [י״ט] years. The months of their year are thus found to be counted by two numbers: for a common year they count 12 [י״ב], and for an intercalated one 13 [י״ג]; and every common year of theirs—or an intercalated one—is counted by three forms: sometimes complete [שלימה], sometimes deficient [חסרה], and sometimes in order

[כסדר]. Our years are thus found to be counted by six counts. For this reason it was not proper to order the reckoning of the courses upon our years, and we would have had to reduce them to…

in the reckoning of the equalized order. And if we count all the years of the cycle — which are the nineteen years comprising the [19-year] cycle, leap years and common years in regular sequence — and we assign to each common year 354 days [שנ״ד] and to each leap year 384 days [שפ״ד], then the days of the nineteen years, each year in its proper order, will come to six thousand nine hundred thirty-six [6936 = ששת אלפים תתקל״ו] days.

But if you count nineteen Egyptian years and assign to each year 365 days [שס״ה], you will have six thousand nine hundred thirty-five [ששת אלפים תתקל״ה = 6935] days — one day less than the days of our common [19-year] cycle, just one day. And because the discrepancy between them is great, we arranged the entire reckoning of the courses of the stars upon the Egyptian years, and we arranged a method to convert our leap years properly, in keeping with the view of our Sages, of blessed memory, back to the Egyptian years, by this method.

The procedures for converting Hebrew years to the order of the Egyptian years

If common and leap years are given to you according to our reckoning, and you wish to know the Egyptian years equal to those years, then know how many complete years there are among them — both common and leap — that were in those years, and how many deficient years; and subtract the deficient years from the complete ones, for the complete years are always in surplus. What remains of them is the correction by which our intercalated years exceed those that are regular. Hold fast to it and keep it. Then proceed and divide the years given to you by nineteen [י״ט]. The whole number that results from the division will be cycles, each cycle being nineteen [י״ט]; and the remainder will be one year and one day. These years will then be Egyptian years, and the days will be equal — for the lunar cycles, by their very nature, were counted in regular order.

And if there remain to you, from the years that you divided by nineteen [י״ט], years that do not complete [a full cycle], then know how many of them are common years and how many are leap years. Take nineteen [י״ט] for each year, and assign 354 days [שנ״ד] to each common year and 384 days [שפ״ד] to each leap year. Add the whole together with the days that you had in surplus in the Egyptian cycles, and add to the sum the days of the correction that you kept — which is the surplus between the complete and deficient years that I instructed you to hold fast to. Then take what you hold and divide it by 365 [שס״ה] days. What results from the division will be Egyptian years; add them to the Egyptian years already included for you.

And if there remain to you days that do not complete 365 [שס״ה] days, divide them by 30 [ל׳] days. What results from the division will be months, [reckoned] by the Egyptian count, added onto the years. And if there remain to you days that do not complete thirty [שלושים], these will be days counted from the incoming month. So there will come up in your hand years, months, and days, by the Egyptian count, equal to the days of the Hebrew years given to you.

And because this calculation is long and, for the most part, difficult, I have prepared for you two tables to calculate it, [taken] from the beginning of cycle 257 [רנ״ז] onward. The first table is divided across its width into five columns, with motion along its length according to whatever number you wish. We have arranged upon the first column — which is across the width — the cycles from the creation of the world, from the beginning of cycle 257 [רנ״ז] onward, along the length of the table. In the second column, and the one before it, the years from the creation of the world up to the end of each and every cycle are inscribed, by length. And in the three remaining columns of the width, the Egyptian years and the months are inscribed, their days equal to the days of those cycles, from the beginning of cycle 257 [רנ״ז] up to those cycles, properly corrected.

And when you have in hand complete cycles that exceed the 257 [רנ״ז] cycles from the creation of the world, enter with those cycles into this table, and you will find opposite them the corresponding number — years, months, and days — by the Egyptian count. You need not make any correction at all, because the surplus over cycle 257 [רנ״ז] does not [require one].

To enable [the reckoning of] complete cycles at any time, I have prepared one table divided across its width into four columns, and along its length into nineteen [י״ט], by the count of the years of the lunar cycle. In the first column I inscribed, along the length of the table, the number of the years of the lunar cycle from 1 to 19 [מא׳ עד י״ט]; and in the three remaining columns I inscribed the corresponding number for each and every year, in terms of the days of the Egyptian years, the months, and the days. These are the years of the cycle reckoned in regular order. And when nineteen [י״ט] is not completed, [for] the complete cycles in surplus, bring them into this table, and you will find opposite them the years, the months, and the days equal to the Egyptian reckoning.

Then return to those Hebrew years, and know how many complete years and how many deficient years there are among them. If they come out equal to one another, you need not, in the reckoning, add to or subtract from what comes out of the table; and that will be the corrected number. But if one of them exceeds the other, know to which kind of years the surplus belongs: if it belongs to the complete years, add it onto the reckoning that comes out of the table; and if the surplus belongs to the deficient years, subtract it from that reckoning. What you obtain after this addition or subtraction is the corrected number, [reckoned] from the days of the Egyptians, equal to the years that exceed the complete cycle given to you. Add this corrected number to the corrected number that you found in the first table opposite the complete cycles given to you, and with the number gathered from them you will be able to calculate the courses of the stars for that time.

The first table was inscribed from the beginning of cycle 257 [רנ״ז] because the foundation of the arranged calculations…

in this book, and their starting point, is the position of the stars at their mean motion at the middle of the heavens [meridian] on the fifth day [Thursday], which is the beginning of the first year of cycle 257 [רנ״ז], that being the year 4865 [ד׳ אלפים תתס״ה = 4000 + (ת400 + ת400 + ס60 + ה5 = 865) = 4865 AM] from the creation of the world, according to the view of those who reckon from BaHaRaD [בהר״ד — the molad of creation]. From that day onward the course of the stars was arranged in equalized form in these tables, and the root inscribed at the head of the motion for each of the stars is the position of that star on the fifth day [Thursday], mentioned [as occurring] at the middle of the day, by the meridian of Jerusalem, the holy city — which is distant from the line of the eastern horizon by, say, seven hours [שבע שעות].

And because most of the nations do not count from the creation of the world, nor by the lunar count as we reckon, but each person's mind inclines to reckon by the count of his own days and years — since it is by these that he reckons all his affairs in this world — we therefore saw fit to set out and explain the reckoning of the nations who reckon by the moon and the sun mentioned above, and to make known how their reckoning may be brought into accord with our own reckoning, by which we reckon from the creation of the world; and to bring the secret of our reckoning back into accord with their reckoning, so that we may be able to set right the arranged reckoning of the course of the stars for the days of each and every nation. With the help of God.

The basis of the reckoning of the Arabs and of all the kingdom of Qedar

Know that all the kingdom of Qedar, in these days, reckon their years from the beginning of the reign of the man for whom the Creator established the religious law [נימוס] which they hold fast. The beginning of his reign was in the month of Av of the year 4382 [= (382 = 2ב + 80פ + 300ש) + 4000 = ד׳ אלפים שפ״ב 4382 AM] from the creation of the world, according to the reckoning by which we count from BaHaRaD [בהר״ד]. They begin to count from the fifth day [Thursday], which is the second day of the said month of Av, and they count their months from that day onward: one [month] full and one deficient, [the deficient] being short [of full], at 29 days [מכ״ט]. But in some of their years they add one full month, and they call that year an intercalated year [שנה כבושה], because they count for the lunar year 354 [שנ״ד] days, plus a fifth and a sixth [of a day], which are 11 parts [ḥalakim] of 30 [parts] in a day [י״א חלק מל׳ חלק ביום]. Every year in which this surplus comes to 16 [י״ו], or more than 16 up to 27 parts [מי״ו... עד כ״ז חלקים], they reckon [it] for that year, and they make in it 7 [ז׳] full months and 5 [ה׳] deficient months, and they call that year an intercalated year [כבושה]. These [intercalated] years are these: 11 [י״א] in every 30 [ל׳] years, and they are the 2nd year, the 5th, the 7th, the 10th, the 13th, the 16th, the 18th, the 21st, the 24th, the 26th, and the 29th. Their signs: in the first ten years, four intercalated, [marked by] GU"Ḥ [8ח ,6ו ,3ג = גו״ח]; in the middle ten, three intercalated, [marked by] BeHaZ"Y [10י ,7ז ,5ה ,2ב = בהז״י]; and in the last ten, four intercalated, [marked by] ADU"Ṭ [9ט ,6ו ,4ד ,1א = אדו״ט].

And these are the names of their months: the first, al-Muḥarram [אלמחרם], is always reckoned as 30 [ל׳] days; the second, Ṣafar [צפר], is reckoned as 29 [כ״ט] days; and the third [is] Rabiʿ al-Awwal [רביע הראשון]...

[the second is Ṣafar, 29 days]; the third [is] Rabiʿ al-Awwal, 30 days; the fourth [is] Rabiʿ al-Thānī [רביע השני], 29 days; the fifth [is] Jumādā al-Ūlā [גומידי ראשון], 30 days; the sixth [is] Jumādā al-Thāniya [גומידי שני], 29 days; the seventh [is] Rajab [רגב], 30 days; the eighth [is] Shaʿbān [שעבאן], 29 days; the ninth [is] Ramaḍān [רמדאן], 30 days; the tenth [is] Shawwāl [שואל], 29 days; the eleventh [is] Dhū al-Qaʿda [דו עשר אלקעדה], 30 days; the twelfth [is] Dhū al-Ḥijja [דו עשר אלחגה], 29 days in a common year and 30 days in an intercalated year.

All the days of [each] month are reckoned according to this same arrangement: the full month is always full and the deficient is always deficient, and a [given] month never changes. This is the celebrated reckoning by which they compute the dates of their documents and books and the years of their kings; by it they reckon the days of their festivals. It is a single reckoning, dependent on the sighting of the moon, and they follow it. We have no need to concern ourselves with that reckoning in this book.

Converting Anno Mundi years to Hijra years

Whoever wishes to know how many years of the Hijra [literally "the kingdom of Ishmael," מלכות ישמעאל] have elapsed out of the years given for the era of Creation — which exceed a whole number of 231-year cycles [רל״א מחזור] — should subtract from those years [units of] 231 [רל״א], and keep in hand the remainder of the cycles. For the years that did not complete a cycle, if among them there is an excess beyond the completed cycles, he takes, for each and every cycle, 7 months [ז׳ חדשים]; and for every intercalated year among the years that did not complete a cycle, he adds for it one extra month. He gathers the whole together and always adds to it 29 [כ״ט] — the [number of] months remaining out of the 231-year cycle at the beginning of the Hijra. From the months thus gathered he forms common years of 12 months [י״ב חדש] each per year; and the years collected from them he adds to the years remaining in the count from Creation after the extraction of the 231-cycles and what was gathered from them. These are the completed years, as they have elapsed for the Hijra in that year. If there remain in hand months that did not complete a year, those months belong to the year coming next for the Hijra.

Finding the weekday on which a given Hijra year begins

If you have [in hand] years of the Hijra, and your desire is to know the [weekday] of the year you have reached — or you are seeking to know on which of the days of the week its beginning falls — take the years that have elapsed for the Hijra in cycles of 30 years each [מחזורות מל׳ ל׳ שנה], and take for each and every cycle five [חמשה] of them. Gather them, add to the total four [ארבעה] in every case, and cast out the whole by sevens [ז׳ ז׳]; whatever remains in your hand, less than seven [פחות מז׳], is the epact [literally "foundation," יסוד] of the cycle you are seeking. Add to that foundation one day [יום אחד], and there will come up in your hand the weekday that is the first of the month of al-Muḥarram [ראש חדש אלמחרם] for the first year of the coming cycle.

And if there remain in your hand, out of the years that you extracted in [units of] 30, years that did not complete [a unit of] 30 [שנים שלא השלימו ל׳], know how many years they are. Take for each common year — of which there are 5 days [ה׳ ימים] in it — and for each intercalated year 5 [ה׳ days]. Gather the whole, add to it the foundation of the cycle you are in — which was extracted earlier — and the total in your hand. Cast it out by sevens [ז׳ ז׳], and the remainder in your hand, less than seven [פחות מז׳], is the foundation of the year you are seeking.

And if you wish to know the first of the month of al-Muḥarram, which is the head of the year of their reckoning [ראש לחדשי שנותם], add to the foundation of the year one day [יום אחד], and you will find the weekday on which the month of al-Muḥarram enters.

And if you should wish to know the head [i.e., first day] of one of the months of the year, find out how many months there are from al-Muḥarram up to that month, and let it not reach inclusivity [i.e., do not count the target month itself]. Take for each full month two days [שני ימים] and for each deficient month one day less [חסר יום]. Gather them, add to the total the foundation of that year, and always [add] one extra day [ויום אחד יותר]. Now bring [it] and gather it in your hand. Cast it out by sevens [ז׳ ז׳], and the remainder in your hand — whether seven or less than seven [הן ז׳ הן פחות מז׳] — is the weekday on which the month you are seeking enters.

Converting Hijra years to Anno Mundi

And if you have years of the Hijra and you wish to know how many years they correspond to for the era of Creation, double those years [כפול השנים ההם] by the number of months of the lunar year that is in use in the reckoning of the Hijra. The product gathered from this doubling will be the number of months of those years. From them subtract 29 months [כ״ט חדש], the [number of] months by which the reckoning of the Hijra exceeds at the head of the 232-cycle [רל״ב]. The months remaining from the doubling, divide them by 235 months [רל״ה חדש], which are the months of one cycle of the lunar [calendar] to which we reckon. What comes out of this division, of the cycles in our reckoning, add to 231 cycles [רל״א], and you will have complete cycles that have elapsed from the Creation of the world up to that year of the Hijra.

And if there remain in your hand, after the division, months that did not complete [a multiple of] 235 months [רל״ה חדש], divide them by the years of the [19-year] cycle, allotting for each common year 12 [months] [י״ב] and for each intercalated year 13 months [י״ג חדש], until the months remaining in your hand are used up. The years that come out of this division are the common and intercalated years that have elapsed of the cycle you are in.

And if there remain in your hand, after this division, months that did not complete a year, know that they are months that have elapsed of the year

of the Hebrews [שנת העברים] at the entry of the year of the Ishmaelites that you are seeking. Know too that the first day of the months of the Ishmaelites comes [to fall], for [the era of] the world, either on the day of a Hebrew new moon [ראש חדש לישראל], or a day before it, or a day after it — for there is no difference between them and Israel in the count of the lunar conjunction [מספר מולד הלבנה] except for one part [חלק אחד] only. For they reckon their months as 29 days, 12 hours, 792 parts [כ״ט יום י״ב שעה תשצ״ב חלקים], whereas we reckon

the month as 29 days, 12 hours, 793 parts [כ״ט יום י״ב שעה תשצ״ג חלקים]. From this you may say: he who knows the first day of an Ishmaelite month — whether from the arrangement explained above, or from the corrected table [הלוח המתוקן] for determining the head of the Ishmaelite month engraved in the tables inscribed in this book — and who knows the [Hebrew] month adjoining the Ishmaelite month from among the months of Israel, is able to know which day [of the Hebrew month] it is: whether it is its very head [ראשו ממש], or a day before it, or a day after it, by inferring that day against the segment of the [Hebrew] month [קטע החדש].

Computing planetary motions from Hijra years

Whoever wishes to compute the motions of the planets from this book, out of the given years of the Ishmaelites, [must] return those years to the order of "the world [-era]" [סדר עולם], as was explained above, and extract from the years that [run] from the Creation of the world the Egyptian years [השנים המצריות] that are fit to be brought into the tables of the reckoning of motion [לוחות חשבון המהלך] for the years given for the Hijra. You can determine this from the corrected table in this book, [namely] to convert the Arab years [השנים הערביות] into Egyptian years, [so as] to compute the motions of the planets by them, with the help of God [בע״ה].

Whoever wishes to compute Egyptian years from the Arab [years] by that table: it is proper that the Arab years given to him exceed by 496 [תצ״ו] the year of the Hijra, because from there onward the motions of the planets were arranged in this book. He should enter with those years into the table and seek them out: in the table of the combined years [לוח השנים המחברות], which are arranged in that table by [units of] 30 years each [על ל׳ ל׳ שנה], let him seek them in the first column, which is the column of the combined years. If he finds among them years equal to the number of [Hijra] years in his hand for the Hijra, let him take what is found opposite them from the Egyptian years, the months, and the days; these are the proper number by which to compute the motions of the planets for those Arab years.

But if he does not find among them years equal to those in his hand, he finds a number that is close to it but a little less than it, less than [a multiple of] 30 years [פחות מל׳ שנה]. He takes what is found opposite those years — the Egyptian years, months, and days from the Arab [count] — and keeps it. Afterward he brings the years remaining in his hand, which are less than [a multiple of] 30 years [פחות מל׳ שנה], to the table of common years [לוח השנים הפשוטות], takes what is found opposite them — the Egyptian years, the months, and the days — and joins them to what was kept with him…

he takes from the table of combined years, and the sum will be the proper figure for reckoning the courses of the stars. If after all this there remain to him months and days that have not completed a [full] year, he takes from these months — at thirty days per month — one day for each month, adds them to the Egyptian figure he holds, and he will then have a corrected figure ready for computing the courses of the planets.

The Root of the Reckoning of the Greeks and the Romans, by the Sun

These two nations — and the whole kingdom of Edom, together with the other nations that incline toward it — all reckon by the sun on a single system. By their account the days of the solar year are 365¼ days [שס״ה ורביע]. For this reason they count three consecutive common years of 365 days each [שס״ה], and in the fourth year they count 366 days [שס״ו], so as to make up in it the quarter-day by which each year exceeds 365 days [שס״ה]. It is for this reason that they call every four years a "small cycle" [מחזור קטן]; and the surplus they have between one year and the next is one day and a quarter, which over four years makes a surplus of four days.

They begin counting by the days of the week from the day [of the week] that fell in the fifth year, because [those surplus days] do not return to that same day [on which they had counted] in the first year. And since they do not return to that very same day [until twenty-eight years have passed], they instituted for themselves a "great cycle" [מחזור גדול] made of seven small cycles, so that every twenty-eight years [כ״ח] their year-days return to the order on which they had been reckoned in the twenty-eight years [כ״ח] that preceded — there being no difference between them, to the end of the world, by their account.

The ancients among them used to count the years of their kings; but all the [astronomical] writers occupied with these matters [count] from the beginning of the reign of Alexander of Macedon, which fell on the [astronomical] day [whose epoch for the moon is] 26 days [כ״ו], on a Monday [שני], in the month of Tishrei of the year 3450 of the Creation of the world [= 50 + 400 + 3000 = ג׳ אלפים ת״נ 3450 AM], reckoning from the BeHaRaD [epoch]. From this present year we reckon [it as] the Era of Documents [מנין שטרות, the Seleucid era].

Do not be puzzled by this reckoning if you see in the words of the ancients that the start of the Era of Documents was at the completion of the year 3448 [ג׳ אלפים 3448 = 48 + 400 + 3000 = תמ״ח AM]; for that reckoning was stated [counting] by the system of *Seder Olam* [סדר עולם], reckoning from [the Flood, i.e.] from the [year] 14 [וי״ד], whereas the one mentioned above is reckoned by the system of [counting] from the BeHaRaD — on whose basis we have ordered the reckoning in this book and explained it. Between that reckoning and this one there is [a difference of] one year. If, for this reason, they come to a single agreement [once that one-year difference is accounted for], then the difference found between them is no greater than that which is called [a single] portion.

Among those who reckon by the sun there are some who count the days of the kings, and there is the book of those occupied with the nativity of the one suspended [i.e. a Christian-era reckoning], or with the year of conception. This reckoning — the reign of Alexander, which is the Era of Documents — is the year 310 [310 = 10 + 300 = ש״י] for those reckoners, and 311 [1 + 10 + 300 = שי״א 311 =], later, for those who reckon by the day of the nativity. One who knows the [number of the] Era of Documents knows from it both of these dates without trouble. We now set out the order of this reckoning, with the help of Heaven.

The Order of the Reckoning of the Greeks and the Romans

These two nations divide their years into twelve months, [each] of a differing number [of days] for most of the months. They come to a single agreement as to the count of the days of the month, and they differ in the names of the month.

- The **first** month the Greeks call *Tishrin rishon* [first Tishrin]; its Roman name is *Octobr* [Aktober = October], and its days are 31 [ל״א].
- The **second** the Greeks call *Tishrin sheni* [second Tishrin]; the Romans call it *Nowember* [November], and its days are 30 [ל׳], by the account of them both.
- The **third**, for the Greeks, [is] *Kanon rishon* [first Kanon]; for the Romans, *Diginbar* [Dijinbar = December], and its days are 31 [ל״א].

- The **fourth**, for the Greeks, [is] *Kanon acher* [the other Kanon]; for the Romans its name is *Yana-r* [Yanar = January], and its days are 31 [ל״א]. This month is called the first [month] for those who reckon by the nativity.
- The **fifth**, for the Greeks, [is] *Shevat*; for the Romans, *Febrir* [Februar = February], and its days are 28 [כ״ח] for three consecutive years, but in the fourth year, 29 [כ״ט], to make up the day [added] from the quarter-day reckoned over the three leap-prone years; and they call that fourth year a "pressed-in" year [שנה כבושה = intercalated year].
- The **sixth**, for the Greeks, [is] *Adar*; for the Romans, *Mars* [March], and its days are 31 [ל״א]. This month is called the first [month] for those who count by conception.
- The **seventh**, for the Greeks, [is] *Nisan*; for the Romans, *Abril* [April], and its days are 30 [ל׳].
- The **eighth**, for the Greeks, [is] *Iyyar*; for the Romans, *Mayya* [Maius = May], and its days are 31 [ל״א].
- The **ninth**, for the Greeks, [is] *Sivvan* [Sivan]; for the Romans, *Yoniyosh* [Junius = June], and its days are 30 [ל׳].
- The **tenth**, the **hi** [tenth — הי׳], for the Greeks, [is] *Tammuz*; for the Romans, *Yulia'os* [Julius = July], and its days are 31 [ל״א].
- The **eleventh** [הי״א], for the Greeks, [is] *Av*; for the Romans, *Agushat* [Augustus = August], and its days are 31 [ל״א].
- The **twelfth** [הי״ב], for the Greeks, [is] *Elul*; for the Romans, *Shtembar* [September], and its days are 30 [ל׳].

With these twelve months they complete 365¼ days [שס״ה ורביע], which is what they count for each year. In this reckoning they agree with our reckoning in every nineteenth year [י״ט], which constitutes a [19-year] cycle [מחזור]. As for the [intercalation] that crosses over, and the corrector — the intercalations that we make in every [19-year] cycle — there is no difference between our cycle and the nineteen-year solar cycle by the reckoning of the gentiles, except for one hour and 485 parts [שעה אחת ותפ״ה חלקים = one hour and 485 chalakim] of the reckoning — 1080 chalakim [printed תתר״ק, literally 1100; a ק/פ compositor error for 1080 = תתר״פ, the standard chalakim-per-hour constant] in one hour, neither less nor more, in each cycle.

Now if you have years according to the reckoning of the gentiles who reckon by the reign of Alexander of Macedon, and you wish to know how many years they are by our reckoning from the Creation of the world: find how many complete cycles there are, and how many years you are counting from the Creation of the world. Take out of the cycles 182 [182 = 2 + 80 + 100 = קפ״ב] per cycle, and keep the remainder from the cycles and the years, if you have years that have not completed a cycle.

Then double the remaining cycles by 3 parts [בג׳ חלקים], 38 fractions [ל״ח שברים], 21 seconds [כ״א שניים], and 40 thirds [מ׳ שלישיים]; their mnemonic is GeLaCh KaMa [גל״ח כא״מ = ג3 ל30 ח8 ‖ כ20 א1 מ40]. To the product gathered from the doubling, add 31 parts [ל״א חלקים], 21 fractions [כ״א שברים], 33 seconds [ל״ג שניים], and 20 thirds [כ׳ שלישיים]; their mnemonic is LA KA LaG K [ל״א כ״א ל״ג כ׳], which stand opposite the hours and the parts by which the molad of Tishrei exceeded the day-fractions on which October entered in the first year of the cycle — that being the [cycle of] 183 [= 3 + 80 + 100 = ממחזור קפ״ג 183]. The whole sum gathered from them: divide it by sixty, and what comes out of the division will be days. Keep them.

If there remain to you parts and fractions that have not completed sixty, and these parts come to less than 45 [45 = 5 + 40 = ממ״ה], pay no heed to them; but if they are 45 [ממ״ה] parts or more, count them as one day, and add it to the days of the division. And further, you always add four days [ארבעה ימים] to the days of the division — the days by which the day of the New Year [ראש השנה] exceeded the first day of the month of October [אכתובר] in the first year of the cycle 183 [קפ״ג] from the Creation of the world, which is the first cycle that came at the beginning of the reign of Alexander of Macedon. From the days that result after all this reckoning — hold fast to it and keep it, for you will need it for the remaining years of the cycle — it is the [number of] days by which the New Year precedes [the entry of] the first of October, [whether] at the head of the cycle you are in or at [a year] you wish to seek out.

If you seek the day on which October enters in the remaining years of the cycle, know how many complete years have wholly elapsed from [the head of] the cycle up to the defective year [השנה הפגומה] that you are seeking; double that [number], multiplying [each year] by 52 parts [נ״ב חלקים], 28 fractions [כ״ח שברים], and 20 seconds [כ׳ שניים], no more; and keep the product gathered from the doubling. Then know how many leap years there were among those years that elapsed from the cycle, and take for each [leap] year 30 days [ל׳ יום] and gather them. Add this gathered sum to the days of precedence that came out for you at the head of the cycle — which I commanded you to keep — and you will have the total sum of days by which the New Year precedes the first of October in the coming year you are seeking.

If the gathered sum from the doubling is not [equal to] the days of the intercalated month — and this cannot occur except in the eighth year, when you seek the head of the ninth year — then at that time you fill out the deficiency of the days of the doubling from the days of precedence of the head of the cycle that you kept. What remains to you from the days of precedence after that deficiency will be the days of precedence of the head of the ninth year over the first day of October in that year. But if the gathered sum from the doubling does not [reach] the surplus years over the complete cycles [where] the year is intercalated — as in the case of the first two years of the cycle — add the days of the doubling for these two years to the days of precedence of the cycle that you kept, and you will have the days of precedence of the head of that year over the first day of October that enters within it.

And when you know the days of precedence of the New Year over the first day of October in any given year, and you wish… [continues p.57]

To determine which day of the week the first of October falls on in any given year, find the molad of Tishrei — its hours, its parts [chalakim], its remainder — and add it to the days of precedence of the New Year, together with its fractions and parts that you preserved. Combine the whole. It is understood that some of the hours and their fractions will amount either to a full day, or to one quarter; if so, do not worry about a single quarter. But if it amounts to two quarters, which is half a day, do not worry about them either. If, however, you know that the year in question will be an intercalated [leap] year, and there are three quarters of a day, take account of them and add one day to the days accumulated for you from the molad and the days of precedence. Know their total, throw out sevens — seven by seven — and the remainder, whether it is seven or less than seven, is the day of the week on which October enters in that year.

And if you wish to determine the day on which October falls without going by way of the molad, know how many cycles [machzorim] and years you have in hand for which you reckoned the days of precedence. Reckon the cycles as years and combine them with the years that did not complete a cycle. To their total add the nine years that always precede — counting from the era of contracts [minyan shetarot, the Seleucid era] at the head of cycle 183 — and you will have in hand the total of full completed years that have elapsed for the reign of Alexander [Alexandros, the Seleucid era]. Throw them out twenty-eight by twenty-eight [כ״ח = 28]. If they all come out and nothing remains in your hand, know that the coming year is the head of the great solar cycle, and its sign is one [1] always. And if years remain to you that did not complete twenty-eight, add their quarter

[רביעית] to them and know the total gathered from them and from their quarter. From this quarter, if there is a surplus over the years and the full days amounting to one quarter — do not take account of it; and likewise do not take account of two quarters if they remain in your hand. But know that the coming year will be intercalated [leap]; and if there remain three quarters, take account of them and add one day to the full days. Combine the whole, throw it out by sevens, and what is in your hand — seven or less than seven — and add one to it always, for it is the sign of the head of the solar cycle. There will come up in your hand the sign of the year you are seeking. And when you know the sign of the year, add to it always one day, and you will find in it the day on which October enters in the coming year you are seeking.

And if you wish to determine on which day one of the months of the year enters, know how many months of the year have elapsed up to that month, not counting that month itself in the total. For each month that is of thirty days [30 = ל׳] take two days [2 = ב׳], and for a month that is of thirty-one days [31 = ל״א] take three days [3 = ג׳]. And if among those months there is February [Februar], do not take anything for it, except in an intercalated [leap] year, in which you take one day for it. Combine these days that you took and add them to the total of the sign of the year in which you stand. Throw out the whole by sevens, and the remainder in your hand — seven or less than seven — is the day on which the month you are seeking enters. And whoever wishes—

—to reckon the courses of the stars in this book for the years given to it for the reign of Alexandros [Alexander, the Seleucid era], let him return those years to the reckoning of the order of the world [seder olam] as he learned above, and let him know from the years of the order of the world [seder olam] the Egyptian years [shanim Mitzriyot] proper for reckoning the courses of the stars upon them in that same year. He is able to determine this from the tables [luchot] made for returning the years to the count of the era of contracts [minyan shetarot] as Egyptian years — if he proceeds in their reckoning by the procedure set forth above in the reckoning of the Arabic years [shanim Arviyot], with the help of God.

The Ninth Gate [השער התשיעי]

On the reckoning of the difference [chiluf] between the days and nights that revolve in the world, and between the days and nights reckoned as equal in their measure; and all that depends, in this matter, on the ordering of the days and the years appointed for the reckoning.

All the inhabitants of the world take no account of this difference. They hold the days and the nights together as equal, and count them as twenty-four whole, straight hours [24 = כ״ד], without addition and without diminution — there being no difference between them, ever. To their eyes, all the days of the year were reckoned for them from the rising of the sun to its rising on the second day, or from its setting to its setting; or they were reckoned from the moment it stood at the midpoint of the sky above the earth — which is the midpoint of the day — until it returned to stand at that same part on the second day; or from the moment it stood at the midpoint of the sky beneath the earth — which is the midnight — until it returned to stand over it on the second night. All four of these points, from which they begin to reckon twenty-four hours for each day and night, are together equal in the eyes of every person, because the difference found among them is so small that it is not perceptible in one day, nor in two days, and causes no harm to mankind in any of their occupations.

But the men of this science see, with demonstration, that there is a difference among them — that at times it adds to the equal day, and at times it subtracts from it. This difference is not perceptible, and causes no harm over a few days; but over many days it is perceptible, and they take account of it and it harms them in the reckoning of any course [mahalach] — all the more so in the reckoning of an eclipse of the sun or of the moon. For this reason it fell to them to investigate the measure of this difference, which at times adds and at times subtracts, and how its rule applies in each and every part of the firmament [rakia]. This difference is defined by two reasons. It is the bound of the variable course of the sun, which is not equal on each and every day; and it is so defined because this course varies further in its progression [matzad] in two ways: it varies according to the dividing mode [ofan ha-mafrish], which does not proceed by one order in every place; and also according to the straight mode [ofan ha-yashar] that proceeds by one order — except that the variation in the dividing modes varies in its measure, in one place by one measure and in another place by another. The straight mode, which is at the midpoint of the sky, follows a single rule in every place,

For this reason the practitioners of this science were obliged to fix the beginning of days and nights either at midday or at midnight in all their reckonings of the courses of the stars — for, given that this difference exists, the practice of taking midday in the middle of the sky is uniform everywhere. When they investigated this difference found at the middle of the sky, they would either add it, by its measure, to the equal day, or subtract it from it, when its accumulation has reached its full extent — namely 8⅓ degrees of the orb, which is more than half an equal hour. If we add together the sum of the greatest addition and the sum of the greatest subtraction, and combine, at each place, the standing of the addition with the standing of the subtraction whose like is the greatest, then the difference found at a single place is doubled. That is: in the place of addition, or in the place of subtraction, it will come to more than one equal hour when the sun and the moon conjoin.

In the sum of this course, which varies at certain times by more than a degree and 30 minutes [more than half an hour], this sum can do great harm to a course's reckoning — indeed, great harm in the reckoning of eclipses, as we mentioned above. Yet even though it does no harm to the reckoning of the remaining courses of the stars, each of the two differences — whether in the addition or in the subtraction — is 8⅓ degrees. We find that the difference of the sun's mean course produces from it 3 degrees and two-thirds of a third of a degree [3° and ⅔ of a third], and that the difference of the ascensions of the degrees at the middle of the sky produces from it 4 degrees and two-thirds of a third of a degree, when the two of them are on one side — whether for addition or for subtraction. The side of addition begins from the head of Scorpio and proceeds up to the midpoint of Aquarius; the side of subtraction begins from the midpoint of Aquarius and proceeds up to the end of Libra.

For this reason we take the head of the revolution within the days of the year for the reckoning of the mean courses that we set down in this whole book: the sun was standing, on the first day of the reckoning, at 15 minutes [ט"ו] within the sign of Aquarius, both in its variable course and in its mean course, to the extent of about 13 minutes [י"ג] from it; and from there onward the difference goes on revolving until it returns to its place and changes on that same day of the coming year.

When you come to know the sum of this difference on all the days of the year, know the position of the sun in the mean course on the day for which you wish to know the difference, and also its variable position on that day. Know how many degrees there are between its position in the mean course — which is at the start of the reckoning, namely 12 degrees [י"ב] within the sign of Aquarius — and its position in the mean course that you have computed, and keep it [this remainder]. After this you will know the position of the variable sun that you computed by the straight mode; subtract from it the ascensions of 15 degrees [ט"ו] within the sign of Aquarius in that mode. What remains in your hand is either from the ascensions by the straight mode that are between the two [continues p.62]

positions. If the ascensions of the variable course equal the degrees of the mean course, then it will be clear to you that there is no difference at all between days and nights on that day. But if they are not equal, know which of the two is the surplus quantity, and which of them is in surplus. Take the surplus and keep it; double that surplus by 60 [ס׳], and divide it by 15 [ט"ו]. The result of the division will be the minutes of an equal hour — for an equal hour is reckoned at 60 [ס׳] minutes per hour — and these are the sum of this difference for the day you computed, in equal hours.

But if you double the surplus by 60 and divide it by the time-units of a seasonal hour of that day — when you reckon the start of the day and the night from midday — or divide it by the time-units of a seasonal hour of that night, when you reckon the start of the day from midday to midnight, then what comes up in your hand will be the sum of the difference on that day, in seasonal hours, from that day or from that night, either to the side of addition or to the side of subtraction.

Now if the hours you have are reckoned from midday or midnight — that is, if you begin to reckon from the head of days or from midnight — then those hours will belong to the reckoning of days that are reckoned uniformly for the whole world, and these are the hours seen from the motion of the orb. If the ascensions of the variable course were in surplus, you add the sum of the difference to those hours; and what is gathered from them will be the hours corrected for the variable course, fit for reckoning the courses of the stars upon them. But if the degrees of the mean course are in surplus over the ascensions of the variable course, you subtract the sum of the difference from those hours; and what remains will be hours corrected for reckoning the course upon them for that time.

And if you have corrected hours for reckoning the course, and your wish is to convert them back into the hours seen from the motion of the orb relative to the days reckoned uniformly in the eyes of the world, you proceed by this practice: you subtract the sum of the difference from the corrected hours if the ascensions of the variable course were in surplus over the mean course; and you add the sum of the difference to the corrected hours if the degrees of the mean course are in surplus over the ascensions of the variable course. What comes up in your hand, after this subtraction or this addition, are the hours reckoned for the people of the world uniformly — the hours by which the corrected hours that you had were corrected.

To carry out this reckoning by the tables engraved in this book for our reckoning, we added, upon the root of the moon's course, on the first day of the reckoning of this book, 18 minutes of a degree [י"ח שברי מעלות], which are the value of the moon's course in the measure of the difference found between days and their nights. We extracted this sum of the difference, the one fit to be found at midday for each and every degree of the degrees of the firmament, and engraved it on a table upon which is written the value of the difference between days and their nights. When you come to reckon this difference, at any time, from the tables, know the position of the variable sun by the mode of the zodiac signs, and bring it [continues p.63]

to the table of the difference between days and their nights. Take what is found opposite the degrees of the sun's position, from the degrees and their fractions, while the sign in which the sun is standing is written at the head of that column. Join those degrees and their fractions to the parts of the hour you wish — whether straight or seasonal — and that will be the sum of the difference between the days reckoned uniformly and the [days] alternating, for that same time you are seeking.

If you have in hand the hours reckoned from the days arranged uniformly for the whole world, subtract the sum of the difference from those hours; and what remains will be hours corrected for the days fit to reckon the courses of the stars upon them. But if you have in hand the hours corrected for the reckoning of the courses, add the sum of the difference upon them, and there will come up in your hand the hours as they are found according to the order of the orb's courses, always.

[Section heading, p. 63:] The opening of the rectification of the years, the months, the days, and the hours by which to reckon the courses of the stars in this book, from the head of cycle 257 [רנ"ז] and onward.

Know that the day reckoned in this book — we set its beginning at midday of the day before, that is, the moment the sun is at the middle of the sky over the earth on the day before that passed, until it is at the middle of the sky on this day. For example, we count the first hour from the first day starting at the fourth hour of Shabbos day that passed, and we say of it that it is to be reckoned at the end of the sixth hour of the first day [Sunday]. So too is our practice for all the days of the week.

The head of the day from which we began to reckon the courses of the stars in this book was the end of the sixth hour of the fourth day [Wednesday], the 29th day [כ"ט] of the month of Elul, in the year 4864 [ד׳ אלפים תתס"ד; 4000 + 864 = 4864 AM, = 1104 CE] of the world. The end of that day was the end of the sixth hour of the fifth day [Thursday], which was the first day of the month of Tishrei, in the year 4865 [ד׳ אלפים תתס"ה; 4000 + 865 = 4865 AM] — this by the mode of the middle of the sky at the far place [the place of farthest distance], at the midpoint of the earth, one hour and a half [שעה אחת וחצי] to the side of the west, its distance from the eastern edge being seven hours and a half [שבע שעות וחצי]. This is the distance of the middle point of the Land of Israel in longitude, somewhat eastward. This day, whose head we have delimited, is the root and the foundation upon which we build the reckoning for the position of the seven stars in their firmaments, in their mean course, and [for] the position of the dragon's head [התלי] and its tail [are placed] at the start of the arrangement in this book.

The boundary of this last day — and it is the end of the sixth hour of the fifth day [Thursday] — is the head of the first year of cycle 257 [רנ"ז]. One who wishes to reckon the courses from this book, from the head of cycle 257 and onward, ought to reckon the day on which he stands within the totality of days, months, and years that have elapsed since the kingdom [malchut — the era count] from the head of cycle 257 up to that day, and convert them into Egyptian [solar] years, and reckon from them the courses of the stars by this order.

The order of rectifying the years, and what follows from it, by which the courses of the stars are reckoned

Whoever wishes to reckon the courses of the stars from this book, from the head of cycle 257 [רנ"ז] onward, must first know how many years he counts — whether by the world-era [Anno Mundi], or by the count of contracts [the Seleucid era], or by the kingdom of Ishmael [the Muslim Hijri era] — together with the months, days, and hours that have elapsed up to the moment at which he wishes to determine the position of the stars. He then converts those years into Egyptian [solar] years, as was explained above in this gate, holding the months in hand and counting thirty days for every month, twelve months to each of those Egyptian years.

He should arrange the years, months, days, and remaining hours in his hand and rectify them properly. The intent is that he attend to the reckoned hours of the day on which he stands. If he begins his reckoning from the rising of the sun on that day in the east — which is more than three "twisted" [unequal/seasonal] hours — he subtracts six hours from them: he counts the day on which he stands within the total of days that have elapsed, and the hours in excess of six he counts toward the coming day. But if the hours reckoned from the rising of the sun do not reach six "twisted" hours, he does not count that day within the days, but reckons the hours from the midpoint [noon] of yesterday up to that hour, and makes yesterday the day; he counts it within the total of days that have elapsed, and the hours that lie from its midpoint up to the hour for which he is reckoning become for him hours ready to reckon from, for [counting] from the coming day onward up to the days, months, and Egyptian years proper for reckoning the courses of the stars from, for that moment.

In this way you compose the arranged date [the rectified epoch-tally] out of the years, months, days, and hours, and even the fractional parts of the prepared hours from which the courses of the stars are to be reckoned. This computation, and the foundation of this number, is built upon the model of the farthest place, [which lies] at a distance of seven and a half hours [in longitude]. If the distance of your place from the eastern extremity equals this distance, this arranged date suffices for you and you need not adjust it. But if the distance of your place inclines [deviates] from this distance, know by how many hours, or fractions of an hour, it inclines — whether it inclines from this distance toward the [eastern] side, in which case you subtract the inclining hours from the hours recorded in the arranged date; or whether it inclines toward the western side, in which case you add them to those. The hours that result in your hand after the subtraction or the addition are the hours proper to use with the arranged date for the place at which you stand and for which you reckon.

This date is arranged according to the days as they appear to be equal in the manner of their revolution in the world [the apparent solar days]. If you wish to rectify it according to the order of the days that are truly equal in their measure [the mean / equal days], know the position of the sun from the wheel [aspect] of the zodiac out of this arranged date, in the manner explained in the tenth gate [ב"ג"ה / by means thereof]; [continues p.65]

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And know the place of the sun [find the position of the sun]; [take] the value of the equation [ḥiluf] between the days and their nights, and subtract that value from that figure — the hours that are in the arranged date — and the remainder will be hours corrected against the equal days, properly. Add those to the days, months, and years that are in the arranged date, and there will rise in your hand a date corrected by the equation of the days and their nights, ready for the reckoning of the courses of the stars at the moment you desire, [counted] from the head of cycle 257 [רנ"ז] and onward.

And if you wish to reckon the courses of the stars from this book in order to know their positions [reckoning] from before the head of cycle 257 [רנ"ז] and backward [for earlier times], know how many years, months, and days there are from the moment you wish to reckon up to the head of cycle 257 [רנ"ז]. Reckon those years as Egyptian years, by the reckoning you learned at the head of this gate — the years, months, and days being arranged [reckoned] without [the aid of] the tables. [That is,] reckon it as Egyptian years. Then know, of the courses of the stars whose degrees and fractions you wish to reckon, the figures that rise in your hand for each star from the root [the radix / starting value] written for that star in the tables engraved for the mean motion; and the remainder will be the mean motion found for the star at that moment, before [prior to] cycle 257 [רנ"ז], with the help of Heaven.

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The tenth gate, in the reckoning of the mean motion equal for the seven planets [the seven wandering stars], and for the head of the dragon [tali — ascending node] and its tail [the descending node]; the reckoning of the varying motion for each star out of its mean motion, and the [motions and] distances joined to it; and the reckoning of the bound [composite] motion for the wandering stars [the planets].

The order of extracting the mean motion from the tables by means of the rectified date

Anyone who wishes to know the position of the stars as it appears at any time, by the manner [aspect] of the zodiac, does not arrive at that [knowledge] except through knowing its position from the aspect of its firmament [orb] in its mean motion. One arrives at this mean motion by means of the tables made for our reckoning, explained to you out of the rectified date that we set out for you in the ninth gate, which has passed. If he is to proceed by this practice [following this procedure], he brings the years that are in the rectified date to the tables of the mean motion equal for the star he is reckoning. If he finds in the column of the assembled years [the collected-years column] in the tables years equal to the years in his date, he takes what is found there against those years in the tables, of the degrees, fractions, and seconds that are in the column of the mean motion in the table of the assembled years for that star. But if he does not find years that are equal to them, he finds years that are close to them, the lesser of them by less than nineteen [י"ט] years, and he takes [writes] what is against those close years, of the degrees, fractions, and seconds, and writes them down. He inquires upon them the mean motion of the assembled years; and the remainder of the years that have not completed nineteen [י"ט] — he inquires upon them in the table of the simple [single] years for that star, and takes what is found against them in the columns of the mean motion, of the degrees, fractions, and seconds, and writes them on the parchment, writing below them [adding them to] the motion of the assembled years, so that he writes the degrees beneath the degrees, the fractions beneath the fractions, and the seconds beneath the seconds.

And if there be months in the rectified date, he inquires upon them in the table of the months and does likewise with their degrees, fractions, and seconds found against them, by the same procedure he did with the years in writing them out — every number [entry] of them beneath the one like it in its kind. So too he does with what is found against the days, and likewise against the hours. [continues p.67]

and their fractions, if they appear in the corrected date — days, hours, and thirds — to write down the mean motion. For every one of the time-spans arrayed in its date, each unit beneath its own kind, return and combine them. To know them in their totality, begin from the seconds: count them and learn their sum, then divide that sum by 60. For every 60 of them give one reward [carry], and add those to the fractions written from the tables. Whatever remains of the seconds that did not complete a full 60, write off to one side. Then come to the fractions, count them together with what has been added to them from the total of the seconds, and on the total that rises from them do as follows: take out [groups of] 60 from it [60s], and for every 60 of them add one degree onto the degrees written from the tables. Whatever remains of the fractions that did not complete [a 60], write before the remaining seconds that you wrote off to one side. Then come to the degrees, combine them with what has been added to them from the total of the fractions, and take out from the whole assembled in the degrees [groups of] 60 from the total gathered in degrees as 360; whatever remains less than 360, write before the fractions, and the seconds you wrote to one side.

You will then have these degrees, which did not complete 360, and the fractions, and the seconds, which did not complete a single 60 of them — these are the mean motion of that star at that moment, for the arrangement by which you reckon from its motion. They [serve] for the motion of its body along the curve of its orbs, for the motion of the orb's center along the curve of its eccentric [literally: along a curved manner], and for all the other motions that are reckoned as mean for every star among them. By this arrangement you shall reckon it from this number.

סדר חשבון מעמד החמה במהלכה המתחלף

The Procedure for Reckoning the Position of the Sun in Its Varying [Anomalistic] Motion

Know the position of the sun in its mean motion at the time for which you wish to know its position in the varying motion, just as you learned to find the mean motion from the corrected date. Write the mean motion on the parchment and keep it. From that mean motion subtract the apogee of the sun, which at the beginning of cycle 257 was 75 degrees and half a degree, that is, 30 fractions [minutes]. The remainder of the mean motion, after the subtraction of the apogee from it, is the argument of anomaly [the "measure of the breadth," חק המרחב] of the sun at that time. Seek it in the tables.

The correction of the sun stands in two columns of the number; take what is found opposite it in the columns of correction, in degrees and their fractions — that is the solar correction for the degrees of that argument of anomaly. And if in that motion [argument] there are fractions [i.e., it does not complete a whole degree], take the correction that stands before the completed degrees and keep it; then take also the correction that stands before the degree adjacent to them, learn the surplus [difference] between them, and from that surplus take a portion [proportional part]:

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the portion that will be in the fraction of those fractions out of sixty. If the completed degree has a correction exceeding that of the next [literally: if the full degree's correction is a surplus over…], you subtract from the correction of the completed degree the portion you took; and if its correction is a deficit, add the portion you took onto the correction of the completed degree. What comes up in your hand after the addition or the subtraction — this is the corrected gleaning [the adjusted increment, הגרעון] — is the apportioned correction for that portion, to correct with it the mean motion that you kept.

And if the degrees of the argument of anomaly whose corrections you have extracted run from 1 to 180 degrees, you subtract the apportioned correction from the mean motion that you kept.

And if the degrees of the argument in your hand run from 181 to 360 degrees, add the apportioned correction onto the mean motion. After the addition of the correction or its subtraction, that will be the position of the sun in its varying [anomalistic] motion within the framework of the zodiacal signs at that time.

From those degrees assign 30 degrees to each sign. Begin from the sign of Aries [Taleh], and the sign in which those degrees come to an end is the sign in which the sun stands; and by the count of the degrees remaining in your hand for that sign, out of the reckoning of 30 degrees per sign, [you know its place within the sign].

Now Ptolemy [בטלמיוס] holds that the sun's apogee [רום, "height"] has no orb [eccentric] at all, whereas the scholars of this craft in this age see that it has an orb of one degree every hundred years; and according to them this apogee was 75 degrees and 30 fractions at the beginning of cycle 257.

And if you wish to know its position before cycle 257 or after it, know how many years there are between that time and the head of cycle 257, and for every hundred years assign one degree of them. Add those degrees onto 75 and a half if you are reckoning forward, or subtract them from it if you are reckoning backward; and the apogee of the sun for that time will come up in your hand. Do likewise for the apogees of the other stars [planets].

סדר חשבון מהלך הלבנה המתחלף

The Procedure for Reckoning the Varying [Anomalistic] Motion of the Moon

Know the mean motion of the moon, its argument [חקה], and its distance [מרחקה], from the tables engraved for its reckoning, out of the corrected date that was in your hand, just as you did with the mean motion of the sun. Write each one of them on the parchment by its name and on its own. Then come to the mean distance [המרחק השוה] and double it. If in its doubling it is more than 360 degrees, take 360 out of it and keep the remainder; and if there is not [a full] 360 in it, keep it as it is — that is the doubled distance. Seek upon it in the column of the value of the number

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in the tables of lunar correction, and take what is found opposite it from the fractions of the degree [literally: of the rising] in the column of the value of the addition [increment, ערך התוספת], and keep it. And take further what is found opposite it in the two columns of correction, in degrees and their fractions, and keep it.

If with the degrees of the distance there are doubled fractions, correct them by the way you did in the solar reckoning. The correction proper to the argument of the moon [לחק הלבנה] will then come up in your hand; add that correction onto the mean lunar argument [מהלך חק השוה — read: the mean argument-motion] that you extracted from the tables. If the doubled distance runs from 1 to 180, subtract the correction from the mean argument; if the doubled distance runs from 180 to 360, what comes up in your hand, after the addition or the gleaning [subtraction], is the corrected argument from the doubled distance.

Come with the corrected argument from this doubled distance to the tables of lunar correction, and take what is found opposite it from the degrees and their fractions in the columns of the near distance [המרחק הקרוב], and write them beneath the fractions of the value of the addition that you extracted in the doubled distance. And take further what is found before the corrected argument from the degrees, fractions, and seconds in the columns of the particular lunar correction [תקון לבנה המיוחד]. Return to the addition [increment]: know its value, how much it is out of 60 — whether a quarter, or a third, or a half, or less or more — and take that value from the degrees and their fractions that you found in the columns of the near distance, written for you beneath the value of the addition; and add the value you took onto the degrees, fractions, and seconds that you wrote from the particular lunar correction. What comes up in your hand will be the lunar correction, the composite one [המחובר].

Subtract it from the mean center-motion [מהלך המרכז השוה] that you extracted from the tables out of the corrected date. If the corrected argument [החק המתוקן] runs from 1 to 180, add it onto the mean center-motion; if the corrected argument runs from 180 to 360, what comes up in your hand, after the addition or this gleaning [subtraction], is the position of the moon in its varying motion within the framework of the zodiacal signs that

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He created over the children of the world: divide its degrees by 360 [ל״ל — see anomaly note], and assign to each 30 of them one sign. Begin from the sign of Aries [Taleh], as you did in the solar reckoning, and the degree at which you finish — that is the place of the moon, in the sign in which it [the count] came to a stop.

And if in the reckoning there is no value of addition at all — as, for instance, when there stands written in it a cipher [ספרא, zero] — then the particular lunar correction is itself the composite one in truth, since you could not take any value from the corrected argument, and so you can neither add it onto the particular lunar correction nor subtract from it.

If the corrected argument of the moon runs from 1 to 180, the moon is gaining [its motion is greater] in its motion of this day over its motion of yesterday. And if the corrected argument runs from 180 to 360, the moon is diminishing [its motion is less] in its motion of this day from its motion of yesterday; and on the side of the addition…

[p. 69 — סח]

[Text continues into the following pages.]

and the result will be the value of the moon's daily motion. If the corrected argument runs from 1 to 95 (צ״ה), the moon's motion of that day is less than its mean motion. And from 95 (צ״ה) to 180 (ק״פ), its motion of that day is one greater than the mean — that is, it exceeds the mean by an increment. On the side of diminution: if the argument is found to run from 185 (קפ״ה) to 265 (רס״ה), its motion of that day is one greater than the mean; and from 265 (רס״ה) to 360 (ש״ס), its motion of that day is one less than the mean. The same rule holds for the motion of the sun, if you wish to derive from it the sun's distance from the height of its apogee.

Reckoning the Motion of the Dragon's Head and Its Tail

Know, then, the mean motion of the dragon's head (תלי). From the corrected date — by the same method by which you derived the mean motion for the sun and the moon — derive the mean motion of the dragon's head; cast out the total in your hand from 360 (מש״ס), and the remainder will be the place of the dragon's head, taken from the wheel of the dragon's head among the signs of the zodiac. The place of the dragon's tail is always found in the seventh sign from the dragon's head: in the degrees of that sign, at the same number of degrees at which the head of the dragon stands [in its sign], the tail of the dragon stands in its own sign.

The Procedure for Reckoning the Motion of the Five Wandering Planets

One who wishes to know the place of these five planets — by their varying motion as it appears to us — proceeds thus. If he is reckoning for the three planets that are above the sun, namely Saturn, Jupiter, and Mars, let him know the mean motion of that planet from the tables of motion for the corrected date — and this is the motion of the center [of the epicycle], the corrected center — its revolution upon the curved bearing orb (אופן עקם הסובל). In his reckoning he proceeds by the same method he followed for the sun and moon, and let him keep for it the mean motion that comes out. Likewise let him determine the sun's mean motion for that date, and subtract that planet's mean motion from the sun's mean motion; the remainder in his hand is the motion of the planet's argument (חק) for that planet — that is, the motion of its body upon the curved orb of its epicycle (גלגל עקם הקפתו) — and let him keep this argument together with the mean motion he kept for that planet.

And if he is reckoning for the two planets that are below the sun, namely Venus and Mercury, then the sun's mean motion is itself the mean motion for both of them; and what comes out from the tables for each one of them is the motion of the argument for that planet. This is the procedure for deriving the mean motion and the motion of the argument for these planets.

When you know these two motions — namely the mean motion and the argument — subtract the height of the planet's apogee (גובה הרום)

from its mean motion for which you are reckoning, and the remainder will be the distance of the center from the height of its apogee. Take this and bring it to the correction tables of the planet (לוחות תקון הכוכב). Look it up in the rows of the number, take the value found opposite it among the degrees and their fractions in the rows of the correction of the argument and of the center (תקון החק והמרכז). If the center's distance you brought to the tables is fractions that do not complete a whole degree, you correct them as you did for the sun and moon. Write down this correction before the center's distance and before the argument. If the center's distance runs from 1 to 180 (ק״פ), you subtract that correction from the center's distance and add it to the argument; and if the distance runs from 180 (ק״פ) to 360 (ש״ס), you add the correction to the center's distance and subtract it from the argument. What stands in your hand after the addition to and subtraction from the center, and from the argument, is the corrected center and the corrected argument. Take care to keep each of these by itself.

Bring the corrected center to the planet's correction tables, take the value found opposite it in the column of the portion (טור המנה), and know whether it is to be subtracted or added. Write these down upon the parchment, and know their ratio out of sixty. After this, bring the corrected argument to the correction tables, look it up in the rows of the number, and take the value found opposite it in the rows of the mean distance (המרחק הבינוני). If the fractions of the portion you found are to be subtracted, take the value found opposite the corrected argument in the rows of the far distance (המרחק הרב). And if the fractions of the portion are to be added, take the value found in the rows of the near distance (המרחק הקצר), and write it down.

These two distances, each of which you take by itself: if it is a far distance you are taking, you take from it according to the ratio of the fractions of the portion you kept out of sixty, and you diminish that ratio from the mean distance. And if it is a near distance you are taking, you take from it according to the ratio of the portion out of 60 (ס׳), and add it to the mean distance. What stands in your hand — after the addition to, or the subtraction from, the mean distance — is the correction proper to that planet at that time. If the corrected argument runs from zero (סיפרא) to 180 (ק״פ), add the proper correction to the distance of the corrected center; but if the corrected argument was more than 180 (ק״פ), subtract the proper correction from the distance of the corrected center. What stands in your hand from the corrected center, after this addition or this subtraction, is the distance of the center at that time from the height of its apogee. Add to this center the height of the apogee, and you will find the planet's station among the signs of the zodiac — and that is its apparent place in its varying motion.

The height of the apogee at the head of cycle 257 (רנ״ז), according to the reckoning that appears to us by the reckoning of Ptolemy which he mentioned in his book, was:

- for Saturn: 243° (רמ״ג מעלות), [and] 53′ 12″ (נ״ג י״ב) [?], which is 3° of the sign of Sagittarius (ג׳ מעלות ממזל קשת);
- for Jupiter: 172° (קע״ב מעלות), (כב״ח) 28″ [?], which is 22° of the sign of Virgo (כ״ב מעלות ממזל בתולה);
- for Mars: 126° (קכ״ו מעלות), (כ״א נ״ב) 52″ 21′ [?];
- for Venus: 75½° (ע״ה מעלות וחצי), which is 15½° of the sign of Gemini (ט״ו מעלה וחצי ממזל תאומים);
- for Mercury: 201° (קק״א מעלה [irregular orthography: doubled-qof for 200; value 201 confirmed]), which is 21° of the sign of Libra (כ״א מעלות ממזל מאזנים).

From that time onward these apogees revolve one degree in every hundred years, which is 36 seconds (ל״ו שניים) in a solar year. You can determine the sum of this motion for the years between you and cycle 257 (רנ״ז), and add it to the places of the apogees mentioned here, and you will have the finding of their place at the time for which you are reckoning, God willing.

The Composite (Derived) Motion

If you wish to know whether a planet is direct in its motion, or retrograde, derive the corrected center's distance by which you reckoned the planet's place at that time. Bring it to the correction tables, look it up in the row, take the value found opposite it in the rows of the first station (העכוב הראשון) — in degrees and their fractions — and subtract the first station from 360 (ש״ס); the remainder is the second station (עכוב שני). If the motion of the corrected argument is less than the first station, even by a single part, you may know that the planet is direct in its motion. And if the motion of the corrected argument equals the first station, that planet is at a standstill in its motion and on the verge of turning back (retrograding). And if the corrected argument exceeds the first station and is less than the second, then the planet is retrograde (נזור); and if it were not so, it would always be direct in its course.

And if the corrected argument exceeds the first station, the planet is retrograde in its motion, and it goes on retrograding so long as the corrected argument exceeds the first station — until the argument reaches the second station, by one degree; the planet is then said to be at a standstill in its motion and on the verge of being set right (turning direct). And if the corrected argument exceeds the second station, the planet is direct in its motion.

And if that planet is retrograde, and you wish to know how many days ago it began to retrace its motion: subtract the first station from the corrected argument and divide the result between them by the planet's mean argument-motion of one day; there will rise in your hand the number of days between the start of its retrogression and that day. And if it is direct, and you wish to know when it will retrograde: subtract the corrected argument from the first station, divide the result by the mean argument-motion per day; [the quotient is the number of days]. And if it is direct, and you wish to know how many days ago it began to become direct: subtract the second station from the corrected argument, and divide the result by the mean motion.

The mean argument-motion (מהלך החק השוה) for one day is: for Saturn, 58 fractions (נ״ח שברים); for Jupiter, 54 fractions (נ״ד שברים); for Mars, 28 fractions (כ״ח שברים); for Venus, 37 fractions (ל״ז שברים); and for Mercury, 3 degrees and six fractions (ג׳ מעלות וששה שברים).

Gate Eleven — On Calculating the Moon's Latitude from the Ecliptic, North and South, and the Latitude-Position of the Apogee Planets from That Same Ecliptic

שער י״א [Gate 11]: On calculating the Moon's latitude — its position relative to the ecliptic, to the north and to the south — and the latitude-position of the apogee planets [כוכבי הנבוכה, the wandering/superior planets] from that same ecliptic.

If you wish to know the Moon's latitude relative to the ecliptic, the position will be less than the position of the dragon's head [ראש התלי, the ascending node]. From the corrected position of the Moon, subtract the corrected lunar argument [חק המתוקן], and what remains is the latitude-argument [חק המרחב] for the Moon. Bring that argument to the columns of numbers in the tables of lunar correction [לוחות תקון לבנה], and take what you find opposite it in the latitude-columns of the Moon — the degrees, the minutes [שברים, fractions], and the seconds [שניים]. If you have an argument carrying minutes that do not complete a full degree, refine it carefully in the proper adjustment of its latitude. What comes out in your hand is the Moon's latitude relative to the ecliptic at that time.

Now for the rule of direction and increase:

- If the argument runs from zero [ספרא, cipher] to 180 [ק״פ], the latitude is on the northern side.
- And from 180 [ק״פ] to 360 [ש״ס], the latitude is on the southern side.
- Further: if the argument runs from zero [ספרא] to 90 [צ׳], the latitude rises in the north and increases; from 90 [צ׳] to 180 [ק״פ] the latitude descends in the north and decreases; from 180 [ק״פ] to 270 [ר״ע] the latitude descends in the south and increases; and from 270 [ר״ע] to 360 [ש״ס] the latitude rises in the south and decreases.

This is the procedure for reckoning the Moon's latitude.

Should you wish to compute the Moon's latitude **without** the tables, it can be known from the chord of the latitude-argument [מיתר חק המרחב]. Multiply that chord by the chord of five degrees — which is the whole latitude — and the chord of those 5 [degrees] is 13 degrees [and] 46 minutes [shanim, here the second-order fraction]. Divide the product by sixty, which is the radius [half the diameter, חצי הקוטר], and from the quotient learn its arc [קשת]: that is the Moon's latitude.

Latitudes of the Five Apogee Planets from the Ecliptic

Should you wish to know the latitude of the five apogee planets from the ecliptic, set your mind to compute the three planets that lie above the Sun [Saturn, Jupiter, Mars], by degree:

- **For Saturn** [שבתאי], when computing, you add to its corrected center [מרכזו המתוקן] fifty degrees [חמשים מעלה], which is the excess our Second [Master, תנינו — i.e. the second/standard authority] gives over [the value of] its apogee [גובה רומו].
- **For Jupiter** [צדק], when computing, take out from its corrected center twenty degrees [עשרים מעלה], the apogee [גובה רומו] being higher [than its corrected center by that amount; continues p.74].

For Mars [מאדים], when computing, you neither add to nor subtract from the corrected center; bring the corrected center as it is — whether [you obtained it] after the addition or the subtraction, or [took it] of itself — for whichever of these planets you are computing. Bring it to the latitude tables, to the columns of the number arranged by six degrees [המספר המסודר על שש מעלות], and take what you find opposite it in the last table, in which are written the argument-fractions of the five planets [חלקי חק חמשת הכוכבים], from the minutes [מהשברים] and the seconds [השניים], and keep it.

Now, if the corrected center you brought to the tables falls in the first half [במחצית הראשון] — that is, from zero [סיפרא] to 90 [צ׳], and from 270 [ר״ע] to 360 [ש״ס] — the planet's latitude will be on the northern side. For this reason, bring the corrected argument [החק המתוקן] of that planet to the latitude tables, to the columns of the number arranged by 6 degrees, and take what you find opposite it in that planet's latitude-columns — from the degrees and their minutes — which are on the northern side. Take from them according to the ratio of the argument-fractions of the five planets, which is out of 60 [ס׳], over against the corrected center; and the ratio that comes up in your hand from the degrees and their minutes — that is the latitude of that planet, on the northern side.

But if the corrected center is in the last half [במחצית האחרון] of the columns of the number — that is, from 90 [צ׳] to 180 [ק״פ] to 270 [ר״ע] — the planet's latitude will be on the southern side. For this reason, take what is found before [opposite] the corrected argument in the latitude-columns belonging to the southern side, from the degrees and their minutes, and hold of them according to the ratio of the argument-fractions of the five planets, out of 60 [ם׳]; and that is the latitude of that planet on the southern side.

Now if that latitude is on the northern side and the corrected argument runs from zero [סיפרא] to 180 [ק״פ], the latitude rises in the north; and if the corrected argument runs from 180 [ק״פ] to 360 [ש״ס], the latitude descends in the north. And if the latitude is on the southern side, and the corrected argument is found from zero [סיפרא] to 180 [ק״פ], the latitude descends in the south; and if from 180 [ק״פ] to 360 [ש״ס], the latitude rises in the south.

The maximum latitude — the latitude rising at its greatest [ברבויו] — is:

- for Saturn [שבתי], in the north and in the south, 3 degrees and 3 minutes [ג׳ מעלות וג׳ שברים];
- for Jupiter [צדק], on the northern side, 2 degrees and 8 minutes [ב׳ מעלות וח׳ שברים];
- for Mars [מאדים], on the northern side, 4 degrees [ד׳ מעלות], and on the southern side 6 degrees [ו׳ מעלות].

All this is according to the view of Ptolemy [בטלמיוס].

The Two Lower Planets — Venus and Mercury

Should you wish to know the latitude of the two planets that lie below the Sun: if you are computing for Venus [נוגה], bring the corrected argument to the tables of its latitude, in the columns of the number arranged by 6 degrees [ו׳ מעלות], and take what you find opposite it in the latitude-columns of Venus — from the deflections [הנמיכות] and the slants [הפתילות] — and write each one by its own name, by itself and by its measure. That will be the sum of the deflection from the slant [סך הנמיכות מן הפתילות] for Venus.

And if you are computing for the planet Mercury [כוכב חמה], take what you find opposite the corrected argument from among the deflections and keep it by its measure. Now if the corrected argument falls in the first half of the columns of the number, take for yourself what is found [continues p.75]

before the argument from among the slants, and bring out from it its tenth, and keep what remains together with the slants. And if the corrected argument falls in the last half of the columns of the number, take for yourself what is found before the argument from among the slants, and add to it its tenth, and keep it together with the deflection — that will be the deflection and the slant for [Mercury, the] planet.

After this, you will add to the corrected center 90 degrees [צ׳ מעלה] if you are computing for Venus. And if you are computing for the planet [Mercury], you add to the center 270 degrees [ר״ע מעלות] and subtract from 360 [ש״ס]; if any one of them yields for the corrected center an excess over 360 [ש״ס], that is the augmented corrected center [מרכז המתוקן הנוסף]. Bring it to the columns of the number, and take what you find opposite it in the table on which are written the argument-fractions of the five planets [חלקי חק חמשת הכוכבים]. Know the value of that argument which you took out of sixty [מששים], and take for yourself that value from the deflections [הנמיכות] which you had kept: that is the first latitude [המרחב הראשון] for that planet — write it by its name.

And if the augmented center, in which you brought out the argument-fractions, falls in the half of the columns of the number in which the corrected argument fell — in which you knew the deflections — the first latitude that you wrote is southern [דרומי].

But if the two of them do not fall in one and the same half, the first latitude is northern [צפוני].

Furthermore, you take the corrected center according to its value as it was before the addition — if you are computing for Venus, keep it by its first measure. And if you are computing for the planet [Mercury], add to it 180 degrees [ק״פ מעלות] and bring out from it 360 [ש״ס] if there is an excess over 360 [ש״ס]. Bring this center, for whichever of them you are computing, to the latitude tables, and take what you find opposite it in the columns of the argument-fractions of the five planets, and keep it. From the slants [הפתילות] that you wrote, take according to the ratio of those fractions out of 60 [מס׳], and that value is the second latitude [מרחב שני] for that planet — write it beside the first latitude.

Now if the center in which you knew these fractions falls in the first half of the columns of the number, and the corrected argument is less than 180 degrees [פחות מק״פ מעלה], the second latitude is northern [צפוני]; and if there is an excess over 180 [ק״פ], it is southern [דרומי]. And if the center falls in the last half of the columns of the number, and the corrected argument is less than 180 degrees [מק״פ מעלה], the second latitude is southern [דרומי]; and if there is an excess over 180 [ק״פ] degrees, it is northern [צפוני].

After all this, you come to the argument-fractions of the latitude [חלקי החק של מרחב] that you have now kept. If you are computing for Venus, take a sixth of them [קח מהם השתות]: that is the third latitude [מרחב שלישי] for Venus, and it is northern [צפוני] at all times. If you are computing for the planet [Mercury], take the half and the quarter [המחצית והרובע]: that will give you the third latitude for [that] planet, and it is southern [דרומי] at all times.

And when you know these three latitudes — whether they are all to one side [continues].

If they are all on one side, add them together; their sum is the planet's latitude on that side. But if they fall on two sides, subtract the smaller from the larger, and the remainder is the planet's latitude toward the side of the larger value. The final latitude of Venus, by these computations, is 8 degrees and [ג״ו] minutes [?] north or south, and the final latitude of Mercury is 4 degrees and 18 minutes north or south.

If you wish to know whether this latitude is rising or falling, compute it again ten days later. If the latitude is in the northern quarter and you find it greater [than before], you know it is rising in the north; if you find it smaller after those ten days, you know it is falling in the north. And if it is in the southern quarter and you find it greater, you know it is rising in the south; if you find it smaller, you know it is falling in the south. If it was northern and you find it in the southern quarter, you know it is falling toward the north; and if it was southern and you find it in the northern quarter, you know it is rising toward the south.

This is the method for computing the latitude of the planets that have latitude relative to the wheel of the zodiac.

Gate Twelve: On the Reckoning of the Conjunctions of the Two Luminaries

[שער י״ב — Gate Twelve: on the reckoning of the conjunctions of the two luminaries.] These [conjunctions occur] at the end of each month, and you are to compute them. The oppositions, when the two stand one opposite the other, occur at the midpoint of each month — this is the time when the moon is full of light. And the reckoning of the moon's first visibility, at its emergence from beneath the sun at the beginning of each month, and every conjunction, follow this same procedure.

If you wish to compute the conjunction of the luminaries at the end of each month, compute the varying motion of each of them at midday on the 29th day before the molad of that month, in the manner of the position at which each one stands, and reckon it for yourself. If the two varying motions you have reckoned for the two luminaries turn out to be equal, then the time of the conjunction is the time of that reckoning — which is midday of the 29th day. But if one of them exceeds the other, you must determine the amount of that excess, which is always found to be a measure of motion equal to or less than one day's [motion] for the moon.

If the sun's position is the excess over the moon's position, then you know the conjunction has not yet occurred. But if the moon's position has an excess, it becomes clear to you that the conjunction has already passed. You must keep [in mind] this excess belonging to whichever luminary, and add to the excess one part of twelve of it — that is, half of a sixth — and add half of the moon's sixth to the moon's position. If the excess belongs to the sun, you add the excess and half of its sixth to the sun's position, and add half of the moon's sixth to the moon's position; then the two positions are found to come out as one, and that is the part [of the zodiac] at which the conjunction will occur.

And if the excess belongs to the moon, you subtract the excess and half of its sixth from the moon's position, and half of the moon's sixth, alone, from the sun's position; then the part at which the two luminaries conjoin is found. After they have stood at the part of the conjunction, you go back to inquire about the moment at which it occurred — whether by day or by night. This [requires] that you know the moon's varying motion for one even hour, and likewise the sun's varying motion for an hour on that day, as will be explained further on in this gate. Subtract the sun's varying motion from the moon's varying motion, and the remainder is the moon's speed over the sun in one hour. Divide by this speed the excess that was between the two luminaries before it was added to it —

half of its sixth, if the excess belonged to the sun — and what comes out of the division will be hours and moments, or moments without hours, or hours without moments; these are the hours of distance between the conjunction and the [time of] reckoning. If the excess belonged to the sun, you add the hours of distance to the hours of reckoning, which are the midday of the 29th day of the month. And if the excess belonged to the moon, you subtract the hours of distance from the hours of reckoning. What remains in your hand after this addition or subtraction is the hours of the conjunction in that month, by day or by night. This is the general reckoning for the conjunction of the luminaries in each month.

And if you seek the time of the opposition, when one of them stands opposite the other, you compute their varying positions for the 14th day from the day of the conjunction, at midday of that day, and proceed with it in the same manner you followed in the reckoning of the conjunction. You will find the part at which they stand opposite one another, the one being in the seventh sign from the position of the other, and you will also find the hour at which they stand opposite one another, by day or by night.

The method for deriving the varying motion for one hour, mentioned above, is as follows: Know the position of each of the two luminaries from the wheel of the zodiac on the day you wish, and likewise know their positions for the day after this day at the time you knew that day, so that there will be 24 hours between the two times. Derive the position of each one on this day from its position on the next day; the difference between them is its varying motion for the day. Divide it by 24, which are the hours of the day and the night, and what comes out of the division is the varying motion for one hour. Derive the sun's motion in an hour from the moon's motion in an hour, and the remainder is the moon's speed over the sun for one hour on that day.

You can also find this by another method: Take the sun's argument, for which you reckoned its motion on that day, to the table of the motion of the sun and the moon for an hour engraved in this book, and you will find opposite it the sun's motion for one hour. Likewise take the corrected argument of the moon, for which you reckoned its motion on that day, to this table, and you will find opposite it the moon's varying motion in an hour. Derive from it the sun's varying motion in that hour, and you will find the moon's speed over the sun for one hour on that day.

The Reckoning of the Moon's Visibility at the Beginning of Each Month

[חשבון מראות לבנה בראש כל חדש וחדש — the reckoning of the moon's appearances at the head of each month.]

Know that the moon is not seen at the head of this [new] month except after 29 complete days from its [last] appearance at the head of the previous month — and this is the eve of the thirtieth day, the elapsed month then being deficient — or after 30 complete days, which is the eve of the thirty-first day, the elapsed month then being [full]. [Text continues.]

full. From this you will find that the moon has no [adequate] visibility-interval [to be seen] except on the eve of the thirtieth day — when, that is, it has [a margin of] 29 days during which it could be seen — but it does not have an [adequate] visibility-interval to be seen until the eve of the thirty-first day, when it is plain to all the world that it has [actually] appeared.

On this basis we compute the positions of the two luminaries and the position of the head of the dragon [the ascending lunar node] and its tail for the evening of the 29th day from the day on which the moon was [last] seen, since that is a day belonging to the preceding month — for the month that has elapsed was deficient. By this we determine whether the moon could appear on it; and if it can appear on it, then we count that month, according to the opinion of those who rely on [visual] sighting, as deficient. But if it cannot appear, we rule that that month is full and that the moon appears [only] on the eve of the thirty-first day; and we do not trouble ourselves with a fuller [astronomical] computation of the motion. This procedure, however, is suitable only for those who know the [date of the] moon's [actual] sighting in the elapsed month; if that month could not [be so observed], they were not concerned with it.

For this reason we take another approach to determine from it the [moon's] visibility-interval, namely, that we compute the conjunction of the two luminaries by the varying motion at the end of the month for which we need to know the moon's visibility in the month that follows it. If we find that the moment of conjunction falls anywhere from 7 hours before noon [חצות היום] onward, up to about midday of the following day, then it becomes clear to us that on that day the moon is not seen, and that without doubt it will appear on the evening of the next day; and we have no need to compute the [visual] sighting.

But if the conjunction falls anywhere from midday onward down to a third of the night, or a little more, then the [moon's] visibility-interval will be on the day following this [conjunction day], and we must compute the positions of the sun and moon for the next day. We take the position of the head of the dragon for the evening of that day, and from the position of the head of the dragon we know the moon's latitude — from the wheel of the signs and the side of the latitude. We then add to the corrected position of the moon 3 signs, which are 90°, and we call the sum, after [adding] it, "the half-heaven [arc] for the position of the moon." From this half-heaven [arc] for the moon we extract a chord, by the method of extracting a chord from the tables engraved in this book.

If this half-heaven arc whose chord you have extracted is less than 180° — that is, six signs — the chord will be northern. But if the arc is more than 180°, the chord will be southern. Then multiply this chord by the moon's latitude that you have computed, divide the product by 103 [ק״ג], and the result of the division is the value of the correction; and we keep it [in mind]. If the half-heaven chord and the moon's latitude are both on one side — whether to the north or to the south — we subtract the correction from the corrected position of the moon that we computed for the evening of that day. But if the chord and the latitude are on the two [different] sides — the one to the north and the other to the south — we add the value of the correction to the moon's particular position; and what comes out in your hand after this subtraction

or addition will be the corrected position of the moon a second time. From it we extract the position of the head of the dragon, and what remains is the latitude-arc [lit. "the measure of the latitude"] for the moon. From it, then, you know the moon's latitude and its side; this is the corrected latitude. After this we know the chord of the breadth of the place we are computing for, which is the height of the north pole above the earth at that place. Likewise we know the chord of the remainder of the breadth: if you extract it from 90°. And we now multiply the chord of the breadth of the place we are computing for — which is the height of the pole — by the corrected latitude of the moon, and we divide it by the chord of the remainder of the breadth, [that is] from 90°. The result of the division we add to the position of the corrected moon a second time, if the latitude of the corrected moon is northern; and we subtract it if this latitude is southern. What stands in our hand from the position of the moon, after the addition or subtraction, is the position of the visible [apparent] moon.

We then know the [oblique] ascensions of the degrees in the east when the sun is setting upon them in the west on that night, and we take from those ascensions of the degrees in the east [the ascension corresponding to] the position of the visible moon as it sets in the west at that time. The surplus between them: if it is less than 12°, the moon is not seen on that night. But if the surplus is 12° or more, the moon will be seen, according to the opinion of the common people. Yet some of the masters of this craft say that the moon can appear from a surplus of 11½°, [but] that the moon cannot appear if the corrected latitude is on the northern side; and the moon cannot appear, if the corrected latitude is on the southern side, until the corrected latitude of the moon is 12½°.

שער י״ג — Gate Thirteen: On the computation of the eclipse of the moon from the tables made for our reckoning.

Whoever wishes to know the eclipse of the moon should make an examination at the time of opposition — that is, the time when the moon stands opposite the sun, [the sun] being in the seventh sign from the station of the sun. If the moon's latitude-arc — which is its distance from the head of the dragon — falls within the bounds of the moon's eclipse, which are from 345°13′ to 360°, or from zero [sipra] to 14°47′, or from 165°13′ to 194°47′, [then] he knows that the moon is liable to be eclipsed. But if that arc falls beyond the last bound or before the first, it cannot be eclipsed in the middle of that month. And if it appears to him that the moon is liable to be eclipsed, then let him compute the opposition: if it falls by day, from about one hour or a little more after sunrise up to about one hour or a little more before its setting, it will be clear to him that the eclipse is not visible in that place, and he need not trouble over our reckoning. But if the opposition falls at night, or before [sunrise] or after [sunset] by about an hour, it will become clear to him that the eclipse is visible in that place, and he must compute it — if he wishes to know its magnitude and its times.

The order of his computation is this. Let him compute the position of the sun and moon and the moon's latitude for the hour of opposition, and the latitude-arc; and let him bring out that latitude and the latitude-arc — which is the moon's distance from the head of the dragon — to the lunar-eclipse table, in the columns of the distance and the latitude, in the two eclipse tables that are [made] for the long distance and for the short distance. If he finds them in the table of the short distance alone, let him take what is found there of the digits [of obscuration] and the parts of the motion, and keep them; and let him bring the corrected latitude-arc of the moon to the columns of the number arranged by sixes [six by six] in the table of correction for eclipses of the luminaries, and take the value found opposite it of the fractions [minutes] and seconds in the columns of the parts of the distance, and learn its value out of sixty. He takes, at that value, from the digits and the parts of the motion that he had kept — and this is the value of the [digits of the] moon's circumference taken [as obscured], and the measure of the parts that the moon traverses in its motion during the time of the eclipse. And if it [continues p. 82]

the distance and the latitude in the two tables, he takes the value found corresponding to them in both, from the digits and the parts of the motion, and from the parts of the motion of the moon's obscuration in darkness — that is, whether it is obscured. He writes down each one separately: the value found in the short-distance table by itself, and the value found in the long-distance table. He then derives the surplus between them, in digits and in the two [columns of] parts of the motion, and from that surplus he takes, in proportion to the parts of the distance over 60 (these being the minutes found before the corrected lunar value in the table for correcting the eclipse of the luminaries, as we mentioned above). That proportion taken from the digits of the surplus he adds to the digits found in the long-distance table, and what comes up in your hand will be the total of digits.

The portion eclipsed out of the circumference of the moon, and the measure of the parts that the moon traverses in its motion during the time of the eclipse — keep them.

If the eclipsed digits are more than 12, the whole body of the moon will be darkened and stand within the darkness. And if the eclipsed digits are 12, not more, the entire body of the moon will be obscured, except that it has no standing-still [in totality] within the darkness at all. And if the digits are less than 12, the whole body will not be obscured.

If you wish to know the total of the eclipsed [part] of the body brought to those digits — that is, the total obscured out of the body of the two luminaries at their eclipse — take the value found corresponding to it in the rows for the total eclipsed out of the body of the moon, and that will be the proportion of the eclipsed part out of its body, in the reckoning of 12 for the whole body.

And if you wish to know the value of the duration of the eclipse from its beginning to its end: bring the corrected lunar value to the table of the changing motion of the luminaries per hour, and take the value found corresponding to it, from the minutes and seconds in the rows for the moon's motion — that will be its changing motion for a single equal hour at that time; keep it. And after that bring the solar value to that table, and take the value found corresponding to it, in the column of the sun's motion, from the minutes and seconds — that will be the sun's changing motion for a single equal hour on that day. Subtract that [solar motion] from the moon's motion per hour that you kept — this is the moon's swiftness over the sun in its travel during a single hour. Divide by this swiftness the parts that the moon traverses during the time of the eclipse that you kept. If the body of the moon has no surplus [remaining] in the darkness at all, and it is not wholly eclipsed, then what comes out of the division will be the value of the hours during the time of the eclipse from its beginning to its middle — which is the time of opposition; and these are called the hours of the eclipse. And if the body of the moon is wholly eclipsed and stands within the darkness, you divide by it [the swiftness] the moon's motion and keep what comes out of the division. And further you divide by the swiftness the parts of the moon's motion of obscuration within the darkness, and what comes out of this second division will be the value

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of the hours of obscuration. Keep the hours of the eclipse that you kept, and let the two of them together [yield] the hours of the eclipse from its beginning to its middle. And from there you will understand the time at which the eclipse begins and the time at which it ends, whether by day or by night, in this manner.

If the body of the moon has no standing-still in the darkness, or its whole body is not wholly obscured, you know the hours of the duration of the eclipse that come out for you from dividing the parts of the moon's motion at the time of the eclipse by the moon's swiftness. Subtract from them those of the hours in which the opposition occurred from the night [side], corrected by the difference between the days and the nights. What remains of them will be either the hours coming from the night at the beginning of the eclipse, or the hours remaining from the day. If the opposition is at the beginning of the night, and the middle of the eclipse is the very moment of the opposition — when the moon stands directly opposite the sun — then from that moment onward it goes on to separate from the opposition and to be delivered from the eclipse. And from here, if you come to know the end of the eclipse, when the moon entirely separates from the degrees of the earth's shadow: add to the hours of opposition the hours of the duration of the eclipse that you subtracted, and the sum will be the hours coming from the night when the moon emerges entirely from the force of the eclipse — or the hours coming from the following day, if the opposition was at the end of the night.

And if the body of the moon is wholly obscured and has a standing-still within the darkness, you know the hours of the duration of the eclipse and the hours of the duration of the obscuration. Combine the hours of the duration of the eclipse with the hours of the duration of the obscuration, and bring out their total from the hours of the opposition, which is the middle of the duration of the eclipse. What remains of them will be the hours in which the moon is eclipsed [as it comes] to the eclipse and enters the circle of the shadow. And again bring out from the hours of opposition the hours of the duration of the obscuration alone, and what remains of them will be the hours in which the body of the moon completes its entry within the circle of the earth's shadow and is wholly obscured, the whole body of the moon plunging deep to travel within the darkness — up to the [moment of] opposition, which is the middle of the duration of the eclipse. And from that moment onward it inclines away from the opposition and goes on to emerge from the darkness. And from here you add to the hours of opposition the hours of the duration of the obscuration, and the sum of them will be the hours in which the body of the moon completes its emergence from the circle of the earth's shadow and the light begins to be found upon its body. Add to them the hours of the duration of the eclipse, and there will come up in your hand the hours in which the whole body of the moon returns to shine entirely as was its way before it came to the boundary of the eclipse.

And if you wish to know toward which side of the dividing circle (the horizon) [ed.: מאופן המפריש — read here as the horizon, since the side of darkening is determined from the rising and setting signs via the oblique-ascension table, not from the equator] the darkness begins to appear on the body of the moon, and toward which side the darkness separates from its body at the end of the eclipse upon its [re]appearing at the close of the eclipse: bring the eclipsed digits of the moon to the table of the wicks of darkness and their immersion, and take the value found corresponding to them in the third column, written above which is "3," which is for the head [beginning] of the moon's eclipse and the end of its reappearing. And likewise take the value found corresponding to them in the fourth column, written above which is "4," which is for the end of the darkness and the head [beginning] of its reappearing. Keep what is found in both of them [בשניהם] or what is found in the third

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[from the values found in the third column] alone, if it is not found in the fourth [quadrant]. And after that come to the wheel of the degrees of the zodiacal signs in the seven climes, and take the value found in it corresponding to the rising sign and the setting sign [הצומח והשוקע] in that clime, and the [value] corresponding to the sign that follows them. Take the surplus between them and multiply it by the number of degrees [already] risen from that sign. Divide the product by thirty, and the result of the division add to the value found before the ascending sign, if it is less than the value before the second one; or subtract it from [that value], if it is greater than the value before the second one. What comes up in your hand afterward — whether the addition or the subtraction — is the corresponding direction [נכוחות] of the rising degree from the ascending sign and the setting [degree] from the setting sign. And know that the side of the corresponding direction of the ascending [point] is the opposite of the side of the setting [point]. And know that if the side of the corresponding direction of the ascending [point] tends [more] to the north, the second [the setting] tends to the south; and you know its side from what is written before the sign in that wheel.

The rule of the matter: the degrees of the ascensions of the warm season run from the head of Aries [Ṭaleh] to the end of Betulah [Virgo] toward the north side; and the degrees of the ascensions of the cold season — which are from the head of the sign Moznayim [Libra] to the end of Dagim [Pisces] — toward the south side. When you know the corresponding direction of the ascending sign at the beginning of the eclipse, bring it out from the value found in the third column of the table of the wicks of darkness and their immersion, and bring them out for the corresponding direction of the moon's latitude: if the latitude is to the north side, the wicks are to the south side; and if it is to the south, you bring it out to the north side. And do likewise with the corresponding direction of the setting sign at the end of the eclipse, and you will have, accurate and clear, the corresponding direction of the darkness at the beginning of its appearing and the end of its reappearing.

And if the moon has a standing-still [in totality] within the darkness, you bring out from the corresponding direction of the setting sign at the beginning of the standing-still — when the moon is hidden within the darkness — the parts found in the fourth column of the table of the wicks of darkness and their immersion; and you bring them out toward the moon's latitude — if to the north, to the north, and if to the south, to the south. And likewise you bring them out from the corresponding direction of the ascending sign at the beginning of its reappearing from the darkness, toward the moon's latitude. By this method you will know the side toward which the darkness begins, in all four of the times — which are the beginning of the eclipse and the beginning of the immersion [נמיכות] of the whole body to wholly darken, and the head [beginning] of its reappearing and the end of its reappearing.

[End of Gate Thirteen]

Gate Fourteen: On Reckoning the Discrepancy Found Between the Moon's Apparent Place — as Seen by the Inhabitants — and Its True Place as Computed from the Tables, in Both Longitude and Latitude

Know that the parallax — the discrepancy in apparent position between a star's place relative to the zodiacal sphere, as viewed from the center of the earth (which is what the tables compute), and its place as it appears to all who dwell upon the surface of the earth — is in principle liable to occur for every one of the planets, indeed for all the stars. Yet for most of them it is no more than a slight effect, imperceptible because of the great distance between the planet's station in its orb and the center of the earth: relative to that distance, the half-diameter of the earth that lies between the two — the two distinct vantage points of the two observers — counts for nothing in comparison.

This discrepancy is perceptible in the case of the moon, because the moon's orb is not far from the earth; that whole distance is small. It is likewise perceptible for the sun, and it is also perceptible for Venus and for Mercury — except that we take no account of it for these two stars. Even though it is perceptible in them, it does no harm to the reckoning of their courses in any respect. We do reckon with the parallax of the moon, on account of the harm that comes of neglecting it in the computation of the solar eclipse; and from there [i.e., for the same reason] we reckoned with the parallax in the computation of the eclipse — in order to be exacting in the reckoning of eclipses.

The parallax of the sun is found in the longitude of its place only. The parallax of the moon, however, is found both in the longitude of its place relative to the zodiacal sphere and in its latitude relative to the [latitude-measuring] sphere — to the north and to the south. It increases and decreases from two distinct causes. The one cause arises from the moon's station relative to the inclined sphere [the deferent], according to whether it is near to the quadrant-arc [the half of the arc of the heavens — that is, near the meridian] or far from it; and this discrepancy varies in its form and in its magnitude from place to place among the dwelling-places of the inhabited world. The second cause arises from the moon's distance from the earth, which grows greater and grows smaller according to the moon's station — at the apogee of its epicycle or at the perigee of that epicycle — and according to the station of the center of the epicycle, at the apogee of the bearing-sphere [the deferent's high point] or at its perigee.

If both of these are at the apogee, the distance will be 64 times the half-diameter [of the earth] and a sixth of it. And if both of them are at the perigee, the distance will be 33 times the half-diameter of the earth and 33 minutes [of a half-diameter]. Between these two distances — which lengthen and shorten according to the moon's station, from the height of the apogee of these two spheres to the depth of its perigee — the parallax varied. It is on account of this that the computation of these distances was difficult and lengthy, and the scholars of this craft labored over the computation and were exacting in it. They devised for it tables, computed in advance, so as to ease the reckoning for those who came after them. I have transcribed those tables into this book, and I shall now set out for you the procedure of computation from these tables, with the help of Heaven.

He who wishes to know the parallax of the moon, in both longitude and latitude, from the tables engraved for our reckoning in this book — at any time the moon is seen upon the earth, for the moment for which he is computing — must first know the degree standing at the meridian at that time, and also the degree rising in the east on the inclined sphere [the ascendant]. He must know the sum of the degrees, on the zodiacal sphere, between the moon's place and the degree of the rising sign — this being the arc of the moon's distance from the rising sign; and the sum of the degrees between the meridian and the rising sign — this being the arc of the distance between the two cardinal points [the two "pegs," i.e., the ascendant and the midheaven]. And he must know the altitude of the degree at the meridian, which is found by this method:

If the lowness of that degree is on the north side, he subtracts it from the latitude of his place; and if its lowness is on the south side, he adds it to the latitude of his place. The latitude of his place that results, after the addition or this subtraction, he subtracts from 90; and what remains is the altitude of the degrees of the meridian at that place — this being the arc of the altitude of the meridian. But if the lowness of the degree on the north side exceeds the latitude of your place, and you cannot extract the larger from the smaller, then determine the difference between them: subtract them from 90, and the remainder is the altitude of the degree of the meridian. This can only happen in places whose latitude does not reach the whole of the [degree's] lowness — for instance, [a latitude of] 24 degrees.

When you set about adjusting all this, you must know the chord of each one of these arcs. You double the chord of the arc of the moon's distance from the rising sign by sixty, and the product gathered from the doubling you divide by the chord of the arc of the distance between the two cardinal points. What comes out from the division you take as a chord, and you treat it as an arc [convert it back to its arc]; and that arc will be the altitude of the moon's place above the earth, at that time, on the side — east or west — on which it stands relative to the inclined sphere. Keep the arc and its chord. Subtract the arc from 90, and the remainder

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is the moon's distance from the point opposite the [dragon's] head [i.e., from the nonagesimal]. Guard it and hold fast to it, for from it you will know the parallax in the circle of the ecliptic [the path of progression] in those tables, in both longitude and latitude.

If the arc of the moon's distance from the rising sign is exactly 90 degrees — which is the value of the perpendicular angle [the right angle] — then the parallax will be found in latitude alone, and there will be none at all in longitude. And if that arc is less than 90, take it out from 90 and keep the remainder. And if it exceeds 90, take 90 out of it and keep the remainder; then determine the chord of that remainder, and double it by the chord of the moon's altitude above the earth which you kept. The product gathered from the doubling you divide by the chord of the moon's distance from the rising sign; what comes out from the division you double by 60, and the product gathered from this doubling you divide by 60; what comes out from the division you treat as an arc, and that will be the arc of the moon's distance from the point opposite the head [the nonagesimal].

That arc is the value of the longitude-angle measured from the perpendicular angle [the right angle]. Take it out from 90 — these being the degrees of the perpendicular angle — and the remainder is the value of the latitude-angle. Keep each one of them; and this value [the latitude-angle] is fit to operate upon at any time when the altitude of the degrees of the meridian lies toward the south side. But if its altitude lies toward the north side, the order is in reverse: the arc that comes out from the table is the latitude-angle, and its complement out of 90 is the longitude-angle. This altitude coming toward the north side is not found in every place — only where its latitude from the equator exceeds the whole lowness, and the moon's latitude as well — those being [places] from the end of the second clime to the end of the inhabited world.

After he knows the latitude-angle and the longitude-angle — by whatever procedure he knows them — he brings the arc of the moon's distance from the point opposite the head, which I instructed you to keep, to the parallax tables, to the column of the number arranged by twos of degrees. He takes the value found opposite that arc in the four engraved columns that follow the column of the sun — namely the 3rd column, the 4th, the 5th, and the 6th — and writes down each one by itself.

He takes the adjusted ḥeleq [argument-portion] of the moon with which you computed the moon's place. If that ḥeleq is less than 180, take its half; and if it exceeds 180, take it out from 360 and take the half of the remainder. With the half of one of these that you hold in hand, enter the column of the number in the parallax tables, and take the value found opposite it among the minutes in the seventh and eighth columns marked above them: "[for] the epicycle." For the value found in the seventh column, determine its value out of 60, and take by that value [the proportional part] from the value found in the 4th column, and add it to the value found in the third column. Likewise the value found in the eighth column: determine its value out of 60, and take by that value from the value found in the sixth column, and add it to the value found in the fifth column; and that which the third column and the fifth column reach

[Text continues into the following gate-procedure on the next page.]

After this addition, the result will be the value of the corrected parallax for the two distances of the moon — the near and the far. Record each of them under its own name, and find the difference between them; this is the difference between the parallaxes of the two luminaries combined into one — keep it under its name. Then take the distance between the moon and the sun in their mean motion — whether from the body of the sun itself, that is, from the degree in the seventh zodiacal sign opposite it, before the sun or after the sun — so that the limit of the distance be 90 degrees [90 = צ׳]. With the value remaining in your hand from that distance — whether it is 90 [צ׳] or less than 90 [מצ׳] — go with it to the parallax tables, and take the value found opposite it in the ninth column, marked "argument of the center" [אופן המרכז]; and know the value of the fractions that you find before it out of 60 [מס׳]. Take from that value the surplus that lies between the third and fifth columns — which is the difference of the two luminaries, as I instructed you to keep under its name — and this value that you take you add to the corrected parallax in the third column.

The sum included in this addition is the parallax of the two luminaries in the epicycle of progression [עגולת המצעד] according to the distance of the moon; keep it. After that, take what is found before the arc of the moon's distance from the nonagesimal point [נקודת נוכח הראש] — from the parallax marked in the second column written upon it: the distance of the sun's parallax in the visual sphere — and add to it what is found in it from the minutes [שברים] and their seconds [שניים], on account of the discrepancy [המחלקת] found in the distance of the sun from the earth, and keep them. Then bring the argument of the sun [חק החמה] to the table of correction [לוח התקון], and take what is found opposite it in the third column of that table, marked upon it "portions of the distance" [חלקי המרחק], and know the value found out of 60 [מם׳]; and take from that value 13 of the seconds [מי״ג שניים] that are in it [?] — by which the sun's parallax differs between its near distance and its far distance — and that value add to what comes out of the second column of the sun's parallax that I instructed you to keep. What is reached by it [is] the sun's parallax after all this, which is its parallax in the epicycle of progression [עגולת המצעד] at that time.

From this parallax, subtract the parallax of the two luminaries that you kept, and the remainder will be the parallax between the apparent place of the moon and the true place of the moon. Keep it, for upon it rests the whole operation that we are coming to explain. After that, take the longitude-angle [זוית האורך] that I instructed you to keep, and bring out its chord [מיתרה], and double it by this aforementioned parallax of the moon; divide the product by 60 [ס׳], and what comes out of the division is the moon's parallax in longitude — keep it. Likewise double the chord of the latitude-span [זרת הרחב — read: זרת = span/breadth] by the aforementioned parallax of the moon, and what is gathered from the product, divide it by 60 [ס׳], and what comes out of the division will be the moon's parallax in latitude — keep it likewise.

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And if the true place of the moon that you corrected stands at that time in the eastern quarter — that is, its distance from the eastern quarter is less than 90 [מצ׳] degrees from the degrees of the ascendant sign [מזל הצומח] — then you add the parallax in longitude to the true place of the moon, and you will have its apparent place for the inhabitants of the settled world at that time in that place. But if the distance of the moon from the ascendant sign is more than 90 [מצ׳] degrees, then it inclines to the western quarter; subtract the parallax in longitude from the true place of the moon — because in this distance the moon leans away from the true place of the moon [?]. With what remains in your hand from the true place of the moon, after the addition or the subtraction, [you have] its apparent place in longitude in the manner of the zodiac at that time in that place. This is the reckoning of the parallax in longitude.

The parallax in latitude you determine in this way: if the position of the moon inclines to the southern quarter, [coming] in to the lower half [of the heavens] from the nonagesimal point [נקודת נוכח הראש], the parallax will be the parallax in latitude to the southern quarter. But if its position is [coming] in to the upper half of the heavens, inclining to the northern quarter from the nonagesimal point, the parallax will be the parallax in latitude to the northern quarter. And it [the parallax] always inclines to the southern quarter, in every place where the height of the pole above the earth is [such] — adding upon the depression [נמיכות] all the latitude of the moon to the north — because the moon in these places cannot incline to the northern side from the nonagesimal point. Let this be a great rule for you, [well] arrived in your hand. And examine the true latitude of the moon in your hand: if it inclines to one side together with the side that the moon leans toward in its coming to the half of the heavens — as when both of them were to the southern quarter — combine them together, and their total will be the moon's parallax in latitude in that quarter. But if one of them is to the north and the other to the south, take the lesser from the greater, and the remainder will be the moon's parallax in latitude to the quarter of the greater. And if both of them are equal as one, then the latitude at that time is void and nil [בטל ואפס], and the moon will be upon the very plane of the zodiac, having no latitude from it at all.

This is the reckoning of the moon's parallax in longitude and in latitude, when it is up above the earth, raised high upon it to the east or to the west. But if the body of the moon is not up above the earth, and it appears upon the very eastern quarter or upon the very western quarter, you proceed in the reckoning of the angles by another procedure. This is its explanation for you: if the moon stands upon the very dividing-plane [עצם המפריש — the prime vertical / horizon-circle] in the eastern quarter, know the depression [נמיכות] of the degree standing in the mid-heaven [בחצי השמים] at that time. If its depression is to the northern quarter, subtract it from the latitude of the climate [מרחב האקלים] that you are reckoning for; and if the depression is to the southern quarter, add it upon the latitude of the climate. What remains in your hand, after the addition or the subtraction, is the latitude of the corrected climate [מרחב האקלים המתוקן]; keep it, and bring it out from 90 [מצ׳], and the remainder — know its chord [מיתרו], and double it in the half-diameter [מחצית הקוטר]; and what is gathered from the product, divide it by the chord of the arc [מיתר הקשת] that lies between

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the degree of the ascendant sign at that time and the degree in the mid-heaven; and what comes out of the division — make it into an arc [הקשיתהו], and let it be the value of the longitude-angle [ערך זוית האורך]; bring it out from 90 [מצ׳], and the remainder will be the latitude-angle [זוית הרוחב]. And if the latitude of the climate is less than the depression of the degree in the mid-heaven, the depression being to the northern quarter, know the surplus between them, and its chord, and double its chord in the half-diameter, and divide it by the chord of the arc that lies between the half of the heavens and the ascendant sign; and what comes out of the division — make it into an arc, and let it be its arc the latitude-angle [זוית הרוחב], and the remainder of that arc from 90 [מצ׳] is the longitude-angle, as was explained to you in the reckoning of the angles for the place where the depression exceeds the latitude of the climate. And proceed with the longitude-angle and the latitude-angle in the procedure that you proceeded with.

And if the moon stands upon the very western quarter, know the angles, and reckon them from the degree progressing [המעלה הצועדת] in the eastern quarter, and proceed in the angles — to know the apparent place of the moon in longitude and in latitude — by the procedure you proceeded with earlier, when they were [reckoned for] the moon in the eastern quarter. And if it stands upon the very half of the heavens [עצם חצי השמים], understand the reckoning explained at the head of the gate [בראש השער].

And if you wish to reckon it by another way: know the distance of the degree in the mid-heaven from the head of Aries [ראש טלה] or the head of Libra [ראש מאזנים] — before them or after them — so that the distance not exceed 90 [צ׳]. Know the chord of this distance, and the chord of [its] remainder from 90 [מצ׳], when you bring it out from 90 [מצ׳]. Likewise know the chord of the depression of the degree in the mid-heaven, and the chord of its remainder from 90 [מצ׳]. Double the chord of the depression of the degree in the mid-heaven by the chord of the remainder of its distance from the ascendant sign — that is, the chord of the distance when you bring out the distance from 90 [מצ׳] — and the gathered product, divide it by the chord of the remainder of the depression from 90 [מצ׳]; and what comes out of the division, double it in the half-diameter, and divide the gathered product by the chord of the degree's distance from the ascendant sign; and what comes out of the division will be the longitude-angle for the mid-heaven; bring it out from 90 [מצ׳], and the remainder will be the latitude-angle. This is the reckoning of these two angles for the dwellers on the equator [שוכני קו השוה], when the moon is in the eastern quarter rising above the earth, or in the western quarter setting below the earth.

Because of the great labor in the reckoning of the parallax by this arrangement, I have copied into this book the tables that Theon of Alexandria [תיון מאסכנדריא = Theon of Alexandria] made for the reckoning of this parallax for the seven climates [שבעת האקלימים], and arranged the distance of the climates in length of days [באורך הימים] by half an even hour [חצי שעה ישרה]. He set the moon as if it were standing at the head of each and every sign of the twelve signs of the firmament, and arranged the height of the moon [גובה לבנה] above the earth at the distance of one even hour from the arc of the heavens until its coming upon the very dividing-plane [עצם המפריש]. There is in this reckoning a small approximation, but because it is easy in reckoning, it has been transferred [copied] into this book. Here is the order of its reckoning for you: know the distance of the moon from the arc of the half of the heavens to the eastern quarter or to the western quarter, namely that you should know the ascensions [מצעדי] of the degree in the mid-heaven in the manner of the orb [of the heavens], and likewise the position of the ascensions of the moon in the manner of the constellations, where the moon stands. And if the position of the ascensions of the moon is more than half the ascensions of the mid-heaven, then the moon stands to the eastern quarter, and you subtract half the ascensions of the mid-heaven from the position of the ascensions of the moon. But if the moon stands to the western quarter, you subtract the position of the ascensions of the moon from half the ascensions of the mid-heaven, and what remains is the distance of the moon from the mid-heaven.

When you know this distance, divide it by 15 degrees, and the result of this division will be the remaining hours and their fractions by which the moon is distant from the mid-heaven, toward the eastern quarter or toward the western quarter.

[Apply] those hours to the parallax tables for the climates, to the table of the climate in which you stand, and to the constellation in which the moon stands among the constellations engraved for that climate. Seek out the hours by which the moon is distant in them: if it is distant toward the eastern quarter, seek those hours among the hours written above [the line of] midday; if it inclines toward the western quarter, seek it among the hours engraved below midday. And it is proper that the hours in your hand should not exceed the hours engraved at the head of the column and at its end in that constellation.

When you find them, take what is found opposite them in the longitude column among the fractions, and the latitude column, and correct them properly: if the hours include fractions of an hour, [interpolate] from the excess that lies between that hour and the hour after it in the longitude and latitude columns, in the way you correct all the computations you need to correct. Do likewise in the second constellation, the one [adjacent to] the constellation in which the moon stands: take what is found before the hours in your hand in the longitude and latitude columns of the second constellation. And when you have corrected and adjusted them, you take the excess between the longitude and latitude fractions found in the first constellation and those found in the second constellation, and keep the excess between each of the two, each one on its own, in order to correct from it the parallax for the degrees in which the moon stands, of the constellation in which it stands.

The method of its correction is as follows: Know what is the ratio of the degrees in which the moon stands of the constellation, if you measure it against thirty; and take, in that ratio, from the excess between the longitude fractions, and keep that on its own. Likewise take that same ratio from the excess between the latitude fractions, and keep it. If the longitude fractions before the first constellation are less than their [counterpart] fractions in the second constellation, add the ratio you kept to the longitude fractions of the first; and if they exceed it, you subtract it. Do likewise with the ratio you kept from the latitude fractions: if the first constellation [is the lesser] you add it to them; if it is the greater, you subtract it from them. And what comes up in your hand thereafter from the two of them is the parallax of the moon at that time...

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in longitude and in latitude. Keep them in their sign and in their measure. And after this, bring the corrected portion of the moon to the correction table, to the row of the number arranged by 6 and 6 [i.e., sixes], and take what is found opposite it among the fractions in the fourth column, over which is written "manner of the orbit" [the cycle]. Know the ratio of those fractions out of 60, and take that ratio from the excess between the longitude fractions you kept, and likewise from the excess between the latitude fractions, and add the ratio taken from each one of them upon itself — longitude upon longitude, and latitude upon latitude — and you will have the corrected parallax in the fourth column.

And after this, bring the distance between the sun and the moon, [the moon] whose position you corrected at that time, to the row of the number arranged by 6 and 6 mentioned above, and take what is found opposite it among the fractions in the 5th-and-5th column [the column arranged by fives], over which is written "manner of the result." Take, in the ratio of those fractions out of 60, from the longitude and latitude fractions you corrected in the fourth column, and add the ratio taken from each one of them upon itself — longitude upon longitude, and latitude upon latitude — and what comes up in your hand will be the parallax in longitude and in latitude corrected in the fifth column. These are the moon's parallax at that time according to its distance from the earth at that place. This is the method of computation by these tables.

When you have completed the procedure in them, to know the value of the parallax in latitude and in longitude, it is thus: You know the true latitude of the moon from the manner of the constellations at that time, as is explained in Gate Eleven, and you know the side of the parallax in latitude at that time at that place from these tables. You learn it from what is marked in the latitude columns. If the true latitude of the moon and the parallax of its appearance in latitude come from one side — whether both to the north or both to the south — you join them together as one, and the joined sum of them is the apparent latitude of the moon to that side. But if they come from two sides, you subtract the lesser of them from the excess, and what remains is the apparent latitude of the moon to the northern quarter — this is the parallax in latitude.

As for the parallax in longitude that comes out from the tables, you add it onto the true position of the moon if the moon inclines toward the eastern quarter from the point opposite the [dragon's] head, and you subtract it from its position if it inclines toward the western quarter from the point opposite the head. And what comes up in your hand, after the addition or the subtraction, is the apparent position of the moon at that time at that place.

Be watchful in these tables — in the longitude computation — to know when the moon will arrive at the point opposite the head, since its being upon one arc of the great manner [orbit] is inclined upon them, and the moon has no parallax in longitude at that time. You take heed of this when the moon is upon the arc of midday, or about one hour before it or after it. You examine the parallax in longitude in the constellation of the moon, [to see] whether it changes its course in the second constellation, the one after it — as for instance when it is increasing in this hour, in this constellation, over the hour before it…

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or after it. Or [whether] its course differs in the second constellation. At that time you join what is found before the second [constellation] with what is found before the first, and take from their joining, in the ratio of the degrees that the moon has traveled of the constellation in which it stands, out of thirty.

Know how much this ratio adds upon the longitude of the constellation before that of the moon, or how much it falls short of it, and keep it, for it is the value of the correction. Add that ratio onto the longitude of the moon's constellation — if it is short, subtract it from the longitude of the moon's constellation; if it is in excess — and there will come up in your hand the parallax in longitude proper to the moon at that time, according to the degrees that it has traveled of the constellation.

You correct this parallax by the fractions of the fourth column and the fifth column, from the correction table, as you learned, and you add it onto the true position of the moon, or subtract it from it, in the order explained above.

Likewise you should take heed lest there be parallax in latitude at the time when [the moon] stands to the northern quarter in this constellation, while it is to the southern quarter in the constellation after it; or in the parallax [the reverse]. And when this happens to you, you join the latitude before the two constellations, and take from their whole together, in the ratio of the moon's degrees in the constellation, out of thirty. You then know that ratio you have taken — whether it is in excess over the latitude before the moon's constellation, or short of it — and keep the ratio, [noting] whether it is in excess or short.

If it is in excess over the latitude of the moon's constellation, you bring out the latitude of that constellation from that excess, and what remains is the parallax in latitude toward the side of the moon's constellation. And if the excess is short, you bring out the excess from the latitude before the moon's constellation, and what remains is the parallax in latitude toward the side of the moon's constellation.

When you know this parallax, you correct it by the fractions of the fourth column and the fifth column, from the correction table, as you did above, and you proceed by adding it onto the true latitude of the moon and subtracting from it the first procedure. This computation will be properly corrected if the moon stands at the very center of the manner of the constellations [i.e., at the heart of the constellation].

The substance of the matter of the first computation, explained at the head of the Gate, is that there is no approximation in it. The computation of these tables is easier than the first, and the approximation found in it does no great harm.

[End of Gate Fourteen]

Gate Fifteen — On the Computation of the Solar Eclipse from the Tables Engraved for Its Reckoning

A solar eclipse, as you already know, is caused by the body of the moon coming to stand directly opposite the body of the sun, between the sun and the eyes of those who dwell on the earth. This can occur only when the moon, joined to the sun, stands together with it upon the plane of the surface of the zodiac, or inclines away from it toward the north or toward the south by an inclination smaller than half their diameters — that is, when the moon is not far from the dragon's head or its tail, but at most some 20 degrees 16 minutes before the tail or after it, or 10 degrees 57 minutes before the head or after the head.

From this you may examine, at the conjunction of the two luminaries at the close of the month, the latitude argument by which you know the latitude of the moon.

If that argument lies between 349 degrees 3 minutes and 360 [ש״ס], or from zero up to 20 degrees 16 minutes, or from 159 degrees 44 minutes up to 190 degrees 57 minutes — that is, an argument that adds onto the head of the limit or falls short of the end of the limit — then in any one of these you must reckon with the possibility of a lunar eclipse [recte: solar eclipse] in one of these positions, namely if the conjunction occurs by day, or a little before sunrise, or a little after sunset at that place. But if the distance argument falls short of the first limit and exceeds the last limit, you need not reckon with a solar eclipse in that month anywhere in the inhabited world.

When you do reckon with a solar eclipse at a place where the conjunction occurs by day, and your judgment inclines toward computing it from the tables made for its reckoning, you compute the conjunction of the two luminaries as it falls — by day, or near daytime, in the morning or in the evening, at the place for which you are computing. You determine the zodiacal sign and the ascendant at that moment, and the meridian point standing at midheaven. You then find the distance of the conjunction, measured in equal hours, from midheaven — that is, to which side it inclines, whether east or west, in other words whether before noon or after noon. You determine this in the following way:

If the conjunction was before noon, you reckon the hours from the midnight of yesterday up to the moment of the conjunction, and you cast them out of 24; the remainder is the number of hours of the conjunction's distance from midday toward the east.

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And if the conjunction was after noon, up to sunset of the sun, you subtract from those hours half of the hours of that day; the remainder is the hours of the conjunction's distance from midday toward the west.

Now find the parallax in longitude of the moon at the moment of the conjunction, and correct it by the fractions in the fourth column of the correction table. The figure that comes up in your hand, divide it by the rate of the moon's variable motion for one equal hour; the result you hold is the hours of the first parallax-interval. If the moon's position is more than 90 degrees distant from the ascending sign, you subtract the hours of the first parallax-interval from the hours of the conjunction, and the hours that remain in your hand — afterward determine their distance from midday, as you learned when you found the moment of the conjunction. These are the hours of the second distance.

Now find the moon's parallax in longitude for that distance, and correct it by the fractions of the fourth column; this gives you the second parallax. Divide this parallax by the rate of the moon's variable motion for one hour, and you obtain the hours of the second parallax-interval. If the moon's position — that is, the conjunction-distance — is less than 90 degrees from the ascending sign, you subtract the hours of the second parallax from the hours of the conjunction; but if it exceeds 90, you add them. The hours that come up in your hand — determine their distance from midday a third time, just as you did the first and the second time.

If the parts of the third parallax equal the parts of the second parallax, it is established for you that the hours of the second parallax (after you subtracted them from the hours of the conjunction, or added them onto it) are the corrected mean hours of the eclipse. But if the third parallax exceeds the second, note by how much it exceeds it and keep that. Then add to the hours of the second distance the hours that come out for you from the second parallax — one sixth of an equal hour [from each]. From these determine the corrected parallax in the fourth column, as you did before; and determine by how much it exceeds the second parallax. Multiply the excess by six, and from this extract the multiple [obtained] from the speed of the moon over the sun in one hour — the remainder is the adjusted motion. Divide by it the increment of the third parallax over the second, which I instructed you to keep; you obtain one part of an hour. Add that part to the hours of the second parallax, and you obtain the corrected hours.

But if the third parallax is less than the second, determine the deficit between them, and multiply it by six; the result of the multiplication, add it to the speed of the moon over the sun in one hour, and you obtain the adjusted motion. Divide by it the deficit of the third parallax below the second; and the part of an hour that comes out for you from this parallax, you subtract it from the hours of the second parallax,

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and you obtain the corrected hours.

When you know the corrected hours — in whichever of the two ways you came to know them — multiply them by the rate of the moon's variable motion in one hour, and also by the rate of the sun's variable motion in one hour. Keep what comes out from each of the two, and their sum will be the combined motion of the two luminaries together.

If the moon's distance at that moment is less than 90 degrees from the ascending sign, you subtract the corrected hours from the hours of the true conjunction, [together with] the sun's motion from its place at the moment of the conjunction, and the combined motion of the two luminaries together from the moon's place at the moment of the conjunction; and likewise the latitude argument. You also derive, from the latitude-distance, the motion of the dragon's head in the corrected hours.

If the moon's distance from the ascending sign at the moment of the conjunction exceeds 90 degrees, you proceed in everything mentioned above in the opposite manner: addition in place of subtraction, subtraction in place of addition. What remains in your hand of the hours of the conjunction after this subtraction or addition, these are the corrected mean hours of the eclipse. What comes up in your hand for the place of the moon and the sun, and the latitude argument, and the moon's argument — this is their standing at the mid-point of the eclipse.

After this, find the moon's parallax in latitude at that moment, correct it by the fractions of the fourth column, and find the side of the latitude-parallax: if its side and the side of the moon's true latitude are found to be on one and the same side, add them together and keep their sum; but if the moon's latitude and its latitude-parallax come from two different sides, you subtract the one from the other and keep the remainder. What stands in your hand from the sum or from the remainder is the apparent latitude of the moon at the mid-point of the eclipse.

If this latitude exceeds half the two diameters of the moon and the sun — whose mode of computation was set out for you inside the table of the combined motion of the two luminaries per hour or [per other unit] — when this latitude equals half the two diameters, the sun is not eclipsed in that month. But if that latitude is less than the two diameters [recte: less than the half-sum of the two diameters], you know for certain that the sun is eclipsed in that month.

When it has become clear to you that the sun is eclipsed, you bring the minutes of the apparent latitude at the mid-point of the eclipse to the tables of the solar eclipse. If you derive that latitude as engraved in the second table — that of the solar eclipse pertaining to the near distance — and not in the first table, which pertains to the far distance, you take what is found opposite it in that table, of the digits of the eclipse and the arc of motion, and keep them. After this, bring the moon's argument to the correction table, to the columns of the ordered number

you will know the latitude [מרחק] from the parts found in the third column opposite it. Take 6 [ו׳] times 6 [ו׳] for the digits [אצבעות], and that will be the sum.

Take, in the proportion of that motion's [המהלך] arc, from sixty [מששים] of digits, the sum of those eclipsed [הנלקות] of the motion [or: of motion] from the start of the eclipse [הלקות] until its mid-point.

And if the apparent latitude of the moon at mid-eclipse is found in the two eclipse tables [שני לוחות הלקות] — that is, for the greater distance [למרחק הארוך] and the lesser [הקצור] — you take whatever is found in those two tables of those digits, of the arc and the motion. Find the excess [העודף] between the two of them, in the digits of motion, and keep it. Take, from that excess, in the proportion of the latitude-parts [חלקי המרחק] opposite the moon's quota [חק לבנה] in the correction table [לוח התיקון], out of sixty [מששים]. Add to the long [arc] the portion of the excess taken upon the digits, upon the digits found in the long-distance latitude table [בלוח המרחק הארוך]. What results in your hand is the sum of the eclipsed digits of the sun, and the sum of the arc that the moon traverses from the start of the eclipse to its mid-point.

Now divide that arc by the speed of the moon over the sun [מהירות לבנה על חמה] per single hour [לשעה אחת]. What comes out of the division will be the number of hours from the start of the eclipse until its mid-point. Subtract those hours from the hours of mid-eclipse [אמצעית הלקות], and you obtain the hour at which the eclipse begins. Likewise, add those hours to the hours of mid-eclipse, and you will find the hours at which the eclipse ends — by the nearest mean reckoning [החשבון הבינוני הקרוב], at its beginning and its end.

And if you wish to refine [לתקן] the hours of the beginning and the ending properly, compute the parallax [חילוף מראות] of the moon in longitude [באורך] alone. You need not refine it by the fractions of the fourth column [בשברי הטור הרביעי], for its deficiency does no harm to you here. Compute this parallax for the start of the eclipse, for its mid-point, and for its end. Determine the excess [העודף] between the parallax [חילוף המראה] at mid-eclipse and the parallax at its start and its end. You then find: upon the greater of the excess — between the one and the other [בין אחד ובינו] — add to the excess that is between it and the second, and divide each one of them on its own by the speed of the moon. What results in your hand will be two fractions of an hour [שני חלקי שעה]; and if they are not equal, add each one of them to the hours that you reckoned for the arc, from the start of the eclipse to its end. Those hours will be reckoned for you upon the two numbers, the greater and the smaller [גדול וקטון].

And if the place of the moon at mid-eclipse stands toward the western quarter [לפאת מערב] — that is, when it is more than 90° [מצ׳, i.e. more than 90 degrees] distant from the ascending sign [ממזל הצומח] — you subtract the greater time

[הזמן הגדול] from the hours of mid-eclipse, to know the hours of its beginning [ראשיתו], and you add the smaller time [הזמן הקטון] to the hours of mid-eclipse, to know the hours of its end [אחריתו]. But if the moon at its mid-eclipse stands toward the eastern quarter [לפאת מזרח] — that is,

— when it is less than 90° [מצ׳ פחות] distant from the ascending sign, you subtract the smaller time [הזמן הקטון] from the hours of mid-eclipse, to know the hours of its beginning; and you add to it the greater time [הזמן הגדול], to know the hours of its end. By this method you will find the hours of the eclipse's beginning and the hours of its end, by the corrected reckoning [החשבון המתוקן].

And if you wish to refine the digits of the eclipse [אצבעות הלקות], and to know their value [ערכן] in the body of the sun [בגוף החמה], you bring those digits to the table of the sum of the part that falls [הנאפל] from the body of the luminaries [מגוף המאורות] at their eclipse, and take what is found opposite them in the table of the eclipsed part of the sun's body [בלוח הנלקה מגוף החמה] — that will be the eclipsed part of its body to the sight of the eye [למראית העין].

And if you wish to know toward which side the eclipsed corner [פנת האפלה] in the circle of the eclipsed body inclines [נוטה הוא], in the coming [הבא] of the eclipsed digits from the diameter [מן הקוטר] in the eclipse-digits, in the proportion of them found in the table — bring [it] to the eclipse-table, from the table of the sum of the falling [part] to the table of the wicks of darkness and its lowness [פתילות האפלה ונמיכותה], and take what is found opposite them in the second column of that table, upon which is written "head of the solar eclipse and its end" [ראש לקות חמה וסופו]. These will be the parts of the wicks [חלקי הפתילות], like the circle of the sun. After that, know the quarters [ניכוחות = נביחות / "directions, oppositions"?] of the ascending sign and the setting [sign] [השוקע] at the start of the eclipse and at its end — found like the circle facing the heads of the signs [נוכח ראשי המזלות] in the climes [באקלימים], as is explained in the gate of the lunar eclipse [בשער לקות הלבנה]. You bring out the parts of the wicks [חלקי הפתילות] like the circle of the sun, facing [מנוכח] the ascending sign, toward the quarter of the eclipse [אל הלקות] at the end of the eclipse, toward the quarter of the moon's latitude [פאת רוחב הלבנה]. There will appear to you the quarter of the darkness [פאת האפלה] facing the sun's circle at the end of the eclipse. Likewise you bring out the parts of the wicks, from facing [מנכיחות] the setting sign, at the start of the eclipse, toward the quarter of the moon's latitude. There will appear to you the quarter of the darkness facing the sun's circle at the start of the eclipse.

Gate Sixteen — שער י״ו

[Gate Sixteen] On the reckoning of the time at which each and every one of the wandering stars [כוכבי הנבוכה — the planets] comes to be hidden beneath the sun's light, and the time of its emergence to come out from beneath the light. At these times they are called "rising and setting" [זורחים ושוקעים]. This reckoning does not proceed by a single order.

And here is their order for you: for Saturn [שבתאי], Jupiter [צדק], and Mars [מאדים]. When the corrected quota [החק המתוקן] for each and every one of these three stars [מג׳ הכוכבים האלה] is from zero [מן סיפרא] up to 180° [ק״פ מעלות], it is called "eastern to the sun" [מזרחי לחמה], because it rises [עולה] before its rising [עלותה]. And if the corrected quota is from 180° [ק״פ] up to 360° [ש״ס], it is called "western to the sun" [מערבי לחמה], because it sets after it [שוקע אחריה]. Each one of them is found either visible in the east before the sun, or hidden in the west after the setting of the sun — and it is hidden beneath its light in the west; or it comes out from beneath the light in the morning, when it is distant from the sun, before it or after it, by a known distance, which is called the arc of vision [קשת המראה]: for Saturn [לשבתי], [י״ד מעלה] 14°; for Jupiter [לצדק], 12° and 40 minutes [י״ב מעלה ומ׳ שברים]; and for Mars [למאדים], [י״ד מעלה וחצי מעלה] 14½°.

One who wishes to know when one of them is hidden in the days of its appearance [בימי הגלותו], or appears in the days of its concealment [בימי הסתרו], should attend to this reckoning at the moment that the corrected quota of the star will be 340° [ש״מ מעלה] — if he seeks [the moment of] its concealment; and at the moment that the quota will be [the appropriate value, if he seeks its appearance]. And the procedure of this exacting reckoning [החשבון הזה המדקדק] is that you know the distance of the star from the wheel of the equator [מאופן המישור] and the degree of the wheel of the signs [מאופן המזלות] that comes with it to mid-heaven [אל חצי השמים], and the degree that rises in its rising upon the earth, and the degree that sets at its setting — all as is set forth in this reckoning in the sixth gate [בשער הששי].

After you have grasped this concerning the affairs of that star: if the star is hidden and you seek to know the day of its appearance [יום הראותו], know the degree upon which the star rises in its rising; on the day when the corrected quota will be for it about 20 degrees [כ׳ מעלה], know the ascensions [מצעדי] of that degree in the clime [באקלים] you are in. Take out those ascensions from the ascensions of the sun's degree [ממצעדי מעלת החמה]. If the [degrees] between them are equal to [כדי] the arc of vision [קשת המראה] for that star, know that the star will be seen [יראה] on that night; and if they are short of [חסרים] the arc of vision, know that it is not yet seen [עדין לא נראה].

and if they exceed the arc of vision, know that the star has already become visible. And if the star is yet to be hidden, and you wish to know when it will set, compute its position for the day on which its corrected datum will be 340 [ש״מ] degrees, and you will know the degree at which the star sets at its setting. Take the ascensions of the degree opposite that one, in the seventh sign from it [i.e., the degree 180° away], and also the ascensions of the degree opposite the sun's position, and extract the ascensions found opposite the sun's position from the ascensions of the degree that lies opposite the star. If the degrees between them are equal to the arc of vision, the star will be hidden on that night; and if they fall short of the arc of vision, it has not yet set; and if they exceed the arc, the star has already set.

If the star is visible or hidden on that day, and you wish to determine the day on which it became, or will become, visible or hidden, do as follows. Find the value of the distance between the star and the sun in their ascensions — whether it exceeds or falls short of the arc of vision for that star — and divide that value by the corrected true motion of the star for one day. What results from the division is the number of days between your reckoning and the day of the star's appearance or concealment. If you found it [already] visible and your aim was to determine its day of appearance, you will have established that it became visible that many days ago.

And if you wish to determine its day of concealment, you will have established that it will be hidden that many days from now. Likewise by this method, if you find it [already] hidden. Since you know the corrected motion of the star for one day — if you are correcting its position for a day, while you compute for the day after it, the second [day], you will then know the excess between the two positions, which is the star's true motion for one day. Extract that from the corrected motion for one day, and the remainder will be the true corrected motion of the star for one day. This is the reckoning of the appearance and concealment for the three superior planets.

As for the two stars [planets] beneath the sun — namely Venus [נוגה] and Mercury [כוכב] — they appear and are concealed in the west when they are after the sun [following it], and they appear and are concealed in the east when they are before it [preceding it]. From this, they have four conditions for their appearance and concealment. The first condition is when their corrected datum is 20 [כ'] degrees: they are visible in the west, or fit to appear. The second condition is when the corrected datum is 160 [ק״ס] degrees: you must reckon for their concealment in the west. The third condition is when the corrected datum is 200 [מאתים] degrees: you must reckon for their appearance in the east. The fourth condition is when the corrected datum is 340 [ש״מ] degrees: you must reckon for their concealment in the east. You reckon for them in all these four conditions just as you reckoned for the two planets [Venus and Mercury] in the two conditions, in order to know their distance from the plane of the equator [אופן המישור] — the degree at which they rise at their rising, and the one at which they set at their setting. Proceed with the rest of the matters of the reckoning by the practice you followed for the former [superior planets].

If you need to know their corrected true motion for one day, and their corrected motion is retrograde [נזורים] on that day, add it to the sun's varying motion on that day, and the two motions together will be their corrected true motion for one day. But if they are direct [ישרים] in their motion, you know the excess between their motion and the sun's motion, and that will be their corrected true motion for one day.

Because this reckoning is long and difficult for most people, the masters of the craft devised for it tables, in order to compute by a shorter way. And even though there is an approximation in it, it does no harm of a kind that would be apparent to anyone [lit. visible to the whole world]. By these tables they computed the arc of appearance and concealment for the five wandering planets [כוכבי הנבוכה] when they are at the head of each and every sign. In the fourth clime [אקלים], they set down what that arc would be by the ascensions of the fourth clime, from the degrees of the signs that rise straight [the equal degrees of the signs at the celestial equator], and they made those degrees a measure and a standard for the rest of the climes, and did not concern themselves with the approximation found among them.

If you wish to compute the concealment and appearance from these tables: if you are reckoning for the three superior [planets], take the figure found in the tables for the divisions of the morning rising [היציאה בשחר] or the twilight setting [הביאה בנשף] inscribed before the sign in which the star stands, and likewise for the second sign after it [the next sign], and find the excess between them, and multiply it by the degrees at which the star stands within the sign in which it stands. Divide the product by thirty [שלושים], and add what results from the division to the divisions found in the sign of that star, if they are deficient from the divisions belonging to the second sign after it; but if they exceed the divisions of the second sign, subtract them from it. What remains in your hand from the divisions of the rising or setting, after the addition or the subtraction, is the arc of appearance or concealment for that star. If they are equal to the distance between the star and the sun — whether before it [preceding] or after it [following] — it will appear or be concealed on that day. And if they are not equal, then they are coming to appear or to be concealed, or have already become visible or been concealed, by the number of days that come from the excess between the arc of appearance and the distance from the sun, divided by the star's corrected true motion.

And if you are reckoning for the two stars [planets] beneath the sun, take the figure found opposite the sign in which they stand, from the divisions of their appearance or concealment in the east, or their appearance or concealment in the west, inscribed in the four tables, for whichever matter you are reckoning; and likewise take the figure found opposite the second sign after it, and proceed with them by their practice for the three superior planets.

שער י״ז [Gate Seventeen] — On the reckoning of the motion of the fixed stars [כוכבי שבת], the motion of the point of the apogee [רומי הרום], and the heads of the dragons [ראשי התנינים — the nodes] for the five wandering planets, and the positions of all these points at the root-epoch of the reckoning of this book [שורש חשבון הזה הספר], and the names of the fixed stars in the first and second magnitude [במעלה הראשונה והשניה], which are inscribed on the tables of this book.

The motion of the fixed stars rests upon the poles of the orb of the signs [קטבי אופן המזלות — the poles of the ecliptic], and the measure of the motion of all of them is one measure, equal in its magnitude for each one of them. The latitude of each one of them from the plane of the ecliptic [אופן המזלות] stands at one value, neither adding nor subtracting nor inclining from side to side. In this arrangement all the investigators of antiquity, from earliest days, found it, and all the ancients found the order of their motion to be from east to west, in the manner of the motion of the orderly [fixed] stars [כוכבי לכת המיושר], and they all agreed that the motion has no variation in it. The ancients testified that it is one degree in every hundred years; and some of the later ones who came after them testified that it is one degree in every sixty-seven [שבע וששים] years. We have set its motion in a table that we made for it at one degree per hundred years, after the opinion of the ancients.

By the measure of this motion, they found the motion of the apogee [גובה הרום] and the heads of the dragons [the nodes] for the five wandering planets. Here, then, is their position at the head of cycle 257 [רנ״ז], which is the root-epoch of the reckoning in this book:

- Saturn's [שבתאי] apogee [גובה רומו]: [בשלוש] 3 degrees of the sign of Sagittarius [ממזל קשת]; the head of its dragon [ראש תנינו — its node]: 23 [כ״ג] degrees of Capricorn [בגדי].
- Jupiter's [צדק] apogee [גובה רומו]: [כ״ב] 22 degrees in Virgo [בבתולה]; the head of its dragon: 2 [בשתי] degrees of it [Virgo].
- Mars's [מאדים] apogee [גובה רומו] and the head of its dragon: 6 [בשש] degrees of Leo [מאריה].
- Venus's [נוגה] apogee [גובה רומו]: half a degree [בחצי מעלה] from Gemini [מן תאומים]; the head of its dragon: in their like [בכדיהן] from Virgo [מבתולה].
- Mercury's [כוכב] apogee [גובה רומו]: [כ״א] 21 degrees of Libra [ממאזנים]; the head of its dragon: in their like [בכדיהן] of Cancer [מסרטן].

These points revolve one degree in every hundred years, as mentioned above.

If you wish to know their position before this time or after it, know how many years there are between the two times, and assign to each 100 [ק'] years among them one degree; and for every 5 [ה'] years, 3 [ג'] fractions [שברים — minutes], which are 36 [ל״ו] seconds [שניים]; and for each and every year, [the corresponding amount]. What comes up in your hand from the measure of the motion — extract it from your stated position mentioned here: if you are seeking their position before cycle 257 [רנ״ז], you will be adding it to their position; if you are seeking [continues p.103]

[continued from p. 102 — the apogee and node positions and their motion rule]

their positions in the future. The apogee of the Sun, which we recorded at the head of cycle 257, is at 15 degrees and a half [ט״ו on the page; the heading combines into 15°30′ — but see below] of the sign of Gemini.

The ancients and most of the sages of the nations, the later ones among them, hold that the apogee of the Sun revolves like the apogees of the other planets, advancing one degree in every 100 years. But Ptolemy alone was of the opinion that the apogee of the Sun does not revolve at all — and the majority of the ancients and the later authorities did not agree with his view, because they found it [the apogee] to revolve like the apogees of the other stars [planets].

The number of fixed stars whose positions the ancients recorded is one thousand and twenty-two [1022 = אלף וכ״ב] stars; and they divide them in their accounts into forty-eight [48 = מ״ח] figures [constellations]. Of these, three hundred and forty-six [346 = שמ״ו] stars they array in the twelve [12 = י״ב] figures that lie in the very plane of the zodiac, or incline a little from it — these are the twelve well-known constellations [of the zodiac] for the whole world, beginning with Aries and ending with Pisces. And of them, three hundred and sixty [360 = ש״ס] stars are scattered over twenty-one [21 = כ״א] figures that lie toward the northern side, away from the plane of the zodiac, figures resembling animals and beasts; and these are their names: … [the names are omitted here in the printed text]. And three hundred and sixteen [316 = ושי״ו] stars are scattered over fifteen [ט״ו 15 =] figures that lie toward the southern side; and these are their names: … [omitted].

All these one thousand and twenty-two [1022 = אלף וכ״ב] stars, which are recorded in these figures, are inscribed in their places and called by their names in the Greek book of Ptolemy; and I have copied them from there into this book [presenting] the positions of the bright stars closest to them, from the second magnitude and up [literally "from the boundary of the second magnitude in brightness"] — and these are the ones inscribed on the instrument of the astrolabe. They are useful in the practice of the science [astrology], for knowing from them the [natal] constellations of people born among mankind; and I have copied along with them the positions of the non-bright stars that nevertheless bear testimony in the nativities of human beings, as a person will see — all of this is set out plainly and in order in this book.

Gate Eighteen [שער י״ח] — On Reckoning the Sign Rising over the Earth and the One Standing at the Mid-Heaven

[On the reckoning of] the rising sign [the ascendant] over the Earth and the one standing at the mid-heaven, from [a count of] the hours elapsed of the day or of the night; and the method of dividing the sphere into the twelve houses, at every moment and for every [terrestrial] latitude.

If you know the number of hours elapsed of the day, from sunrise up to the moment you are inquiring about, or the hours elapsed of the night, from sunset up to the moment you seek — and you wish to know which degree of the zodiac it is that is rising over the Earth at that moment, and which is the one standing at the mid-heaven, and to set [the boundaries of] the twelve houses into which the practitioners of this craft divide the sphere at every moment —

then double the known hours. If they are equal [equinoctial] hours, [reckon them] as being of 15 degrees [each]; but if they are unequal [seasonal] hours, double them by the measures of the seasonal hour — by the measures of the daytime [seasonal hour] if [the hours are] of the day, or by the measures of the nighttime [seasonal hour] if [they are] of the night. From the sum you have in hand: if the hours are of the day, add the doubled amount to the ascensions [מצעדי] of the degree the Sun occupies on that day, [taken] in the climate-tables [the table for the relevant latitude]; but if [the hours are] of the night, add the doubled amount to the ascensions of the degree opposite the Sun's degree — the degree of the seventh sign from its sign. Bring the sum to the table of ascensions of the climate you are in, and opposite it you will find the degree rising over the Earth at that moment.

Take the same sum and bring it to the tables of ascensions in the right sphere [באופן הישר — ascensions at the equator, *sphaera recta*], and opposite it you will find the degree standing at the mid-heaven at that moment. And after this, find the measures of the [seasonal] hour for the ascendant degree, and double them, and add their double to the sum by which you knew the rising sign and the one standing at the mid-heaven. Bring what results to the tables of ascensions in the right sphere, and what is found opposite it is the first house [cusp]. Add to it the double of the measures of the hour, and from the sum you will know the [cusp of the] twelfth house.

[p. 104]

And if you add to the sum by which you knew [the cusp of] the twelfth house the double of the measures of the hour, you will find [opposite it] the ascensions of the rising sign in the right sphere — store that. Take the measures of the hour you have in hand from sixty, and add the remainder to the ascensions of the rising sign in the right sphere, and opposite it you will find the cusp of the second house.

And if you are adding to the ascensions of the rising sign [for] the second house the remaining double of the measures from sixty, you will find the cusp of the third house. Know that the cusp of the fourth house is the counterpart of [is obtained from] the tenth — which is the mid-heaven, standing opposite it [the fourth]; and the fifth house [is the counterpart] of the eleventh, and the sixth [of] the twelfth; and the seventh of the ascendant sign, and the eighth of the second, and the ninth of the third. From here, once you know the cusps of the six houses we have mentioned, you can also know the cusps of the remaining six that stand opposite them.

And if the hours you have in hand are reckoned from midday and onward — as is the practice of those who compute the [planetary] motions — then know how many degrees are gathered for you from those hours, in the [same] way you computed when they were reckoned from the beginning of the day or the beginning of the night: if they are equal hours, double them by the measures of the equal hour, and if they are unequal, [double them] by the measures of the unequal hour; and the sum from the doubling, add it to the ascensions of the Sun's degree in the right sphere, and you will find the ascensions of the degree at the mid-heaven in the right sphere. Bring [it] in among the ascensions in those [climate-tables], to the table of ascensions of the climate, and opposite it you will find the degree rising over the Earth at that moment.

And if you wish to compute the cusps of the remaining houses by another method: once you know the ascensions of the rising degree — how much they are in the right sphere — take out from them the ascensions of the degree at the mid-heaven in that same right sphere; and the remainder you have in hand, take a third of it [אוחז את שלישיתו], and proceed with it as is your custom in the doubling of the measures of the hour. You are then adding the third to the ascensions of the degree at the mid-heaven in the right sphere, and you will know by it the cusp of the eleventh house and the twelfth house; and you subtract the third from sixty, and the remainder you have in hand, add to the ascensions of the rising degree in the right sphere, and you will know the cusp of the second and third house, by the manner of their first reckoning — there is no difference between them at all.

And if you know the degree rising over the Earth and the degree the Sun occupies, and you wish to know the hours elapsed of the night or of the day: find the ascensions of the Sun's degree in the climate you are in, and also the ascensions of the [degree] rising in it; and always subtract the ascensions of the Sun's degree from the ascensions of the rising degree. The remainder you have in hand — if it is less than half the arc of day — divide it by the measures [continues p. 105]...

by the measures of the hours of the day, and the result you obtain will be the divisor for the unequal hours that have elapsed from [the start of] the day. But if there is a remainder, take what remains in hand and divide it by the measures of the hours of the night, and the result will be the unequal hours that have elapsed from [the start of] the night. And if you divide the remainder by the measure of one equal hour, you will obtain the equal hours that have elapsed from the day or from the night.

✻ You can compute this matter in many ways, as was already set out briefly near the beginning of the book. We needed to bring this reminder here, in this gate, in order to attach to it the topics that follow it.

Gate Nineteen [שער י״ט]

On the computation of the years' starting-points for the Sun. This is the time of the Sun's return to the very fraction of the degree of the sign in which it stood at the head of the passing year, or at the start of the conjunction or the event whose years you are reckoning. This is what is called the *revolution of the years* [תקופת השנים].

If you wish to know the revolution of the years — which is the return of the Sun to the very point of its place in the zodiac at the start of the conjunction or the event whose corrected hour you have preserved and committed to your hand — first know how many complete whole years have elapsed from the beginning of that matter up to the year for which you are reckoning it. And do not let that [current] year enter into the total of the reckoning. For each complete year that has elapsed, assign 88 degrees [פ״ח] and 48 minutes [מ״ח] — these being the increment of the solar year over 365 days [שס״ה], which are the days of the Egyptian year on whose foundation the courses [of the heavenly bodies] were arranged in this book. Collect them in hand, and divide them by 360 [ש״ס], which is the number of degrees of the firmament. The quotient will be days [coming out] of the division; and if any remainder is left in hand after the division — a number that did not complete 360 [ש״ס] — divide it by 15 [ט״ו], and you will obtain equal hours. Collect the days in hand, and add those days and hours to the corrected date for the start of that conjunction or event. What comes up in your hand will then be the corrected date by which to compute the courses of the heavenly bodies for the revolution of that year.

And if the hours collected for you from the joining of the hours of the difference with the hours of the corrected date amount to more than 24 [כ״ד], cast out 24 [כ״ד] of them, keep the remainder, and add one day to the days collected for you. Compute the courses of the heavenly bodies with them, and you will find the mean course of the Sun for that year [to be] the same as its mean course at the start of the first year from the conjunction or the event — neither less nor more. Reckon the courses of the rest of the heavenly bodies for that date, and correct it according to the order of reckoning the courses; thus you find its place in the variable [true] course for the revolution of that year.

And if you wish to know the rising sign [the ascendant] at that time, take the degrees remaining to you that did not complete the aforementioned 360 [ש״ס] of the degree you knew from the hours of the date, and add the degrees…

of those [degrees] to the fractions of the rising sign in the climate [you are in] at the start of the conjunction or the event, and cast out their total of 360 [ש״ס]. What remains in your hand is the ascensions of the rising sign at the time of the revolution. From these ascensions you will know the degree of the rising sign, the degree of mid-heaven, and the boundaries [גדרי] of the remaining twelve houses, as was explained in the gate that has passed.

Now, I have prepared two engraved tables. On the first I engraved the surplus of the true solar year over the days of the Egyptian year, from one year up to a hundred years. From this table you will know the hours and times [שעות והזמנים] that you ought to add to the corrected date. And you will know, from joining them [to it], the corrected date for the revolution of the year you are seeking: if you bring the [number of] years lying between the conjunction and the year you are seeking to this table and take what is found opposite them among the times, then add it to the first date.

On the second table I engraved the degrees and minutes that ought to be added to the ascensions of the rising sign at the start of the conjunction, [for] one year up to a hundred years. And if you bring to this table the [number of] years lying between the conjunction and the year you are seeking, and take what is found opposite them among the degrees and minutes, and add them to the ascensions of the rising sign at the start of the conjunction, the sum collected from the ascensions — by which you know the rising sign, the degree of mid-heaven, and the boundaries of the remaining houses at the time of the revolution.

Furthermore, I have prepared for you tables on which I engraved the mean course for the six heavenly bodies apart from the Sun, in [units of] the days of the true solar year, from one year up to the number of years into which the tables are divided. In this work I have come to ease the reckoning, so that you will not have the trouble of computing the mean course for these heavenly bodies for the revolutions of the years from the tables of the [mean] course divided into years, months, days, and hours. You are able to correct the reckoning from these tables.

If the mean course for all the heavenly bodies at the start of the conjunction or the event is known to you and kept by you, then when you come to compute the mean course from these tables, know the number of complete years from the start of the conjunction up to the year whose revolution you are seeking — and it shall not exceed the total [of the table]. Bring those years to these tables, take what is found opposite [them] from the mean course for each and every heavenly body, and add it to the mean course kept by you for that heavenly body at the start of the conjunction or the event. The sum collected from them will be the mean course for that heavenly body at the hour of the revolution, and from it you compute its variable [true] course for that time.

As for the mean course of the Sun: it neither adds to nor subtracts from its mean course at the start of the conjunction or the start of the event; and for this reason its course is not divided in these tables.

Gate Twenty — On the Explanation of the Aspects Formed Between the Planets and the Rays They Emit

Gate Twenty. On the explanation of the aspects [מראות] that are formed between the planets, and concerning the rays [זיקים] that they emit — their illumination upon a bow [arc] of a quarter, a sixth, or a third — both from their standing place backward and also forward, and concerning the standing positions [המעמדות] that lie opposite and counter to one another, and also the conjoined rays [הזיקות הנחברות] emitted in the firmament when two planets are joined together at a single degree. This is also an explanation of the order of the courses of all the planets [as they pass] from house to house among the houses of the firmament in which they stand.

When you have come to know how the firmament is divided into the twelve signs, you will find that this number — that is, the number into which the firmament is divided, so that the count should be a whole, complete number to make its reckoning come out evenly — is not [an arbitrary one]. For the years that are [actually] counted, you may count by it six times; the four [that are counted into it] you count three times; the three [that are counted into it] you count four times; the six [that are counted into it] you count twice. [?] From here came the foundations of the aspect [סודי המראה] that occurs between the planets, which come about [according to] four configurations [תמונות]. None of them is counted among them as a configuration of conjunction.

The first of these configurations is the configuration of opposition [הנגוד], which occurs when between the two planets there are six full signs, that is 180 [ק״פ] degrees, and each of them stands facing his fellow, directly opposite. **The second** is the configuration of the square [הרביע], which is half of the opposition; it occurs when between the two planets there are three full signs and 90 [וצ׳] degrees, and it comes from two sides: if the planet is before [ahead of] it [the reference planet], it is in the tenth sign from it, and this is called the first square and the right-hand square; and if it is behind it, it is in the fourth house, and this is called the second square and the left-hand square. This aspect is called "the square" [תרביע] because it draws in the firmament the likeness of a figure that has four sides: the first, a line going out from the planet to the tenth [house] relative to it; the second, the line going out from it to the fourth house; the third, the line going out from the fourth to the seventh; and the fourth, the line going out from the seventh to the tenth.

The third configuration is called the trine [תשליש], which occurs when between the two planets there are four full signs, that is 120 [ק״כ] degrees; and it too comes from two sides: if the planet is before [ahead of] that [reference] planet, in the ninth sign, it is called the first trine and the right-hand trine; and if it is behind it, in the fifth sign, it is called the second trine and the left-hand trine. They named this aspect "the trine" because it draws in the firmament the likeness of a triangle [a three-sided figure].

The fourth configuration is the configuration of the sextile [השתות], which occurs when between them there are two signs, that is 60 [ס׳] degrees; and it too comes from two sides: if the planet is before [ahead of] it, in the eleventh sign, it is called the first and the right-hand [sextile]; and if it is behind, the planet is in the third sign, and it is called the second and the left-hand [sextile]. This configuration draws in the firmament the likeness of a figure that has six sides.

Any two planets that are joined together [in conjunction], or that stand in one of these four configurations — of these you say that they appear to one another and are partners; they testify together, and are agreeable and bound, this one with that one. But if the distance between them is one sign, or five signs, or seven, or eleven [signs], of these you say that they do not appear to one another and are not partners by way of the rays [מדרך הניצוצות], according to the opinion of all the masters of the craft.

They [the planets] have yet another kind of partnership, which is neither by way of the aspect nor by way of the rays. It is this: any two degrees of the firmament that are equidistant from one of the points of the [seasonal] figure — namely the head of the sign Capricorn [ראש מזל גדי] and the head of the sign Cancer [ראש מזל סרטן] — are called degrees that agree [are in accord] by one power [כח אחד],

because the day-arc [קשת היום] at the one degree is equal to the day-arc at the other degree. For example, the tenth degree of Capricorn agrees by one power with the twentieth degree of Sagittarius [ממזל קשת], because they are equidistant — at one [equal] distance — from the head of the sign Capricorn. And likewise you say of the tenth degree of the sign Cancer that it agrees by one power with the twentieth degree of the sign Gemini [ממזל תאומים], because they are equidistant — at one [equal] distance — from the head of the sign Cancer: the one before it and the other after it. And so with every case like them.

And likewise you say of any two degrees that are equidistant from one of the two points of equinox [נקודות היושר] — namely the head of the sign Aries [ראש מזל טלה] and the head of the sign Libra [ראש מזל מאזנים] — they are called "agreeing by one nature" [מסכימות על טבע אחד], because their ascensions [מצעדיהן] are equal in their rising along the manner that divides [the day] in every clime [אקלים], and in their standing at mid-heaven [בחצי השמים]. For example, the tenth degree of Aries agrees by one nature with the twentieth degree of Pisces [מדגים], since each of them is at an equal distance from the head of Aries — the one before it and the other after it. And likewise you would say of two points that are equidistant from the head of Libra — at one [equal] distance. From this [principle] you may say of two planets among the wandering stars [the planets] that stand upon these two degrees that they agree [are in accord] by power or by nature, even though they appear as [if] one is seen above the other and are partners.

And sometimes the two planets will be [both] aided [בעזרים] and bound upon the configuration of a trine or a square or one of the other configurations mentioned above, while [at the same time] they agree by one nature or one power by way of their distance from the head of one of the signs of reversal [ממזלות התמורה] or the signs of the straight [path] [מזלות היושר]. For example, the 15th degree [ט״ו מעלה] of the sign Taurus [ממזל שור] agrees by one nature with the 15th degree of the sign Aquarius [ממזל דלי], and [also] agrees with the 15th degree of the sign Leo [ממזל אריה] by one power [על כח אחת]; and it is seen by both of them upon the configuration

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of a square [or] sextile. And likewise the head of the sign Pisces [ראש מזל דגים] and the head of the sign Taurus [ראש מזל שור] agree by one nature, and they are seen [together] upon a bow of a sixth [sextile]; and likewise the head of the sign Taurus and the head of the sign Virgo [ראש מזל בתולה] are seen upon the configuration of a trine, and they agree by one power. And the head of Cancer

[ראש סרטן] and the head of Capricorn [ראש גדי] are oppositions [נגודים] lying opposite [one another], each one of them facing the other, and they agree by one power. And in many [such cases] you find them like this.

Now this agreement — whether by power or by nature — is not reckoned with [taken into account] for the fixed stars [בכוכבי שבת]. It belongs to the craft of [judicial] divination [ניסיון] alone, while the aspect and the configuration and the bringing-down of the rays [מוריד הניצוצות] belong to the craft of observation [חזיון] and trial [נסיון]. It is fitting to take account of it for the planets and for the fixed stars whose rays of light fall upon the earth and are scattered over the firmament. For the planets, you must adjust the distance, and the ray [ניצוץ] that has fallen upon it [must be reckoned] in each and every configuration of the aspect-configurations; but for the fixed stars, we do not take account of their aspects, [though] we do take account of their conjunction with the planets, and all the more so with the two luminaries [the Sun and Moon]; and likewise we take account of their standing at the four pegs of the sphere [יתדי הגלגל — the four cardinal points/angles] and at the [lunar] births [conjunctions], and the like.

Now I shall explain to you the order of the configuration of the aspect for the planets, and I say: The full opposition [הניגוד המלא], which is according to its [proper] manner, occurs when each of the two planets stands upon the very body of the [ecliptic] circle [עצם אופן המזלות — the plane of the signs]. At that time the distance between them will be a full 180 [ק״פ] degrees, which is half the sphere. Their opposition is likewise full if both of them are spread out [in latitude] from the circle [נרחבים מן האופן] by an equal latitude to the two sides — the one to the north and the other to the south. But if one of them is upon the very body of the circle and the other is inclined from the circle, whether to the north or to the south, then the distance between them will be shorter than the known opposition-arc, which is 180 degrees, by the [amount of the] latitude of that planet from the circle. And if both of them are spread out [in latitude] from it, the arc will be shorter than the known arc by the [amount of the] latitude of both of them.

As for the configuration of the square: it keeps one distance, namely 90 [צ׳] degrees, whether the two planets were upon the very body of the circle, or were spread out from the circle to one side or to both sides. The arc of this distance neither adds nor subtracts.

And the configuration that is upon a bow of a sixth [the sextile]: if the planet is upon the very body of the circle, its sextile-arc [קשת שתותו] falls at a distance of 60 [ס׳] degrees, and its trine-arc [קשת תשלישו] falls at a distance of 120 [ק״כ]

degrees. But if it is moving away from the circle [in latitude], its sextile-arc falls at less than 60 [מס׳] degrees, and its trine-arc falls at more than 120 [מק״כ] degrees, by the [amount of the] deficiency of the sextile-arc from 60 [מס׳].

and subtract that star's latitude. Here is the procedure for this subtraction calculation: First find the value of that star's latitude. Take in hand the chord of [the latitude's] complement, and the remainder of 90 minus that latitude is the chord that will serve as the full side. If you know the latitude through which the star passes, take the chord of the full latitude. Raise the chord of the full half [latitude] and double it; that doubled value is known for two purposes, [serving] for that latitude — keep it. And you know the square of this full chord; keep its square as well.

After that, you double the chord of the full side mentioned above [taken] at 60 degrees, and from the doubling [you have] the assembled [product]. To it you add the square of the chord of the full latitude that you kept, and from their combination you obtain the foundation [root-base]: that foundation will certainly exceed 60. Take the [square of the] excess by which that foundation exceeds 60. Divide 60 by the chord of the full side, and the result of that division will be the corrected chord of the latitude.

Again you take the square of the excess over the foundation, the 60 by which you divided the chord of the side, and you double it [together] with the chord of the full side, and from the doubling [you have] the assembled [product]. You divide that by the chord of the full side, and from this division you obtain the corrected latitude — keep it with you. After that, take the square of the chord of the full latitude that you kept and subtract from it the square of the side of the sextile, which is sixty and its square is three thousand six hundred [$60^2 = 3600$]. Find its foundation and subtract the foundation, the [established] rule of that latitude — keep it. Again subtract the square of the chord of the full latitude, three thousand six hundred [3600] once more, and from the remainder divide it. The remainder will be the corrected chord of the second side — keep it.

Then again subtract the square of the chord of the full latitude — three thousand six hundred [3600] once more — and divide that by the corrected rule you kept, and the result of the division is the chord sought. Make it the chord of a full arc, taking its half, which is the half-arc; and you will know that half-arc, and you double it: its double will be the sextile-arc proper to the star's position from the wheel of the zodiac. Subtract it from 180, and what remains will be the trine-arc.

Subtract each of these two arcs from the star's place, and add both of them to the star's place. Thus you will know the degree you reach by this addition and this subtraction from the wheel of the zodiac. You will have two places of subtraction — the parts of the figure of the trine and the right-hand sextile of that star — and two places of addition, the parts of [the figure of] their left-hand [trine and sextile]. This is the method of reckoning the places of the rays of light cast by stars falling upon the firmament at the four figures of the aspects mentioned above.

The order of their true full conjunction is when the two stars stand encamped at a single degree on the body of the wheel itself, or are spread apart from it by one [degree] of latitude — this[?] latitude — and at that moment they are joined in longitude and in breadth. The lower of them opposes the upper. But if their latitude is not equal — and all the more so [continues p.112]

if the latitude is on the two sides — there is no full conjunction between them; rather they are reckoned as joined or appear [conjoined] more than the other figures, so long as the distance between them does not exceed the measure of their bodies. This is the method of reckoning the rays of light for the planets [כוכבי לכת]. It is the corrected method, ordered upon the foundation of this science. Many of the masters of this craft have varied and corrupted methods for their reckoning, which I have not seen fit to mention, because they are not worthy to be relied upon.

The order of reckoning the courses of the stars throughout the houses of the firmament

The ascensions of the zodiacal signs have already been set out for you, [house] by house, at the meridian-circle [the dividing circle of every place], and at those ascensions occurs the course of the stars that stand encamped on the body of the rising sign [ascendant], and likewise the stars that stand encamped on the body of the setting sign. So too the ascensions of the signs at the right sphere [sphaera recta], in the arc of half the heaven, have been set out for you; and at those ascensions occurs the course of the stars that stand encamped on the body of the tenth house and the fourth house. The ascensions of the signs in the other houses of the wheel do not follow the practice of [either of] these two ascensions; rather, they pass over them by mixed ascensions, blended of these two [kinds of] ascensions.

And I will set out the method of combining them, saying: If the star is inclined away from [one of] the four cardinal points of the wheel — which are the setting sign, the ascendant sign, the tenth house, and the fourth house — you find that star's distance from the tenth cardinal point or the fourth cardinal point, and you find their distance [reckoning] in this way:

If that star, or the degree whose distance you seek, stands encamped [at a point] from the ascendant up to the fourth house, or from the fourth house up to the setting sign, know that its encampment is beneath the earth, and you must seek its distance from the fourth cardinal point, which is the cardinal point of the half-heaven beneath the earth. And if it stands encamped from the setting sign up to the tenth house, or from the tenth house up to the ascendant, know that its encampment is above the earth, and you must seek its distance from the tenth house, which is the cardinal point of the half-heaven above the earth — as I shall explain to you.

If the star, or the degree, falls between the tenth house and the ascendant sign — and this quadrant is called "manifest and rising" [revealed and ascending] — you subtract the ascensions of the tenth house at the right sphere from the ascensions of the star, or the degree, at that [given] sphere, and the excess between them will be the star's distance from the cardinal point of the half-heaven above the earth. And if it falls between the setting sign and the tenth house — and this quadrant is called "manifest and descending" — you subtract the ascensions of the star at the right sphere from the ascensions of the half-heaven, and the excess [continues p.113]

between them will be the star's distance from the cardinal point mentioned. When you know the distance from the half-heaven by one of these two orderings, divide that distance by the [seasonal] hour-periods of the day, for the degree at which the star stands encamped from the wheel of the zodiac, and the result of the division will be the sixth-part [shitut] of the distance from that cardinal point.

And if the star stands encamped between the ascendant sign and the fourth house — and this quadrant is called "hidden and rising" — you subtract the ascensions of the star at the right sphere from the ascensions of the fourth house, and the excess between them is the distance from the fourth cardinal point. And if it stands encamped between the fourth house and the setting sign — and this quadrant is called "hidden and descending" — you subtract the ascensions of the fourth house from the ascensions of the star at the right sphere, and the excess between them will be the distance from the fourth cardinal point. Divide that distance by the [seasonal] hour-periods of the night for that distance, and the result of the division will be the hours of distance from the fourth cardinal point.

And if from this distance you wish to know the distance of the star from the ascendant sign or the setting sign, you subtract its distance from the cardinal point of the half-heaven that is above the earth or beneath it. Subtract whichever of them is in your hand from six hours [6 = שש שעות hours], and the remainder will be the star's distance from the ascendant sign or the setting sign. If the star was in the two ascending quadrants of the wheel — which are from the fourth house up to the ascendant — the remainder will be its distance from the ascendant sign. And if it was in the two descending quadrants opposite them, the remainder will be its distance from the setting sign.

When you know the hours of the star's distance from the cardinal point, and you wish to know the distance between that star and another star that comes after it: if the hours of distance that you found are taken [reckoned] from the cardinal point of the half-heaven, and the star stands encamped in the ascending half of the wheel — as when it is set out above, from the fourth house toward the ascendant sign up to the tenth house — you find the ascensions of the first star and the second [star] in the climate [clime], and the excess between the two ascensions is the value of the distance between them by the ascensions of the climate. Keep it. After that, find the ascensions of the first [star] at the right sphere, and subtract from it the ascensions of the second star at the right sphere; the excess between the two ascensions is the value of the distance between two stars by the ascensions of the right sphere. If this distance is equal to the distance by the ascensions of the climate, that distance is the corrected distance [continues p.114]

between two stars. If the two distances are not equal, you find the excess between them. Take a sixth of that excess and multiply it by the hours of the distance from the cardinal point; the sum from this multiplication is the correction of the distance. If the distance by the ascensions of the right sphere exceeds the distance by the ascensions of the climate, keep that correction: you subtract the correction you kept from the distance by the ascensions of the right sphere; but if there is a deficit from the distance by the ascensions of the climate, you add the correction you kept to the distance by the ascensions of the right sphere. Whatever results in your hand, after the addition or the subtraction, is the corrected distance between the two stars.

If the hours of the distance from the cardinal point fall within the [degrees] held by the rising sign, you add the product you kept onto the distance by the ascensions of the climate if it is a deficit from the distance by the ascensions of the right sphere, and you subtract it if it is an excess. These two computations come out to a single value of the distance, with no difference between them.

Now if the star was located in the half of the orb from the tenth house down to the setting point, on to the fourth house, you find the ascensions of the setting of the degree at which the first star is located — these are the ascensions of the rising of the degree opposite it — as though the star were located in the sign of Aries. You take its ascensions in the sign of Libra and subtract them from the ascensions of the second star, which follow this same procedure; the excess between them is the distance between the two stars by the ascensions of the climate. After this you will likewise know the ascensions of the first star in the right sphere and subtract them from the ascensions of the second star in that same right sphere; the excess between them is the value of the distance between the two stars by the ascensions of the right sphere. If the two distances are equal, that distance is the corrected distance between them. But if they are not equal, you find the excess between them, take a sixth of it, and multiply it by the hours of the star's distance from the cardinal point; the sum from the multiplication is the correction of the distance between the two stars. You proceed by adding this correction onto the distance between them, or subtracting it from it, according to the procedure you followed when the star was in the rising half of the orb.

Now I give you an example. Suppose the sun and Venus were located in the sign of Taurus at 16 degrees, and their position in the firmament was in the climate over which the height of the pole is 36 degrees, in the eleventh house; the rising sign was 20 degrees of the sign of Cancer, and the degree at mid-heaven was 5 degrees [ה׳] of the sign of Aries; and Saturn was located at 25 degrees [כ״ה] of Gemini. [Stage-5: mid-heaven corrected from 25° to 5° (ה׳) — confirmed on the scan; the prior pass had duplicated Saturn's (25°) כ״ה onto the mid-heaven.] We wish to know the distance between the sun and Saturn. We first investigate the hours of the sun's distance from the cardinal point that precedes it or that follows it, and we find its distance in this climate, at its position, by

this present [point]. The distance from the cardinal point of mid-heaven is 90 degrees and a quarter, since the ascensions of mid-heaven are 94 [צ״ד] degrees, 35 minutes, and the ascensions of the sun's degree in the right sphere are 133 degrees, 26 minutes [קל״ג מעלות כ״ו שברים], and between them are 38 degrees, 51 minutes [ל״ח מעלות נ״א שברים]. If you divide them by 17 degrees, 18 minutes [י״ז מעלות י״ח שברים] — which are the time-units of one hour of the sun's degree at the climate — the result of the division is 2 and a quarter; and that is the distance of the sun from the cardinal point of mid-heaven. If you subtract these from six hours, the remainder is 4 hours less a quarter, and that is the distance of the sun from the rising sign.

We now come to know the distance between the sun and Saturn. We subtract the ascensions of the sun's degree in the right sphere, which are 133 degrees, 26 minutes [קל״ג מעלות כ״ו שברים], from the ascensions of Saturn in that same sphere, which are 174 degrees, 30 minutes [קע״ד מעלות ל׳ שברים]; the excess is the distance between the two stars by the ascensions of the right sphere, and it is 41 degrees, 4 minutes between them [ר' ביניהם מ״א מעלה ד׳ שברים]. We then likewise subtract the sun's ascensions at the climate, which are 30 degrees, 52 minutes [ל׳ מעלה נ״ב שברים], from Saturn's ascensions at the climate, which are 15 degrees, 8 minutes [ט״ו מעלה ח׳ שברים]; the remainder is 15 degrees, 16 minutes [י״ה מעלה י״ו שברים], and that is the distance between the two stars by the ascensions of the climate — and these [degrees] are deficient at the climate from the first distance of 5 degrees, 48 minutes [ה' מעלות מ״ח שברים]. The sixth-part of this number is 58 minutes [נ״ח שברים].

If you multiply them [the 5°48′] by two and a quarter, which is the number of hours of the sun's distance from the cardinal point of mid-heaven, the product is 2 degrees and about a sixth of a degree [ב׳ מעלות וכגון שתות מעלה]. And since the distance by the ascensions of the right sphere exceeds the distance by the ascensions of the climate, we subtract this product from the distance by the ascensions of the right sphere, which is 41 degrees, 4 minutes [מ״א מעלה ד׳ שברים]; the remainder will be 38 degrees, 54 minutes [ל״ח מעלות ונ״ד שברים], and that is the corrected distance in hours of the distance from the cardinal point of mid-heaven. Had you instead multiplied the sixth-part, which is 6 degrees, 28 minutes [ו׳ מעלות כ״ח שברים], by the hours of the distance from the rising sign, which are four less a quarter, the product is 3 degrees, 28 minutes [ג' מעלה כ״ח שברים]; and if you add this product onto the distance by the ascensions of the climate — which are deficient from the distance by the ascensions of the right sphere — namely 35 degrees, 16 minutes [ל״ה י״ו], the combined sum will be 38 degrees, 44 minutes [ל״ח מ״ד]; and if the corrected distance in hours of the distance from the rising sign equals the corrected distance in hours of the distance from the cardinal point of mid-heaven, then you may rely in your computation on either of them, and it will suffice for you.

We give another example, and we say: in the figure we mentioned above, suppose the moon were located in the

fifth house — which is Scorpio — at 20 degrees, and Mars [is] from it at 2 degrees of the sign of Sagittarius. We come to know the corrected distance between the moon and Mars, and we first investigate the distance of the moon from the fourth cardinal point. We subtract the ascensions of the fourth house, which are 4 degrees, 45 minutes [ד' מעלות מ״ה שברים], from the ascensions of [the moon's degree] in Libra and their ascensions in the right sphere, which are 274 [degrees], 20 [minutes] [רע״ד כ׳], from the ascensions of the moon's degree, which are 316 [degrees], 28 [minutes] [שי״ו כ״ח]; the remainder will be 43 [degrees], 8 [minutes] [מ״ג ח׳]. We divide this number by the time-units of the night for the moon's degrees, which are 17, 17 [י״ז י״ז]; the division comes out to about 2 and a half [ב׳ וחצי], and these are the hours of the moon's distance from the cardinal point of mid-heaven below the earth.

After this we come to know the corrected distance between the moon and Mars. We subtract the moon's ascensions in the right sphere, mentioned just above, from the ascensions of Mars, which are 349 [degrees], 5 [minutes] [שמ״ט וה׳]; the remainder will be 31 [degrees], 37 [minutes] [ל״א ל״ז], and that is the distance between them by the ascensions of the right sphere. We then subtract the ascensions of the degree opposite the moon's degrees at the climate — which is 20 degrees of Taurus, and their ascensions are 33 [degrees], 46 [minutes] [ל״ג מ״ו] — from the ascensions opposite Mars's degree, which is 20 degrees of Gemini, and their ascensions are 60 degrees, 41 minutes [ס' מעלות מ״א]; the [resulting] distance is the distance between them by the ascensions of the climate, and the excess between them is 26 [degrees], 55 [minutes] [כ״ו נ״ה]. The excess between the two distances is 4 [degrees], 42 minutes [ד' מ״ב], and the sixth-part of this number is 47 minutes [מ״ז]. We multiply it by two and a half [ב׳ פעמים וחצי], according to the number of hours of the distance from the cardinal point; this product will be 2 degrees less about 2 minutes [ב' מעלות פחות ב׳ שברים בקרוב]. We subtract this product from the distance between them by the ascensions of the right sphere — since it [the right-sphere distance] is in excess — and the corrected distance will be 29 [degrees], 29 [minutes] [כ״ט כ״ט].

This is the procedure for finding the corrected distance when the two stars, or the degrees, are in one of the four quadrants of the orb, if both of them are in a single half — the rising half or the descending half.

But if one of them is in the rising half and the other in the descending half, you find the corrected distance between them in this way. If the [earlier] star or the [earlier] degree that precedes is in the descending half, you find the corrected distance from the cardinal point of mid-heaven above the earth, which is the tenth house; and if it is located in the rising half, you find its corrected distance from the cardinal point of the fourth house, by the procedure mentioned above. And when you know this corrected distance, you join to it

the distance between the cardinal point of the tenth or the cardinal point of the fourth and the later star by the ascensions of the [right] sphere…

the right [sphere] — and that will be the corrected distance between the two stars. We have no need to know these distances and to correct them except in the case of nativities, when we come to correct the courses [of the planets] that lie between the two stars at the births of human beings, or at their seasons in their several years; and by way of [these distances we proceed]. ✻

This concludes the exposition of the Reckoning of the Courses according to Ptolemy the Greek, foremost of the sages of Greece in this art, in the book called the *Four Discourses* [the *Tetrabiblos* / *Quadripartitum*], upon which one may rely. And you will find this reckoning expounded by way of many methods, none of which can be relied upon as can this one.

I have here set the seal upon my words, and praise to my Creator who has helped me: may His name be praised forever, Selah.

[Closing colophon, in parentheses in the source:]

(Herewith is completed this book, namely the Reckoning of the Courses, [composed] by our prince, the great sage, our Rabbi Avraham [bar Ḥiyya] — may the memory of the righteous be a blessing — son of the prince, the great [sage], Rabbi Ḥiyya — may his memory be a blessing. And I, Shalom bar [the son of] Shelomo [?], called [recte: the one surnamed] the Jerusalemite, completed it on Thursday, the 28th day of the month of Second Adar of the year 5644 [תרמ״ד = 5644 = 400+200+40+4 + 5000 AM; 5644 − 3760 = 1884 CE] [?], in the city of Syracuse, which is on the island of Sicily [סרקוסה דאי בצקליאה — Syracuse, Sicily].

May the Merciful One make me worthy to meditate upon it and upon the like of it. Amen, and so may it be His will [אוי״ר = אמן וכן יהי רצון].)

תיקוני טעות — Corrections of Errors (Errata)

The errata block is printed at the foot of p. 117 [קי״ז, the page bearing the running-header folio mark] as two side-by-side sub-tables, each reading right-to-left: a column of the corrected word/form followed by a single numeric column. There is **no printed header row**; the source does not label the numeral as page or line. The Hebrew terms listed are the astronomy vocabulary words at issue. [Verified at 600 DPI, further.md.]

Right-hand sub-table (reads right-to-left in source):

Word (printed form)	No.
מצעד [ascensions]	22
חלקם [their portion/parts]	23
סדר [order/arrangement]	38
המזל [the zodiacal sign]	45

Left-hand sub-table (reads right-to-left in source):

Word (printed form)	No.
מצעד [ascensions]	12
הלקם [recte: חלקם, "their portion/parts"]	23
סדר. [order/arrangement]	1
המזלות [the zodiacal signs]	6